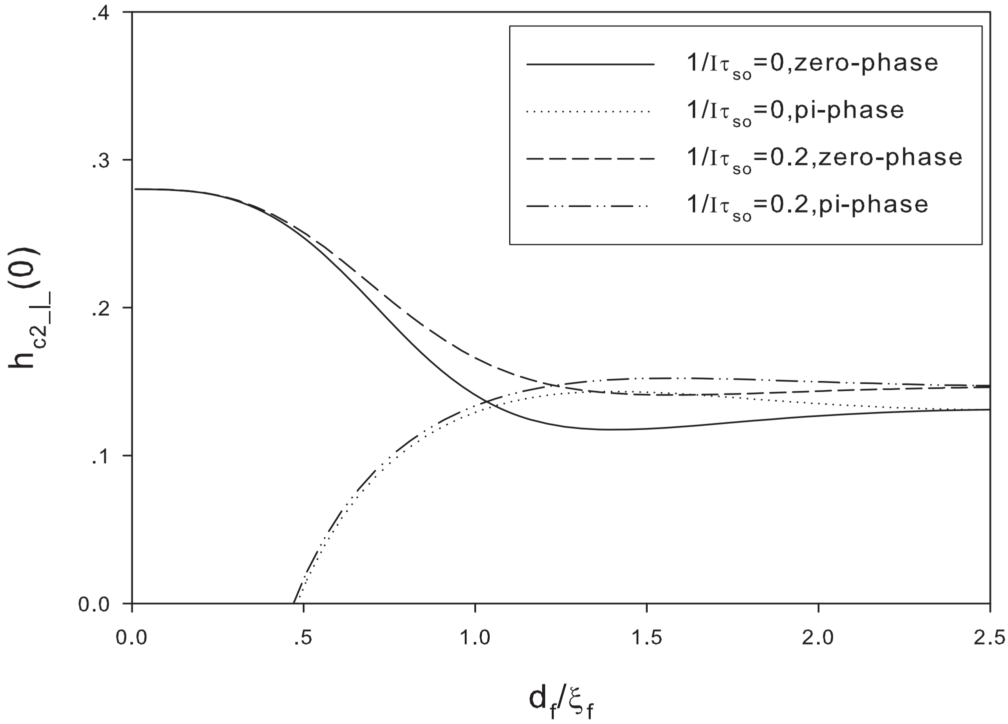
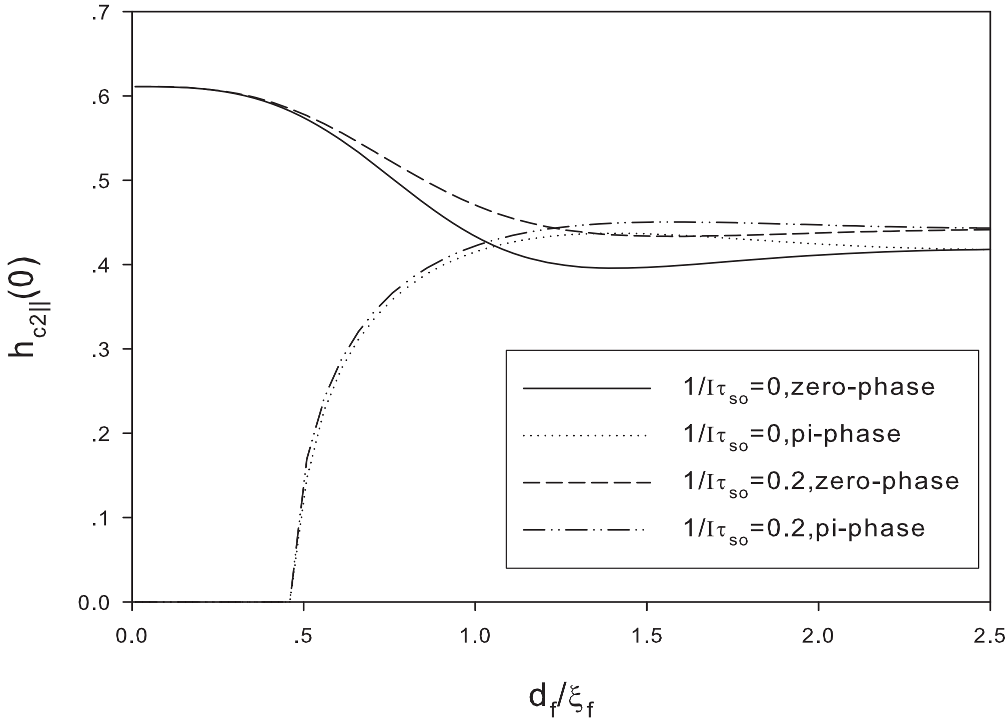


รูปที่ 14 การประชันเฟสระหว่างเฟสศูนย์และเฟสพายของ $h_{c2\perp}(0)$ โดยผันแปร $1/I\tau_{so}$ ค่าพารามิเตอร์อื่นๆ คือ $I/\pi T_{cs}=10, \gamma=0.1, \gamma_b=0, d_s/\xi_s=3$



รูปที่ 15 การประชันเฟสระหว่างเฟสศูนย์และเฟสพายของ $h_{c2||}(0)$ ค่าพารามิเตอร์เหมือนกับรูปที่ 14

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Upper critical fields of ferromagnet/superconductor layered structures

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Abstract

The temperature dependence of the upper critical fields, both perpendicular $H_{c2\perp}$ and parallel $H_{c2\parallel}$ to layer planes of ferromagnet/superconductor bi- and multilayers, is theoretically investigated. The secular equation of the superconducting order parameter for determining the phase diagram (H, T) is obtained by solving exactly the linearized Usadel equations in the multimode method taking into account the material parameter values. For the bilayers system, the influence of the boundary resistivity on the critical fields, and the dimensional crossover behavior of $H_{c2\parallel}(T)$ are studied in details. For the multilayered structure, the effects of the π-phase state on both the superconducting transition temperature T_c and the upper critical fields ($H_{c2\perp}$, and $H_{c2\parallel}$) are also considered. The nonmonotonic T_c behaviors are predicted. The interplay between 0- and π-phases leading to the strong oscillations of T_c as well as the temperature dependence of the zero temperature critical fields on the ferromagnetic layer thickness are investigated theoretically.

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Keywords: Superconductivity; Ferromagnetism; Proximity effect; Transition temperature; Critical field

1. Introduction

In recent years, extensive studies of artificially layered structures, consisting of superconductor (S) and ferromagnet (F), have attracted much attention [1–17]. The competition between ferromagnetism and superconductivity is feasible through the proximity effect. The one interesting feature is nonmonotonic critical temperature T_c with increasing the F layer thickness (d_f) while the S thickness (d_s) is kept fixed. The another remarkable feature is the temperature dependence of upper critical fields H_{c2} whose the dimensionality specifies the superconducting layers are either coupled (3D) or decoupled (2D). The occurrence of a 3D to 2D crossover in $H_{c2\parallel}$ near T_c may indicate the coupling phenomena via the proximity effect.

The evidences of the nonmonotonic T_c were reported on S/F bilayers (Nb/Fe [19], Nb/CuNi [20]), F/S/F trilayers (Fe/Nb/Fe [21,22], Fe/Pb/Fe [23], CuNi/Nb/CuNi [13,25], Fe/V/Fe [24]), and S/F mutilayers (Nb/Gd [26], Nb/FeCu [27], Nb/Co and V/Co [28], Nb/CuMn [29]), but some studies showed the contradicting results [30–33]. Attempts to explain these discrepancies have been made. Strunk et al. [31] proposed that the obtained $T_c(d_f)$ behavior is due to the transition from a paramagnetic to a ferromagnetic state in magnetic films. Mühge et al. [21] interpreted the nonmonotonic $T_c(d_f)$ dependence occurs due to the existence of magnetically dead layers near the interface. Jiang et al. [26,34] suggested that the pronounced oscillatory $T_c(d_f)$ dependence is the evidence of the π coupling in S/F multilayers which had been predicted theoretically by Radović et al. [35]. Aarts et al. [36] presented an experimental evidence of the interface transparency using the Kupriyanov–Lukichev boundary conditions [37] in the dirty limit at the S/F interface. Fominov et al. [11,12] argued that the nonmonotonic $T_c(d_f)$ is caused by the interface transparency even in the weak ferromagnet. From the experimental facts, the onset of ferromagnetism marks the

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onset of the nonmonotonic $T_c(d_f)$ curve on the length scale of the magnetic stiffness length $\xi_I = v_F/I$, where v_F is the Fermi velocity and I the exchange splitting of conduction band. For $d_f < \xi_I$, the magnetic layer becomes a paramagnet and then this is called the intermixed layer. The influence of alloying on an oscillation of T_c has been investigated theoretically by Vodopyanov et al. [10]. Upon the onset of ferromagnetism ($d_f \geqslant \xi_I$) there exist two long-range orders, the first one is the Cooper pairs with opposite spins and the second one is the exchange field which tries to align the electron spins and suppresses superconductivity. However, the situation changes if the spin-singlet pairs acquire an admixture of spin-triplet part. The spin-triplet pair is generated due to the interplay between the spin-singlet superconductivity and the magnetization in F [14–17] and is the so called odd triplet superconductivity [18].

Another crucial parameter needed to elucidate the coupling phenomenon of the magnetic layer is the upper critical field H_{c2} which gives the information on the coherence length and the dimensionality, because H_{c2} plays an important role of the pair-breaking effect. Close to T_c, the temperature dependence of the parallel upper critical field of a single S film shows a square-root like behavior, $H_{c2\parallel} \propto \sqrt{1 - T/T_c}$, (2D) whereas for the perpendicular field, $H_{c2\perp} \propto 1 - T/T_c$ which is the 3D behavior. Nevertheless, $H_{c2\parallel}$ of a contacted S/F films revealed 3D behavior due to the strength I and the range d_f of the pair breaking in F layers. The manifestation of $H_{c2\parallel}$ is a 2D–3D crossover at the temperature T^* ($T^* < T_c$). For $T^* \leqslant T \leqslant T_c$, deviations from the square-root like behavior were observed [30–32,38–42] which indicated the typical of 3D nature or the coupled regime. Below the crossover temperature T^* ($T < T^*$), the $H_{c2\parallel}$ still exhibits a 2D behavior then within this temperature range the system is in the decoupled regime.

On theoretical considerations, a proximity theory of upper critical fields based on de-Gennes's correlation function method [43] was formulated by Takahashi and Tachiki [44] for the S/N superlattices (N is normal metal). A numerical studies on S/F superlattices have been performed by Kuboya and Takanaka [45]. This theory explained how T_c and H_{c2} are related to the pairing potential, the density of states at Fermi surface, the diffusion coefficient, and the exchange field of F layers. Later on, another S/F proximity effect study came from Radović et al. [35,46–48] who applied Usadel's equations [49] to evaluate T_c and H_{c2} analytically. Of course, in their model, the strong ferromagnet exchange field I ($\gg T_{cs}$, where T_{cs} is the bulk transition temperature in the absence of an external field), is solely considered and this implies that the effect of an orbital magnetic field in F layers can be neglected completely and as a result the propagating momentum is approximated to be frequency independent. In the works of Refs. [35,48] T_c and $H_{c2\perp}$ of S/F superlattices were solved exactly with the predictions that an oscillatory dependence of T_c and $H_{c2\perp}$ on the F-layer thickness have been attributed to the π-phase difference between neighboring S layers. However, in the case of the parallel magnetic field $H_{c2\parallel}$, they considered only the S layer embedded in F metal [47].

In this paper, we present the theoretical results of the temperature dependence of $H_{c2\perp}$ and $H_{c2\parallel}$ for F/S bilayers and superlattices based on the exact multimode linearized Usadel's equations taking into account the finite value of the interface transparency boundaries and the finite thickness of the layers and also the arbitrary strength of an exchange field I which is not merely restricted to the strong value. In Section 2, the formulation for the linearized Usadel's equations near the transition point with the gauge choices, and the proper set of boundary conditions are presented. In Section 3 we give the detail calculations of the upper critical fields ($H_{c2\perp}$, $H_{c2\parallel}$) for the F/S bilayers and show the reductions of the secular equations in the single-mode approximation. In Section 4 we give the results for the F/S superlattices and pay attentions to the effect of π-phase shift on the critical fields. Our results recover the S/N case (when we take $I = 0$) of the previous works [44]. Finally, conclusions are presented in Section 5.

