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ในแบบจำลองคوار์ก

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ຜູ້ວິຈัย ນາຍເຂັ້ມ ພຸ່ມສະອາດ

ສັງກັດການວິຊາພິສິກສົ່ງ ຄະນະວິທາຍາຄາສົດ ມາວິທາຍາລັຍຄຣີນຄຣິນທຣວິໂຮຣມ

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## บทคัดย่อ

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งานวิจัยนี้ทำการศึกษาถึงฟอร์มแฟกเตอร์เชิงแม่เหล็กไฟฟ้าของไฮเพอรอนโดยใช้แบบจำลองไครัลค่าวาร์ก ในแบบจำลองนี้แบริอันถูกพิจารณาให้เป็นสถานะที่มีข้อบเขตซึ่งประกอบด้วยค่าวาร์กที่ถูกปรับแต่งด้วยกลุ่มหมอกของซูโดสเกลาร์เมฆอน ในขั้นแรก ลักษณะจีบินนี้สามารถนำไปใช้ในการปรับแต่งค่าวาร์กโดยกลุ่มหมอกของซูโดสเกลาร์เมฆอนและสถานะหนักอื่นๆ โดยใช้เทคนิคการคำนวณแบบอินฟราเรด ไดเมนชันนัล เรกวัลารีเซ็นสำหรับแผนภาพที่เป็นถูก จากนั้นตัวดำเนินการการแปลงซึ่งได้รับการปรับแต่งจะถูกนำมาใช้ในการคำนวณ samaชิกเมทริกซ์ของแบริอัน วิธีการพารามิเตอร์ไรเซ็นสำหรับฟอร์มแฟกเตอร์ของแบริอันได้ถูกนำมาใช้โดยจีบินให้อยู่ในรูปของฟอร์มแฟกเตอร์ของค่าวาร์กในทฤษฎีกลุ่ม SU(6) พารามิเตอร์ต่างๆ ที่ได้จากการพิจารณาในกรณีของสมบัติเชิงแม่เหล็กไฟฟ้าของนิวเคลียนได้ถูกนำมาใช้เพื่อเป็นค่าเริ่มต้นสำหรับการคำนวณฟอร์มแฟกเตอร์ของไฮเพอรอนและโมเมนต์แม่เหล็กของนิวเคลียนและไฮเพอรอน

**Keywords:** สมมาตรไครัล, ลักษณะจีบินยังผล, แบบจำลองค่าวาร์กเชิงสัมพัทธภาพ, ฟอร์มแฟกเตอร์และโมเมนต์แม่เหล็กของนิวเคลียนและไฮเพอรอน

## Abstract

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**Project Code :** MRG5180066

**Project Title :** Baryon Properties and Meson Cloud Effects in the Quark Model

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We study electromagenetic form factors of hyperons using a chiral quark model. In this model baryons are bound states of constituent quarks dressed by a cloud of pseudoscalar mesons. In a first step, this Lagrangian can be used to perform a dressing of the constituent quarks by a cloud of light pseudoscalar mesons and other heavy states using the calculational technique of infrared dimensional regularization of loop diagrams. Then the dressed transition operators are used to calculate the baryon matrix elements. We use the parameterization of baryon form factors in terms of quark form factors in the SU(6). The parameters fitted from the nucleon electromagnetic properties are used as an input for the calculation of hyperons electromagenetic form factors and their magnetic moments.

**Keywords:** chiral symmetry, effective Lagrangian, relativistic quark model, nucleon and hyperons form factors and magnetic moments

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## ความสำคัญและที่มาของปัญหา

แบริโอน (baryon) คืออนุภาคที่ประกอบขึ้นมาจากการ์ก (quark) จำนวนสามตัว ตัวอย่างของแบริโอนที่เรารู้จักคุ้นเคยกันดีก็คือ โปรตอน (proton) และนิวตรอน (neutron) ซึ่งมีชื่อเรียกโดยรวมว่า นิวเคลียน (nucleon) ในปัจจุบันเรามีข้อมูลเกี่ยวกับสมบัติโดยรวมของแบริโอนชนิดต่างๆ ซึ่งสามารถวัดได้จากการทดลองจากห้องปฏิบัติการต่างๆ ทั่วโลก อย่างไรก็ตามในทางทฤษฎีนั้น เรา ยังขาดซึ่งความเข้าใจในเชิงลึกถึงโครงสร้างของการกระจายตัวของควร์กภายในแบริโอน ความรู้ ความเข้าใจเกี่ยวกับโครงสร้างของแบริโอนจึงมีความสำคัญเป็นอย่างยิ่งเนื่องจากในการนำความรู้ ไปประยุกต์ใช้นั้น เราจำเป็นต้องเข้าใจถึงรายละเอียดของโครงสร้างต่างๆ อย่างลึกซึ้ง

ในทางทฤษฎีนั้นระบบของควร์กจะสามารถอธิบายได้ด้วยทฤษฎีควอนตัมคอร์โนมไดนามิกส์ (Quantum Chromodynamics หรือ QCD) อย่างไรก็ตาม QCD สามารถใช้ได้เฉพาะในบริเวณที่ พลังงานของระบบมีค่าสูงมากๆ สำหรับระดับพลังงานในช่วงของนิวเคลียนนั้น เราไม่สามารถใช้ QCD ได้เนื่องจากปัญหาทางเทคนิคในการคำนวณที่เราไม่สามารถนำเอาวิธีการเพอร์เทอร์เบชัน (perturbation) ซึ่งใช้ได้ในกลศาสตร์คุณตั้มมาใช้ในการคำนวณได้ อย่างไรก็ตาม ยังมีวิธีอื่นๆ ใน การที่จะศึกษาถึงสมบัติและโครงสร้างของแบริโอน เช่น Lattice QCD, Effective Field Theory (EFT) หรือแบบจำลองควร์ก (Quark Model)

ในส่วนของแบบจำลองควร์กนั้น ได้มีการนำเสนอแบบจำลองต่างๆ อย่างมากมาย แต่ละ แบบจำลองก็มีข้อดีและข้อเสียที่แตกต่างกันไป แต่ผลโดยรวมที่สามารถสรุปได้ก็คือ ใน การอธิบายถึง สมบัติและโครงสร้างของแบริโอนนั้น นอกจากส่วนที่มาจากการ์กแล้วยังมีส่วนสำคัญอีกส่วนหนึ่ง ที่มาจากการกลุ่มหมอกของควร์กและแอนติควร์ก (antiquark) ซึ่งเราสามารถพิจารณาให้เป็นกลุ่ม หมอกเมzon (Meson Cloud) เนื่องจาก

เมzonนั้นเป็นอนุภาคที่ประกอบขึ้นมาจากการ์กและแอนติควร์กนั้นเอง

ในการทำการวิจัยครั้งนี้จะศึกษาถึงบทบาทของกลุ่มหมอกเมzonที่มีต่อแบบจำลองควร์ก ซึ่งในปัจจุบันนี้ยังไม่มีคำอธิบายที่ชัดเจนว่ากลุ่มหมอกเมzonนี้ส่วนสำคัญต่อสมบัติต่างๆ ของแบริ โอนมากน้อยแค่ไหน ความเข้าใจเกี่ยวกับควร์กและกลุ่มหมอกเมzonนี้จะนำไปสู่แบบจำลองควร์ก ที่สมบูรณ์ยิ่งขึ้น และสามารถนำไปประยุกต์ใช้ในการคำนวณสมบัติอื่นๆ ที่แม่นยำได้ต่อไป

## วัตถุประสงค์

เพื่อศึกษาและวิจัยถึงผลของกลุ่มหมอกเมzonที่มีต่อสมบัติต่างๆ ของแบริโอนชนิดต่างๆ เช่น นิวเคลียน ไฮเพอรอน และเดลตา(1232) เป็นต้น โดยใช้แบบจำลองควร์กเป็นพื้นฐานในการศึกษา

# 1. Introduction

The basic building blocks of the atomic nuclei, proton and neutron, play an crucial role in subatomic physics. The fully understanding of their properties and structure will probably lead us to a deeper understanding of the mechanism of the strong interaction in nature. Since the masses of the proton ( $M_p = 938.27$  MeV) and the neutron ( $M_n = 939.57$  MeV) are nearly identical, one considers both of them as two different states of the same particle, the nucleon. Experiments point out that the nucleon is not a point-like particle but contains a subtle structure. One of the very first evidence came from the measurement of the magnetic moment of the proton. A deviation of the proton magnetic moment from the value of the point-like particle was observed, hence the introduction of an anomalous magnetic moment. Other evidence for the structure of the nucleon arises from the rich nucleon excitation spectrum. An important tool to study the electromagnetic structure of nucleon is an elastic electron scattering. Deep inelastic scatterings of electrons on the nucleon lead to the evidence for point-like scattering centers in the nucleon and the existence of quark and gluon degrees of freedom.

The knowledge of the electromagnetic structure of the nucleon which tells us how the charge and the current are distributed within the nucleon is very important. The subject is actively studied both on theoretical and experimental sides. As a result of a new technology, modern experiments [1, 2, 3], utilizing polarized beams and targets significantly improved the previous data based on the Rosenbluth separation technique. Recently, the improved measurement on low- $Q^2$  electromagnetic form factors data of nucleon, which is one of the ongoing programs for the complete measurement of the electromagnetic form factors of the nucleon, has reported the improved data [4]. This will lead to more precise data, which is important for the theoretical study.

Quarks were proposed by Gell-Mann [5] and, independently, by Zweig [6] as an elementary particle within the strong interaction particle, hadrons. By assigning  $u$ ,  $d$  and  $s$  quarks and their antiparticles  $\bar{u}$ ,  $\bar{d}$  and  $\bar{s}$  as the fundamental representations 3 and  $\bar{3}$  of SU(3), respectively, hadrons can be constructed from these representations. Therefore, hadrons are believed to be composed of quarks and antiquarks. Baryons are composed of three valence quarks and mesons are composed

of a quark-antiquark pair. These quark-antiquark combinations are constructed such that the correct quantum numbers associated with the corresponding hadrons are achieved. For example, the quark flavor contents of the proton and the neutron are  $uud$  and  $udd$ , respectively.

Experiments reveal the existence of heavier quarks, i.e. the  $c$ ,  $b$  and  $t$  quarks. Therefore, there are six quark flavors along with their antiquarks. However, the free quark state, with the fractional electric charge, was never observed in nature. From this fact it is deduced that there exists a mechanism, named confinement, preventing that free quarks exist. This point is directly lead to a new degree of freedom called “color”, originally introduced to restore the Pauli exclusion principle in the  $\Delta^{++}$  system with the quark content  $uuu$ . For each quark flavor there are three color degrees of freedom, namely, “red”, “blue” and “green”. The non-observation of free quarks is therefore consistent with the proposal that hadrons contain no net color i.e they are color singlets.

Quantum Chromodynamics (QCD) is believed to be the correct theory for describing the physics of the strong interaction. The basic particles in QCD are quarks and their interactions are mediated by exchange of gluons which are the gauge quanta of the color fields. Two important properties of QCD are the asymptotic freedom and the color confinement. The asymptotic freedom is related to the experimental result that in the high energy regime or at small distances the interaction between the quarks is small. In this regime the coupling constant between quarks and gluons is therefore small and a perturbative method can be applied to evaluate QCD. However, in the low energy regime where the strong running coupling constant is large, at the order of one, the perturbative method cannot be applied and one has to deal with a non-perturbative approach.

In QCD, at the scale of 1 GeV, the masses of the light quarks ( $u$ ,  $d$  and  $s$  quarks) are much smaller than the nucleon mass. When we neglect the small quark masses and consider light quarks as massless particles, another important global symmetry in the low energy regime arises in the strong interaction, the so-called “chiral symmetry”. This symmetry is not perfect and spontaneously broken which results in the existence of massless particles called the “Goldstone bosons”. Pions, which considered as Goldstone bosons, however, are massive. Therefore, the finite

value for the pion mass is due to the explicit breaking of chiral symmetry when the quarks has small physical mass values.

Perturbative technique works well for high-energy regime in QCD. However, study of nucleon structure cannot be solved analytically from perturbative QCD. Alternative approaches in order to study the nucleon structure were proposed, for example, QCD sum rule, lattice QCD,  $1/N_c$  expansion, etc. However, one of the most method for the treatment of light hadrons at small energies is “Chiral Perturbation Theory” (ChPT) [7, 8, 9], which is considered as an effective field theory of the strong interaction. For mesons, ChPT works very well, especially in the description of pion-pion interactions, but this is not the case for nucleon. Many versions of ChPT are introduced, for example, Heavy Baryon Chiral Perturbation Theory (HBChPT) [10] and even a manifestly Lorentz invariant version of ChPT [11, 12, 13, 14].

Note, that ChPT is formulated on the hadronic level, therefore the important features of low-energy QCD, such as confinement and hadronization are not considered in ChPT. Alternatively techniques formulated on the quark level and take into account chiral symmetry argument are exist. One on these is the Chiral quark models which describe the elementary baryon, the nucleon, as a bound state of valence quarks supplied by the sea-quark excitation in form of the pions. Depending on the philosophy for each quark model, both perterbative and non-perterbative techniques are employed.

In this report, the so-called Perturbative Chiral Quark Model (PCQM) is used as a tool in order to get the information for the description of low-energy properties of baryons. The latest development of the PCQM is a manifestly Lorentz covariant approach [15, 16]. The main idea is to dress the quark operators by using the chiral Lagrangian taken from baryon ChPT. The dressed quark operators are calculated and the physical observables are obtained from the matrix elements projected on the baryonic level. Constraints of the model can be fixed by using the symmetries of the system and the matching to the original ChPT. According to this matching, the Low Energy Constants (LECs) which are parameters of the model and can be adjusted to fit the various related physical observables. Parameters obtained from the consideration of electromagnetic form factors will be used to further analysis of the electromagnetic form factors of hyperons.

We proceed as follows. First, in Section 2, we discuss the basic theory of strong interaction, QCD and Chiral symmetry when light quark masses are vanish. In

addition, the basic idea of ChPT are presented. The chiral Lagrangian motivated by baryon ChPT [17]-[24] and their formulation in terms of quark and mesonic degrees of freedom are shown in Section 3, together with the discussion of the electromagnetic form factors of nucleon and the extension to the case of hyperons. Next, the results for electromagnetic properties of hyperons are present in Section 4, together with a conclusion.

## 2. QCD, Chiral Symmetry and Chiral Perturbation Theory

The elementary theory of the strong interaction is Quantum Chromodynamics (QCD). QCD is a local gauge theory which describes the interaction of quarks and gluons. The quarks and gluons possess color which is the basic quantum number associated with QCD. Another important symmetry of the QCD Lagrangian is chiral symmetry. This symmetry is only on the level of approximation and is fulfilled only in the limit of massless quarks. Approximate chiral symmetry is widely manifest in low-energy hadron phenomenology and is therefore an important constraint in the derivation of phenomenological approaches motivated by QCD. We briefly discuss the basic notions of the QCD Lagrangian and the aspects of chiral symmetry, including its explicit and spontaneous breaking. Finally, the effective field theory for the strong interaction at low energies— Chiral Perturbation Theory (ChPT)— will be briefly reviewed.

### 2.1 The QCD Lagrangian

The quark fields  $q_f^c(x)$  which are Dirac particles are the matter fields in QCD. They have two specific quantum numbers, color ( $c$ ) and flavor ( $f$ ). Their free Lagrangian is written as

$$\mathcal{L} = \bar{q}_f^c(x)(i\cancel{\partial} - m_f)q_f^c(x). \quad (2.1)$$

The slash notation is defined as :  $\cancel{\partial} \equiv \partial_\mu \gamma^\mu$ . For each quark flavor  $f = u, d, s, c, b$  and  $t$ , it contains three additional quantum numbers, the color charge,  $c = r, g, b$ .

The color charges of the quarks form a fundamental representation related to the generators of  $SU(3)_c$  i.e. the Gell-Mann matrices  $\lambda_a^c$ . The explicit forms of  $\lambda_a^c$  are

$$\begin{aligned}
\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, & \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
\lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, & \lambda_8 &= \sqrt{\frac{1}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},
\end{aligned}$$

where the color indices ( $c$ ) are suppressed. The free Lagrangian of Eq. (2.1) is invariant under the “global” transformation of the color degrees of freedom,

$$q_f^c(x) \mapsto U[\theta] q_f^c(x), \quad (2.2)$$

where

$$U[\theta] = \exp\left(-i \sum_{a=1}^8 \frac{\lambda_a}{2} \theta_a\right) \equiv \exp\left(-i \frac{\lambda_a}{2} \theta_a\right), \quad (2.3)$$

and  $\theta = (\theta_1, \dots, \theta_8)$  are arbitrary constants. Note, that we employ the Einstein summation convention here i.e. summation over the same indices is implied.

The interaction of the matter fields with the gauge fields which are the mediators of the interaction can be generated from the requirement of the local gauge invariance principle applied to the free Lagrangian of the matter fields. Interactions in QCD can be constructed by extension of the global transformations to the “local” transformations according to

$$q_f^c(x) \mapsto U[\theta(x)] q_f^c(x), \quad (2.4)$$

where  $\theta(x)$  is now space-time dependent. In order to maintain the invariance of the Lagrangian of Eq. (2.1) under this local gauge transformation one has to introduce the gauge fields which interact with the quark fields. The usual way is to replace the normal space-time derivative,  $\partial_\mu q_f^c(x)$ , of the free quark Lagrangian by the so-called “covariant derivative”,  $D_\mu q_f^c(x)$

$$\partial_\mu q_f^c(x) \mapsto D_\mu q_f^c(x). \quad (2.5)$$

This covariant derivative is constructed such that it has the same transformation property as the quark fields, i.e.

$$D_\mu q_f^c(x) \mapsto U[\theta] D_\mu q_f^c(x) = \exp\left(-i \frac{\lambda_a}{2} \theta_a(x)\right) D_\mu q_f^c(x). \quad (2.6)$$

As in QED, Eq. (2.6) can be fulfilled by the introduction of the gauge fields  $\mathcal{A}_\mu^a(x)$  in the covariant derivative

$$D_\mu q_f^c(x) = (\partial_\mu - i g \mathcal{A}_\mu(x)) q_f^c(x) \quad (2.7)$$

where  $\mathcal{A}_\mu(x) = \frac{\lambda_a \mathcal{A}_\mu^a(x)}{2}$  and  $g$  is a coupling constant related the strong interaction.

These  $\mathcal{A}_\mu^a(x)$  are the gluon fields which considered as the gauge fields of the strong interaction. Under the gauge group  $SU(3)_c$ , the gluon fields transformation is

$$\mathcal{A}_\mu(x) \mapsto U[\theta(x)] \mathcal{A}_\mu(x) U^\dagger[\theta(x)] - \frac{i}{g} \partial_\mu U[\theta(x)] U^\dagger[\theta(x)] \quad (2.8)$$

One can defined the field strength tensor  $\mathcal{G}_{\mu\nu,a}(x)$  in QCD with the explicit form

$$\mathcal{G}_{\mu\nu,a}(x) = \partial_\mu \mathcal{A}_{\nu,a}(x) - \partial_\nu \mathcal{A}_{\mu,a}(x) + g f_{abc} \mathcal{A}_{\mu,b}(x) \mathcal{A}_{\nu,c}(x) \quad (2.9)$$

where  $f_{abc}$  are the structure constants of  $SU(3)$

$$\begin{aligned} f_{123} &= 1, \\ f_{147} &= -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2}, \\ f_{458} &= f_{678} = \frac{\sqrt{3}}{2}. \end{aligned}$$

The last term in Eq. (2.9) originates from the non-Abelian properties of  $SU(3)$ . The transformation of the field strength tensor is simpler if one defines the tensor  $\mathcal{G}_{\mu\nu}(x)$  such that

$$\mathcal{G}_{\mu\nu}(x) \equiv \frac{\lambda_a}{2} \mathcal{G}_{\mu\nu,a}(x) \mapsto U[\theta(x)] \mathcal{G}_{\mu\nu}(x) U^\dagger[\theta(x)]. \quad (2.10)$$

In terms of  $\mathcal{G}_{\mu\nu}(x)$ , the free gluonic Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{2} \text{Tr}[\mathcal{G}_{\mu\nu}(x) \mathcal{G}^{\mu\nu}(x)]. \quad (2.11)$$

The full QCD Lagrangian is therefore

$$\mathcal{L}_{QCD} = \sum_{f=u,d,s,c,b,t} \bar{q}_f^c(x) (i \not{D} - m_f) q_f^c(x) - \frac{1}{2} \text{Tr}[\mathcal{G}_{\mu\nu}(x) \mathcal{G}^{\mu\nu}(x)]. \quad (2.12)$$

As a consequence of the non-Abelian nature of the group  $SU(3)$ , gluon fields can interact with themselves in addition to the coupling between the quarks and gluons. There exist the three- and four-gluonic self-coupling terms which are proportional to  $g$  and  $g^2$ , respectively. This is not the case for the electromagnetic fields in QED, but in QCD the gluon fields are “charged” i.e. they carry “color”, whereas the photon carries no (electric) charge.

