



รายงานวิจัยฉบับสมบูรณ์

โครงการการออกแบบระบบโลจิสติกส์ที่เหมาะสมที่สุดของ
โครงข่ายการส่งสินค้าที่มีผู้ส่งสินค้าและลูกค้าหลายราย

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มิถุนายน 2556

สัญญาเลขที่ MRG5380086

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สินค้าที่มีผู้ส่งสินค้าและลูกค้าหลายราย

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ปัญหาการกำหนดเส้นทางเดินรถที่เหมาะสม (Vehicle Routing Problem; VRP) ถือเป็นปัญหาที่ผู้ขนส่งสินค้าทางถนนทุกรายต้องประสบและมีหลายปัจจัยที่ต้องคำนึงถึง เช่น ระยะทางของเส้นทางที่จะวิ่ง จำนวนลูกค้า ตำแหน่งของลูกค้า ชนิดและปริมาณของสินค้า เวลาที่ลูกค้าสะดวกให้ไปส่ง จำนวนและชนิดของรถส่งของ ฯลฯ จากงานวิจัยด้าน VRP ที่ผ่านมามักเน้นไปเพื่อประโยชน์ของผู้ขนส่งสินค้ารายเดียว และให้ความสำคัญกับประโยชน์และความต้องการของลูกค้าผู้รับสินค้าน้อยกว่า และเป็นความจริงที่ว่า ลูกค้าแต่ละรายไม่ได้รอรับสินค้าจากผู้ขนส่งสินค้าแค่รายเดียว แต่มีรอรับสินค้าจากผู้ส่งสินค้าหลายๆ รายด้วยกัน ผู้วิจัยจึงคิดที่จะพัฒนาวิธีการแก้ปัญหา VRP ในโครงข่ายการส่งสินค้าที่มีผู้ขนส่งสินค้าหลายราย โดยวิธีนี้จะพยายามออกแบบเส้นทางของการขนส่งสินค้าที่ให้บริการต่อทุกฝ่ายในโครงข่าย โดยผู้วิจัยได้พัฒนาแบบจำลองทางคณิตศาสตร์สำหรับปัญหาการกำหนดเส้นทางเดินรถที่เหมาะสมสำหรับโครงข่ายการส่งสินค้าที่มีผู้ส่งสินค้าและลูกค้าหลายรายเป็นผลสำเร็จและได้พัฒนาวิธีการคำตอบแบบฮิวริสติกเพื่อแก้ปัญหานี้ โดยวิธีฮิวริสติกส์ที่พัฒนาขึ้นมาอาศัยวิธีการค้นแบบสุ่ม (Random Search) และหาคำตอบใหม่โดยการสุ่มเลือกลูกค้า 1 รายจากรถบรรทุกคันแรกไปแทรกอยู่ในลำดับการส่งสินค้าของรถบรรทุกคันที่สอง จากผลการทดลองเปรียบเทียบประสิทธิภาพด้วยปัญหาจำลองจำนวน 15 ปัญหา พบว่าวิธีฮิวริสติกส์สามารถหาคำตอบที่มีต้นทุนรวมต่ำกว่าในเวลาที่สั้นกว่าวิธีการหาค่าที่ดีที่สุดด้วยโปรแกรม ILOG Cplex

คำหลัก : แบบจำลองทางคณิตศาสตร์, วิธีฮิวริสติกส์, ปัญหาการกำหนดเส้นทางเดินรถ

Abstract

Project Code : MRG5380086

**Project Title : On Optimal Design of Collaborative Logistics System for Multiple
Suppliers – Multiple Customers Network**

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Vehicle routing problem (VRP) is a common problem faced by all companies who are required to deliver their products on road network. The problem has many factors involved such as route distance, number of customers, customer's location, type and quantity of products, time availability of customers, type and capacity of trucks, etc. Most previous works on VRP focus on the benefit of the only one supplier, and ignore the reality that customers who receive the delivery. In fact, each customer does not generally receive products from one supplier but many. Thus, this research aims to study the VRP for the multiple suppliers – multiple customers network. The mathematical model of this problem is proposed. A heuristics based on random search is also developed. The heuristics can find new solutions by randomly choosing one customer from the first truck and move it to the second truck in any random sequence. Based on the computational experiments with 15 test problems, the heuristics could find the solution with the lower total cost in the relatively shorter period of time than the optimization method by ILOG Cplex.

Keywords : Mathematical Model, Heuristics, Vehicle Routing Problem

Executive Summary

1. ความสำคัญและที่มาของปัญหาที่ทำการวิจัย

ในปัจจุบันกลุ่มอุตสาหกรรมต่างๆในประเทศไทยกำลังพบกับปัญหาในการดำเนินงานมากมาย ไม่ว่าจะเป็นค่าใช้จ่ายในการดำเนินธุรกิจหรือการผลิตที่พุ่งสูงขึ้น เช่น ค่าน้ำมัน ค่าแรงงาน และค่าวัตถุดิบ ทั้งยังต้องเผชิญกับการแข่งขันที่สูงขึ้นที่เกิดจากแรงผลักดันทั้งในประเทศและต่างประเทศ โดยเฉพาะอย่างยิ่งจากบริษัทต่างชาติหรือที่มีเทคโนโลยีที่เหนือกว่า และเงินทุนที่สูงกว่า อุตสาหกรรมไทยจึงต้องพยายามพัฒนาและปรับปรุงการดำเนินงานของตัวเองเพื่อเพิ่มขีดความสามารถในการแข่งขัน เป็นที่ทราบกันดีว่า ประเทศไทยมีต้นทุนด้านโลจิสติกส์สูงมากเมื่อเทียบกับประเทศอื่นๆ การลดต้นทุนด้านโลจิสติกส์จึงมีความสำคัญต่อการระดับการแข่งขันของประเทศไทย การขนส่งสินค้าถือเป็นกิจกรรมหลักที่ก่อให้เกิดต้นทุนด้านโลจิสติกส์ในระดับที่สูง โดยเฉพาะอย่างยิ่งปริมาณการขนส่งสินค้าทางถนนในประเทศไทยนั้นมีปริมาณมากกว่าการขนส่งสินค้าทางเรือ รถไฟ และเครื่องบินรวมกัน โดยสาเหตุนี้ผู้วิจัยจึงสนใจที่จะทำการวิจัยเกี่ยวกับการขนส่งสินค้าทางถนน

ปัญหาการกำหนดเส้นทางเดินรถที่เหมาะสม (Vehicle Routing Problem; VRP) ถือเป็นปัญหาที่ผู้ขนส่งสินค้าทางถนนทุกรายต้องประสบ และถือเป็นปัญหาที่มีความซับซ้อนมาก เพราะมีหลายปัจจัยที่ต้องคำนึงถึง เช่น ลักษณะของถนนและเส้นทางที่รถส่งของต้องวิ่งผ่าน จำนวนลูกค้าที่ต้องขนส่งสินค้าไปส่ง ที่อยู่หรือตำแหน่งของลูกค้า ชนิดและปริมาณของสินค้าที่ลูกค้าแต่ละรายที่ต้องการ เวลาที่ลูกค้าสะดวกให้ไปส่ง จำนวนและชนิดของรถส่งของที่มีให้ใช้ไปส่ง ฯลฯ และเนื่องจากปัญหา VRP เป็นปัญหาที่พบบ่อยมากที่สุดในการจัดการขนส่งสินค้าบนถนน จึงมีงานวิจัยจำนวนมากที่เกี่ยวข้องกับปัญหา VRP

จากการทบทวนงานวิจัยที่ผ่านมา ผู้วิจัยพบว่างานวิจัยส่วนใหญ่มักเน้นไปเพื่อประโยชน์ของผู้ขนส่งสินค้าเป็นหลัก คือเพื่อลดต้นทุนการขนส่งสำหรับผู้ทำการขนส่งสินค้าเท่านั้น และให้ความสำคัญกับประโยชน์และความต้องการของลูกค้าผู้รับสินค้าน้อยกว่า และเป็นความจริงที่ว่า ลูกค้าแต่ละรายไม่ได้รอรับสินค้าจากผู้ขนส่งสินค้าแค่รายเดียว แต่มีรอรับสินค้าจากผู้ส่งสินค้าหลายๆ รายด้วยกัน และเป็นไปได้ว่า ผู้ส่งสินค้า 2 รายขึ้นไปจะมาส่งสินค้าให้ลูกค้ารายหนึ่งในเวลาเดียวกัน ซึ่งต้องทำให้ผู้ส่งสินค้าอีกรายหนึ่งต้องเสียเวลาจากการรอเพราะลูกค้าอาจไม่สามารถรับสินค้าจากผู้ส่งพร้อมๆ กันได้ ด้วยเหตุนี้ ผู้วิจัยจึงคิดที่จะพัฒนาวิธีการแก้ปัญหา VRP ในลักษณะใหม่ คือเป็นปัญหา VRP ในโครงข่ายการส่งสินค้าที่มีผู้ขนส่งสินค้าหลายราย โดยวิธีนี้จะพยายามออกแบบเส้นทางของการขนส่งสินค้าที่ให้ประโยชน์ต่อทุกฝ่ายในโครงข่าย คือผู้ขนส่งสินค้าจะสามารถส่งสินค้าโดยใช้ต้นทุนที่ต่ำและเสียเวลาในการเดินทางและรอน้อย และลูกค้าจะสามารถรับสินค้าได้จากผู้ขนส่งสินค้าหลายๆ รายในเวลาอันรวดเร็วขึ้น วิธี

แก้ปัญหาที่จะพัฒนานี้สามารถนำไปใช้กับอุตสาหกรรมหลายประเภท เช่น กลุ่มธุรกิจโรงแรม และภัตตาคารซึ่งประกอบไปด้วยผู้ส่งวัตถุดิบและอุปกรณ์ที่ใช้ในการปรุงอาหาร เช่น เนื้อสัตว์ ผักสด น้ำแข็ง เครื่องดื่ม เป็นต้น ส่วนโรงแรมและภัตตาคารคือลูกค้าผู้รับสินค้า วิธีนี้ยังสามารถนำไปประยุกต์ใช้กับสถานประกอบการจำพวก ตลาดสด ตลาดนัด หรือศูนย์การค้า ที่มีผู้ส่งสินค้าและลูกค้าจำนวนมาก

การแก้ปัญหา VRP สำหรับโครงข่ายการส่งสินค้าที่มีผู้ส่งสินค้าและลูกค้าหลายรายมีแนวคิดตรงกับหลักการของการจัดการโซ่อุปทาน คือต้องคำนึงถึงทุกฝ่ายที่เกี่ยวข้องกับห่วงโซ่อุปทานตั้งแต่ต้นห่วงถึงปลายห่วง ไม่ใช่คำนึงถึงแค่ฝ่ายใดฝ่ายหนึ่ง หากกลุ่มอุตสาหกรรมไหนมีการนำวิธีที่พัฒนานี้ไปใช้ จะทำให้ต้นทุนการขนส่งสินค้าโดยรวมของอุตสาหกรรมลดต่ำลง ทำให้ทุกฝ่ายในอุตสาหกรรมเติบโตไปพร้อมๆกัน ส่งผลให้ต้นทุนโลจิสติกส์ลดลงและเพิ่มขีดความสามารถในการแข่งขันของกลุ่มอุตสาหกรรมนั้นได้

