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โครงการ การออกแบบตัวจับชิ้นงานเพื่อจับยึดงานที่ต้องการความแม่นยำสูง

A theoretical approach to a robust fixture design

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บทคัดย่อ

ในกระบวนการผลิตที่ต้องอาศัยความเที่ยงตรงสูง (high precision machining) เช่น ชิ้นส่วนเครื่องยนต์ แม่พิมพ์ และคอมพิวเตอร์ ผลจากความผิดพลาดเพียงเล็กน้อยอาจก่อให้เกิดความเสียหายเป็นมูลค่าสูง ก่อนเริ่มกระบวนการผลิต ชิ้นงานถูกจับยึดบนเครื่องจักรด้วยฟิกซ์เจอร์ซึ่งมีความสำคัญเป็นอย่างมากในการควบคุมคุณภาพของชิ้นงาน หากตำแหน่งชิ้นงานเบี่ยงเบนไปจากที่ควรจะเป็น ตำแหน่งการตัดก็จะผิดพลาดไปด้วย งานวิจัยนี้มีจุดประสงค์คือ สร้างแบบจำลองทางคณิตศาสตร์สำหรับทำนายความคลาดเคลื่อนของตำแหน่งการตัดบนชิ้นงานก่อนเริ่มต้นกระบวนการตัด โดยแบ่งการวิเคราะห์ค่าความคลาดเคลื่อนของตำแหน่งการตัดเป็นสองขั้นตอนด้วยกันคือ 1. การประเมินในขั้นตอนการกำหนดตำแหน่งชิ้นงาน ซึ่งผู้วิจัยได้นำหลักการการเปลี่ยนแปลงทางเรขาคณิต (geometric transformation) มาทำการวิเคราะห์เพื่อตรวจจับตำแหน่ง (position) และลักษณะการจัดวาง (orientation) ของชิ้นงานที่เบี่ยงเบนไปในขณะที่วางอยู่ในฟิกซ์เจอร์ และ 2. การประเมินในขั้นตอนการจับยึดชิ้นงาน ผู้วิจัยได้นำระเบียบวิธีการทางไฟไนต์เอลิเมนต์มาทำการวิเคราะห์การเสียรูปของชิ้นงานที่มีสาเหตุมาจากแรงในการจับยึด และส่งผลให้เกิดความคลาดเคลื่อนของตำแหน่งการตัด ผลการวิเคราะห์ที่ได้จากงานวิจัยแสดงถึงลักษณะความคลาดเคลื่อนของตำแหน่งการตัดพื้นผิวด้านบนของชิ้นงานที่ทำให้เกิดความผิดพลาดในการตัดชิ้นงานตามเส้นทางการเดินใบมีดที่ออกแบบไว้ โดยค่าความคลาดเคลื่อนที่ประเมินได้นี้สามารถนำไปใช้ในการออกแบบตำแหน่งและทิศทางการตัดของใบมีดให้ชดเชยกับความคลาดเคลื่อนนั้นเพื่อเพิ่มความแม่นยำและเป็นแนวทางในการออกแบบตัวจับชิ้นงานที่มีประสิทธิภาพในกระบวนการผลิตต่อไป

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ABSTRACT

Errors even of very small fraction in a precision manufacturing process could cause significant losses. Fixturing workpiece is an important procedure as it plays a crucial role in workpiece quality control. Whenever the actual workpiece position is displaced from its nominal position, machining errors are expected. This research aims to develop a mathematical approach to assess cutting deviation of a fixtured workpiece prior to machining. Geometric transformation was employed to capture rigid workpiece movements as results of locating errors. Local deformation exerted by clamping forces was subsequently analyzed by a finite element method. Displacement of surface coordinates on the workpiece as influenced by both locating and clamping errors is clearly shown. Without correction measures, quality of machined features would be impaired. Results from this research can be used to redesign cutting tool paths to accommodate such displacement and hence, increase feature accuracy. In addition, a more effective fixturing design can also be established.

Keywords workpiece displacement, fixture; locator; clamp; cutting deviation

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CHAPTER 1

INTRODUCTION

1.1 Background of Problem

Manufacturing process variability results in dimensional variability of the workpiece and ultimately means additional production costs. This variability is not acceptable for precision manufacturing. In the automobile industry, for instance, a diameter of a shaft must be precisely controlled to allow functional assembly in a gear hole. If the dimension is off tolerance, failure in assembly will result and the unfit components must be reworked or rejected, hence loss in time and productivity. To prevent such loss, a scheme to control potential machining errors is needed.

One of key determinants for workpiece geometrical and dimensional accuracy is the relative position and orientation of a workpiece with respect to that of a machine. As shown in Figure 1, the accuracy of a cylindrical feature is deteriorated by a displacement of the workpiece. When the workpiece position relative to the machine coordinate system is displaced from P_w to P'_w , the feature position with respect to the workpiece is then changed from the nominal position, P_f to P'_f , thus introducing dimensional errors. The workpiece displacement can result from several factors such as machine vibration, clamping forces, and locating errors. In manufacturing process, fixture is a device used to locate, hold and support the workpiece through a sequence of operations. The workpiece displacement can be minimized or even eliminated by an efficient fixturing system. Generally, fixture design amounts on average to 10-20% of the total production cost [1]; hence it is a crucial part of the manufacturing system. However, the effects of fixturing method on workpiece displacement are yet to be completely analyzed.

Fixturing system typically utilizes locators and clamps to locate and restrain the workpiece. The workpiece could primarily be displaced from its minimal position in a fixture by locating errors, which is consequent on inaccuracy of locators [2] and of the workpiece dimension and geometry [3-8]. Since inaccuracy in many occasions is inevitable the goal of this project is to account for it. After the workpiece is located at a desired position, clamps will be used to restrain the workpiece stationary but not to deform the workpiece. Though a secondary displacement apparently occurs as affected by clamping forces, the preceding errors from locating method will contribute in this displacement as well. The workpiece deviation related to clamping has been widely discussed [9-13]. However, assuming negligible locating error when

the primary displacement is comparatively significant such assumption can lead to imprecise determination about fixturing parameters.

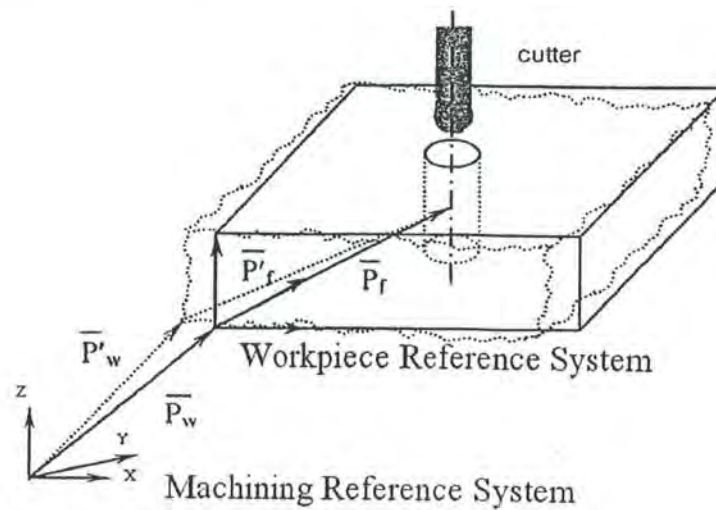


Figure 1-1. The impact of workpiece displacement on the errors of the machined feature

To acquire an efficient fixture design, this study will firstly assess the potential primary displacement before evaluation of the secondary displacement. The method to determine a resultant error as a combination of both sources will be established. A fixture design algorithm will then be developed upon the analyses of both types of displacement. This algorithm will find fixturing parameters needed to formulate a minimum resultant workpiece displacement. Even the solution derived may not completely eliminate the displacement, it would help significantly not only in fixture design but also in tool path optimization. Machining parameters can then be selected based on the knowledge gained from this study.

1.2 Objectives

- 1.2.1 To assess workpiece displacement prior to machining, as a result of locating errors and clamping parameters
- 1.2.2 To propose methods for fixturing design in order to minimize displacement

1.3 Literature review

The effects of locating and clamping errors on the displacement of the workpiece were discussed widely as follows

- 1.3.1 The effect of locating errors

1.3.1 The effect of locating errors

Locating errors is a consequence of both errors in workpiece and locators. For the workpiece, quality of its surface error is dependent on the preceding processes and the process parameters, as shown in Figure 1-2. If the preceding process does not provide a surface with acceptable surface errors, rough machining of the contact areas is advised. To minimize the effect of surface errors, datum targets are established to improve the repeatability of workpiece location and orientation. Datum targets (see Figures) are expected contact points (or areas) and are typically selected such that the surface irregularities are minimal. They are more functional than the use of entire surfaces for the establishment of datums [14]. However, the use of datum targets will not completely eliminate the effect of surface errors on workpiece locating, therefore, it will not guarantee successful mapping of the workpiece coordinate system with that of the fixture. It is thus suggested to understand the impact of the variability on a resultant workpiece quality. The allowable surface error or variability may not severely affect the overall production process; however, it could introduce significant errors during processing. There are several publications relevant to fixturing errors in manufacturing processes. Only a few researchers investigated the displacement of workpiece's location and orientation in a fixture caused by surface errors at contact points.

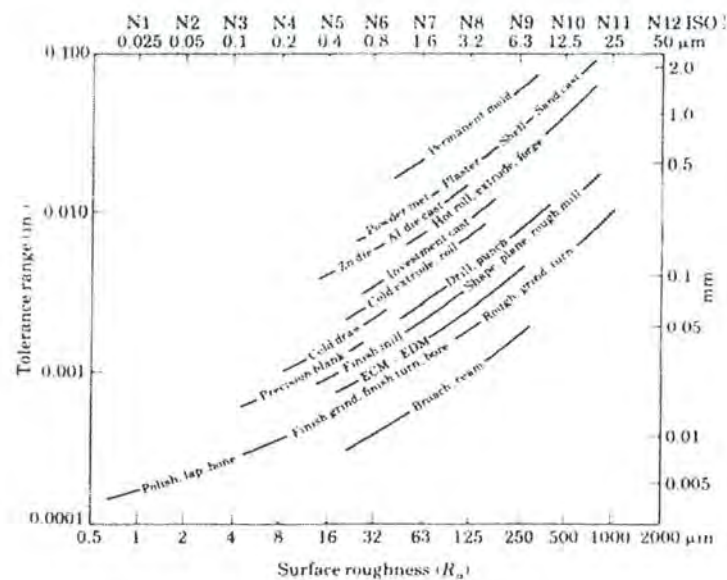


Figure 1-2 Tolerances and surface roughness obtained in various manufacturing processes.

These tolerances apply to 25 mm workpiece dimension [15].

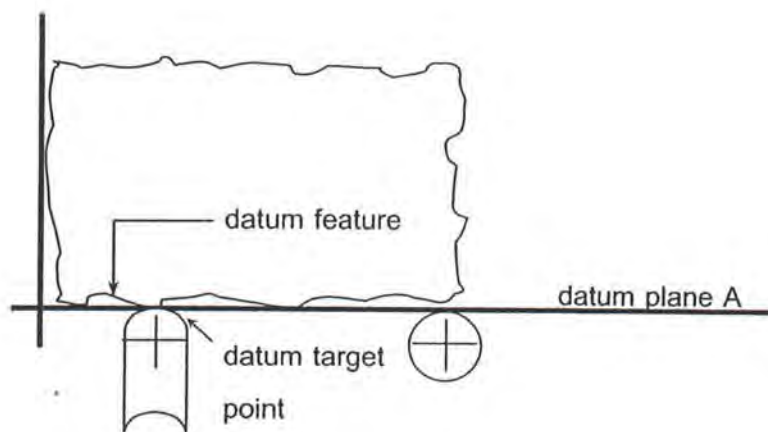


Figure 1-3. The use of a datum target point and line in locating a rough surface workpiece

Rong and Bai [6] analyzed a dependent relationship of operational dimensions to estimate machining errors resulted from different setup planning approaches. The errors of linear and angular dimensions were analyzed deterministically and randomly. The deterministic machining errors were caused by position error of fixture locating components, datum variations of the workpiece, tool-fixture alignment errors, tool wear and other deterministic errors in the machining process, while the clamping deformation, cutting force and thermal deformation are classified as random error sources. The author proposed that the error of a resultant dimension was a relationship of those of relevant dimensions and the relationship pattern was differentiated by setup planning. The effects of locating methods on the error determination were illustrated. Cai et al [7] proposed a method to conduct a robust fixture design to minimize the workpiece positional errors as results of workpiece surface and fixture setup errors. A condition of deterministic locating was defined as when the rank of Jacobian of the constraint equations equaled the degrees of freedom of the workpiece. Similarly, a total fixturing condition could be achieved when the workpiece maintained the deterministic locating condition under clamping and machining loading. A simulation module was developed to numerically compute optimal fixturing parameters i.e. locators, clamps position and clamping forces. However, the algorithm established can be applied only when the workpiece is assumed rigid. Salisbury and Peters [16] gave closer attention at the impact of surface errors on the deviation of workpiece location and orientation. They developed a model to predict the deviation of a prismatic workpiece located by 3-2-1 fixturing method. A step-wise approach was used to calculate the workpiece final location and orientation before workpiece displacement was determined.

Simulations were carried out to verify theoretical results. It was found that maximum displacement was not necessarily caused by maximum possible errors at all contact points. Later, the mathematical model of the same purpose for a cylindrical workpiece was developed by Sangnui and Peters [4]. An algorithm was created in order to study a relationship between the workpiece and fixture coordinate systems. Surface error model was established. The error was positive when the contact point was located beyond the nominal surface and it was negative otherwise. Newton-Raphson was used to solve for location and orientation of a cylindrical workpiece when the errors were present at the contact points. Experiments were also conducted to determine validity of the model and repeatability of the equipment. To be more realistic, instead of holding the errors constant, Sangnui and Peters [5] formulated a model which accounted for variability of the errors. The distributions of the workpiece's final location and orientation were formulated, after positioned in a fixture, as a relationship of its variability in dimension and geometry. Bhat and De Meter [8] carried out a simulation analysis to determine the effect of datum establishment methods on the geometric errors of machine features. The methods included 3-2-1, sequential least-squares and simultaneous least-squares. The study indicated that datum established by mean of the simultaneous least-squares results in significantly less hole-position error than the other methods. The effects of locator error were discussed in Chouduri and De Meter [2]. They developed a method for modeling and analyzing the impact of a locator tolerance scheme on the potential datum related, geometric errors of linear, machined features.

1.3.2 The effect of clamping parameters

The role of clamping forces at contact regions between the fixture and the workpiece has been discussed in several articles. Some of the forces considered are the cutting force exerting during machining, gravity force and clamping force. Before and operation starts, displacement of the workpiece due to clamping forces can be a significant source of geometric error. De Meter [9] analyzed a placement of the workpiece in terms of friction. Workpiece restraints can be theoretically classified into two types, form closed restraint and force closed restraint. The nonnegative linear hull concept was employed to develop the model to evaluate the capability of the fixture in workpiece restraint. De Meter [10] formulated a model to predict and impact of locator and clamp placement on the workpiece displacement during machining. The author adopted a scheme of Coulombic friction which assumed the workpiece and the fixture were rigid bodies. Loads at the locator contact regions were derived from both clamping and machining forces. Such loads were assumed to be uniformly distributed. The author applied three constraint concepts to develop the model: contact region wrench constraints, actuator

intensity constraints, and workpiece static equilibrium constraints. Convex hull was employed to establish boundary equations to find the minimum contact region loads necessary to maintain the workpiece in position throughout all clamping and machining operation. Based on the model, the author describe an algorithm for determining locator and clamp positions, and clamp actuation intensities that result in minimum loading throughout a series of cuts [11]. Since workpiece displacement depends upon loads applied at the contact regions, in the paper, boundary equations were established to satisfy kinematic restraint and total restraint requirement. The equations were then solved to attain a feasibly optimal fixture configuration. De Meter and Hockenberger [13] proposed a technique to determine workpiece displacement during clamping. The first technique applied eddy current displacement sensors to measure workpiece displacement online. An analytical model was created to predict workpiece displacement during clamp actuation. Hockenberger and De Meter [17] applied quasi-static analysis and meta modeling to predict the impact of fixture design parameters on prismatic workpiece displacement. The workpiece displacement resulting from forces exerted by locators and clamps were analyzed separately. In such study, the workpiece was assumed to displace quasi-statically.

Finite element analysis has been used to solve fixturing problems. Liao and Hu [18] presented a technique to dynamically model and analyzed the fixture-workpiece system subjected to time-varying machining loads. The authors combined the advantages of a finite element analysis and a nonlinear rigid body dynamics to analyze influential factors such as clamping forces, machining loads, and contact friction exerting on the workpiece restrained in a 3-2-1 fixturing scheme. De Meter [12] proposed a model called the Fast Support Layout Optimization (FSLO) to determine support locations that will minimize the maximum displacement to tolerance ratio of a set of workpiece feature subject to machining loads. This model utilized a finite element analysis to characterize workpiece stiffness. Roy discussed the issues of integrating finite element modeling in a CAD environment, including automated fixture design which implements an intelligent interfacing mechanism. Pham and Lazaro [19] employed a finite element study to determine the elastic behavior of a workpiece in a fixture and to locate the points of maximum deflection to minimize the errors. The information obtained from this study was then fed to an expert system for jig and fixture design.

From the past work, it can be seen that most researchers focused on minimizing the displacement while disregarded the inborn workpiece errors. The degree of such errors depends on the preceding process as shown in the previous section which in many occasions is relatively high compared to the accuracy required in the current process. Since the workpiece

surface errors are inevitable, the method to characterize the design parameters of the fixture that results in the feasible least workpiece displacement is necessary.

1.4 Methodology

Considered in this model are the effects of locating errors and clamping parameters since they strongly influence the behavior of the workpiece before the operation. In the first part, evaluation of the effect of locating errors will be carried out under geometric constraints of the workpiece and the fixture. This error is named herein a primary displacement. A secondary displacement occurs when clamps are applied to the workpiece. At this moment, the workpiece is subject to loading from both clamps and locators. The workpiece will start to deform once the amount of loads surpasses its yield strength. The secondary displacement is greatest at the contact regions and diminished in areas away from such regions. It can be described as a local displacement as opposed to the primary displacement that affects globally. Due to their distinct characteristics, both displacements will be analyzed separately. A finite element method will be used in the second part to investigate the workpiece behavior under clamping forces. After the method to determine the resultant error from the combination of both sources is established, an algorithm to find parameters of a feasible fixturing design that will lead to the least overall displacement prior to machining will be developed. This study focuses on displacement prior to machining of a prismatic workpiece fixture in a 3-2-1 fixturing method.

In summary, the procedure of this research is listed as follows.

- Review literature to gather all necessary information related to the role of factors affecting workpiece displacement.
- Evaluate the primary displacement as affected by locating errors on the displacement by using a geometric analysis.
- Evaluate the secondary displacement as resulted from clamping parameters by using a finite element method.
- Based on the research findings , propose guidelines for fixture design.
- Conclude the study and prepare a final report.

1.5 Research Benefit

It is beneficial if the process planner and the fixture designer have a better perception on the workpiece behavior while it is held in a fixture. Fixture handbooks exist to give these designers and planners qualitative advice to design a functional fixture. The algorithm

developed in this work; however, will provide rather specific information. Workpiece displacement as a result of each design can be quantified. Based on this information, the fixture could be redesigned in a way that it will repeatedly position the workpiece closest to its theoretical location and orientation. The algorithm will help locate contact points to areas of the workpiece offering the most accurate workpiece positioning. This study also helps in specifying tolerances needed for the preceding process and to define reasonable tolerances of descendant features. By understanding the interaction between the workpiece and the fixture, manufacturers will be able to establish an efficient and economical process plan, and ultimately reduce production cost.

1.6 Report Organization

This report is organized in a chapter format which each chapter contains a manuscript of a published article.

In Chapter 2, development of an algorithm to determine variant final locations of a displaced workpiece given normally distributed errors at contact points is described. Resultant geometric variation of workpiece location reveals interesting information which is beneficial in tolerance planning.

In Chapter 3, a mathematical model that is able to predict geometric variation of a resultant-machined surface when the tolerance of a datum feature is given is developed. A model for a 2-D scheme was initially established that subsequently expanded to a more realistic case in a 3-D scenario. Results from the model show that when variability of a datum feature is allowed, a paraboloid-shaped variational function of a machined surface is detected.

In Chapter 4, three types of distribution, which are a normal, a beta, and a uniform distribution, were used to represent variability of surface errors at contact points. From the simulation, it is found that the variability of the machined feature is obviously dependent on the distribution selected. Aside from mathematical complexity, this affected feature variability is another factor to be considered in choosing an appropriate surface distribution. The method developed herein will help a designer to impose more efficient tolerancing system and reduce production cost eventually.

In Chapter 5, workpiece displacement as a result of clamping error is analyzed. A finite element method (FEM) is used to explain how the workpiece is deformed by clamping forces. The Hertz theory is used to validate reliability of the simulation. Once the displacement is quantified, methods to alleviate effects of workpiece displacement are suggested.

Chapter 6 is a conclusion of this research.

1.7 References

1. Fuh, J.Y.H., C.-H. Chang and M.A. Melkanoff, An integrated fixture planning and analysis system for machining process. *Robotics and Computer-integrated Manufacturing*, 1993. 10: p 339-353.
2. Choudhuri, S.A. and E.C. De-Meter, Tolerance analysis of machining fixture locators. *Journal of Manufacturing Science and Engineering*, 1999. 121: p.273-81
3. Sangnui, S., The impact of surface errors on location and orientation of a workpiece fixture in datum locators, in *Industrial and Manufacturing Systems Engineering*. 1998, Iowa State University: Ames, IA.
4. Sangnui, S. and F.E. Peters, The impact of surface errors on the location and orientation of a cylindrical workpiece in a fixture. *Journal of Manufacturing Science and Engineering*, 2001. 123: p. 325-330
5. Sangnui, S. and F.E. Peters, Prediction of fixture workpiece location when there are variation surface errors at fixturing points. in *Proceedings of the 4th Asia-Pacific conference on Industrial Engineering and Management Systems*. 2002. Taipei, Taiwan
6. Rong, Y. and Y. Bai, Machining accuracy analysis for computer-aided fixture design verification. *Journal of Manufacturing Science and Engineering*, 1996. 118: p. 289-300.
7. Cai, W., S.J. Hu, and J.X. Yuan, A variational method of robust fixture configuration design for 3-D workpieces. *Journal of Manufacturing Science and Engineering*, 1997. 119: p. 593-602.
8. Bhat, V. and E.C. De-Meter, An analysis of the effect of datum establishment methods on the geometric errors of machined features. *International Journal of Machine Tools & Manufacture*, 2000. 40: p. 1951-1975.
9. De-Meter, E.C., Restraint analysis of fixtures which rely on surface contact. *Journal of Engineering for Industry*, 1994. 116: p. 207-215.
10. De-Meter, E.C., The min-max load criteria as a measure of machining fixture performance. *Journal of Engineering for Industry*, 1994. 116: p. 500-507.
11. De-Meter, E.C., Min-max load model for optimizing machining fixture performance . *Journal of Engineering for industry*, 1995. 117: p.186-193.
12. De-Meter, E.C., Fast support layout optimization. *International Journal of Machine Tools and Manufacture*, 1998. 38: p. 1221-1239.

13. De-Meter, E.C. and M.J. Hockenberger, The application of tool path compensation for the reduction of clamping induced geometric errors. *International Journal of Production Research*, 1997. 35: p. 3415-3432.
14. Meadows, J.D., *Geometric dimensioning and tolerancing: applications and techniques for use in design, manufacturing and inspection*. 1995, New York, NY: Marcel Dekker, Inc.
15. Kalpakjian, S., *Manufacturing Processes for Engineering Materials*, 2nd ed. 1991: Addison Wesley Publishing Company, Inc.
16. Salisbury, E.J. and F.E. Peters, The impact of surface errors on fixture workpiece location and orientation. *NAMRC Transactions*, 1998. 26: p. 323-328.
17. Hockenberger, M.J. and E.C. De-Meter, The application of meta functions to the quasi-static analysis of workpiece displacement within a machining fixture. *Journal of Manufacturing Science and Engineering*, 1996, 118: p. 325-331.
18. Liao, Y.J.G. and S.J. Hu, Flexible multibody dynamics based fixture-workpiece analysis model for fixturing stability. *International Journal of Machine Tools and Manufacture*, 2000. 40: p. 343-362.
19. Pham, D.T. and A.D.S. Lazaro, Finite element study of a workpiece in a machining fixture. *Mathematical Computation Modeling*, 1990. 14: p. 1024-1028.
20. *Handbook of Jig and Fixture Design*. 2nd ed, ed. W.E. Boyes. 1989, Dearborn, MI: Society of Manufacturing Engineers.

CHAPTER 2

RESULTANT GEOMETRIC VARIATION OF A FIXTURED WORKPIECE: A SIMULATION

2.1 Nomenclature

$\bar{n}_p, \bar{n}_s, \bar{n}_t$	Nominal vectors normal to a primary, secondary and tertiary datum features, respectively
$\bar{n}_{pe}, \bar{n}_{se}, \bar{n}_{te}$	Actual Vectors normal to a primary, secondary, and tertiary datum features, respectively
$\bar{P}_i$	Nominal contact point when $i=1,2,..6$
$\bar{P}_{ie}$	Actual contact point when $i=1,2,..6$
D_p, D_s, D_t	Nominal Distance from the origin to a primary, secondary, and tertiary datum features, respectively
D_{pe}, D_{se}, D_{te}	Actual Distance from the origin to a primary, secondary, and tertiary datum features, respectively
α, β, γ	Rotational angle about Z, Y, and X axes
dx, dy, dz	Translational distance along X, Y, and Z axes

2.2 Introduction

Variability existing in a production line is obstructive to gain an efficiently precise control over manufacturing operations. Typically, among causes of variability are machine malfunctions, operator and equipment errors and inherent properties of raw materials. Discussed in this paper is fixturing or locating error. In a manufacturing process, a fixture is used to locate and restrain a workpiece from movements during machining or inspection. A reference to locate the workpiece at its nominal position and orientation is called a datum, which can be established from fixture or machine components as shown in Figure 2-1. A realistic part of the workpiece which is used to map with a datum is called 'a datum feature'. When the workpiece is fixtured for an operation, drilling a hole for example, exact mapping between the datum feature and its counterpart is desirable as it defines the hole accuracy. Since in many occasions a datum feature is a workpiece surface, any surface deviation from a perfect topography would lead to inaccuracy of final products.

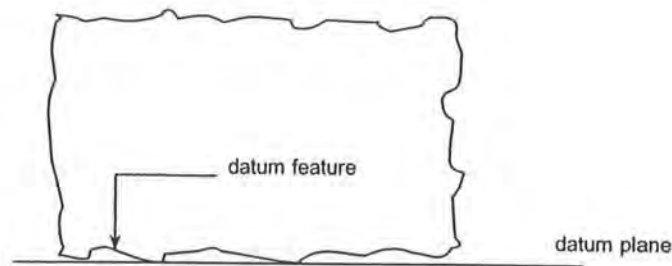


Figure 2-1 The mapping between a datum plane and a workpiece datum feature

The impact of erratic fixturing on the feature accuracy is illustrated in Figure 2-2. It shows a perfect workpiece at its theoretical location and an actual workpiece with errors in locating. It is assumed that the nominal workpiece's coordinate system, XYZ , is located coincidently with that of the machine. The workpiece is drilled at a location defined by $\bar{L}$ with respect to the machine's or the nominal workpiece system. When the workpiece is displaced, the hole location will be moved to $\bar{L}'$ with respect to the workpiece, XYZ' .

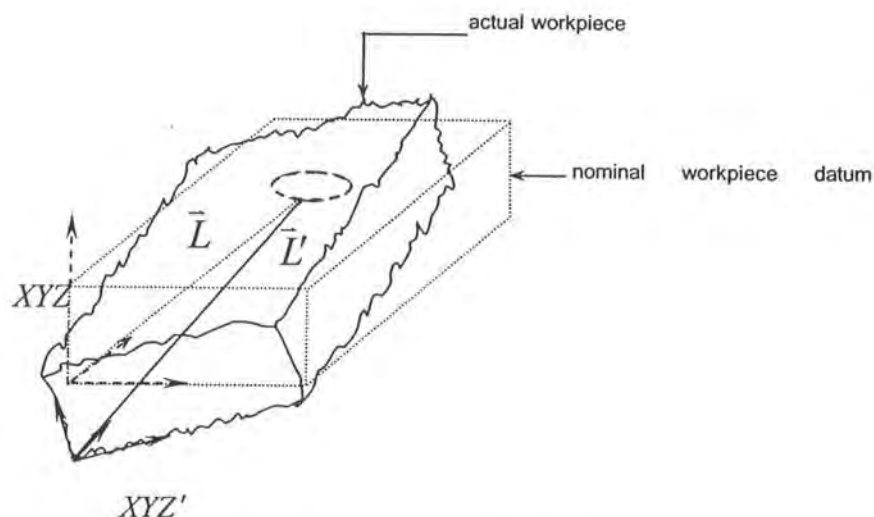


Figure 2-2. The workpiece and feature displacement

Workpiece displacement because of locating or fixturing errors is a threat in an early stage of precision machining. It has been discussed widely either in the workpiece deviation determination or fixture component placement optimization. Researchers working in the fixturing area have developed algorithms that provide complete constraint of the workpiece (King and Hutter 1993; Trappey and Matrubhutam 1993; Brost and Goldberg 1996; Dai, Nee et al. 1997; Jeng and Gill 1997; Meter 1998; Wu, Rong et al. 1998), determined sufficient support, performance of the fixture, and accessibility (Pham and Lazaro 1990; Fuh, Chang et al. 1993;

Meter 1994; Meter 1994; Meter 1995; Ong and Nee 1998). However, less focus has been placed on determining the initial displacement of the workpiece in the fixture before clamping. Rong and Bai (1996) analyzed a dependent relationship of operational dimensions to estimate machining errors in terms of linear and angular dimensions of the workpiece. Cai et al. (1997) proposed a method to conduct robust fixture design to minimize the workpiece positional errors as a result of workpiece surface and fixture set-up errors. A method for modeling and analyzing the impact of a locator tolerance scheme on the potential datum related, geometric errors of linear, machined features was developed by Choudhuri and De Meter (1999). There were also algorithms to predict a deviation of a prismatic workpiece located by 3-2-1 fixturing method (Salisbury and Peters 1998) and a cylindrical workpiece in a v-block (Sangnui and Peters 2001). The algorithms were established under a static case scenario. The results indicated that the largest deviation did not necessarily come from the largest errors at the contact points. Djurdjanovic and Ni (2003) developed procedures for determining the influence of errors in fixtures, locating datum features and measurement datum features on dimensional errors in machining. However, only a few researchers employed a variational model to evaluate fixture performance. Results of the models in absence of workpiece variability consideration would not be applicable and functional in most cases. Bhat and De Meter (2000) carried out a simulation analysis to evaluate the performance of three different datum-establishment methods: 3-2-1, sequential least-squares (SQLS) and simultaneous least-squares (SMLS). The authors assumed variant errors of datum features. Only a magnitude of relative positional workpiece displacement was presented but variation in workpiece position was yet to be analyzed. A probabilistic fixturing model was also developed by Sangnui (2002) and geometric variation of a resultant feature was determined. In this paper, an approach to assess prismatic workpiece displacement resulted from misalignment in locating is introduced. When errors of a datum feature are assumed probabilistic, the resultant workpiece geometric variation reveals a significant systematic pattern. Unlike previous studies, a step-wise analysis is considered unnecessary and the mathematical formulation is simplified. By utilizing the information obtained from this method, a designer can calculate the variation of subsequent processes, hence appropriate tolerances. Tool path can be more efficiently compensated in accordance with deviated workpiece location. Ultimately, the fixture can be designed to reduce effects of inevitably erratic workpiece locating.

2.3 Methodology

2.3.1 Fixturing Model

Fixturing is a critical part of a manufacturing system. If surface variation exists at the contact points of the datum feature, deviation of the workpiece from its nominal location is expected. The deviation will lead to errors in geometry of a feature to be produced. A typical 3-2-1 fixturing method is composed of six locators forming three datum planes used to locate the workpiece. The first plane, the primary datum plane as shown in Figure 2-3 is constructed by the first three locators (no. 1, 2, 3), while the secondary datum plane, perpendicular to the first plane, is established from the contact of the locators no. 4 and 5. Finally, the tertiary datum plane is the plane perpendicular to the preceding ones in which the last contact point (no. 6) lies. Note that, the workpiece surfaces in contact with the datum planes are called workpiece datum features. Given a distribution of the surface errors at the contact locations, statistics allows us to derive variability of final position and orientation of the fixtured workpiece.

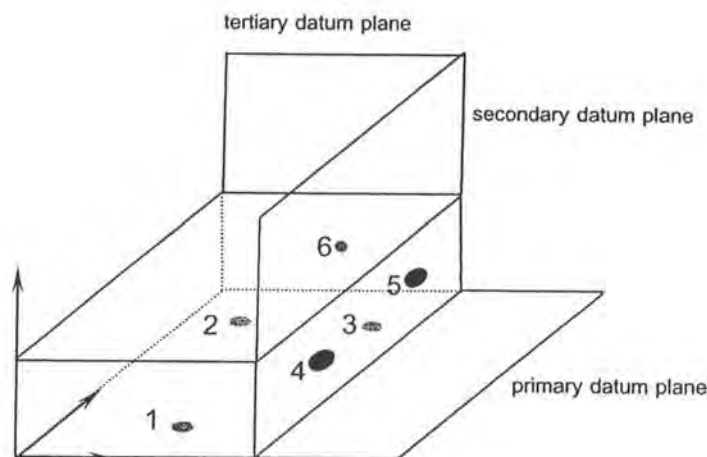


Figure 2-3 Datum planes in 3-2-1 fixturing method

2.3.2 Assumptions

The following assumptions regarding the fixture and the workpiece were used.

- 1) A workpiece is considered a rigid body. Deformation of the workpiece during transformation is not allowed.
- 2) Errors at contact points are assumed to be normally distributed with mean μ and variance σ^2 , or $N(\mu, \sigma^2)$.
- 3) The errors are measured perpendicularly to the datum features. The determination of the surface error sign is shown in Figure 2-4.

