รายงานการวิจัยฉบับสมบูรณ์

โครงการ

การพัฒนาแบบจำลองโดยใช้ Damage Mechanics เพื่อทำนายพฤติกรรมของคอนกรีตภายใต้แรงกดทั้งแบบคงที่ (Static) และ แบบอัดกระแทก (Impact)

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สัญญาเลขที่ MRG4580039

รายงานวิจัยฉบับสมบูรณ์

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สนับสนุนโดยทบวงมหาวิทยาลัยและสำนักงานกองทุนสนับสนุนการวิจัย

กิตติกรรมประกาศ

ผู้วิจัยขอขอบคุณ ทบวงมหาวิทยาลัย และ สำนักงานกองทุนสนับสนุนการวิจัย (สกว.) ให้ทุนสนับสนุนการวิจัยครั้งนี้ (ทุนพัฒนาศักยภาพในการทำการวิจัยของอาจารย์รุ่นใหม่ประจำปี 2545)

บทคัดย่อ

รหัสโครงการ MRG4580039

ชื่อโครงการ การพัฒนาแบบจำลองโดยใช้ Damage Mechanic เพื่อทำนายพฤติกรรมของคอนกรีตภายใต้แรงกดทั้งแบบคงที่ (Static) และ แบบอัดกระแทก (Impact)

ชื่อนักวิจัยและสถาบัน ดร. ปิติ สุคนธสุขกุล
ภาควิชาวิศวกรรมโยธา สถาบันเทคโนโลยีพระจอมเกล้าพระนครเหนือ

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ระยะเวลาโครงการ 1 ปี

การศึกษาครั้งนี้เป็นการนำทฤษฎี Damage Mechanics มาประยุกต์ใช้ในการหาค่าความเสียหาย (Damage) ที่เกิดขึ้น เพื่อนำมาสร้างแบบจำลองใช้ในการทำนายพฤติกรรมของคอนกรีตภายใต้แรงกระทำแบบคงที่ (Static) และแบบอัดกระแทก (Impact) ทฤษฎี Damage Mechanics เป็นทฤษฎีที่ค่อนข้างง่ายและไม่ซับซ้อน มีความสามารถในการอธิบายพฤติกรรมของคอนกรีตโดยอาศัยตัวแปรเพียงหนึ่งตัวที่เรียกว่าความเสียหาย (D) แนวทางในการหาความเสียหายของคอนกรีตนั้นมีมากมายหลายวิธี ที่นำมาใช้ในการศึกษาครั้งนี้มี 2 แนวทางขึ้นกับประเภทของแรงกระทำ ในกรณีของคอนกรีตรับแรงกระทำแบบคงที่ เราใช้แนวทางที่เรียกว่าการเปลี่ยนแปลงของค่าโมดูลัสยืดหยุ่น (Variations of E) ส่วนในกรณีของคอนกรีตรับแรงกระทำแบบกระแทก แนวทางที่นำมาใช้คือการเปลี่ยนแปลงของอัตราการเครียด (Variations of Strain Rate)

เราพบว่าระดับความเสียหายของคอนกรีตที่จุดสูงสุดนั้นสัมพันธ์โดยตรงกับอัตราการเค้น (Stress Rate) โดยเราพบว่าระดับความเสียหายของคอนกรีตจะสูงขึ้นตามการเพิ่มขึ้นของอัตราการเค้น นอกจากนี้แบบจำลองที่ได้ในทั้งสองกรณีสามารถทำนายพฤติกรรมของคอนกรีตได้ในระดับหนึ่ง โดยเมื่อเปรียบเทียบกับพฤติกรรมจริงของคอนกรีตที่ได้จากการทดลองจะมีความแม่นยำช่วงต้นและช่วงปลาย ส่วนในช่วงกลางจะทำนายได้ต่ำกว่าความเป็นจริง ทั้งนี้เชื่อว่าสาเหตุน่าจะมาจากการที่ทฤษฎีของ Damage Mechanics นั่นเองที่ละเลยและไม่สามารถจับพฤติกรรมของคอนกรีตที่ความซับซ้อนในระดับอนุภาคได้

ABSTRACT

Project Code	MRG4580039
Project Title	Development of Damage Mechanics Model to Predict Compressive Response of Concrete under both Static and Impact Loading
Investigator	Dr. Piti Sukontasukkul Department of Civil Engineering, King Mongkut's Institute of Technology-North Bangkok
Email address	piti@kmitnb.ac.th, piti_sk@hotmail.com
Project Period	1 year

In this study, the theory of Damage Mechanics was used to determine the damage and create a model to predict the behaviour of concrete under both static and impact loading. The theory of Damage Mechanics is known to its simplicity to describe the behaviour of loaded concrete using a single dimensionless parameter called D (Damage). There are several ways to determine damage. In this study, depending on the type of loading, damage was determined using two different approaches: in the case of static loading, the approach called 'Variation of Elastic Modulus (E)' was used, and in the case of impact loading, the 'Variation of Strain Rate' approach was used.

Results indicated that the damage of concrete was, in fact, a rate of loading dependent. The level of damage at the peak was found to increase with increasing rate of loading. The responses of concrete obtained from the proposed models were found to agree fairly well to the actual responses. However, the models seemed to under-predict the response of concrete in the mid-range. One reason is believed to be because of the fundamental of the theory of Damage Mechanics itself that ignore the complexity of the microstructures of concrete during loading.

Chapter 1
Introduction

It is known that most of the civil engineering materials researches are experimental based researches. In this kind of research, many people do not realize that, in addition to obtaining fine test results, one of the most important steps is to develop a model based on those results. Modeling is another step to make use of the experimental results and, in turn, extend the possibility of those test results to be used in practice.

However, in the real world, experimental based researchers have no intention to develop a model from their test results (perhaps, due to the lack of interest), at the same time, simulation based researchers also have no intention to do any full experiment. As a result, many of those experimental results are abandoned without any further development to a practical model. Furthermore, many mathematical models created by researchers who have little experience in the experiment are too complex and not easy to understand or use. Therefore, there are a lot of needs on this matter, researchers who are capable of both conducting research and creating a model.

In this study, we are trying to use the theory of continuum damage mechanics to model the behavior (response) of concrete under both impact and static loading. The major advantage of damage mechanics is its simplicity to describe the material behavior using a single dimensionless parameter called 'Damage (D)'. The review of the theory is given in Chapter 3. As for the experimental part, the static tests were carried out at KMITNB, while the impact tests were carried out at the University of British Columbia. Results from both tests are given in Chapter 2.

Our objective of this study is to develop a simple mathematical model that is capable of predicting the response of concrete subjected to two different types of loadings. Results and obtained models can also be used as a teaching material.

Chapter 2
Experimental Procedure and Results

2.1 Experimental Program

2.1.1 Specimen Preparation

The specimens were cast using the following materials:

Cement:	CSA Type 10 Normal Portland Cement (ASTM Type I)
Fine aggregates:	Clean river sand with a fineness modulus of about 2.7
Coarse aggregates:	Gravel with a 10 mm maximum size

The mix proportions shown in Table 2.1 were used.

Table 2.1 Mix Proportions, by weight

Cement	Water	Fine Agg.	Coarse Agg.
1	0.5	2	2.5

Concrete with mix proportion above was prepared in the form of cylinder (Dia-100 x 200 mm.). Prior to the mix, the water content in sand and aggregates was determined to adjust the amount of mixing water. The concrete was mixed using a pan type mixer, placed in oiled steel molds in three layers, then roughly compacted each layer with a shovel before being covered with polyethylene sheets. After 24 hours, the specimens were demoulded and transferred to storage in a water tank for 30 days.