2. วัตถุประสงค์ของโครงการ

1. เพื่อพัฒนาแบบจำลองทางคณิตศาสตร์สำหรับปัญหาการกำหนดเส้นทางเดินรถที่เหมาะสมสำหรับโครงข่ายการส่งสินค้าที่มีผู้ส่งสินค้าและลูกค้าหลายราย
2. เพื่อพัฒนาวิธีหาคำตอบแบบฮิวริสติกสำหรับปัญหาการกำหนดเส้นทางเดินรถที่เหมาะสมสำหรับโครงข่ายการส่งสินค้าที่มีผู้ส่งสินค้าและลูกค้าหลายราย

3. ระเบียบวิธีวิจัย

1. ศึกษาและทบทวนวรรณกรรมที่เกี่ยวข้องกับปัญหาการกำหนดเส้นทางเดินรถที่เหมาะสม (VRP) และการจัดการการขนส่งสินค้าทางถนน
2. สร้างแบบจำลองทางคณิตศาสตร์ของแก้ปัญหา VRP สำหรับโครงข่ายการส่งสินค้าที่มีผู้ส่งสินค้าและลูกค้าหลายราย
3. ปรับปรุงแก้ไขแบบจำลองทางคณิตศาสตร์ให้มีความถูกต้อง
4. พัฒนาอัลกอริทึมแบบฮิวริสติกส์สำหรับปัญหา VRP สำหรับโครงข่ายการส่งสินค้าที่มีผู้ส่งสินค้าและลูกค้าหลายราย
5. เขียนโปรแกรมคอมพิวเตอร์ด้วยอัลกอริทึมที่ได้พัฒนามา
6. ทำการทดลองการประมวลผล (Computational Experiment) ของโปรแกรมคอมพิวเตอร์ เพื่อพิสูจน์ความถูกต้องและประสิทธิภาพของแบบจำลองและอัลกอริทึมที่พัฒนาขึ้นมา (Validation of the model and algorithm with empirical data)
7. จัดทำ Manuscript ของผลการศึกษาเพื่อส่งตีพิมพ์ใน International Journal

4. ผลการวิจัยของโครงการ

ผู้วิจัยสามารถสร้างแบบจำลองทางคณิตศาสตร์ของแก้ปัญหา VRP สำหรับโครงข่ายการส่งสินค้าที่มีผู้ส่งสินค้าและลูกค้าหลายรายโดยเรียกชื่อเป็นภาษาอังกฤษว่า Multiple-Supplier Vehicle Routing Problem (MS-VRP) ซึ่งได้รับแนวความคิดมาจากแบบจำลองทางคณิตศาสตร์ของปัญหา VRP แบบ Time Windows โดยสมการเป้าหมายจะเป็นค่าต้นทุนรวมระหว่างต้นทุนค่าขนส่งคิดตามระยะทางที่รถบรรทุกทุกคันเดินทางเป็นกิโลเมตรและต้นทุนค่าพนักงานทุกคนคิดตามระยะเวลาเป็นนาที

ผู้วิจัยสามารถพัฒนาอัลกอริทึมแบบฮิวริสติกส์สำหรับแก้ปัญหา MS-VRP โดยวิธีฮิวริสติกส์ที่พัฒนาขึ้นมาอาศัยวิธีการค้นแบบสุ่ม(Random Search) และหาคำตอบใหม่โดยการสุ่มเลือกลูกค้า 1 รายจากรถบรรทุกคันแรกไปแทรกอยู่ในลำดับการส่งสินค้าของรถบรรทุกคันที่สอง

ผู้วิจัยได้ทำการเปรียบเทียบประสิทธิภาพของอัลกอริทึมแบบฮิวริสติกส์ที่พัฒนาขึ้นมากับการหาคำตอบด้วยวิธี Optimization ด้วยโปรแกรม ILOG Cplex จากผลการทดลองเปรียบเทียบประสิทธิภาพด้วยปัญหาจำลองจำนวน 15 ปัญหา พบว่าวิธีฮิวริสติกส์สามารถหาคำตอบที่มีต้นทุนรวมต่ำกว่าในเวลาที่สั้นกว่าวิธีการหาค่าที่ดีที่สุดด้วยโปรแกรม ILOG Cplex

1. INTRODUCTION

A common objective of the traditional vehicle routing problem (VRP) is to identify the minimal-distance routes of a group of trucks to deliver goods from a depot of a single supplier to customers at various locations, where each customer needs different amounts of goods (Lai *et al.*, 2012). VRP is one of the most frequently found problems in the delivery service industry, and one of the most studied problems due to its various real-world applications. Examples of the single objective of VRP are to minimize the total delivery cost, the number of trucks used, or the total travel distance. Some situations however require multiple objectives (Geiger, 2001; Coello-Coello *et al.*, 2002). The real constraints of VRP make the problem even more complex and, consequently, more difficult to solve. For example, delivery trucks are of different capacities (Jozefowicz *et al.*, 2009), goods may have many different types with different stock keeping units (SKU), customers may require goods to be delivered in certain duration, so-called time-window constraint (Hong and Park, 1999; Ombuki *et al.*, 2006), to name a few. Most research works on VRP consider suppliers or people who deliver goods more important than customers who receive goods (Du *et al.*, 2005; Hwang, 2005; Confessore *et al.*, 2008).

Minimizing the total travel distance is not always a practical objective if customers are located in a heavy-traffic city area. Shorter distance of a route could result in the unexpected longer driving time due to heavy traffic congestion. It is obvious that minimizing both travel distance and time is more practical especially for intra-city delivery. Generally, customers are likely to place orders to many suppliers. If each supplier plans its delivery routes without knowledge of other suppliers' delivery information, it could happen that many suppliers might arrive at one customer location at the same time or while one supplier has already arrived and not yet finished unloading. This situation could cause other suppliers to have to wait until the customer is free especially if the customer can accommodate only one supplier unloading at a time. This incurs the cost of waiting time for the suppliers whose trucks are idle and not fully utilized. In fact, it is common for trucks to wait idly at many customer locations when each truck competes for its priority over other trucks to unload goods.

Obviously, it would be best if all suppliers and all customers in the logistics network could share the delivery information and develop a master delivery route plan together so that all parties can benefit from a more efficient delivery system with the shorter total service time. To our knowledge, there is no study about the VRP that considers "multiple suppliers" with the aim of finding so-called coordinated delivery routes for all suppliers involved in the collaborative logistics network. There are only some previous works focusing on the single-supplier problems with multiple depots (Malonia and Benton, 1997; Wasner and Zäpfel, 2004; Onoyama *et al.*, 2006; Selim *et al.*, 2008; Silva *et al.*, 2009). Hence, this research work aims to study a new VRP, namely, the multiple-supplier VRP (MS-VRP), when there is a collaborative logistics network consisting of many suppliers and many customers, and the customers are served by several suppliers. Its objective is to develop the coordinated delivery routes for all suppliers' trucks in order to minimize the total delivery cost for the entire network. Note that the total delivery cost is the sum of total travel cost and total time cost for all suppliers. The time duration that all trucks spend in traveling, waiting, and unloading is included in the calculation of time cost.

Readers might find the MS-VRP similar to the multiple-depot vehicle routing problem or MDVRP (Ho *et al.*, 2008; Mirabi *et al.*, 2010; Sumichras and Markham, 1995). However, both problems are significantly different in many ways. MDVRP aims to minimize the total cost of a single supplier who has many depots and many trucks. But MDVRP does not aim to minimize the total cost of any single supplier. But it aims to minimize the total cost of all suppliers and customers in the logistics network through collaboration. All depots of MDVRP belong to one supplier. All depots of MS-VRP do not belong to one supplier but they belong to different suppliers. All trucks of MDVRP belong to only one supplier, and each truck must be based at one depot. All trucks of MS-VRP do not belong to only one supplier. Each supplier can have more than one truck. Each truck must be based at each supplier's depot. All products of MDVRP are available at all depots of this supplier. All products of MS-VRP are not available at all depots as each supplier's product is different. And it is impossible that all depots will have all products from all suppliers. Customers of MDVRP will receive goods from only one truck based at one depot. Customers of MS-VRP will receive goods from many trucks where each truck belongs to different supplier. In MDVRP, the single supplier is the sole decision maker. He or she has to decide which truck based at which depot will make a delivery to which customer. He ignores the fact that his customers may receive goods from other suppliers. He only concerns about his business and does not give attention to the overall logistics network. In MS-VRP, all suppliers and customers in the logistics network will take part in making decisions. They have to decide which truck from which supplier will make a delivery to which customer. In this single supplier of MDVRP's view, there is no conflict as he or she believes that his truck can always

start to unload as soon as it arrives each customer. In reality, there could be many conflicts as two or more trucks of different suppliers can arrive at a customer during the same time. The truck that comes first will unload first. The other trucks have to wait idly.

The solutions (delivery routes) of MS-VRP and MDVRP are also very much different. For example in Figure 1 a solution of MDVRP (1 supplier; 3 depots: D1, D2, D3; 5 customers: c1, c2, c3, c4, c5; 3 trucks – 1 truck/depot) differs greatly from a solution of MS-VRP (3 suppliers: S1, S2, S3; 5 customers; 6 trucks – 2 trucks/supplier) since each supplier has one depot and all customers place an order to all suppliers. That is, each supplier must make a delivery to all customers.