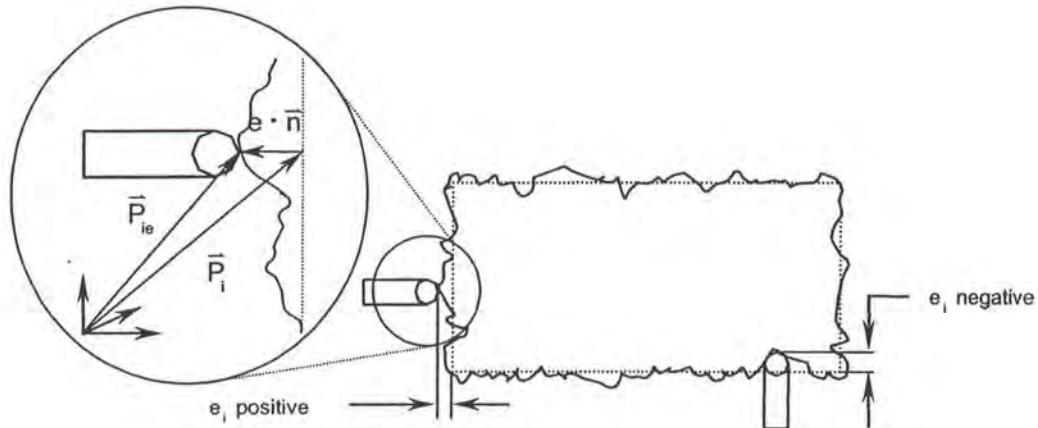


Figure 2- 4 Error determination

2.3.3 Analysis

When an imperfect workpiece is fixtured for an operation, variation at the contact point between the workpiece and the fixture would be imparted to the feature to be produced. The impact of such errors on the feature can be analyzed through the workpiece deviation from its nominal location. The deviation is explained by a concept of geometric transformation: translation and rotation. The workpiece and the feature behavior after located in a fixture will be captured and mathematically analyzed. First of all, a relationship of workpiece geometry and the contact points must be developed. Workpiece location in the fixture is represented by an arbitrary point, which could be the location of a feature of interest. Workpiece orientation in this paper is defined by vectors normal to the primary and secondary datum features. A normal vector to the primary datum feature $\vec{n}_p$ is derived from

$$\vec{n}_p = \vec{P}_{31} \times \vec{P}_{21} \quad (2-1)$$

where $\vec{P}_{21} = \vec{P}_2 - \vec{P}_1$
 $\vec{P}_{31} = \vec{P}_3 - \vec{P}_1$

Once an orientation of the primary datum feature is computed, a vector normal to the secondary datum feature can be obtained from the following equation. Note that the secondary datum feature must stay perpendicular to the preceding one. Therefore,

$$\bar{n}_s = \bar{P}_{s4} \times \bar{n}_p \quad (2-2)$$

where $\bar{P}_{s4} = \bar{P}_s - \bar{P}_4$

When the errors at the contact point P_i are considered in the equations, Eq. (2-1) and (2-2) become

$$\bar{n}_{pe} = \bar{P}_{31e} \times \bar{P}_{21e} \quad (2-3)$$

$$\bar{n}_{se} = \bar{P}_{54e} \times \bar{n}_{pe} \quad (2-4)$$

where $\bar{P}_{21e} = \bar{P}_{2e} - \bar{P}_{1e}$
 $\bar{P}_{31e} = \bar{P}_{3e} - \bar{P}_{1e}$
 $\bar{P}_{54e} = \bar{P}_{5e} - \bar{P}_{4e}$

The algorithm is developed in this paper to find a set of transformations matching a nominal workpiece with an actual one once located in a fixture. At the nominal status, the workpiece is initially placed and oriented at its nominal state (Figure 1-5.). At this status, the primary and secondary workpiece datum features align with XY and YZ planes of the machine coordinate system. Since errors are assumed at the contact areas, the workpiece will be disoriented and dislocated to allow a complete contact with the six locators. To guarantee such kind of contact, the following constraints must be satisfied.

$$\bar{n}_{pe} - R_1(\alpha, \beta, \gamma, dx, dy, dz)\bar{n}_p = 0 \quad (2-5)$$

$$\bar{n}_{se} - R_1(\alpha, \beta, \gamma, dx, dy, dz)\bar{n}_s = 0 \quad (2-6)$$

$$\bar{n}_{pe} \cdot \bar{P}_{1e} - D_p = 0 \quad (2-7)$$

$$\bar{n}_{se} \cdot \bar{P}_{4e} - D_s = 0 \quad (2-8)$$

$$\bar{n}_{te} \cdot \bar{P}_{6e} - D_t = 0 \quad (2-9)$$

where

$$R_1 = \begin{bmatrix} \cos(\alpha)\cos(\beta) & -\sin(\alpha)\cos(\beta) & \sin(\beta) & dx \\ \sin(\alpha)\cos(\gamma) + \cos(\alpha)\sin(\beta)\sin(\gamma) & \cos(\alpha)\cos(\gamma) - \sin(\alpha)\sin(\beta)\sin(\gamma) & -\cos(\beta)\sin(\gamma) & dy \\ \sin(\alpha)\sin(\gamma) - \cos(\alpha)\sin(\beta)\cos(\gamma) & \cos(\alpha)\sin(\gamma) + \sin(\alpha)\sin(\beta)\cos(\gamma) & \cos(\beta)\cos(\gamma) & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α, β, γ are rotational angles about Z, Y, and X axes, respectively

dx, dy, dz are a translational distance along X, Y , and Z axes, respectively

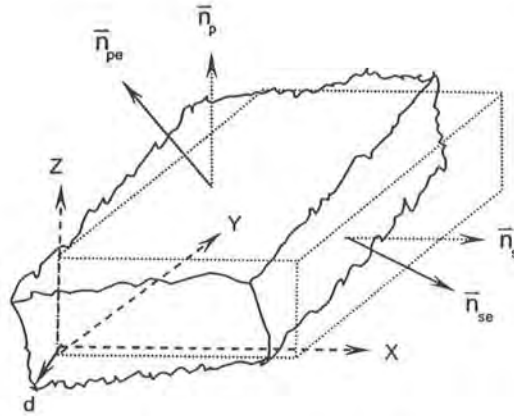


Figure 2-5 Workpiece location affected by variability at workpiece surface

A system of the above equations is solved by breaking down the procedure into 2 major steps. The first step is to find an appropriate rotation which matches $\bar{n}_p$ with $\bar{n}_{pe}$ and $\bar{n}_s$ with $\bar{n}_{se}$ by using a Newton-Raphson method. A set of objective functions is in the following equations.

$$\begin{aligned} f_1 : \bar{n}_{xp} - \bar{n}_{xpe} &= 0 \\ f_2 : \bar{n}_{yp} - \bar{n}_{ype} &= 0 \\ f_3 : \bar{n}_{zp} - \bar{n}_{zpe} &= 0 \\ f_4 : \bar{n}_{xs} - \bar{n}_{xse} &= 0 \\ f_5 : \bar{n}_{ys} - \bar{n}_{yse} &= 0 \\ f_6 : \bar{n}_{zs} - \bar{n}_{zse} &= 0 \end{aligned} \quad (2-10)$$

where $\bar{n}_{ie} = R_2(\alpha, \beta, \gamma) * \bar{n}_i$

$$R_2 = \begin{bmatrix} \cos(\alpha)\cos(\beta) & -\sin(\alpha)\cos(\beta) & \sin(\beta) \\ \sin(\alpha)\cos(\gamma) + \cos(\alpha)\sin(\beta)\sin(\gamma) & \cos(\alpha)\cos(\gamma) - \sin(\alpha)\sin(\beta)\sin(\gamma) & -\cos(\beta)\sin(\gamma) \\ \sin(\alpha)\sin(\gamma) - \cos(\alpha)\sin(\beta)\cos(\gamma) & \cos(\alpha)\sin(\gamma) + \sin(\alpha)\sin(\beta)\cos(\gamma) & \cos(\beta)\cos(\gamma) \end{bmatrix} \quad (2-11)$$

Following the Newton-Raphson procedure, the algorithm will be executed repeatedly until it finds a solution (α, β, γ) which makes Eq.(2-10) true. Once a rotation to transform $\bar{n}_p$ to $\bar{n}_{pe}$ and $\bar{n}_s$ to $\bar{n}_{se}$ is found, it will be used to transform the workpiece and the workpiece now is at the final orientation. However, the final state is not yet reached, a translation vector is

needed to bring the workpiece to its proper location (Figure 2-6). It is calculated by the difference in an existing and final location along the corresponding normal vectors, $\bar{n}_{pe}$, $\bar{n}_{se}$, and $\bar{n}_{te}$. A translation vector bringing the workpiece to make contact with all locators will be calculated from Eq. (2-12)

$$\bar{T} = \bar{n}_{pe} \cdot \Delta D_p + \bar{n}_{se} \cdot \Delta D_s + \bar{n}_{te} \cdot \Delta D_t \quad (2-12)$$

$$\Delta D_p = D_{pe} - D_p$$

$$\Delta D_s = D_{se} - D_s$$

$$\Delta D_t = D_{te} - D_t$$

(2-13)

where D_t is the perpendicular distance from the origin to the corresponding plane after the rotation.

D_{te} is the perpendicular distance from the origin to the corresponding plane after the workpiece makes contact with all locators.

The affected final location of the workpiece is then

$$\bar{L}_t = R_2(\alpha, \beta, \gamma) \bar{L} + \bar{T} \quad (2-14)$$

where $\bar{L}$ is a point of interest.

$\bar{L}_t$ is the point after transformation.

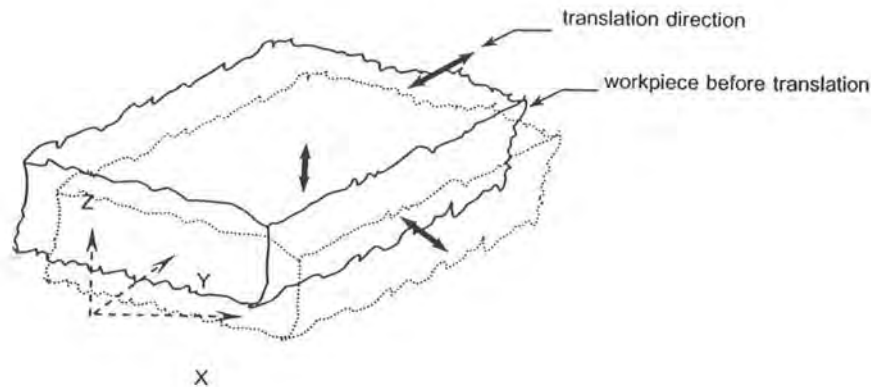


Figure 2-6 Workpiece translation

Note that the algorithm simultaneously solves for the transformation satisfying the objective functions. It may sound contradictory to the practical use of a 3-2-1 fixture, in which the workpiece must be sequentially located starting from the primary, secondary and tertiary datum plane. It is also realized that different orders of locating the planes will be associated

with different contact locations. However, it is considered in this paper the sequential process is not necessary since only variability of the workpiece location is estimated, not its exact configuration. Besides, the true contact points are impossible to obtain in practice, unless sophisticated equipment such as a probe or sensor is employed.

2.4 Results and Discussion

Once the algorithm to characterize the workpiece behavior was developed, a Monte Carlo simulation was conducted to assess the variability of workpiece location by using the following method.

- 1) A set of normal or Gaussian distributed random numbers represented the errors at each contact point was generated.
- 2) The samples were used in the algorithm and the results were collected.
- 3) A multivariate statistical process was used to evaluate the variability of the sample distribution.

Examples of fixturing constraints are shown in Table 2-1. Table 2-2 contains the results which are final locations of a point of interest under the constraints.

Table 2-1. Coordinates and distributions of errors at each locator (unit: mm).

Plane	Locators coordinates	Errors Distribution				
		1	2	3	4	5
Primary Plane	1 [30 30 0]	N(0.0, 4.00)	N(-1.0, 1.21)	N(0.0, 3.61)	N(1.5, 1.44)	N(0.0, 4.00)
	2 [120 30 0]	N(0.0, 2.25)	N(2.0, 3.24)	N(2.0, 2.89)	N(2.3, 1.69)	N(-1.0, 2.25)
	3 [75 120 0]	N(0.0, 1.44)	N(0.0, 1.69)	N(-1.2, 1.44)	N(0.0, 1.44)	N(-1.5, 1.44)
Secondary Plane	4 [150 30 15]	N(0.0, 1.69)	N(-1.2, 2.56)	N(0.0, 0.81)	N(-1.5, 3.61)	N(0.0, 1.69)
	5 [150 120 15]	N(0.0, 3.61)	N(2.0, 2.25)	N(2.0, 4.41)	N(-1.0, 4.00)	N(2.3, 3.61)
Tertiary Plane	6 [75 150 15]	N(0.0, 2.25)	N(0.0, 1.44)	N(-1.0, 2.25)	N(0.0, 3.24)	N(1.5, 2.25)

Table 2-2. Results obtained from the constraints shown in Table 2-1.

Error set	μ			σ_x^2	σ_y^2	σ_z^2	σ_{xy}	σ_{xz}	σ_{yz}
1	0.0040	-0.0285	0.2120	4.6756	6.5716	10.1982	-3.2095	-1.1812	-1.2336
2	-1.6142	2.4725	-1.8671	5.5041	5.1305	2.9554	-3.9664	-0.1632	-0.1694
3	-0.4418	0.4104	0.0755	2.1175	6.1266	8.2082	-2.3401	-0.3508	-0.9357
4	-1.6176	0.0155	1.9616	7.3417	10.0237	4.7778	-5.6818	-0.7146	-0.5029
5	-0.8809	3.2380	0.5825	3.0245	5.8589	9.8903	-2.5032	-1.1441	-0.8828

From the scatter plots (Figure 2-7 and 2-8), it can be seen that when a number of samples is greater (i.e. 300) the pattern of distribution is clearly shown. The elliptical-shaped point cloud preliminary indicates a normal or Gaussian distribution pattern. In 3-dimensional system; however, it does not guarantee the global multivariate normal relationship among the three variables.

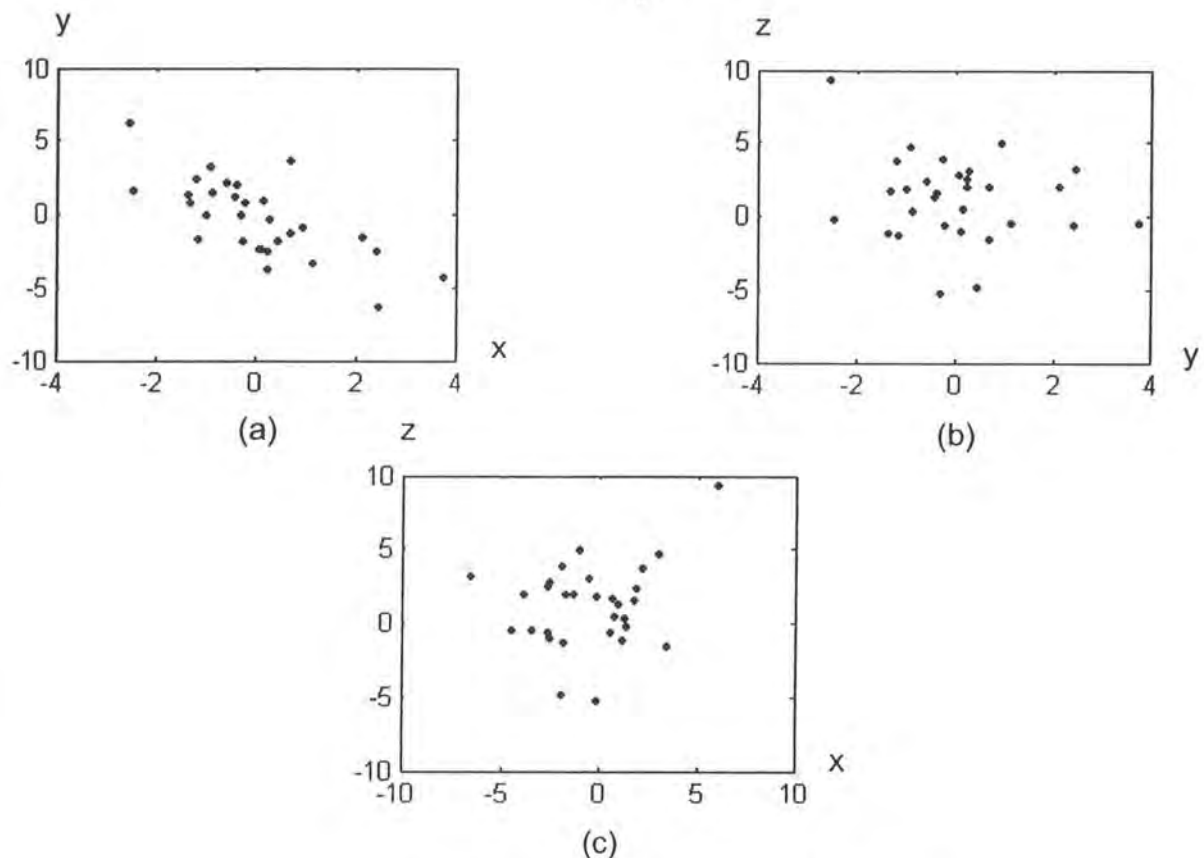


Figure 2-7. Variability in workpiece location when $n=30$

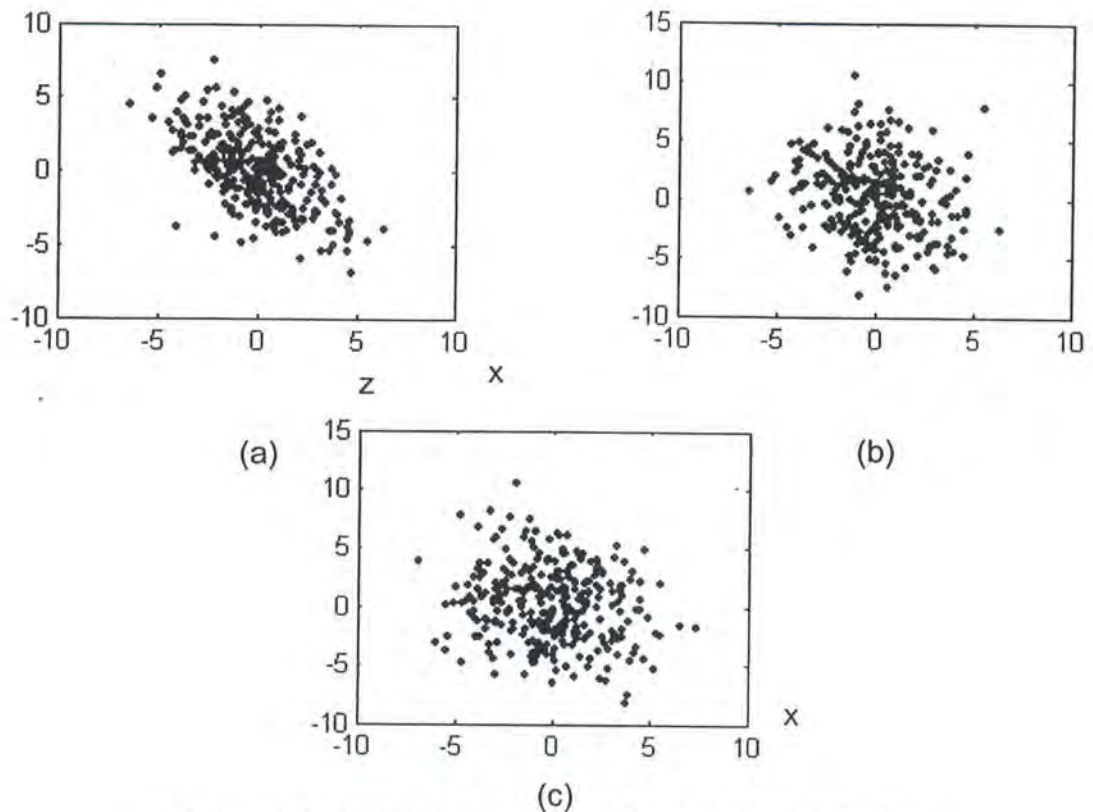


Figure 2- 8 Variability in workpiece location when $n=300$

To evaluate the distribution of multivariate data, the normality test is performed by constructing a chi-square plot (see detail in (Johnson and Wichern 1998)). If the data are drawn from a multivariate normal distribution, the plot should be similar to a straight line through the origin with slope equal to one. Any systematic curves indicate lack of normality and points far from the line suggest outlying observations. The data from the simulation were plotted in Figure 9, is reasonably assumed a multivariate normal distribution. This statistical results are beneficial in tolerance analysis and will be discussed later in future work.

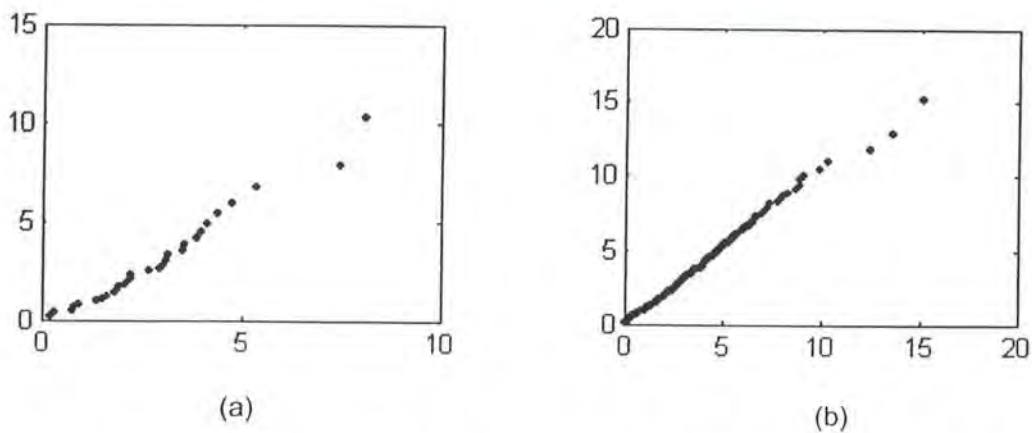


Figure 2- 9 Normality plot (a) when a sample size is 30 (b) when a sample size is 300

Relationships among the components of the locations of interest in Table 2 are in well agreement with the scatter plot. It can be seen that strong correlations between a component 'x' and 'y' of workpiece location is notable, while the other pairs ('y' and 'z', 'x' and 'z') do not demonstrate any significant evidence of correlations. The distinct correlation between 'x' and 'y' suggests extra care if a circular positional tolerance is in use. How much a proportion in the elliptical area should be included to establish the circular tolerance is decisive and needs further study.

2.5 Conclusion

This study proposes an analysis of the variability in location of the workpiece as affected by variant errors at contact areas. The error samples are assumed normally distributed. Unlike previous studies, a step-wise analysis is considered unnecessary and the mathematical formulation is simplified. The repetitive Newton-Raphson is employed to obtain a transformation which matches a theoretical with an actual position and orientation. Multivariate statistical techniques were used to analyze resultant variability. The information gained is beneficial to manufacturers as it reveals how surface variability becomes influential on geometric variation of the fixtured workpiece. It is recommended for a designer to account for the variation before tolerancing will be established. From the results, an elliptical-shaped point cloud is captured suggesting imposing a circular positional tolerance would require extra care. Tighter tolerance ensures a functional assembly; however, usually is associated with high cost. It is a responsibility of a designer to find the way that would benefit the production the most. Implementing the concept proposed in this work would help the designer impose the tolerance efficiently, and consequently reduce manufacturing cost and improve product quality.

2.6 Acknowledgement

The authors would like to acknowledge and thank the financial support provided by the Thailand Research Fund (TRF).

2.7 References

1. Bhat, V. and E.C. De Meter, An analysis of the effect of datum-establishment methods on the geometric errors of machined features. *International Journal of Machine Tools & Manufacture*, 2000. 40: p. 1951-1975.
2. Brost, R.C. and K.Y. Goldberg, *A complete algorithm for designing planar fixtures using modular components*. *IEEE Transactions on Robotics and Automation*, 1996. 12: p. 31-45.

3. Cai, W., S.J. Hu, and J.X. Yuan, *A variational method of robust fixture configuration design for 3-d workpieces*. Journal of Manufacturing Science and Engineering, 1997. **119**: p. 593-602.
4. Choudhuri, S.A. and E.C. De-Meter, *Tolerance analysis of machining fixture locators*. Journal of Manufacturing Science and Engineering, 1999. **121**: p. 273-81.
5. Dai, J.R., et al., *An approach to automating modular fixture design and assembly*. Proceedings of Institution of Mechanical Engineers, 1997. **211 Pt.B**: p. 509-521.
6. De Meter, E.C., *Fast support layout optimization*. International Journal of Machine Tools and Manufacture, 1998. **38**: p. 1221-1239.
7. De Meter, E.C., *Min-max load model for optimizing machining fixture performance*. Journal of Engineering for Industry, 1995. **117**: p. 186-193.
8. De Meter, E.C., *Restraint analysis of fixtures which rely on surface contact*. Journal of Engineering for Industry, 1994. **116**: p. 207-15.
9. De Meter, E.C., *The min-max load criteria as a measure of machining fixture performance*. Journal of Engineering for Industry, 1994. **116**: p. 500-507.
10. Djurdjanovic, D. and J. Ni, *Dimensional errors of fixtures, locating and measurement datum features in the stream of variation modeling in machining*. Journal of Manufacturing Science and Engineering, Transactions of the ASME, 2003. **v 125(n 4)**: p. p 716-730.
11. Fuh, J.Y.H., C.-H. Chang, and M.A. Melkanoff, *An integrated fixture planning and analysis system for machining processes*. Robotics and Computer-integrated Manufacturing, 1993. **10**: p. 339-353.
12. Jeng, Y.C. and K.F. Gill, *A CAD-based approach to the design of fixtures for prismatic parts*. Proceedings of Institution of Mechanical Engineers, 1997. **211 Pt.B**: p. 523-538.
13. King, L.S.-B. and I. Hutter, *Theoretical approach for generating optimal fixturing locations for prismatic workparts in automated assembly*. Journal of Manufacturing Systems, 1993. **12**: p. 409-416.
14. Ong, S.K. and A.Y.C. Nee, *A systematic approach for analyzing the fixturability of parts for machining*. Journal of Manufacturing Science and Engineering, 1998. **120**: p. 401-407.
15. Pham, D.T. and A.d.S. Lazaro, *Finite element study of a workpiece in a machining fixture*. Mathematical Computation Modelling, 1990. **14**: p. 1024-1028.
16. Rong, Y. and Y. Bai, *Machining accuracy analysis for computer-aided fixture design verification*. Journal of Manufacturing Science and Engineering, 1996. **118**: p. 289-300.
17. Salisbury, E.J. and F.E. Peters, *The impact of surface errors on fixtured workpiece location and orientation*. NAMRC Transactions, 1998. **26**: p. 323-328.

18. Sangnui, S., *The impact of fixturing error on feature tolerance allocation*, in *Industrial and Manufacturing Systems Engineering*. 2002, Ph. D. Dissertation, Iowa State University: Ames
19. Sangnui, S. and F.E. Peters, *The impact of surface errors on the location and orientation of a cylindrical workpiece in a fixture*. *Journal of Manufacturing Science and Engineering*, 2001. **123**: p. 325-330.
20. Trappey, J.C. and S. Matrubhutam, *Fixture configuration using projective geometry*. *Journal of Manufacturing Systems*, 1993. **12**: p. 486-495.
21. Wu, Y., et al., *Automated modular fixture planning geometric analysis*. *Robotics and Computer-integrated Manufacturing*, 1998. **14**: p. 1-15.

CHAPTER 3

MODELING OF POSITIONAL VARIABILITY OF A FIXTURED WORKPIECE DUE TO LOCATING ERRORS

3.1 Nomenclature

- j, k Unit vectors $[0 \ 1 \ 0]$ and $[0 \ 0 \ 1]$, respectively
 e_i Errors at contact point i
 $d(x)$ Workpiece displacement at x
 L Distance between locators
 C_i Locator i
 $P(x)$ Workpiece position at distance x from C_i
 $P_e(x)$ Displaced workpiece position at distance x from C_i
 σ_i^2 Variability of errors at contact point i
 μ_i Mean of errors at contact point i
 GV Geometric variation at a feature

3.2 Introduction

A datum is a theoretically exact point, axis, or plane specified in a drawing that functions as a reference for dimensioning or locating features on a workpiece. However, in actuality, a feature is indeed located with respect to "a datum feature", the datum's actual counterpart. Various factors such as machine vibration, tool-path errors, tool deflection, etc., are some of the causes differentiating a datum feature from a datum. Realizing undesirable effects of such inevitable causes, a designer sometimes allows variation of a datum by imposing an appropriate tolerance. When the workpiece is fixtured for machining operation, a datum feature is used to locate a feature position, and errors at the datum feature will be passed on to a machined feature through a locating method. Throughout this article, errors in locating which result from imperfect datum features will be called "locating errors". A number of past published works related to effects of locating errors on part quality can be found. Rong and Bai (1996) analyzed a dependent relationship of operational dimensions to estimate machining errors in terms of linear and angular dimensions of a workpiece. Cai et al. (1997) proposed a method to conduct a robust fixture design to minimize workpiece positional errors as a result of workpiece surface and fixture setup errors. There are also algorithms to predict a deviation of a prismatic

workpiece located by 3-2-1 fixturing method (1998) and a cylindrical workpiece in a V-block (2001). Djurdjanovic and Ni (2003) developed procedures for determining the influence of errors in fixtures, locating datum features and measurement datum features on dimensional errors in machining. These studies were conducted when a static case was assumed. More realistic variational models used to explain distributions of errors in locating have also been discussed. Bhat and De Meter (2000) carried out a simulation analysis to evaluate performance of three different datum-establishment methods. A probabilistic fixturing model was also developed and resultant feature errors were determined by Sangnui and Peters (2002). To avoid mathematical complication due to such respective locating scheme, Chaiprapat and Rujikietgumjorn (2006) proposed a system of equations that could be simultaneously solved for workpiece positional errors. In (2006), feature errors resulting from different types of distributions used to explain irregularities of a locating surface were compared. Carlson (2001) evaluated errors propagated from perturbation in locators by using a quadratic sensitivity equation that could better handle larger errors, in comparison to a linear approximation.

Effects of locating errors as discussed by the above studies evolve in the areas of workpiece displacement and its consequences on machining errors. For machined parts to be acceptable, such errors must be bounded within a range imposed by a designer called 'tolerance'. Tolerance analysis is carried out extensively in fixturing. Kang et al. (2003) proposed two different approaches for fixture tolerance analysis. From the given locator displacement and deformation, the deviation of affected machined surfaces can be predicted. On the contrary, with an allowable error range of the surfaces defined, tolerance of the locators can also be calculated. Establishment of relationship between machined feature geometric error and locator tolerance was also studied by Choudhuri and DeMeter (1999). Whenever an initial fixturing layout does not lead to acceptable errors, rearranging the layout should be considered. Optimal fixturing layouts were suggested through a number of approaches (Wang 2002; DeMeter 1995; Li B and Melkote 2001; DeMeter 1998; Asada 1985). However, none of them explicitly presents how errors associative with the proposed layout can be quantified.

In this study, a pattern of variability over a feature placement surface is investigated under the influence of locating probabilistic errors. Since it is sometimes technically or economically infeasible to eliminate errors from locating, understanding how a workpiece through a datum feature interacts with locators is crucial because it helps design and manufacturing engineers find ways to create more accurate features.

3.3 Analysis

3.3.1 Locating method

There are typically two different coordinate systems involved in machining operation. One is a machine reference system, which is used to position a cutter. However, after machining, a feature will be inspected with respect to another system, which is constructed from workpiece locating surfaces. As a result, a coincident or certain relationship between both systems is a requirement for the accuracy of a machined feature. It is realized that the errors at the contact areas between workpiece datum feature and fixture locators will often cause misalignment between both systems, and thus inducing errors in feature location and orientation. The workpiece in this study is assumed to be non-deformable, prismatic, and restrained by a 3-2-1 fixturing method. Figure 3-1 shows determination of an error at a contact location. The error will be positive when the contact point is located outside a nominal workpiece boundary, and negative otherwise.

In (Sangnui and Peter 2002) and (Chaiprapat and Rujikietgumjorn 2006), resultant geometric variation of a point representing a feature location was calculated. However, it is always found that most features fabricated on the manufacturing shop floor, i.e., slot, pocket, shoulder cut, and milled face, cannot be represented by a single location. For such features, the method was applicable only when uncertainty in workpiece displacement was consistent across a feature placement area. When this assumption does not hold, a variational profile of the placement area should be investigated. The following sections will demonstrate how the displacement is distributed over the workpiece surface and how this irregular displacement affects tolerance establishment of a feature.

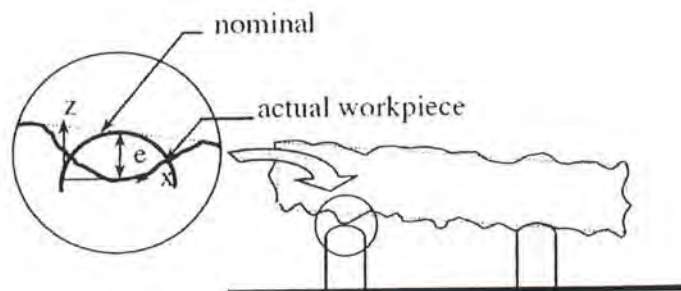


Figure 3-1 Error determination

3.3.2 Displacement profile: Two-dimensional case

A workpiece datum surface in this case is supported by two locating pins. The displaced workpiece, resulting from errors at contact points, is represented by a dotted line as

shown in Figure 3-2. Note that the errors of the datum surface can be represented by errors of locators at given contact points. When the errors (e_1 and e_2) are small compared to the distance between the locating pins (L), the displacement $d(x)$ can be obtained from

$$d(x) = e_1 + \frac{x}{L}(e_1 - e_2) \quad (3-1)$$

where $d(0) = e_1$ and $d(L) = e_2$

and the workpiece position at x , $P(x)$, will be translated to

$$P_e(x) = P(x) + d(x) \bullet j \quad (3-2)$$

where $P_e(x)$ is $P(x)$ after displacement

The displacement function in Eq. (3-1) expresses a linear relationship with 'x'. A magnitude of the greatest displacement along the datum line is equal to the larger error of the two. Since existing uncertainty in manufacturing processes makes an assumption of static contact errors invalid in most cases, a probabilistic error model is chosen in this study. Because of the number of parameters involved, however, an actual distribution of the errors is almost impracticable to determine. Once derived, the calculation may be too complicated and give inapplicable results. When true characteristics of the surface are not revealed, normally distributed errors will be used as an example.

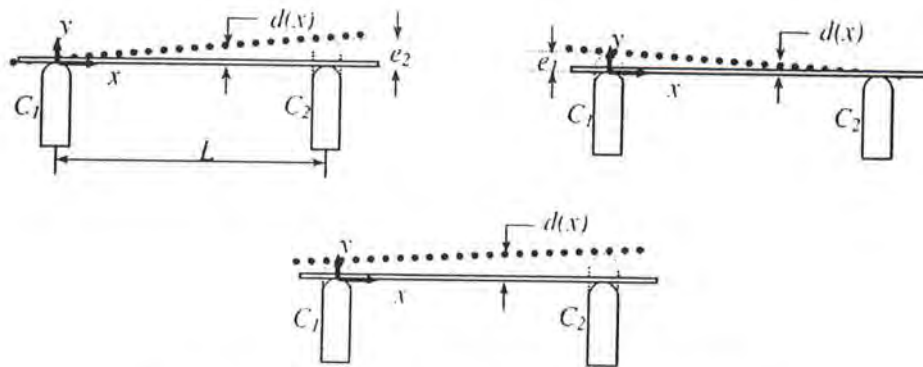


Figure 3- 2 Workpiece displacement in a 2-D case

$$e_i \sim N(\mu_i, \sigma_i^2) \quad (3-3)$$

From Equation (3-2) and (3-3), statistically, a variability of $P_e(x)$ is equal to that of $d(x)$ which throughout this article will be represented by σ_x^2

$$\sigma_x^2 = \left(1 - \frac{x}{L}\right)^2 \sigma_1^2 + \left(\frac{x}{L}\right)^2 \sigma_2^2 \quad (3-4)$$

where e_1 and e_2 are independent

Equation (3-4) shows the variability in the position of the point of interest varies nonlinearly as a function of variability at both locators and the distance from such point to the locators (also see Figure 3-3). By differentiating Eq. (3-4), the minimum variability can be found at

$$x_{\min} = \frac{L\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \quad (3-5)$$

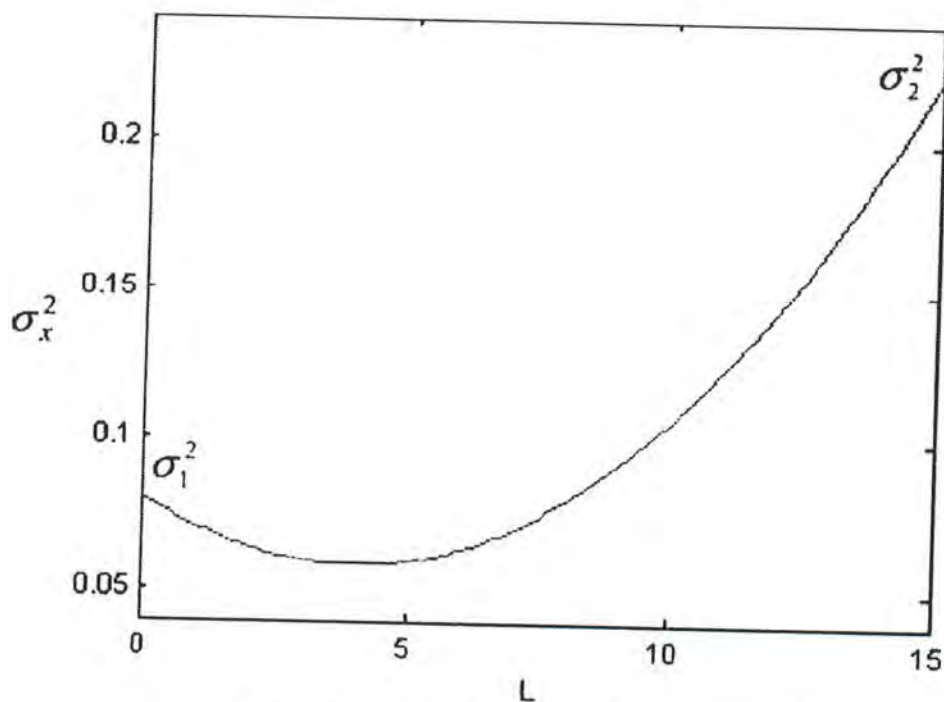


Figure 3-3 Change in σ_x^2 over the distance L

When $\sigma_1^2 = \sigma_2^2$, the minimum is at the midpoint between both locators. Theoretically, the feature should be placed where the minimum variability is located. However, when design constraints do not allow this, the locators may have to be rearranged as necessary to obtain minimal feature variability.