2. Formulation

We begin with the linearized integral equation for the superconducting order parameter, $\Delta(\mathbf{r})$, [44]

$$\Delta(\mathbf{r}) = \pi T N(\mathbf{r}) V(\mathbf{r}) \sum_{\omega} \int \mathrm{d}\mathbf{r}' K_{\omega}(\mathbf{r},\mathbf{r}')\Delta(\mathbf{r}'), \tag{1}$$

where $\omega = (2n+1)\pi T$, with an integer n, the kernel $K_{\omega}(\mathbf{r},\mathbf{r}')$ satisfies the Usadel equation

$$\left[-\frac{1}{2}D(\mathbf{r})\Pi^2 + (|\omega| + \mathrm{i}I(\mathbf{r})\,\mathrm{sgn}\,\omega)\right]K_{\omega}(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \tag{2}$$

and the parameters $N(\mathbf{r})$, $V(\mathbf{r})$, $D(\mathbf{r})$, and $I(\mathbf{r})$ are all position dependent which represent the electronic density of states, the pairing interaction, the diffusion coefficient, and the exchange field, respectively. The gauge-invariant operator $\Pi = \nabla - (2\pi\mathrm{i}/\phi_0)\mathbf{A}$ is expressed in terms of a fluxoid ϕ_0 and a magnetic vector potential $\mathbf{A}$. The anomalous Usadel function $F(\mathbf{r},\omega)$ is simply an integral equation of the kernel $K_{\omega}(\mathbf{r},\mathbf{r}')$;

$$F(\mathbf{r},\omega) = \int \mathrm{d}^3 r' K_{\omega}(\mathbf{r},\mathbf{r}')\Delta(\mathbf{r}'). \tag{3}$$

The layered structures under our consideration consist of superconductor- and ferromagnet-films located in the y–z plane so the proximity effect will be taken along the x-axis. Within the well-known relation $\mathbf{H} = \nabla \times \mathbf{A}$ for a uniform applied magnetic field we may choose $\mathbf{A} = (0,0,Hy)$ for the magnetic field perpendicular to the layer plane $\mathbf{H} = H\hat{x}$ and so the gauge-invariant gradient reads as

$$\Pi_{\perp} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} - \frac{2\pi\mathrm{i}Hy}{\phi_0}\right).$$

In the case of parallel magnetic field $\mathbf{H} = H\hat{z}$, the gauge choice $\mathbf{A} = (0, Hx, 0)$ implies

$$\Pi_{\parallel} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y} - \frac{2\pi i H x}{\phi_0}, \frac{\partial}{\partial z}\right).$$

In the Radović's fashion [48], the parameters in each layer are treated separately. The exchange potential exists only in F layers whereas the pairing interaction and the order parameter are set equal to zero because the exchange field quenches the superconductivity. The Usadel equations are

$$(\Pi^2 - k_f^2)F_f(\mathbf{r},\omega) = 0 \tag{4}$$

for F, and

$$(\Pi^2 - k_s^2)F_s(\mathbf{r},\omega) = -\frac{1}{\pi T_{cs}\xi_s^2}\Delta(\mathbf{r}), \tag{5}$$

$$\Delta(\mathbf{r}) = \pi T\lambda \sum_{\omega} F(\mathbf{r},\omega) \tag{6}$$

for S. Therefore

$$k_f = \frac{1}{\xi_f}\sqrt{\frac{|\omega| + iI\,\mathrm{sgn}\,\omega}{\pi T_{cs}}}, \quad k_s = \frac{1}{\xi_s}\sqrt{\frac{|\omega|}{\pi T_{cs}}} \tag{7}$$

are the propagating momenta in each layer, $\xi_{s(f)} = \sqrt{D_{s(f)}/2\pi T_{cs}}$ is the coherence length and λ the dimensionless BCS coupling constant. The pair amplitudes F_s and F_f are related through the boundary conditions at F/S interfaces [37]

$$\xi_s \nabla F_s = \gamma \xi_f \nabla F_f, \tag{8}$$

$$F_s = F_f - \gamma_b \xi_f \mathbf{n}_f \cdot \nabla F_f \tag{9}$$

with $\gamma = \rho_s\xi_s/\rho_f\xi_f, \gamma_b = 2D_f/v_f T_f \xi_f$. The parameters ρ_s and ρ_f stand for the resistivity of each metal, usually $\rho_f > \rho_s$ or $\gamma < 1$, this means that the pairing induced in F is weak. γ_b is called the boundary resistivity since it represents a jump of Cooper pairs at interfaces where $\mathbf{n}_f$ is the unit vector outward normal to the interface, v_f is the Fermi velocity inside F and T_f the interface transparency parameter, $T_f \in [0,\infty]$. The limit $T = \infty$ ($\gamma_b = 0$) corresponds to a perfectly transparency interface.

The structures of F/S proximity systems in the bilayer case consist of F and S layers occupy the regions $-d_f \leqslant x \leqslant 0$ and $0 \leqslant x \leqslant d_s$, respectively. At the outer surfaces ($x < -d_f$ for F and $x > d_s$ for S) F_f and F_s satisfy the conditions of no pairing current pass through vacuum

$$\frac{d}{dx}F_f|_{x=-d_f} = 0, \quad \frac{d}{dx}F_s|_{x=d_s} = 0, \tag{10}$$

the F/S bilayer structure can be considered as a unit cell with length $d_s + d_f$ so a superlattice is created from an infinite stack of F/S bilayers. Instead of (10), the function $F_{s,f}$ is subject to the Bloch condition due to the periodicity of the superlattice [35,48]

$$F(x + d_s + d_f) = e^{i\varphi}F(x), \tag{11}$$

where φ is the phase shift between adjacent layers. The stable ground state corresponds to $\varphi = 0$, however, the candidate state, $\varphi = \pi$, is energetically favorable.

3. F/S bilayers

To determine the upper critical fields H_{c2} or the superconducting transition temperature T_c, the main task is to find F_s and Δ self-consistent. Since the Usadel equation for F_f can be solved directly and so the boundary conditions reduce to the prescribed values of F_s at the edges of the S layer. The boundary conditions for the F/S bilayer, from (8) and (9), are

$$\xi_s \frac{d}{dx}F_s(0) = \gamma\xi_f \frac{d}{dx}F_f(0), \tag{12}$$

$$F_s(0) = F_f(0) + \gamma_b\xi_f \frac{d}{dx}F_f(0), \tag{13}$$

together with (10). In the following, the orientation of magnetic field, either perpendicular or parallel, will be considered separately.

3.1. Perpendicular upper critical field $H_{c2\perp}$

According to Radović's method [48] the pair amplitudes F_s and F_f are assumed to be of the form

$$F_{s,f}(\mathbf{r},\omega) = f(y,z)g_{s,f}(x,\omega), \tag{14}$$

where $f(y,z)$ is finite in the entire film plane and ω independent. The lowest state of $f(y,z)$ provides the perpendicular upper critical field $H_{c2\perp}$;

$$\frac{2\pi}{\phi_0}H_{c2\perp} = p_f^2 - k_f^2, \tag{15}$$

here p_f is the constant of separation and satisfies

$$\frac{d^2}{dx^2}g_f(x,\omega) = p_f^2 g_f(x,\omega). \tag{16}$$

Thus for F_f, using (14)–(16) and (4) (with $\Pi_\perp$) becomes

$$\frac{d^2}{dx^2}F_f(x,\omega) - p_f^2F_f(x,\omega) = 0. \tag{17}$$

The solution of (17) subject to the outer boundary condition at $x = -d_f$, (10), is given by

$$F_f(x,\omega) = C\cosh(p_f[x + d_f]). \tag{18}$$

For S, applying (14) and (15) to (5), the result is

$$\frac{d^2}{dx^2}F_s(x,\omega) - k_{s\perp}^2F_s(x,\omega) = -\frac{1}{\pi T_{cs}\xi_s^2}\Delta(x), \tag{19}$$

where $k_{s\perp} = \sqrt{k_s^2 + (2\pi/\phi_0)H_{c2\perp}}$. Now making use (18), the boundary conditions (12) and (13) can be written as

$$\xi_s \frac{d}{dx}F_s(0) = W_\perp(\omega)F_s(0) \tag{20}$$

with

$$W_\perp(\omega) = \frac{\gamma}{\gamma_b + \frac{1}{p_f\xi_f}\coth(p_f d_f)}.$$

To solve (19) exactly we employ (2) in the form

$$\left(\frac{d^2}{dx^2}-k_{s\perp}^2\right)K_\omega(x,x')=-\frac{1}{\pi T_{cs}\xi_s^2}\delta(x-x') \tag{21}$$

with the same boundary conditions as F_s, namely

$$\xi_s\frac{d}{dx}K_\omega(0,x')=W_\perp(\omega),\quad \frac{d}{dx}K_\omega(d_s,x')=0. \tag{22}$$

By virtue of the Fourier method

$$K_\omega(x,x')=\sum_{m=-\infty}^{\infty}K_\omega(q_m,x')\cos(q_m x), \tag{23}$$

$$K_\omega(q_m,x')=\frac{1}{d_s}\int_0^{d_s}dx K_\omega(x,x')\cos(q_m x) \tag{24}$$

with the eigenmode $q_m=m\pi/d_s$, (21) can be transformed which leads to the result

$$K_\omega(q_m,x')=\frac{\cos(q_m x')}{d_s\pi T_{cs}\xi_s^2(k_{s\perp}^2+q_m^2)}-\frac{\Omega_\perp(\omega)}{d_s\pi T_{cs}}\sum_{l=-\infty}^{\infty}\frac{\cos(q_l x')}{\xi_s^4(k_{s\perp}^2+q_m^2)(k_{s\perp}^2+q_l^2)}, \tag{25}$$

where

$$\Omega_\perp(\omega)=\frac{W_\perp(\omega)}{(d_s/\xi_s)\left(1+\frac{W_\perp(\omega)}{k_{s\perp}\xi_s}\coth(k_{s\perp}d_s)\right)}. \tag{26}$$

Consequently, the self-consistency equation (6) becomes

$$\Delta(q_m)=\lambda\pi T\sum_{m'=-\infty}^{\infty}\sum_\omega\Delta(q_{m'})\int_0^{d_s}dx' K_\omega(q_m,x')\cos(q_{m'}x'). \tag{27}$$

Using (25) we obtain the secular equation for (27) as follows:

$$\det|\delta_{mm'}-\lambda\frac{T}{T_{cs}}\sum_\omega L^{bi}_{mm'\perp}(\omega)|=0, \tag{28}$$

where

$$L^{bi}_{mm'\perp}(\omega)=\frac{\delta_{mm'}}{\xi_s^2(k_{s\perp}^2+q_m^2)}-\frac{\Omega_\perp(\omega)}{\xi_s^4(k_{s\perp}^2+q_m^2)(k_{s\perp}^2+q_{m'}^2)}. \tag{29}$$

The perpendicular upper critical field $H_{c2\perp}(T)$ corresponds to the largest temperature for a fixed magnetic field which is a solution of (28).

We will show that (28) and (29) reduce to the Abrikosov–Gorkov like-formula [50] in the single-mode approximation (SMA). It is sufficient to take only the $(0,0)$ element of $L_{mm'}$ with the assumptions that (i) the S layer thickness is very thin $d_s/\xi_s\ll 1$, and (ii) the exchange field inside the F layer is so strong $I/\pi T_{cs}\gg 1$, or $k_f\xi_f\approx\sqrt{iI/\pi T_{cs}}$. As a result, we obtain

$$\ln t=\psi\left(\frac{1}{2}\right)-\mathrm{Re}\psi\left(\frac{1}{2}+\frac{\varrho}{2t}\right), \tag{30}$$

where $t=T/T_{cs}$ is the reduced temperature, $\psi(x)$ is the digamma function, and the complex pair-breaking parameter

$$\varrho(t)=\varrho^{bi}(t_c)+h_{c2\perp}(t), \tag{31}$$

containing the contributions from the strong ferromagnetic exchange field through

$$\varrho^{bi}(t_c)=\frac{W}{(d_s/\xi_s)}=\frac{\gamma(\xi_s/d_s)}{\gamma_b+\coth(k_f d_f)/k_f\xi_f}, \tag{32}$$

and the (dimensionless) orbital field effect $h_{c2\perp}=(2\pi/\phi_0)H_{c2\perp}\xi_s^2$.

It should be noted that the function W is obtained from $W_\perp(\omega)$ by taking k_f to be ω-independent and neglecting the orbital magnetic field in F.

At $t=t_c$, $h_{c2\perp}(t_c)=0$, and (30) reduces to the equation of the variation of the superconducting critical temperature t_c on the material parameters such that the layer thicknesses is either d_s/ξ_s in S, or d_f/ξ_f in F, and the boundary resistivity γ_b. The F/S structure coincides with the F/S/F triple layers when the relation $d_s^{tri}=2d_s^{bi}$ is used. An investigation of T_c behavior has already been reported in Ref. [51].

