รายงานวิจัยฉบับสมบูรณ์

โครงการ การพัฒนาทฤษฎีสำหรับวิเคราะห์อัตราแพ็กเกตผิดพลาดของระบบโทรศัพท์เคลื่อนที่เซลลูลาร์แบบซีดีเอ็มเอ

โดย นายพงศธร เศรษฐีธร

16 มีนาคม 2553

สัญญาเลขที่ MRG5280130

โครงการ การพัฒนาทฤษฎีสำหรับวิเคราะห์อัตราแพ็กเกตผิดพลาดของระบบโทรศัพท์เคลื่อนที่เซลลูลาร์แบบซีดีเอ็มเอ

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ภาควิชาวิศวกรรมไฟฟ้า คณะวิศวกรรมศาสตร์

มหาวิทยาลัยมหิดล

สนับสนุนโดยสำนักงานคณะกรรมการอุดมศึกษา

และสำนักงานกองทุนสนับสนุนการวิจัย

กิตติกรรมประกาศ

ขอขอบคุณสำนักงานคณะกรรมการอุดมศึกษาและสำนักงานกองทุนสนับสนุนการวิจัย สำหรับเงินทุนสนับสนุนโครงการวิจัยนี้ ขอขอบคุณ รศ. ดร. ลัญฉกร วุฒิสิทธิกุลกิจ นักวิจัยพี่เลี้ยง ผู้ให้คำปรึกษาอันเป็นประโยชน์อย่างยิ่งต่อการดำเนินงานวิจัย ตลอดจนคณาจารย์ ภาควิชาวิศวกรรมไฟฟ้า คณะวิศวกรรมศาสตร์ มหาวิทยาลัยมหิดล ที่ให้คำแนะนำต่างๆ อย่างดีตลอดมา

หัวหน้าโครงการวิจัย

นายพงศธร เศรษฐีธร

Abstract

Project Code : MRG5280130

Project Title : Development of a Theoretical Analysis for Packet Error Rate of CDMA Mobile Cellular Systems

Investigator : Mr. Pongsatorn Sedtheetorn

Department of Electrical Engineering, Faculty of Engineering

Mahidol University

E-mail Address : egpse@mahidol.ac.th

Project Period : 1 Year

This project presents an original and exact theoretical analysis for the packet error rate in CDMA cellular systems. The signals are accurately analyzed chip-by-chip and then the expressions of the cumulative probability density function of the signal to interference plus noise ratio (SINR) for both BPSK and QPSK-modulated CDMA are introduced. This facilitates formulating new accurate closed-form expressions of packet error rates that consider bit-to-bit error dependence, optimum transmitted power level and distribution of users in a 3G cellular multipath fading environment. As a result, the proposed expressions lead to accurate evaluation for the system performance of CDMA in terms of both packet error rate and throughput. It is important to say that the proposed expressions can be developed into technical software for any CDMA network operator.

Keywords : Packet error rate, CDMA, Cellular systems

บทคัดย่อ

รหัสโครงการ : MRG5280130

ชื่อโครงการ : การพัฒนาทฤษฎีสำหรับวิเคราะห์อัตราแพกเกตผิดพลาดของระบบโทรศัพท์เคลื่อนที่เซลลูลาร์ซีดีเอ็มเอ

ชื่อนักวิจัย : นายพงศธร เศรษฐีธร

ภาควิชาวิศวกรรมไฟฟ้า คณะวิศวกรรมศาสตร์ มหาวิทยาลัยมหิดล

อีเมล์ : egpse@mahidol.ac.th

ระยะเวลาโครงการ : 1 ปี

โครงการนี้นำเสนอทฤษฎีใหม่ในการวิเคราะห์อัตราแพ็กเกตผิดพลาดอย่างแม่นยำ ในระบบเซลลูลาร์ซีดีเอ็มเอ โดยจะวิเคราะห์สัญญาณอย่างละเอียดทีละชิป เป็นผลให้ได้สมการของฟังก์ชันการแจกแจงความน่าจะเป็นสะสมของสัดส่วนของสัญญาณต่อสัญญาณแทรกสอดบวกนอยส์ของทั้งซีดีเอ็มเอที่มอดูเลตแบบ BPSK และ QPSK ทำให้สามารถนำเสนอสมการรูปปิดของอัตราแพ็กเกตผิดพลาดที่พิจารณาความอ้างอิงระหว่างบิต ระดับกำลังส่งดีที่สุด และการแจกแจงผู้ใช้ในระบบเซลลูลาร์ซีดีเอ็มเอ และท้ายที่สุด เราสามารถนำสมการดังกล่าวไปใช้ในการตรวจสอบสมรรถนะของระบบซีดีเอ็มเอทั้งในรูปแบบของอัตราแพ็กเกตผิดพลาดและค่าวิสัยสามารถ โดยกล่าวได้ว่า สมการที่นำเสนอนี้สามารถนำไปใช้เป็นซอฟท์แวร์สำหรับผู้ให้บริการระบบซีดีเอ็มเอได้ด้วย

คำหลัก *:* อัตราแพ็กเกตผิดพลาด ซีดีเอ็มเอ ระบบเซลลูลาร์

Output จากโครงการวิจัยที่ได้รับทุนจาก สกอ. และ สกว.

ผลงานตีพิมพ์ในวารสารวิชาการนานาชาติ

1. วารสารวิชาการนานาชาติ (International Transaction Journal) ในหัวข้อเรื่อง “Accurate packet error rate analysis of VSG-CDMA and MCD-CDMA wireless communication networks” ในวารสาร IEEE VTC (International Electrical and Electronics Engineering Vehicular Technology Communication) โดยได้การตอบรับให้ตีพิมพ์จากผู้พิจารณาบทความ (*reviewer*) *2* จำนวน ท่านจากทั้งสิ้น *3* ท่าน ซึ่งในขณะนี้กำลังรอประกาศเวลาตีพิมพ์

2. วารสารจดหมายวิชาการนานาชาติ (International Transaction Letter) ในหัวข้อเรื่อง “MGF-based Performance Analysis for Wireless Systems in Nakagami Fading” ในวารสาร IET Electronics Letter ขององค์กร The Institute of Engineering and Technology (The IET) โดยในขณะนี้อยู่ระหว่างรอผลการพิจารณา

3. วารสารจดหมายวิชาการนานาชาติ ในหัวข้อเรื่อง “Theoretical analysis on bit error rate of VSG CDMA in Nakagami Fading” ในวารสาร IEEE Information Theory Letter ขององค์กร IEEE Information Theory Society โดยในขณะนี้อยู่ระหว่างรอผลการพิจารณา

การนำผลงานวิจัยไปใช้ประโยชน์

- เชิงวิชาการ ผู้ทำวิจัยได้นำทฤษฎีที่ได้จากโครงการวิจัยนี้ไปใช้ในการเรียนการสอนวิชา EGEE322 Digital Communications และ EGEE429 Modern Wireless Communications

ภาคผนวก โดยจัดเรียงเอกสารตามลำดับต่อไปนี้

1. วารสารวิชาการนานาชาติ (International Transaction Journal*)* ในหัวข้อเรื่อง “Accurate packet error rate analysis of VSG-CDMA and MCD-CDMA wireless communication networks”

2. ผลการพิจารณาบทความ *“*Accurate packet error rate analysis of VSG-CDMA and MCD-CDMA wireless communication networks”

3. วารสารจดหมายวิชาการนานาชาติ (International Transaction Letter) ในหัวข้อเรื่อง “MGF-based Performance Analysis for Wireless Systems in Nakagami Fading”

4. จดหมายยืนยันการส่งบทความ “MGF-based Performance Analysis for Wireless Systems in Nakagami Fading”

5. วารสารจดหมายวิชาการนานาชาติในหัวข้อเรื่อง “Theoretical analysis on bit error rate of VSG CDMA in Nakagami Fading”

6. จดหมายยืนยันการส่งบทความ “Theoretical analysis on bit error rate of VSG CDMA in Nakagami Fading”

เนื้อหางานวิจัย

บทนำ

ระบบโทรศัพท์เคลื่อนที่เซลลูลาร์ซีดีเอ็มเอ เป็นระบบที่ออกแบบสำหรับบริการข้อมูลความเร็วสูง โดยที่สมรรถนะ (performance) ในการรับส่งข้อมูลสามารถวัดได้จากค่าวิสัยสามารถ (throughput) ซึ่งขึ้นอยู่กับอัตราเร็วของข้อมูลหรือแบนด์วิดท์และอัตราการส่งแพ็กเกตผิดพลาด สำหรับค่าอัตราการส่งแพ็กเกตผิดพลาดนั้น เป็นค่าที่เกิดจากสภาพแวดล้อมภายนอกที่ไม่สามารถคาดคะเนได้ นอกจากนี้ยังเป็นปัจจัยที่สำคัญมากต่อค่าวิสัยสามารถหรือสมรรถนะของระบบโทรศัพท์เคลื่อนที่เซลลูลาร์ซีดีเอ็มเอ โดยอัตราการส่งแพกเกตผิดพลาดนี้จะมีค่ามากหรือน้อยขึ้นอยู่กับจำนวนสัญญาณรบกวนเข้าถึงหลายทาง (multiple access interference, MAI) ที่ได้รับมาจากผู้ใช้อื่น ๆ หรือเซลล์ข้างเคียง ดังนั้น งานวิจัยทางด้านการคำนวณหาค่าอัตราการส่งแพ็กเกตผิดพลาด จึงเป็นสิ่งสำคัญอย่างยิ่งสำหรับการพัฒนาโทรศัพท์เคลื่อนที่เซลลูลาร์ซีดีเอ็มเอ ให้มีประสิทธิภาพสูงขึ้น

อย่างไรก็ตาม ในส่วนของงานวิจัยในสาขานี้ ถือว่ายังมีน้อยโดยเฉพาะอย่างยิ่งในประเทศไทย เนื่องเทคโนโลซีดีเอ็มเอเป็นเทคโนโลยีที่ค่อนข้างซับซ้อน นักวิชาการตลอดจนผู้ให้บริการโครงข่ายจำนวนมากยังขาดความรู้ความเข้าใจในเทคโนโลยีอย่างจริงจัง ถึงแม้ว่า จะเป็นเพียงการรับเทคโนโลยีจากต่างประเทศเข้ามาใช้ในประเทศ แต่เราก็ต้องเข้าใจถึงเทคโนโลยีและต้องทำการวิจัยและพัฒนา (research and develop, R&D) อยู่ตลอดเวลาเพื่อให้การใช้เทคโนโลยีเกิดประโยชน์สูงสุด

จากปัญหาที่กล่าวข้างต้น ผู้ทำวิจัยจึงเสนอโครงการนี้ โดยมุ่งหวังให้เกิดการสร้างองค์ความรู้ใหม่ในการพัฒนาเทคโนโลยีโทรศัพท์เคลื่อนที่เซลลูลาร์ซีดีเอ็มเอ ซึ่งแน่นอนว่า องค์ความรู้ใหม่ที่ได้นี้จะสามารถนำไปใช้ได้จริงในเชิงปฏิบัติ

วัตถุประสงค์

โครงการงานวิจัยนี้เป็นการสร้างองค์ความรู้ใหม่ในการพัฒนาทฤษฎีสำหรับการวิเคราะห์และคำนวณอัตราการส่งแพ็กเกตผิดพลาดอย่างแม่นยำบนระบบโทรศัพท์เคลื่อนที่เซลลูลาร์ซีดีเอ็มเอ โดยองค์ความรู้ (ทฤษฎี) ที่ได้จะเป็นประโยชน์อย่างมากในทางประยุกต์ กล่าวคือสามารถนำไปพัฒนาเป็นซอฟท์แวร์สำหรับให้ผู้ปฏิบัติการโครงข่าย (network operator) ใช้ในการบำรุง (maintenance) การบริการรับส่งข้อมูลของโครงข่ายไร้สายเซลลูลาร์ซีดีเอ็มเอให้มีมาตรฐานในการให้บริการตามที่กำหนด

ระเบียบวิธีการวิจัย (วิธีการทดลอง)

งานวิจัยมีกำหนดระยะเวลาการทำงานประมาณ 12 เดือน โดยแบ่งช่วงการทำงานเป็น 5 ช่วง คือ

<u>ช่วงที่ 1</u> (มี.ค.-มิ.ย. 2552)

- พัฒนาทฤษฎีสำหรับการวิเคราะห์อัตราแพ็กเกตผิดพลาดบนระบบ VSG CDMA โดยขอบเขตของงานวิจัยจะอยู่ที่ระบบซีดีเอ็มเอที่อยู่ในสภาพแวดล้อมของตัวเมือง (urban area) โดยจะเน้นการสื่อสารขาขึ้น (uplink) จากตัวโทรศัพท์เคลื่อนที่ไปยังสถานีฐาน โดยทฤษฎีดังกล่าวจะใช้ความน่าจะเป็นและเอกลักษณ์ทางคณิตศาสตร์เข้ามาช่วยในการวิเคราะห์ (ดูรายละเอียดในบทความ)

<u>ช่วงที่ 2</u> (ก.ค.-ก.ย. 2552)

- ขยายผลทฤษฎีการวิเคราะห์ไปสู่ระบบ MCD CDMA ซึ่งเป็นเทคโนโลยีที่ใช้หลายๆรหัสในการส่งข้อมูลที่มีอัตราเร็วสูงจากนั้นผู้ทำวิจัยจะทำการทดสอบทฤษฎีที่ได้ทั้งในเชิงคณิตศาสตร์และเชิงซิมมูเลชันร่วมกับทางอาจารย์นักวิจัยพี่เลี้ยงคือ รศ ดร ลัญฉกร วุฒิสิทธิกุลกิจ
- เขียนบทความวารสารนานาชาติ ในหัวข้อเรื่อง “Accurate packet error rate analysis of VSG-CDMA and MCD-CDMA wireless communication networks” ส่งไปที่วารสาร IEEE VTC (International Electrical and Electronics Engineering Vehicular Technology Communication)

<u>ช่วงที่ 3</u> (ต.ค.-ธ.ค. 2552)

- ขยายขอบเขตงานวิจัยไปสู่การวิเคราะห์สมรรถนะของโครงข่ายไร้สายในสภาพแวดล้อมที่มีการจางหายของสัญญาณแบบพหุวิถีชนิดนากากามิ ซึ่งการวิเคราะห์ดังกล่าวต้องใช้เอกลักษณ์ทางคณิตศาสตร์บางเอกลักษณ์ โดยได้รับคำแนะนำจากผู้ทรงคุณวุฒิต่างประเทศ Dr. Khairi Hamdi, Senior Lecture, The University of Manchester
- รวบรวมผลงานวิจัยส่งวารสารจดหมายวิชาการนานาชาติ (International Transaction Letter) ในหัวข้อเรื่อง “Theoretical analysis on bit error rate of VSG CDMA in Nakagami Fading” ในวารสาร IEEE Information Theory Letter *ขององค์กร* IEEE Information Theory Society

<u>ช่วงที่ 4</u> (ม.ค.-ก.พ. 2553)

- พัฒนาทฤษฎีการวิเคราะห์สมรรถนะของระบบ CDMA ด้วยเทคนิคเชิงความน่าจะเป็นด้วยวิธีฟังก์ชันโมเมนต์ก่อกำเนิด (Moment generating function, MGF)

– รวบรวมผลงานวิจัยส่งวารสารจดหมายวิชาการนานาชาติ ในหัวข้อเรื่อง “MGF-based Performance Analysis for Wireless Systems in Nakagami Fading” ในวารสาร IET Electronics Letter ขององค์กร The Institute of Engineering and Technology (The IET)

<u>ช่วงที่ 5</u> (มี.ค.2553) สรุปและจัดทำรายงานโครงงานวิจัยฉบับสมบูรณ์

ผลการวิจัยที่ได้ (ผลการทดลอง)[1]

จากกระบวนการดำเนินการวิจัยที่กล่าวมา จะได้ผลงานวิจัยดังนี้

1. ทฤษฎีการหาอัตราแพ็กเกตผิดพลาดอย่างแม่นยำบนระบบสื่อสารไร้สายซีดีเอ็มเอแบบอัตราขยายแปรเปลี่ยน (variable spreading gain CDMA, VSG CDMA) และแบบหลายรหัส (multicode CDMA, MCD CDMA)

2. การวิเคราะห์สมรรถภาพในรูปแบบของประสิทธิภาพสเปกตรัม (spectral efficiency) และอัตราบิตผิดพลาด (bit error rate) บนโครงข่ายไร้สายโดยใช้ทฤษฎีความน่าจะเป็นเรื่องฟังก์ชันก่อกำเนิด (moment generating function, MGF) มาช่วยให้การวิเคราะห์แม่นยำและใช้เวลาในการประมวลผลอย่างมีประสิทธิภาพมากขึ้น

3. การวิเคราะห์หาอัตราบิตผิดพลาดของระบบโทรศัพท์เคลื่อนที่ซีดีเอ็มเอในสภาพแวดล้อมที่มีการจางหายของสัญญาณแบบพหุวิถีชนิดนากากามิ (Nakagami multipath fading)

บทวิจารณ์

องค์ความรู้เชิงทฤษฎีที่ได้จากงานวิจัยนี้สามารถขยายขอบเขตไปสู่ระบบสื่อสารไร้สายอื่นที่นอกเหนือจากระบบซีดีเอ็มเอ ยกตัวอย่างเช่น ระบบหลายอินพุตหลายเอาต์พุต (multiple input multiple output, MIMO) หรือระบบที่ใช้ตัวรับแบบ Rake (Rake receivers) ที่ประกอบไปด้วยอาร์เรย์ของแมทช์ฟิลเตอร์ (array of match filters) หรือที่เราเรียกว่า N-finger Rake receivers โดยระบบการสื่อสารทั้งสองรูปแบบ กำลังเป็นหัวข้อที่น่าสนใจเป็นอย่างยิ่งสำหรับวงการวิจัยของระบบโทรคมนาคมทั้งแบบเซลลูลาร์ 4 จีและบรอดแบนด์ไร้สาย Wimax

นอกจากนี้ทฤษฎีดังกล่าวยังสามารถนำไปขยายสมมติฐานของการทำวิจัยให้สามารถหาค่าอัตราแพ็กเกตผิดพลาดได้ทั้งในแนวสัญญาณ (LOS) และนอกแนวสัญญาณ (NLOS) ซึ่งต้องใช้การโมเดลด้วย

[1] ทฤษฎีนำเสนอตลอดจนผลการทดลองที่ใช้ในการเปรียบเทียบสามารถดูรายละเอียดไปที่บทความในภาคผนวก

ตัวแปรสุ่มแบบไรซ์ (Rice) หรือนากากามิ (Nakagami) ที่มีความซับซ้อนมาก จากการสำรวจบทความที่เกี่ยวข้อง ผู้ดำเนินการวิจัยยังมิพบการวิเคราะห์หาค่าอัตราแพ็กเกตผิดพลาดด้วยโมเดลดังกล่าว

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Decision Letter (VT-2009-00938)

From: eeeugshl@cityu.edu.hk

To: egpse@mahidol.ac.th, pak2520@yahoo.com

CC: egpse@mahidol.ac.th, pak2520@yahoo.com, k.hamdi@manchester.ac.uk, hamdi@ieee.org, wlunchak@chula.ac.th, eeeugshl@cityu.edu.hk, ieeetvt@ecemail.uwaterloo.ca

Subject: IEEE TVT - Decision on VT-2009-00938

Body: Dear Dr. sedtheetorn:

The review of paper

VT-2009-00938: Accurate Packet Error Rate Analysis of VSG-CDMA and MCD-CDMA Wireless Communication Networks

has been completed. Attached below please find the reviewers' comments.