## 2.2 Chiral Symmetry

In the limit where the light quark masses vanish, the QCD Lagrangian of Eq. (2.12) has another important symmetry. This is the so-called “chiral symmetry”. This symmetry is only approximate since in reality quarks possess a small but finite mass. The sector of light quarks is composed of the  $u$ ,  $d$  and  $s$  quarks with the estimated masses [25]

$$m_u = 1.5 - 4 \text{ MeV}, \quad m_d = 4 - 8 \text{ MeV} \quad \text{and} \quad m_s = 80 - 130 \text{ MeV}.$$

The  $c$ ,  $b$  and  $t$  quarks are considered as heavy quarks with masses  $\geq 1$  GeV. In the low-energy regime the heavy quarks do not play a role due to their large masses. Since the  $u$ ,  $d$  and  $s$  quarks are much lighter than the hadronic mass scale of 1 GeV this suggests that one can treat the current quark masses as a small perturbation. Therefore, for the low-energy regime and in the chiral limit, where  $m_u, m_d, m_s \rightarrow 0$ , the appropriate QCD Lagrangian reduced from Eq. (2.12) to becomes

$$\mathcal{L}_{QCD}^0 = \sum_{f=u,d,s} \bar{q}_f^c(x) i \not{D} q_f^c(x) - \frac{1}{2} \text{Tr} [\mathcal{G}_{\mu\nu}(x) \mathcal{G}^{\mu\nu}(x)]. \quad (2.13)$$

The symmetry of the Lagrangian (2.13) can be made explicit if one decomposes the quark fields in terms of left- and right-handed components. This can be achieved through the projection operators  $P_R$  and  $P_L$  defined by

$$P_R = \frac{1}{2}(1 + \gamma_5), \quad P_L = \frac{1}{2}(1 - \gamma_5), \quad (2.14)$$

where  $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$  is the usual gamma matrix in the Dirac theory. With these operators the right- and left-handed components of the quark fields can be written as

$$q_{R,f}^c(x) = P_R q_f^c(x), \quad q_{L,f}^c(x) = P_L q_f^c(x). \quad (2.15)$$

Consequently, Eq. (2.13) can be rewritten as

$$\mathcal{L}_{QCD}^0 = \bar{q}_R(x) i \not{D} q_R(x) + \bar{q}_L(x) i \not{D} q_L(x) - \frac{1}{2} \text{Tr} [\mathcal{G}_{\mu\nu}(x) \mathcal{G}^{\mu\nu}(x)], \quad (2.16)$$

where we represent the right- and left-handed quark fields in terms of the column vectors

$$q_R(x) = \begin{pmatrix} q_{R,u}(x) \\ q_{R,d}(x) \\ q_{R,s}(x) \end{pmatrix}, \quad q_L(x) = \begin{pmatrix} q_{L,u}(x) \\ q_{L,d}(x) \\ q_{L,s}(x) \end{pmatrix}, \quad (2.17)$$

and we simplify the notation by dropping the color index. We consider the “global” unitary transformation of the quark fields of Eq. (2.17) with

$$q_L \mapsto U_L q_L, \quad q_R \mapsto U_R q_R, \quad (2.18)$$

and

$$U_L = \exp\left(-i \frac{\lambda_a}{2} \theta_a^L\right), \quad U_R = \exp\left(-i \frac{\lambda_a}{2} \theta_a^R\right), \quad (2.19)$$

where  $\theta_a^{L(R)}$  are independent, real parameters. The group of this transformation is denoted by  $SU(3)_R \times SU(3)_L$ . Obviously, Eq. (2.16) is invariant under such transformations and hence is referred to as the “chiral symmetry” of QCD. Since  $U_L$  and  $U_R$  contain altogether 16 real parameters, the symmetry, due to Noether’s theorem, results in 16 conserved currents associated with the transformation of Eq. (2.18). These conserved currents are

$$R_a^\mu = \bar{q}_R \gamma^\mu \frac{\lambda_a}{2} q_R, \quad L_a^\mu = \bar{q}_L \gamma^\mu \frac{\lambda_a}{2} q_L, \quad (2.20)$$

with

$$\partial_\mu R_a^\mu = 0, \quad \partial_\mu L_a^\mu = 0. \quad (2.21)$$

Instead of working with the left- and right-handed currents, one conventionally considers the linear combinations

$$V_a^\mu = R_a^\mu + L_a^\mu = \bar{q} \gamma^\mu \frac{\lambda_a}{2} q, \quad (2.22)$$

and

$$A_a^\mu = R_a^\mu - L_a^\mu = \bar{q} \gamma^\mu \gamma_5 \frac{\lambda_a}{2} q, \quad (2.23)$$

together with

$$\partial_\mu V_a^\mu = 0, \quad \partial_\mu A_a^\mu = 0 \quad (2.24)$$

These are the vector and axial currents. Note, that a simple phase transformations of  $q_L$  and  $q_R$  also results in an invariance of  $\mathcal{L}_{QCD}^0$ . The corresponding group of transformations are referred to as  $U(1)_V$  and  $U(1)_A$ , if  $q_L$  and  $q_R$  transform with the same and the opposite phases, respectively. Consequently, there exist two additional conserved currents

$$V^\mu = \bar{q} \gamma^\mu q, \quad A^\mu = \bar{q} \gamma^\mu \gamma_5 q, \quad (2.25)$$

with  $\partial_\mu V^\mu = \partial_\mu A^\mu = 0$ .

Transition from classical fields consideration to the case of quantum fields, there arises extra terms referred to as “anomalies”, which presented in the axial currents as

$$\partial_\mu A^\mu = \frac{3g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} G_a^{\mu\nu} G_a^{\rho\sigma} \quad (2.26)$$

where  $\epsilon_{\mu\nu\rho\sigma}$  is the totally antisymmetric tensor with  $\epsilon_{0123} = 1$ . Therefore,  $A^\mu$  is no longer conserved. Furthermore, nonvanishing current quark masses will contribute to Eq. (2.26) as well. Moving from the level of classical to quantum fields the global  $U(3)_R \times U(3)_L$  symmetry of  $\mathcal{L}_{QCD}^0$  is reduced to a global  $SU(3)_R \times SU(3)_L \times U(1)_V$  symmetry.

Finally, after quantization, we note that corresponding to the conserved currents  $V_a^\mu$ ,  $A_a^\mu$  and  $V^\mu$  we have the conserved charge operators  $Q_V^a$ ,  $Q_A^a$  and  $Q_V$ . These operators form the algebra

$$\begin{aligned} [Q_V^a, Q_V^b] &= i f_{abc} Q_V^c, \\ [Q_V^a, Q_A^b] &= i f_{abc} Q_A^c, \\ [Q_A^a, Q_A^b] &= i f_{abc} Q_V^c, \\ [Q_V^a, Q_V] &= [Q_A^a, Q_V] = 0 \end{aligned} \quad (2.27)$$

The algebra which is constructed from the currents themselves is known as “current algebra”. Before QCD, where the elementary origin of chiral symmetry was not understood yet, current algebra was already applied to the study of low-energy hadronic processes.

## 2.3 Chiral Symmetry Breaking

Previously, we have shown that in the chiral limit the Lagrangian  $\mathcal{L}_{QCD}^0$  has a  $SU(3)_R \times SU(3)_L \times U(1)_V$  symmetry which results in the conserved charge operators  $Q_V^a$ ,  $Q_A^a$  and  $Q_V$ . If  $\mathcal{H}_{QCD}^0$  is the Hamiltonian corresponding to  $\mathcal{L}_{QCD}^0$ , this means that

$$[\mathcal{H}_{QCD}^0, Q_V^a] = [\mathcal{H}_{QCD}^0, Q_A^a] = [\mathcal{H}_{QCD}^0, Q_V] = 0 \quad (2.28)$$

By considering the symmetry of the vacuum state  $|0\rangle$ , above symmetry of the Lagrangian can be realized in two modes. The first realization relies on the assumption that the vacuum has exactly the same symmetry as the Lagrangian. As a consequence the vacuum state is annihilated by the conserved charge operators

$$Q_V^a |0\rangle = Q_A^a |0\rangle = 0 \quad (2.29)$$

This realization in which the Lagrangian and the vacuum share the same symmetry is called the “Wigner-Weyl” mode of chiral symmetry. As a consequence the hadronic spectrum of positive and negative parity states built upon the vacuum is degenerate resulting in parity doublets. However, this is not the case in the observed spectrum of hadrons, e.g. the light pseudoscalar ( $J^\pi = 0^-$ ) mesons have masses much lower than those of the lightest scalar ( $J^\pi = 0^+$ ) mesons.

Another realization of chiral symmetry is achieved when the vacuum state of the system does not share the symmetry of the Lagrangian. This realization is called the “Nambu-Goldstone” mode of chiral symmetry and the symmetry is said to be “hidden” or “spontaneously broken”. Since the approximate validity of  $SU(3)$  flavor symmetry suggests that  $Q_V^a |0\rangle = 0$ , in the Nambu-Goldstone realization we are left with

$$Q_A^a |0\rangle \neq 0 \quad (2.30)$$

As a result of the spontaneously broken symmetry there exist massless particles the so-called “Goldstone bosons”, as evident from Goldstone’s theorem. In nature, chiral symmetry is realized in the Nambu-Goldstone mode, since the observed hadron spectrum contains the rather light pseudoscalar mesons ( $\pi, K, \eta$ ) in comparison to the scale set by the nucleon mass of  $\sim 1$  GeV. Hence the low-lying pseudoscalar mesons are interpreted as Goldstone bosons. The finite but small masses of the  $\pi$ ,  $K$  and  $\eta$  mesons arise from the fact that the quarks have a nonvanishing current mass. Then, “explicit” symmetry breaking due to the quark masses is responsible for the finite masses of the  $\pi$ ,  $K$  and  $\eta$  mesons. Therefore, the  $SU(3)_R \times SU(3)_L \times U(1)_V$  symmetry is spontaneously broken down to the  $SU(3)_V \times U(1)_V$  symmetry.

The spontaneous breaking of chiral symmetry is closely related to the nonvanishing of the order parameter, the “quark condensate”, which is defined as

$$\langle 0 | \bar{q}q | 0 \rangle \equiv \langle \bar{q}q \rangle = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle + \langle \bar{s}s \rangle \quad (2.31)$$

The condition  $Q_V^a |0\rangle = 0$  suggests that  $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle = 0$ , whereas  $Q_A^a |0\rangle \neq 0$  results in

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle \neq 0 \quad (2.32)$$

The spontaneous breaking of chiral symmetry induces a rearrangement of the ground state such that it is populated by scalar quark-antiquark pairs with nonzero expectation values.

Nevertheless, the current quark masses, although, they are small, do not vanish. The finite values of the quark masses give rise to explicit breaking of chiral symmetry due to the presence of a quark mass term in the QCD Lagrangian

$$\mathcal{L}_M = -\bar{q}(x) \mathcal{M} q(x) \quad (2.33)$$

where  $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$  is the quark mass matrix. Including the explicit quark mass terms the divergence of the various currents becomes

$$\begin{aligned} \partial_\mu V_a^\mu &= i \bar{q} \left[ \mathcal{M}, \frac{\lambda_a}{2} \right] q, \\ \partial_\mu A_a^\mu &= i \bar{q} \left\{ \mathcal{M}, \frac{\lambda_a}{2} \right\} \gamma_5 q, \\ \partial_\mu V^\mu &= 0, \\ \partial_\mu A^\mu &= 2i \bar{q} \mathcal{M} \gamma_5 q + \frac{3g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathcal{G}_a^{\mu\nu} \mathcal{G}_a^{\rho\sigma}, \end{aligned}$$

where the anomaly of Eq. (2.26) is taken into account for completeness. Note, that  $V^\mu$  is always conserved, whereas  $V_a^\mu$  is only conserved when all the quark masses are equal. However,  $A_a^\mu$  is not conserved and this is the microscopic origin of the so-called Partially Conserved Axial-vector Current (PCAC).

## 2.4 Chiral Perturbation Theory

Unfortunately, perturbative methods in QCD cannot be applied directly to hadronic systems in the low-energy regime due to the large coupling constant of the strong interaction. However, phenomena in the low-energy region can be studied in terms of Effective Field Theory (EFT) proposed by Weinberg [7] in 1979. The link between QCD and the EFT can be employed through the generating functional. In the presence of external fields the QCD Lagrangian in Eq. (2.13) reads

$$\mathcal{L}_{ext} = \mathcal{L}_{QCD}^0 + \bar{q} \gamma^\mu (v_\mu + \gamma_5 a_\mu) q - \bar{q} (s - i \gamma_5 p) q, \quad (2.34)$$

where,  $v_\mu$ ,  $a_\mu$ ,  $s$  and  $p$  are the external fields concerning vector, axial vector, scalar and pseudoscalar currents, respectively. The generating functional  $Z$  is related to  $\mathcal{L}_{ext}$  and can be considered as the vacuum to vacuum transition amplitude in the presence of external fields i.e.

$$\exp(iZ[v, a, s, p]) = \langle 0 | T \exp \left[ i \int d^4x \mathcal{L}_{ext}(x) \right] | 0 \rangle = \langle 0_{out} | 0_{in} \rangle_{v, a, s, p}. \quad (2.35)$$

In terms of EFT with some asymptotic hadron fields as the relevant degrees of freedom rather than the quark and gluon fields, the low-energy representation of the generating functional  $Z$  can be obtained by the use of an effective Lagrangian  $\mathcal{L}_{eff}$ .

In the path-integral formalism this can be written as

$$\exp(iZ[v, a, s, p]) = N \int [dU] \exp \left( i \int d^4x \mathcal{L}_{eff}(U, v, a, s, p) \right), \quad (2.36)$$

where  $U$  is a matrix containing the asymptotic fields. This leads to the development of Chiral Perturbation Theory (ChPT) [7, 8, 9], which is the EFT of strong interactions at low energies. ChPT was first applied to the study of the system of Goldstone bosons which originate from the spontaneous breaking of chiral symmetry of the QCD Lagrangian. In ChPT, instead of considering the quark and gluon fields as the elementary degrees of freedom of the theory, the active degrees of freedom in ChPT are the asymptotically observed states, the hadrons. In the mesonic sector, the effective Lagrangian is composed of the string of terms as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots, \quad (2.37)$$

where the subscripts refer to the order in the momentum and quark mass expansion. The lowest-order effective Lagrangian  $\mathcal{L}_2$  which contains two derivatives and one quark mass term is

$$\mathcal{L}_2 = \frac{F^2}{4} \text{Tr} \left[ D_\mu U (D^\mu U)^\dagger + \chi U^\dagger + U \chi^\dagger \right] \quad (2.38)$$

The matrix  $U$  contains the Goldstone boson fields. The covariant derivative  $D_\mu$  is composed of the usual derivative and terms concerning the coupling of the Goldstone boson fields to external fields. The current quark mass is hidden in the definition of  $\chi$ , i.e.  $\chi = 2B(s + ip)$ , where  $B$  is related to the quark condensate parameter and  $F$ , in the SU(2) sector, is the pion decay constant in the chiral limit. Higher-order terms in the Lagrangian can be constructed and each term contains coefficients, the so-called Low Energy Constants (LEC). In case one could solve the fundamental theory from first principles, one can map the LECs of the EFT to the fundamental parameters of the underlying theory. However, since QCD cannot be solved analytically in the low-energy region, we consider the LECs as free parameters, which at this point can be extracted from physical observables.

After the most general effective Lagrangian is constructed one also needs a method to classify the order of the diagram built from the effective Lagrangian. Weinberg's power counting scheme offers such a method for labelling the specific order  $D$ , the chiral dimension, of the diagram of interest and it can be obtained from

$$D = 2 + 2N_L + \sum_{k=1}^{\infty} 2(k-1)N_{2k}$$

where  $N_L$  is the number of independent loop momenta and  $N_{2k}$  is the number of vertices originating from the Lagrangian  $\mathcal{L}_{2k}$ . In ChPT loop diagrams also contain a divergent part, which has to be renormalized. However, ChPT is not a renormalizable theory in the traditional sense since the infinities cannot be reabsorbed into parameters of the lowest-order Lagrangian, e.g.  $B$  and  $F$ . A consistent removal of infinities can be done by redefinition of the fields and the LECs. The extension to include the

nucleon in ChPT is also possible and was done in Ref. [26]. In the SU(2) sector, the effective Lagrangian describing the interaction of  $\pi$  and  $N$  can be written as

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \dots \quad (2.39)$$

The lowest order Lagrangian  $\mathcal{L}_{\pi N}^{(1)}$  is of the form

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\psi}_0 \left( i\gamma^\mu \partial_\mu - m_0 - \frac{1}{2} \frac{\overset{\circ}{g}_A}{F_0} \gamma^\mu \gamma_5 \tau^a \partial_\mu \pi_0^a \right) \psi_0 \quad (2.40)$$

where  $\psi_0$  and  $\pi_0$  denote a doublet and a triplet of bare nucleon and pion fields, respectively. The constants  $m$ ,  $\overset{\circ}{g}_A$  and  $F$  denote the nucleon mass, the axial vector coupling and the pion decay constant in the chiral limit, which arise after renormalization. The new scale associated with the mass of the nucleon, which does not vanish in the chiral limit as opposed to the case of the Goldstone bosons, brings a new difficulty when demanding consistency in the power counting of the specific diagram. Namely, loop diagrams involving the nucleon contribute also to lower order diagrams and therefore a consistent perturbative picture collapses. The first attempt to remedy this deficiency was formulated in terms of the Heavy Baryon Chiral Perturbation Theory (HBChPT). The basic idea of HBChPT is the separation of the nucleon momenta into a part which is close to the on-shell kinematics and a soft residual part, i.e.  $p = mv + k_p$  where  $v^2 = 1, v^0 \leq 1$ . The nucleon field is then expressed in terms of

$$\psi(x) = e^{-imv \cdot x} (\mathcal{N}_v + \mathcal{H}_v)$$

where  $\mathcal{N}_v = e^{+imv \cdot x} \frac{1}{2} (1 + \gamma) \psi$  and  $\mathcal{H}_v = e^{+imv \cdot x} \frac{1}{2} (1 - \gamma) \psi$ . As a consequence in HBChPT the power counting as in the mesonic sector is restored. The disadvantages of HBChPT are that higher order terms in the Lagrangian due to the  $\frac{1}{m}$  expansion become increasingly complicated and not all the scattering amplitudes resulting from such a Lagrangian show the correct analytic behavior in the low-energy region.

Recently, the formulation of the manifestly Lorentz-invariant baryon ChPT was developed in Refs. [11, 12, 13, 14]. It is constructed to utilize the advantages of the mesonic ChPT and HBChPT while at the same time avoiding their disadvantages. The technique associated with this formulation is the so-called “infrared regularization”. The basic idea of this technique is to separate the loop integral containing the nucleon into two parts, an infrared singular and regular part. The infrared singular part contains fractional powers of the meson masses, whereas the infrared regular part involves fractional powers of the nucleon mass. The power counting is valid for the infrared singular part, but not for the infrared regular part. One therefore surmounts the problem of power counting by absorbing the infrared regular part into a redefinition of the LECs. Another renormalization technique is also available as proposed in Refs. [27, 28, 29], namely, the “Extended On-Mass-Shell” (EOMS) formalism.

