A characterization of maximal non-k-factor-critical graphs

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Abstract

A graph G of order p is k-factor-critical, where p and k are positive integers with the same parity, if the deletion of any set of k vertices results in a graph with a perfect matching. G is called maximal non-k-factor-critical if G is not k-factor-critical but $G+e$ is k-factor-critical for every missing edge $e \notin E(G)$. A connected graph G with a perfect matching on $2n$ vertices is k-extendable, for $1 \leq k \leq n-1$, if for every matching M of size k in G there is a perfect matching in G containing all of edges of M. G is called maximal non-k-extendable if G is not k-extendable but $G+e$ is k-extendable for every missing edge $e \notin E(G)$. A connected bipartite graph G with a bipartitioning set (X, Y) such that $|X| = |Y| = n$ is maximal non-k-extendable bipartite if G is not k-extendable but $G + xy$ is k-extendable for any edge $xy \notin E(G)$ with $x \in X$ and $y \in Y$. A complete characterization of maximal non-k-factor-critical graphs, maximal non-k-extendable graphs and maximal non-k-extendable bipartite graphs is given.

Keywords: matching, k-factor-critical graphs, k-extendable graphs

1. Introduction

All graphs considered in this paper are finite, connected, loopless and have no multiple edges. For the most part our notation and terminology follows that of Bondy and Murty [2]. Thus G is a graph with vertex set $V(G)$, edge set $E(G)$ and minimum degree $\delta(G)$. For $V' \subseteq V(G)$, $G[V']$ denotes the subgraph induced by V'. Similarly, $G[E']$ denotes the subgraph induced by the edge set E' of G. $N_G(u)$ denotes the neighbour set of u in G and $\overline{N}_G(u)$ the non-neighbours of u. Note that $\overline{N}_G(u) = V(G)\backslash(N_G(u) \cup \{u\})$. The join $G \vee H$ of disjoint graphs G and H is the graph obtained from $G \cup H$ by joining each vertex of G to each vertex of H.

[1]Worked supported by the Thailand Research Fund grant # BRG4680019
[2]Worked supported by the Western Australian Centre of Excellence in Industrial Optimisation (WACEIO)
[3]Worked supported by the Thailand Research Fund grant # BRG/15/2545