รายงานวิจัยฉบับสมบูรณ์

โครงการ แผนภูมิควบคุมค่าเฉลี่ยถ่วงน้ำหนักเอ็กซ์โปเนนเชียล สำหรับกระบวนการผลิตที่มีการแจกแจงแบบ ปัวส์ซองวางนัยทั่วไป

โดย เนรัญชรา เกตุมี

เมษายน 2561

สัญญาเลขที่ MRG5980104

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เนรัญชรา เกตุมี ม.เทคโนโลยีราชมงคลล้านนา ตาก

สนับสนุนโดยสำนักงานกองทุนสนับสนุนการวิจัยและ
ม.เทคโนโลยีราชมงคลล้านนา ตาก

บทคัดย่อ

รหัสโครงการ : MRG5980104

ชื่อโครงการ : แผนภูมิควบคุมค่าเฉลี่ยถ่วงน้ำหนักเอ็กซ์โปเนนเชียลสำหรับกระบวนการผลิตที่มีการแจกแจงแบบปัวส์ซองวางนัยทั่วไป

ชื่อนักวิจัย : นางเนรัญชรา เกตุมี ม.เทคโนโลยีราชมงคลล้านนา ตาก

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ระยะเวลาโครงการ : 2 ปี

บทคัดย่อ: งานวิจัยนี้มีวัตถุประสงค์เพื่อสร้างแผนภูมิควบคุมค่าเฉลี่ยถ่วงน้ำหนักเอ็กซ์โปเนนเชียล (*EWMA-chart*) สำหรับกระบวนการผลิตที่มีการแจกแจงแบบปัวส์ซองวางนัยทั่วไป (*GPD*) และกระบวนการผลิตที่มีการแจกแจงแบบปัวส์ซองวางนัยทั่วไปที่มีศูนย์มาก(*ZIGPD*) ภายใต้อัตราส่วนล็อกไลค์ลิฮูด (log-likelihood ratio) โดยประสิทธิภาพของแผนภูมิควบคุมจะพิจารณาที่ค่าความยาววิ่งเฉลี่ย (*ARL*) การเปรียบเทียบประสิทธิภาพของแผนภูมิควบคุมที่สร้างขึ้นใหม่กับแผนภูมิคุมจำนวนรอยตำหนิต่อหน่วย *(c-chart)*

คำหลัก: แผนภูมิควบคุมค่าเฉลี่ยถ่วงน้ำหนักเอ็กซ์โปเนนเชียล การแจกแจงแบบปัวส์ซองวางนัยทั่วไป การแจกแจงแบบปัวส์ซองวางนัยทั่วไปที่มีศูนย์มาก อัตราส่วนล็อกไลค์ลิฮูด ค่าความยาววิ่งเฉลี่ย

Abstract

Project Code : MRG5980104

Project Title : The EWMA charts for the Generalized Poisson Process

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Project Period : Two years

Abstract : The purpose of this project is to construct new exponentially-weighted moving average (*EWMA*) chart for the generalized Poisson distribution (*GPD*) and Zero-Inflated Generalized Poisson (*ZIGP*) distribution are based on a log-likelihood ratio. The performance of the control charts are considered from the Average Run Length (*ARL*). The comparison of the new control charts with *c-chart*

Keywords : exponentially-weighted moving average, generalized Poisson distribution, Zero-Inflated Generalized Poisson, log-likelihood ratio, Average Run Length

วัตถุประสงค์ : สร้างแผนภูมิควบคุมค่าเฉลี่ยถ่วงน้ำหนักเอ็กซ์โปเนนเชียลขึ้นใหม่ โดยสร้างภายใต้อัตราส่วนล็อกไลค์ลิฮูด (log-likelihood ratio) เพื่อหาแผนภูมิควบคุมที่มีประสิทธิภาพในการตรวจจับการเปลี่ยนแปลงในกระบวนการผลิตที่มีการแจกแจงแบบปัวส์ซองวางนัยทั่วไป และกระบวนการผลิตที่มีการแจกแจงแบบปัวส์ซองวางนัยทั่วไปที่มีศูนย์มาก เพื่อการตีพิมพ์ในวารสารระดับนานาชาติที่อยู่ในฐานข้อมูล Science Citation Index (SCI) ของ ISI Web of Science Q1-Q4 จำนวน 2 บทความ

ระเบียบวิธีวิจัย :

1. ค้นคว้าหาเอกสารที่เกี่ยวข้องกับการสร้างแผนภูมิควบคุมค่าเฉลี่ยถ่วงน้ำหนักเอ็กซ์โปเนนเชียลที่สร้างภายใต้อัตราส่วนล็อกไลค์ลิฮูด
2. สร้างแผนภูมิควบคุมค่าเฉลี่ยถ่วงน้ำหนักเอ็กซ์โปเนนเชียลขึ้นใหม่ ภายใต้อัตราส่วนล็อกไลค์ลิฮูด
3. พิมพ์และส่งผลงานวิจัยตีพิมพ์ในวารสารนานาชาติ
4. สรุปผลโครงการ

ผลการวิจัย :

Paper 1

This paper developed the exponentially-weighted moving average (*EWMA-chart*) based on the log-likelihood ratio for the Generalized Poisson process. The developed *EWMA-chart* is called the $EWMA_{DV}$*-chart*. The $EWMA_{DV}$*-chart* is used for detecting parameter shifts in the mean of nonconformities (λ) where there is a condition of over-dispersion. The different methods of charts for comparison with the $EWMA_{DV}$*-chart* are called c_{GP}*-chart*, $\square_{or}$ *-CUSUM chart* and $EWMA_{GP}$*-chart*. The c_{GP}*-chart* is constructed based on *GP* distribution. The $\square_{or}$ *-CUSUM chart* is constructed based on the cumulative sum (*CUSUM*) chart. The $EWMA_{GP}$*-chart* is based on the *EWMA-chart*. The performance of these charts was considered from simulation of the average run length (*ARL*). This study showed that the $EWMA_{DV}$*-chart* is more effective when there are high levels of parameter shift and variance.

Paper 2

This paper presented the new exponentially-weighted moving average (*EWMA-chart*) based on the Zero-Inflated Generalized Poisson (*ZIGP*) process when there is situation of over-dispersion. The two different methods resulting in two new *EWMA-charts* are called the *EWMA*$_\lambda$*-chart* and *EWMA*$_T$*-chart*. The new *EWMA-chart*s are sensitive to small parameter shifts, we presented a *EWMA* statistics based on log-likelihood ratio *for* plotting on the new *EWMA-chart*. The *EWMA*$_\lambda$*-chart* for detecting shift in only parameter □, which is a case study of *EWMA*$_\lambda$*-chart* based on *ZIGP* process. We also presented the *EWMA*$_T$*-chart* for detecting shift in simultaneous of λ, φ and ω. The performance of *EWMA-charts* were considered from simulation of the average run length (*ARL*). We presented in-control parameter of $\square_0$ = 1 and 2, and $\square_0$ = 0.1, 0.2, 0.3 and 0.4, $\square_0$ = 1.1, and $\square_1 = \square_0$ + 1 to be detections shifts in λ is desired. The results showed that the Shewhart *c-chart* (*c*$_Z$*-chart*) is unsuitable in detecting small shift in λ. The performance of the *EWMA*$_\lambda$*-chart* is most suitable for detecting small changes in λ. *Moreover,* the *EWMA*$_T$*-chart* is the best for monitoring of simultaneous shifts in λ, φ and ω of *ZIGP* processes. Finally, we presented case study to illustrate for used in a manufacturing process.