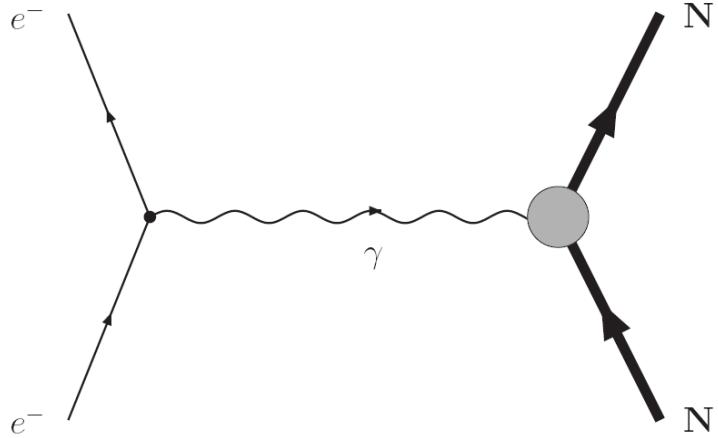
The mesonic ChPT, especially in the  $\pi\pi$  interaction, has achieved impressive success as the EFT of the strong interaction at low energies. In baryonic ChPT, the recent development of the manifestly Lorentz-invariant technique has tremendously improved the previous analysis of ChPT. The electromagnetic form factors of baryons as well as other baryonic properties have been studied. The further inclusion of vector mesons in baryon ChPT successfully improved the description of the electromagnetic nucleon form factors up to approximately  $Q^2 \approx 0.4 \text{ GeV}^2$  as shown in Ref. [29]. Open questions concerning the inclusion of other additional degrees of freedom like the  $\Delta(1232)$  resonance are currently studied with the hope to further extend the kinematic region, where ChPT is applicable.

### 3. Electromagnetic Form Factors of Nucleon and Hyperons

In this section, we present the approach used in order to study nucleon and hyperons. Restricted to the electromagnetic interaction, the important observables that can be studied are the electromagnetic form factors. Other observables, such as, magnetic moment of baryons, the charge and magnetic radii are related to such form factors. In case of nucleon, we follow the consideration in Ref. [30]. Extension to hyperons cases are presented.

#### 3.1 Electromagnetic Form Factors of Nucleon

The lowest-order elastic electron-nucleon scattering process as an important tool in order to study the electromagnetic structure of nucleon is shown in Fig. 3.1. The four-momenta of the incident and scattered electron are  $p = (\varepsilon, \vec{p})$  and  $p' = (\varepsilon', \vec{p}')$ , respectively.  $P = (E, \vec{P})$  and  $P' = (E', \vec{P}')$  are the four-momenta of the nucleon in the initial and final state. The four-momentum transfer carried by a photon is  $q = p - p' = P' - P$ . The characteristics of this scattering process is such that the square of the four-momentum transfer is space-like, i.e.  $q^2 < 0$ . Usually one defines a quantity  $Q^2$ , which is positive, i.e.  $Q^2 = -q^2 > 0$ .



**Fig. 3.1** Lowest-order electron-nucleon scattering.

The invariant amplitude of this process is of the form

$$\mathcal{M} \sim \bar{u}_e(p') \gamma_\mu u_e(p) \frac{e^2}{q^2} \langle N(P') | J_{em}^\mu(0) | N(P) \rangle, \quad (3.1)$$

where  $u_e(p), \bar{u}_e(p')$  refers to the electron Dirac spinors and  $\langle N(P')|J_{em}^\mu(0)|N(P)\rangle$  is the nucleon current matrix element. From considerations of Lorentz covariance, charge and parity conservation, the most general form of the nucleon current matrix element is

$$\langle N(P')|J_{em}^\mu(0)|N(P)\rangle = \bar{u}_N(P') \left[ \gamma^\mu F_1^N(Q^2) + \frac{i}{2M_N} \sigma^{\mu\nu} q_\nu F_2^N(Q^2) \right] u_N(P), \quad (3.2)$$

where  $F_1^N(Q^2)$  and  $F_2^N(Q^2)$  are the Dirac and Pauli form factors, respectively. Their normalizations are such that at zero recoil ( $Q^2 = 0$ )  $F_1^N(Q^2)$  is the charge of the nucleon (in units of the elementary charge), whereas  $F_2^N(Q^2)$  is the anomalous magnetic moment ( $\kappa_N$ ) of the nucleon

$$\begin{aligned} F_1^p(0) &= 1, & F_2^p(0) &= \kappa_p = 1.793, \\ F_1^n(0) &= 0, & F_2^n(0) &= \kappa_n = -1.913 \end{aligned} \quad (3.3)$$

where  $\kappa_N$  is given in units of the nuclear magneton.

In the laboratory frame, where the target nucleon is at rest, and neglecting the small mass of the electron, the energy  $\varepsilon'$  of the outgoing electron scattered by an angle  $\theta$  off the target of mass  $M$  is

$$\varepsilon' = \frac{\varepsilon}{1 + \frac{2\varepsilon}{M} \sin^2 \frac{\theta}{2}}$$

with the momentum transfer squared as,  $Q^2 = 4\varepsilon\varepsilon' \sin^2 \frac{\theta}{2}$ . For the simplest case of a spinless, point-like target the differential cross section reduced to the “Mott” differential cross section with the inclusion of the recoil factor  $\frac{\varepsilon'}{\varepsilon}$  as

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \frac{\alpha^2}{4\varepsilon^2 \sin^4 \frac{\theta}{2}} \frac{\varepsilon'}{\varepsilon} \cos^2 \frac{\theta}{2} \quad (3.4)$$

Extension to the case of spin- 1/2 target particle, but still point-like, leads to the well-known modification of the Mott formula

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ 1 + \frac{Q^2}{4M^2} 2 \tan^2 \frac{\theta}{2} \right] \quad (3.5)$$

The term proportional to  $\tan^2 \frac{\theta}{2}$  results in an increase of the differential cross section at backward angles. It is due to the magnetic scattering of the spin of both projectile and target. For a spin-1/2 target with an extended structure and an anomalous

magnetic moment, as is the case for the nucleon, the differential cross section is referred to as “Rosenbluth cross section” [31]

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ (F_1^N(Q^2))^2 + \frac{Q^2}{4M_N^2} \left\{ (F_2^N(Q^2))^2 + 2(F_1^N(Q^2) + F_2^N(Q^2))^2 \tan^2 \frac{\theta}{2} \right\} \right] \quad (3.6)$$

where  $F_1^N(Q^2)$  and  $F_2^N(Q^2)$  are the Dirac and Pauli form factors.

Instead of working with  $F_1^N(Q^2)$  and  $F_2^N(Q^2)$ , it is convenient to consider linear combinations constructed as

$$\begin{aligned} G_E^N(Q^2) &= F_1^N(Q^2) - \frac{Q^2}{4M_N^2} F_2^N(Q^2), \\ G_M^N(Q^2) &= F_1^N(Q^2) + F_2^N(Q^2) \end{aligned} \quad (3.7)$$

which are the “Sachs form factors”. With Eq. (3.3) the normalizations for the Sachs form factors of the nucleon are

$$\begin{aligned} G_E^p(0) &= 1, & G_M^p(0) &= \mu_p = 2.793, \\ G_E^n(0) &= 0, & G_M^n(0) &= \mu_n = -1.913 \end{aligned} \quad (3.8)$$

where  $\mu_N$  are the nucleon magnetic moments. As for  $F_1^N(Q^2)$  and  $F_2^N(Q^2)$ , the Sachs form factors can be related to the current matrix elements of Eq. (3.2). The interpretation of the Sachs form factors become simple when we restrict to a specific frame of reference, namely, the “Breit frame”. For the elastic electron-nucleon scattering process the Breit frame coincides with the center-of-mass frame. In this particular frame the energy transfer vanishes and thus the photon carries the four-momentum  $q^\mu = (0, \vec{q})$  and therefore  $Q^2 = \vec{q}^2$ . The incoming electron has momentum  $\vec{p} = +\frac{\vec{q}}{2}$  and the incoming nucleon has opposite momentum  $\vec{P} = -\frac{\vec{q}}{2}$ , while in the final state the outgoing electron and nucleon have momenta  $\vec{p}' = -\frac{\vec{q}}{2}$  and  $\vec{P}' = +\frac{\vec{q}}{2}$ , respectively. In the Breit frame, the corresponding matrix elements of Eq. (3.2) are

$$\begin{aligned} \left\langle N\left(\frac{\vec{q}}{2}, s'\right) \middle| J_{em}^0(0) \middle| N\left(-\frac{\vec{q}}{2}, s\right) \right\rangle &= G_E^N(Q^2) \delta_{s's} \\ \left\langle N\left(\frac{\vec{q}}{2}, s'\right) \middle| \bar{J}_{em}(0) \middle| N\left(-\frac{\vec{q}}{2}, s\right) \right\rangle &= G_M^N(Q^2) \chi_{s'}^\dagger \frac{i\vec{\sigma} \times \vec{q}}{2M_N} \chi_s \end{aligned} \quad (3.9)$$

where  $s$  and  $s'$  are the spin orientations of the incoming and outgoing nucleon, respectively, while  $\chi_s$  and  $\chi_{s'}$  refer to the two-component Pauli spinors. In terms of the Sachs form factors the Rosenbluth formula for elastic scattering of an electron on the nucleon target becomes

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ \frac{\left( G_E^N(Q^2) \right)^2 + \frac{Q^2}{4M_N^2} \left( G_M^N(Q^2) \right)^2}{1 + \frac{Q^2}{4M_N^2}} + \frac{Q^2}{2M_N^2} \left( G_M^N(Q^2) \right)^2 \tan^2 \frac{\theta}{2} \right] \quad (3.10)$$

Elastic electron-nucleon scattering is the basic tool in order to extract the electromagnetic form factors of the nucleon. Due to the finite lifetime of the neutron, one faces the difficulty in constructing free neutron targets. Instead, deuteron or  $^3\text{He}$  targets have been used in which an additional subtraction of the effect due to the presence of the protons is needed in the analysis. As known from early experiments, the electromagnetic form factors of the nucleon, except for the neutron charge form factor  $G_E^N(Q^2)$ , are well described by the dipole parameterization

$$G_E^p(Q^2) \approx \frac{G_M^p(Q^2)}{\mu_p} \approx \frac{G_M^n(Q^2)}{\mu_n} \approx G_D(Q^2) \quad (3.11)$$

where the dipole form factor is

$$G_D(Q^2) = \frac{1}{\left[ 1 + \frac{Q^2}{(0.71 \text{ GeV}^2)} \right]^2} \quad (3.12)$$

The electromagnetic proton form factors can be directly obtained by measuring the differential cross section of the elastic electron-proton scattering process. Alternatively, Hand, Miller and Wilson [32] suggested the extraction of  $G_E^p(Q^2)$  and  $G_M^p(Q^2)$  rather than the Dirac and Pauli form factors from the differential cross section by rewritten Eq. (3.10) as

$$\sigma_R \equiv \frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{\text{Mott}}} = \varepsilon(1 + \tau) = \tau \left( G_M^p(Q^2) \right)^2 + \varepsilon \left( G_E^p(Q^2) \right)^2 \quad (3.13)$$

where  $\sigma_R$  is the reduced cross section,  $\tau = \frac{Q^2}{4M_p^2}$  and the linear polarization of the virtual photon is

$$\varepsilon = \left[ 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \right]^{-1} \quad (3.14)$$

By fixing  $Q^2$ , the plots of the measured quantities  $\sigma_R$  and  $\varepsilon$  for different combinations of  $(\theta, \varepsilon)$  can be fitted by a linear polynomial in which the slope is  $(G_E^p(Q^2))^2$  and the intercept on the  $\sigma_R$ -axis is. This method is referred to as “Rosenbluth separation technique”.

However, at large  $Q^2$  the Rosenbluth separation for  $G_E^p(Q^2)$  suffers from the increasing systematic uncertainties with increasing values of  $Q^2$ .  $\frac{G_M^p}{(\mu_p G_D)}$  can be well measured up to  $Q^2 \sim 30 \text{ GeV}^2$ , whereas the data for  $\frac{G_E^p}{G_D}$  scatter and have large uncertainties for values above  $Q^2 \sim 1 \text{ GeV}^2$ .

Akhiezer, Rozentsweig and Shumshkevich [33] already showed in 1958 that a considerable increase in accuracy of the nucleon charge form factor measurement can be achieved by scattering polarized electrons off a polarized nucleon target. However, it took several decades before such experiments were technically feasible. In the polarization transfer experiment, e.g.  $\vec{e} p \rightarrow e \vec{p}$ , the polarization of the final proton is measured in addition. The longitudinal part  $P_l$  parallel to the proton momentum and the transverse part  $P_t$  of the proton polarization are given by

$$\begin{aligned} P_l &= \frac{\varepsilon + \varepsilon'}{M_p I_0} \sqrt{\tau(1+\tau)} (G_M^p)^2 \tan^2 \frac{\theta}{2} \\ P_t &= -\frac{2}{I_0} \sqrt{\tau(1+\tau)} G_E^p G_M^p \tan \frac{\theta}{2} \end{aligned} \quad (3.15)$$

with

$$I_0 = (G_E^p)^2 + \tau \left[ 1 + 2(1+\tau) \tan^2 \frac{\theta}{2} \right] (G_M^p)^2 \quad (3.16)$$

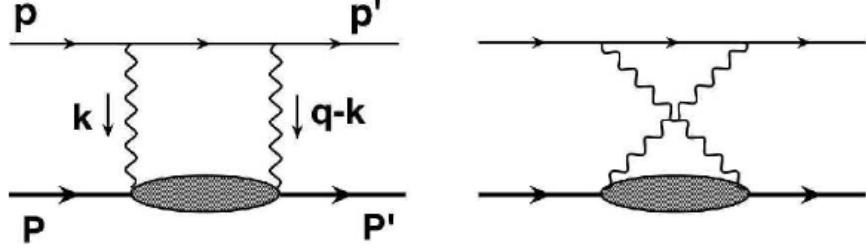
Both  $P_l$  and  $P_t$  can be measured by the polarimeter and their ratio gives rise to

$$\frac{G_E^p}{G_M^p} = -\frac{\varepsilon + \varepsilon'}{2M_p} \left( \frac{P_t}{P_l} \right) \tan \frac{\theta}{2} \quad (3.17)$$

In this way the systematic uncertainties in extracting the ratio of  $\frac{G_E^p}{G_M^p}$  are minimized.

Obviously, the polarization measurements lead to a significant improvement of the experimental data. An important feature detected by the polarization transfer experiments is the observed linear decline of  $\frac{\mu_p G_E^p}{G_M^p}$  as  $Q^2$  increases. This is in clear contradiction to the results obtained by the Rosenbluth separation technique. Due to occurrence of large systematic errors of  $G_E^p(Q^2)$  at large  $Q^2$  with the Rosenbluth extraction, attempts have been made in order to improve the data. A careful reanalysis of the old Rosenbluth data was done. Results from a high-precision Rosenbluth extraction especially designed for the measurement in Hall A at Jefferson Lab were reported by Qattan et al. [34]. All of these recent analyses of the Rosenbluth data showed agreement with the previous Rosenbluth results. Therefore, the origin for the discrepancy of results between Rosenbluth separation and polarization technique must

be due to other mechanisms. One believes that such mechanisms is the hard two-photon exchange as shown in Fig. 3.2



**Fig. 3.2** Feynman diagrams for the two-photon exchange.

### 3.2. Chiral Quark Lagrangian

Motivated from ChPT, the chiral quark Lagrangian  $\mathcal{L}_{qU}$  (up to order  $p^4$ ), which dynamically generates the dressing of the constituent quarks by mesonic degrees of freedom, consists of two primary pieces  $\mathcal{L}_q$  and  $\mathcal{L}_U$ :

$$\mathcal{L}_{qU} = \mathcal{L}_q + \mathcal{L}_U, \quad \mathcal{L}_q = \mathcal{L}_q^{(1)} + \mathcal{L}_q^{(2)} + \mathcal{L}_q^{(3)} + \mathcal{L}_q^{(4)} + \dots, \quad \mathcal{L}_U = \mathcal{L}_U^{(2)} + \dots \quad (3.18)$$

The superscript  $(i)$  attached to  $\mathcal{L}_{q(U)}^{(i)}$  denotes the low energy dimension of the Lagrangian:

$$\begin{aligned} \mathcal{L}_U^{(2)} &= \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle, \\ \mathcal{L}_q^{(1)} &= \bar{q} \left[ i \not{D} - m + \frac{1}{2} g \not{u} \gamma^5 \right] q, \\ \mathcal{L}_q^{(2)} &= -\frac{c_2}{4m^2} \langle u_\mu u_\nu \rangle \left( \bar{q} D^\mu D^\nu q + \text{h.c.} \right) + \frac{c_4}{4} \bar{q} i \sigma^{\mu\nu} [u_\mu, u_\nu] q + \frac{c_6}{8m} \bar{q} \sigma^{\mu\nu} F_{\mu\nu}^+ q + \dots, \\ \mathcal{L}_q^{(3)} &= \frac{id_{10}}{2m} \bar{q} [D^\mu, F_{\mu\nu}^+] D^\nu q + \text{h.c.} + \dots, \\ \mathcal{L}_q^{(4)} &= \frac{e_6}{2} \langle \chi_+ \rangle \bar{q} \sigma^{\mu\nu} F_{\mu\nu}^+ q + \frac{e_7}{4} \bar{q} \sigma^{\mu\nu} \{F_{\mu\nu}^+, \hat{\chi}_+\} q + \frac{e_8}{2} \bar{q} \sigma^{\mu\nu} \langle F_{\mu\nu}^+ \hat{\chi}_+ \rangle q \\ &\quad - \frac{e_{10}}{2} \bar{q} [D^\alpha, [D_\alpha, F_{\mu\nu}^+]] \sigma^{\mu\nu} q + \dots, \end{aligned}$$

where  $\hat{\chi}_+ = \chi_+ - \frac{1}{3} \langle \chi_+ \rangle$ , the symbols  $\langle \rangle$ ,  $[\ ]$  and  $\{ \}$  denotes the trace over flavor matrices, commutator and anticommutator, respectively. We show here only the terms

involved in the calculation of the dressed electromagnetic quark operator. Also we have included the vector mesons and the detailed form of the chiral Lagrangian can be found in Ref. [30]. The couplings  $m$  and  $g$  denote the quark mass and axial charge in the chiral limit,  $c_i$ ,  $d_i$  and  $d_i$  are the second-, third- and fourth-order low-energy coupling constants, respectively, which encode the contributions of heavy states. Parameter  $m$  is counted as a quantity of order  $O(1)$  in the chiral expansion.

The quark field is  $q$ , and the octet of pseudoscalar fields are

$$\phi = \sum_{i=1}^8 \phi_i \lambda_i = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \quad (3.19)$$

which is contained in the non-linear representation of  $SU(3)$  matrix  $U = u^2 = \exp\left(\frac{i\phi}{F}\right)$ , where  $F$  is the octet decay constant. We introduce the standard notations [35, 36, 37]

$$\begin{aligned} D_\mu &= \partial_\mu + \Gamma_\mu, \\ \Gamma_\mu &= \frac{1}{2} [u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger R_\mu u - \frac{i}{2} u L_\mu u^\dagger, \\ u_\mu &= i u^\dagger \nabla_\mu U u^\dagger, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\ \chi &= 2B\mathcal{M} + \dots \end{aligned}$$

The fields  $R_\mu$  and  $L_\mu$  include external fields (electromagnetic  $A_\mu$ , weak, etc.):  $R_\mu = eQA_\mu + \dots$ ,  $L_\mu = eQA_\mu + \dots$  where  $Q = \text{diag}\{2/3, -1/3, -1/3\}$  is the quark charge matrix. The tensor  $F_{\mu\nu}^+$  is defined as  $F_{\mu\nu}^+ = u^\dagger F_{\mu\nu} Q u + u F_{\mu\nu} Q u^\dagger$  where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the conventional photon field strength tensor. Here  $\mathcal{M} = \text{diag}\{\hat{m}, \hat{m}, \hat{m}_s\}$  is the mass matrix of current quarks (we work in the isospin symmetry limit with  $\hat{m}_u = \hat{m}_d = \hat{m} = 7$  MeV and the mass of the strange quark  $\hat{m}_s$  is related to the nonstrange one as  $\hat{m}_s = 25\hat{m}$ ).

