Final Report

Project Title: Robust Motion Estimation in Image Sequences

Dr. Toshiaki Kondo, Assistant Professor

Sirindhorn International Institute of Technology
Thammasat University

From May 2008 to May 2010

Contract No. MRG5180364

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Project Granted by the Thailand Research Fund (MRG5180364)

Abstract

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Project Title: Robust motion estimation in image sequences

Investigators:
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1. Abstract

The project is concerned with robust motion estimation in image sequences, especially under varying lighting conditions. Since conventional approaches are based on image intensities or gradients, they are sensitive to variations of lighting conditions. To overcome this problem, we propose to use gradient orientation information in place of image intensities and gradients because gradient orientation is known to be notably invariant to varying illuminations. In this project, we have developed three motion estimation techniques: the gradient orientation-based spatio-temporal gradient method (GOGM), the gradient orientation structure tensor method (GOSTM), and gradient orientation pattern matching technique (GOPM). They were compared with their respective original methods: the spatio-temporal gradient method (GM), the gradient structure tensor method (GSTM), and correlation-based methods such as the sum-of-absolute differences (SAD) matching, the sum-of-squared differences (SSD) matching, and the zero-mean normalized cross-correlation (ZNCC). Our simulation results show that the proposed methods perform motion estimation very well regardless of irregular lighting conditions, while the conventional approaches fail to work under such conditions.

Keywords: 3-5 words

Motion estimation, motion vectors, template matching, the spatio-temporal gradient method, and gradient structure tensor

2. Executive summary

We have developed three novel techniques for estimating motions in image sequences that work robustly under irregular lighting conditions. The first two techniques are based on a well-known constraint, the optical flow constraint equation (OFCE), while the third belongs to correlation-based approaches. The first technique developed is the gradient orientation based spatio-temporal gradient method (GOGM) that is originated in the classical spatio-temporal gradient method (GM). The second is the gradient orientation structure tensor method (GOSTM) that is an improved version of the gradient structure tensor method (GSTM). Finally, the third is the gradient orientation pattern matching technique (GOPM) that is related to conventional template (or block) matching techniques, including sum-of-absolute differences (SAD) matching, sum-of-squared differences (SSD) matching, and zero-mean normalized cross-correlation (ZNCC). A common feature among the three proposed techniques is that they employ gradient orientation information instead of image intensities and gradients that are used by the existing methods mentioned above. Both the GM and GSTM are image gradients-based, while SAD, SSD, and ZNCC use image intensities. Since gradient orientation is remarkably invariant to changing lighting conditions, all the developed techniques, i.e., GOGM, GOSTM, and GOPM, work robustly under irregular illuminations, unlike the original techniques that they are based on.

3. Objective

The objective of this project is to develop a technique for estimating motions in image sequences that work robustly under varying lighting conditions.

4. Research methodology

Gradient-based motion estimation techniques (e.g. aforementioned GM, GSTM) are based on the optical flow constraint equation (OFCE) that assumes the intensity of an image is constant along its motion trajectory over time. In place of the constraint, we have introduced a new assumption, that is, the gradient orientation of an image is constant along its motion trajectory. Following the new assumption, we devised two novel gradient-based techniques, GOGM and GOSTM. The derivation of those techniques are fully described in the journal paper, “Robust motion estimation methods using gradient orientation information”, published in *Science Asia*, Vol. 35, No. 2, pp. 196-202, June 2009, together with the review of the existing methods, GM and GSTM. Please see Appendix A for more details.

Correlation-based motion estimation techniques make use of image intensities or gradients. In place of image intensities or gradients, we utilize gradient orientation information by means of unit gradient vectors (or normalized gradient vectors). The unit gradient vector can be expressed with two scalars, i.e., the x and y components of the vector. These two scalars can be regarded as a certain signal as intensities. This means that we obtain two intensity patterns from one image. We call them gradient orientation patterns (GOPs). Those GOPs can be used in any correlation-based techniques, such as popular sum-of-absolute differences (SAD) matching, sum-of-squared differences (SSD) matching, zero-mean normalized cross correlation (ZNCC) and so on. We have tested the motion estimation performance of this concept using GOPs in the SAD matching strategy. The proposed method named gradient orientation pattern matching method (GOPM) is described in the refereed conference paper, "A block matching technique using unit gradient vectors", published in the proceeding of the *IAPR Conference on Machine Vision Applications* (MVA2009), pp. 390-393, Japan, May 2009. Please see Appendix B for more details.

5. Results

As mentioned above, the simulation results of the proposed techniques, the GOGM and GOSTM, are shown in the paper attached as Appendix A, while those of the GOPM are given in the paper attached as Appendix B. We have also achieved a real-time implementation of the GOPM in C language with the OpenCV library where the proposed matching technique is used for tracking a human eye at the video rate during eye surgery. Please see Appendix C for more details.

6. Conclusions and discussion

Conclusive remarks on the GOGM and GOSTM: The gradient orientation based spatio-temporal gradient method (GOGM) and the gradient orientation structure tensor method (GOSTM) are based on the spatio-temporal gradient method (GM) and the gradient structure tensor method (GSTM), respectively. Unlike the conventional approaches utilizing image gradients, we make use of gradient orientation information (GOI) by means of unit gradient vectors. Since GOI is insensitive to changes of image intensities, the proposed methods have achieved a significant robustness to varying lighting conditions. They also perform better than the conventional methods when encountering the aperture problem. The implementation of the proposed methods is straightforward because image gradients are commonly computed at an early stage in image sequence processing and computer vision applications and are readily available.

Conclusive remarks on the GOPM: Most existing approaches for image matching are based on either image intensities or gradients. Consequently, it is inevitable that these conventional techniques are susceptible to varying image intensities caused by irregular lighting conditions. To cope with this illumination problem, we have presented a novel matching technique that is based on gradient orientation patterns (GOPs) that can be obtained as the *x* and *y* components of unit gradient vectors. We do not use the angular values θ (rad) of gradient vectors directly to avoid modulo computation, which enables a fast implementation of the proposed method. Simulation results on both synthetic and real image sequences have revealed that the proposed technique, GOPM, works much more robustly than SAD matching with varying image intensities. The motion estimation performance of the GOPM is comparable to that of ZNCC with uniformly changing intensities and also non-uniformly but smoothly varying intensities. Furthermore, it is a significant advantage of the GOPM over ZNCC that it can cope with non-uniform and rapid changes of image intensities that may occur in outdoor environment. We have also shown that the computational cost of the GOPM is less than that of ZNCC. Gradient vectors are generally computed at an early stage of various image processing and computer vision applications, and are readily available. The normalization of the gradient vectors to obtain the unit gradient vectors can be performed prior to the computation for image correspondence. Therefore, the GOPM will be well-suited to real-time applications and also hardware implementation (see Appendix C).

7. List of Appendixes

Please see the following papers attached to the end of this report.

A) Toshiaki Kondo and Waree Kongprawechnon, “Robust motion estimation methods using gradient orientation information”, *Science Asia*, Vol. 35, No. 2, pp. 196-202, June 2009.

B) Toshiaki Kondo and Waree Kongprawechnon, "A block matching technique using unit gradient vectors", *IAPR Conference on Machine Vision Applications* (MVA2009), pp. 390-393, Japan, May 2009.

C) Wattanit Hotrakook, Prarinya Siritanawan, and Toshiaki Kondo, "A real-time eye-tracking method using time-varying gradient orientation patterns", accepted by *ECTI-CON 2010*, Chiang Mai, Thailand, May 19-21, 2010.

8. Outputs

8.1 International journal publications

§ Toshiaki Kondo and Waree Kongprawechnon, "Robust motion estimation methods using gradient orientation information", *Science Asia*, Vol. 35, No. 2, pp. 196-202, June 2009.

§ Pramuk Boonsieng, Toshiaki Kondo, and Waree Kongrawechnon, "Unit gradient vectors based motion estimation techniques", submitted to *ECTI Transactions on Electrical Eng., Electronics, and Communications* (ECTI-EEC), May 2009.

8.2 National journal publication

§ Toshiaki Kondo and Waree Kongprawechnon, "A matching technique using gradient orientation patterns", *Thammasat International Journal of Science and Technology*, Vol. 14, No. 3, pp. 41-55, July-September 2009.

8.3 Proceedings of refereed international conferences

§ Toshiaki Kondo, Pramuk Boonsieng, and Waree Kongprawechnon, "Improved gradient-based methods for motion estimation in image sequences", *SICE International Annual Conference on Instrumentation, Control and Information Technology*, Tokyo, August 2008.

§ Toshiaki Kondo and Waree Kongprawechnon, "A block matching technique using unit gradient vectors", *IAPR Conference on Machine Vision Applications* (MVA2009), pp. 390-393, Japan, May 2009.

§ Wattanit Hotrakook, Prarinya Siritanawan, and Toshiaki Kondo, "A real-time eye-tracking method using time-varying gradient orientation patterns", accepted by *ECTI-CON 2010*, Chiang Mai, Thailand, May 19-21, 2010.

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Appendixes A, B, and C

A) Toshiaki Kondo and Waree Kongprawechnon, "Robust motion estimation methods using gradient orientation information", *Science Asia*, Vol. 35, No. 2, pp. 196-202, June 2009.

B) Toshiaki Kondo and Waree Kongprawechnon, "A block matching technique using unit gradient vectors", *IAPR Conference on Machine Vision Applications* (MVA2009), pp. 390-393, Japan, May 2009.

