# On The Adjacency Properties of Generalized Paley Graphs

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## Abstract

Let m and n be non–negative integers and k a positive integer. A graph G is said to have property P(m,n,k) if for any m + n distinct vertices of G there are at least k other vertices, each of which is adjacent to the first m vertices but not adjacent to any of the latter n vertices. We know that almost all graphs have property P(m,n,k). However, for the case m, n $\geq$ 2, almost no graphs have been constructed, with the only known examples being Paley graphs which defined as follows. For $q \equiv 1 \pmod 4$ a prime power, the Paley graph $G_q$ of order q is the graph whose vertices are elements of the finite field $F_q$; two vertices a and b are adjacent if and only if their difference is a quadratic residue. By using higher order residues on finite fields we can generate other classes of graphs which we refer to as generalized Paley graphs. For any m, n and k, we show that all sufficiently large (order) graphs obtained by taking cubic and quadruple residues satisfy property P(m,n,k).

## 1. Introduction

All graphs considered in this paper are finite, loopless and have no multiple edges. For the most part, our notation and terminology follows that of Bondy and Murty [10]. Thus G is a graph with vertex set V(G), edge set E(G), $\nu(G)$ vertices and $\varepsilon(G)$ edges.

Let m and n be non–negative integers and k a positive integer. A graph G is said to have property P(m,n,k) if for any disjoint sets A and B of vertices of G with $|A| = m$ and $|B| = n$ there exist at least k other vertices, each of which is adjacent to every vertex of A but not adjacent to any vertex of B. The class of graphs having property P(m,n,k) is denoted by $\mathcal{G}(m,n,k)$. The cycle $C_\nu$ of length $\nu$ is a member of $\mathcal{G}(1,1,1)$ for every $\nu \geq 5$. The well–known Petersen graph is a member of $\mathcal{G}(1,2,1)$ and also of $\mathcal{G}(1,1,2)$. The class

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