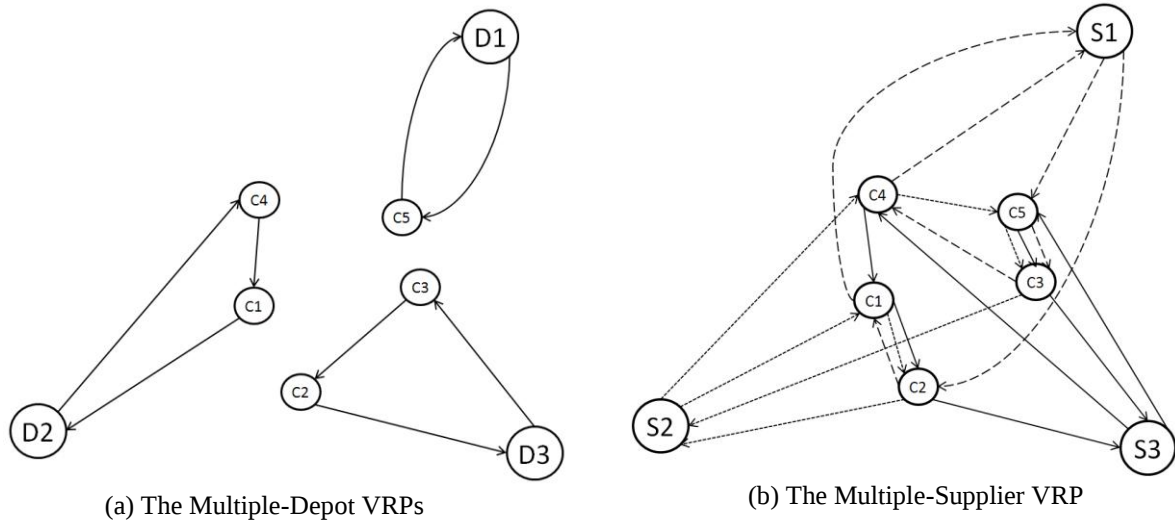


Figure 1. Comparison of delivery routes: (a) from MDVRPs, and (b) from MS-VRP

The remainder of this paper is organized as follows. Section 2 describes the multiple-supplier logistics network and the need for a master delivery schedule plan. Section 3 presents a mathematical model of the MS-VRP and how some non-linear constraints are linearized. A numerical example is illustrated in Section 4.1, followed by the computational experiment on 15 test problems comparing MS-VRP and traditional VRP in Section 4.2. The heuristics based on the random search technique to solve MS-VRP is described in Section 5. The paper ends with the conclusion (Section 6).

2. COLLABORATIVE LOGISTICS NETWORK

The application of the multiple-supplier vehicle routing problem (MS-VRP) exists in most delivery logistics networks that involve shopping malls, supermarkets, superstores, retailers, distribution centers, hotels, or in the area that are densely clustered by convenience stores and restaurants. One main property of this type of networks is that it consists of many suppliers who manufacture or sell the same type of goods or products and deliver to the same group of customers, for example, the manufacturers or distributors of office supplies, consumer products, and produces. These suppliers have to deliver their products to retail stores, convenience stores, or restaurants that are scattered around the city.

Since these suppliers have to make the delivery around the city, each of their trucks has to endure heavy city traffic before they can visit all customers. Additionally, there are other problems that they frequently encounter at the customer location. Since most customers are retail stores, convenience stores, or restaurants, their premises are usually small or medium size, with small loading/unloading facility or none at all. That is, the customer can usually accommodate only one supplier at a time. If one supplier has already arrived and not yet finished unloading its goods, other suppliers who might have arrived at the same customer cannot start unloading their goods until the on-going unloading of goods is finished. If there are no dedicated parking/waiting spaces at the customer location, the waiting trucks have to find appropriate parking spaces on their own or they might have to circle around the block to avoid parking tickets. Thus, the delays caused by having to wait for their turns to unload goods not only affect the delivery services at other customers along the delivery routes but also increase the delivery cost. This situation is expected to be quite common especially when each customer is served by several suppliers in each workday and unloading time can be lengthy due to lack of supporting facilities, manpower, and mechanized materials handling equipment.

Figure 2 shows delivery routes and schedules of a hypothetical multiple-supplier logistics network in which the suppliers develop their delivery routes independently. This is often the case when no information sharing is practiced in a supply chain system. Suppose there are 11 suppliers and 5 customers, and each customer has to receive goods

from a number of suppliers. Each row represents delivery services performed at each customer. The length of each block roughly indicates the length of unloading time. The number shown in each block indicates the time mark at which the unloading can be started. For example, customer 1 (the first row) receives goods from 5 suppliers in the following sequence: S2, S5, S10, S4 and S1. Let us consider supplier S1, a delivery truck of supplier S1 arrives at customer 1 at time mark 63 min. However, supplier S4 is still unloading its goods at customer 1. Therefore, the truck has to wait until the on-going unloading activity is finished (at time mark 74.8 min) before it can start its unloading. This results in a delay of 11.8 min. That is not all for supplier S1. Prior to its arrival at customer 1, the truck has to deliver goods to customer 2. Notice that at customer 2, the truck from supplier 1 arrives at customer 2 at time mark 35 min, during which supplier S4 is still unloading its goods. Supplier S1 can start unloading its goods at this customer at time mark 44.1 min, after 9.1 min of waiting.

When the suppliers develop their delivery routes without consulting others, such arrival conflicts are quite inevitable. It is logical to assume that the more suppliers and customers there are in the logistics network, the more arrival conflicts and the longer waiting time; thus, causing inefficient delivery services. Had the suppliers shared the delivery information and planned their delivery route collaboratively, the arrival conflicts could have been minimized (if not completely avoided) and the logistics network service improved.

Through improved IT and internet technology, collaboration among suppliers and customers can be achieved. All parties in the logistics network have to change their mindset and understand that their businesses and operations can be improved if they collaborate. Solving the MS-VRP is an example to prove that collaboration works for overall network.

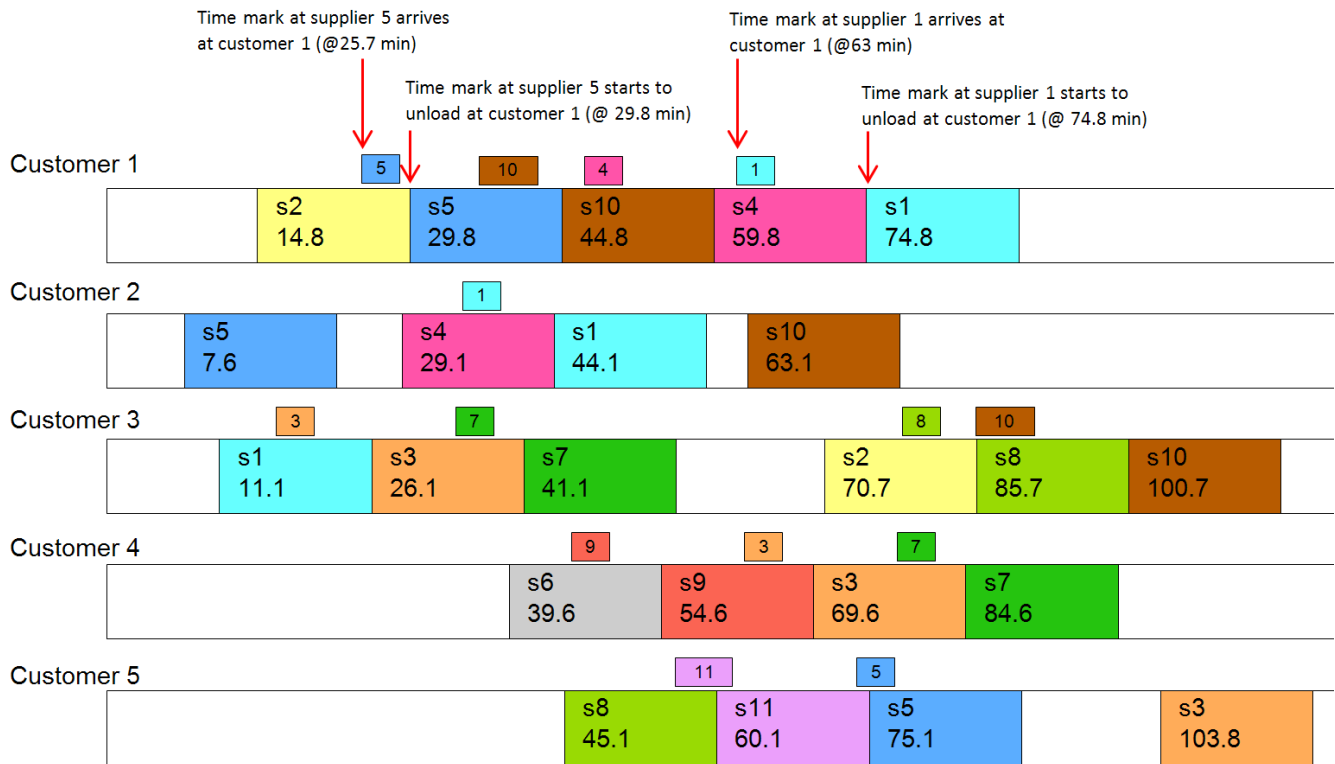


Figure 2. Delivery routes and schedules with arrival conflicts resulting in unloading delays

3. MULTIPLE-SUPPLIER VEHICLE ROUTING PROBLEM

The multiple-supplier vehicle routing problem (MS-VRP) is an extension of the vehicle routing problem involving only one supplier that is normally intended to develop optimal delivery routes for their trucks to deliver goods to only its customers. Here, the MS-VRP considers the logistics network having multiple suppliers serve a set of customers. Each customer is served by several suppliers. For each supplier, the delivery information including a list of customers to be served, customer requirements, number of delivery trucks, and truck capacities is known and shared among all suppliers in the same logistics network. The coordinated delivery routes for all involved suppliers are developed for the collaborative logistics network so as to minimize the total delivery cost that consists of the travel cost and delay cost.

In this section, a mixed integer linear programming model for solving the MS-VRP is presented. The model is initially developed based on the formulation of the VRP with time windows (Kallehauge *et al.*, 2005). This is owed to the fact that the formulation of constraints that prevent two trucks from unloading at any customer during the same time

originates from an idea of formulating the constraints of arrival time in the VRP with time windows. However, there is no time-window restriction in the MS-VRP. The trucks can arrive any customer at any time. However, if there is one supplier's truck performing unloading, another truck that has just arrived has to wait until the earlier truck finishes its unloading. The assumptions and notation used for formulating the MS-VRP are as follows.

3.1 Assumptions

The assumptions used in the formulation of the MS-VRP model are as follows.

- *Truck*: Only the weight capacity of the truck is considered. The trucks of different suppliers can have different capacities. The number of trucks utilized by each supplier can be unequal.
- *Goods*: Only the weight of goods ordered by each customer to each supplier is considered. Each customer is not required to order goods from all suppliers. Only one truck of each supplier can visit a customer. That is, the weight of one order by a customer must not exceed the capacity of any supplier's truck.
- *Supplier*: Each supplier has only one depot. Each truck of each supplier must leave its depot and return to its depot exactly once. Trucks of one supplier cannot visit the depot of any other supplier. Each truck of each supplier must make a delivery to at least one customer.
- *Customer*: A customer can receive goods from only one supplier's truck at a time. That is, one supplier's truck can unload goods at a time. Other suppliers' truck(s) have to wait until the customer is free.
- *Unloading Time*: The time durations that individual trucks spend in unloading goods at any customer are equal.
- *Travel time and cost*: The travel times that different trucks move from any location among customers and suppliers are equal for a whole day. However, each supplier can have different costs of truck drivers and delivery crews per minute.
- *Travel distance and cost*: The distances that different trucks move from any location among customers and suppliers are equal among all suppliers' trucks. However, each supplier can have different costs of travel per kilometer.