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Robust motion estimation methods using gradient orientation information

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ABSTRACT: Changing lighting conditions cause temporal variations of image intensities and make most existing motion estimation techniques ineffective. As a solution to this problem, we use gradient orientation information, which depends very little on changes of image intensities, in place of commonly used image features such as intensities and gradients. By employing gradient orientation, two conventional motion estimation techniques (the spatio-temporal gradient method and the gradient structure tensor method) can be transformed into methods that are far more robust to changing image intensities. Simulation results on both synthetic and real image sequences show that the proposed methods perform motion estimation remarkably well irrespective of time-varying image intensities. In addition, the proposed methods also cope better with the aperture problem in which only unidirectional gradients are available and large erroneous motion estimates are produced.

KEYWORDS: motion vectors, optical flow, spatio-temporal gradient method, gradient structure tensor

INTRODUCTION

Motion estimation is an important task in the fields of image sequence processing and computer vision. It has various applications, including object-based video coding (e.g. MPEG-4), object detection and tracking, scene change detection for video editing, image stabilization for camcorders, and dynamic 3-d scene analysis for autonomous navigation. Motion estimation techniques in the spatial domain may be classified as being either gradient-based or correlation-based methods (also commonly referred to as block matching or template matching)[1]. Gradient-based techniques can be further divided into spatio-temporal gradient methods (often simply referred to as gradient methods)[1–3] and gradient structure tensor methods (also referred to as gradient square tensor methods or 3-d structure tensor methods)[1,4–7]. Gradient-based methods are in general used to obtain a dense optical flow field or motion vectors. These techniques are effective especially when the displacement between images over time is small, typically a few pixels. On the other hand, correlation-based methods may be the most intuitive approach as they search for similar patterns between two images[1,8–10]. Since they can handle larger displacements, they are used not only for motion estimation, but also for establishing correspondence between images captured at different viewpoints such as stereo vision. They are, however, not suitable for computing dense motion vectors owing to their high computational cost.

This paper is concerned with gradient-based methods. They are based on the optical flow constraint equation which assumes that image intensities are constant along motion trajectories. This assumption, however, is often violated by changes of lighting conditions that is a common occurrence in outdoor environments. To circumvent this varying illumination problem, it is reasonable to employ a feature that is less dependent on image intensities. Gradient orientation (or direction) is an attractive feature because it is not sensitive to variations in illumination[11–14]. In this paper, we propose the use of gradient orientation information, instead of the commonly used image features such as intensities and gradients, to achieve a robust performance for motion estimation.

OPTICAL FLOW CONSTRAINT EQUATION

An optical flow field, showing dense motion vectors within an image, is generally estimated under the assumption that the image intensities of an object are constant over time. By assuming that the image intensities I at points (x, y) at time t are constant over time $t + \mathrm{d}t$, we have

$$I(x, y, t) = I(x + \mathrm{d}x, y + \mathrm{d}y, t + \mathrm{d}t), \qquad (1)$$

where $\mathrm{d}x$ and $\mathrm{d}y$ denote the displacements in the x and y directions between two images recorded at times t and $t + \mathrm{d}t$. By taking the first-order Taylor series expansion of the right-hand side of (1), we obtain

$$I(x+\mathrm{d}x, y+\mathrm{d}y, t+\mathrm{d}t) \approx I(x,y,t) + I_x\mathrm{d}x + I_y\mathrm{d}y + I_t\mathrm{d}t, \quad (2)$$

where the subscripts denote partial derivatives. From (1) and (2), one can derive the well-known optical flow constraint equation (OFCE),

$$I_x u + I_y v + I_t = 0, \quad (3)$$

in which $(u,v) \equiv (\mathrm{d}x/\mathrm{d}t, \mathrm{d}y/\mathrm{d}t)$ are motion vectors. Eq. (3) is apparently underdetermined because it contains two unknowns, u and v. There are two popular approaches to solve (3) for (u,v), leading to two motion estimation techniques, namely, the spatio-temporal gradient method and the gradient structure tensor method.

THE SPATIO-TEMPORAL GRADIENT METHOD

The spatio-temporal gradient method, which we will refer to as the gradient method (GM), is a traditional optical flow estimation method[1–3]. It is based on regression analysis where the two unknowns (u,v) are computed by the least-squares method assuming that I_t is a dependent (or target) variable and I_x and I_y are independent variables. To solve the OFCE for (u,v) we may introduce either a global or local smoothness constraint. The former constraint assumes that the optical flow changes smoothly over the entire image[2], while the latter assumes that the optical flow in a small region is constant[3]. To avoid the possibly time-consuming iterative procedure involved in the former approach, we employed the latter. Under the local smoothness constraint, motion vectors (u,v) can be straightforwardly determined by minimizing the quadratic cost-function,

$$F = \sum (I_x u + I_y v + I_t)^2, \quad (4)$$

where the summation is performed over a small region or block.

The procedure of the GM is equivalent to determining of the best-fitting plane to a given set of points (I_x, I_y, I_t) by minimizing the sum of the squares of the distances between the points and the plane along the I_t axis. The normal vector of the plane gives the motion vector (u,v) of the small region. The least-squares solution of (4), which we denote by $(\tilde{u}, \tilde{v})$, is given by

$$\tilde{u}_{\mathrm{GM}} = \frac{(\sum I_x I_y)(\sum I_y I_t) - (\sum I_y^2)(\sum I_x I_t)}{(\sum I_x^2)(\sum I_y^2) - (\sum I_x I_y)^2}, \quad \tilde{v}_{\mathrm{GM}} = \frac{(\sum I_x I_y)(\sum I_x I_t) - (\sum I_x^2)(\sum I_y I_t)}{(\sum I_x^2)(\sum I_y^2) - (\sum I_x I_y)^2}. \quad (5)$$

The implementation of (5) is easy and fast, and is often used for obtaining dense motion vectors.

THE GRADIENT STRUCTURE TENSOR METHOD

The gradient structure tensor method (GSTM) is a newer approach[1,4–7]. With the rapid advance of computer technology, the GSTM has been employed for real-time motion estimation in recent years[15,16]. The GSTM is based on orthogonal regression (or total least-squares fitting) using principal component analysis (PCA). The gradient structure tensor T of the image intensities I is defined as

$$T = \begin{pmatrix} \sum I_x^2 & \sum I_x I_y & \sum I_x I_t \\ \sum I_y I_x & \sum I_y^2 & \sum I_y I_t \\ \sum I_t I_x & \sum I_t I_y & \sum I_t^2 \end{pmatrix}, \quad (6)$$

where $\sum$ indicates the summation within a local 3-d region such as a cube. T can be viewed as a covariance matrix of the spatio-temporal image gradients in PCA. PCA can be used to find the best-fitting plane to a given set of points (I_x, I_y, I_t) in the 3-d space spanned by the I_x, I_y, and I_t axes. The GSTM determines the best-fitting plane by minimizing the sum of the squares of the orthogonal offsets of the points from the plane. In contrast, in the GM, errors are defined as the sum of the squared vertical offsets from the data points to the fitted plane. In other words, errors are measured along the I_t axis because I_t is considered as a variable dependent on the other two variables, I_x and I_y. Such fitting model of the GM is often criticized for its unequal treatment of spatial and temporal derivatives[1]. When there is no clear relationship among variables, whether they are independent or dependent variables, it makes more sense to measure errors by treating all three variables equally and minimize the orthogonal offsets of the points to the plane. Our previous work also revealed that the GSTM performs motion estimation much better than the GM largely owing to this difference in fitting models[17].

Another interpretation of the GSTM is that when a small object P is moving at a constant speed in a certain direction in a series of time-sequential images, the trajectory of the moving object P forms a tubular structure in 3-d spatio-temporal space. By applying PCA to (I_x, I_y, I_t), we obtain three eigenvectors $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ and their corresponding eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ where $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant 0$. The first two eigenvectors will point in directions that are perpendicular to the trajectory of P, while the third eigenvector $\mathbf{e}_3 = [x_3\, y_3\, t_3]^{\mathrm{T}}$, associated with the smallest eigenvalue λ_3, will indicate the direction along the trajectory. Hence, motion vectors may be estimated from

$$\tilde{u}_{\mathrm{GSTM}} = x_3/t_3, \qquad \tilde{v}_{\mathrm{GSTM}} = y_3/t_3. \tag{7}$$

It is relevant to note that we may evaluate the confidence of estimated motion vectors by analysing the relationship between the three eigenvalues of the tensor[1,4,6,7,15]. For instance, if $\lambda_1 \approx \lambda_2 \gg \lambda_3 \geqslant 0$, the trajectory forms a linear structure in the spatio-temporal space and the confidence is high. When $\lambda_1 \gg \lambda_2 \approx \lambda_3 \geqslant 0$, the motion trajectory is plane-like, which indicates that there is an aperture problem where we can estimate motion only along the gradient vector available. When $\lambda_1 \approx \lambda_2 \approx \lambda_3 > 0$, the corresponding local region may have impulsive noise or motion discontinuities, and the resultant motion vectors may be inaccurate. Finally, if $\lambda_1 \approx \lambda_2 \approx \lambda_3 \approx 0$, there is no gradient information and motion estimation is impossible.

GRADIENT ORIENTATION INFORMATION

As a robust image feature, we propose using gradient orientation information (GOI) rather than the conventional image features such as intensities and gradients. We now describe how to extract GOI and how to use it in a computationally efficient manner.

