III. Simulation Results and Discussion

A. Current Density

The simulation result of the current density distribution in each section at the same time is shown in Fig. 5. The four distributions from up to down indicate respectively the current density distributions at $z = L/16$, $3L/16$, $5L/16$, and $7L/16$. For L = 20 m, these four distributions clearly show the coupling phenomena (see Fig. 5(a)). On the contrary, for L = 20 μm, these figures show the perfectly decoupling phenomena (see Fig. 5(c)).

Moreover, these results are more interesting for L = 20 mm because the coupling currents in each section are different (see Fig. 5(b)). We find that the maximum total current in one of the superconducting square bar is at $z = L/16$, the coupling current fills the whole cross-section of the bar. This total current decreases respectively at $z = 3L/16$, $5L/16$, and $7L/16$. By using the formulation in [4] to calculate theoretically the value of the critical length (L_c), we find that L_c = 20 mm. This is for the typical values as shown in Table 1 (corresponding to a circular filament of diameter 7 μm used for the LHC main magnets at CERN [7]).

TABLE I. Typical Values

Symbol	Quantity	Value
a	Square bar half-width	3.5×10^{-6} m
J_c	Critical current density (NbTi at 5 T)	2×10^{9} A/m^2
ρ	Copper resistivity (OFHC copper at 5 T)	3.6×10^{-10} Ω·m
$\dot{B}$	Rate of change of field	0.1 T/s

The value of the critical length obtained from the analytical formulation in [4] (theoretical results) is in harmony with the results obtained from the numerical simulations.

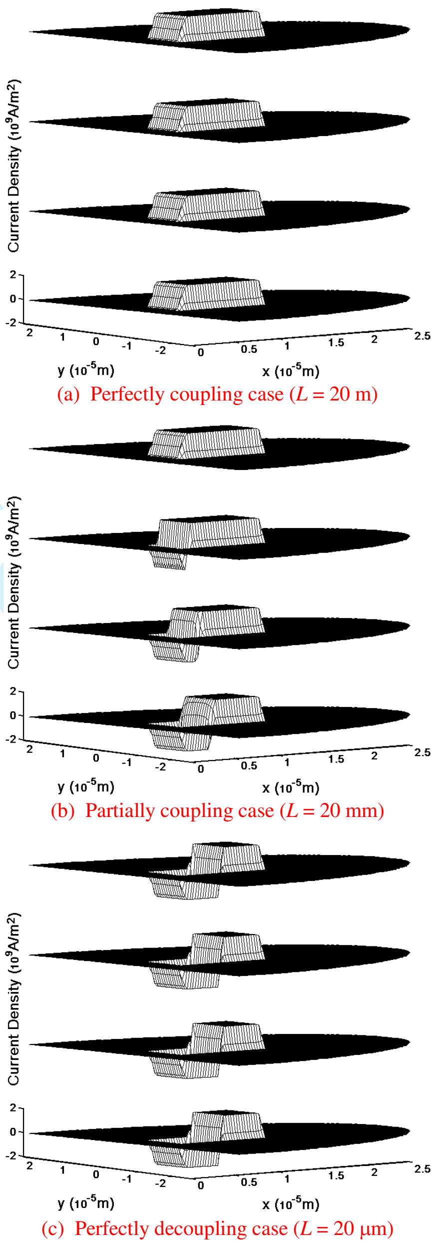
B. Magnetic Field

Fig. 6 shows at the same time the simulation result of the magnetic field distribution at $z = 7L/16$. We can observe the changes of the magnetic field direction inside the square bar. Obviously, these figures can illustrate the direction of the current in the bar, for instance only one direction for L = 20 m (Fig. 6(a)), and two directions for L = 20 μm (Fig. 6(c)) and 20 mm (Fig. 6(b)).

C. Magnetization

The magnetization value is calculated by using the obtained value of the current density. And a complete cycle of magnetization is obtained by taking the values of the magnetization and of the magnetic induction. Fig. 7 compares the magnetization cycles of each section for three strand lengths. For L = 20 m and 20 μm, all sections give the same magnetization cycle. However, the maximum value of the magnetization is very important for L = 20 m (perfectly coupling case). But for L = 20 mm (Fig. 7(b)), the sizes of the magnetization cycle decrease. This is from the section at $z = L/16$ to the section at $z = 7L/16$, or from near the middle of the strand to near the end of the strand.

The total magnetization of the strand can be obtained by calculating the average value of the magnetization of all sections at a given length. In Fig. 8, we find the result as in [2] that the values of the total magnetization in the case of partially coupling are clearly between those in the perfectly coupling and perfectly decoupling cases. These simulation results of the simple problem in [4] indicate that the superconducting filaments in composites of several kilometers of length into the superconducting magnet are always fully coupled. For this reason, the strand is always twisted with a pitch appreciably lower than the critical length [3].



(a) Perfectly coupling case (L = 20 m)

(b) Partially coupling case (L = 20 mm)

(c) Perfectly decoupling case (L = 20 μm)

Fig. 5. Current density distributions in the modeled domain in each section.

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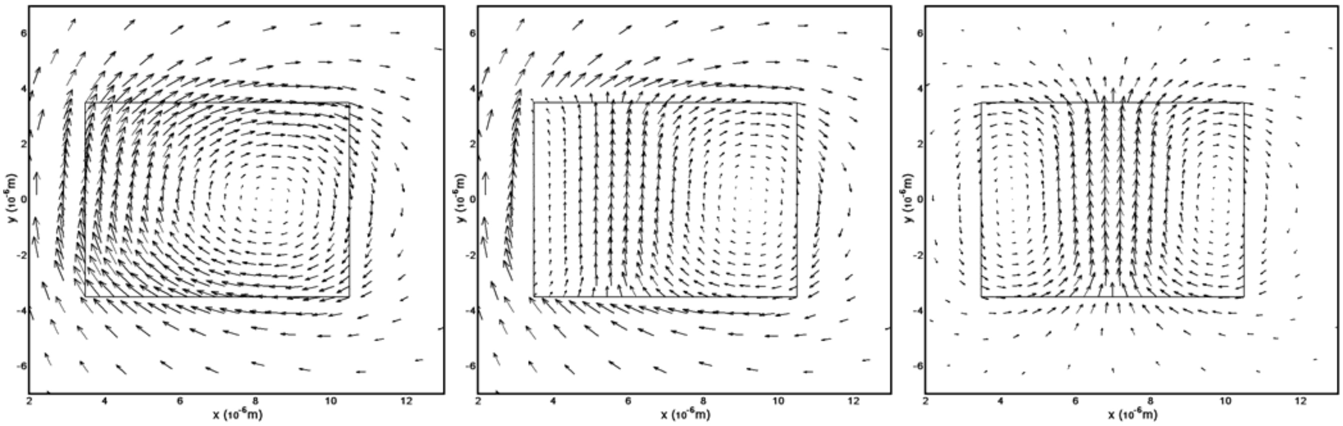
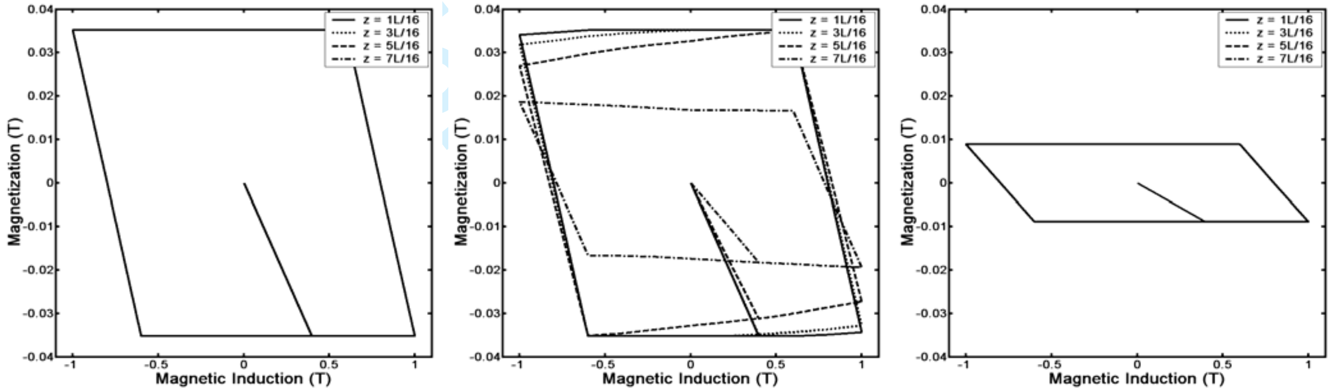


(a) Perfectly coupling case (L = 20 m) (b) Partially coupling case (L = 20 mm) (c) Perfectly decoupling case (L = 20 µm)

Fig. 6. Magnetic field distributions in the modeled domain.



(a) Perfectly coupling case (L = 20 m) (b) Partially coupling case (L = 20 mm) (c) Perfectly decoupling case (L = 20 µm)

Fig. 7. Comparison of the magnetization cycles of each section for 3 lengths.

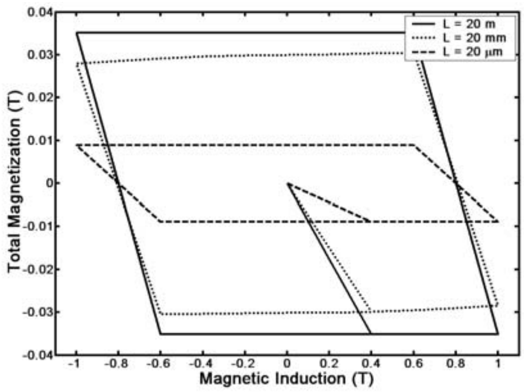


Fig. 8. Comparison of the total magnetization cycles of each strand length.

I. Conclusion

In this paper, we developed a 2D numerical code to study the coupling between superconductor strands. The code permits to take into account the parameters influencing the coupling: not only the length of the filaments but also its radius, the critical current density, the rate of change of the field or the resistivity of the conductor matrix. The simulation results show that the superconducting filaments in composites of several kilometers of length into the superconducting magnet are always fully coupled. For this reason, the strand is always twisted with a pitch lower than the critical length. In order to see appear the differences of simulation results; an extension of this work is then to divide each strand into more sections.

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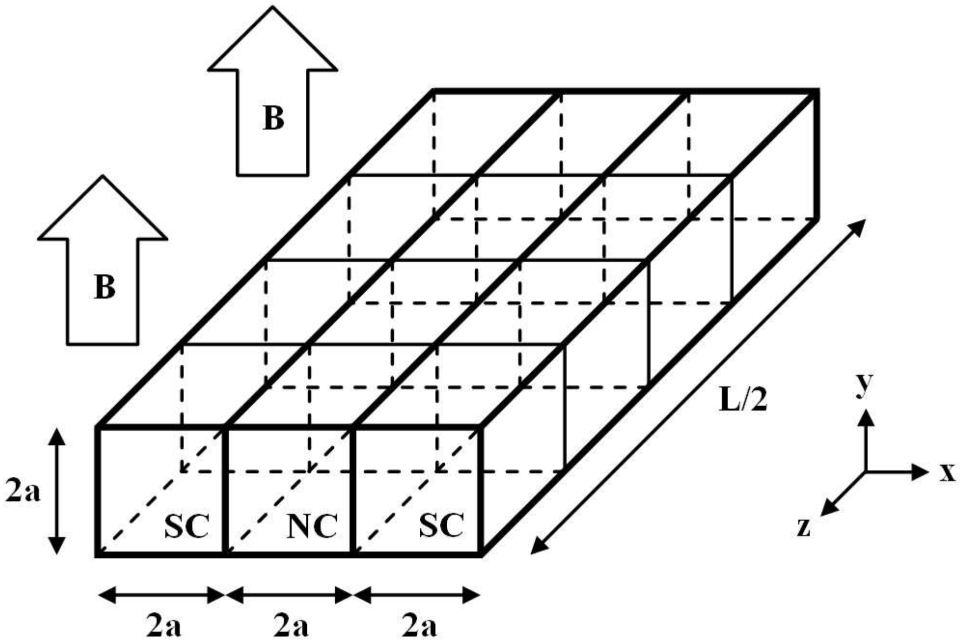

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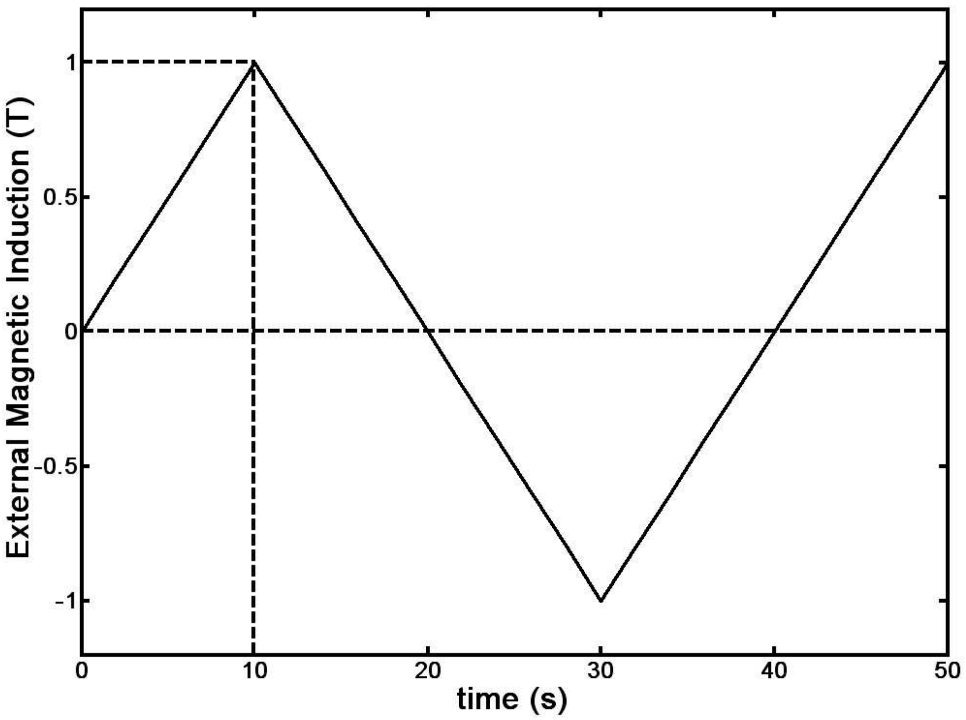

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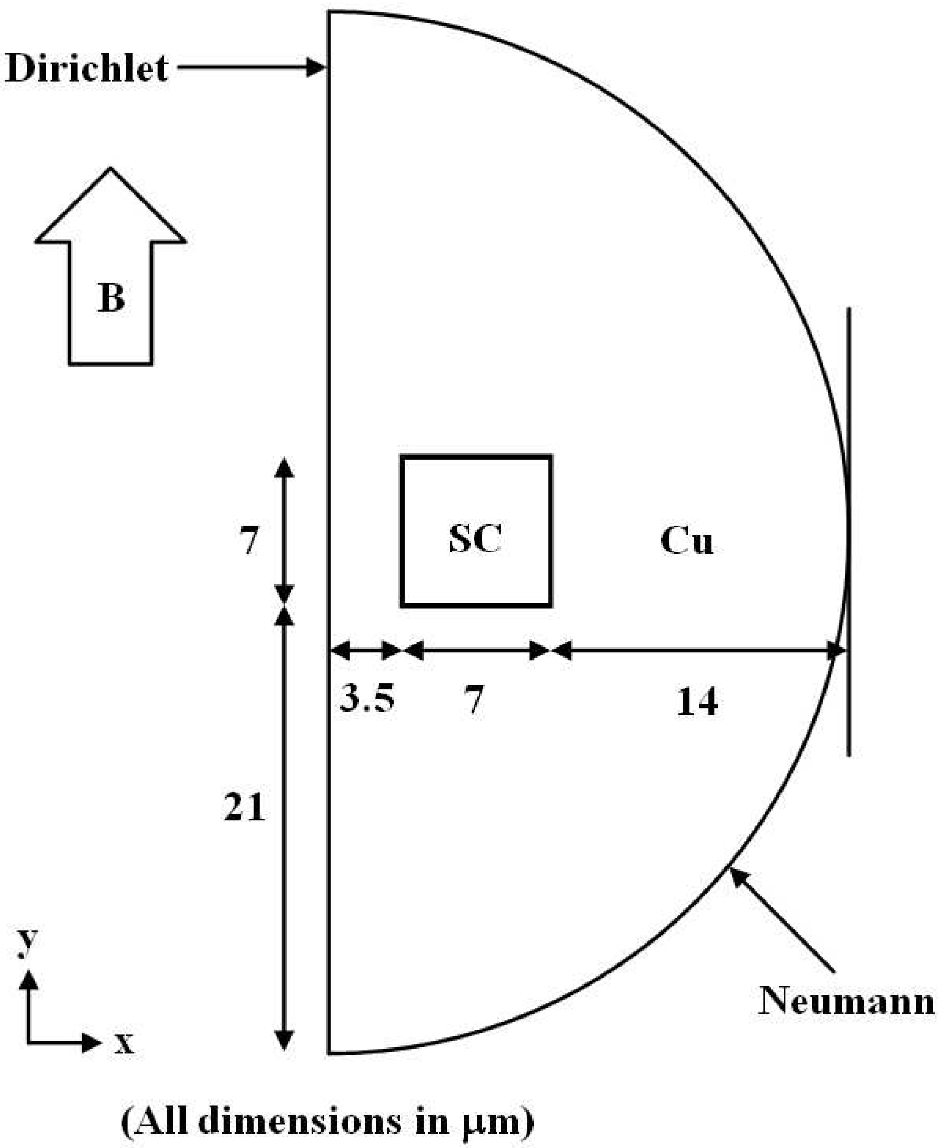