3.2 Model Variables and Parameters

C	a set of customers, where $c = 1, 2, \dots, C $
C_s^d	cost (Thai baht/km) of operating the trucks of supplier s
C_s^t	cost (Thai baht/min) of truck drivers and delivery crews of supplier s
d_{ij}	travel distance (km) of traveling directly from node i to node j
i_c	node i that represents the location of customer c (i.e., $i_c \in N_{cus}$)
j_c	node j that represents the location of customer c (i.e., $j_c \in N_{cus}$)
i_s	node i that represents the depot of supplier s (i.e., $i_s \in N_{dpo}$)
M	maximum duration (min) that any truck spends from leaving its depot and returning to its depot (Note that M is used to linearize constraints (9), (10), and (12).)
N	a set of all nodes where $N = N_{cus} \cup N_{dpo}$
N_{cus}	a set of nodes representing the locations of all customers in C
N_{dpo}	a set of nodes representing the depots of all suppliers in S
o_{sc}	= 1 if customer c has placed an order to supplier s . That is, supplier s has to deliver its order and visit customer c . = 0 otherwise.
Q_{st}	capacity limit (kg) of goods loaded in truck t of supplier s
q_{sc}	quantity (kg) of goods that customer s orders from supplier s
S	a set of suppliers, where $s = 1, 2, \dots, S $
T_s	a set of trucks owned by supplier s
t_{ij}	time duration (min) of traveling directly from node i to node j
$\bar{u}$	maximum time duration (min) of unloading goods at any customer for any supplier
θ	number of customers in C plus 1 (i.e., $\theta = C +1$)

3.3 Decision Variables

$b_{j_c s_1 s_2}$	a binary decision variable used in the linearized model of MS-VRP to remove the absolute value operator in constraint (14)
F_{st}	time duration (min) since truck t of supplier s leaves the depot of supplier s until it returns to the depot, or the finished time that this truck finishes its delivery and return to its depot
r_{ist}	time mark (min) that truck t of supplier s currently staying at node i begins to unload (note that node i could either represents a customer location or depot). This truck could arrive at this node long before the time mark r_{ist} but r_{ist} is the time mark that the truck can start to unload goods.

- y_{ijst} a binary decision variable where $y_{ijst} = 1$ if truck t of supplier s travels from node i to node j , and $y_{ijst} = 0$ otherwise.
- z_{sit} a decision variable that represents the ranking or sequence of node i that is visited by truck t of supplier s . All nodes visited by supplier s will have different value of z_{sit} . The lower the value of z_{sit} , the earlier time truck t visiting node i . This variable is used in the constraint for breaking sub-tours. z_{sit} has no constraint to force it to be integer but the feasible solution always yields an integer value for z_{sit} .

3.4 Mathematical Model

The multiple-supplier vehicle routing problem for a collaborative logistics network can be formulated as shown below.

$$\text{Minimize} \quad \sum_{i \in N} \sum_{j \in N} \sum_{s \in S} \sum_{t \in T_s} C_s^d d_{ij} y_{ijst} + \sum_{s \in S} \sum_{t \in T_s} C_s^t F_{st}, \quad \dots \quad (1)$$

subject to

$$F_{st} \geq r_{j_cst} + (\bar{u} + t_{j_c i_s}) y_{j_c i_s st}, \quad j_c \in N_{cus}, i_s \in N_{dpo}, c \in C, t \in T_s, s \in S, \quad \dots \quad (2)$$

$$\sum_{i \in N} \sum_{t \in T_s} y_{ij_cst} = o_{sc}, \quad j_c \in N_{cus}, c \in C, s \in S, \quad \dots \quad (3)$$

$$\sum_{c \in C} \sum_{j \in N} q_{cs} y_{icjst} \leq Q_{st}, \quad t \in T_s, s \in S, \quad \dots \quad (4)$$

$o_{sc}=1, i_c \in N_{cus}$

$$\sum_{j \in N_{cus}} y_{i_s jst} = 1, \quad i_s \in N_{dpo}, t \in T_s, s \in S, \quad \dots \quad (5)$$

$$\sum_{j \in N_{cus}} y_{j i_s st} = 1, \quad i_s \in N_{dpo}, t \in T_s, s \in S, \quad \dots \quad (6)$$

$$\sum_{i \in N} y_{ibst} - \sum_{i \in N} y_{bist} = 0, \quad b \in N, t \in T_s, s \in S, \quad \dots \quad (7)$$

$$\sum_{q \in N_{dpo}} \sum_{j \in N_{cus}} y_{qjst} = 0, \quad i_s \in N_{dpo}, t \in T_s, s \in S, \quad \dots \quad (8)$$

$q \neq i_s$

$$y_{ijst}(r_{ist} + \bar{u} + t_{ij} - r_{jst}) \leq 0, \quad i \neq j, j \in N, i \in N_{cus}, t \in T_s, s \in S, \quad \dots \quad (9)$$

$$y_{i_s jst}(t_{i_s j} - r_{jst}) \leq 0, \quad j \in N_{cus}, i_s \in N_{dpo}, t \in T_s, s \in S, \quad \dots \quad (10)$$

$$r_{j_cst} \leq o_{sc} \times M, \quad j_c \in N_{cus}, c \in C, t \in T_s, s \in S, \quad \dots \quad (11)$$

$$|r_{j_c s_1 t_1} - r_{j_c s_2 t_2}| \geq \bar{u}, \quad o_{s_1 c} = 1, o_{s_2 c} = 1, t_1 \in T_{s_1}, t_2 \in T_{s_2}, s_1 < s_2, \quad \dots \quad (12)$$

$s_1 \in S, s_2 \in S, j_c \in N_{cus}, c \in C,$

$$(\theta - 3)y_{jst} + (\theta - 1)y_{ijst} + z_{sit} \leq z_{s jt} + (\theta - 2), \quad i \in N_{cus}, j \in N_{cus}, t \in T_s, s \in S, \quad \dots \quad (13)$$

$$y_{ijst} = \{0,1\}, \quad i \in N, j \in N, t \in T_s, s \in S. \quad \dots \quad (14)$$

The objective of this model (1) is to minimize the total cost which is the sum of total travel cost and total time cost. The total travel cost is defined as the sum of all distances (km) traveled by all trucks multiplied by the cost of operating the trucks (C_s^d , Thai baht/km) which can be different among suppliers. The total time cost is the sum of time durations that all trucks spend in traveling, waiting, and unloading multiplied by the cost of truck drivers and delivery crews (C_s^t), which can also be different among suppliers. Constraint (2) states that the finish time of any truck of any supplier (the time mark that this truck finishes its delivery and returns to its depot) must be greater than or equal to the start time of unloading at the last customer plus the unloading time plus the travel time from the last customer to the depot of this particular supplier. Constraint (3) states that the truck of any supplier must visit its customer if that particular customer has placed an order to that supplier, and must not visit the customer if no order is placed.

Constraint (4) is the weight capacity limit forcing that the sum of quantities of all orders delivered by a truck must not exceed its capacity. In this model, trucks of different suppliers can have different capacities. Constraints (5) and (6) confirm that each supplier's truck must leave its depot and return to its depot exactly once. Constraint (7) is the conservation of flow. That is, once a truck arrives at a customer, it has to leave that customer. Constraint (8) does not allow the truck of any supplier to depart from the depot that the supplier does not own. That is, the truck must depart from its depot only, not from other supplier's depots.

Constraint (9) is a natural time constraint between two consecutive customer locations. It states that, for each supplier, considering any two consecutive customer locations in the delivery route of a truck of this supplier, the truck cannot begin to unload at the latter during the time interval starting at the time mark when the truck begins to unload at the former plus the unloading time at the former plus the required travel time between the two customer locations. Constraint (10) does not allow any truck to unload at the first customer of its delivery route after departure from its depot during the time interval that is shorter than the length of travel time from its depot to the first customer. Constraint (11) stipulates that the beginning of unloading of a supplier's truck at a particular customer is zero if that customer does not place an order from that supplier. Constraint (12) does not allow any two trucks of any two different suppliers to begin unloading at the same time or while the truck that arrives earlier still has not finished its unloading. Constraint (13) is for breaking sub-tours that might occur in each truck's delivery route using the technique called MTZ sub-tour elimination by Miller *et al.* (1960). Constraint (14) is a binary constraint for the decision variable y_{ijst} .

3.5 Linearizing Non-linear Constraints

Non-linearity in constraints (9) and (10) can be tackled by the application of a large value, namely M , such as the arrival-time constraint in the VRP with time windows (Kallehauge *et al.*, 2005). We define M as the maximum duration possible that any truck spends from leaving its depot until returning to its depot. Hence, constraints (9) and (10) can be linearized and become constraints (9*) and (10*):

$$r_{ist} + \bar{u} + t_{ij} - M(1 - y_{ijst}) \leq r_{jst}, \quad i \neq j, j \in N, i \in N_{cus}, t \in T_s, s \in S, \quad \dots \quad (9^*)$$

$$t_{isj} - M(1 - y_{isjst}) \leq r_{jst} \quad j \in N_{cus}, i_s \in N_{dpo}, t \in T_s, s \in S, \quad \dots \quad (10^*)$$

Note that the value of M is problem-specific. The larger the travel distance, the longer the travel time, the higher the number of customers in a problem, the bigger the value of M is required. To solve any MS-VRP, It is recommended that firstly a very large value of M be randomly selected. Next, the MS-VRP is solved. Given a short period of computation time, if a feasible solution is found, we can decrease M until a feasible solution takes relatively long time to be obtained.

As for constraint (12), it is non-linear because of the use of absolute value operator. Here, the technique of removing the absolute value operator illustrated by Lp_solve (2012) is used. Briefly, one constraint in constraint (12) can be linearized by replacing it with two equivalent constraints with one binary variable, say $b_{jcs_1s_2}$, and also M . They are:

$$r_{jcs_1} - r_{jcs_2} + M(b_{jcs_1s_2}) \geq u_t, \text{ and } -(r_{jcs_1} - r_{jcs_2}) + M(1 - b_{jcs_1s_2}) \geq u_t, \quad \dots \quad (12^*)$$

These two constraints successfully remove the absolute value operator. While this technique substantially doubles the number of constraints and increases the number of integer decision variables, its increasing chance of obtaining a global optimal solution is worth adding more constraints and binary variables to the model.