The EWMA chart for the Generalized Poisson Process

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Abstract

This paper aimed to develop an exponentially-weighted moving average (*EWMA-chart*) based on the generalized Poisson (*GP*) process. This developed chart is effective for detecting shifts in the mean of nonconformities (λ) and is called the *EWMA*$_{\mathrm{DV}}$*-chart*. The *EWMA*$_{\mathrm{DV}}$*-chart* was constructed based on the log-likelihood ratio and compared with the Shewhart control chart based on *GP* distribution (c_{GP}*-chart*), the cumulative sum (*CUSUM*) chart based on a *GP* distribution ($□_{\mathrm{or}}$ *-CUSUM chart*), and the *EWMA-chart* based on *GP* distribution (*EWMA*$_{GP}$*-chart*). The evaluation of performance was considered from the average run length (*ARL*). This study found that the *EWMA*$_{\mathrm{DV}}$*-chart* was effective for all levels of shift when there was a shift in λ for high levels of parameters.

Keywords: generalized Poisson distribution, cumulative sum chart, exponentially weighted moving average, Shewhart control chart

2000 Mathematics Subject Classification: 62P30, 62H10, 68U20, 65C60

Introduction

The classical Shewhart control chart (*c-chart*) has been used for monitoring the number of nonconformities per unit in a Poisson Process. The Poisson distribution is defined by parameter □ when □ is the mean number of nonconformities. The mean and variance of a Poisson random variable are both equal to □. When the ratio of the variance to the mean is equal to 1, it can be called equi-dispersion. If the Poisson variable does not meet this ratio equal to 1, then either finding of the ratio of variance to the mean is less than 1 and it is called under-dispersion. Conversely, a ratio of variance to the mean that is greater than 1 is called over- dispersion (Lambert [6]; Xie et al. [22]; Ramirez and Cantell [11]). The researchers tried to develop the Poisson distribution for suitable dispersion during these events.

Consul and Jain [15] developed the generalized Poisson distribution (*GPD*). They added the dispersion (□) into the classic Poisson distribution. The *GPD* is an appropriate alternative for the Poisson process, which is not in accordance with Poisson assumptions. They are an extension of Poisson distribution with mixed dispersion.

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Famoye and Singh [8] developed the zero-inflated, generalized Poisson (*ZIGP*) distribution. It was developed from generalized Poisson distribution where the *GPD* has two parameters, □ and □. They are extensions of the *GPD* with a mixture of the proportion of zero nonconformity(□). For the corresponding characteristic function of the Poisson with *ZIGP* distribution, if defined that □=1 and □=0, then the *ZIGP* distribution reduces to the Poisson distribution. The authors studied about *ZIGP* distribution e.g. Gupta et al. [16] studied a score test for testing over-dispersion of the *ZIGP* regression. Famoye and Singh [9] investigated the maximum likelihood method of parameters in the *ZIGP* regression model. Moreover, they applied a *ZIGP* regression model for domestic violence. The results showed that the *ZIGP* model sufficiently fit the domestic violence data for large zeros.

The most commonly used tool for monitoring the Poisson process is the *c-chart* (Montgomery [5], and Liu [23]). However, the *c-chart* is an unsuitable control chart when occurring in a dispersion situation. It has tighter control limits, which leads to a large number of false alarm rates (He et al [2]; Sim and Lim [4]). This paper is focused on alternative charts, which are the Exponentially-Weighted Moving Average (*EWMA chart*) and Cumulative Sum chart (*CUSUM chart*) for efficient monitoring of shifts in the process. (Zaman et al. [19]; Lianjie [17]; Haq et al. [1]).

Borror et al. [3] proposed extending the Poisson *EWMA chart* for the Poisson process, which calculates ARL's by using a Markov chain approach. Testik et al. [13] studied the effect of estimating the mean on the performance of the Poisson *EWMA* control chart, which used a Markov chain approach for estimating run lengths in the control chart. Page [7] was the first researcher who proposed the *CUSUM* chart, followed by many other authors including Ewan [20], Gan [10], Lucus [12], and Woodall and Adams [21]. He et al.[2] developed the *CUSUM chart* for the Zero Inflated Poisson (*ZIP*) process. They constructed both the □ - *CUSUM chart* and□- *CUSUM chart* based on the *CUSUM chart*. These charts are used to detect individual parameter shifts. The □ - *CUSUM chart* is used for detecting single parameter □ shifts. The □- *CUSUM chart* is used for detecting single parameter□shifts. Katemee and Mayureesawan [14] developed the *CUSUM chart* for the Zero Inflated Generalized Poisson (*ZIGP*) process. In this paper, the *CUSUM chart* for the *ZIGP* process was introduced. The four charts are □ - *CUSUM chart*, □- *CUSUM chart*, □- *CUSUM chart* and T - *CUSUM chart*. This paper developed the □ - *CUSUM chart*, □- *CUSUM chart* and □- *CUSUM chart* to detect changes in individual parameters of □, □ and φ, respectively. However, the T-*CUSUM chart* detected changes together in parameters □, □and □. The results showed that the □ - *CUSUM chart*, □- *CUSUM chart* and □- *CUSUM chart* were efficient for detecting changes in the individual parameters of □, □ and □, respectively. However, the T-*CUSUM chart* was efficient for both the detection of changes in individual parameters of □and detection of changes in combined parameters □, □and □. Gan [10] proposed a modified version of the *EWMA chart* for monitoring the □ of the Poisson process. He found that the *EWMA chart* was efficient for detecting shifts in □.

This paper aims to present the developed *EWMA* statistics constructed base on log-likelihood ratio for plotting on the *EWMA chart* for the Generalized Poisson (*GP*) process when there is a condition of over-dispersion. The developed *EWMA* statistics are used for detecting shifts in the individual parameters. Moreover, we also studied the influence of the Cumulative Sum chart (*CUSUM*) and the *c-chart* in monitoring shifts in the process parameters. The performance of the goal charts is the average run length (*ARL*).

Materials and Methods

The Generalized Poisson (*GP*) distribution

The probability function is given by: (Consul and Jain [15])

$$P(Y=y)=\exp(-(\square+y(\square-1)))\frac{\square(\square+y(\square-1))^{y-1}}{\square^{y}y!}, \quad y=0,1,2,... \qquad (1)$$

Where Y is the random variables of nonconformities in a sample unit,

□ is the mean of nonconformities in a sample unit based on the *GP* distribution,

□ is the over dispersion for *GP* distribution, and

$$E(Y)=\square \text{ and } V(Y)=\square^{2}\square. \qquad (2)$$

The Shewhart control chart of nonconformities is based on *GP* distribution (c_{GP}-*chart*)

The c_{GP}-*chart* is the same as a *c-chart* based on the *GP* distribution for monitoring in shift parameter. The upper control limit (*UCL*) of the c_{GP}-*chart* is $c+L\sqrt{c}$, where c is assumed to be the mean number of nonconformities if the mean of the probability distribution is known and L is the coefficient of control limit of *c-chart*. The c_{GP}-*chart* will signals when any observations of nonconformities (y_i) is greater than H_c , where H_c is the *UCL* where the c_{GP}-*chart* that is selected matching the desired in-control performance.

