รายงานวิจัยฉบับสมบูรณ์

โครงการ เทคนิควิเคราะห์และ Design Curves
สำหรับ Pickett-Potter Horn-Reflector antenna **ที่ดีที่สุด**

โดย ดร. พิเชษฐ กิจธารา และคณะ

พฤศจิกายน 2550

สัญญาเลขที่ MRG5080003

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สนับสนุนโดยสำนักงานคณะกรรมการการอุดมศึกษา และสำนักงานกองทุนสนับสนุนการวิจัย

บทคัดย่อ

เรานำเสนอรูปร่างและประสิทธิภาพของ Potter Horn สำหรับการใช้งานในช่วงความยาวคลื่นระดับมิลลิเมตรและสั้นกว่ามิลลิเมตร Horn ที่เรานำเสนอนี้ผ่านการออกแบบโดยใช้ Genetic Algorithm ซึ่งเป็นส่วนหนึ่งของซอฟแวร์ออกแบบที่เราพัฒนาขึ้นเอง Horn ที่ได้มีรูปร่างที่ง่ายต่อการผลิตและมีคุณสมบัติทางคลื่นแม่เหล็กไฟฟ้าที่ดีเช่น มี Beam Circularity ที่ดี มี Cross Polarization ที่ต่ำ มี Bandwidth ถึง 15% สำหรับการใช้งานในหลายรูปแบบ พลังงานของคลื่นของ Mode ลำดับที่สูงถูกกระตุ้นโดยการความไม่ต่อเนื่องของท่อหรือมุมที่เปลี่ยนไปอย่างกะทันหันที่บริเวณโคนท่อ Horn ในแต่ละกรณีเราได้สร้างกราฟสำหรับช่วยในการออกแบบเพื่อหาตัวแปรที่ดีที่สุดสำหรับการใช้งานที่ความถี่หนึ่งๆ ช่วงค่าของตัวแปรต่างๆที่แสดงไว้ในกราฟนั้นครอบคลุมช่วงที่มีการใช้งานปกติของกล้องโทรทัศน์ กราฟที่ได้แสดงให้เห็นชัดเจนว่าการออกแบบโดยการเปลี่ยนมุมของกรวยของ Horn นั้นจะให้ผลที่ดีกว่าการออกแบบโดยการเปลี่ยนรัศมีแบบที่ใช้กันอยู่ใน Potter Horn แบบดั้งเดิม การคำนวณทั้งหมดทำที่ความยาวคลื่นระดับมิลลิเมตร แต่ผลการทดลองก็สามารถนำไปสเกลเพื่อใช้ที่ความความต่ำหรือสูงกว่านี้ได้เช่นกัน ส่วนประกอบหลักของซอฟแวร์ทีใช้ออกแบบก็คือส่วนที่ค้นหาค่าของตัวแปรต่างๆ เช่น ขนาดของมุมของกรวย ที่ดีที่สุดสำหรับการนำไปใช้ เราได้พัฒนาซอฟแวร์โดยใช้ Modal Matching, Genetic Algorithm และ Downhill Simplex ในการออกแบบ เราใช้ Genetic Code สำหรับค้นหาตำแหน่งของจุดที่ดีที่สุดอย่างคร่าวๆ ก่อนจะใช้ Downhill Simplex Method ค้นหาให้ได้ความละเอียดแม่นยำยิ่งขึ้น

Keywords: Potter horn, Horn Antenna, Genetic Algorithm

Abstract

We describe the design and performance of Potter horns at millimetre and submillimetre wavelength employing a novel software package that we have developed, using Genetic Algorithm. The horn is easy to fabricate and exhibits excellent beam circularity and low cross polarization over a 15% bandwidth which is sufficient for many applications. Excitation of the required higher order modes is done by either a step or a flare discontinuity at the horn throat. In each case we provide design curves that give the optimum parameters of the horn geometry as a function of frequency and beamwidth. The range of values provided covers the parameters required for the design of horns for telescope feeds and various other instruments. The design curves show clearly that the flare-step performance is superior to the traditional groove-step Potter horn. The simulations for designing these horns were carried out at millimetre and submillimetre wavelengths but the results can be scaled to lower or higher frequencies. A key component in the design method is the optimization software that searches for the correct magnitude and location of the flare discontinuities. We have developed a software package based on the combination of modal matching, a genetic algorithm (GA) and downhill simplex optimization. The genetic code is first used to locate the proximity of the global minimum. The set of parameters obtained are then used as a starting point for the simplex method, which refines the parameters to the required accuracy.