The quark vacuum condensate parameter is denoted by  $B = -\langle 0|\bar{u}u|0\rangle/F^2$ . To distinguish between constituent and current quark masses we attach the symbol  $\hat{\phantom{a}}$  (“hat”) when referring to the current quark masses. We rely on the standard picture of chiral symmetry breaking ( $B \gg F$ ). In leading order of the chiral expansion the masses of pseudoscalar mesons are given by  $M_\pi^2 = 2\hat{m}B$ ,  $M_K^2 = (\hat{m} + \hat{m}_s)B$ ,  $M_\eta^2 = \frac{2}{3}(\hat{m} + 2\hat{m}_s)B$ . In the numerical analysis we will use:  $M_\pi = 139.57 \text{ MeV}$ ,  $M_K = 493.677 \text{ MeV}$  (the charged pion and kaon masses),  $M_\eta = 574.75 \text{ MeV}$  and the canonical set of differentiated decay constants:  $F_\pi = 92.4 \text{ MeV}$ ,  $\frac{F_K}{F_\pi} = 1.22$  and  $\frac{F_\eta}{F_\pi} = 1.3$  [38].

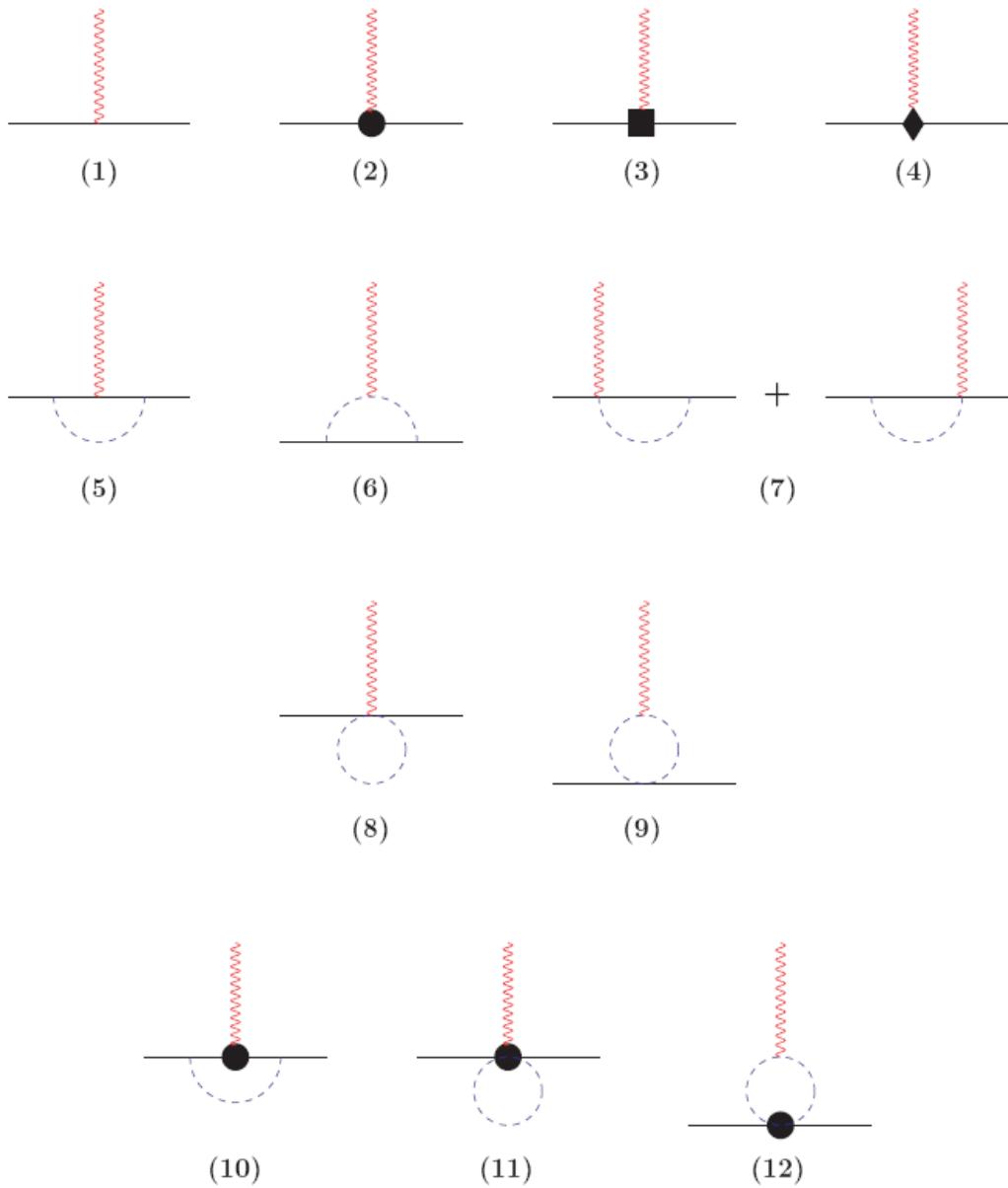
### 3.3. Dressing of the Quark Operators

Any bare quark operator (both one- and two-body) can be dressed by a cloud of pseudoscalar mesons and heavy states in a straightforward manner by use of the effective chirally-invariant Lagrangian  $\mathcal{L}_{qU}$ .

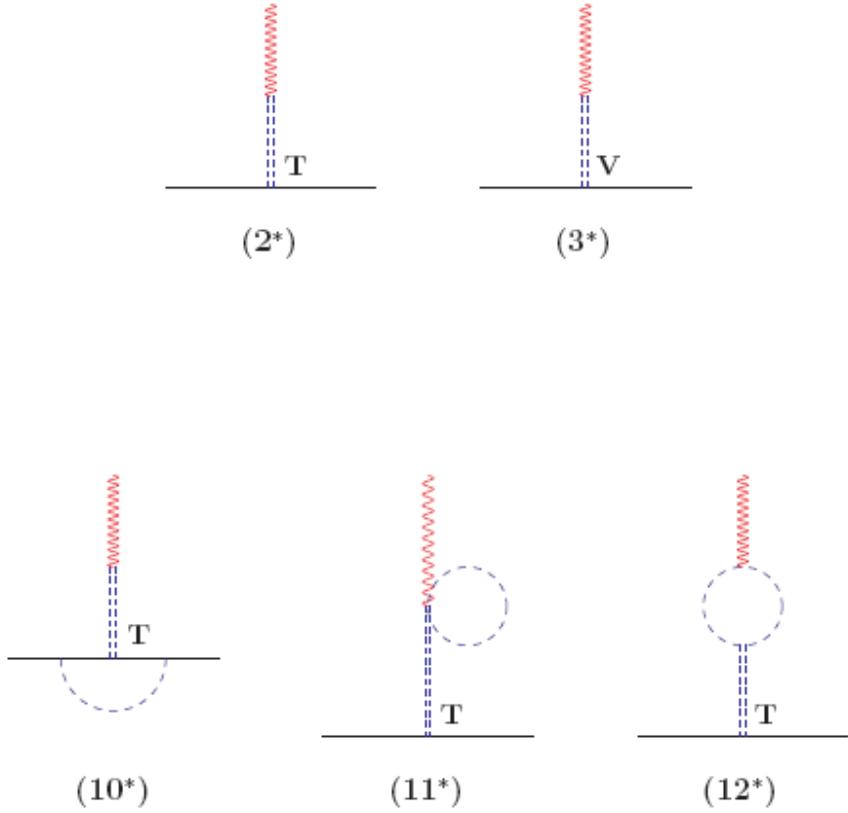
To illustrate the idea of such a dressing we consider the Fourier-transform of the electromagnetic quark operator:

$$\begin{aligned} J_{\mu,em}^{bare}(q) &= \int d^4x e^{-iqx} J_{\mu,em}^{bare}(x), \\ J_{\mu,em}^{bare}(x) &= \bar{q}(x)\gamma_\mu Q q(x) \end{aligned} \tag{3.20}$$

In Fig. 3.3 we display the tree and loop diagrams which contribute to the dressed electromagnetic operator  $J_{\mu,em}^{dress}$  up to fourth order, which come from the chiral quark Lagrangian. Additional diagrams including the vector-meson contributions are shown in Fig.3.4.



**Fig. 3.3** Diagrams including pseudoscalar meson contributions to the electromagnetic quark transition operator up to fourth order. Solid, dashed and wiggly lines refer to quarks, pseudoscalar mesons and the electromagnetic field, respectively. Vertices denoted by a black filled circle, box and diamond correspond to insertions from the second, third and fourth order chiral Lagrangian.



**Fig. 3.4** Diagrams including vector-meson contributions to the electromagnetic quark transition operator. Double-dashed lines correspond to vector mesons. The symbols V and T refer to the vectorial and tensorial couplings of vector mesons to quarks.

Note, here we restrict our consideration to the one-body quark operator. The dressed quark operator  $J_{\mu, em}^{dress}(x)$  and its Fourier transform  $J_{\mu, em}^{dress}(q)$  have the following forms

$$\begin{aligned}
 J_{\mu, em}^{dress}(x) &= \sum_{q=u,d,s} \left\{ f_D^q(-\partial^2) [\bar{q}(x) \gamma_\mu q(x)] + \frac{f_P^q(-\partial^2)}{2m_q} \partial^\nu [\bar{q}(x) \sigma_{\mu\nu} q(x)] \right\}, \\
 J_{\mu, em}^{dress}(q) &= \int d^4x e^{-iqx} J_{\mu, em}^{dress}(x) \\
 &= \int d^4x e^{-iqx} \sum_{q=u,d,s} \bar{q}(x) \left\{ \gamma_\mu f_D^q(q^2) + \frac{i}{2m_q} \sigma_{\mu\nu} q^\nu f_P^q(q^2) \right\} q(x)
 \end{aligned}$$

where  $m_q$  is the dressed constituent quark mass generated by the chiral Lagrangian (see details in Ref. [30]);

Here  $f_D^u(q^2)$ ,  $f_D^d(q^2)$ ,  $f_D^s(q^2)$  and  $f_P^u(q^2)$ ,  $f_P^d(q^2)$ ,  $f_P^s(q^2)$  are the Dirac and Pauli form factors of  $u$ ,  $d$  and  $s$  quarks and can be calculated directly from

diagrams in Fig. 3.3 and Fig. 3.4. Here we use the appropriate sub- and superscripts with a definite normalization of the set of  $f_D^q(0) \equiv e_q$  (quark charges) due to charge conservation. Note, that the dressed quark operator satisfies current conservation:  $\partial^\mu J_{\mu,em}^{dress}(x) = 0$ . Evaluation of the diagrams in Fig.3.3 is based on the infrared dimensional regularization (IDR) suggested in Ref. [35] to guarantee a straightforward connection between loop and chiral expansion in terms of quark masses and small external momenta.

To calculate the electromagnetic  $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$  transitions between baryons we project the dressed quark operator between the corresponding baryon states. The master formula is:

$$\begin{aligned}
\langle B(p') | J_{\mu,em}^{dress}(q) | B(p) \rangle &= (2\pi)^4 \delta^4(p' - p - q) \bar{u}_B(p') \left\{ \gamma_\mu F_1^B(q^2) + \frac{i}{2m_B} \sigma_{\mu\nu} q^\nu F_2^B(q^2) \right\} u_B(p) \\
&= (2\pi)^4 \delta^4(p' - p - q) \sum_{q=u,d,s} \left\{ f_D^q(q^2) \langle B(p') | J_{\mu,q}^{bare}(0) | B(p) \rangle \right. \\
&\quad \left. + i \frac{q^\nu}{2m_q} f_P^q(q^2) \langle B(p') | J_{\mu\nu,q}^{bare}(0) | B(p) \rangle \right\}
\end{aligned} \tag{3.21}$$

where  $F_1^B(q^2)$  and  $F_2^B(q^2)$  are the Dirac and Pauli baryon form factors,  $B(p)$  and  $u_B(p)$  are the baryon state and spinor, respectively, normalized as

$$\begin{aligned}
\langle B(p') | B(p) \rangle &= 2E_B (2\pi)^3 \delta^3(\bar{p} - \bar{p}'), \\
\bar{u}_B(p) u_B(p) &= 2m_B,
\end{aligned} \tag{3.22}$$

with  $E_B = \sqrt{m_B^2 + \bar{p}^2}$  being the baryon energy and  $m_B$  the baryon mass. We express the matrix elements of the dressed quark operator in terms of the matrix elements of the bare operators for vector  $J_{\mu,q}^{bare}(0)$  and tensor  $J_{\mu\nu,q}^{bare}(0)$  currents defined as

$$\begin{aligned}
J_{\mu,q}^{bare}(0) &= \bar{q}(0) \gamma_\mu q(0), \\
J_{\mu\nu,q}^{bare}(0) &= \bar{q}(0) \sigma_{\mu\nu} q(0)
\end{aligned} \tag{3.23}$$

In general, due to Lorentz and gauge invariance, the matrix elements in Eq. (3.21) can be written as

$$\begin{aligned}
\langle B(p') | J_{\mu,q}^{bare}(0) | B(p) \rangle &= \bar{u}_B(p') \left\{ \gamma_\mu F_1^{Bq}(q^2) + \frac{i}{2m_B} \sigma_{\mu\nu} q^\nu F_2^{Bq}(q^2) \right\} u_B(p), \\
i \frac{q^\nu}{2m_q} \langle B(p') | J_{\mu\nu,q}^{bare}(0) | B(p) \rangle &= \bar{u}_B(p') \left\{ \gamma_\mu G_1^{Bq}(q^2) + \frac{i}{2m_B} \sigma_{\mu\nu} q^\nu G_2^{Bq}(q^2) \right\} u_B(p),
\end{aligned} \tag{3.22}$$

where  $F_{1(2)}^{Bq}(q^2)$  and  $G_{1(2)}^{Bq}(q^2)$  are the Pauli and Dirac form factors describing the distribution of quarks of flavor  $q = u, d, s$  in the baryon  $B$ . Finally, the baryon form factors  $F_i^B(q^2)$  with  $i = 1, 2$  can be separated into a bare part  $F_i^{Bbare}(q^2)$  and a meson cloud part  $F_i^{Bcloud}(q^2)$  as

$$\begin{aligned}
F_i^B(q^2) &= F_i^{Bbare}(q^2) + F_i^{Bcloud}(q^2), \\
F_i^{Bbare}(q^2) &= \sum_{q=u,d,s} e_q F_i^{Bq}(q^2), \\
F_i^{Bbare}(q^2) &= \sum_{q=u,d,s} [(f_D^q(q^2) - e_q) F_i^{Bq}(q^2) + f_P^q(q^2) G_i^{Bq}(q^2)]
\end{aligned} \tag{3.23}$$

where  $e_q$  are the electric quark charges.

Eqs. (3.21)-(3.23) contain our main result: we perform a model-independent factorization of the effects of hadronization and confinement contained in the matrix elements of the bare quark operators  $J_{\mu,q}^{bare}(0)$  and  $J_{\mu\nu,q}^{bare}(0)$  and the effects dictated by chiral symmetry (or chiral dynamics) which are encoded in the relativistic form factors  $f_D^q(q^2)$  and  $f_P^q(q^2)$ . Due to this factorization the calculation of  $f_D^q(q^2)$  and  $f_P^q(q^2)$ , on one side, and the matrix elements of  $J_{\mu,q}^{bare}(0)$  and  $J_{\mu\nu,q}^{bare}(0)$ , on the other side, can be done independently. In particular, in a first step we derived a model-independent formalism based on the ChPT Lagrangian, which is formulated in terms of constituent quark degrees of freedom, for the calculation of  $f_D^q(q^2)$  and  $f_P^q(q^2)$  (see their explicit forms in Appendix C of Ref. [30]).

The calculation of the matrix elements of the bare quark operators one can utilize the quark models based on specific assumptions about hadronization and confinement. Here we considered a treatment of valence quark degrees of freedom by using a parameterization of the bare quark distributions in the baryon (nucleon) with taking into account model-independent constraints dictated by certain symmetries:

gauge, isospin and chiral invariance, as was done in Ref. [30]. Another possibility as we have done is in Ref. [39] when we calculated valence quark form factors explicitly with the use of relativistic quark model [40]-[42] based on a specific ansatz of quarks in baryons. In this report we complete our analysis started in Ref. [30] where we presented a comprehensive analysis of electromagnetic nucleon properties including magnetic moments, radii and form factors. However, for the case of hyperons we restricted only to the calculation of magnetic moments.

### 3.4. Matrix Elements of the Bare Quark Operators

We will modeled the matrix elements of the bare quark operators using certain symmetry constraints leading to a set of relationships between the nucleon and corresponding  $u$ -,  $d$ - and  $s$ -quark form factors at zero momentum transfer. This is an extension of the original idea of Ref. [30] to be applied to the case of hyperon form factors.

In case of nucleons one can derive the constraints on the form factors arising from charge conservation, isospin invariance and infrared-singular structure of QCD [30]:

$$\begin{aligned} F_1^{pu}(0) &= F_1^{nd}(0) = 2, \\ F_1^{pd}(0) &= F_1^{nu}(0) = 1, \\ G_1^{Nq}(0) &= 0, \\ F_2^{pu}(0) &= F_2^{nd}(0), \\ F_2^{pd}(0) &= F_2^{nu}(0), \\ G_2^{pu}(0) &= G_2^{nd}(0), \\ G_2^{pd}(0) &= G_2^{nu}(0), \end{aligned}$$

and

$$\begin{aligned} 1 + F_2^{pu}(0) - F_2^{pd}(0) &= G_2^{pu}(0) - G_2^{pd}(0) = \left( \frac{g_A}{g} \right)^2 \frac{\overset{\circ}{m}_N}{\overset{\circ}{m}}, \\ 1 + F_2^{nd}(0) - F_2^{nu}(0) &= G_2^{nd}(0) - G_2^{nu}(0) = \left( \frac{g_A}{g} \right)^2 \frac{\overset{\circ}{m}_N}{\overset{\circ}{m}} \end{aligned}$$

where  $g_A$  and  $\bar{m}_N$  are the axial charge and the mass of the nucleon in the chiral limit and  $\bar{m} = m_u = m_d$  is the dressed non-strange constituent quark mass in the isospin limit.

Restricting our consideration to the one-body quark operator and by using SU(6)-symmetry relations one can relate the Dirac and Pauli form factors describing the distribution of quarks of flavor  $q = u, d, s$  in the baryon “ $B$ ”, that is  $F_{1(2)}^{Bq}$  and  $G_{1(2)}^{Bq}$  to the bare (or valence) quark form factors. In particular, one can introduce the bare Dirac ( $F_1^q, G_1^q$ ) and Pauli ( $F_2^q, G_2^q$ ) form factors of the quark of flavor  $q$  as

$$\begin{aligned} \langle q(p') | J_{\mu,q}^{bare}(0) | q(p) \rangle &= \bar{u}_q(p') \left\{ \gamma_\mu F_1^q(q^2) + \frac{i}{2m_q} \sigma_{\mu\nu} q^\nu F_2^q(q^2) \right\} u_q(p), \\ i \frac{q^\nu}{2m_q} \langle q(p') | J_{\mu\nu,q}^{bare}(0) | q(p) \rangle &= \bar{u}_q(p') \left\{ \gamma_\mu G_1^q(q^2) + \frac{i}{2m_q} \sigma_{\mu\nu} q^\nu G_2^q(q^2) \right\} u_q(p), \end{aligned} \quad (3.24)$$

The Sachs form factors of the quark of flavor  $q$  are

$$\begin{aligned} \mathcal{F}_q^E &= F_q^E(t) + G_q^E(t), \\ \mathcal{F}_q^M &= F_q^M(t) + G_q^M(t) \end{aligned} \quad (3.25)$$

where

$$\begin{aligned} F\{G\}_q^E(t) &= F\{G\}_1^q(t) - \frac{t}{4m^2} F\{G\}_2^q(t), \\ F\{G\}_q^M(t) &= F\{G\}_1^q(t) + F\{G\}_2^q(t), \\ t &= -q^2 \end{aligned} \quad (3.26)$$

are the contributions to the Sachs form factors associated with the expectation values of the vector and tensor currents, respectively. Finally, the baryonic form factors  $F_i^{Bq}(t)$  are expressed in term of quark form factors  $F_q^{E(M)}(t)$  and  $G_q^{E(M)}(t)$  by :

$$\begin{aligned}
F_1^{Bq}(t) &= \frac{1}{1 + \tau_B} \left\{ \alpha_E^{Bq} F_q^E(t) + \alpha_M^{Bq} \chi^{Bq} F_q^M(t) \tau_B \right\}, \\
F_2^{Bq}(t) &= \frac{1}{1 + \tau_B} \left\{ -\alpha_E^{Bq} F_q^E(t) + \alpha_M^{Bq} \chi^{Bq} F_q^M(t) \right\}, \\
G_1^{Bq}(t) &= \frac{1}{1 + \tau_B} \left\{ \alpha_E^{Bq} G_q^E(t) + \alpha_M^{Bq} \chi^{Bq} G_q^M(t) \tau_B \right\}, \\
G_2^{Bq}(t) &= \frac{1}{1 + \tau_B} \left\{ -\alpha_E^{Bq} G_q^E(t) + \alpha_M^{Bq} \chi^{Bq} G_q^M(t) \right\},
\end{aligned} \tag{3.27}$$

where  $F_q^E(t)$  and  $F_q^M(t)$  are the quark Sachs form factors and  $\tau_B = \frac{t}{4m_B^2}$ . In addition

to the strict evaluation of SU(6) we have introduced the additional parameter  $\chi^{Bq}$  for each quark of flavor  $q$ . The interpretation for adding these factors is such that to allow the quark distributions for hyperons to be different from that for the nucleons. In the case of the nucleons we set  $\chi^{Bq} = 1$ . The values for  $\alpha_E^{Bq}$  and  $\alpha_M^{Bq}$  for the baryon octet as derived from SU(6)-symmetry relations are given in Table 3.1.

