Abstract

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Project Title:
Interior fixed points of map extensions

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Abstract: Schirmer proved that there is a class of smooth self-maps of the unit sphere in Euclidean n-space with the property that any smooth self-map of the unit ball that extends a map of that class must have at least one fixed point in the interior of the ball. We generalize Schirmer's result by proving that a smooth self-map of Euclidean n-space that extends a self-map of the unit sphere of that class must have at least one fixed point in the interior of the unit ball.

Let X be a compact smooth n-manifold, with or without boundary, and let A be an $(n-1)$-dimensional smooth submanifold of the interior of X. Let $\phi\colon A \to A$ be a smooth map and $f\colon (X, A) \to (X, A)$ be a smooth map whose restriction to A is ϕ. If $p \in A$ is an isolated fixed point of f that is a transversal fixed point of ϕ, that is, the linear transformation $d\phi_p - I_A\colon T_pA \to T_pA$ is nonsingular, then the fixed point index of f at p satisfies the inequality $|i(X, f, p)| \leq 1$. It follows that if ϕ has k fixed points, all transverse, and the Lefschetz number $L(f) > k$, then there is at least one fixed point of f in $X\backslash A$. Examples demonstrate that these results do not hold if the maps are not smooth.

Mathematics Subject Classification: 55M20.

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