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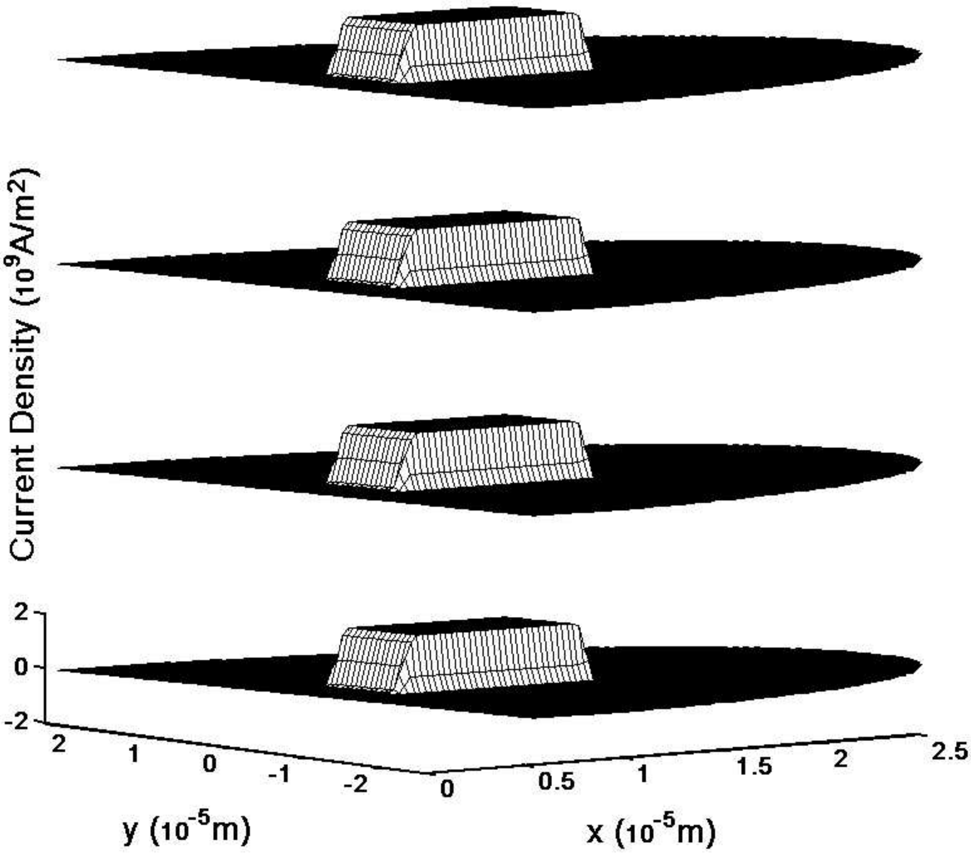

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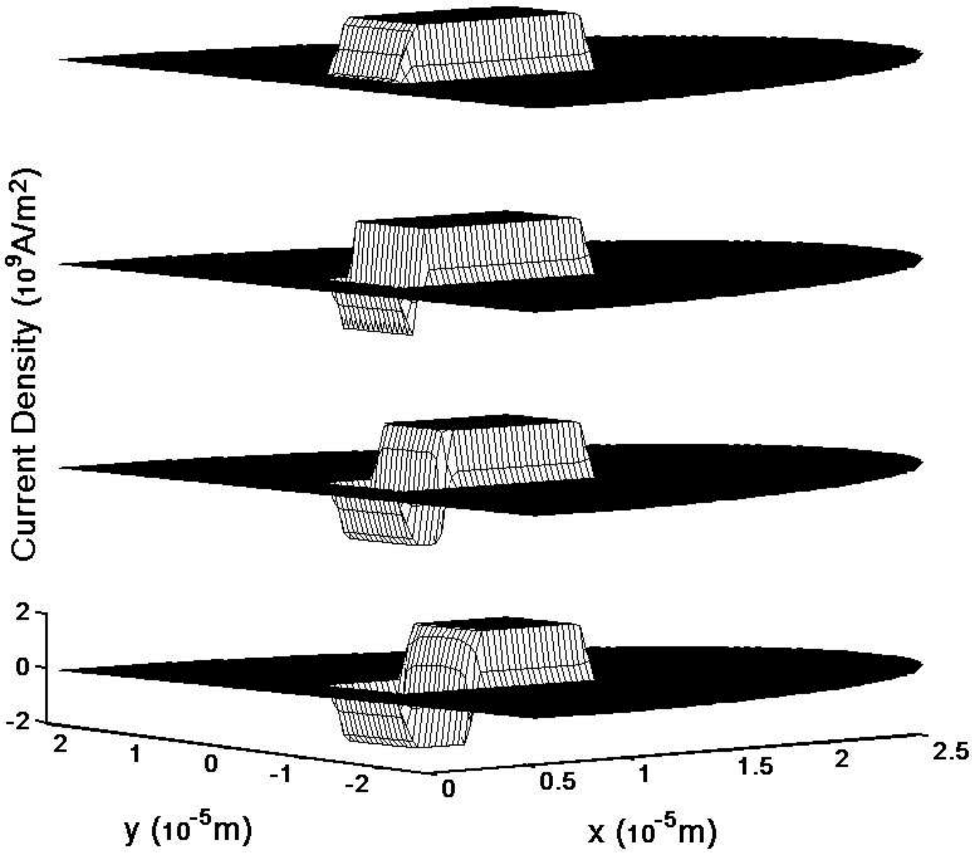

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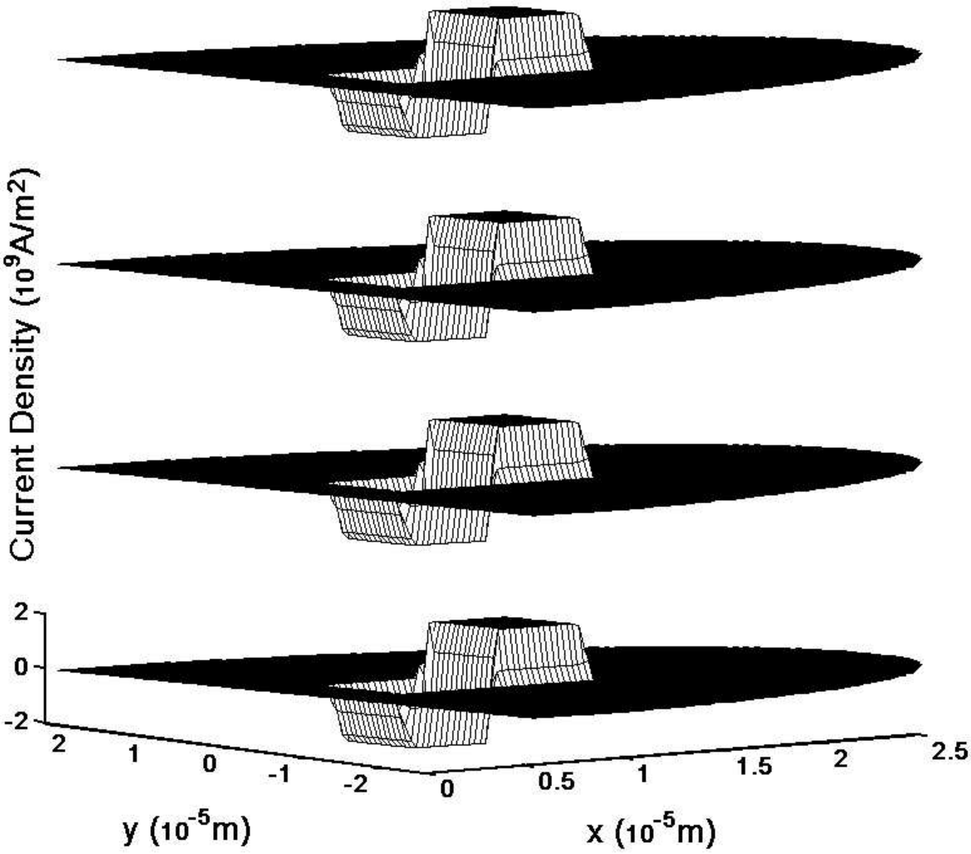

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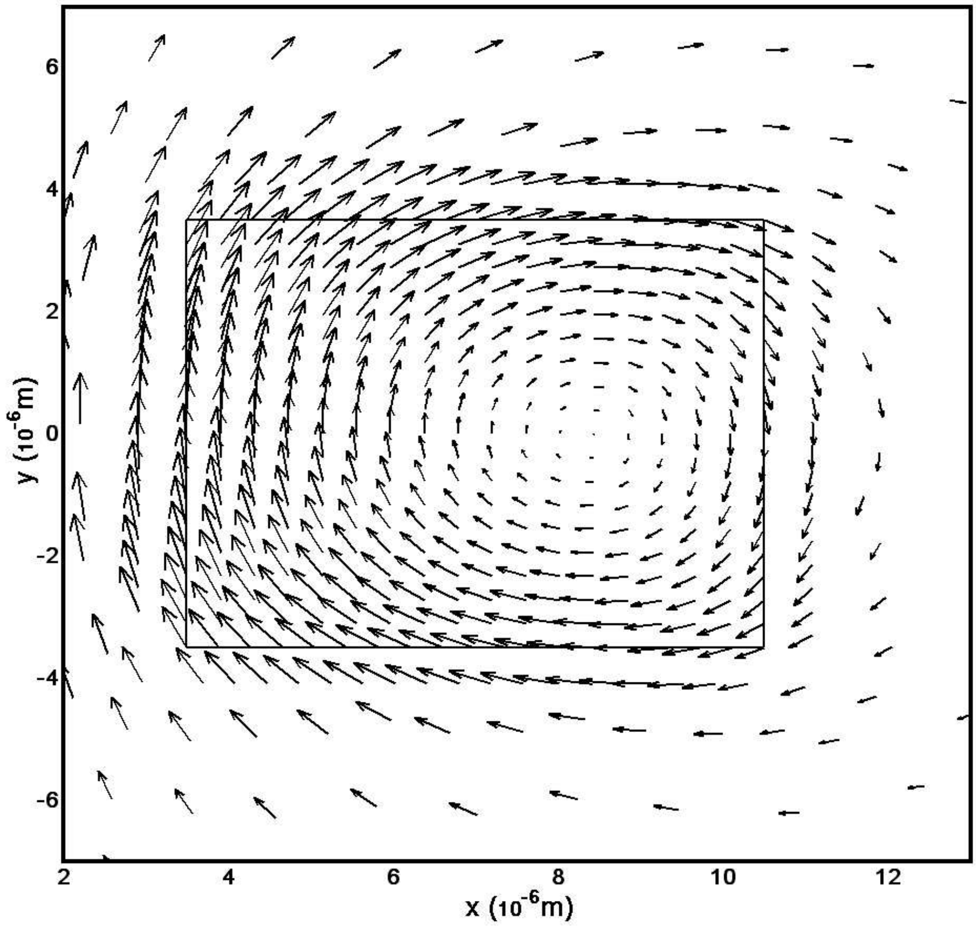

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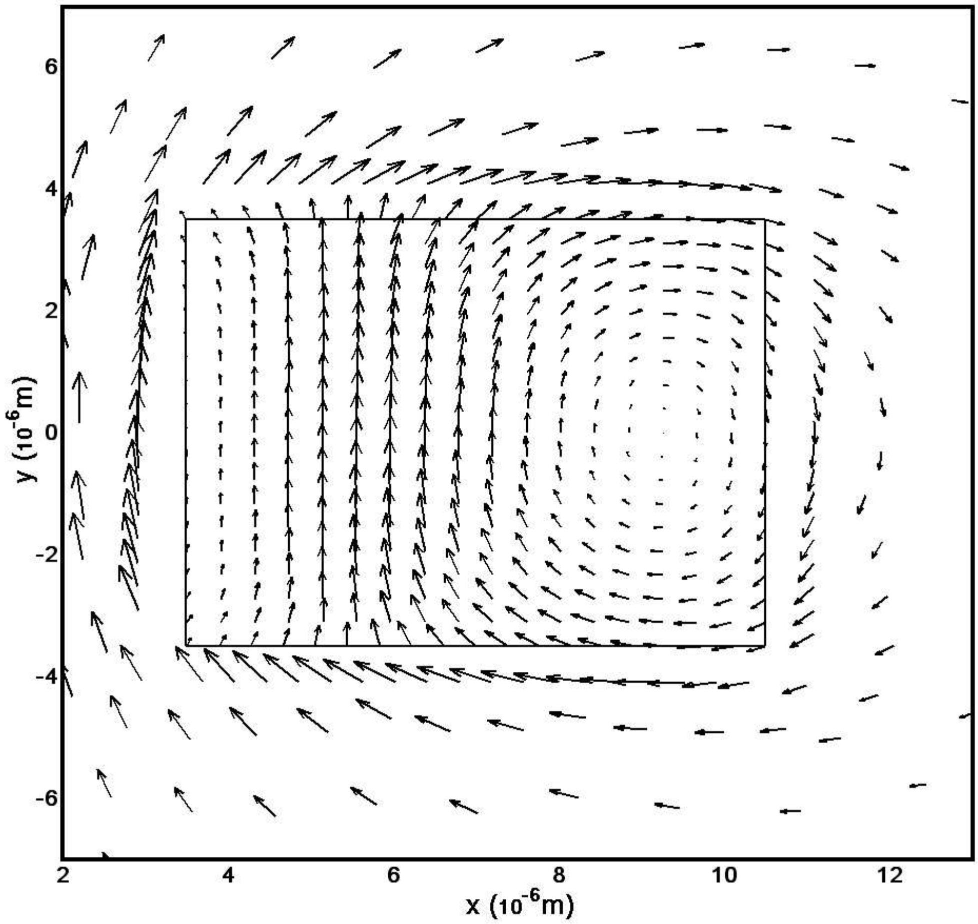

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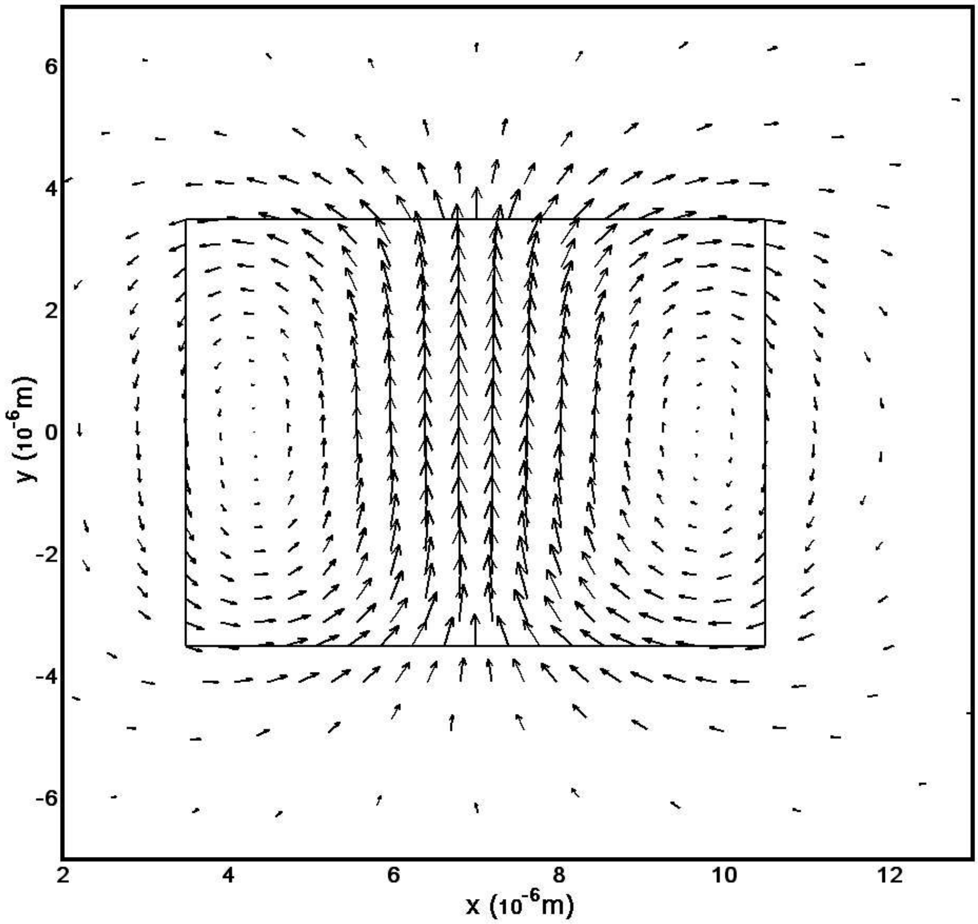