	$\alpha_E^{Bu}$	$\alpha_E^{Bd}$	$\alpha_E^{Bd}$	$\alpha_M^{Bu}$	$\alpha_M^{Bd}$	$\alpha_M^{Bs}$
$p$	2	1	0	$\frac{4}{3}$	$-\frac{1}{3}$	0
$n$	1	2	0	$-\frac{1}{3}$	$\frac{4}{3}$	0
$\Lambda^0$	1	1	1	0	0	1
$\Sigma^+$	2	0	1	$\frac{4}{3}$	0	$-\frac{1}{3}$
$\Sigma^0$	1	1	1	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$
$\Sigma^-$	0	2	1	0	$\frac{4}{3}$	$-\frac{1}{3}$
$\Xi^-$	0	1	2	0	$-\frac{1}{3}$	$\frac{4}{3}$
$\Xi^0$	1	0	2	$-\frac{1}{3}$	0	$\frac{4}{3}$
$\Sigma^0 \Lambda^0$	0	0	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	0

**Table 3.1** SU(6) couplings  $\alpha_E^{Bi}$  and  $\alpha_M^{Bi}$

## 4. Results and Conclusion

In Ref. [30] we considered only nucleon form factors and therefore, the Sachs form factors of  $u$  - and  $d$  -quark. In particular, we modeled the  $u$  - and  $d$  -quark form factors by the dipole characteristics with damping functions of an exponential form. This phenomenological form is required to reproduce the deviation of the electromagnetic form factors of the nucleon from the dipole fit as evident from recent experimental measurements. In this paper we use the same parametrization for  $s$ -quark. Therefore, for the Sachs form factors of  $u$  -,  $d$  - and  $s$ -quark we use the parameterization

$$F_q^E(t) = \frac{\rho_q^E(t)}{\left[1 + \frac{t}{\Lambda_{qE}^2}\right]^2}, \quad F_q^M(t) = \mu_q^F \frac{\rho_q^M(t)}{\left[1 + \frac{t}{\Lambda_{qM}^2}\right]^2},$$

$$G_q^E(t) = \gamma_q \rho_q^E(t) \frac{\frac{t}{\Lambda_{qE}^2}}{\left[1 + \frac{t}{\Lambda_{qE}^2}\right]^3}, \quad G_q^M(t) = \mu_q^G \frac{\rho_q^M(t)}{\left[1 + \frac{t}{\Lambda_{qM}^2}\right]^2},$$

where  $\rho_q^E(t) = \exp\left(-\frac{t}{\lambda_{qE}^2}\right)$  and  $\rho_q^M(t) = \exp\left(-\frac{t}{\lambda_{qM}^2}\right)$ . Note, that in Ref. [34] a similar

parametrization of the nucleon form factors has been considered. In Ref. [35] the damping functions  $\rho(t)$  have been parametrized with constant values. For convenience we suppose that the quark Sachs form factors degenerate at zero recoil according to SU(6) symmetry. In other words all effects of possible SU(6) symmetry-breaking are encoded in the coefficients  $\chi^{Bq}$ . Therefore, the parameters  $\mu_q^F$  and  $\mu_q^G$  are fixed by the SU(6) symmetry and by the set of other symmetry constraints as:

$$\mu_q^F = \mu_q^G = \frac{3}{5} \left( \frac{g_A}{g} \right)^2 \frac{\overset{\circ}{m}_N}{\overline{m}}$$

The remaining parameters  $\gamma_q$ ,  $\Lambda_{qE(M)}$  and  $\lambda_{qE(M)}$  are free parameters. In the case of  $u$  - and  $d$  -quark the corresponding parameters  $(\gamma_u, \gamma_d, \Lambda_{uE(M)}$  and  $\lambda_{dE(M)})$  have been fixed from the consideration of the full momentum dependence of the nucleon

electromagnetic form factors at intermediate and high value of momentum transfer squared:

$$\begin{aligned}\lambda_{uE} &= 2.0043, & \lambda_{dE} &= 0.9996, & \lambda_{uM} &= 7.3367, & \lambda_{dM} &= 2.2954, \\ \Lambda_{uE} &= 0.8616, & \Lambda_{dE} &= 0.9234, & \Lambda_{uM} &= 0.9278, & \Lambda_{dM} &= 1.0722 \\ \gamma_u &= 1.081, & \gamma_d &= 2.596.\end{aligned}$$

The remaining parameters relevant for the strange quark  $\Lambda_{sE}, \Lambda_{sM}, \lambda_{sE}, \lambda_{sM}$  and  $\gamma_s$  can be fixed or varied using the following arguments. A general remark is that an information about hyperon form factors (and as consequence about strange valence form factors) is very poor: we know only their normalization due charge conservation or from a knowledge of magnetic moments. Therefore, for some parameters we use typical values. In particular, we fix  $\gamma_s = 1$  (which is typical for  $u$  and  $d$  quarks). For the cutoff parameters  $\Lambda_{sE}$  and  $\Lambda_{sM}$  we use typical value 1 GeV guaranteeing the correct  $\frac{1}{t^2}$  scaling of the baryon form factors at large  $t$ . A more nontrivial situation is with parameters  $\lambda_{sE}$  and  $\lambda_{sM}$  controlling a deviation of strange quark (or hyperons) form factors from the dipole fit. To our knowledge based on analysis of nucleon form factors, these parameters can be roughly varied from 1 to 10 GeV. This gives a major ambiguity in the description of hyperons form factors.

Finally we specify the parameters  $\chi^{Bq}$  encoding the effects of SU(6) symmetry breaking and in the chiral quark Lagrangian. They have been fixed in Ref. [30] from the description of magnetic moments of the baryon octet hyperons and nucleon slopes. In particular, in Ref. [30] we considered two scenarios: SU(6) symmetric case (Set I) and beyond SU(6) symmetry (Set II). In case of the Set I the couplings  $\chi^{Bq}$  are trivially equal to 1. For the Set II we got:

$$\begin{aligned}\chi^{\Sigma u} &= \chi^{\Sigma d} = 0.963, & \chi^{\Sigma s} &= 0.259, \\ \chi^{\Xi u} &= \chi^{\Xi d} = 0.633, & \chi^{\Xi s} &= 0.694, \\ \chi^{\Lambda u} &= \chi^{\Lambda d} = 0.988\end{aligned}$$

relying on isospin symmetry. In the isotriplet  $\Sigma^+, \Sigma^0$  and  $\Sigma^-$  shares the same set of  $\chi^{Bq}$  for the quark of flavor  $q$ , while  $\Xi^0$  and  $\Xi^-$  contains the same set of  $\chi^{\Xi q}$ . The

parameters  $\chi^{\Sigma\Lambda u}$  and  $\chi^{\Sigma\Lambda d}$  are directly related to the  $\Sigma - \Lambda$  magnetic transition moment.

In the numerical calculations we use the same set of parameters in chiral quark Lagrangian as fixed in Ref. [30]. In particular, for both sets (Set I and Set II) we used the unified set of parameters  $g, m, c_2, c_4, \bar{d}_{10}$  and  $\bar{e}_{10}$ :

$$\begin{aligned} g &= 0.9, \\ m &= 0.42 \text{ GeV}, \\ c_2 &= 2.502 \text{ GeV}^{-1}, \\ c_4 &= 1.693 \text{ GeV}^{-1}, \\ \bar{d}_{10} &= 1.110 \text{ GeV}^{-2}, \\ \bar{e}_{10} &= 0.039 \text{ GeV}^{-3} \end{aligned}$$

For the parameters  $\tilde{c}_6, \bar{e}_7$  and  $\bar{e}_8$  we used a slightly different values in Set I and Set II because they have been fixed from the fit of the magnetic moments of proton, neutron and  $\Lambda$ -hyperon:

Set I

$$\tilde{c}_6 = 0.593, \quad \bar{e}_7 = -0.473 \text{ GeV}^{-3}, \quad \bar{e}_8 = 0.013 \text{ GeV}^{-3}$$

Set II

$$\tilde{c}_6 = 0.569, \quad \bar{e}_7 = -0.649 \text{ GeV}^{-3}, \quad \bar{e}_8 = 0.031 \text{ GeV}^{-3}$$

Here  $\tilde{c}_6 = c_6 - 16m(2\hat{m} + \hat{m}_s)B\bar{e}_6$  and the couplings  $\bar{d}_{10}, \bar{e}_6, \bar{e}_7, \bar{e}_8$  and  $\bar{e}_{10}$  refer to the renormalized coupling constants (see details in Ref. [30]).

We present here the obtained results for electromagnetic properties of hyperons. For completeness we also present our results for magnetic moments and slopes of nucleons. The resulting values for the magnetic moments of the baryon octet for this case (Set I) are shown in Table 4.1, where reasonable agreement with data is obtained.

	Set I			Set II			Exp.
	3q	Meson Cloud	Total	3q	Meson Cloud	Total	
$\mu_p$	2.357	0.436	2.793	2.357	0.436	2.793	2.793
$\mu_n$	-1.571	-0.342	-1.913	-1.571	-0.342	-1.913	-1.913
$\mu_{\Lambda^0}$	-0.786	0.173	-0.613	-0.518	-0.095	-0.613	$-0.613 \pm 0.004$
$\mu_{\Sigma^+}$	2.357	0.317	2.674	2.085	0.373	2.458	$2.458 \pm 0.010$
$\mu_{\Sigma^0}$	0.786	0.005	0.791	0.570	0.073	0.643	...
$\mu_{\Sigma^-}$	-0.786	-0.306	-1.092	-0.935	-0.225	-1.160	$-1.160 \pm 0.025$
$\mu_{\Xi^0}$	-1.571	0.136	-1.435	-1.058	-0.192	-1.250	$-1.250 \pm 0.014$
$\mu_{\Xi^-}$	-0.7855	0.2921	-0.4934	-0.5580	-0.0927	-0.6507	$-0.6507 \pm 0.003$
$ \mu_{\Sigma^0 \Lambda^0} $	1.36	0.27	1.63	1.34	0.27	1.61	$1.61 \pm 0.08$

**Table 4.1** Magnetic moments of the baryon octet (in units of the nucleon magneton)

Due to our analysis, meson cloud contributions to the total values of the magnetic moments are about 5 – 30% depending on the baryon.

Finally, the charged and magnetic form factors of hyperons are present in Fig. 4.1 and Fig. 4.2, respectively.

In conclusion, we have reported the study of the electromagnetic properties of hyperons in the Perturbative Chiral Quark Model (PCQM). By following the same technique as was done in Ref.[30] for the electromagnetic properties of nucleon, this serves as an input for further study on hyperons where available data are not completed. Meson cloud shows significant contribution as expected from previous analysis, however, this contribution strongly dependent on the type of hyperons.

For the hyperons magnetic moments, two sets of parameters are reported. One of these sets are chosen so that the exact total magnetic moments reproduced the available experimental data, while another set relaxes these constraints. However, results obtained from these two sets are not much different. Furthermore, we have predicted the value of the magnetic moment of  $\Sigma^0$ , which is not reported.

The electromagnetic form factors of hyperons are also reported. Due to the lack of experimental data, the best we can do is to model the possibility of the form factors by varying the cutoff parameters introduced in our analysis.

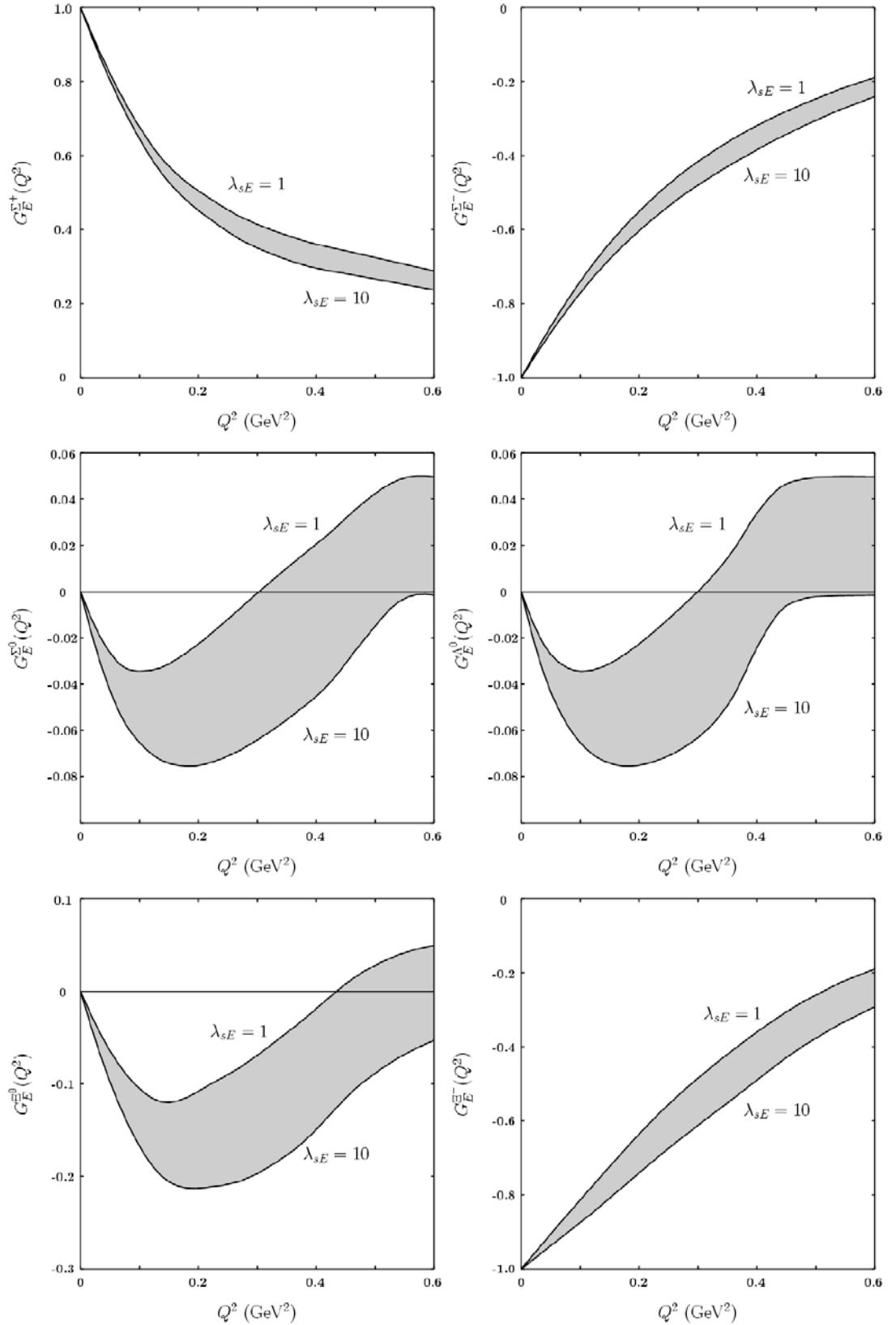


Fig. 4.1 The charge form factors of  $\Sigma^+$ ,  $\Sigma^-$ ,  $\Sigma^0$ ,  $\Lambda^0$ ,  $\Xi^0$  and  $\Xi^-$  baryons. The shaded region shows the range of the form factors with the parameter  $\lambda_{sE}$  being varied in the interval from 1 to 10 in units of  $\text{GeV}$ .

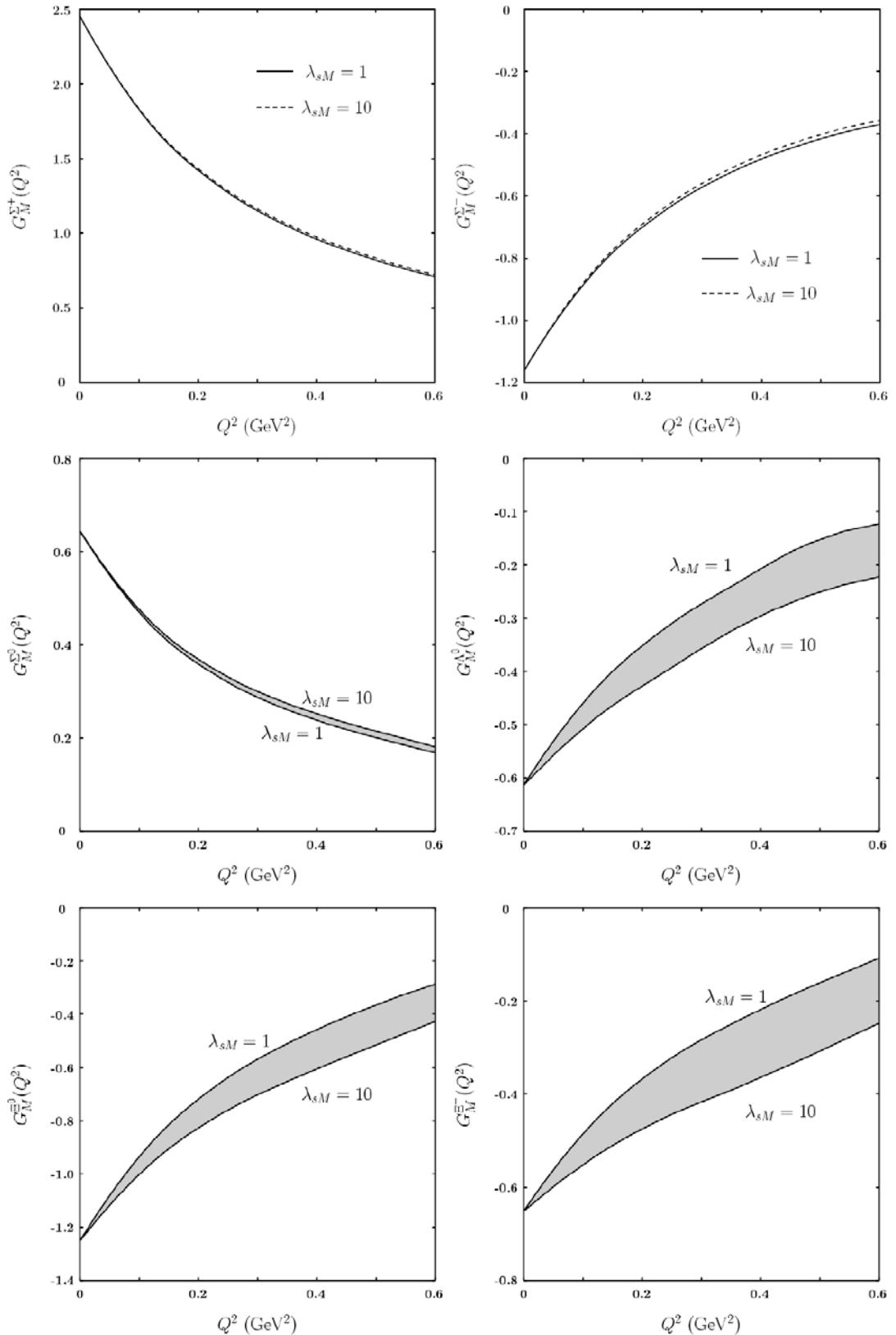


Fig. 4.2 The magnetic form factors of  $\Sigma^+$ ,  $\Sigma^-$ ,  $\Sigma^0$ ,  $\Lambda^0$ ,  $\Xi^0$  and  $\Xi^-$  baryons. The shaded region shows the range of the form factors with the parameter  $\lambda_{sM}$  being varied in the interval from 1 to 10 in units of GeV.