Keywords: Potter horn, Horn antenna, Genetic algorithm

Executive Summary

1. ชื่อโครงการ

(ภาษาไทย) เทคนิคการวิเคราะห์และ Design Curves สำหรับ Pickett-Potter Horn-Reflector antenna ที่ดีที่สุด

(ภาษาอังกฤษ) Optimization Techniques and Design Curves for Pickett-Potter Horn-Reflector antenna

2. ชื่อหัวหน้าโครงการ หน่วยงานที่สังกัด ที่อยู่ หมายเลขโทรศัพท์ โทรสาร และ e-mail

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5. **สาขาวิชาที่ทำการวิจัย**

- Computational Physics : Optimization techniques
- Computational Physics : Electromagnetic simulations
- Feeds for submillimetre-wavelength detectors

6. **ผลงานและประโยชน์ที่ได้รับ**

จัดตีพิมพ์ในวารสาร *Int J. Infrared millimetre waves* ซึ่งเป็นวารสารด้านนี้โดยเฉพาะ
ชื่อเรื่อง The Design of Potter Horns for THz Applications Using a Genetic Algorithm

7. **ปัญหาที่ทำการวิจัย และความสำคัญของปัญหา**

ในการศึกษาวิจัยในระดับปริญญาเอก ข้าพเจ้าได้ทำการออกแบบ Horn จำนวน 2 ประเภทคือ Pickett-Potter Horn และ Corrugated Horn พบว่ากว่าที่จะได้ Horn ที่มีคุณสมบัติที่ดีตามต้องการนั้นต้องใช้เวลามากหลายสัปดาห์ในการปรับค่าของตัวแปรต่างๆ (Parameters) เช่น ความกว้างของ Aperture และความยาวของ Horn เป็นต้น จึงเห็นว่าโปรแกรมที่จะทำการปรับปรุงและคัดเลือกตัวแปรค่าต่างๆโดยอัตโนมัตินั้นมีความสำคัญอย่างยิ่งต่อวงการการออกแบบ Horn เพื่อใช้เป็น Feed สำหรับ Receiver ในย่านความถี่ของ Submillimetre Waves โครงการวิจัยนี้จึงได้มีการพัฒนาเทคนิคและโปรแกรมแล้วจะสามารถสำหรับลดเวลาในการออกแบบและทำให้มั่นใจยิ่งขึ้นว่า Horn ที่ได้มีคุณภาพสูงสุดจริงๆ

8. **วัตถุประสงค์ของโครงการ**

- มีโปรแกรมสำหรับออกแบบ Horn โดยอัตโนมัติ
- นำเสนอเทคนิคและวิธีการที่จะใช้วิเคราะห์คุณสมบัติของ Horn และ Horn-Reflector

9. **ระเบียบวิธีวิจัย**

- **การดัดแปลงโปรแกรมคำนวณคุณสมบัติของ Horn และ Horn-Reflector**
  เป็นโปรแกรมที่มีอยู่แล้วที่เขียนขึ้นเองทั้งหมดโดย ดร. พิเชษฐ กิจธารา สามารถนำมาดัดแปลงเพิ่มเติมได้ทันที โปรแกรมนี้ได้ผ่านการทดสอบผลการคำนวณเทียบกับการทดลองจริงที่ความถี่ 15 GHz และ 700 GHz พบว่าถูกต้องแม่นยำ

- **การสร้างโปรแกรมส่วน Optimization Routines**
  เป็นส่วนของโปรแกรมที่จะต้องสร้างขึ้นมาใหม่ ซึ่งจะทำการปรับค่า Parameters ต่างๆโดยอัตโนมัติเพื่อให้แน่ใจว่าค่าที่ได้เป็นค่าที่ดีที่สุดจริงๆ (Global Optimization) และเพื่อป้องกัน Local Minimum อันเนื่องมาจากระบบนี้มี Non-linearity ค่อนข้างสูง โปรแกรมส่วนนี้จะต้องประกอบด้วย 2 ส่วนย่อยคือ

    - **Genetic Algorithm** ใช้ในการหาบริเวณของ Global Minimum และเทคนิคด้าน Multivariable Optimization with Inequality constraints สำหรับใช้ในกรณีที่ Genetic Algorithm อาจจะช้าเกินไป เทคนิคหนึ่งที่อาจจะนำมาใช้คือ Generalized reduced gradient method

- **Simplex Method** หลังจากที่ได้บริเวณที่เป็น Global Minimum จาก Genetic Algorithm แล้ว โปรแกรมจะทำการค้นหาค่า Parameters ที่ละเอียดยิ่งขึ้นไปอีกด้วย Simplex Method

- **การนิยามคุณสมบัติของ Horn เพื่อการ Optimization**

  งานวิจัยได้มีการจะนำเสนอวิธีการนิยาม Error Function .ให้เป็นรูปธรรมที่โปรแกรมส่วน Optimization Routines จะนำไปใช้ทำการ Minimize ได้ โดยจะนำคุณสมบัติหลักๆของ Radiation Patterns เช่น Beam Circularity และ Cross-polarization Level มาพิจารณา

  Error functions ทั้งหมดรวมทั้งแนวคิดในการใช้ Error Functions แบบนี้ไม่เคยมีการนำเสนอที่ใดมาก่อน

Output ที่ได้จากโครงการ

1. ชื่อโครงการ

(ภาษาไทย) เทคนิคการวิเคราะห์และ Design Curves สำหรับ Pickett-Potter Horn-Reflector antenna ที่ดีที่สุด

(ภาษาอังกฤษ) Optimization Techniques and Design Curves for Pickett-Potter Horn-Reflector antenna

2. ชื่อหัวหน้าโครงการ

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3. Output

จัดตีพิมพ์ในวารสาร *Int J. Infrared millimetre waves* ซึ่งเป็นวารสารด้านนี้โดยเฉพาะ