Extraction of gradient orientation information

Let $I(x, y)$ be the image intensities at pixel coordinates (x, y). The gradient vectors are then (I_x, I_y). By convention, the upper left corner of the image is the origin, the x-axis is directed downwards, and the y-axis is horizontal. The unit gradient vectors (n_x, n_y) are obtained by dividing (I_x, I_y) by their magnitudes where we assign zeros to $n_x(x, y)$ and $n_y(x, y)$ if the magnitude is zero. Notice that unit gradient vectors carry rich spatial information even in relatively low-contrast areas (Fig. 1d). As the unit gradient vectors (n_x, n_y) are represented by two scalars, n_x and n_y, ranging from -1 to 1, they can be treated as two separate intensity patterns. We call them gradient orientation patterns and make use of them as a robust image feature for motion estimation. It should be emphasised that the use of unit gradient vectors is

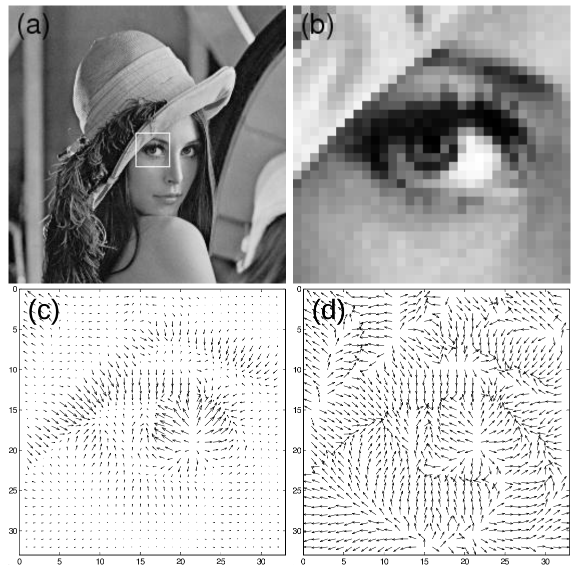

Fig. 1 (a) An 8-bit grey-scale 256×256 pixel image – the Lena image (b) an enlarged (32×32 pixel) subimage (c) subimage gradient vectors (d) subimage unit gradient vectors.

preferable to using angular values θ directly because angular values require modulo calculations (e.g., the difference between two angles cannot exceed π)[14].

Intensity-invariance of gradient orientation

One way to model the intensity variations is to assume $I' = aI + b$ where I and I' are image intensities before and after a lighting condition is changed, while a and b are scalar constants[18]. Gradient-based, including second order derivative-based[19], methods can work irrespective of additive variations of intensities ($b \neq 0$). They are, however, susceptible to multiplicative variations ($a \neq 0$). Fig. 2a shows the same subimage as in Fig. 1b, except that the intensities of the upper half of it are reduced by 50% (i.e., $a = 0.5$, $b = 0$). Fig. 2b shows the gradient vectors of the subimage. They differ considerably from those of Fig. 1c because of the multiplicative changes of image intensities. On the other hand, Fig. 2c shows unit gradient vectors that are identical to those of Fig. 1d, except at the boundary between the halves. Since the boundary acts as a step edge, it disturbs its neighbouring gradient orientations. Therefore, except at such boundaries, we may state that unit gradient vectors are insensitive to both additive and multiplicative changes of image intensities and thus can maintain gradient orientation patterns well regardless of varying illumination.

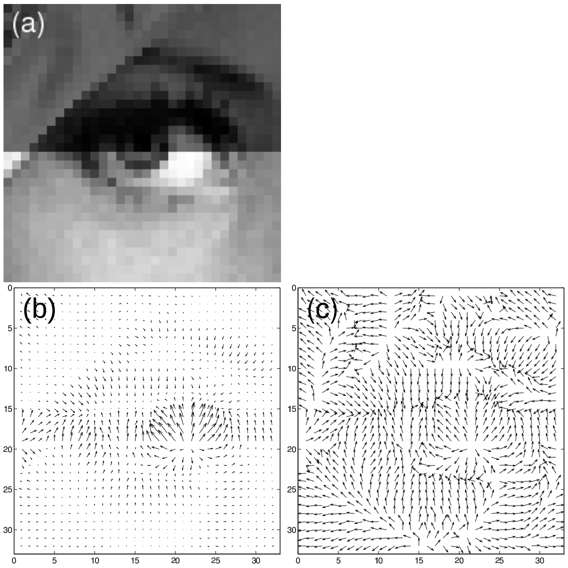

Fig. 2 (a) Partially shaded subimage (b) gradient vectors (c) unit gradient vectors.

APPLICATION OF GRADIENT ORIENTATION INFORMATION TO MOTION ESTIMATION

Gradient orientation based gradient method

In the first proposed approach, we use the unit gradient vectors (n_x, n_y), instead of image intensities I, and assume that (n_x, n_y) are constant over time:

$$\begin{aligned} n_x(x,y,t) &= n_x(x+\mathrm{d}x, y+\mathrm{d}y, t+\mathrm{d}t),\\ n_y(x,y,t) &= n_y(x+\mathrm{d}x, y+\mathrm{d}y, t+\mathrm{d}t). \end{aligned} \tag{8}$$

From (8), we obtain two modified OFCEs:

$$\begin{aligned} X_x u_1 + X_y v_1 + X_t = 0,\\ Y_x u_2 + Y_y v_2 + Y_t = 0, \end{aligned} \tag{9}$$

where $X \equiv n_x$ and $Y \equiv n_y$. We have used the Sobel operators for computing the partial derivatives because they yield good approximations of the derivatives with minimal computation[11]. In place of (4) we have the following two cost functions

$$\begin{aligned} F_1 &= \sum (X_x u_1 + X_y v_1 + X_t)^2,\\ F_2 &= \sum (Y_x u_2 + Y_y v_2 + Y_t)^2. \end{aligned} \tag{10}$$

From (10), we then obtain two motion estimates, $(\tilde{u}_{\mathrm{GOGM}x}, \tilde{v}_{\mathrm{GOGM}x})$ and $(\tilde{u}_{\mathrm{GOGM}y}, \tilde{v}_{\mathrm{GOGM}y})$ using the GM.

Because there should be only one motion vector per block, we need to unify those two motion estimates into one. For this we introduce some weighting

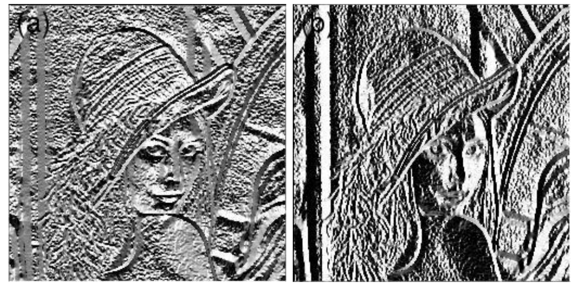

Fig. 3 Gradient orientation patterns of the Lena image: (a) n_x (b) n_y.

factors that are dependent on the gradient orientation patterns. Figs. 3a and 3b show gradient orientation patterns n_x and n_y of the Lena image in Fig. 1a, scaled between 0 and 255 for visualization purposes. Vertical gradients (i.e., horizontal lines) are rich in n_x, whereas horizontal gradients (vertical lines) are more dominant in n_y. This observation indicates that n_x is more suitable for computing vertical motion u and n_y is better for horizontal motion v.

The reliability of each motion estimate may be judged from the diversity of gradient orientations (directions) in its local area. The diversity of gradient orientations can be measured by the two eigenvalues of the covariance matrix $\mathbf{C}$ between n_x and n_y,

$$\mathbf{C} = \begin{bmatrix} \sum n_x^2 & \sum n_x n_y \\ \sum n_x n_y & \sum n_y^2 \end{bmatrix}, \tag{11}$$

where $\lambda_1 \geqslant \lambda_2 \geqslant 0$ are the eigenvalues of $\mathbf{C}$. This approach resembles the analysis of eigenvalues from gradient covariance[1,4,6,7,15], but we can focus on the analysis of gradient orientations regardless of variations of image intensities by employing unit gradient vectors. If $\lambda_1 \gg \lambda_2 \approx 0$, the gradients are unidirectional and we can only estimate motion along the direction parallel to the gradients. This is the so-called aperture problem. In this case, vertical and horizontal motion estimation should rely heavily on the pattern of n_x and n_y, respectively. Meanwhile, if $\lambda_1 \approx \lambda_2 > 0$, there are omni-directional gradients and similar weights can be used for motion estimates from n_x and n_y as they are considered equally reliable. In the proposed method this weight control is achieved by using the following unified motion estimates

$$\begin{aligned} \tilde{u}_{\mathrm{GOGM}} &= \frac{\lambda_1}{\lambda_1+\lambda_2}\tilde{u}_{\mathrm{GOGM}x} + \frac{\lambda_2}{\lambda_1+\lambda_2}\tilde{u}_{\mathrm{GOGM}y},\\ \tilde{v}_{\mathrm{GOGM}} &= \frac{\lambda_2}{\lambda_1+\lambda_2}\tilde{v}_{\mathrm{GOGM}x} + \frac{\lambda_1}{\lambda_1+\lambda_2}\tilde{v}_{\mathrm{GOGM}y}. \end{aligned} \tag{12}$$

We call this approach the gradient orientation based gradient method (GOGM).