3.2. Parallel upper critical field $H_{c2\|}$

In this case we try the pair amplitude $F_{s,f}(\mathbf{r},\omega)$ in the form

$$F_{s,f}(\mathbf{r},\omega)=\exp\left(i\frac{2\pi Hx_0}{\phi_0}y\right)F_{s,f}(x,\omega), \tag{33}$$

here x_0 denotes a free parameter which indicates the position of the center of superconducting nucleus. As usual by applying (33) to (4)–(6) with the replacement $\Pi\to\Pi_\|$, the Usadel equations read

$$\frac{d^2}{dx^2}F_f(x,\omega)-\left[k_f^2+\left(\frac{2\pi H}{\phi_0}\right)^2(x-x_0)^2\right]F_f(x,\omega)=0, \tag{34}$$

in F $(-d_f\leqslant x\leqslant 0)$, and

$$\frac{d^2}{dx^2}F_s(x,\omega)-\left[k_s^2+\left(\frac{2\pi H}{\phi_0}\right)^2(x-x_0)^2\right]F_s(x,\omega)=-\frac{\Delta(x)}{\pi T_{cs}\xi_s^2}, \tag{35}$$

$$\Delta(x)=\lambda\pi T\sum_\omega F_s(x,\omega), \tag{36}$$

in S $(0\leqslant x\leqslant d_s)$. The general solution of (34) is expressed in terms of the confluent hypergeometric functions $\Phi(a,b;z)$ [52] as

$$F_f(x,\omega)=C_1u_1(x)+C_2u_2(x) \tag{37}$$

with

$$u_1(x)=\exp\left(-\frac{\pi H}{\phi_0}(x-x_0)^2\right)\Phi\left(\frac{1}{4}+\frac{\phi_0 k_f^2}{8\pi H},\frac{1}{2};\frac{2\pi H}{\phi_0}(x-x_0)^2\right),$$

and

$$u_2(x)=\sqrt{\frac{2\pi H}{\phi_0}}(x-x_0)\times\exp\left(-\frac{\pi H}{\phi_0}(x-x_0)^2\right)\Phi\left(\frac{3}{4}+\frac{\phi_0 k_f^2}{8\pi H},\frac{3}{2};\frac{2\pi H}{\phi_0}(x-x_0)^2\right).$$

Eliminating the constants C_1 and C_2, with denoting $u'_{1,2}=\xi_f\mathrm{d}u_{1,2}/\mathrm{d}x$, to obtain the boundary conditions for F_s

$$\frac{\mathrm{d}}{\mathrm{d}x}F_s(d_s)=0,\quad \xi_s\frac{\mathrm{d}}{\mathrm{d}x}F_s(0)=W_\parallel(\omega)F_s(0), \tag{38}$$

$$W_\parallel(\omega)=\frac{\gamma}{\gamma_b+B_\parallel(\omega)},$$

$$B_\parallel(\omega)=\frac{u_1(0)u'_2(-d_f)-u_2(0)u'_1(-d_f)}{u'_1(0)u'_2(-d_f)-u'_2(0)u'_1(-d_f)}. \tag{39}$$

Repeating the same arguments that have been discussed in the previous subsection, we obtain the secular equation for determining the parallel upper critical field $H_{c2\parallel}(T)$ as a function of temperature

$$\det\left|\delta_{mm'}-\lambda\frac{T}{T_{cs}}\sum_\omega L^{bi}_{mm'\parallel}(\omega)\right|=0. \tag{40}$$

Therefore the matrix element

$$\begin{aligned}L^{bi}_{mm'\parallel}(\omega)=&\frac{\delta_{mm'}}{\xi_s^2(k_{s\parallel}^2+q_m^2)}-\frac{\Omega_\parallel(\omega)}{\xi_s^4(k_{s\parallel}^2+q_m^2)(k_{s\parallel}^2+q_{m'}^2)}\\&+\frac{2\Omega_\parallel(\omega)h_{c2\parallel}^2}{\xi_s^2(k_{s\parallel}^2+q_m^2)}\\&\times\sum_{l=-\infty}^{\infty}\left(\frac{x_0}{d_s}v_l^++\left(1-\frac{x_0}{d_s}\right)(-1)^l v_l^-\right)L^{bi}_{lm'\parallel}\\&-\frac{2h_{c2\parallel}^2}{\xi_s^2(k_{s\parallel}^2+q_m^2)}\\&\times\sum_{l\neq m}\left(\frac{x_0}{d_s}+\left(1-\frac{x_0}{d_s}\right)(-1)^{m+l}\right)\frac{L^{bi}_{lm'\parallel}}{\xi_s^2(q_m-q_l)^2}\end{aligned} \tag{41}$$

has the self-consistency manner due to the parallel orientation of magnetic field causes the electron eigenstates coupling to each others. Here

$$\Omega_\parallel(\omega)=\frac{W_\parallel(\omega)}{(d_s/\xi_s)\left(1+\frac{W_\parallel(\omega)}{k_{s\parallel}\xi_s}\coth(k_{s\parallel}d_s)\right)}, \tag{42}$$

$$v_l^\pm=\sum_{m\neq l,m=-\infty}^{\infty}\frac{(\pm1)^m}{\xi_s^4(q_m-q_l)^2(k_{s\parallel}^2+q_m^2)}, \tag{43}$$

and

$$k_{s\parallel}^2=k_s^2+\left(\frac{h_{c2\parallel}d_s}{\xi_s^2}\right)^2\left(\frac{1}{3}-\frac{x_0}{d_s}+\left(\frac{x_0}{d_s}\right)^2\right), \tag{44}$$

where $h_{c2\parallel}=2\pi H\xi_s^2/\phi_0$ is the dimensionless parallel magnetic field.

In the SMA scheme, the pair-breaking parameter, including the orbital field effect reads as

$$\varrho(t)=\varrho^{bi}(t_c)+h_{c2\parallel}^2\left(\frac{d_s}{\xi_s}\right)^2\left[\frac{1}{3}-\frac{x_0}{d_s}+\left(\frac{x_0}{d_s}\right)^2\right], \tag{45}$$

where $\varrho^{bi}(t_c)$ is given by (32), in deriving this expression an asymptotic behavior of $u_{1,2}(x)$ function has been used i.e., $u_{1,2}(x)=\exp(\pm k_f x)$. It can be seen that the vortex nucleation is also accounted in the pair-breaking parameter which is the generalization of Ref. [46].

We may reproduce the work of Radović et al. [46] by considering the F/S/F system and using the relation $d_s^{bi}=d_s^{tri}/2$, the explicit form of $\varrho^{tri}(t)$ is

$$\varrho^{tri}(t)=\frac{2\gamma(\xi_s/d_s^{tri})}{\gamma_b+\coth(k_f d_f)/k_f\xi_f}+h_{c2\parallel}^2\left(\frac{d_s^{tri}}{2\xi_s}\right)^2\left[\frac{1}{3}-\frac{2x_0}{d_s^{tri}}+\left(\frac{2x_0}{d_s^{tri}}\right)^2\right]. \tag{46}$$

By taking: (1) no boundary resistivity $\gamma_b=0$, (2) a thin S film embedded in a ferromagnet $k_f d_f\gg1$, and (3) no vortices appear in the decoupled S layer so the superconductivity nucleation starting in the middle of the film $x_0=0$, thus $\varrho^{tri}(t)$ becomes

$$\varrho^{tri}(t)=\frac{2\gamma(k_f\xi_f)}{d_s^{tri}/\xi_s}+\frac{1}{12}h_{c2\parallel}^2\left(\frac{d_s^{tri}}{\xi_s}\right)^2, \tag{47}$$

which is identical to Radović et al.'s result. Also, for the case of thin S film in vacuum, $\gamma=0$, and $T_{cs}=T_c$, (30) and (47) imply the laminar nucleation field near the critical temperature T_c, $H_{c2\parallel}=\sqrt{3}\phi_0/\pi d_s\xi_{GL}(T)$, where $\xi_{GL}(T)=(\pi/2)\xi_s(1-T/T_c)^{-1/2}$, is the Ginzburg–Landau coherence length [53].

We perform the numerical calculations of $h_{c2\perp}$ and $h_{c2\parallel}$ vs. t by varying the boundary resistivity γ_b (Figs. 1–3) and the position of superconducting nucleus x_0 (Figs. 4 and 5). Here the orbital field effect in F is neglected and the vortex nucleation is confined in S ($0\leqslant x_0/d_s\leqslant1$). We limit ourself to the case of weak ferromagnet ($I/\pi T_{cs}=10$) and weak proximity effect ($\gamma=0.2$). In Figs. 1–3, we take $d_s/\xi_s=3$ and $d_f/\xi_f=2$ (Figs. 1 and 2), and 0.5 (Fig. 3).

The perpendicular upper critical field $h_{c2\perp}$ shows the linear temperature dependence near T_c for any γ_b. For the parallel upper critical fields $h_{c2\parallel}$, we find $x_0/d_s=0.55$ gives the maximum of $h_{c2\parallel}$ at all temperature ranges and the square-root temperature dependence of $h_{c2\parallel}$ in the vicinity of T_c is illustrated in Figs. 2 and 3 for different values of γ_b. The comparison shows that the 2D behavior is not caused by the size of the F layer (thick or thin). Therefore we can see that the critical fields, in both directions, increase as γ_b increases, and the F layer thicknesses become important only for small γ_b. In Figs. 4 and 5 the thicknesses of the S layer are taken as $d_s/\xi_s=1.5$, and 4.0, and vary a parameter x_0/d_s in each

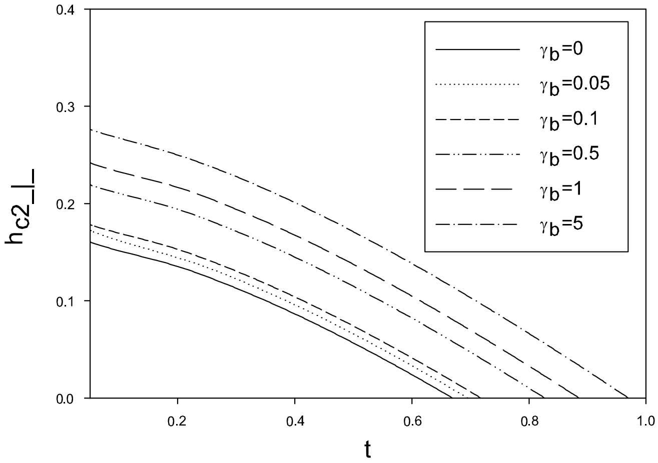

Fig. 1. Reduced perpendicular upper critical field $h_{c2\perp}$ as a function of reduced temperature t for several values of γ_b. $I/\pi T_{cs} = 10$, $d_s/\xi_s = 3$, $d_f/\xi_f = 2$, and $\gamma = 0.2$.

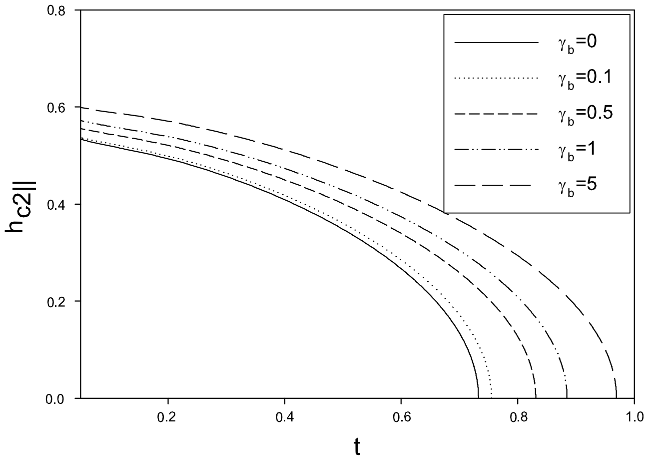

Fig. 2. Reduced parallel upper critical field $h_{c2\parallel}$ vs. reduced temperature t with varying γ_b. $x_0/d_s = 0.55$, and other parameters are the same as in Fig. 1.

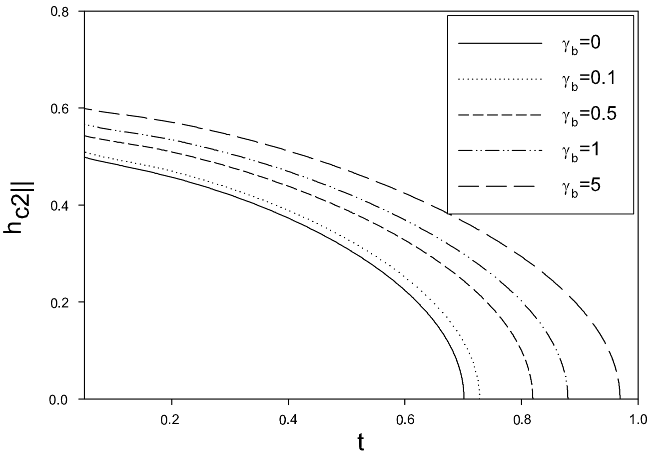

Fig. 3. Reduced parallel upper critical field $h_{c2\parallel}$ vs. reduced temperature t for different values of γ_b. $d_f/\xi_f = 0.5$. The values of other parameters are the same as in Fig. 2.