After careful consideration of the reviewers' comments, the decision has been made not to publish this paper in the IEEE Transactions on Vehicular Technology.

However, you may take into consideration the comments from the reviewers and re-submit it as a new submission within 6 months. If you choose to resubmit your paper, please refer to this original paper number (VT-2009-00938) when submitting and include a summary of the changes you have made in response to the AE and reviewers. The manuscript will undergo a new review process.

Thank you for submitting your manuscript to the TRANSACTIONS. We look forward to the possibility of receiving more submissions on your future technical work.

Sincerely yours,

Prof. Shu-Hung Leung
Associate Editor, IEEE Transactions on Vehicular Technology

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Reviews for VT-2009-00938

*Please note that there may be attached files that contain additional comments from the reviewers. To see these files, please log on to your "Author Center" at the Manuscript Central web site (http://mc.manuscriptcentral.com/tvt-ieee) and select this paper to see and download the files.
==

Reviewer: 1
ADDITIONAL COMMENTS TO THE AUTHOR (will be returned to the author):
The authors present an analysis for PER in VSG and MD CDMA systems. While the techniques presented in the paper are interesting, application and feasibility of the analysis is doubtful. The claim is cross-layer analysis, but seriously only a routine PER analysis is considered. The expressions are too complex to be called "closed-form". A complex integral can really not be called closed form. Moreover, their analysis for multi-cell does not exactly capture the power control performed in each cell. While this can be overlooked for the sake of avoiding complexity in the analysis, not mentioning it at all is surprising. Assuming perfect power control for multi-rate packet applications is also not practical. In practice, rate control is performed on multimedia applications and power control is reserved only for voice. I could overlook that assumption too as a means of simplifying the analysis. Yet the techniques mentioned in the paper are interesting and tha paper is worth publishing. However, the authors must change the claims. The expressions (especially those in (50), (51) etc are so complex that the authors must avoid the use of the term closed form for those. There are some typos (HSPDA for HSDPA) which must be corrected.

Although some of the assumptions made are impractical, there are some interesting insights available from the analysis. If the authors remove some claims (e. g. they claim intensive literature survey, it is hardly intensive) and calling huge complex expressions as closed form, **then I do recommend acceptance with minor changes.**

Reviewer: 2
ADDITIONAL COMMENTS TO THE AUTHOR (will be returned to the author):
The paper analyzes packet error rate and throughput for variable spreading gain CDMA and multicode CDMA systems. The bit error dependency cross bits in a packet is considered. An expression of accumulative probability function of SIR is obtained, which yields the expression of packet success rate. **The paper can be published with minor revision.**

1) The paper considers only uncoded packet transmission, which should be stated at the beginning of the system description.
2) When the system is large, in particular for the MCD-CDMA where each user transmits simultaneously a number of packets each spread by a random sequence, the result in [A] can be applied to obtain the constant SIR. Authors should address this result in the large system case.
3) page 4 line 22: Should L×R1 be L/R1?
4) page 4 line 26: “a unique signature sequence with rate G” is not clear. If G is the processing gain or chip rate, then state so. The same problem exists for page 4 line 40.
5) page 4 line 47: “It” should be it.
6) page 4 line 51: ,
7) page 5 line 41: rect(t/(Tb/m)-i) can be written as rect(t-(Tb/m)i) for brevity and easiness of understanding. The same problem exists for page 6 line 13.
8) page 6 line 13: “where G is the processing gain and equal to the number of chips per bit duration.” should be presented when G appears the first time.
9) page 6 line 17: “equivalent” should read “equal”.
10) page 6 line 26: “r(t) by” should read “by r(t)”.
11) page 7 line 44: remove the formula because it does not present anything new as (9) has presented.
12) page 12 line 8: erfc should be defined, or the accumulative probability function of a standard Gaussian random variable is used.
13) page 12 line 8: a pair of parentheses are missing.

[A] S. Shamai (Shitz) and S. Verdú, “The impact of frequency-flat fading on the spectral efficiency of CDMA,” IEEE Trans. Inf. Theory, vol. 47, no. 4, pp. 1302-1327, May 2001.

Reviewer: 3
ADDITIONAL COMMENTS TO THE AUTHOR (will be returned to the author):
The paper is restricted to uncoded systems whereas all practical CDMA systems are coded - typically turbo codes are used for redundancy.

The paper makes an assumption regarding independent bit errors with the same SINR. This will typically not hold unless the fading is very slow.

Additionally, multipath is not explicitly modeled. Multipath is the main reason why MCD CDMA would have loss of orthogonality. Otherwise the M signatures from one user would usually be orthogonal.

All three issues (uncoded system analyzed, assumptions that are only valid for slow fading environments, and lack of multipath modeling) need to addressesed. The paper needs to be reanalyzed and reworked and a new paper written.

The changes required in the analysis are significant. **The recommendation is therefore to reject the paper in its current form and not reconsider for publication.**

Date Sent: 22-Jan-2010

Accurate Packet Error Rate Analysis of VSG-CDMA and MCD-CDMA Wireless Communication Networks

Pongsatorn Sedtheetorn*, Khairi Hamdi**, and Lunchakorn Wuttisittikuljit***

Abstract

This paper presents a system level theoretical analysis of two different types of multirate code division multiaccess (CDMA) wireless communication systems, which include both variable spreading gain (VSG) CDMA and multicode (MCD) CDMA systems. Based on accurate interference models, we derive unified expressions of the cumulative probability density function of the signal to interference plus noise ratio (SINR) in the cases of both binary and quaternary multirate CDMA schemes. These lead to new unified accurate expressions for the packet error rates which take into account the bit-to-bit error dependence in a slotted packet transmission system. These are used to compare between the system-level performance of VSG and MCD based multicellular CDMA networks over Rayleigh fading environment.

Index Terms

Multicode CDMA, packet error rate, Rayleigh fading, variable spreading gain CDMA

I. Introduction

This paper is concerned with the system level performance analysis of multirate DS/CDMA cellular systems, and presents new accurate analysis of packet error rates of uplink variable spreading gain (VSG) CDMA and multicode (MCD) CDMA systems.

*The author is with the Department of Electrical Engineering, Mahidol University, Thailand, Tel. +66 2 889 2138 ext. 6522, Email: egpse@mahidol.ac.th. **The author is with the School of Electrical & Electronic Engineering, The University of Manchester, United Kingdom, Tel. +44 161 306 4728, Email: k.hamdi@manchester.ac.uk. ***The author is with the Department of Electrical Engineering, Chulalongkorn University, Thailand, Tel. +66 2 218 6908, Email: wlunchak@chula.ac.th.

Most previous research on performance analysis of multirate CDMA have been limited to bit error rate analysis (e.g., [1]-[8]). In [1], Fan *et al* have presented an analytical method for bit error performance of VSG CDMA and MCD CDMA in uplink transmissions. Instead of employing the standard Gaussian approximation (SGA) [2]-[3] which is optimistically inaccurate (compared with the actual error rate), the analysis is based on improved Gaussian approximation (IGA) [4]. However, IGA technique requires very high numerical effort to compute multiple convolutions (K-fold convolutions) of K interferences with respect to the reference signal. The proposed method is therefore confined to a dual-rate CDMA system with a few users.

To reduce the complexity of IGA, Holtzman [5] has proposed simplified Gaussian approximation (SIGA) technique which uses an expansion in differences (Stirling formula) to estimate the K-fold integrations rather than in derivatives. SIGA is a good trade-off between SGA and IGA in terms of accuracy and simplicity [6]. However, there are some exact expressions which have been proposed recently (e.g. [7]-[8]) without any penalty on the accuracy of error rate approximation.

For instance, Lee and Beaulieu [7] have introduced a non-Gaussian approximation for exact bit error rate of MCD-CDMA in Rayleigh fading. The final expression of the error rate is presented in a single-integral form where the integrands are the characteristic functions of interferences. Nevertheless, the derivation of the integrands requires full knowledge of the signature sequences. This makes the computation of the final expression very intractable especially when the spreading gain is larger than 32. Moreover, this technique does not lend itself to packet error rate approximation.

Hamdi [9] has proposed an accurate analysis on packet error performance for a single rate CDMA in Rayleigh fading. Based on IGA, the expression of cumulative probability density function of signal-to-interference-plus-noise ratio (SINR) is derived. This leads to a new explicit expression of packet error rate that accounts for bit-to-bit error dependence. However, this work is limited to a conventional CDMA single-cell system in which the data rate of every user is assumed to be identical.

According to our literature survey, most of the previous work is implemented based on the performance of physical layer. To analyze the performance of higher layers such as the packet error rate, one needs to account for the impact of bit-to-bit error dependence within each transmitted packet [10]. This very complicates the analysis, though little of the related work (e.g.

[9]) could approximate the packet error rate but it cannot be applied to such multirate packet CDMA systems.

On the other hand, packet level performance analysis can be used as a key constraint for cross-layer optimization problems. For instance, Wong *et al* [11] have formulated a joint throughput maximization for VSG CDMA across 3 layers– connection layer, packet layer, and link layer– where the probability of packet loss is set as a constraint of the problem. Similarly in [12]-[13], the outage probability of packet error is defined as a criterion of the proposed optimized scheduling technique for uplink DS-CDMA systems.

In [14], Yu *et al* have presented an optimal joint admission control policy for WLAN and CDMA networks, where the constraint of the policy is subject to the average delay of packet transmissions. Assaad *et al* [15] have also introduced a new analytical model of packet throughput for a high-speed downlink packet access (HSDPA) system in Nakagami fading environment. The proposed model is used to evaluate the performances of 3 different packet scheduling techniques, namely proportional fair, score-based, and Max C/I. As in the literature mentioned above, we positively say that the issue of packet-level performance recently gains considerable research interests.

In this paper, we go deeper into theoretical analysis of exact packet error rate for uplink in the cases of both VSG and MCD CDMA in Rayleigh fading. The analysis leads us to introduce new expressions of the packet error rate that considers bit-to-bit error dependence, optimum transmitted power levels and random users' distributions for both BPSK and QPSK-modulated schemes. These allow us to comprehensively study the impacts of channel's conditions, power allocations, and the characteristics of random users' distributions on the packet error performance of multirate CDMA in Rayleigh fading.

The rest of this paper is organized as follows. Section II explains the system model of packet-based VSG and MCD CDMA systems. Section III and IV are concerned with SINR analysis for VSG and MCD CDMA in the cases of binary and quaternary modulation. This leads us to introduce new explicit expressions of packet error rate and throughput for both VSG and MCD CDMA in Section V. In Section VI, we extend the packet error rate analysis to multicell CDMA systems. Section VII presents some numerical as well as simulation results, and Section VIII finally draws the conclusions.

II. The Packet Layer Model

In this section, we present the packet formats of VSG and MCD CDMA. On the other hand, detailed description of the PHY model for both systems is presented in Sections III-IV.

We consider a multirate communication system where users' data streams are classified into M different classes having different transmission rates. Let class-m rate be denoted by R_m. Without any loss of generality, the class–1 rate is regarded as the basic (lowest) rate and therefore $R_m = mR_1,\ m \in 1, 2, ..., M$.

We assume a slotted packet multiple access scheme, where the time is divided into slots of equal length and a packet transmission can start only at the beginning of a time slot. We assume that the slot duration is enough to transmit a single class-1 packet. That is, if the packet length is L bits. Then the slot duration equals to L/R_1 seconds, where R_1 is the basic rate.

1) VSG-CDMA Packets: In VSG scheme, an m-rate data bit stream is directly modulated with the carrier, spreaded by a unique signature sequence with spreading gain G (which is equal to number of chips per bit duration), and then transmitted at the antenna front end (see Figure 1). Therefore, in VSG scheme, class–1, class–2,..., class–M signals have different spreading gains, i.e. $G_1 = G$, $G_2 = \frac{G}{2}$,..., $G_M = \frac{G}{M}$, respectively. In the case of an m–rate transmission, a class–m user is able to transmit m packets (L-bit long each) in one time slot (see Figure 3). It is essential to assume that the relative delay and gain of all m packets remain constant over the slot duration. As a result, bit errors become conditionally independent given the signal-to-interference-plus-noise-ratio [4].

2) MCD-CDMA Packets: In the case of MCD CDMA, an m-rate data bit stream is split into m basic-rate sub-streams each of which is independently modulated, spreaded with spreading gain G, and synchronously sent to the channel (see Figure 2). As a result, there are m basic-rate signals (MCD sub-signals) with equal spreading gain G. As seen in Figure 3, a class–m MCD user transmits m packets in parallel, each of which is independently spreaded by unique signature sequences and transmitted at the basic rate. Likewise, it is assumed that the relative delay and gain of the m–parallel transmission are unchanged over the slot period.

In this paper, slow power control is assumed. In multipath fading environments, there are generally two types of power control, namely fast and slow power control. The fast power control seems superior in terms of rapid feedback and report on channel measurements. However, it could be difficult to obtain accurate measurements on the required time scale especially when the processing and propagation delays are taken into account [20]. Then the slow power control is more reasonable in practice. Consequently, in this paper we use slow power control as a model to analyze the error rate in a Rayleigh fading environment. Notice that this assumption is widely used in CDMA system models [21]-[26] and well-matched to Rayleigh fading channel [22].

In the next sections (Section III-IV), we go down to the detail of the physical (PHY) layer model where the signal-to-interference-plus-noise-ratio (SINR) of each individual bit, in the cases of both BPSK and QPSK, is analyzed. Later on, with the packet models and the PHY-layer analysis, we formulate new exact xpressions of packet error rate as well as packet throughput for VSG CDMA and MCD CDMA in Section V.

III. SINR Analysis for VSG CDMA

In this section, we describe the physical-layer models of a multiclass uplink VSG CDMA in the cases of BPSK and QPSK-modulated. Thereafter the signal to interference plus noise ratios (SINRs) are accurately analyzed based on improved Gaussian approximation (IGA) technique.

A. Binary VSG CDMA

Consider a multiclass VSG CDMA system in an Additive White Gaussian Noise (AWGN) Rayleigh fading channel. Assume that users' signals are transmitted asynchronously and signature codes are independent and randomly generated. Denote $m \in 1, 2, ..., M$ by the normalized transmission rate of class–m traffic (with respect to class–1 rate or so-called the basic rate). Let $b_{k,m}(t)$ be the bit-stream signal of user k class m with the waveform function

$$b_{k,m}(t) = \sum_{i=-\infty}^{\infty} b_{i,k,m}\text{rect}(t - \frac{T_b}{m}i) \tag{1}$$

where $k = 1, 2, ..., K_m$ with K_m being the total number of class–m signals (users). T_b/m is the bit duration of class–m. $b_{i,k,m}$ is a sequence of independently and identically distributed random variables (i.i.d RVs) that takes values in $\{+1, -1\}$ with equal probability. rect(t) is an unit-amplitude rectangular pulse for $0 \leq t < 1$ and rect$(t) = 0$ for otherwise.

Here we consider uncoded CDMA packet transmission, where bit-stream signals are multiplied directly to their signature sequences. Accordingly, a class–m signal received at the targeted base station is

$$v_{k,m}(t) = \sqrt{S_m} h_{k,m} b_{k,m}(t) a_{k,m}(t) \tag{2}$$

where S_m is the class–m power. Since perfect power control at the base station is assumed, the received powers from the users in the same class are identical, e.g. $S_{1,m} = S_{2,m} = ... = S_{K_m,m} = S_m$. It is important to state that this assumption is commonly used in CDMA system model analysis, for example [1]-[9].

$\{h_{k,m}\}$, $k = 1, 2, ..., K_m$, $m = 1, 2, ..., M$ are the complex channel gains for slow-nonselective Rayleigh fading. It is assumed that $\{h_{k,m}\}$ are mutually independent zero-mean complex Gaussian random variables with $E[h_x h_y^*] = 1$ only if $x = y$ and equals 0 otherwise.

$a_{k,m}(t) = \sum_{i=-\infty}^{\infty} a_{i,k,m}\text{rect}(t - \frac{T_b}{G} i)$ is the signature sequence signal. T_b/G is the chip interval which is equal for all classes. The signature sequences $\{a_{i,k,m}\}$ are modeled as a sequence of i.i.d RVs with $\Pr\{a_{i,k,m} = +1\} = \Pr\{a_{i,k,m} = -1\} = 1/2$. It is clear to say that each bit of a transmitted signal is spreaded by multiplying with a newly chosen random signature sequence. This model is widely employed in [1]-[8], [24] and [25].

Let the receiver be locked onto an arbitrary reference signal of a class–n signal (user), say signal 0, in the presence of other multiple access interferences (MAIs). Let $r(t)$ be the composite signal at the matched-filter receiver front end, we obtain

$$r(t) = v_{0,n}(t) + \sum_{m=1}^{M} \sum_{k=1}^{K_m} v_{k,m}(t - \tau_{k,m}) + \eta(t), \qquad n \in 1, 2, ..., M \tag{3}$$

where $\tau_{k,m}$ represents the relative propagation delay of the signal k class–m with respect to the reference signal. $\eta(t)$ is the low-pass complex value additive white Gaussian noise with two sided spectral density $N_0/2$.

Let $Z_{0,n}$ be the normalized decision variable. At $t = T_b/n$, the output of a conventional matched-filter receiver is expressed as [2]

$$Z_{0,n} = \text{Re}\left\{ \pm |h_{0,n}| \sqrt{S_n} + \sum_{m=1}^{M} \sum_{k=1}^{K_m} \sqrt{S_m} e^{-\mathrm{i} \arg h_{0,n}} h_{k,m} W_{k,m} + \eta \right\} \tag{4}$$

where $\arg h_{0,n}$ represents the phase of the complex channel gain $h_{0,n}$. η is the complex Gaussian random variable with variance $\frac{N_0}{2(T_b/n)}$.

Based on improved Gaussian approximation (IGA) technique [4], $W_{k,m}$ is the multiple access interference component in the interval $[iT_b/n, (i+1)T_b/n]$ of the reference ith bit respect to the interfering signal k class–m [3] which is given by

$$W_{k,m} = \frac{1}{T_b/n} \int_{iT_b/n}^{(i+1)T_b/n} b_{k,m}(t - \tau_{k,m}) a_{k,m}(t - \tau_{k,m}) a_{0,n}(t) dt. \tag{5}$$

It is shown in [3]-[4] that, when $G >> 1$, $\{W_{k,m}\}$ can be accurately approximated by a set of conditionally independent zero-mean Gaussian random variables (conditioned on the relative time offset) with the conditional variance [3]

$$\text{Var}\left(W_{k,m} \mid \{\delta_{k,m}\}\right) = \frac{1}{G/n}\left[1 + 2\delta_{k,m}\left(\delta_{k,m} - 1\right)\right] \tag{6}$$

where

$$\delta_{k,m} = \tau_{k,m} - \frac{T_b}{G}\left\lfloor \frac{\tau_{k,m}}{T_b/G} \right\rfloor \tag{7}$$

is the time offset which is uniformly distributed over $[0, 1]$.

With (6), we obtain, when we condition on $\{h_{k,m}, \delta_{k,m}\}$, the conditionally distributed Gaussian variance of the decision variable by

$$\text{Var}\left[Z_{0,n} \mid \{h_{k,m}, \delta_{k,m}\}\right] = \frac{1}{G/n}\sum_{m=1}^{M}\sum_{k=1}^{K_m} S_m \,\text{Re}^2\left(h_{k,m}\right)\left[1 + 2\delta_{k,m}(\delta_{k,m} - 1)\right] + \frac{N_0}{2(T_b/n)}. \tag{8}$$

Accordingly, SINR, which is a random variable, experienced by a class–n signal is

$$\begin{aligned} \text{SINR}_n &= \frac{1}{2}\frac{\text{E}^2\left[Z_{0,n}\right]}{\text{Var}\left[Z_{0,n}\right]} \\ &= \frac{S_n\left|h_{0,n}\right|^2}{\frac{2}{G/n}\sum_{m=1}^{M}\sum_{k=1}^{K_m} S_m \,\text{Re}^2\left(h_{k,m}\right)\left[1 + 2\delta_{k,m}(\delta_{k,m} - 1)\right] + \frac{nS_1}{(E_1/N_0)}} \end{aligned} \tag{9}$$

where $E_1 = S_1 T_b$ is the energy per bit interval of class–1 traffic.