2.1.2 Testing Program

2.1.2.1 Static tests

All static tests were carried out at the Department of Civil Engineering, King Mongkut's Institute of Technology-North Bangkok (KMITNB) using a 1500 kN universal testing machine[Ψ] (Fig. 2.1). Specimens were placed at the center of the machine. The data were collected by a PC-based data acquisition system.

[Ψ] Instron 'Fast-Track 8800'

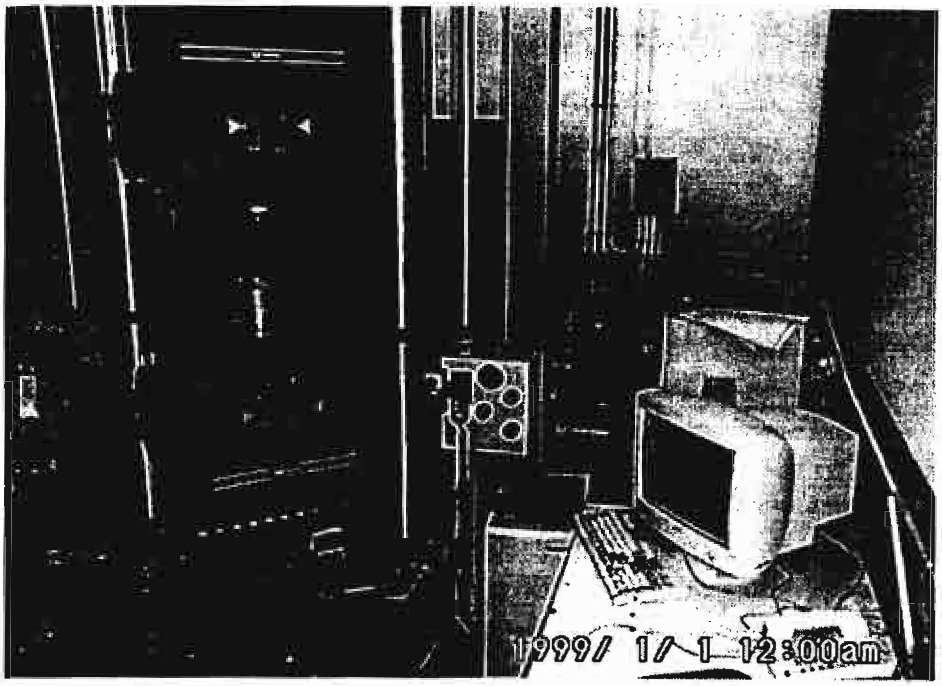


Fig. 2.1 Universal testing machine

2.1.2.2 Impact testing

All impact tests were carried out using an instrumented, drop-weight impact apparatus designed and constructed in the Department of Civil Engineering, University of British Columbia (UBC), and having the capacity of dropping a 578 kg mass from heights of up to 2500 mm on to the target specimen. The impact machine is shown schematically in Fig. 2.2. The velocity of the falling mass was obtained by:

$$v_h = \sqrt{2(0.91g)h} \qquad (2.1)$$

where v_h = the velocity of the falling hammer

g = the gravitational acceleration

h = the drop height

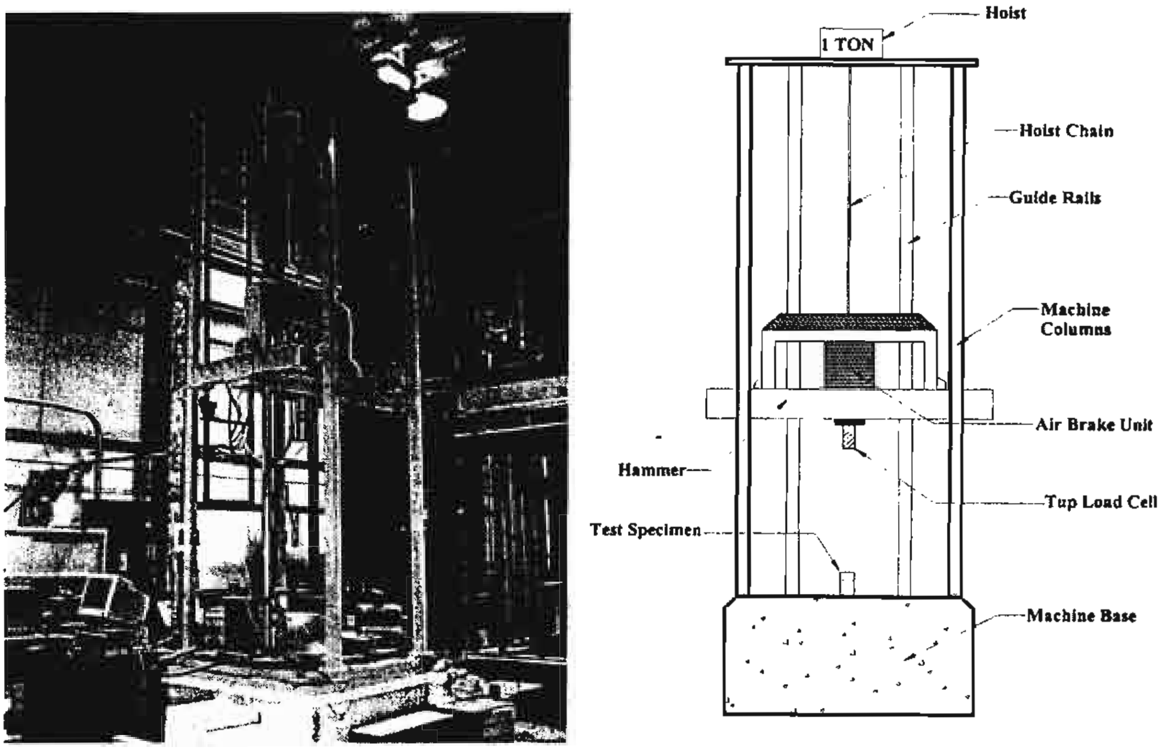


Fig. 2.2 Impact Machine at UBC

The specimen was placed vertically on a 100*100 mm rigid steel base located at the center of the impact machine as shown in Fig 2.2 and 2.3. The hammer was dropped from 250 and 500 mm heights to provide two different striking velocities of 2.21 and 3.13 m/s, respectively, and impact energies of 1417 and 2835 J, respectively. The PCB accelerometer and the Aromat Laser Analog Sensor (ALAS) were used to measure the specimen deformation. The PCB accelerometer was mounted on the hammer, while the ALAS was mounted on the machine with the laser beam pointing vertically at an extension part of the hammer. The testing program is summarized Table 2.2.