Impact Factor: 0.326

P. Kittara, A. Jiralucksanawong, G. Yassin, S. Wangsuya, J. Leech (2007). The Design of Potter Horns for THz Applications Using a Genetic Algorithm. *Int J Infrared Milli Waves* (2007) 28:1103–1114

reprint ดังแนบในภาคผนวก

Int J Infrared Milli Waves (2007) 28:1103–1114
DOI 10.1007/s10762-007-9290-0

The Design of Potter Horns for THz Applications Using a Genetic Algorithm

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Received: 25 May 2007 / Accepted: 14 September 2007 /
Published online: 26 September 2007

Abstract We describe the design and performance of Potter horns at millimetre and submillimetre wavelength employing a novel software package that we have developed, using Genetic Algorithm. The horn is easy to fabricate and exhibits excellent beam circularity and low cross polarization over a 15% bandwidth which is sufficient for many applications. Excitation of the required higher order modes is done by either a step or a flare discontinuity at the horn throat. In each case we provide design curves that give the optimum parameters of the horn geometry as a function of frequency and beamwidth. The range of values provided covers the parameters required for the design of horns for telescope feeds and various other instruments. The design curves show clearly that the flare-step performance is superior to the traditional groove-step Potter horn. The simulations for designing these horns were carried out at millimetre and submillimetre wavelengths but the results can be scaled to lower or higher frequencies. A key component in the design method is the optimization software that searches for the correct magnitude and location of the flare discontinuities. We have developed a software package based on the combination of modal matching, a genetic algorithm (GA) and downhill simplex optimization. The genetic code is first used to locate the proximity of the global minimum. The set of parameters obtained are then used as a starting point for the simplex method, which refines the parameters to the required accuracy.

Keywords Potter horn · Horn antenna · Genetic algorithm

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1 Introduction

A high-quality feed usually consists of a conical corrugated horn, collimated by a dielectric lens or profiled to a cylindrical section. The presence of azimuthal corrugations in the horn presents isotropic surface boundary conditions to the electric and magnetic fields on the wall, resulting in the propagation of a “hybrid mode”. This configuration produces a circular beam with low cross polarization and sidelobes over a substantial fractional bandwidth, which is required for many applications. Corrugated horns however require the fabrication of several corrugations per wavelength, which can be technically complicated and expensive at THz frequencies.

A Potter horn [1] has many of the desired properties of a corrugated horn and yet is much simpler to construct. Conventionally, it consists of a conical horn with a *single* step discontinuity at the horn throat whose dimensions are chosen in order to excite the TM_{11} mode at a carefully selected amplitude (~16%) with respect to the amplitude of the incident TE_{11} mode. The “dual-mode” is then made to propagate through a cylindrical “phasing section” to make the two modes arrive at the horn aperture in phase. This results in sidelobe cancellation and low cross polarization in the horn radiation pattern.

A simplified version of the conventional Potter horn was first presented by Pickett *et al.* [2]. The phasing section of the Potter horn was removed, resulting in a more compact and easier to machine horn as shown in Fig. 1. The phasing of the modes was then obtained during propagation through the flared horn section. In other words, the simplification is achieved by sacrificing one degree of freedom in choosing the horn dimensions. Realization of this design at submillimetre wavelengths was also reported where a Pickett-Potter horn was shown to have good performance at 700 GHz, over a 15% bandwidth [3].

An alternative method of exciting the TM_{11} mode is a sudden change in the horn flare angle as shown in Fig. 2. Here, a second section is formed by changing the flare angle when the horn radius is r_1, at an axial distance L_1 from the start of the horn.

Although this method was reported previously [4], it was not thoroughly investigated or commonly used in conjunction with conical horns. Our simulations (see Fig. 8) show that the pattern obtained with a flare-step potter horn is in fact superior to the pattern of a groove-step horn. The choice between the two types of horn could also depend on the method of fabrication. The availability of programmable lathes for precision machining makes the fabrication of the flare discontinuity type fairly straightforward, in particular if

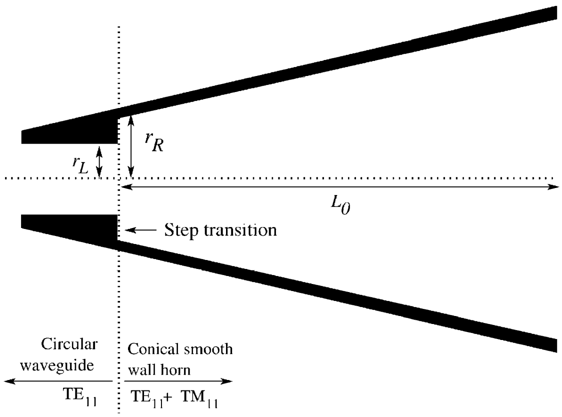

Fig. 1 Cross section of the single-groove Potter horn (Pickett-Potter horn). r_L and r_R are the two radii at the groove. L_0 is the length of the conical phasing section. The step transition generates appropriate amount of TM_{11} mode.