Since $\left|h_{0,n}\right|^2$ is a negative exponential random variable (recall that $h_{0,n}$ is a complex Gaussian RV), For any $z > 0$, the probability that $\text{SINR}_n > z$ is

$$\Pr(\text{SINR}_n > z \mid \{h_{k,m}, \delta_{k,m}, K_m\}) = e^{-\left(\frac{2}{G/n}\sum_{m=1}^{M}\sum_{k=1}^{K_m} S_m \,\text{Re}^2\left(h_{k,m}\right)\left[1+2\delta_{k,m}(\delta_{k,m}-1)\right] + \frac{nS_1}{(E_1/N_0)}\right)}. \tag{10}$$

Owing to the fact that $\text{Re}^2\left(h_{k,m}\right)$ is a Chi-squared random variable, we remove the condition on $\{h_{k,m}\}$ by letting

$$\begin{aligned}\Pr(\text{SINR}_n &> z \mid \{\delta_{k,m}, K_m\}) \\ &= e^{-\frac{nS_1/S_n}{E_1/N_0}z} \prod_{m=1}^{M} \prod_{k=1}^{K_m} \mathrm{E}\left[e^{-z\left(\frac{2}{G/n}\frac{S_m}{S_n}\mathrm{Re}^2\left(h_{k,m}\right)\left[1+2\delta_{k,m}(\delta_{k,m}-1)\right]\right)} \mid \{\delta_{k,m}, K_m\} \right] \\ &= e^{-\frac{nS_1/S_n}{E_1/N_0}z} \prod_{m=1}^{M} \prod_{k=1}^{K_m} \frac{1}{\sqrt{1+2\left\{\frac{2z}{G/n}\frac{S_m}{S_n}\left[1+2\delta_{k,m}(\delta_{k,m}-1)\right]\right\}}}. \end{aligned} \tag{11}$$

Next remove the condition on the uniform random variable $\{\delta_{k,m}\}$, finally we have

$$\begin{aligned}\Pr(\text{SINR}_n &> z \mid K_1, K_2, ..., K_M) \\ &= e^{-\frac{nS_1/S_n}{E_1/N_0}z} \prod_{m=1}^{M} \left[\mathrm{V}\left(\frac{nz}{S_n/S_m} \right) \right]^{K_m} \end{aligned} \tag{12}$$

where

$$\mathrm{V}(z) = \frac{1}{2}\sqrt{\frac{G}{z}} \ln\left(\frac{\sqrt{G+2z}+\sqrt{z}}{\sqrt{G+2z}-\sqrt{z}} \right). \tag{13}$$

B. Quaternary VSG CDMA

A QPSK class–m signal is given by [24]-[25]

$$v_{k,m}(t) = \sqrt{S_m} h_{k,m} \sum_{i=\infty}^{-\infty} \left[b_{i,k,m}^{(I)} a_{k,m}^{(I)}(t) - \mathrm{i} b_{i,k,m}^{(Q)} a_{k,m}^{(Q)}(t) \right] \mathrm{rect}\left(\frac{t}{T_b/m} - i \right) \tag{14}$$

where I and Q represent the in-phase and quadrature phase components. Assume that $\left\{a_{k,m}^{(I)}, a_{k,m}^{(Q)}\right\}$ and $\left\{b_{k,m}^{(I)}, b_{k,m}^{(Q)}\right\}$ are respectively independent signature sequences and data bit streams.

Analogous to BPSK, the variance of $\{W_{k,m}\}$ converges (when $G >> 1$) into a set of independent Gaussian random variables, conditioned on $\{h_{k,m}, \delta_{k,m}\}$. Then SINR, for QPSK-based VSG CDMA, at receiver front end is

$$\text{SINR}_n = \frac{S_n \left|h_{0,n}\right|^2}{\frac{2}{G/n} \sum_{m=1}^{M} \sum_{k=1}^{K_m} S_m \left|h_{k,m}\right|^2 \left[1+2\delta_{k,m}(\delta_{k,m}-1)\right] + \frac{nS_1}{E_1/N_0}} \tag{15}$$

where $\left|h_{k,m}\right|^2,\ k = 1, 2, ..., K_m, m = 1, 2, ..., M$ stand for independent exponential random variables (due to the fact that individual channel gains of I and Q phase are complex Gaussian random variables).

Again, since $|h_{0,n}|^2$ is negatively exponentially distributed, we have

$$\Pr(\text{SINR}_n > z \mid \{h_{k,m}, \delta_{k,m}, K_m\}) = e^{-\left(\frac{2}{G/n}\sum_{m=1}^{M}\sum_{k=1}^{K_m} S_m \left|h_{k,m}\right|^2 \left[1+2\delta_{k,m}(\delta_{k,m}-1)\right] + \frac{nS_1}{(E_1/N_0)}\right)}. \quad (16)$$

Remove the condition on $|h_{k,m}|^2$, then

$$\Pr(\text{SINR}_n > z \mid \{\delta_{k,m}, K_m\}) = e^{-\frac{nS_1/S_n}{E_1/N_0}z} \prod_{m=1}^{M}\prod_{k=1}^{K_m} \frac{1}{1+\left\{\frac{2z}{G/n}\frac{S_m}{S_n}\left[1+2\delta_{k,m}(\delta_{k,m}-1)\right]\right\}}. \quad (17)$$

As a result, we can express $\Pr\left(\text{SINR}_n > z\right)$ for QPSK as

$$\Pr\left(\text{SINR}_n > z \mid K_1, K_2, ..., K_M\right) = e^{-\frac{nS_1/S_n}{E_1/N_0}z} \prod_{m=1}^{M}\left[\text{V}_Q\left(\frac{nz}{S_n/S_m}\right)\right]^{K_m} \quad (18)$$

where

$$\text{V}_Q(z) = \frac{G}{\sqrt{z(G+z)}}\tan^{-1}\sqrt{\frac{z}{G+z}}. \quad (19)$$

IV. SINR Analysis for MCD CDMA

In the case of MCD CDMA, an m–rate data stream is split into m low-rate sub-streams. Each sub-stream is spreaded by a unique signature sequence and then all the sub-streams are transmitted synchronously in parallel. Theoretically, these m transmissions are orthogonal to one another. In practice, due to the different propagation delay of each transmission, the synchronization is lost and consequently the orthogonality among m transmissions could be broken. We call this phenomenon as “self-interference”.

In this section, we show the SINR analysis of MCD CDMA both in the theory, when perfect orthogonality is assumed, and in practice where self-interference is taken into account.

A. Orthogonal MCD CDMA

As mentioned above, a class–m MCD signal can be given by

$$v_{k,m}(t) = \sqrt{S}h_m \sum_{l=1}^{m} b_{k,l}(t)a_{k,l}(t) \quad (20)$$

where $b_{k,l}(t) = \sum_{i=-\infty}^{\infty} b_{i,k,l}\text{rect}(\frac{t}{T_b} - i)$ and $a_{k,l}(t) = \sum_{i=-\infty}^{\infty} a_{i,k,l}\text{rect}(\frac{t}{T_b/G} - i)$ are respectively the bit waveform function and signature sequence of the lth spreaded sub-signal.

Assume that all n codes of any class–n signal, $n \in 1, 2, ..., M$, are synchronous and perfectly orthogonal. At the receiver front end, the decision variable of an arbitrary code of the reference signal (signal 0 class–n) becomes

$$Z_{0,n} = \text{Re}\left\{ \pm |h_{0,n}| \sqrt{S} + \sum_{m=1}^{M} \sum_{k=1}^{K_m} \sqrt{S} e^{-\text{i} \arg h_{0,n}} h_{k,m} \sum_{l=1}^{m} W_{l,k,m} + \eta \right\} \tag{21}$$

where $W_{l_1,k_1,m}$ and $W_{l_2,k_2,m}$ are mutually independent when $k_1 \neq k_2$, but dependent for any k, m when $l_1 \neq l_2$ (since $a_{l_1,k,m}(t)$ is orthogonal to $a_{l_2,k,m}(t)$).

Fortunately, it is found that $W_{l_1,k,m}$ and $W_{l_2,k,m}$, when $l_1 \neq l_2$, are independent when conditioned on $\{\delta_{k,m}\}$ [26]. We have utilized this conditional independence property and obtain

$$\text{Var}\left(\sum_{l=1}^{m} W_{l,k,m} \mid \{\delta_{k,m}\} \right) = m \left[1 + 2\delta_{k,m} \left(\delta_{k,m} - 1\right)\right]. \tag{22}$$

With (22), the SINR$_n$ is given by

$$\text{SINR}_n = \frac{|h_{0,n}|^2}{\frac{1}{G} \sum\limits_{m=1}^{M} \sum\limits_{k=1}^{K_m} m \, \text{Re}^2\left(h_{k,m}\right) \left[1 + 2\delta_{k,m}(\delta_{k,m} - 1)\right] + \frac{1}{2(E_1/N_0)}}. \tag{23}$$

Owing to the fact that $\text{Re}^2\left(h_{k,m}\right)$ and $\delta_{k,m}$ are Chi-squared and uniform random variables respectively, we compute $\Pr(\text{SINR}_n > z \mid K_1, K_2, ..., K_M)_{\text{MCD}}$ by the same procedure as shown in (9) - (12). Then we have

$$\begin{aligned} \Pr(\text{SINR}_n > z \mid K_1, K_2, ..., K_M)_{\text{MCD}} &= e^{-\frac{z}{E_1/N_0}} \prod_{m=1}^{M} \prod_{k=1}^{K_m} \text{E}\left[e^{-\frac{2zm}{G} \text{Re}^2\left(h_{k,m}\right)\left[1+2\delta_{k,m}(\delta_{k,m}-1)\right]} \right] \\ &= e^{-\frac{z}{E_1/N_0}} \prod_{m=1}^{M} \text{V}^{K_m}\left(mz\right) \end{aligned} \tag{24}$$

where V(z) is already given in (13) for BPSK and in (19) for QPSK, respectively.

B. Imperfectly Orthogonal MCD CDMA

As the impact of self interferences is concerned, m sub-stream transmissions are not fully orthogonal to one another. The loss of orthogonality can be modeled by the orthogonality factor

$\sigma \in [0,1]$ where $\sigma = 0$ stands for perfect orthogonality and $\sigma = 1$ corresponds to fully losing orthogonality [27]. Denote signal 0 of class–n by the reference signal. The SINR$_n$ is

$$\text{SINR}_n = \frac{|h_{0,n}|^2}{\sigma(n-1)|h_{0,n}|^2 + \frac{1}{G}\sum_{m=1}^{M}\sum_{k=1}^{K_m} m\,\text{Re}^2(h_{k,m})\left[1+2\delta_{k,m}(\delta_{k,m}-1)\right] + \frac{1}{2(E_1/N_0)}} \tag{25}$$

where $\sigma(n-1)|h_{0,n}|^2$ represents the self interferences (the reference signal is interfered by other $n-1$ signals which are transmitted from the same user).

Due to the fact that the desired signal are dependent to the self interferences ($|h_{0,n}|^2$ exists both in the numerator and denominator), the resultant expression of $\Pr(\text{SINR}_n > z)_{\text{MCD}}$ cannot be obtained by exploiting the independence property of the interference as assumed in the previous sections.

Fortunately, we have found that when x is an exponential random variable, then for any $y, z > 0$ and $0 \le a \le 1$ we obtain

$$\Pr\left(\frac{x}{ax+y} > z\right) = \Pr\left(x > \frac{z}{1-az}y\right) = \begin{cases} 1, & z > 1/a \\ 0, & z = 1/a \\ e^{-\frac{z}{1-az}y}, & z < 1/a. \end{cases} \tag{26}$$

With (26), the expression of $\Pr(\text{SINR}_n > z)_{\text{MCD}}$, conditioned on $K_1, K_2, ..., K_M$, is given by

$$\Pr(\text{SINR}_n > z \mid K_1, K_2, ..., K_M)_{\text{MCD}} = \begin{cases} 1, & z > \frac{1}{2\sigma(n-1)} \\ 0, & z = \frac{1}{2\sigma(n-1)} \\ e^{-\frac{z}{[1-2\sigma(n-1)z](E_b/N_0)}} \prod_{m=1}^{M} \text{V}^{K_m}(mz), & z < \frac{1}{2\sigma(n-1)}. \end{cases} \tag{27}$$

V. Packet Success Rate and Throughput

In this section, we derive new unified accurate expressions for the packet success rates for the different systems described in the previous sections.

A. *Non-random Number of Users*

Denote the number of class–1, class–2, ..., class–M users by $K_1, K_2, ..., K_M$. Based on the packet models mentioned in Section II, here the system performance in terms of packet success rate and throughput can be analyzed as follows.

To evaluate the packet throughput of both VSG and MCD, let us consider the packet success rate for each class–n packet in a general form

$$\mathrm{P}_{\mathrm{s},n}\left(K_1, K_2, ..., K_M\right) = \mathrm{E}\left[\left(1 - \frac{1}{2}\mathrm{erfc}\left(\sqrt{\mathrm{SINR}_n}\right)\right)^L \mid K_1, K_2, ..., K_M\right] \tag{28}$$

where $\mathrm{erfc}(z) = \frac{2}{\pi}\int_z^\infty e^{-x^2}dx$ is the complementary error function.

Average out (28) with respect to SINR_n and use the rules of integration by parts, then we have

$$\mathrm{P}_{\mathrm{s},n}\left(K_1, K_2, ..., K_M\right) = \frac{1}{2^L} + \frac{L}{2^L}\int_0^\infty \frac{e^{-z}}{\sqrt{\pi z}}\left[1 + \mathrm{erf}(\sqrt{z})\right]^{L-1} \Pr\left(\mathrm{SINR}_n > z\right) dz \tag{29}$$

where $\mathrm{erf}(z) = 1 - \mathrm{erfc}(z)$ is the error function.

With (29), the expressions of packet success rate for both VSG and MCD can be obtained by placing the corresponding $\Pr(\mathrm{SINR}_n > z)$ directly into the equation. For example, substituting (12) into (29) yields the packet success rate expression for BPSK-modulated VSG CDMA,

$$\mathrm{P}_{\mathrm{s},n}^{(\text{B-VSG})}\left(K_1, K_2, ..., K_M\right) = \frac{1}{2^L} + \frac{L}{2^L}\int_0^\infty \frac{e^{-z\left(1+\frac{nS_1/S_n}{E_b/N_0}\right)}}{\sqrt{\pi z}}\left[1 + \mathrm{erf}(\sqrt{z})\right]^{L-1} \prod_{m=1}^M \mathrm{V}^{K_m}\left(\frac{nz}{S_n/S_m}\right) dz \tag{30}$$

while replacing with (24) gives the expression for BPSK-based orthogonal MCD CDMA

$$\mathrm{P}_{\mathrm{s},n}^{(\text{B-MCD})}\left(K_1, K_2, ..., K_M\right) = \frac{1}{2^L} + \frac{L}{2^L}\int_0^\infty \frac{e^{-z\left(1+\frac{1}{E_b/N_0}\right)}}{\sqrt{\pi z}}\left[1 + \mathrm{erf}(\sqrt{z})\right]^{L-1} \prod_{m=1}^M \mathrm{V}^{K_m}\left(mz\right) dz. \tag{31}$$

Similarly, by applying (27) to (29), the packet success rate for imperfectly orthogonal MCD is

$$\mathrm{P}_{\mathrm{s},n}^{(\text{im B-MCD})}\left(K_1, K_2, ..., K_M\right) = \frac{1}{2^L} + \frac{L}{2^L}\int_0^{\frac{1}{2\sigma(n-1)}} \left\{\frac{e^{-z\left(1+\frac{1}{[1-2\sigma(n-1)z]\left(E_b/N_0\right)}\right)}}{\sqrt{\pi z}} \left[1 + \mathrm{erf}(\sqrt{z})\right]^{L-1} \prod_{m=1}^M \mathrm{V}^{K_m}\left(mz\right) dz\right\}. \tag{32}$$

Note that the expressions for QPSK-based VSG/MCD are simply achieved by placing $\mathrm{V}_Q(\cdot)$ (readily given in (19)) instead of $\mathrm{V}(\cdot)$.

As a result, the throughput is expressed as

$$C = \sum_{m=1}^{M} m K_m \mathrm{P}_{\mathrm{s},m}. \tag{33}$$

B. Random Number of Users

In Section IV-A, the packet error rate is analyzed based on the assumption that every user transmits their packets all time. Therefore we can say that the number of users $K_1, K_2, ..., K_M$ are also the number of class–1, class–2,..., class–M packets, which are conditionally constant.

However, packet transmissions do not occur all time in practice. A packet is transmitted only when a user is active. In contrast, there is no transmission during the silent period. In this case, the number of class–n packets can be modeled as a binomial random variable which is analyzed as follows.

1) Independent Binomial Distribution: Given the total number of class–n users by K_n, the probability that there are k active users (k transmitted packets) is

$$\Pr\left(K_n = k\right) = \binom{K_n}{k} q_n^k \left(1 - q_n\right)^{K_n - k} \tag{34}$$

where $q_n \in [0, 1]$ is the probability of packet transmission of a class–n user.

Let us consider the case of BPSK VSG. In this case $\Pr(\mathrm{SINR}_n > z)$ can be computed by

$$\begin{aligned} \Pr(\mathrm{SINR}_n > z) &= \mathrm{E}\left[\Pr(\mathrm{SINR}_n > z \mid K_1, K_2, ..., K_M)\right] \\ &= e^{-\frac{nS_1/S_n}{E_b/N_0} z} \prod_{m=1}^{M} \mathrm{E}\left[\mathrm{V}^{K_m}\left(\frac{nz}{S_n/S_m}\right)\right]. \end{aligned} \tag{35}$$

To find the average $\mathrm{E}\left[\mathrm{V}^{K_m}\left(\cdot\right)\right]$, we apply the fact that, when K_m is binomially distributed, $\mathrm{E}\left[\mathrm{V}^{K_m}\left(\cdot\right)\right] = \left[1 - q_m + q_m \mathrm{V}\left(\cdot\right)\right]^{K_m}$ for any function $\mathrm{V}(\cdot)$. Then we have

$$\Pr(\mathrm{SINR}_n > z) = e^{-\frac{nS_1/S_n}{E_b/N_0} z} \prod_{m=1}^{M} \left[1 - q_m + q_m \mathrm{V}\left(\frac{nz}{S_n/S_m}\right)\right]^{K_m}. \tag{36}$$

Note that we can substitute $\mathrm{V}_Q(\cdot)$, see (19), into (36) for the $\Pr(\mathrm{SINR}_n > z)$ of QPSK VSG.

Accordingly the average packet success rate can be achieved by applying (36) to (29). Then the average throughput is given by

$$C_{\text{Binomial}} = \sum_{m=1}^{M} m q_m K_m \mathrm{P}_{\mathrm{s},m}. \tag{37}$$

Hint that $\Pr(\text{SINR}_n > z)$ of MCD CDMA can be similarly obtained by replacing (36) with (24) for orthogonal MCD and with (27) for imperfect orthogonal MCD, respectively. As a result, the MCD throughput is given by (37).