The Cumulative Sum chart based on a *GP* distribution ($\square_{or}$ -*CUSUM chart*)

The $\square_{or}$ -*CUSUM chart* is the same as a *CUSUM chart* based on a *GP* distribution for detecting shifts in parameter. The cumulative sum statistics for plotting on the $\square_{or}$ -*CUSUM chart* (L_i) defined as:

$$L_i=\max(0, y_i-k+L_{i-1}), \quad i=1,2, ... \qquad (3)$$

Where y_i is the observations of y taken at the time *i*,

k is the reference value,

The head start value of the cumulative sum statistics $(L_0)=0$. The $\square_{or}$ -*CUSUM chart* will signals in the process when $L_i>H_{or}$, where H_{or} is the *UCL* of the $\square_{or}$ -*CUSUM chart* that is determined based on require in-control performance.

The Exponentially Weighted Moving Average chart based on *GP* distribution (*EWMA$_{GP}$-chart*)

The *EWMA$_{GP}$-chart* is the same as a *EWMA-chart* based on detect changes in a parameter of the *GP* distribution. The *EWMA* statistics for plotting on the *EWMA$_{GP}$-chart* (E_i) defined as (Roberts [18])

$$E_i= \square y_i+(1-\square)\, E_{i-1}, \quad i=1,2, ... \qquad (4)$$

Where □ is a constant that determines must satisfy $0<\square\leq 1$,

y_i is the observations of y taken at the time i, The head start value of the *EWMA* statistics $(E_0) = \square_0$.The *EWMA*$_{GP}$*-chart* will signals in the process when $E_i > H_{GP}$, where H_{EM} is the *UCL* of the *EWMA*$_{GP}$*-chart* that is determined based on require in-control performance.

Development of the Exponentially Weighted Moving Average chart based on *GP* distribution (*EWMA*$_{DV}$*-chart*)

The *EWMA*$_{DV}$*-chart* is a development of the *EWMA-chart* based on *GP* distribution for monitoring shifts in a parameter. The *EWMA* statistics for plotting on the *EWMA*$_{DV}$*-chart* constructed based on log-likelihood ratio (Z_i) defined as

$$Z_i = (\square)\,\square_i + (1-\square)Z_{i-1}, \quad i = 1,2,... \tag{5}$$

Where $\square_i$ is the log-likelihood ratio of *GP* distribution,

ξ is a constant that determines must satisfy $0 < \square \leq 1$,

The $\square_i$ is the log-likelihood ratio of *GP* distribution defined as follows:

$$\square_i = \square_0 - \square_1 + \ln\left(\frac{\square_1}{\square_0}\right) + (y_i - 1)\ln\left(\frac{(\square_1 + y_i(\square_0 - 1))}{(\square_0 + y_i(\square_0 - 1))}\right), \quad y_i = 0,1,2,... \tag{6}$$

Where y_i is the observations of y taken at the time i,

$\square_0$ is the in-control value of the mean number of nonconformities for *GP* distribution,

$\square_1$ is the out-of-control values of the mean number of nonconformities for *GP* distribution,

$\square_0$ is the in-control value of the over dispersion for *GP* distribution,

The head start value of the *EWMA* statistics $(Z_0) = \square_0$. The *EWMA*$_{DV}$*-chart* will signals in the process when $Z_i > H_{DV}$, where H_{DV} is the *UCL* of the *EWMA*$_{DV}$*-chart* that is determined based on require in-control performance.

Simulation Results

We present the simulation code in situations where the process has the mean number of nonconformities of: $(\square_0)$ = 1 and 2. The over dispersion is: $(\square_0)$ = 1.1 (0.1) 1.5. The out-of-control of the mean number of nonconformities*:* $\square_1 = \square_0 + \square$. The criteria for evaluating the performance of control charts is the average run length (*ARL*). The research process has the following steps:

1. The R program is used to simulate the number of nonconforming items for a *GP* distribution where the parameters are $(n, \square_0, \square_0)$.

2. The value of the upper control limit with *ARL*$_0$ = 370 matching for all of the charts, including the value of H_{or} for the $\square_{or}$ *-CUSUM chart*, the value of H_{GP} for the *EWMA*$_{GP}$*-chart* and the value of H_{DV} for the *EWMA*$_{DV}$*-chart* and the value of H_C for c_{GP}*-chart*. Specifications for the shift sizes of $\square_1$ that we are interested in for all charts have rapid detection based on 100,000 replications that are in each level of the parameters.

3. The calculation of statistics for plotting in the control chart is based on the log-likelihood ratios. The *EWMA* statistics are plotted in the *EWMA*$_{DV}$*-chart*, calculating $\square_i$ value from (6) for Z_i value from (5). The cumulative sum statistics are plotted in $\square_{or}$ *-CUSUM chart*, the numbers of simulation of nonconforming (y_i) for L_i value from (3). For the *EWMA* statistics

plotted in the *EWMA*$_{GP}$-*chart*, the y_i value for E_i value is from (4). For the y_i are used for c_{GP}-*chart*.

4. An investigation examined the *CUSUM* and *EWMA* statistics with a control limit for each chart to find the run length (*RL*). The □$_{or}$ -*CUSUM chart* examined L_i value with H_{or} value. The *EWMA*$_{GP}$-*chart* examined E_i value with H_{GP} value. The *EWMA*$_{DV}$-*chart* examined Z_i value with H_{DV} value, while the c_{GP}-*chart* examined y_i value with H_C value. Consider examining the L_i, E_i, Z_i and y_i as out-of-control points. If there are points that fall outside the control limit, then they will be stored in the observations before a point indicated as out-of-control for run length (*RL*) calculation. If they are at *i* statistics indicating out-of-control, then *RL* = *i* -1.

5. Steps 3 to 4 were repeated in 100,000 replications to compute the average run length (*ARL*) for each of the charts.

6. Comparison of the performance of control charts gives a low *ARL*, meaning that the control charts are efficient.

7. Changing during parameters value in the study to completely.

Results

The summary of efficient control charts is shown for all levels of parameters and all levels of shifts.

