

With these weighting functions, the crossover frequency of the desired system is 2 rad/s. Singular value plots of open loop transfer function for the nominal plant and shaped plant are shown in Fig. 3 (a). As shown in this figure, the shaped plant has large gains at low frequencies. This makes the designed system good in terms of performance tracking and disturbance rejection. By using (3), the optimal stability margin ( $\varepsilon_{opt}$ ) is founded to be 0.361. This value indicates that the selected weights are compatible with robust stability requirement in the problem. By the conventional  $H_\infty$  loop shaping procedure,  $\varepsilon$  is selected as 0.328, which is less than the optimal value. Based on the procedure described in section 2.1,  $H_\infty$  loop shaping controller (HLS) can be synthesized as following.

$$K(HLS) = W_1 K_\infty W_2 = W_1 \begin{bmatrix} K_{11\infty} & K_{12\infty} \\ K_{21\infty} & K_{22\infty} \end{bmatrix} W_2 \quad (12)$$

The controller  $K_\infty$  (full order controller) in (12) is 15<sup>th</sup> order and complicated, making it difficult to implement practically. In order to design a more practical controller, full order controller is reduced in order after it is designed. In this example, a 5<sup>th</sup> order  $K_\infty$  controller (reduced order controller) is further designed by using the balanced truncation method [8, 9]. By substituting this controller into (4), the stability margin obtained from the reduced order controller is found to be 0.263. Clearly, the stability margin obtained from the reduced order controller is less than that of the full order controller. This indicates that the robustness of the designed system is decreased when the reduced order controller is applied.

Next, two fixed-structure robust controllers are designed. The structure of controllers is selected as PID with first-order derivative filter. Accordingly, these controllers are simple and easy to implement in real applications. These controllers are expressed in (13) and (14) for centralized and decentralized controllers, respectively.  $K_p$ ,  $K_i$ ,  $K_d$ , and  $\tau_d$  are parameters to be evaluated.

#### Centralized controller:

$$K(p) = \begin{bmatrix} K_{p1} + \frac{K_{i1}}{s} + \frac{K_{d1}s}{\tau_d s + 1} & K_{p2} + \frac{K_{i2}}{s} + \frac{K_{d2}s}{\tau_d s + 1} \\ K_{p3} + \frac{K_{i3}}{s} + \frac{K_{d3}s}{\tau_d s + 1} & K_{p4} + \frac{K_{i4}}{s} + \frac{K_{d4}s}{\tau_d s + 1} \end{bmatrix} \quad (13)$$

$$K_p = \begin{bmatrix} K_{p1} & K_{p2} \\ K_{p3} & K_{p4} \end{bmatrix}, K_i = \begin{bmatrix} K_{i1} & K_{i2} \\ K_{i3} & K_{i4} \end{bmatrix}, K_d = \begin{bmatrix} K_{d1} & K_{d2} \\ K_{d3} & K_{d4} \end{bmatrix}$$

**Decentralized controller:**

$$K(p) = \begin{bmatrix} K_{p1} + \frac{K_{i1}}{s} + \frac{K_{d1}s}{\tau_d s + 1} & 0 \\ 0 & K_{p4} + \frac{K_{i4}}{s} + \frac{K_{d4}s}{\tau_d s + 1} \end{bmatrix} \quad (14)$$

$$K_p = \begin{bmatrix} K_{p1} & 0 \\ 0 & K_{p4} \end{bmatrix}, K_i = \begin{bmatrix} K_{i1} & 0 \\ 0 & K_{i4} \end{bmatrix}, K_d = \begin{bmatrix} K_{d1} & 0 \\ 0 & K_{d4} \end{bmatrix}$$

In the optimization problem, the upper and lower bounds of control parameters and PSO parameters are set as follows:  $K_p \in [-10, 10]$ ,  $K_i \in [-10, 10]$ ,  $K_d \in [-5, 5]$ ,  $\tau_d \in [0.01, 1]$ , population size = 300, minimum and maximum velocities are 0 and 2 respectively, acceleration coefficients = 2.1, minimum and maximum inertia weights are 0.6 and 0.9, respectively, maximum iteration = 80. As shown in the above mentioned control parameter ranges, the selection of upper and lower bounds is easily carried out by observing the performance weight  $W_1$ . After running the PSO for 80 iterations, the optimal control parameters are found to be

**Case I: Centralized controller**

$$K_p = \begin{bmatrix} 2.6003 & -0.75154 \\ -1.1158 & -2.4684 \end{bmatrix}, K_i = \begin{bmatrix} 0.87399 & -0.28553 \\ 0.096274 & -0.96078 \end{bmatrix}, \quad (15)$$

$$K_d = \begin{bmatrix} 0.76181 & -0.30821 \\ -1.4953 & -0.028287 \end{bmatrix}, \tau_d = 0.14945$$

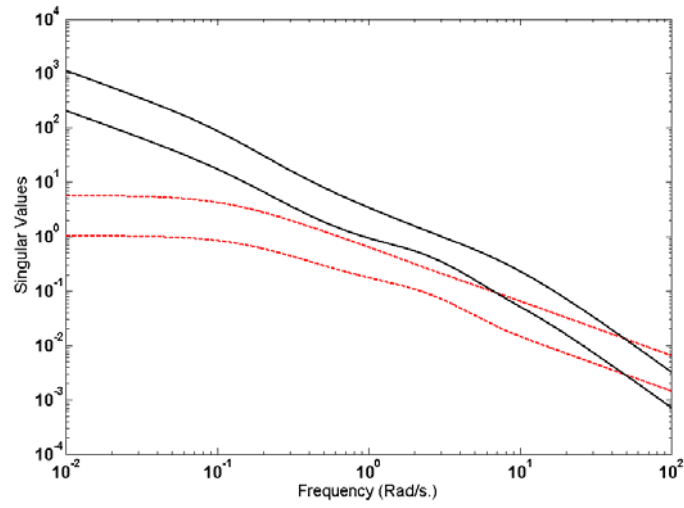
**Case II: Decentralized controller**

$$K_p = \begin{bmatrix} 1.9277 & 0 \\ 0 & -2.4511 \end{bmatrix}, K_i = \begin{bmatrix} 0.71976 & 0 \\ 0 & -0.74521 \end{bmatrix}, \quad (16)$$

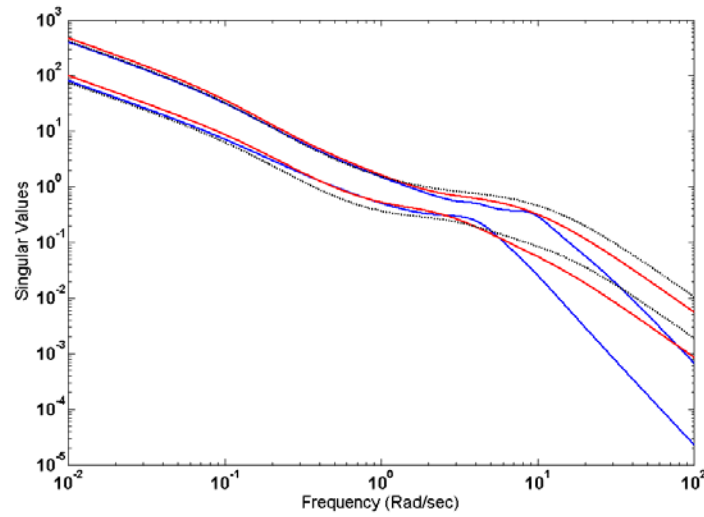
$$K_d = \begin{bmatrix} 0.80265 & 0 \\ 0 & -1.1439 \end{bmatrix}, \tau_d = 0.07566$$

Note that all of the designed controllers in this paper are the controllers in positive feedback control system. By using (4), the stability margins obtained from the proposed centralized and decentralized controllers are 0.343 and 0.274, respectively. The singular values are plotted to verify the proposed algorithm. Fig. 3 (b) shows comparison of the loop shapes by the proposed controllers and HLS. As seen in this figure, all loop shapes are close to the desired loop shape. It is verified that the proposed technique and conventional  $H_\infty$  loop shaping control are efficient methods to design a robust controller. Fig. 4 shows plots of convergence of

objective function (stability margin) versus iterations by PSO. As shown in the figure, the optimal robust PID controllers provide satisfactory stability margins at 0.343 and 0.274 for centralized and decentralized controllers, respectively.



(a)

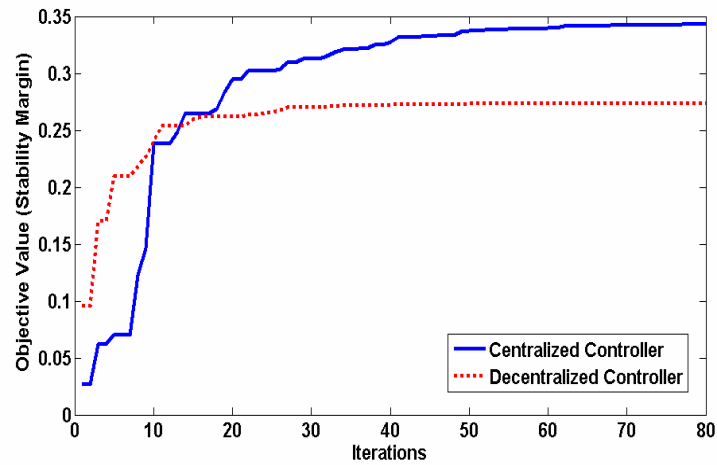


(b)

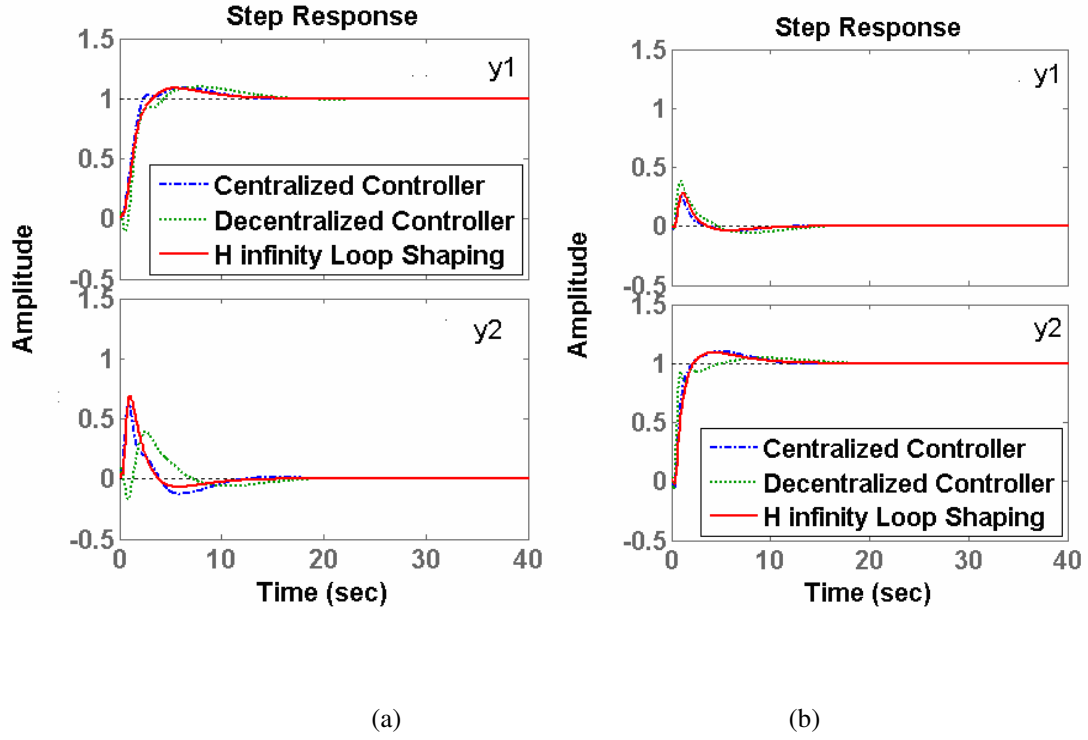
**Fig. 3.** Singular value plots of open loop transfer function of (a) the nominal plant (dash line) and shaped plant (solid line) (b) the loop shape by the proposed controllers (Red line: Centralized PID, dash line: Decentralized PID) and HLS (Blue line).

Based on the concept of  $H_\infty$  loop shaping, stability margin is used to indicate performance and robustness of the designed system. By the results, the stability margin of the system from decentralized controller ( $\varepsilon=0.274$ ) is less than that of the proposed centralized controller. However, if such stability margin is satisfied for the control system (more than the specified stability margin), decentralized controller can be applied to control the plant and the advantages of using decentralized control scheme such as simple controller, less complexity, etc. can be achieved.

To evaluate the tracking performance of the proposed system, step responses of the close loop system from conventional  $H_\infty$  loop shaping (HLS), proposed centralized and decentralized controllers are investigated. As seen in Fig. 5, the responses from the proposed controllers are almost the same as response from the HLS controller.



**Fig. 4** Convergence of the fitness value.

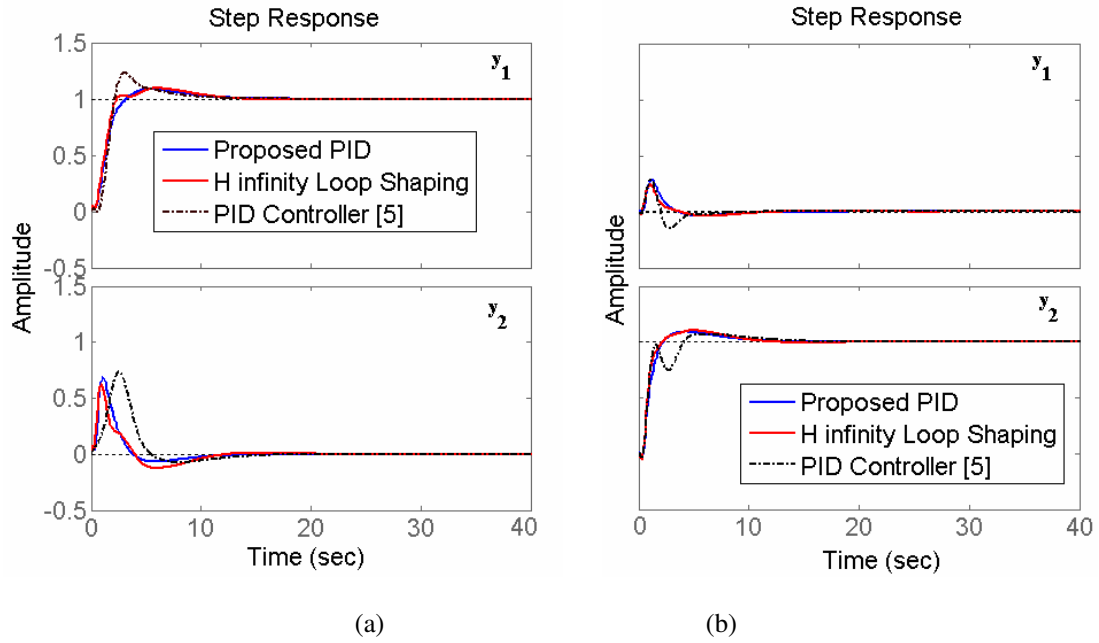


**Fig. 5** Responses from the proposed PID controllers and HLS controller when the unit step commands are applied to control the outputs (a)  $y_1$  and (b)  $y_2$ .

In simulation studies, the robustness and performance of the proposed controller and HLS controller are compared with those of the controller obtained from [5], that is.

$$K_p = \begin{bmatrix} 2.4719 & -1.2098 \\ -1.1667 & -2.4766 \end{bmatrix}, K_i = \begin{bmatrix} 0.4657 & -0.31 \\ -0.2329 & -0.487 \end{bmatrix}, K_d = \begin{bmatrix} 0.0534 & -0.0072 \\ -0.015 & -0.0434 \end{bmatrix}, \tau_d = \frac{1}{16.61} \quad (17)$$

By using (4), the stability margin obtained by the above controller is 0.246. Fig. 6 shows the time domain responses from the proposed centralized controller, HLS controller, and the controller in [5]. As seen in this figure, responses from the proposed centralized and HLS controllers have a small maximum overshoot and less coupling compared to the robust PID controller in [5].



**Fig. 6.** Responses from the proposed centralized controller, HLS controller and the controller in [5], when the step command is applied to control (a)  $y_1$  and (b)  $y_2$ .

Table 1 shows the results of stability margins obtained by the designed controllers. Clearly, the stability margin of the proposed centralized PID controller is much better than that of both the centralized PID controller in [5] and the reduced order HLS controller.