To avoid the repetition of presentation, in the rest of this section, we raise the system model of BPSK VSG as an example for the analysis. Therefore, one should keep in mind that the performance analyzes of the other system models (i.e. QPSK VSG, perfectly and imperfectly orthogonal MCD) can be accomplished by the same procedure.

2) Dependent Multinomial Distribution: Based on the previous assumption, each user is confined to transmit its packets at one particular rate. Now we consider a different case where every user can operate at various rates $1, 2, ..., M$. In this case, the numbers of class–1, class–2,..., class–M packets are hence dependent and the analysis can be shown as follows.

Consider a multirate VSG CDMA system serving K users, each of which is able to transmit its packets (one packet at a time) at rates $1, 2, ..., M$. p_0 is the probability when a user is inactive and p_m is declared as the probability of rate–m transmission such that $\sum_{m=0}^{M} p_m = 1$. In this scenario, the numbers of class–1, class–2,..., class–M users can be modeled as dependent multinomial random variables. Denote $K_1, K_2, .., K_M$ by the number of active users class–1, class–2,..., class–M and K_0 by the number of inactive users. When $K_0, K_1, .., K_M$ are dependent multinomial distributed, for any function V($\cdot$) we have

$$\text{E}\left[\text{V}_1^{K_1}\text{V}_2^{K_2}...\text{V}_M^{K_M}\right] = \left[p_0 + p_1\text{V}_1 + p_2\text{V}_2 + ... + p_M\text{V}_M\right]^K \tag{38}$$

which is called as the multinomial expansion with the restriction $K = \sum_{m=0}^{M} K_m$ [28] .

Apply (38), then $\Pr(\text{SINR}_n > z)$ for multinomial distribution becomes

$$\Pr(\text{SINR}_n > z) = e^{-\frac{nS_1/S_n}{E_b/N_0}z}\left[p_0 + p_1\text{V}\left(\frac{nz}{S_n/S_1}\right) + ... + p_M\text{V}\left(\frac{nz}{S_n/S_M}\right)\right]^K. \tag{39}$$

As a result, the probability of packet success is given by

$$\text{P}_{\text{s},n} = \frac{1}{2^L} + \frac{L}{2^L}\int_0^\infty \frac{e^{-z\left(1+\frac{nS_1/S_n}{E_b/N_0}\right)}}{\sqrt{\pi z}}\left[1+\text{erf}(\sqrt{z})\right]^{L-1} \left[p_0 + \sum_{m=1}^{M} p_m\text{V}\left(\frac{nz}{S_n/S_m}\right)\right]^K dz \tag{40}$$

and the throughput is

$$C_{\text{Multinomial}} = K\sum_{m=1}^{M} mp_m\text{P}_{\text{s},m}. \tag{41}$$

3) Infinite Number of Users: Consider the special case when $K_1, K_2, ..., K_M \rightarrow \infty$ and $q_1, q_2, ..., q_M \rightarrow 0$, the numbers of transmitted packets become Poisson random variables with probability density function

$$\Pr(K_m = k) = \frac{\lambda_m^k}{k!} e^{-\lambda_m}; \qquad m \in 1, 2, ..., M \tag{42}$$

where $\lambda_m = q_m K_m$ is the average of class–m packets.

Due to the fact that for any Poisson random variable Y with mean λ, we have $\mathrm{E}[e^{-zY}] = \exp\left[\lambda\left(e^{-z} - 1\right)\right]$. Recall (12), it can be straightforwardly shown that

$$\Pr\left(\mathrm{SINR}_n > z\right) = e^{-\frac{nS_1/S_n}{E_b/N_0} z} e^{-\sum_{m=1}^{M} \lambda_m \left[1 - \mathrm{V}\left(\frac{nz}{S_n/S_m}\right)\right]}. \tag{43}$$

As a result, the probability of success is obtained by replacing (43) into (29) and the average throughput is given by

$$C_{\text{Poisson}} = \sum_{m=1}^{M} m \lambda_m \mathrm{P}_{\mathrm{s},m}. \tag{44}$$

VI. Multi-Cellular System

In this section, we extend the scope to multi-cellular scenario where the interferences from the neighbor cells are also taken into account. Let us consider a VSG CDMA multi-cellular system as shown in Figure 4.

Denote $P_{m,j}$ by the power transmitted from a class–m user (the interfering user) in a neighbor cell, say cell j. The received power at the reference base station, situated at cell 0, can be given by

$$S_{m,j} = P_{m,j} \left[r^2 + D^2 - 2rD\cos\theta\right]^{-\frac{\beta}{2}} \tag{45}$$

where $D = \sqrt{3}$ (for CDMA systems) is the distance, normalized by the cell radius, of two adjacent base stations. $r \in [0, 1]$ is the normalized distance between the interfering user and its home base station. $\theta \in [0, 2\pi]$ is the angle between D and r. β is path loss exponent [29]. Note that small path loss exponent ($\beta < 3$) represents open propagation area, whereas large β stands for obstructed environment.

Based on the assumption of perfect power control on every cell, the received power of class–1, class–2,..., class–M at the home base station can be expressed as S_1, S_2,..., S_M respectively.

Therefore, the recieved power of class–m at the base station j (see BS$_j$ in Figure 4) is presented by

$$S_m = P_{m,j} r^{-\beta}. \tag{46}$$

With (46), (45) is thus reduced to $S_{m,j} = S_m d^{-\beta}(r,\theta)$ where

$$d(r,\theta) = \sqrt{1 + (D/r)^2 - 2\,(D/r)\cos\theta}. \tag{47}$$

To simplify the presentation, we assume that every cell has identical number of class–m users where $m \in 1, 2, ..., M$. Accordingly, the variance of decision variable for multicell is expressed as

$$\begin{aligned}\text{Var}\left[Z_{0,n} \mid \{\delta_{k,m}\}\right] = \frac{1}{G/n} \sum_{m=1}^{M} \sum_{k=1}^{K_m} S_m \,\text{Re}^2\left(h_{k,m}\right)\left[1 + 2\delta_{k,m}(\delta_{k,m} - 1)\right] \\ + \frac{1}{G/n} \sum_{j=1}^{J} \sum_{m=1}^{M} \sum_{k=1}^{K_m} S_m d_{k,m,j}^{-\beta}(r,\theta)\,\text{Re}^2\left(h_{k,m,j}\right)\left[1 + 2\delta_{k,m,j}(\delta_{k,m,j} - 1)\right] \\ + \frac{nS_1}{2\,(E_b/N_0)}.\end{aligned} \tag{48}$$

Follow the analysis that leads to (12), $\Pr\left(\text{SINR}_n > z\right)$ becomes

$$\begin{aligned}\Pr\left(\text{SINR}_n > z \middle|\, r,\theta\right) = e^{-\frac{nS_1/S_n}{E_b/N_0} z} \prod_{m=1}^{M} \text{V}^{K_m}\left(\frac{nz}{S_n/S_m}\right) \\ \cdot \left[\prod_{m=1}^{M} \text{E}\left[\text{V}^{K_m}\left(\frac{nz}{S_n/S_m} d^{-\beta}(r,\theta)\right)\right]\right]^J.\end{aligned} \tag{49}$$

Relax the conditions on r and θ , then we have

$$\begin{aligned}\Pr\left(\text{SINR}_n > z\right) = e^{-\frac{nS_1/S_n}{E_b/N_0} z} \prod_{m=1}^{M} \text{V}^{K_m}\left(\frac{nz}{S_n/S_m}\right) \\ \cdot \left[\prod_{m=1}^{M}\left(\frac{1}{\pi}\int_0^{2\pi}\int_0^1 \text{V}\left(\frac{nz}{S_n/S_m} d^{-\beta}(r,\theta)\right) r dr d\theta\right)^{K_m}\right]^J.\end{aligned} \tag{50}$$

Replacing (50) in (29), we achieve the probability of packet success. Then the average throughput is given by (33).

Analogously, we can compute the packet success rate of MCD by the same way. For instance, recall (27), the packet success rate of imperfectly orthogonal MCD is

$$\mathrm{P}_{\mathrm{s},n}^{(\mathrm{MCD})} = \frac{1}{2^L} + \frac{L}{2^L}\left\{ \int_0^{\frac{1}{2\sigma(n-1)}} \frac{e^{-z\left(1+\frac{1}{[1-2\sigma(n-1)z](E_b/N_0)}\right)}}{\sqrt{\pi z}} \left[1+\mathrm{erf}(\sqrt{z})\right]^{L-1} \prod_{m=1}^{M} \mathrm{V}^{K_m}(mz) \right.$$
$$\left. \cdot \left[\prod_{m=1}^{M} \left(\frac{1}{\pi} \int_0^{2\pi}\int_0^{1} \mathrm{V}\left[mzd^{-\beta}(r,\theta)\right] r dr d\theta \right)^{K_m} \right]^J dz \right\} \quad (51)$$

and the throughput expression is readily given in (33).

VII. Numerical and Simulation Results

This section presents some new numerical and simulation results which are divided into 4 parts. In the first part, the optimal power selection for multirate VSG CDMA is discussed. The second part shows different scenarios of random users' distributions versus the system performance. The third part addresses the impact of self interferences on the overall throughput of multirate MCD CDMA. Finally, the degradation on the system throughput, resulting from intercell interferences, is mentioned in the fourth part.

In this paper, we use packet throughput as the key parameter to measure the system performance. Here the throughput is, previously mentioned in (33), defined by the average number of transmitted packets from class–1, class–2, ...,class–M.

As seen in (33), the throughput is a function of packet success rate. Hence it is important to firstly assess the accuracy of our proposed expressions of the packet success rate. In Figure 5, we compare our numerical results, derived from (30) for VSG and (31) for MCD, with the simulation results in (28) which is the average error function of SINR. In our simulation, we average out over 2,000,000 samples of SINR in order to obtain such reliable results. From the figure, it is obvious that both numerical and simulation results match up very closely. This allows us to confirm the accuracy of our proposed explicit expressions.

A. *Optimal Power Selection for VSG CDMA*

Let us firstly consider VSG CDMA systems. Refer to (9), the equation can be given in a shorter form by

$$\mathrm{SINR}_n = \frac{G}{n} \frac{S_n}{\frac{1}{|h_{0,n}|^2} \sum \sum S_m \cdot \cdot}$$

where G and n are respectively the spreading gain and the class–n transmission rate with respect to class–1 rate. S_n is the power of the desired signal and the denominator represents interferences and background noise. We can see that SINR_n is proportional to G and S_n but inverse-proportional to the transmission rate n. Hence, the increase of transmission rate results in deterioration of SINR_n. In this regard, most of the previous work (e.g. [31]-[33]) straightforwardly compensates SINR by proportionally increasing the transmission power S_n, i.e. $S_n = nS_1$. As a result, every class retains the same bit energy which is defined by the ratio $\frac{S_1}{R_1} = \frac{S_2}{R_2} = ... = \frac{S_n}{R_n} = E_1 = E_2 = ... = E_n$ [34]. However, the efficiency of this allocation is quite controversial. Accordingly, it is very of interest to draw a discussion on this issue.

Let us consider a dual-class VSG CDMA system, where class–1 rate represents the basic rate and class–2 stands for the high rate (m times higher than the basic rate). Denote the spreading gain $G = 500$, the number of users $K_1 = K_2 = 20$, packet length $L = 100$ bits, the activity probabilities $q_1 = q_2 = 1$. E_1/N_0 is the bit energy to Gaussian noise spectral density ratio of class–1 users which implies the level of background noise (high E_1/N_0 corresponds to low background noise). Let S_2/mS_1 be the power ratio normalized by rate m and be generally given by $S_2/mS_1 = 1$ in the literature [31]-[33].

Figure 6 shows the throughput plotted against the normalized power ratio S_2/mS_1 for a wide range of background noise levels (represented by E_1/N_0) in the case of both BPSK and QPSK VSG CDMA. Assume that class–2 rate is twice as class–1 then the power ratio becomes $S_2/2S_1$.

In the figure, we vary the power ratio from 0.01 to 5. It is found that the optimum ratio (where the throughput is maximum) is not fixed at 1 but depends on the background noise level. For instance, when the background noise is high (e.g. $E_1/N_0 = 10$ dB), the optimum power ratio goes around 3. In contrast, the optimum ratio is approximately 1 in low-noise environment ($E_1/N_0 = 30$ dB). Therefore, we can say that the throughput is maximized, in a noisy environment, when a larger amount of bit energy is allocated to high-rate transmissions. Otherwise, the equal-bit-energy allocation is the optimum choice when the channel noise is low.

B. Random Number of Users

In the previous section, the numerical results are computed based on the assumption where the numbers of users are binomially distributed (see (34)). Here we alternatively show the system performance in the cases of independent Poisson and dependent multinomial, respectively.

1) Independent Poisson Distribution: Consider a dual-class BPSK-based VSG CDMA with $G = 500$, $L = 100$, $E_1/N_0 = 30$ dB and $S_2/mS_1 = 1$ where $m = 2$. Figure 7 shows the throughput (obtained from (44)) versus the average appearances of class–2 users (λ_2) for different arrival rates of class–1 (λ_1).

From the figure, we can determine the optimum class–2 arrival rate (where the throughput is maximized) for each given class–1 rate (given λ_1). For example, when $\lambda_1 = 5$, the optimum λ_2 is 40. This presentation seems straightforward but is actually very useful for system operators to perform optimal admission so that the overall throughput is maximized.

2) Dependent Multinomial Distribution: Assume there are K users in the system and each of which can transmit its packets at rates $1, 2, ..., M$. Therefore, the numbers of packets distributing among class–$1, ...,$class–M are dependently multinomially distributed.

Consider a dual-class VSG with $K = 40$, $G = 500$, $L = 100$ and $E_1/N_0 = 30$ dB. Define p_1 and p_2 as the probabilities of transmitting class–1 and class–2 packets respectively, then $p_0 + p_1 + p_2 = 1$ where p_0 is the probability of being inactive.

In Figure 8, we study the effect of rate ratio, i.e. $m = R_2/R_1$, on the optimum power ratio (opt S_2/S_1). The throughput is plotted against the range of S_2/mS_1. We varies the rate ratio by letting $m = 2, 3,$ and 5, then we have 3 different curves in the figure. We can observe that the optimum points (where the throughput is maximum) of these curves are differently located, i.e. at $m = 2$ Opt $S_2/mS_1 = 0.5$, at $m = 3$ Opt $S_2/mS_1 = 0.7$ and at $m = 5$ Opt $S_2/mS_1 = 1$. The result implies that the optimum power ratio goes to unity when the rate ratio is large (that means when $m \to \infty$ then $S_2/S_1 = m = R_2/R_1$ and $E_{2,\text{opt}} = S_2/R_2 = S_1/R_1 = E_1$). Therefore we can conclude that, with conditioned E_1/N_0 (fixed E_1/N_0), the traditional power allocation by letting $\frac{S_m}{mS_1} = 1$ is efficient only when m is large, otherwise this allocation seems excessive.

Figure 9 shows the plot of throughput against the number of users (K) for a 5-class VSG CDMA system with $p_0 = 0.4$ and $p_1 = p_2 = ... = p_5$. The normalized rate of class–1, class–2,..., and class–5 are $m = 1, 2, 3, 4$ and 5 respectively. The other parameters are unchanged from those of Figure 8. From the figure, the throughput drops when the number of users in the system is excessive. We further observe that the maximum throughput is located differently for individual E_1/N_0s, for instance the maximum throughput occurs at $K = 35$ when $E_1/N_0 = 10$ dB whereas it is at $K = 50$ when $E_1/N_0 = 20$ dB. Straightforwardly, the system can accommodate more users when the background noise is lower. We further say that this plot also benefits us graphically selection of the optimum number of users, that maximizes the throughput, at a wide range of background noise levels.

C. The Impact of Self Interferences on MCD CDMA

Here we study the impact of orthogonality among m sub-streams for MCD CDMA communications. Figure 10 shows the overall throughput against the background noise (E_1/N_0) for different values of orthogonality factor $\sigma = 0.1, 0.3, 0.5$ and 1.0. As previously mentioned in Section III, $\sigma = 1$ stands for fully losing orthogonality whereas $\sigma \to 0$ represents perfect orthogonality. From the figure, we can see that the throughput drops over 40% when σ turns to unity. This is because, in multipath environment, the lost of orthogonality causes large amplitude variations on the received MCD sub-signals [35]. Consequently, the amplitude variations leads to high resultant error rate (when the precoding scheme at the receiver front end is not applied).

On the result in Figure 10, we further observe that the slope of throughput goes to zero when E_1/N_0 grows larger. This finding corresponds to that of [36] in the case of match-filter receivers. This is because a match filter is interference-limited. Based on multiuser information theory [36]-[37], in a large system (high number of users) where multiple-access interferences act as a Gaussian random variable, the capacity (or throughput) converges to a limit (which is a function of E_1/N_0) regardless of fading distribution. Though the number of users in Figure 10 is not that large, we can see the trend of the curves.

D. Multi-Cellular Environment

This section illustrates the effect of intercell interferences on the system throughput (see Figure 11). Consider a multicell dual-class BPSK-based VSG CDMA system with parameters $G = 500$, $L = 100$, $K_1 = 10$, $q_1 = q_2 = 1$, $S_2/mS_1 = 1$, $E_b/N_0 = 30$ dB, the path loss exponent $\beta = 3$, and the number of adjacent cells $J = 6$.

Compared with the single scenario, the system throughput of multicell drops by half when the intercell interferences are taken into account. It is obvious to say that the system performance in single scenario is quite optimistic in practice. Multicell environment is therefore a recommended issue to be included in any research on cellular communications.

VIII. CONCLUSIONS

In this paper, we have presented an original analytical technique for packet error rate in the cases of both uplink VSG CDMA and MCD CDMA on Rayleigh fading environment. Based on the accurate improved Gaussian approximation (IGA), we have derived expressions for the cumulative probability density function of SINR for binary and quaternary multirate CDMA packet communications. This has facilitated achieving new exact expressions of packet error rate as well as throughput that consider bit-to-bit error dependence, optimal power selection and random users' distributions in multicellular environment. The proposed expressions have introduced a new efficient way to accurately investigate the system performance of multirate CDMA in Rayleigh fading channel. The numerical results have revealed that in a low-noise environment, for both BPSK and QPSK-based VSG CDMA, the throughput is optimized when every class retains the same bit energy, i.e. $\frac{S_1}{R_1} = \frac{S_2}{R_2} = ... = \frac{S_M}{R_M} = E_1 = E_2 = ... = E_M$. However, in a noisy channel, the throughput is maximized when a larger amount of bit energy is allocated to high-rate transmissions, e.g. $\frac{S_1}{R_1} < \frac{S_2}{R_2}$ when $R_1 < R_2$. In the case of MCD CDMA, the results have shown that, in multipath fading with a matched-filter receiver, self interferences cause a considerable degradation on overall throughput, i.e. the throughput drops to 40% when the orthogonality among spreaded sub-signals is fully lost.