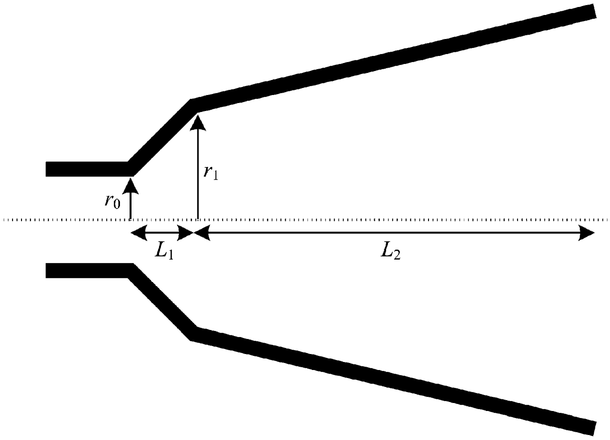

Fig. 2 Geometry of the Potter horn with a flare angle discontinuity. r_0 is the radius of the circular waveguide. r_1 and L_1 are the radius and the length of the first conical section. L_2 is the length of the second conical section.

electroforming is used. If, on the other hand, the horn is machined from a split-block of metal then turning a single groove at the throat with a reasonably large flare angle is also easy.

The conventional analysis of either type of Potter horn, which relied on an analytical method, assumed that only the TE_{11} and TM_{11} propagate to the horn aperture. This assumption, however, is not realistic, since higher order modes that are evanescent at the throat can later become oscillatory and carry energy to the aperture. Moreover it turns out that weak excitation of additional higher order modes (e.g. TE_{12}) at carefully selected amplitude and phase can in fact broaden the useful bandwidth of the horn. Thus the analysis described in the original Potter horn papers cannot accurately predict the horn radiation patterns. Higher order modes can, however, be easily accounted for if the modal matching method [5, 6] is used to propagate the electromagnetic fields components numerically from the waveguide to the aperture. This method yields precise and fast converging solutions for the analysis of lossless horns with rectangular or cylindrical geometry. As we shall see later, the designer only needs to request the software to search for the optimum design in a specified bandwidth. The amplitude and phase of the higher order modes used to construct the radiation pattern are returned in an output array.

Similarly, predicting the horn dimensions that yield optimum performance is not straightforward. Consequently, an optimization (minimization) technique is required to calculate the optimum horn parameters, for a required performance [7]. Each point in the search space represents a set of parameters of the horn geometry that we want to optimize against a specified radiation pattern criteria. The choice of parameters and optimization method are described in later sections.

In this paper, we shall present a new technique of reliable and efficient minimization based on a genetic algorithm (GA) [8] combined with a downhill simplex technique. First the approximate position of the global minimum is found using the GA and then this is used as a starting point for the iterative simplex procedure which accurately locates the position of the minimum. The analysis is not restricted to Potter horns only but can be used to generate designs with a larger number of horn sections [9].

The above discussion illustrates that accurate design of dual-mode horns requires intensive numerical computations. An important objective of this work therefore, is to compile design curves that give optimized horn parameters for a required radiation pattern.

These include the input waveguide radii, the step dimensions, the input waveguide radii and the aperture radii, all normalized to wavelength, as a function of far-field beamwidth. Our curves will cover the practical range of radiation patterns used for feeds at submillimetre wavelengths.

2 Method of computation

2.1 The general scheme

The design method of the Potter horns is done by using a combination of analysis and synthesis packages. The analysis package is a modal matching routine that computes the radiation pattern and the electrical properties of a given horn geometry and the synthesis package which consists of an optimization routine that searches for a new improved horn design. We have written modal matching and genetic algorithm minimization software and combined these into a single horn design package (***hornsynth***) with a graphical user interface.

Finding the optimum horn geometry that satisfies a pre-determined specification is done using a genetic algorithm (GA), which is a powerful computational method for solving optimization problems. It employs a natural selection process which is similar to biological evolution. One usually begins by choosing a random set of parameters which form the *chromosomes*. A set of chromosomes is called an *individual*. A *population* consists of many individuals. The fitness of an individual is computed from *the cost function* by using the corresponding chromosomes of that individual. The cost function is a test function that measures the deviation of the fitness of an individual from a desired value of fitness. The fitness of an individual is defined to be inversely proportional to the cost function. The members are then randomly paired to form parents that produce offspring that forms a new generation. Members with high fitness are more likely to be selected. The algorithm is motivated by the expectation that the new generation will be fitter than the old one and so this evolutionary process will eventually yield a solution consisting of the fittest population.

The mating process at the heart of the genetic algorithm involves selecting two parents to give two offspring. The chromosomes for the two offspring are generated by the use of

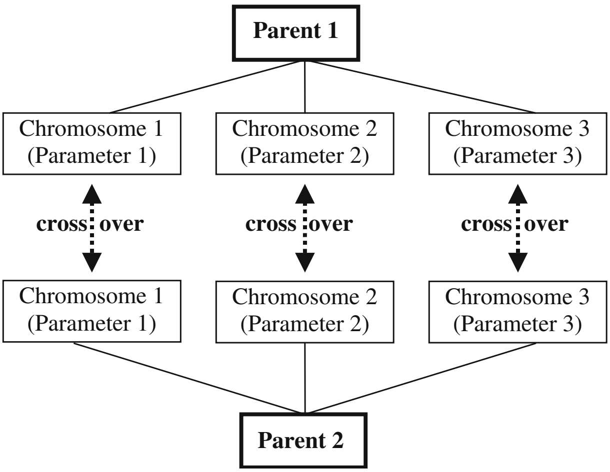

Fig. 3 Crossover occurs between corresponding chromosomes of the individual parents in the GA optimization. For a single-groove Potter horn, the geometry parameters 1, 2 and 3 are r_L, r_R and L_0 as described in Fig. 1.

crossover and mutation, which are responsible for the genetic diversity of the offspring population. In crossover, each parent chromosome is divided at the same random point along its length, and then two offspring chromosomes are formed by swapping over each end of the two parent chromosomes and then joining them back together. After the offspring are formed by crossover, the individual bits of each offspring chromosome may be flipped with small probability, thus mimicking random mutation in biological natural selection.