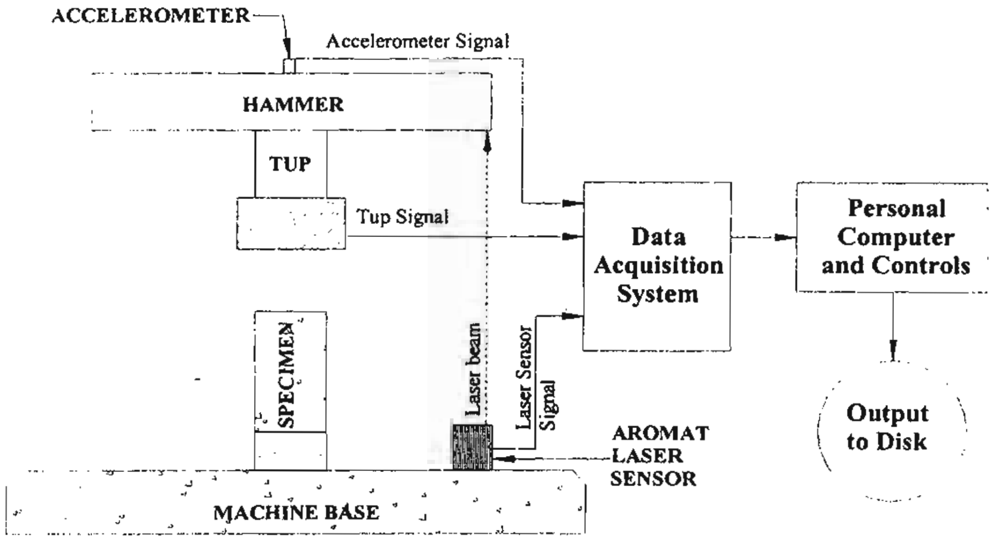


Fig. 2.3 Schematic Illustration of the Impact Testing Setup

Table 2.2 Testing Program

Designation	Description	Drop Height (mm)	Number of Specimen
Static Test			
PLN	Concrete subjected to static loading	-	3
Impact Test			
PL250	Concrete subjected to impact loading	250	3
PL500	Concrete subjected to impact loading	500	3

2.2 Test Results

2.2.1 Failure Modes

2.2.1.1 Static loading

The typical failures modes of concrete specimens under static loading are shown in Fig. 2.4. Since the tests were carried out using steel loading platens with a relatively high frictional restraint at the contact surface between the loading platens and the specimen, the plain concrete specimens clearly showed an "hour glass (101) or "shear-cone (119)" failure mode (Fig. 2.4).

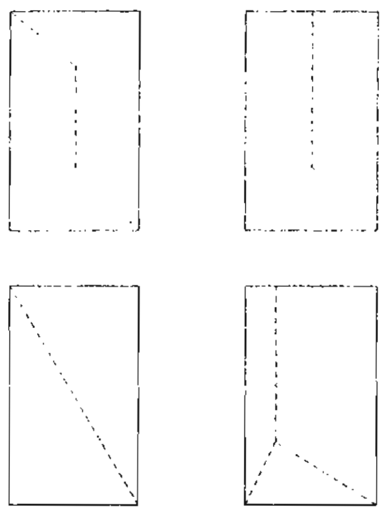


Fig. 2.4 Failure Patterns of Concrete under Static Compressive Loading

For concrete subjected to uniaxial compression in a machine in which there is significant and friction on the specimens due to the stiff steel platens on the top and bottom, a triaxial state of stress occurs near the contact zones which is in turn due to the differences in Poisson's ratio between the steel ($\nu = 0.33$) and the concrete ($\nu = 0.20$). This end restraint "strengthens" the specimen ends, leading to failure initiating near the mid-height of the specimen. This results in the typical shear-cone type of failure, as was found in this study. Two main diagonal cracks initiate, intersecting at the mid-height of the specimen. Fewer (but longer) cracks are found.

2.2.1.2 Impact loading

Under impact loading, the failure was quite catastrophic and only the bottom portion of the specimen remained on the base after impact; clearly, it was a shear cone type of failure (Fig. 2.5). An increased loading rate did not alter the failure pattern of plain concrete, though, failure then occurred more rapidly.

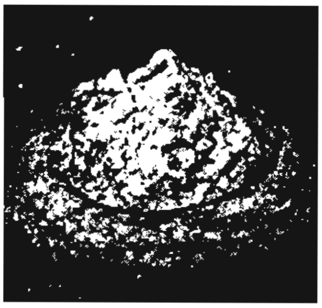


Fig. 2.5 Failure Patterns of Concrete Subjected to Impact Loading from a 250mm Drop Height

2.2.2 Stress-Strain ($\sigma - \varepsilon$) Response

2.2.2.1 Static loading

The typical stress-strain responses of concrete cylinders subjected to short-term static loading are shown in Fig. 2.6. It was found that the linear portion of the $\sigma - \varepsilon$ curve extended up to about 40-60% of peak load. Although a small amount of creep or other non-linearity might occur in this linear portion, the deformation was essentially recoverable. At about 80-95% of the peak load, internal disruption began to occur, and small cracks might appear on the outer surface. These small cracks continued to propagate and interconnect until the specimen was completely fractured into several separate pieces. The formation and coalescence of the small cracks has long been recognized as the prime cause of fracture and failure of concrete, and of the marked non-linearity of the stress-strain curve (1). The deformation associated with cracks is completely irrecoverable, and may in some cases be considered as a quasi-plastic deformation. The load (or stress) at which the cracking became severe enough to cause a distinct non-linearity in the $\sigma - \varepsilon$ curve is often referred to as the point of "discontinuity" (2). At the peak load, the failure of concrete occurred in catastrophic manner, due to the large amount of energy that was released from the testing machine; thus, the full descending branch of the $\sigma - \varepsilon$ curve could not be obtained.

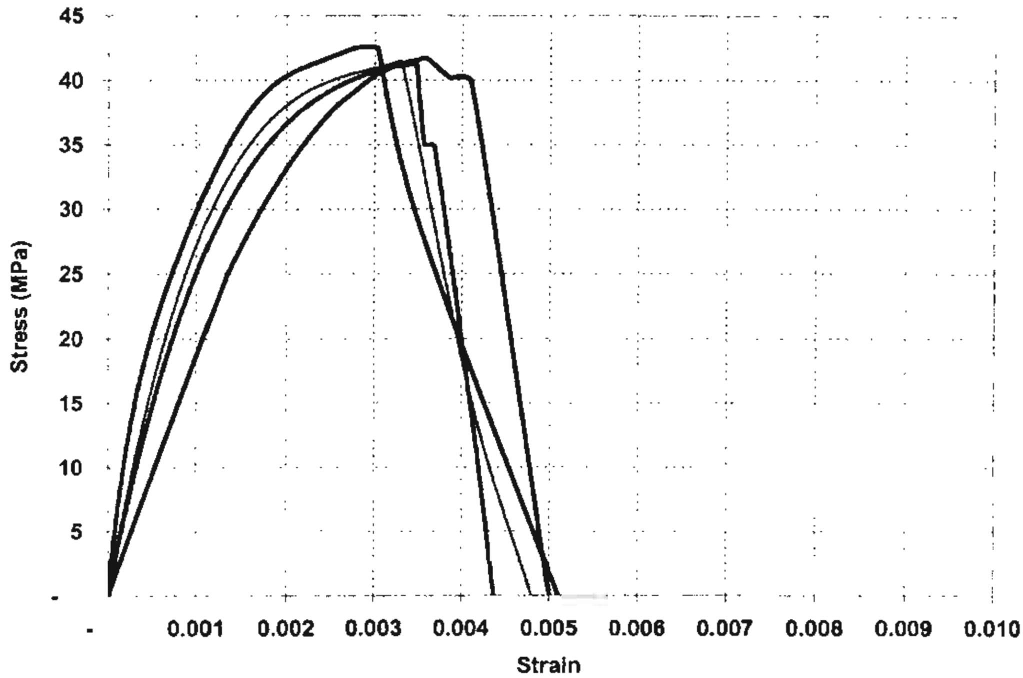


Fig. 2.6 Responses of concrete subjected to static loading

2.2.2.2 Impact loading

The concrete subjected to impact loading exhibited a quite different behavior from that subjected to static loading. The material behaved in a more brittle manner, and increases in strength, toughness, and modulus of elasticity were found as the rate of loading increased. Typical responses are given in Fig. 2.7

For the specimens tested under impact loading, the linear portion of the $\sigma - \varepsilon$ curve extended to higher stress values than for specimens tested under static loading. This linear portion was also found to increase with increased rates of loading, as seen in the specimen tested at the highest rate of loading. Theoretically, the small cracks that cause the non-linearity in the static case are forced to propagate much more quickly under impact loading. Thus, the cracks tend to propagate through rather than around aggregate particles, leading to an increase in strength and toughness, and a decrease in the non-linear portion of the $\sigma - \varepsilon$ curve.