4. NUMERICAL EXAMPLE AND COMPUTATION EXPERIMENT COMPARING TRADITIONAL VRP WITH MS-VRP

4.1 Numerical Example

To illustrate the benefits of applying the MS-VRP as opposed to the traditional VRP and how it can be implemented, let us consider a hypothetical logistics network problem. The problem consists of 3 suppliers and 5 customers. All 5 customers are assumed to be located in the inner city with relatively heavy traffic while the 3 suppliers are located outside the city. Each customer has placed orders to all 3 suppliers ($o_{sc} = 1$ for $\forall s, c$). Each supplier has only 2 trucks which are all identical and have a weight capacity of 1,000 kg ($Q_{st} = 1,000$ for $\forall s, t$). All unloading times are assumed to be 30 min, irrespective of the suppliers and customers. The cost of operating the trucks is assumed to be 3 Thai baht/km for all trucks ($C_s^d = 3$ for $\forall s, d$). (The currency exchange rate is approximately 30 Thai baht per 1 USD.) The cost of hiring truck drivers and crews is also assumed to be 3 Thai baht/min for all suppliers ($C_s^t = 3$ for $\forall s, t$). The travel distance (d_{ij} , km) and travel time (t_{ij} , min) among customers and suppliers are shown in Tables 1 and 2,

respectively. The order quantity (q_{sc} , kg) that each customer orders from each supplier is given in Table 3. The appropriate value of M for this problem is 400 min.

Table 1. Travel distance (d_{ij} , km) among the 5 customers (C1-C5) and 3 suppliers (S1-S3)

From\To	C1	C2	C3	C4	C5	S1	S2	S3
C1	-	3.4	11.4	12.1	14.3	32.9	42.0	28.9
C2	3.4	-	11.7	15.3	15.8	32.9	43.7	26.2
C3	11.4	11.7	-	14.2	6.0	44.3	32.9	23.4
C4	12.1	15.3	14.2	-	11.3	39.3	34.0	37.1
C5	14.3	15.8	6.0	11.3	-	46.8	27.9	28.7
S1	32.9	32.9	44.3	39.3	46.8	Not Applicable		
S2	42.0	43.7	32.9	34.0	27.9			
S3	28.9	26.2	23.4	37.1	28.7			

Table 2. Travel time (t_{ij} , min) among the 5 customers (C1-C5) and 3 suppliers (S1-S3)

From\To	C1	C2	C3	C4	C5	S1	S2	S3
C1	-	15	51	66	96	56	77	57
C2	18	-	81	84	105	75	69	49
C3	72	66	-	69	36	74	46	42
C4	66	81	96	-	60	64	63	85
C5	60	93	39	57	-	103	60	62
S1	49	65	96	91	72	Not Applicable		
S2	60	74	48	46	47			
S3	66	42	44	73	45			

Table 3. Order quantity (q_{sc} , kg) that each customer orders from each supplier

Customer\Supplier	S1	S2	S3
C1	385	177	239
C2	501	590	227
C3	245	354	427
C4	485	490	519
C5	196	136	229

Table 4. Comparison of the optimal solutions obtained from the 3 traditional VRPs and MS-VRP

Comparison	Traditional VRPs	MS-VRP	Difference (+/-)
Total travel distance (km)	477.2	485.7	8.5
Total travel cost (Thai baht)	1,431.6	1,457.1	25.5
Total service time (min)	1,603	1,503	-100
Total time cost (Thai baht)	4,809	4,509	-300
Total Cost (Thai baht)	6,240.6	5,966.1	-274.5

An IBM ILOG CPLEX v.12.1.0 optimization software program is utilized to solve the MS-VRP to optimality using a personal computer with CPU speed of 2.67 GHz and 1.99 GB of RAM. Firstly, the problem is modeled as 3 separate VRPs for the 3 suppliers and then solved independently to obtain 3 sets of optimal solutions. The results thus show what will happen when no delivery information is shared among suppliers and no collaboration exists in the network. The resulting delivery routes can be called uncoordinated delivery routes. Next, it is assumed that delivery information is shared among the suppliers to find an optimal master delivery routes solution. That is, the same problem

is modeled as the MS-VRP. The optimal solutions from both viewpoints of logistics network are shown in Table 4. Note that for the traditional VRP approach, the results are the sums of the three optimal solutions (obtained from solving the 3 VRPs independently).

Readers can see that the MS-VRP is able to yield an optimal solution with a lower total cost than the sum of the 3 independent solutions. While the MS-VRP's total travel cost is a little higher than that of the traditional VRPs due to its longer travel distance, its total service time cost is however much lower resulting in a reduced total cost. When solving the traditional VRP for each supplier, the objective function is formulated such that the total travel distance is to be minimized. The service performance of each supplier is considered by neglecting possible interference from other suppliers. For a collaborative logistics network in which the network service is collectively optimized (as measured by both travel distance and service time), the 3 suppliers' service performances are concurrently considered. This could cause some supplier to sacrifice its short travel distance to enhance the entire network service.

Specifically, it can be seen that the total cost resulting from considering and solving the problem as 3 separate traditional VRPs (one VRP for one supplier) with the objective of minimizing the total travel distance only is higher than that resulting from solving the problem as the MS-VRP by 274.5 Thai baht. The total travel distance of the traditional VRPs is lower than that of the MS-VRP by 8.5 km, but the total service time of the MS-VRP is lower by 100 min. If the cost of operating the trucks (C_s^d) and the cost of drivers and delivery crews (C_s^t) in this problem are not equal, the optimal solution would then be different. If we assume that C_s^d is constant but C_s^t increases, then solving the problem as the MS-VRP will result in a more superior result than solving it as the traditional VRPs in terms of the total cost. However, if C_s^t is reduced to zero, there is no significant difference between solving the problem as the traditional VRPs or as the MS-VRP. A major benefit of solving the MS-VRP is that the decision maker can see and measure the impact of reducing the total cost if the time cost is considered in the logistics network under consideration. The trade-off between travel distance and service time truly depends on their costs.

Table 5 shows the breakdown of the optimal solutions from the traditional VRPs and the MS-VRP into delivery routes for the 6 trucks. By comparing the solutions of the traditional VRPs and the MS-VRP, it is seen that delivery routes of some trucks are different between the two solution approaches. For example, consider truck No. 2 of supplier S1, the sequence of its deliveries changes from S1→C3→C5→C4→S1 in the former approach to S1→C5→C3→C4→S1 in the latter approach, thereby increasing the travel distance by 5.4 km but reducing the time by 6 min. Note that, for truck No. 1 of supplier S2, routes (a) and (b) are totally different even though both routes have the same travel distance. That is, for route (a), the travel distance of route S2→C1→C2→C5→S2 is 42.0+3.4+15.8+27.9 = 89.1 km while the travel distance of route (b) S2→C1→C2→S2 is 42.0+3.4+43.7 = 89.1 km. However, the service time of route (b) is substantially shorter than that of route (a) by 126 min. The delivery routes of both trucks of supplier S3 are unchanged, so are their travel distances. Nevertheless, the service time does change slightly. That is, for its truck No. 1, the service time is increased by 3 min; for truck No. 2, it is reduced by 12 min.

Table 5. Comparison of delivery routes obtained from the traditional VRPs and the MS-VRP

Supplier	Truck	Delivery Route		Travel Distance (km)			Service Time (min)		
		Traditional VRPs	MS-VRP	Traditional VRPs	MS-VRP	Diff. (+/-)	Traditional VRPs	MS-VRP	Diff. (+/-)
S1	1	C2→C1	C2→C1	69.2	69.2	0	199	199	0
	2	C3→C5→C4	C5→C3→C4	100.9	106.3	5.4	343	337	-6
S2	1	C1→C2→C5 ^(a)	C1→C2 ^(b)	89.1 ^(a)	89.1 ^(b)	0	330	204	-126
	2	C3→C4	C4→C5→C3	81.1	84.2	3.1	240	281	41
S3	1	C4→C1→C2	C4→C1→C2	78.8	78.8	0	293	296	3
	2	C5→C3	C5→C3	58.1	58.1	0	198	186	-12
Total				477.2	485.7	8.5	1603	1503	-100

Note: All trucks must leave from and return to their designated depots.

To further explain the advantages of the MS-VRP, Table 6 shows a comparison of the service time components (i.e., travel time, unloading time, and waiting time) between the solutions obtained from the traditional VRPs and MS-VRP. The result reveals that the travel time is the main contributor for the 100-min decrease in total service time. Specifically, when solving the problem as the MS-VRP instead of the traditional VRPs, the total travel time is decreased by 94 min and the total waiting time by 6 min. In some trucks, their travel times decrease (e.g., truck No. 2 of supplier S1 and truck No. 1 of supplier S2). On the other hand, for truck No. 2 of supplier S2, its travel time increases. The unloading times of both trucks of supplier S2 are changed as both trucks exchange their customers. The waiting time of truck No. 2 of supplier S3 decreases while those of trucks No. 1 of suppliers S1 and S3 increase.

Note that to compute the waiting time not only for the traditional VRP but also for the MS-VRP, first the feasible or optimal solution must be obtained. Then the Gantt chart as shown in Figure 1 has to be drawn manually using the delivery routes for each truck of each supplier generated by ILOG CPLEX. The trucks that arrive at each customer

busy with another truck have to wait until the customer become free. The waiting time is the sum of all durations that each truck needs to wait for unloading at all customers whom that particular truck has to visit.

In brief, collaboration among suppliers by sharing the delivery information (i.e., solving the problem as the MS-VRP) will help to enhance the service performance by decreasing the total service time for the network. As seen in Table 6, both the total travel time and total waiting time decrease when the logistics network problem is globally not locally solved. Interestingly, the optimal solution of the MS-VRP is able to obtain the master delivery routes that result in a shorter total travel time than that of the traditional VRPs.

Figure 3 depicts delivery routes of all trucks of the 3 suppliers based on the individualized (traditional VRPs) and collaborative logistics networks.