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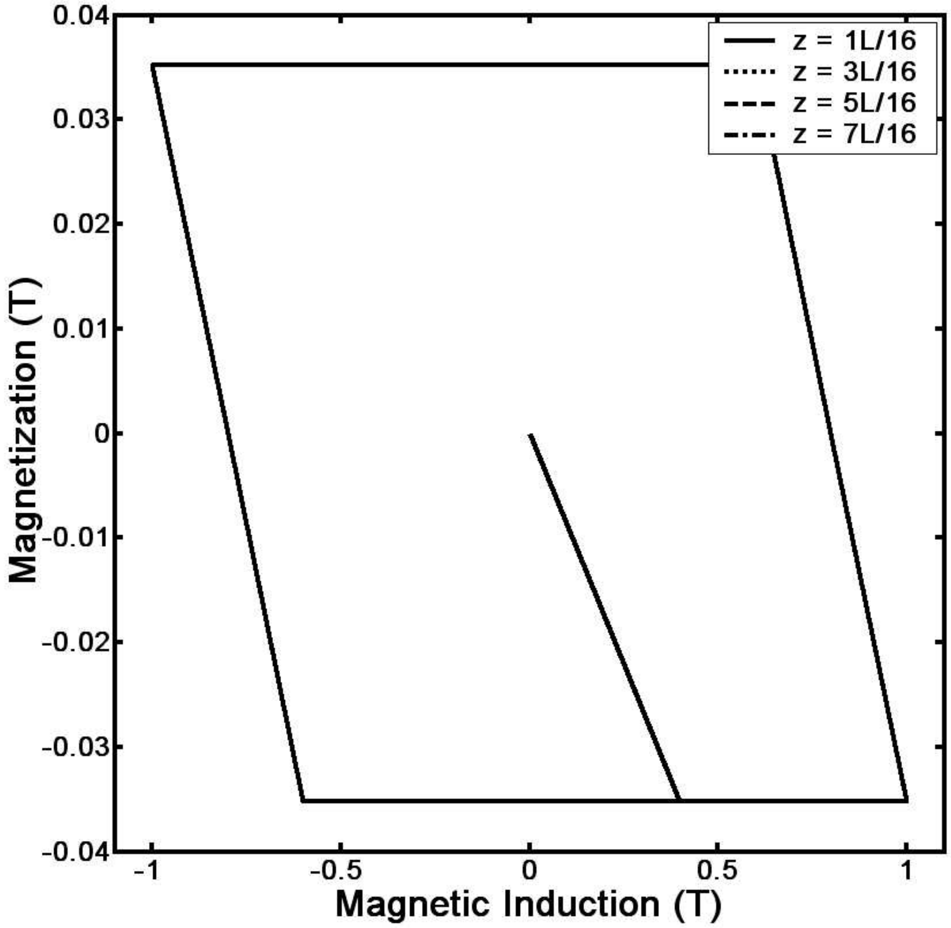

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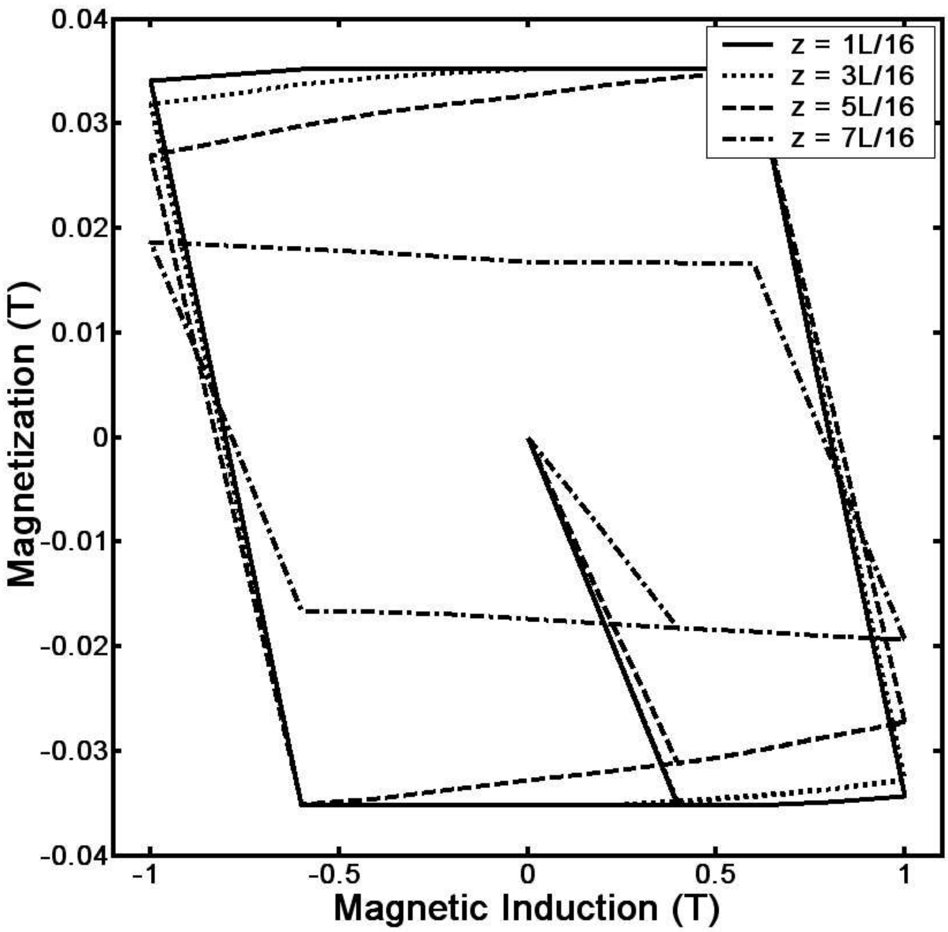

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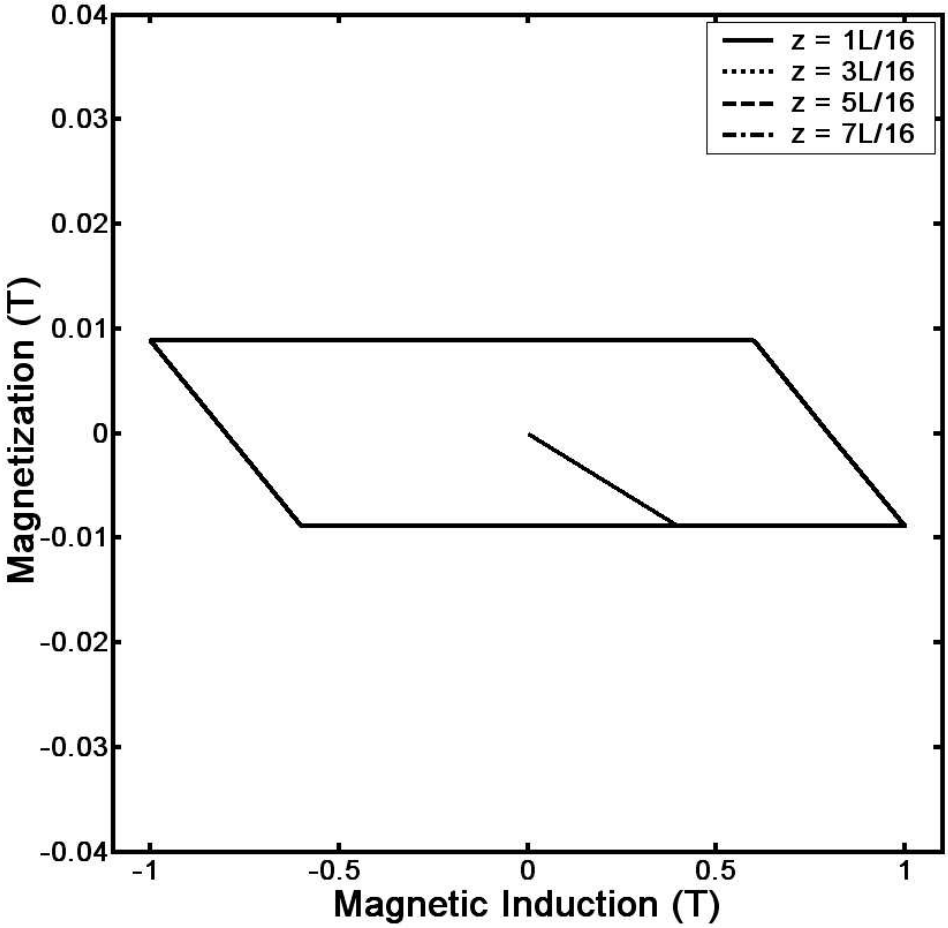

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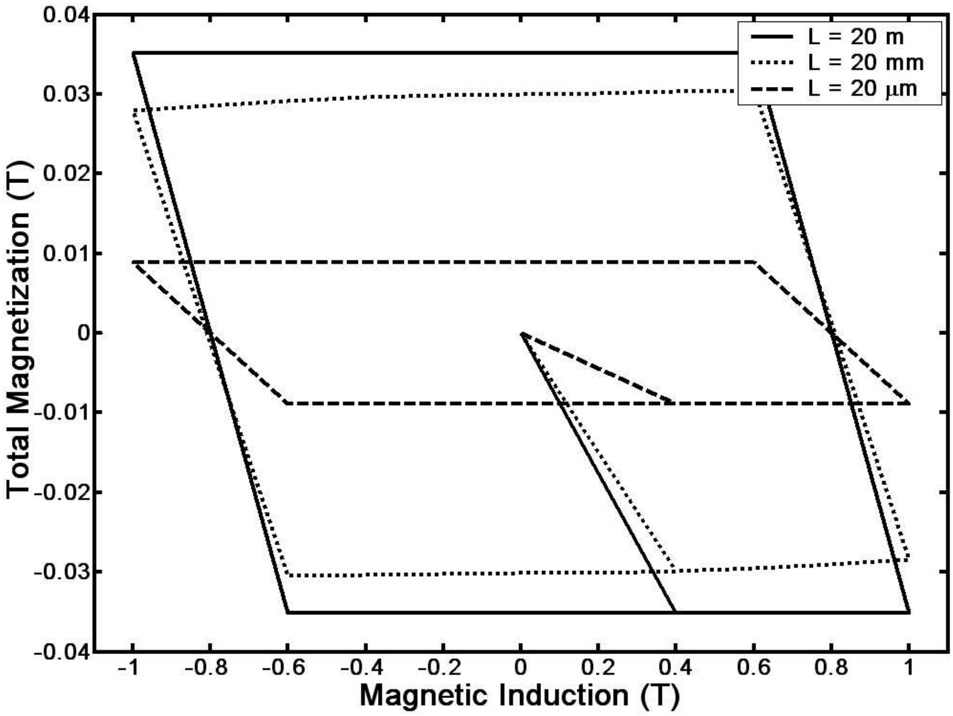

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Magnetization Modeling of Superconducting Magnet

Thitipong SATIRAMATEKUL [1] and Frédéric BOUILLAULT [2]

[1] Faculty of Engineering, Kasetsart University, Kamphaengsaen, 73140, Nakhon Pathom, Thailand

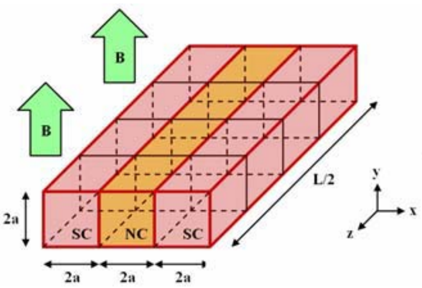
[2] Laboratoire de Génie Electrique de Paris, Supélec, Plateau de Moulon, 91192, Gif/Yvette Cedex, France

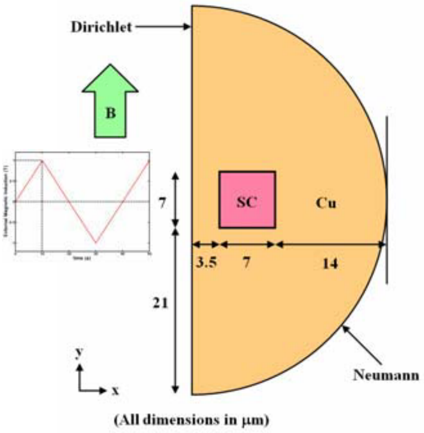
e-mail: thitipong.s@ku.ac.th

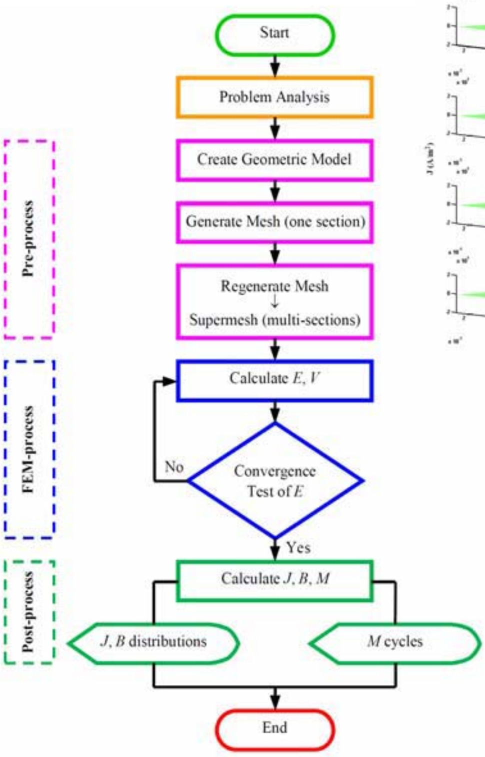
ABSTRACT

The computation of magnetization of a strand constituted of two superconducting square bars immersed in a uniform magnetic field is studied. The numerical results are obtained from the program developed with the FEM. Bean's critical state model is carried out to characterize the electrical behavior law of superconducting materials. The magnetization cycle, the magnetic field and the current density distributions are presented. The influence of strand length on the electromagnetic coupling phenomena in the superconducting square bars is also considered. The obtained results are in good agreement with the theoretical results.

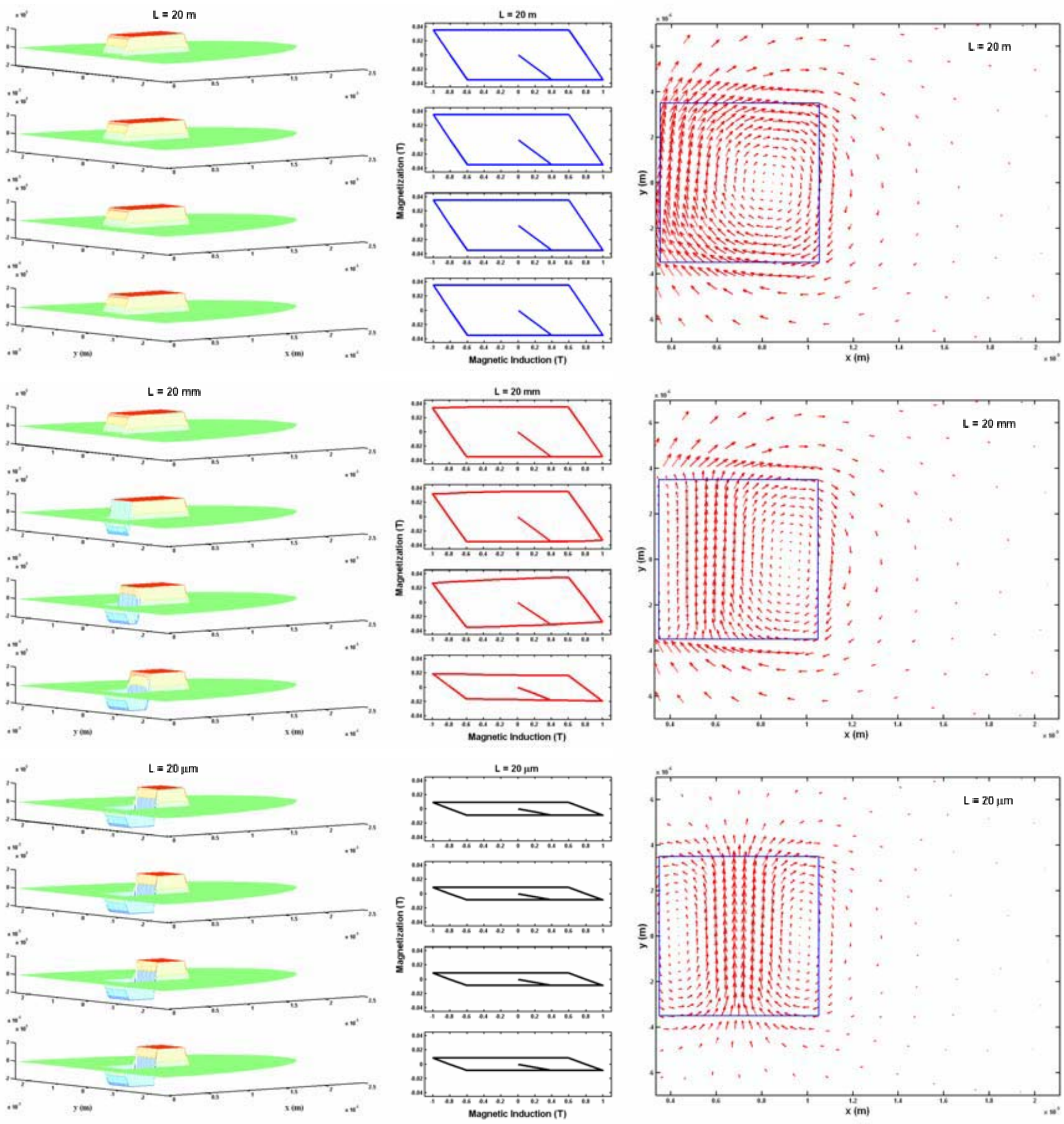
PROBLEM PRESENTATION



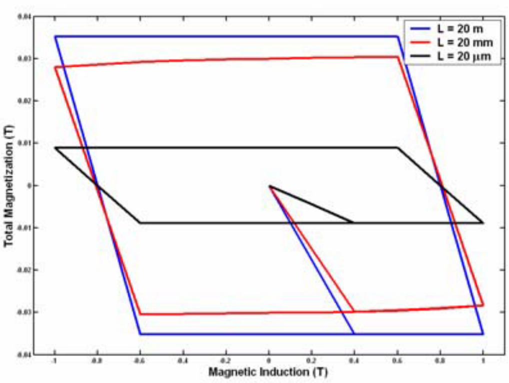




SIMULATION RESULTS



In order to approach the real problem, it is appropriate to model the problem in 3D. However, to avoid a very high computing time needed for solving the large number of elements required by 3D modeling, a novel 2D modeling technique is proposed. This is to divide the superconducting strand into elementary sections and to consider what happens in each section. The relations of the current inside the superconducting square bar, the current crossing the copper matrix and the voltage for each section are obtained directly from Kirchhoff's and Ohm's laws. Finally, we can express the relation between the current and the electric field in the superconducting square bar.