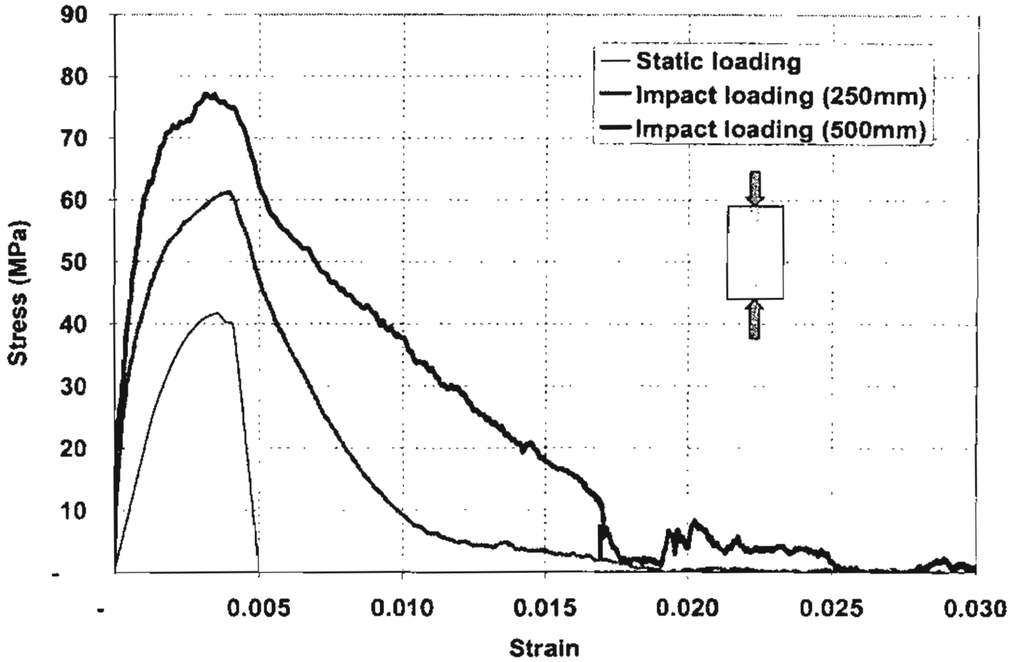


Fig. 2.7 Stress-Strain Curves of Plain Concrete Subjected to Static and Impact Loading

Chapter 3

Review: Theory of Scalar Damage Mechanics

In this chapter, the basic theory of scalar continuum damage mechanics is reviewed, and different approaches are introduced and selected to predict the pre-peak behavior of concrete under both static and impact loading conditions.

3.1 Kachanov Concept (3)

Different materials deform differently when loaded, depending on factors such as their atomic structure, composition, rate of loading and temperature. To understand the deformation characteristics of each individual material completely would require an extensive knowledge of its atomic and molecular structure. But, in practice, the general constitutive equations of the continuum model relate materials and their deformations without considering the atomic structure of the real material. Variables such as stress, strain and elastic modulus are used. Under applied loads, the material structure eventually begins to disintegrate and the load carrying capacity is reduced. The state of deterioration of a material was characterized by Kachanov (3) using a dimensionless, scalar variable denoted as damage (D).

Lemaitre (4) has stated that the damage of a material is the progressive physical process by which its breaks. There are three stages in the fracture of a material. First, at the micro-scale, damage occurs due to the accumulation of localized microcracks and the breaking of bonds. Then, the growth and coalescence of microcracks can initiate a single crack at the meso-scale. Finally, at the macro-scale, the growth of that crack is considered. The first two stages may be studied by means of the damage of the continuous medium, whereas the third stage is usually studied using fracture mechanics. All materials, regardless of their composition and structure, show elastic behavior, yielding, plastic or pseudo-plastic strain, damage by monotonic loading or fatigue, and crack growth under static or dynamic loading. This suggests that the properties common to all materials can somehow be explained using the same theory. This is the main reason why it is possible to study material behavior at the meso-scale using the mechanics of continuous media, which can explain the behavior of the material without considering in detail the complexity of the microstructure.

3.2 Macromechanical or one dimensional damage model (5)

Consider a damaged body (an apple) and a Representative Volume Element (RVE (4)) at a point M oriented in a plane defined by its normal $\vec{n}$ and its abscissa x along the direction $\vec{n}$ (Fig. 3.1). If δA represents the cross-sectional area of the RVE and δA_d represent the area of microvoids or microcracks that lie in δA, the value of damage, D(M, $\vec{n}$), is:

$$D(M,\vec{n}) = \frac{\delta A_d}{\delta A} \tag{3.1}$$

From the above expression, it is clear that the value of the scalar variable D is bounded by 0 and 1:

$$0 \le D \le 1 \tag{3.2}$$

where D = 0 corresponds to the intact or undamaged RVE

D = 1 corresponds to the completely fractured RVE

In the case of a simple one-dimensional homogeneous material, eq. 3.1 is simplified to:

$$D = \frac{A_d}{A} \tag{3.3}$$

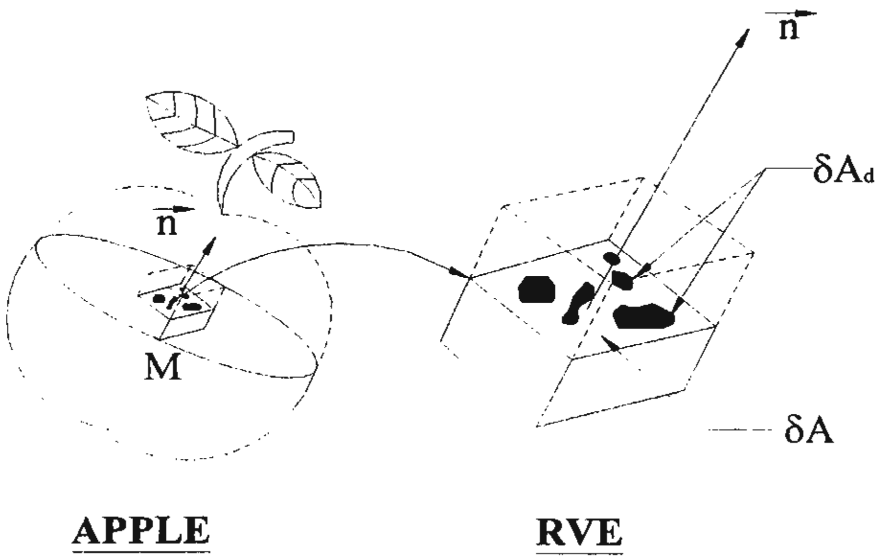


Fig. 3.1 A Damaged Body and RVE, after Lemaitre (4)

3.3 Effective stress concept (6)

One application of continuum damage mechanics is the effective stress concept. For an RVE with a cross-sectional area of A and loaded by a force F, the uniaxial stress (σ) is:

$$\sigma = \frac{F}{A} \tag{3.4}$$

If the cracked area on the representative surface of the RVE is A_d, then the effective cross-sectional area becomes $A-A_d$ and the effective stress (σ_{eff}) is:

$$\sigma_{eff} = \frac{F}{A - A_d} \tag{3.5}$$

Substituting the damage variable (D) from 3.3 into 3.5 yields,

$$\sigma_{eff} = \frac{F}{A(1 - A_{eff})} \quad \text{or} \quad \sigma_{eff} = \frac{\sigma}{1-D} \tag{3.6}$$

3.4 Strain equivalence principle

The same concept can also be applied to strain. According to Lemaitre (7), the constitutive strain equation for a damaged material may be derived in the same way as for a virgin material, except that the normal stress is replaced by the effective stress.

$$\varepsilon = f\left(\frac{\sigma}{1-D},\ldots\right) = f(\sigma_{eff},\ldots) \tag{3.7}$$

3.5 Relationship between strain and damage

Combining Eq. 3.7 with Hooke's law, the relationship between strain (ε_e) and damage can be derived as follow:

$$\varepsilon_e = \frac{\sigma}{E(1-D)} \tag{3.8}$$

where E is the elastic modulus of the undamaged material.