Optimization using GA is illustrated schematically in Fig. 3. The cost function of the individuals is calculated as described in the next section and a subset of individuals is then chosen for pairing, with the individuals with higher fitness (lower cost function) are favorably selected. The next generation is then produced using the mating process involving crossover, yielding a new population with an improved fitness. The process is repeated to form a third generation and so on until the change in the average cost function of the population becomes negligible. At this stage the process is terminated and the *optimum* solution of the problem is found by searching for the individual with the lowest cost function (i.e. search for global minima). There are other criteria for terminating the process of evolution, e.g. by monitoring the cost function of each individual. In practice, one finds good results by choosing populations with at least 3N individuals, where N is the number of parameters to be optimized. We find that running between 500 and 1000 generations is sufficient to generate a well optimized Potter horn design.

Application of (GA) to horn design is made by associating a chromosome to each key geometry parameter. The initial population of individuals (horns) is generated randomly, with the values of each horn parameter allowed to vary between some sensibly chosen external values. For example, the horn in Fig. 1 has three parameters namely the radius of the input waveguide r_L, the radius of the horn at the step, r_R and the length of the horn L_0. It will therefore constitute an individual with three chromosomes. The float value of the parameters is encoded into a binary string with a certain number of (1-0) bits using a standard encoding procedure [8]. The binary integers are converted to their Gray codes where each successive integer value differs by only a single bit, and thus a crossover event between two parent chromosomes which are numerically similar will result in an offspring with chromosomes which are also numerically similar. During crossover, the binary string representing the chromosomes of each two parent is divided into two parts and the four parts are then recombined by joining the left hand part from one parent string to the right hand of the other, to produce two offspring represented by two new binary strings. The binary strings are then decoded back into floats and the cost functions for the new population is computed using the formulas derived in the next section. In our case this will be done by calculating the radiation pattern of the horns with various geometries and comparing them with the specifications (e.g. beam circularity and cross polarization) of a desired pattern by some application. After terminating the process, the *fittest* horn with the lowest cost function is found using a search routine.

An important feature of the GA is that it avoids convergence to a local extremum by exploring the search space. However, searching for the final solution is very time consuming hence the GA is only used to find the proximity of the global optimum. This output is then refined by feeding it to a downhill simplex which is an efficient search iterative minimization algorithm for problems that do not have a very large number of variables.

2.2 Optimization parameters and criteria for the Potter horns

The quality of the radiation pattern of a horn is normally characterised by several parameters, depending on the application. An antenna feed for example is expected to have good beam circularity (a beamwidth independent on the polarization), low cross polarization and sidelobes, low return loss and high beam efficiency, over a given bandwidth. Some of these properties are generally related. For example, good beam circularity is associated with low cross polarization level and high beam efficiency corresponds to low sidelobe level. However, considering that we are dealing with trade-offs between the quality of these parameters, we found it necessary to optimize several of them at the same time. Since the return loss of Potter horns is normally low we did not include it in the optimization parameters. Also, the beam efficiency was excluded since it requires a large number of time consuming integrations. We therefore have chosen to optimize the horn designs for beam circularity and cross polarization. We would like to emphasize however that different criteria can easily be incorporated into our software.

The problem constraints include specifying the frequency band edges and the central frequency, in addition to the geometry parameters such as the waveguide radius and the horn flare angle. The problem variables include the discontinuity magnitude (e.g. the step height) and location. These parameters are incorporated in the software using a spectral cost function δ_f with a weighting function w_p for a given frequency f. The spectral cost function is defined as

$$\delta_f^2 = (w_X)\left[\sum_{P=-1}^{P=-30}\left(\frac{\sigma_P}{\sigma_P^{av}}\right)^2 w_P\right]$$

where

P	power level in dB,
$w_P=$	$10^{\frac{P}{15}}$, the weighting function for beam circularity
w_X	peak cross polar power relative to the main beam peak power.
θ_P^E	width of the E-plane at the P dB power level,
θ_P^H	width of the H-plane at the P dB power level,
σ_P	the difference between the widths at P dB, $\sigma_P = \left\|\theta_P^E - \theta_P^H\right\|$
σ_P^{av}	the average width, $\left(\frac{\theta_P^E+\theta_P^H}{2}\right)$.

These parameters are also illustrated in Fig. 4 for clarity.