Table 6. Comparison of time duration of all routes generated by the traditional VRPs and MS-VRP in Problem S1

Supplier	Truck	Travel Time (min)			Unloading Time (30 min/customer)			Waiting Time (min)		
		Traditional VRPs	MS-VRP	Diff. (+/-)	Traditional VRPs	MS-VRP	Diff. (+/-)	Traditional VRPs	MS-VRP	Diff. (+/-)
S1	1	139	139	0	60	60	0	0	0	0
	2	253	244	-9	90	90	0	0	3	3
S2	1	240	144	-96	90	60	-30	0	0	0
	2	180	191	11	60	90	30	0	0	0
S3	1	203	203	0	90	90	0	0	3	3
	2	126	126	0	60	60	0	12	0	-12
Total		1,141	1,047	-94	450	450	0	12	6	- 6

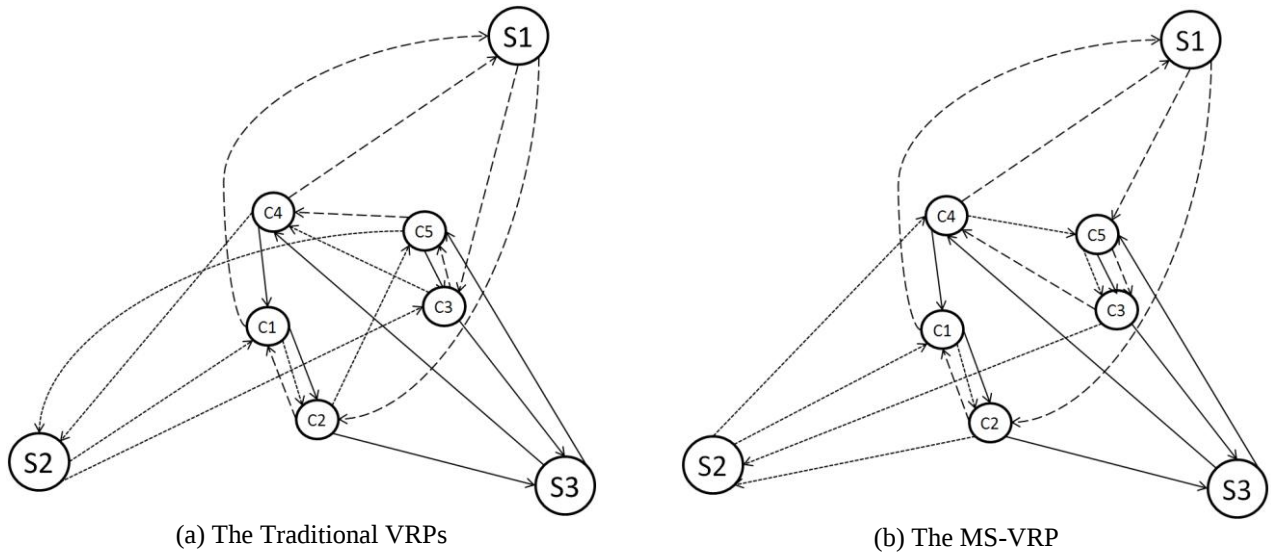


Figure 3. Comparison of delivery routes: (a) from traditional VRPs, and (b) from MS-VRP

4.2 Computation Experiment

4.2.1 Test Problems

Fifteen test problems with 3 different sizes are randomly generated for the computation experiment. All 15 problems have 5 customers but differ in the number of suppliers. The numbers of suppliers are 3, 4, and 5 suppliers for test problems S1 – S5, M1 – M5, and L1 – L5, respectively. Locations of all customers and suppliers are also randomly generated. The travel distance is calculated by $1.5 \times$ the diagonal distance between two locations. The travel times between suppliers and customers are generated by assuming that the truck's travel speed follows a uniform distribution with the minimum and maximum speeds of 25-45 km/h. Similarly, the travel times among customers are generated by assuming that the truck speed is uniformly distributed between 8-15 km/h, as the traffic is more congested in the city area.

Similar to the numerical example in Section 4, it is assumed that each customer places orders to all suppliers ($o_{sc} = 1$ for $\forall s, c$). Each supplier has only 2 trucks which are all identical and have a weight capacity of 1,000 kg ($Q_{st} = 1,000$ for $\forall s, t$). The unloading times are fixed at 30 min for all deliveries. The cost of operating the trucks is 3 Thai baht/km ($C_s^d = 3$ for $\forall s, d$). The cost of drivers and delivery crews is also 3 Thai baht/min ($C_s^t = 3$ for $\forall s, t$). The order quantity q_{sc} is randomly distributed between 100-400 kg. The initial feasible value of M for each test problem is identified prior to the experiments.

4.2.2 Results

The same personal computer used in solving the numerical example in Section 4 is also utilized in the computation experiment. All 15 test problems are formulated as the traditional VRPs and the MS-VRP and solved to optimality by the IBM ILOG CPLEX v.12.1.0 optimization software program. The results are summarized in Table 7.

Note that LB is the best lower bound value (Thai baht); UB is the best upper bound or the optimal solution value (Thai baht); % *Gap* is the gap percentage = $(UB-LB) \times 100/UB$; *Termination Time* is the computation time terminated because (1) the optimal solution is found (*Opt*), (2) ILOG CPLEX runs out of memory in ILOG (*Out*), or (3) ILOG CPLEX exceeds the time limit of 2,000 s (*Ter*); *UB of Traditional VRP* is the upper bound value of solving the test problem as the traditional VRPs, i.e., only minimizing the total travel distance; % *Difference* is the difference percentage between the upper bound value from solving the test problem as the MS-VRP and that from solving the test problem as the traditional VRPs (i.e., % *Difference* is the reduction percentage of total cost from solving as the MS-VRP instead of solving as the traditional VRPs).

Table 7. Summary of results from the 15 test problems

Test Problem	S1	S2	S3	S4	S5	M1	M2	M3	M4	M5	L1	L2	L3	L4	L5
Suppliers	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5
M	400	800	500	400	400	500	500	500	500	700	400	400	400	400	500
Number of Constraints	555	555	555	555	555	820	820	820	820	820	1125	1125	1125	1125	1125
No. of Binary Variables	390	390	390	390	390	640	640	640	640	640	950	950	950	950	950
LB	5966	11066	3910	7119	6854	8171	4496	4310	6063	12437	6158	8881	8874	7447	9047
UB	5966	11066	3910	7119	6854	9662	5750	6237	8804	12437	11326	12834	13018	11228	12653
% Gap	0.0	0.0	0.0	0.0	0.0	15.4	21.8	30.9	31.1	0.0	45.6	30.8	31.8	33.7	28.5
Termination Time (s)	197	293	859	1387	1796	2000	2000	2000	2000	1049	2000	2000	1269	2000	2000
Cause of Termination	<i>Opt</i>	<i>Opt</i>	<i>Opt</i>	<i>Opt</i>	<i>Opt</i>	<i>Ter</i>	<i>Ter</i>	<i>Ter</i>	<i>Ter</i>	<i>Opt</i>	<i>Ter</i>	<i>Ter</i>	<i>Out</i>	<i>Ter</i>	<i>Ter</i>
UB of Traditional VRP	6241	11389	4058	7269	6912					12827					
% Difference	4.4	2.8	3.6	2.1	0.9					3.0					

From the 15 test problems, ILOG CPLEX could guarantee the optimality within the time limit of 2,000 s for only 6 problems (S1-S5 and M5). Eight problems are terminated after exceeding the time limit (M1-M4, L1, L2, L4, and L5). Only one test problem, L3, is terminated because ILOG CPLEX runs out of its memory. The number of constraints increases from 555 to 1,125 constraints for 3 different sizes of test problems. The number of binary decision variables for the 3 problem sizes are 390, 640, and 950, respectively. For the small-sized test problems (S1-S5), the optimality could be guaranteed within the time limit. If these 5 test problems are solved as the traditional VRPs, the total cost (which is sum of the travel cost and the time cost) would increase from the total cost of the MS-VRP by 0.9 up to 4.4%. Again, the range of % *Difference* would be larger if C_s^t increases. For 9 test problems in medium and large sizes, the values of *UB of Traditional VRP* and % *Difference* are not shown because the optimal solution value, UB , are not known due to either exceeding the time limit (*Ter*) or running out of memory (*Out*). If shown, the values will not reflect the actual efficiency of the MS-VRP versus the traditional VRP obtained from the computational experiments.

As the problem size increases, ILOG CPLEX finds it more difficult to obtain the optimal solution. In fact, there is only one test problem (M5) in the medium-sized and large-sized problems that the optimal solution can be guaranteed. The gap between the best known upper bound value and the lower bound value reported by ILOG CPLEX for test problems M1-M4 ranges from 15.4% to 31.1%. This gap could be as high as 45.6% for large-sized problems. This truly reflects the complexity of the MS-VRP. The traditional VRP has been known to be *NP*-hard and is difficult to solve when the problem size is large. The MS-VRP thus is even more difficult to solve. The problem of minimizing the travel time of one supplier's truck has some similarity to the bin packing problem, which is also a classical *NP*-hard

problem. Hence, to minimize the sum of total times of all trucks in the collaborative logistics network can be as difficult as solving many bin packing problems at the same time.

As witnessed in our experiment, ILOG CPLEX fails to solve the test problems of 5 customers and 5 suppliers. In reality, a logistics network could have many more customers and suppliers. The more efficient way to solve the MS-VRP is the implementation of quick and greedy heuristics together with the evolutionary approach such as genetic algorithm as discussed in Lai *et al.* (2012).

5. THE PROPOSED HEURISTICS

The proposed heuristics is based upon the random neighborhood search technique. The whole process consists of 2 steps. First, the initial solution is created by the initialization step. For the 2nd step which is the improvement step, it randomly moves one customer from one truck to another truck as shown in Figure 4. The improvement step is repeated until a number of times and then the initialization step is repeated. The whole process (Step 1 + Step2) is also repeated until a number of times. The longer time the process continues, the better solution can be achieved. The heuristics was coded in Visual Basic Application in Microsoft Excel. Based on the computational experiment with previous 15 test problems, the heuristics can yield the lower-cost solution of many test problems in the relatively shorter computational time when compared with the optimization method of ILOG CPLEX.

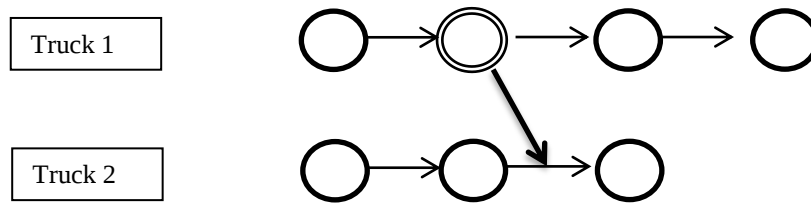


Figure 4. For the improvement step of the proposed heuristics, the current solution is improved by randomly choosing one customer from the first truck and move it to the second truck by inserting between any position of the second truck's customer sequence.

6. CONCLUSION

In this paper, a multiple-supplier vehicle routing problem (MS-VRP) is proposed. As the logistics network is considered to be collaborative (i.e., delivery information is shared among suppliers), the delivery routes of all suppliers' trucks are simultaneously generated instead of being generated independently for individual suppliers. In any typical logistics network in which each supplier operates on one's own, the trucks might have to wait idly and queue up at the customer location due to limited unloading space/facility. The increased travel time and waiting time can incur significant amount of cost which is usually ignored in the traditional VRP. Hence, the objective of the MS-VRP is to minimize the sum of total travel cost and service time cost.

The mathematical model of the MS-VRP can be linearized by using a large-value constant variable, namely M , and by replacing one constraint with absolute value operator by two equivalent constraints. From the given numerical example, it is seen that for the collaborative logistics network, the MS-VRP considers all delivery information simultaneously and generates the coordinated delivery routes for all trucks of all suppliers in order to optimize the service performance of the entire logistics network. Some of the suppliers might have to sacrifice their optimal performance for the sake of the network performance. Nevertheless, it is expected that the increased delivery service quality of the entire network will lead to the overall supplier-customer satisfaction.

From the computation experiment involving 15 test problems, ILOG CPLEX is able to solve only 6 out of 15 problems successfully to obtain their optimal solutions. For the rest of the test problems, ILOG CPLEX is terminated either because the given time limit is reached or it runs out of memory. This result clearly reflects the complexity of the MS-VRP. It also stresses the need for a more efficient solution procedure for solving the MS-VRP.