Table 1 defines the levels for the parameters shifts when $\lambda_0 = 1$, $\varphi_0 = 1.1$ and □= 0.7(0.2)1.7

Levels of Shifts	1	2	3	4	5	6
The shift in λ □$_1$ =□$_0$ +□	1.7	1.9	2.1	2.3	2.5	2.7

Table 2 the upper control limit H_{or} , H_{GP} and H_{DV} were matching with the desired in-control performance 370 for monitoring for shift in parameter □

□$_0$	□$_0$	□$_{or}$ -*CUSUM*		*EWMA*$_{GP}$		*EWMA*$_{DV}$	
		K	H_{or}	ξ	H_{GP}	ξ	H_{DV}
1	1.1	1	19	0.1	1.7000	0.3100	0.6194
	1.2	1	20	0.1	1.7900	0.3860	0.6300
	1.3	1	23	0.1	1.8800	0.4309	0.5852
	1.4	1	25	0.1	1.9660	0.4700	0.5330
	1.5	1	27	0.1	2.0600	0.5600	0.5250
2	1.1	2	28	0.1	2.9582	0.5500	0.8930
	1.2	2	31	0.1	3.0650	0.5680	0.7370
	1.3	2	33	0.1	3.1830	0.5700	0.6700
	1.4	2	36	0.1	3.3000	0.5900	0.5350
	1.5	2	38	0.1	3.4200	0.6400	0.4600

Table 3 the ARL_1 of the $\square_{or}$ *-CUSUM chart*, $EWMA_{GP}$*-chart* and $EWMA_{DV}$*-chart* for shift in parameter $\square$

$\square_0$	$\square_0$	$\square_1$	$\square_{or}$ *-CUSUM*	$EWMA_{GP}$	$EWMA_{DV}$
1	1.1	1.7	24.463	22.017	**17.977**
		1.9	18.831	14.265	**8.588**
		2.1	13.320	10.488	**4.753**
		2.3	10.169	8.061	**2.814**
		2.5	8.558	6.565	**1.828**
		2.7	7.037	5.454	**1.164**
	1.2	1.9	24.251	21.330	**20.222**
		2.1	17.674	14.595	**10.785**
		2.3	13.504	10.951	**7.3452**
		2.5	10.488	8.655	**4.013**
		2.7	8.626	7.111	**2.644**
		2.9	7.561	6.005	**1.795**
	1.3	2.1	23.130	20.502	**19.580**
		2.3	16.757	14.877	**11.342**
		2.5	13.178	11.315	**6.789**
		2.7	10.433	9.097	**4.498**
		2.9	8.714	7.617	**3.105**
		3.1	7.171	6.474	**2.124**
	1.4	2.3	23.869	20.082	**19.270**
		2.5	17.418	14.865	**11.675**
		2.7	14.341	11.788	**7.623**
		2.9	11.115	9.575	**5.063**
		3.1	9.123	8.013	**3.538**
		3.3	7.558	6.863	**2.621**
	1.5	2.9	14.868	12.081	**11.137**
		3.1	11.422	9.996	**8.080**
		3.3	9.484	8.441	**5.919**
		3.5	8.189	7.308	**4.529**
		3.7	7.234	6.415	**3.446**
		3.9	6.205	5.720	**2.666**
2	1.1	2.4	13.280	11.276	**10.452**
		2.6	11.672	9.150	**6.749**
		2.8	10.423	7.655	**4.287**
		3.0	9.385	6.583	**3.030**
		3.2	8.630	5.692	**2.284**
		3.4	7.872	5.080	**1.543**
	1.2	2.7	13.173	10.772	**9.704**
		2.9	11.760	9.019	**6.363**
		3.1	10.580	7.682	**4.607**
		3.3	9.340	6.679	**3.150**
		3.5	7.836	5.859	**2.313**
		3.7	6.747	5.236	**1.761**
	1.3	2.9	13.590	11.466	**10.627**
		3.1	12.322	9.624	**6.731**
		3.3	10.920	8.353	**4.698**

Table 3 (Continued)

	3.5	9.594	7.284	**3.263**
	3.7	8.442	6.417	**2.216**
	3.9	7.182	5.782	**1.536**
1.4	3.0	14.791	13.209	**12.467**
	3.2	13.330	11.107	**8.626**
	3.4	11.378	9.609	**5.669**
	3.6	10.182	8.386	**3.542**
	3.8	8.797	7.442	**2.888**
	4.0	7.953	6.598	**2.095**
1.5	3.4	14.272	11.732	**10.024**
	3.6	12.380	10.210	**7.194**
	3.8	10.671	8.905	**4.997**
	4.0	8.987	7.905	**3.898**
	4.2	8.138	7.174	**2.831**
	4.4	7.242	6.389	**2.207**

Table 1 defines the levels for the parameter shifts when $\lambda_0 = 1$, $\varphi_0 = 1.1$ and $\square$= 0.7(0.2)1.7 Table 2 shows the upper control limit of $\square_{or}$ -*CUSUM chart* (H_{or}), *EWMA*$_{GP}$-*chart* (H_{GP}) and *EWMA*$_{DV}$-*chart* (H_{DV}), which were matching with the ARL_0 = 370. However, the c_{GP}-*chart* does not match with the required ARL_0. Therefore, this chart was not discussed. It can be seen that in all values of $\square_0$, the values of the H_{or} and H_{GP} are higher when the values of $\square_0$ are higher. However, the values of the H_{DV} are lower when the values of $\square_0$ are higher. Table 3 shows the ARL_1 of the $\square_{or}$ -*CUSUM chart*, *EWMA*$_{GP}$-*chart* and *EWMA*$_{DV}$-*chart* for shift in parameter $\square$ only. For the $\square_0$ = 1, $\square_0$ = 1.1 when $\square$= 0.7(0.2)1.7, $\square_0$ = 1.2 when $\square$= 0.9(0.2)1.9, $\square_0$ = 1.3 when $\square$= 1.1(0.2)2.1, $\square_0$ = 1.4 when $\square$= 1.0(0.2)3.0 and $\square_0$ = 1.5 when $\square$= 2.4(0.2)3.4, the *EWMA*$_{DV}$-*chart* returns the lowest values for ARL_1. That means the *EWMA*$_{DV}$-*chart* is able to detect shift faster than other charts, while the $\square_{or}$ -*CUSUM chart* usually detects slower than other charts. Moreover, the *EWMA*$_{DV}$-*chart* and *EWMA*$_{GP}$-*chart* return similar low values of ARL_1 for the levels of parameters shifts 1. For the levels of parameter shifts 3-6, the *EWMA*$_{GP}$-*chart* return low values of ARL_1, which can be seen from Figure 1. For $\square_0$ = 2, $\square_0$ = 1.1 when $\square$= 0.4(0.2)1.4, $\square_0$ = 1.2 when $\square$= 0.7(0.2)1.7, $\square_0$ = 1.3 when $\square$= 0.9(0.2)1.9, $\square_0$ = 1.4 when $\square$= 1.0(0.2)2.0 and $\square_0$ = 1.5 when $\square$= 1.4(0.2)2.4, the *EWMA*$_{DV}$-*chart* returns the lowest values of ARL_1. That means the *EWMA*$_{DV}$-*chart* is able to detect shift faster than other charts. Moreover, the *EWMA*$_{DV}$-*chart* and *EWMA*$_{GP}$-*chart* return similar low values of ARL_1 for the levels of parameter shifts 1. However, the $\square_{or}$ -*CUSUM chart* detects slower than other charts when the levels of all parameter shifts, as can be seen from Figure 2.

