A NOTE ON CONSTRUCTING DIGRAPHS WITH PRESCRIBED PROPERTIES

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ABSTRACT

Let n be non-negative integer and k a positive integer. A digraph D is said to has property Q(n,k) if for every subset of n vertices of D is dominated by at least k other vertices. For $q \equiv 5 \pmod 8$ be a prime power. Define a quadruple Paley digraph $D_q^{(4)}$ as follows. The vertices of $D_q^{(4)}$ are the elements of the finite field F_q. Vertex a joins to vertex b by an arc if and only if $a - b = y^4$ for some $y \in F_q$. In this paper, we show for sufficiently large q, $D_q^{(4)}$ has property Q(n,k).

1. INTRODUCTION

In this paper, our graphs are directed. For our purpose, all digraphs are finite and strict. If (x, y) is an arc in a digraph D, then we say vertex x **dominates** vertex y. A set of vertices A dominates a set of vertices B if every vertex of A dominates every vertex of B. A digraph D is said to have property Q(n,k) if every subset of n vertices of D is dominated by at least k other vertices. Further, a digraph D is said to have property Q(m,n,k) if for any set of m + n distinct vertices of D there exist at least k other vertices each of which dominates the first m vertices and is dominated by the latter n vertices.

A special digraph arises in round robin tournaments. More precisely, consider a tournament T_q with q players 1, 2, ..., q in which there are no draws. This gives rise to a digraphs in which either (a, b) or (b, a) is an arc for each pair a, b. Tournaments with property Q(n, k) have been studied by Ananchuen and Caccetta [2] Bollobás [3] and Graham and Spencer [4].

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