CONCLUSION

We developed a 2D numerical code to study the coupling between superconductor strands. The code permits to take into account the parameters influencing the coupling: not only the length of the filaments but also its radius, the critical current density, the rate of change of the field or the resistivity of the conductor matrix. The simulation results show that the superconducting filaments in composites of several kilometers of length into the superconducting magnet are always fully coupled. For this reason, the strand is always twisted with a pitch lower than the critical length.

บทคัดย่อ บทความ และโปสเตอร์

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Numerical Modelling of Superconducting Filaments for Coupled Problem

T. Satiramatekul [1] and F. Bouillault [2]
[1] Faculty of Engineering, Kasetsart University, Kamphaengsaen Campus, Nakhon Pathom, 73140, Thailand
[2] LGEP, UMR 8507 CNRS, SUPELEC, Gif sur Yvette Cedex, 91192, France
thitipong.s@ku.ac.th

Abstract — **This paper deals with two-dimensional modelling of a multifilamentary wire composed of two superconducting filaments in a conducting matrix. In order to avoid the three-dimensional problem solving, a novel technique is proposed to solve the coupled problem in two-dimensions. For that, it is enough to divide the filaments in several sections in the direction of the length of the wire. The difference of the currents in the superconducting filaments between two successive sections is equal to the current which losses in the copper matrix. The relation between the currents crossing the matrix and the electric fields in the filaments obtained by the analytical method is implemented in the finite element program. For a given geometry, the critical length of the wire where the filaments are coupled can be found. The numerical simulation results present the distributions of the current density in the modelled domain. The influence of the wire length on the total magnetization is also considered.**

I. INTRODUCTION

A better understanding of the electromagnetic coupling phenomena in the superconducting filaments can be done by numerical simulation. For that, a finite element program has been developed at the LGEP in France for modelling the superconducting materials.

In [1], we proposed new methods for solving the problem of partially coupled superconducting filaments in two-dimensions. In order to approach the three-dimensional problem with better behaviour, we propose in this paper a new technique by dividing the superconducting filaments into several sections and searching the relation between the currents crossing the copper matrix and the electric fields in the filaments.

II. PROBLEM ANALYSIS

In order to solve the problem, it is well to take the problem of several partially coupled filaments in [1], but by treating the problem with the minimum hypothesis. For that, we have considered a multifilamentary wire constituted of two superconducting filaments embedded in a copper matrix. The current density is supposed parallel to the wire axis in each section and invariant along this axis. It only depends on x, y and t. The magnetic induction is thus parallel to the x-y plane and also depends on x, y and t.

To avoid the large number of elements required by three-dimensional modelling, we have then divided the wire into elementary sections of L/n length, where L and n are the wire length and the number of sections respectively. In a simple case, we have taken $n = 8$. The two-dimensional presentation of the currents and voltages of two filaments along a wire length L shows in Fig. 1. Because of the symmetry of the problem, only the half of length has been studied in order to reduce the number of sections. Due to the low conductivity of the matrix compared to that of the superconductor, some small currents can circulate between the filaments. The difference of the currents in the filaments for two successive sections is equal to the current circulating in a section of matrix [2]. By using Kirchhoff's current law, we have obtained the relation between the currents inside the superconducting filaments (i) and the currents crossing the copper matrix (i_m) for the section k as follows:

$$i_k = \begin{cases} i_{k+1} + i_{mk} & ; \ k = 1,2,\dots,(n/2-1) \\ i_{mk} & ; \ k = n/2 \end{cases} \tag{1}$$

where $i_{m0} = 0$ A.

And by using Ohm's law, we have obtained the relation between the currents and the voltages for the section k as follows:

$$i_{mk} = \begin{cases} 2v_k/R & ; \ k = 1,2,\dots,(n/2-1) \\ v_k/R & ; \ k = n/2 \end{cases} \tag{2}$$

where $v_0 = 0$ V and R is representative of the resistance of a section of L/n length, its value could be determined by an electrokinetic formulation. It is interesting to note that, the resistance of the last section is twice because its length is two times smaller.

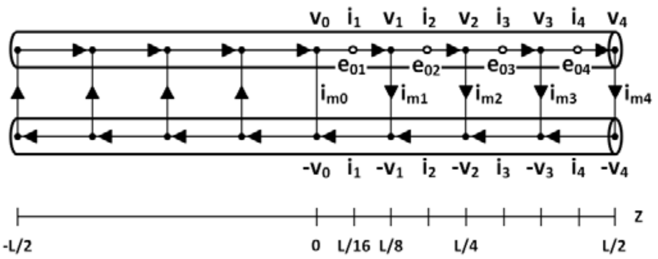


Fig. 1. 2D presentation of two filaments over a wire length *L*.

The equation of the electric fields (e_0) defined in Fig. 1 could be obtained as in [2].

$$e_{0k} = \frac{(v_{k-1} - v_k)}{(L/n)} \quad \text{at } z = \frac{(2k-1)L}{2n} \quad ; \ k = 1,2,\dots,n/2 \tag{3}$$

Equation (3) can be rewritten as follows:

$$v_k = -\frac{L}{n}\sum_{i=1}^{k} e_{0i} \quad ; \ k = 1,2,\dots,n/2 \tag{4}$$

By substituting (2) and (4) in (1), we have obtained the relation between the currents and the electric fields.

$$i_k = \begin{cases} i_{k+1} - \frac{2L}{nR}\sum_{i=1}^{k} e_{0i} & ; \ k = 1,2,\dots,(n/2-1) \\ -\frac{L}{nR}\sum_{i=1}^{k} e_{0i} & ; \ k = n/2 \end{cases} \tag{5}$$

Equation (5) can be rewritten in the following matrix form:

$$I = [C]E_0 \tag{6}$$

where $I = [i_1 \ i_2 \ \cdots \ i_{(n/2)}]^t$, $E_0 = [e_{01} \ e_{02} \ \cdots \ e_{0(n/2)}]^t$ and $[C]$ is a matrix of constant value whose dimension is $(n/2) \times (n/2)$. For $n = 8$, we have found that:

$$[C] = \begin{bmatrix} 8c & 6c & 4c & 2c \\ 6c & 6c & 4c & 2c \\ 4c & 4c & 4c & 2c \\ c & c & c & c \end{bmatrix} \tag{7}$$

where $c = -L/8R$.

III. NUMERICAL MODELLING

The domain of the problem shows in Fig. 2 (in 2D). To reduce the computing time, by looking at the symmetries of the problem, we have only modelled a quarter of the domain. A technique proposed for this modelling is to execute $n/2$ computations of finite element in parallel. For that, we have just created a mesh and then regenerated a super mesh with $n/2$ times more elements and $n/2$ times more nodes. The coupling between the calculations of finite element would be done via (6).

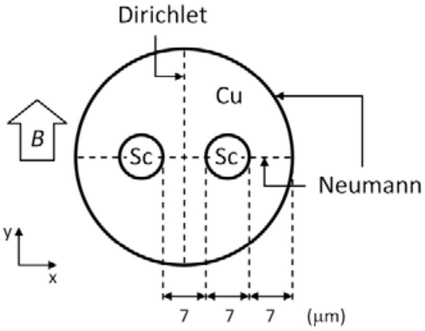


Fig. 2. Domain of the problem in 2D with given boundary conditions.

Bean's critical state model [3], which is replaced by a nonlinear function defined in [4], have been selected to characterize the electrical behaviour law of superconducting materials. As in [1], [5], in this case we have found that the matrix systems obtained by using the finite element method are as follows:

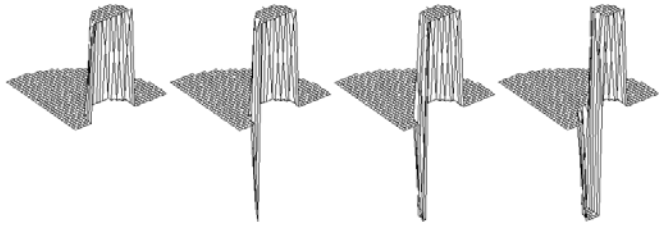
$$[M]\partial_t j_k + [A_e]e_k + [A_{ev}]e_{0k} = F \quad ; \quad k = 1,2,\ldots,n/2 \tag{8}$$

$$[A_{ev}]^t e_k + [A_v]e_{0k} = \partial_t i_k \quad ; \quad k = 1,2,\ldots,n/2 \tag{9}$$

where j is the current density. $[M]$ is the matrix of dimension $m \times m$. $[A_e]$, $[A_{ev}]$ and $[A_v]$ are the matrices of dimension $m \times m$, $m \times (n/2)$ and $(n/2) \times (n/2)$ respectively (m, n are the number of unknowns and the number of sections). F is the source vector.

IV. SIMULATION RESULTS

The simulation results have been obtained by using a code based on the finite element method and Bean's model. A series of numerical simulations has been realized for the superconducting filaments immersed in an external magnetic induction varying sinusoidally in time. Figure 3 shows the results of the current density distributions in the modelled domain at $t = T/8$ where T is the period of the applied magnetic induction. The four subfigures from left to right indicate respectively the current density distributions at $z = L/16, 3L/16, 5L/16, 7L/16$. In the case that the wire length (L) is equal to the critical length (L_c), we have obtained the results in the case of partially coupled (Fig. 3(a)). We have observed that the total current in the middle of the wire is equal to $J_c \pi r^2$, where r is the radius of the filament and J_c is the critical current density (3×10^9 A/m^2). These results show clearly the electromagnetic coupling phenomena in the superconducting filaments and are in agreement with those in [6]. For $L \gg L_c$ and $L \ll L_c$, the obtained results show the case of perfectly coupled (Fig. 3(b)) and perfectly decoupled (Fig. 3(c)) respectively. We could observe that the total current in the filament in Fig. 3(b) is the maximum value and the same in all sections. But in Fig. 3(c), this total current is zero and the same in all sections too. These results are in agreement with those shown in [1], [5]. In addition, the total magnetization of the wire could be computed by using the obtained value of the current density.



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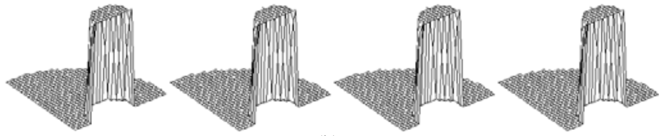

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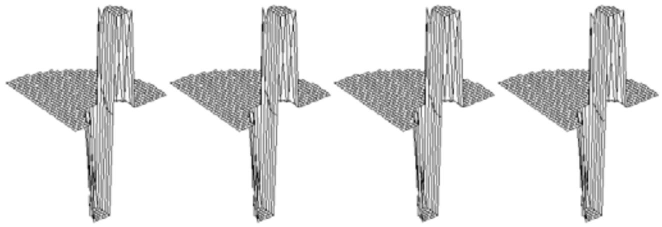

(c)

Fig. 3. Current density distributions in the modelled domain: (a) partially coupled case, (b) perfectly coupled case, and (c) perfectly decoupled case.

V. REFERENCES

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Numerical Modeling of Superconducting Filaments for Coupled Problem

Thitipong Satiramatekul[1] and Frédéric Bouillault[2]

[1]Faculty of Engineering, Kasetsart University, Kamphaengsaen Campus, Nakhon Pathom, 73140, Thailand
[2]LGEP, UMR 8507 CNRS, SUPELEC, Paris VI & Paris XI Universities, Gif sur Yvette Cedex, 91192, France

This paper deals with two-dimensional modeling of a multifilamentary wire composed of two superconducting filaments in a conducting matrix. In order to avoid the three-dimensional problem solving, a novel technique is proposed to solve the coupled problem in two-dimensions. For that, it is enough to divide the filaments in several sections in the direction of the length of the wire and to impose a relation between the currents crossing the matrix and the electric fields in the different sections of the filaments. The numerical simulation results present the distributions of the current density in the modeled domain. The influence of the wire length on the total magnetization is also considered.

***Index Terms*—Electromagnetic coupling, finite element method, modeling, superconducting filaments.**

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I. Introduction

A better understanding of the electromagnetic coupling phenomena in the superconducting filaments can be done by numerical simulation. For that, a finite element program has been developed at the LGEP in France for modeling the superconducting materials.

In [1], we proposed methods for solving the problem of partially coupled superconducting filaments in two-dimensions. In order to approach the three-dimensional problem with better behavior, we propose in this paper a new technique by dividing the superconducting filaments into several sections and introducing the relation between the currents crossing the copper matrix and the electric fields in the filaments.

II. Problem Analysis

In order to solve the problem, it is well to take the problem of several partially coupled filaments in [1], but by treating the problem with the minimum hypothesis. For that, we consider a multifilamentary wire constituted of two superconducting filaments embedded in a copper matrix. The current density in each section is supposed parallel to the wire axis and invariant along this axis. It only depends on x, y and t. The magnetic induction is thus parallel to the x-y plane and also depends on x, y and t.

To avoid the large number of elements required by three-dimensional modeling, we divide the wire into elementary sections of L/n length, where L and n are the wire length and the number of sections respectively. In a simple case, we take n = 8. The two-dimensional presentation of the currents and voltages of two filaments along a wire length L is shown in Fig. 1. Because of the symmetry of the problem, only the half of length is studied in order to reduce the number of sections. Due to the low conductivity of the matrix compared to that of the superconductor, some small currents can circulate between the filaments. The difference of the currents in the filaments for two successive sections is equal to the current circulating in a section of matrix [2]. By using Kirchhoff's current law, we obtain the relation between the currents inside the superconducting filaments (i) and the currents crossing the copper matrix (i_m) for the section k as follows:

$$i_k = \begin{cases} i_{k+1} + i_{mk} & ; k = 1,2,\ldots,(n/2-1) \\ i_{mk} & ; k = n/2 \end{cases} \tag{1}$$

where i_{m0} = 0 A.

And by using Ohm's law, we can obtain the relation between the currents and the voltages for the section k as follows:

$$i_{mk} = \begin{cases} 2v_k / R & ; k = 1,2,\ldots,(n/2-1) \\ v_k / R & ; k = n/2 \end{cases} \tag{2}$$

where v_0 = 0 V and R is representative of the resistance of a section of L/n length, its value could be determined by an electrokinetic 2D modeling. It is interesting to note that, the resistance of the last section is twice because its length is two times smaller.

FIG. 1 HERE

The equation of the electric fields (e_0) defined in Fig. 1 could be obtained as in [2]:

$$e_{0k} = \frac{(v_{k-1} - v_k)}{(L/n)} \quad at \quad z = \frac{(2k-1)L}{2n} \quad ; k = 1,2,\cdots,n/2 \cdot \tag{3}$$

Equation (3) can be rewritten as follows:

$$v_k = -\frac{L}{n}\sum_{i=1}^{k} e_{0i} \quad ; k = 1,2,\cdots,n/2 \cdot \tag{4}$$

By substituting (2) and (4) in (1), we obtain the relation between the currents and the electric fields:

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$$i_k = \begin{cases} i_{k+1} - \dfrac{2L}{nR}\sum_{i=1}^{k} e_{0i} & ;\ k = 1,2,\ldots,(n/2-1) \\ -\dfrac{L}{nR}\sum_{i=1}^{k} e_{0i} & ;\ k = n/2 \end{cases} \quad (5)$$

Equation (5) can be rewritten in the following matrix form:

$$I = [C]E_0 \quad (6)$$

where $I = [i_1 \quad i_2 \quad \cdots \quad i_{(n/2)}]^t$, $E_0 = [e_{01} \quad e_{02} \quad \cdots \quad e_{0(n/2)}]^t$ and $[C]$ is a matrix of constant value whose dimension is $(n/2)\times(n/2)$. For $n = 8$, we find that:

$$[C] = \begin{bmatrix} 8c & 6c & 4c & 2c \\ 6c & 6c & 4c & 2c \\ 4c & 4c & 4c & 2c \\ c & c & c & c \end{bmatrix} \quad (7)$$

where $c = -L/(8R)$.

III. Numerical Modeling

The domain of the problem shows in Fig. 2 (in 2D). To reduce the computing time, by looking at the symmetries of the problem, we model only a quarter of the domain. A technique proposed for this modeling is to execute $n/2$ computations of finite element in parallel. For that, we just create a mesh and then regenerate a super mesh with $n/2$ times more elements and $n/2$ times more nodes. The coupling between the calculations of finite element is done via (6).

FIG. 2 HERE

Bean's critical state model [3], which is replaced by a nonlinear function defined in [4], is selected to characterize the electrical behavior law of superconducting materials. As in [1], [5], in this case we find that the matrix systems obtained by using the finite element method are as follows:

$$[M]\partial_t j_k + [A_e]e_k + [A_{ev}]e_{0k} = F \quad ;\ k = 1,2,\ldots,n/2 \quad (8)$$
$$[A_{ev}]^t e_k + [A_v]e_{0k} = \partial_t i_k \quad ;\ k = 1,2,\ldots,n/2 \quad (9)$$

where j is the current density. $[M]$ is the matrix of dimension $m\times m$. $[A_e]$, $[A_{ev}]$ and $[A_v]$ are the matrices of dimension $m\times m$, $m\times(n/2)$ and $(n/2)\times(n/2)$ respectively (m, n are the number of unknowns and the number of sections). F is the source vector.