The heuristics based on the random search technique is developed. The heuristics consists of 2 steps: initialization and improvement. Based on the computational experiment, the heuristics can yield the lower-cost solution in the relatively shorter computational time when compared with ILOG CPLEX.

7. INDUSTRY APPLICATION

For intra-city goods delivery, suppliers are normally faced with more problems than when hauling large loads of goods to deliver to large, modern warehouses or distribution centers. To name a few, truck size has to be small to negotiate narrow city streets. The suppliers have to be responsible for unloading goods at the customer locations since customers are likely to be retail stores, grocery stores, and restaurants. Since mechanized materials handling equipment such as industrial trucks are usually not available, goods must be unloaded and moved manually. More importantly, due to having small premises, the customers usually can accommodate only one supplier at a time for unloading. While waiting for their turn to unload, the other supplier's truck is either parked idly or must be driven around due to a lack of parking spaces. Such uncoordinated delivery arrivals are likely to occur when the suppliers generate their delivery routes without knowledge of other suppliers' delivery information. This conflict is owed to the fact that suppliers do not share their delivery information among others even when they operate within the same city. Such independent operations can negatively affect the delivery service efficiency of the entire logistics network and lead to unsatisfied suppliers and customers.

The MS-VRP discussed in this paper is a result of considering that the deliveries of goods should be collaborative instead of being individualized. When all delivery information is shared, an optimal master delivery schedule plan can be generated by solving one integrated mathematical model (MS-VRP) not several independent models (traditional VRPs). Such master plan would definitely enhance the quality of providing delivery service for the entire logistics network. The proposed heuristics is also applicable and faster to solve MS-VRP in real situation than the traditional optimization technique as in ILOG CPLEX.

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Output จากโครงการวิจัยที่ได้รับทุนจาก สกว.

1. ผลงานตีพิมพ์ในวารสารวิชาการนานาชาติ (ระบุชื่อผู้แต่ง ชื่อเรื่อง ชื่อวารสาร ปี เล่มที่ เลขที่ และหน้า) หรือผลงานตามที่คาดไว้ในสัญญาโครงการ

ถึงตอนนี้ยังไม่มีผลงานตีพิมพ์ใดๆ แต่ผู้วิจัยได้เคยส่งบทความไปที่ International Journal of Industrial Engineering: Theory, Applications and Practice (IJETAP) แต่ได้ถูกปฏิเสธกลับมา ตอนนี้ผู้วิจัยอยู่ในระหว่างการแก้ไขบทความเพื่อส่งไปในวารสารวิชาการนานาชาติฉบับอื่นต่อไป

2. การนำผลงานวิจัยไปใช้ประโยชน์

ถึงตอนนี้ยังไม่มีผลการนำผลงานวิจัยไปใช้ประโยชน์ แต่ผู้วิจัยตั้งใจจะนำความรู้ที่ได้จากโครงการนี้ มาใช้เป็นส่วนหนึ่งของการเรียนการสอนของผู้วิจัยในอนาคต และยังตั้งใจจะนำวิธีฮิวริสติกส์ที่พัฒนาได้มาทำเป็นโปรแกรมคอมพิวเตอร์หรือเว็บไซต์ที่สามารถแก้ปัญหาการกำหนดเส้นทางเดินรถที่เหมาะสม (VRP) เพื่อให้ทางภาครัฐกิจและภาคอุตสาหกรรมสามารถนำไปใช้งานได้สะดวก

3. อื่น ๆ (เช่น ผลงานตีพิมพ์ในวารสารวิชาการในประเทศ การเสนอผลงานในที่ประชุมวิชาการ หนังสือ การจดสิทธิบัตร)

ผู้วิจัยได้นำผลการวิจัยบางส่วน of โครงการไปนำเสนอในการประชุมวิชาการนานาชาติ the 7th International Congress on Logistics and SCM Systems (ICLS2012) ในระหว่างวันที่ 9-7 มิถุนายน 2555 ความในภาคผนวก ก รุ้งโซล ประเทศเกาหลีใต้ โดยได้แนบบท

ภาคผนวก

On Mathematical Models of Multiple-Supplier Vehicle Routing Problems

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Abstract

This research work aims to study a new vehicle routing problem when there is a logistics network consisting of many suppliers and many customers, namely the multiple-supplier vehicle routing problem (msVRP). A supplier has to deliver goods to many customers. A customer also receives goods from many suppliers and can handle one supplier at a time. The objective of msVRP is to produce the master delivery schedule for all suppliers and customers that minimize the total cost of all parties in the network, not just a single supplier as in the regular VRP.

This paper proposes two different mathematical models of msVRP. Both models can be categorized as the integer linear programming. One test problem based on real geographical data is used in computational experiments. The results of computational experiments are presented. The advantages and disadvantages of both models are discussed.

Keywords: Vehicle Routing Problem, Mathematical Model, Integer Linear Programming

1. Introduction

The basic model of the Vehicle Routing Problem (VRP) is to find the appropriate routes of a group of delivery trucks that are required to deliver goods from a depot of a supplier to a group of customers located at different locations, where each customer needs different amount of goods [1]. VRP is one of the most frequently found problems in the delivery activity, and one of the most researched problems due to its various applications. The examples of single objective of VRP are to minimize the total delivery cost, the number of trucks used, or the total distance. Some situations require multiple objectives [2,3]. The real constraints of VRP make the problem even more complex. For example, delivery trucks are of different capacity [1]; goods may have many different types with different stock keeping units (SKU); customer may requires goods to be

delivered in a certain duration, so-called time-window constraint [7,9].

Most research works on VRP consider suppliers or people who deliver goods more important than customers who receive goods [4,5,6]. Many times, customers will need to have other goods delivered by other suppliers, and if each supplier plans its delivery without considering the customers' availability for delivery, it may happen that many suppliers arrive a customer at the same time causing some suppliers need to wait until the customer is free. In fact, all suppliers and customers in the delivery network should share the delivery information and plan the master delivery schedule together so that all parties can have the more efficient delivery system. To our knowledge [8,10,11,12,13], there is no study about the VRP that considers multiple suppliers and customers with the aim of finding the appropriate delivery schedule for all parties.

This research work aims to study a new VRP namely the Multiple-Supplier VRP (msVRP) when there is a logistics network consisting of many suppliers and many customers. Section 2 describes the two potential mathematical models for solving msVRP. Both models are still in the process of development. They have different objectives: the total finished time vs. the total distance. In the future, the authors wish to develop the model with the objective function and constraints that are most suitable for msVRP. A test problem is created in order to test both models. Section 3 describes the details of the test problem. Section 4 shows the computational results and discusses the pros and cons of both models.

2. Mathematical Models

In this section, two mathematical models in progress for solving msVRP are presented in details. The first model is based upon the nature of the assignment problem. Three dimensions of the assignment are customer, supplier, and period. The objective of the assignment model is to minimize the total finished time of all suppliers' trucks in the unit of time periods where 1 period equals 10

minutes. In contrast, the second model is a slight adaptation of the standard form of vehicle routing problem with time windows. This time-window model aims to minimize the total distance travelled by all suppliers' trucks in the unit of kilometers.

Both models differ in their objective functions and associated constraints. But they equally provide possible or feasible solutions for msVRP. In the future, the authors aim to develop the most suitable mathematical model for msVRP. The idea behind both developing models could lead the authors to discover the final version for msVRP. Next, the notations and equations for both models are described.

2.1 The Assignment Model

Notations of the Assignment Model for msVRP are described as follows.

C is a set of customers, where $c = 1, 2, \dots, |C|$.

S is a set of suppliers, where $s = 1, 2, \dots, |S|$.

P is a set of periods, $p = 1, 2, \dots, |P|$. For the test problem, one period equals to 10 minutes.

d_{ij} is the driving distance (unit of kilometer) of traveling directly from the location of Customer i or the depot of Supplier i , to the location of Customer j or the depot of Supplier j .

t_{ij} is the time of traveling (unit of period) directly from the location of Customer i or the depot of Supplier i , to the location of Customer j or the depot of Supplier j , where $t_{ij} = \left\lceil \frac{60d_{ij}}{45 \times 10} \right\rceil + 1$ for the test problem.

u_a is the additional time duration (unit of period) of unloading products at any customer for any supplier. For the test problem, let $u_a = 1$.

o_{sc} is equal to 1 if Customer c has placed an order to Supplier s . That is, Supplier s has to deliver its order and visit Customer c , 0 otherwise.

F_s is the decision variable: the time duration that the truck of Supplier s returns to its depot.

x_{pcs} is the decision variable: $x_{pcs} = 1$ if the truck of Supplier s visits and begin to unload for Customer c at Period p , 0 otherwise.

The assignment model for msVRP is as follows.

$$\text{Minimize} \quad \sum_{s \in S} F_s \quad (a1)$$

subject to

$$F_s \geq px_{pcs} + u_a + t_{cs}, \quad p \in P, c \in C, s \in S, (a2)$$

$$\sum_{p \in P} x_{pcs} = o_{sc}, \quad c \in C, s \in S, \quad (a3)$$

$$\sum_{s \in S} x_{pcs} \leq 1, \quad c \in C, p \in P, \quad (a4)$$

$$\sum_{c \in C} x_{pcs} \leq 1, \quad s \in S, p \in P, \quad (a5)$$

$$1 - x_{p_1 c_1 s} \geq x_{p_2 c_2 s}, \quad \begin{aligned} &p_1 < p_2 \leq p_1 + u_a + t_{c_1 c_2}, \\ &p_2 \in P, p_1 \in P, \\ &o_{sc_1} = 1, o_{sc_2} = 1, c_1 \neq c_2, \\ &c_1 \in C, c_2 \in C, s \in S, \end{aligned} \quad (a6)$$

$$x_{pcs} = 0, \quad p \leq t_{sc}, p \in P, c \in C, s \in S, (a7)$$

$$x_{pcs} \in \{0,1\}, \quad p \in P, c \in C, s \in S, (a8)$$

The objective of this model (Equation a1) is to minimize the total sum of the time that each truck returns to its depot, or finish its delivery. This objective value is integer since the finished time of each supplier, F_s , is also integer. Constraint a2 states that the finished time period of each supplier must be greater than or equal to the begin-loading time period of the last customer including the unloading time and the traveling time from the last customer to the depot of that particular supplier.

Constraint a3 confirms that the truck of any supplier must visit its customer if that particular customer has placed an order to that supplier. Constraint a4 and a5 are the one-to-one assignment condition, i.e., at each period, one customer can handle not more than one supplier's truck, and one supplier's truck can visit not more than one customer.

Constraint a6 stipulates that, for each supplier, considering the two consecutive customers in the delivery route of this supplier's truck, the truck cannot begin to unload the 2nd customer in the period that the truck still park at the 1st customer for unloading, and in the period that the truck has still travelled from the 1st customer but still has not arrived the 2nd customer. Constraint a7 simply does not allow any truck to unload at the 1st customer after its departure from its depot at the period that is earlier than the time period required to travel from its depot to the 1st customer.