The elastic modulus of the damaged material or the effective elastic modulus (E_{eff}) is then defined as:

$$E_{eff} = E(1-D) \tag{3.9}$$

The degradation of E as expressed by E_{eff} in Eq. 3.9 is the key ingredient to predicting the behavior or response of concrete under load.

3.6 Damage measurement

Damage measurements can be made in by several ways: direct measurements, variations of the elastic modulus (E), energy based methods, variations of the microhardness, variations of the density, and so on. If the stress-strain curve is known (from tests), methods such as the variation of E and energy based measurements seem to be most practical. In this study, the variation of the elastic modulus (as in Fig. 3.2) is used to determine the damage variable (D). The variation of the elastic modulus is a measurement based on the influence of damage on E as expressed by Eq. 3.9. Provided that the E of the undamaged material is known, the damage value can be derived from the measurement of E_{eff}.

$$D = 1 - \frac{E_{eff}}{E} \tag{3.10}$$

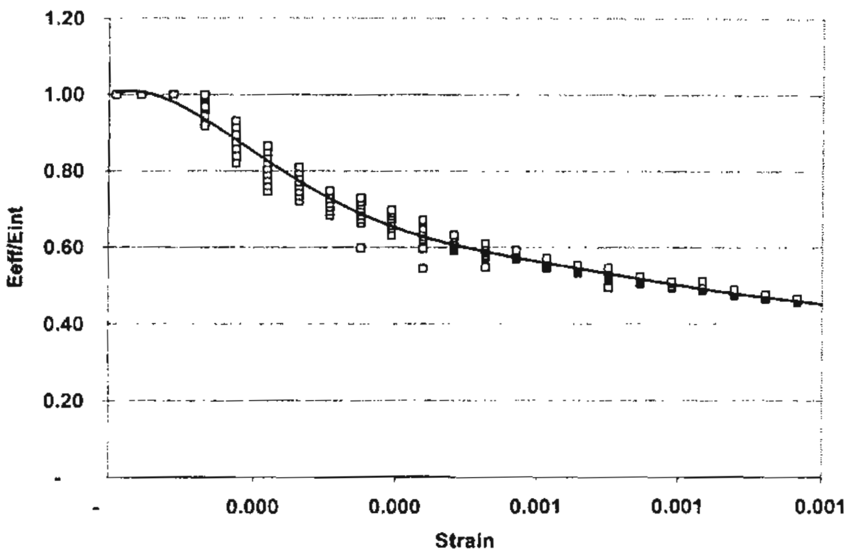


Fig. 3.2 Variation of E with Damage of Plain Concrete under Static Compression

Chapter 4
Proposed Models

The modeling begins with the determination of a damage-strain (D-ε) relationship for loaded unconfined concrete based on the actual damage. The actual damage for the static case is measured directly from the test results using the procedure described next in section 4.1.1, and is expressed in terms of the degradation of the initial E. The results indicate that the response obtained from the model proposed here agrees well with the actual response.

In the case of impact loading, a new set of stress-strain relationships is proposed, again based on scalar damage mechanics. However, in the case of impact loading, the variation in strain rate with time is used as an indicator of the damage. Details are described in section 4.2.1.

4.1 Static Response

4.1.1 Approach and Proposed Model

This section describes the use of CDM to predict the response of concrete under static compressive loading. For a brittle material such as concrete, a modification of equation 3.10 is necessary to make it more suitable. In ductile materials, E_{eff} is basically the change of E with respect to strain (or damage) after the loading goes beyond the linear stage and enters the plastic region (Fig. 4.1a). However, for plain concrete, the response is different; after reaching peak load, failure occurs abruptly (Fig. 4.1b). Therefore, to use CDM in concrete, the undamaged E is replaced by the initial E (E_{int}), while for the damaged E (E_{eff}), the secant E is used instead. Eq. 3.10 is then rewritten as:

$$D = 1 - \frac{E_{sec}}{E_{int}} \tag{4.1}$$

The damage curve obtained from the test results can be assumed to have the following form:

$$D = A\ln(\varepsilon) + B \tag{4.2}$$

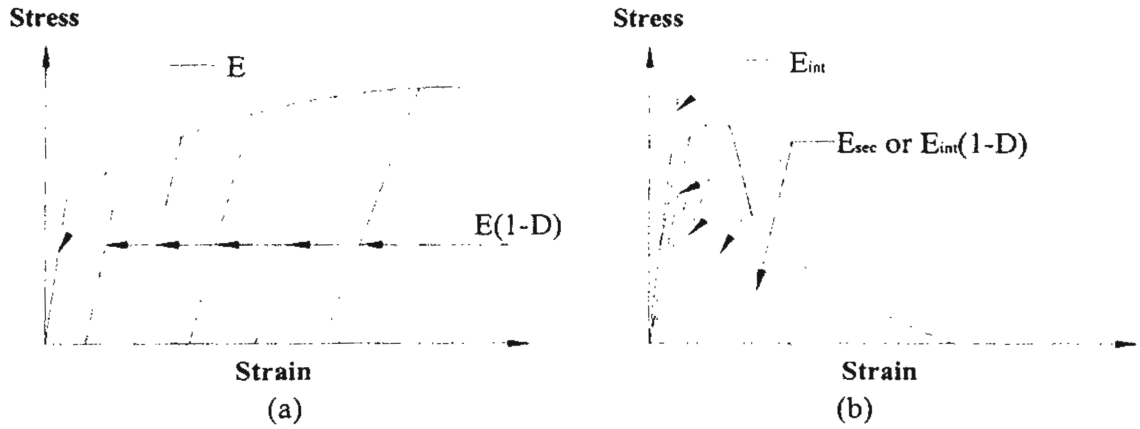


Fig. 4.1 Measurement of E for (a) Ductile Material and (b) Brittle Material

Applying the following conditions: at D = 0, $\varepsilon = \varepsilon_0$ and at D = D_{ult}, $\varepsilon = \varepsilon_c^{'}$, yields:

$$0 = A\ln(\varepsilon_0) + B \qquad (4.3)$$

$$D_{ult} = A\ln(\varepsilon_c^{'}) + B \qquad (4.4)$$

From Eq. 4.3,

$$B = -A\ln(\varepsilon_0) \qquad (4.5)$$

Substituting Eq. 4.5 into Eq. 4.4, and solving for A:

$$A = \frac{D_{ult}}{\ln(\varepsilon_c^{'}/\varepsilon_0)} \qquad (4.6)$$

Substituting A in to Eq. 4.5, one obtains B equal to:

$$B = \frac{-D_{ult}}{\ln(\varepsilon_c^{'}/\varepsilon_0)}\ln\varepsilon_0 \qquad (4.7)$$

where D is the damage

ε_0 is the assumed initial strain (>0)

D_{ult} is the damage at peak load

$\varepsilon_c^{'}$ is the strain at peak load

D_{ult} is the damage value at the peak and can be obtained directly from the test results.