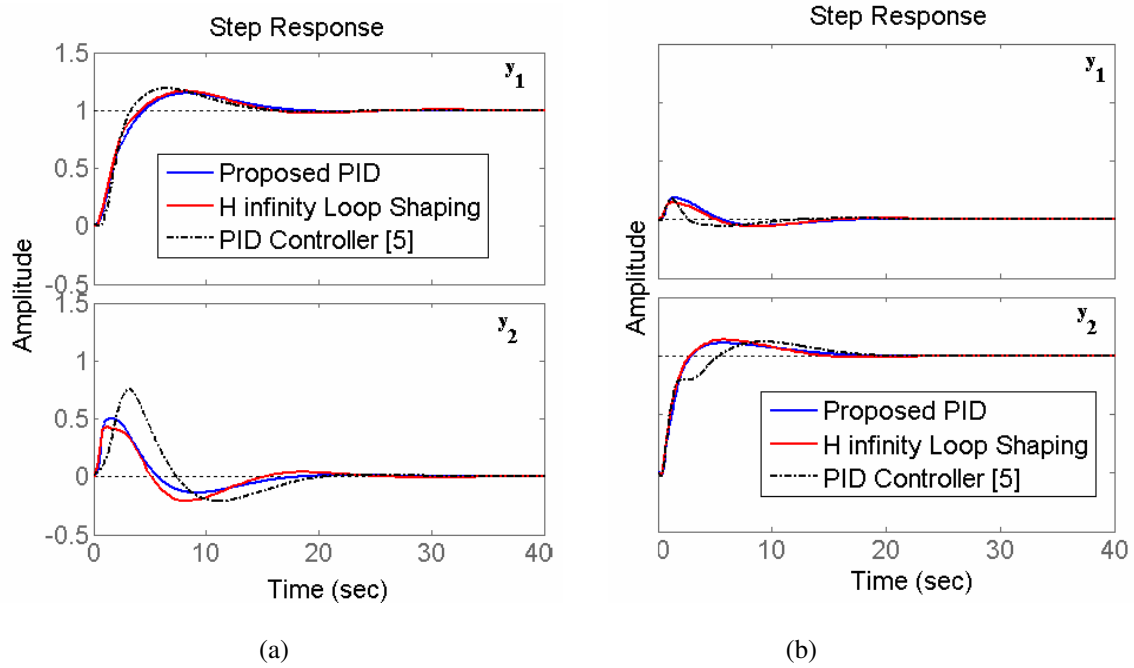
**Table 1** The results of stability margin of the designed systems in Example 1.

Controller	Stability Margin
1. Proposed Controller:	
1.1 Centralized PID Controller	0.343
1.2 Decentralized PID Controller	0.274
2. Robust Centralized PID Controller designed by BMI optimization [5].	0.246
3. Reduced Order $H_\infty$ loop shaping controller	0.263

To verify the robust performance of the system, the responses from HLS, proposed centralized PID and robust PID [5] are investigated in the following perturbed plant.

$$G_0 = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} t_{17} \\ t_4 \end{bmatrix} = \begin{bmatrix} \frac{-2.2e^{-s}}{12s+1} & \frac{1.3e^{-0.3s}}{12s+1} \\ \frac{-2.8e^{-1.8s}}{14.5s+1} & \frac{4.3e^{-0.35s}}{14.2s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (18)$$

As shown in (18), time constants of the plant are different from the nominal plant. Fig. 7 shows the responses from the above mentioned controllers in the perturbed plant. As seen in this figure, all responses are almost the same as the responses at the nominal plant. This signifies that all designed controllers are robust against the parameter changes. Clearly, the settling time of the responses and couplings from the proposed PID controller and HLS controller are less than those of the PID controller designed by the method in [5].

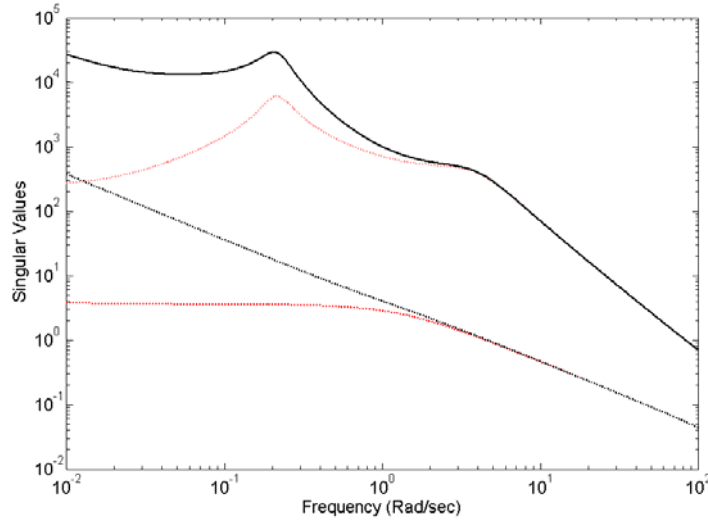


**Fig. 7** Responses from the proposed centralized controller, HLS controller, and the controller in [5] when the step input command is applied to control (a)  $y_1$  and (b)  $y_2$  and plant is perturbed.

**Example 2:** In this example, the design of pitch axis controller for an experimental highly maneuverable airplane, HIMAT, is studied. The dynamic model of this plant is taken from the  $\mu$ -synthesis and analysis toolbox user's guide [12]. The state vector of this plant consists of the four variables which are forward velocity, angle-of-attack, pitch rate, and pitch angle. The control inputs are the elevon and the canard. The measured variables are angle-of-attack and pitch angle. The details of this plant are given in appendix B. The design objective is to reject disturbances up to about 1 rad/s in the presence of substantial plant uncertainty above 100 rad/s [5]. In this problem, the pre- and post-compensator weights are chosen as [5].

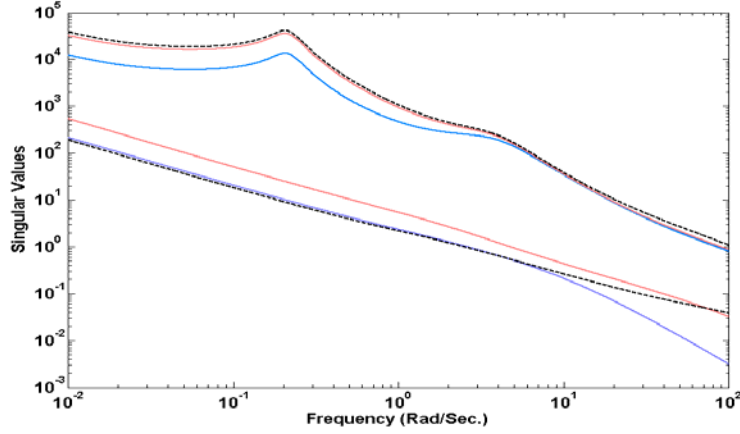
$$W_1 = \begin{bmatrix} \frac{s+1}{s+0.001} & 0 \\ 0 & \frac{s+1}{s+0.001} \end{bmatrix}, \quad W_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (19)$$

Singular values of HIMAT and desired loop shape are plotted in Fig. 8 (a). As seen in this figure, the bandwidth and performance are significantly improved by the compensator weights. The shaped plant has large gains at low frequencies for performance and small gains at high frequencies for noise attenuation. With these weighting functions, the robust requirement is satisfied.



(a)





(b)

**Fig. 8.** Open loop shape of (a) the nominal plant (Red line) and shaped plant (Black line) (b) the loop shape by the proposed controllers (Red line: Centralized PID, dash line: Decentralized PID) and HLS (Blue line).

By using (3), the optimal stability margin of the shaped plant is found to be 0.436. This value indicates that the selected weights are compatible with robust stability requirement in the problem. To design the HLS controller, stability margin 0.3964 is selected. As a result, the final controller (full order HLS controller) is 7<sup>th</sup> order and complicated. In this example, a 4<sup>th</sup> order controller (reduced order HLS controller) is further designed by using the balanced truncation method [8, 9]. By solving (4), the stability margin from the reduced order controller is found to be 0.387 which is less than that of the full order controller.

Next, both centralized controller in (13) and decentralized controller in (14) are also designed for this plant. By observing the weight  $W_l$ , the upper and lower bound values of control parameters are the same as those used in example 1. The PSO parameters used in example 1 are also used in this example. After running the PSO for 80 iterations, the optimal control parameters are found to be

**Case I:** Centralized controller.

$$K_p = \begin{bmatrix} 0.52601 & 0.14172 \\ 0.47833 & -0.70387 \end{bmatrix}, K_i = \begin{bmatrix} 1.96082 & -0.28352 \\ 2.2261 & -1.224 \end{bmatrix}, K_d = \begin{bmatrix} 0.007434 & 0.006844 \\ -0.006272 & 0.0016058 \end{bmatrix}, \tau_d = 0.0068479$$

(20)

**Case II:** Decentralized controller.

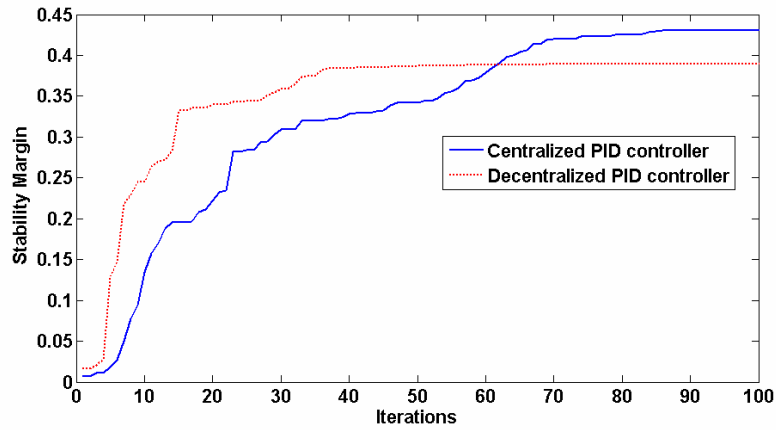
$$K_p = \begin{bmatrix} 0.5367 & 0 \\ 0 & -0.5659 \end{bmatrix}, K_i = \begin{bmatrix} 1.488 & 0 \\ 0 & -0.4909 \end{bmatrix}, K_d = \begin{bmatrix} 0.01569 & 0 \\ 0 & -0.00499 \end{bmatrix}, \tau_d = 0.0085855 \quad (21)$$

Fig. 8 (b) shows comparison of the loop shapes by the proposed controllers and HLS. As seen in this figure, all loop shapes are close to the desired loop shape. Fig. 9 shows plots of convergence of objective function (stability margin) versus iterations by PSO. As seen in this figure, the stability margins obtained from the proposed centralized and decentralized controllers are 0.432 and 0.389, respectively. These values indicate that the robust stability and performance of the designed systems are satisfied.

In this simulation studies, the robustness and performance of the proposed controllers are compared with those of the controller obtained from [5], that is.

$$K_p = \begin{bmatrix} 1.3074 & -0.0601 \\ 1.3414 & -1.3123 \end{bmatrix}, K_i = \begin{bmatrix} 1.2729 & -0.0795 \\ 1.3609 & -1.2921 \end{bmatrix}, K_d = \begin{bmatrix} 0.0077 & 0.0043 \\ -0.0069 & -0.0039 \end{bmatrix}, \tau_d = \frac{1}{99.5724} \quad (22)$$

The stability margin obtained by the above controller is 0.309. Clearly, the stability margin of the proposed centralized controller is also much better than that of both the controller in [5] and the reduced order controller.



**Fig. 9** Convergence of the fitness value.

Table 2 summarizes the results of stability margin obtained from the proposed controller and others. As the results, the stability margin from the proposed centralized PID controller is better than that of other controllers.

Accordingly, the proposed technique is an efficient method to design a fixed-structure robust loop shaping controller. Note that the design of decentralized PID controller was not presented in the previous work [5].

**Table 2** Comparisons of the stability margins obtained from the controllers in Example 2.

Controller	Stability Margin
1. Proposed Controller:	
1.1 Centralized PID Controller	0.432
1.2 Decentralized PID Controller	0.389
2. Robust Centralized PID Controller designed by BMI optimization [5]	0.309
3. Reduced Order $H_\infty$ loop shaping controller	0.387

#### 4. Conclusions

In this paper, a PSO based fixed-structure  $H_\infty$  loop shaping controller for MIMO system is proposed. Based on the concept of conventional  $H_\infty$  loop shaping, only a single index, stability margin,  $\varepsilon$ , is used to indicate performance and robustness of the designed controller. This index is utilized as the objective function of the proposed technique. The resulting stability margins indicate that the proposed controllers are compatible with the specified open loop shape and also guarantee robustness. Moreover, the structure of controller is not restricted to PID. The controller  $K(p)$  can be replaced by any fixed-structure controller and the proposed algorithm can still be applied functionally. As shown in simulation results, the stability margin from the proposed PID controller is much better than that of the reduced order controller. By comparison with the previous work [5], the optimal stability margin obtained by the proposed technique is also much better than that of the method in [5] and the problem of local minima is reduced by the proposed algorithm. In conclusion, by combining the two approaches of PSO and  $H_\infty$  loop shaping; fixed-structure  $H_\infty$  loop shaping controller can be designed. Although the design of fixed-structure robust controller is difficult because of its inherently non-convex nonlinear problem, the PSO simplifies the problem by searching the optimal solution. Simulation results demonstrate that the proposed technique is valid and flexible.

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## Appendix A

Given a shaped plant  $G_s$  and  $A, B, C, D$  represent the shaped plant in the state-space form. To determine  $\varepsilon_{opt}$ , there is a unique method as follows [8].

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = (1 + \lambda_{\max}(XZ))^{1/2}$$

where  $X$  and  $Z$  are the solutions of two Riccati in (A.1) and (A.2) respectively,  $\lambda_{\max}$  is the maximum eigenvalue.

$$(A - BS^{-1}D^TC)Z + Z(A - BS^{-1}D^TC)^T - ZC^TR^{-1}CZ + BS^{-1}B^T = 0 \quad (\text{A.1})$$

$$(A - BS^{-1}D^TC)^TX + X(A - BS^{-1}D^TC) - XBS^{-1}B^TX + C^TR^{-1}C = 0 \quad (\text{A.2})$$

where  $S = I + D^TD, R = I + DD^T$

## Appendix B

The state vector of HIMAT model consists of vehicle's rigid body variables.

$$x^T = [\delta v, \alpha, q, \theta]$$

where  $\delta v$  is the forward velocity,  $\alpha$  is angle between velocity vector and aircraft's longitudinal axis,  $q$  is rate-of-change of aircraft attitude angle, and  $\theta$  is the aircraft attitude angle. The state space of HIMAT can be written as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

The control inputs are the elevon and the canard. The measured variables are angle-of-attack and pitch angle. In this paper, the linearized model is taken from [12], that is

$$A = \begin{bmatrix} -0.0226 & -36.6 & -18.9 & -32.1 \\ 0 & -1.9 & 0.983 & 0 \\ 0.0123 & -11.7 & -2.63 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -0.414 & 0 \\ -77.8 & 22.4 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 57.3 & 0 & 0 \\ 0 & 0 & 0 & 57.3 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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# FIXED-STRUCTURE $\mathcal{H}_\infty$ LOOP SHAPING CONTROL OF VOLTAGE CONTROLLER FOR AC/DC BUCK CONVERTER USING PARTICLE SWARM OPTIMIZATION

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**Abstract—** Being complex and high-order, robust controllers designed by  $\mathcal{H}_\infty$  loop shaping are difficult to implement in practice. To overcome this problem, we propose an algorithm, *Particle Swarm Optimization (PSO) based fixed-structure  $\mathcal{H}_\infty$  loop shaping control*, to design a robust controller. PSO algorithm is used to solve the  $\mathcal{H}_\infty$  loop shaping design problem under a structure-specified controller. Additionally, in the proposed technique, the performance weighting function, which is normally difficult to obtain, is determined by using PSO. The performance and robustness of the proposed controller are investigated in a buck converter in comparison with those of the controller designed by conventional  $\mathcal{H}_\infty$  loop shaping method. Results of simulation demonstrate the advantages of simple structure and robustness against plant perturbations and disturbances of the proposed controller. Experiments are performed to verify the effectiveness of the proposed technique.

**Index Terms—**  $\mathcal{H}_\infty$  loop shaping , Particle Swarm Optimization , buck converter

## 1 INTRODUCTION

DC-DC converters have been widely used in computer hardware and industrial applications. Controlling of these converters is a challenging field because of their intrinsic nature of nonlinear, time-variant systems [1]. In previous research works, the

linear models of these converters were derived by using linearization method [2-3]. Some linear control techniques were applied to these converters based on the linear models [1, 4-5]. NAIM, R., *et.al.*[4], applied the  $\mathcal{H}_\infty$  control to a boost converter. Three controllers of different mode which are voltage mode, feed-forward, and current mode control were investigated and compared in performance. G.C. Ioannidis and S.N. Manias [5] applied the  $\mathcal{H}_\infty$  loop shaping control schemes for a buck converter. In their paper, the  $\mu$ -analysis was used to examine the robust features of the designed controllers. Simone Buso [1] adopted the robust  $\mu$ -synthesis to design a robust voltage controller for a buck-boost converter with current mode control. The parameter variations in the converter's transfer function were described in terms of perturbations of linear fraction transformations (LFT) class.