IV. Simulation Results

The simulation results are obtained by using a code based on the finite element method and Bean's model. A series of numerical simulations is realized for the superconducting filaments immersed in an external magnetic induction varying sinusoidally in time. The direction of this applied magnetic induction is parallel to the y axis. Figure 3 shows the results of the current density distributions in the modeled domain at $t = T/8$ where T is the period of the applied magnetic induction. The four subfigures from left to right indicate respectively the current density distributions at $z = L/16$, $3L/16$, $5L/16$, $7L/16$. In the case that the wire length (L) is equal to the critical length (L_c), we obtain the results in the case of partially coupled (Fig. 3(a)). We observe that the total current in the middle of the wire is equal to $J_c\pi r^2$, where r is the radius of the filament and J_c is the critical current density (3×10^9 A/m^2). These results show clearly the electromagnetic coupling phenomena in the superconducting filaments and are in agreement with those in [6]. For $L >> L_c$ and $L << L_c$, the obtained results show the cases of perfectly coupled (Fig. 3(b)) and of perfectly decoupled (Fig. 3(c)) respectively. We can observe that the total current in the filament in Fig. 3(b) is the maximum value and the same in all sections. But in Fig. 3(c), this total current is zero and the same in all sections too. These results are in agreement with those shown in [1], [5].

FIG. 3 HERE

In addition, the magnetization of each section can be computed by using the obtained value of the current density. Taking the values of the magnetization and of the magnetic induction, a complete cycle of magnetization can be obtained numerically. Figure 4 shows the magnetization cycles of each section. These cycles correspond with the current density distributions in Fig. 3. In order to explain the results in the perfectly coupled (Fig. 4(b)) and perfectly decoupled cases (Fig. 4(c)), it is interesting to consider the maximum value of the magnetization (at saturation) of a filament calculated analytically as follows:

$$\vec{M} = \int_L \int_S (\vec{r}_1 + \vec{r}_2) \times \vec{j}\, ds\, dz \quad (10)$$

where S is the cross-sectional area of a superconducting filament, $\vec{r}_1$ and $\vec{r}_2$ are defined in Fig. 5.

In the perfectly coupled case, when the wire is saturated in current, we have:

$$\int_S \vec{j}\, ds = \pm J_c S \vec{u}_z = \pm I_c \vec{u}_z \quad (11)$$

where $\vec{u}_z$ is the unit vector in the direction of the wire axis, we have then:

$$\vec{M} = \int_L (\vec{r}_1 \times I_c \vec{u}_z + \int_S \vec{r}_2 \times \vec{j}\, ds)\, dz \cdot \quad (12)$$

Due to the symmetry of the problem at saturation, the second term is thus zero and the first term is equal to $x_1 I_c \vec{u}_y$ where x_1 is defined in Fig. 5 (in this case $x_1 = 2r = 7$ μm). The total magnetization per unit volume of superconductor of the wire at saturation can be computed as follows:

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$$M_{total} = 2\mu_0 M / S_{total} = \mu_0 x_1 J_c \quad (13)$$

where μ_0 is the magnetic permeability of vacuum and S_{total} is the total cross-sectional area of the superconductor (in this case $S_{total} = 2\pi r^2$).

FIG. 4 HERE
FIG. 5 HERE

For the perfectly decoupled case, from (10) and the symmetry of the problem, we have:

$$\vec{r}_1 \times \int_S \vec{j} ds = 0 \cdot \quad (14)$$

So the magnetization of one filament in a set of superconducting filaments does not depend on its position in the arrangement of the wire:

$$\vec{M} = \int_L \int_S \vec{r}_2 \times \vec{j} dsdz \cdot \quad (15)$$

As in [1], the total magnetization per unit volume of superconductor of the wire is equal to that of one filament [6]:

$$M_{total} = 4\mu_0 J_c r / (3\pi) \cdot \quad (16)$$

In both cases; perfectly coupling and perfectly decoupling, the numerical and analytical values are in good agreement, the differences between two values are only 0.2 % and 1.2 % for perfectly coupled and perfectly decoupled cases respectively.

Figure 4(a) shows the situation of partially coupling. In this case, the total magnetization values at saturation of each section in four subfigures reduce from left to right, or from near perfectly coupling in the section at the middle of the wire approach to near perfectly decoupling in the section at the end of the wire. These results are in harmony with those presented in [6].

For all cases, because of the symmetry of the current density distributions at saturation in the direction of the x axis (perpendicular to the external magnetic field) shown in Fig. 5, the total magnetization per unit volume of superconductor of the wire is about zero (see in [7]).

Finally, the average value of the total magnetization per unit volume of superconductor of the wire can be obtained numerically as follows:

$$\overline{M}_{total} = \left(\sum_{k=1}^{n/2} M_{total,k}\right) / (n/2) \cdot \quad (17)$$

Figure 6 compares three cycles of total magnetization obtained by using the numerical method (FEM) for three different wire lengths. It shows clearly the deformation of the cycle in the case of partially coupling. Its values are less than those of perfectly coupled case and more than those of perfectly decoupled case. This is to confirm that our results in [1] and [2] are correct.

In order to see appear the effect of the wire length on the maximum value of the total magnetization, we realize a new series of numerical simulations for several wire lengths. From (6) and (7), it shows that a good parameter to characterize the effect of coupling for a given geometry and a given frequency is: L/R. So to know the value of the critical length of wire which leads to a situation of partially coupling, it has to know the value of the resistance R. This one can be evaluated by a 2D electrokinetic code. If the currents in the matrix are assumed as in Fig. 7, this value is approximately equal to:

$$R = n / (2\sigma L) \quad (18)$$

where σ is the electrical conductivity of the conductive matrix.

For this given expression of R, it appears that a good intrinsic parameter to characterize the effect of coupling is: σL^2. Figure 8 presents the maximum total magnetization per unit volume of superconductor as a function of the value of σL^2 at a given frequency of the applied magnetic field (fixed at 1 Hz). The value of σL^2 is varied from 10^{-2} to 10^8 m/Ω. We find that the phenomenon of partially coupling appears when the value of σL^2 is approximately between 10^2 and 10^4 m/Ω.

In the case that the conductive matrix is OFHC copper ($\sigma \cong 10^{10}$ S/m at 4.2 K) [2] and $n = 8$, the maximum length of wire (L_c) that the superconducting filaments are still perfectly decoupled is about 100 μm.

FIG. 6 HERE
FIG. 7 HERE
FIG. 8 HERE

If we plot the distribution of the electrical potential along the axis of the filament as in Fig. 9, the results show that for partially coupled case the variation is linear and prove the assumption used in [1]. With this fact, the electric field (e_0) is constant and the relation between this field and the total current which flows in the matrix (i_m) is:

$$i_m = (nL) / (4R) \times e_0 \cdot \quad (19)$$

So the distribution of the potential in the section at the middle of the filament ($z = 0$) can be calculated by a classical 2D approach and by imposing this relation between the current in the filament and the electric field [1]. The value of the critical length using this model is in harmony with that calculated previously in this paper.

FIG. 9 HERE

V. Conclusion

In order to approach to the 3D coupled problem, a novel technique is proposed for solving a coupled problem of superconducting filaments in 2D. The numerical simulation results are obtained by using the finite element code and

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Bean's model. These results show clearly the electromagnetic coupling phenomena in the superconducting filaments.

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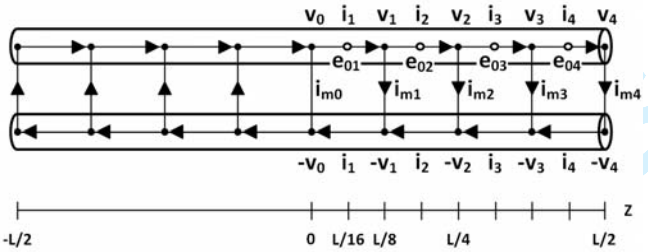


Fig. 1. 2D presentation of two filaments over a wire length *L*.

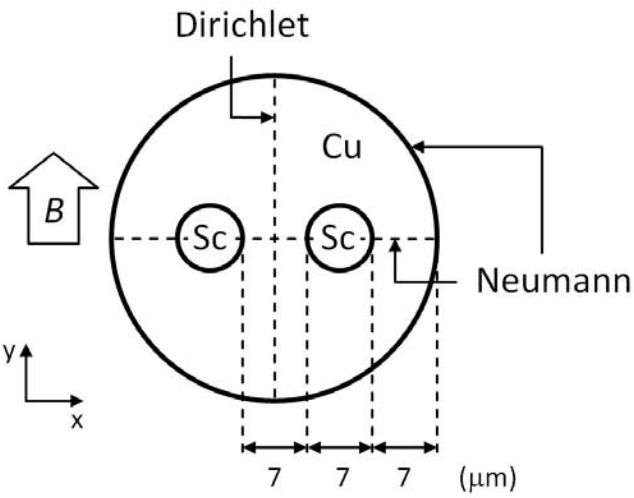
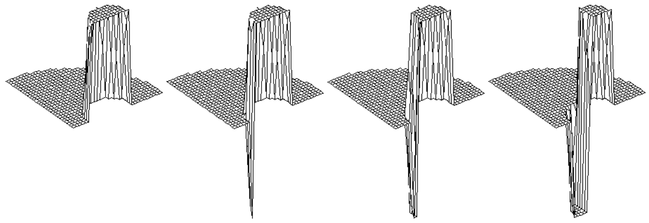
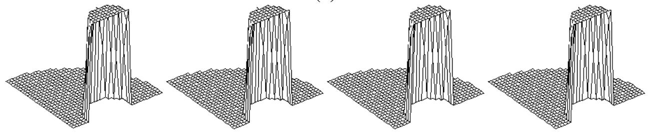


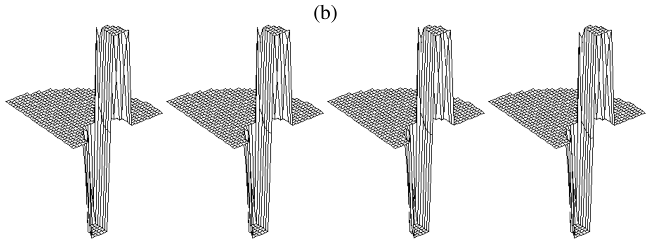
Fig. 2. Domain of the problem in 2D with given boundary conditions.



(a)

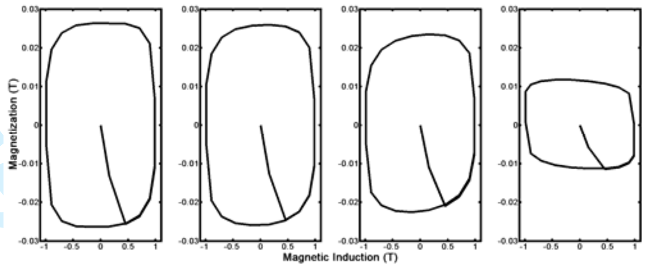


(b)

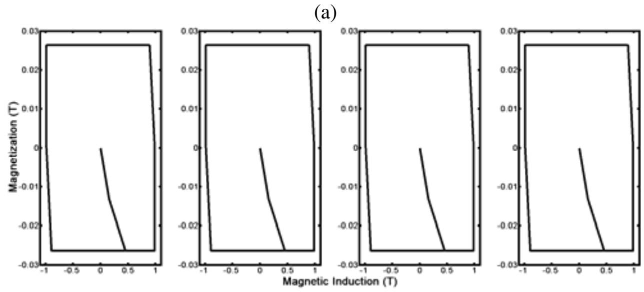


(c)

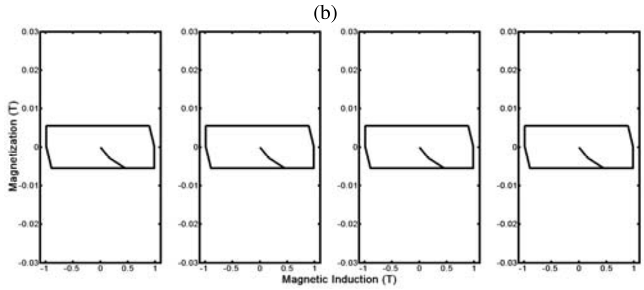
Fig. 3. Current density distributions in the modeled domain: (a) partially coupled case, (b) perfectly coupled case, and (c) perfectly decoupled case.



(a)



(b)



(c)

Fig. 4. Magnetization cycles of each section related to Fig. 3: (a) partially coupled case, (b) perfectly coupled case, and (c) perfectly decoupled case.

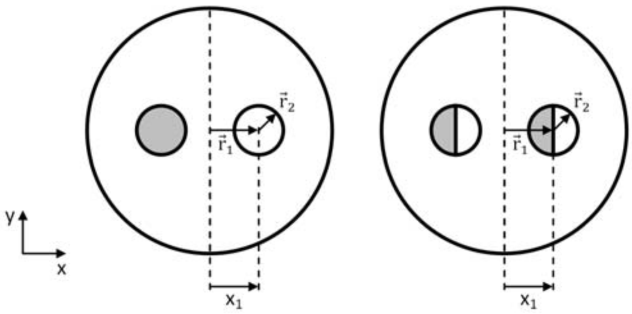


Fig. 5. Current density distributions at saturation for two cases studied.

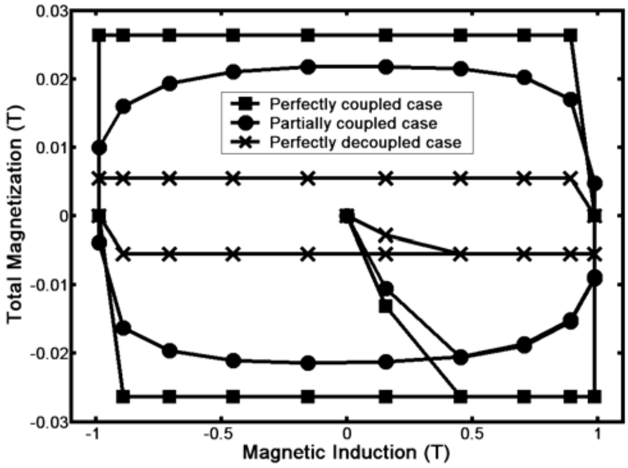


Fig. 6. Comparison of the cycles of average total magnetization for perfectly coupled, partially coupled, and perfectly decoupled cases.

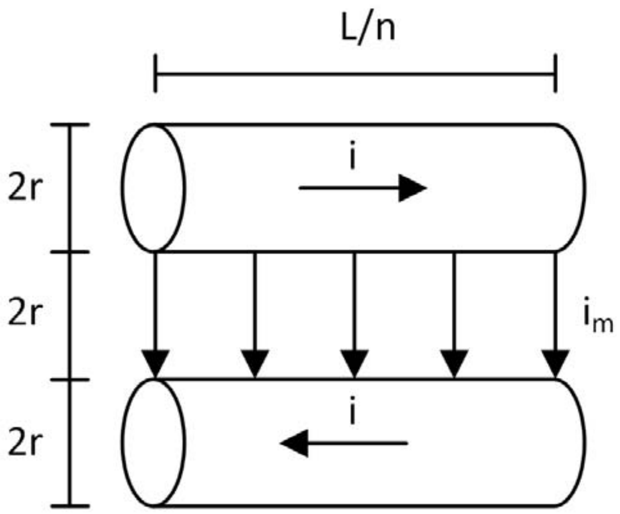


Fig. 7. Distribution of the currents in the conductive matrix.

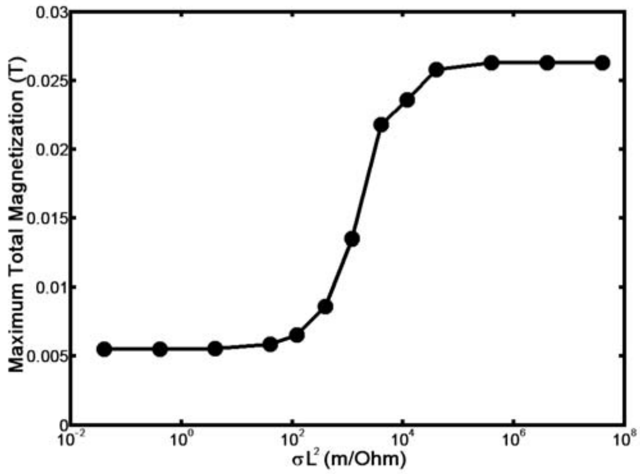


Fig. 8. Maximum total magnetization versus the value of σL^2.