For the elastic modulus degradation, D_{ult} is equal to $1 - E_c/E_{int}$ where E_c is the secant modulus at peak load and E_{int} is the initial modulus. Substituting D_{ult} into Eqs. 4.6 and 4.7, A and B can be rewritten as:

$$A = \frac{1 - E_c/E_{int}}{\ln\left(\varepsilon_c^{'}/\varepsilon_0\right)} \tag{4.8}$$

$$B = \frac{E_c/E_{int} - 1}{\ln\left(\varepsilon_c^{'}/\varepsilon_0\right)} \ln \varepsilon_0 \tag{4.9}$$

To obtain A and B, the secant modulus at peak load (E_c), the initial modulus (E_{int}), the strength ($f_c^{'}$) and the strain at peak ($\varepsilon_c^{'}$) must be known. These values are quite easy to obtain as they are well documented in the literature or can be obtained directly from experiments. After obtaining the corresponding damage (D) for any strain, the stress can then be determined as:

$$\sigma = E_{int}(1 - D)\varepsilon \tag{4.10}$$

4.1.2 Obtained Damage

The damage obtained by Eq. 4.1 was plotted against the stress Fig. 4.2, it may be seen that the damage of plain concrete can be represented by a nonlinear curve up to the peak and a linear curve after the peak (as it increases nonlinearly with strain from 0.0 up to 0.65 when it begins to slow down and, right after the peak strain, it forms another straight line).

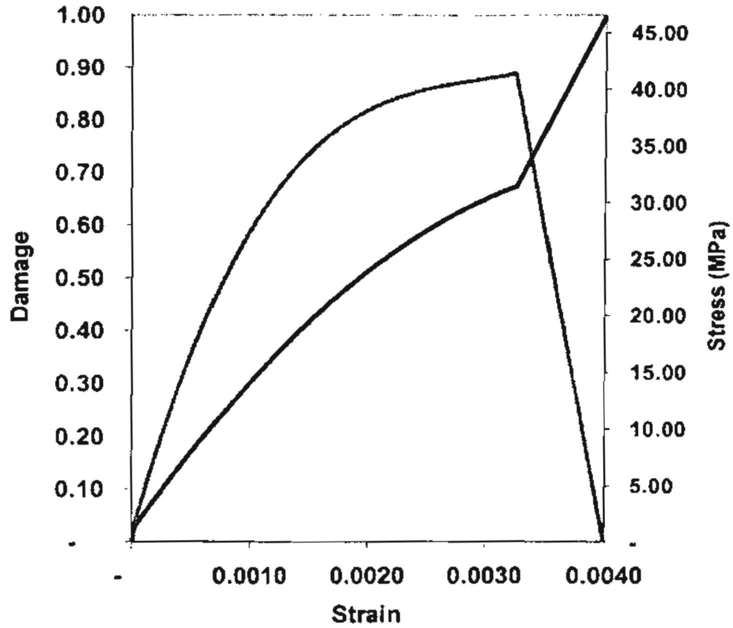


Fig. 4.2 Damage of Concrete Subjected to Static Loading

The damage of plain concrete under static loading was found to be around 0.65 at the peak load. Upon the fracture (at peak), because of the brittleness, the amount of energy in the machine was released immediately, this resulted in a sudden and catastrophic failure. The catastrophic failure also led to a quick increase of damage up to 1 without loading.

4.1.3 Predicted Response vs Actual Response

To predict the response, the following values were assumed (based on actual experimental results):

$$E_{int} = 35.4 \text{ GPa}, \; f_c^{'} = 41.6 \text{ MPa and } \varepsilon_c^{'} = 0.0036$$

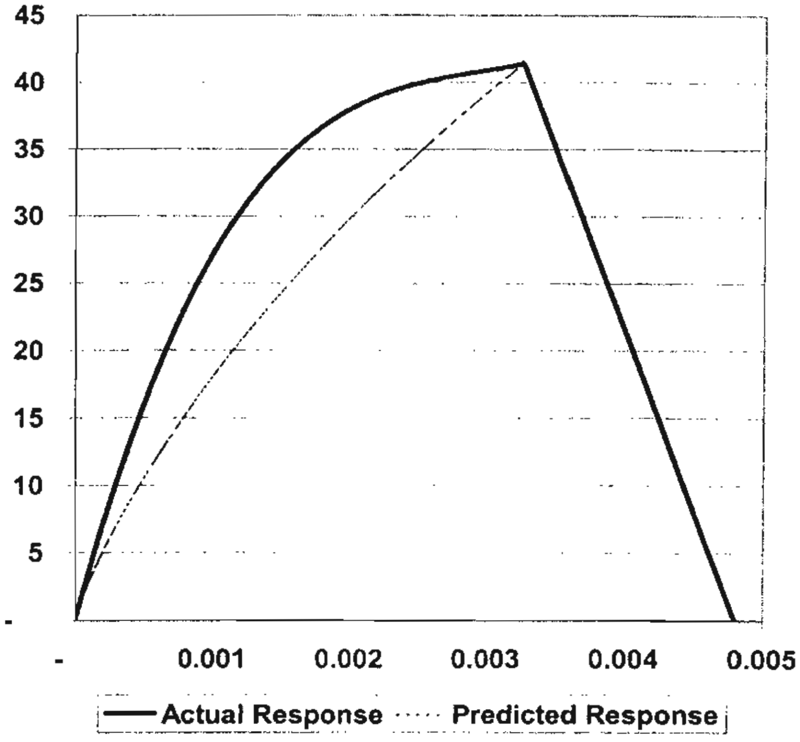


Fig. 4.3 Actual and Predicted Response of Concrete Subjected to Static Loading

The comparison between the actual responses and the predicted responses were given in Fig. 4.3. From Fig. 4.3, it may be seen that the damage of plain concrete can be represented by a nonlinear curve, as it increases linearly with strain from 0.0 up to 0.001 when it begins to slow down and then turns into a nonlinear line. The proposed model gave a slightly lower value in the middle of the loading event; however, the overall prediction was in fair agreement with the actual response. The under-prediction of the values in the middle is believed to cause by the complexity in the occurrence of micro-cracks which is essentially ignored by the theory of damage mechanics.

4.2 Impact Response

4.2.1 Approach and Proposed Model

In this section, CDM is used to predict the impact compressive response of concrete. The damage parameters A and B used to predict the static response are inadequate, because they do not include the strain rate parameter that becomes important during the impact event.

Therefore, a new set of stress-strain models using CDM was proposed to make use of the variation in strain rate during the impact event.

For the drop-weight impact test, the increase in strain up to the peak load was assumed to be linear (constant strain rate) in order to simplify the analysis. However, the actual test results indicated that the strain rate was, in fact, not constant (Fig. 4.6). The strain rate was slower at the beginning of the impact event, and then began to increase with the accumulation of damage (Fig. 4.7).