Although the higher control techniques mentioned above are powerful design tools, they are difficult to implement in converters due to the controller's high-order structure. To solve this problem, the design of fixed-structure robust controller has been proposed. Fixed-structure robust controller has become an interesting area of research because of its simple structure and acceptable controller order. However, the design of this controller by using analytical methods remains difficult. To simplify the problem, searching algorithms such as genetic algorithm, particle swarm optimization technique, tabu-search, etc., can be employed. Several approaches to design a fixed-structure robust controller were proposed in [6-8, 10-12]. In [6], a robust  $\mathcal{H}_\infty$  optimal control problem with structure specified controller was solved by using genetic algorithm (GA). As concluded in [6], genetic algorithm is a simple and efficient tool to design a fixed-structure  $\mathcal{H}_\infty$  optimal controller. Bor-Sen.Chen. *et. al.*[7], proposed a PID design algorithm for mixed  $\mathcal{H}_2/\mathcal{H}_\infty$  control. In their paper, PID control parameters were tuned in the stability domain to achieve mixed  $\mathcal{H}_2/\mathcal{H}_\infty$  optimal control. A similar work was proposed in [8] by using the



intelligent genetic algorithm to solve the mixed  $\mathcal{H}_2/\mathcal{H}_\infty$  optimal control problem. The techniques in [6-8] are based on the concept of  $\mathcal{H}_\infty$  optimal control which two appropriate weights for both the uncertainty of the model and the performance are essentially chosen. A difficulty with the  $\mathcal{H}_\infty$  optimal control approach is that the appropriate selection of close-loop objectives and weights is not straightforward [9]. Alternatively, the robust controller can be designed by  $\mathcal{H}_\infty$  loop shaping control [9] which is a simple and efficient technique. Uncertainties in this approach are modeled as normalized co-prime factors; this uncertainty model does not represent actual physical uncertainty, which usually is unknown in real problems. This technique requires only two specified weights, pre- and post-compensator weights, for shaping the nominal plant so that the desired open loop shape is achieved. Fortunately, the selection of such weights is based on the concept of classical loop shaping which is a well known technique in the controller design. By the reasons mentioned above, this technique is simpler and more intuitive than other robust control techniques. However, the controller designed by  $\mathcal{H}_\infty$  loop shaping is still complicated. To overcome this problem, PSO based fixed-structure  $\mathcal{H}_\infty$  loop shaping is proposed to synthesize a fixed-structure  $\mathcal{H}_\infty$  loop shaping controller for Average Current-Mode Control (ACMC) buck converter system. In this paper, PSO is employed to find the parameters of the weighting functions and those of the controllers. Simulation results show that the controller designed by the proposed approach has a good performance and robustness as well as simple structure. This allows our designed controller to be implemented practically and reduces the gap between the theoretical and the practical approach.

The remainder of this paper is organized as follows. Section 2 covers the converter modeling. In section 3, conventional  $\mathcal{H}_\infty$  loop shaping and the proposed technique are

discussed as well as PSO algorithm. The design examples and results are demonstrated in section 4. And in section 5 the paper is summarized.

## 2 CONVERTER MODELING

Current mode control (CMC) is widely used in the DC-DC converter. The control system of this control scheme is typically two-loop system (voltage loop and current loop). The current loop is used to maintain the inductor current equal to reference current. This reference current is obtained from the output of the voltage control loop. The voltage loop is used to compare the reference voltage (command) and the output voltage of a buck converter. In continuous conduction mode of CMC, the popular control schemes are PCMC (Peak Current Mode Control) or ACMC (Average Current Mode Control). ACMC has several advantages over PCMC such as robustness against the noise, high degree of accuracy to track the average inductor current. In ACMC, the current loop control is typically designed as PI controller. In this paper, the design of this current loop gain is adopted from [13-14]. Consider the equation in [13], the controller gain of current loop can be expressed as

$$G_{CA} = \frac{K_c(1 + \frac{s}{\omega_z})}{s(1 + \frac{s}{\omega_p})} \quad (1)$$

Where  $K_c = \frac{1}{R_l(C_{fp} + C_{fe})}$ ,  $\omega_z = \frac{1}{R_f C_{fe}}$ ,  $\omega_p = \frac{C_{fe} + C_{fp}}{R_f C_{fe} C_{fp}}$

$G_{CA}$  can be approximate as  $R_f/R_l$  at the switching frequency [13], and can be expressed as

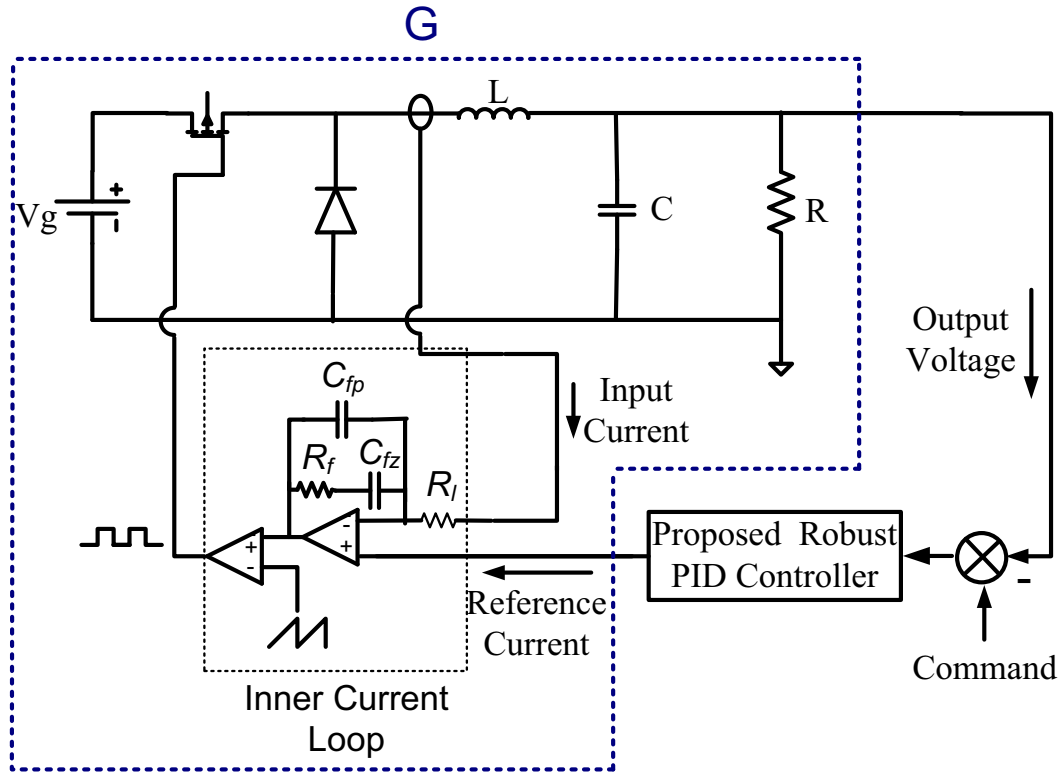
$$G_{CA} \cong \frac{R_f}{R_l} < \min\left\{\frac{2V_m f_s L}{V_g R_s}, \frac{V_m f_s L}{V_0 R_s}\right\} \quad (2)$$

Where  $V_m$  is the peak-to-peak voltage of ramp signal

$R_s$  is the small current sensing resistance

$V_g$  is the input voltage

$R_f, R_l, L$  is shown in Fig. 1.



**Fig.1.** Buck converter with average current mode

Equation (1) and (2) set the criteria for choosing  $R_f$  and  $R_l$ . In this paper,  $\omega_z$  is placed at one-half of  $\omega_0$  which  $\omega_0 = 1/\sqrt{LC}$  and  $\omega_p$  is placed at one-half of switching frequency ( $f_s$ ).

The dynamic model of this converter from the current reference ( $V_c$ ) to the output voltage ( $V_o$ ) is given by [13].

$$\frac{V_o(s)}{V_c(s)} = \frac{K_m(1 + r_cCs)[G_{CA} + 1]G_{dv}(s)}{1 + T_c(s)} \quad (3)$$

Where  $K_m = 1/V_m$ ,  $G_{dv}$ ,  $r_c$  and  $T_c$  are the transfer functions from the duty cycle to the output voltage, current loop gain and equivalent series resistor (ESR) respectively. In this paper, neglecting the ESR of the output capacitor.  $G_{dv}$  and  $T_c$  can be written as

$$G_{dv}(s) = \frac{(1 + r_C Cs)V_g}{R + (L + RCr_C)s + (RLC + r_C LC)s^2} \quad (4)$$

$$T_c(s) = \frac{R_S K_m V_g [1 + (R + r_C)Cs][1 + G_{CA}]}{R + (L + RCr_C)s + (RLC + r_C LC)s^2} \quad (5)$$

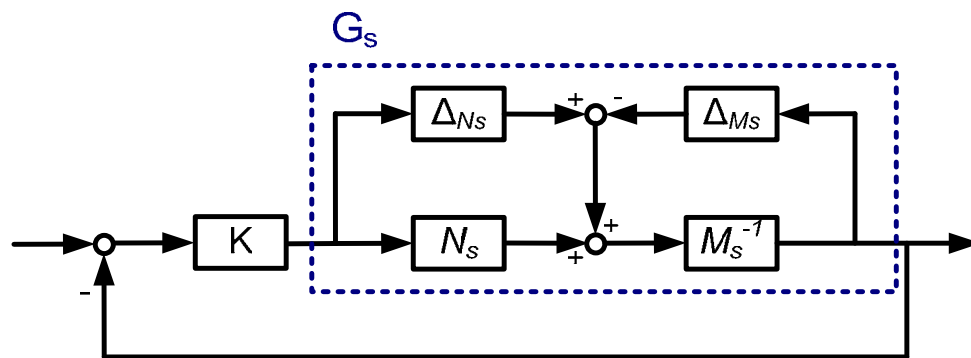
Nominal plant in (3) is used as the plant to be controlled in this paper.

### 3 $\mathcal{H}_\infty$ LOOP SHAPING CONTROL AND PROPOSED TECHNIQUE

This section illustrates the concepts of the standard  $\mathcal{H}_\infty$  loop shaping control and the proposed technique.

#### 3.1 Standard $\mathcal{H}_\infty$ Loop Shaping

$\mathcal{H}_\infty$  loop shaping control [9] is an efficient method to design a robust controller. This approach requires only a desired open loop shape in frequency domain. Two weighting functions,  $W_1$  (pre-compensator) and  $W_2$  (post-compensator), are specified to shape the original plant  $G$ . In this approach, the shaped plant is formulated as normalized coprime factor, which separates the plant  $G_s$  into normalized nominator  $N_s$  and denominator  $M_s$  factors. Fig. 2 shows the coprime perturbed plant and robust stabilization used in this approach.



**Fig. 2.** Co-prime factor robust stabilization problem.

If the shaped plant  $G_s = W_2 G_o W_1 = N_s M_s^{-1}$ , then a perturbed plant is written as [15]

$$G_\Delta = (N_s + \Delta_{N_s})(M_s + \Delta_{M_s})^{-1} \quad (6)$$

Where  $\Delta_{N_s}$  and  $\Delta_{M_s}$  are stable, unknown representing the uncertainty

satisfying  $\|\Delta_{N_s}, \Delta_{M_s}\|_\infty \leq \varepsilon$ ,

$\varepsilon$  is the uncertainty boundary, called stability margin.

According to the standard procedure of  $\mathcal{H}_\infty$  loop shaping, the following steps can be applied to design the  $\mathcal{H}_\infty$  loop shaping controller.

**Step 1** Shape the singular values of the nominal plant  $G_o$  by using a pre-compensator  $W_1$  and/or a post-compensator  $W_2$  to get the desired loop shape.  $W_2$  can be chosen as an identity matrix, since we can neglect the sensor noise effect when the use of good sensor is assumed [16]. There are some guidelines for the weight selection in [16]. In SISO system, the weighting functions  $W_1$  and  $W_2$  can be chosen as

$$W_1 = K_w \frac{s + a}{s + b} \text{ and } W_2 = 1 \quad (7)$$

Where  $K_w$ ,  $a$  and  $b$  are positive values

**Step 2** Minimize  $\infty$ -norm of the transfer matrix  $T_{zw}$  over all stabilizing controllers  $K$ , to obtain an optimal cost  $\gamma_{opt}$ , as [16]

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = \inf_{stab K} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + G_s K)^{-1} M_s^{-1} \right\|_\infty \quad (8)$$

To determine  $\varepsilon_{opt}$ , there is a unique method explained in appendix A.  $\varepsilon_{opt} \ll 1$  indicates that  $W_1$  or  $W_2$  designed in step 1 are incompatible with robust stability requirement. If  $\varepsilon_{opt}$  is not satisfied ( $\varepsilon_{opt} \ll 1$ ), then return to step 1, adjust  $W_1$ .

**Step 3** Select  $\varepsilon < \varepsilon_{opt}$  and then synthesize a controller  $K_\infty$  that satisfies

$$\|T_{zw}\|_{\infty} = \left\| \begin{bmatrix} I \\ K_{\infty} \end{bmatrix} (I + G_s K_{\infty})^{-1} M_s^{-1} \right\|_{\infty} \leq \varepsilon^{-1} \quad (9)$$

Controller  $K_{\infty}$  is obtained by solving the optimal control problem. See [15] for more details.

**Step 4** Final controller ( $K$ ) follows

$$K = W_1 K_{\infty} W_2 \quad (10)$$

### 3.2 PSO based Fixed-Structure $\mathcal{H}_{\infty}$ Loop Shaping Optimization

In the proposed technique, PSO is adopted to design a weighting function and a fixed-structure robust controller. PSO was first proposed by Eberhart and Kennedy [17]. This technique is a population-based optimization problem-solving algorithm. Fig. 3 shows the swarm's movement which is the basic idea of PSO. As seen in this figure, a bird represents the particle and the position of each particle represents the candidate solution. Population is formed by a number of particles. In the PSO, particles fly around the problem space until the stopping criteria are met. This algorithm is simple, fast and can be programmed easily. During the flight, the velocity and position of each particle are updated according to its own and its companion's fitness value. To illustrate the strategies of PSO, the following equation is shown.

$$v_i(iter+1) = Qv_i(iter) + \alpha_1[\gamma_{1i}(X_{pi} - X_i(iter))] + \alpha_2[\gamma_{2i}(G - X_i(iter))] \quad (11)$$

where  $Q$  is the momentum coefficient,  $v_i$  is the velocity of  $i^{th}$  particle,  $iter$  is the iteration count,  $\alpha_1$  and  $\alpha_2$  are the specified acceleration coefficients,  $X_{pi}$  is the best position found by  $i^{th}$  particle,  $G$  is the best position found by swarm (global best),  $\gamma_{1i}$  and  $\gamma_{2i}$  are the random numbers in the range  $[0,1]$ . Note that the velocity must be within the specified range  $[V_{min}, V_{max}]$ . If not, set it to the limiting values. As shown in (11), there are three terms in the equation. By these terms, the advantages of local minimum searching, global minimum searching, local optima avoidance and the information sharing among particles

are achieved and the particle can reach the best solution. The details of PSO are available in [17].



Fig. 3. The movement of a swarm.

In the proposed technique, although the controller is structured, it still retains the entire robustness and performance guarantee as long as a satisfactory uncertainty boundary  $\varepsilon$  is achieved. The proposed algorithm is explained as follows. Assume that the predefined structure controller  $K(p)$  has satisfied parameters  $p$  and the performance weighting function  $W_1(x)$  has satisfied the parameter  $x$ . Based on the concept of  $\mathcal{H}_\infty$  loop shaping, optimization goal is to find the parameters  $p$  in the controller  $K(p)$  and the parameters  $x$  in the weighting function  $W_1(x)$  that minimize infinity norm from disturbances  $w$  to states  $z$ ,  $\|T_{zw}\|_\infty$ . In the proposed technique, the final controller  $K$  is defined as

$$K = K(p)W_2 \quad (12)$$

Assuming that  $W_1$  are invertible, from (10) then it is obtained that

$$K_\infty = W_1^{-1}(x)K(p) \quad (13)$$

In many cases, the weight  $W_2$  is selected as identity matrix  $I$ . However, if  $W_2$  is a transfer function matrix, then the final controller is the controller  $K(p)$  in series with the

weight  $W_2$ . By substituting (13) into (9), the  $\infty$ -norm of the transfer function matrix from disturbances to states,  $\|T_{zw}\|_{\infty}$ , which is subjected to be minimized can be written as

$$J_{\cos t} = \gamma = \|T_{zw}\|_{\infty} = \left\| \begin{bmatrix} I \\ W_1^{-1}(x)K(p) \end{bmatrix} (I - W_2 G_0 K(p))^{-1} [I \quad G_s] \right\|_{\infty} \quad (14)$$

The optimization problem can be written as

$$\text{Maximize } \left\| \begin{bmatrix} I \\ W_1^{-1}(x)K(p) \end{bmatrix} (I - W_2 G_0 K(p))^{-1} [I \quad G_s] \right\|_{\infty}^{-1}$$

$$\text{Subject to } p_{i,\min} < p_i < p_{i,\max} ,$$

$$x_{i,\min} < x_i < x_{i,\max} ,$$

$$\left. \begin{array}{l} \text{Maximum Overshoot} < OV \\ \text{Settling Time} < ST \\ \text{Steady State Error} < SE \end{array} \right\} \text{ (Constrains in time domain)}$$

$$\left. \begin{array}{l} \text{Bandwidth} > BW \\ \text{Gain}(\omega_{\text{low freq}}) > LG \\ \text{Gain}(\omega_{\text{high freq}}) < HG \end{array} \right\} \text{ (Constrains in frequency domain)}$$

where

$OV$  is the acceptable maximum overshoot.