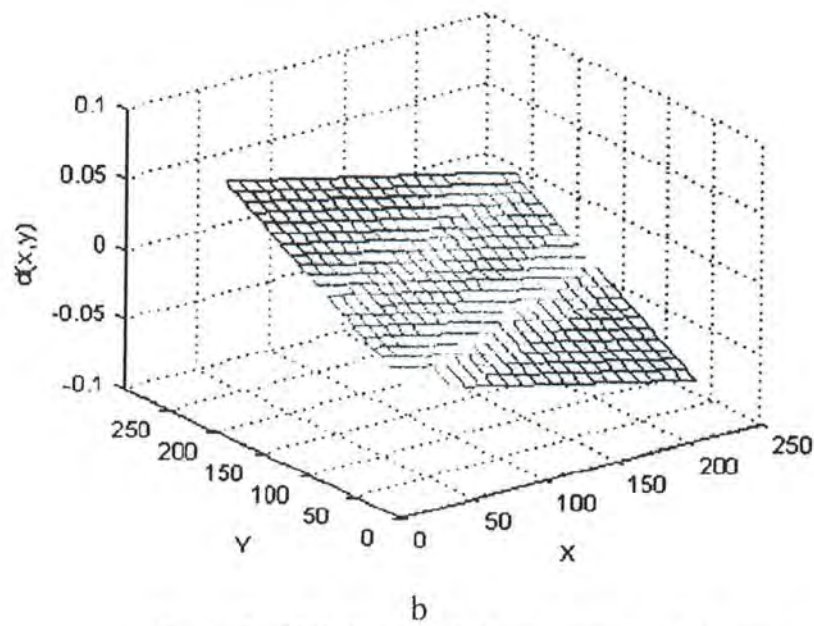
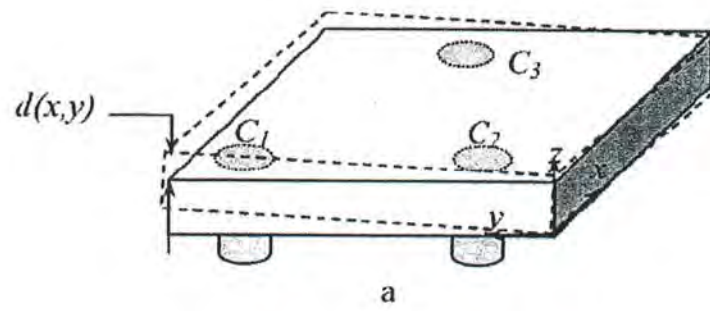
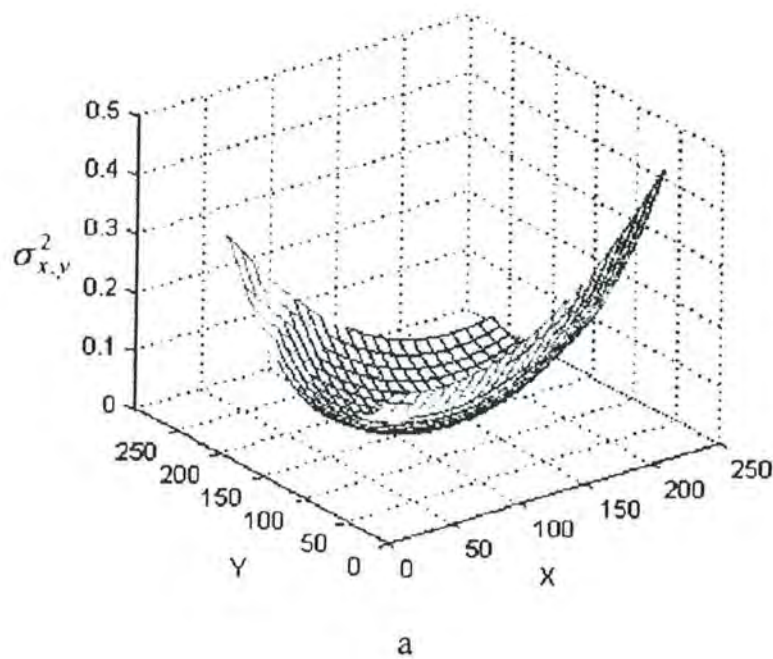


Figure 3- 4 Workpiece displacement in a 3-D case



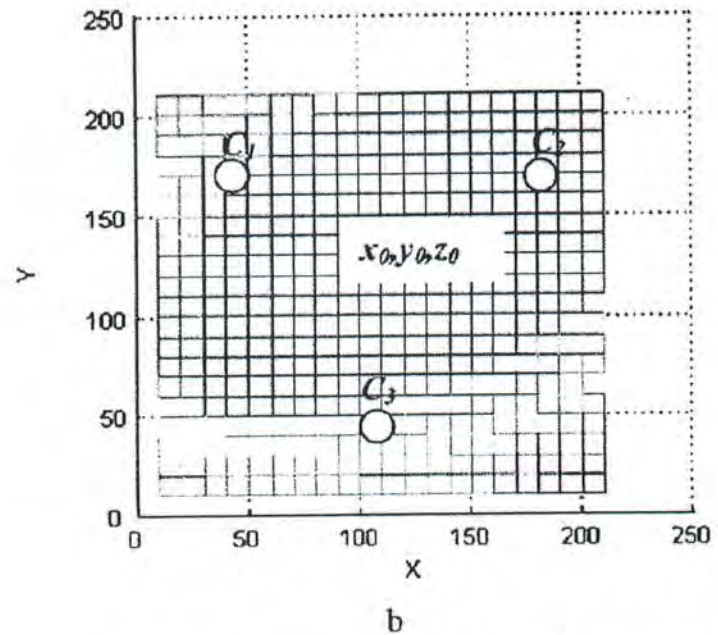


Figure 3- 5 A variational profile in a 3-D case

3.3.3 Displacement profile: Three-dimensional case

Using a 3-2-1 fixturing method, the workpiece's surfaces must contact with 3, 2, and 1 locator of the primary, secondary, and tertiary datum planes of the fixture, respectively. Errors at the contact points in the primary datum plane will rotate and move the workpiece away from its nominal position (Figure 3-4), leading to displacement in all directions (x,y,z) with that in z direction shown to be the most prominent. Errors of the secondary datum plane will rotate the workpiece around the normal vector of the primary datum plane. Finally, the errors of the last plane will only affect the translational displacement of the workpiece while maintaining the same orientation (Chaiprapat and Rujikietgumjorn2006). From such analysis, the errors of the secondary and tertiary datum planes play a little role in determining the accuracy of features like shoulder cut and milled face, when their placement plane parallel to the primary datum feature as illustrated in section 2.3.5

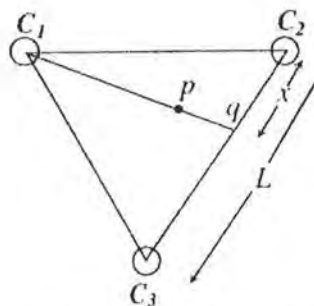


Figure 3- 6 Determination of variation at "p" on the workpiece placement face

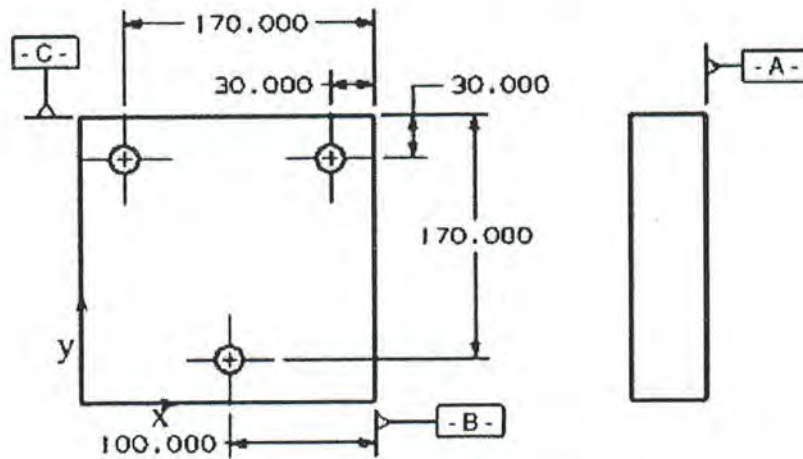


Figure 3- 7 A sample workpiece

Given surface errors at the contact points, one can determine the displacement and obtain the actual workpiece position and orientation by using the methodology developed in (Chaiprapat and Rujikietgumjorn2006). Similar to the displacement in a 2-D case, the displacement of 'P' in a 3-D environment can also be explained by

$$P_e(x, y) = P(x, y) + d(x, y) \bullet k \quad (3-6)$$

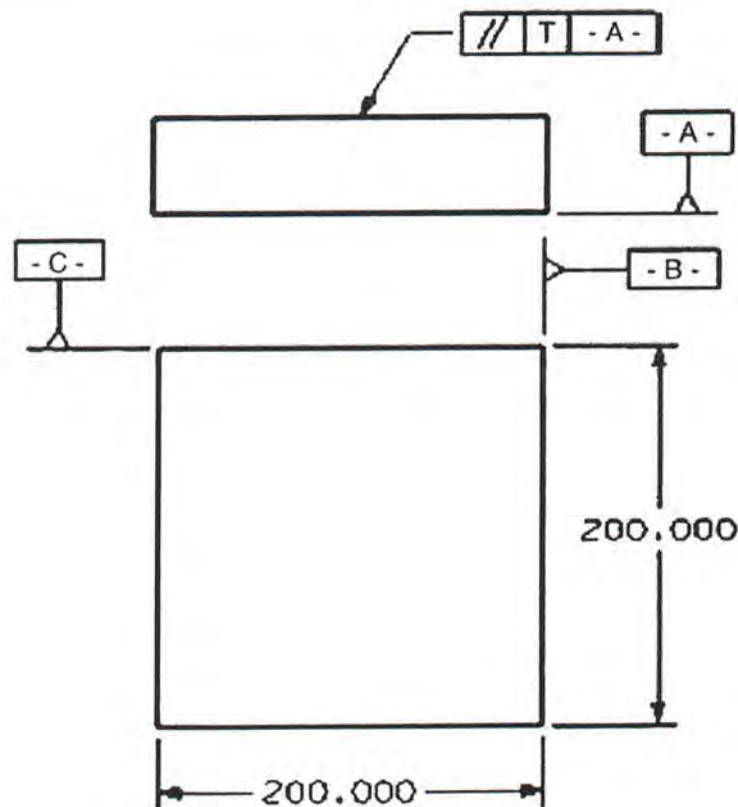


Figure 3- 8 Geometric description of the workpiece

Table 3-1 Geometric variation (GV) of a faced feature when the datum 'A' is tolerated by 1 and 2 mm

Datum tolerance	Data set					Mean
	1	2	3	4	5	
1 mm	1.6288	1.5715	1.5704	1.6166	1.5863	1.5947
2 mm	3.3406	3.4409	3.3331	3.4551	3.3781	3.3896

3.3.4 Datum feature variability

When the errors are small enough and normally distributed, $\sigma_{x,y}^2$ is preliminarily evaluated by a Monte Carlo simulation and plotted in a 3-D space. It can be seen as a curved surface resembling an elliptic paraboloid surface shown in Figure 3-5a and approximated by the paraboloid equation explained below.

$$\sigma_{x,y}^2 = \left\{ \frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} \right\} \quad (3-7)$$

where $[x_0, y_0, z_0]$ is a center of the surface.

The paraboloid vertex, $[x_0, y_0, z_0]$, is pointing where the lowest point, in the other words, the minimum variability of the surface is located. To derive $[x_0, y_0, z_0]$, an optimization tool in Matlab may be employed. However, how precisely the algorithm can locate the true vertex depends on how well an initial guess is established. This geometry resembles an ellipse when observed from the top and when sectioned vertically will be seen as a parabola. Variation on the placement face therefore can also be approximated by using the scheme developed for the 2-D variational profile as shown in Figure. 3-6.

According to Figure 3-7, to obtain the variation at "p" using Eq. (3-4), the variations at the end of the line on which "p" is lying must be quantified. Where the variation at C_1 or σ_1^2 is given, σ_q^2 can be calculated from the following parabolic equation.

$$\sigma_q^2 = \left(1 - \frac{x}{L}\right)^2 \sigma_2^2 + \left(\frac{x}{L}\right)^2 \sigma_3^2 \quad (3-8)$$

Once σ_q^2 is known, σ_p^2 can also be evaluated by the same equation.

It is not surprising that when distributions of errors at the contact points are the same, the vertex of the paraboloid is positioned equally apart from all three locators (Fig. 3-5b). When

it is inevitable to position the locators by using surfaces from different setup, extra care would have to be taken. Especially in precision machining when tight tolerancing becomes an issue it is recommended for a designer to seriously take into account an irregular pattern of the variability across the placement surface.

3.3.5 Feature geometric variation

If the displacement is certain over the workpiece surface, the tool path can be designed accordingly to compensate the errors in order to obtain more accurate features. Likewise, if the variability is constant, a designer could impose a tolerance to the feature effortlessly. However, it is shown in the previous section that the displacement is changed linearly over the placement surface according to the errors at the contacts. This circumstance leads to a parabolic-shaped variability pattern. What a process designer can possibly do in this case is to design manufacturing procedures conforming to such imperfect incoming workpiece while expecting the best from it. Before a tolerance can be imposed to a feature, a designer has to be aware of how uncertainty in the manufacturing process would contribute to variability of a feature. The degree of differentiation in geometry from part to part in the same production lot is called "geometric variation (GV)". Where geometric tolerance is a range in which a feature is allowed to be varied, geometric variation, on the other hand, reflects its actual variation. Therefore, geometric variation must be kept smaller than the corresponding tolerance. In this study, only geometric variation resulting from locating errors is discussed. The following is an example of workpiece's locating details. Datum features 'A', 'B', and 'C' are as labeled in Figure 3-7. Locators on the datum 'A' or the primary datum feature are positioned as indicated.

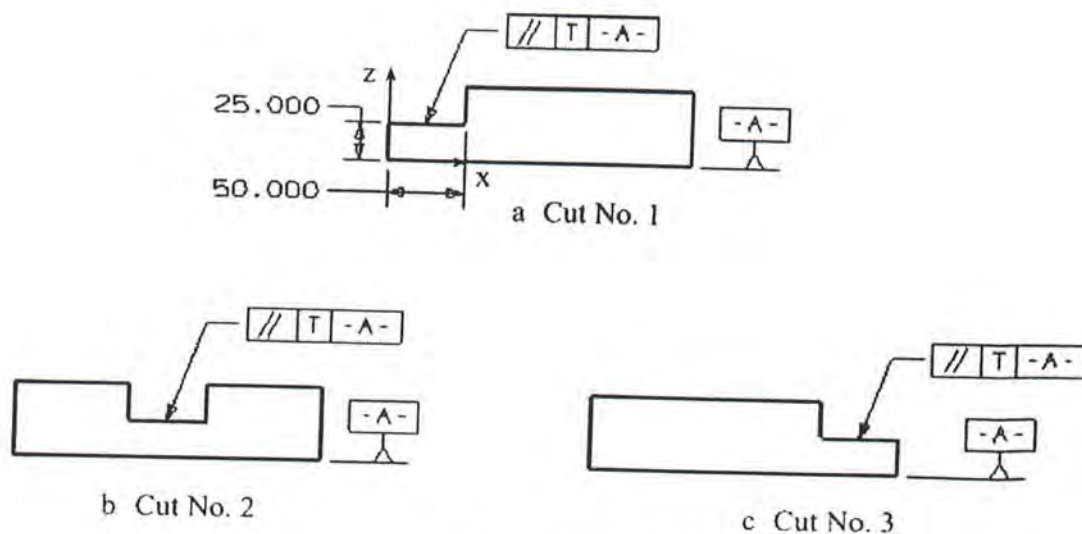


Figure 3- 9 Slots used in the study

Table 3-2 Geometric variation (GV) of slots placed at different locations along X-axis

Datum tolerance	Geometric variation (mm)			Difference in variation between cut no.1 and 3
	Cut no.1 (x=0-50 mm)	Cut no.2 (x=75-125 mm)	Cut no.3 (x=150-200 mm)	
1 mm	1.5238	1.2304	1.5947	0.0709
2 mm	3.3386	2.5977	3.3705	0.0319

1) Geometric variation of facing

The top surface of the workpiece shown in Figure 3-8 is to be faced and geometrically constrained according to the tolerance frame indicated below. From the drawing, the top surface must be parallel to the datum 'A' and is allowed to be varied within the range of 'T'.

At any given point (x,y) on the placement plane, it is assumed that feature variability following the central limit theorem exhibits a normal distribution pattern. A geometric variation (GV) herein is defined by a range containing six times of standard deviation (6σ) of the feature variability and that can be approximated by

$$GV = 6 * \sqrt{\sigma_{x,y}^2} \quad (3-9)$$

Since GV of any surface is not consistent over the entire area of the cut, only the maximum value of GV of each simulation trial is presented in Table 3-1. According to ANSI Y14.5M, a parallel tolerance constraining the surface as shown in Figure 3-8 is a distance between two theoretical planes parallel to the datum in which the cut surface is allowed to be varied. Therefore, to ensure that the machined surface is kept well within tolerance, GV should not exceed the range of 'T'.

2) Geometric variation of slotting

Inconsistent variation distributed across the surface will have a crucial role in tolerancing a feature partially cut on the surface. In Figure. 3-5, variation at the surface perimeter is significantly higher and lower at the locations approaching the center. To minimize surface variation resulting from variability at contact locations, a feature should be placed around the center of a supporting triangle (a triangle formed by three locators shown in Figure. 3-5 and 3-6). However, when the feature is tightly toleranced and impossible to be located at the optimum position for example in the case of side slotting, it is advised to look more closely at a magnitude of the variation over the target area. In this study, GV of three slots placed at different locations shown in Figure 9 are calculated and presented in Tables 3-2 and 3-3. The

cut of 50 mm in width and 25 mm in depth is placed at the left, the middle, and the right of the placement surface.

The above cuts lie longitudinally along X-axis. It appears that geometric variation is declining when approaching the center of the supporting triangle and getting larger in the area farther away. The cuts on both sides of the surface ($y=0-50$ mm and $y=150-200$ mm) have the similar degree of geometric variation.

As the cuts are relocated to be parallel to the Y-axis, it can be seen that the pattern of geometric variation is slightly different as shown in Figure 3-10. The variation near the supporting triangle's center is lowest and rising to the highest around the perimeter. Therefore, two corners of the surface (cut no. 1) have higher variability than the other two (cut no. 3). Examining the locator layout, the greatest variability can be spotted at the corners far from any locators, which in Fig. 8 are the corners along the X-axis. It can be explained by using Eqs. (3-7) and (3-8) that variability of an arbitrary point is a function of a squared distance from such point to the surface vertex or the center of the supporting triangle. This observation is in agreement with [13], which stated that the errors of the feature decrease in the central region but increase in the other region.

Table 3-3 Geometric variation (GV) of slots placed at different locations along Y-axis

Datum tolerance	Geometric variation (mm)			Difference in variation between cut no.1 and 3
	Cut no.1 ($x=0-50$ mm)	Cut no.2 ($x=75-125$ mm)	Cut no.3 ($x=150-200$ mm)	
1 mm	1.5947	1.2232	1.3289	0.2658
2 mm	3.3896	2.6287	2.8243	0.5653

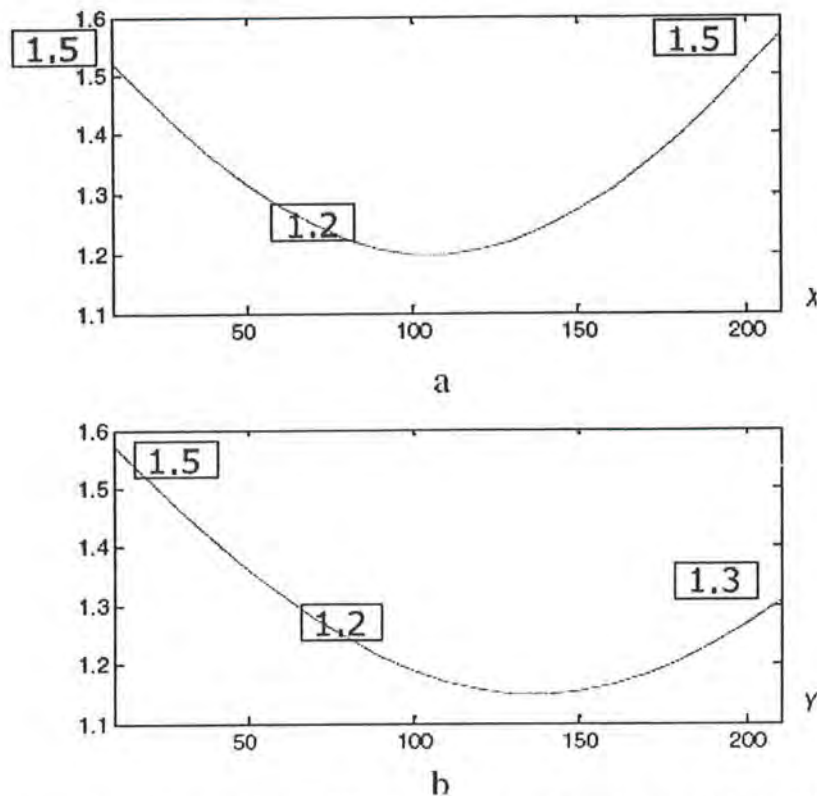


Figure 3- 10 Variation at different position along X-axis (a) and Y-axis (b)

When it is impossible to place the feature at the center, for example in the case of side slotting, positioning it along the edge closer to the center will help keeping the feature variation minimal. The geometric variations shown in the tables indicate how much the feature varies with respect to datum features; therefore, tolerances applied to the feature should not be less than these geometric variations. Note that this variation results from locating errors only. Although only parallel tolerancing is described as the example, the proposed approach can be as well applied to other types of tolerances.

3.4 Conclusions

When the workpiece is fixtured for a facial machining, conventionally a tolerance of the resultant surface is imposed, assuming that errors are consistent across the entire surface. However, results from this study indicated that variability is found greatest at the boundary of the surface and lowest at the equi-distance from all supporting locators. Although the difference in geometric variation over the entire placement surface seems minimal, in the application of precision machining such as injection mold machining this inconsistency is crucial. When tight tolerance is required, a feature should not be positioned far away from the centroid of the supporting triangle. Larger variability is expected with a feature located off the optimized

position. This study has shown that features located on the same placement surface are not necessarily tolerated by the same value. Before a tolerance can be imposed to any feature efficiently, a design engineer should be able to comprehend and preferably quantify potential feature variability. Features with too tight tolerance are hard (or too costly) to produce. On the other hand, wide tolerance opens up chances of assembly or functional difficulties. As a design engineer is the one who is responsible for imposing a tolerancing system, he or she has to ensure that the tolerated feature will be economically producible and maintain its functional use at the same time. Balancing the mechanics of tolerancing therefore requires knowledge of how the workpiece geometry would vary under a certain manufacturing environment.

3.5 Acknowledgements

The authors would like to acknowledge the financial support provided by the Thailand Research Fund (TRF) Grant No. MRG 4780070.

3.6 References

1. Rong Y, Bai Y (1996) Machining accuracy analysis for computer-aided fixture design verification.
2. J Manuf Sci Eng 118:289–3002. Cai W, Hu SJ et al (1997) A variational method of robust fixture configuration design for 3-D workpieces. J Manuf Sci Eng 119:593–602
3. Salisbury EJ, Peters FE (1998) The impact of surface errors on fixtured workpiece location and orientation. NAMRC Trans 26:323–328
4. Sangnui S, Peters FE (2001) The impact of surface errors on the location and orientation of a cylindrical workpiece in a fixture. J Manuf Sci Eng 123:325–330
5. Djurdjanovic D, Ni J (2003) Dimensional errors of fixtures, locating and measurement datum features in the stream of variation modeling in machining. J Manuf Sci Eng Trans ASME 125(4):716–730
6. Bhat V, DeMeter EC (2000) An analysis of the effect of datum establishment methods on the geometric errors of machined features. Int J Mach Tools Manuf 40:1951–1975
7. Sangnui S, Peters FE (2002) Prediction of fixtured workpiece location when there are variable surface errors at fixturing points. Proceedings of the 4th Asia-Pacific conference on industrial engineering and management systems, Taipei, Taiwan
8. Chaiprapat SS, Rujikietgumjorn S (2006) Resultant geometric variation of a fixtured workpiece: a simulation. Songklanakarin J Sci Technol 31:182–190

9. Chaiprapat SS, Rujikietgumjorn S (2006) A method to analyze effects of surface variational model on positional geometric variability. *Songklanakarin J Sci Technol* 31:170–179
10. Carlson JS (2001) Quadratic sensitivity analysis of fixtures and locating schemes for rigid parts. *J Manuf Sci Eng* 123:462–472
11. Kang Y, Rong Y et al (2003) Computer-aided fixture design verification. Part 2. Tolerance analysis. *Int J Adv Manuf Technol* 21:836–841
12. Choudhuri SA, DeMeter EC (1999) Tolerance analysis of machining fixture locators. *J Manuf Sci Eng* 121:273–281
13. Wang MY (2002) Tolerance analysis for fixture layout design. *Assem Autom* 22(2):153–162
14. DeMeter EC (1995) Min-max load model for optimizing machining fixture performance. *J Eng Ind* 117:186–193
15. Li B, Melkote SN (2001) Optimal fixture design accounting for the effect of workpiece dynamics. *Int J Adv Manuf Technol* 18:701–707
16. DeMeter EC (1998) Fast support layout optimization. *Int J Mach Tools Manuf* 38:1221–1239
17. Asada H, By AB (1985) Kinematic analysis of workpart fixturing for flexible assembly with automatically reconfigurable fixtures. *IEEE Trans Robot Autom* RA-1(2):86–94

CHAPTER 4

A METHOD TO ANALYZE EFFECTS OF SURFACE VARIATIONAL MODEL ON POSITIONAL GEOMETRIC VARIABILITY

4.1 Introduction

Irregularities of a machined surface topography can be broken down to surface roughness, waviness, and form error. For example, a structure of turned surface is a combination of a waviness component generated by a feeding process and a random roughness component caused by a built-up edge on a cutting tool (Stout, Davis et al. 1990). Unlike turned surfaces, ground surfaces under microscope show gouge marks which are resulted from sharp grits attached to a grinding wheel periphery. It appears that type of process and process parameters can differentiate manufactured surface characteristics as shown in Figure 1. After machining, a workpiece is left with irregular topographies which may affect functional applications of the surface. Particularly when the surface is selected as a locating reference, any deviations from its perfect geometry will lead to consequent workpiece inaccuracy. Eliminating such deviations to improve the workpiece accuracy is not always an economical solution. However, their undesirable effects on the workpiece quality can be controlled if we know how the deviations are distributed across the entire surface.

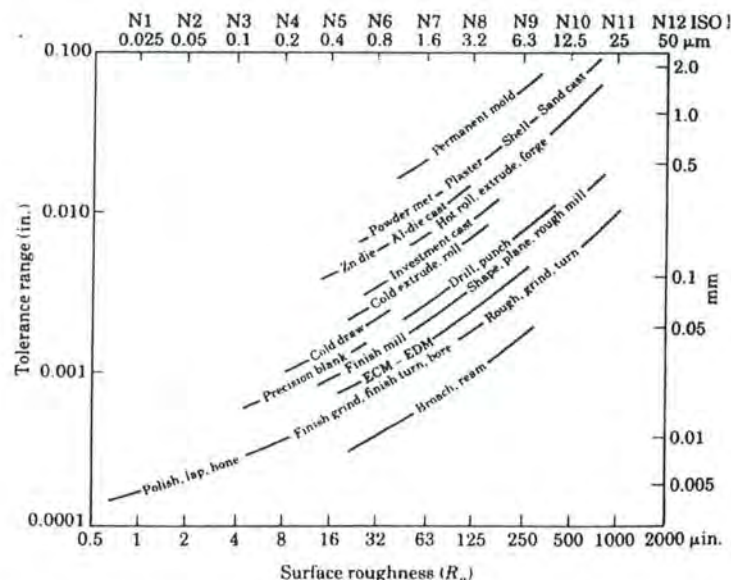


Figure 4- 1 Tolerances and surface roughness obtained in various manufacturing processes. These tolerances apply to 25 mm workpiece dimension. (Kalpakjian 1991).

Because uncountable parameters involved, actual distribution of a machined surface is almost impracticable to determine. Once derived; however, the calculation would be too complicated giving inapplicable results. Variational models widely found in published work are usually based on a Gaussian or normal distribution. Nassef and ElMaraghy (1995) assumed variability of manufactured surfaces was distributed as multinormal. Multinormal random surface points were generated and estimated surfaces were constructed by interpolating to the points. The authors then investigated the simulated surfaces to determine geometric deviation of the surfaces before calculating optimal tolerance zones that would result in desired probability of rejection. The drawback is that systematic errors may produce an asymmetric distribution, which is not suitable to explain with the Gaussian or normal function. However, as a sample size increases, the distribution can be assumed Gaussian following the central limit theorem. Besides a Gaussian distribution, a beta distribution is also suggested because of its advantageous applications in representing manufacturing variability as following:

- 1) The beta distribution is more flexible. It has a capability to conform various types of variability, when the Gaussian distribution is only applied to a symmetric function.
- 2) The beta distribution has finite upper and lower limit, which is favorable in computation.

The beta distribution is also chosen over a normal or Gaussian distribution because of its capability of supporting the calculation of an inverse probability distribution function (Lin, Wang et al. 1997). He (1991) used the beta distribution to explain variability of dimension in manufacturing processes. Since the variability is distributed varying from process to process and application to application, the author then proposed a beta distribution as it has a capability to cover all kinds of distributions from rectangular to normal distributions as well as asymmetrical distributions. Treacy, Ochs et al. (1991) carried out an analysis on tolerance distribution of an assembly of mechanical parts. For the sake of its flexibility, the authors decided on a beta distribution to represent variability of resultant tolerance chain of the assembly though the model may be associated with calculation complexity. Because practically every sum of interacting dimensions gives an expected variability that falls between the results from a worst case and root sum of square approaches, Di Stefano (2003) developed a mean shift model modified from Mansoor (1964) where a factor "f" was imposed to determine a process bias. The factor is a function of a mean shift ratio, a confidence level, and a number of dimensions in an assembly. This model is applicable to Gaussian ($f = 0$) to non-Gaussian ($f = 1$) distributed dimensions. Unlike Gaussian and beta distributions, a uniform distribution is far less popular. ElMaraghy, Gadalla et al. (1995) employed the distribution to represent the

variation explained by different tolerance types. Choudhuri and De-Meter (1999) carried out a simulation to analyze an impact of locator tolerance scheme on the potential datum related, geometric errors of linear and machined features. They used a set of equally spaced numbers to simulate locator variability. The results revealed a linear relationship between some variables, which is not very surprising when the analysis was done over a small data range. There are also works on datum establishment which left the controversial issue of a distribution function unspecified. Nassef and ElMaraghy (1997) proposed a new criterion, mismatch probability, to determine appropriate tolerancing scheme where production cost was kept minimum. Because the procedure used with dimensional tolerancing is not applicable to geometric tolerancing, a new algorithm based on genetic algorithm was invented to evaluate each tolerancing candidate. Though computational time was relatively long, the benefits of choosing the appropriate tolerances were paid off later on. Roy and Li (1998) developed mathematical models representing real world form variations. They proposed two strategies used to construct the approximated form of surfaces. The first technique was for simulating curved surfaces from randomly generated surface points. In order to create approximated planar surfaces, the second technique employed a linear regression analysis from the random points to estimate plane parameters. Besides the form variation, the authors also presented a computational scheme to represent size, orientation and position tolerance zones. Variational models of these tolerances were established relying on random data points. Bhat and Meter (2000) conducted a simulation analysis to evaluate efficiency in accurate locating among three different datum establishment methods. The sample workpiece and its features were constrained by geometric tolerances. Random variables were simulated variation according to each tolerance value generating variant feature position and orientation. The datum establishment method giving the least error which means that it is unlikely to be affected by such erratic geometry of the feature was considered the most functional method. It can be seen that these research were more intensified on development of methods of reference establishment without taking into account an actual pattern of surface variability. The reasons involved in choosing an appropriate surface distribution should be more than computational complexity. This study proposes another aspect of tolerancing analysis. A method to analyze effects of chosen surface distributions on feature positional tolerancing is discussed in the following sections.

4.2 Analysis

In this section, the effects of variational models representing datum feature variability on positional errors of a feature to be produced are discussed. As shown in Figure 4-2, a

datum feature surface is allowed to deviate from its perfect geometry by d_i , where $|d_i| \leq \frac{T}{2}$; T is a tolerance constraining the datum feature. A prismatic workpiece with a hole and tolerated datum features (Figure 4-3) is used as an example in the analysis. Problems arise when the hole location is specified with respect to a misplaced reference frame, which is established from the deviated datum feature.

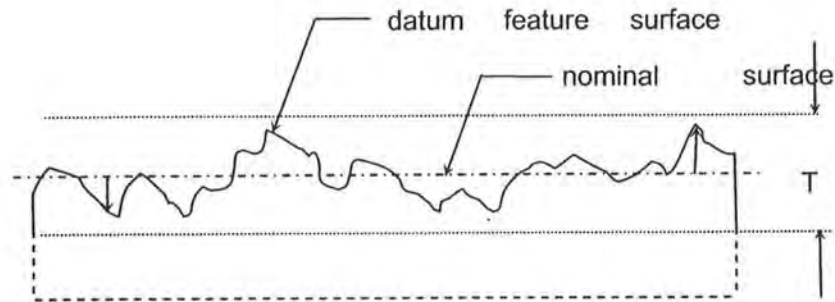


Figure 4- 2 Datum feature and tolerance width, T

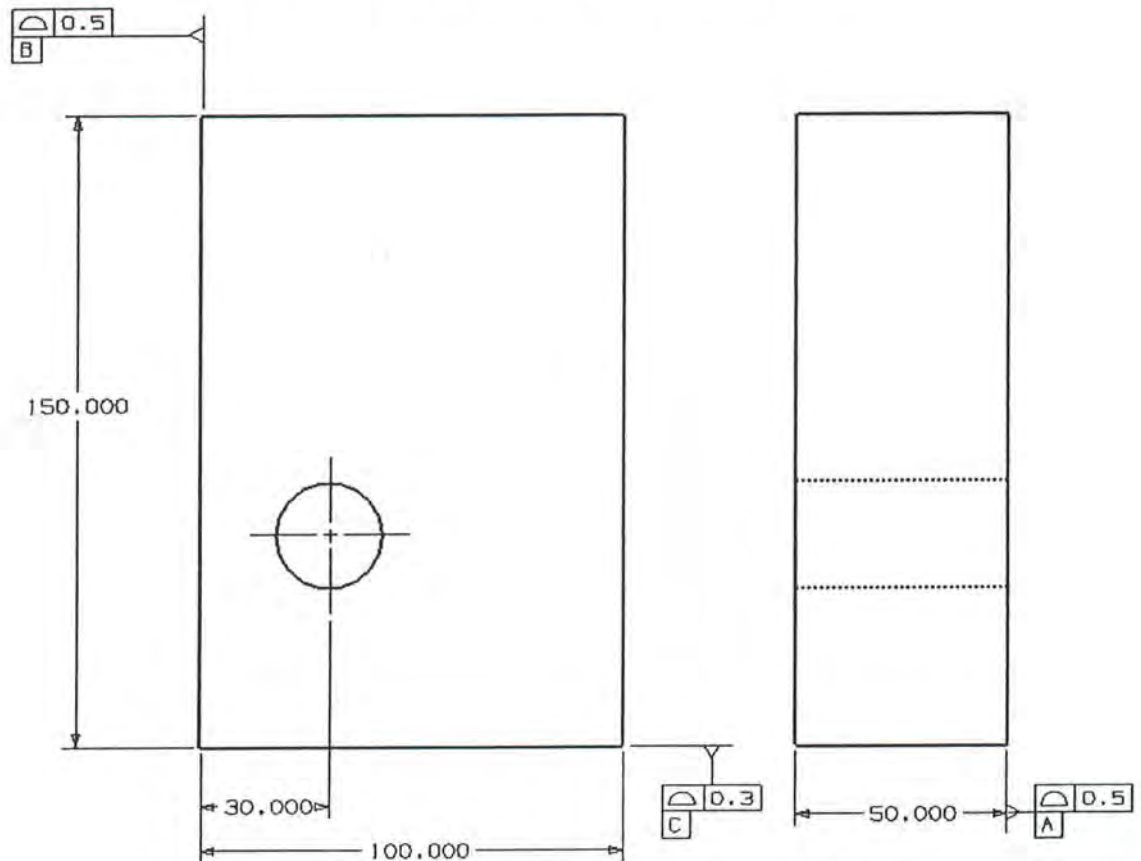


Figure 4- 3 A sample workpiece used in simulations

4.2.1 Tolerance Distributions

Three types of distributions usually found used to explain machined surface variability, d_i , are a normal, a beta and a uniform distributions. In this study, it is assumed that the distribution curve will span over the entire range of the tolerance width.

1) Gaussian or normal distribution

A unit normal probability density function is given as follows

$$f(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \quad -\infty < u < \infty \quad (4-1)$$

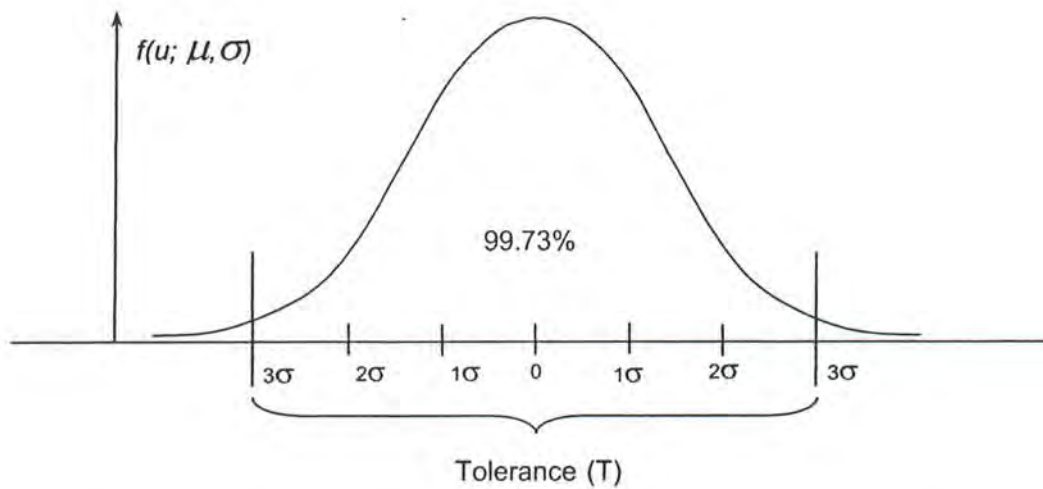


Figure 4- 4 Tolerance determination when a normal distribution is assumed

The normal distribution curve spreads infinitely and symmetry about its mean as shown in Figure 2-4. Since it is impossible to define the end of its coverage, 99.73% containment is assumed. When $N(0, 1)$ represents a unit normal distribution which has its mean at zero and standard deviation of one, the surface points assimilating each datum feature are distributed as

$$d \sim \left(\frac{T}{6}\right) \cdot N(0,1) \quad (4-2)$$

2) Uniform Distribution

A unit uniform probability density function is given as follows:

$$f(u) = 1, \quad 0 \leq u \leq 1 \quad (4-3)$$

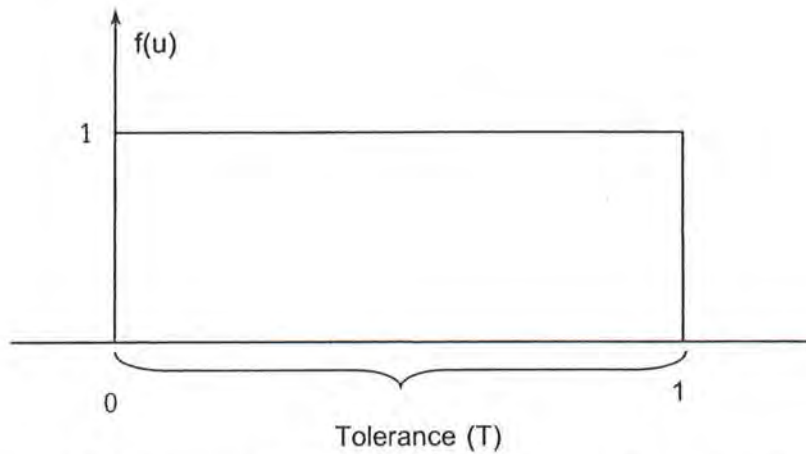


Figure 4-5 Tolerance determination when a uniform distribution is assumed

When $U(0,1)$ represents a unit beta distribution which spans over the range from zero to one, the surface points assimilating each datum feature are distributed as

$$d \sim T \cdot U(0,1) - \frac{T}{2} \quad (4-4)$$

3) Beta Distribution

A unit beta probability density function is given as follows

$$f(u; \alpha, \beta) = \begin{cases} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} u^{\alpha-1} (1-u)^{\beta-1}, & 0 \leq u \leq 1, \alpha \geq 0, \beta \geq 0 \\ 0, & \text{elsewhere} \end{cases} \quad (4-5)$$

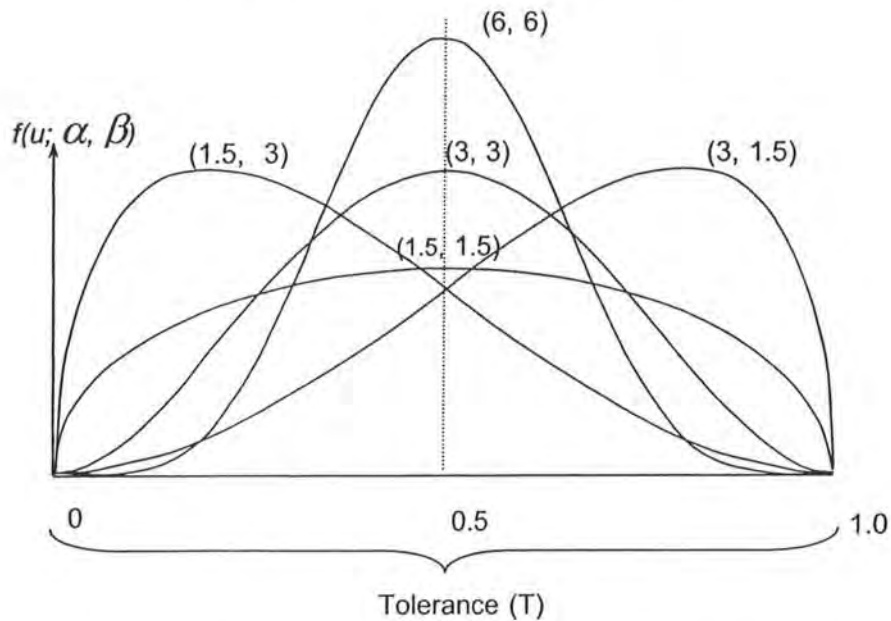


Figure 4-6 Tolerance determination when a beta distribution is assumed

Since the study assumes a symmetric surface variability pattern over the tolerance zone, the values of shape controlling parameters, α and β , used in the simulations therefore are chosen to be equal. When $B(\alpha, \beta)$ represents a unit beta distribution covering the region from zero to one, the surface points assimilating each datum feature are distributed as

$$d \sim T \cdot B(\alpha, \beta) - \frac{T}{2} \quad (2-6)$$

4.2.2 Datum Establishment Methods

In this study, the method used to establish a reference frame is 4-2-1 method. Following ASME Y14.5M, the primary datum plane is imposed from three highest points on the primary datum feature. Orthogonal with the primary plane, the secondary plane lies on two extreme points. The last plane, the tertiary plane, is established from the outermost point while maintaining perpendicular to the preceding two planes as shown in Figure 4-7. By implementing this method, it is inevitable the frame constructed would be sensitive to the pattern the datum feature irregularities are distributed.