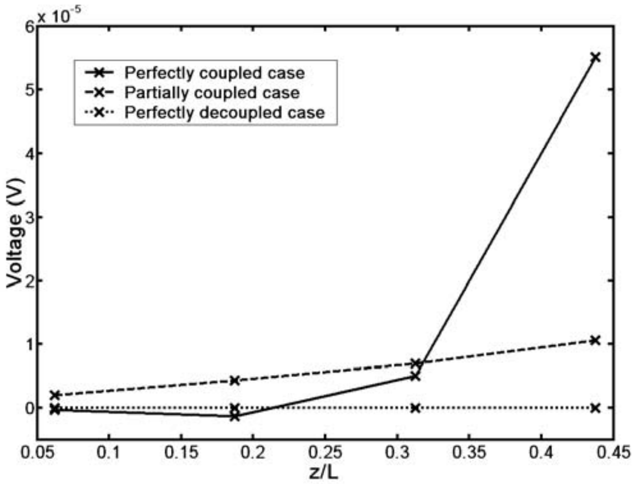
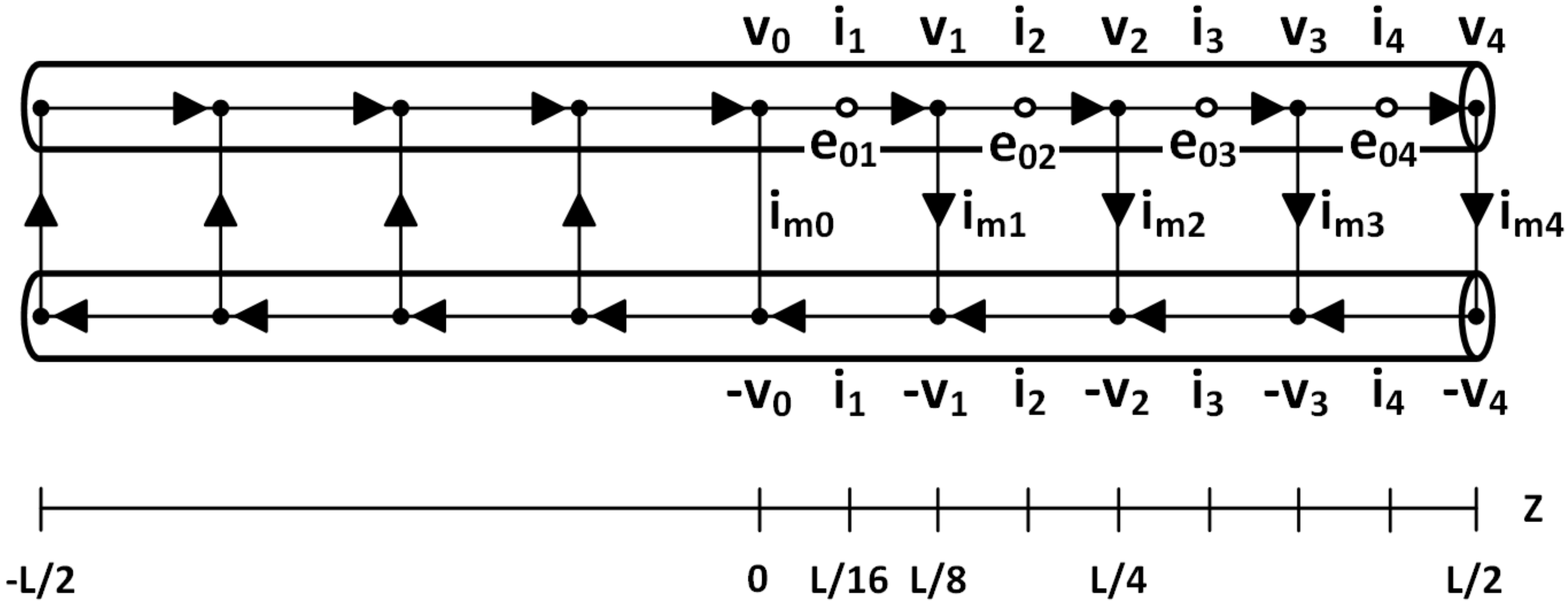
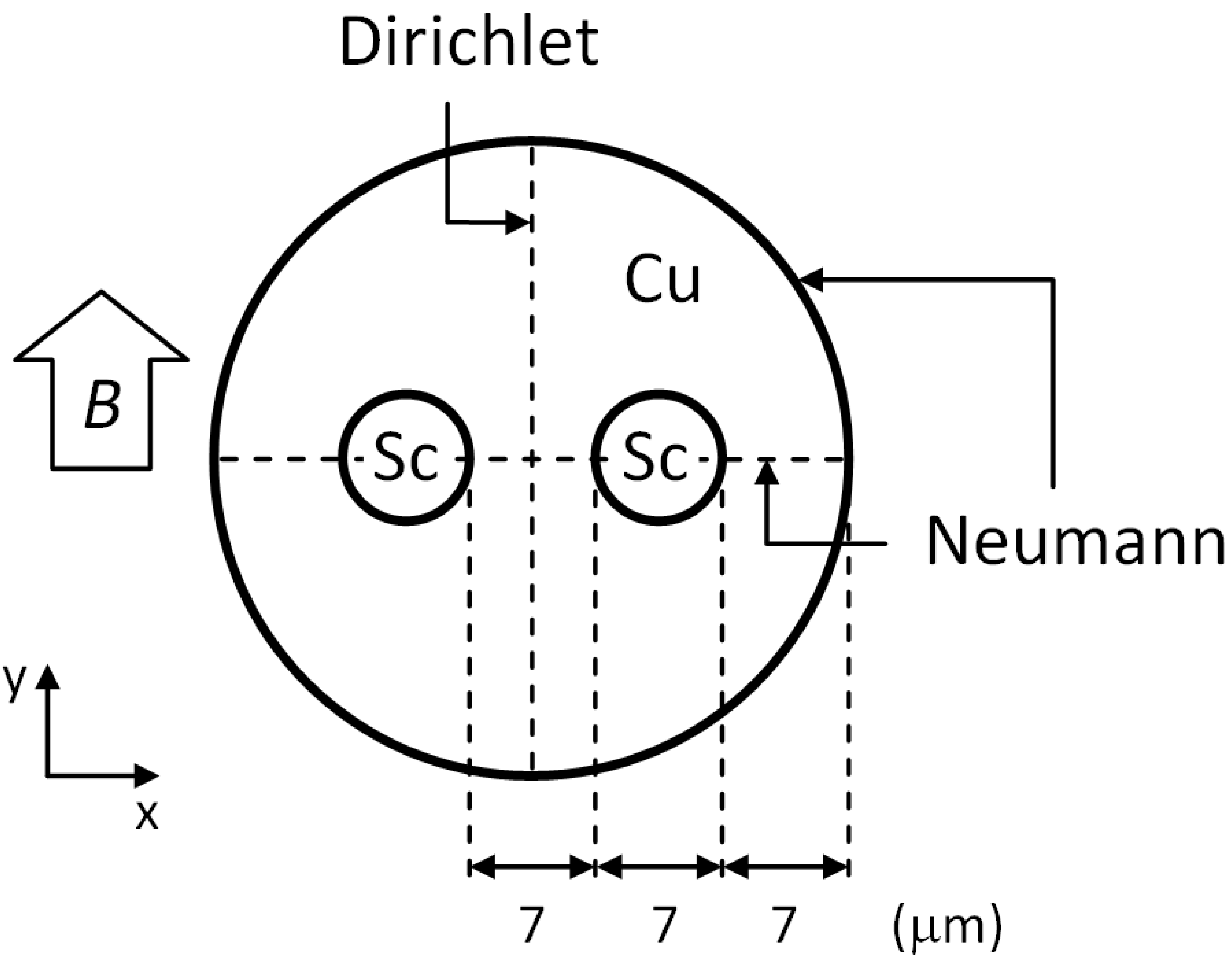
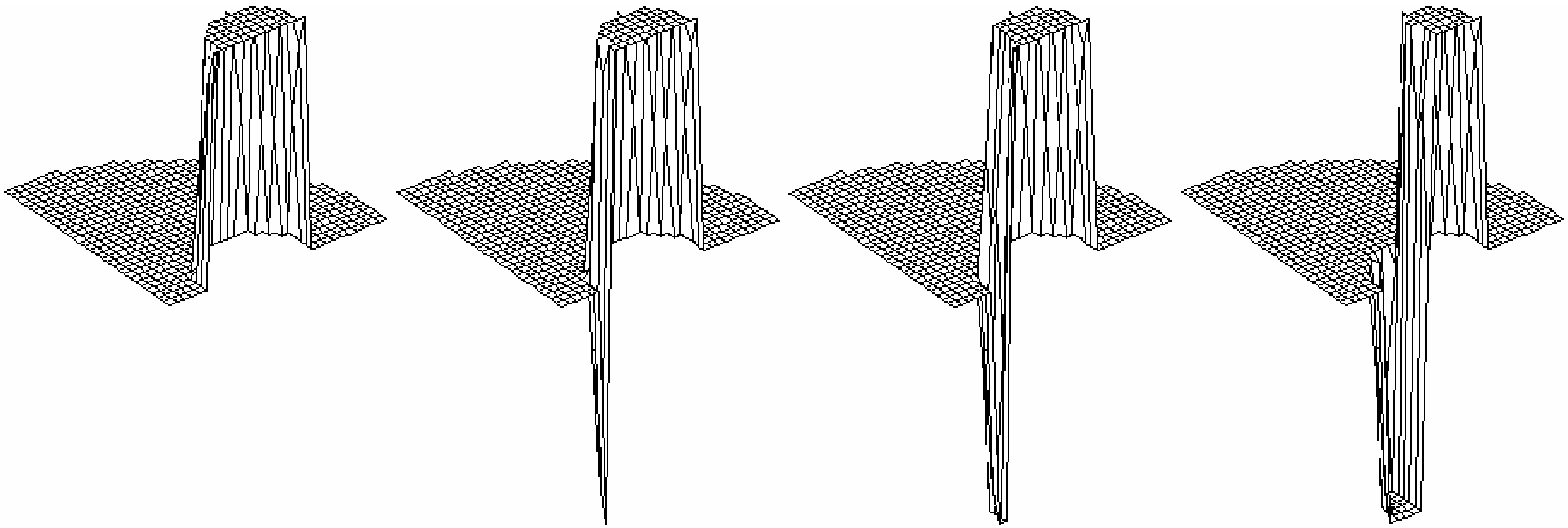
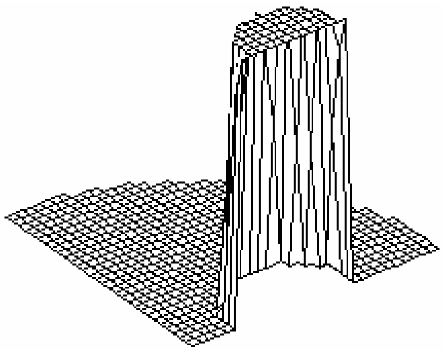


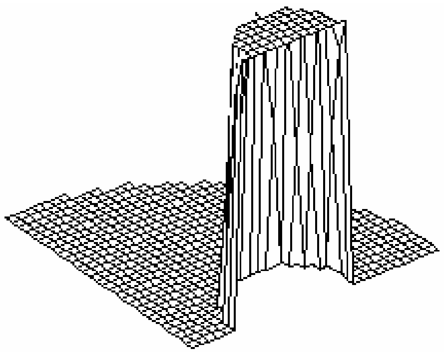
Fig. 9. Distribution of the potential along the axis of the filament.

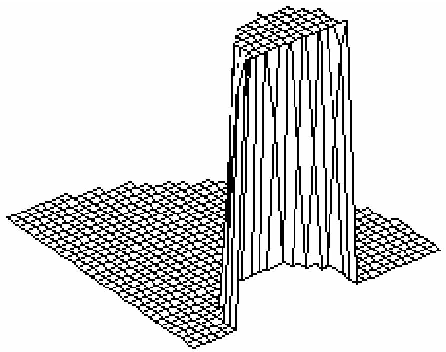

v0 i1 v1 i2 v2 i3 v3 i4 v4
e01 e02 e03 e04
im0 im1 im2 im3 im4
-v0 i1 -v1 i2 -v2 i3 -v3 i4 -v4
z
-L/2 0 L/16 L/8 L/4 L/2

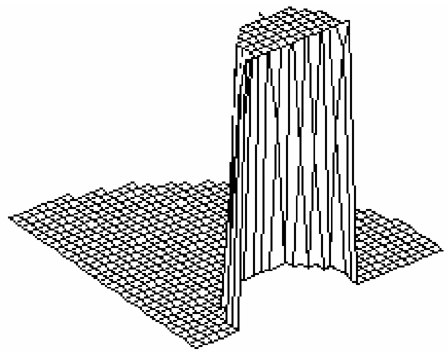

Dirichlet
Cu
B
Sc
Sc
Neumann
y
x
7
7
7
(μm)