Gradient orientation structure tensor method

In the second proposed method we use (n_x, n_y) instead of the image intensities I. Thus, as in (6), we have two local tensors T_X and T_Y

$$T_X = \begin{pmatrix} \sum X_x^2 & \sum X_x X_y & \sum X_x X_t \\ \sum X_y X_x & \sum X_y^2 & \sum X_y X_t \\ \sum X_t X_x & \sum X_t X_y & \sum X_t^2 \end{pmatrix}, \quad T_Y = \begin{pmatrix} \sum Y_x^2 & \sum Y_x Y_y & \sum Y_x Y_t \\ \sum Y_y Y_x & \sum Y_y^2 & \sum Y_y Y_t \\ \sum Y_t Y_x & \sum Y_t Y_y & \sum Y_t^2 \end{pmatrix}. \tag{13}$$

We use the 3-d Sobel operators for computing the partial derivatives[20]. By applying PCA to T_X and T_Y separately, we obtain two estimates of image motions $(\tilde{u}_{\mathrm{GOSTM}x}, \tilde{v}_{\mathrm{GOSTM}x})$ and $(\tilde{u}_{\mathrm{GOSTM}y}, \tilde{v}_{\mathrm{GOSTM}y})$. Following the same procedure described above, we integrate these two motion estimates depending on the diversity of local gradient orientations by using

$$\tilde{u}_{\mathrm{GOSTM}} = \frac{\lambda_1}{\lambda_1 + \lambda_2} \tilde{u}_{\mathrm{GOSTM}x} + \frac{\lambda_2}{\lambda_1 + \lambda_2} \tilde{u}_{\mathrm{GOSTM}y}, \quad \tilde{v}_{\mathrm{GOSTM}} = \frac{\lambda_2}{\lambda_1 + \lambda_2} \tilde{v}_{\mathrm{GOSTM}x} + \frac{\lambda_1}{\lambda_1 + \lambda_2} \tilde{v}_{\mathrm{GOSTM}y}. \tag{14}$$

We call this approach the gradient orientation structure tensor method (GOSTM).

RESULTS AND DISCUSSION

We evaluate the motion estimates by GOGM and GOSTM by comparing their performances with those of conventional approaches on four standard test images (Fig. 1a, Fig. 4) converted to 8-bit grey-scale 256×256 pixel images.

Synthetic time-sequential images were produced by translating the four test images by 2 pixels in both the vertical and horizontal directions. Gaussian noise was added to all images (signal-to-noise ratio $\sim$ 40 dB). For evaluation, we assume that the estimated motion vectors are correct when they satisfy the condition,

$$\max \max(|\Delta x|, |\Delta y|) \leqslant 0.5, \tag{15}$$

where Δx and Δy denote the deviations (in pixels) between true and estimated motions. Hence, if rounded motion vectors are equal to the true motion, those motion estimates are considered successful. Note that the motion estimates are not integers, but real numbers.

Fig. 4 Standard test images: (a) girl (b) cameraman (c) house.

Gradient orientation based gradient method

Prior to motion estimation it is always necessary to apply a certain low-pass filter for gradient-based methods because the computation of gradients is sensitive to high frequency components in an image. A Gaussian low-pass filter of size 13×13 pixels whose standard deviation is half of the filtering mask (σ=13/2) was applied to all the four image sequences[17]. We then computed 14×14 motion vectors for each sequence with the blocks of size 16×16 pixels.

We first tested the GM and GOGM under mildly noisy (40 dB) and constant lighting conditions. It is apparent that the GOGM achieves much higher success rates than the GM for all the four image sequences (Table 1). Next, the intensities of the second frame were uniformly reduced by 10% to simulate a time-varying lighting condition. Note that this is a robustness test to temporal variation of image intensities that is different from a spatial variation of image brightness shown in Fig. 2a. The GM completely breaks down whereas the GOGM performs motion estimation regardless of the variation of image intensities (Table 1).

Gradient orientation structure tensor method

Next we compare the GOSTM with the GSTM in the same settings as above. Both methods work equally well under constant lighting conditions (Table 1). For varying lighting conditions, the stark contrast between

Table 1 Percentage success rates of 196 motion estimates of the true motion (2,2) by the GM, GOGM, GSTM, and GOSTM under constant (Sim 1) and time-varying (Sim 2) lighting conditions.

Image name	GM		GOGM		GSTM		GOSTM	
Sim:	1	2	1	2	1	2	1	2
Lena	38.3	0.5	62.8	64.3	97.4	0.0	98.0	98.0
Girl	56.1	4.1	77.6	77.0	99.0	3.1	96.4	94.4
Cameraman	42.9	5.1	60.7	57.7	90.8	6.1	86.2	81.6
House	27.6	0.0	48.0	49.0	77.6	0.5	77.0	73.5

the GSTM and GOSTM clearly shows that even a slight change of image intensities makes the GSTM completely ineffective, while the GOSTM performs motion estimation irrespective of varying image intensities. Notice that the successful motion estimate rates in the cameraman and house images tend to be low for every approach. This is because these images contain large extremely low-contrast areas, such as sky and wall, where no gradient information is available.

Improved performance to the aperture problem

A close observation of the motion estimates by the four methods also reveals that the proposed techniques tend to work better where only one-directional gradients are available in a local area (the aperture problem). For this comparison, the block size of the GSTM and GOSTM has been reduced to 8×8 pixels to highlight the differences between them. This modification is necessary because both methods produce equally high success rates when a block size of 16×16 pixels is used (Table 1). Under the new setting, the success rates of the GSTM and GOSTM are respectively 67.5% and 77.6%. In Fig. 5a, most of the motion vectors along the vertical edges on the left-hand side are incorrect. Similarly, Fig. 5c shows several large erroneous motion vectors along the curved lines located on the right-hand side. In both cases the incorrect motion vectors tend to have large motion estimation errors in the direction tangent to the edges. They are typical faulty motion vectors resulting from the aperture problem. Many of these unsuccessful motion estimations, however, have been improved or corrected by the GOGM and GOSTM (Fig. 5b,d). The improved performance may be attributed to the weighting factors based on the diversity of local gradient orientations. To summarize, large erroneous motion estimates are well suppressed by the weighted sum of two motion estimates in (12) and (14).

Motion estimation on a real-image sequence

Finally, we demonstrate the feasibility of the GOGM and GOSTM on a real image sequence of size 240×320 pixels that contain local motions. The sequence shows a walking woman with stationary background. We have reduced the image intensities uniformly by 10% in one frame in the sequence. The size of the block is set at 8×8 pixels. Since there is no ground truth data for this sequence, the proposed methods GOGM and GOSTM are compared with the GM and GSTM based on visual inspection. It is obvious that the conventional approaches GM (Fig. 6a) and GSTM (Fig. 6c) cannot produce reliable motion estimates at all under the time-varying lighting condition. They produce numerous faulty motion estimates in the background where there is no motion. On the other hand, both GOGM (Fig. 6b)

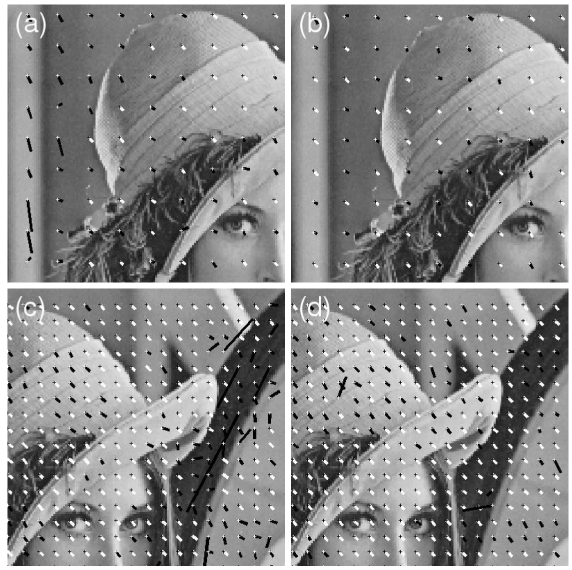

Fig. 5 Parts of the motion vectors estimated under mildly noisy constant lighting conditions using (a) GM (b) GOGM (c) GSTM (d) GOSTM. White vectors indicate correctly estimated motions, i.e., displacements of (2,2) pixels after rounding. Black vectors show incorrect estimations.

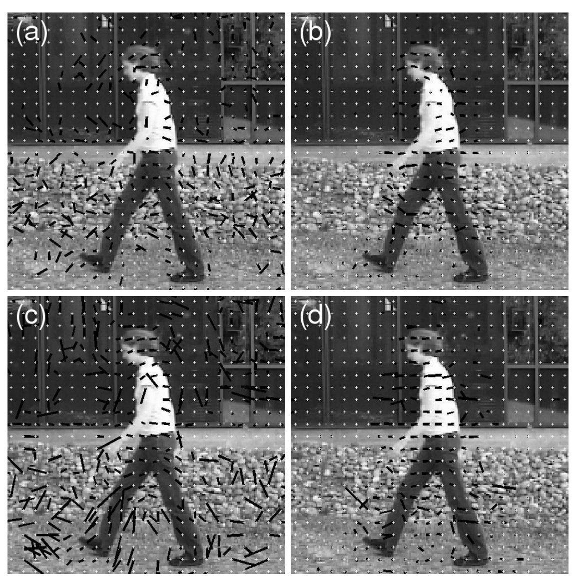

Fig. 6 Parts of the motion estimation results on a real image sequence under time-varying lighting conditions using (a) GM (b) GOGM (c) GSTM (d) GOSTM.

and GOSTM (Fig. 6d) work reasonably well under the same condition, and there are far less faulty responses to the stationary background. Therefore, it is confirmed that the proposed techniques outperform their conventional counterparts, especially under time-varying illumination conditions.