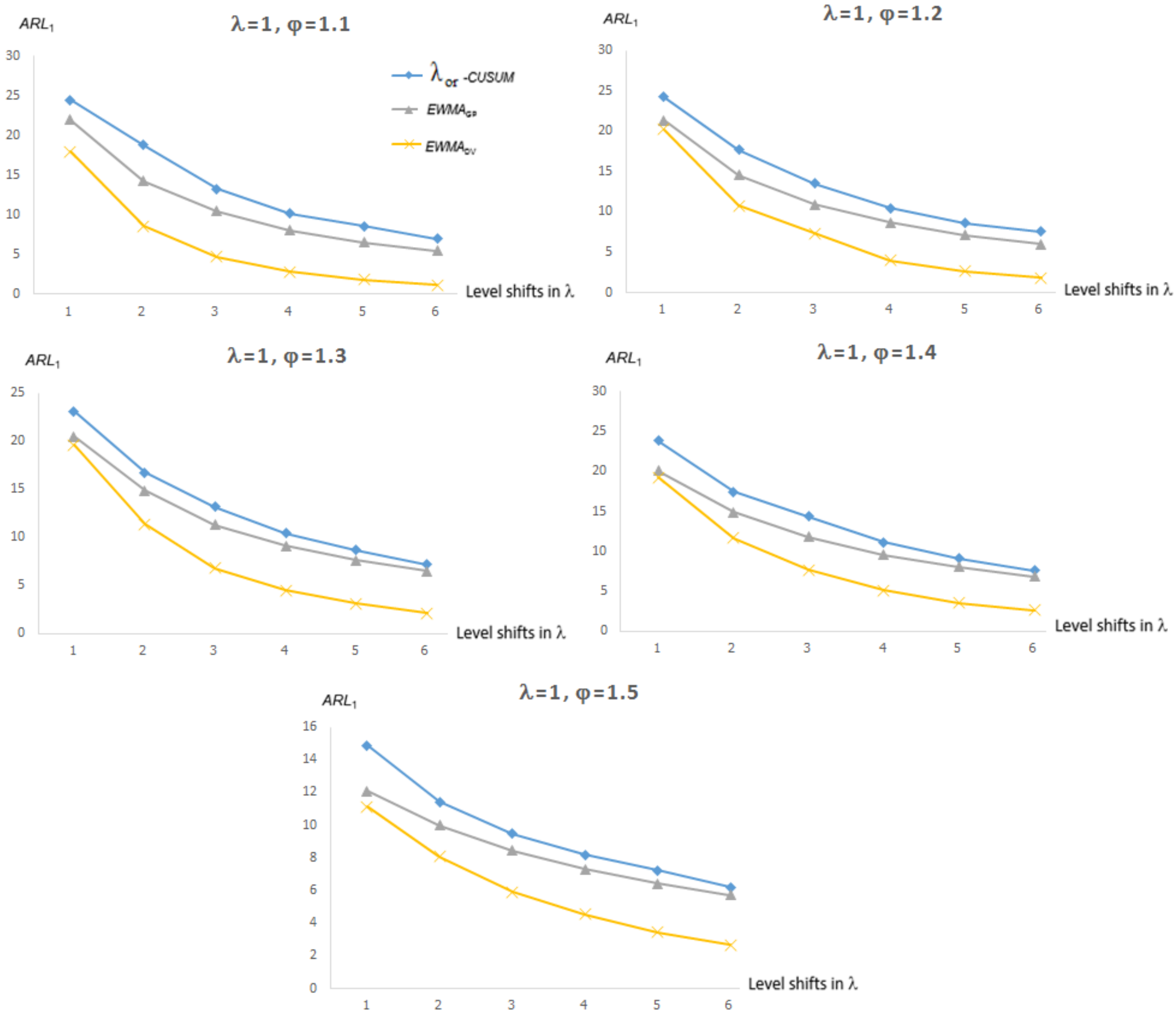

Fig. 1 the ARL_1 of the □$_{\text{or}}$ -*CUSUM chart* , *EWMA$_{GP}$-chart* and *EWMA*$_{\text{DV}}$-*chart* for shift in parameter □=1

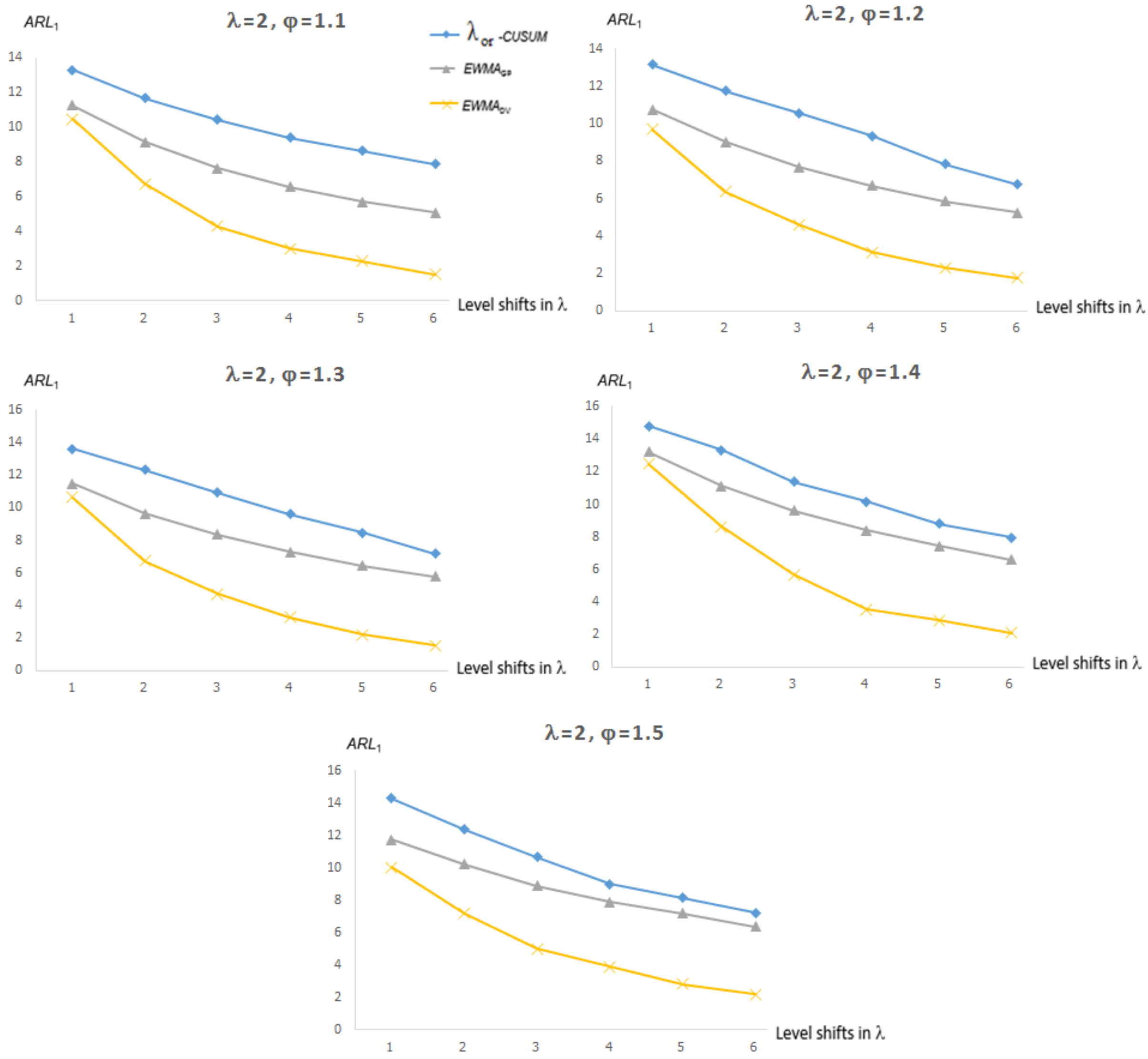

Fig. 2 the ARL_1 of the λ_{or} *-CUSUM chart*, *EWMA*$_{GP}$*-chart* and *EWMA*$_{DV}$*-chart* for shift in parameter $\lambda=2$