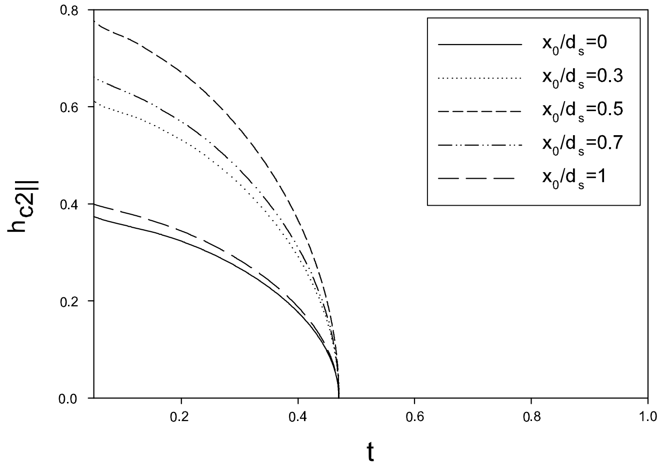

Fig. 4. Reduced parallel upper critical field $h_{c2\parallel}$ as a function of reduced temperature t for several values of x_0/d_s. $d_s/\xi_s = 1.5$, $d_f/\xi_f = 2$, $\gamma = 0.2$, $\gamma_b = 0.1$, and $I/\pi T_{cs} = 10$.

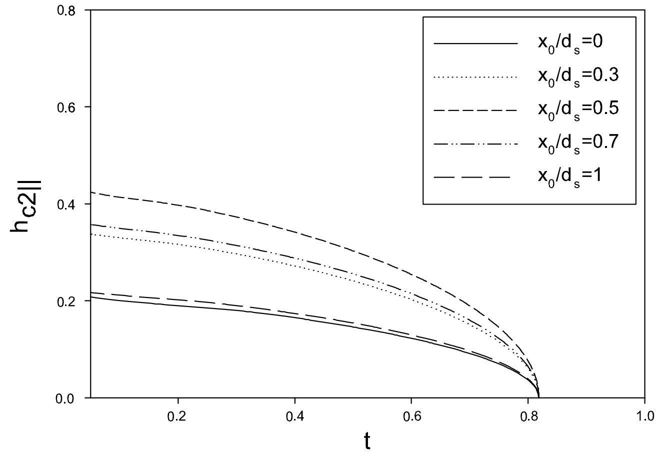

Fig. 5. Reduced parallel upper critical field $h_{c2\parallel}$ as a function of reduced temperature t for several values of x_0/d_s. $d_s/\xi_s = 4$. The values of other parameters are the same as in Fig. 4.

curve. It is clear that when $d_s/\xi_s = 4$, all curves in Fig. 5 exhibit a 2D behavior, this means that $h_{c2\parallel}$ is in the decoupled regime and so the Cooper pairs in the S layer do not penetrate through the F layer. In the case of the thinner S layer (Fig. 4), a 3D-like feature is seen around the mid-film, even for weak ferromagnets. The linearity diminishes as the center of superconducting nuclei shifts away from the middle point. As a result, the dimensional crossover from 3D to 2D reaches to T_c. On the other hand, a 2D behavior is still kept when superconducting nuclei are formed near the film boundaries with the lower critical fields.

4. F/S superlattices

As already mentioned in Section 2 that the superlattice consists of a repeated structure of bilayers. This means the set of Usadel's equations and their procedures that lead to the secular equation can be applied directly except only the boundary conditions must be changed. We can write down two pairs of boundary conditions by means of (8), (9) and (11) as

$$F_s(0) = e^{-i\varphi}\left[F_f(d_s+d_f) + \gamma_b\xi_f\frac{d}{dx}F_f(d_s+d_f)\right],\quad F_s(d_s) = F_f(d_s) - \gamma_b\xi_f\frac{d}{dx}F_f(d_s), \tag{48}$$

and

$$\xi_s\frac{d}{dx}F_s(0) = e^{-i\varphi}\gamma\xi_f\frac{d}{dx}F_f(d_s+d_f),\quad \xi_s\frac{d}{dx}F_s(d_s) = \gamma\xi_f\frac{d}{dx}F_f(d_s). \tag{49}$$

We use (48) to evaluate the constant coefficients of the general solution of F_f in terms of F_s and utilize them with (49) to obtain the boundary conditions for F_s at $x = 0$ and d_s. As a result, the desired formulas are given by

$$\xi_s\frac{d}{dx}F_s(0) = \gamma[PF_s(0) + e^{-i\varphi}QF_s(d_s)],\quad \xi_s\frac{d}{dx}F_s(d_s) = \gamma[e^{i\varphi}RF_s(0) + SF_s(d_s)], \tag{50}$$

where the functions P, Q, R, and S will be shown explicitly later. The secular equations still have the forms like (28) or (40) but the matrix elements $L_{mm'}$ are more complicated than the bilayer case.

In the case of the perpendicular field, we have

$$L^{sl}_{mm'\perp} = \frac{\delta_{mm'}}{\xi_s^2(k_{s\perp}^2+q_m^2)} - \frac{\gamma X_{mm'\perp}}{(d_s/\xi_s)Y_\perp\xi_s^4(k_{s\perp}^2+q_m^2)(k_{s\perp}^2+q_{m'}^2)}, \tag{51}$$

where

$$X_{mm'\perp} = [P_\perp - (-1)^{m+m'}S_\perp] + [(-1)^{m'}e^{-i\varphi}Q_\perp - (-1)^m e^{i\varphi}R_\perp] + \frac{\gamma[1+(-1)^{m+m'}]}{k_{s\perp}\xi_s\sinh(k_{s\perp}d_s)}(\cosh(k_{s\perp}d_s) - (-1)^m)(Q_\perp R_\perp - P_\perp S_\perp), \tag{52}$$

$$Y_\perp = 1 + \frac{\gamma}{k_{s\perp}\xi_s\sinh(k_{s\perp}d_s)}((P_\perp - S_\perp)\cosh(k_{s\perp}d_s) + e^{-i\varphi}Q_\perp - e^{i\varphi}R_\perp) + \left(\frac{\gamma}{k_{s\perp}\xi_s}\right)^2(Q_\perp R_\perp - P_\perp S_\perp), \tag{53}$$

$$P_\perp = -S_\perp = p_f\xi_f[\coth(p_f d_f) + \gamma_b p_f\xi_f]/M_\perp, \tag{54}$$

$$Q_\perp = -R_\perp = -p_f\xi_f/(M_\perp\sinh(p_f d_f)), \tag{55}$$

$$M_\perp = 1 + 2\gamma_b p_f\xi_f\coth(p_f d_f) + \gamma_b^2(p_f\xi_f)^2. \tag{56}$$

In the parallel field case, we have

$$\begin{aligned} L^{sl}_{mm'\parallel} = {} & \frac{\delta_{mm'}}{\xi_s^2(k_{s\parallel}^2+q_m^2)} - \frac{\gamma X_{mm'\parallel}}{(d_s/\xi_s)Y_\parallel\xi_s^4(k_{s\parallel}^2+q_m^2)(k_{s\parallel}^2+q_{m'}^2)} \\ & + \frac{2\gamma h_{c2\parallel}^2}{(d_s/\xi_s)Y_\parallel\xi_s^2(k_{s\parallel}^2+q_m^2)} \times \sum_{l=-\infty}^{\infty}\left(\frac{x_0}{d_s}Z^+_{ml} + \left(1-\frac{x_0}{d_s}\right)(-1)^l Z^-_{ml}\right)L^{sl}_{lm'\parallel} \\ & - \frac{2h_{c2\parallel}^2}{\xi_s^2(k_{s\parallel}^2+q_m^2)} \times \sum_{l\neq m}\left(\frac{x_0}{d_s} + \left(1-\frac{x_0}{d_s}\right)(-1)^{m+l}\right)\frac{L^{sl}_{lm'\parallel}}{\xi_s^2(q_m-q_l)^2}, \end{aligned} \tag{57}$$

here, $X_{mm'\parallel}$ and $Y_\parallel$ are obtained by making the substitution the subscript $\perp$ by $\parallel$ in $X_{mm'\perp}$ and $Y_\perp$, respectively. The remaining functions are therefore

$$Z^\pm_{ml} = [P_\parallel - (-1)^m e^{i\varphi}R_\parallel]v_l^\pm + [e^{-i\varphi}Q_\parallel - (-1)^m S_\parallel]v_l^\mp + \frac{\gamma[v_l^\pm + (-1)^m v_l^\mp]}{k_\parallel\xi_s\sinh(k_{s\parallel}d_s)}(\cosh(k_{s\parallel}d_s) - (-1)^m)\Big(Q_\parallel R_\parallel - P_\parallel S_\parallel\Big), \tag{58}$$

$$P_\parallel = ([u_2(d_s) - \gamma_b u_2'(d_s)]u_1'(d_s+d_f) - [u_1(d_s) - \gamma_b u_1'(d_s)]u_2'(d_s+d_f))/M_\parallel, \tag{59}$$

$$Q_\parallel = \big(u_1(d_s+d_f)u_2'(d_s+d_f) - u_1'(d_s+d_f)u_2(d_s+d_f)\big)/M_\parallel, \tag{60}$$

$$R_\parallel = (u_1'(d_s)u_2(d_s) - u_1(d_s)u_2'(d_s))/M_\parallel, \tag{61}$$

$$S_\parallel = ([u_1(d_s+d_f) + \gamma_b u_1'(d_s+d_f)]u_2'(d_s) - [u_2(d_s+d_f) + \gamma_b u_2'(d_s+d_f)]u_1'(d_s))/M_\parallel, \tag{62}$$

$$M_\parallel = [u_1(d_s+d_f) + \gamma_b u_1'(d_s+d_f)][u_2(d_s) - \gamma_b u_2'(d_s)] - [u_2(d_s+d_f) + \gamma_b u_2'(d_s+d_f)][u_1(d_s) - \gamma_b u_1'(d_s)]. \tag{63}$$

At this stage we would emphasize that (51)–(56) are the generalizations of Refs. [35,48] for determining T_c and $H_{c2\perp}$ of S/F superlattices in the exact multimode calculation, while the set of the secular equations for the parallel field orientation, (57)–(63), allows one to investigate the existence of the unusual, $\varphi \neq 0$, ground state.

We now treat both field orientations in the SMA. It is obvious that there is a need for only the exchange field contribution to the pair-breaking parameter, $\varrho(t_c)$, and after the straightforward algebra, we obtain

$$\varrho^{sl}(t_c,\varphi) = \left\{\frac{2\gamma}{(d_s/\xi_s)}(P_\perp - \cos\varphi R_\perp) + \gamma^2(P_\perp^2 - R_\perp^2)\right\}\Bigg|_{T=T_c}, \tag{64}$$

where it is understood that at $T = T_c$, $H_{c2\perp} = 0$, and then $p_f \to k_f$ in the strong ferromagnetic field limit. We can see that the phases $\varphi = 0$, and π are the most stable ground states with $\varrho^{sl}(t_c, \varphi = 0) < \varrho^{sl}(t_c, \varphi = \pi)$, and so $T_c(\varphi = 0) < T_c(\varphi = \pi)$ in some parameter ranges due to the oscillatory characteristic of T_c. There exist some domain stabilities which compete against the two phases.