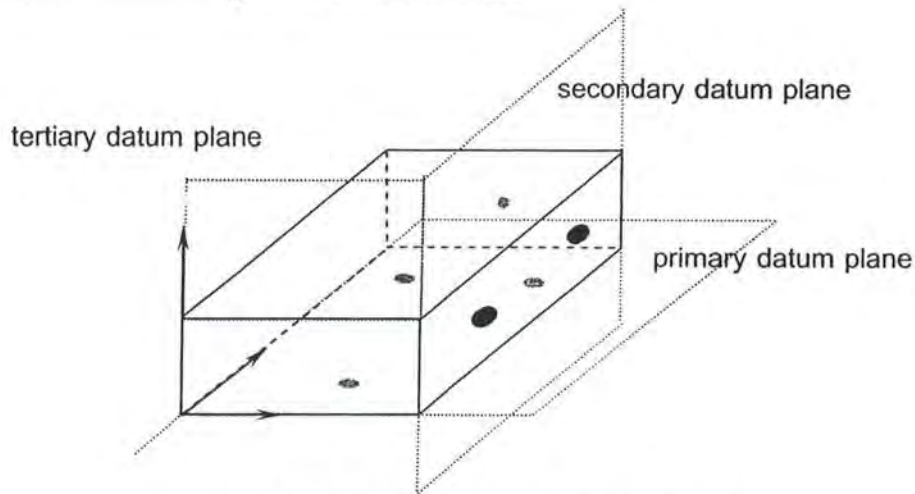


Figure 4-7 The system of datum planes

4.2.3 Datum Planes Definitions

Planes as elements of the reference frame must be calculated according to the datum establishment method described in section 3.2.2. The hole which we use as an example will be located with respect to these planes.

An algebraic equation of a plane is

$$\frac{Ax + By + Cz - D}{\sqrt{A^2 + B^2 + C^2}} = 0 \quad (4-7)$$

where $[A B C]$ is a vector normal to the plane

D is an orthogonal distance from the origin to the plane

To locate an arbitrary plane, its normal vector and a distance from the plane to the origin must be defined.

Primary datum feature

The normal vector of the primary datum feature ($\vec{N}_p$) can be obtained from

$$\vec{N}_p = (\vec{P}_{pm1} - \vec{P}_{pm2}) \times (\vec{P}_{pm3} - \vec{P}_{pm2}) \quad (4-8)$$

and the orthogonal distance is

$$D_p = (\vec{n}_p \bullet \vec{P}_p) \quad (4-9)$$

where $\vec{P}_{pmi}$ are the extremities of the primary datum feature

$\vec{n}_p$ is a unit vector of $\vec{N}_p$

$\vec{P}_p$ is a point on the primary datum feature

Secondary datum feature

To comply with the principle of 3-2-1 method, the secondary datum feature must be perpendicular to the primary one while passing through 2 highest surface points. Therefore,

$$\vec{N}_s = \vec{N}_p \times (\vec{P}_{sm2} - \vec{P}_{sm1}) \quad (4-10)$$

where $\vec{P}_{smi}$ are the extremities of the secondary datum feature

Tertiary datum feature

The vector normal to the tertiary datum feature is defined by

$$\vec{N}_t = \vec{N}_p \times \vec{N}_s \quad (4-11)$$

The determination of location and orientation of the planes are also shown in Figure 4-8

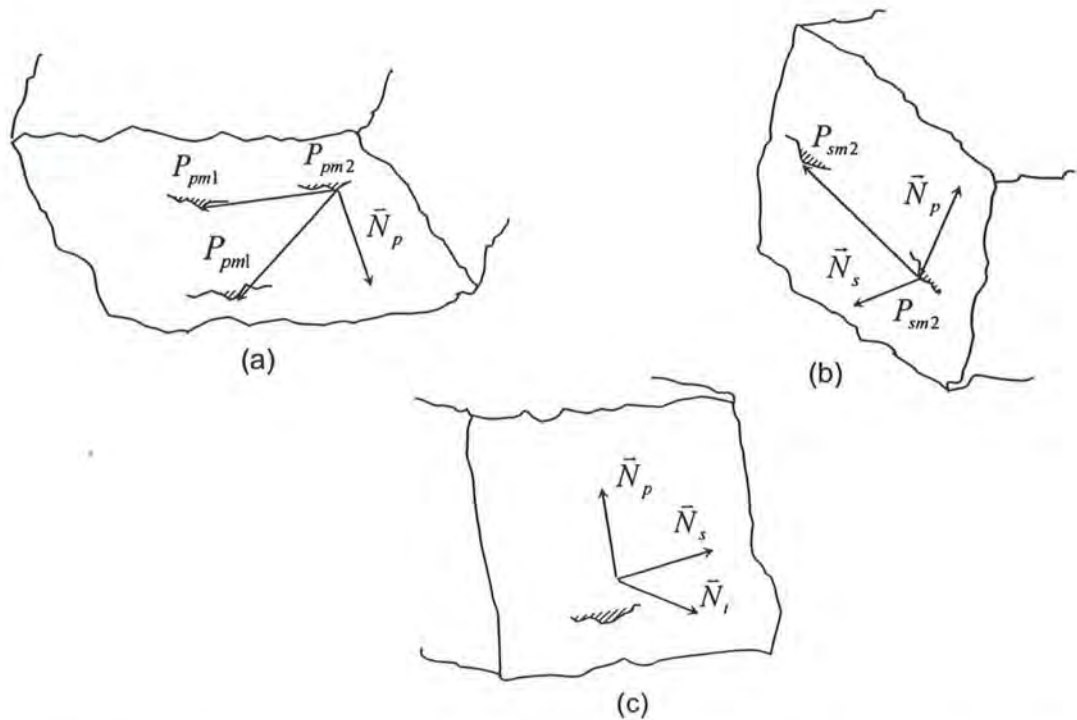


Figure 4- 8 Datum planes location and orientation determination (a) primary datum plane

(b) secondary datum plane (c) tertiary datum plane

The distances from the origin to the secondary and tertiary datum feature, D_s and D_t , are calculated the same way shown in Eq. 4-9. After the reference system imposed from the workpiece datum feature is established, coordinates of the hole relative to the nominal and actual systems are compared. The effects of variational surface model will then be analyzed. Please be noted that the nominal reference system is obtained when datum features are in absence of any irregularities.

4.2.4 Positional Error Determination

Coordinates of an arbitrary point in a rectangular or Cartesian system is a Euclidean perpendicular distance from the point to a corresponding axis. By means of geometric transformation, a coordinate of a point P in a reference system F is related to its coordinate with respect to another reference system, M , as shown in Eq. 4-12 and Figure 4-9.

$$[P]^F = A[P]^M + L \quad (4-12)$$

where $[P]^F$ = a point coordinate in F reference system

$[P]^M$ = a point coordinate in M reference system

L is a distance vector in which M system located away from F system

and

$$A = \begin{bmatrix} f^1 \bullet m^1 & f^1 \bullet m^2 & f^1 \bullet m^3 \\ f^2 \bullet m^1 & f^2 \bullet m^2 & f^2 \bullet m^3 \\ f^3 \bullet m^1 & f^3 \bullet m^2 & f^3 \bullet m^3 \end{bmatrix} \quad (4-13)$$

where f^i is a direction cosine of i^{th} axis of F reference system

m^i is a direction cosine of i^{th} axis of M reference system

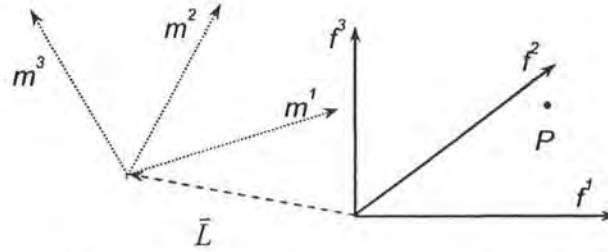


Figure 4- 9 F ref. system relative to M ref. system

From Figure 4-7 and Eq. 4-13, we obtain

$$A = \begin{bmatrix} i \bullet \bar{n}_s & i \bullet \bar{n}_t & i \bullet \bar{n}_p \\ j \bullet \bar{n}_s & j \bullet \bar{n}_t & j \bullet \bar{n}_p \\ k \bullet \bar{n}_s & k \bullet \bar{n}_t & k \bullet \bar{n}_p \end{bmatrix} \quad (4-14)$$

In the current model, the system established from imperfect datum features is represented by the M reference system which is deviated from the nominal or F reference system. When the hole is produced, its location is specified relative to the machine or F system. Unfortunately, once the workpiece is removed from the machine, its accuracy is determined based on its actual or M system. The errors, e , of feature position, P , as resulted from such discrepancy between both systems are calculated as follows.

$$e = P - (A \bullet P + L) \quad (4-15)$$

4.2.5 Positional Geometric Variation Region

According to ASME Y14.5M, a positional tolerance is the permissible variation in the location of a feature about its exact true position. For a cylindrical feature like a hole, it is a diameter of the tolerance zone with which the axis of the feature must lie, where the center of the tolerance zone being at the exact true position. In this study, the positional geometric

variability is defined the similar way except that it circumscribes actual locations as affected by undesirable factors. Following the algorithm explained in section 3.2.1-3.2.4, the feature's center is located. Because of irregular locating datum surface, inconsistency of the feature location is anticipated and can be illustrated as a cloud of points after a number of trials carried out (Figure 4-10). A circle bounding a specified proportion of possible feature locations (positional geometric variation region) is then calculated (See Sangnui (2002) for details). How big the circle is indicates how much the feature is varied in its position, and it should not exceed the tolerance associated with the hole.

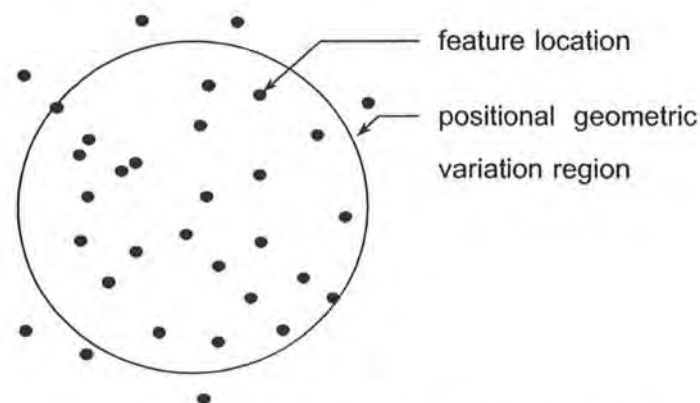


Figure 4- 10 Positional geometric variation region

4.3 Results and Discussion

The objective of this study is to present the impact of surface variational model on feature positional tolerancing. Three different distributions; a uniform, a normal, and a beta distribution are chosen to represent surface irregularities. Certain parameter values (α, β) of a beta distribution are selected to create various shapes of the distribution. Shown in Figure 6, the distribution is leaning towards uniformity when the parameters are approaching 1, and as the parameters are greater the distribution will become more centrally concentrated. Details of the analysis conducted are described in the following steps.

- 1) Random numbers representing workpiece surface points are generated according to the tolerance distributions of interest (normal, uniform and beta distributions). See section 2.1.
- 2) Datum feature surfaces are partitioned into a number of small rectangle patches.
- 3) To simulate a deviated surface, generated random numbers are assigned to the appropriate direction (i.e. for the primary datum feature, the surface errors will be in the z direction) at grid points connecting the patches as shown in Figure 4-11.

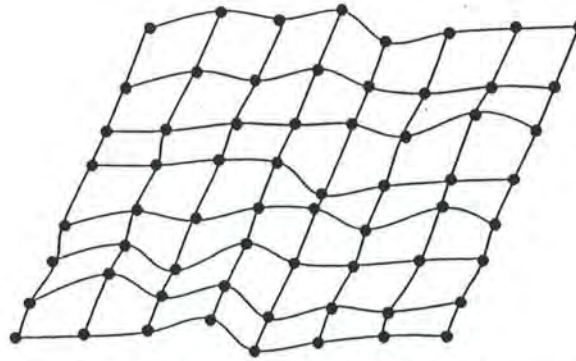


Figure 4- 11 A simulated datum feature

4) Following the algorithm explained in section 3.2, the actual feature locations are calculated.

5) Tolerance regions associated with a desired probability of acceptance are calculated for each surface distribution pattern based on 300 simulation runs. When the number is too large the resultant distribution tends to follow the normal distribution pattern, according to the central limit theorem, and the true information will be masked. Too small number of sample runs, on the other hand, is also not recommended since the results will be unclear.

6) An average of 50 radiuses of circular positional region is calculated and shown in Table 4-1.

7) Results from each distribution are compared.

Table 4-1 The averaged radiuses of positional geometric variation region at 95% containment.

Data	Datum Feature Tolerance Specifications	Radiuses of Feature Positional Geometric Variation Region (mm.)				
		Uniform	Normal	Beta		
				$\alpha, \beta = 1.5$	$\alpha, \beta = 4$	$\alpha, \beta = 7$
1	(Tp = 3, Ts = 3, Tt = 3)	0.0202	0.2993	0.0694	0.2568	0.2894
2	(Tp = 3, Ts = 5, Tt = 3)	0.0196	0.2864	0.0706	0.2521	0.2780
3	(Tp = 3, Ts = 5, Tt = 5)	0.0295	0.4125	0.1037	0.3660	0.4000
4	(Tp = 3, Ts = 3, Tt = 5)	0.0288	0.3888	0.0980	0.3421	0.4099
5	(Tp = 5, Ts = 5, Tt = 5)	0.0323	0.4982	0.1163	0.4057	0.5062
6	(Tp = 5, Ts = 3, Tt = 5)	0.0336	0.4965	0.1137	0.4345	0.4946
7	(Tp = 5, Ts = 3, Tt = 3)	0.0233	0.3693	0.0858	0.3291	0.3753
8	(Tp = 5, Ts = 5, Tt = 3)	0.0246	0.3665	0.0825	0.3470	0.4066

From the data in Table 4-1, it can be seen that the radiuses of positional geometric variation obtained from all eight simulation runs exhibit the same pattern across different

surface distributions. As the surface variability uniformly spreads over the allowable range (in the case of the uniform distribution), the resultant positional variation is expectedly smaller compared to one derived from more centrally concentrated distributions like the normal or the beta distributions ($\alpha=\beta$). It is clearly shown that feature positional geometric variation is dependent on variational model chosen to explain datum feature variability. The difference in positional tolerance radiuses is getting clearer as the surface tolerances becomes wider. Be noted that, this conclusion is regarded only when the datum establishment method is 3-2-1.

4.4 Conclusion

When the workpiece is subject to a machining operation, a feature to be produced is located with respect to a machine reference system. The system must have a certain relationship with a workpiece reference system which is established from the workpiece datum feature. By that way, the feature will be precisely located. However, with the existence of uncertainties in manufacturing processes, the relationship is usually hard or impossible to identify. If the pattern of irregularities is characterized, such discrepancy will be quantified, the undesirable displacement will be resolved, and ultimately an accuracy of the workpiece will be improved.

There are several variational models proposed to explain the pattern of irregularities of machined surfaces. Only the models' advantages and drawbacks in computation handling were reported. This study has investigated another interesting aspect, which is their effect over resultant positional tolerancing of a feature when referencing features are characterized by different variational models. From the analysis, the results indicated a distinct arrangement from one distribution to another. The area circumscribing possible feature locations is growing bigger as the surface variability is distributed apart from uniformity and when surface irregularities are evenly distributed, smaller feature variability is obtained. As results, the variability of the feature is obviously dependent on a distribution selected to explain irregular referencing surfaces. This source of variability should not be disregarded and left out of a system of tolerancing. It is well understood that the system is a critical element in a manufacturing process. An efficient tolerancing system not only bridges a gap between a design and a manufacturing department, but it does also help in production cost reduction.

4.5 References

1. Bhat, V. and E. C. D. Meter (2000). "An analysis of the effect of datum-establishment methods on the geometric errors of machined features." *International Journal of Machine Tools & Manufacture* 40: 1951-1975.
2. Choudhuri, S. A. and E. C. De-Meter (1999). "Tolerance analysis of machining fixture locators." *Journal of Manufacturing Science and Engineering* 121: 273-81.
3. Di Stefano, P. (2003). "Tolerance analysis and synthesis using the mean shift model." *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* v 217(n 2): p 149-160.
4. ElMaraghy, W. H., M. A. Gadalla, et al. (1995). Relating ANSI GD&T standards and symbols to variations on primitives. *Proceedings of the 1995 ASME International Mechanical Engineering Congress & Exposition, American Society of Mechanical Engineers (Paper)*.
5. He, J. R. (1991). "Estimating the distributions of manufactured dimensions with the beta probability density function." *International Journal of Machine Tools & Manufacture* 31(3): 38.-396.
6. Kalpakjian, S. (1991). *Manufacturing Processes for Engineering Materials*, Addison-Wesley Publishing Company, Inc.
7. Lin, S.-S., H.-P. Wang, et al. (1997). "Statistical tolerance analysis based on beta distributions." *Journal of Manufacturing Systems* 16: 150-8.
8. Mansoor, E. M. (1964). "The application of probability to tolerance used in engineering designs." *Proceedings of the Institution of Mechanical Engineers* 178 Pt1(1).
9. Nassef, A. O. and H. A. ElMaraghy (1997). "Allocation of geometric tolerances: New criterion and methodology." *CIRP Annals - Manufacturing Technology* 46(1): 101-106.
10. Roy, U. and R. Li (1998). "Representation and interpretation of geometric tolerances for polyhedral objects - I. Form tolerances." *Computer Aided Design* 30(2): 151-161.
11. Sangnui, S. (2002). *The impact of fixturing error on feature tolerance allocation. Industrial and Manufacturing Systems Engineering*. Ames, IA, Iowa State University: 83.
12. Stout, K. J., E. J. Davis, et al. (1990). *Atlas of machined surfaces*. New York, NY., Chapman and Hall.
13. Treacy, P., J. B. Ochs, et al. (1991). "Automated tolerance analysis for mechanical assemblies modeled with geometric features and relational data structure." *Computer Aided Design* 23(6): 444-453.

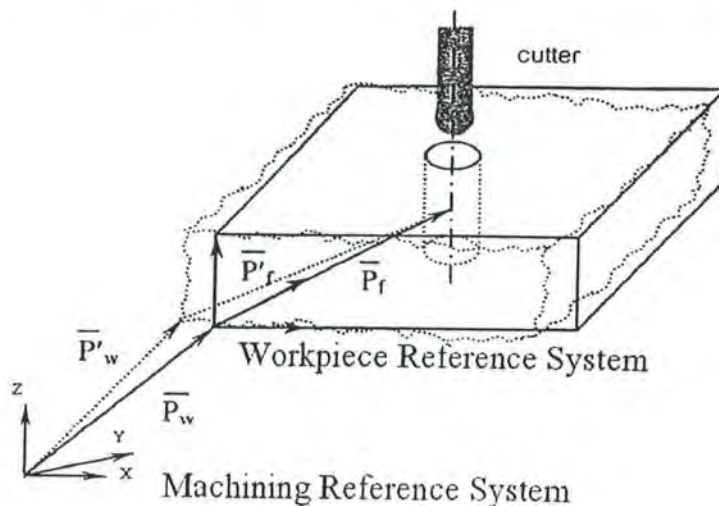
CHAPTER 5

A MODEL TO EVALUATE DEVIATION OF CUTTING POSITIONS ON A FIXTURED WORKPIECE PRIOR TO MACHINING

แบบจำลองประเมินความคลาดเคลื่อนของตำแหน่งการตัดบนชิ้นงานที่ถูกจับยึดด้วยตัวจับชิ้นงาน ก่อนกระบวนการตัด

5.1 บทนำ

ก่อนเริ่มขั้นตอนการผลิต ชิ้นงานถูกจับยึดบนตัวรองรับชิ้นงาน (workholder) หรืออีกชื่อหนึ่งคือฟิกซ์เจอร์ ประกอบด้วยตัวกำหนดตำแหน่ง (locator) และตัวจับยึด (clamp) เพื่อให้มั่นใจว่าชิ้นงานอยู่ในตำแหน่งที่ต้องการและถูกจับยึดมั่นคงเพียงพอที่จะไม่สั่นไถลในระหว่างการผลิต จากนั้นเครื่องจักรหรือใบมีดจะถูกควบคุมให้ตัดชิ้นงานตามตำแหน่งที่กำหนดไว้ การประเมินความถูกต้องของตำแหน่งการตัดนั้นอ้างอิงกับระบบพิกัดของชิ้นงาน (workpiece reference system) แต่การควบคุมใบมีดให้เคลื่อนที่อ้างอิงกับระบบพิกัดของเครื่องจักร (machine reference system) ดังนั้นเมื่อไรก็ตามที่ชิ้นงานถูกจับยึดเบี่ยงเบนไปจากที่ควรจะเป็น ทำให้ตำแหน่งของชิ้นงานที่อ้างอิงกับระบบพิกัดของเครื่องจักรจาก $\bar{P}_w$ กลายเป็น $\bar{P}'_w$ ($\bar{P}_w$ และ $\bar{P}'_w$ คือ เวกเตอร์แสดงตำแหน่งของชิ้นงานที่ควรจะเป็นและผิดพลาดตามลำดับ) ส่งผลให้ตำแหน่งการตัดผิดพลาดไป จาก $\bar{P}_f$ กลายเป็น $\bar{P}'_f$ ($\bar{P}_f$ และ $\bar{P}'_f$ คือ เวกเตอร์ตำแหน่งการตัดชิ้นงานที่ควรจะเป็นและผิดพลาดตามลำดับ) ดังแสดงใน รูปที่ 5-1 การเบี่ยงเบนของตำแหน่งชิ้นงานนี้ส่งผลกระทบต่อความถูกต้องของขนาดและรูปร่างของผลิตภัณฑ์ที่ผ่านการตัด



รูปที่ 5-1 การเบี่ยงเบนของตำแหน่งชิ้นงานส่งผลให้เกิดความคลาดเคลื่อนของตำแหน่งการตัด

จากการสำรวจพบว่ามิงงานวิจัยที่ทำการศึกษเกี่ยวกับสาเหตุที่ทำให้ตำแหน่งชิ้นงานเบี่ยงเบนไปจากที่ควรจะเป็นในขั้นตอนก่อนกระบวนการผลิต ซึ่งสามารถแบ่งตามสาเหตุได้คือ

1) ความไม่แน่นอนของพื้นผิวชิ้นงานที่เป็นผลมาจากกระบวนการผลิต ยกตัวอย่างเช่น Wang และคณะ (2006) ศึกษาถึงสาเหตุที่ทำให้พื้นผิวของชิ้นงานที่ผ่านการตัดมีความผันแปร โดยสาเหตุส่วนหนึ่งเกิดจากชิ้นงานที่มีรูปร่างไม่สมมาตรทำให้จับยึดชิ้นงานยากและจับยึดชิ้นงานไม่แน่น เมื่อขณะทำการผลิตชิ้นงานจะทำให้ชิ้นงานมีการหมุนเปลี่ยนตำแหน่งได้ นอกจากนี้ Chaiprapat และ Rujikietgumjorn (2006) ได้พัฒนากระบวนการที่ใช้ในการคำนวณหาความไม่แน่นอนของตำแหน่งชิ้นงานที่ถูกจับยึด ภายใต้สมมติฐานว่าชิ้นงานมีความผิดพลาด ณ จุดสัมผัส ซึ่งผลจากแบบจำลองทำให้สามารถนำไปใช้ในการกำหนดระยะพิถีพิถัน เป็นต้น

2) ความผิดพลาดของตัวกำหนดตำแหน่งชิ้นงาน ที่เกิดจากความบกพร่องของตัวกำหนดตำแหน่งที่เสื่อมสภาพและถูกวางในตำแหน่งที่ไม่เหมาะสม ยกตัวอย่างเช่น Cai และคณะ (1996, 1007) ใช้หลักการ N-2-1 ของตัวกำหนดตำแหน่ง ซึ่งเป็นวิธีสำหรับออกแบบฟิกซ์เจอร์ที่เหมาะสม โดยใช้วิธีการวิเคราะห์ทางไฟไนต์เอลิเมนต์และวิธีการไม่เชิงเส้น (non-linear) หา N ดีที่สุดที่ทำให้ชิ้นงานเกิดการเสียรูปน้อยสุดและได้เปรียบเทียบกับหลักการ 3-2-1 ที่ถูกยอมรับอย่างกว้างขวางสำหรับชิ้นงานวัสดุแข็ง (rigid) และได้พัฒนาโปรแกรมซึ่งเรียกว่า RFix Design ใช้สำหรับออกแบบลักษณะการจับยึดของฟิกซ์เจอร์เพื่อให้เกิดความผิดพลาดน้อยที่สุด นอกจากนี้ Chaiprapat และ Rujikietgumjorn (2007) ได้ทำการพัฒนาแบบจำลองทางคณิตศาสตร์ที่สามารถทำนายการเปลี่ยนแปลงทางเรขาคณิตซึ่งมีสาเหตุมาจากตัวกำหนดตำแหน่ง โดยจำลองชิ้นงานที่วางบนตัวกำหนดตำแหน่งแสดงเป็นแบบสองมิติในขั้นต้น และนำความสัมพันธ์ไปอธิบายได้จนถึงในกรณีสามมิติ ผลจากแบบจำลองนี้สรุปได้ว่าความเบี่ยงเบนของตำแหน่งชิ้นงานส่งผลให้เกิดความแปรปรวนของผิวชิ้นงานที่ผ่านการตัดเป็นฟังก์ชันพาราโบลอยด์ เป็นต้น

3) ความผิดพลาดของการจับยึดชิ้นงาน สาเหตุนี้ส่วนใหญ่ที่พบคือ ความไม่เหมาะสมของแรงและตำแหน่งในการจับยึด ยกตัวอย่างเช่น DeMeter (1994) แสดงแบบจำลองเชิงเส้นสำหรับทำนายผลกระทบจากตัวกำหนดตำแหน่งและตัวจับยึดชิ้นงานที่ส่งผลให้ชิ้นงานเคลื่อนตำแหน่งในขณะทำการผลิตชิ้นงาน ต่อมา DeMeter (1995) ได้มีการนำข้อมูลนี้ไปวิเคราะห์เพื่อหาตำแหน่งที่เหมาะสมของตัวกำหนดตำแหน่งและตัวจับยึดชิ้นงาน สรุปได้ว่าแรงตัวจับยึดที่กระทำกับชิ้นงานขณะทำการผลิตส่งผลให้ชิ้นงานเกิดการหมุนขึ้น และหลังจากนั้น DeMeter (2001) ได้ทำการแสดงแบบจำลองเชิงเส้นของแรงในการจับยึดชิ้นงาน (LCPL) เพื่อใช้ในการคำนวณหาแรงที่จำเป็นน้อยที่สุดในการจับยึดที่ไม่ทำให้เกิดการเสียดี ณ จุดสัมผัสกับตัวจับยึดขณะทำการผลิตชิ้นงาน Yeh และ Liou (1999) ได้สร้างแบบจำลองวิเคราะห์การจับยึดชิ้นงานที่จุดสัมผัสในขณะทำการผลิตโดยใช้วิธีการวิเคราะห์ของสปริงเอลิเมนต์และวิธีทางไฟไนต์เอลิเมนต์บนพื้นฐานทฤษฎี Hertz's contact เพื่อการออกแบบระบบฟิกซ์เจอร์ Li และ Melkote (2001) เสนอวิธีใหม่สำหรับการหาแรงในการจับยึดชิ้นงานที่เหมาะสม กรณีมีตัวจับยึดหลายตัวและเป็นระบบฟิกซ์เจอร์แบบ 3-2-1 พิจารณาในช่วงเวลาที่แรงของเครื่องจักรกระทำคงที่ บริเวณจุดสัมผัสเป็นแบบยึดหยุ่น และใช้วิธีทางไฟไนต์เอลิเมนต์วิเคราะห์หาความสัมพันธ์ระหว่างจุดสัมผัสชิ้นงานกับฟิกซ์เจอร์เพื่อออกแบบหาแรงที่เหมาะสมของตัวจับยึดชิ้นงาน Satyanarayana และ Melkote (2004) ได้วิเคราะห์จุดสัมผัสระหว่างฟิกซ์เจอร์กับชิ้นงาน โดยใช้วิธีทางไฟไนต์เอลิเมนต์วิเคราะห์

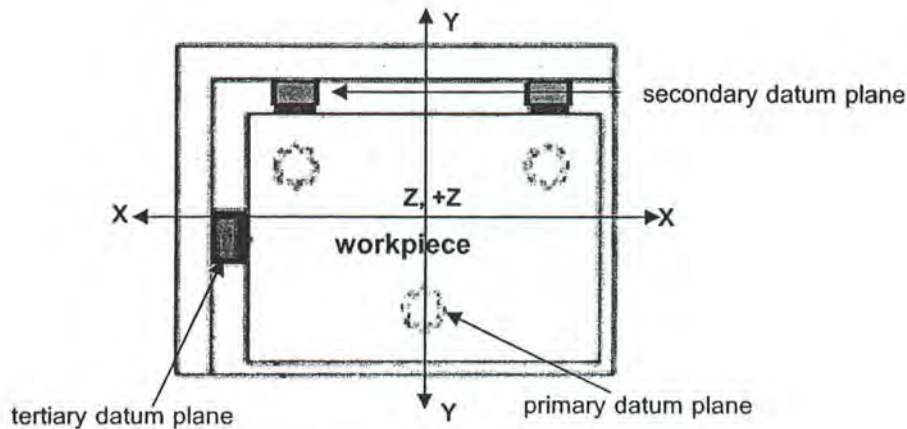
พิจารณาหนึ่งตำแหน่งจุดสัมผัส กรณีบริเวณจุดสัมผัสระหว่างฟิกซ์เจอร์กับชิ้นงานเป็นแบบยึดหมุน เป็นต้น จากการสำรวจงานวิจัยข้างต้น แสดงให้เห็นว่ามีหลายสาเหตุที่ทำให้ตำแหน่งชิ้นงานเบี่ยงเบนไปจากที่ควรจะเป็นในขั้นตอนก่อนกระบวนการผลิต เช่น ความไม่แน่นอนของพื้นผิวชิ้นงาน ฟิกซ์เจอร์ที่เสื่อมคุณภาพ ลักษณะและแรงในการจับยึดชิ้นงาน อย่างไรก็ตามเมื่อพิจารณาในรายละเอียดจะพบว่าโอกาสที่จะเกิดความผิดพลาดจากตัวกำหนดตำแหน่งนั้นมีน้อยมากเมื่อเทียบกับอีกสองสาเหตุ เนื่องจากในกระบวนการผลิตอุปกรณ์จับยึดจะต้องมีการตรวจสอบคุณภาพที่เข้มงวด ผ่านกระบวนการทางความร้อนเพื่อปรับคุณสมบัติของวัสดุให้มีความทนทานต่อการใช้งานอีกทั้งยังมีการกำหนดระยะพิกัดเผื่อที่ค่อนข้างแคบกว่ากระบวนการผลิตชิ้นงาน ดังนั้นสาเหตุหลักที่ส่งผลต่อการเบี่ยงเบนของตำแหน่งชิ้นงานจึงมาจากความไม่แน่นอนของพื้นผิวชิ้นงานและการเสีรูปร่างของชิ้นงานเนื่องจากลักษณะการจับยึดของฟิกซ์เจอร์ งานวิจัยนี้จึงได้ทำการสร้างแบบจำลองทางคณิตศาสตร์สำหรับทำนายความคลาดเคลื่อนของตำแหน่งการตัดบนชิ้นงานก่อนกระบวนการตัดอันเป็นผลมาจากสาเหตุข้างต้น โดยแบ่งการวิเคราะห์ค่าความคลาดเคลื่อนของตำแหน่งการตัดเป็นสองขั้นตอนด้วยกันคือ 1. การประเมินในขั้นตอนการกำหนดตำแหน่งชิ้นงาน และ 2. การประเมินในขั้นตอนการจับยึดชิ้นงาน ผลการประเมินค่าความคลาดเคลื่อนของตำแหน่งการตัดนั้นสามารถนำไปใช้เป็นข้อมูลในการปรับปรุงกระบวนการตัดให้แม่นยำและมีประสิทธิภาพมากขึ้น

5.2 ข้อมูลเบื้องต้น

5.2.1 หลักการทำงานของฟิกซ์เจอร์

ฟิกซ์เจอร์คือ อุปกรณ์ที่ประกอบด้วยตัวกำหนดตำแหน่งและตัวจับยึดชิ้นงานซึ่งหน้าที่ในการจับยึดชิ้นงานให้ติดแน่นอยู่กับลำตัวฟิกซ์เจอร์เพื่อให้ชิ้นงานอยู่ในตำแหน่งที่ต้องการ และไม่ทำให้ชิ้นงานเกิดการลื่นไถลระหว่างกระบวนการผลิต โดยสามารถแบ่งขั้นตอนการจับยึดชิ้นงานด้วยฟิกซ์เจอร์ออกเป็นสองส่วนดังนี้

1) ขั้นตอนการกำหนดตำแหน่งคือ การกำหนดตำแหน่งของชิ้นงานที่จะทำให้ทราบแน่นอนว่าตำแหน่งการตัดอยู่ตรงตำแหน่งไหน งานวิจัยนี้ได้พิจารณาเลือกชิ้นงานรูปทรงสี่เหลี่ยมผืนผ้าเป็นกรณีศึกษา และเลือกการกำหนดตำแหน่งแบบ 3-2-1 ซึ่งมีตัวกำหนดตำแหน่งทั้งหมด 6 ตัว สามารถจำกัดระดับความเป็นอิสระได้ 9 ทิศทาง ซึ่งในการกำหนดตำแหน่งด้วยฟิกซ์เจอร์นั้น เริ่มต้นด้วยการวางชิ้นงานบน primary datum plane ซึ่งมีตัวกำหนดอยู่ 3 ตำแหน่ง สามารถจำกัดการเคลื่อนที่ของชิ้นงานได้ 5 ทิศทาง ต่อจากนั้นจะต้องทำการดันชิ้นงานให้ไปสัมผัสกับ secondary datum plane ซึ่งเป็นระนาบขนานกับระนาบอ้างอิง Y-Z มีตัวกำหนดอยู่ 2 ตำแหน่งสามารถจำกัดการเคลื่อนที่ของชิ้นงานได้ 3 ทิศทาง และสุดท้ายเลื่อนชิ้นงานไปสัมผัสกับ tertiary datum plane ซึ่งเป็นระนาบขนานกับระนาบอ้างอิง X-Z มีตัวกำหนดอยู่ 1 ตำแหน่งสามารถจำกัดการเคลื่อนที่ของชิ้นงานได้ 1 ทิศทาง ดังรูปที่ 5-2

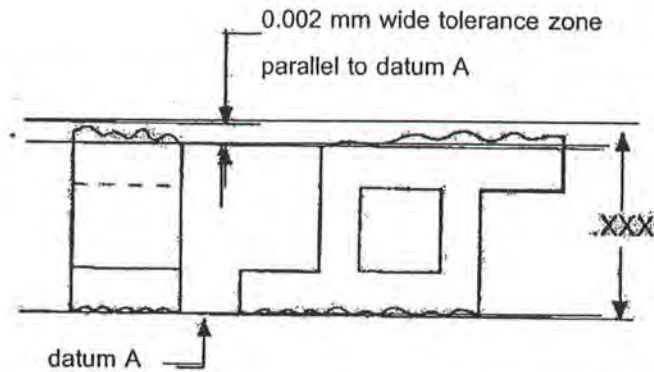


รูปที่ 5- 2 ลักษณะการกำหนดตำแหน่งแบบ 3-2-1

2) ขั้นตอนการยึดจับคือ การยึดชิ้นงานไว้ไม่ให้สั่นคลอนหรือเคลื่อนตำแหน่งไปจากจุดที่กำหนดไว้ในขณะทำการตัดชิ้นงาน ฉะนั้นตัวยึดจับชิ้นงานจะต้องมีความแข็งแรงเพียงพอที่จะต้านทานแรงที่เกิดจากการตัด แต่จะต้องไม่มากพอที่จะทำให้แรงในการยึดจับนั้นส่งผลเสียต่อชิ้นงานได้ โดยตำแหน่งของตัวยึดจับจะต้องสัมผัสกับชิ้นงานตรงจุดที่ชิ้นงานมีความแข็งแรง

5.2.2 การพิจารณาความคลาดเคลื่อนจากการผลิตชิ้นงาน

เนื่องจากความไม่แน่นอนในการผลิตชิ้นงานเป็นสิ่งที่หลีกเลี่ยงได้ยาก ฉะนั้นเพื่อเป็นการควบคุมความไม่แน่นอนดังกล่าว จึงต้องมีการกำหนดระยะพิสัย (tolerancing) ของรูปร่างและขนาดชิ้นงาน ซึ่งมาตรฐานที่ใช้อย่างแพร่หลายในการควบคุมความคลาดเคลื่อนในกระบวนการผลิต คือ ANSI Y14.5M Geometric Dimensioning and Tolerancing (GD&T) ในที่นี้ขอยกตัวอย่างการวิเคราะห์ความคลาดเคลื่อนของทิศทาง (orientation deviation) แบบการขนานกันของพื้นผิว (parallelism deviation) ซึ่งเป็นการพิจารณาเกี่ยวกับระยะห่างของความขนานของพื้นผิว ศูนย์กลางของระนาบเทียบกับระนาบอ้างอิง เช่น การกำหนดค่าระยะพิสัยเพื่อการขนานกันในช่วง 0.002 mm จะเป็นพิจารณาช่วงระยะห่างกัน 0.002 mm ระหว่างระนาบขนานกันสองระนาบที่ขนานกับระนาบอ้างอิง ซึ่งระนาบของชิ้นงานจริงต้องตั้งอยู่ระหว่างระนาบขนานกันสองระนาบนั้นด้วย ดังรูปที่ 5-3