Table 4 Recommended on choosing an appropriate chart based on the variance with over-dispersion for the Generalized Poisson process

Variance	□	□	Type of recommended chart	Variance	□	□	Type of recommended chart
1.331 - 1.452	1	1.1	$□_{or}$ *-CUSUM chart*	2.541 - 3.630	2	1.1	$□_{or}$ *-CUSUM chart*
1.572 - 1.694			$EWMA_{GP}$*-chart*	3.753 - 3.873			$EWMA_{GP}$*-chart*
1.815 - 6.050			$EWMA_{\mathrm{DV}}$*-chart*	3.990 - 7.261			$EWMA_{\mathrm{DV}}$*-chart*
1.583 - 1.871		1.2	$□_{or}$ *-CUSUM chart*	3.023 - 4.467		1.2	$□_{or}$ *-CUSUM chart*
2.016 - 2.305			$EWMA_{GP}$*-chart*	4.607 - 4.898			$EWMA_{GP}$*-chart*
2.448 - 7.200			$EWMA_{\mathrm{DV}}$*-chart*	5.042 - 8.645			$EWMA_{\mathrm{DV}}$*-chart*
1.859 - 2.198		1.3	$□_{or}$ *-CUSUM chart*	3.023 - 4.750		1.3	$□_{or}$ *-CUSUM chart*
2.368 - 2.873			$EWMA_{GP}$*-chart*	4.898 - 5.042			$EWMA_{GP}$*-chart*
3.041 - 8.451			$EWMA_{\mathrm{DV}}$*-chart*	5.186 - 8.645			$EWMA_{\mathrm{DV}}$*-chart*
2.155 - 2.742		1.4	$□_{or}$ *-CUSUM chart*	3.549 - 5.576		1.4	$□_{or}$ *-CUSUM chart*
2.940 - 3.527			$EWMA_{GP}$*-chart*	5.743 - 5.919			$EWMA_{GP}$*-chart*
3.723 - 9.806			$EWMA_{\mathrm{DV}}$*-chart*	6.085 - 10.139			$EWMA_{\mathrm{DV}}$*-chart*
2.474 - 3.146		1.5	$□_{or}$ *-CUSUM chart*	4.118 - 7.876		1.5	$□_{or}$ *-CUSUM chart*
3.376 - 5.174			$EWMA_{GP}$*-chart*	8.105 - 8.548			$EWMA_{GP}$*-chart*
5.396 - 11.247			$EWMA_{\mathrm{DV}}$*-chart*	8.766 - 11.256			$EWMA_{\mathrm{DV}}$*-chart*

Table 4 shows recommendation for choosing an appropriate chart based on variance for the Generalized Poisson process. When $□_0 = 1$ for all levels of $□_0$ and low variance exists, the $□_{or}$ *-CUSUM chart* recommended using monitor shift in the process. However, the $EWMA_{\mathrm{DV}}$*-chart* is recommended for high variance. When $□_0 = 2$ all levels of $□_0$ for a low-to-moderate variance, the $□_{or}$ *-CUSUM chart* recommended using monitor shift in process. However, the $EWMA_{\mathrm{DV}}$*-chart* is recommended for high variance.

Conclusion

This paper developed the exponentially-weighted moving average (*EWMA-chart*) based on the log-likelihood ratio for the Generalized Poisson process. The developed *EWMA-chart* is called the $EWMA_{\mathrm{DV}}$*-chart*. The $EWMA_{\mathrm{DV}}$*-chart* is used for detecting parameter shifts in the mean of nonconformities (λ) where there is a condition of over-dispersion. The different methods of charts for comparison with the $EWMA_{\mathrm{DV}}$*-chart* are called c_{GP}*-chart*, $□_{or}$ *-CUSUM chart* and $EWMA_{GP}$*-chart*. The c_{GP}*-chart* is constructed based on *GP* distribution. The $□_{or}$ *-CUSUM chart* is constructed based on the cumulative sum (*CUSUM*) chart. The $EWMA_{GP}$*-chart* is based on the *EWMA-chart*. The performance of these charts was considered from simulation of the average run length (*ARL*). This study showed that the $EWMA_{\mathrm{DV}}$*-chart* is more effective when there are high levels of parameter shift and variance.

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EWMA Charts for Monitoring a Zero-Inflated Generalized Poisson Process

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Abstract

The zero-inflated generalized Poisson (*ZIGP*) process is more appropriate than the generalized Poisson (*GP*) process when there is an unusually excess number of zeros. This model can be used to large numbers of zeros-defect in manufacturing processes. In this paper, we proposed a two control charting method using a log-likelihood ratio of two exponentially-weighted moving average (*EWMA*) charts are called $EWMA_{\lambda}$*-chart* and $EWMA_T$*-chart*. The $EWMA_{\lambda}$*-chart* is proposed for monitoring individual shift in the mean of nonconformities (λ) of the *ZIGP* process. Furthermore, we presented a $EWMA_T$*-chart* for monitoring simultaneous shift in λ, φ (over dispersion) and ω (zero inflation). The evaluation of performance was considered from the average run length (*ARL*). Comparisons between the $EWMA_{\lambda}$*-chart* and $EWMA_T$*-chart* with a traditional *EWMA chart* show that the $EWMA_{\lambda}$*-chart* is better than traditional *EWMA chart* when there are only shift in parameter λ. The $EWMA_T$*-chart* is the best for monitoring simultaneous shift in λ, φ and ω of the *ZIGP* process. Finally, we presented case study from defect in ceramics of a manufacturing process.