The numerical calculations are shown in Figs. 6–11. In Fig. 6 the transition temperature variations t_c vs. d_f/ξ_f in a zero-phase characterize various types of t_c oscillations, as well as in F/S/F trilayers [51], depend on the values of γ_b. At a very low γ_b, superconductivity is rapidly destroyed near boundary interfaces. The nonmonotonic decay of t_c, including the reentrant behavior, at a moderate γ_b, t_c has a minimum at some d_f/ξ_f and it rises again. The region of t_c suppression is at $0.5 < d_f/\xi_f < 1.5$, i.e., about a monolayer

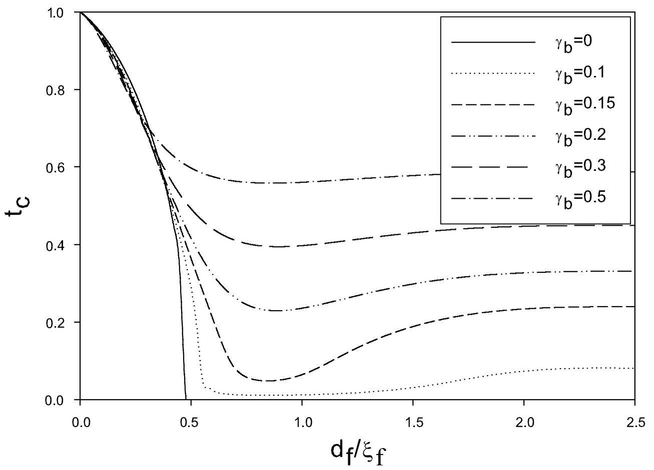

Fig. 6. The reduced superconducting transition temperature t_c as a function of the reduced ferromagnetic layer thickness d_f/ξ_f with varying γ_b. $I/\pi T_{cs} = 10$, $d_s/\xi_s = 3$, $\gamma = 0.2$, and $\varphi = 0$ (zero-phase).

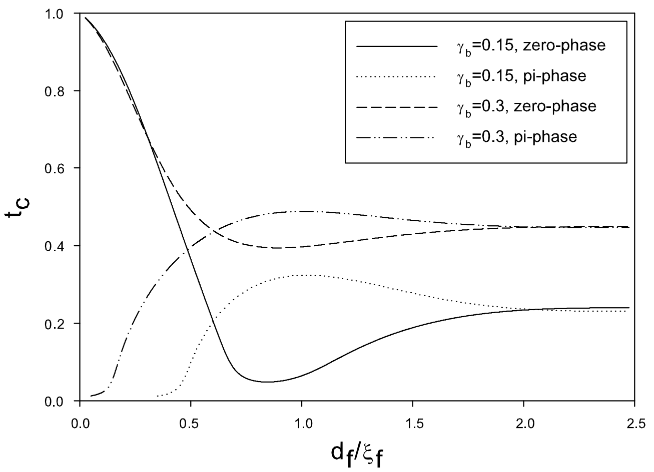

Fig. 7. $t_c(d_f/\xi_f)$ curves between zero-phase ($\varphi = 0$) and pi-phase ($\varphi = \pi$) for $\gamma_b = 0.15$, and 0.3. The values of other parameters are the same as in Fig. 6.

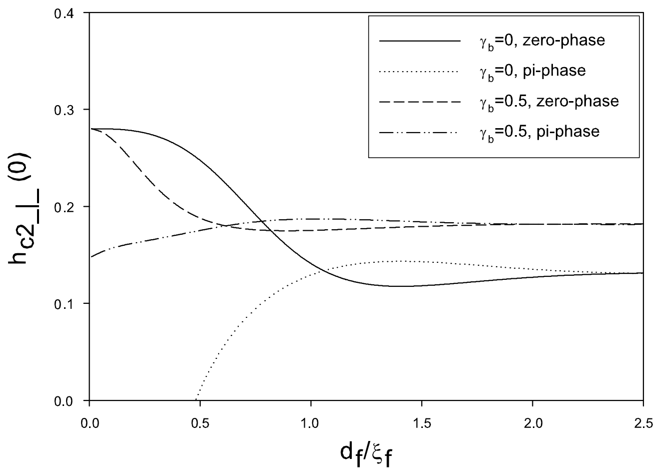

Fig. 8. Reduced perpendicular upper critical field at zero temperature $h_{c2\perp}(0)$ as a function of reduced ferromagnetic layer thickness d_f/ξ_f between zero-phase and pi-phase for two values of $\gamma_b = 0$, and 0.5. The other parameters are $I/\pi T_{cs} = 10$, $d_s/\xi_s = 3$, $\gamma = 0.1$.

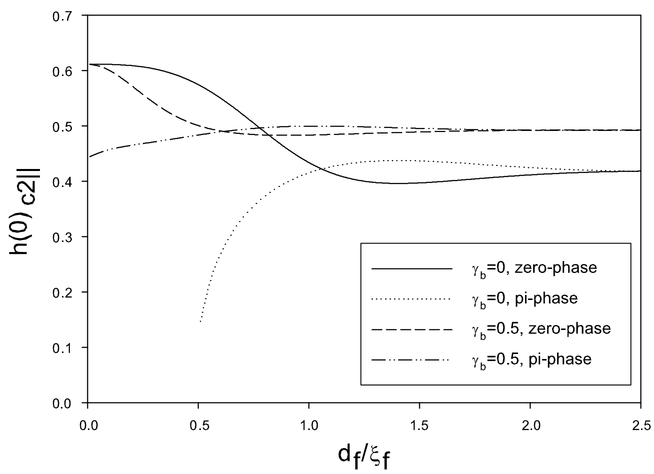

Fig. 9. $h_{c2\parallel}(0)$ vs. d_f/ξ_f between zero-phase and pi-phase for $\gamma_b = 0$, and 0.5. The other parameters are the same as in Fig. 8.

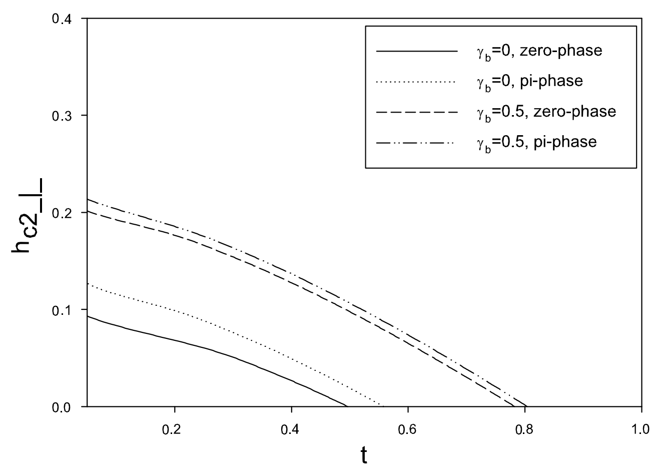

Fig. 10. The linear temperature dependence of reduced perpendicular upper critical field $h_{c2\perp}$ with varying γ_b. $d_f/\xi_f = 1$, and other parameters are taken from Fig. 8.

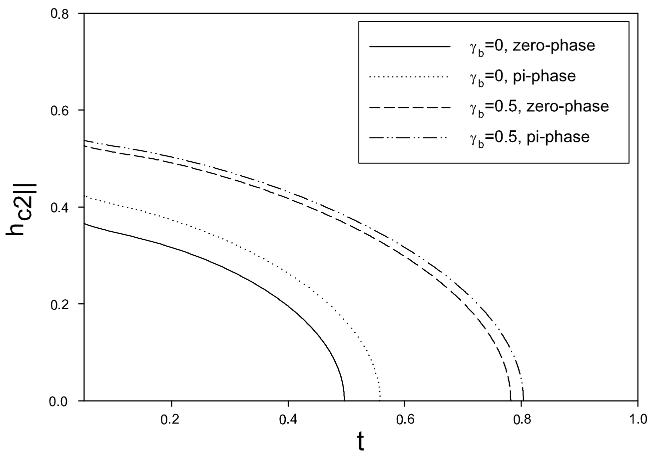

Fig. 11. Reduced parallel upper critical field $h_{c2\parallel}$ vs. t. Parameters are the same as in Fig. 10.

of F films. The interplay between the zero-phase ($\varphi = 0$) and the pi-phase ($\varphi = \pi$) is drawn in Fig. 7, their stability domain exists at some ranges of d_f/ξ_f. The crossed-curves indicate the possibility of pi-phases and is feasible when $d_f/\xi_f < 2$. The difference of t_c between the two phases decreases as γ_b increases and the maximum widths lie at $d_f/\xi_f \approx 1$, or the single F film.

Similar to t_c oscillations, the critical fields at zero temperature, $h_{c2\perp}(0)$ and $h_{c2\parallel}(0)$, can exhibit oscillatory behaviors inside the F layers (Figs. 8 and 9). In the region of strong oscillations the pi-phase state is energetically favorable at all temperature ranges (Figs. 10 and 11). Although, the cross-section curves for each γ_b become broadened when γ_b increases.

5. Conclusions

We have investigated the temperature dependence of the upper critical fields, $H_{c2\perp}$ and $H_{c2\parallel}$, of F/S layered structures. The phase diagram (H, T) is obtained from the secular equation of the linearized self-consistent order parameter equation which has been solved by the exact multimode method. The reduction equations in the single-mode approximation are given. Our attention mainly involves the influence of the boundary resistivity γ_b on critical fields.

For the F/S bilayers the perpendicular upper critical field shows the linear temperature dependence for any γ_b whereas the parallel field reveals a 2D behavior independent of the F layer thickness. The factors that decide the dimensionality are the thickness of the S layer (d_s/ξ_s) and the position of superconducting nucleation (x_0/d_s). We find for the thicker S layer, there is no vortex regime at all temperature ranges.

For the F/S superlattices, we have investigated the possibility of the pi-phase state. The obtained results show that it is more energetically favorable than the zero-phase in a certain region of d_f/ξ_f. The oscillatory t_c behavior in the zero-phase is of the same feature as in F/S/F trilayers. Also, the critical fields at zero temperature are predicted to oscillate as a function of the F layer thickness.

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Pair breaking effects on upper critical fields in ferromagnet/superconductor layered structures

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Abstract

The influence of magnetic and spin–orbit impurity scattering on the upper critical fields H_{c2} in ferromagnet/superconductor (F/S) hybrid structures is theoretically investigated. The generalized Usadel equations which are a pair of coupled equations containing additional spin–orbit and magnetic impurity scattering are shown to be decoupled in spite of the coupling interaction between the spin exchange field and the spin–orbit interaction in the F layer. The temperature dependence of the parallel upper critical field in the F/S bilayer is shown to be less prominent in the presence of the spin–orbit and the spin–flip scattering processes. The interplay between the zero- and pi-phases is analyzed in the case of the F/S multilayers. The pi-phase formation is found to be feasible through the strong oscillation of the zero temperature upper critical fields versus the thickness of the ferromagnetic layer. Our results which are valid in wide range of parameters may be very important for comparison with experimental data.

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1. Introduction

Nowadays it is known that the interplay between superconductor (S) and ferromagnet (F) provides several striking phenomena such as the nonmonotonic behaviors of the critical temperature and the Josephson current as a function of the ferromagnetic layer thickness [1–4]. Although they are two antagonistic orderings which suppress each other. Nevertheless when they are spatially separated, the coexistence may be realized in F/S layered structures and this is the so-called proximity effect. The main assumption of the proximity effect theory is characterized by the existence of the pair amplitude inside F, even though there is no pairing interaction and consequently the vanishing of the order parameter. The presence of the ferromagnetic exchange field plays the role that provokes an oscillation of the pair amplitude in F. Thus this system is unlike an S/N proximity effect or a bulk impure superconductor in which the critical temperature decays monotonously as the N layer thickness increases, because of the impurity depairing effect.

The theoretical description of the proximity effect based on the Usadel equations [5] is the simplified version of the Eilenberger quasi-classical theory [6] when the electron mean free path is shorter than the superconducting coherence length, which is the so-called diffusive regime. Another corresponding parameter is given by the condition when the exchange field is much less than the inverse of the momentum relaxation time.

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It is well known that in the vicinity of the second-order phase transition, the nonlinear Usadel equations can be retained up to the linear term and the order parameter is obtained in a self-consistent way. The pair amplitudes in each layer are related through the boundary conditions at the F/S interfaces. One interesting feature is the occurrence of the pi-phase state as the superconducting order parameter changes its sign relative to the adjacent layers. The evidence of the pi-phase state has recently been confirmed experimentally [7]. Another remarkable feature is the dimensional crossover behavior of the temperature dependent upper critical field which indicates the coupling phenomena via the proximity effect. In the previous paper [8], we have shown that the dimensionality of the system is determined by the thickness of the superconducting layer and the position of the superconducting nucleation. We have also shown that the upper critical fields are predicted to oscillate spatially inside the F layers and the stability domain of the zero-phase versus the pi-phase states is feasible only for a monolayer.

Now in a dirty ferromagnet apart from the spin exchange field and the orbital diamagnetism, other pair-breaking mechanisms such as the spin-flip and spin–orbit interactions are also important. The spin–orbit scattering has been considered by Demler et al. [9] and Oh et al. [10] in an attempt to resolve the contradicting results between theory and experiment. They found that the oscillatory behavior of the superconducting transition temperature is reduced by the spin–orbit scattering and strongly depends on the material parameters. However, the interfacial boundary resistivity had not been considered in their work. Very recently, the influence of both the spin–flip and the spin–orbit scattering on the nonmonotonic dependence of T_c has been investigated by Fauré et al. [11] who showed that the spin–flip scattering can modify the T_c significantly.