รูปที่ 5-3 แสดงการพิจารณาความคลาดเคลื่อนการขนานกันของพื้นผิว (parallelism tolerance)
(Foster, 1994)

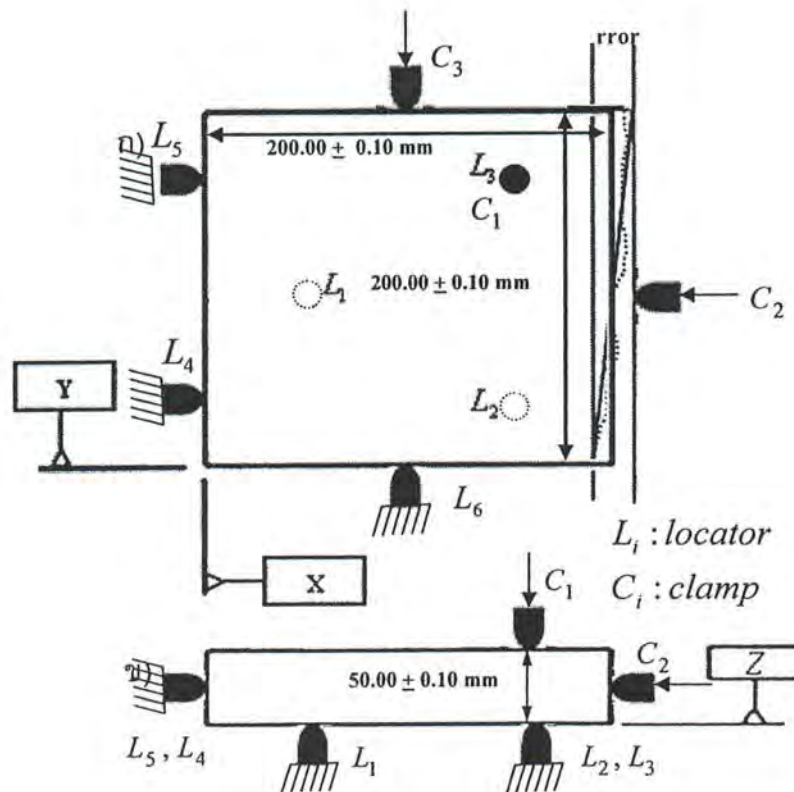
5.3 การพัฒนาแบบจำลองทางคณิตศาสตร์

5.3.1 สมมติฐานและเงื่อนไข

งานวิจัยนี้ได้ทำการศึกษาความผิดพลาดของการกำหนดตำแหน่งและการจับยึดชิ้นงานที่ส่งผลต่อความคลาดเคลื่อนของตำแหน่งการตัดในขั้นตอนก่อนกระบวนการตัด ซึ่งมีสมมติฐานที่ใช้ในการวิเคราะห์ดังนี้

- 1) ความผิดพลาด ณ จุดสัมผัส (error) วัดจากระยะตั้งฉากเทียบจากระนาบอ้างอิง ณ จุดสัมผัสระหว่างชิ้นงานกับตัวกำหนดตำแหน่งดังแสดงในรูปที่ 5-4
- 2) ชิ้นงานมีลักษณะรูปทรงสี่เหลี่ยมผืนผ้า
- 3) การกำหนดตำแหน่งของฟิกซ์เจอร์เป็นแบบ 3-2-1 มีการสัมผัสแบบจุดตามตำแหน่งต่างๆ จำนวน 6 ตำแหน่ง ดังแสดงในรูปที่ 5-4
- 4) การจับยึดชิ้นงานมีการสัมผัสแบบจุดตามตำแหน่งต่างๆ จำนวน 3 ตำแหน่ง ดังแสดงในรูปที่ 5-4
- 5) แรงในการจับยึดชิ้นงานอยู่ในรูปของแรงดันคงที่ (constant pressure)
- 6) การจับยึดระหว่างชิ้นงานกับฟิกซ์เจอร์ในแต่ละจุดสัมผัส มีการสัมผัสแบบแตะกันอยู่ (touching)

ในรูปที่ 5-4 ได้แสดงลักษณะขนาดและตำแหน่งการจับยึดชิ้นงานด้วยฟิกซ์เจอร์ ซึ่งรูปที่ 5-4 (ก) แสดงในมุมมองด้านบน และรูป 5-4 (ข) แสดงในมุมมองด้านหน้า โดย L1,..., L6 คือ ตำแหน่งตัวกำหนดตำแหน่ง และ C1,...,C3 คือ ตำแหน่งตัวจับยึด



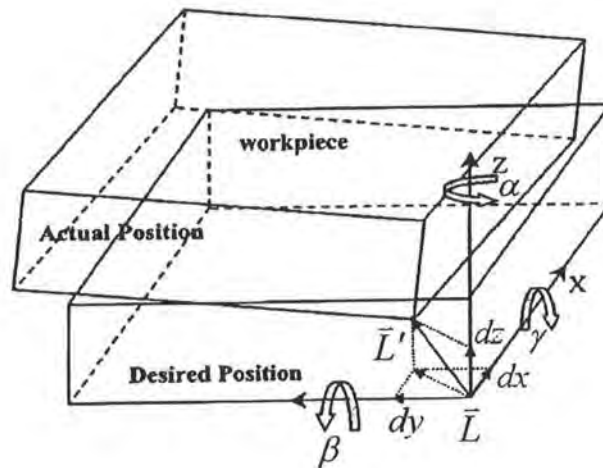
รูปที่ 5- 4 ขนาดและตำแหน่งการจับยึดชิ้นงานด้วยฟิกซ์เจอร์สำหรับงานวิจัย

5.3.2 การคำนวณค่าความคลาดเคลื่อนของตำแหน่งการตัด

งานวิจัยนี้ได้ทำการศึกษาความผิดพลาดของการกำหนดตำแหน่งและการจับยึดชิ้นงานที่ส่งผลต่อความคลาดเคลื่อนของตำแหน่งการตัดในขั้นตอนก่อนกระบวนการตัด ซึ่งสามารถวิเคราะห์ได้โดยแบ่งเป็นสองส่วนด้วยกันคือ

1) การคำนวณหาค่าความคลาดเคลื่อนของตำแหน่งการตัดที่เกิดขึ้นจากความผิดพลาดในขั้นตอนการกำหนดตำแหน่ง

การคำนวณในส่วนนี้ เป็นการหาค่าความคลาดเคลื่อนของตำแหน่งการตัด ที่เกิดจากความผิดพลาด ณ จุดสัมผัสระหว่างชิ้นงานกับตัวกำหนดตำแหน่ง ซึ่งมีสาเหตุมาจากความไม่แน่นอนของชิ้นงานและความผิดพลาดของตัวกำหนดตำแหน่ง ผู้วิจัยได้ใช้กระบวนการทางคณิตศาสตร์ในการวิเคราะห์หาค่าความคลาดเคลื่อนของตำแหน่งการตัด โดยอ้างอิงจากงานวิจัยของ Chaiprapat และ Rujikietgumjorn. (2006) ซึ่งได้นำสมการความคลาดเคลื่อนของตำแหน่งในรูปแบบทั่วไป โดยสมการนี้เป็นสมการที่คิดมาจากหลักการเปลี่ยนแปลงทางเรขาคณิต (geometric transformation) ประกอบด้วย การเปลี่ยนแปลงแบบหมุน (rotation transformation) และการเปลี่ยนแปลงแบบเลื่อน (translation transformation) ซึ่งผลจากวิเคราะห์ทำให้ทราบค่า $\gamma, \beta, \alpha, dx, dy, dz$ ดังแสดงในรูปที่ 5-5 และสามารถสรุปสมการได้ คือ



รูปที่ 5- 5 ลักษณะการเปลี่ยนแปลงทางเรขาคณิตของชิ้นงานจากขั้นตอนการกำหนดตำแหน่ง

$$\bar{L}' = R(\gamma, \beta, \alpha, dx, dy, dz)\bar{L} \quad (5-1)$$

เมื่อ $\bar{L}$ คือตำแหน่งที่สนใจบนชิ้นงาน

$\bar{L}'$ คือตำแหน่งดังกล่าวหลังจากการเปลี่ยนแปลงทางเรขาคณิต

$R(\gamma, \beta, \alpha, dx, dy, dz)$ คือเมตริกซ์หลักการเปลี่ยนแปลงทางเรขาคณิต

γ, β, α คือมุมของการเปลี่ยนแปลงทางแบบหมุนรอบแกน X, Y, Z ตามลำดับ

dx, dy, dz คือตำแหน่งของการเปลี่ยนแปลงทางเรขาคณิตแบบเลื่อนตามแนวแกน X, Y, Z

ตามลำดับ

2) การคำนวณหาค่าความคลาดเคลื่อนของตำแหน่งการตัดที่เกิดจากการจับยึดชิ้นงาน

การคำนวณในส่วนนี้ เป็นการหาค่าความคลาดเคลื่อนของตำแหน่งการตัด ที่เกิดจากแรงและตำแหน่งในการจับยึดชิ้นงานที่ไม่เหมาะสม ผู้วิจัยได้ใช้ระเบียบวิธีทางไฟไนต์เอลิเมนต์ซึ่งเป็นระเบียบวิธีเชิงตัวเลขวิธีหนึ่งที่ใช้สำหรับแก้สมการเชิงอนุพันธ์หรือการแก้ระบบสมการเชิงพีชคณิต ด้วยการแบ่งชิ้นส่วนที่ต้องการศึกษาออกเป็นส่วนย่อยๆ อย่างต่อเนื่องตามรูปลักษณะที่แท้จริงของชิ้นส่วน โดยเรียกแต่ละส่วนย่อยว่าเอลิเมนต์ (element) หลังจากนั้นจะทำการวิเคราะห์แต่ละเอลิเมนต์ ผลเฉลยที่ได้รับจะเป็นผลเฉลยที่จุดต่อ (node) ของแต่ละเอลิเมนต์นำมารวมเข้าด้วยกันเป็นผลเฉลยของระบบ ซึ่งสามารถแบ่งการวิเคราะห์ออกเป็นสองส่วนคือ

2.1) การวิเคราะห์ด้วยระเบียบวิธีทางไฟไนต์เอลิเมนต์ ณ จุดสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์ เนื่องจากในงานวิจัยนี้ผู้วิจัยได้กำหนดแรงของตัวจับยึดที่กระทำกับชิ้นงาน (F'_j) และค่าคงที่ความยืดหยุ่น ณ จุดสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์ (k'_j) ดังนั้นทำให้สามารถหาความคลาดเคลื่อนของตัวจับยึดและส่งผลให้ทราบแรงและความคลาดเคลื่อน ณ จุดสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์ (d'_j) ได้ดังรูปที่ 5-6 และสามารถเขียนให้อยู่ในรูปสมการเมตริกซ์ได้ดังสมการที่ (5-2) โดยอ้างอิงจากงานวิจัย ของ Li และ Melkote (2001)

$$\begin{Bmatrix} F'_x \\ F'_y \\ F'_z \end{Bmatrix} = \begin{bmatrix} k'_x & 0 & 0 \\ 0 & k'_y & 0 \\ 0 & 0 & k'_z \end{bmatrix} \begin{Bmatrix} d'_x \\ d'_y \\ d'_z \end{Bmatrix} \quad (5-2)$$

เมื่อ F'_j คือแรงที่ฟิกซ์เจอร์กระทำกับชิ้นงาน ณ จุดสัมผัสต่างๆ

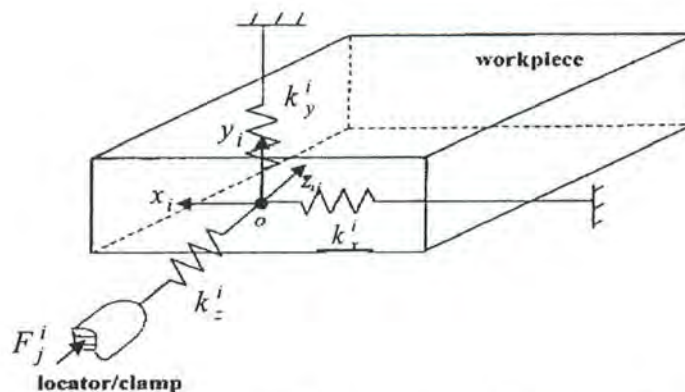
k'_j คือค่าคงที่ความยืดหยุ่น ณ จุดสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์ (workpiece – fixture contact stiffness)

d'_j คือความคลาดเคลื่อนของตำแหน่ง ณ จุดสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์

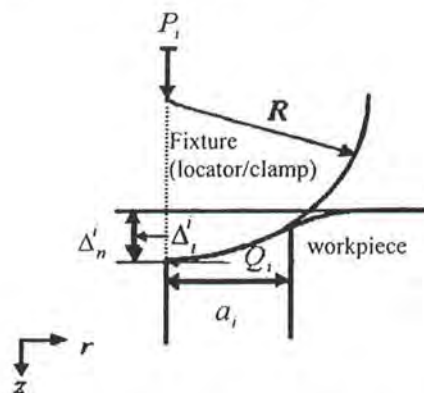
i คือตำแหน่งอ้างอิง ณ จุดสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์

j คือแกนอ้างอิง X Y และ Z ตามลำดับ

Li และ Melkote (2001) ได้นำหลักทฤษฎี Hertz's contact (2007) ซึ่งเป็นทฤษฎีใช้สำหรับพิจารณาการสัมผัสกันของวัตถุสองชิ้น (การจับยึดชิ้นงานด้วยฟิกซ์เจอร์ ณ จุดสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์) มาประยุกต์ใช้หาค่า k'_j ดังแสดงในรูปที่ 5-7 ซึ่งเป็นการนำเสนอสมการการเสียรูป ณ จุดสัมผัสระหว่างวัตถุสองชิ้นทั้งในแนวตั้งฉาก (normal deformation) และแนวเส้นสัมผัส (tangential deformation) ดังสมการที่ (5-3) และ (5-4)



รูปที่ 5-6 แบบจำลองลักษณะการจับยึดชิ้นงาน ณ จุดสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์ (2001)



รูปที่ 5-7 ลักษณะการสัมผัสกันของวัตถุสองชิ้นตามหลัก ทฤษฎี Hertz's contact

สมการระยะเสียรูป ณ จุดสัมผัสระหว่างตัวกำหนดตำแหน่งและตัวจับยึดกับชิ้นงานในแนวตั้งฉาก (Δ_n')

$$\Delta_n' = \left(\frac{9(P')^2}{16R(E^*)^2} \right)^{1/3} \quad (5-3)$$

โดย $\frac{1}{E^*} = \frac{1-\nu_w^2}{E_w} + \frac{1-\nu_f^2}{E_f}$

สมการระยะเสียรูป ณ จุดสัมผัสระหว่างตัวกำหนดตำแหน่งและตัวจับยึดกับชิ้นงานในแนวเส้นสัมผัส (Δ_t')

$$\Delta_t' = \frac{Q'}{8a_i} \left(\frac{2-\nu_f}{G_f} + \frac{2-\nu_w}{G_w} \right) \quad (5-4)$$

โดย $a_i = \left(\frac{3P'R}{4} \left(\frac{1-\nu_f}{E_f} + \frac{1-\nu_w}{E_w} \right) \right)^{1/3}$

เมื่อ P' คือแรงกระทำตั้งฉากกับชิ้นงาน ณ ตำแหน่งใดๆ

Q' คือแรงกระทำแนวเส้นสัมผัสกับชิ้นงาน ณ ตำแหน่งต่างๆ

R คือรัศมีตัวกำหนดตำแหน่งและตัวจับยึด

a_i คือระยะสัมผัสระหว่างตัวกำหนดตำแหน่งและตัวจับยึดกับชิ้นงานตามแนวแกน r เทียบกับจุดสัมผัสเริ่มต้น ณ ตำแหน่งต่างๆ

Δ_n', Δ_t' คือระยะเสียรูป ณ จุดศูนย์กลางการสัมผัสกันระหว่างตัวกำหนดตำแหน่งและตัวจับยึดกับชิ้นงานตามแนวแกน z และ r เทียบจุดสัมผัสที่ยังไม่เสียรูป

E^* คือค่ามอดุลัสรวมของการสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์

E_f, E_w คือค่ามอดุลัสความยืดหยุ่นของฟิกซ์เจอร์และชิ้นงาน ตามลำดับ

G_f, G_w คือค่ามอดุลัสเฉือนของฟิกซ์เจอร์และชิ้นงาน ตามลำดับ

ν_f, ν_w คือค่าอัตราส่วนปัวซองของฟิกซ์เจอร์และชิ้นงาน ตามลำดับ

จากความสัมพันธ์ $k = \frac{F}{d}$ (k คือค่าคงที่ความยืดหยุ่น F คือแรงที่กระทำ และ d คือระยะการเสียรูป) สามารถเปลี่ยนสมการที่ (5-3) และ (5-4) ให้อยู่ในรูปสมการค่าคงที่ความยืดหยุ่น (contact stiffness) ดังแสดงในสมการที่ (5-5) และ (5-6) ตามลำดับ

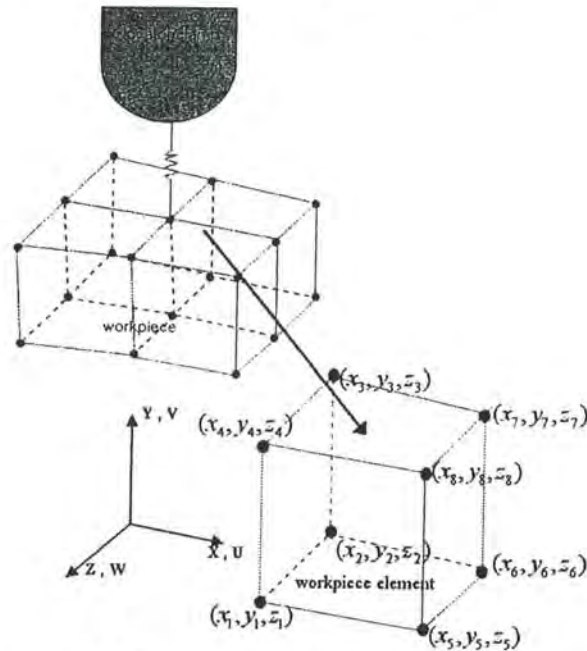
$$k_z' = 8.82 \left(\frac{16R(E^*)^2}{9} \right)^{1/3} \quad (5-5)$$

$$k'_x = k'_y = \frac{4}{E^*} \left(\frac{2-\nu_f}{G_f} + \frac{2-\nu_w}{G_w} \right)^{-1} k'_z \quad (5-6)$$

โดยค่า 8.82 เป็นค่าคงที่จากการใช้วิธีการจัดรูปเป็นสมการเส้นตรง (least-squares fit) โดยมีแรง P^i กระทำอยู่ในช่วง 0 ถึง 1,000 N

2.2) การวิเคราะห์ด้วยระเบียบวิธีทางไฟไนต์เอลิเมนต์ของตัวชิ้นงาน

จากการวิเคราะห์ในหัวข้อที่ 2.1) ทำให้ทราบถึงลักษณะการเบี่ยงเบนของชิ้นงาน ณ จุดสัมผัสระหว่างชิ้นงานกับฟลักซ์เจอร์ และสามารถนำข้อมูลนี้ไปวิเคราะห์หาความเบี่ยงเบนของชิ้นงานในตำแหน่งอื่นๆด้วยระเบียบวิธีทางไฟไนต์เอลิเมนต์และได้ทำการแบ่งเอลิเมนต์ของชิ้นงานออกเป็นเอลิเมนต์ทรงหกหน้า (hexahedral element) ดังแสดงในรูปที่ 5-8



รูปที่ 5- 8 แบบจำลองทางไฟไนต์เอลิเมนต์ของการจับยึดชิ้นงาน ณ จุดสัมผัสและลักษณะเอลิเมนต์ชิ้นงาน

ในงานวิจัยนี้ผู้วิจัยได้ใช้โปรแกรม MSC Marc ซึ่งเป็นโปรแกรมที่สามารถจำลองลักษณะชิ้นงานและทำการวิเคราะห์ด้วยระเบียบวิธีทางไฟไนต์เอลิเมนต์ โดยยกตัวอย่างการจำลองลักษณะการจับยึดชิ้นงานบนฟลักซ์เจอร์ ตามหัวข้อ 4.4.1 โดยกำหนดคุณสมบัติของชิ้นงานและฟลักซ์ ดังแสดงในตารางที่ 5-1 และกำหนดเงื่อนไขการจับยึดชิ้นงานบนฟลักซ์เจอร์ ดังแสดงในตารางที่ 5-2 และ 5-3 โดยมีสัมประสิทธิ์แรงเสียดทานระหว่างชิ้นงานกับฟลักซ์เจอร์ (μ) เท่ากับ 0.39 (Xie et al., 2000)

ตารางที่ 5-1 คุณสมบัติของชิ้นงานและฟิกซ์เจอร์

Property	Workpiece	Fixture
Material	Aluminium 6061-T6	Steel AISI 1144
Elastic modulus	70 GPa	201 GPa
Poisson ratio	0.334	0.292
Mass density	2700 kg/m^3	7900 kg/m^3
Geometry	prismatic block size = 200x200x50 mm	spherical tip radius = 6 mm

ตารางที่ 5-2 ตำแหน่งความผิดพลาด ณ จุดสัมผัสระหว่างตัวกำหนดตำแหน่งกับชิ้นงาน

Plane	Locators	Position at the contact points of fixture-workpiec (mm)		
		X	Y	Z
primary	1	30.00	100.00	<u>0.03</u>
	2	170.00	30.00	<u>0.05</u>
	3	170.00	170.00	<u>0.09</u>
secondary	4	0.00	<u>30.10</u>	25.00
	5	0.00	<u>170.04</u>	25.00
tertiary	6	<u>100.07</u>	0.00	25.00

ตารางที่ 5-3 ตำแหน่งและแรงในการจับยึด

Clamp	Force (N)	Position at the contact points of fixture-workpiec (mm)		
		X	Y	Z
1	800	170.00	170.00	50.00
2	450	200.00	100.00	25.00
3	300	100.00	200.00	25.00

โดยในการกำหนดค่าความผิดพลาด ณ จุดสัมผัสระหว่างตัวกำหนดตำแหน่งกับชิ้นงานได้นำข้อมูลความไม่แน่นอนของพื้นผิวชิ้นงานที่เกิดจากกระบวนการผลิตชิ้นงานให้เป็นรูปร่าง (shape mill) (Kalpakjian, 1992) และตำแหน่งของตัวกำหนดตำแหน่งได้กำหนดให้อยู่ห่างจากกันให้มากที่สุดเพื่อลดความผิดพลาดในการกำหนดตำแหน่งให้น้อยลง (Chaiprapat and Rujikietgumjorn, 2007) นอกจากนี้ในการตำแหน่งของตัวจับยึดได้กำหนดให้วางในตำแหน่งที่ไม่กีดขวางเส้นทางเดินใบมีดและสามารถจำกัดการเคลื่อนที่ของชิ้นงาน โดยแรงที่ใช้การจับยึดจะต้องเป็นแรงที่น้อยที่สุดที่ไม่ทำให้ชิ้นงานเกิดการ

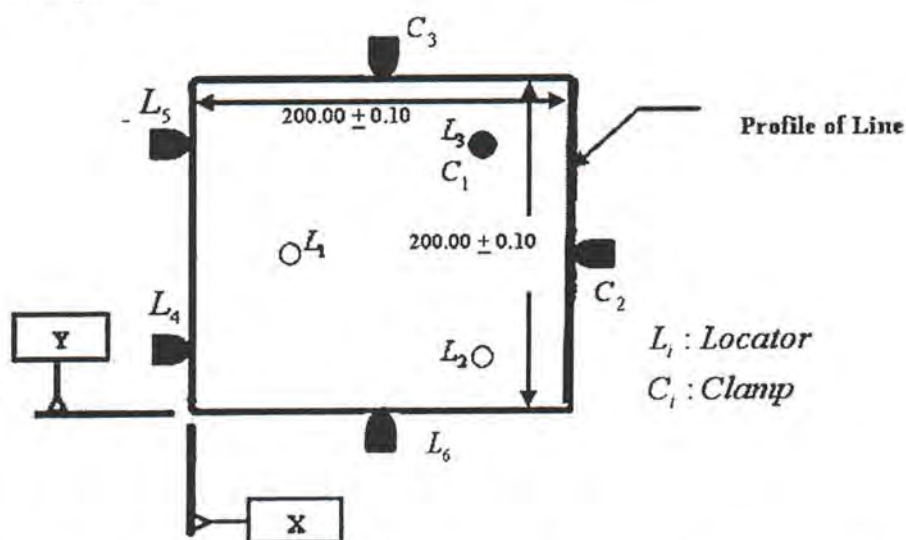
บิดหรือสั่นไถลขณะทำการผลิต ซึ่งในการกำหนดแรงนั้นจะต้องนำค่าสัมประสิทธิ์แรงเสียดทานระหว่างชิ้นงานกับฟิกซ์เจอร์และแรงการตัดชิ้นงานมาพิจารณาด้วย (DeMeter et al., 2001)

5.4 ผลการวิเคราะห์

จากการจำลองลักษณะการจับยึดชิ้นงานบนฟิกซ์เจอร์อันเป็นผลมาจากการกำหนดตำแหน่งที่ผิดพลาดและการการจับยึดของฟิกซ์เจอร์ ทำให้สามารถทำนายลักษณะความคลาดเคลื่อนของตำแหน่งการตัดบนชิ้นงานในขั้นตอนก่อนกระบวนการตัด ซึ่งสามารถอธิบายผลการวิเคราะห์ได้ดังนี้คือ

5.4.1 วิเคราะห์ความคลาดเคลื่อนตำแหน่งการตัด

ในส่วนนี้ได้พิจารณาผลความคลาดเคลื่อนของตำแหน่งการตัดที่ได้จากการทำนายโดยเปรียบเทียบกับตำแหน่งที่ควรจะเป็นในแกน X Y และ Z ในรูปของค่าเฉลี่ย (average) และค่าความคลาดเคลื่อนมาตรฐาน (SE) ดังตารางที่ 4 ตามเส้นทาง (profile of line) ในส่วนพื้นผิวด้านบนของชิ้นงานดังแสดงในรูปที่ 5-9



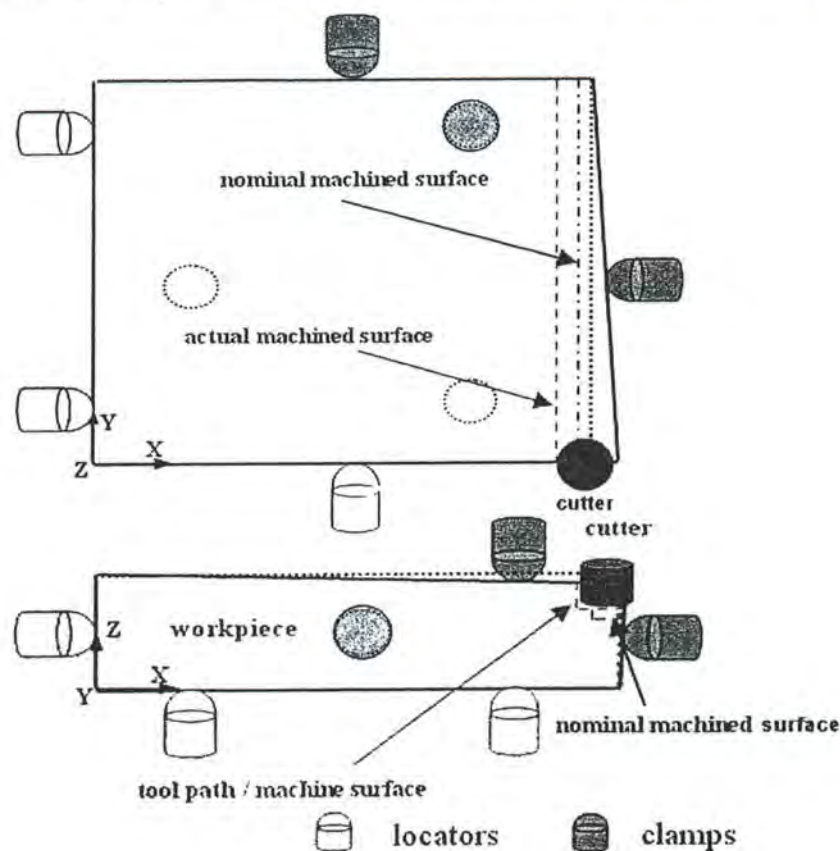
รูปที่ 5- 9 เส้นทางที่ใช้ในการพิจารณาความคลาดเคลื่อนของตำแหน่งการตัด

ตารางที่ 5-4 ค่าความคลาดเคลื่อนตำแหน่งการตัดในแต่ละแนวแกน

Axis	Cutting positions displacement (mm)			
	Locating		Clamping	
	average	SE	average	SE
X	0.0233	0.0017	0.0015	0.0014
Y	0.1923	0.0000	-0.0005	0.0002
Z	0.1479	0.0005	-0.0045	0.0003
Absolute	0.2437	-	-0.0048	-

จากการวิเคราะห์พบว่าค่าความคลาดเคลื่อนของตำแหน่งการตัดส่วนใหญ่จะเกิดขึ้นจากสาเหตุการกำหนดตำแหน่งมากกว่าการจับยึดชิ้นงานถึง 98 % และเมื่อพิจารณาค่าความคลาดเคลื่อนของ

ตำแหน่งการตัดในระนาบ X-Y พบว่าค่าความคลาดเคลื่อนของตำแหน่งการตัดที่เกิดจากการจับยึดมีค่าน้อยมากเมื่อเทียบกับระนาบ Y-Z เนื่องจากแรงในการจับยึดของตัวจับยึดที่ 1 จะกระทำกับชิ้นงานตามแนวแกน Z ซึ่งมีความหนาแน่นน้อยกว่าแกน X และ Y ทำให้ชิ้นงานมีการเสียรูปในค่า Z มากกว่าค่า X และ Y ดังนั้นงานวิจัยของ Satyanarayana และ Melkote (2004) ได้วิเคราะห์ผลของแรงจับยึด ณ จุดสัมผัสระหว่างฟิกซ์เจอร์กับชิ้นงาน โดยใช้วิธีการทางไฟไนต์เอลิเมนต์ในการพิจารณาถึงปัจจัยความหนาของชิ้นงานที่ส่งผลต่อการเสียรูปของชิ้นงาน พบว่าเมื่อชิ้นงานมีความหนามากจะมีการเสียรูป ณ จุดสัมผัสมีค่าน้อยกว่าชิ้นงานที่มีความหนาแน่นน้อยกว่า ฉะนั้นหากนำลักษณะการจับยึดชิ้นงานนี้ไปใช้ในกระบวนการผลิตที่ต้องการความแม่นยำสูง โดยไม่มีการชดเชยความคลาดเคลื่อนของตำแหน่งการตัด จะทำให้เกิดความผิดพลาดของการตัดชิ้นงานขึ้นได้ เนื่องจากพื้นผิวที่ต้องการให้ถูกตัด (nominal machined surface) ไม่เป็นพื้นผิวเดียวกับพื้นผิวที่ถูกตัดจริง (actual machined surface) ดังแสดงในรูปที่ 5-10

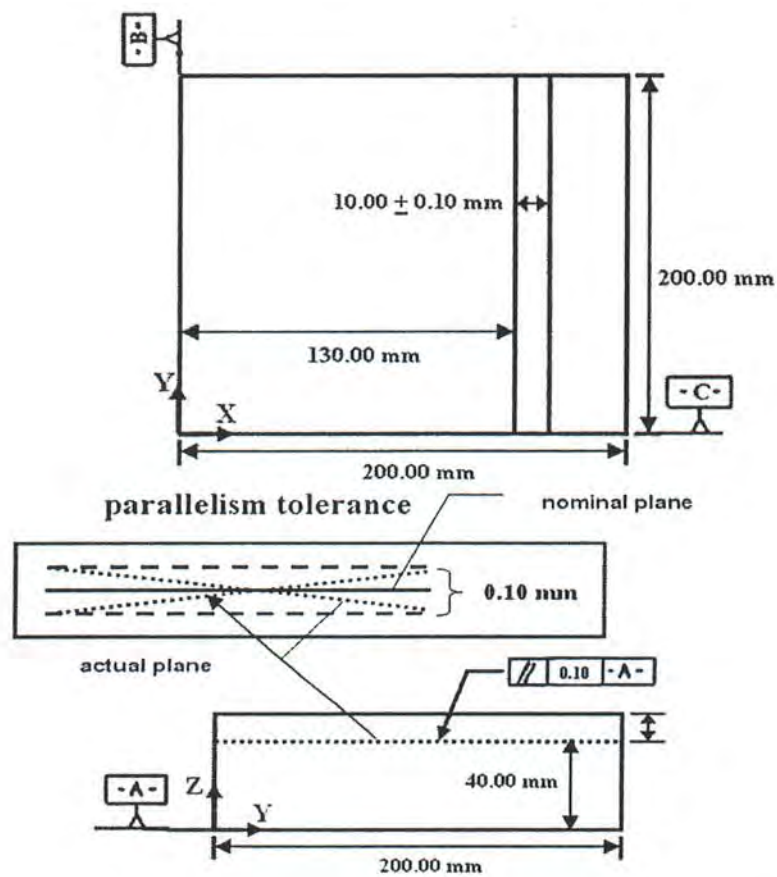


รูปที่ 5- 10 ลักษณะความผิดพลาดของการตัดชิ้นงาน

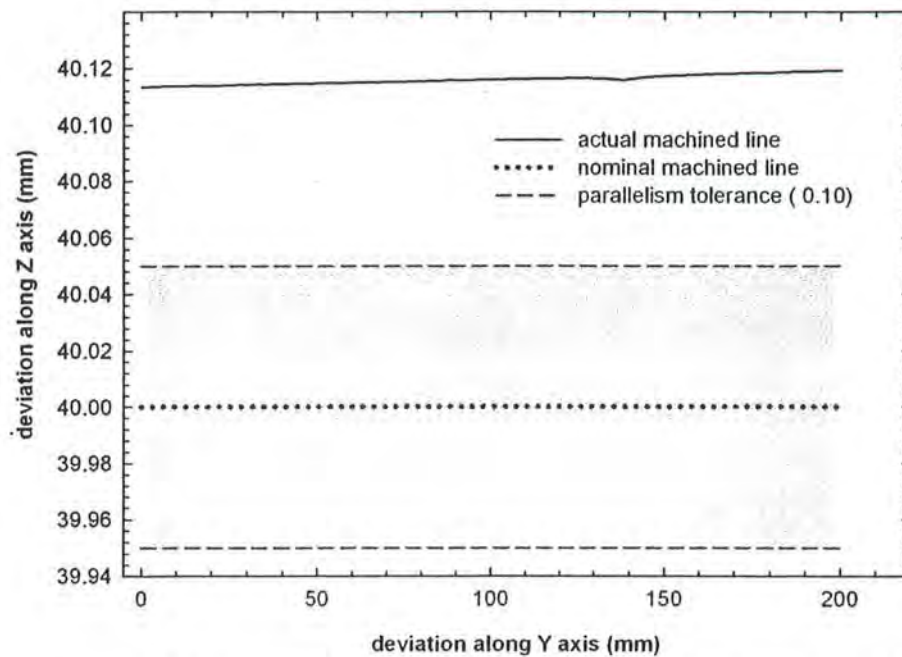
5.4.2 วิเคราะห์ความเบี่ยงเบนของพื้นผิวที่ถูกตัด

ผู้วิจัยได้พิจารณาความเบี่ยงเบนของพื้นผิวที่ถูกตัดตามรายละเอียดในรูปที่ 5-11 ซึ่งในกระบวนการกัดผิว ได้กำหนดให้พื้นผิวดังกล่าวถูกควบคุมโดยมีความคลาดเคลื่อนแบบการขนานกันของพื้นผิว (parallelism deviation) เปรียบเทียบกับระนาบอ้างอิง A (ระนาบ X-Y) อยู่ในช่วงขอบเขต 0.10 mm พบว่าเมื่อมีความคลาดเคลื่อนของตำแหน่งการตัดตามหัวข้อ 4.1 ไม่นิดไม่สามารถตัดชิ้นงานใน

ตำแหน่งที่ถูกต้องได้ ส่งผลให้พื้นผิวที่ถูกควบคุมมีลักษณะเบี่ยงเบนออกไปจากพื้นผิวที่ต้องการให้ถูกต้อง ดังแสดงในรูปที่ 5-12