CONCLUSIONS

We have presented two motion estimation techniques, the gradient orientation based gradient method and the gradient orientation structure tensor method that are based, respectively, on the spatio-temporal gradient method and the gradient structure tensor method. Unlike the conventional approaches utilizing image gradients, we make use of gradient orientation information (GOI) by means of unit gradient vectors. Since GOI is insensitive to changes of image intensities, the proposed methods have achieved a significant robustness to time-varying lighting conditions. They also perform better than the previous methods when encountering the aperture problem. The implementation of the proposed methods is straightforward because image gradients are commonly computed at an early stage in image sequence processing and computer vision applications and are readily available. At present, we are testing a correlation-based method using GOI. We plan to evaluate the performances of these GOI-based approaches further on more real image sequences.

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A Block Matching Technique Using Unit Gradient Vectors

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Abstract

Irregular lighting causes temporal variations of image intensities, which makes most existing block matching techniques ineffective. For this, we propose a novel matching technique based on gradient orientation that is known to be insensitive to variations of intensities. We show that gradient orientation information can be effectively utilized by means of two intensity patterns that are obtained as the x and y components of unit gradient vectors. Simulation results show the proposed technique is remarkably robust to both spatially uniform and non-uniform changes of image intensities.

1. Introduction

Establishment of correspondence between two or more images is an important task in image sequence processing and computer vision applications. For instance, image correspondence is an essential step for estimating motions and depths. Motion estimation is concerned with the correspondence between time-sequential images such as video sequences. It finds a variety of applications, including object-based video coding (e.g. MPEG-4), object detection for surveillance systems, scene changes detection for video editing, and image stabilization technology for image acquisition devices.

Techniques for image correspondence in the spatial domain may be classified into two categories; gradient-based methods and matching methods. This paper is concerned with the latter approach that is also widely referred to as block matching, template matching, or correlation-based methods [1]-[5]. In either approach, the intensities of objects in an image are assumed to be constant over time. This assumption, however, is often violated by changes of lighting conditions that is a common incident in outdoor environment. To circumvent this irregular illumination problem, it is reasonable to employ a feature that is less dependent on image intensities or gradients.

This paper presents a novel block matching technique using gradient orientation information, rather than relying on conventional image features such as intensities and gradients, because gradient orientation is known to be insensitive to variations in illumination [6]-[9]. A comparative study with conventional block matching techniques reveals that the proposed method is remarkably robust to both uniformly and non-uniformly varying image intensities.

2. Method

2.1 Gradient orientation information

Let $I(x, y)$ be the image intensities at pixel coordinates (x, y). The gradient vectors of I may be expressed by (I_x, I_y) where I_x and I_y are the partial derivatives of I in x and y directions. Gradient orientation information (GOI) can then be expressed using unit gradient vectors (n_x, n_y) that are obtained by dividing (I_x, I_y) by their norms as

$$\left.\begin{aligned} n_x(x,y) &= I_x(x,y) \Big/ \sqrt{I_x^2(x,y) + I_y^2(x,y)} \\ n_y(x,y) &= I_y(x,y) \Big/ \sqrt{I_x^2(x,y) + I_y^2(x,y)} \end{aligned}\right\}, \tag{1}$$

where we assign zero to $n_x(x, y)$ and $n_y(x, y)$ when the denominator is zero to avoid zero-division.

Fig. 1(a) shows $I(x, y)$ of a test image of size 256 by 256 pixels with 256 gray levels. The upper left corner of the image is the origin, and vertical and horizontal axes are respectively denoted as x and y axes. The small region of size 32 by 32 pixels encompassed by a white square in Fig. 1(a) is cropped and enlarged in Fig. 1(b). Fig. 1(c) shows the gradient vectors (I_x, I_y) within the cropped region while Fig. 1(d) shows the unit gradient vectors (UGVs) in the same region. Note that UGVs carry rich local gradient information even in relatively low-contrast areas.

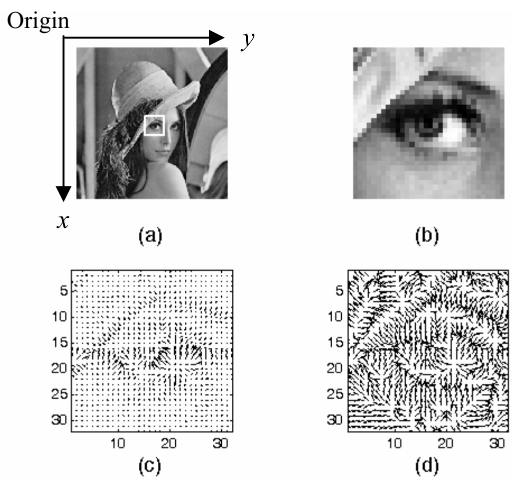
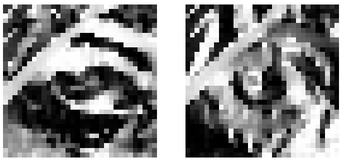


Figure 1. (a) A test image, (b) a cropped and enlarged subimage, (c) gradient vectors and (d) unit gradient vectors within the subimage.

Since UGVs are represented by two scalars n_x and n_y ranging from −1 to 1, we may easily utilize GOI by treating these scalars as intensities. Fig. 2(a) shows the gradient orientation pattern n_x corresponding to the subimage in Fig. 1(b) while Fig. 2(b) shows the gradient orientation pattern n_y. Both n_x and n_y are scaled and visualized as 8-bit intensity patterns.



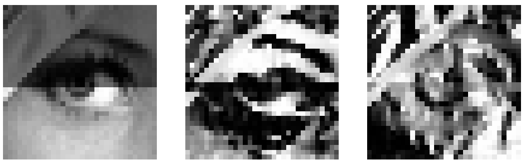
(a) (b)

Figure 2. (a) Gradient orientation pattern n_x and (b) gradient orientation pattern n_y.

It should be stressed that the use of UGVs is computationally more efficient than using angular values θ (rad) because UGVs require no modulo calculations [9].

2.2 Intensity-invariance of gradient orientation

Gradient orientation is known to be insensitive to variations of lighting conditions [6]-[9]. This is because the order of image intensities in a local area is well preserved under varying lighting conditions. For instance, the black pupil is darker than the brown iris irrespective of illumination changes. Fig. 3 demonstrates such intensity invariance of gradient orientation. Fig. 3(a) shows the same subimage as in Fig. 1(b), except that the intensities of the upper half of it are reduced by 50%. Figs. 3(b) and 3(c) show the gradient orientation patterns n_x and n_y. The comparison between the patterns in Fig. 2 and those in Figs. 3(b) and 3(c) shows that gradient orientation patterns remain unchanged before and after shading occurs, except for slight changes along the border of the shade.



(a) (b) (c)

Figure 3. (a) Subimage whose upper half is shaded, (b) gradient orientation pattern n_x and (c) gradient orientation pattern n_y within the subimage.

2.3 Block matching technique with GOI

Instead of image intensities, we make use of gradient orientation patterns as inputs to a conventional block matching technique with the widely used matching metric, the sum of absolute differences (SAD) criterion:

$$\left.\begin{aligned} &GOPM\_n_x(\vec{p},\vec{d}) = \\ &\sum_{j=-N/2}^{N/2}\sum_{i=-N/2}^{N/2}\left|n_{x1}(x+i,y+j)-n_{x2}(x+i+u,y+j+v)\right| \\ &GOPM\_n_y(\vec{p},\vec{d}) = \\ &\sum_{j=-N/2}^{N/2}\sum_{i=-N/2}^{N/2}\left|n_{y1}(x+i,y+j)-n_{y2}(x+i+u,y+j+v)\right| \end{aligned}\right\} \quad (2)$$

where $\vec{p}$ denotes a point (x, y) in the image coordinate, $\vec{d}$ a displacement (u,v) from that point, equivalent to the motion vector between two time-sequential images being compared, N the block size, n_{x1} and n_{y1} gradient orientation patterns of the first frame (reference image), and n_{x2} and n_{y2} the second frame where a best-matching block is being searching for. Finally, these two may be combined into one measure

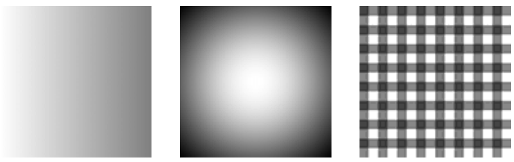
$$\begin{aligned} &GOPM(\vec{p},\vec{d}) = \\ &GOPM\_n_x(\vec{p},\vec{d}) + GOPM\_n_y(\vec{p},\vec{d}) \end{aligned} \quad (3)$$

The position of the best matching is indicated by the minimum of Eq. (3). We call this proposed method the gradient orientation pattern matching technique (GOPM). Note that we also have evaluated the sum of squared differences (SSD) criterion, but there was no noticeable improvement of performance.