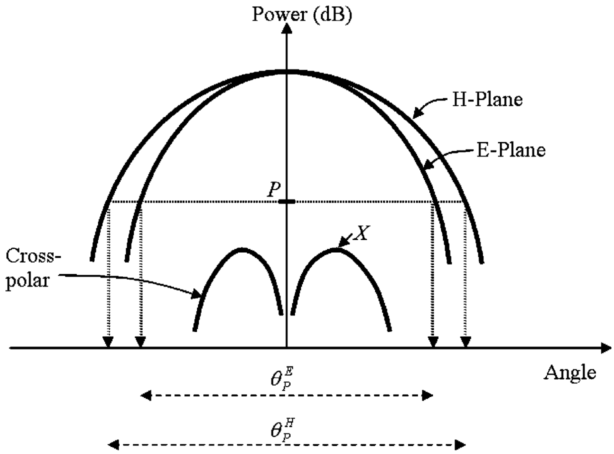

Fig. 4 A sketch of radiation patterns and variables used in the cost function. When the cross-polarization is small and the beam is circular $\left(\sigma_P^E = \sigma_P^H\right)$, the cost function becomes small.

Notice that the form of the cost functions allows for the fact that the beam circularity near the centre of the radiation pattern needs to be much better than that at low power levels. The minimum power level at which good circularity needs to be controlled depends on the application. In our case we requested the software to optimize the beam circularity down to the −30 dB level which is achieved by making w_P vary slowly with power. Optimization against cross polarization is done using the weight w_X. This choice also represents the view that a −10 dB drop in the peak cross-polarization level results in a 10 times better performance. It nevertheless should be noted that a higher order of w_X compromises the beam circularity.

Finally, we include optimization across a frequency bandwidth, centred at f_0 was done by choosing the cost function given by

$$\delta^2 = \sum_f \delta_f^2 w_f.$$

Here $w_f = \exp\left(\frac{-(f-f_0)^2}{2\sigma_f^2}\right)$ is the weight at frequency f and $\sigma_f = f_U - f_L$ is the difference between the upper (f_U) and lower (f_L) frequencies in the required bandwidth.

The choice of a Gaussian weight emphasizes the symmetry with the respect to the band centre. This choice, which is natural for corrugated horns, may not in fact be the best for Potter horns, because the performance deterioration of Potter horns is much sharper at the low frequency band than at the high band edge. Clearly, an alternative choice to a Gaussian distribution can be used or alternatively, the specified centre frequency could be made lower than intended in the real horn. We found that a larger value of σ_f broadened the bandwidth, but worsened the centre frequency. The number of frequency steps used in the summation depends on the value of σ_f, but should not be less than 5. We found that a step frequency of $0.01f_0$ works well.

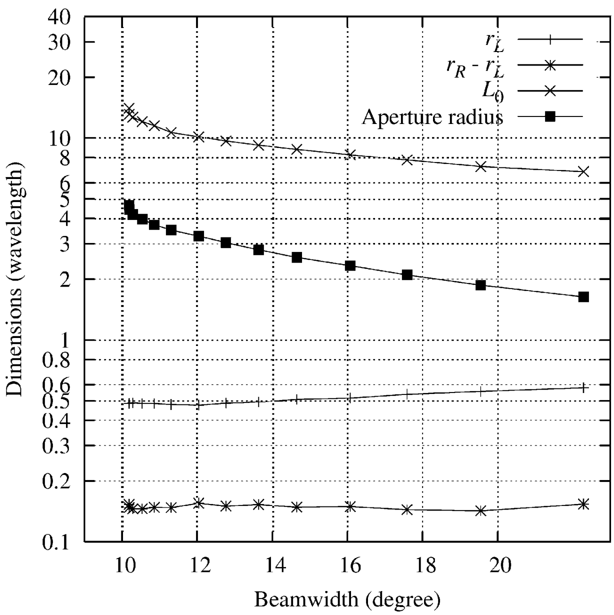

Fig. 5 A plot of horn dimensions vs. beamwidths at the centre frequency. To design a single-groove Potter horn, first choose the required beamwidth and then obtain the other dimensions from this plot.

Table 1 The geometry of the single-groove horn with aperture radius of 3.50 wavelengths or 1.5 mm. The beamwidth of this horn is 12.30 degree.

Initial waveguide radius, r_L	0.2058 mm
Radius after the step, r_R,	0.2693 mm
Size of the step , r_R–r_L	0.0635 mm
Conical length , L_0	4.564 mm
Aperture radius	1.500 mm
Semiflare angle	15.09 degrees

3 Design of potter horns

In the previous sections we have described the principle of operation of the software package ***hornsynth*** which we created using a combination of genetic algorithm and modal matching software. In what follows we shall apply the software to synthesize the design of the two types of Potter horns, namely the single groove and the single flare step configurations.

3.1 Designing a single groove Potter horns using *hornsynth*

The centre frequency for this horn was chosen at 700 GHz in conjunction with bandwidth of 15%. During an optimization run, the frequency band and the aperture of the horn were constrained while the rest were left as optimization variables. The software therefore returned the dimensions and the radiation patterns of the horn that yields an optimized performance corresponding to these constraints. The results are summarized in Fig. 5 where we present simulation curves that can be used to design Potter horns of this type. The figure shows the horn dimensions as function of beamwidth. The dimensions are normalized to the unit wavelength, so that they can be easily scaled to other frequencies. As expected, a diffraction limited horn with a small beamwidth can be obtained by increasing its aperture size. As a consequence, a narrower beamwidth (larger aperture) requires longer conical section since a small increase in the flare angle of the horn alters substantially, the relative amplitude of the excited TE_{11} and TM_{11} modes at the discontinuity.