## References

- [1] M. K. Jones et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 84, 1398 (2000) [arXiv:nucl-ex/9910005].
- [2] O. Gayou et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 88, 092301 (2002) [arXiv:nucl-ex/0111010].
- [3] V. Punjabi et al., Phys. Rev. C 71, 055202 (2005) [Erratum-ibid. C 71, 069902 (2005)] [arXiv:nucl-ex/0501018].
- [4] X. Zhan et al., Phys. Lett. B 705, 59-64 (2011).
- [5] M. Gell-Mann, Phys. Lett. 8, 214 (1964).
- [6] G. Zweig, CERN-8419-TH-412, 1964.
- [7] S. Weinberg, PhysicaA 96, 327 (1979).
- [8] J. Gasser and H. Leutwyler, Annals Phys. 158, 142 (1984).
- [9] J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985).
- [10] E. Jenkins and A. V. Manohar, Phys. Lett. B 255 558 (1991); V. Bernard, N. Kaiser, J. Kambor and U. G. Meissner, Nucl. Phys. B 388 315 (1992).
- [11] T. Becher and H. Leutwyler, Eur. Phys. J. C 9 643 (1999) [arXiv:hepph/9901384]; JHEP 0106 017 (2001) [arXiv:hep-ph/0103263].
- [12] P. J. Ellis and H. B. Tang, Phys. Rev. C 57 3356 (1998) [arXiv:hepph/9709354] H. B. Tang, arXiv:hep-ph/9607436.

[13] J. Gegelia and G. Japaridze, Phys. Rev. D 60, 114038 (1999) [arXiv:hepph/9908377].

[14] M. R. Schindler, J. Gegelia and S. Scherer, Phys. Lett. B 586, 258 (2004) [arXiv:hep-ph/0309005].

[15] A. Faessler, T. Gutsche, V. E. Lyubovitskij and K. Pumsa-Ard, Prog. Part. Nucl. Phys. 55, 12 (2005).

[16] A. Faessler, T. Gutsche, V. E. Lyubovitskij and K. Pumsa-ard, arXiv:hepph/0511319.

[17] B. Kubis and U. G. Meissner, Eur. Phys. J. C 18 747 (2001) [arXiv:hepph/0010283].

[18] J. L. Goity, D. Lehmann, G. Prezeau and J. Saez, Phys. Lett. B 50421 (2001) [arXiv:hep-ph/0101011]; D. Lehmann and G. Prezeau, Phys. Rev. D 65 016001 (2002) [arXiv:hep-ph/0102161].

[19] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).

[20] S. Theberge, A. W. Thomas and G. A. Miller, Phys. Rev. D 22 2838 (1980) [Erratum-ibid. D 23 2106 (1981)]; Phys. Rev. D 24 216 (1981); S. Theberge and A. W. Thomas, Nucl. Phys. A 393 252 (1983); A. W. Thomas, Adv. Nucl. Phys. 13 1 (1984).

[21] E. Oset, R. Tegen and W. Weise, Nucl. Phys. A 426 456 (1984) [Erratumibid. A 453 751 (1986)]; R. Tegen, Annals Phys. 197 439 (1990); S. A. Chin, Nucl. Phys. A 382 355 (1982).

[22] T. Gutsche and D. Robson, Phys. Lett. B 229, 333 (1989).

[23] D. Diakonov and V. Y. Petrov, Nucl. Phys. B 245 259 (1984); Nucl. Phys. B 272 457 (1986); D. Diakonov, V. Y. Petrov and M. Praszalowicz, Nucl. Phys. B 323 53 (1989).

[24] T. Gutsche, Ph.D. thesis, Florida State University, 1987.

[25] S. Eidelman et al. [Particle Data Group Collaboration], Phys. Lett. B 592 1 (2004).

[26] J. Gasser, M. E. Sainio and A. 'Svarc, Nucl. Phys. B 307 779 (1988).

[27] T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D 68, 056005 (2003) [arXiv:hep-ph/0302117]; Eur. Phys. J. A 19, 35 (2004) [arXiv:hepph/0309234].

[28] T. Fuchs, M. R. Schindler, J. Gegelia and S. Scherer, Phys. Lett. B 575, 11 (2003) [arXiv:hep-ph/0308006].

[29] M. R. Schindler, J. Gegelia and S. Scherer, Eur. Phys. J. A 26, 1 (2005) [arXiv:nucl-th/0509005].

[30] A. Faessler, T. Gutsche, V. E. Lyubovitskij and K. Pumsa-ard, Phys. Rev. D 73, 114021 (2006) [arXiv:hep-ph/0511319].

[31] M. N. Rosenbluth, Phys. Rev. 79 615 (1950).

[32] L. N. Hand, D. G. Miller and R. Wilson, Rev. Mod. Phys. 35, 335 (1963).

[33] A. I. Akhiezer, L. N. Rozentsweig and I. M. Shmushkevich, Sov. Phys. JETP 6, 588 (1958).

[34] I. A. Qattan et al., Phys. Rev. Lett. 94, 142301 (2005) [arXiv:nuclex/0410010].

[35] T. Becher and H. Leutwyler, Eur. Phys. J. C 9, 643 (1999) [arXiv:hep-ph/9901384]; JHEP 0106, 017 (2001) [arXiv:hepph/0103263].

[36] J. Gasser, M. E. Sainio and A. 'Svarc, Nucl. Phys. B 307, 779 (1988).

[37] N. Fettes, U. G. Meissner and S. Steininger, Nucl. Phys. A 640, 199 (1998) [arXiv:hep-ph/9803266].

[38] J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985).

[39] A. Faessler, T. Gutsche, B. R. Holstein, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, arXiv:hep-ph/0608015.

[40] M. A. Ivanov, M. P. Locher and V. E. Lyubovitskij, Few Body Syst. 21, 131 (1996).

[41] M. A. Ivanov, V. E. Lyubovitskij, J. G. Koerner and P. Kroll, Phys. Rev. D 56, 348 (1997) [arXiv:hep-ph/9612463]; M. A. Ivanov, J. G. Koerner and V. E. Lyubovitskij, Phys. Lett. B 448, 143 (1999) [arXiv:hep-ph/9811370]; M. A. Ivanov, J. G. Koerner, V. E. Lyubovitskij, M. A. Pisarev and A. G. Rusetsky, Phys. Rev. D 61, 114010 (2000) [arXiv:hepph/9911425].

[42] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, Phys. Rev. D 73, 094013 (2006) [arXiv:hep-ph/0602193].

# Electromagnetic form factors of hyperons in a Lorentz covariant chiral quark approach

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We study electromagentic form factors of hyperons using a chiral quark model. In this model baryons are bound states of constituent quarks dressed by a cloud of pseudoscalar mesons. In a first step, this Lagrangian can be used to perform a dressing of the constituent quarks by a cloud of light pseudoscalar mesons and other heavy states using the calculational technique of infrared dimensional regularization of loop diagrams. Then the dressed transition operators are used to calculate the baryon matrix elements. We use the parameterization of baryon form factors in terms of quark form factors in the SU(6). The parameters fitted from the nucleon electromagnetic properties are used as an input for the calculation of hyperons electromagenetic form factors and their magnetic moments.

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## I. INTRODUCTION

Analysis of the electromagnetic form factors of light baryons helps to understand their internal structure. In particular, baryons as extended objects are characterized by a set of electromagnetic properties: magnetic moments, radii and form factors. At present, most of these quantities are well known for nucleons: proton and neutron. For hyperons data rarely exist with the exception of the magnetic moments. A few years ago, the charge radius of the  $\Sigma^-$  has been measured by the SELEX Collaboration at FNAL [1] and WA89 Collaboration at CERN [2]. It gave a first estimate of the charge form factor of the hyperon at low momentum transfers. On the other hand, a systematic study of electromagnetic radii and form factors of hyperons can help to investigate an impact of strangeness on the hadronic properties. Therefore, a comprehensive theoretical study of electromagnetic properties of hyperons is important and promising task. Calculation of electromagnetic properties of hyperons have been carried out in the framework of different approaches: QCD sum rules [3], Lattice QCD [4], Chiral Perturbation Theory (ChPT) [5], QCD string approach [6],  $1/N_c$ -expansion [7], different types of soliton and quark models [8]-[20], etc.

In the manuscript we proceed as follows. First, in Section II, we discuss basic notions of our approach. We derive the chiral Lagrangian motivated by baryon ChPT [21]-[28], and formulate it in terms of quark and mesonic degrees of freedom. Next, we use this Lagrangian to perform a dressing of the constituent quarks by a cloud of light pseudoscalar mesons and by other heavy states, using the calculational technique developed in Ref. [21]. We derive dressed transition operators within a proper chiral expansion, which are in turn relevant for the interaction of quarks with external fields in the presence of a virtual meson cloud. Then we discuss the calculation of matrix elements of dressed quark operators between baryons states using a model-independent formalism based on symmetry constraints applied previously for the case of nucleon form factors [19]. In Section III, we present our results for electromagnetic properties of hyperons. In Section IV we present a short summary of our results.

## II. APPROACH

### A. Chiral Lagrangian

The chiral quark Lagrangian  $\mathcal{L}_{qU}$  (up to order  $p^4$ ), which dynamically generates the dressing of the constituent quarks by mesonic degrees of freedom, consists of two primary pieces  $\mathcal{L}_q$  and  $\mathcal{L}_U$ :

$$\mathcal{L}_{qU} = \mathcal{L}_q + \mathcal{L}_U, \quad \mathcal{L}_q = \mathcal{L}_q^{(1)} + \mathcal{L}_q^{(2)} + \mathcal{L}_q^{(3)} + \mathcal{L}_q^{(4)} + \dots, \quad \mathcal{L}_U = \mathcal{L}_U^{(2)} + \dots. \quad (1)$$

The superscript  $(i)$  attached to  $\mathcal{L}_{q(U)}^{(i)}$  denotes the low energy dimension of the Lagrangian:

$$\begin{aligned} \mathcal{L}_U^{(2)} &= \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle, \quad \mathcal{L}_q^{(1)} = \bar{q} \left[ i \not{D} - m + \frac{1}{2} g \not{\gamma}^5 \right] q, \\ \mathcal{L}_q^{(2)} &= -\frac{c_2}{4m^2} \langle u_\mu u_\nu \rangle (\bar{q} D^\mu D^\nu q + \text{h.c.}) + \frac{c_4}{4} \bar{q} i \sigma^{\mu\nu} [u_\mu, u_\nu] q + \frac{c_6}{8m} \bar{q} \sigma^{\mu\nu} F_{\mu\nu}^+ q + \dots, \\ \mathcal{L}_q^{(3)} &= \frac{id_{10}}{2m} \bar{q} [D^\mu, F_{\mu\nu}^+] D^\nu q + \text{h.c.} + \dots, \\ \mathcal{L}_q^{(4)} &= \frac{e_6}{2} \langle \chi_+ \rangle \bar{q} \sigma^{\mu\nu} F_{\mu\nu}^+ q + \frac{e_7}{4} \bar{q} \sigma^{\mu\nu} \{F_{\mu\nu}^+, \hat{\chi}_+\} q + \frac{e_8}{2} \bar{q} \sigma^{\mu\nu} \langle F_{\mu\nu}^+, \hat{\chi}_+ \rangle q - \frac{e_{10}}{2} \bar{q} [D^\alpha, [D_\alpha, F_{\mu\nu}^+]] \sigma^{\mu\nu} q + \dots, \end{aligned} \quad (2)$$

where  $\hat{\chi}_+ = \chi_+ - \frac{1}{3} \langle \chi_+ \rangle$ , the symbols  $\langle \rangle$ ,  $[\cdot]$  and  $\{ \cdot \}$  occurring in Eq. (2) denotes the trace over flavor matrices, commutator and anticommutator, respectively. In Eq. (2) we display only the terms involved in the calculation of the dressed electromagnetic quark operator. Also include vector mesons (see details in Ref. [19]). The detailed form of the chiral Lagrangian can be found in Ref. [19]. The couplings  $m$  and  $g$  denote the quark mass and axial charge in the chiral limit,  $c_i$ ,  $d_i$  and  $e_i$  are the second-, third- and fourth-order low-energy coupling constants, respectively, which encode the contributions of heavy states. Parameter  $m$  is counted as a quantity of order  $O(1)$  in the chiral expansion.

Here  $q$  is the quark field, and the octet of pseudoscalar fields

$$\phi = \sum_{i=1}^8 \phi_i \lambda_i = \sqrt{2} \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} \quad (3)$$

is contained in the SU(3) matrix  $U = u^2 = \exp(i\phi/F)$  where  $F$  is the octet decay constant. We introduce the standard notations [21, 23, 25]

$$D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}u^\dagger R_\mu u - \frac{i}{2}u L_\mu u^\dagger, \quad (4)$$

$$u_\mu = iu^\dagger \nabla_\mu U u^\dagger, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B\mathcal{M} + \dots$$

The fields  $R_\mu$  and  $L_\mu$  include external fields (electromagnetic  $A_\mu$ , weak, etc.):  $R_\mu = eQ A_\mu + \dots$ ,  $L_\mu = eQ A_\mu + \dots$  where  $Q = \text{diag}\{2/3, -1/3, -1/3\}$  is the quark charge matrix. The tensor  $F_{\mu\nu}^+$  is defined as  $F_{\mu\nu}^+ = u^\dagger F_{\mu\nu}^+ Qu + u F_{\mu\nu}^+ Qu^\dagger$  where  $F_{\mu\nu}^+ = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the conventional photon field strength tensor. Here  $\mathcal{M} = \text{diag}\{\hat{m}_u, \hat{m}_d, \hat{m}_s\}$  is the mass matrix of current quarks (we work in the isospin symmetry limit with  $\hat{m}_u = \hat{m}_d = \hat{m} = 7$  MeV and the mass of the strange quark  $\hat{m}_s$  is related to the nonstrange one as  $\hat{m}_s = 25\hat{m}$ ).

The quark vacuum condensate parameter is denoted by  $B = -\langle 0|\bar{u}u|0\rangle/F^2 = -\langle 0|\bar{d}d|0\rangle/F^2$ . To distinguish between constituent and current quark masses we attach the symbol  $\hat{\phantom{a}}$  ("hat") when referring to the current quark masses. We rely on the standard picture of chiral symmetry breaking ( $B \gg F$ ). In leading order of the chiral expansion the masses of pseudoscalar mesons are given by  $M_\pi^2 = 2\hat{m}B$ ,  $M_K^2 = (\hat{m} + \hat{m}_s)B$ ,  $M_\eta^2 = (2/3)(\hat{m} + 2\hat{m}_s)B$ . In the numerical analysis we will use:  $M_\pi = 139.57$  MeV,  $M_K = 493.677$  MeV (the charged pion and kaon masses),  $M_\eta = 574.75$  MeV and the canonical set of differentiated decay constants:  $F_\pi = 92.4$  MeV,  $F_K/F_\pi = 1.22$  and  $F_\eta/F_\pi = 1.3$  [32].

## B. Dressing of quark operators

Any bare quark operator (both one- and two-body) can be dressed by a cloud of pseudoscalar mesons and heavy states in a straightforward manner by use of the effective chirally-invariant Lagrangian  $\mathcal{L}_{qU}$ . To illustrate the idea of such a dressing we consider the Fourier-transform of the electromagnetic quark operator:

$$J_{\mu, \text{em}}^{\text{bare}}(q) = \int d^4x e^{-iqx} j_{\mu, \text{em}}^{\text{bare}}(x), \quad j_{\mu, \text{em}}^{\text{bare}}(x) = \bar{q}(x) \gamma_\mu Q q(x). \quad (5)$$

In Fig.1 we display the tree and loop diagrams which contribute to the dressed electromagnetic operator  $J_{\mu, \text{em}}^{\text{dress}}$  up to fourth order. Additional diagrams including the vector-meson contributions are shown in Fig.2 of Ref. [19]. Note, here we restrict our consideration to the one-body quark operator. An extension of our method to two-body quark operators will be done in future.

The dressed quark operator  $j_{\mu, \text{em}}^{\text{dress}}(x)$  and its Fourier transform  $J_{\mu, \text{em}}^{\text{dress}}(q)$  have the following forms

$$j_{\mu, \text{em}}^{\text{dress}}(x) = \sum_{q=u,d,s} \left\{ f_D^q(-\partial^2) [\bar{q}(x) \gamma_\mu q(x)] + \frac{f_P^q(-\partial^2)}{2m_q} \partial^\nu [\bar{q}(x) \sigma_{\mu\nu} q(x)] \right\} \quad (6)$$

$$J_{\mu, \text{em}}^{\text{dress}}(q) = \int d^4x e^{-iqx} j_{\mu, \text{em}}^{\text{dress}}(x) = \int d^4x e^{-iqx} \sum_{q=u,d,s} \bar{q}(x) \left[ \gamma_\mu f_D^q(q^2) + \frac{i}{2m_q} \sigma_{\mu\nu} q^\nu f_P^q(q^2) \right] q(x),$$

where  $m_q$  is the dressed constituent quark mass generated by the chiral Lagrangian (2) (see details in Ref. [19]);  $f_D^u(q^2)$ ,  $f_D^d(q^2)$ ,  $f_D^s(q^2)$  and  $f_P^u(q^2)$ ,  $f_P^d(q^2)$ ,  $f_P^s(q^2)$  are the Dirac and Pauli form factors of  $u$ ,  $d$  and  $s$  quarks. Here we use the appropriate sub- and superscripts with a definite normalization of the set of  $f_D^q(0) \equiv e_q$  (quark charges) due to charge conservation. Note, that the dressed quark operator satisfies current conservation:  $\partial^\mu j_{\mu, \text{em}}^{\text{dress}}(x) = 0$ . Evaluation of the diagrams in Fig.1 is based on the *infrared dimensional regularization* (IDR) suggested in Ref. [21] to guarantee a straightforward connection between loop and chiral expansion in terms of quark masses and small external momenta. We relegate the discussion of the calculational technique to Ref. [19].