$ST$  is the acceptable settling time.

$SE$  is the acceptable steady state error.

$BW$  is the bandwidth of the desired loop shape.

$LG$  is the gain in low frequency range of the desired loop shape.



$HG$  is the gain in high frequency range of the desired loop shape.

$p_{i,\min}$ ,  $x_{i,\min}$  and  $p_{i,\max}$ ,  $x_{i,\max}$  are the lower and upper bound values of the parameter  $p_i$  and  $x_i$  in the parameter vector  $p$  and  $x$ , respectively.

As shown in the constraints of the above optimization problem, the performance specifications are specified in terms of algebraic or functional inequalities. For example, in the step response, the system may be required to have a rise-time less than 1000  $\mu s$ ., a steady state error less than 0.01%, and an overshoot less than 1%. In this paper, the performance specifications are evaluated by plotting the desired open-loop shape and time domain response of the candidate of controller and weighting function. Thus, the fitness function in the controller synthesis can be written as

$$\text{Fitness (J)} = \begin{cases} \left( \left\| \begin{bmatrix} I \\ W_1^{-1}(x)K(p) \end{bmatrix} (I - W_2 G_0 K(p))^{-1} [I \quad G_s] \right\|_{\infty} \right)^{-1} & \text{if the constraints are met} \\ 0.0001 & \text{otherwise} \end{cases} \quad (15)$$

The fitness is set to a small value (in this case is 0.0001) if  $K$  does not stabilize the plant. Our proposed algorithm is summarized as follows.

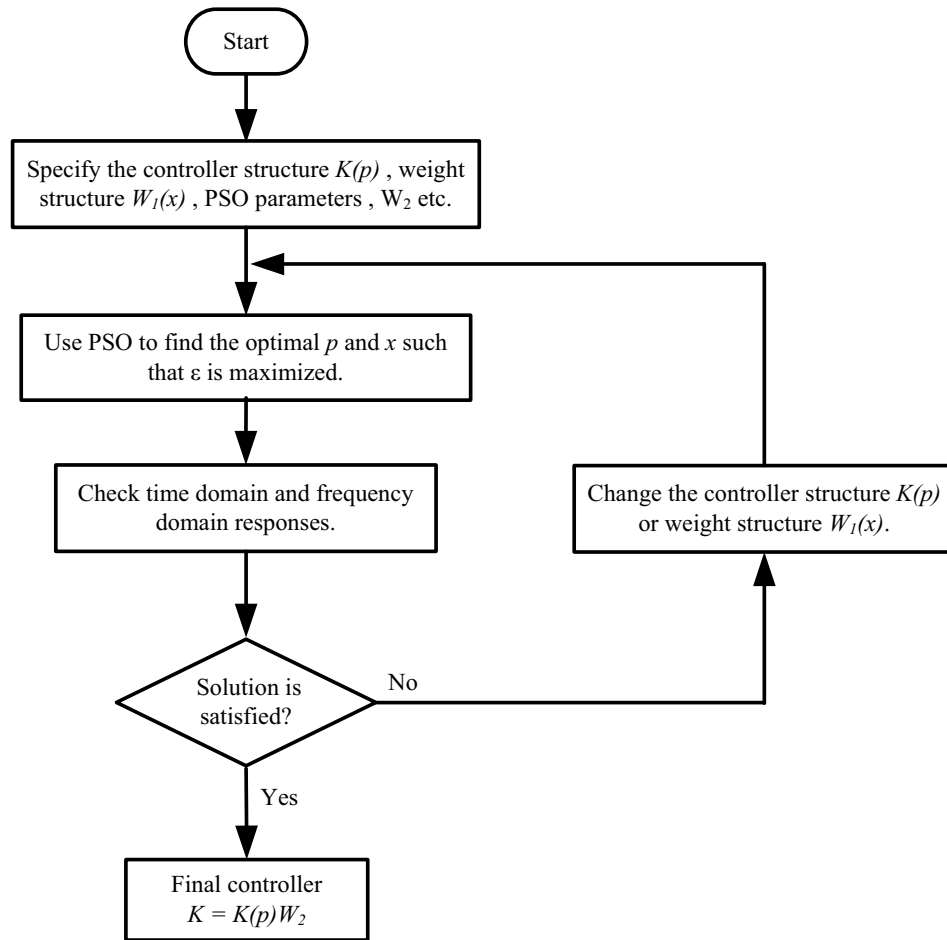
**Step 1** Specify controller structure  $K(p)$ , weighting function structure  $W_1(x)$  and PSO parameters.

**Step 2** Specify the control and weighting function parameter ranges.

**Step 3** Initialize several sets of parameters  $p$  and  $x$  as swarm in the 1<sup>st</sup> iteration. In this case, each  $[p, x]$  is a particle.

**Step 4** Use the PSO to find the optimal control parameter,  $p$  and optimal weighting function parameter,  $x$ .

**Step 5** Check performances in both frequency and time domains. If the performance is not satisfied such as too low  $\varepsilon$  (too low fitness function), then go to step 2 to find the optimal control parameter and optimal weighting function.



**Fig. 4.** Flow chart of the proposed design procedure.

Standard PSO algorithm used in step 4 of the proposed technique is briefly described as follows.

Specify the parameters in PSO such as population size ( $n$ ), upper and lower bound values of problem space, fitness function ( $J$ ), maximum and minimum velocity of particles ( $V_{max}$  and  $V_{min}$ , respectively), maximum and minimum inertia weights ( $Q_{max}$  and  $Q_{min}$ , respectively).

1. Initialize  $n$  particles with random positions within upper and lower bound values of the problem space. Set iteration count as  $iter = 1$ .
2. Evaluate the fitness function ( $J$ ) of each particle using (15).

3. For each particle, find the best position found by particle  $i$  call it  $X_{pi}$  and let the fitness value associated with it be  $J_{pbesti}$ . At first iteration, position of each particle and its fitness value of  $i^{th}$  particle are set to  $X_{pi}$  and  $J_{pbesti}$ , respectively.
4. Find a best position found by swarm call it  $G$  which is the position that maximum fitness value is obtained. Let the fitness value associated with it be  $J_{Gbest}$ . To find  $G$  the following algorithm described by pseudo code is adopted.

(At first iteration set  $J_{Gbest}=0$ )

For  $i = 1$  to  $n$  do

    If  $J_{pbesti} > J_{Gbest}$ , then

$$G = X_{pi}, J_{Gbest} = J_{pbesti}$$

end;

5. Update the inertia weight by following equation

$$Q = Q_{\max} - \frac{Q_{\max} - Q_{\min}}{iter_{\max}} iter$$

where  $Q$  is inertia weight,  $iter$  and  $iter_{\max}$  are the iteration count and maximum iteration, respectively.

6. Update the velocity and position of each particle. For the particle  $i$ , the updated velocity and position can be determined by following equations.

$$\begin{aligned} v_i(iter+1) &= Qv_i(iter) + \alpha_1[\gamma_{1i}(X_{pi} - X_i(iter))] + \alpha_2[\gamma_{2i}(G - X_i(iter))] \\ X_i(iter+1) &= X_i(iter) + v_i(iter+1) \end{aligned}$$

7. Increment iteration for a step. ( $iter = iter+1$ )
8. Stop if the convergence or stopping criteria are met, otherwise go to step 2.

#### 4 SIMULATION AND EXPERIMENTAL RESULTS

A buck converter's parameters and considered variation ranges used in this paper are given in Table 1.

**Table 1** Chosen converter's parameter and considered variation ranges

Parameter	Name	Nominal Value
$R_L$	Load Resistant	20 $\Omega$
$V_o$	Output Voltage	6 V
$V_i$	Input Voltage	12 V
$L$	Inductance	100 $\mu\text{H}$
$C$	Capacitor	680 $\mu\text{F}$
$f_{sw}$	Switching frequency	100 kHz

The selected and designed current loop elements in this paper are according to the method illustrated in section 2. ( $R_f = 10\text{k}\Omega$ ,  $R_l = 1\text{k}\Omega$ ,  $C_{fp} = 2\text{nF}$ ,  $C_{fz} = 50\text{nF}$ ) By (3), the nominal transfer function is found to be

$$\frac{V_o(s)}{V_c(s)} = G = \frac{1.2 \times 10^{-15} s^5 + 7.3 \times 10^{-10} s^4 + 3.6 \times 10^{-5} s^3 + 0.076 s^2 + 531.9 s + 923000}{6.8 \times 10^{-22} s^7 + 7.9 \times 10^{-17} s^6 + 6.8 \times 10^{-12} s^5 + 2.4 \times 10^{-7} s^4 + 5.9 \times 10^{-4} s^3 + 3.674 s^2 + 6943 s + 461000} \quad (16)$$

To simplify the controller synthesis, insignificant small terms are neglected. Therefore, the reduced-order plant is found to be

$$\frac{V_o(s)}{V_c(s)} = G = \frac{531.9 s + 923100}{3.674 s^2 + 6943 s + 461500} \quad (17)$$

Both  $\mathcal{H}_\infty$  loop shaping control and our proposed technique are applied to this converter. Firstly, we design the controller by the proposed technique. In this case, weighting function ( $W_1$ ) are expressed in (18).  $x_1$  and  $x_2$  are parameters that will be evaluated.

$$W_1(x) = \frac{x_1 s + x_2}{s + 0.001} \quad (18)$$

$W_2$  is chosen as 1 since we neglect the sensor noise effect when the use of good sensor is assumed.

$$W_2 = 1 \quad (19)$$

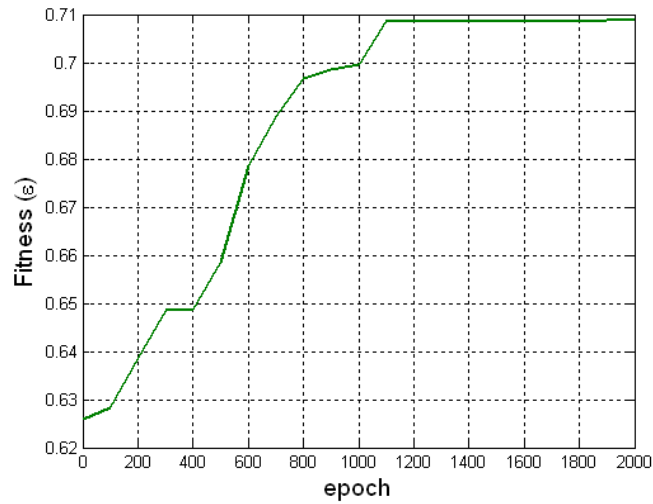
Next, PID controller is investigated as a fixed-structure controller. The controller structure is expressed in (20).  $K_p$ ,  $K_i$ ,  $K_d$  and  $t_d$  are parameters that will be evaluated.

$$K(p) = K_p + \frac{K_i}{s} + \frac{K_d s}{t_d s + 1} \quad (20)$$

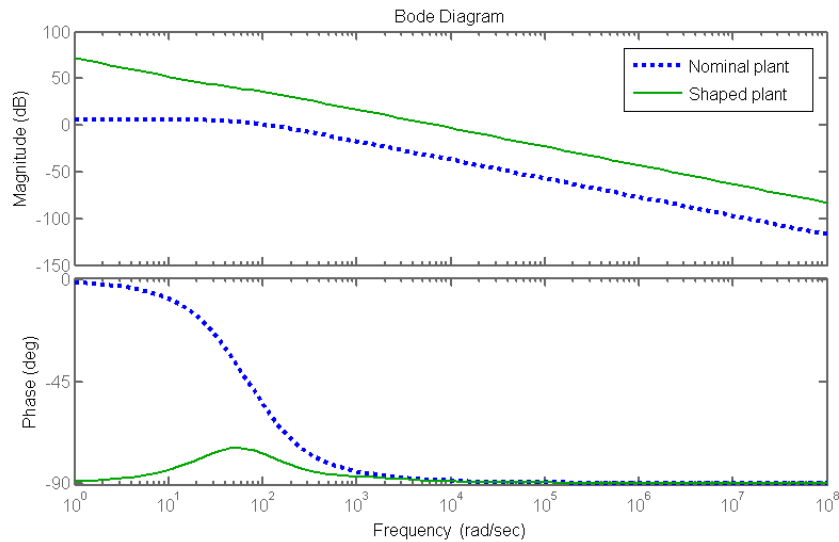
Select the controller parameters, weighting function parameters, their ranges, and Particle Swarm Optimization parameters as follows:  $K_p \in [0, 100]$ ,  $K_i \in [0, 4000]$ ,  $K_d \in [0, 0.01]$ ,  $t_d \in [0, 0.01]$ ,  $x_1 \in [0, 200]$ ,  $x_2 \in [0, 2000]$ , population size = 24, minimum and maximum velocities are 0 and 2 respectively, acceleration coefficients = 2.1, minimum and maximum inertia weights are 0.6 and 0.9, respectively, maximum iteration = 2000. An optimal solution is obtained after 1100 generations. Fig. 5 shows plots of convergence of cost function  $J_{cost}$  versus generations by PSO algorithm. The optimal solution is shown in (21), which has stability margin ( $\varepsilon$ ) of 0.70873 ( $\gamma = 1.4109$ ).

$$K(p) = 49.18 + \frac{2283.4}{s} + \frac{0.00010s}{0.00099s + 1} \quad (21)$$

$$W_1(x) = \frac{49.50s + 1586}{s + 0.001} \quad (22)$$



**Fig. 5.** Convergence of the fitness value.



**Fig. 6.** Bode plots of the nominal plant and the shaped plant

Fig. 6 shows the plot of open loop shape of nominal plant and shaped plant. As seen in this figure, at low frequency, the open loop gain of shaped plant is much larger than that of the nominal plant. This makes the system good in terms of disturbance attenuation and performance tracking. The bandwidth of the nominal plant is about 100 rad/sec. With these weighting functions, bandwidth of the desired control system is increased to 9,000 rad/sec. Significant performances and robustness improvement are carried out by these weighting functions.