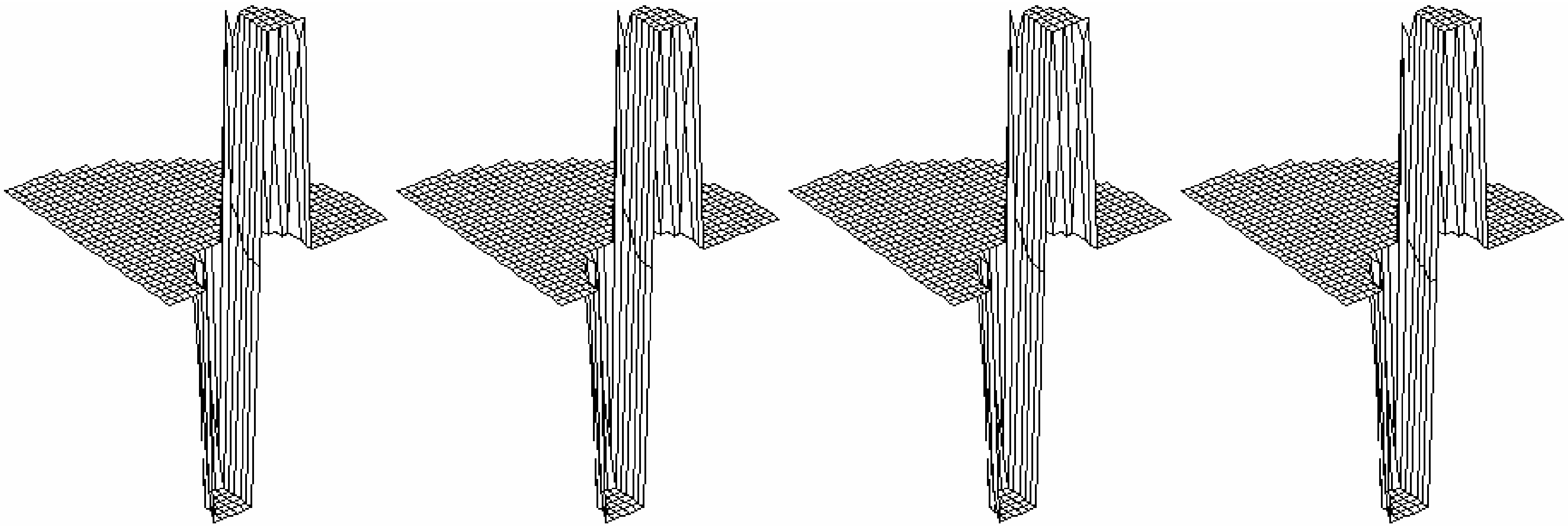


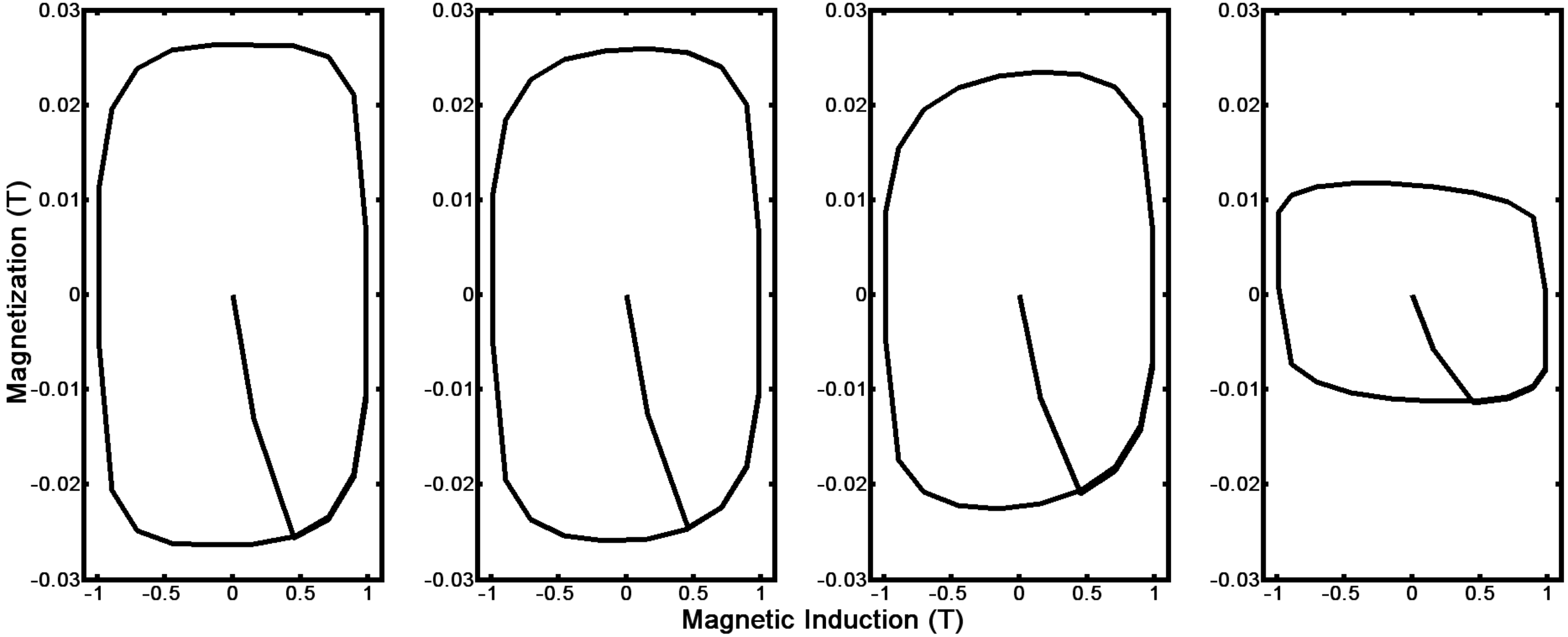


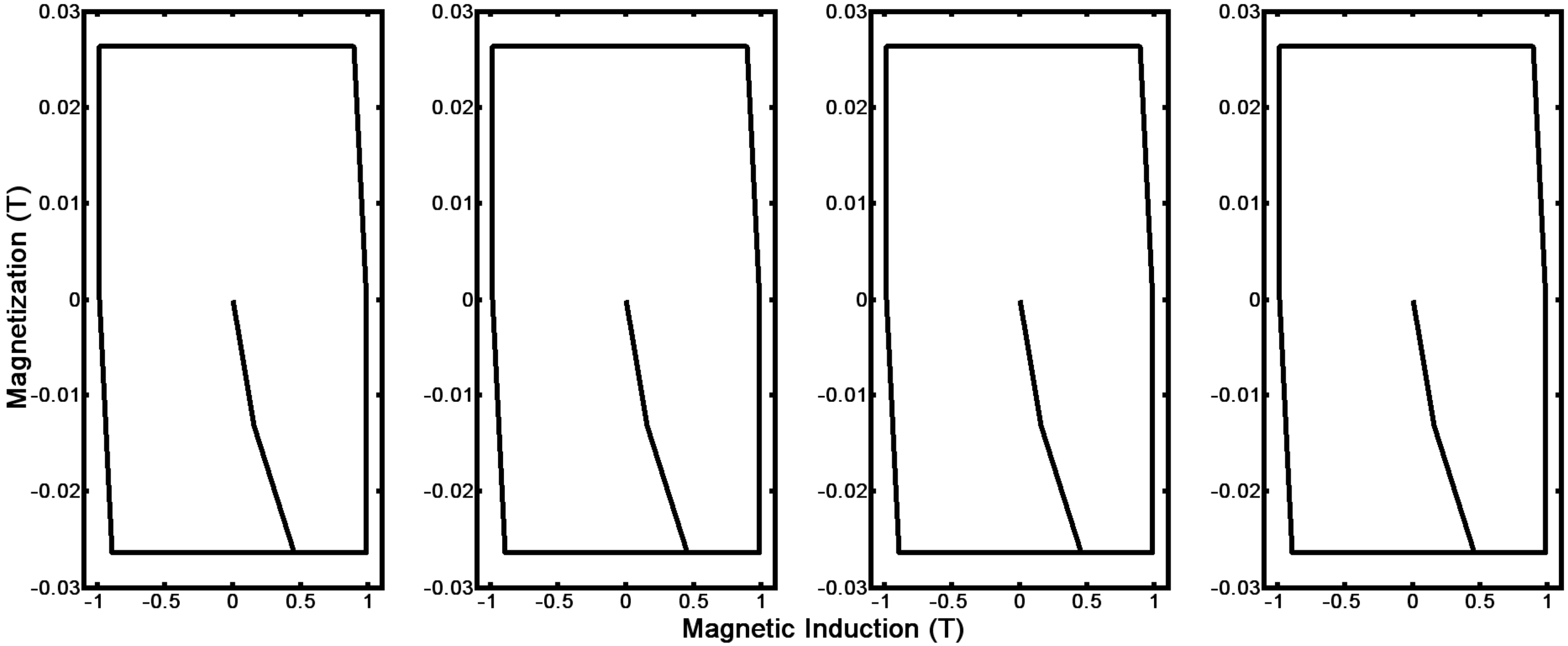









Magnetization (T)
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-1
-0.5
0
0.5
1
Magnetic Induction (T)


Magnetization (T)
Magnetic Induction (T)
0.03
0.02
0.01
0
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-0.02
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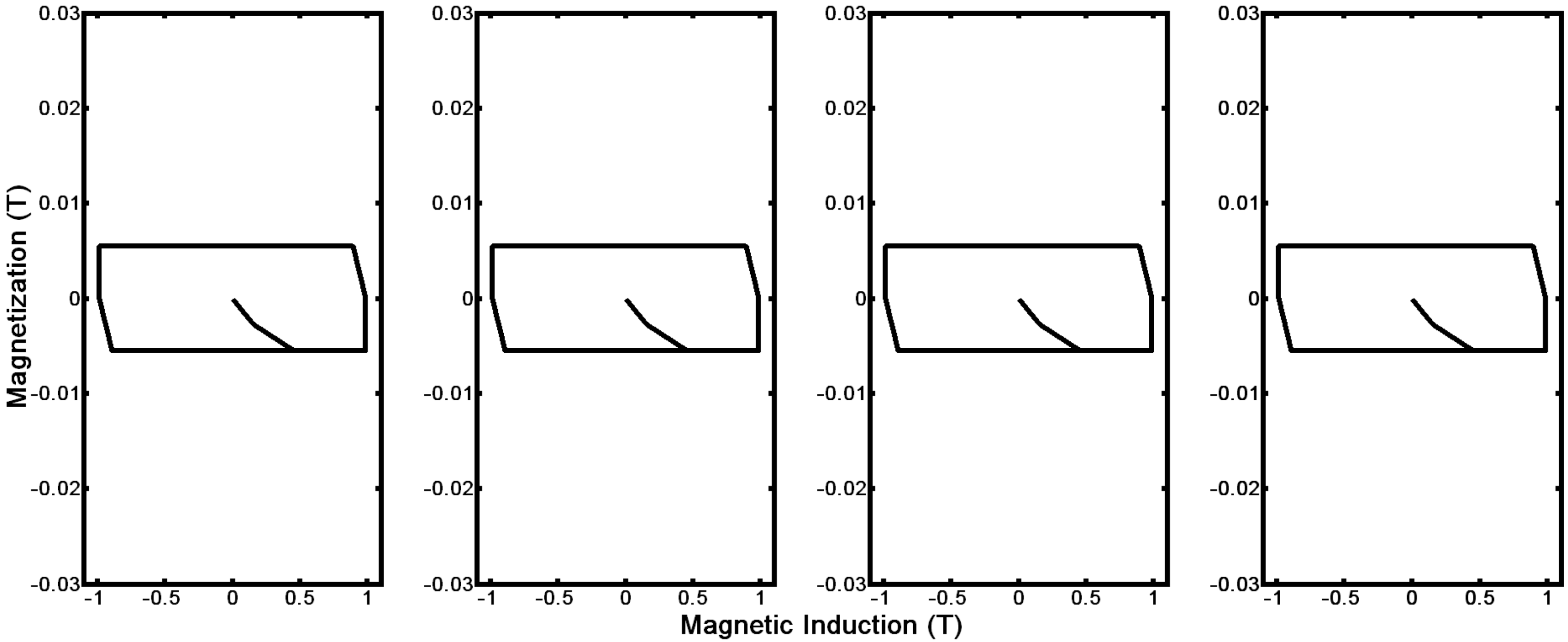
-1
-0.5
0
0.5
1


Magnetization (T)
Magnetic Induction (T)
0.03
0.02
0.01
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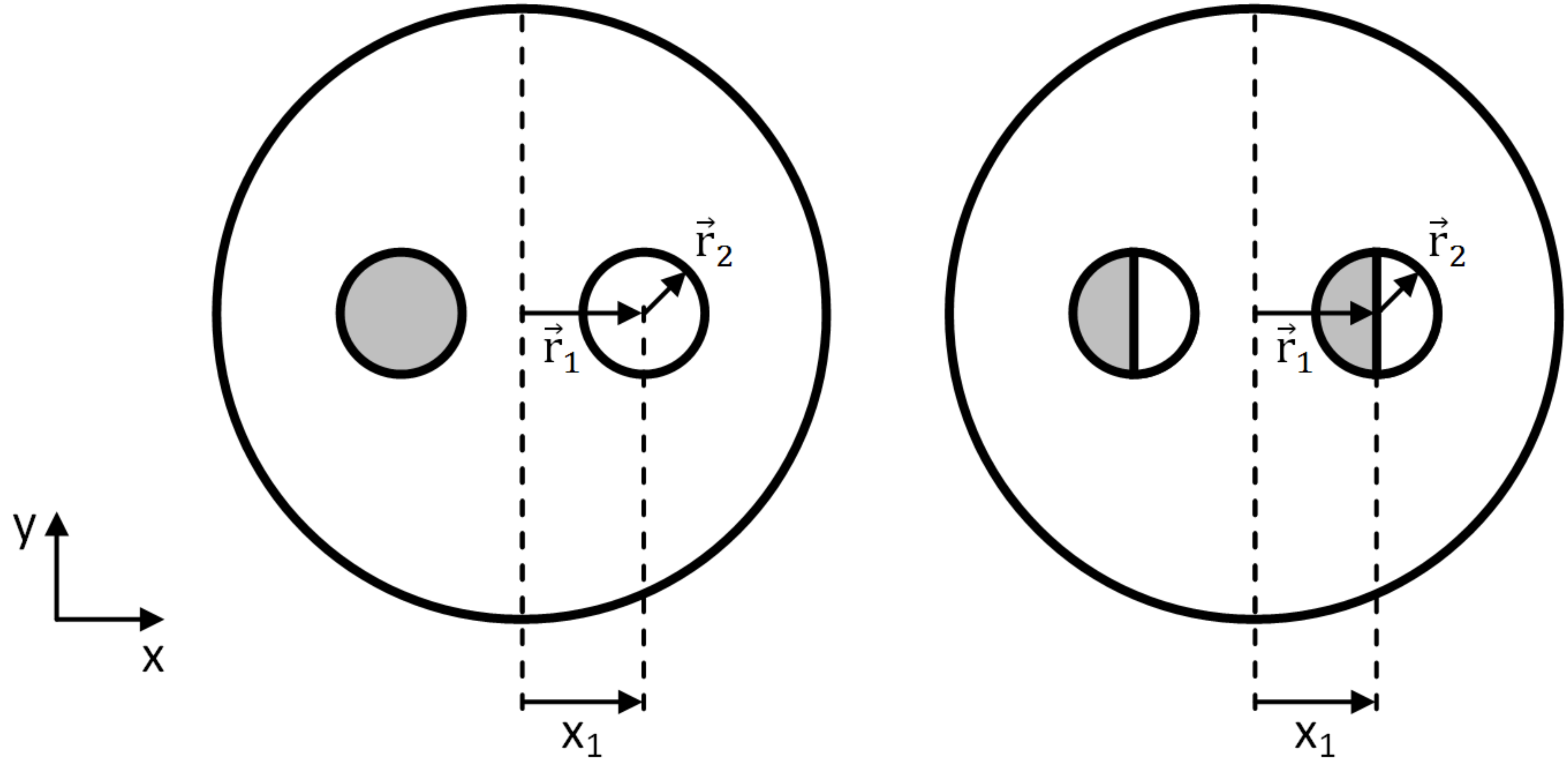
-0.01
-0.02
-0.03
-1
-0.5
0
0.5
1