To calculate the electromagnetic transitions between baryons we project the dressed quark operator between the corresponding baryon states. The master formula is:

$$\langle B(p') | J_{\mu, \text{em}}^{\text{dress}}(q) | B(p) \rangle = (2\pi)^4 \delta^4(p' - p - q) \bar{u}_B(p') \left\{ \gamma_\mu F_1^B(q^2) + \frac{i}{2m_B} \sigma_{\mu\nu} q^\nu F_2^B(q^2) \right\} u_B(p)$$

$$= (2\pi)^4 \delta^4(p' - p - q) \sum_{q=u,d,s} \left\{ f_D^q(q^2) \langle B(p') | j_{\mu,q}^{\text{bare}}(0) | B(p) \rangle + i \frac{q^\nu}{2m_q} f_P^q(q^2) \langle B(p') | j_{\mu\nu,q}^{\text{bare}}(0) | B(p) \rangle \right\}, \quad (7)$$

where  $F_1^B(q^2)$  and  $F_2^B(q^2)$  are the Dirac and Pauli baryon form factors,  $B(p)$  and  $u_B(p)$  are the baryon state and spinor, respectively, normalized as

$$\langle B(p') | B(p) \rangle = 2E_B (2\pi)^3 \delta^3(\vec{p} - \vec{p}'), \quad \bar{u}_B(p) u_B(p) = 2m_B \quad (8)$$

with  $E_B = \sqrt{m_B^2 + \vec{p}^2}$  being the baryon energy and  $m_B$  the baryon mass. Eq. (7) deals with the diagonal  $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$  transitions. The extension to the nondiagonal transitions and transitions involving higher spin states like the  $\Delta(1232)$  isobar is straightforward (see Ref. [20]). In Eq. (7) we express the matrix elements of the dressed quark operator in terms of the matrix elements of the bare operators for vector  $j_{\mu,q}^{\text{bare}}(0)$  and tensor  $j_{\mu\nu,q}^{\text{bare}}(0)$  currents defined as

$$j_{\mu,q}^{\text{bare}}(0) = \bar{q}(0) \gamma_\mu q(0), \quad j_{\mu\nu,q}^{\text{bare}}(0) = \bar{q}(0) \sigma_{\mu\nu} q(0). \quad (9)$$

In general, due to Lorentz and gauge invariance, the matrix elements in Eq. (7) can be written as

$$\begin{aligned} \langle B(p') | j_{\mu,q}^{\text{bare}}(0) | B(p) \rangle &= \bar{u}_B(p') \left\{ \gamma_\mu F_1^{Bq}(q^2) + \frac{i}{2m_B} \sigma_{\mu\nu} q^\nu F_2^{Bq}(q^2) \right\} u_B(p), \\ i \frac{q^\nu}{2m_q} \langle B(p') | j_{\mu\nu,q}^{\text{bare}}(0) | B(p) \rangle &= \bar{u}_B(p') \left\{ \gamma_\mu G_1^{Bq}(q^2) + \frac{i}{2m_B} \sigma_{\mu\nu} q^\nu G_2^{Bq}(q^2) \right\} u_B(p), \end{aligned} \quad (10)$$

where  $F_{1(2)}^{Bq}(q^2)$  and  $G_{1(2)}^{Bq}(q^2)$  are the Pauli and Dirac form factors describing the distribution of quarks of flavor  $q = u, d, s$  in the baryon  $B$ . Finally, the baryon form factors  $F_i^B(q^2)$  with  $i = 1, 2$  can be separated into a bare part  $F_i^B \text{bare}(q^2)$  and a meson cloud part  $F_i^B \text{cloud}(q^2)$  as

$$\begin{aligned} F_i^B(q^2) &= F_i^B \text{bare}(q^2) + F_i^B \text{cloud}(q^2) \\ F_i^B \text{bare}(q^2) &= \sum_{q=u,d,s} e_q F_i^{Bq}(q^2), \\ F_i^B \text{cloud}(q^2) &= \sum_{q=u,d,s} [(f_D^q(q^2) - e_q) F_i^{Bq}(q^2) + f_P^q(q^2) G_i^{Bq}(q^2)] \end{aligned} \quad (11)$$

where  $e_q$  are the electric quark charges.

Eqs. (7)-(11) contain our main result: we perform a model-independent factorization of the effects of hadronization and confinement contained in the matrix elements of the bare quark operators  $j_{\mu,q}^{\text{bare}}(0)$  and  $j_{\mu\nu,q}^{\text{bare}}(0)$  and the effects dictated by chiral symmetry (or chiral dynamics) which are encoded in the relativistic form factors  $f_D^q(q^2)$  and  $f_P^q(q^2)$ . Due to this factorization the calculation of  $f_D^q(q^2)$  and  $f_P^q(q^2)$ , on one side, and the matrix elements of  $j_{\mu,q}^{\text{bare}}(0)$  and  $j_{\mu\nu,q}^{\text{bare}}(0)$ , on the other side, can be done independently. In particular, in a first step we derived a model-independent formalism based on the ChPT Lagrangian, which is formulated in terms of constituent quark degrees of freedom, for the calculation of  $f_D^q(q^2)$  and  $f_P^q(q^2)$  (see their explicit forms in Appendix C of Ref. [19]).

The calculation of the matrix elements of the bare quark operators (10) can then be relegated to quark models based on specific assumptions about hadronization and confinement. In preceding publications we considered two possibilities for a treatment of valence quark degrees of freedom. In particular, in Ref. [19] we used a parametrization of the bare quark distributions in the baryon (nucleon) with taking into account model-independent constraints dictated by certain symmetries: gauge, isospin and chiral invariance. In Ref. [20] we calculated valence quark form factors explicitly with the use of relativistic quark model [29]-[31] based on a specific ansatz of quarks in baryons. In this paper we complete our analysis started in Ref. [19] where we presented a comprehensive analysis of electromagnetic nucleon properties including magnetic moments, radii and form factors. For the case of hyperons we restricted to the calculation of magnetic moments. Here we present the results for hyperon radii and form factors. As we said we will use a parametrization for the bare quark distribution in baryons.

### C. Matrix elements of the bare quark operators

The matrix elements of the bare quark operators should be calculated using specific model-dependent assumptions about hadronization and confinement. As we mentioned before this possibility has been considered by us in Ref. [20] where we presented a detailed analysis of magnetic moments of light baryons and properties of  $N \rightarrow \Delta\gamma$  transition. In preceding paper [19] we modelled matrix elements of the bare quark operators using certain symmetry constraints leading to a set of relationships between the nucleon and corresponding  $u$ - and  $d$ -quark form factors at zero momentum transfer. Here we extend the idea of Ref. [19] on the case of hyperon form factors.

In case of nucleons one can derive the constraints on the form factors arising from charge conservation, isospin invariance and infrared-singular structure of QCD [19]:

$$\begin{aligned} F_1^{nu}(0) &= F_1^{nd}(0) = 2, & F_1^{pd}(0) &= F_1^{nu}(0) = 1, & G_1^{Nq}(0) &= 0, \\ F_2^{nu}(0) &= F_2^{nd}(0), & F_2^{pd}(0) &= F_2^{nu}(0), & G_2^{nu}(0) &= G_2^{nd}(0), & G_2^{pd}(0) &= G_2^{nu}(0) \end{aligned} \quad (12)$$

and

$$1 + F_2^{pu}(0) - F_2^{pd}(0) = G_2^{pu}(0) - G_2^{pd}(0) = \left(\frac{g_A}{g}\right)^2 \frac{m_N}{\bar{m}}, \quad (13)$$

$$1 + F_2^{nd}(0) - F_2^{nu}(0) = G_2^{nd}(0) - G_2^{nu}(0) = \left(\frac{g_A}{g}\right)^2 \frac{m_N}{\bar{m}} \quad (14)$$

where  $g_A$  and  $\overset{\circ}{m}_N$  are the axial charge and the mass of the nucleon in the chiral limit and  $\bar{m} = m_u = m_d$  is the dressed nonstrange constituent quark mass in the isospin limit.

Restricting our consideration to the one-body quark operator and by using SU(6)-symmetry relations one can relate the Dirac and Pauli form factors describing the distribution of quarks of flavor  $q = u, d, s$  in the baryon "B", that is  $F_{1(2)}^{Bq}$  and  $G_{1(2)}^{Bq}$  to the bare (or valence) quark form factors (see details in Ref. [19]). In particular, one can introduce the bare Dirac ( $F_1^q, G_1^q$ ) and Pauli ( $F_2^q, G_2^q$ ) form factors of the quark of flavor  $q$ :

$$\begin{aligned} \langle q(p') | j_{\mu,q}^{\text{bare}}(0) | q(p) \rangle &= \bar{u}_q(p') \left\{ \gamma_\mu F_1^q(q^2) + \frac{i}{2m_q} \sigma_{\mu\nu} q^\nu F_2^q(q^2) \right\} u_q(p), \\ i \frac{q^\nu}{2m_q} \langle q(p') | j_{\mu\nu,q}^{\text{bare}}(0) | q(p) \rangle &= \bar{u}_q(p') \left\{ \gamma_\mu G_1^q(q^2) + \frac{i}{2m_q} \sigma_{\mu\nu} q^\nu G_2^q(q^2) \right\} u_q(p). \end{aligned} \quad (15)$$

The Sachs form factors of the quark of flavor  $q$  are:

$$\mathcal{F}_q^E(t) = F_q^E(t) + G_q^E(t), \quad \mathcal{F}_q^M(t) = F_q^M(t) + G_q^M(t), \quad (16)$$

where

$$F\{G\}_q^E(t) = F\{G\}_1^q(t) - \frac{t}{4m_q^2} F\{G\}_2^q(t), \quad F\{G\}_q^M(t) = F\{G\}_1^q(t) + F\{G\}_2^q(t), \quad t = -q^2, \quad (17)$$

are the contributions to the Sachs form factors associated with the expectation values of the vector and tensor currents, respectively. Finally, the baryonic form factors  $F_i^{Bq}(t)$  are expressed in term of quark form factors  $F_q^{E(M)}(t)$  and  $G_q^{E(M)}(t)$  by [19]:

$$\begin{aligned} F_1^{Bq}(t) &= \frac{1}{1 + \tau_B} \left\{ \alpha_E^{Bq} F_q^E(t) + \alpha_M^{Bq} \chi^{Bq} F_q^M(t) \tau_B \right\}, \\ F_2^{Bq}(t) &= \frac{1}{1 + \tau_B} \left\{ -\alpha_E^{Bq} F_q^E(t) + \alpha_M^{Bq} \chi^{Bq} F_q^M(t) \right\}, \\ G_1^{Bq}(t) &= \frac{1}{1 + \tau_B} \left\{ \alpha_E^{Bq} G_q^E(t) + \alpha_M^{Bq} \chi^{Bq} G_q^M(t) \tau_B \right\}, \\ G_2^{Bq}(t) &= \frac{1}{1 + \tau_B} \left\{ -\alpha_E^{Bq} G_q^E(t) + \alpha_M^{Bq} \chi^{Bq} G_q^M(t) \right\}, \end{aligned} \quad (18)$$

where  $F_q^E(t)$  and  $F_q^M(t)$  are the quark Sachs form factors,  $\tau_B = t/(4m_B^2)$ . In addition to the strict evaluation of SU(6) we have introduced the additional parameter  $\chi^{Bq}$  for each quark of flavor  $q$ . The interpretation for adding these factors is such that to allow the quark distributions for hyperons to be different from that for the nucleons. In the case of the nucleons we set  $\chi^{Bq} = 1$ . The values for  $\alpha_E^{Bq}$  and  $\alpha_M^{Bq}$  for the baryon octet as derived from SU(6)-symmetry relations are given in Table I.

In Ref. [19] we considered only nucleon form factors and therefore, the Sachs form factors of  $u$ - and  $d$ -quark. In particular, we modeled the  $u$ - and  $d$ -quark form factors by the dipole characteristics with damping functions of an exponential form. This phenomenological form is required to reproduce the deviation of the electromagnetic form factors of the nucleon from the dipole fit as evident from recent experimental measurements. In this paper we use the same parametrization for  $s$ -quark. Therefore, for the Sachs form factors of  $u$ -,  $d$ - and  $s$ -quark we use the parameterization

$$\begin{aligned} F_q^E(t) &= \frac{\rho_q^E(t)}{[1 + t/\Lambda_{qE}^2]^2}, & F_q^M(t) &= \mu_q^F \frac{\rho_q^M(t)}{[1 + t/\Lambda_{qM}^2]^2}, \\ G_q^E(t) &= \gamma_q \rho_q^E(t) \frac{t/\Lambda_{qE}^2}{[1 + t/\Lambda_{qE}^2]^3}, & G_q^M(t) &= \mu_q^G \frac{\rho_q^M(t)}{[1 + t/\Lambda_{qM}^2]^2}, \end{aligned} \quad (19)$$

where  $\rho_q^E(t) = \exp(-t/\lambda_{qE}^2)$  and  $\rho_q^M(t) = \exp(-t/\lambda_{qM}^2)$ . Note, that in Ref. [34] a similar parametrization of the nucleon form factors has been considered. In Ref. [35] the damping functions  $\rho(t)$  have been parametrized with constant values. For convenience we suppose that the quark Sachs form factors degenerate at zero recoil according to SU(6) symmetry. In other words all effects of possible SU(6) symmetry-breaking are encoded in the coefficients  $\chi^{Bq}$  [see Eq. (18)]. Therefore, the parameters  $\mu_q^F$  and  $\mu_q^G$  are fixed by the SU(6) symmetry and by the set of other symmetry constraints [see Eqs. (13), (12) and Ref. [19]] as:

$$\mu_q^F = \mu_q^G = \frac{3}{5} \left( \frac{g_A}{g} \right)^2 \frac{m_N}{\bar{m}}. \quad (20)$$

The remaining parameters  $\gamma_q$ ,  $\Lambda_{qE(M)}$  and  $\lambda_{qE(M)}$  are free parameters. In the case of  $u$ - and  $d$ -quark the corresponding parameters ( $\gamma_u$ ,  $\gamma_d$ ,  $\Lambda_{uE(M)}$  and  $\lambda_{dE(M)}$ ) have been fixed from the consideration of the full momentum dependence of the nucleon electromagnetic form factors at intermediate and high value of momentum transfer squared:

$$\begin{aligned} \lambda_{uE} &= 2.0043, & \lambda_{dE} &= 0.9996, & \lambda_{uM} &= 7.3367, & \lambda_{dM} &= 2.2954, \\ \Lambda_{uE} &= 0.8616, & \Lambda_{dE} &= 0.9234, & \Lambda_{uM} &= 0.9278, & \Lambda_{dM} &= 1.0722 \\ \gamma_u &= 1.081, & \gamma_d &= 2.596. \end{aligned} \quad (21)$$

The remaining parameters relevant for the strange quark  $\Lambda_{sE}$ ,  $\Lambda_{sM}$ ,  $\lambda_{sE}$ ,  $\lambda_{sM}$  and  $\gamma_s$  can be fixed or varied using the following arguments. A general remark is that an information about hyperon form factors (and as consequence about strange valence form factors) is very poor: we know only their normalization due charge conservation or from a knowledge of magnetic moments. Therefore, for some parameters we use typical values. In particular, we fix  $\gamma_s = 1$  (which is typical for  $u$  and  $d$  quarks). For the cutoff parameters  $\Lambda_{sE}$  and  $\Lambda_{sM}$  we use typical value 1 GeV guaranteeing the correct  $1/t^2$  scaling of the baryon form factors at large  $t$ . A more nontrivial situation is with parameters  $\lambda_{sE}$  and  $\lambda_{sM}$  controlling a deviation of strange quark (or hyperon) form factors from the dipole fit. To our knowledge based on analysis of nucleon form factors, these parameters can be roughly varied from 1 to 10 GeV. This gives a major ambiguity in the description of hyperon form factors.

Finally we specify the parameters  $\chi^{Bq}$  encoding the effects of  $SU(6)$  symmetry breaking and in the chiral Lagrangian (2). They have been fixed in Ref. [19] from the description of magnetic moments of the baryon octet hyperons and nucleon slopes. In particular, in Ref. [19] we considered two scenarios:  $SU(6)$  symmetric case (Set I) and beyond  $SU(6)$  symmetry (Set II). In case of the Set I the couplings  $\chi^{Bq}$  are trivially equal to 1. For the Set II we got [19]:

$$\begin{aligned} \chi^{\Sigma u} &= \chi^{\Sigma d} = 0.963, & \chi^{\Sigma s} &= 0.259, \\ \chi^{\Xi u} &= \chi^{\Xi d} = 0.633, & \chi^{\Xi s} &= 0.694, \\ \chi^{\Sigma \Lambda u} &= \chi^{\Sigma \Lambda d} = 0.988 \end{aligned} \quad (22)$$

relying on isospin symmetry. In the isotriplet  $\Sigma^+$ ,  $\Sigma^0$ , and  $\Sigma^-$  shares the same set of  $\chi^{\Sigma q}$  for the quark of flavor  $q$ , while  $\Xi^0$  and  $\Xi^-$  contains the same set of  $\chi^{\Xi q}$ . The parameters  $\chi^{\Sigma \Lambda u}$  and  $\chi^{\Sigma \Lambda d}$  are directly related to the  $\Sigma - \Lambda$  magnetic transition moment.

In the numerical calculations we use the same set of parameters in chiral Lagrangian (2) as fixed in Ref. [19]. In particular, for both sets (Set I and Set II) we used the unified set of parameters  $g$ ,  $m$ ,  $c_2$ ,  $c_4$ ,  $\bar{d}_{10}$  and  $\bar{e}_{10}$ :

$$g = 0.9, \quad m = 0.42 \text{ GeV}, \quad c_2 = 2.502 \text{ GeV}^{-1}, \quad c_4 = 1.693 \text{ GeV}^{-1}, \quad \bar{d}_{10} = 1.110 \text{ GeV}^{-2}, \quad \bar{e}_{10} = 0.039 \text{ GeV}^{-3}. \quad (23)$$

For the parameters  $\tilde{c}_6$ ,  $\bar{e}_7$  and  $\bar{e}_8$  we used a slightly different values in Set I and Set II because they have been fixed from the fit of the magnetic moments of proton, neutron and  $\Lambda$ -hyperon:

Set I

$$\tilde{c}_6 = 0.593, \quad \bar{e}_7 = -0.473 \text{ GeV}^{-3}, \quad \bar{e}_8 = 0.013 \text{ GeV}^{-3}. \quad (24)$$

Set II

$$\tilde{c}_6 = 0.569, \quad \bar{e}_7 = -0.649 \text{ GeV}^{-3}, \quad \bar{e}_8 = 0.031 \text{ GeV}^{-3}. \quad (25)$$

Here  $\tilde{c}_6 = c_6 - 16m(2\hat{m} + \hat{m}_s)B\bar{e}_6$  and the couplings  $\bar{d}_{10}$ ,  $\bar{e}_6$ ,  $\bar{e}_7$ ,  $\bar{e}_8$  and  $\bar{e}_{10}$  refer to the renormalized coupling constants (see details in Ref. [19]).

### III. RESULTS

Now we discuss obtained results for electromagnetic properties of hyperons. For completeness we also present our results for magnetic moments and slopes of nucleons. Detailed analysis of the nucleon electromagnetic form factors done in Ref. [19]).

The resulting values for the magnetic moments of the baryon octet for this case (Set I) are shown in Table II, where reasonable agreement with data is obtained. Meson cloud contributions to the total values of the magnetic moments are about 5 – 30% depending on the baryon.

Finally, the charged and magnetic form factors of hyperons are present in Fig.2 and Fig.3, respectively.

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[1] M. I. Adamovich *et al.* [WA89 Collaboration], *Eur. Phys. J. C* **8**, 59 (1999).

[2] I. Eschrich *et al.* [SELEX Collaboration], *Phys. Lett. B* **522**, 233 (2001) [arXiv:hep-ex/0106053].

[3] B. L. Ioffe and A. V. Smilga, *Phys. Lett. B* **133**, 436 (1983); T. M. Aliev, A. Ozpineci and M. Savci, *Phys. Rev. D* **66**, 016002 (2002) [Erratum-*ibid. D* **67**, 039901 (2003)] [arXiv:hep-ph/0204035].

[4] D. B. Leinweber, R. M. Woloshyn and T. Draper, *Phys. Rev. D* **43**, 1659 (1991); F. X. Lee, R. Kelly, L. Zhou and W. Wilcox, *Phys. Lett. B* **627**, 71 (2005) [arXiv:hep-lat/0509067]; S. Boinepalli, D. B. Leinweber, A. G. Williams, J. M. Zanotti and J. B. Zhang, arXiv:hep-lat/0604022.

[5] E. Jenkins, M. E. Luke, A. V. Manohar and M. J. Savage, *Phys. Lett. B* **302**, 482 (1993) [Erratum-*ibid. B* **388**, 866 (1996)] [arXiv:hep-ph/9212226]; B. R. Holstein, *Int. J. Mod. Phys. E* **9**, 359 (2000) [arXiv:hep-ph/0010125]; B. Kubis and U. G. Meissner, *Eur. Phys. J. C* **18**, 747 (2001) [arXiv:hep-ph/0010283]; M. J. Savage, *Nucl. Phys. A* **700**, 359 (2002) [arXiv:nucl-th/0107038]. D. Arndt and B. C. Tiburzi, *Phys. Rev. D* **68**, 094501 (2003) [arXiv:hep-lat/0307003].

[6] B. O. Kerbikov and Yu. A. Simonov, *Phys. Rev. D* **62**, 093016 (2000) [arXiv:hep-ph/0001243].

[7] A. J. Buchmann and R. F. Lebed, *Phys. Rev. D* **67**, 016002 (2003) [arXiv:hep-ph/0207358].