As a result, the damage of the material subjected to time-dependent loading can be expressed as (4):

$$D = 1 - \left(\frac{\dot{\varepsilon}_{\text{int}}}{\dot{\varepsilon}(t)} \right)^{1/N} \qquad (4.12)$$

where $\dot{\varepsilon}_{\text{int}}$ is the minimum or initial strain rate (sec^{-1})

$\dot{\varepsilon}(t)$ is the strain rate at any time t (sec^{-1})

N is a temperature-dependent constant (assume equal to 1)

The stress can then be obtained using Eq. 4.10, with E_{int} replaced by the initial impact elastic modulus (E^{d}_{int})

$$\sigma = E^{d}_{\text{int}}(1 - D)\varepsilon \qquad (4.13)$$

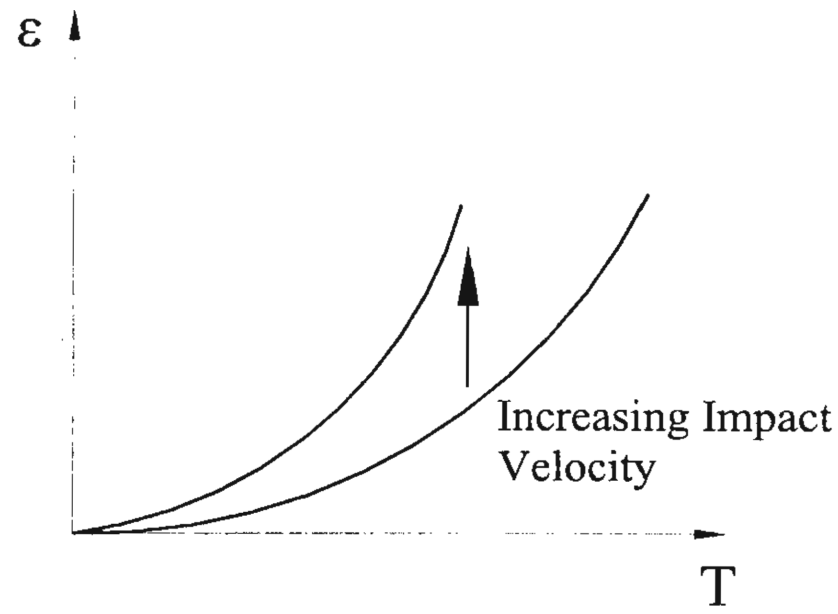


Fig. 4.6 Change of Strain Rate with Time for Plain Concrete under Impact Compressive Loading

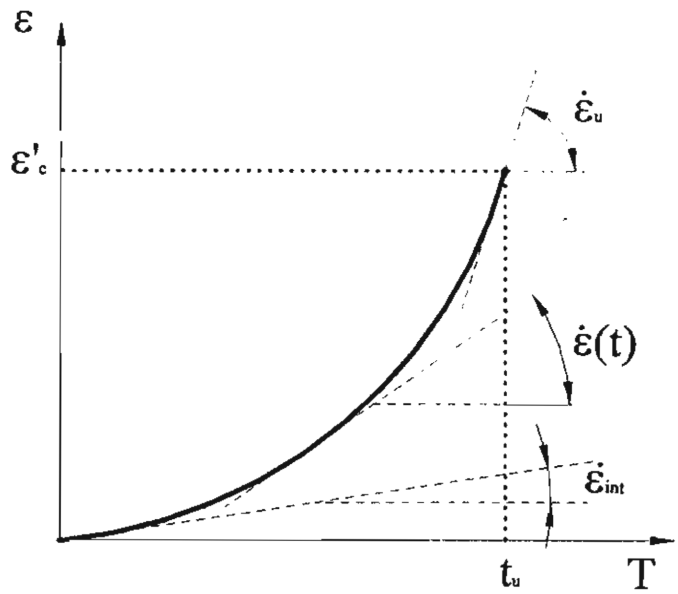


Fig. 4.7 Schematic Illustration of the Initial and the Ultimate Strain Rate, and the Strain Rate at Any Time t

According to Fig. 4.6 and 4.7, the general form of the strain-time relationship can be assumed to be a polynomial function:

$$\varepsilon = AT^3 + BT^2 + CT + D \qquad (4.14)$$

Applying the following conditions: (1) $T = 0 \rightarrow \varepsilon(t) = 0$, (2) $T = t_u \rightarrow \varepsilon(t) = \varepsilon_c^{'}$, (3) $T = 0 \rightarrow \dot{\varepsilon}(t) = \dot{\varepsilon}_{\text{int}}(t)$, *and* (4) $T = t_u \rightarrow \dot{\varepsilon}(t) = \dot{\varepsilon}_u(t)$, where t_u is the time at the peak load and $\dot{\varepsilon}_u(t)$ is the strain rate at the peak load or time t_u, yields:

$$A = \frac{\dot{\varepsilon}_u t_u + \dot{\varepsilon}_{\text{int}} t_u - 2\varepsilon_c^{'}}{T_u^{\,3}} \cong \frac{\dot{\varepsilon}_u + \dot{\varepsilon}_{\text{int}} - 2\dot{\varepsilon}_{avg}}{T_u^{\,2}}$$

$$B = \frac{3\varepsilon_c^{'} - \dot{\varepsilon}_u t_u - 2\dot{\varepsilon}_{\text{int}} t_u}{T_u^{\,2}} \cong \frac{3\dot{\varepsilon}_{avg} - \dot{\varepsilon}_u - 2\dot{\varepsilon}_{\text{int}}}{T_u} \qquad (4.15)$$

$$C = \dot{\varepsilon}_{\text{int}} \quad \text{and} \quad D = 0$$

where $\dot{\varepsilon}_{avg}$ is the average strain rate over the entire impact event equal to $\frac{\varepsilon_c^{'}}{t_u}$.

4.2.2 Obtained Damage

Based on the test results, the change of strain with respect to time was plotted as shown in Fig. 4.8. As mentioned earlier, under impact loading, the strain rate was actually changed with time; this can be seen clearly in Fig. 4.8. At undamaged stage, the specimen was highly stiff and strong, thus the rate of deformation (strain) was quite low. However, with further loading, the stiffness decreased gradually and the rate of deformation started to jump to a faster rate.

Once the strain rate corresponding to each strain was established, the damage can also be determined using the procedure described in 4.2.1. Results are plotted in Fig. 4.9