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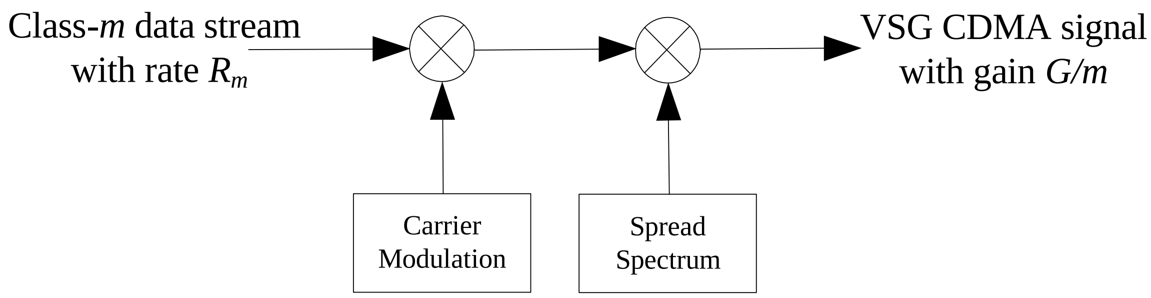


Fig. 1. VSG CDMA transmission scheme

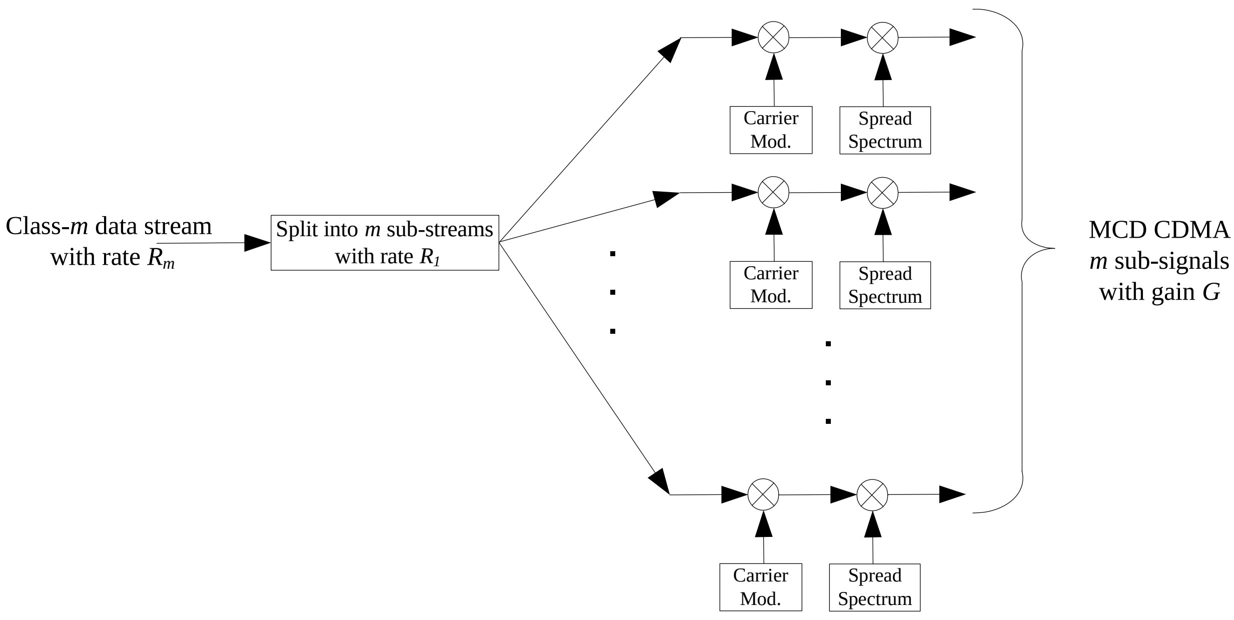


Fig. 2. MCD CDMA transmission scheme

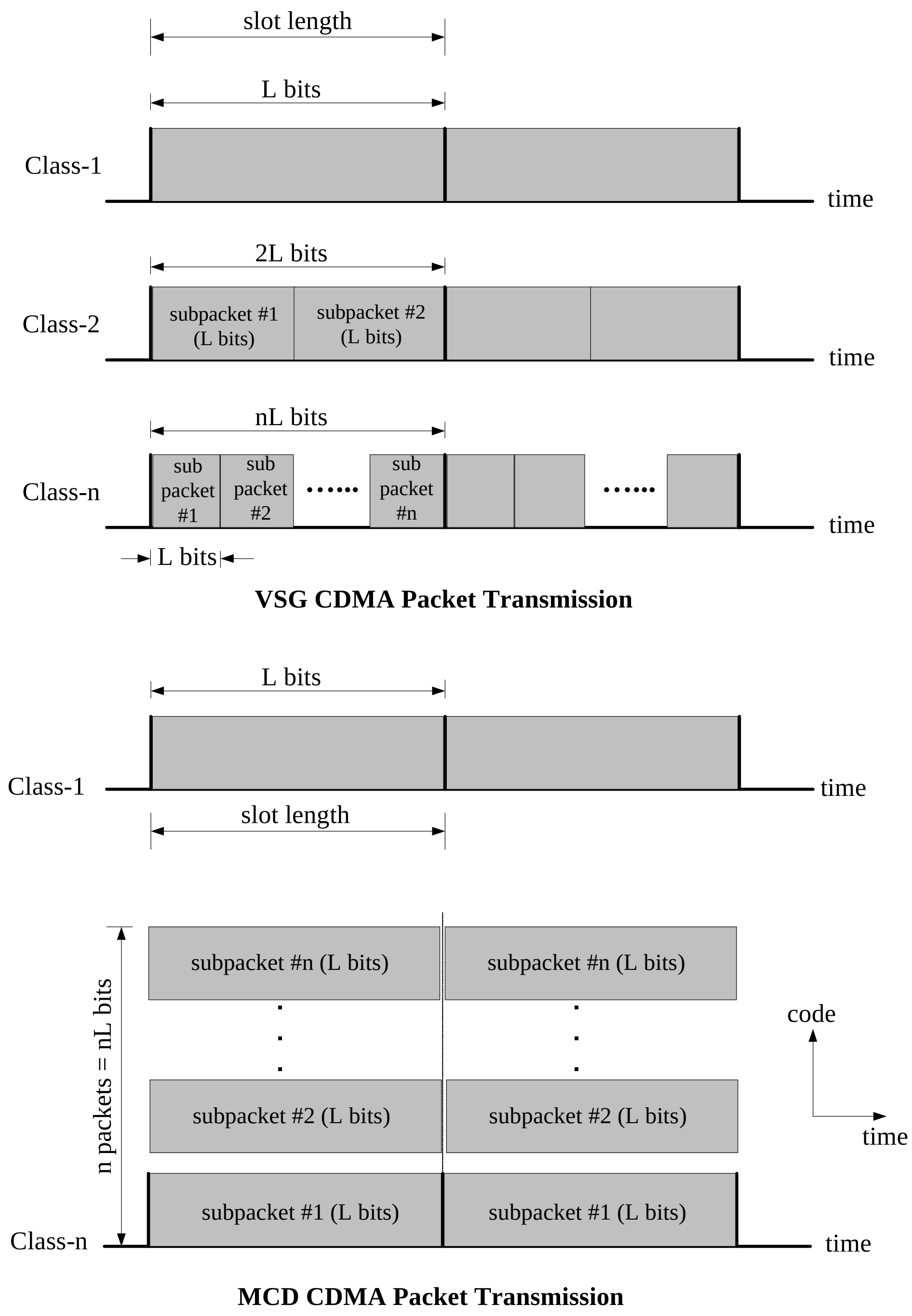


Fig. 3. Packet Transmissions of Slotted VSG and MCD CDMA

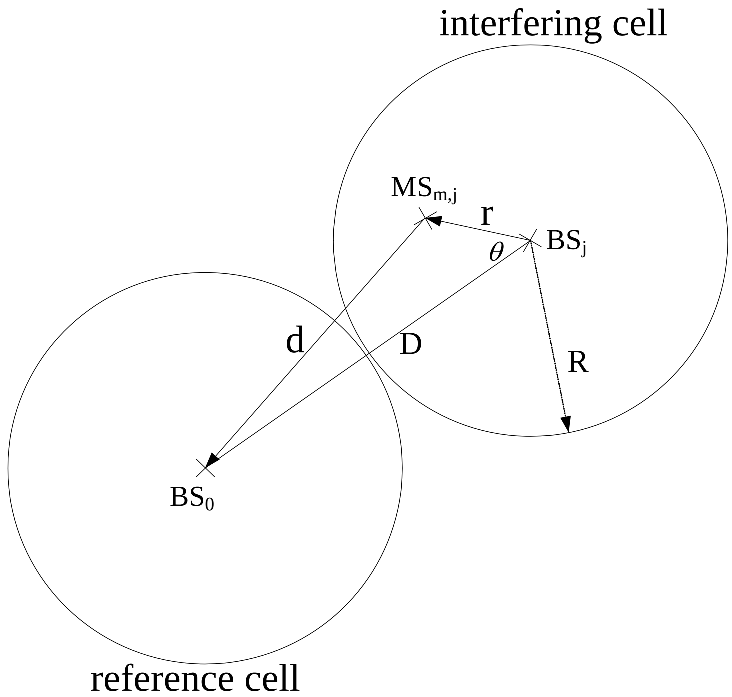
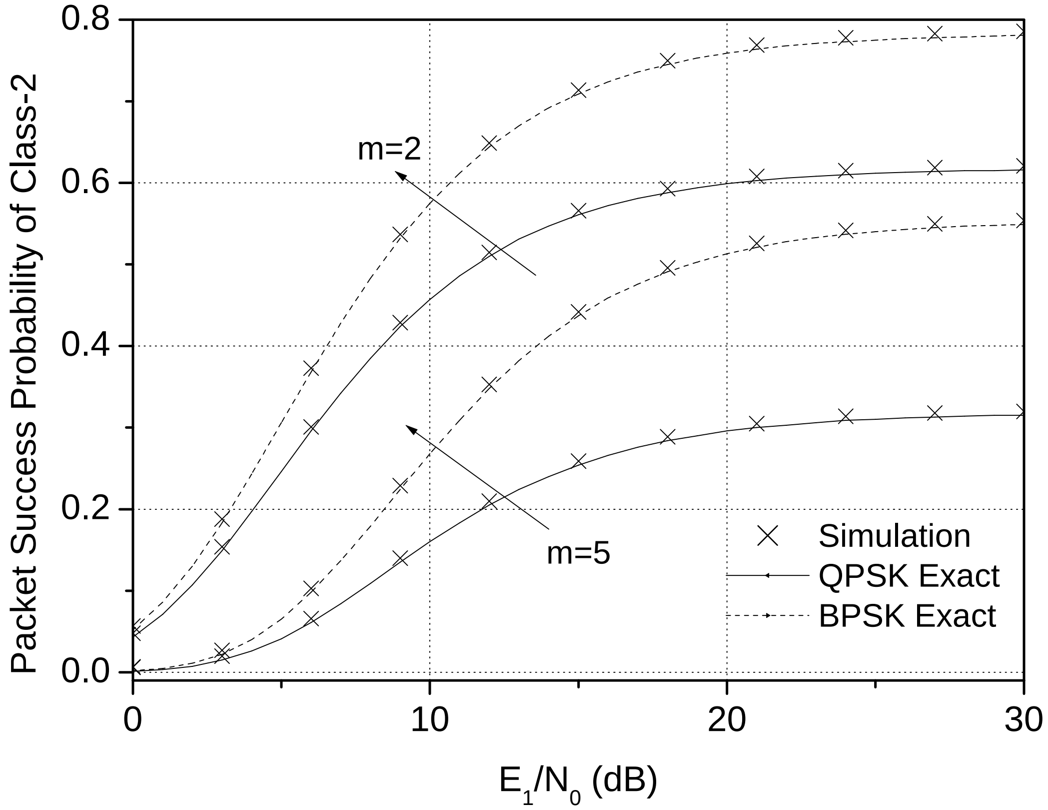


Fig. 4. Uplink Propagation Model: $\mathrm{MS}_{m,j}$ is the interfering class–m user in cell j. BS_0 is the targeted base station in the reference cell, say cell 0.



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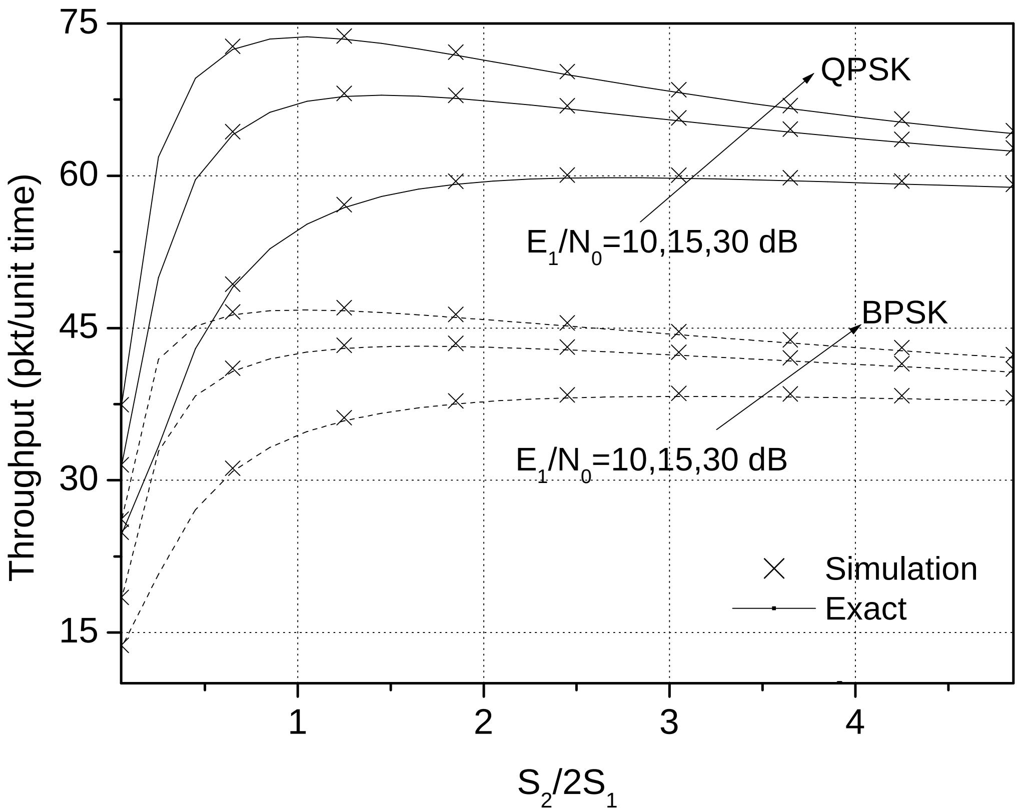

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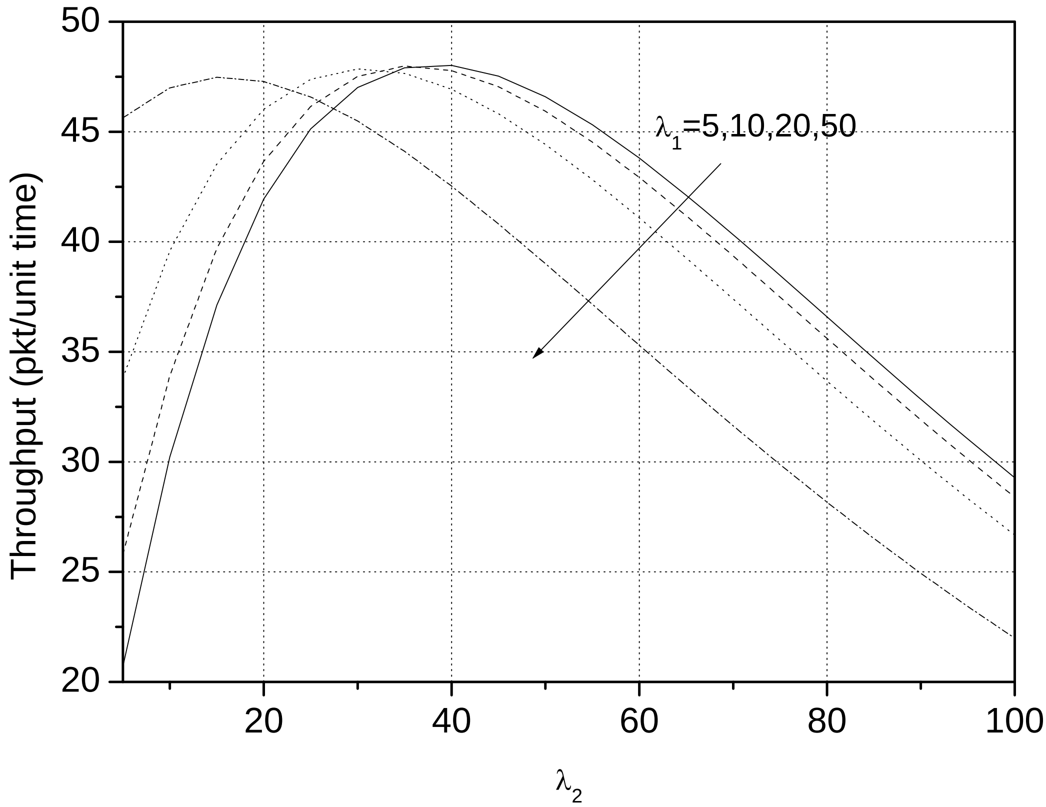

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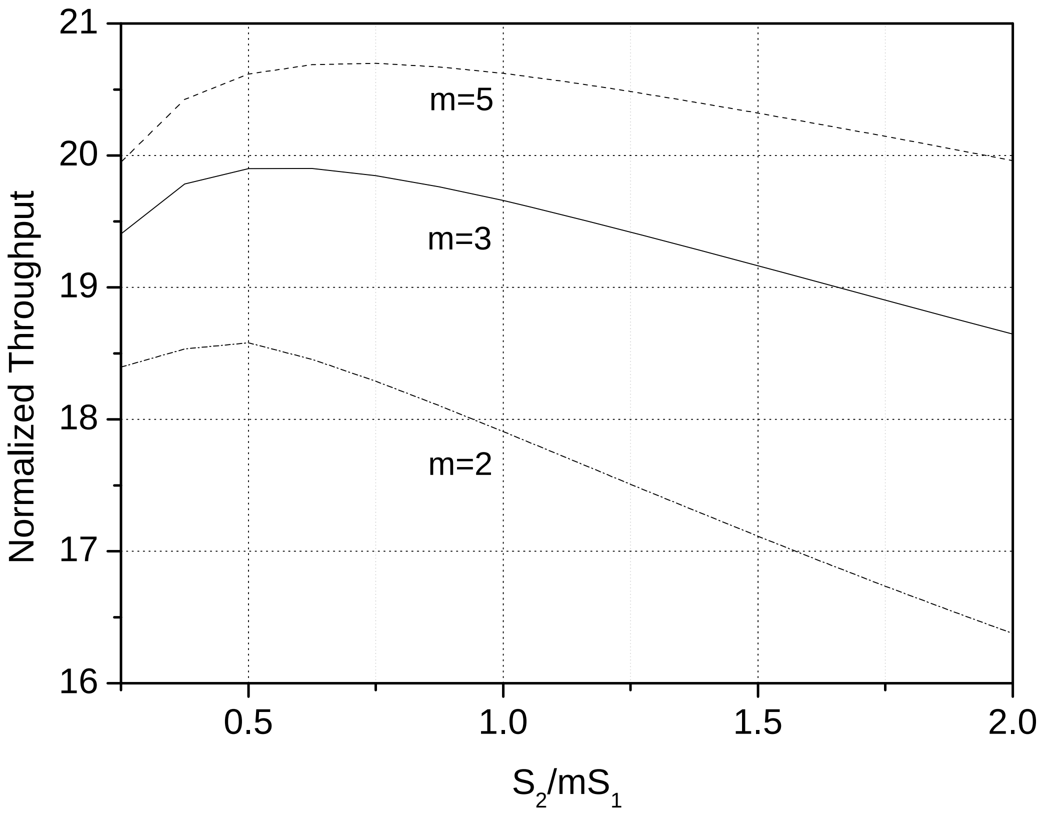