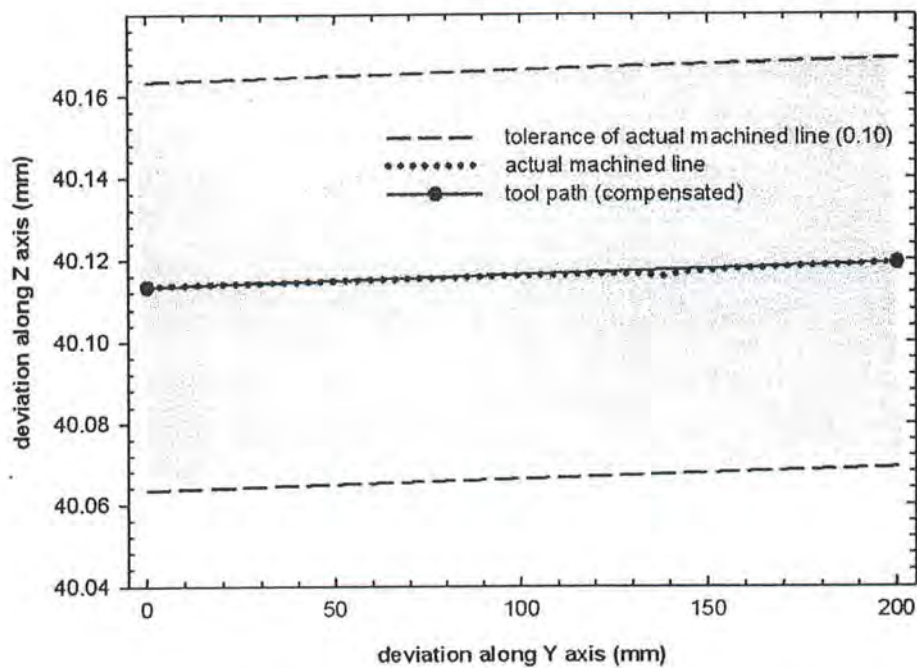


รูปที่ 5- 11 การออกแบบตำแหน่งการตัดชิ้นงาน



รูปที่ 5- 12 การเบี่ยงเบนของพื้นผิวที่ถูกตัดเทียบกับระนาบอ้างอิง A

จากรูปที่ 5-12 เมื่อพิจารณาการเบี่ยงเบนของพื้นผิวที่ถูกตัดกับช่วงขอบเขตความคลาดเคลื่อนการขนานกันของระนาบอ้างอิง A (deviation along Z axis) พบว่าความคลาดเคลื่อนของพื้นผิวที่ถูกตัดเมื่อเทียบกับเส้นทางที่ควรจะเป็น (nominal machined line) มีค่าเฉลี่ยเท่ากับ 0.1161 mm และมีค่าส่วนเบี่ยงเบนมาตรฐานเท่ากับ 0.0016 ซึ่งค่าความคลาดเคลื่อนของพื้นผิวที่ถูกตัดไม่อยู่ในช่วงค่าระยะพิสัยเผื่อของช่วงความคลาดเคลื่อนการขนานกันเกือบทั้งหมด ผู้ปฏิบัติงานควรควบคุมความไม่แน่นอนของชิ้นงานให้มีค่าน้อยกว่าความต้องการของพื้นผิวชิ้นงานที่ถูกตัด จึงจะทำให้ความผิดพลาดที่เกิดขึ้นอยู่ในช่วงที่ยอมรับได้ นอกจากนี้หากต้องการลดความผิดพลาดที่เกิดขึ้นก่อนกระบวนการผลิตโดยไม่สามารถแก้ไขความผิดพลาดของการกำหนดตำแหน่งและการจับยึดชิ้นงานด้วยฟิกซ์เจอร์ ผู้ปฏิบัติงานควรชดเชยเส้นทางเดินใบมีดเพื่อเพิ่มความแม่นยำในการผลิตชิ้นงาน ซึ่งผู้วิจัยได้ยกตัวอย่างการชดเชยเส้นทางเดินใบมีดในระนาบ Z-Y โดยใช้ข้อมูลของการทำนายในการกำหนดเส้นทางเดินใบที่จุดเริ่มต้นและจุดปลายเท่านั้น โดยมีการชดเชยในส่วนเริ่มต้นของการเดินใบมีด จากพิกัด $X = 135.00$ $Y = 0.00$ และ $Z = 40.00$ mm ให้เปลี่ยนเป็นตำแหน่ง $X = 135.05$ $Y = 0.17$ และ $Z = 40.11$ mm และในส่วนสุดท้ายของการเดินใบมีดจากพิกัด $X = 135.00$ $Y = 200.00$ และ $Z = 40.00$ mm ให้เปลี่ยนเป็นตำแหน่ง $X = 134.97$ $Y = 200.17$ และ $Z = 40.12$ mm ตามลำดับดังรูปที่ 5-13



รูปที่ 5- 13การชดเชยเส้นทางเดินใบมีดช่วงระนาบ Y-Z

จากรูปที่ 5-13 พบว่าการกำหนดเส้นทางเดินใบมีดเพียงแค่สองตำแหน่ง ก็มีความใกล้เคียงกับเส้นความคลาดเคลื่อนของตำแหน่งการตัดและอยู่ในช่วงของค่าระยะพิสัยที่ต้องการเนื่องจากความคลาดเคลื่อนของตำแหน่งการตัดจากลักษณะการจับยึดนี้ส่วนใหญ่เกิดจากความผิดพลาดในขั้นตอนการกำหนดตำแหน่งซึ่งทำให้ความคลาดเคลื่อนของตำแหน่งการตัดเป็นรูปแบบเชิงเส้น แต่ถ้ามีการกำหนดค่าช่วงระยะพิสัยที่แคบหรือทำการจับยึดชิ้นงานบาง ผลของการตัดชิ้นงานจะมีความถูกต้องมากเพียงใดนั้นจะขึ้นอยู่กับลักษณะของการกำหนดชดเชยเส้นทางเดินใบมีดที่กำหนดช่วงระยะห่างต่างๆ ของตำแหน่งการเดินใบมีดมากน้อยเพียงใด เนื่องจากความคลาดเคลื่อนของตำแหน่งการตัดบริเวณตัวจับยึดจะมีความคลาดเคลื่อนแบบไม่เป็นเชิงเส้น ดังนั้นหากกำหนดช่วงระยะห่างของตำแหน่งที่แคบก็จะเพิ่มความถูกต้องในการตัดมากยิ่งขึ้น (ทั้งนี้ขึ้นอยู่กับลักษณะความคลาดเคลื่อนของตำแหน่งการตัดที่จะทำการชดเชยเส้นทางเดินใบมีด)

จากการประเมินความคลาดเคลื่อนของตำแหน่งการตัดบนชิ้นงานที่ถูกจับยึดด้วยตัวจับชิ้นงาน ก่อนกระบวนการตัดสรุปได้ว่า หากมีการผลิตชิ้นงานโดยไม่ตรวจสอบความคลาดเคลื่อนของตำแหน่งชิ้นงานที่เกิดจากการจับยึดชิ้นงานด้วยฟิกซ์เจอร์ลักษณะนี้ และยังทำการกำหนดลักษณะการผลิตแบบเดิม ส่งผลให้ชิ้นงานที่ผลิตออกมามีความเสียหายและไม่สามารถนำไปใช้งานในกระบวนการถัดไปได้ ฉะนั้นหากต้องการให้การผลิตชิ้นงานอยู่ในช่วงความคลาดเคลื่อนที่ยอมรับได้ ผู้ปฏิบัติงานควรให้ความสำคัญกับความผิดพลาดของการกำหนดตำแหน่งการจับยึดชิ้นงานและลักษณะการจัดวางของชิ้นงานบนเครื่องจักร ซึ่งในการออกแบบฟิกซ์เจอร์นั้น ผู้ออกแบบจะต้องคำนึงถึงค่าความผิดพลาดของชิ้นงานที่ยอมรับได้ด้วย โดยตามกฎหมายฯ ไป ค่าความผิดพลาดของฟิกซ์เจอร์จะต้องมีค่าระหว่าง 20-50%

ของค่าความผิดพลาดที่ยอมรับได้ของชิ้นงานที่ถูกผลิต (Wattanuchariya, 2008) และความถูกต้องแม่นยำของการตัดชิ้นงานจะต้องสูงกว่า 20-35% ของค่าความผิดพลาดที่ยอมรับได้ของชิ้นงานที่ถูกผลิต (Kang et al., 2003)

5.5 สรุป

งานวิจัยนี้มีจุดประสงค์เพื่อสร้างแบบจำลองทางคณิตศาสตร์สำหรับทำนายความคลาดเคลื่อนของตำแหน่งการตัดบนชิ้นงานก่อนเริ่มต้นกระบวนการตัดอันเป็นผลมาจากการกำหนดตำแหน่งที่ผิดพลาดจากความไม่แน่นอนของพื้นผิวชิ้นงานที่และการเสียรูปของชิ้นงานเนื่องจากลักษณะการจับยึดของฟิกซ์เจอร์ ผู้วิจัยสามารถสร้างแบบจำลองการจับยึดชิ้นงานด้วยฟิกซ์เจอร์โดยแบ่งการวิเคราะห์ค่าความคลาดเคลื่อนของตำแหน่งการตัดเป็นสองขั้นตอนด้วยกันคือ 1. การประเมินในขั้นตอนการกำหนดตำแหน่งชิ้นงาน ในส่วนนี้ผู้วิจัยได้ใช้กระบวนการทางคณิตศาสตร์ซึ่งอาศัยพื้นฐานของหลักการเปลี่ยนแปลงทางเรขาคณิตสำหรับทำนายโดยใช้โปรแกรม Matlab ผลจากการทำนายพบว่าความคลาดเคลื่อนของตำแหน่งการตัดจะมีลักษณะการเคลื่อนตำแหน่งไปทุกส่วนในทิศทางเดียวกันโดยแบ่งเป็นความคลาดเคลื่อนของตำแหน่งการตัดแบบเลื่อนซึ่งถูกแสดงด้วยเวกเตอร์ (dx, dy, dz) และความคลาดเคลื่อนของตำแหน่งการตัดแบบหมุนซึ่งถูกแสดงด้วยเวกเตอร์ (γ, β, α) และ 2. การประเมินในขั้นตอนการจับยึดชิ้นงาน ซึ่งเกิดจากแรงและตำแหน่งในการจับยึดของตัวจับยึดชิ้นงาน ในการทำนายความคลาดเคลื่อนของตำแหน่งการตัดในส่วนนี้ผู้วิจัยได้ใช้โปรแกรมทางระเบียบวิธีไฟไนต์เอลิเมนต์ MSC Marc ผลจากการทำนายส่งผลให้ชิ้นงานมีการเสียรูปบริเวณ ณ จุดสัมผัสระหว่างชิ้นงานกับฟิกซ์เจอร์ และทำให้ตำแหน่งการตัดมีความคลาดเคลื่อนในรูปแบบเลื่อนและหมุนเช่นเดียวกับสาเหตุแรกแต่มีปริมาณน้อยกว่า จากการวิเคราะห์ผลที่ได้จากการทำนายลักษณะความคลาดเคลื่อนของตำแหน่งการตัดพื้นผิวด้านบนของชิ้นงานจากทั้งสองขั้นตอนสามารถหาความคลาดเคลื่อนของตำแหน่งการตัดรวมทั้งหมดในขั้นตอนก่อนกระบวนการตัดได้ โดยค่าความคลาดเคลื่อนของตำแหน่งการตัดส่วนใหญ่จะเกิดจากสาเหตุการกำหนดตำแหน่งมากกว่าการจับยึดชิ้นงานสูงถึง 90 เปอร์เซ็นต์ (ทั้งนี้ขึ้นอยู่กับขนาดความไม่แน่นอนของชิ้นงานและแรงในการจับยึดของฟิกซ์เจอร์) นอกจากนี้ผลความคลาดเคลื่อนของตำแหน่งการตัด สามารถนำไปใช้เป็นข้อมูลสำหรับการชดเชยเส้นทางเดินใบมีดหรือนำข้อมูลไปปรับปรุงแก้ไขร่วมกับผลการทำนายความคลาดเคลื่อนของตำแหน่งการตัดในขณะทำการตัด การกำหนดลักษณะการจัดวางของชิ้นงานบนเครื่องจักร การออกแบบฟิกซ์เจอร์ และการกำหนดค่าระยะพิกัดเผื่อที่เหมาะสมของชิ้นงานที่นำมาผลิตเพื่อช่วยลดความผิดพลาดของตำแหน่งชิ้นงานและเพิ่มประสิทธิภาพในการผลิต สำหรับงานวิจัยในอนาคตจะเป็นการเปรียบเทียบผลจากการทดลองวัดความคลาดเคลื่อนที่เกิดขึ้นจริงกับผลความคลาดเคลื่อนที่วิเคราะห์ได้จากการสร้างแบบจำลองการจับยึดชิ้นงานด้วยฟิกซ์เจอร์ที่ได้นำเสนอไปแล้วข้างต้น

5.6 กิตติกรรมประกาศ

ขอขอบคุณ สำนักงานกองทุนสนับสนุนการวิจัย (สกว.) สัญญาเลขที่ MRG4780070 และ มหาวิทยาลัยสงขลานครินทร์ สัญญาเลขที่ ENG530153S ที่ให้การสนับสนุนงานวิจัยในครั้งนี้

5.7 เอกสารอ้างอิง

1. Y. Wang, X. Chen, and N. Gindy, "Surface error decomposition for fixture development," The International Journal of Advanced Manufacturing Technology, vol. 31 pp. 948-956, 2006.
2. S. Chaiprapat, and S. Rujikietgumjorn, "Resultant geometric variation of a fixtured workpiece part I: a simulation," Songklanakarin Journal Science Technology, vol. 28 pp. 181-190, 2006.
3. W. Cai, S.J. Hu, and J.X. Yuan, "Deformable sheet metal fixturing: principles, algorithms, and simulations," Journal of Manufacturing Science and Engineering, vol. 118(3) pp. 318-324, 1996.
4. W. Cai, S.J. Hu, and J.X. Yuan, "A variation method of robust fixture configuration design for 3-D workpieces," Journal of Manufacturing Science and Engineering vol. 119 pp. 593-602, 1997.
5. S. Chaiprapat, and S. Rujikietgumjorn, "Modeling of positional variability of a fixtured workpiece due to locating errors," The International Journal of Advanced Manufacturing Technology, vol. 36 pp. 724-731, 2007.
6. E.C. DeMeter, "Max-min load criteria as a measure of machining fixture performance," Journal of Engineering for Industry, vol. 116 p. 500-507, 1994.
7. E.C. DeMeter, "Min-max load model for optimizing machining fixture performance," Journal of Engineering for Industry, vol. 117(2) pp. 186-193, 1995.
8. E.C. DeMeter, "A model to predict minimum required clamp pre-loads in light of fixture-workpiece compliance," International Journal of Machine Tools & Manufacture, vol. 41 pp. 1031-1054, 2001.
9. J.H. Yeh, and F.W. Liou, "Contact condition modelling for machining fixture setup processes," International Journal of Machine Tools & Manufacture, vol. 39 pp. 787-803, 1999.
10. B. Li, and S.N. Melkote, "Fixture clamping force optimisation and its Impact on workpiece location accuracy," The International Journal of Advanced Manufacturing Technology, vol. 17 pp. 104-113, 2001.

11. S. Satyanarayana, and S.N. Melkote, "Finite element modeling of fixture-workpiece contacts: single contact modeling and experimental verification," *International Journal of Machine Tools & Manufacture*, vol. 44 pp. 903-913, 2004.
12. Foster, L.W., *GEO-METRICS III.*, Addison-Wesley, Massachusetts, 1994, pp. 71-75
13. A.C. Fischer-Cripps, *Introduction to Contact Mechanics*. Second ed., Springer, 2nd Edition, USA, pp. 77-106, 2007.
14. W. Xie, E.C. De Meter, and M.W. Trethewey, "An experimental evaluation of coefficients of static friction of common workpiece-fixture element pairs", *International Journal of Machine Tools & Manufacture*, vol. 40, pp. 467-488, 2000.
15. S. Kalpakjian, "Manufacturing process for engineering materials" Addison-Wesley. 2nd Edition, pp. 205, 1992,
16. E.C. DeMeter, W. Xie, and S.Choudhuri, "A model to predict minimum required clamp pre-load in light of fixture-workpiece compliance.", *International Journal of Machine Tools & Manufacture*, vol. 41, pp. 1031-1054, 2001.
17. W. Wattanutchariya, 2008,"Workholding device",[online]. Available at http://www.ie.cmu.edu/wattnutchariya/tool_engineering/workholding-device.pps, Accessed 20 December 2008
18. Y. Kang, Y. Rong, and J.C. Yang,, "Computer-aided fixture design verification part 2. tolerance analysis", *The International Journal of Advanced Manufacturing Technology*, Vol. 21, pp. 836-841, 2003.

CHAPTER 6

CONCLUSION

When a workpiece is fixtured for a machining or inspection operation, an accuracy of the operation is mainly determined by an efficiency of fixturing method. Variability in manufactured workpiece is hardly avoidable. When such variability is found at contact areas between the workpiece and the fixture, errors in locating are expected. The errors will affect quality of features to be produced. In this research development of an algorithm to determine variant final locations of a displaced workpiece given normally distributed errors at contact points is described. Resultant geometric variation of workpiece location reveals interesting information which is beneficial in tolerance planning.

Once variational pattern of a specific location on the workpiece is known, it is challenging to further explore how this variability is distributed over the workpiece surface. A mathematical model used to predict geometric variation of a resultant-machined surface when a tolerance of a datum feature is given is then developed. A model for a 2-D scheme was initially established that subsequently expanded to a more realistic case in a 3-D scenario. Results from the model show that when variability of a datum feature is allowed, a paraboloid-shaped variational function of a machined surface is detected. This information is crucial that it helps a designer to position fixturing elements in such a way that displacement of a target position will be minimized.

Beside locating error, workpiece displacement before machining can be resulted from clamping error. A finite element method and the Hertz theory are used to explain the deformation induced by clamping forces. To derive the total displacement, global displacement from locating error and local displacement from clamping error are combined. At the end of this research, methods to alleviate the effect of workpiece displacement are suggested. The best solution to the workpiece displacement is to relocate fixturing elements that the target position will be least affected. However, when it is inevitable to eliminate the displacement, toolpath compensation is worth considering. Calculating the toolpath will be a little more complicate as it will take into account the displacement pattern. Whenever toolpath compensation is impossible, a designer should be aware of this deviation and resultant inaccuracy that may occur. More reasonable tolerancing will then be needed to accommodate the variation.

ผลลัพธ์ของโครงการวิจัย

ผลงานที่ตีพิมพ์ในวารสารวิชาการนานาชาติ

S. Chaiprapat, and S. Rujikietgumjorn, "Modeling of positional variability of a fixtured workpiece due to locating errors," The International Journal of Advanced Manufacturing Technology, vol. 36 pp. 724-731, 2007.

การนำผลงานวิจัยไปใช้ประโยชน์

การสร้างนักวิจัยใหม่ ผู้วิจัยได้นำผลงานจากงานวิจัยนี้ไปต่อยอดเพื่อศึกษาผลกระทบของการเบี่ยงเบนของตำแหน่งชิ้นงาน (workpiece displacement) ที่มีต่อความแม่นยำและการกำหนดระยะพิงัดเผื่อ (tolerance establishment) ของการตัด พร้อมทั้งนำเสนอวิธีการปรับปรุงแก้ไขเพื่อให้กระบวนการตัดมีความแม่นยำมากขึ้น โดยได้กำหนดเป็นหัวข้อวิทยานิพนธ์แก่นักศึกษาระดับปริญญาโท (นส.กุลภัทร์ ทองแก้ว)

อื่นๆ

ผลงานตีพิมพ์ในวารสารวิชาการในประเทศ

S. Chaiprapat, and S. Rujikietgumjorn, "Resultant geometric variation of a fixtured workpiece part I: a simulation," Songklanakarin Journal Science Technology, vol. 28 pp. 181-190, 2006.

S. Chaiprapat, and S. Rujikietgumjorn, "A method to analyze effects of surface variational model on positional geometric variability" Songklanakarin Journal Science Technology, vol. 28 pp. 169-179, 2006.

กุลภัทร์ ทองแก้ว สุภาพรรณ ไชยประพัทธ์ และ เจริญยุทธ เดชวายุกุล, "แบบจำลองประเมินความคลาดเคลื่อนของตำแหน่งการตัดบนชิ้นงานที่ถูกจับยึดด้วยตัวจับชิ้นงานก่อนกระบวนการตัด" วารสารวิชาการพระจอมเกล้าพระนครเหนือ, ปีที่ 21 ฉบับที่ 1 เดือน มกราคม-เมษายน 2554

ภาคผนวก ก.

MODELING OF POSITIONAL VARIABILITY OF A FIXTURED WORKPIECE DUE TO
LOCATING ERRORS

Modeling of positional variability of a fixtured workpiece due to locating errors

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Abstract In manufacturing practice, fixtures play an important role in locating and restraining a workpiece. It is clear that locating error is one of the factors that significantly contribute to workpiece quality in the early stage of machining. In this study, we develop a mathematical model that is able to predict geometric variation of a resultant-machined surface when the tolerance of a datum feature is given. A model for a 2-D scheme was initially established that subsequently expanded to a more realistic case in a 3-D scenario. Results from the model show that when variability of a datum feature is allowed, a paraboloid-shaped variational function of a machined surface is detected.

Keywords Locating error · Feature variability · Geometric variation · Fixture

1 Introduction

A datum is a theoretically exact point, axis, or plane specified in a drawing that functions as a reference for dimensioning or locating features on a workpiece. However, in actuality, a feature is indeed located with respect to “a

datum feature”, the datum’s actual counterpart. Various factors such as machine vibration, tool-path errors, tool deflection, etc., are some of the causes differentiating a datum feature from a datum. Realizing undesirable effects of such inevitable causes, a designer sometimes allows variation of a datum by imposing an appropriate tolerance. When the workpiece is fixtured for machining operation, a datum feature is used to locate a feature position, and errors at the datum feature will be passed on to a machined feature through a locating method. Throughout this article, errors in locating which result from imperfect datum features will be called “locating errors”. A number of past published works related to effects of locating errors on part quality can be found. Rong and Bai [1] analyzed a dependent relationship of operational dimensions to estimate machining errors in terms of linear and angular dimensions of a workpiece. Cai et al. [2] proposed a method to conduct a robust fixture design to minimize workpiece positional errors as a result of workpiece surface and fixture setup errors. There are also algorithms to predict a deviation of a prismatic workpiece located by 3-2-1 fixturing method [3] and a cylindrical workpiece in a V-block [4]. Djurdjanovic and Ni [5] developed procedures for determining the influence of errors in fixtures, locating datum features and measurement datum features on dimensional errors in machining. These studies were conducted when a static case was assumed. More realistic variational models used to explain distributions of errors in locating have also been discussed. Bhat and De Meter [6] carried out a simulation analysis to evaluate performance of three different datum-establishment methods. A probabilistic fixturing model was also developed and resultant feature errors were determined by Sangnui and Peters [7]. To avoid mathematical complication due to such respective locating scheme, Chaiprapat and Rujikietgumjorn [8] proposed a system of equations that

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could be simultaneously solved for workpiece positional errors. In [9], feature errors resulting from different types of distributions used to explain irregularities of a locating surface were compared. Carlson [10] evaluated errors propagated from perturbation in locators by using a quadratic sensitivity equation that could better handle larger errors, in comparison to a linear approximation.

Effects of locating errors as discussed by the above studies evolve in the areas of workpiece displacement and its consequences on machining errors. For machined parts to be acceptable, such errors must be bounded within a range imposed by a designer called 'tolerance'. Tolerance analysis is carried out extensively in fixturing. Kang et al. [11] proposed two different approaches for fixture tolerance analysis. From the given locator displacement and deformation, the deviation of affected machined surfaces can be predicted. On the contrary, with an allowable error range of the surfaces defined, tolerance of the locators can also be calculated. Establishment of relationship between machined feature geometric error and locator tolerance was also studied by Choudhuri and DeMeter [12]. Whenever an initial fixturing layout does not lead to acceptable errors, rearranging the layout should be considered. Optimal fixturing layouts were suggested through a number of approaches [13–17]. However, none of them explicitly presents how errors associative with the proposed layout can be quantified.

In this study, a pattern of variability over a feature placement surface is investigated under the influence of locating probabilistic errors. Since it is sometimes technically or economically infeasible to eliminate errors from locating, understanding how a workpiece through a datum feature interacts with locators is crucial because it helps design and manufacturing engineers find ways to create more accurate features.

2 Analysis

2.1 Locating method

There are typically two different coordinate systems involved in machining operation. One is a machine reference system, which is used to position a cutter. However, after machining, a feature will be inspected with respect to another system, which is constructed from workpiece locating surfaces. As a result, a coincident or certain relationship between both systems is a requirement for the accuracy of a machined feature. It is realized that the errors at the contact areas between workpiece datum feature and fixture locators will often cause misalignment between both systems, and thus inducing errors in feature location and orientation. The workpiece in this study is assumed to

be non-deformable, prismatic, and restrained by a 3-2-1 fixturing method. Figure 1 shows determination of an error at a contact location. The error will be positive when the contact point is located outside a nominal workpiece boundary, and negative otherwise.

In [7] and [8], resultant geometric variation of a point representing a feature location was calculated. However, it is always found that most features fabricated on the manufacturing shop floor, i.e., slot, pocket, shoulder cut, and milled face, cannot be represented by a single location. For such features, the method was applicable only when uncertainty in workpiece displacement was consistent across a feature placement area. When this assumption does not hold, a variational profile of the placement area should be investigated. The following sections will demonstrate how the displacement is distributed over the workpiece surface and how this irregular displacement affects tolerance establishment of a feature.

2.2 Displacement profile: Two-dimensional case

A workpiece datum surface in this case is supported by two locating pins. The displaced workpiece, resulting from errors at contact points, is represented by a dotted line as shown in Fig. 2. Note that the errors of the datum surface can be represented by errors of locators at given contact points. When the errors (e_1 and e_2) are small compared to the distance between the locating pins (L), the displacement $d(x)$ can be obtained from

$$d(x) = e_1 + \frac{x}{L}(e_2 - e_1) \text{ where } d(0) = e_1 \text{ and } d(L) = e_2 \quad (1)$$

and the workpiece position at x , $P(x)$, will be translated to $P_e(x) = P(x) + d(x) \bullet j$ where $P_e(x)$ is $P(x)$ after displacement (2)

The displacement function in Eq. (1) expresses a linear relationship with ' x '. A magnitude of the greatest displacement along the datum line is equal to the larger error of the two. Since existing uncertainty in manufacturing processes

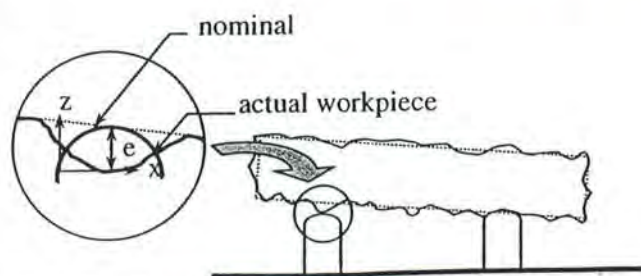
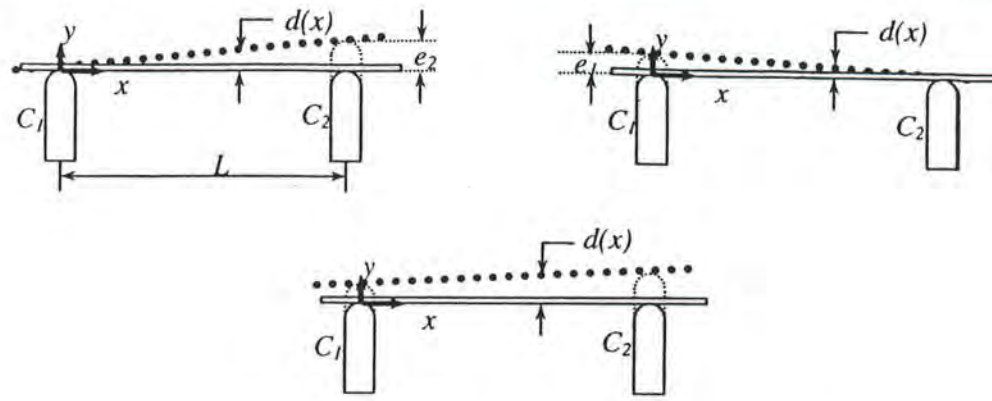


Fig. 1 Error determination

Fig. 2 Workpiece displacement in a 2-D case



makes an assumption of static contact errors invalid in most cases, a probabilistic error model is chosen in this study. Because of the number of parameters involved, however, an actual distribution of the errors is almost impracticable to determine. Once derived, the calculation may be too complicated and give inapplicable results. When true characteristics of the surface are not revealed, normally distributed errors will be used as an example.

$$e_i \sim N(\mu_i, \sigma_i^2) \quad (3)$$

From Eqs. (2) and (3), statistically, a variability of $P_e(x)$ is equal to that of $d(x)$, which throughout this article will be represented by σ_x^2 .

$$\sigma_x^2 = \left(1 - \frac{x}{L}\right)^2 \sigma_1^2 + \left(\frac{x}{L}\right)^2 \sigma_2^2; \text{ when } e_1 \text{ and } e_2 \text{ are independent} \quad (4)$$

Equation (4) shows the variability in the position of the point of interest varies nonlinearly as a function of variability at both locators and the distance from such

point to the locators (also see Fig. 3). By differentiating Eq. (4), the minimum variability can be found at

$$x_{\min} = \frac{L\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \quad (5)$$

When $\sigma_1^2 = \sigma_2^2$, the minimum is at the mid point between both locators. Theoretically, the feature should be placed where the minimum variability is located. However, when design constraints do not allow this, the locators may have to be rearranged as necessary to obtain minimal feature variability.

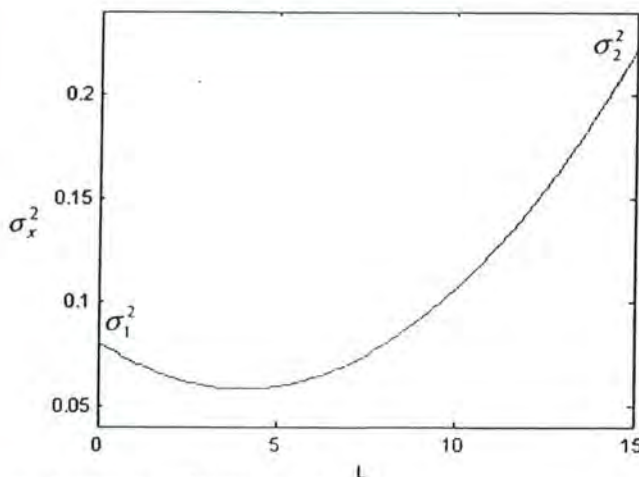
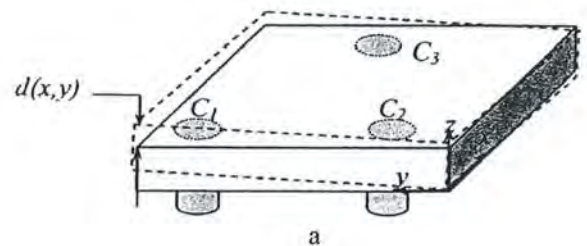


Fig. 3 Change in σ_x^2 over the distance L

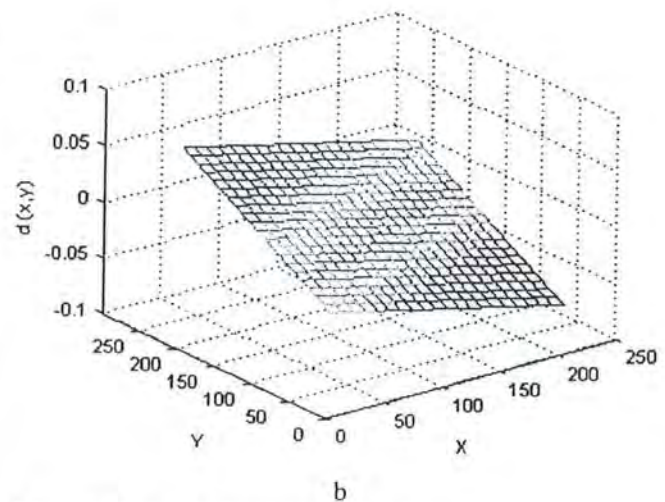
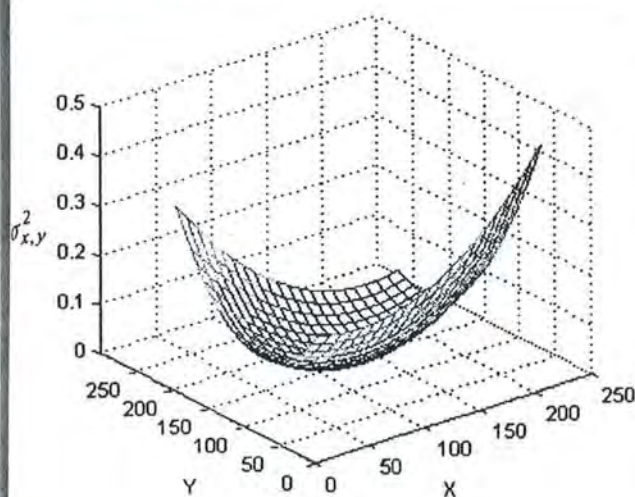
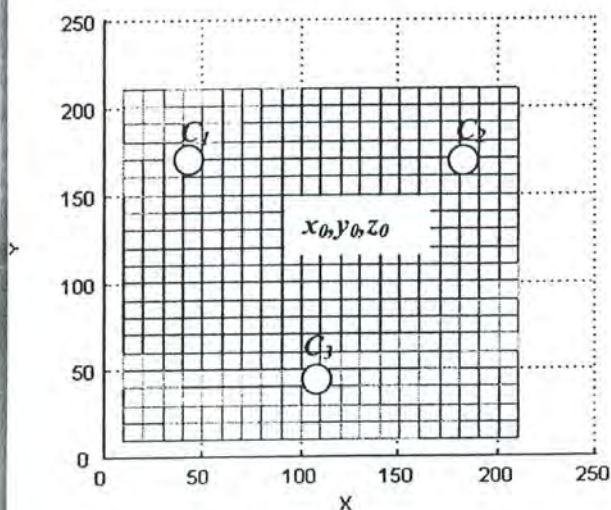


Fig. 4 Workpiece displacement in a 3-D case



a



b

Fig. 5 A variational profile in a 3-D case

2.3 Displacement profile: Three-dimensional case

Using a 3-2-1 fixturing method, the workpiece's surfaces must contact with 3, 2, and 1 locator of the primary, secondary, and tertiary datum planes of the fixture, respectively. Errors at the contact points in the primary

Fig. 6 Determination of variation at "p" on the workpiece placement face

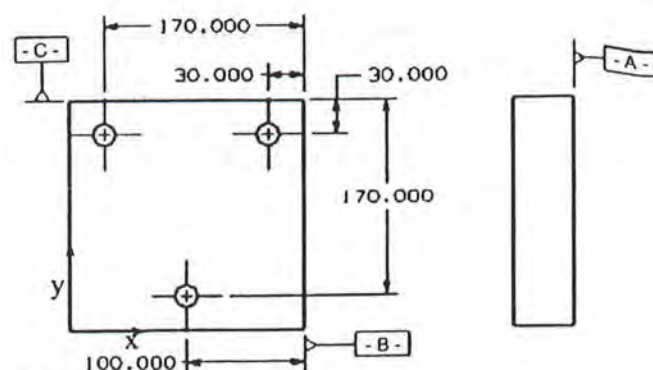
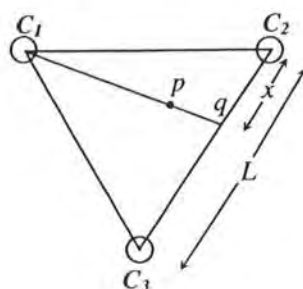


Fig. 7 A sample workpiece

datum plane will rotate and move the workpiece away from its nominal position (Fig. 4), leading to displacement in all directions (x,y,z) with that in z direction shown to be the most prominent. Errors of the secondary datum plane will rotate the workpiece around the normal vector of the primary datum plane. Finally, the errors of the last plane will only affect the translational displacement of the workpiece while maintaining the same orientation [8]. From such analysis, the errors of the secondary and tertiary datum planes play a little role in determining the accuracy of features like shoulder cut and milled face, when their placement plane parallel to the primary datum feature as illustrated in section 2.5.

Given surface errors at the contact points, one can determine the displacement and obtain the actual workpiece

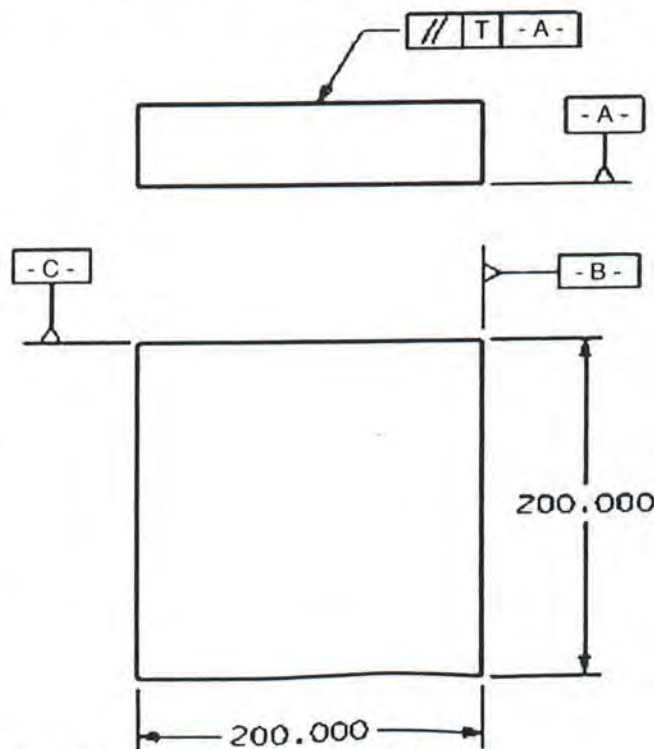


Fig. 8 Geometric description of the workpiece

Table 1 Geometric variation (*GV*) of a faced feature when the datum 'A' is tolerated by 1 and 2 mm

Datum tolerance	Data set					Mean
	1	2	3	4	5	
1 mm	1.6288	1.5715	1.5704	1.6166	1.5863	1.5947
2 mm	3.3406	3.4409	3.3331	3.4551	3.3781	3.3896

position and orientation by using the methodology developed in [8]. Similar to the displacement in a 2-D case, the displacement of 'P' in a 3-D environment can also be explained by

$$P_e(x, y) = P(x, y) + d(x, y) \cdot k \quad (6)$$

2.4 Datum feature variability

When the errors are small enough and normally distributed, $\sigma_{x,y}^2$ is preliminarily evaluated by a Monte Carlo simulation and plotted in a 3-D space. It can be seen as a curved surface resembling an elliptic paraboloid surface shown in Fig. 5a and approximated by the paraboloid equation explained below.

$$\sigma_{x,y}^2 = \left\{ \frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} \right\} c + z_0 \quad (7)$$

where $[x_0, y_0, z_0]$ is a center of the surface.

The paraboloid vertex, $[x_0, y_0, z_0]$, is pointing where the lowest point, in the other words, the minimum variability of the surface is located. To derive $[x_0, y_0, z_0]$, an optimization tool in Matlab may be employed. However, how precisely the algorithm can locate the true vertex depends on how well an initial guess is established. This geometry resembles an ellipse when observed from the top and when sectioned vertically will be seen as a parabola. Variation on the placement face therefore can also be approximated by

using the scheme developed for the 2-D variational profile as shown in Fig. 6.

According to Fig. 7, to obtain the variation at "p" using Eq. (4), the variations at the end of the line on which "p" is lying must be quantified. Where the variation at C_1 or σ_1^2 is given, σ_q^2 can be calculated from the following parabolic equation.

$$\sigma_q^2 = \left(1 - \frac{x}{L}\right)^2 \sigma_2^2 + \left(\frac{x}{L}\right)^2 \sigma_3^2 \quad (8)$$

Once σ_q^2 is known, σ_p^2 can also be evaluated by the same equation.