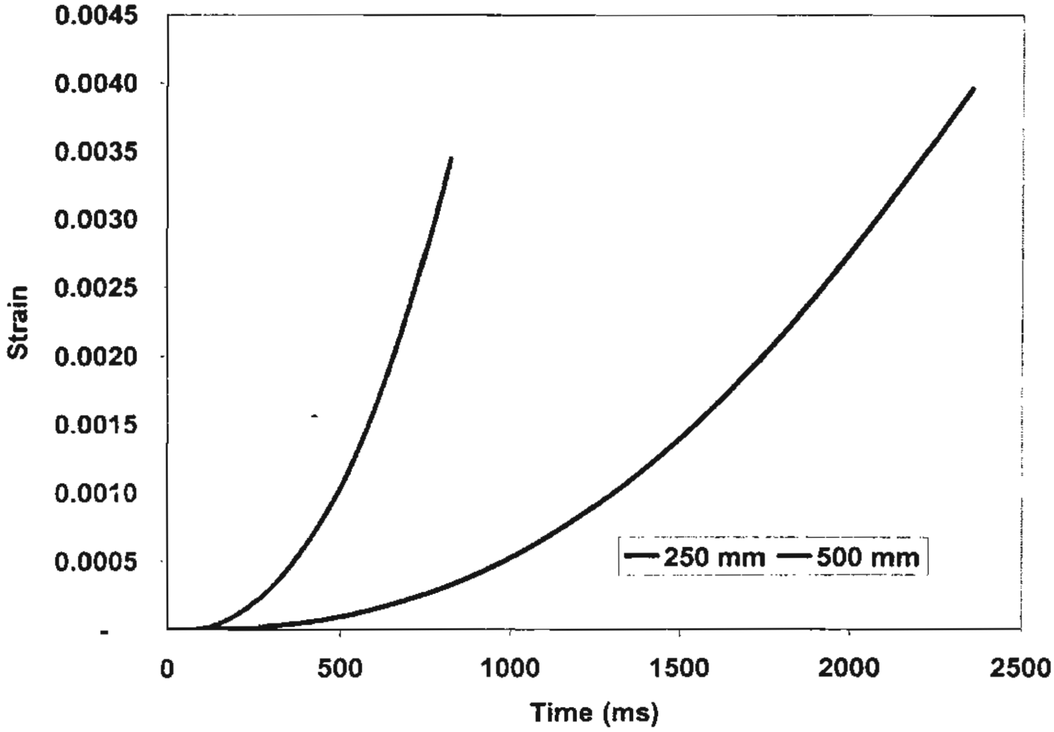
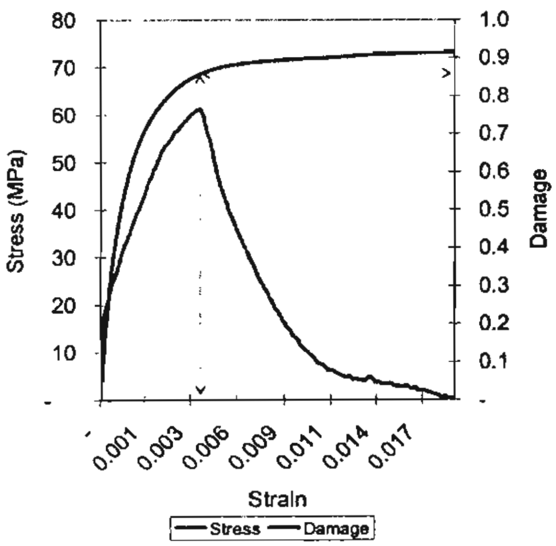
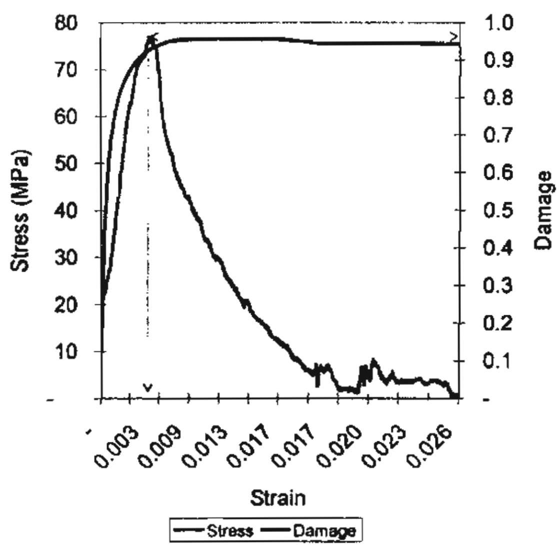


Fig. 4.8 Change of Strain with Time of Concrete Subjected to Impact Loading



(a)



(b)

Fig. 4.9 Relationship between Damage (strain rate) and Strain for Concrete Subjected to Impact Loading at (a) 250 mm and (b) 500 mm Drop Height

Prior to the peak, the damage obtained by the strain rate variations approach (0.858 and 0.930) were quite similar to that obtained by the variations of E approach (0.855 and 0.952). Both were found increase with stress in the similar manner and up to the peak, the values of D were drawn to near one. However, the difference was that, in this case, the value of D after peak would never reach the value of one even at the complete fracture. This was believed to cause by a limitation of the Eq. 4.12 itself. For any given initial strain rate, the last term of Eq. 4.12 which was the ratio between initial strain rate and strain rate at any time t, would not turn into zero. Therefore, it is recommended here that this method should be used only to determine the damage up to the peak.

The damage (value of D) was also found to be affected by the impact velocity (i.e., energy). Consider Fig. 4.9a and 4.9b, the value of D at the peak of specimen subjected to higher impact velocity (500mm) was larger than that subjected to lower impact velocity (250mm). This gave us an idea that, with higher impact velocity, the specimen was tortured at higher level (seen by larger value of D) and pushed closer to a complete fractured state.

Even though, the stain rate variation was selected as an approach to predict the response of concrete subjected to impact loading, a comparison between both method (variations of E and strain rate) was still carried out in order to point out the advantage and disadvantage between two approaches. Results were given in Appendix A.

4.2.3 Predicted Response vs Actual Response

For a plain concrete tested under impact loading from a 250 mm drop height (impact energy 1417 J), the initial impact elastic modulus was 125 GPa, the initial strain rate ($\dot{\varepsilon}_{int}$) was 0.6 sec^{-1}, the ultimate strain rate was 2.61 sec^{-1}, the duration of impact up to the peak load (t_u) was 0.00214 sec, and ultimate strain ($\varepsilon_c^{'}$) was 0.00257. The predicted response is shown in Fig. 4.8 below.

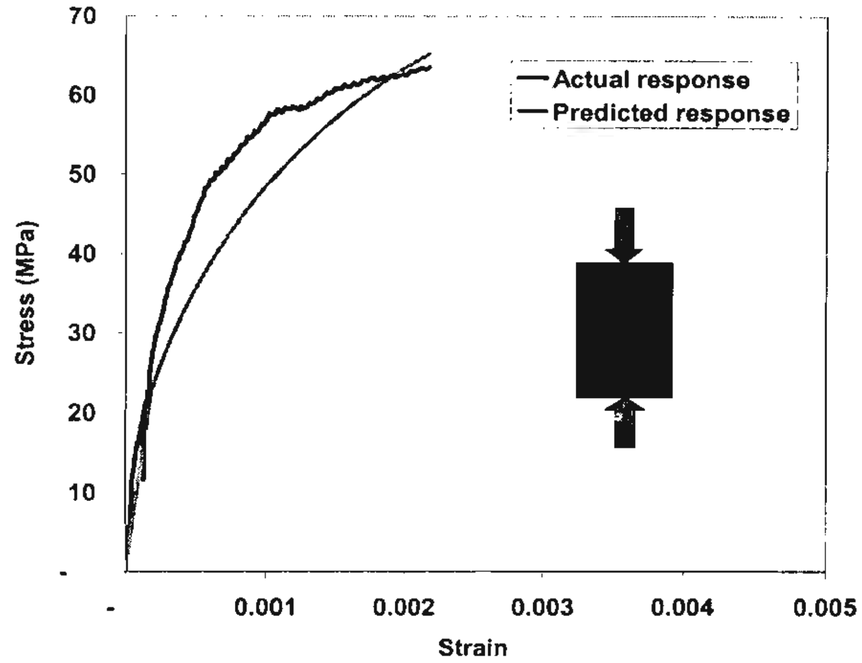


Fig. 4.8 Actual and Predicted Response of Plain Concrete under Impact Compressive Loading at 250mm Drop Height (Impact Energy of 1417 J)

For impact loading from a 500mm drop height (impact energy of 2835J), the initial impact elastic modulus was 150 GPa, the initial strain rate ($\dot{\varepsilon}_{int}$) was 1.53 sec^{-1}, ultimate strain rate ($\dot{\varepsilon}_{u}$) was 10.16 sec^{-1}, time to peak (t_u) was 0.000824 sec, and ultimate strain (ε_c') was 0.00344. The predicted response, compared to the actual response, is shown is Fig. 4.9.

It was found that, for both drop heights, the predicted responses of plain concrete subjected to impact loading exhibited less of a linear portion and a slightly higher peak load than the actual response (actual strength of 63.5 kN and predicted strength of 65.5 kN). The lack of agreement in the shape of the curve result from the micro-mechanisms of failure that is more complex than can be described by a simple scalar damage mechanics model.