3. Results and Discussion

3.1 Simulations on synthetic image sequences

We compare GOPM with SAD block matching (SAD) and zero-mean normalized cross-correlation (ZNCC) on four synthetic image sequences. Four standard test images of size 256 by 256 pixels with 256 gray levels are used as the first frames or references. The second frames are then generated by translating them by 5 pixels both horizontally and vertically. We have computed 225 (15 by 15) motion vectors in each sequence. The size of the block is fixed at 16 by 16 pixels. The range for searching for the best matching position in the second image is set at ±8 pixels both horizontally and vertically. When a motion vector points a correct pixel, it is considered as a successful motion estimate. To make the simulation realistic, zero-mean Gaussian noise is randomly generated and added to every image where the SNR is set at approximately 40dB. Further, to test the robustness to varying lightings, the intensities of the second image are modified in four ways. One is a uniform reduction of intensities, and the other three are non-uniform modifications of intensities achieved by multiplying the masks shown in Fig. 4. Figs. 4(a) and 4(b) show realistic smooth linear and Gaussian shadings. Fig. 4(c), on the other hand, shows rapidly varying shadows that may simulate the case that a spot light is flashed on an object.



(a) (b) (c)

Figure 4. (a) Linear shadow mask, (b) Gaussian shadow mask and (c) checkerboard shadow mask.

Table 1 shows the successful motion estimation rates (%) by the three methods in which the second images are subject to the uniform intensity reduction by 20% (Simulation 1). It is evident that SAD does not work at all while ZNCC and GOPM work nearly perfectly. Tables 2 and 3 show the success rates under non-uniform but smooth changes of image intensities. In Simulation 2, linear shading is applied to the second images where intensities are linearly reduced from the left to the right end of the image up to 50% (Fig. 4(a), Table 2). In Simulation 3, Gaussian shading is applied so that intensities are reduced from the center of the image following the profile of a Gaussian function (Fig. 4(b), Table 3). Under non-uniform but smooth variations of intensities, both ZNCC and GOPM achieve high success rates. Unsuccessful motion estimates are due to the lack of gradient information (e.g. sky) and the aperture problem.

In Simulation 4, rapid and non-uniform shading is applied in which the image intensities in vertical and horizontal stripes are reduced to 50% and the intensities in the areas where two stripes overlap are reduced to 25% (Fig. 4(c)). ZNCC can cope with both additive and multiplicative variations of intensities when those variations occur uniformly within a block. By contrast, GOPM can handle such rapid and non-uniform intensity changes within a block, which is highlighted in Table 4.

Table 1. Success rates under a mildly noisy condition with uniformly varying image intensities.

Simulation 1	SAD	ZNCC	GOPM
Lena	24.9	100	100
Girl	27.1	100	100
Cameraman	40.9	97.8	98.2
House	4.90	99.6	96.4

Table 2. Success rates under a mildly noisy condition with linear shading.

Simulation 2	SAD	ZNCC	GOPM
Lena	25.8	99.1	100
Girl	27.1	100	100
Cameraman	39.1	98.7	99.1
House	4.89	97.3	96.0

Table 3. Success rates under a mildly noisy condition with Gaussian shading.

Simulation 3	SAD	ZNCC	GOPM
Lena	44.9	97.3	99.6
Girl	21.3	100	99.6
Cameraman	36.9	98.2	97.3
House	41.8	94.2	92.9

Table 4. Success rates under a mildly noisy condition with rapid and non-uniform shading.

Simulation 4	SAD	ZNCC	GOPM
Lena	10.2	20.0	97.3
Girl	13.8	26.2	100
Cameraman	13.3	33.3	91.6
House	2.2	5.33	88.0

3.2 Simulations on real image sequences

We next evaluate the performances of SAD, ZNCC, and GOPM on two real image sequences, A and B. Since there is no ground truth data (i.e., true motion vectors) available for these real sequences, we use the motion vectors estimated under a constant illumination as references shown in the left column of Fig. 5. Fig. 5 shows the image sequence A in which the camera tracks a walking man. The motion vectors in background are supposed to point rightward while those on the man are small. Under such ideal lighting condition, the motion estimates by the three methods are similar to each other. We then apply the same four intensity modifications as in 3.1. The robustness of motion estimation is evaluated in terms of the means and variances (m, σ^2) of the differences between the references and the motion vectors estimated under varying lighting conditions. Table 5 shows the differences when the second image is subject to a uniform change of intensities as described in Simulation 1. SAD shows large variances while those of ZNCC and GOPM are far smaller, indicating that the latter two methods work robustly with uniform variations of intensities. Tables 6 and 7 show the differences when the second image is subject to non-uniform but smooth changes of intensities as depicted in Simulations 2 and 3. SAD fails to estimate motion reliably. Conversely, ZNCC and GOPM withstand such lighting conditions. Finally, Table 8 shows the differences when the second image is subject to rapid and non-uniform changes of intensities as in Simulation 4. As shown in the right column of Fig. 5, SAD and ZNCC fail to work properly under such condition, while GOPM still estimates reasonably accurate motion vectors.

Table 5. Differences in the estimated motion vectors before and after uniform shading is applied.

Sim 1	SAD	ZNCC	GOPM
Image A	(5.02, 24.0)	(0.016, 0.02)	(0.067, 0.25)
Image B	(3.51, 21.5)	(0.067, 0.55)	(0.19, 1.77)

Table 6. Differences in the estimated motion vectors before and after linear shading is applied.

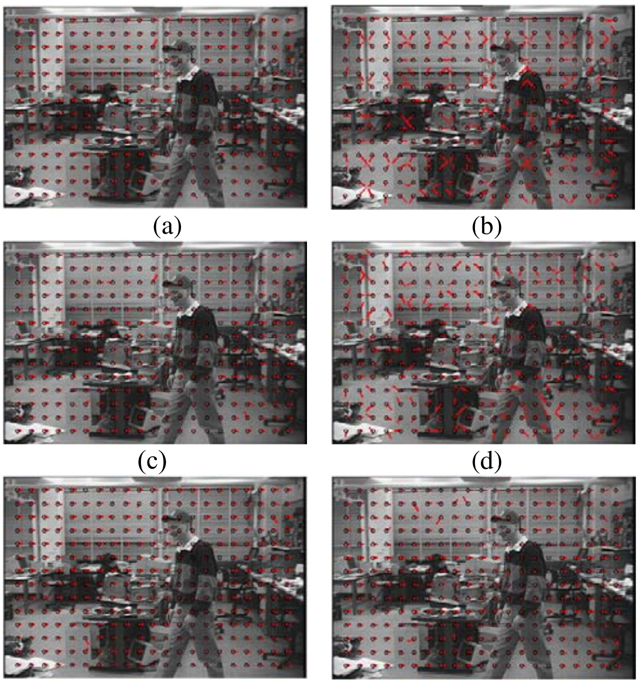
Sim 2	SAD	ZNCC	GOPM
Image A	(6.19, 31.1)	(0.013, 0.63)	(0.11, 0.41)
Image B	(3.67,20.3)	(0.36, 3.46)	(0.34, 2.79)

Table 7. Differences in estimated motion vectors before and after Gaussian shading is applied.

Sim 3	SAD	ZNCC	GOPM
Image A	(5.77, 22.5)	(0.78, 4.74)	(0.22, 0.84)
Image B	(4.19, 28.1)	(0.98, 9.62)	(0.70, 5.44)

Table 8. Differences in estimated motion vectors before and after rapid and non-uniform shading is applied.

Sim 4	SAD	ZNCC	GOPM
Image A	(6.99, 17.9)	(5.93, 24.7)	(0.88, 4.11)
Image B	(5.21, 20.5)	(3.69, 23.5)	(0.77, 5.30)



(a) (b) (c) (d) (e) (f)

Figure 5. Motion vectors estimated by (a), (b) SAD, (c), (d) ZNCC and (e), (f) GOPM under constant (left column) and varying lighting conditions (right column).

3.3 Computational cost

We have compared the computational costs of SAD, ZNCC, and GOPM. Table 9 shows the computation time of each method for computing 225 motion vectors. The three techniques are implemented in MATLAB (Ver. 7.0) and executed on a PC with the Pentium 4 (2.80GHz) and 1GB of RAM. SAD is the fastest method among the three owing to its simplest similarity measure whereas ZNCC is the slowest because of the complexity of its computation [4]. GOPM is slower than SAD because GOPM requires an extra computation of Eq. (1) and also two sums-of-differences have to be computed as in Eq. (2). It is important to note that GOPM is faster than ZNCC. GOPM can be computed more efficiently than ZNCC because the similarity measure of GOPM is the same as that of SAD which is much simpler than that of ZNCC. Another advantage of GOPM is that it allows the use of integers while real numbers are necessary for ZNCC.

Table 9: Computation times of SAD, ZNCC, and GOPM.

	SAD	ZNCC	GOPM
Computation time (sec)	0.94	3.97	1.47

4. Conclusions

Most existing approaches for image matching are based on either image intensities or gradients. Consequently, it is inevitable that these conventional techniques are susceptible to varying image intensities caused by irregular lighting conditions. To cope with this illumination problem, we have presented a novel matching technique that is based on gradient orientation patterns that can be obtained as the x and y components of unit gradient vectors. We do not use the angular values θ (rad) of gradient vectors directly to avoid modulo computation, which enables a fast implementation of the proposed method. Simulation results on both synthetic and real image sequences have revealed that the proposed technique, GOPM, works much more robustly than SAD with varying image intensities. The motion estimation performance of GOPM is comparable to that of ZNCC with uniformly changing intensities and also non-uniformly but smoothly varying intensities. Furthermore, it is a significant advantage of GOPM over ZNCC that it can cope with non-uniform and rapid changes of image intensities that may occur in outdoor environment. We have also shown that the computational cost of GOPM is less than that of ZNCC. Gradient vectors are generally computed at an early stage of various image processing and computer vision applications, and are readily available. The normalization of the gradient vectors to obtain the unit gradient vectors can be performed prior to the computation for image correspondence. Therefore, GOPM will be well-suited to real-time applications and also hardware implementation.

