ON THE ADJACENCY PROPERTIES OF GENERALIZED PALEY GRAPHS

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ABSTRACT

Let m and n be non–negative integers and k a positive integer. A graph G is said to have property P(m,n,k) if for any m + n distinct vertices of G there are at least k other vertices, each of which is adjacent to the first m vertices but not adjacent to any of the latter n vertices. We know that almost all graphs have property P(m,n,k). However, for the case m, n ≥ 2, almost no graphs have been constructed, with the only known examples being Paley graphs which defined as follows. For $q \equiv 1 \pmod 4$ a prime power, the Paley graph G_q of order q is the graph whose vertices are elements of the finite field F_q; two vertices a and b are adjacent if and only if their difference is a quadratic residue. By using higher order residues on finite fields we can generate other classes of graphs which we refer to as generalized Paley graphs. For any m, n and k, we show that all sufficiently large (order) graphs obtained by taking cubic and quadruple residues satisfy property P(m,n,k).

1. INTRODUCTION

All graphs considered in this paper are finite, loopless and have no multiple edges. For the most part, our notation and terminology follows that of Bondy and Murty [10]. Thus G is a graph with vertex set V(G), edge set E(G), $\nu(G)$ vertices and $\varepsilon(G)$ edges.

Let m and n be non–negative integers and k a positive integer. A graph G is said to have property P(m,n,k) if for any disjoint sets A and B of vertices of G with $|A| = m$ and $|B| = n$ there exist at least k other vertices, each of which is adjacent to every vertex of A but not adjacent to any vertex of B. The class of graphs having property P(m,n,k) is denoted by $\mathcal{G}(m,n,k)$. The cycle C_ν of length ν is a member of $\mathcal{G}(1,1,1)$ for every $\nu \geq 5$. The well–known Petersen graph is a member of $\mathcal{G}(1,2,1)$ and also of $\mathcal{G}(1,1,2)$. The class

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