In all theoretical works mentioned above the underlying physics of F/S proximity hybrids is mostly concerned with the oscillations of the critical temperature and the critical current for the variation of the ferromagnetic layer thickness. There is, to the best of our knowledge, no study on how the proximity effect the spin–flip and spin–orbit scattering processes can influence the dimension dependent behavior of the upper critical fields. In particular, the dimensional crossover from 3D to 2D in the parallel upper critical field is of very fundamental interest, theoretically and experimentally. Therefore in this paper we will attack this problem by extending the theoretical study of the upper critical fields, $H_{c2\perp}$ and $H_{c2\|}$, of our previous work so as to include the influence of various pair-breaking effects, especially, the spin–orbit interaction. For the F/S bilayers, attention will be paid to the dimensional crossover phenomenon which can be seen through the $(H_{c2\|}, T)$ phase diagram. For the F/S superlattices, we will focus on the effect of pi-phase shift on the zero temperature critical fields and the critical temperatures. As in Ref. [8], the method used here is based on the exact multimode linearized Usadel's equations.

2. Model and formulation

Assuming that the dirty-limit conditions are fulfilled, the F/S proximity effect problems are well described by the Usadel equations [5] or equivalently, by the Takahashi–Tachiki formalism [12]. The latter theory was extended by Auvil et al. [13] to include the effects of the spin–orbit and magnetic impurity scattering. Because the spin–orbit scattering mixes the up- and down-electron spins and the magnetic impurity scattering flips electron's spin states. This lead to the condition that the superconducting kernels $Q_\omega(\mathbf{r},\mathbf{r}')$, and $R_\omega(\mathbf{r},\mathbf{r}')$ are coupled together. The resulting set of coupled differential equations reads

$$\left[|\omega| - \frac{1}{2}D(\mathbf{r})\Pi^2 + \frac{1}{\tau_m}\right]F^+(\mathbf{r},\omega) + \mathrm{i}I(\mathbf{r})\mathrm{sgn}(\omega)F^-(\mathbf{r},\omega) = \Delta(\mathbf{r}), \tag{1}$$

$$\left[|\omega| - \frac{1}{2}D(\mathbf{r})\Pi^2 + \frac{1}{\tau_m} + \frac{1}{\tau_{so}}\right]F^-(\mathbf{r},\omega) + \mathrm{i}I(\mathbf{r})\mathrm{sgn}(\omega)F^+(\mathbf{r},\omega) = 0, \tag{2}$$

$$\Delta(\mathbf{r}) = \pi T N(\mathbf{r})V(\mathbf{r})\sum_\omega F^+(\mathbf{r},\omega), \tag{3}$$

where $\omega = (2n+1)\pi T$, with an integer n, $D(\mathbf{r})$ is the diffusion coefficient, $\Pi = \nabla - (2\pi\mathrm{i}/\phi_0)\mathbf{A}$ the gauge-invariant operator which is expressed in terms of a fluxoid ϕ_0 and a magnetic vector potential $\mathbf{A}$, $I(\mathbf{r})$ is the spin exchange field, τ_m is the spin–flip scattering time and τ_{so} is the spin–orbit scattering time. $N(\mathbf{r})$ and $V(\mathbf{r})$ are the position-dependent electronic density of states and pairing interaction, respectively. The anomalous Usadel functions $F^+(\mathbf{r},\omega)$ and $F^-(\mathbf{r},\omega)$ are introduced as the integral equations of the kernels $Q_\omega(\mathbf{r},\mathbf{r}')$ and $R_\omega(\mathbf{r},\mathbf{r}')$;

$$F^+(\mathbf{r},\omega) = \int \mathrm{d}^3r' Q_\omega(\mathbf{r},\mathbf{r}')\Delta(\mathbf{r}'), \tag{4}$$

$$F^-(\mathbf{r},\omega) = \int \mathrm{d}^3r' R_\omega(\mathbf{r},\mathbf{r}')\Delta(\mathbf{r}'). \tag{5}$$

The scattering rates can be related to an averaging the temperature Green functions over impurity configurations in the Born approximation and are defined as [14]

$$\frac{1}{\tau_m} = \frac{2\pi}{3}N(0)n_{imp}S(S+1)\int\frac{\mathrm{d}\Omega}{4\pi}|u_m|^2, \tag{6}$$

$$\frac{1}{\tau_{so}} = \frac{\pi}{3}N(0)n_{imp}\int\frac{\mathrm{d}\Omega}{4\pi}|u_{so}|^2\sin^2\theta, \tag{7}$$

where n_{imp} is the impurity concentration, S denotes the isotropic spin of the localized magnetic moment, u_m and u_{so} are the magnetic impurity and the spin–orbit scattering potentials, respectively.

To deal with the problems of F/S proximity effects, the parameter values are specified individually in each layer. According to the Radović model [15], the pair amplitude $F^\pm(\mathbf{r},\omega)$ exists even the order parameter $\Delta(\mathbf{r})$ vanishes inside the ferromagnet which is the result of the proximity of the superconductor. Let us observe that when we take $\Delta(\mathbf{r}) = 0$ in (1), the pair functions in the F layer $F^+(\mathbf{r},\omega)$

and $F^{-}(\mathbf{r},\omega)$ are proportional to each other. Then by introducing a new parameter α_ω via a relation $F_{\rm f}^{-}(\mathbf{r},\omega)=\alpha_\omega F_{\rm f}^{+}(\mathbf{r},\omega)$ [10], and doing some simple manipulations of (1) and (2), we find

$$(\Pi^2-\widetilde{k}_{\rm f}^{\,2})F_{\rm f}(\mathbf{r},\omega)=0, \tag{8}$$

with

$$\widetilde{k}_{\rm f}=\frac{1}{\xi_{\rm f}}\sqrt{\frac{|\omega|+1/\tau_{\rm m}+{\rm i}I\alpha_\omega}{\pi T_{\rm cs}}}, \tag{9}$$

where

$$\alpha_\omega=\operatorname{sgn}(\omega)\sqrt{1-\frac{1}{(I\tau_{\rm so})^2}}-{\rm i}\frac{1}{I\tau_{\rm so}}, \tag{10}$$

is the coupling parameter which comes from the fact that the spin–orbit interaction dissipates the exchange field in such a way that it reduces the pair-breaking effect. Meanwhile, the set of the Usadel equations for the S layer that needs to be considered is

$$(\Pi^2-k_{\rm s}^2)F_{\rm s}(\mathbf{r},\omega)=-\frac{1}{\pi T_{\rm cs}\xi_{\rm s}^2}\Delta(\mathbf{r}), \tag{11}$$

$$\Delta(\mathbf{r})=\pi T\lambda\sum_{\omega}F_{\rm s}(\mathbf{r},\omega), \tag{12}$$

where

$$k_{\rm s}=\frac{1}{\xi_{\rm s}}\sqrt{\frac{|\omega|}{\pi T_{\rm cs}}}. \tag{13}$$

$k_{\rm s}$ and $\widetilde{k}_{\rm f}$ are the frequency-dependent wave vectors in each layer, and the corresponding coherence length given by $\xi_{\rm s(f)}=\sqrt{D_{\rm s(f)}/2\pi T_{\rm cs}}$, with $T_{\rm cs}$ is the bulk critical temperature. λ denotes a dimensionless BCS coupling constant. Note that the superscript "+" has been dropped out of the pair amplitude $F_{\rm s,f}(\mathbf{r},\omega)$. As was first pointed out by Demler et al. [9], the oscillation of the pair amplitude is restricted in the range $0<1/\tau_{\rm so}<I$. In the limit where both $1/\tau_{\rm m}$, and $1/\tau_{\rm so}$ tend to zero we recover the standard Usadel equations as expected.

An explicit form of the gauge-invariant operator depends on an orientation of an external magnetic field. Given a layered film lying on the y–z plane, the gauge choices, $\mathbf{A}=(0,0,H_y)$ for the perpendicular applied magnetic field and $\mathbf{A}=(0,H_x,0)$ for the parallel one, may be chosen, where H is the strength of the applied field. Then we have

$$\Pi_\perp=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}-\frac{2\pi{\rm i}H_y}{\phi_0}\right).$$

and

$$\Pi_\parallel=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}-\frac{2\pi{\rm i}H_x}{\phi_0},\frac{\partial}{\partial z}\right).$$

The structures of F/S proximity systems under our consideration occupy the regions $-d_{\rm f}\leqslant x\leqslant 0$ for F and $0\leqslant x\leqslant d_{\rm s}$ for S. In the case of the perpendicular upper critical field, only the lowest Landau level is needed [16] then (11) takes the form

$$\frac{{\rm d}^2}{{\rm d}x^2}F_{\rm s}(x,\omega)-k_{\rm s\perp}^2F_{\rm s}(x,\omega)=-\frac{\Delta(x)}{\pi T_{\rm cs}\xi_{\rm s}^2}, \tag{14}$$

where $k_{\rm s\perp}=(k_{\rm s}^2+h_{\rm c2\perp}/\xi_{\rm s}^2)^{1/2}$, and $h_{\rm c2\perp}=(2\pi/\phi_0)H_{\rm c2\perp}\xi_{\rm s}^2$ is the dimensionless magnetic field. While the parallel field orientation causes all states coupled together, thus by introducing an extra parameter x_0 as the position of superconducting nucleation (11) can be written as

$$\frac{{\rm d}^2}{{\rm d}x^2}F_{\rm s}(x,\omega)-\left[k_{\rm s}^2+\left(\frac{2\pi H}{\phi_0}\right)^2(x-x_0)^2\right]F_{\rm s}(x,\omega)=-\frac{\Delta(x)}{\pi T_{\rm cs}\xi_{\rm s}^2}. \tag{15}$$

In both field orientations the orbital field effect in F layers may be omitted from (8) since we have assumed that the vortex nucleation is confined only in the S layer so that the general solution is immediately given by

$$F_{\rm f}(x,\omega)=C_1\cosh(\widetilde{k}_{\rm f}x)+C_2\sinh(\widetilde{k}_{\rm f}x), \tag{16}$$

where the unknown coefficients C_1 and C_2 can be eliminated by means of the boundary conditions that connect the pair amplitudes $F_{\rm s}$ and $F_{\rm f}$.

For the bilayered structure $F_{\rm s}$ and $F_{\rm f}$ satisfy

$$\frac{\rm d}{{\rm d}x}F_{\rm f}(-d_{\rm f})=0=\frac{\rm d}{{\rm d}x}F_{\rm s}(d_{\rm s}), \tag{17}$$

$$\xi_{\rm s}\frac{\rm d}{{\rm d}x}F_{\rm s}(0)=\gamma\xi_{\rm f}\frac{\rm d}{{\rm d}x}F_{\rm f}(0), \tag{18}$$

$$F_{\rm s}(0)=F_{\rm f}(0)+\gamma_{\rm b}\xi_{\rm f}\frac{\rm d}{{\rm d}x}F_{\rm f}(0). \tag{19}$$

Putting (16) into (17)–(19), we arrive at the boundary conditions of the S film

$$\xi_{\rm s}\frac{\rm d}{{\rm d}x}F_{\rm s}(d_{\rm s})=0, \tag{20}$$

$$\xi_{\rm s}\frac{\rm d}{{\rm d}x}F_{\rm s}(0)=W(\omega)F_{\rm s}(0), \tag{21}$$

$$W(\omega)=\frac{\gamma}{\gamma_{\rm b}+\coth(\widetilde{k}_{\rm f}d_{\rm f})/\widetilde{k}_{\rm f}\xi_{\rm f}}. \tag{22}$$

For the superlattice system, we have two pairs of boundary conditions

$$F_{\rm s}(0)={\rm e}^{-{\rm i}\varphi}\left[F_{\rm f}(d_{\rm s}+d_{\rm f})+\gamma_{\rm b}\xi_{\rm f}\frac{\rm d}{{\rm d}x}F_{\rm f}(d_{\rm s}+d_{\rm f})\right], \tag{23}$$

$$F_{\rm s}(d_{\rm s})=F_{\rm f}(d_{\rm s})-\gamma_{\rm b}\xi_{\rm f}\frac{\rm d}{{\rm d}x}F_{\rm f}(d_{\rm s}), \tag{24}$$

and

$$\xi_{\rm s}\frac{\rm d}{{\rm d}x}F_{\rm s}(0)={\rm e}^{-{\rm i}\varphi}\gamma\xi_{\rm f}\frac{\rm d}{{\rm d}x}F_{\rm f}(d_{\rm s}+d_{\rm f}), \tag{25}$$

$$\xi_{\rm s}\frac{\rm d}{{\rm d}x}F_{\rm s}(d_{\rm s})=\gamma\xi_{\rm f}\frac{\rm d}{{\rm d}x}F_{\rm f}(d_{\rm s}). \tag{26}$$

Applying (16) to the first pair of the boundary conditions, (23) and (24), to determine the constants C_1 and C_2, and then inserting them back to the second pair, (25) and (26), the equations are recast in the form

$$\xi_s \frac{d}{dx} F_s(0) = \gamma[PF_s(0) - e^{-i\varphi} RF_s(d_s)], \tag{27}$$

$$\xi_s \frac{d}{dx} F_s(d_s) = \gamma[e^{i\varphi} RF_s(0) - PF_s(d_s)], \tag{28}$$

with

$$P = \widetilde{k}_f \xi_f [\coth(\widetilde{k}_f d_f) + \gamma_b \widetilde{k}_f \xi_f]/M, \tag{29}$$

$$R = \widetilde{k}_f \xi_f / (M \sinh(\widetilde{k}_f d_f)), \tag{30}$$

$$M = 1 + 2\gamma_b \widetilde{k}_f \xi_f \coth(\widetilde{k}_f d_f) + \gamma_b^2 (\widetilde{k}_f \xi_f)^2, \tag{31}$$

are the material functions of the F layers. The parameters γ and γ_b represent a leakage of paired electrons from S to F, and a jump at the interfaces due to the transparency of materials, respectively [17]. The phase shift φ has been introduced to account for the pi-phase state.