Next, we design a controller by the conventional  $\mathcal{H}_\infty$  loop shaping procedure where  $W_I$  evaluated by PSO is used to synthesize the controller designed by conventional  $\mathcal{H}_\infty$  loop shaping. The shaped plant is written as

$$G_s = W_2 G W_1 = \left( \frac{531.9s + 923100}{3.674s^2 + 6943s + 461500} \right) \frac{49.18s + 1586}{s + 0.001} \quad (23)$$

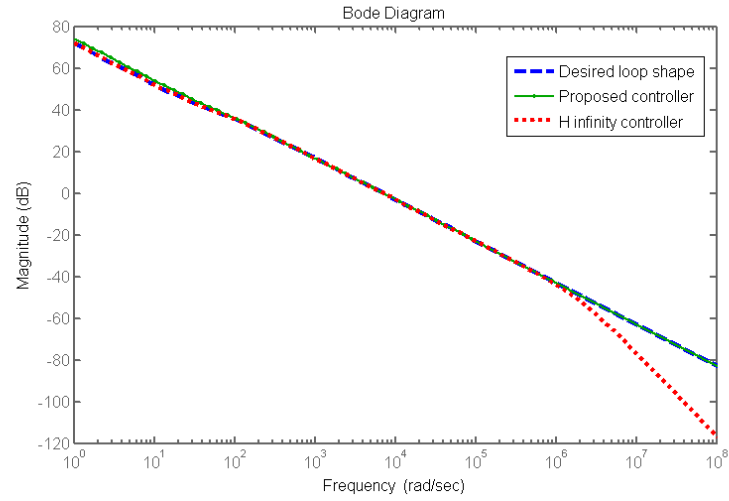
By applying the  $\mathcal{H}_\infty$  loop shaping method, the optimal stability margin ( $\varepsilon_{opt}$ ) is founded at 0.71165 ( $\gamma = 1.4051$ ). This value indicates that the selected weighting function is compatible with the robust stability requirement. The  $\varepsilon = 0.7100$  ( $\gamma = 1.4084$ ), which is less than the optimal stability margin, is chosen to synthesize the controller. Based on the conventional technique in section 3, the conventional  $\mathcal{H}_\infty$  loop shaping controller is synthesized as follows.

$$K(s) = W_1 K_\infty W_2 = \frac{(49.50s + 1586)}{(s + 0.001)} \frac{1.505 \times 10^6 (s + 1765)(s + 32.39)}{(s + 1.532 \times 10^6)(s + 1739)(s + 32.04)} \quad (24)$$

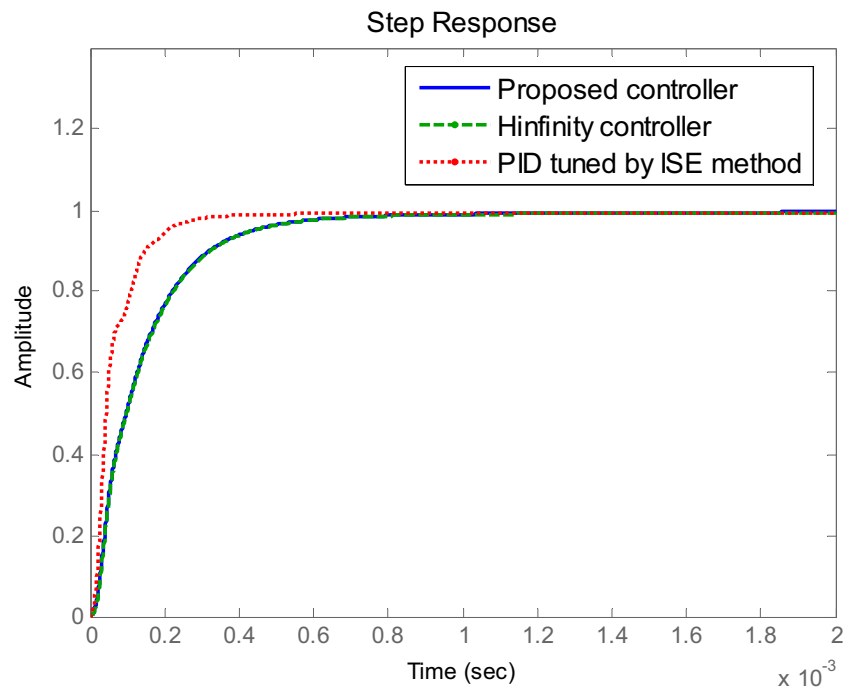
As shown in (24), the controller is 4<sup>th</sup> order and complicated in structure.

In addition, to compare performance, a PID controller designed by ISE method with model reference [18] are investigated. This resulting PID is shown in following equation.

$$K_{ISE} = 101.27 + \frac{1013}{s} + \frac{0.00032s}{0.00001s + 1} \quad (25)$$



(a) Desired loop shape and loop shape by conventional  $\mathcal{H}_\infty$  loop shaping and proposed controller



(b) Step responses of proposed and  $\mathcal{H}_\infty$  controllers

**Fig. 7.** Bode diagram of (a) desired loop shape and loop shape by proposed controller and (b) step responses of controllers.

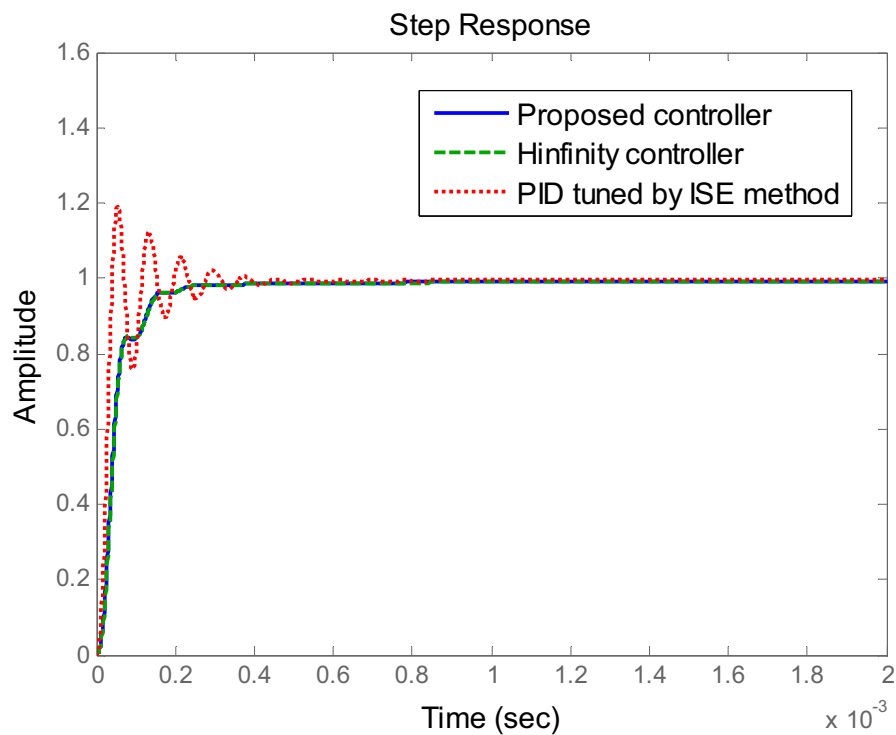


Open loop bode diagrams are plotted in Fig. 7(a) to verify the proposed algorithm. It is clearly shown that the proposed technique performs as a robust controller. Fig. 7(b) shows the step responses of the proposed robust PID, the conventional  $\mathcal{H}_\infty$  controller, and PID tuned by ISE method. The settling time of all responses is about 700  $\mu\text{sec}$ . To verify the robust performance, we change the converter's parameters as:

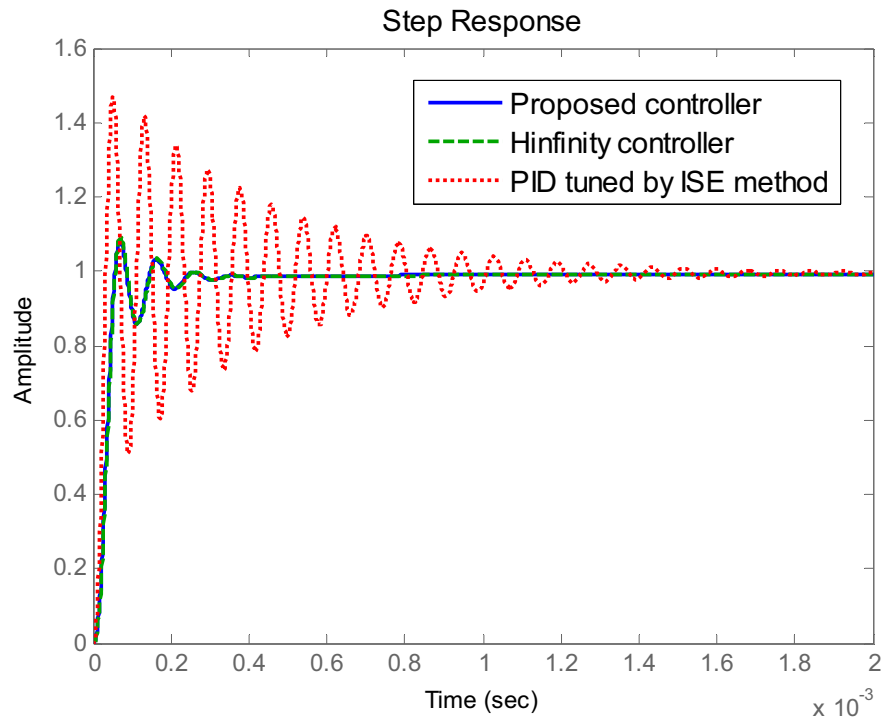
Case I :  $R_L = 16 \Omega$ ,  $L = 110 \mu\text{H}$  and  $C = 330 \mu\text{F}$

Case II :  $R_L = 14 \Omega$ ,  $L = 130 \mu\text{H}$  and  $C = 250 \mu\text{F}$

Obviously, these conditions (increase L and decrease load and C) are worse than the nominal condition. The designed controllers in (21), (24) and (25) are adopted to control this perturbed plant. The step responses are shown in Fig. 8.



(a) Step responses in the perturbed plant. (Case I)



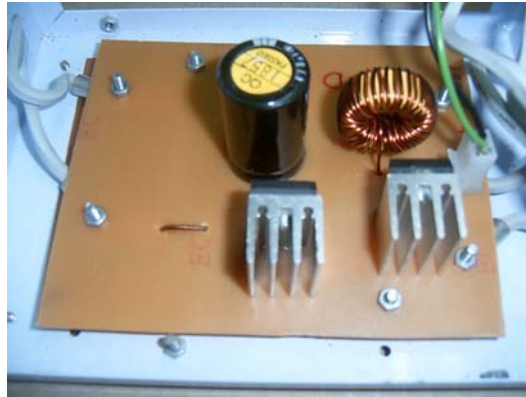
(b) Step responses in the perturbed plant. (Case II)

**Fig.8.** the step responses of all controllers in the perturbed plants (a) case I (b) case II.

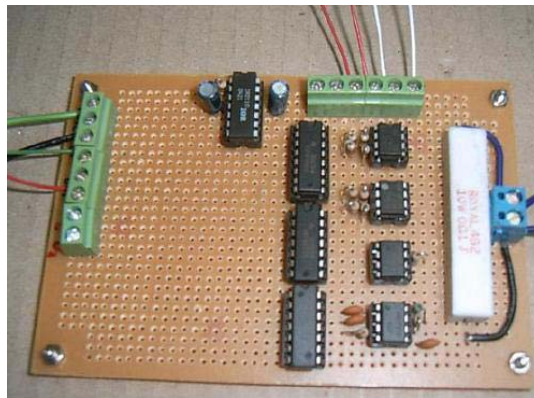
The responses in the perturbed plant of the proposed controller and  $\mathcal{H}_\infty$  loop shaping controller are almost the same as those in the nominal plant with some difference in the settling time and oscillation. The results show that the designed system from the proposed controller and  $\mathcal{H}_\infty$  loop shaping has a good performance and robustness.

As for the controllers designed by ISE method, the settling time and rise time of the nominal plant are slightly smaller than those of the proposed controllers and  $\mathcal{H}_\infty$  controllers. There is neither overshoot nor oscillation in the responses of the nominal plant; however, the response of the perturbed plant has large overshoot and oscillation. The maximum overshoot is about 30% in case I, and 60% in case II. The response of the perturbed plant is much different from that of the nominal plant, displaying poor robust performance. When applying the  $\mathcal{H}_\infty$  control and the proposed controller, the nominal

plant and the perturbed plant have similar responses. As the results indicated,  $\mathcal{H}_\infty$  control and the proposed design technique yields better robust performance than ISE method.



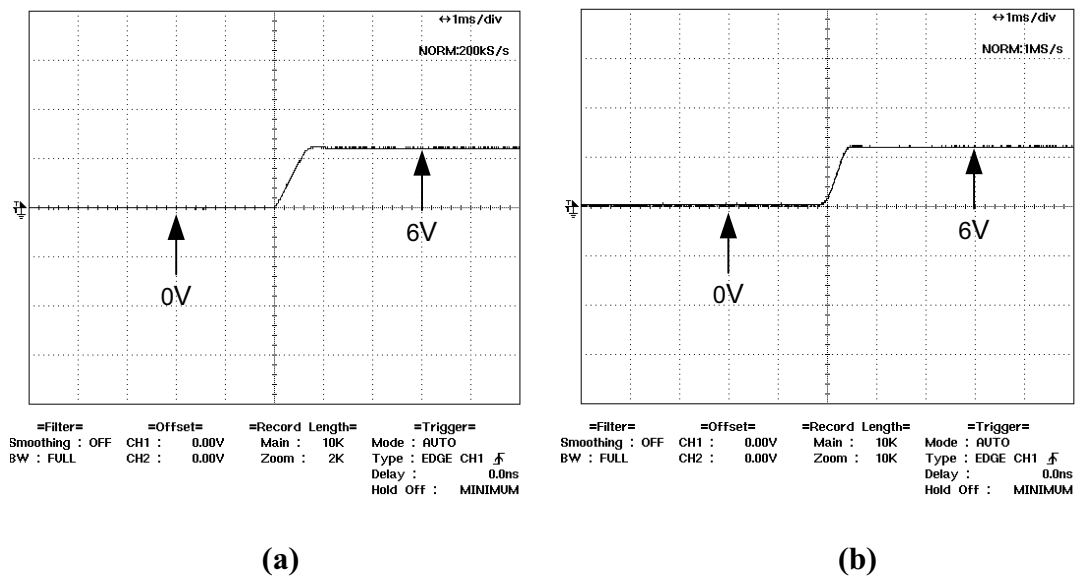
**Fig. 9.** Buck converter circuit



**Fig. 10.** Average Current Mode Control (ACMC) circuit

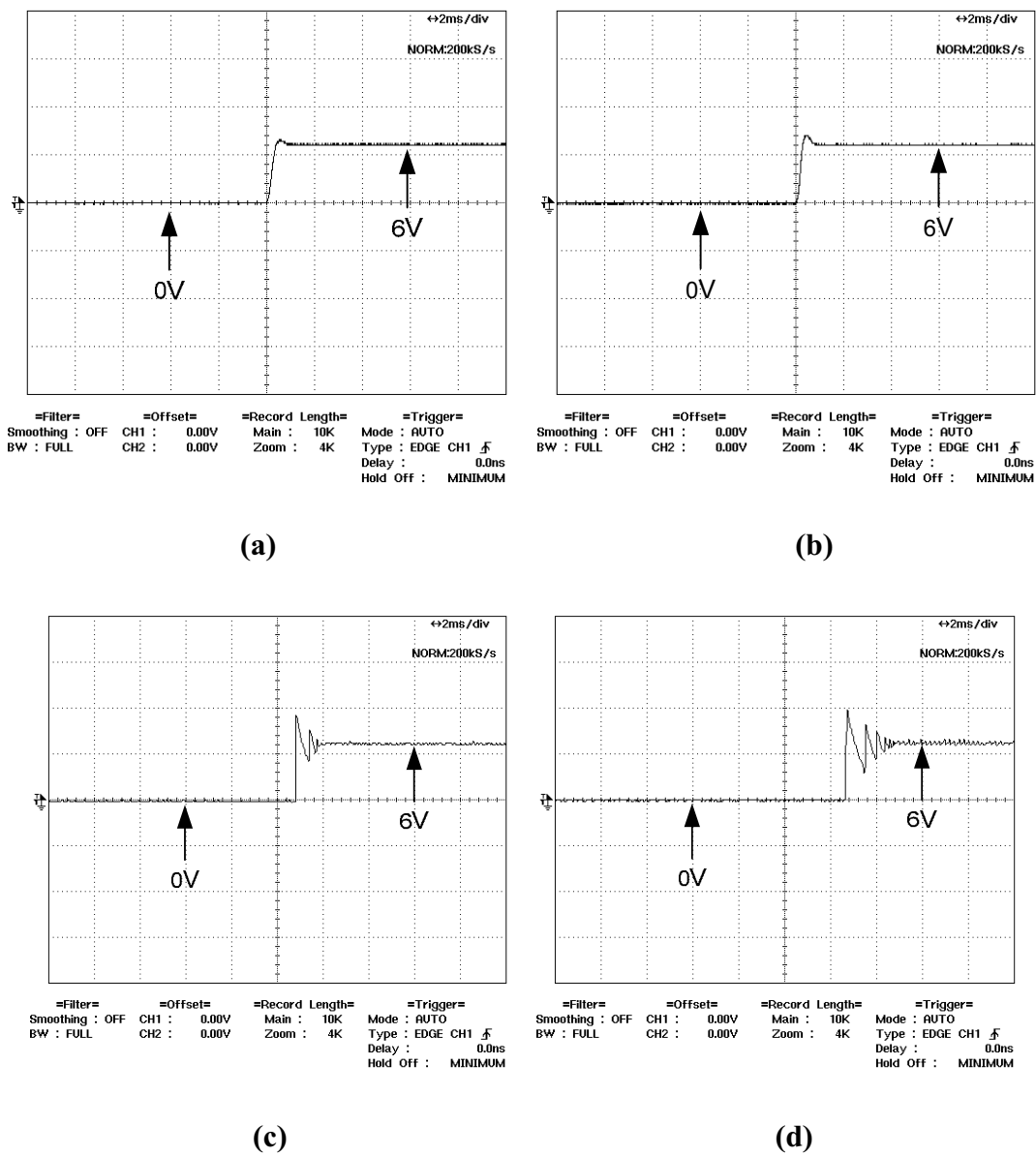
Some experiments are performed to verify the effectiveness of the proposed controller. The nominal values in Table 1 are used to design a buck converter with current mode control. Fig. 9 and 10 show the circuits used in the experiment. A proposed controller, robust PID controller in (21) and PID tuned by ISE method in (25) are used to control the converter. Fig. 11 shows the experimental results of the step response in nominal condition of the proposed controller and PID tuned by ISE method. As seen in Fig. 11 and Fig. 7, the responses of experimental results are almost the same as those of the simulation results. Fig.11 confirms that the time domain response at the nominal plant

of the proposed controller achieves the performance specification as well as the controller designed by ISE method.