It is not surprising that when distributions of errors at the contact points are the same, the vertex of the paraboloid is positioned equally apart from all three locators (Fig. 5b). When it is inevitable to position the locators by using surfaces from different setup, extra care would have to be taken. Especially in precision machining when tight tolerancing becomes an issue it is recommended for a designer to seriously take into account an irregular pattern of the variability across the placement surface.

2.5 Feature geometric variation

If the displacement is certain over the workpiece surface, the tool path can be designed accordingly to compensate the errors in order to obtain more accurate features. Likewise, if the variability is constant, a designer could impose a tolerance to the feature effortlessly. However, it is shown in the previous section that the displacement is changed linearly over the placement surface according to the errors at the contacts. This circumstance leads to a parabolic-shaped variability pattern. What a process designer can possibly do in this case is to design manufacturing procedures conforming to such imperfect incoming workpiece while expecting the best from it. Before a tolerance can be imposed to a feature, a designer has to

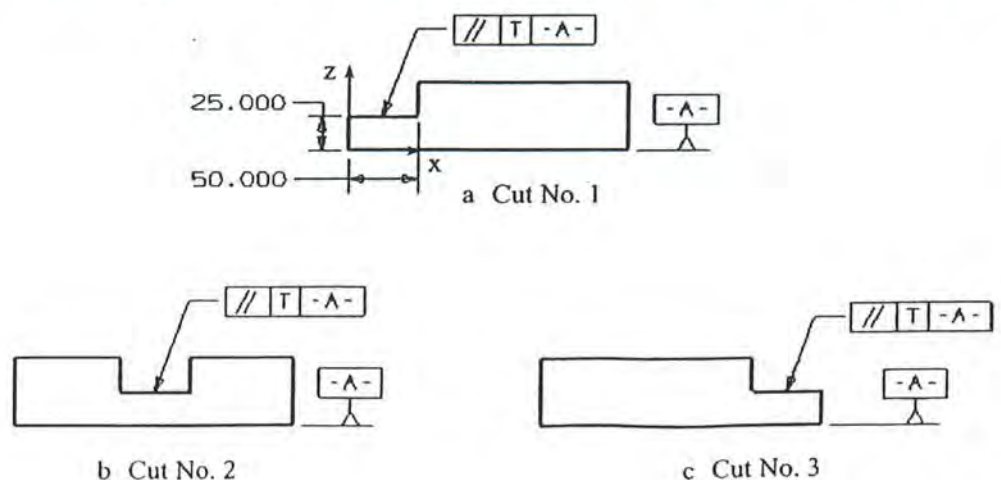
Fig. 9 Slots used in the study

Table 2 Geometric variation (*GV*) of slots placed at different locations along X-axis

Datum tolerance	Geometric variation (mm)			Difference in variation between cut no. 1 and 3
	Cut no. 1 (x=0–50 mm)	Cut no. 2 (x=75–125 mm)	Cut no. 3 (x=150–200 mm)	
1 mm	1.5238	1.2304	1.5947	0.0709
2 mm	3.3386	2.5977	3.3705	0.0319

be aware of how uncertainty in the manufacturing process would contribute to variability of a feature. The degree of differentiation in geometry from part to part in the same production lot is called “geometric variation (*GV*)”. Where geometric tolerance is a range in which a feature is allowed to be varied, geometric variation, on the other hand, reflects its actual variation. Therefore, geometric variation must be kept smaller than the corresponding tolerance. In this study, only geometric variation resulting from locating errors is discussed. The following is an example of workpiece’s locating details. Datum features ‘A’, ‘B’, and ‘C’ are as labeled in Fig. 7. Locators on the datum ‘A’ or the primary datum feature are positioned as indicated.

2.5.1 Geometric variation of facing

The top surface of the workpiece shown in Fig. 8 is to be faced and geometrically constrained according to the tolerance frame indicated below. From the drawing, the top surface must be parallel to the datum ‘A’ and is allowed to be varied within the range of ‘T’.

At any given point (x,y) on the placement plane, it is assumed that feature variability following the central limit theorem exhibits a normal distribution pattern. A geometric variation (*GV*) herein is defined by a range containing six times of standard deviation (6σ) of the feature variability and that can be approximated by

$$GV = 6 * \sqrt{\sigma_{x,y}^2} \quad (9)$$

Since *GV* of any surface is not consistent over the entire area of the cut, only the maximum value of *GV* of each simulation trial is presented in Table 1. According to ANSI Y14.5M, a parallel tolerance constraining the surface as shown in Fig. 8 is a distance between two theoretical planes

parallel to the datum in which the cut surface is allowed to be varied. Therefore, to ensure that the machined surface is kept well within tolerance, *GV* should not exceed the range of ‘T’.

2.5.2 Geometric variation of slotting

Inconsistent variation distributed across the surface will have a crucial role in tolerancing a feature partially cut on the surface. In Fig. 5, variation at the surface perimeter is significantly higher and lower at the locations approaching the center. To minimize surface variation resulting from variability at contact locations, a feature should be placed around the center of a supporting triangle (a triangle formed by three locators shown in Figs. 5 and 6). However, when the feature is tightly toleranced and impossible to be located at the optimum position for example in the case of side slotting, it is advised to look more closely at a magnitude of the variation over the target area. In this study, *GV* of three slots placed at different locations shown in Fig. 9 are calculated and presented in Tables 2 and 3. The cut of 50 mm in width and 25 mm in depth is placed at the left, the middle, and the right of the placement surface.

The above cuts lie longitudinally along X-axis. It appears that geometric variation is declining when approaching the center of the supporting triangle and getting larger in the area farther away. The cuts on both sides of the surface (y=0–50 mm and y=150–200 mm) have the similar degree of geometric variation.

As the cuts are relocated to be parallel to the Y-axis, it can be seen that the pattern of geometric variation is slightly different as shown in Fig. 10. The variation near the supporting triangle’s center is lowest and rising to the highest around the perimeter. Therefore, two corners of the surface (cut no. 1) have higher variability than the other two (cut no.

Table 3 Geometric variation (*GV*) of slots placed at different locations along Y-axis

Datum tolerance	Geometric variation (mm)			Difference in variation between cut no. 1 and 3
	Cut no. 1 (y=0–50 mm)	Cut no. 2 (y=75–125 mm)	Cut no. 3 (y=150–200 mm)	
1 mm	1.5947	1.2232	1.3289	0.2658
2 mm	3.3896	2.6287	2.8243	0.5653

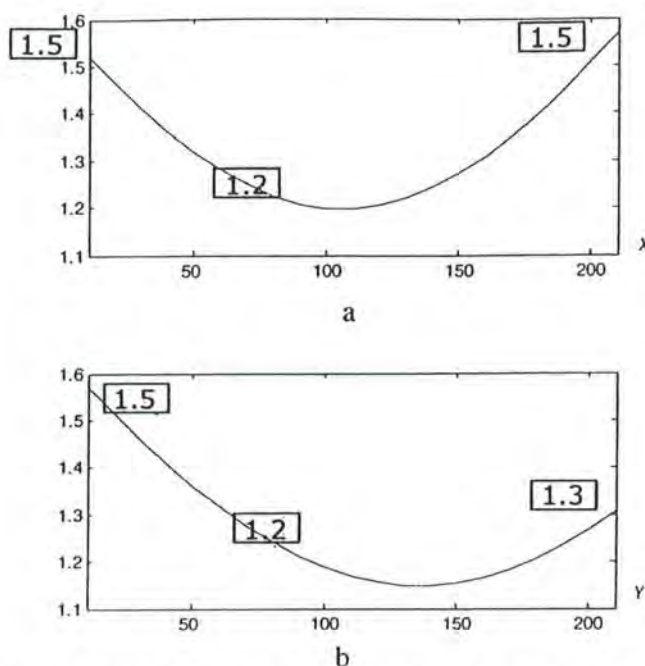


Fig. 10 Variation at different position along X-axis (a) and Y-axis (b)

3). Examining the locator layout, the greatest variability can be spotted at the corners far from any locators, which in Fig. 8 are the corners along the X-axis. It can be explained by using Eqs. (7) and (8) that variability of an arbitrary point is a function of a squared distance from such point to the surface vertex or the center of the supporting triangle. This observation is in agreement with [13], which stated that the errors of the feature decrease in the central region but increase in the other region.

When it is impossible to place the feature at the center, for example in the case of side slotting, positioning it along the edge closer to the center will help keeping the feature variation minimal. The geometric variations shown in the tables indicate how much the feature varies with respect to datum features; therefore, tolerances applied to the feature should not be less than these geometric variations. Note that this variation results from locating errors only. Although only parallel tolerancing is described as the example, the proposed approach can be as well applied to other types of tolerances.

3 Conclusions

When the workpiece is fixtured for a facial machining, conventionally a tolerance of the resultant surface is imposed, assuming that errors are consistent across the entire surface. However, results from this study indicated that variability is found greatest at the boundary of the surface and lowest at the equi-distance from all supporting locators. Although the difference in geometric variation

over the entire placement surface seems minimal, in the application of precision machining such as injection mold machining this inconsistency is crucial. When tight tolerance is required, a feature should not be positioned far away from the centroid of the supporting triangle. Larger variability is expected with a feature located off the optimized position. This study has shown that features located on the same placement surface are not necessarily tolerated by the same value. Before a tolerance can be imposed to any feature efficiently, a design engineer should be able to comprehend and preferably quantify potential feature variability. Features with too tight tolerance are hard (or too costly) to produce. On the other hand, wide tolerance opens up chances of assembly or functional difficulties. As a design engineer is the one who is responsible for imposing a tolerancing system, he or she has to ensure that the tolerated feature will be economically producible and maintain its functional use at the same time. Balancing the mechanics of tolerancing therefore requires knowledge of how the workpiece geometry would vary under a certain manufacturing environment.

Nomenclature

j, k	unit vectors $[0 \ 1 \ 0]$ and $[0 \ 0 \ 1]$, respectively
e_i	errors at contact point i
$d(x)$	workpiece displacement at x
L	distance between locators
C_i	locator i
$P(x)$	workpiece position at distance x from C_i
$P_e(x)$	displaced workpiece position at distance x from C_i
σ_i^2	variability of errors at contact point i
μ_i	mean of errors at contact point i
GV	geometric variation at a feature

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References

1. Rong Y, Bai Y (1996) Machining accuracy analysis for computer-aided fixture design verification. *J Manuf Sci Eng* 118:289–300
2. Cai W, Hu SJ et al (1997) A variational method of robust fixture configuration design for 3-D workpieces. *J Manuf Sci Eng* 119:593–602
3. Salisbury EJ, Peters FE (1998) The impact of surface errors on fixtured workpiece location and orientation. *NAMRC Trans* 26:323–328
4. Sangnui S, Peters FE (2001) The impact of surface errors on the location and orientation of a cylindrical workpiece in a fixture. *J Manuf Sci Eng* 123:325–330
5. Djurdjanovic D, Ni J (2003) Dimensional errors of fixtures, locating and measurement datum features in the stream of

- variation modeling in machining. *J Manuf Sci Eng Trans ASME* 125(4):716–730
6. Bhat V, DeMeter EC (2000) An analysis of the effect of datum-establishment methods on the geometric errors of machined features. *Int J Mach Tools Manuf* 40:1951–1975
 7. Sangnui S, Peters FE (2002) Prediction of fixtured workpiece location when there are variable surface errors at fixturing points. *Proceedings of the 4th Asia-Pacific conference on industrial engineering and management systems*, Taipei, Taiwan
 8. Chaiprapat SS, Rujikietgumjorn S (2006) Resultant geometric variation of a fixtured workpiece: a simulation. *Songklanakarin J Sci Technol* 31:182–190
 9. Chaiprapat SS, Rujikietgumjorn S (2006) A method to analyze effects of surface variational model on positional geometric variability. *Songklanakarin J Sci Technol* 31:170–179
 10. Carlson JS (2001) Quadratic sensitivity analysis of fixtures and locating schemes for rigid parts. *J Manuf Sci Eng* 123:462–472
 11. Kang Y, Rong Y et al (2003) Computer-aided fixture design verification. Part 2. Tolerance analysis. *Int J Adv Manuf Technol* 21:836–841
 12. Choudhuri SA, DeMeter EC (1999) Tolerance analysis of machining fixture locators. *J Manuf Sci Eng* 121:273–281
 13. Wang MY (2002) Tolerance analysis for fixture layout design. *Assem Autom* 22(2):153–162
 14. DeMeter EC (1995) Min-max load model for optimizing machining fixture performance. *J Eng Ind* 117:186–193
 15. Li B, Melkote SN (2001) Optimal fixture design accounting for the effect of workpiece dynamics. *Int J Adv Manuf Technol* 18:701–707
 16. DeMeter EC (1998) Fast support layout optimization. *Int J Mach Tools Manuf* 38:1221–1239
 17. Asada H, By AB (1985) Kinematic analysis of workpart fixturing for flexible assembly with automatically reconfigurable fixtures. *IEEE Trans Robot Autom* RA-1(2):86–94



ภาคผนวก ข.

RESULTANT GEOMETRIC VARIATION OF A FIXTURED WORKPIECE PART I: A SIMULATION

Resultant geometric variation of a fixtured workpiece Part I: a simulation

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Abstract

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Resultant geometric variation of a fixtured workpiece Part I: a simulation

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When a workpiece is fixtured for a machining or inspection operation, the accuracy of an operation is mainly determined by the efficiency of the fixturing method. Variability in manufactured workpiece is hardly inevitable. When such variability is found at contact areas between the workpiece and the fixture, errors in location are expected. The errors will affect quality of features to be produced. This paper developed an algorithm to determine variant final locations of a displaced workpiece given normally distributed errors at contact points. Resultant geometric variation of workpiece location reveals interesting information which is beneficial in tolerance planning.

Key words : fixture, datum, workpiece displacement, Newton-Raphson

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บทคัดย่อ

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รูปแบบความไม่แน่นอนของชิ้นงานที่ถูกจับยึดด้วยฟิกซ์เจอร์
ส่วนที่ 1 แบบจำลองทางคณิตศาสตร์

ว. สงขลานครินทร์ วทท. 2549 28(1) : 181-190

ปกติแล้วในกระบวนการผลิต ชิ้นงานจะถูกจับยึดบนเครื่องจักรด้วยตัวจับชิ้นงานหรือฟิกซ์เจอร์ (Fixture) ฟิกซ์เจอร์มีความสำคัญเป็นอย่างมากในการควบคุมคุณภาพของผลิตภัณฑ์ ผลิตภัณฑ์ที่ผ่านกระบวนการผลิตจะมีความเที่ยงตรงแม่นยำถูกต้องตามลักษณะที่กำหนดไว้เพียงใดขึ้นอยู่กับประสิทธิภาพในการจับยึดชิ้นงานของฟิกซ์เจอร์ อย่างไรก็ตาม ในกระบวนการผลิตมักจะพบว่ามีความไม่แน่นอนเกิดขึ้นเสมอและหลีกเลี่ยงได้ยาก ซึ่งอาจเกิดจากความผิดพลาดของผู้ปฏิบัติงาน การเสื่อมสภาพของเครื่องจักร อุปกรณ์ต่างๆ เป็นต้น ทำให้ผลิตชิ้นงานที่มีขนาดและรูปร่างที่ผิดเพี้ยนไปจากที่กำหนดไว้ เมื่อนำชิ้นงานนั้นมาจับยึด ความผิดพลาดที่จุดสัมผัสระหว่างฟิกซ์เจอร์กับชิ้นงานจะถูกส่งผ่านไปยังฟีทเจอร์ (Feature) ที่จะทำการผลิต งานวิจัยนี้มีจุดประสงค์เพื่อที่จะพัฒนาระบบวิธีที่ใช้ในการคำนวณหาความไม่แน่นอนของตำแหน่งชิ้นงานที่ถูกจับยึด ภายใต้สมมติฐานที่ว่าชิ้นงานมีความผิดพลาด ณ จุดสัมผัสโดยมีลักษณะการกระจายเป็นแบบปกติ ผลจากการทดสอบแบบจำลองทำให้ทราบว่า ความไม่แน่นอนของฟีทเจอร์ที่จะทำการผลิตมีลักษณะเป็นรูปแบบที่น่าสนใจ และจากข้อมูลนี้สามารถนำไปใช้ในการกำหนดระยะพิสัยเพื่อให้มีประสิทธิภาพในกระบวนการผลิตต่อไป

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Workpiece displacement because of locating or fixturing errors is a threat in the early stage of precision machining. It has been discussed widely either in the workpiece deviation determination or fixture component placement optimization. Researchers working in fixturing area have developed algorithms providing complete constraint of a workpiece (King and Hutter, 1993; Trappey and Matrubhutam, 1993; Brost and Goldberg, 1996; Dai, Nee *et al.*, 1997; Jeng and Gill, 1997; De Meter, 1998; Wu, Rong *et al.*, 1998), determining sufficient support, performance of the fixture, and accessibility (Pham and Lazaro, 1990; Fuh, Chang *et al.*, 1993; De Meter, 1994; De Meter, 1994; De Meter, 1995; Ong and Nee, 1998). Rong and Bai (1996) analyzed a dependent relationship of operational dimensions to estimate machining errors in terms of linear and angular dimensions of a workpiece. Cai *et al.* (1997) proposed a method to conduct robust fixture design to minimize workpiece positional errors as a result of workpiece surface and fixture set-up errors. A method for modeling and analyzing the impact of a locator

tolerance scheme on geometric errors of machined features was developed by Choudhuri and De Meter (1999). There were also algorithms to predict a deviation of a prismatic workpiece located by 3-2-1 fixturing method (Salisbury and Peters, 1998) and a cylindrical workpiece in a v-block (Sanguin and Peters, 2001). The results from Salisbury and Peters (1998) and Sanguin and Peters (2001) indicated that the largest deviation did not necessarily come from the largest errors at contact points. Djurdjanovic and Ni (2003) developed procedures for determining the influence of errors in fixtures, locating datum features and measurement datum features on dimensional errors in machining. These studies were conducted when a static case is assumed. Although variability existing in a production line is playing an obstructive role in gaining an efficiently precise control over manufacturing operations, only a few researchers employed a variational model to evaluate fixture performance. A model in the absence of workpiece variability consideration would not be applicable and functional in most cases. Bhat and De Meter

(2000) carried out a simulation analysis to evaluate performance of three different datum-establishment methods: 3-2-1, sequential least-squares (SQLS) and simultaneous least-squares (SMLS). The authors assumed variant errors of datum features. However, only the magnitude of relative positional workpiece displacement was presented. Variation in workpiece position was yet to be analyzed. A probabilistic fixturing model was also developed by Sangnui (2002) and geometric variation of a feature was determined. Because the analytical model was established according to a sequential locating scheme of 3-2-1 fixturing method, calculation was somewhat tedious.

In this paper, an approach to assess prismatic workpiece displacement resulting from misalignment in locating is introduced. When errors of a datum feature are assumed probabilistic, the resultant workpiece geometric variation reveals a significant systematic pattern. Unlike Sangnui (2002), a step-wise analysis is considered unnecessary and the mathematical formulation is simplified. By implementing the method developed in this study, a designer will be able to determine variation of subsequent processes as affected by fixturing errors.

Methodology

1. Fixturing Model

In a manufacturing process, a fixture is used to locate and restrain a workpiece from movements during machining or inspection. A reference to locate the workpiece at its nominal position and orientation is called a datum, which can be established from fixture or machine components as shown in Figure 1. A realistic part of the work-

piece used to map with a datum is called 'a datum feature'. When the workpiece is fixtured for an operation, drilling a hole for example, exact mapping between a datum and a datum feature is desirable. However, in case that a workpiece surface is used as a datum feature, any surface deviations from a perfect topography would lead to workpiece displacement, and therefore inaccuracy of final products.

The impact of erratic fixturing on a feature accuracy is illustrated in Figure 2. A perfect workpiece is shown located at its theoretical location as opposed to an actual workpiece. It is assumed that the machine coordinate system, XYZ , is positioned coincidentally with that of the nominal workpiece. The workpiece is to be drilled at a location defined by $\bar{L}$ with respect to the machine coordinate system. When the workpiece is displaced from its theoretical position, the hole will be moved to $\bar{L}'$ with respect to the workpiece, XYZ' . Thus, discrepancy in location of the hole occurs.

If surface variation exists at contact points between a workpiece and a fixture, deviation of the workpiece from its nominal location is expected. A typical 3-2-1 fixturing method is composed of six locators forming three mutually perpendicular datum planes. The primary datum plane as shown in Figure 3 is constructed from the first three locators (no. 1, 2, 3). Perpendicular to the primary datum plane, the secondary datum plane is established from the contacts of the locator no. 4 and 5. Finally, the tertiary datum plane is the plane perpendicular to the preceding ones in which the last contact point (no. 6) lies. Note that the workpiece surfaces in contact with the datum planes are called workpiece datum features. Given a distribution of surface errors at the contact locations,



Figure 1. The mapping between a datum plane and a workpiece datum feature.

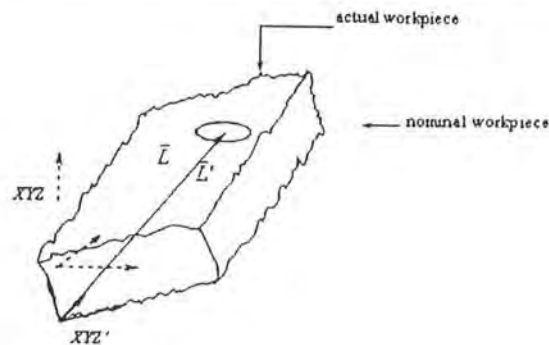


Figure 2. The workpiece and feature displacement.

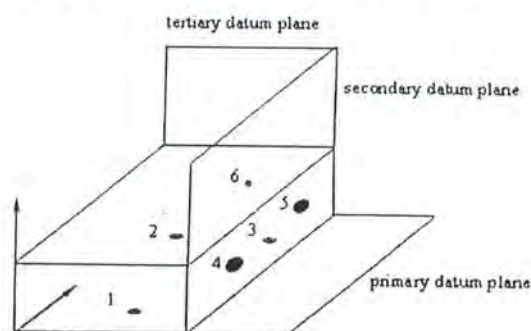


Figure 3. Datum planes in 3-2-1 fixturing method.

statistics allows us to determine variability of final position of the fixtured workpiece.

2. Assumptions

The following assumptions regarding the fixture and the workpiece were used.

1. A workpiece is considered a rigid body. Deformation of the workpiece during transformation is not allowed.

2. The workpiece surface errors are measured perpendicularly to the perfect form of datum features at the contact points. The determination of the surface error sign is shown in Figure 4.

3. The errors are assumed to be normally distributed with mean μ and variance σ^2 , or $N(\mu, \sigma^2)$.

3. Analysis

When an imperfect workpiece is fixtured for an operation, variation at the contact point between

the workpiece and the fixture would be imparted to a feature to be produced. The impact of such errors on the feature can be analyzed through workpiece deviation from its nominal location. The deviation is explained by means of geometric transformation: translation and rotation. Workpiece location and orientation after located in a fixture will be mathematically analyzed. Workpiece location in the fixture can be represented by an arbitrary point, which could be a location of a feature of interest. Workpiece orientation in this paper is defined by vectors normal to the primary and secondary datum features. A normal vector to the primary datum feature $\bar{n}_p$ is derived from

$$\bar{n}_p = \bar{P}_{31} \times \bar{P}_{21} \quad (1)$$

where $\bar{P}_{21} = \bar{P}_2 - \bar{P}_1$

$$\bar{P}_{31} = \bar{P}_3 - \bar{P}_1$$

Once the orientation of the primary datum feature is computed, a vector normal to the second-

ary datum feature, $\bar{n}_s$, can be obtained from Eq.(2). According to the right-hand rule, $\bar{n}_s$ must stay perpendicular to the vector normal to the primary datum plane, $\bar{n}_p$. Therefore,

$$\bar{n}_s = \bar{P}_{54} \times \bar{n}_p \quad (2)$$

where $\bar{P}_{54} = \bar{P}_5 - \bar{P}_4$

When the errors at the contact point $\bar{P}_i$ are considered in the equations, Eq. (1) and (2) become

$$\bar{n}_{pe} = \bar{P}_{31e} \times \bar{P}_{21e} \quad (3)$$

$$\bar{n}_{se} = \bar{P}_{54e} \times \bar{n}_{pe} \quad (4)$$

where $\bar{P}_{21e} = \bar{P}_{2e} - \bar{P}_{1e}$

$$\bar{P}_{31e} = \bar{P}_{3e} - \bar{P}_{1e}$$

$$\bar{P}_{54e} = \bar{P}_{5e} - \bar{P}_{4e}$$

Because of errors at the contact areas, the workpiece will be disoriented and dislocated from nominal location and orientation to allow complete contacts with the six locators. To guarantee such contacts, the following constraints must be satisfied.

$$\bar{n}_{pe} - R_1(\alpha, \beta, \gamma, dx, dy, dz) \bar{n}_p = 0 \quad (5)$$

$$\bar{n}_{se} - R_1(\alpha, \beta, \gamma, dx, dy, dz) \bar{n}_s = 0 \quad (6)$$

$$\bar{n}_{pe} \cdot \bar{P}_{1e} - D_p = 0 \quad (7)$$

$$\bar{n}_{se} \cdot \bar{P}_{4e} - D_s = 0 \quad (8)$$

$$\bar{n}_{te} \cdot \bar{P}_{6e} - D_t = 0 \quad (9)$$

where R_1 is a homogeneous transformation matrix represented by

$$R_1 = \begin{bmatrix} \cos(\alpha)\cos(\beta) & -\sin(\alpha)\cos(\beta) & \sin(\beta) & dx \\ \sin(\alpha)\cos(\gamma) + \cos(\alpha)\sin(\beta)\sin(\gamma) & \cos(\alpha)\cos(\gamma) - \sin(\alpha)\sin(\beta)\sin(\gamma) & -\cos(\beta)\sin(\gamma) & dy \\ \sin(\alpha)\sin(\gamma) + \cos(\alpha)\sin(\beta)\cos(\gamma) & \cos(\alpha)\sin(\gamma) + \sin(\alpha)\sin(\beta)\cos(\gamma) & \cos(\beta)\cos(\gamma) & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The system of the above equations is solved by breaking down the procedure into 2 steps. The first step is to find an appropriate rotation which matches $\bar{n}_p$ with $\bar{n}_{pe}$ and $\bar{n}_s$ with $\bar{n}_{se}$ by using the *Newton-Raphson* method. A set of objective functions is in the following equations.

$$\begin{aligned} f_1: \bar{n}_{xp} - \bar{n}_{xpe} \\ f_2: \bar{n}_{yp} - \bar{n}_{ype} \\ f_3: \bar{n}_{zp} - \bar{n}_{zpe} \\ f_4: \bar{n}_{xs} - \bar{n}_{xse} \\ f_5: \bar{n}_{ys} - \bar{n}_{yse} \\ f_6: \bar{n}_{zs} - \bar{n}_{zse} \end{aligned} \quad (10)$$

where $\bar{n}_{te} = R_2(\alpha, \beta, \gamma) \bar{n}_t$

$$R_2 = \begin{bmatrix} \cos(\alpha)\cos(\beta) & -\sin(\alpha)\cos(\beta) & \sin(\beta) \\ \sin(\alpha)\cos(\gamma) + \cos(\alpha)\sin(\beta)\sin(\gamma) & \cos(\alpha)\cos(\gamma) - \sin(\alpha)\sin(\beta)\sin(\gamma) & -\cos(\beta)\sin(\gamma) \\ \sin(\alpha)\sin(\gamma) + \cos(\alpha)\sin(\beta)\cos(\gamma) & \cos(\alpha)\sin(\gamma) + \sin(\alpha)\sin(\beta)\cos(\gamma) & \cos(\beta)\cos(\gamma) \end{bmatrix} \quad (11)$$

Following the *Newton-Raphson* procedure, the algorithm will be executed repeatedly until it finds a solution (α, β, γ) which makes Eq.(10) true. Once such a rotation is found, it will be used to

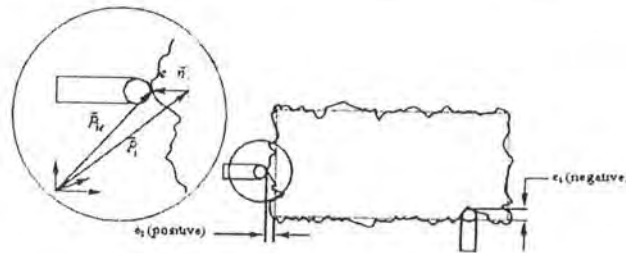


Figure 4. Error determination.

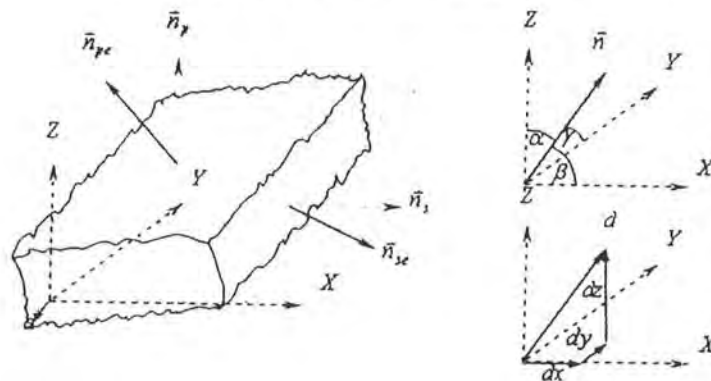


Figure 5. Workpiece location affected by variability at workpiece surface.

transform the workpiece to its final orientation. The second step is to calculate a translation vector needed to bring the workpiece to its final location while maintaining the orientation obtained from the first step (Figure 6). The vector is derived from the difference in the distance from the origin, D , of the nominal and actual datum features along corresponding normal vectors, $\bar{n}_{pe}$, $\bar{n}_{se}$ and $\bar{n}_{te}$. In summary, a translation vector bringing the workpiece to make contact with all locators will be calculated from Eq. (12).

$$\bar{T} = \bar{n}_{pe} \cdot \Delta D_p + \bar{n}_{se} \cdot \Delta D_s + \bar{n}_{te} \cdot \Delta D_t \quad (12)$$

$$\begin{aligned} \Delta D_p &= D_{pe} - D_p \\ \Delta D_s &= D_{se} - D_s \\ \Delta D_t &= D_{te} - D_t \end{aligned} \quad (13)$$

The final location of the workpiece is then

$$\bar{L}' = R_2(\alpha, \beta, \gamma) \bar{L} + \bar{T} \quad (14)$$

where $\bar{L}'$ is a point of interest.

$\bar{L}$ is the point after transformation.

Note that the algorithm simultaneously solves for the transformation satisfying the objective functions (Eq. 5-9). It may sound contradictory to the practical use of a 3-2-1 fixture, in which the workpiece must be sequentially located starting from the primary, secondary and tertiary datum planes. It is also realized that different orders of locating the planes will be associated with different contact locations. However, it is considered in this paper that the sequential process is not necessary since only variability of the workpiece location is estimated, not its exact arrangement.

Results and Discussion

Once the algorithm to characterize the workpiece behavior was developed, a Monte Carlo simulation was conducted to assess the variability of workpiece location by using the following method.

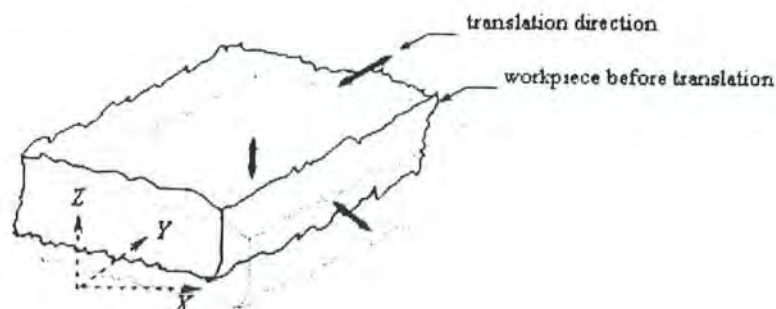


Figure 6. Workpiece translation.

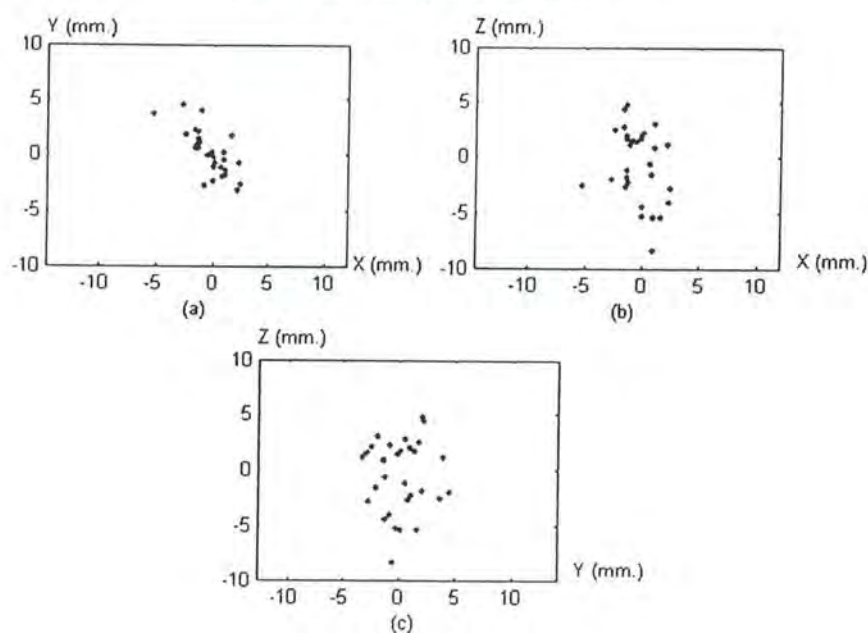


Figure 7. Variability in workpiece location when $n=30$.

1. A set of normal or Gaussian distributed random numbers represented the errors at each contact point was generated.

2. The error samples were used in the algorithm and the results were collected.

3. A multivariate statistical process was used to evaluate the variability of the sample distribution.

Examples of fixturing constraints are shown in Table 1. Table 2 contains the results which are the distributions of hole location under the constraints.

From the scatter plots (Figure 7 and 8), it can be seen that when the number of samples is greater (i.e. 300) the pattern of distribution is clearly shown. The elliptical-shaped point cloud preliminary indicates a normal or Gaussian distribution pattern. In 3-dimensional system, however, it does not guarantee a global multivariate normal relationship among the three axes.

To evaluate the distribution of multivariate data, a normality test is performed by constructing a chi-square plot (see detail in (Johnson and Wichern, 1998)). If the data are drawn from a

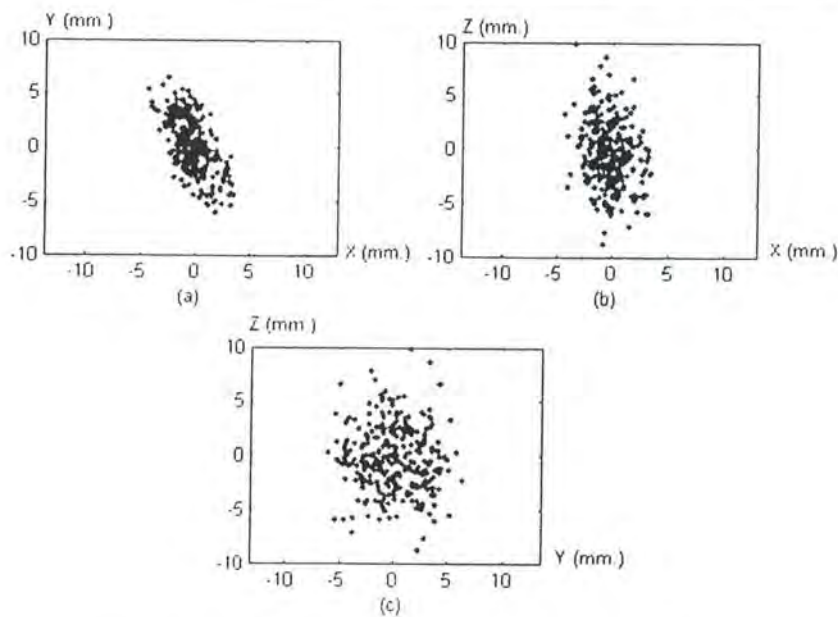


Figure 8. Variability in workpiece location when $n=300$.

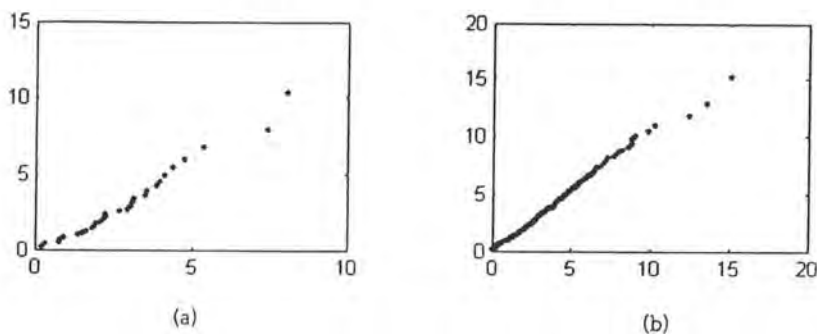


Figure 9. Normality plot (a) when a sample size is 30 (b) when a sample size is 300.

multivariate normal distribution, the plot should be similar to a straight line through the origin with slope equal to one. Any systematic curves indicate lack of normality and points far from the line suggest outlying observations. The data from the simulation plotted in Figure 9 is reasonably assumed a multivariate normal distribution. These statistical results are beneficial in tolerance analysis and will be discussed later in future work.

Relationships among the components of the locations of interest in Table 2 are in well agreement with the scatter plot. It can be seen that strong correlations between a component 'x' and 'y' of

workpiece location is notable, while the other pairs ('y' and 'z', 'x' and 'z') do not demonstrate any significant evidence of correlations. The distinct correlation between 'x' and 'y' suggests extra care if a circular positional tolerance is in use. How much a proportion in the elliptical area should be included to establish the circular tolerance is decisive and needs further study.

Conclusion

This study proposes an analysis of the variability in location of the workpiece as affected

Table 1. Coordinates and distributions of errors at each locator.