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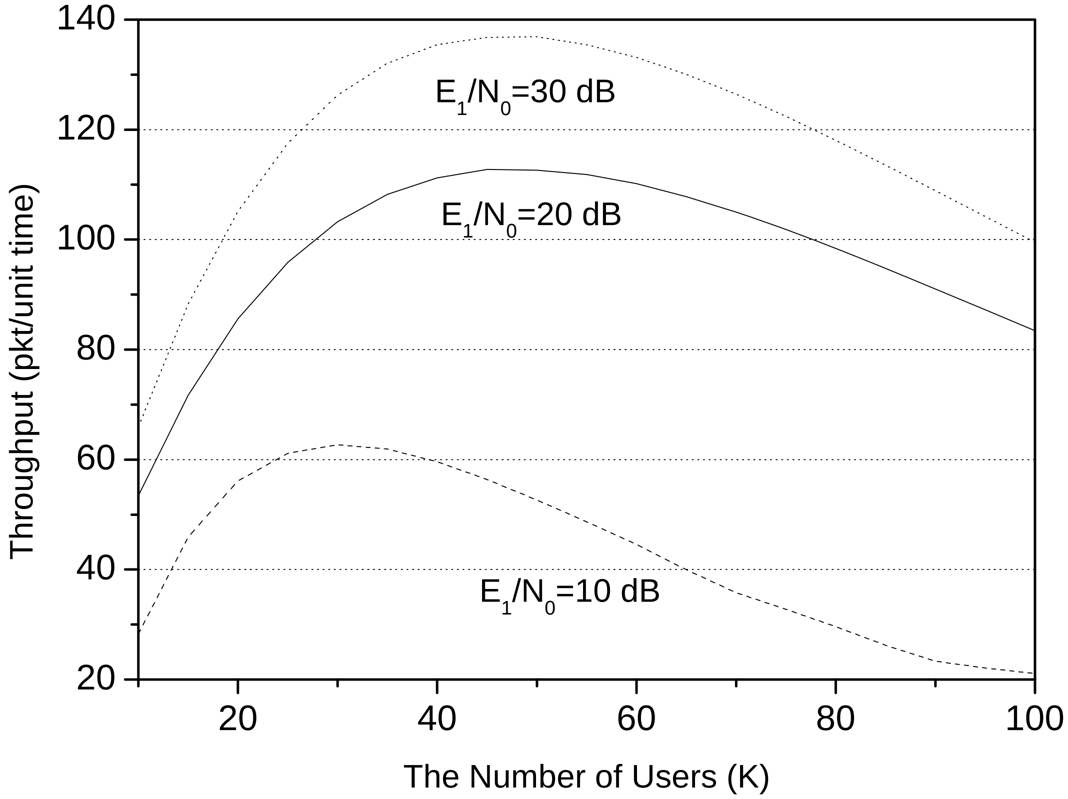

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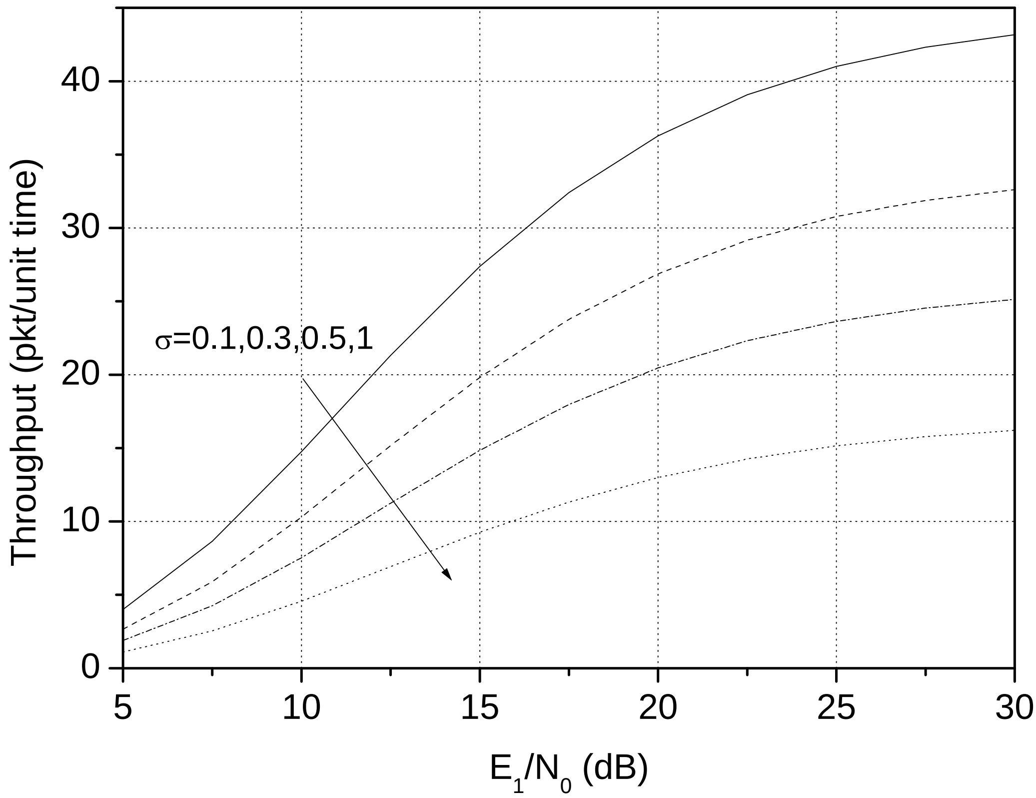

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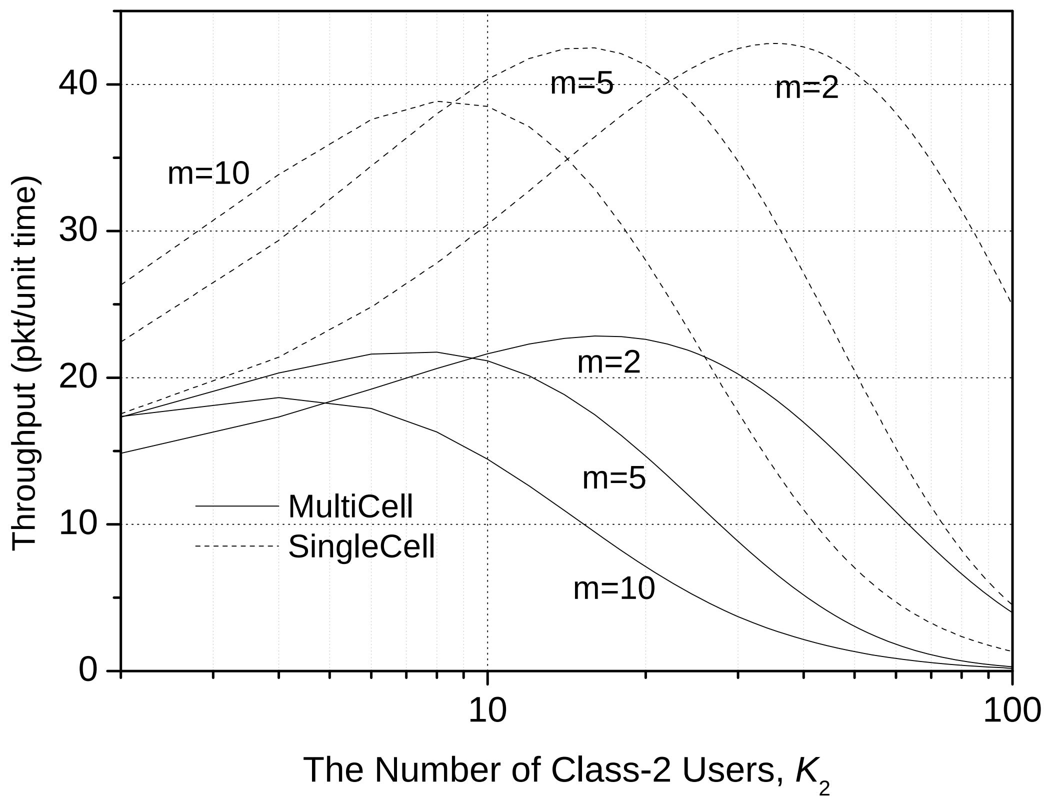

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MGF-based Performance Analysis for Wireless Systems in Nakagami Fading

Pongsatorn Sedtheetorn* and Lunchakorn Wuttisittikuljit**

Abstract

This paper proposes an efficient moment generating function (MGF) approach for performance analysis, in terms of bit error rate and spectral efficiency, for modern wireless systems in Nakagami fading. Validated by Monte Carlo simulation, the presented approach provides low-numerical complexity as well as accurate computation on the performances.

I. Introduction

This paper presents an efficient technique to compute the system performances, in terms of bit error rate and spectral efficiency, for modern wireless communications. According to the system models, signal-to-noise-plus-interference-ratio (SINR) is a key parameter affecting the performances. In general, SINR is defined as a ratio of SINR$= \frac{s_0}{\sum_{k=1}^{K-1} s_k+\eta}$ where η is additive white Gaussian noise (AWGN) with two-sided spectrum $N_0/2$. $s_0, s_1, ..., s_{K-1}$ are the set of mutually independent non-negative random processes that represent the received normalised powers of the desired signal (s_0) and $K-1$ interfering signals $s_1, s_2, ..., s_{K-1}$. As a result, the expressions of bit error rate and spectral efficiency are given respectively by [1] $\mathrm{P_e} = \frac{1}{2}\mathrm{E}[\mathrm{erfc}(\sqrt{\mathrm{SINR}})]$ and $C = K\mathrm{E}[\log_2(1+\mathrm{SINR})]$ which is in the unit of bits/Hz/second.

The direct approach, to average out $\mathrm{P_e}$ and C, requires knowledge of the probability density functions (pdf) of SINR. The resultant error rate and efficiency can be achieved by integrating K-fold convolution. It is obvious that high numerical effort is needed to compute $\mathrm{P_e}$. Alternatively, in this work, we introduce an effective approach, by using moment generating function (MGF), to deal with such the problem. Firstly, let us introduce MGF of any random variable X, with pdf $f_X(x)$, by $\phi_X(t) = \mathrm{E}\left[e^{tX}\right] = \int_{-\infty}^{\infty} e^{tx} f_X(x)dx$. Then, the average of X is simply achieved by computing $\phi'_X(0)$ [2]. This is a clear example showing

*The author is with the Department of Electrical Engineering, Mahidol University, Thailand, Tel. +66 2 889 2138 ext. 6522, Email: egpse@mahidol.ac.th. **The author is with the Department of Electrical Engineering, Chulalongkorn University, Thailand, Tel. +66 2 218 6908, Email: wlunchak@chula.ac.th.

that MGF is a very useful technique and widely used to calculate means (and variances) of such random variables.

Recently, there has been large amount of research on performance evaluation for wireless communications [3]-[8]. Due to the fact that SINR is a mixture of multi random variables, most of previous evaluation techniques (e.g. [3]-[5]) are based on the 1[st] of average, e.g.

$$\mathrm{P_e} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\mathrm{E}[s_0]}{(K-1)\,\mathrm{E}[s_k] + N_0/2}}\right)$$

which is classified as the lower bound of exact average. Some of past work [6] has proposed efficient method to accurately estimate $\mathrm{P_e}$ by letting $\mathrm{P_e} = \Pr\left(\frac{N_0}{2} > s_0 - \sum_{k=1}^{K-1} s_k\right) = \mathcal{G}\left(s_0 - \sum_{k=1}^{K-1} s_k\right)$ which is the characteristic function of s_k's that can be approximated by Fourier series. The results therein have shown that small numerical effort is required to obtain the evaluation. This technique has been developed recently in [7]-[8]. However, it does not lend itself to derive the average error rate directly from the fractional SINR.

In this paper, we propose a new MGF-based technique to accurately calculate bit error rate and spectral efficiency of general wireless communication systems in Nakagami fading. The organisation of this paper is as follows. In Section II, we derive exact expressions for bit error rate in the cases of both Rayleigh and Nakagami fading. Thereafter, explicit expressions of spectral efficiency are introduced in Section III. Section IV shows some new numerical results as well as discussions. Finally the conclusions are drawn in Section V.

II. Bit Error Rate

Consider a modern wireless system in which SINR is readily defined in Section I. Without the loss of generality, we assume that s_k for any $k = 0, 1, ..., K-1$ is a unit-mean gamma random variable with probability density function $f_{s_k}(z) = \frac{z^{m-1}}{\Gamma(m)} m^m e^{-mz}$ where $\frac{1}{2} \le m < \infty$ is Nakagami fading index and $\Gamma(m) = (m-1)!$ is the gamma function. In the case that $m = 1$, the probability density function becomes exponential $f_{s_k}(z) = e^{-z}$, for $k = 0, 1, ..., K-1$ which models Rayleigh fading [1].

In this section, we firstly derive expressions of the cumulative probability functions of SINRs in the cases of Rayleigh and Nakagami fading. This benefits us formulating new exact expressions for bit error rate of both cases.

A. The Special Case: Rayleigh Fading

Let us consider the special case of Nakagami fading when $m = 1$. In this case, the received power $s_0, s_1, ..., s_{K-1}$ become independent exponential random variables. Then the cumulative probability function, conditioned on $s_1, ..., s_{K-1}$, is

$$\Pr\left(\text{SINR} > z \mid s_1, ..., s_{K-1}\right) = \Pr\left\{\frac{s_0}{\sum_{k=1}^{K-1} s_k + \eta} > z\right\} = e^{-z\left[\sum_{k=1}^{K-1} s_k + N_0/2\right]} \tag{1}$$

which follows the fact that $\Pr\left(X > x\right) = e^{-x}$ when x is an exponential random variable. We can compute (1), when we remove the condition on $s_1, ..., s_{K-1}$, by MGF of the exponential random variables $s_1, ..., s_{K-1}$

$$\Pr\left(\text{SINR} > z\right) = e^{\frac{-zN_0}{2}} \prod_{k=1}^{K-1} \text{V}(z) \tag{2}$$

where

$$\text{V}(z) = \text{E}\left[e^{-zs_k}\right] = \frac{1}{1+z}. \tag{3}$$

Here we can express bit error rate in the case of Rayleigh fading as

$$\text{P}_\text{e} = \frac{1}{2}\left[\int_0^\infty \text{erfc}\left(\sqrt{z}\right) f_z(z)dz\right] = \frac{1}{2}\left[1 - \int_0^\infty \frac{e^{-z}}{\sqrt{\pi z}} \Pr\left(\text{SINR} > z\right)dz\right] \tag{4}$$

where $d\Pr\left(\text{SINR} > z\right) = -f_z(z)dz$. Accordingly, bit error rate for Rayleigh fading environment can be obtained by placing (1) in (4).

B. General Nakagami Fading

Now we concern the general case when s_0 is a unit-mean gamma random variable (due to the fact that the channel exhibits Nakagami fading). Condition on $\mathbf{I} = \sum_{k=1}^{K-1} s_k + N_0/2$, then the bit error rate for Nakagami fading is

$$\text{P}_\text{e} = \frac{1}{2}\int_0^\infty \text{erf c}\left(\sqrt{z}\right) \frac{z^{m-1}}{\Gamma\left(m\right)} \left(m\mathbf{I}\right)^m e^{-zm\mathbf{I}} dz. \tag{5}$$

Integrate by parts m times, as a result we achieve the bit error rate

$$\text{P}_\text{e} = \frac{1}{2}\left[1 + \int_0^\infty \psi_m\left(z\right) \phi(mz)dz\right] \tag{6}$$

where $\phi(mz)$ is the MGF with respect to $\mathbf{I}$ and

$$\psi_m\left(z\right) = -\frac{2}{\pi}\frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma\left(m\right)}\frac{e^{-z}}{\sqrt{z}}\ {}_1\text{F}_1\left(1-m; \frac{3}{2}; z\right) \tag{7}$$

with ${}_1\text{F}_1\left(a; b; z\right)$ being the hypergeometric function [9].

It is worth saying that the expression of $\phi(mz)$ depends on the random distribution of the interferences $s_1, ..., s_{K-1}$. For instance, we can determine $\phi(mz)$, when $s_1, ..., s_{K-1}$ are independently exponentially distributed, by

$$\phi(mz) = \text{E}\left[e^{-zm\left(\sum_{k=1}^{K-1} s_k + N_0/2\right)}\right] = e^{-\frac{zmN_0}{2}} \prod_{k=1}^{K-1} \text{V}(z) \tag{8}$$

where $\text{V}(z)$ is readily given in (3).

III. Spectral Efficiency

The spectral efficiency, for Rayleigh fading, can be expressed as

$$C = K\int_0^{\infty} \log_2(1+z) f_{\text{SINR}}(z)dz = K\log_2 e\int_0^{\infty} \Pr\left(\text{SINR} > z\right)\frac{dz}{1+z} \tag{9}$$

where $\Pr\left(\text{SINR} > z\right)$ is already mentioned in (2).

In the case of Nakagami fading, we compute (9) by using the following lemma.

Lemma 1: Let u be a random variable having a pdf $f\left(u\right) = \frac{u^{m-1}}{\Gamma(m)} m^m e^{-mu}$ (which is a unit-mean gamma random variable with parameter m) and v be any non-negative random variable that is "independent" of u. Denote $z = u/v$, then

$$\text{E}\left[\ln\left(1+z\right)|v\right] = \int_0^{\infty} \frac{1}{z}\left[1 - \frac{1}{\left(1+z\right)^m}\right]\phi\left(mz\right)dz \tag{10}$$

where $\phi\left(mz\right)$ is the MGF of the random variable v.

Proof: Recall Lemma 1 and apply the rules of integration by parts m times, then

$$\text{E}\left[\ln\left(1+\frac{u}{v}\right)|v\right] = \int_0^{\infty}\left\{\frac{1}{\Gamma(m)}\frac{d^m}{dz^m}z^{m-1}\ln\left(1+z\right)\right\}e^{-zmv}dz. \tag{11}$$

Consider ([9];15.1.3) which is $\ln\left(1+z\right) = z\ {}_2F_1\left(1,1;2;-z\right)$ with ${}_2F_1\left(.,.;.;.\right)$ being the Gauss hypergeometric function. Next apply ([9];15.2.3) to get

$$\begin{aligned} \frac{1}{\Gamma(m)}\frac{d^m}{dz^m}z^{m-1}\ln\left(1+z\right) &= \frac{1}{\Gamma\left(m\right)}\frac{d^m}{dz^m}z^m\ {}_2F_1\left(1,1;2;-z\right) = m\ {}_2F_1\left(1+m,1;2;-z\right) \\ &= m\int_0^1 \left(1+tz\right)^{-(m+1)}dt = \frac{1}{z} - \frac{1}{z\left(1+z\right)^m} \end{aligned} \tag{12}$$

where ([9];15.3.1) was used in the third line of (12). Eventually we place (12) into (11). ■

Apply (10), then the resultant spectral efficiency, in Nakagami fading channel, is

$$C = K\left(\log_2 e\right)\text{E}\left[\ln\left(1+z\right)|\mathbf{I}\right] = K\log_2 e\int_0^{\infty}\frac{1}{z}\left[1 - \frac{1}{\left(1+z\right)^m}\right]\phi\left(mz\right)dz \tag{13}$$

where $\phi\left(mz\right)$ is the MGF of $\mathbf{I} = \sum_{k=1}^{K-1} s_k + N_0/2$.

IV. Numerical and Simulation Results

This section presents some new numerical and simulation results. Here the major concern is to validate our proposed expressions in (6) and (13) with the Monte Carlo simulation for bit error rate and spectral efficiency, respectively. To achieve such reliable results, we average out over 2,000,000 samples of SINR of a wireless network in which signal-to-noise ratio (SNR= s_0/N_0), which represents the background noise, is 3 dB.