**Fig. 11.** Step response in the nominal conditions of (a) proposed controller and (b) PID tuned by ISE method.

To verify the robust performance of the system, an experiment is performed. The component values and operating points of converter are changed to case I and case II. The controllers from the previous experiment are used to control the converter in this perturbed plants. The performance is verified by using the step response. Fig.12(a) and 12(b) show the response of the perturbed plant. The step response is almost the same as the response in nominal conditions. This response is underdamped with a small difference in the settling time. Little overshoots occur in both case I and case II. Experimental results verify that the proposed controller can be applied for the ACMC buck converter to achieve a good robust performance.



**Fig. 12.** Step response in the perturbed conditions of (a) proposed controller, Case I, (b) proposed controller, Case II, (c) PID tuned by ISE method, Case I and (d) PID tuned by ISE method, Case II.

As shown in Fig. 12, in both cases, in the perturbed conditions, the proposed controller has almost the same step response as that in the nominal condition with little overshoot; while the perturbed plant of the controller tuned by ISE method has large overshoot and oscillation. This is similar to the simulation result. As shown in the

experimental result, the system designed by the proposed method has a better robust performance than the one designed by ISE method.

## 5 CONCLUSION

Both of  $\mathcal{H}_\infty$  loop shaping and the proposed technique can be applied to design a robust controller for a buck converter. However, the proposed approach significantly improves in practical control viewpoint by simplifying the controller structure, reducing the controller order, and retaining the robust performance. Although the proposed controller is structured, it still retains the entire robustness and performance guarantee as long as a satisfactory uncertainty boundary  $\varepsilon$  is achieved. Structure of controller in the proposed technique is selectable. This is desirable, especially in DC-DC converter which analog circuit is normally used to design the controller. As shown in the simulation and experimental results, the robust performance obtained from the proposed technique and  $\mathcal{H}_\infty$  loop shaping is better than that of the conventional PID designed by ISE method. In conclusion, by combining of the approaches, genetic algorithms and  $\mathcal{H}_\infty$  loop shaping; robust fixed-structure controller can be designed. Implementation in buck-boost converter assures that the proposed technique is valid and flexible.

## APPENDIX A

Given a shaped plant  $G_s$  and  $A$ ,  $B$ ,  $C$ ,  $D$  represent the shaped plant in the state-space form. To determine  $\varepsilon_{opt}$ , there is a unique method as follows [15].

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = (1 + \lambda_{\max}(XZ))^{1/2} \quad (\text{A.1})$$

where  $X$  and  $Z$  are the solutions of two Riccati in (A.2) and (A.3) respectively,  $\lambda_{\max}$  is the maximum eigenvalue.

$$(A - BS^{-1}D^T C)Z + Z(A - BS^{-1}D^T C)^T - ZC^T R^{-1}CZ + BS^{-1}B^T = 0 \quad (\text{A.2})$$

$$(A - BS^{-1}D^T C)^T X + X(A - BS^{-1}D^T C) - XBS^{-1}B^T X + C^T R^{-1}C = 0 \quad (\text{A.3})$$

where  $S = I + D^T D$ ,  $R = I + DD^T$

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**ภาคผนวก ค**  
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Chapter #

# **FIXED STRUCTURE ROBUST LOOP SHAPING CONTROLLER FOR A BUCK-BOOST CONVERTER USING EVOLUTIONARY ALGORITHM**

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Abstract

In this paper, we propose a new technique used to design a robust controller that is not as high-order and complicated as the ones designed by conventional  $\mathcal{H}_\infty$  loop shaping method. The proposed algorithm is called *Genetic Algorithm (GA) based fixed-structure  $H_\infty$  loop shaping control*. In the approach, GA is adopted to solve the  $H_\infty$  loop shaping design problem under a structure specified controller. The performance and robustness of the proposed controller are investigated in a buck-boost converter in comparison with the controllers designed by conventional  $H_\infty$  loop shaping and conventional ISE method. Results of simulations demonstrate the advantages of the proposed controller in terms of simple structure and robustness against plant perturbations and disturbances. Experiments are performed to verify the effectiveness of the proposed technique.

Keywords:  $H_\infty$  loop shaping , robust controller , Genetic algorithm , buck-boost converter

## **1. INTRODUCTION**

DC-DC converters have been widely used in computer hardware and industrial applications. Controlling of these converters is a challenging field because of their intrinsic nature of nonlinear, time-variant systems (Simone

Buso, 1999). In previous research works, the linear models of these converters were derived by using linearization method (R. B. Ridley, 1989; J. G. Kassakian et. al, 1989). Some linear control techniques were applied to these converters based on the linear models (Simone Buso, 1999; NAIM, R. et. al., 1995; G.C. Ioannidis and S.N. Manias, 1999). NAIM, R., et.al.(1995) applied the  $\mathcal{H}_\infty$  control to a boost converter. Three controllers; voltage mode, feed-forward and current mode control were investigated and compared the performance. G.C. Ioannidis and S.N. Manias (1999) applied the  $\mathcal{H}_\infty$  loop shaping control schemes for a buck converter. In their paper, the  $\mu$ -analysis was used to examine the robust features of the designed controllers. Simone Buso (1999) adopted the robust  $\mu$ -synthesis to design a robust voltage controller for a buck-boost converter with current mode control. The parameter variations in the converter's transfer function were described in term of perturbations of linear fraction transformations (LFT) class.

The controllers in DC to DC converters are usually designed using analog circuit. Although the controllers designed by the techniques mentioned earlier are robust, and high performance, they are complicated and high order, thus they are difficult to be implemented in the converters. Nevertheless, the design of analog circuit for these controllers is not feasible. To solve this problem, fixed-structure controller is investigated. Fixed-structure robust controllers have become an interesting area of research because of their simple structure and acceptable controller order. However, it is difficult to design this controller using analytical method. Algorithms such as genetic algorithm, particle swarm optimization, and gradient method can be employed to simplify the design of this controller.

Several approaches to design a robust control for structure specified controller were proposed in (Bor-Sen Chen and Yu-Min Cheng, 1998; Bor-Sen Chen, et. al, 1995; Shinn-Jang Ho et. al, 1995). In (Bor-Sen Chen and Yu-Min Cheng, 1998), a robust  $\mathcal{H}_\infty$  optimal control problem with structure specified controller was solved by using genetic algorithm (GA). As concluded in this paper, genetic algorithm is a simple and efficient tool to design a structure specified  $\mathcal{H}_\infty$  optimal controller. Bor-Sen Chen, et. al.(1995), proposed a PID design algorithm for mixed  $\mathcal{H}_2/\mathcal{H}_\infty$  control. In their paper, PID controller parameters were tuned in the stability domain to achieve mixed  $\mathcal{H}_2/\mathcal{H}_\infty$  optimal control. A similar work was proposed in (Shinn-Jang Ho et. al, 1995 ) by using the intelligent genetic algorithm to solve the mixed  $\mathcal{H}_2/\mathcal{H}_\infty$  optimal control problem.

The techniques mentioned above are based on the concept of  $\mathcal{H}_\infty$  optimal control which two appropriate weights for both the uncertainty of the model and the performance are essentially chosen. A difficulty with the  $\mathcal{H}_\infty$  optimal control approach is that the appropriate selection of close-loop objectives and weights is not straightforward. In robust control,  $\mathcal{H}_\infty$  loop shaping which is a

simple and efficient technique for designing a robust controller can be alternatively used to design the robust controller for the system. Uncertainties in this approach are modeled as normalized co-prime factors; this uncertainty model does not represent actual physical uncertainty, which usually is unknown in real problems. This technique requires only two specified weights, pre-compensator and post-compensator, for shaping the nominal plant so that the desired open loop shape is achieved. Fortunately, the selection of such weights is based on the concept of classical loop shaping which is a well known technique in controller design. By the reasons mentioned above, this technique is simpler and more intuitive than other robust control techniques. However, the controller designed by  $\mathcal{H}_\infty$  loop shaping is still complicated and has high order. To overcome this problem, in this paper, we propose a fixed-structure  $\mathcal{H}_\infty$  loop shaping control to design a robust controller for a buck-boost converter. In the proposed technique, the controller structure is first specified and the genetic algorithm is then used to evaluate the control's parameters. Simulation and experimental results show the advantages of simple structure, lower order and robustness of the proposed controller.

The remainder of this paper is organized as follows. Converter dynamics are described in section II.  $\mathcal{H}_\infty$  loop shaping and the proposed technique are discussed in section III. Section IV demonstrates the design example and results. Finally, section V concludes the paper with some final remarks.

## 2. CONVERTER MODELING

A typical circuit of buck-boost converter with current mode control is shown in Fig. 1. The dynamic model of this converter from the current reference ( $i_r$ ) to output voltage ( $u_o$ ) is given by (R. B. Ridley, 1989; J. G. Kassakian et. al, 1989).

$$\frac{du_o}{di_r} = R_L \frac{V_i}{V_i + 2V_o} \frac{(1 - \frac{s \cdot L}{R_L} \cdot \frac{V_o}{V_i} \cdot \frac{V_o + V_i}{V_o})}{(1 + s \cdot C \cdot R_L \cdot \frac{V_o + V_i}{2V_o + V_i})} \quad (1)$$

Where  $R_L$  is the nominal load resistant,  $V_o$  is the nominal output voltage,  $V_i$  is the nominal input voltage,  $L$  is the inductance of an inductor used in the circuit,  $C$  is the capacitance,  $f_{sw}$  is the switching frequency. The accuracy of this model has been proved to be accepted, at least in frequency of interest in this application.

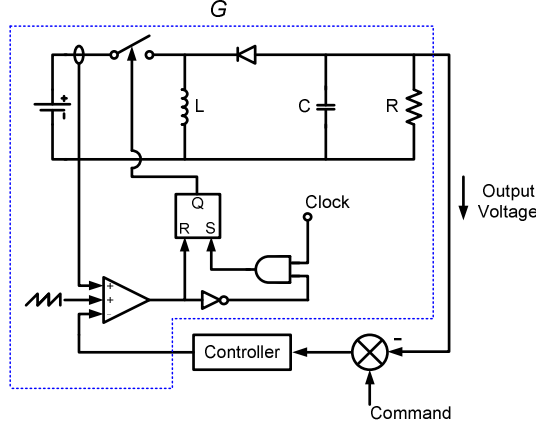


Figure #-1. Buck-boost converter with current mode control.

### 3. $\mathcal{H}_\infty$ LOOP SHAPING CONTROL AND PROPOSED TECHNIQUE

This section illustrates the concepts of the standard  $\mathcal{H}_\infty$  loop shaping control and the proposed technique. Consider the system described by the block diagram (Fig. 2), where the plant  $G$  and the controller  $K$  are real rational and proper.  $y$  is the output,  $u$  is the control input,  $w$  is the vector signal including noises, disturbances, and reference signals, and  $z$  is the vector signal including all controlled signals and tracking errors. The  $\mathcal{H}_\infty$  optimal control problem is to find admission controller  $K(p)$  such that  $\|T_{zw}\|_\infty$  is minimized, where  $\|T_{zw}\|_\infty$  is the maximum norm of the transfer function from  $w$  to  $z$ , and the admission controller is the controller that internally stabilizes the system (McFarlane, D.C. and K. Glover, 1992).

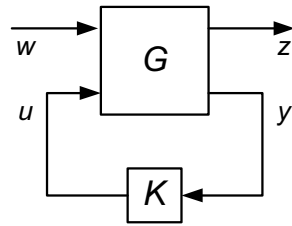


Figure #-2. General block diagram in robust control problem.

### 3.1 Standard $\mathcal{H}_\infty$ Loop Shaping

$\mathcal{H}_\infty$  loop shaping control (McFarlane, D.C. and K. Glover, 1992) is an efficient method to design a robust controller. This approach requires only a desired open loop shape in frequency domain. Two weighting functions,  $W_1$  (pre-compensator) and  $W_2$  (post-compensator), are specified to shape the original plant  $G$ . In this approach, the shaped plant is formulated as normalized coprime factor, which separates the plant  $G_s$  into normalized nominator  $N_s$  and denominator  $M_s$  factors. If the shaped plant  $G_s = W_1 G W_2 = N_s M_s^{-1}$ , then a perturbed plant can be written as

$$G_\Delta = (N_s + \Delta_{N_s})(M_s + \Delta_{M_s})^{-1} \quad (2)$$

Where  $\Delta_{N_s}$  and  $\Delta_{M_s}$  are stable, unknown representing the uncertainty satisfying  $\|\Delta_{N_s}, \Delta_{M_s}\|_\infty \leq \varepsilon$ ,  $\varepsilon$  is the uncertainty boundary, called stability margin.

According to the standard procedure of  $\mathcal{H}_\infty$  loop shaping, the following steps can be applied to design the  $\mathcal{H}_\infty$  loop shaping controller.

**Step 1** Shape the singular values of the nominal plant  $G_o$  by using a pre-compensator  $W_1$  and/or a post-compensator  $W_2$  to get the desired loop shape.  $W_2$  can be chosen as an identity matrix, since we can neglect the sensor noise effect when the use of good sensor is assumed (Kemin Zhou and Jhon C. Doyle, 1998). Weight selection is very important for the design. There are some guidelines for the weight selection in (Sigurd Skogestad, Ian Postlethwaite, 1996). In SISO system, the weighting functions  $W_1$  and  $W_2$  can be chosen as

$$W_1 = K_w \frac{s+a}{s+b} \quad \text{and} \quad W_2 = \frac{c}{s+c} \quad (3)$$

Where  $K_w$ ,  $a$ ,  $b$  and  $c$  are positive values.

**Step 2** Minimize  $\infty$ -norm of the transfer matrix  $T_{zw}$  over all stabilizing controllers  $K$  to obtain an optimal cost  $\gamma_{opt}$ , as

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = \inf_{stab K} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + G_s K)^{-1} M_s^{-1} \right\|_\infty \quad (4)$$

To determine  $\varepsilon_{opt}$ , there is a unique method explained in (Sigurd Skogestad and Ian Postlethwaite, 1996).  $\varepsilon_{opt} \ll 1$  indicates that  $W_1$  or  $W_2$



designed in step 1 are incompatible with robust stability requirement. If  $\varepsilon_{opt}$  is not satisfied ( $\varepsilon_{opt} < 1$ ), then return to step 1, adjust  $W_1$ .

**Step 3** Select  $\varepsilon < \varepsilon_{opt}$  and then synthesize a controller  $K_\infty$  that satisfies

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + G_s K_\infty)^{-1} M_s^{-1} \right\|_\infty \leq \varepsilon^{-1} \quad (5)$$

Controller  $K_\infty$  is obtained by solving the optimal control problem. More detail is available in (Sigurd Skogestad and Ian Postlethwaite, 1996).

**Step 4** Final controller ( $K$ ) follows

$$K = W_1 K_\infty W_2 \quad (6)$$

### 3.2 Genetic Algorithm based Fixed-Structure $\mathcal{H}_\infty$ Loop Shaping Optimization

Practical implementation of the controller derived from  $\mathcal{H}_\infty$  loop shaping method is difficult because the order of the controller is quite high. In this paper, the genetic searching algorithm is adopted to solve this problem. Although the proposed controller is structured, it still retains the entire robustness and performance guarantee as long as a satisfactory uncertainty boundary  $\varepsilon$  is achieved. The proposed algorithm is explained as following.

Assume that the predefined structure controller  $K(p)$  has a satisfied parameters  $p$ . Based on the concept of  $\mathcal{H}_\infty$  loop shaping, optimization goal is to find parameters  $p$  in controller  $K(p)$  that minimize infinity norm  $\|T_{zw}\|_\infty$ . In the proposed technique, the final controller  $K$  is defined as

$$K = K(p)W_2 \quad (7)$$

Assuming that  $W_1$  is invertible, from (6) then it is obtained that

$$K_\infty = W_1^{-1} K(p) \quad (8)$$

In many cases, the weight  $W_2$  is selected as identity matrix  $I$ . However, if  $W_2$  is a transfer function matrix, then the final controller is the controller  $K(p)$  in series with the weight  $W_2$ . By Substitution of (8) into (5), then the  $\infty$ -norm of the transfer function matrix from disturbances to states,  $\|T_{zw}\|_\infty$ , which is subjected to be minimized can be written as

$$J_{cost} = \gamma = \|T_{zw}\|_{\infty} = \left\| \begin{bmatrix} I \\ W_1^{-1}K(p) \end{bmatrix} (I + G_s W_1^{-1}K(p))^{-1} M_s^{-1} \right\|_{\infty} \quad (9)$$

In this paper, GA is adopted to find the optimal control parameters  $p^*$  in the stabilizing controller  $K(p)$  such that the  $\|T_{zw}\|_{\infty}$  is minimized.