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A Real-time Eye-tracking Method Using Time-varying Gradient Orientation Patterns

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***Abstract*—Eye tracking is a computer vision technique widely used in many fields especially in a medical field. This paper introduced the new eye tracking method based on gradient orientation pattern matching and automatic template update technique. This proposed method can be implemented in real-time application and can provide high robustness to changing lighting condition during tracking process.**

Keywords- eye tracking, real-time tracking, template matching, gradient orientation pattern, template update.

I. Introduction

Eye tracking is one of the fields in computer vision, used for measuring a focusing or an eye movement. The technique of eye tracking can be applied into several other fields such as cognitive study, human interface devices, developing disable support equipments, and developing diagnosis and treatment equipments in opthalmology.

There are many methods proposed for eye tracking, such as gaze tracking method[1], Hough transform method[2], fixation precision in non-contact eye-gaze tracking[3]. These techniques provide very reliable result with low error, however, their require lots of computation and therefore hardly implemented in real-time application. Others newly proposed technique such as horizontal and vertical projection [4],[5], eye detection based on color information[6], and eye-tracking using gray prediction[7]. These technique aimed to improve the speed of eye tracking algorithm so that can be used in real-time application. But these techniques still have their limitations. These real-time techniques used intensity data of the images as an input for analyzing, therefore they are very sensitive to changing lighting condition and obstacle and result in miss-matching. This paper proposes a new eye-tracking method using an gradient orientation pattern matching and automatic template update technique together. The gradient orientation pattern matching (GOPM) is the newly introduced template matching technique that use only the direction of images' gradient for tracking, so that it can provide the robustness to changing lighting condition[8]. The GOPM is also fast enough to be implemented in real-time applications[9]. The automatic template update algorithm can also be applied to improve the correctness of the matching result. By using this method, the error in matching result due to lighting condition is much more reduced.

The remaining parts of this paper are organized as follows: section II briefly introduces the GOPM, section III describes about template matching algorithm, section IV describes the implementation method. Finally, we discuss and the the simulation results in section V.

II. Gradient Orientation Pattern Matching

The gradient orientation pattern matching (GOPM) is a matching technique based on gradient vectors of image. Unlike the others traditional template matching techniques, such as sum-of-squared differences(SSD) or cross-correlation (CC) that use intensity information of the images, GOPM uses unit gradient vectors, obtained through the normalization of image's gradient vector, for establishing image correspondences. A main advantage of GOPM is that it can provide a robustness to lighting-condition changes and yields a very good matching result.

The GOPM uses x-axis and y-axis unit gradient vector as an inputs, obtained through the normalization equation

$$\begin{aligned} n_x(x,y) &= I_x(x,y)/\sqrt{I_x^2(x,y)+I_y^2(x,y)+\epsilon} \\ n_y(x,y) &= I_y(x,y)/\sqrt{I_x^2(x,y)+I_y^2(x,y)+\epsilon} \end{aligned} \tag{1}$$

where I_x and I_y are the gradient of the image at (x, y) coordinates. The ϵ is a small constant that is added to prevent zero-division during normalization. Then the normalized gradient vectors n_x and n_y are used separately using the sum-of-squared differences as

$$\begin{aligned} GOPM\_n_x(\vec{p},\vec{d}) &= \textstyle\sum_{j=-N/2}^{N/2}\sum_{i=-N/2}^{N/2} \\ &\quad (n_{x1}(x+i,y+j)-n_{x2}(x+i+u,y+j+v))^2 \\ GOPM\_n_y(\vec{p},\vec{d}) &= \textstyle\sum_{j=-N/2}^{N/2}\sum_{i=-N/2}^{N/2} \\ &\quad (n_{y1}(x+i,y+j)-n_{y2}(x+i+u,y+j+v))^2 \end{aligned} \tag{2}$$

where $\vec{p}$ denotes a point (x, y) in the image coordinate, $\vec{d}$ denotes a intensity displacement (u, v) of the corresponding points between two images, N the block size, I_1 a block partitioned from the template, and I_2 the block partitioned from image being searched. Both n_{x1} and n_{y1} are the unit gradient vectors of template I_1 and n_{x2},n_{y2} are the unit gradient vectors from the image I_2. Gradient orientation pattern matching can be obtained by combining these two terms.

$$GOPM(\vec{p},\vec{d}) = GOPM\_n_x(\vec{p},\vec{d}) + GOPM\_n_y(\vec{p},\vec{d}) \quad (3)$$

The point that shows the lowest gradient displacement $\vec{d}$ is the best matching point in its local search region. Since GOPM uses only the direction information of gradient, it does not depend on intensities and can provide robustness to varying lighting-condition.It should be noted that the difference between two unit vectors can be also evaluated by the angular dispersion θ between them. However, it requires a trigonometric calculations and modulo calculations (e.g. difference between two angles cannot exceed π) [8]. It is computationally more advantageous to decompose the unit vector into two scalars (i.e., the x and y components) and compute the difference using the standard matching metric such as SAD, SSD, and CC.

III. Automatic Template Update

When GOPM is implemented for eye tracking, sometimes the appearance of pupil and iris is changed, so that no exactly identical pattern of the template appears in the input images. This error cause the miss-matching problems in matching result. In this section, an automatic template update algorithm is introduced in order to increase the performance and reliability of the tracking result. The main concept of automatic template update technique is to reconstruct a new template in every frame using the best-matching data corresponding to the previous input image [10]. By using this technique, the template will be very similar to the current input image, thus the matching result will be improved.

A. Correct-matching Criterion

The usage of automatic template update technique alone has a disadvantage that in case miss-matching occurred, the template will be reconstruct using the miss-matching result, and then yield the totally fail result onward. In order to implement this technique, we need a constrains to check whether the matching result is acceptable or not before we can use it data to reconstruct the template. By using the assumption that the video sequence is fast enough so that no any suddenly change in a location of the corresponding image, these constrains are proposed to limit the acceptable range of the matching results.

$$\vec{\delta} = \sum_{i,j}((n_{x,n+1} - n_{x,n})^2 - (n_{y,n+1} - n_{y,n})^2) \leq \tau \quad \text{and} \quad (4)$$

$$\vec{D} = \sqrt{(x_n - x_c)^2 + (y_n - y_c)^2} \leq T \quad (5)$$

where $n_{x,n}$ and $n_{y,n}$ are the normalized gradient of a current template, $n_{x,n+1}$ and $n_{y,n+1}$ the normalized gradient of new template corresponding to the best-matching position of the current frame, x_n and y_n are the coordinates of the best-matching position of the current frame, x_c and y_c are the coordinates of the best-matching position of latest correct matching result, $\vec{\delta}$ is a difference value between current the template and the new created template, $\vec{D}$ is a Euclidean distance between the best-matching positions of the current frame and latest correct matching frame, τ is the threshold for Eq.(4), and T is the threshold for Eq.(5). Both τ and T can be any threshold function, and these functions determine the efficiency of this technique. In this paper, the value of τ is 1000 and T is 30 thresholds, based on empirical method on authors' video samples. and If the new template corresponding to the matching result has a difference from the currently used template larger than τ, this result should be considered as miss-matching frame by Eq.(4). The matching result that its distance from the last correct result larger than T is too far away and should also be considered as miss-matching frame by Eq.(5). The matching result of every frame need to pass both Eq.(4) and Eq.(5) to be accepted as correct matching result and then be used as new x_c and y_c. If the matching result is considered as miss-matching frame, the new template corresponding to this frame will be discard and the previous template is used instead.

B. Template Update Algorithm

The traditional template matching technique is mostly used for finding a part of the image that are identical to the interesting pattern that called template. In general, for real-time template matching, we use the same template to perform template matching for all images in a video sequence. However, it is difficult to design the template that can suitable to entire the video sequence. Mostly the pre-defined template will be created from the first frame of the entire sequence or the general pattern for that application. But this pre-define template is not guaranteed that it can yield the good result when a sequence passed along. So when the condition of the images in the sequence change, the template will be out-of-date and cannot find the corresponding point any more. To solve this problem, the automatic template update is implemented to update the template along with the video sequence.

Automatic template update algorithm uses the best-matching position which passed the correct-matching criterion to construct the new template for every frame. The algorithm is as be shown in Fig. 1

For every frame, the template matching technique is applied to input image, yield the best-matching location that the corresponding image the template. This position is then tested with the correct-matching criterion. If the position fails the criterion and considered as miss-matching, then this miss-matching position is rejected. If the position pass the criterion, then a portion of input image is cropped corresponding to this position . This crop region with the same size to the old current template will be used as a new template for the next frame. By using this technique, the conditions of template will be almost the same to the conditions of the current frame, helping the matching result to be more success. However, the use of template update technique may make the template to slightly shift from the desire position so the appropriate threshold in correct-matching criteria is needed to prevent this shifting to occured.

IV. Procedure

A. Simulation Environment

The method proposed in this paper is originally aimed to be an eye-tracking system used for ophthalmic surgery, the objective of this simulation is to show the robustness to changing-lighting condition and the improved matching efficiency. In order to response the surgeon work flow, all implementation are developed in real-time. The program is developed using OpenCV library based on C/C++ programming running on computer with Intel Core2 Duo 1.66 GHz processors and 4 GB of memory [11],[12],[13]. For this simulation, we use sample video sequences in various situation including real ophthalmic surgery video. The sample input from the surgery camera and our sample video sequence are shown in Fig.2

The algorithm (Fig.3) consists of four steps: Preprocessing, Gradient Orientation Pattern Matching, correct-matching criterion, and automatic template update. Before running this algorithm, the initial template is needed to be initialized.