Keywords: average run length; $EWMA_{\lambda}$*-chart*; $EWMA_T$*-chart*; log-likelihood ratio; manufacturing process; *ZIGP* process

2000 Mathematics Subject Classification: 62P30, 62H10, 68U20, 65C60

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1. Introduction

The traditional Shewhart *c-chart* for monitoring quality characteristics by take on the number of nonconformities based on the Poisson process. If in situation where the process is either not finding any number of nonconformities or else the process finds an excess number of zero nonconformities on a product, then the equal of mean or variance for assumption of Poisson is changed. That is, the ratio of variance to the mean is greater than 1 (over dispersion) (Lambert [7]; Xie et al. [19]; Ramirez and Cantell [13]). When the mean tends to underestimate that result the *c-chart* has narrow down of control limits (Sim and Lim [4]), which leads to a higher number of false alarm rates in processes. For the reason that the Poisson distribution is inappropriate where an excess of nonconformities in processes, this paper focus the alternative of distribution to suitable apart from the Poisson distribution that is called the zero-inflated generalize Poisson (*ZIGP*) distribution. The *ZIGP* distribution, was first developed by Famoye and Singh [9] to applied from generalized Poisson distribution (*GPD*) by mixture of the zero inflation of nonconformity (ω) with two parameters λ and φ, where λ is the mean of nonconformities in a sample unit and φ is the dispersion. For the corresponding of characteristic function of the Poisson with the *ZIGP* distribution, if defined that $\varphi = 1$ and $\omega = 0$ then the *ZIGP* distribution reduces to the Poisson distribution. The authors have studied about a *ZIGP* distribution e.g. Gupta et al. [22] studied a score test for testing over-dispersion for *ZIGP* regression. Famoye and Singh [10] investigated the maximum likelihood method of parameters in *ZIGP* regression model. Moreover, they applied a *ZIGP* regression model for domestic violence. The result shows that the *ZIGP* model sufficiently fits the domestic violence data for large zeros.

The cumulative sum chart (*CUSUM chart*) was first proposed by Page [8] and many authors have since studied this chart, see Ewan [27], Gan [11], Lucus [14], and Woodall and Adams [28]. The *CUSUM chart* is faster in detecting small shifts in process, Montgomery [6]. Lucus [14] is in agreement with efficiency of the *CUSUM chart* for Poisson processes. He found this chart was efficient for detecting small shifts in parameters. Moreover, He et al. [2] introduced the *CUSUM chart* for the *ZIP* distribution, which was constructed of two charts based on the *CUSUM chart* that were $\omega - CUSUM\ chart$ and $\lambda - CUSUM\ chart$. These control charts are used to detect individual parameter shifts. The $\omega - CUSUM\ chart$ is used to detect parameter ω shifts. The $\lambda - CUSUM\ chart$ is used to detect parameter λ shifts. If the process uses the single chart to detect shifts together in parameters ω and λ, then it cannot tell which parameter in the process has changed. Katemee and Mayureesawan [21] developed the *CUSUM chart* for the Zero Inflated Generalized Poisson (*ZIGP*) process. In this paper, the *CUSUM chart* for the *ZIGP* process was introduced. The four charts are □ - *CUSUM chart*, □- *CUSUM chart*, □- *CUSUM chart* and T - *CUSUM chart*. This paper developed the □ - *CUSUM chart*, □- *CUSUM chart* and □- *CUSUM chart* to detect changes in individual parameters of □, □ and φ, respectively. However, the T-*CUSUM chart* detected changes together in parameters □, □and □. The results showed that the □ - *CUSUM chart*, □- *CUSUM chart* and □- *CUSUM chart* were efficient for detecting changes in the individual parameters of □, □ and □, respectively. However, the T-*CUSUM chart* was efficient for both the detection of changes in individual parameters of □and detection of changes in combined parameters □, □and □.

The Exponential Weighted Moving Average (*EWMA*) control charts are effective tools

for used to monitor various aspects of production processes. The *EWMA* chart was first introduced by Roberts (26), and is an alternative to Shewhart control charts more especially efficient in detecting small and moderate in process shift (Montgomery[5]. Various approaches on the enhancements of Poison *EWMA* chart to monitor discrete count data have been propose. (Babar [29]; Lianjie [25]; Haq et al. [1]). Borror et al. [3] proposed extending the Poisson *EWMA* chart for the Poisson process, which calculates ARL's by using a Markov chain approach. Testik et al. [17] studied the effect of estimating the mean on the performance of the Poisson *EWMA* control chart, which used a Markov chain approach for estimating run lengths in the control chart. Gan [11] proposed a modified version of the *EWMA chart* for monitoring the □ of the Poisson process. He found that the *EWMA chart* was efficient for detecting shifts in □. The *EWMA* commonly used to monitor continuous data is an alternative to the *CUSUM* control chart that used for controlling discrete count data. See, for example, Zhang et al. [16], Sparks et al. [23], Shu et al. [15] and Abujiya, et al. [18]. For recent Katemee [20] studied on the *EWMA* control chart to detect parameter shifts of mean for the *ZIGP* process with the situation of the over-dispersion. The result show that the $EWMA_Z$-*chart* is the best for all level of □, □and □.

In this paper, we propose a *EWMA* statistics constructed based on log-likelihood ratio where plotting on the new *EWMA chart* for efficient monitoring of changes in a parameter of a *ZIGP* process. The new *EWMA chart*s are for detecting shifts in parameters for both of the individual and the simultaneous parameters shifts. The following two *EWMA chart*s were developed: the $EWMA_\lambda$-*chart* for use in detecting shifts in the mean of nonconformities (λ), the $EWMA_T$-*chart* is used to detect simultaneous shifts in □, □and □. The performance of the propose charts is average run length (*ARL*) and then compared with the *c-chart* based on *ZIGP* distribution (c_Z-*chart*) and the classical *EWMA chart* based on *ZIGP* distribution ($EWMA_G$-*chart*)

2. Materials and Methods

2.1 The Zero-Inflated Generalized Poisson (*ZIGP*) distribution. The *ZIGP* was applied from generalized Poisson distribution (*GPD*) by mixture of the zero inflation of nonconformity$(\omega)$with two parameters λ and φ. The *ZIGP* distribution is useful for the analysis of count data with a large amount of zeros (see e.g. Famoye and Singh [9], Gupta et al. [22], Joe and Zhu [12], Bae et al. [24]). The probability function is given by:

$$P(Y=y)=\begin{cases}\omega+(1-\omega)\exp(-\lambda\varphi), & y=0\\ (1-\omega)\exp(-\dfrac{1}{\varphi}(\lambda+y(\varphi-1)))\dfrac{\lambda(\lambda+y(\varphi-1))^{y-1}}{\varphi^{y}y!}, & y>0\end{cases} \tag{1}$$

Where Y is the random variables of nonconformities in a sample unit,

□is the mean of nonconformities in a sample unit based on the *ZIGP* distribution,

□ is a measure of the zero inflation of nonconformity in a sample unit,

□is the dispersion for *ZIGP* distribution, and

$$E(Y)=(1-\omega)\lambda \text{ and } V(Y)=(1-\omega)\lambda(n^2+\lambda\omega). \tag{2}$$

2.2 The Shewhart *c-chart* of nonconformities is based on *ZIGP* distribution (c_Z-*chart*)

The c_Z-*chart* is the same as a *c-chart* based on the *ZIGP* distribution. Upper control limit (*UCL*) of the c_Z-*chart* is $c+L\sqrt{c}$, where *c* is assumed to be the mean number of nonconformities if the mean of the probability distribution is known and *L* is the coefficient of control limit of *c-chart*. The c_Z-*chart* will signals when any observations of nonconformities (y_i) is greater than H_c, where H_c is the *UCL* where the c_Z-*chart* that is selected matching the desired in-control performance.