### 3.2.1 Genetic Algorithms

GA is well known as a biologically inspired class of algorithms that can be applied to any nonlinear optimization problem. This algorithm applies the concept of chromosomes, and the operations of crossover, mutation and reproduction. At each step, called generation, fitness values of all chromosomes in population are calculated. Chromosome, which has the maximum fitness value (minimum cost value), is kept as a solution in the current generation and passed to the next generation. The new population of the next generation is obtained by performing the genetic operators such as crossover, mutation, and reproduction. Crossover randomly selects a site along the length of two chromosomes, and then splits the two chromosomes into two pieces by breaking them at the crossover site. The new chromosomes are then formed by matching the headpiece of one chromosome with the tailpiece of the other. Mutation operation forms a new chromosome by randomly changing value of a single bit in the chromosome. Reproduction operation forms a new chromosome by just copying the old chromosome. Chromosome selection in genetic algorithm depends on the fitness value; high fitness value means high chance to be selected. Operation type selection; mutation, reproduction, or crossover, depends on the pre-specified operation's probability. Chromosome in genetic population is coded as binary number. However, for the real number problem, decoding binary number to floating number is applied (Chris Houck, et. al., 1995). Our proposed algorithm is summarized as

**Step 1** Shape the singular values of the nominal plant  $G$  by  $W_1$  and  $W_2$ . Then evaluate the  $\varepsilon_{opt}$  using (4). If  $\varepsilon_{opt} < 0.25$ , then go to step 1 to adjust the weight  $W_1$ .

**Step 2** Select a controller structure  $K(p)$  and initialize several sets of parameters  $p$  as population in the 1<sup>st</sup> generation. Define the genetic parameters such as initial population size, crossover and mutation probability, maximum generation, etc.

**Step 3** Evaluate the cost function  $J_{cost}$  of each chromosome using (9). Assign  $J_{cost} = 100$ , or large number if  $K(p)$  does not meet the constraints in our optimization problem. The fitness value is assigned as  $1/J_{cost}$ . Select the

chromosome with minimum cost function as a solution in the current generation. For the first generation,  $\text{Gen} = 1$ .

**Step 4** Increment the generation for a step.

**Step 5** While the current generation is less than the maximum generation, create a new population using genetic operators and go to step 3. If the current generation is the maximum generation, then stop.

**Step 6** Check performances in both frequency and time domains. If the performance is not satisfied, such as too low  $\varepsilon$  (too low fitness function), then go to step 3 to change the control structure. Low  $\varepsilon$  indicates that the selected control structure is not suitable for the problem.

#### 4. SIMULATION AND EXPERIMENTAL RESULTS

In this paper, a buck-boost converter designed for a photovoltaic system is studied. Converter's parameters and considered variation ranges used in this paper are given in Table 1.

Table #1. Converter's parameters and considered variation ranges.

Parameter	Name	Nominal Value
$R_L$	Load Resistant	40 $\Omega$
$V_o$	Output Voltage	30 V
$V_i$	Input Voltage	12 V
$L$	Inductance	100 $\mu\text{H}$
$C$	Capacitor	470 $\mu\text{F}$
$f_{sw}$	Switching frequency	100 kHz

By (1), the nominal transfer function is found to be

$$G = \frac{-0.0042s + 480}{0.7896s + 72} \quad (10)$$

Both  $\mathcal{H}_\infty$  loop shaping control and our proposed technique are applied to this converter. Firstly, we design a controller by the conventional  $\mathcal{H}_\infty$  loop shaping procedure. Based on the concept of  $\mathcal{H}_\infty$  loop shaping,  $W_1$  and  $W_2$  can be selected as

$$W_1 = 30 \frac{(s + 26.7)}{(s + 0.001)}, \quad W_2 = \frac{100000}{s + 100000} \quad (11)$$

Fig. 4(a) shows the plot of open loop shape of nominal plant and shaped plant. As seen in this figure, the bandwidth of the nominal plant is about 600

rad/sec. With these weighting functions, bandwidth of the desired control system is increased to 20,000 rad/sec. Significant improvement in terms of performances and robustness is carried out by these weighting functions. The shaped plant is written as

$$G_s = W_1 G W_2 = 30 \frac{(s+26.7)}{(s+0.001)} \frac{(-0.0042s+480)}{(0.7896s+72)} \frac{(100000)}{(s+100000)} \quad (12)$$

By applying the  $\mathcal{H}_\infty$  loop shaping method, the optimal stability margin ( $\varepsilon_{opt}$ ) is founded at 0.612 ( $\gamma_{opt} = 1.6338$ ). This value indicates that the selected weighting function is compatible with the robust stability requirement. The  $\varepsilon = 0.590$  ( $\gamma=1.6949$ ), which is less than the optimal stability margin, is chosen to synthesize the controller. Based on the conventional technique in section III, the conventional  $\mathcal{H}_\infty$  loop shaping controller is synthesized as following

$$K(s) = W_1 K_\infty W_2 = 30 \frac{(s+26.7)}{(s+0.001)} \frac{(261841)(s+1.002 \times 10^5)(s+26.9)}{(s^2+3.265 \times 10^5 s+3.608 \times 10^{10})(s+26.7)} \frac{(100000)}{(s+100000)} \quad (13)$$

As shown in (13), the controller is 5<sup>th</sup> order controller resulting in a complicated structure. Next, PI controller is investigated as a fixed-structure controller. The controller structure is expressed in (14).  $K_p$  and  $K_i$  are parameters that will be evaluated.

$$K(p) = K_p + \frac{K_i}{s} \quad (14)$$

The controller parameters, their ranges, and genetic algorithms parameters are selected as follows:  $K_p \in [0,200]$ ,  $K_i \in [0,1000]$ , population size = 100, crossover probability = 0.7, mutation probability = 0.25, and maximum generation = 30. An optimal solution is obtained after 18 generations. The optimal solution is shown in (15), which has stability margin ( $\varepsilon$ ) of 0.586 ( $\gamma=1.7064$ ).

$$K(p)^* = 21.84 + \frac{597.6}{s} \quad (15)$$

The final controller ( $K$ ) is shown in (16).

$$K = \left( 21.84 + \frac{597.6}{s} \right) \left( \frac{100000}{s+100000} \right) \quad (16)$$

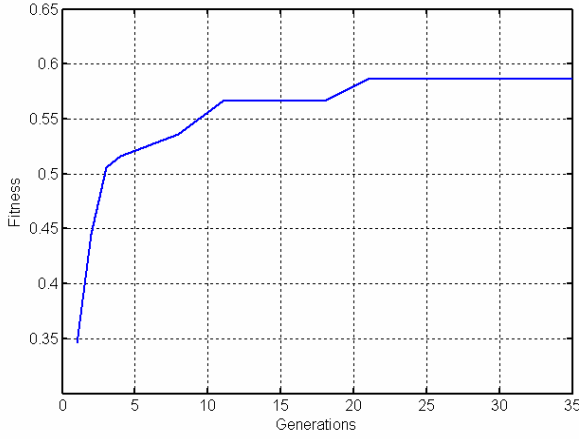


Figure #-3. Fitness functions versus iterations in genetic algorithm.

Fig. 3 shows plots of convergence of cost function  $J_{cost}$  versus generations by genetic algorithm. As seen in this figure, the optimal fixed-structure controller provides the satisfied stability margin at 0.586 ( $\gamma=1.7064$ ). The open loop bode diagrams of the nominal and shaped plants are shown in Fig. 4(a). As shown in this figure, at low frequency, the open loop gain of shaped plant is much larger than that of the nominal plant. This makes the system good in term of performance tracking and disturbance rejection. Open loop bode diagrams are plotted in Fig. 4(b) to verify the proposed algorithm. It is clearly shown that the loop shapes of  $\mathcal{H}_\infty$  control and proposed PI controller are close to the desired loop shape. Fig. 4(c) shows the step responses of the optimal solutions from the proposed robust PI and the conventional  $\mathcal{H}_\infty$  controllers. As shown in this figure, the settling time of all responses is about 500  $\mu\text{sec}$ .

In addition, in this paper, we apply the conventional PI controller based on the ISE method. In this method, the controller parameter is tuned in such a way that the integral of square error between output and desired response is minimized. However, to prevent the oscillations in the response, ISE with the model reference can be applied (Jietae Lee and Thomas F. Edgar , 2004). In this paper, we adopted the ISE with model reference to design a PI controller to make the settling time close to 500  $\mu\text{sec}$ . The resulting controller is,

$$K_{ISE} = 19 + \frac{2800}{s} \quad (17)$$

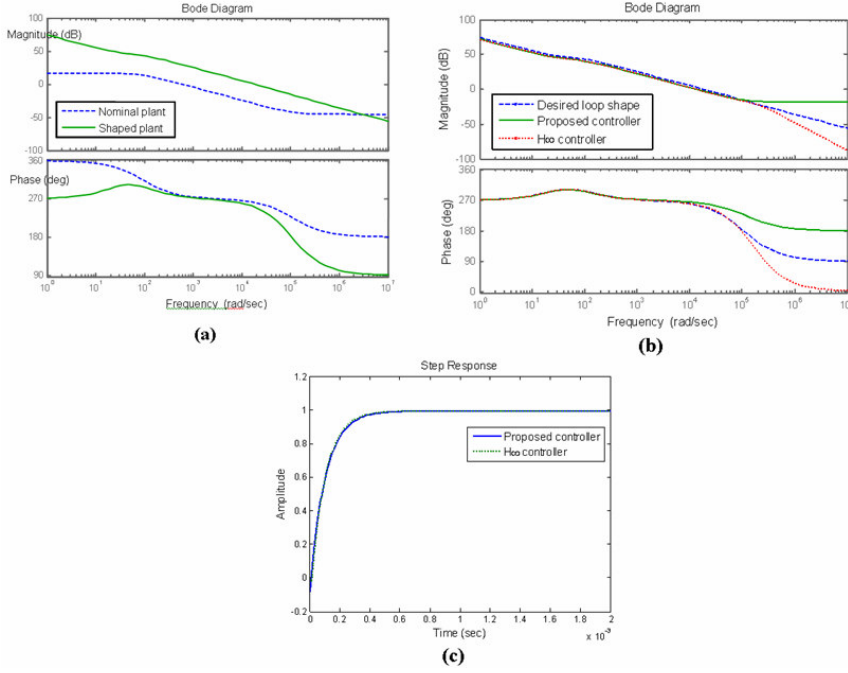


Figure #4. (a) Bode plots of the nominal plant and the shaped plant (desired loop shape) (b) The desired loop shape and the loop shape by the conventional  $H_\infty$  loop shaping and the proposed PI, (c) Step responses by the proposed PI and  $H_\infty$  loop shaping controllers.

The step responses of proposed and  $H_\infty$  controllers in the nominal plant and perturbed plant are shown in Fig. 5(a) and Fig. 5(b), respectively. As shown in the figures, the responses in perturbed plant are almost the same as the responses in the nominal plant with some different in the setting time. The results show that the designed system from the proposed controller and  $H_\infty$  loop shaping has a good performance and robustness. To verify the robust performance, we change the converter's parameters as:  $R_L = 10 \Omega$ ,  $V_i = 10.8 \text{ V}$ ,  $L = 130 \mu\text{H}$  and  $C = 2200 \mu\text{F}$ . The designed controllers in (16) and (17) are adopted to control the perturbed plant. Obviously, this condition (increase  $L$  and  $C$  and decrease the load resistance and input voltage) is worse than the nominal condition, because in this case the gain and phase of the plant are decreased in the crossover region.

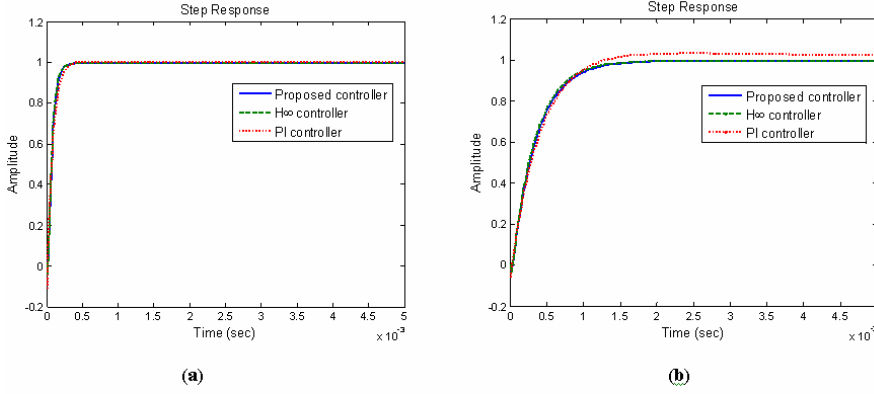


Figure #5. (a) Step responses in the nominal plant. (b) Step responses in the perturbed plant.

Some experiments are performed to verify the effectiveness of the proposed controller. The nominal values in Table 1 are used to design a buck-boost converter with current mode control. A proposed robust PI controller in (16) and PI controller tuned by ISE method in (17) are used to control the converter. As seen in Fig. 5(a) and Fig. 6, the response of experimental result is almost the same as that of the simulation result.

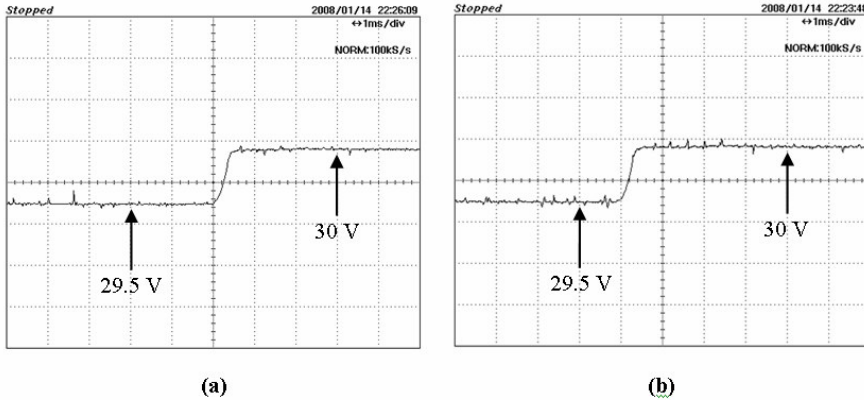


Figure #6. Step responses in nominal condition. (a) Proposed PI controller. (b) PI tuned by ISE method.

To verify the robust performance of the system, an experiment is performed. The component values and operating point of the converter are changed to:  $R_L = 10 \, \Omega$ ,  $V_i = 10.8 \, \text{V}$ ,  $L = 130 \, \mu\text{H}$  and  $C = 2200 \, \mu\text{F}$ . The performance is verified by using the step response. As shown in Fig. 7, the

step response of the proposed controller is almost the same as the response in nominal conditions while the step response of the PI controller tuned by ISE method has an overshoot when the system parameters change. This can be verified that the robust performance of the proposed technique is better than that of PI controller by ISE method.

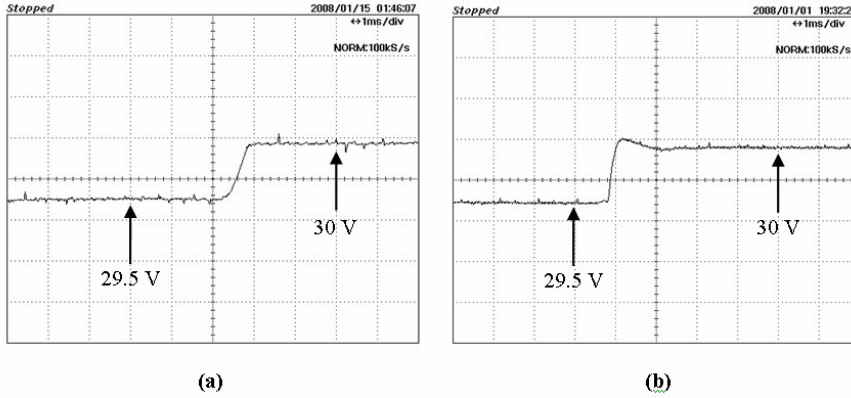


Figure #-7. Step response in the closed loop in perturbed conditions. (a) Proposed PI controller. (b) PI tuned by ISE method.

To verify the robust against the sudden change of load, an experiment were performed. As shown in Fig.8, when the load is abruptly changed, the proposed controller can maintain the desired voltage.