Figure 1 and 2 show respectively the bit error rate and spectral efficiency versus the number of users at a wide range of Nakagami fading indices m's where m is the degree of freedom of Nakagami distribution. It is important saying that $m = 1$ represents Rayleigh fading in which no line-of-sight exists between the transmitter and the receiver. In contrast, the fading effect subsides when m goes large. From the figures, our exact numerical results match up the simulation results very closely. This allows us to confirm the accuracy of our proposed expressions in (6) and (13).

V. Conclusions

In this paper, we have proposed an effective approach to accurately evaluate bit error rate and spectral efficiency of a modern wireless system in Nakagami fading. Based on MGF technique, we have derived cumulative probability functions of SINRs which has facilitated us to introduce original exact expressions for bit error rate and spectral efficiency. Validated with Monte Carlo simulation, the proposed expressions have required low numerical effort, compared with the known direct approach, to compute such the performances of wireless systems in Nakagami fading channel.

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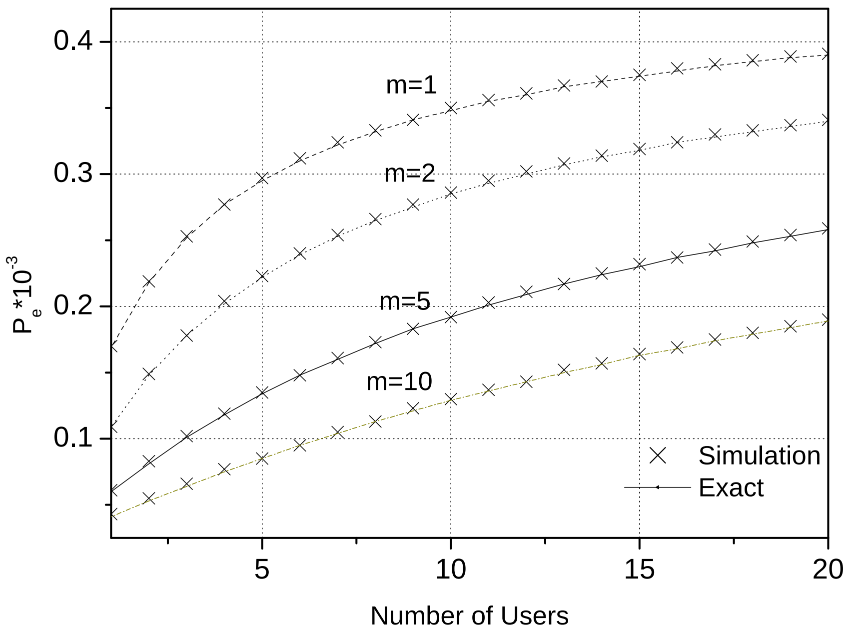

Fig. 1. Bit Error Rate (P_e) versus Number of Users for Nakagami fading index $m = 1, 2, 5$ and 10

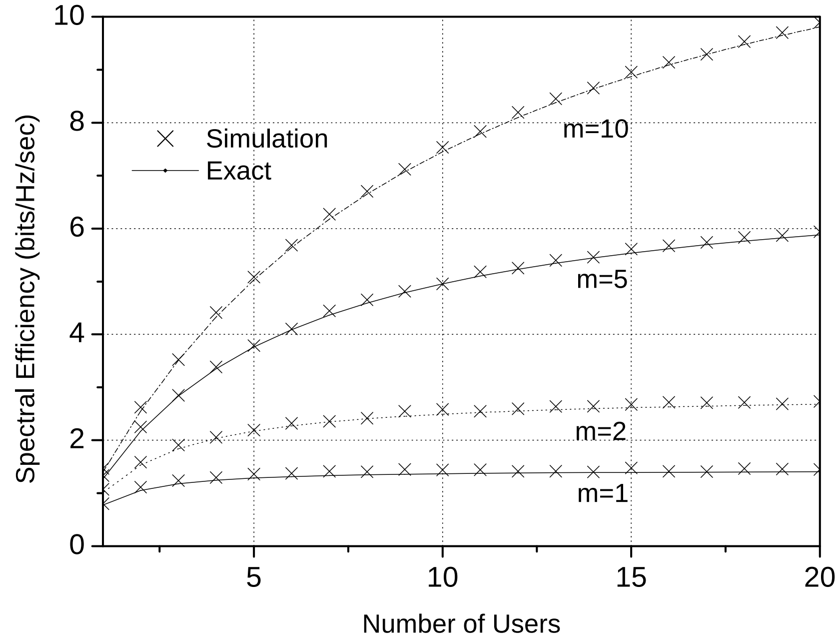

Fig. 2. Spectral Efficiency versus Number of Users for Nakagami fading index $m = 1, 2, 5$ and 10

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Theoretical Analysis on Bit Error Rate of VSG CDMA in Nakagami Fading

Pongsatorn Sedtheetorn*, Khairi Ashour Hamdi**, and Lunchakorn Wuttisitikuljit***

Abstract

This paper presents an efficient analytical method for bit error rate of variable spreading gain code division multiple access (VSG CDMA) in Nakagami fading environment. Based on improved Gaussian approximation (IGA) technique, the signal-to-interference-plus-noise-ratio (SINR) is firstly derived. Then we introduce an indirect method to express the bit error rate in a closed form. Validated by the simulation, the proposed expression benefits us to accurately investigate the impacts of channel's conditions, power allocations, and characteristics of random users' distributions on the bit error performance.

Index Terms

bit error rate, VSG CDMA, Nakagami fading, wireless communications

I. Introduction

In this paper, we focus on theoretical analysis on bit error rate of VSG CDMA in Nakagami fading channel. Over the past decades, there have been many publications on the related research area as follows.

We start from the pioneers on this research area [1]-[3]. Bit error rate for CDMA systems is analyzed in [1]-[2] and then standard Gaussian approximation (SGA) technique is proposed in [3] to estimate the error rate. This technique has been used widely (e.g. [4]-[5]) due to its simplicity. However, SGA becomes very optimistic compared with the actual error rate especially in the cases of low spreading gain or few interferers [6]. Morrow *et al* has introduced improved Gaussian approximation (IGA) technique [6] where bit error rate can be accurately obtained by integrating K–fold over the probability density functions of K multiple access interferences (MAIs). This technique nontheless requires very high numerical effort. To reduce the numerical comlexity, Holtzman [7] has presented simplified improved Gaussian approximation (SIGA) in which the K-fold integral is approximated by the well-known expansion in differences (Stirling formula). It is found that SIGA is a good trade-off between SGA and IGA in terms of accuracy and simplicity.

In [8]-[10], a precise bit error rate (non-Gaussian) approximation in Nakagami fading has been proposed. The error rate expression is in a single-integral form where the integrand is the characteristic function of MAIs. The proposed expression seems simple but actually needs considerable numerical computation. This is because the characteristic function of each interference is a summation of the recursive functions of spreading sequences (see [8, eq. (7)-(10)]). Consequently, this technique is only practical to some small systems with a few subscribers.

Without any penalty on the accuracy, there is an approximation technique, which has been proposed recently in [11]-[12]. Furthermore, this technique is applicable to both non-packet and packet-based communication (e.g. [13]-[14]). However, it is confined to a special case of multipath fading, i.e. Rayleigh fading. Based on our literature survey, there has been no work dealing with bit error rate evaluation for multirate variable spreading gain CDMA (VSG CDMA).

This paper introduces an efficient method to analyze and then propose a new closed-form expression for bit error rate of VSG CDMA in Nakagami fading. Validated with Monte Carlo simulation, the proposed

*The author is with the Department of Electrical Engineering, Mahidol University, Thailand, Tel. +66 2 889 2138 ext. 6522, Email: egpse@mahidol.ac.th. **The author is with the School of Electrical & Electronic Engineering, The University of Manchester, United Kingdom, Tel. +44 161 306 4728, Email: k.hamdi@manchester.ac.uk. ***The author is with the Department of Electrical Engineering, Chulalongkorn University, Thailand, Tel. +66 2 218 6908, Email: lunchakorn_w@ee.eng.chula.ac.th.

expression facilitates us to accurately study the effects of channel's conditions, power allocation, and the characteristics of users' distributions on the bit error performance.

The organization for the rest of this paper is as follows. Section II shows the system model of VSG CDMA in Nakagami fading. In Section III, an efficient analytical method is introduced and then a new closed-form expression of bit error rate is proposed. The analysis is also extended to consider different characteristics of users' random distributions in Section IV. In Section V, the numerical and simulation results are discussed. Finally the conclusions are presented in Section VI.

II. System Model

Consider a multirate (multi-class) VSG CDMA network in Nakagami fading environment. There are L classes of users with different transmission rates $R_1, R_2, ..., R_L$. Without loss of generality, class–1 rate is defined as the lowest rate, which is so-called the basic rate, and class–l rate is then given by $R_l = lR_1$, $l = 1, 2, ..., L$.

For the sake of perfect power control, the received power at the base station from a class–l user is a gamma random variable (which models Nakagami fading), given by the transmitted power S_l, $l = 1, 2, ..., L$. Let the matched-filter receiver of a mobile user be locked onto an arbitrary reference signal. Denote $r(t)$ be the composite signal at the receiver front end, we have

$$r(t) = v_{m,0}(t) + \sum_{l=1}^{L}\sum_{k=1}^{K_l} v_{l,k}(t - \tau_{l,k}) + n(t) \tag{1}$$

where $v_{m,0}(t)$ is the reference class–m signal 0. $v_{l,k}(t - \tau_{l,k})$, $k = 1, 2, ..., K_l$ is the interfering signal from class–l user k with $\tau_{l,k}$ being the multipath delay with respect to $v_{m,0}(t)$. $n(t)$ is the additive white Gaussian noise with two-sided power spectral density $N_0/2$.

In the case of binary VSG CDMA, the reference signal can be expressed as [3]

$$v_{m,0}(t) = h_{m,0}\sqrt{S_m} b_{m,0}(t) a_{m,0}(t) \tag{2}$$

where $h_{m,0}$ is the complex channel gain that represents Nakagami fading with $\mathrm{E}[h_x h_y^*] = 1$ for $x = y$ and $\mathrm{E}[h_x h_y^*] = 0$ for $x \neq y$. $b_{m,0}(t) = \sum_{i=-\infty}^{\infty} b_i \mathrm{rect}(t - iT_b/m)$ is the rectangular waveform of class–m data bit stream. Note that $\{b_i\}$ is a sequence of independent and identically distributed random variables (i.i.d RVs) that take values on $\{-1, +1\}$ with equal probability. T_b is the bit transmission time.

$a_{m,0}(t) = \sum_{i=-\infty}^{\infty} a_i \psi(t - iT_b/G)$ are the signature sequence signals. G is the processing gain which is equal to the number of chips per bit duration. T_b/G is the chip interval which is identical for all classes. $\psi(t)$ is an arbitrary chip waveform function which is assumed to be time limited in $[0, T_b/G]$ and normalized such that $\frac{1}{T_b/G}\int_0^{T_b/G} |\psi(t)|^2\, dt = 1$.

Accordingly, the normalized decision variable of bit i of the reference signal is

$$Z_{m,0,i} = \mathrm{Re}\{\pm |g_{m,0}| \sqrt{2S_m} + \sum_{l=1}^{L}\sum_{k=1}^{K_l} \sqrt{2S_l} e^{-j\arg(h_{m,0})} g_{l,k} W_{l,k} + \eta\}$$

where η is the complex Gaussian random variable with variance $\frac{N_0}{2(T_b/m)}$. $g_{m,0}$ is the power gain of class–m signal 0 which is a unit-mean gamma i.i.d RV. $W_{l,k}$, $l = 1, ..., L$; $k = 1, ..., K_l$ is the multiple access interference component in the interval $[iT_b/m, (i+1)T_b/m]$ of the reference bit i with respect to the interfering class–l signal k [3], which is equal to

$$W_{l,k} = \frac{1}{T_b/m}\int_{iT_b/m}^{(i+1)T_b/m} b_{l,k}(t - \tau_{l,k}) a_{l,k}(t - \tau_{l,k}) a_{m,0}(t) dt. \tag{3}$$

Based on improved Gaussian approximation technique (IGA) [3]-[6], when $G >> 1$, $W_{l,k}$ can be accurately approximated by a set of conditionally independent zero-mean Gaussian random variables with the conditional variance [3]

$$\text{Var}\left(W_{l,k} \mid \{\delta_{l,k}\}_{L,K}\right) = \frac{1}{G/n}\left[\rho^2\left(\delta_{l,k}\right) + \hat{\rho}^2\left(\delta_{l,k}\right)\right] \tag{4}$$

where

$$\delta_{l,k} = \tau_{l,k} - \frac{T_b}{G}\left\lfloor \frac{\tau_{l,k}}{T_b/G} \right\rfloor \tag{5}$$

is the time offset which is uniformly distributed over $[0, 1]$. $\rho(\cdot)$ and $\hat{\rho}(\cdot)$ are defined as the continuous partial correlation functions of the chip wave form, i.e. $\hat{\rho}(\tau) = \frac{1}{T_b/G}\int_{\tau}^{T_b/G}\psi(t)\psi(t-\tau)dt$ and $\rho(\tau) = 1 - \hat{\rho}(\tau)$.

With (4), we obtain, when we condition on $\{h_{l,k}, \delta_{l,k}\}_{L,K}$, the conditionally distributed Gaussian variance of the decision variable by

$$\text{Var}\left[Z_{m,0} \mid \{g_{l,k}, \delta_{l,k}\}_{L,K}\right] = \frac{1}{G/m}\sum_{l=1}^{L}\sum_{k=1}^{K_l} S_l g_{l,k}\left[\rho^2\left(\delta_{l,k}\right) + \hat{\rho}^2\left(\delta_{l,k}\right)\right] + \frac{N_0}{2(T_b/m)}. \tag{6}$$

Therefore, signal-to-interference-plus-noise-ratio (SINR) experienced by a class–m signal is

$$\begin{aligned} \text{SINR}_m &= \frac{1}{2}\frac{\text{E}^2\left[Z_{m,0}\right]}{\text{Var}\left[Z_{m,0}\right]} \\ &= \frac{S_m g_{m,0}}{\frac{2}{G/m}\sum\limits_{l=1}^{L}\sum\limits_{k=1}^{K_l} S_l g_{l,k}\left[\rho^2\left(\delta_{l,k}\right) + \hat{\rho}^2\left(\delta_{l,k}\right)\right] + \frac{mS_1}{(E_1/N_0)}}. \end{aligned} \tag{7}$$

where $E_1 = S_1 T_b$ is the class–1 energy per bit interval.

III. The Analysis

In this section, we introduce an original analysis on bit error rate (BER) of VSG CDMA in Nakagami fading. Firstly, let us define the expression of bit error rate as

$$\text{P}_\text{e} = \frac{1}{2}\text{E}\left[\text{erfc}\left(\sqrt{\text{SINR}}\right)\right] \tag{8}$$

where SINR is readily given in (7).

The direct approach to compute (8) requires full knowledge of the probability density functions (pdfs) of the power gains. The resultant expression can be achieved by $KL-$fold integrations over (8) which consume very high numerical effort. In this paper, we propose an efficient approach to obtain a simple-form expression of the error rate as follows.

Let $\mathcal{M} = \frac{2}{G/m}\sum_{l=1}^{L}\sum_{k=1}^{K}\frac{S_l}{S_m}g_{l,k}\left[\rho^2\left(\delta_{l,k}\right) + \hat{\rho}^2\left(\delta_{l,k}\right)\right] + \frac{S_1}{S_m}\frac{m}{\text{SNR}}$ where $\text{SNR} = E_1/N_0$ stands for signal-to-noise ratio. Then, SINR_m becomes conditionally unit-mean gamma distributed (due to $g_{m,0}$) with pdf

$$f_{\text{SINR}}\left(z \mid \mathcal{M}\right) = \frac{z^{u-1}}{\Gamma\left(u\right)}u^u e^{-uz} \tag{9}$$

where $\frac{1}{2} < u < \infty$ is Nakagami fading index (which is assumed to be integer in this work) and $\Gamma\left(u\right) = (u-1)!$ is the gamma function. Accordingly, (8) is given by

$$\frac{1}{2}\text{E}\left[\text{erfc}\left(\sqrt{\text{SINR}}\right)\middle|\, \mathcal{M}\right] = \frac{1}{2}\int_0^\infty \text{erfc}\left(\sqrt{z}\right)\frac{z^{u-1}}{\Gamma\left(u\right)}\left(u\mathcal{M}\right)^u e^{-zu\mathcal{M}}dz. \tag{10}$$

Apply the rules of integration by parts u times, then we have

$$\frac{1}{2}\mathrm{E}\left[\mathrm{erfc}\left(\sqrt{\mathrm{SINR}}\right)\Big|\,\mathcal{M}\right]=\frac{1}{2}\left\{\sum_{j=0}^{u-1}\left[\alpha_j(z)\left(u\mathcal{M}\right)^{u-j-1}e^{-zu\mathcal{M}}\right]_0^\infty+\int_0^\infty\alpha_u(z)e^{-zu\mathcal{M}}dz\right\} \tag{11}$$

where

$$\alpha_j(z)=\frac{1}{\Gamma\left(u\right)}\frac{d^j}{dz^j}z^{u-1}\,\mathrm{erfc}\left(z\right). \tag{12}$$

Due to that fact that $\mathrm{erfc}\left(z\right)=1-\frac{2z}{\sqrt{\pi}}\;{}_1\mathrm{F}_1\left(\frac{1}{2};\frac{3}{2};-z^2\right)$ with ${}_1\mathrm{F}_1\left(a;b;z\right)$ being the hypergeometric function [15]. It is straightforward to prove that $\alpha_j(z)=0$ for $j=0,1,..,u-1$ and

$$\begin{aligned}\alpha_u(z)&=\frac{1}{\Gamma\left(u\right)}\frac{d^u}{dz^u}z^{\frac{1}{2}+u-1}\;{}_1\mathrm{F}_1\left(\frac{1}{2};\frac{3}{2};-z\right)\\&=-\frac{2}{\pi}\frac{\Gamma\left(u+\frac{1}{2}\right)}{\Gamma\left(u\right)}\frac{e^{-z}}{\sqrt{z}}\;{}_1\mathrm{F}_1\left(1-u;\frac{3}{2};z\right)\end{aligned} \tag{13}$$

where the second line is reduced to the third line by Kummer transformation [15].

Therefore the bit error rate of class–l is

$$\mathrm{P}_{\mathrm{e}|l}=\frac{1}{2}\left[1+\int_0^\infty\alpha_u(z)\mathrm{MGF}(\mathcal{M})dz\right] \tag{14}$$

where $\mathrm{MGF}(\mathcal{M})=\mathrm{E}\left[e^{-zu\mathcal{M}}\right]$ is the moment generating function of $\mathcal{M}$.

Next we calculate $\mathrm{MGF}(\mathcal{M})$. Assume that the chip waveform function is rectangular, then (4) becomes [3]

$$\mathrm{Var}_{\mathrm{rect}}\left(W_{l,k}\mid\{\delta_{l,k}\}_{L,K}\right)=\frac{1}{G/m}\left[1+2\delta_{l,k}\left(\delta_{l,k}-1\right)\right]. \tag{15}$$

At this point, we have

$$\begin{aligned}\mathrm{MGF}\left(\mathcal{M}\mid\{\delta_{l,k}\}_{L,K}\right)&=\mathrm{E}\left[e^{-zu\left(\frac{2}{G/m}\frac{S_l}{S_m}g_{l,k}\left[1+2\delta_{l,k}(\delta_{l,k}-1)\right]+\frac{mS_1}{S_m\mathrm{SNR}}\right)}\right]\\&=e^{-\frac{mS_1}{S_m\mathrm{SNR}}zu}\prod_{l=1}^{L}\prod_{k=1}^{K_l}\frac{1}{\left(1+\frac{2zu}{G/m}\frac{S_l}{S_m}\left[1+2\delta_{l,k}(\delta_{l,k}-1)\right]\right)^u}\end{aligned} \tag{16}$$

which is based on the fact that $g_{l,k}$'s are unit-mean gamma random variables with index u.