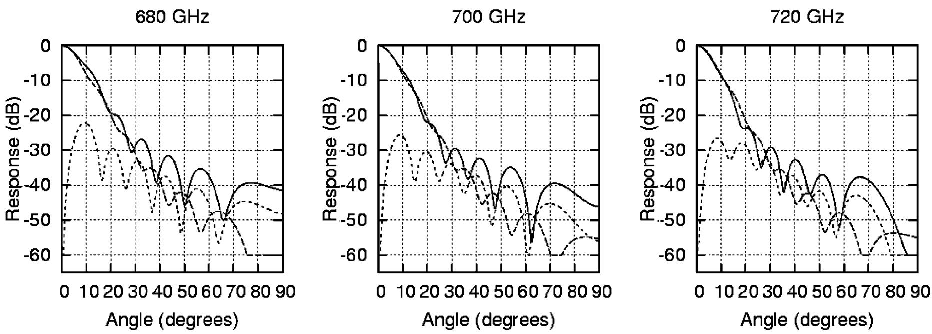

Fig. 6 E-plane (dashed), H-plane (solid) and cross polar (dashed-dot) levels for a GA and Simplex optimised single-groove Potter horn with the aperture radius of 1.50 mm, having a beamwidth of 11.31 degree at 700 GHz.

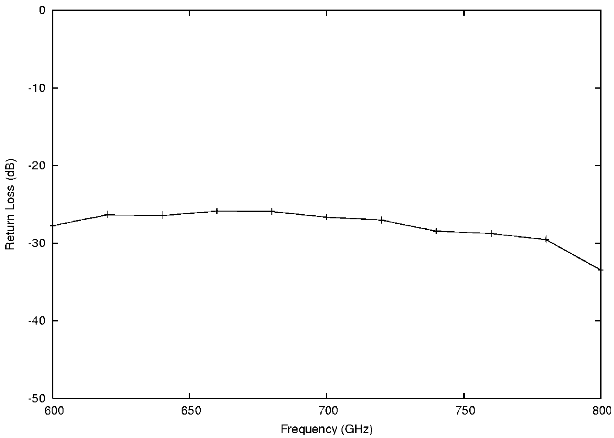

Fig. 7 The return loss of the Pickett-Potter horn with a 1.5 mm aperture radius.

In summary, the design of this type of horn proceeds as follows:

1. Choose the beamwidth according to specifications and use Fig. 5 to determine the horn dimensions scaled to the required centre frequency.
2. If possible, run modal matching software to examine the radiation patterns across the band and refine the design.

To illustrate the method we have chosen a horn with an aperture radius of 3.50 λ. Table 1 describes the geometry of a single-groove Potter horn scaled to the centre frequency 700 GHz. In Fig. 6 we plotted the radiation pattern of the horn, using the modal matching software at 680, 700 and 720 GHz. The patterns exhibit good beam circularity, and low cross polarization (less than −24 dB) across the band. The horn also has a cross polar level below −20 dB between 660 and 750 GHz, giving a useable bandwidth of around 13%.

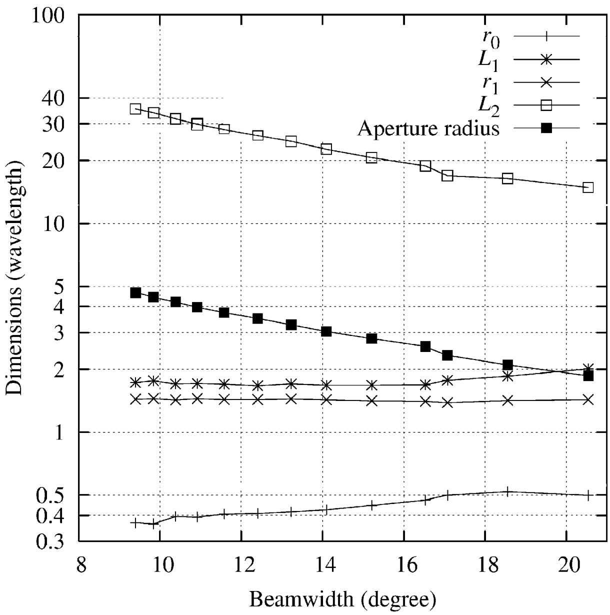

Fig. 8 A plot of horn dimensions vs. beamwidths at the centre frequency. To design a two-angle horn, first choose the required beamwidth and then obtain the other dimensions from this plot. r_0, L_1, r_1 and L_2 are defined in Fig. 2. The beamwidths are in degrees, and the horn dimensions are in wavelengths.

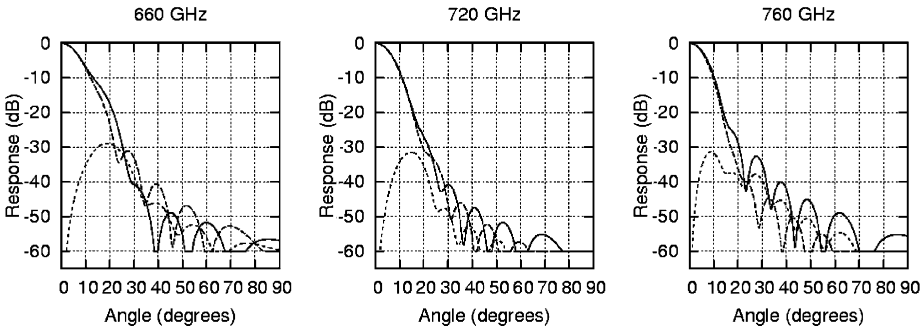

Fig. 9 E-plane (dashed), H-plane (solid) and cross polar (dashed-dot) levels for the Potter horn consisting of two conical sections with a discontinuity in flare angle.