In order to determine the upper critical fields and the superconducting critical temperatures, we have to solve the above equations for $F_s(x,\omega)$ and $\Delta(x)$ self-consistently subject to the boundary conditions. Instead of $F_s(x,\omega)$, let us consider the diffusive kernel $Q_\omega(x,x')$ for the S layer, and its differential equations. The exact multimode solution can be found by employing the method of eigenfunction expansion, that is

$$Q_\omega(x,x') = \sum_{m=-\infty}^{\infty} Q_\omega(q_m, x') \cos(q_m x), \tag{32}$$

with $q_m = m\pi/d_s$. The nontrivial solution of the transformed $\Delta(x)$ equation obeys the secular equation

$$\det \left| \delta_{mm'} - \lambda \frac{T}{T_{cs}} \sum_\omega L_{mm'}(\omega) \right| = 0, \tag{33}$$

where we have denoted

$$L_{mm'}(\omega) = \pi T_{cs} \int_0^{d_s} dx' Q_\omega(q_m, x') \cos(q_{m'} x'). \tag{34}$$

Having obtained $Q_\omega(q_m, x')$, by the Fourier transformation of the differential equation of $Q_\omega(x,x')$, $L_{mm'}(\omega)$ is easily evaluated and it final form depends on the field orientations and the boundary conditions of the layered structures. The straightforward calculations of the matrix element $L_{mm'}(\omega)$ in the parallel field orientation yield

$$\begin{aligned} L^{bi}_{mm'\parallel}(\omega) = {} & \frac{\delta_{mm'}}{\xi_s^2(k_{s\parallel}^2 + q_m^2)} - \frac{\Omega_\parallel(\omega)}{\xi_s^4(k_{s\parallel}^2 + q_m^2)(k_{s\parallel}^2 + q_{m'}^2)} \\ & + \frac{2\Omega_\parallel(\omega) h_{c2\parallel}^2}{\xi_s^2(k_{s\parallel}^2 + q_m^2)} \\ & \times \sum_{l=-\infty}^{\infty} \left(\frac{x_0}{d_s} v_l^+ + \left(1 - \frac{x_0}{d_s}\right)(-1)^l v_l^- \right) L^{bi}_{lm'\parallel} \\ & - \frac{2h_{c2\parallel}^2}{\xi_s^2(k_{s\parallel}^2 + q_m^2)} \\ & \times \sum_{l \neq m} \left(\frac{x_0}{d_s} + \left(1 - \frac{x_0}{d_s}\right)(-1)^{m+l} \right) \frac{L^{bi}_{lm'\parallel}}{\xi_s^2(q_m - q_l)^2}, \end{aligned} \tag{35}$$

for the bilayer, where

$$\Omega_\parallel(\omega) = \frac{W(\omega)}{(d_s/\xi_s)\left(1 + \frac{W(\omega)}{k_{s\parallel}\xi_s} \coth(k_{s\parallel} d_s)\right)}, \tag{36}$$

$$v_l^\pm = \sum_{m \neq l, m=-\infty}^{\infty} \frac{(\pm 1)^m}{\xi_s^4 (q_m - q_l)^2 (k_{s\parallel}^2 + q_m^2)}, \tag{37}$$

$$k_{s\parallel}^2 = k_s^2 + \left(\frac{h_{c2\parallel} d_s}{\xi_s^2} \right)^2 \left(\frac{1}{3} - \frac{x_0}{d_s} + \left(\frac{x_0}{d_s} \right)^2 \right), \tag{38}$$

with $h_{c2\parallel} = 2\pi H \xi_s^2/\phi_0$ is the dimensionless parallel orbital magnetic field, and

$$\begin{aligned} L^{sl}_{mm'\parallel} = {} & \frac{\delta_{mm'}}{\xi_s^2(k_{s\parallel}^2 + q_m^2)} \\ & - \frac{\gamma X_{mm'\parallel}}{(d_s/\xi_s) Y_\parallel \xi_s^4 (k_{s\parallel}^2 + q_m^2)(k_{s\parallel}^2 + q_{m'}^2)} \\ & + \frac{2\gamma h_{c2\parallel}^2}{(d_s/\xi_s) Y_\parallel \xi_s^2 (k_{s\parallel}^2 + q_m^2)} \\ & \times \sum_{l=-\infty}^{\infty} \left(\frac{x_0}{d_s} Z_{ml}^+ + \left(1 - \frac{x_0}{d_s}\right)(-1)^l Z_{ml}^- \right) L^{sl}_{lm'\parallel} \\ & - \frac{2h_{c2\parallel}^2}{\xi_s^2(k_{s\parallel}^2 + q_m^2)} \\ & \times \sum_{l \neq m} \left(\frac{x_0}{d_s} + \left(1 - \frac{x_0}{d_s}\right)(-1)^{m+l} \right) \frac{L^{sl}_{lm'\parallel}}{\xi_s^2(q_m - q_l)^2}, \end{aligned} \tag{39}$$

for the superlattice, where

$$\begin{aligned} X_{mm'\parallel} = {} & [1 + (-1)^{m+m'}]P - [(-1)^{m'} e^{-i\varphi} + (-1)^m e^{i\varphi}]R \\ & + \frac{\gamma[1 + (-1)^{m+m'}]}{k_{s\parallel}\xi_s \sinh(k_{s\parallel} d_s)} \left(\cosh(k_{s\parallel} d_s) - (-1)^m\right)\left(P^2 - R^2\right), \end{aligned} \tag{40}$$

$$\begin{aligned} Y_\parallel = {} & 1 + \frac{2\gamma}{k_{s\parallel}\xi_s \sinh(k_{s\parallel} d_s)} (P \cosh(k_{s\parallel} d_s) - \cos\varphi R) \\ & + \left(\frac{\gamma}{k_{s\parallel}\xi_s} \right)^2 (P^2 - R^2), \end{aligned} \tag{41}$$

$$\begin{aligned} Z_{ml}^\pm = {} & (v_l^\pm + (-1)^m v_l^\mp)P - ((-1)^m e^{i\varphi} v_l^\pm + e^{-i\varphi} v_l^\mp)R \\ & + \frac{\gamma[v_l^\pm + (-1)^m v_l^\mp]}{k_\parallel \xi_s \sinh(k_{s\parallel} d_s)} (\cosh(k_{s\parallel} d_s) - (-1)^m)(P^2 - R^2). \end{aligned} \tag{42}$$

The analogous formulae for the perpendicular field orientation are easily obtained by the following procedure: (i) replace $k_\parallel$ by $k_\perp$, (ii) change the subscript $\parallel$ in $\Omega_\parallel$, $X_{mm'\parallel}$, and $Y_\parallel$, to $\perp$, and (iii) take $L_{lm'}(\omega)$ to be zero.

An analytical approach using the secular Eq. (33) in the single mode approximation leads to the Abrikosov–Gorkov like-formula [18] and to the determination of the phase diagram (H,T) under the assumptions that (i) the S layer is thin $d_s/\xi_s \ll 1$, and (ii) the propagating momentum in F can be approximated to be frequency independent, so

$$\widetilde{k}_f \approx \frac{1}{\xi_f}\sqrt{\frac{1/\tau_m + 1/\tau_{so} + i\sqrt{I^2 - (1/\tau_{so})^2}}{\pi T_{cs}}}.$$

Then the $(0,0)$ element of (33) yields the simple formula

$$\ln t = \psi\left(\frac{1}{2}\right) - \mathrm{Re}\,\psi\left(\frac{1}{2} + \frac{\varrho(t)}{2t}\right), \tag{43}$$

where $t = T/T_{cs}$ is the reduced temperature, $\psi(x)$ is the digamma function, and the complex pair-breaking parameters

$$\varrho(t) = \varrho(t_c) + h_{c2\perp}(t), \tag{44}$$

$$\varrho(t) = \varrho(t_c) + h^2_{c2\parallel}(t)\left(\frac{d_s}{\xi_s}\right)^2\left(\frac{1}{3} - \frac{x_0}{d_s} + \left(\frac{x_0}{d_s}\right)^2\right), \tag{45}$$

for the perpendicular and the parallel magnetic fields, respectively. Eqs. (44) and (45) contain both contributions from the strong ferromagnetic exchange field $\varrho(t_c)$ and the orbital field effects depending on the orientations of applied magnetic fields either perpendicular or parallel to the layered planes. The orbital field completely vanishes at a transition point $t = t_c$ so (43) implies the equation for the variation of the superconducting transition temperature as a function of material parameters,

$$\varrho^{bi}(t_c) = \frac{\gamma(\xi/d_s)}{\gamma_b + \coth(\widetilde{k}_f d_f)/\widetilde{k}_f\xi_f}, \tag{46}$$

$$\varrho^{sl}(t_c, \varphi) = \frac{2\gamma}{(d_s/\xi_s)}(P - \cos\varphi R) + \gamma^2(P^2 - R^2). \tag{47}$$

Near T_c, we find that the universal ratio $h^2_{c2\parallel}/h_{c2\perp}$ does not depend on temperature, material parameters of the F film, pair-breaking scattering rate, number of layers, and phase shift angle, i.e.

$$\frac{h^2_{c2\parallel}}{h_{c2\perp}} = \frac{1}{(d_s/\xi_s)^2\left(\frac{1}{3} - \frac{x_0}{d_s} + \left(\frac{x_0}{d_s}\right)^2\right)}. \tag{48}$$

By taking the nucleation center at the mid S film, $x_0/d_s = 1/2$, which corresponds to the decoupled regime, we reobtain the result of Radović et al. [19].

3. Numerical results

We present the numerical calculations of the upper critical fields and critical temperatures of F/S bilayers (Figs. 1 and 2) and F/S multilayers (Figs. 3–6). For the bilayered structure the $(h_{c2\parallel}, t)$ phase diagrams are drawn to investigate the dimensional crossover behavior by varying the parameters associated with the spin–orbit scattering $1/I\tau_{so}$, and the spin–flip scattering $1/I\tau_m$, in Figs. 1 and 2, respectively. We consider the case of weak ferromagnet ($I/\pi T_{cs} = 10$), weak proximity effect ($\gamma = 0.2$), and low boundary resistivity ($\gamma_b = 0.1$). The position of superconducting nucleus $x_0/d_s = 0.55$ corresponds to the highest field $h_{c2\parallel}(t)$,

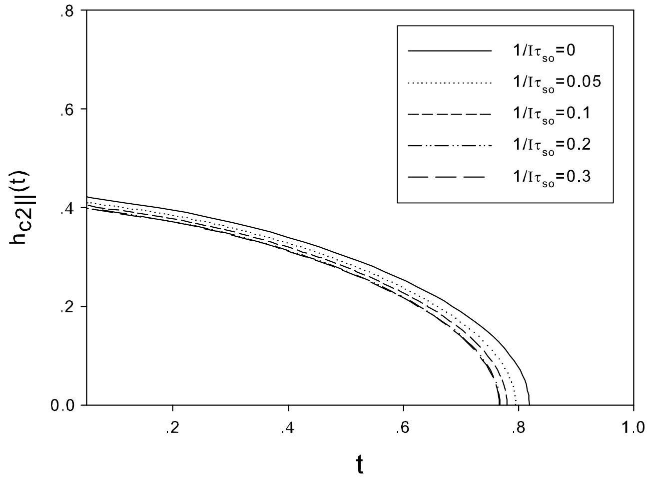

Fig. 1. Reduced parallel upper critical field $h_{c2\parallel} = 2\pi H\xi_s^2/\phi_0$ as a function of reduced temperature $t = T/T_{cs}$ for several values of $1/I\tau_{so}$. $I/\pi T_{cs} = 10$, $d_s/\xi_s = 4$, $d_f/\xi_f = 2$, $\gamma = 0.2$, $\gamma_b = 0.1$, $x_0/d_s = 0.55$, and $1/I\tau_m = 0$.