To remove the condition on $\{\delta_{l,k}\}_{L,K}$, one needs to integrate the fraction $1/[1+\frac{2zu}{G/m}\frac{S_l}{S_m}\{1+2\delta_{l,k}(\delta_{l,k}-1)\}]^u$, with respect to $\delta_{l,k}$, over $[0,1]$. Mathematically, it is impossible to derive a closed-form expression of $\mathrm{MGF}(\mathcal{M})$ for $u=1,2,...$ (see the solution in [16] pp. 149-150). Some past work (e.g. [21]) therefore considers the special case where $g_{l,k}$'s are negative exponential random variables (which models Rayleigh fading). Accordingly, the exponent u becomes unity and the closed form of $\mathrm{MGF}(\mathcal{M})$ is hence obtained. To the best of our knowledge, there is no work presenting an explicit closed-form expression of $\mathrm{MGF}(\mathcal{M})$.

In this work, we instead take the first order average of uniform RV $\delta_{l,k}$ in (15)

$$\mathrm{Var}_{\mathrm{rect}}\left(W_{l,k}\right)=\frac{1}{G/m}\mathrm{E}\left[1+2\delta_{l,k}\left(\delta_{l,k}-1\right)\right]=\frac{2m}{3G}. \tag{17}$$

Then, the average value is placed into (16). This yields a closed-form expression of the moment generating function of MAIs as

$$\mathrm{MGF}(\mathcal{M})=e^{-\frac{mS_1}{S_m\mathrm{SNR}}zu}\prod_{l=1}^{L}\mathrm{V}^{K_l}\left(\frac{S_l/S_m}{G/m}z\right) \tag{18}$$

where

$$\mathrm{V}\left(z\right)=\frac{1}{\left(1+4zu/3\right)^u}. \tag{19}$$

Note that the approximation in (17) has been used widely in CDMA performance analysis e.g. [17]-[20].

As a result, the expression of bit error rate is obtained by placing (18) in (14).

IV. USERS' DISTRIBUTIONS

In the previous section, the numbers of users $K_1, K_2, ..., K_L$ are assumed to be conditionedly constant (fixed). Here we consider more practical cases as follows.

A. Independent Binomial Distribution

In practice, users do not transmit their signals all time. A signal transmission occurs only when the user is active. Let $q_l \in [0,1]$ be the active probability of a class–l user. Assume $K_1, K_2, ..., K_L$ are mutually independent, then the probability that there are k active class–l users is

$$\Pr\left(K_l = k\right) = \binom{K_l}{k} q_l^k \left(1 - q_l\right)^{K_l - k} \tag{20}$$

which is binomially distributed.

In this case, (18) is

$$\mathrm{MGF}_{\mathrm{Bin}}(\mathcal{M}) = e^{-\frac{mS_1}{S_m \mathrm{SNR}} zu} \prod_{l=1}^{L} \mathrm{E}\left[\mathrm{V}^{K_l}\left(\frac{S_l/S_m}{G/m} z\right)\right]. \tag{21}$$

To find the average $\mathrm{E}\left[\mathrm{V}^{K_l}\left(\cdot\right)\right]$, we apply the fact that, when K_l is binomially distributed, $\mathrm{E}\left[\mathrm{V}^{K_l}\left(\cdot\right)\right] = \left[1 - q_l + q_l \mathrm{V}\left(\cdot\right)\right]^{K_l}$ for any function $\mathrm{V}(\cdot)$. Then we have

$$\mathrm{MGF}_{\mathrm{Bin}}(\mathcal{M}) = e^{-\frac{mS_1}{S_m \mathrm{SNR}} zu} \prod_{l=1}^{L} \left[1 - q_l + q_l \mathrm{V}\left(\frac{S_l/S_m}{G/m} z\right)\right]^{K_l}. \tag{22}$$

Accordingly, bit error rate, in the case of binomial distribution, can be achieved by placing (22) into (14).

B. Dependent Multinomial Distribution

Based on the previous assumption, each user is confined to transmit its signals at one particular rate. Now we consider a different case where every user can operate at various rates $R_1, R_2, ..., R_L$. In this case, $K_1, K_2, ..., K_L$ are hence dependent and the analysis can be shown as follows.

Consider a multirate VSG CDMA network serving K users, each of which is able to operate at rates $R_1, R_2, ..., R_L$. p_0 is the probability when a user is inactive and p_l is the probability of a class–l transmission such that $\sum_{l=0}^{L} p_l = 1$. In this regard, $K_1, K_2, ..., K_L$ can be modeled as dependent multinomial random variables. Denote K_0 by the number of inactive users. When $K_0, K_1, .., K_L$ are dependent multinomial distributed, for any function $\mathrm{V}(\cdot)$ we have

$$\mathrm{E}\left[\mathrm{V}_1^{K_1} \mathrm{V}_2^{K_2} ... \mathrm{V}_L^{K_L}\right] = \left[p_0 + p_1 \mathrm{V}_1 + p_2 \mathrm{V}_2 + ... + p_L \mathrm{V}_L\right]^K \tag{23}$$

which is called as the multinomial expansion [22] with the restriction $K = \sum_{l=0}^{L} K_l$.

Apply (23) to (18), then we have

$$\mathrm{MGF}_{\mathrm{Mul}}(\mathcal{M}) = e^{-\frac{mS_1}{S_m \mathrm{SNR}} zu} \left[p_0 + p_1 \mathrm{V}\left(\frac{S_1/S_m}{G/m} z\right) + p_2 \mathrm{V}\left(\frac{S_2/S_m}{G/m} z\right) + ... + p_L \mathrm{V}\left(\frac{S_L/S_m}{G/m} z\right)\right]^K. \tag{24}$$

C. Poisson Distribution

Consider the special case when $K_1, K_2, ..., K_L \to \infty$ and $q_1, q_2, ..., q_L \to 0$, the numbers of users become Poisson random variables with pdf

$$\Pr(K_l = k) = \frac{\lambda_l^k}{k!} e^{-\lambda_l}; \qquad l \in 1, 2, ..., L \tag{25}$$

where $\lambda_l = q_l K_l$ is the average of class–l users.

Due to the fact that for any Poisson random variable Y with mean λ, we have $\mathrm{E}[e^{-zY}] = \exp\left[\lambda\left(e^{-z} - 1\right)\right]$. Recall (18), it can be straightforwardly shown that

$$\mathrm{MGF}_{\mathrm{Poi}}(\mathcal{M}) = e^{-\frac{mS_1}{S_m \mathrm{SNR}} zu} e^{-\sum_{l=1}^{L} \lambda_l \left[1 - \mathrm{V}\left(\frac{S_l/S_m}{G/m} z\right)\right]}. \tag{26}$$

V. Numerical and Simulation Results

In this section, we presents some new numerical results, compared with simulation results, and discussions. Consider a dual-class VSG CDMA network in Nakagami fading environment. Class–2 rate is assumed to be twice as class–1 rate (the basic rate). Also class–2 power is double to class–1, i.e. $S_2 = 2S_1$, such that bit energy of both classes retains the same level, $E_1 = \frac{S_1}{R_1} = \frac{S_2}{R_2} = E_2$ [23]-[24]. Based on this configuration where $E_1 = E_2$, our numerical and simulation results reveal that the bit error rates of class–1 and class–2 are identical (the graphical presentation is omitted due to the limit of submission pages). Therefore, in Figure 1-2, we instead show the overall error rate along with the system parameters such as SNRs, fading indices, and random users distributions. Finally, in Figure 3, we study the impact of power allocation ($S_2$and S_1) on the error rates of class–1 and class–2.

Assume the numbers of class–1 and class–2 users are binomially distributed where $K_1 = K_2 = 10$ and $q_1 = q_2 = 0.5$. Figure 1 shows the bit error rate versus SNR for different Nakagami fading index $u = 1, 2, 3, 5$. It is well known that small fading indices correspond to severe fading whereas $u \to \infty$ stands for non-fading channels. Furthermore, SNR represents the condition of channel, i.e. high SNR means low background noise in the channel and vice versa. From the figure, our results positively support the mentioned statements. However, the major concern of Figure 1 is to validate the numerical results from our proposed expression in (14) and (22) with the simulation. Based on our simulation, we average out (8) from 2,000,000 samples of SINRs so that the outcomes are accurate and reliable. It is obviously found that the numerical results match up very closely to the simulation.

Figure 2 compares the bit error rates of 3 systems with different random users distributions, i.e. Binomial, Poisson, and Multinomial distributions. For the sake of fairness, every system, in the same environment at SNR = 10 dB, has equal offered load as follows; $q_1 = q_2 = 0.5$ with $K_1 = K_2 = 10$ for the Binomial system, $\lambda_1 = \lambda_2 = 5$ for Poisson, and $p_1 = p_2 = 0.5$ with $K = 20$ for Multinomial. From the results, the error rates of the Binomial and Poisson systems, where the users are independent distributed, are nearly identical. However, the error rate of the Multinomial system is obviously higher than the other systems due to the dependency of class–1 and class–2 transmissions.

Figure 3 illustrates the impact of class–2 power allocation on the error rates. As mentioned earlier, the error rates of class–1 and class–2 are identical when the bit energy of both classes remains the same. Here the class–1 power is fixed at S_1 and then the class–2 power S_2 is varied as the multiple times of S_1. Let Nakagami fading index $u = 3$ and SNR = 10 dB. Then we keep the other system parameters as the same as in Figure 1. The result shows that the bit error performance of class–2 can be improved by increasing the power allocation on S_2. Inevitably, this power adjustment degrades the bit error performance of class–1. Consequently, we can conclude that, in VSG CDMA, the quality of service, in terms of bit error rate, of each individual class can be arbitrarily controlled by power allocation. It is worth saying that the impact of power allocation on bit error performance of VSG CDMA is never mentioned in the literature.

VI. Conclusions

This paper has proposed an efficient (indirect) method for bit error rate analysis of a VSG CDMA network in Nakagami fading channel. Based on IGA, we have derived the SINR and then introduced a non-conventional method to compute the bit error rate. This has facilitated us to express the error rate in a new closed form. Validated by the simulation, the proposed expressions have yielded some new accurate and useful numerical results. The results have revealed that the dependency among class–$1, \ldots,$class–l users degrades the overall bit error performance. We have further found that the error rates of individual classes can be controlled by the power allocation. This benefits us to arbitrarily direct the quality of service (in terms of bit error rate) of each class.

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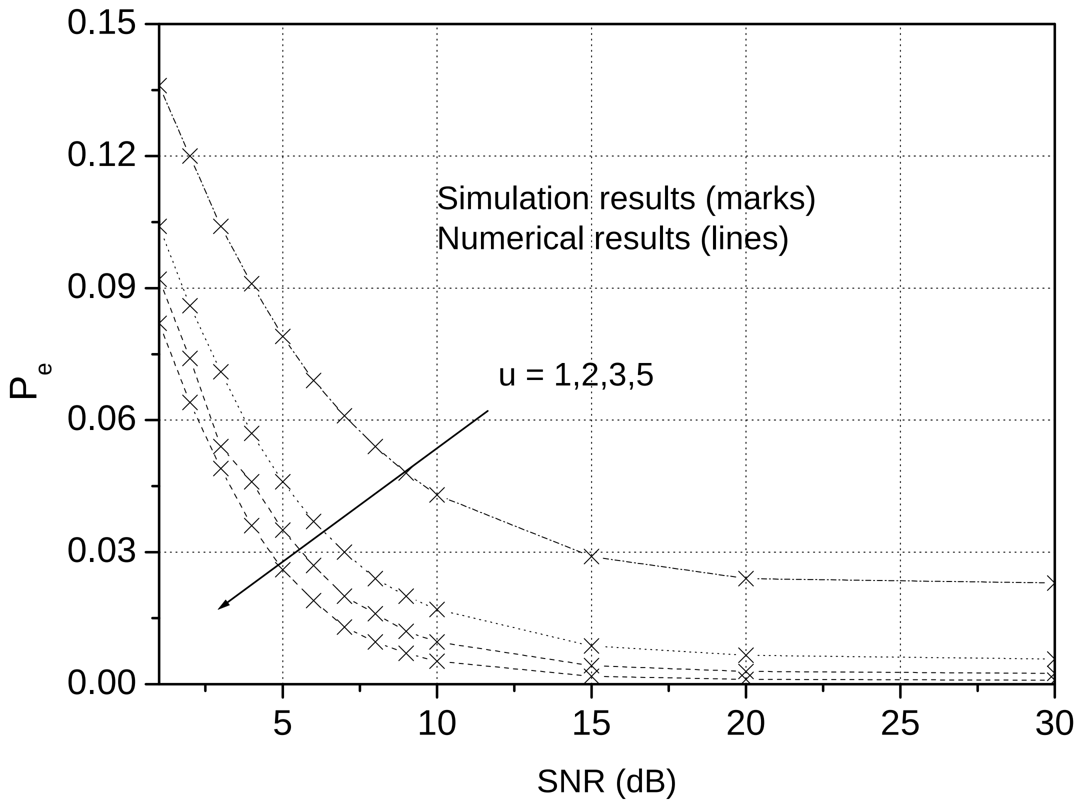

Fig. 1. The bit error rate against signal-to-noise-ratio (SNR) for a wide range of Nakagami fading index $u = 1, 2, 3, 5$ with $G = 200$.

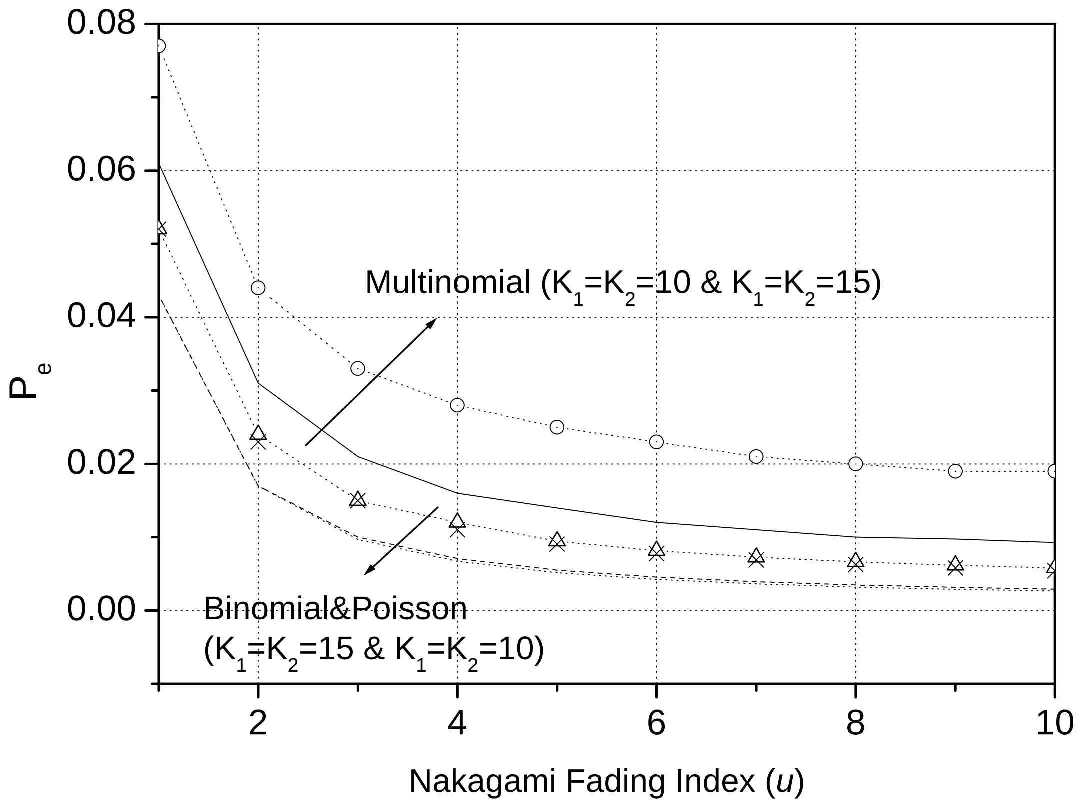

Fig. 2. The bit error rate against Nakagami fading index u with $G = 200$,SNR = 10 dB.

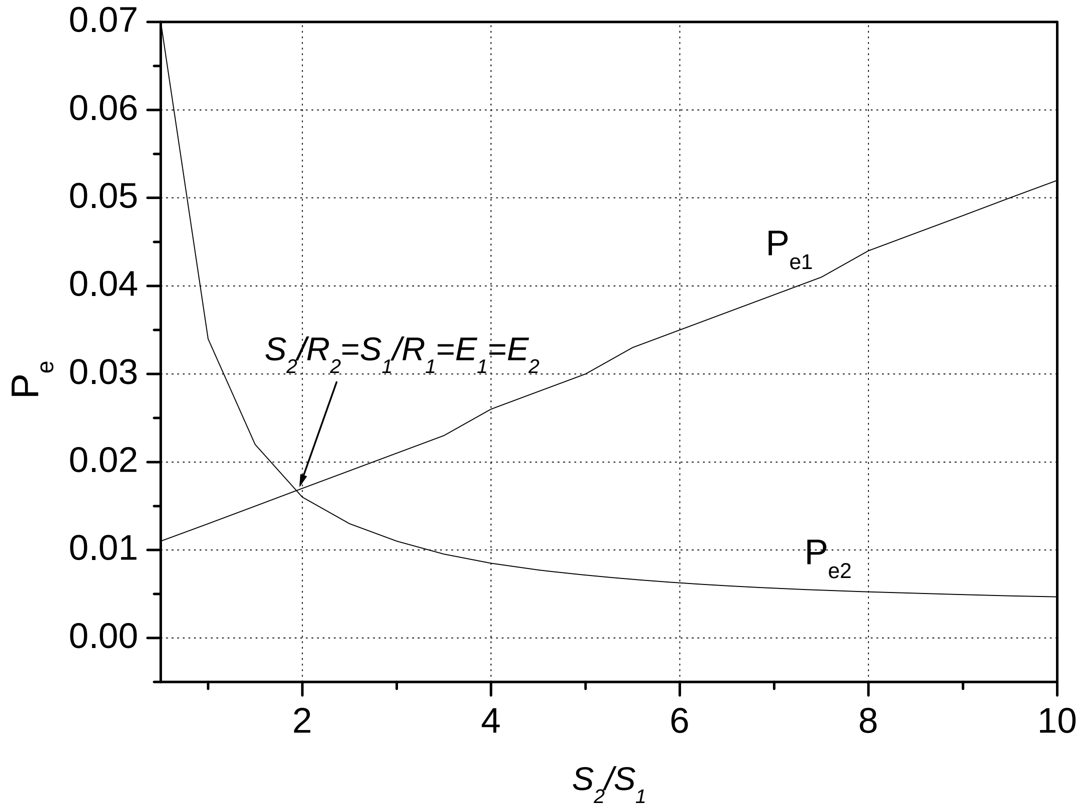

Fig. 3. The bit error rates of class–1 and class–2 against S_2/S_1 for Nakagami fading index $u = 3$, SNR = 10 dB and $G = 200$.