[8] J. Bijnens, H. Sonoda and M. B. Wise, *Phys. Lett. B* **140**, 421 (1984); J. Kunz and P. J. Mulders, *Phys. Rev. D* **41**, 1578 (1990); J. Kunz, P. J. Mulders and G. A. Miller, *Phys. Lett. B* **255**, 11 (1991); N. W. Park and H. Weigel, *Nucl. Phys. A* **541**, 453 (1992);

[9] H. C. Kim, A. Blotz, M. V. Polyakov and K. Goeke, *Phys. Rev. D* **53**, 4013 (1996) [arXiv:hep-ph/9504363]; C. V. Christov *et al.*, *Prog. Part. Nucl. Phys.* **37**, 91 (1996) [arXiv:hep-ph/9604441].

[10] T. A. DeGrand, R. L. Jaffe, K. Johnson and J. E. Kiskis, *Phys. Rev. D* **12**, 2060 (1975).

[11] N. Isgur and G. Karl, *Phys. Rev. D* **21**, 3175 (1980).

[12] S. Theberge and A. W. Thomas, *Nucl. Phys. A* **393**, 252 (1983).

[13] N. Barik and B. K. Dash, *Phys. Rev. D* **34**, 2803 (1986).

[14] M. Warns, W. Pfeil and H. Rollnik, *Phys. Lett. B* **258**, 431 (1991).

[15] G. Wagner, A. J. Buchmann and A. Faessler, *Phys. Lett. B* **359**, 288 (1995) [arXiv:nucl-th/9507032].

[16] M. S. Bae and J. A. McGovern, *J. Phys. G* **22**, 199 (1996) [arXiv:hep-ph/9509366].

[17] T. Van Cauteren, D. Merten, T. Corthals, S. Janssen, B. Metsch, H. R. Petry and J. Ryckebusch, *Eur. Phys. J. A* **20**, 283 (2004) [arXiv:nucl-th/0310058].

[18] S. Cheedket, V. E. Lyubovitskij, T. Gutsche, A. Faessler, K. Pumsa-ard and Y. Yan, *Eur. Phys. J. A* **20**, 317 (2004) [arXiv:hep-ph/0212347].

[19] A. Faessler, T. Gutsche, V. E. Lyubovitskij and K. Pumsa-ard, *Phys. Rev. D* **73**, 114021 (2006) [arXiv:hep-ph/0511319].

[20] A. Faessler, T. Gutsche, B. R. Holstein, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, arXiv:hep-ph/0608015.

[21] T. Becher and H. Leutwyler, *Eur. Phys. J. C* **9**, 643 (1999) [arXiv:hep-ph/9901384]; *JHEP* **0106**, 017 (2001) [arXiv:hep-ph/0103263].

[22] P. J. Ellis and H. B. Tang, *Phys. Rev. C* **57**, 3356 (1998) [arXiv:hep-ph/9709354]; H. B. Tang, arXiv:hep-ph/9607436.

[23] J. Gasser, M. E. Sainio and A. Švarc, *Nucl. Phys. B* **307**, 779 (1988).

[24] E. Jenkins and A. V. Manohar, *Phys. Lett. B* **255**, 558 (1991); V. Bernard, N. Kaiser, J. Kambor and U. G. Meissner, *Nucl. Phys. B* **388**, 315 (1992).

[25] N. Fettes, U. G. Meissner and S. Steininger, *Nucl. Phys. A* **640**, 199 (1998) [arXiv:hep-ph/9803266].

[26] J. Gegelia and G. Japaridze, *Phys. Rev. D* **60**, 114038 (1999) [arXiv:hep-ph/9908377]; M. R. Schindler, J. Gegelia and S. Scherer, *Phys. Lett. B* **586**, 258 (2004) [arXiv:hep-ph/0309005].

[27] B. Kubis and U. G. Meissner, *Nucl. Phys. A* **679**, 698 (2001) [arXiv:hep-ph/0007056]; *Eur. Phys. J. C* **18**, 747 (2001) [arXiv:hep-ph/0010283].

[28] T. Fuchs, J. Gegelia and S. Scherer, *Eur. Phys. J. A* **19**, 35 (2004) [arXiv:hep-ph/0309234]; M. R. Schindler, J. Gegelia and S. Scherer, *Eur. Phys. J. A* **26**, 1 (2005) [arXiv:nucl-th/0509005].

[29] M. A. Ivanov, M. P. Locher and V. E. Lyubovitskij, *Few Body Syst.* **21**, 131 (1996).

[30] M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner and P. Kroll, *Phys. Rev. D* **56**, 348 (1997) [arXiv:hep-ph/9612463]; M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, *Phys. Lett. B* **448**, 143 (1999) [arXiv:hep-ph/9811370]; M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, M. A. Pisarev and A. G. Rusetsky, *Phys. Rev. D* **61**, 114010 (2000) [arXiv:hep-ph/9911425]; M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and A. G. Rusetsky, *Phys. Rev. D* **60**, 094002 (1999) [arXiv:hep-ph/9904421]; M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and A. G. Rusetsky, *Phys. Lett. B* **476**, 58 (2000) [arXiv:hep-ph/9910342]; A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, *Phys. Lett. B* **518**, 55 (2001) [arXiv:hep-ph/0107205].

[31] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, *Phys. Rev. D* **73**, 094013 (2006) [arXiv:hep-ph/0602193].

[32] J. Gasser and H. Leutwyler, *Nucl. Phys. B* **250**, 465 (1985).

[33] M. A. B. Bég and A. Zepeda, *Phys. Rev. D* **6**, 2912 (1972).

[34] J. J. Kelly, *Phys. Rev. C* **66**, 065203 (2002) [arXiv:hep-ph/0204239].

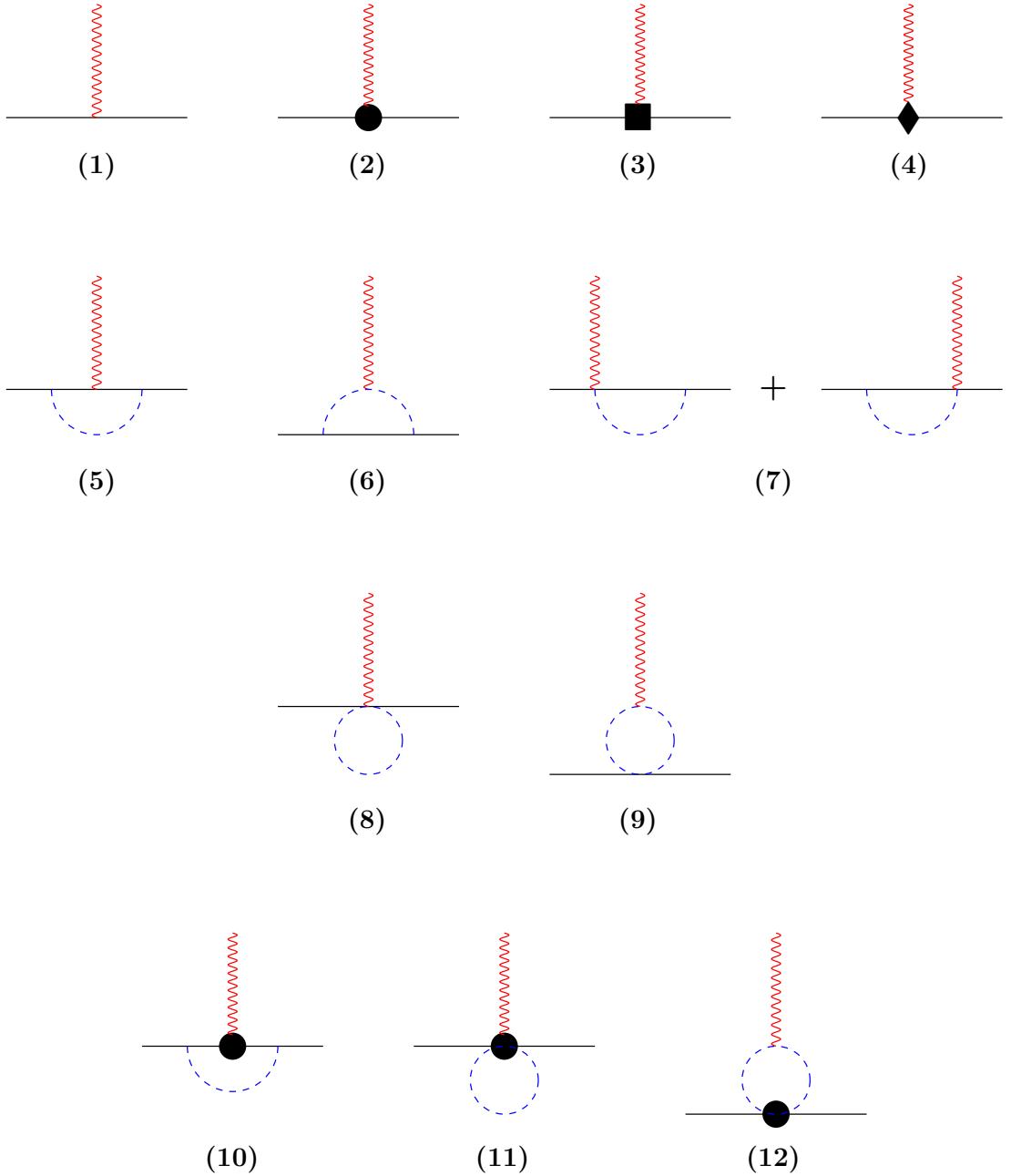
[35] J. Friedrich and T. Walcher, *Eur. Phys. J. A* **17**, 607 (2003) [arXiv:hep-ph/0303054].

**Table I.** SU(6) couplings  $\alpha_E^{Bi}$  and  $\alpha_M^{Bi}$ .

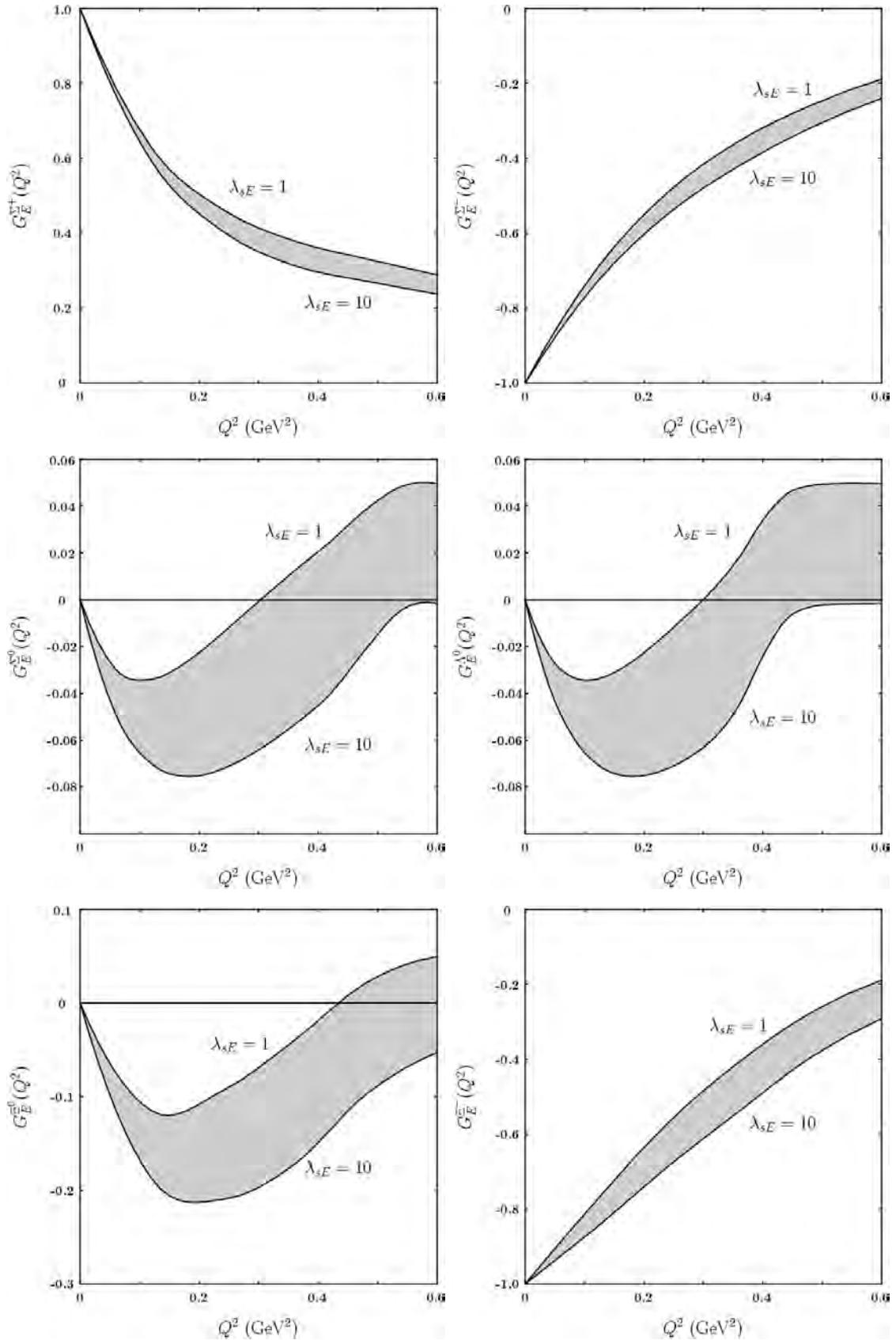
	$\alpha_E^{Bu}$	$\alpha_E^{Bd}$	$\alpha_E^{Bd}$	$\alpha_M^{Bu}$	$\alpha_M^{Bd}$	$\alpha_M^{Bs}$
$p$	2	1	0	$\frac{4}{3}$	$-\frac{1}{3}$	0
$n$	1	2	0	$-\frac{1}{3}$	$\frac{4}{3}$	0
$\Lambda^0$	1	1	1	0	0	1
$\Sigma^+$	2	0	1	$\frac{4}{3}$	0	$-\frac{1}{3}$
$\Sigma^0$	1	1	1	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$
$\Sigma^-$	0	2	1	0	$\frac{4}{3}$	$-\frac{1}{3}$
$\Xi^-$	0	1	2	0	$-\frac{1}{3}$	$\frac{4}{3}$
$\Xi^0$	1	0	2	$-\frac{1}{3}$	0	$\frac{4}{3}$
$\Sigma^0 \Lambda^0$	0	0	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	0

**Table II.** Magnetic moments of the baryon octet (in units of the nucleon magneton  $\mu_N$ )

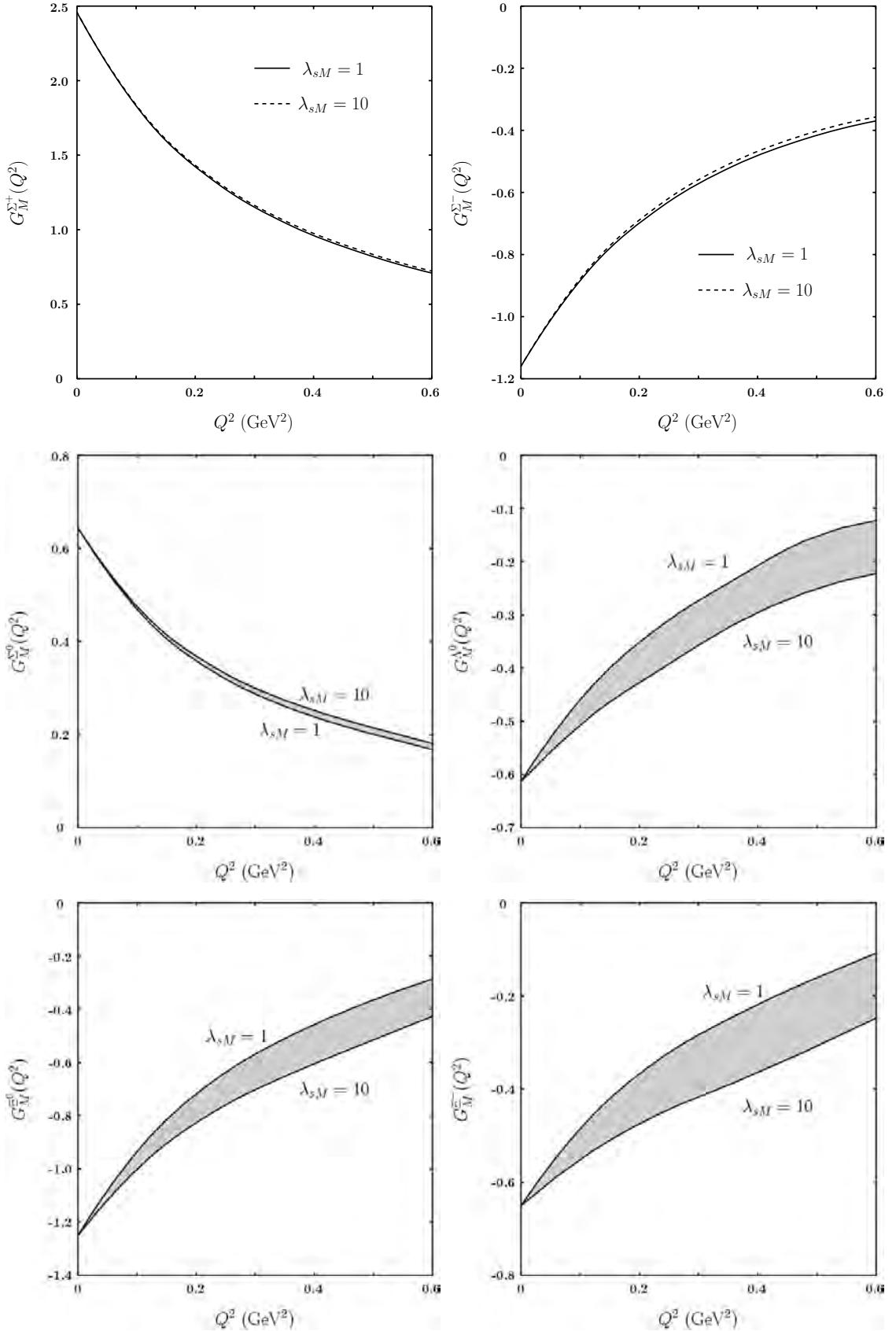
	Set I			Set II			Exp.
	3q	Meson Cloud	Total	3q	Meson Cloud	Total	
$\mu_p$	2.357	0.436	2.793	2.357	0.436	2.793	2.793
$\mu_n$	-1.571	-0.342	-1.913	-1.571	-0.342	-1.913	-1.913
$\mu_{\Lambda^0}$	-0.786	0.173	-0.613	-0.518	-0.095	-0.613	$-0.613 \pm 0.004$
$\mu_{\Sigma^+}$	2.357	0.317	2.674	2.085	0.373	2.458	$2.458 \pm 0.010$
$\mu_{\Sigma^0}$	0.786	0.005	0.791	0.570	0.073	0.643	...
$\mu_{\Sigma^-}$	-0.786	-0.306	-1.092	-0.935	-0.225	-1.160	$-1.160 \pm 0.025$
$\mu_{\Xi^0}$	-1.571	0.136	-1.435	-1.058	-0.192	-1.250	$-1.250 \pm 0.014$
$\mu_{\Xi^-}$	-0.7855	0.2921	-0.4934	-0.5580	-0.0927	-0.6507	$-0.6507 \pm 0.003$
$ \mu_{\Sigma^0 \Lambda^0} $	1.36	0.27	1.63	1.34	0.27	1.61	$1.61 \pm 0.08$



**Fig. 1.** Diagrams including pseudoscalar meson contributions to the EM quark transition operator up to fourth order. Solid, dashed and wiggly lines refer to quarks, pseudoscalar mesons and the electromagnetic field, respectively. Vertices denoted by a black filled circle, box and diamond correspond to insertions from the second, third and fourth order chiral Lagrangian.



**Fig. 2.** The charge form factors of  $\Sigma^+$ ,  $\Sigma^-$ ,  $\Sigma^0$ ,  $\Lambda^0$ ,  $\Xi^0$  and  $\Xi^-$  baryons. The shaded region shows the range of the form factors with the parameter  $\lambda_{sE}$  being varied in the interval from 1 to 10 in units of GeV.



**Fig. 3.** The magnetic form factors of  $\Sigma^+$ ,  $\Sigma^-$ ,  $\Sigma^0$ ,  $\Lambda^0$ ,  $\Xi^0$  and  $\Xi^-$  baryons. The shaded region shows the range of the form factors with the parameter  $\lambda_{sM}$  being varied in the interval from 1 to 10 in units of GeV.

# **Output**

To be submitted to Journal of Physics G.