B. Preprocesses

Before performing the tracking process in each frame, preprocessing step can be applied to improve the efficiency of the tracking process. This preprocessing step consists of downsampling and low-pass filtering steps. Downsampling is applied to the input video sequence and the template in order to reduce the computation time of the template matching. Since the template matching processes on every single pixel, when the sizes of the input and template decrease, the computation time is reduced [14],[15]. In here, we reduce the size of the input video sequence and template to 50 percent of the height and width. This downsampling has no effect to the matching result since both video and template are downsampled with the same ratio.

Low-pass Filtering is applied in order the remove small noise due to video input/output device or camera quality [16]. This small noise are hard to noticed visually, but can make an large effect on gradient pattern which create an error on the matching result of GOPM. In this simulation, the $3x3$ rectangular low-pass filter is applied to both input video sequence and template (Eq.(6)).

$$H(x,y) = \begin{bmatrix} 0.11 & 0.11 & 0.11 \\ 0.11 & 0.11 & 0.11 \\ 0.11 & 0.11 & 0.11 \end{bmatrix} \quad (6)$$

C. Gradient Orientation Pattern Matching

Gradient Orientation Pattern Matching (GOPM) is applied to perform template matching between input video sequences and the template. The input image's and template's gradient information are extracted into x and y direction as shown in Fig.4. The result of GOPM is the sum of squared differences between gradient of image to those of template in both x and y directions. The point which has the lowest difference will yield the (x, y) coordinates of the best-matching position.

D. Correct-matching Criterion

The best-matching position's coordinates from previous step is checked with this correct-matching criterion (Eq.(4) and Eq.(5)). If the position can satisfy these criterion, then this best-matching position is consider as correct-matching and can be use further. If the position cannot satisfy the criterion, this matching result is considered as miss-matching and be rejected.

E. Automatic Template Update

The best-matching position that can satisfy the previous criterion will be used to update the new template which is to use in step IV-C for the next frame. The algorithm for automatic template update is already described in section III-B.

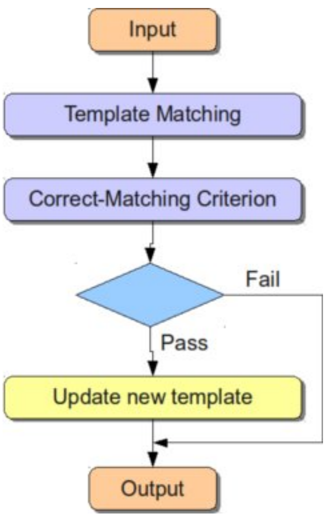
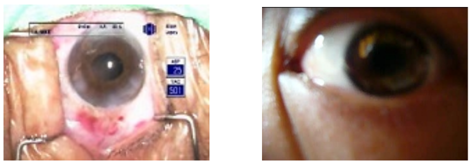


Figure 1: Template update algorithm.



(a) (b)

Figure 2: Sample inputs from (a) a real surgery (b) a light-varying video sequence.

Table I: Precision Error(%) of SSD, standart GOPM, and time-varying GOPM

Input	Standard SSD		Standard GOPM		Time-varying GOPM	
	Average computation time (ms)	Precision Error (%)	Average computation time (ms)	Precision Error (%)	Average computation time (ms)	Precision Error (%)
Video sequence1 under constant lighting condition (Resolution 320x240 px)	78.35	1.33	62.48	0	13.81	0
Video sequence2 under changing lighting condition (Resolution 320x240 px)	78.63	40.47	63.09	12.87	12.92	0

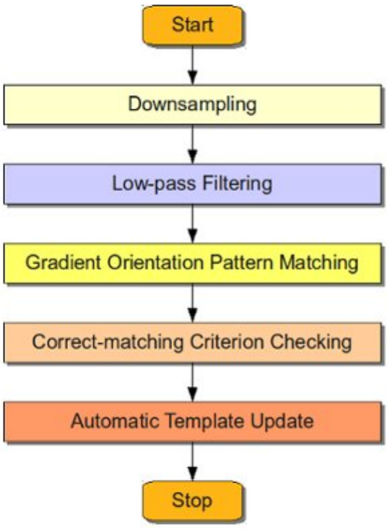
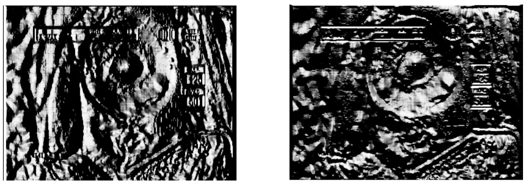


Figure 3: Algorithm of proposed eye-tracking method.



(a) (b)

Figure 4: Gradient infomation of sample frame in (a) x direction and (b) y direction

V. Results

Most of the contents on this paper focus on the precision of tracking and computational time. In Table. I, we compare the recognition performance of SSD, standard GOPM and time-varying GOPM in various conditions; constant lighting condition, changing lighting condition and video sequence from an actual surgery. The precision error is defined as a number of frames that its best matching position goes out of pupil area over the total number of frames.

A. Changing Light Condition Robustness Using GOPM

Firstly, the simulation of three techniques under the uniformly varying or constant lighting condition provide very well performance (i.e. 1.33% precision error of SSD and 0% precision error for both GOPM and time-varying GOPM). However the performance of SSD is rather poor under the changing lighting condition, almost half of the tracking (i.e. 40.47%) failed to estimate the eye's centroid position. Hence standard GOPM still keeps tracking eye's centroid with precision error of 12.87%.

Although standard GOPM yields higher precision than the SSD under the changing lighting condition, the precision error is relatively far higher than the standard GOPM under constant lighting condition. Fig.5b and 5d shows result maps after performing GOPM. In Fig.5b, which is the result map of corrected matching frame (Fig.5a) shows an obviously best-matching peak (darkest spot) whereas the result map of mis-matching frame(Fig.5a) shows many of low amplitude peaks. This implies that matching technique might found the similar pattern of gradient in the unexpected places. By inspection, source of the problems came from the uncontrollable factor such as responsiveness or luminous sensitivity of camera. Some frame might loss significant content and cause an adjustment in gradient information i.e. the template is out-of-date.

B. Error Rejection Using the Automatic Template Update template and Correct Matching Criteria

In simulation, correct-matching criterion individually could handle almost mis-matching frame. However the criteria is to unable indicate all of the existence mis-matching frame. As ideas presented in the previous section, we made use of the template update algorithm with correct matching criterion to prevent the mis-matching from a large dissimilarity between template and sample component in difference time. As a result in Table tab:comparetable, time-varying GOPM reduces precision error of standard GOPM from 16.01% to 0.884% which imply the great influence of template update algorithm and correct-matching criterion on GOPM efficiency.

However in some circumstances, template update may make the template which consists of no eye at all. This

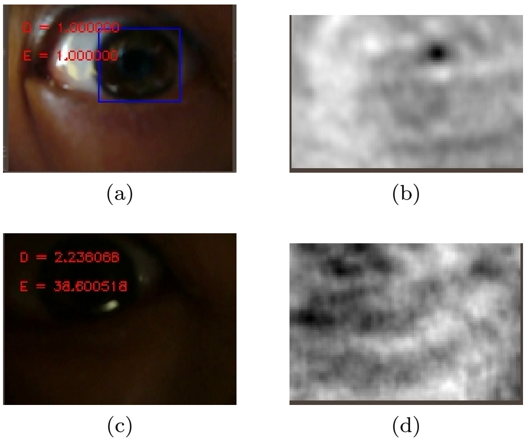


Figure 5: Matching results and corresponding GOPM map (a) correct matching result, (b) corresponding GOPM map, (c) mis-matching result, and (d) corresponding GOPM map.

fault template can fail the tracking result. In this case, the template is needed to be reset to the initial template before continue tracking process.

C. Computation Time

By performing downsampling to the video sequence, the computation time of the proposed technique is far faster than previous one. Table I clearly shows that the computation time is reduced from about 60ms to about 15ms in standard SSD and GOPM after downsampling is applied. After downsampling, the computation times decrease to about 1/4 of the original-size video. With this computation time, time-varying GOPM can process at more than 50 frames per second, with is now enough for implement to the camera which normally operate at 25 frames per second.

VI. Conclusion

We previously showed that a gradient orientation pattern matching technique (GOPM) works remarkably well under varying lighting conditions [8], [9]. In this paper, we presented a time-varying GOPM for human eye tracking. The proposed method updates the template over time, which enhances the tolerance of the tracking method to the variations in image acquisition conditions (in addition to lighting conditions), including slight changes of the image sharpness, scales, and the camera view-points. The template is updated only when two conditions are satisfied, which ensures that the two templates before and after updating are geometrically close enough to each other and also resemble enough to avoid template drift. Another notable improvement over the previous work is that we have significantly shortened the computation time by introducing a down-sampling step of the input video sequences. Thus, the proposed method can work very robustly and smoothly in real time. To summarize, we have achieved a higher performance of real-time eye-tracking at a reduced computation cost. For further improvement of the technique, we plan to make the conditions for template updating adaptive rather than the fixed criteria that are currently used.

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