In Fig. 7, we plotted the return loss, calculated using modal matching, for the horn. It can clearly be seen the return loss is below −25 dB across the band.

3.2 Designing a Potter horn with a flare discontinuity angles using *hornsynth*

Here, the higher order modes in the horn are excited by a sudden change in a flare angle as shown in Fig. 2. In the previously reported work [4] the design relied on analytical formulas that assumed the excitation of the TE_{11} and TM_{11} only, hence the location and the magnitude of the flare discontinuity were restricted. The first angle (which transformed the waveguide into a horn) was chosen to be small enough to allow the propagation of the fundamental mode TE_{11} only while the second angle discontinuity excited the higher order mode. Our designs however are analyzed with modal matching computation hence have no such restriction and can in fact benefit from the excitation of additional modes, as we have indicated previously. The location and magnitude of the angles are chosen to maximize the useful bandwidth of the horn. Trials with the genetic algorithm illustrated that a large bandwidth can be achieved if the TM_{11} is in fact excited at the angular discontinuity while the length of the second horn section, L_2, is used to fine-tune and optimize the bandwidth and the phasing.

The geometry of optimized two-sections horns for different beamwidths are shown in Fig. 8. Here also, a single set of parameters was obtained by specifying the frequency band and the aperture of the horns, while the other dimensions were left as optimization variables.

Table 2 The geometry of the two-angle horn with aperture radius of 3.50 wavelengths.

Initial waveguide radius, r_0	0.1752 mm
Length of the 1st conical section, L_1	0.7163 mm
Radius of the 1st conical section, r_1	0.6145 mm
Length of the 2nd conical section, L_2	11.256 mm
Aperture radius	1.500 mm

The dimensions are scaled to the centre frequency 700 GHz. See Fig. 2 for the description of each dimension. The beamwidth of this horn is 12.40 degrees.

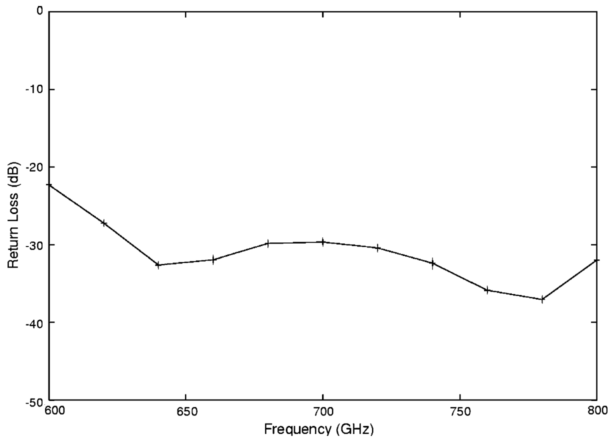

Fig. 10 The return loss of the horn consisting of 2 conical sections with an aperture radius of 1.5 mm described in the text.

Example beam patterns of the two-sections horn centred at 700 GHz are shown in Fig. 9, while Table 2 gives the parameters of the horn determined by the genetic algorithm. The beam circularity is good and the cross polar levels are below −28 dB across a bandwidth of 15%. The calculated return loss for the horn is shown in Fig. 10.

As we have mentioned previously, the radiation patterns obtained by a flare-step horn are significantly better than those obtained by the conventional groove-step Potter horn. Comparing Figs. 9 with 6 we find that the patterns of the former have better beam circularity, substantially lower cross polarization at the lower frequency end and lower sidelobes, in a wider bandwidth. Also the return loss obtained by the flare step horn is −30 db which is very impressive even when compared with conical corrugated horns. Close examination of the modes excited at the discontinuity indicates that this effect may be attributed to the fact that the ratio of the amplitudes TM_{11}/TE_{11} at the discontinuity varies much more slowly with frequency in the first case. Also it seems that the amplitude of the inevitably excited higher order modes, in particular the TE_{12}, are more difficult to control in the groove-step excitation

4 Conclusion

We have presented a new method for the design of easy to fabricate Potter horns at submillimetre wavelengths. Two types were discussed, the first employs a step discontinuity to excite higher order modes and the second employs a flare discontinuity, forming a two section horn. In each case we supplied design curves that can be used to design Potter horns at a wide range of beamwidths and frequencies. Synthesis of the horn geometry was done using the software package *hornsynth* which we have developed for this purpose. The software combines a minimization package using Genetic Algorithm and a horn analysis package using the modal matching method. The useful bandwidth of these horns is around 15% which, at THz frequencies, is sufficient for many applications. Increasing the bandwidth can easily be achieved using *hornsynth* in conjunction with more steps [9]. This work is now in progress.

Acknowledgements This work has been supported by Thailand Research Fund and the UK STFC.

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