y
x
$\vec{r}_1$
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x_1
$\vec{r}_1$
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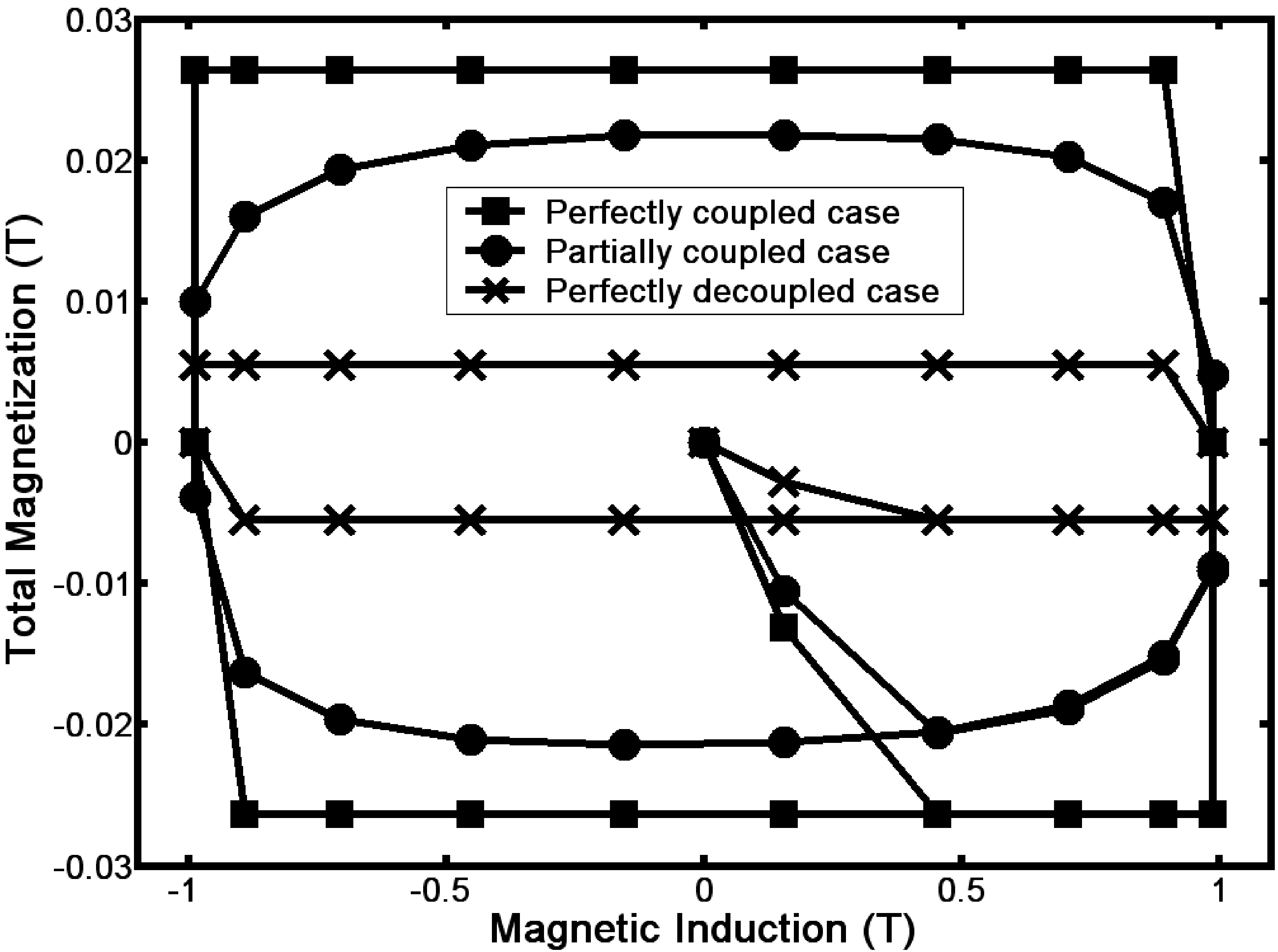
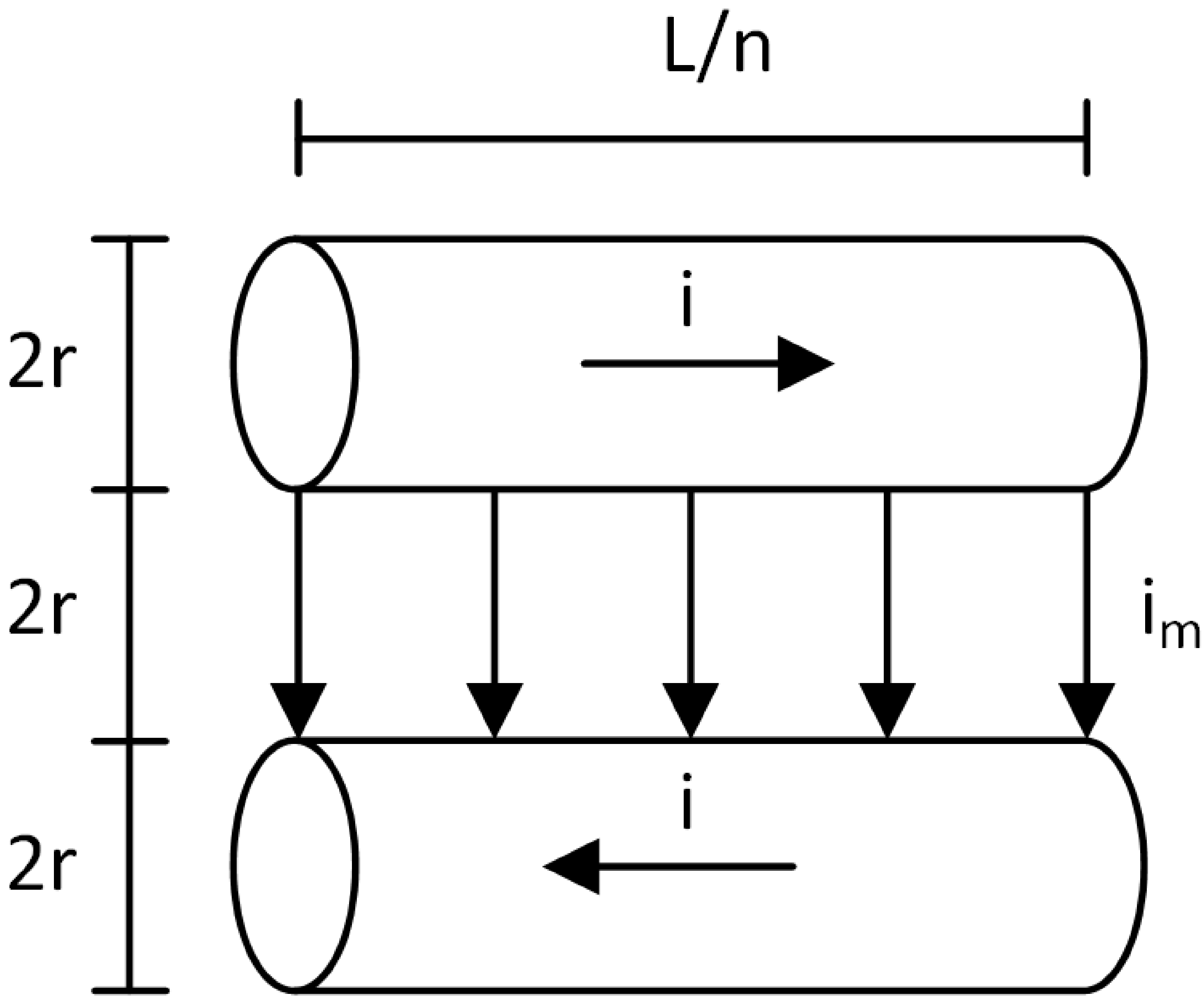
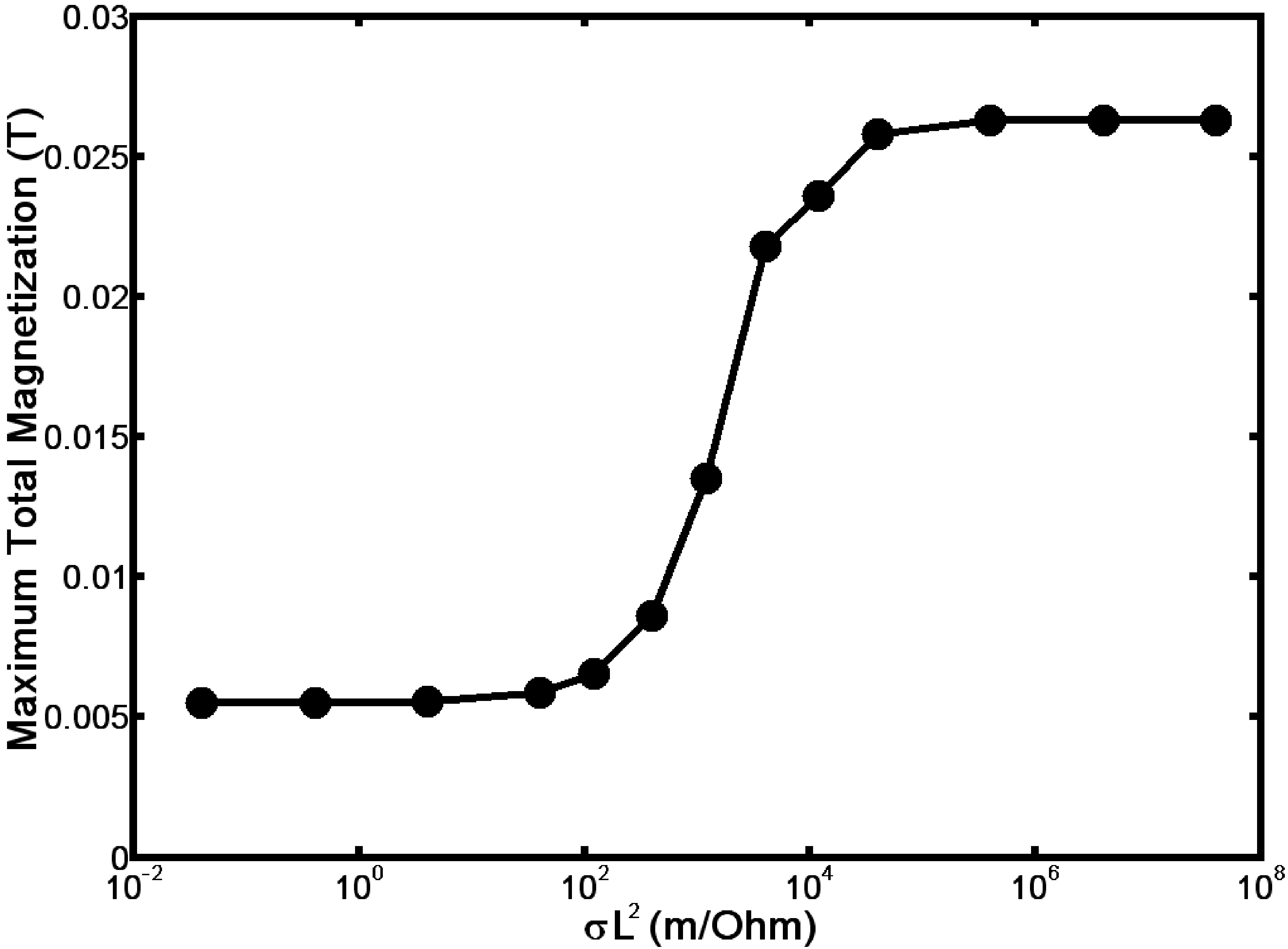
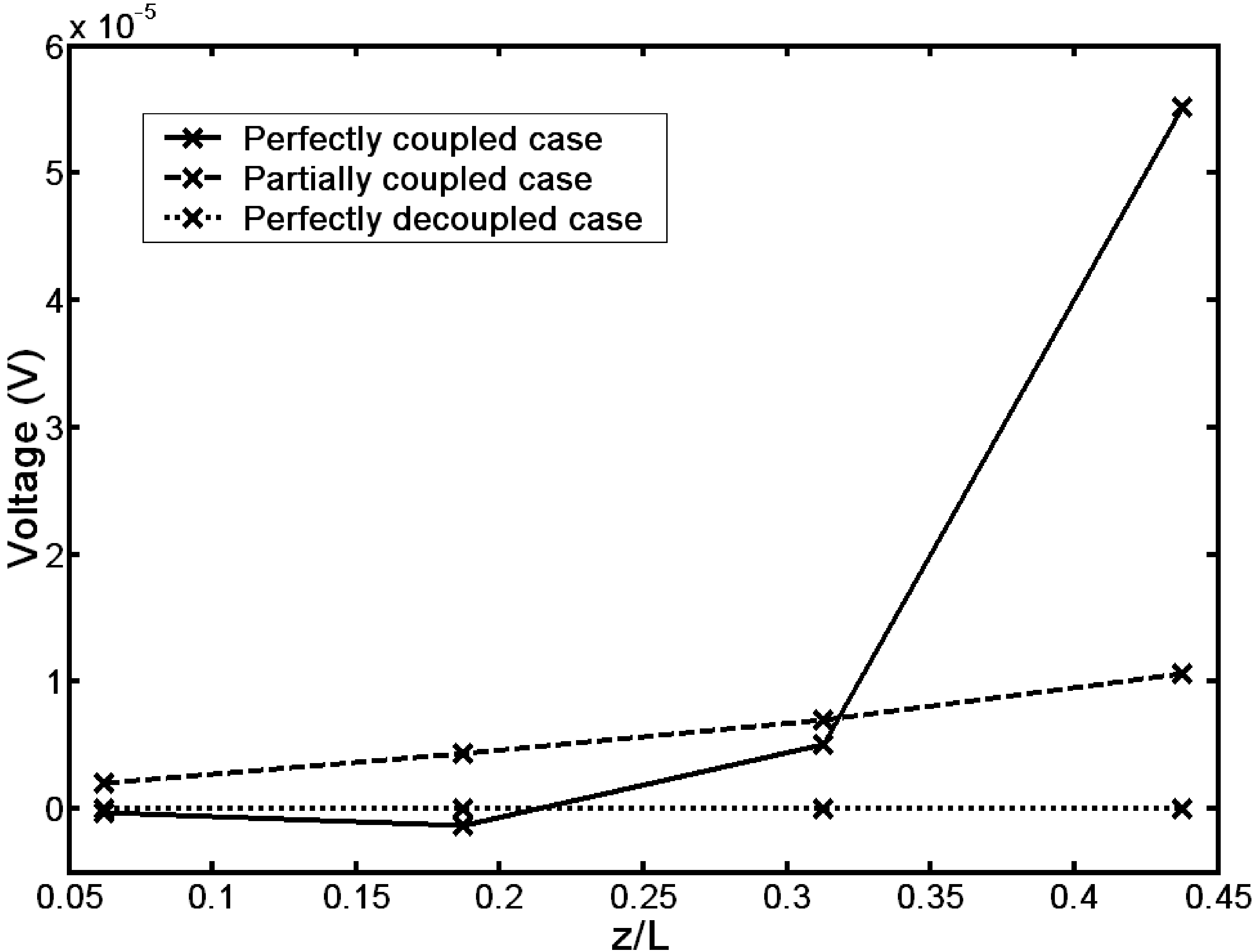
x_1


Perfectly coupled case
Partially coupled case
Perfectly decoupled case
Total Magnetization (T)
Magnetic Induction (T)
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-1
-0.5
0
0.5
1


L/n
2r
2r
2r
i
i
i_m



Maximum Total Magnetization (T)
0.03
0.025
0.02
0.015
0.01
0.005
0
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
10^{8}
σL^2 (m/Ohm)


x 10^-5
Voltage (V)
z/L
Perfectly coupled case
Partially coupled case
Perfectly decoupled case
6
5
4
3
2
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

Contact Information

Dr. Thitipong Satiramatekul **(Corresponding Author)**
Address: Department of Computer Engineering, Faculty of Engineering, Kasetsart University, Kamphaengsaen Campus, Kamphaengsaen, Nakhon Pathom, 73140, Thailand
Tel: +66-(0)34-352-853
Fax: +66-(0)34-351-842
E-mail: thitipong.s@ku.ac.th

Prof. Frédéric Bouillault
Address: Laboratoire de Génie Electrique de Paris, Plateau du Moulon, Supélec, Gif sur Yvette Cedex, 91192, France
Tel: +33-(0)1-6985-1631
Fax: +33-(0)1-6941-8318
E-mail: bouillault@lgep.supelec.fr

For Review Only

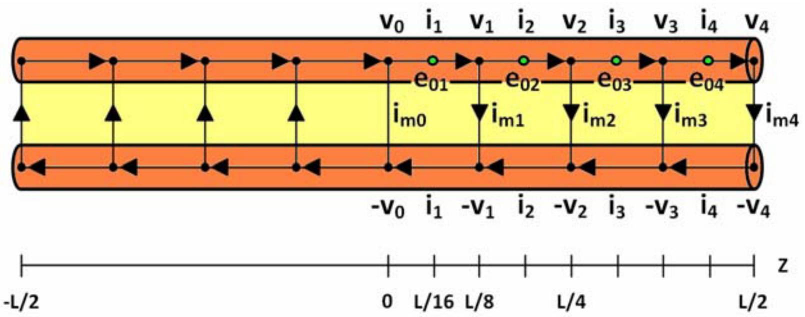
Numerical Modelling of Superconducting Filaments for Coupled Problem

Thitipong SATIRAMATEKUL [1] and Frédéric BOUILLAULT [2]
[1] Faculty of Engineering, Kasetsart University, Kamphaengsaen, 73140, Nakhon Pathom, Thailand
[2] Laboratoire de Génie Electrique de Paris, Supélec, Plateau de Moulon, 91192, Gif/Yvette Cedex, France
e-mail: thitipong.s@ku.ac.th

INTRODUCTION

A better understanding of the electromagnetic coupling phenomena in the superconducting filaments can be done by numerical simulation. For that, a finite element program has been developed at the LGEP for modelling the superconducting materials. In order to approach the three-dimensional problem with better behaviour, we propose in this work a new technique by dividing the superconducting filaments into several sections and introducing the relation between the currents crossing the copper matrix and the electric fields in the filaments.

PROBLEM ANALYSIS



$$I = [C]E_0$$

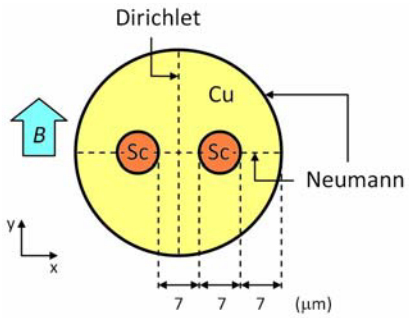
$$I = [i_1 \; i_2 \; \cdots \; i_{(n/2)}]^t$$

$$E_0 = [e_{01} \; e_{02} \; \cdots \; e_{0(n/2)}]^t$$

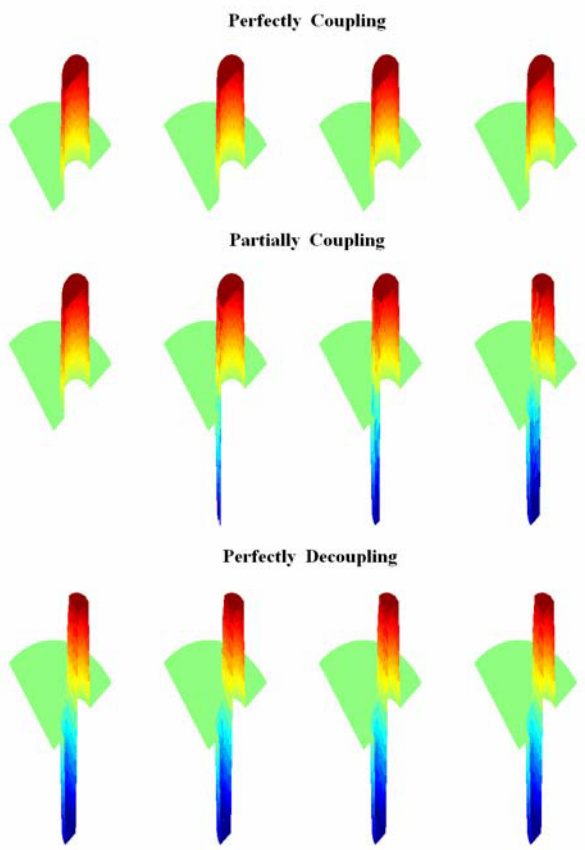
$$[C] = \begin{bmatrix} 8c & 6c & 4c & 2c \\ 6c & 6c & 4c & 2c \\ 4c & 4c & 4c & 2c \\ c & c & c & c \end{bmatrix}$$

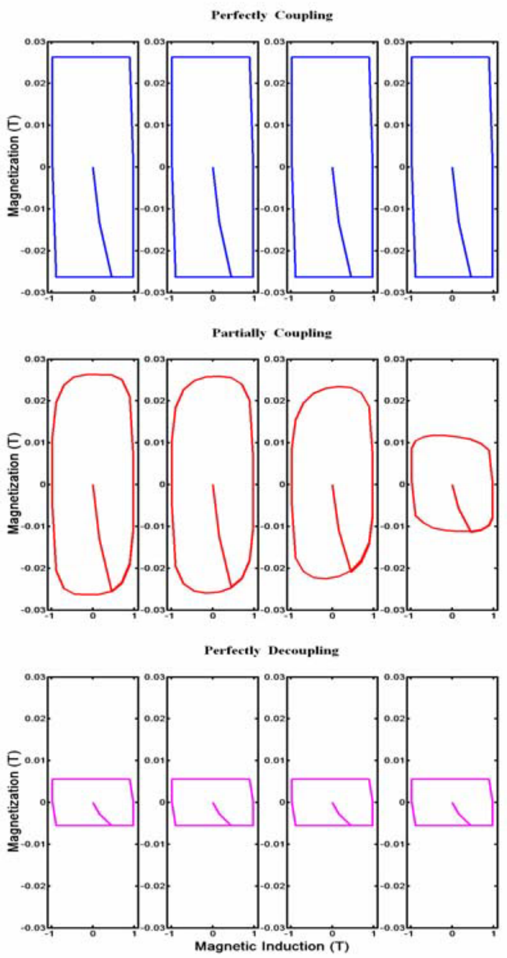
$$c = -L/8R$$

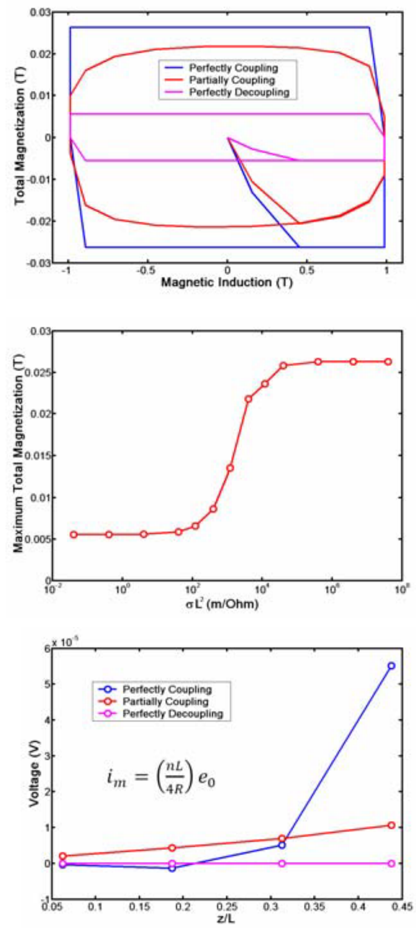
NUMERICAL MODELLING



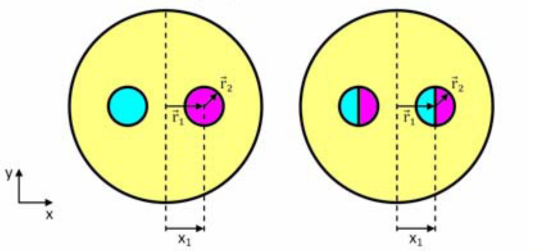
SIMULATION RESULTS



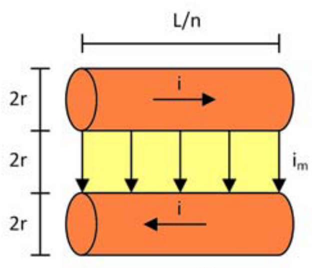




$$\vec{M} = \int_L \int_S (\vec{r}_1 + \vec{r}_2) \times \vec{j} ds dz$$



$M_{total} = \mu_0 x_1 J_c$ $\qquad$ $M_{total} = 4\mu_0 J_c r / 3\pi$



$$R = \frac{n}{2\sigma L}$$

CONCLUSION

In order to approach to the 3D coupled problem, a novel technique is proposed for solving a coupled problem of superconducting filaments in 2D. The numerical simulation results are obtained by using the finite element code and Bean's model. These results show clearly the electromagnetic coupling phenomena in the superconducting filaments.