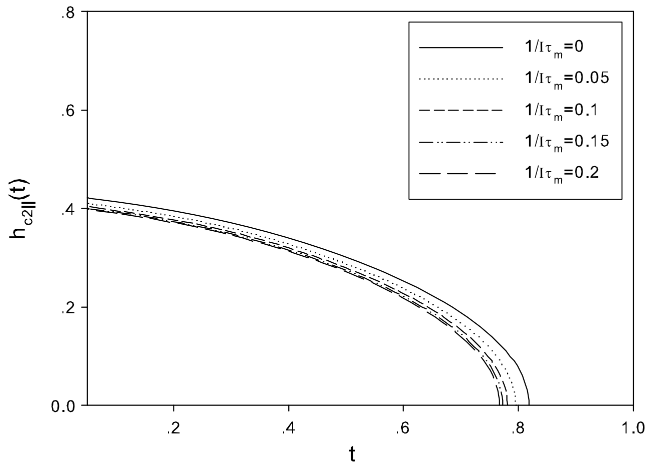

Fig. 2. Reduced parallel upper critical field $h_{c2\parallel}$ versus reduced temperature t with varying $1/I\tau_m$. $1/I\tau_{so} = 0$, and other parameters are the same as in Fig. 1.

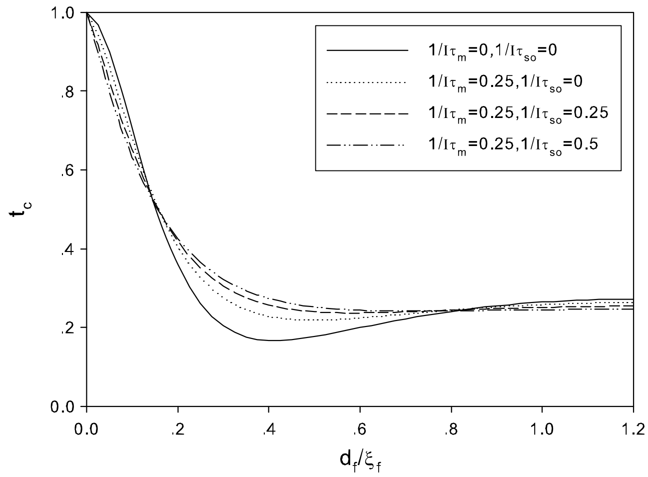

Fig. 3. The reduced superconducting transition temperature $t_c = T_c/T_{cs}$ as a function of the reduced ferromagnetic layer thickness d_f/ξ_f with varying $1/I\tau_{so}$, and $1/I\tau_m$. $I/\pi T_{cs} = 30$, $d_s/\xi_s = 3$, $\gamma = 0.2$, $\gamma_b = 0.3$, and $\varphi = 0$ (zero-phase).

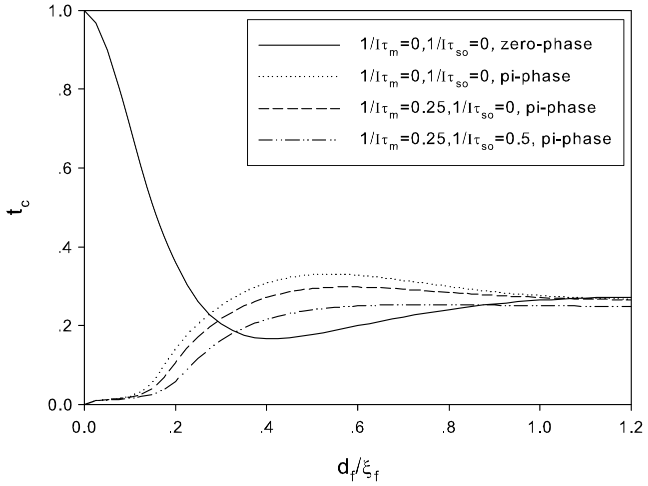

Fig. 4. $t_c(d_f/\xi_f)$ curves between zero-phase ($\varphi = 0$) and pi-phase ($\varphi = \pi$) for several values of $1/I\tau_{so}$, and $1/I\tau_m$. The values of other parameters are the same as in Fig. 3.

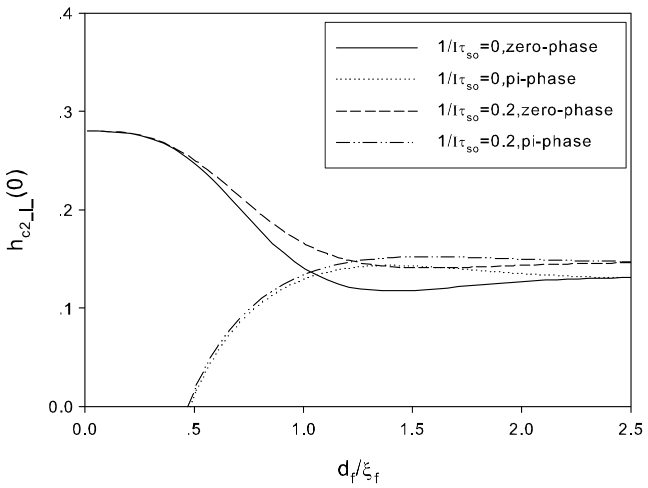

Fig. 5. Reduced perpendicular upper critical field at zero temperature $h_{c2\perp}(0)$ as a function of reduced ferromagnetic layer thickness d_f/ξ_f between zero-phase and pi-phase for two values of $1/I\tau_{so} = 0$, and 0.2. The other parameters are $I/\pi T_{cs} = 10$, $d_s/\xi_s = 3$, $\gamma = 0.1$, and $\gamma_b = 0$.

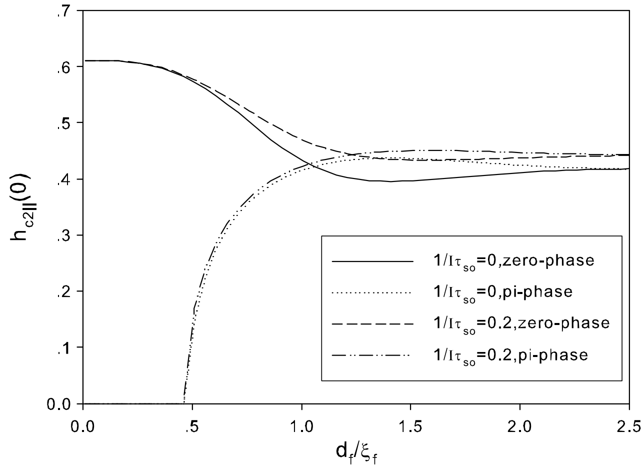

Fig. 6. Reduced parallel upper critical field at zero temperature $h_{c2\parallel}(0)$ as a function of reduced ferromagnetic layer thickness d_f/ξ_f between zero-phase and pi-phase for two values of $1/I\tau_{so} = 0$, and 0.2. The other parameters are the same as in Fig. 5.

when the film thicknesses are $d_s/\xi_s = 4$ for S and $d_f/\xi_f = 2$ for F. Since for the thicker S layer the parallel upper critical field $h_{c2\parallel}(t)$ displays a 2D feature independent of the exchange field strength and the F layer thickness, we find that a 2D behavior is still retained for any values of the scattering rates either spin–orbit or spin–flip. Both pair-breaking scatterers provide identical results, and in general, $h_{c2\parallel}(t)$ decreases as $1/\tau_{so}$ or $1/\tau_m$ increases and becomes saturated when $1/\tau_{so}$, $1/\tau_m \approx 0.2I$. This means we cannot distinguish which pair-breaking mechanism is responsible for the reduction of the parallel critical field $h_{c2\parallel}(t)$. Though the spin–orbit term is coupled to the exchange field through the coupling parameter α_ω which suppresses the oscillations of the pair amplitude in F as defined by the imaginary part of $\widetilde{k}_f$, whereas its real part which also contains the spin–flip term implies an extra decay of the pair amplitude. As already mentioned above, a 2D behavior is not altered in any way when the exchange field changes, therefore the only important factor which causes the effect will come from the real part of $\widetilde{k}_f$ which in turn makes the Cooper pair amplitude decay spatially and becomes conduction electrons in the F region and consequently this results in the reduction of $h_{c2\parallel}(t)$.

For multilayer systems, we examine the influence of the spin-dependent scattering on both the critical temperature and the zero temperature upper critical fields as a function of the ferromagnetic layer thickness. In Fig. 3, we show the variations of $t_c(d_f)$ in a zero-phase ($\varphi = 0$) for several values of the pair-breaking parameters $1/I\tau_{so}$, and $1/I\tau_m$. The influence of the scattering processes emerge when the exchange field is sufficiently strong ($I/\pi T_{cs} = 30$). Therefore the S layer thickness is not supposed to be thin ($d_s/\xi_s = 3$). In the absence of scattering processes with a perfect transparency ($\gamma_b = 0$), $t_c(d_f)$ exhibits the nonmonotonic decay within a single F layer. Upon the introduction of the spin–orbit and the spin–flip scattering, t_c decreases at first near the interface. An enhancement of t_c occurs later inside F with less pronounced nonmonotonicity. As is well known [9], the spin–orbit interaction produces a smaller exchange field and gives a higher critical temperature resulting from an increase of the oscillation period of the pair amplitude. However, the spin–flip scattering plays the same role as the spin–orbit does. Furthermore, Fig. 4 demonstrates the competition between a zero-phase and a pi-phase. Unfortunately, the highest critical temperature in the pi-phase state is most probable in the absence of the pair-breaking processes. Our obtained result is therefore in contrast to the previous study by Oh et al. [10] who argued that the pi-phase shift solution yields higher critical temperatures as the spin–orbit scattering increases. We would like to point out here that our solutions for t_c and $h_{c2\perp}$ are the correct ones because our formulae are found to agree perfectly with the case of an ordinary ferromagnet i.e., without the spin-dependent scattering process [15,16].

We next consider the reduced upper critical fields at zero temperature $h_{c2\perp}(0)$, and $h_{c2\|}(0)$ as a function of the normalized F layer thickness (d_f/ξ_f) with varying values of the spin–orbit scattering rate $1/I\tau_{so}$. Unlike the t_c oscillations, both perpendicular and parallel critical fields reveal an interesting interplay between the zero-and pi-phases, Figs. 5 and 6. The physical solution corresponds of course to the maximum field. Thus the pi-phase state is predicted to be most pronounced for certain values of ferromagnetic layer thicknesses. We note also that the difference between the critical fields for each parameter set becomes less when $1/\tau_{so}$ increases.

4. Conclusion

We have investigated the influence of the pair-breaking effects on the superconducting critical temperature and the upper critical fields in F/S hybrid structures. The generalized Usadel equations which include the spin–orbit and spin–flip interactions are solved in closed forms. The secular equations for the determination of the (H, T) phase diagrams are obtained.

For the F/S bilayers, the parallel upper critical field shows a 2D behavior as the S layer thickness increases and the critical field is lowered as the scattering rate increases. This feature is caused by the decrease of the decay length. Therefore the spin–orbit and the spin–flip interactions play exactly the same role in reducing the parallel upper critical field and they do not affect the dimensionality of the system in any way.

For the F/S multilayers attention is paid to the interplay between the zero-phase and the pi-phase. We have found that the evolution of t_c in the presence of the pair-breaking scattering processes is suitable for sufficiently strong ferromagnets. In the zero-phase the t_c is enhanced with the increase of the pair-breaking scattering rates in which the nonmonotonic character tends toward the monotonic decay. For a certain value of d_f, the pi-phase state is feasible only when the scattering processes are absent. Conversely, the zero temperature upper critical fields $h_{c2\perp}(0)$, and $h_{c2\|}(0)$ are found to exhibit the oscillatory behavior inside F. We predict that the pi-phase can exist within some ranges of d_f/ξ_f. This suggests the influence of the pair-breaking scattering effects which provokes the transition from the zero- to pi-state can be seen through $h_{c2}(0)$.

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