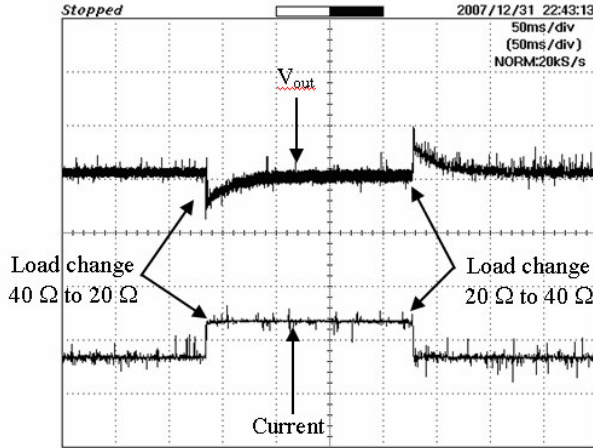


Figure #-8. Transient response of propose controller when the load change  $40\ \Omega$  to  $20\ \Omega$ .

## 5. CONCLUSION

Both  $\mathcal{H}_\infty$  loop shaping and *GA based fixed-structure  $H_\infty$  loop shaping control*, are applicable in designing a robust controller for a current mode buck-boost converter. However, the proposed approach significantly improves the controller design in practical control viewpoint by simplifying the controller structure, reducing the controller order and retaining the robust performance. Structure of controller in the proposed technique is selectable. This is desirable, especially in the DC-DC converter which analog circuit is normally used to design the controller. In conclusion, by combining of these two approaches, genetic algorithms and  $\mathcal{H}_\infty$  loop shaping; fixed-structure controller design can be achieved. Implementation in buck-boost converter assures that the proposed technique is valid and flexible.

## 6. ACKNOWLEDGMENT

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**ภาคผนวก ง**

**รางวัลที่ได้รับ (Best Paper Award การประชุม  
วิชาการระดับนานาชาติ ICCA ณ.ประเทศ ฮ็องกง)**

## Best Paper Award จากการประชุม ICCA2007 Hong Kong

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A Compact Magnetic Bearing System for Axial Flow Blood Pump  
**Certificate of Merit for The 2007 IAENG International Conference on Control and Automation: Dr. Somyot Kaitwanidvilai**  
**Multi-Objective Genetic Algorithms Based Mixed Robust/Model Reference Control**  
Certificate of Merit (Student) for The 2007 IAENG International Conference on Control and Automation: Yanliang Zhang  
Non-linear Sliding Mode Control for a Rotary Inverted Pendulum  
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Need of Customer Orientation in IT Domain  
Certificate of Merit for The 2007 IAENG International Conference on Computer Science: Dr. Walter Dosch  
On the High-Level Synthesis of Combinational Circuits for Contiguous Pattern Matching

## ภาคผนวก จ

### ตัวอย่างบทความประชุมวิชาการระดับนานาชาติที่ได้ นำเสนอในที่ประชุมระดับนานาชาติ (จำนวน 3 เรื่อง จาก ทั้งหมด 5 เรื่อง)

1. S. Kaitwanidvilai, P. Olarnthichachart, “Genetic based Robust  $H_\infty$  Loop Shaping PID Control for a Current-Mode Boost Converter”, *International Conference on Electric Machine and System ICEMS 2006*, November 2006, Nagasaki, Japan.
2. S. Kaitwanidvilai, P. Olarnthichachart, “Multi-Objective Genetic Algorithms based Mixed Robust/Model Reference Control”, *International Conference on Control and Automation*, Hongkong, 21-23 March 2007.
3. “Particle Swarm Optimization based Fixed-Structure  $H_\infty$  Loop Shaping Control of MIMO System”, *International Conference of Modeling, Identification and Control*, Austria, 11-13 February 2008.
4. “Structured Robust Loop shaping control for HIMAT SYSTEM using SWARM INTELLIGENT APPROACH”, *International Conference of Control and Automation*, HK 2008.
5. “Fixed Structure Robust Loop Shaping Controller for a Buck-Boost Converter using Genetic Algorithm”, *International Conference of Electrical Engineering*, HK 2008.

The 2006 International Conference on Electrical Machines and Systems

# **ICEMS2006**

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*November 20 – 23, 2006*

Session Paper

## **LS3D**

Power Converters

# Genetic based Robust $H_\infty$ Loop Shaping PID Control For a Current-Mode Boost Converter

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**Abstract**— $H_\infty$  control is one of popular techniques for controlling system. Robust controller, which designed by conventional  $H_\infty$  control results in high order and complicated structure. It is not easy to implement this controller in practical works. To solve this problem, we propose a genetic algorithm based  $H_\infty$  loop shaping PID control to design a robust controller. The infinity norm of the transfer function from disturbance to output is subjected to be minimized by genetic algorithm. The simple structure and robust controller can be achieved by the proposed techniques. In this paper, the designed controller is implemented on an average current mode control of boost converter. Robustness against parameters changing and disturbance is clearly shown in experimental results. Simulation results are also shown for comparison and prove the effectiveness of our technique.

## I. INTRODUCTION

Boost converter plays an important role in the power electronic circuit. This converter is applied in many applications such as battery charger, maximum power-point tracking in PV system, switching mode power supply, etc. Many techniques were applied in this converter by many researchers [1-7]. Simone Buso [1] adopted the robust  $\mu$ -synthesis to design a robust voltage controller for a buck-boost converter (PCMC: Peak Current Mode Control). An inner current control loop is applied to limit current and switching protection. M. Ahmed *et al.* [2] presented a standard procedure for modeling the basic converters. In their work, analytic methods and simple PID control was applied to a general converter control topology. Buck converter was also implemented. In [3], the standard procedure and sliding mode control were applied to control a converter. The comparison between PID and sliding mode control method was shown in [4].

Robust control is widely used to control many systems. In DC/DC converter, many researchers designed and implemented the robust control to this circuit. Sliding mode control in [5] was implemented in a boost converter to achieve the robustness. In [6], a linear robust approach to DC/DC converter was proposed. Passivity based robust control was implemented in [7] to a boost converter circuit.

There are two modes for voltage controlling in boost converter, voltage and current mode control. Current mode control gains many advantages such as current limiting, switching protection and better dynamic response. There are an inner current-loop and outer voltage-loop in this controller [2].

From previous researches in control system,  $H_\infty$  control is one of popular techniques to control in many systems. Various  $H_\infty$  controls are able to design the controller to guarantee the robustness and performance of the closed loop system. However, controller designed by conventional  $H_\infty$  control technique [8] results in the complex structure and high order controller. To solve this problem, Somyot and Manukid [8] proposed the genetic based fixed structure robust  $H_\infty$  loop shaping control. This technique designs the simple structure but achieve a good robust performance. In this paper, this technique is adopted to design a current mode controller for a boost converter.

This paper is organized as follows. In section 2, we describe the dynamic model and system identification procedure of a boost converter circuit. Section 3 presents a review of  $H_\infty$  loop shaping. Our proposed algorithm, robust PID controller that evolved by genetic algorithms is described in this section. Section 4 details the design, simulation and experimental results. Finally, Sections 5 summarizes and concludes the paper.

## II. CONVERTER MODELING AND SYSTEM IDENTIFICATION

Current mode control (CMC) in DC-DC converter is typically two-loop system (voltage loop and current loop). The current loop is used to maintain the inductor current equal to reference current. This reference current is obtained from the output of voltage control loop. The voltage loop is used to compare the reference voltage (command) and output voltage (output) and control the output voltage of boost converter. In continuous conduction mode of CMC, the popular control schemes are PCMC (Peak Current Mode Control) or an ACMC (Average Current Mode Control). ACMC has several advantages over PCMC [9] such as robust against the noise, high degree of accuracy to track the average inductor current. In ACMC, the current loop control is typically designed as PI controller. In this paper, the design of this current loop gain is adopted from [9-10]. Consider the equation in [9], the controller gain of current loop can be expressed as

# STRUCTURED ROBUST LOOP SHAPING CONTROL FOR HIMAT SYSTEM USING SWARM INTELLIGENT APPROACH

Somyot Kaitwanidvilai and Manukid Parnichkun

**Abstract**— Robust loop shaping control is a feasible method for designing a robust controller; however, the controller designed by this method is complicated and difficult to implement practically. To overcome this problem, in this paper, a new design technique of a fixed-structure robust loop shaping controller for a highly maneuverable airplane, HIMAT, is proposed. The performance and robust stability conditions of the designed system satisfying  $H_\infty$  loop shaping control are formulated as the objective function in the optimization problem. Particle Swarm Optimization (PSO) technique is adopted to solve this problem and to achieve the control parameters of the proposed controller. Simulation results demonstrate that the proposed approach is numerically efficient and leads to performance comparable to that of the other method.

**Index Terms**—  $H_\infty$  loop shaping control, robust control, particle swarm optimization, HIMAT system, fixed-structure controller.

## I. INTRODUCTION

In the past decades, many immense developments in robust control techniques have been proposed and the results of those are utilized in many control systems. As shown in previous works,  $H_\infty$  optimal control is a powerful technique to design a robust controller for system under conditions of uncertainty, parameter change, and disturbance. However, the order of controller designed by this technique is much higher than that of the plant. It is not easy to implement this controller in practical applications. In industrial applications, structures such as PID, lead-lag compensators are widely used because their structures are simple, tuning parameters are fewer, and they are lower order. Unfortunately, tuning of control parameters of such controllers for achieving a good performance and robustness is difficult. To solve this problem, the design of fixed-structure robust controller has been proposed. Fixed-structure robust controller has become an interesting area of research because of its simple structure and acceptable controller order. However, the design of this controller by using analytical methods remains difficult. To simplify the problem, searching algorithms such as genetic algorithm, particle swarm optimization technique, tabu-search, etc., can be employed. Several approaches to design a fixed-structure robust controller were proposed in

[1-3, 5-7]. In [1], a robust  $H_\infty$  optimal control problem with structure specified controller was solved by using genetic algorithm (GA). As concluded in [1], genetic algorithm is a simple and efficient tool to design a fixed-structure  $H_\infty$  optimal controller. Bor-Sen.Chen. *et. al.*[2], proposed a PID design algorithm for mixed  $H_2/H_\infty$  control. In their paper, PID control parameters were tuned in the stability domain to achieve mixed  $H_2/H_\infty$  optimal control. A similar work was proposed in [3] by using the intelligent genetic algorithm to solve the mixed  $H_2/H_\infty$  optimal control problem. The techniques in [1-3] are based on the concept of  $H_\infty$  optimal control which two appropriate weights for both the uncertainty of the model and the performance are essentially chosen. A difficulty with the  $H_\infty$  optimal control approach is that the appropriate selection of close-loop objectives and weights is not straightforward [4]. Moreover, especially in MIMO system, it is not easy to specify the uncertainty weight in practice. Alternatively, MIMO controller can be designed by using  $H_\infty$  loop shaping control [4] which is a simple and efficient technique for designing a robust controller. Uncertainties in this approach are modeled as normalized co-prime factors; this uncertainty model does not represent actual physical uncertainty, which usually is unknown in real problems. This technique requires only two specified weights, pre- and post-compensator weights, for shaping the nominal plant so that the desired open loop shape is achieved. Fortunately, the selection of such weights is based on the concept of classical loop shaping which is a well known technique in the controller design. By the reasons mentioned above, this technique is simpler and more intuitive than other robust control techniques. However, the controller designed by  $H_\infty$  loop shaping is still complicated. To overcome this problem, several approaches have been proposed to design a fixed-structure  $H_\infty$  loop shaping controller, such as a state-space approach by A. Umut. Genc in 2000 [5], genetic algorithms based fixed-structure  $H_\infty$  loop shaping by Somyot and Manukid in 2004 [6], etc. The method in [5] is based on the concept of state space approach and BMI optimization. Unfortunately, the chance of reaching a satisfactory solution of this approach depends on the initial controller chosen and the problem of the local minima is often occurred. In [6], a global optimization method was adopted to design the fixed-structure robust  $H_\infty$  loop shaping controller; however, the designed controllers in [6] were only implemented on a pneumatic servo system which is a SISO system. In [7], the same technique as [6] was adopted to design a robust controller of a boost converter; however, this application is also a SISO system. In this paper, PSO is proposed to synthesize a fixed-structure  $H_\infty$  loop shaping controller for HIMAT system. Based on the concept of PSO technique, the choosing of initial controller required in the method in [5] is not necessary and the problem of local minima is reduced. Structure of controller in the proposed technique is selectable;

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# Fixed Structure Robust Loop Shaping Controller for a Buck-Boost Converter using Genetic Algorithm

Somyot Kaitwanidvilai, Piyapong Olanthichachat and Manukid Parnichkun

**Abstract**— Robust controller designed by  $H_\infty$  loop shaping is complicated and its order is much higher than that of the plant. It is not easy to implement this controller in practical engineering applications. To overcome this problem, we propose an algorithm, *GA based fixed-structure  $H_\infty$  loop shaping control*, to design a robust controller. Genetic algorithm is used to solve the  $H_\infty$  loop shaping design problem under a structure specified controller. The performance and robustness of the proposed controller are investigated in a buck-boost converter in comparison with the controller designed by conventional  $H_\infty$  loop shaping method. Results of simulation demonstrate the advantages of simple structure and robustness against plant perturbations and disturbances of the proposed controller. Experiments are performed to verify the effectiveness of the proposed technique.

**Index Terms**—  $H_\infty$  loop shaping , genetic algorithm , buck boost converter

## I. INTRODUCTION

DC-DC converters have been widely used in computer hardware and industrial applications. Controlling of these converters is a challenging field because of their intrinsic nature of nonlinear, time-variant systems [1]. In previous research works, the linear models of these converters were derived by using linearization method [2-3]. Some linear control techniques were applied to these converters based on the linear models [1, 4-5]. NAIM, R., *et al.* [4], applied the  $H_\infty$  control to a boost converter. Three controllers; voltage mode, feed-forward and current mode control were investigated and compared the performance. G.C. Ioannidis and S.N. Manias [5] applied the  $H_\infty$  loop shaping control schemes for a buck converter. In their paper, the  $\mu$ -analysis was used to examine the robust features of the designed controllers. Simone Buso [1] adopted the robust  $\mu$ -synthesis to design a robust voltage controller for a buck-boost converter with current mode control. The parameter variations in the converter's transfer function were described in term of perturbations of linear fraction transformations (LFT) class.

In DC to DC converter, normally, the controller is designed by using analog circuit. Although the higher control

techniques mentioned above are powerful techniques for designing the high performance and robust controller; however, the structure of these controllers is complicated with a high order. It is not easy to implement these controllers in the converters. Nevertheless, the design of analog circuit for these controllers is not feasible. To overcome this problem, fixed-structure controller is investigated. Fixed-structure robust controllers have become an interesting area of research because of their simple structure and acceptable controller order. However, the design of this controller by using the analytical method remains difficult. To simplify this problem, the searching algorithms such as genetic algorithm, particle swarm optimization technique, gradient method, etc., can be employed.

Several approaches to design a robust control for structure specified controller were proposed in [6-8]. In [6], a robust  $H_\infty$  optimal control problem with structure specified controller was solved by using genetic algorithm (GA). As concluded in [6], genetic algorithm is a simple and efficient tool to design a structure specified  $H_\infty$  optimal controller. Bor-Sen.Chen. *et al.* [7], proposed a PID design algorithm for mixed  $H_2/H_\infty$  control. In their paper, PID controller parameters were tuned in the stability domain to achieve mixed  $H_2/H_\infty$  optimal control. A similar work was proposed in [8] by using the intelligent genetic algorithm to solve the mixed  $H_2/H_\infty$  optimal control problem. The techniques in [6-8] are based on the concept of  $H_\infty$  optimal control which two appropriate weights for both the uncertainty of the model and the performance are essentially chosen. A difficulty with the  $H_\infty$  optimal control approach is that the appropriate selection of close-loop objectives and weights is not straightforward. In robust control,  $H_\infty$  loop shaping which is a simple and efficient technique for designing a robust controller can be alternatively used to design the robust controller for the system. Uncertainties in this approach are modeled as normalized co-prime factors; this uncertainty model does not represent actual physical uncertainty, which usually is unknown in real problems. This technique requires only two specified weights, pre-compensator and post-compensator, for shaping the nominal plant so that the desired open loop shape is achieved. Fortunately, the selection of such weights is based on the concept of classical loop shaping which is a well known technique in controller design. By the reasons mentioned above, this technique is simpler and more intuitive than other robust control techniques. However, the controller designed by  $H_\infty$  loop shaping is still complicated and has high order. To overcome this problem, in this paper, we propose a fixed-structure  $H_\infty$  loop shaping control to design a robust controller for a buck boost converter. In the proposed technique, the controller structure is firstly specified and the genetic algorithm is then

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