2.2 The Time-Window Model

The additional and new variables are required for this model.

Q is the number of customers in C where $Q = |C|$.

S is a set of suppliers, where $s = 1, 2, \dots, |S|$.

M is the maximum period or $M = |P|$. For the test problem, $M = |P| = 30$.

N is a set of all nodes where $N = N_{cus} \cup N_{dpo}$. Let

N_{cus} be a set of nodes representing the locations of all customers in C , and N_{dpo} be a set of nodes representing the depots of all suppliers in S .

i_s is Node i that represents the depot of Supplier s .

That is, $i_s \in N_{dpo}$.

j_c is Node j that represents the location of Customer c . That is, $j_c \in N_{cus}$.

d_{ij} is the driving distance (unit of kilometer) of traveling directly from Node i to Node j .

t_{ij} is the time duration of traveling (unit of period) directly from Node i to Node j , where $t_{ij} = \left\lceil \frac{60d_{ij}}{45 \times 10} \right\rceil + 1$ for the test problem.

u_t is the total time duration (unit of period) of unloading products at any customer for any supplier. For the test problem, let $u_t = 2$. Because, in the assignment model, the unloading time of 1 period (10 minutes) is already included in the period that $x_{pcs} = 1$, unlike the time-window model. This makes $u_t = u_a + 1$.

r_{is} is the point of time (unit of period) that the truck of Supplier s arrives at Node i and begin to unload (note that Node i could either present a customer's location or a supplier's depot).

y_{ijs} is the decision variable: $y_{ijs} = 1$ if the truck of Supplier s travels from Node i to Node j , 0 otherwise.

z_{si} is the decision variable: the integer value that can represent the ranking or sequence of Node i that is visited by the truck of Supplier s . All nodes visited by Supplier s will have different value of z_{si} . The lower z_{si} means the earlier the truck visits Node i . This variable is used in the constraint of breaking sub tours.

The time-window model for msVRP is as follows.

$$\text{Minimize} \quad \sum_{i \in N} \sum_{j \in N} \sum_{s \in S} d_{ij} y_{ijs} \quad (t1)$$

subject to

$$\sum_{i \in N, i \neq j_c} y_{ijcs} = o_{sc}, \quad j_c \in N_{cus}, c \in C, s \in S, \quad (t2)$$

$$\sum_{j \in N_{cus}} y_{ijs} = 1, \quad i_s \in N_{dpo}, s \in S, \quad (t3)$$

$$\sum_{i \in N} y_{ibs} - \sum_{i \in N} y_{bis} = 0, \quad b \in N, s \in S, \quad (t4)$$

$$\sum_{j \in N_{cus}} y_{jis} = 1, \quad i_s \in N_{dpo}, s \in S, \quad (t5)$$

$$\sum_{q \in N_{dpo}, q \neq i_s} \sum_{j \in N_{cus}} y_{qjs} = 0, \quad i_s \in N_{dpo}, s \in S, \quad (t6)$$

$$y_{ijs}(r_{is} + u_t + t_{ij} - r_{js}) \leq 0, \quad i \neq j, j \in N, i \in N_{cus}, s \in S, \quad (t7)$$

$$y_{ijs}(t_{ijs} - r_{js}) \leq 0, \quad j \in N_{cus}, i_s \in N_{dpo}, s \in S, \quad (t8)$$

$$r_{jcs} \leq o_{sc} \times M, \quad j_c \in N_{cus}, c \in C, s \in S, \quad (t9)$$

$$\begin{aligned} |r_{jcs_1} - r_{jcs_2}| &\geq u_t, \quad o_{s_1c} = 1, o_{s_2c} = 1, \\ s_1 < s_2, s_1 &\in S, s_2 \in S, \\ j_c &\in N_{cus}, c \in C, \end{aligned} \quad (t10)$$

$$(Q-3)y_{jis} + (Q-1)y_{ijs} + z_{si} \leq z_{sj} + (Q-2), \quad i \in N_{cus}, j \in N_{cus}, s \in S, \quad (t11)$$

$$y_{ijs} = \{0,1\}, \quad i \in N, j \in N, s \in S, \quad (t12)$$

$$z_{si} \in \text{Integer}, \quad i \in N, s \in S, \quad (t13)$$

The objective of this model (Equation t1) is to minimize the total sum of the distance that each truck travels to deliver its products to all of its customers, not the finished time period. However, the finished time of each supplier, F_s , of any feasible/optimal solution can be calculated manually. Constraint t2 states that the truck of any supplier must visit its customer if that particular customer has placed an order to that supplier, and must not visit if no order is placed. Constraint t3 and t5 confirms that each supplier's truck must leave its depot and return to its depot exactly once. Constraint t4 is of the standard condition for balancing the in and out degree. Constraint t6 does not allow the truck of any supplier to depart from the depot that the supplier does not own, i.e., one must depart from its depot only, not other supplier's depots.

Constraint t7 is similar to Constraint a6 of the assignment model, which states that, for each supplier, considering the two consecutive customers in the delivery route of this supplier's truck, the truck cannot begin to unload the 2nd customer in the period that the truck still park at the 1st customer for unloading, and in the period that the truck has still travelled from the 1st customer but still has not arrived the 2nd customer. Note that Constraint t7 can be linearized and replaced by $r_{is} + u_t + t_{ij} - M(1 - y_{ijs}) \leq r_{js}$. Constraint t8 is similar to Constraint a7 of the assignment model, which does not allow any truck to unload at the 1st customer after its departure from its depot at the period that is earlier than the time period required to travel from its depot to the

1st customer. Also Constraint $t8$ can be linearized and becomes $t_{i_s j} - M(1 - y_{i_s j s}) \leq r_{j s}$.

Constraint $t9$ stipulates that the begin-unloading time of a supplier's truck at a particular customer is zero if that customer does not place an order of that supplier. Constraint $t10$ is unique for only the time-window model. It does not allow the two trucks of two suppliers to begin to unload at the same time or while the truck that arrives earlier still does not finish unloading. The non-linearity in Constraint $t10$ due to the absolute difference can become linearized by replacing with the two following constraints and a binary variable $b_{j_c s_1 s_2}$, i.e.,

$$\begin{aligned} r_{j_c s_1} - r_{j_c s_2} + M(b_{j_c s_1 s_2}) &\geq u_t, \text{ and} \\ -(r_{j_c s_1} - r_{j_c s_2}) + M(1 - b_{j_c s_1 s_2}) &\geq u_t. \end{aligned}$$

Constraint $t11$ is for breaking sub-tours that might occur in each truck's delivery route.

3. The Test Problem

One test problem is generated to test the proposed mathematical models. The specifications of this test problem are as follows.

- There are 11 suppliers and 6 customers. These become 17 nodes for the time-window model.
- The locations of each supplier and customer are randomly selected from Bangkok and nearby areas with the real latitudes and longitudes.
- The approximate distance in kilometer between among all suppliers and customers are obtained from Google map which provide road directions and the driving distance.
- Traveling time is calculated under the assumption that all trucks travel at the equal and constant speed of 45 km/hour. Travel time in minutes is also rounded up to the next biggest integer travel time periods where 1 period equals to 10 minutes.
- It is assumed that the truck has unlimited capacity.
- It is assumed that one supplier has only one truck and one type of product.
- $M = 30$, i.e., the maximum time period to be considered in the test problem is 300 periods, i.e., 10 minutes for 1 period.
- The unloading time for all customers and suppliers is equal to 20 minutes, or equal to 2 periods. u_t is the total time duration (unit of period) of unloading products for the time-window model. u_a is the additional time of unloading products since the unloading time of 1 period is already

considered.

- One supplier has the order from 3 up to 6 customers.

4. Computational Results and Discussions

Both proposed models were programmed in an integer linear programming solver namely LINGO. The computational results of the test problem formulated as both msVRP models are displayed in Table 1. In term of the total distance of all suppliers' trucks (which is the objective of the time-window model), the assignment model is inferior to the time-window model by $= 749.8 - 697.5 = 52.3$ km. However, the assignment model is superior to the time-window model in term of total finished time of all suppliers' trucks by $302 - 223 = 79$ periods (or 790 minutes). The total waiting time of the assignment model is also much less than that of the time-window model (48 periods versus 107 periods). Waiting time is the time that each supplier's truck has to wait before it can unload goods. A waiting truck still cannot begin to unload if the customer that the truck just arrives is still occupied by another truck that arrives before the waiting truck. That is, waiting time is the begin-to-unload time minus the arrival time. The finished time of each supplier's truck (F) is also given in Table 1.

In term of computer run time, it is found that the time-window model could quickly yield its optimal solution in 6 minutes, i.e., the total distance of 697.5 km is the minimal. However, the assignment model had a great difficulty proving the optimality. The authors decided to terminate LINGO at 1,088 minutes yielding a feasible solution of the total finished time of 223 periods. Surprisingly, the number of integer decision variables of the time-window model is much larger than that of the assignment model but the assignment model took much longer time to solve for this test problem. The number of constraints in the assignment model is also much larger than that of the time-window model, but it was harder to solve the assignment model than the time-window model.

One great difficulty in formulating the mathematical model for msVRP is to include the waiting time into the model. To the author's knowledge, there is still no possible way to incorporate any variable representing the waiting time of a supplier's truck when visiting any customer into the model. Further additional techniques in constructing equations and variables are very much needed. However, the authors are in the process of combining the two different objective functions in both developing models into one objective function. The possible idea is to

convert both time period and traveled distance into the same unit, say a cost unit such as in dollars.

Table 1 Comparisons on the two proposed models

Results from LINGO	Proposed Model for msVRP	
	Assignment	Time-Window
Total distance (t_1)	749.8 km	697.5 km
Total finished time (α_1)	223 periods	302 periods
Total waiting time (periods)	48	107
Computer Run time (minutes)	1,088 (non-optimal*)	6 (optimal)
Finished time of each supplier's truck (periods)	F(1) = 27 F(2) = 12 F(3) = 26 F(4) = 16 F(5) = 15 F(6) = 24 F(7) = 18 F(8) = 18 F(9) = 18 F(10) = 33 F(11) = 16	F(1) = 27 F(2) = 28 F(3) = 30 F(4) = 18 F(5) = 26 F(6) = 30 F(7) = 29 F(8) = 29 F(9) = 24 F(10) = 35 F(11) = 26
Number of integer decision variables	1,783	5,236
Number of constraints	14,006	2,291

* LINGO was terminated before the optimal solution was obtained.

5. Conclusions

Two possible mathematical models for solving msVRP are presented; they are the assignment model and the time-window model. Both models are in the process of development, and it is hoped that the final model for msVRP can be constructed. One test problem is used to test both models. The assignment model outperforms the time-window model in term of the total finished time and the total waiting time but its weaknesses are the total distance and the computer run time.

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