Plane	Locators (coordinates)	Errors Distribution				
		1	2	3	4	5
Primary Plane	1 (30 30 0)	N(0.0, 4.00)	N(-1.0, 1.21)	N(0.0, 3.61)	N(1.5, 1.44)	N(0.0, 4.00)
	2 (120 30 0)	N(0.0, 2.25)	N(2.0, 3.24)	N(2.0, 2.89)	N(2.3, 1.69)	N(-1.0, 2.25)
	3 (75 120 0)	N(0.0, 1.44)	N(0.0, 1.69)	N(-1.2, 1.44)	N(0.0, 1.44)	N(-1.5, 1.44)
Secondary Plane	4 (150 30 15)	N(0.0, 1.69)	N(-1.2, 2.56)	N(0.0, 0.81)	N(-1.5, 3.61)	N(0.0, 1.69)
	5 (150 120 15)	N(0.0, 3.61)	N(2.0, 2.25)	N(2.0, 4.41)	N(-1.0, 4.00)	N(2.3, 3.61)
Tertiary Plane	6 (75 150 15)	N(0.0, 2.25)	N(0.0, 1.44)	N(-1.0, 2.25)	N(0.0, 3.24)	N(1.5, 2.25)

Table 2. Results obtained from the constraints shown in Table 1.

Error Set	μ			σ_x^2	σ_y^2	σ_z^2	σ_{xy}	σ_{xz}	σ_{yz}
1	0.0040	-0.0285	0.2120	4.6756	6.5716	10.1982	-3.2095	-1.1812	-1.2336
2	-1.6142	2.4725	-1.8671	5.5041	5.1305	2.9554	-3.9664	-0.1632	-0.1694
3	-0.4418	0.4104	0.0755	2.1175	6.1266	8.2082	-2.3401	-0.3508	-0.9357
4	-1.6176	0.0155	1.9616	7.3417	10.0237	4.7778	-5.6818	-0.7146	-0.5029
5	-0.8809	3.2380	0.5825	3.0245	5.8589	9.8903	-2.5032	-1.1441	-0.8828

by variant errors at contact areas. The errors are assumed normally distributed. Unlike previous studies, a step-wise analysis is considered unnecessary and the mathematical formulation is simplified. The repetitive *Newton-Raphson* is employed to obtain a transformation which matches a theoretical with an actual position and orientation. Multivariate statistical techniques were used to analyze resultant variability. The information gained is beneficial to manufacturers as it reveals how surface variability becomes influential on location variation of the fixtured workpiece. It is recommended that such surface variability should be accounted in tolerancing establishment. Tighter tolerance ensures a functional assembly; however, usually associated with higher cost. It is the responsibility of a designer to find ways that would benefit the production the most. Implementing the concept proposed in this work would help the designer impose the tolerances more efficiently, and consequently reduce manufacturing cost and improve product quality.

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Nomenclature

$\bar{n}_p, \bar{n}_s, \bar{n}_t$	Nominal vectors normal to primary, secondary and tertiary datum features, respectively
$\bar{n}_{pe}, \bar{n}_{se}, \bar{n}_{te}$	Actual vectors normal to primary, secondary, and tertiary datum features, respectively
$\bar{P}_i$	Nominal contact points when $i = 1, 2, \dots, 6$
$\bar{P}_{ie}$	Actual contact points when $i = 1, 2, \dots, 6$
D_p, D_s, D_t	Nominal distances from the origin to primary, secondary, and tertiary datum features, respectively
D_{pe}, D_{se}, D_{te}	Actual distances from the origin to primary, secondary, and tertiary datum features, respectively

- α, β, γ Rotational angles about Z, Y, and X axes, respectively
- dx, dy, dz Translational distances along X, Y, and Z axes, respectively

References

- Bhat, V. and De Meter, E.C. 2000. An analysis of the effect of datum establishment methods on the geometric errors of machined features. *Int. J. Mach. Tool Manu.*, 40: 1951-1975.
- Brost, R.C. and Goldberg, K.Y. 1996. A complete algorithm for designing planar fixtures using modular components. *IEEE T. Robotic Autom.*, 12: 31-45.
- Cai, W., Hu, S.J. and Yuan, J.X. 1997. A variational method of robust fixture configuration design for 3-d workpieces. *J. Manuf. Sci. E.-T ASME*, 119: 593-602.
- Choudhuri, S.A. and De Meter, E.C. 1999. Tolerance analysis of machining fixture locators. *J. Manuf. Sci. E.-T ASME*, 121: 273-281.
- Dai, J.R., Nee, A.Y.C., Fuh, J.Y.H. and Kumar, S. 1997. An approach to automating modular fixture design and assembly. *P.I. Mech. Eng. B.- J. Eng.*, 211: 509-521.
- De Meter, E.C. 1998. Fast support layout optimization. *Int. J. Mach. Tool Manu.*, 38: 1221-1239.
- De Meter, E.C. 1995. Min-max load model for optimizing machining fixture performance. *J. Eng. Ind.-T ASME*, 117: 186-193.
- De Meter, E.C. 1994. Restraint analysis of fixtures which rely on surface contact. *J. Eng. Ind.-T ASME*, 116: 207-15.
- De Meter, E.C. 1994. The min-max load criteria as a measure of machining fixture performance. *J. Eng. Ind.-T ASME*, 116: 500-507.
- Djurdjanovic, D. and Ni, J. 2003. Dimensional errors of fixtures, locating and measurement datum features in the stream of variation modeling in machining. *J. Manuf. Sci. E.-T ASME*, v 125(n 4): 716-730.
- Fuh, J.Y.H., Chang, C.H. and Melkanoff, M.A. 1993. An integrated fixture planning and analysis system for machining processes. *Robot. CIM-Int. Manuf.*, 10: 339-353.
- Jeng, Y.C. and Gill, K.F. 1997. A CAD-based approach to the design of fixtures for prismatic parts. *P. I. Mech. Eng. B.- J. Eng.*, 211: 523-538.
- King, L.S.-B. and Hutter, I. 1993. Theoretical approach for generating optimal fixturing locations for prismatic workparts in automated assembly. *J. Manuf. Syst.*, 12: 409-416.
- Ong, S.K. and Nee, A.Y.C. 1998. A systematic approach for analyzing the fixturability of parts for machining. *J. Manuf. Sci. E.-T ASME*, 120: 401-407.
- Pham, D.T. and Lazaro, A.S. 1990. Finite element study of a workpiece in a machining fixture. *Math. Comp. Model.*, 14: 1024-1028.
- Rong, Y. and Bai, Y. 1996. Machining accuracy analysis for computer-aided fixture design verification. *J. Manuf. Sci. E.-T ASME*, 118: 289-300.
- Salisbury, E.J. and Peters, F.E. 1998. The impact of surface errors on fixtured workpiece location and orientation. *NAMRC Trans.*, 26: 323-328.
- Sangnui, S. 2002. The impact of fixturing error on feature tolerance allocation, in *Industrial and Manufacturing Systems Engineering*. Ph.D. Dissertation, Iowa State University, Iowa
- Sangnui, S. and Peters, F.E. 2001. The impact of surface errors on the location and orientation of a cylindrical workpiece in a fixture. *J. Manuf. Sci. E.-T ASME*, 123: 325-330.
- Trappey, J.C. and Matrubhutam S. 1993. Fixture configuration using projective geometry. *J. Manuf. Syst.*, 12: 486-495.
- Wu, Y., Rong, Y., Ma, W. and LeClari, S.R. 1998. Automated modular fixture planning geometric analysis. *Robot. CIM-Int. Manuf.*, 14: 1-15.

ภาคผนวก ค.

A METHOD TO ANALYZE EFFECTS OF SURFACE VARIATIONAL MODEL ON
POSITIONAL GEOMETRIC VARIABILITY

A method to analyze effects of surface variational model on positional geometric variability

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Abstract

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A method to analyze effects of surface variational model on positional geometric variability

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After machining, a processed workpiece is left with irregular topographies which may affect functional applications of its surface. Particularly when the workpiece is restrained again for a consequent machining operation and the surface is selected as a locating reference, any surface deviations from its perfect geometry will lead to inaccuracy of a machined feature. This study proposes a method to analyze effects of distributions chosen to explain irregularities of locating surfaces on feature positional tolerancing. Three types of distribution, a normal, a beta and a uniform distributions, were used in the analysis. From the simulation, it is found that the variability of the machined feature is obviously dependent on the distribution selected. Aside from mathematical complexity, this affected feature variability is another factor to be considered in choosing an appropriate surface distribution. The method developed herein will help a designer to impose more efficient tolerancing system, reduce production cost eventually.

Key words : datum establishment, tolerance, feature variability, surface distribution

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บทคัดย่อ

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ระเบียบวิธีในการวิเคราะห์ผลกระทบของตัวแบบความไม่แน่นอนของพื้นผิวอ้างอิง
ที่มีต่อการกำหนดตำแหน่งของการตัด

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โดยทั่วไปแล้ว พื้นผิวของชิ้นงานที่ผ่านการตัดมักจะมีรูปแบบเฉพาะตัวที่มีลักษณะไม่สม่ำเสมอ (irregular) และเมื่อนำชิ้นงานมาทำการจับยึดเพื่อที่จะสร้างฟีทเจอร์ (feature) โดยใช้พื้นผิวที่ผ่านการตัดมาแล้วนั้นเป็นพื้นผิวอ้างอิง ก็จะทำให้การกำหนดตำแหน่งของฟีทเจอร์ตานั้นผิดพลาดไป งานวิจัยนี้เป็นการนำเสนอระเบียบวิธีในการวิเคราะห์ผลกระทบของรูปแบบการกระจาย (distribution) ของพื้นผิวอ้างอิงที่มีต่อความแม่นยำในการวางตำแหน่งของฟีทเจอร์ โดยเปรียบเทียบระหว่างการกระจายแบบปกติ แบบเบต้า และแบบยูนิฟอร์ม ผลสรุปที่ได้ระบุว่าความไม่แน่นอนของตำแหน่งฟีทเจอร์มีความสัมพันธ์กับรูปแบบการกระจายของพื้นผิวอ้างอิงนั้น จากวิธีการดังกล่าวนี้จะช่วยให้นักออกแบบสามารถที่จะตัดสินใจในการกำหนดระยะพิสัยกัดเผื่อของฟีทเจอร์ที่มีประสิทธิภาพได้

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Irregularities of a machined surface topography can be broken down to surface roughness, waviness, and form error. For example, a structure of turned surfaces is a combination of a waviness component generated by a feeding process and a random roughness component caused by a built-up edge on a cutting tool (Stout *et al.*, 1990). Unlike turned surfaces, ground surfaces under microscope show gouge marks which result from sharp grits attached to a grinding wheel periphery. It appears that type of process and process parameters can differentiate manufactured surface characteristics as shown in Figure 1. After machining, a workpiece is left with irregular topographies which may affect functional applications of the surface. Particularly when the surface is selected as a locating reference, any deviations from its perfect geometry will lead to consequent workpiece inaccuracy. Eliminating such deviations to improve the workpiece accuracy is not always an economical solution. However, their undesirable effects on the workpiece quality can be controlled if we know how the deviations are distributed across the entire surface.

Because of the uncountable parameters involved, the actual distribution of a machined surface is almost impracticable to determine. Once derived; however, the calculation would be too

complicated giving inapplicable results. Variational models widely found in published work are usually based on a Gaussian or normal distribution. Nassef and ElMaraghy (1995) assumed variability of manufactured surfaces was distributed as multinormal. Multinormal random surface points were generated and estimated surfaces were constructed by interpolating to the points. The authors then investigated the simulated surfaces to determine geometric deviation of the surfaces before calculating optimal tolerance zones that would result in desired probability of rejection. The drawback is that systematic errors may produce an asymmetric distribution, which is not suitable to explain with the Gaussian or normal function. However, as a sample size increases, the distribution can be assumed Gaussian following the central limit theorem. Besides a Gaussian distribution, a beta distribution is also suggested because of its advantageous applications in representing manufacturing variability as following:

1. The beta distribution is more flexible. It has the capability to conform various types of variability, when the Gaussian distribution is applied only to a symmetric function.

2. The beta distribution has finite upper and lower limits, which is favorable in computation.

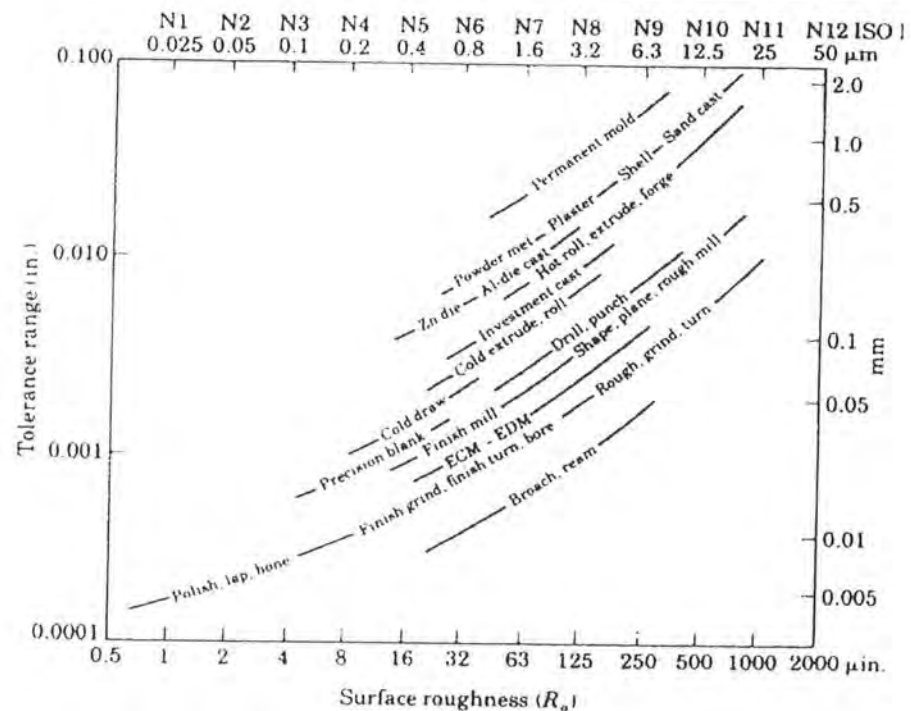


Figure 1. Tolerances and surface roughness obtained in various manufacturing processes. These tolerances apply to 25 mm workpiece dimension. (Kalpakjian, 1991).

The beta distribution is also chosen over a normal or Gaussian distribution because of its capability of supporting the calculation of an inverse probability distribution function (Lin *et al.*, 1997). He (1991) used the beta distribution to explain variability of dimension in manufacturing processes. Since the variability is distributed varying from process to process and application to application, the author then proposed a beta distribution as it has the capability to cover all kinds of distributions from rectangular to normal distributions as well as asymmetrical distributions. Treacy *et al.* (1991) carried out an analysis on tolerance distribution of an assembly of mechanical parts. For the sake of its flexibility, the authors decided on a beta distribution to represent variability of resultant tolerance chain of the assembly though the model may be associated with calculation complexity. Because practically every sum of interacting dimensions gives an expected variability that falls between the results from a worst case and root sum of square approaches,

Di Stefano (2003) developed a mean shift model modified from Mansoor (1964) where a factor "f" was imposed to determine a process bias. The factor is a function of a mean shift ratio, a confidence level, and a number of dimensions in an assembly. This model is applicable to Gaussian ($f = 0$) to non-Gaussian ($f = 1$) distributed dimensions. Unlike Gaussian and beta distributions, a uniform distribution is far less popular. ElMaraghy *et al.* (1995) employed the distribution to represent the variation explained by different tolerance types. Choudhuri and De Meter (1999) carried out a simulation to analyze the impact of locator tolerance scheme on the potential datum related, geometric errors of linear and machined features. They used a set of equally spaced numbers to simulate locator variability. The results revealed a linear relationship between some variables, which is not very surprising when the analysis was done over a small data range.

There are also works on datum establishment which left the controversial issue of a distribution

function unspecified. Nassef and ElMaraghy (1997) proposed a new criterion, mismatch probability, to determine appropriate tolerancing scheme where production cost was kept minimum. Because the procedure used with dimensional tolerancing is not applicable to geometric tolerancing, a new algorithm based on genetic algorithm was invented to evaluate each tolerancing candidate. Though computational time was relatively long, the benefits of choosing the appropriate tolerances paid off later on. Roy and Li (1998) developed mathematical models representing real world form variations. They proposed two strategies used to construct the approximated form of surfaces. The first technique was for simulating curved surfaces from randomly generated surface points. In order to create approximated planar surfaces, the second technique employed a linear regression analysis from the random points to estimate plane parameters. Besides the form variation, the authors also presented a computational scheme to represent size, orientation and position tolerance zones. Variational models of these tolerances were established relying on random data points. Bhat and De Meter (2000) conducted a simulation analysis to evaluate efficiency in accurate locating among three different datum establishment methods. The sample workpiece and its features were constrained by geometric tolerances. Random variables were generated to simulate variation according to each tolerance value generating variant feature position and orientation. The datum establishment method giving the least error which means that it is unlikely to be affected by such erratic geometry of the feature was considered the most functional method.

It can be seen that these research were more intensified on development of methods of reference establishment without taking into account an actual pattern of surface variability. The reasons involved in choosing an appropriate surface distribution should be more than computational complexity. This study proposes another aspect of tolerancing analysis. A method to analyze effects of chosen surface distributions on feature positional tolerancing is discussed in the following sections.

Analysis

In this section, the effects of variational models representing datum feature variability on positional errors of a feature to be produced are discussed. As shown in Figure 2, a datum feature surface is allowed to deviate from its perfect geometry by d_i where $|d_i| \leq \frac{T}{2}$; T is a tolerance constraining the datum feature. A prismatic workpiece with a hole and toleranced datum features (Figure 3) is used as an example in the analysis. Problems arise when the hole location is specified with respect to a misplaced reference frame, which is established from the deviated datum feature.

1. Tolerance Distributions

Three types of distributions usually found used to explain machined surface variability, d_i , are a normal, a beta and a uniform distributions. In this study, it is assumed that the distribution curve will span over the entire range of the tolerance width.

1.1 Gaussian or normal distribution

A unit normal probability density function is given as follows:

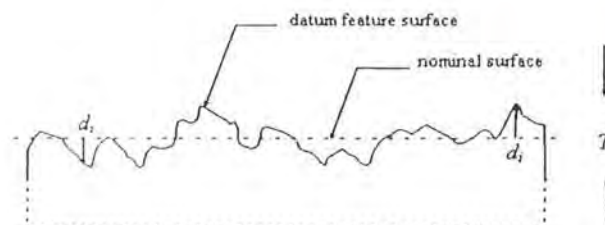


Figure 2. Datum feature and tolerance width, T

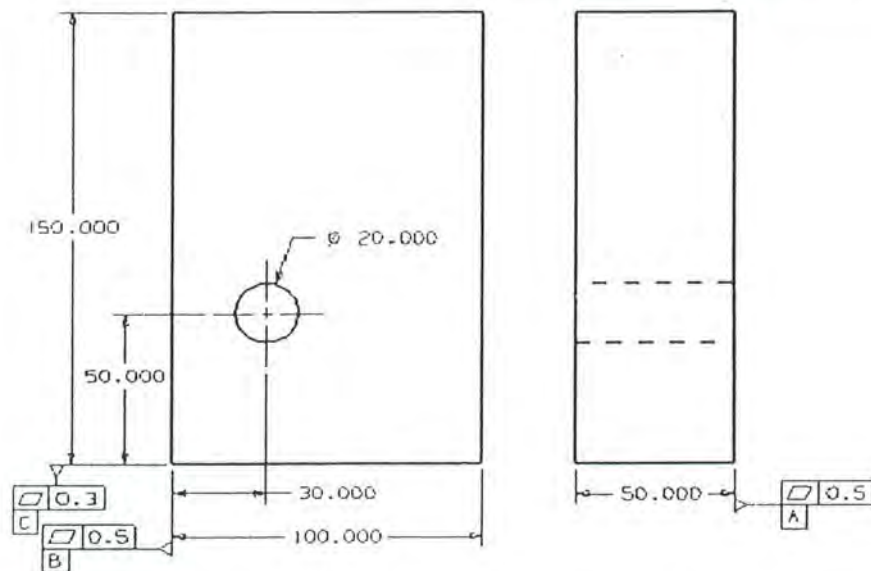


Figure 3. A sample workpiece used in simulations.

$$f(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \quad -\infty < u < \infty \quad (1)$$

The normal distribution curve spreads infinitely and symmetry about its mean as shown in Figure 4. Since it is impossible to define the end of its coverage, a 99.73% containment is assumed. When $N(0,1)$ represents a unit normal distribution which has its mean at zero and standard deviation of one, the surface points assimilating each datum feature are distributed as

$$d \sim \left(\frac{T}{6}\right) \cdot N(0,1) \quad (2)$$

1.2 Uniform Distribution

A unit uniform probability density function is given as follows:

$$f(u) = 1, \quad 0 \leq u \leq 1 \quad (3)$$

When $U(0,1)$ represents a unit beta distribution which spans over the range from zero to one, the surface points assimilating each datum feature are distributed as

$$d \sim T \cdot U(0,1) - \frac{T}{2} \quad (4)$$

1.3 Beta Distribution

A unit beta probability density function is given as follows:

$$f(u; \alpha, \beta) = \begin{cases} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} u^{\alpha-1} (1-u)^{\beta-1}, & 0 \leq u \leq 1, \alpha \geq 0, \beta \geq 0 \\ 0, & \text{elsewhere} \end{cases} \quad (5)$$

Since the study assumes a symmetric surface variability pattern over the tolerance zone, the values of shape controlling parameters, α and β , used in the simulations therefore are chosen to be equal. When $B(\alpha, \beta)$ represents a unit beta distribution covering the region from zero to one, the surface points assimilating each datum feature are distributed as

$$d \sim T \cdot B(\alpha, \beta) - \frac{T}{2} \quad (6)$$

2. Datum Establishment Methods

In this study, the 3-2-1 locating method is used to establish a reference frame. Following ASME Y14.5M, the primary datum plane is imposed from three highest points on the primary datum feature. Orthogonal with the primary plane, the secondary plane lies on two extreme points.

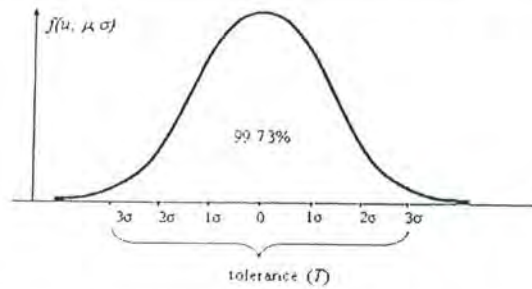


Figure 4. Tolerance determination when a normal distribution is assumed.

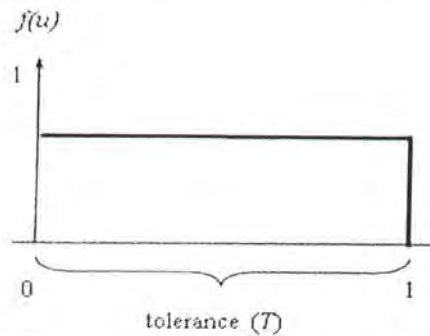


Figure 5. Tolerance determination when a uniform distribution is assumed.

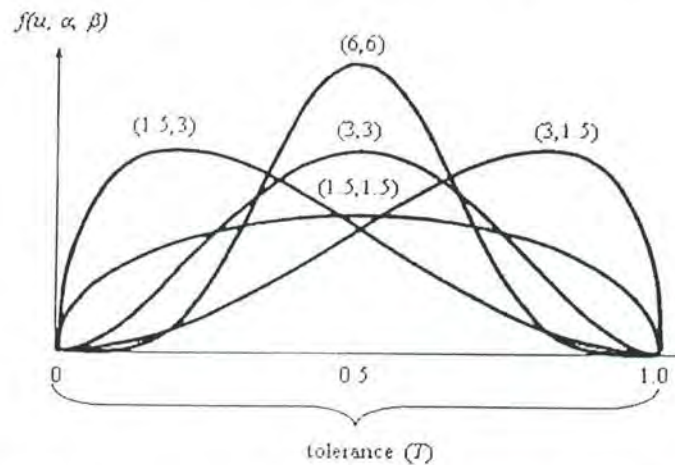


Figure 6. Tolerance determination when a beta distribution is assumed.

The last plane, the tertiary plane, is established from the outermost point while maintaining perpendicular to the preceding two planes as shown in Figure 7. By implementing this method, it is inevitable the frame constructed would be sensitive to the pattern the datum feature irregularities are distributed.

3. Datum Planes Definitions

Planes as elements of the reference frame must be calculated according to the datum establishment method described in section 2.2. The hole which we use as an example will be located with respect to these planes.

An algebraic equation of a plane is

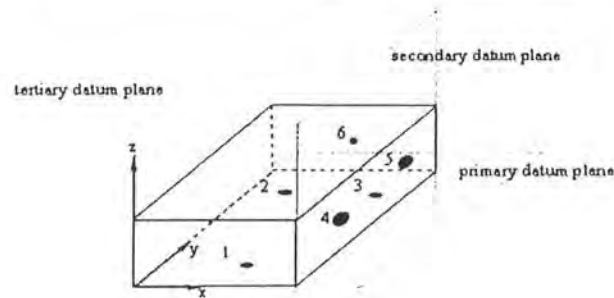


Figure 7. The system of datum planes.

$$\frac{Ax + By + Cz - D}{\sqrt{A^2 + B^2 + C^2}} = 0 \quad (7)$$

where $[A \ B \ C]$ is a vector normal to the plane
 D is an orthogonal distance from the origin to the plane

To locate an arbitrary plane, its normal vector and a distance from the plane to the origin must be defined.

Primary datum feature

The normal vector of the primary datum feature ($\vec{N}_p$) can be obtained from

$$\vec{N}_p = (\vec{P}_{pm1} - \vec{P}_{pm2}) \times (\vec{P}_{pm3} - \vec{P}_{pm2}) \quad (8)$$

and the orthogonal distance is

$$D_p = (\vec{n}_p \cdot \vec{P}_p) \quad (9)$$

where $\vec{P}_{pmi}$ are the extremities of the primary datum feature

$\vec{n}_p$ is a unit vector of $\vec{N}_p$

$\vec{P}_p$ is a point on the primary datum feature

Secondary datum feature

To comply with the principle of 3-2-1 method, the secondary datum feature must be perpendicular to the primary one while passing through 2 highest surface points. Therefore,

$$\vec{N}_s = \vec{N}_p \times (\vec{P}_{sm2} - \vec{P}_{sm1}) \quad (10)$$

where $\vec{P}_{smi}$ are the extremities of the secondary datum feature

Tertiary datum feature

The vector normal to the tertiary datum feature is defined by

$$\vec{N}_t = \vec{N}_p \times \vec{N}_s \quad (11)$$

The determination of location and orientation of the planes are also shown in Figure 8.

The distances from the origin to the secondary and tertiary datum feature, D_s and D_t , are calculated the same way as shown in Eq. 9. After the reference system imposed from the workpiece datum feature is established, coordinates of the hole relative to the nominal and actual systems are compared. The effects of variational surface model will then be analyzed. Please note that the nominal reference system is obtained when datum features are devoid in absence of any irregularities.

4. Positional Error Determination

Coordinates of an arbitrary point in a rectangular or Cartesian system is a Euclidean perpendicular distance from the point to a corresponding axis. By means of geometric transformation, a coordinate of a point P in a reference system F is related to its coordinate with respect to another reference system, M , as shown in Eq. 12 and Figure 9.

$$[P]^F = A[P]^M + L \quad (12)$$

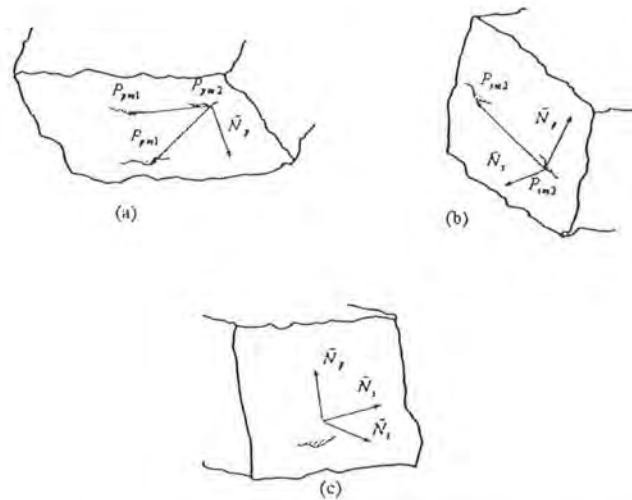


Figure 8. Datum planes location and orientation determination (a) primary datum plane (b) secondary datum plane (c) tertiary datum plane.

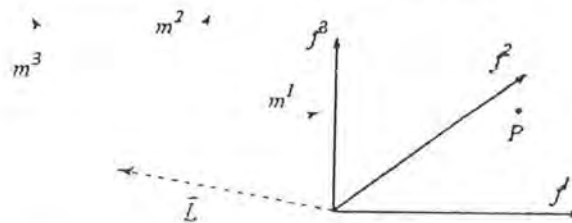


Figure 9. *F* ref. system relative to *M* ref. system.

where $[P]^F$ = a point coordinate in *F* reference system

$[P]^M$ = a point coordinate in *M* reference system

L is a distance vector in which *M* system located away from *F* system

and

$$A = \begin{bmatrix} f^1 \bullet m^1 & f^1 \bullet m^2 & f^1 \bullet m^3 \\ f^2 \bullet m^1 & f^2 \bullet m^2 & f^2 \bullet m^3 \\ f^3 \bullet m^1 & f^3 \bullet m^2 & f^3 \bullet m^3 \end{bmatrix} \quad (13)$$

where f^i is a direction cosine of *i*th axis of *F* reference system

m^i is a direction cosine of *i*th axis of *M* reference system

From Figure 7 and Eq. 13, we obtain

$$A = \begin{bmatrix} i \bullet \bar{n}_s & i \bullet \bar{n}_t & i \bullet \bar{n}_p \\ j \bullet \bar{n}_s & j \bullet \bar{n}_t & j \bullet \bar{n}_p \\ k \bullet \bar{n}_s & k \bullet \bar{n}_t & k \bullet \bar{n}_p \end{bmatrix} \quad (14)$$

In the current model, the system established from imperfect datum features is represented by the *M* reference system which is deviated from the nominal or *F* reference system. When the hole is produced, its location is specified relative to the machine or *F* system. Unfortunately, once the workpiece is removed from the machine, its accuracy is determined based on its actual or *M* system. The errors, e , of feature position, P , as resulted from such discrepancy between both systems are calculated as follows.

$$e = P - (A \bullet P + L) \quad (15)$$

5. Positional Geometric Variation Region

According to ASME Y14.5M, a positional tolerance is the permissible variation in the location of a feature about its exact true position. For a cylindrical feature like a hole, the positional tolerance is a diameter of the tolerance zone within which the axis of the feature must lie. In this study, the positional geometric variability is defined in a similar way except that it circumscribes actual locations as affected by undesirable factors. Following the algorithm explained in section 2.1-2.4, the feature's center is located. Because of irregular locating datum surface, inconsistency of the feature location is anticipated and can be illustrated as a cloud of points after a number of trials carried out (Figure 10). A circle bounding a specified proportion of possible feature locations (positional geometric variation region) is then calculated (See Sangnui (2002) for details). How big the circle is indicates how much the feature is varied in its position, and it should not exceed the tolerance associated with the hole.

Results and Discussion

The objective of this study is to present the impact of surface variational model on feature positional tolerancing. Three different distributions; a uniform, a normal, and a beta distribution are chosen to represent surface irregularities. Certain parameter values (α , β) of a beta distribution are selected to create various shapes of the distribution. Shown in Figure 6, the distribution is leaning towards uniformity when the parameters are approaching 1, and as the parameters are

greater the distribution will become more centrally concentrated. Details of the analysis conducted are described in the following steps.

1. Random numbers representing work-piece surface points are generated according to the tolerance distributions of interest (normal, uniform and beta distributions). See section 2.1.

2. Datum feature surfaces are partitioned into a number of small rectangle patches.

3. To simulate a deviated surface, generated random numbers are assigned to the appropriate direction (i.e. for the primary datum feature, the surface errors will be in the z direction) at grid points connecting the patches as shown in Figure 11.

4. Following the algorithm explained in section 2, the actual feature locations are calculated.

5. Tolerance regions associated with a desired probability of acceptance are calculated for each surface distribution pattern based on 300 simulation runs. When the number is too large the resultant distribution tends to follow the normal distribution pattern, according to the central limit theorem, and the true information will be masked. Too small number of sample runs, on the other hand, is also not recommended since the results will be unclear.

6. An average of 50 radiuses of circular positional region is calculated and shown in Table 1.

7. Results from each distribution are compared.

From the data in Table 1, it can be seen that the radii of positional geometric variation obtained from all eight simulation runs exhibit the same

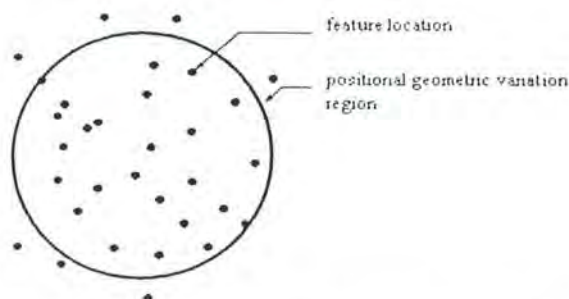


Figure 10. Positional geometric variation region.



Figure 11. A simulated datum feature.

Table 1. The averaged radiuses of positional geometric variation region at 95% containment

Data	Datum Feature Tolerance Specifications	Radiuses of Feature Positional Geometric Variation Region (mm.)				
		Uniform	Normal	Beta		
				$\alpha, \beta = 1.5$	$\alpha, \beta = 4$	$\alpha, \beta = 7$
1	(Tp = 3, Ts = 3, Tt = 3)	0.0202	0.2993	0.0694	0.2568	0.2894
2	(Tp = 3, Ts = 5, Tt = 3)	0.0196	0.2864	0.0706	0.2521	0.2780
3	(Tp = 3, Ts = 5, Tt = 5)	0.0295	0.4125	0.1037	0.3660	0.4000
4	(Tp = 3, Ts = 3, Tt = 5)	0.0288	0.3888	0.0980	0.3421	0.4099
5	(Tp = 5, Ts = 5, Tt = 5)	0.0323	0.4982	0.1163	0.4057	0.5062
6	(Tp = 5, Ts = 3, Tt = 5)	0.0336	0.4965	0.1137	0.4345	0.4946
7	(Tp = 5, Ts = 3, Tt = 3)	0.0233	0.3693	0.0858	0.3291	0.3753
8	(Tp = 5, Ts = 5, Tt = 3)	0.0246	0.3665	0.0825	0.3470	0.4066

pattern across different surface distributions. As the surface variability uniformly spreads over the allowable range (in the case of the uniform distribution), the resultant positional variation is expectedly smaller compared to one derived from more centrally concentrated distributions like the normal or the beta distributions ($\alpha = \beta$). It is clearly shown that feature positional geometric variation is dependent on the variational model chosen to explain datum feature variability. The difference in positional tolerance radii is getting clearer as the surface tolerances becomes wider. Noted that, this conclusion is regarded only when the datum establishment method is 3-2-1.

Conclusion

When the workpiece is subject to a machining operation, a feature to be produced is located with respect to a machine reference system. The system must have a certain relationship with a workpiece reference system which is established from the workpiece datum feature. In that way, the feature will be precisely located. However, with the existence of uncertainties in manufacturing processes, the relationship is usually hard or impossible to identify. If the pattern of irregularities is characterized, such discrepancy will be quantified, the undesirable displacement will be resolved, and ultimately the accuracy of the workpiece will be improved.

There are several variational models proposed to explain the pattern of irregularities of machined surfaces. Only the models' advantages and drawbacks in computation handling were reported. This study has investigated another interesting aspect, which is their effect over resultant positional tolerancing of a feature when referencing features are characterized by different variational models. From the analysis, the results indicated a distinct arrangement from one distribution to another. The area circumscribing possible feature locations is growing bigger as the surface variability is distributed apart from uniformity and when surface irregularities are evenly distributed, smaller feature variability is obtained. As a results, the variability of the feature is obviously dependent on the distribution selected to explain irregular referencing surfaces. This source of variability should not be disregarded and left out of a system of tolerancing. It is well understood that the system is a critical element in a manufacturing process. An efficient tolerancing system not only bridges a gap between a design and a manufacturing department, but it does also help in production cost reduction.

References

- Bhat, V., and De Meter, E.C. 2000. An analysis of the effect of datum establishment methods on the geometric errors of machined features. *Int. J. Mach. Tools Manu.*, 40: 1951-1975.

- Choudhuri, S.A., and De Meter, E.C. 1999. Tolerance analysis of machining fixture locators. *J. Manuf. Sci. E-T ASME*, 121: 273-281.
- Di Stefano, P. 2003. Tolerance analysis and synthesis using the mean shift model. *P.I. Mech. Eng. B-J. Eng.*, 217: 149-160.
- ElMaraghy, W.H., Gadalla, M.A., Valluri, S.R., and Skubnik, B.M. 1995. Relating ANSI GD&T standards and symbols to variations on primitives. *Proc. of the 1995 ASME Int. Mech. Eng. Cong. & Exp. ASME (Paper)*.
- He, J.R. 1991. Estimating the distributions of manufactured dimensions with the beta probability density function. *Int. J. Mach. Tool Manu.*, 31: 383-396.
- Kalpakjian, S. 1991. *Manufacturing Processes for Engineering Materials*. Addison-Wesley Publishing Company, Inc.
- Lin, S., Wang, H., and Zhang, C. 1997. Statistical tolerance analysis based on beta distributions. *J. Manuf. Syst.*, 16: 150-158.
- Mansoor, E.M. 1964. The application of probability to tolerance used in engineering designs. *P.I. Mech. Eng. B-J. Eng.*, 178: 29-51.
- Nassef, A.O. and ElMaraghy, H.A. 1997. Allocation of geometric tolerances: New criterion and methodology. *CIRP Annals - Man. Tech.* 46: 101-106.
- Roy, U. and Li, B. 1998. Representation and interpretation of geometric tolerances for polyhedral objects - I. Form tolerances. *Comput. Aided Des.* 30: 151-161.
- Sangnui, S. 2002. The impact of fixturing error on feature tolerance allocation. *Industrial and Manufacturing Systems Engineering*, Iowa State University, Ames, IA.
- Stout, K.J., Davis, E.J. and Sullivan, P.J. 1990. *Atlas of machined surfaces*. Chapman and Hall, New York.
- Treacy, P., Ochs J.B., Ozsoy T.M. and Wang, N. 1991. Automated tolerance analysis for mechanical assemblies modeled with geometric features and relational data structure. *Comput. Aided Des.* 23: 444-453.