2.3 The Exponentially Weighted Moving Average chart based on *ZIGP* distribution (*EWMA$_G$-chart*)

The *EWMA$_G$-chart* is the same as a *EWMA-chart* based on detect changes in a parameter of the *ZIGP* distribution. The *EWMA* statistics for plotting on the *EWMA$_G$-chart* (E_i) defined as (Roberts [18])

$$E_i = \square y_i + (1-\square) E_{i-1}, \quad i = 1, 2, \ldots \tag{3}$$

Where $\square$ is a constant that determines must satisfy $0 < \square \leq 1$,

y_i is the observations of y taken at the time i,

The head start value of the *EWMA* statistics $(E_0) = \square_0$.The *EWMA$_G$-chart* will signals in the process when $E_i > H_G$, where H_G is the *UCL* of the *EWMA$_G$-chart* that is determined based on require in-control performance.

2.4 The new Exponentially Weighted Moving Average chart based on a *ZIGP* distribution (*EWMA$_\lambda$-chart*)

The *EWMA$_\lambda$-chart* is a development of the *EWMA-chart* based on *ZIGP* distribution for monitoring only shifts in a parameter $\square$. The *EWMA* statistics for plotting on the *EWMA$_\lambda$-chart* constructed based on log-likelihood ratio (Z_i) defined as

$$Z_i = (\square)\square_i + (1-\square) Z_{i-1}, \quad i = 1, 2, \ldots \tag{4}$$

Where $\square_i$ is the log-likelihood ratio of *ZIGP* distribution,

ξ is a constant that determines must satisfy $0 < \square \leq 1$,

The $\square_i$ is the log-likelihood ratio of *ZIGP* distribution defined as follows:

$$\psi_i = \psi(y_i) = \begin{cases} \ln \dfrac{\omega_0 + (1-\omega_0)\, e^{(-\lambda_1 \varphi_0)}}{\omega_0 + (1-\omega_0)\, e^{(-\lambda_0 \varphi_0)}}, & y_i = 0 \\ \dfrac{\lambda_0 - \lambda_1}{\varphi_0} + \ln\left(\dfrac{\lambda_1}{\lambda_0}\right) + (y_i - 1)\ln \dfrac{(\lambda_1 + y_i(\varphi_0 - 1))}{(\lambda_0 + y_i(\varphi_0 - 1))}, & y_i > 0 \end{cases} \tag{5}$$

Where y_i is the observations of y taken at the time *i*,

$\square_0$ is the in-control value of the mean number of nonconformities for *ZIGP* distribution,

$\square_1$ is the out-of-control values of the mean number of nonconformities for *ZIGP* distribution,

ω_0 is the in-control value of the proportion of zero nonconformity for *ZIGP* distribution,

$\square_0$ is the in-control value of the over dispersion for *ZIGP* distribution,

The head start value of the *EWMA* statistics $(Z_0) = \square_0$. The *EWMA$_\lambda$-chart* will signals in the process when $Z_i > H_\square$, where $H_\square$ is the *UCL* of the *EWMA$_\lambda$-chart* that is determined based on require in-control performance.

2.5 The new Exponentially Weighted Moving Average chart based on a *ZIGP* distribution (*EWMA*$_T$*-chart*)

The *EWMA*$_T$*-chart* is a development of the *EWMA-chart* based on *ZIGP* distribution for monitoring together shifts in parameters □, □and □. The *EWMA* statistics for plotting on the *EWMA*$_T$*-chart* constructed based on log-likelihood ratio (Z_i) defined as

$$\delta_i = (\xi)\,\kappa_i + (1-\xi)\,\delta_{i-1},\quad i=1,2,3,\ldots \tag{6}$$

Where $□_i$ is the log-likelihood ratio of *ZIGP* distribution,

ξ is a constant that determines must satisfy $0 < □ \leq 1$,

The $□_i$ is the log-likelihood ratio of *ZIGP* distribution defined as follows:

$$\kappa_i = \kappa(y_i) = \begin{cases} \ln \dfrac{\omega_1 + (1-\omega_1)\, e^{(-\lambda_1 \varphi_1)}}{\omega_0 + (1-\omega_0)\, e^{(-\lambda_0 \varphi_0)}}, & y_i = 0 \\ \ln \dfrac{(1-\omega_1)}{(1-\omega_0)} + (\dfrac{1}{\varphi_0}(\lambda_0 + y_i(\varphi_0 - 1))) - (\dfrac{1}{\varphi_1}(\lambda_1 + y_i(\varphi_1 - 1))) + \ln(\dfrac{\lambda_1}{\lambda_0}) + (y_i - 1)\ln \dfrac{(\lambda_1 + y_i(\varphi_1 - 1))}{(\lambda_0 + y_i(\varphi_0 - 1))} + y_i \ln(\dfrac{\varphi_0}{\varphi_1}), & y_i > 0 \end{cases} \tag{7}$$

Where $□_1$ is the out-of-control value of the over dispersion for *ZIGP* distribution,

The head start value of the *EWMA* statistics $(\delta_0) = \lambda_0$. The *EWMA*$_T$*-chart* will signals in the process when $\delta_i > H_T$, where H_T is the *UCL* of the *EWMA*$_T$*-chart* that is determined based on require in-control performance.

3. Simulation Results

We present the simulation code in situations where the process has the mean number of nonconformities of: $(□_0)$ = 1, 2 and 3. The over dispersion is: $(□_0)$ = 1.1. The shift sizes of pre-determined for the *EWMA charts* to have fast detections: $\lambda_1 = \lambda_0 + 1$, $\varphi_1 = \varphi_0 + 0.1$ and $□_1 = □_0 + 0.01$. The out-of-control of the mean number of nonconformities*:* $□ = □_0 + □$ where: $□$ = 0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3 and 1.5. The out-of-control of the dispersion of nonconformities: $□ = □_0 + \theta$ where: θ = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8. The out-of-control of the zero inflation of nonconformities: $\omega = \omega_0 + \Omega$ where: $□$ = 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07 and 0.08**.** The criteria for evaluating the performance of control charts is the average run length (*ARL*). The research process has the following steps:

1. The R program is used to simulate the number of nonconforming items for a *ZIGP* distribution where the parameters are $(n, \lambda_0, \varphi_0, \omega_0)$.
2. The value of the upper control limit with ARL_0 = 370 matching for all of the charts, including the value of H_G for the *EWMA*$_G$*-chart*, the value of $H_□$ for the *EWMA*$_\lambda$*-chart*, the and the value of H_T for the *EWMA*$_T$*-chart* and the value of H_C for c_Z*-chart*. Specifications for the shift sizes of $□_1$ that we are interested in for all charts have rapid detection based on 100,000 replications that are in each level of the parameters.
3. The calculation of statistics for plotting in the control chart is based on the log-likelihood ratios. The *EWMA* statistics are plotted in *EWMA*$_\lambda$*-chart*, calculating $□_i$ value from (5) for Z_i value from (4). The *EWMA* statistics are plotted in the *EWMA*$_T$, calculating $□_i$ value from (7)

for $\square_i$ value from (6). For the *EWMA* statistics plotted in the *EWMA$_G$-chart*, the y_i value for E_i value is from (3). For the y_i are used for *c_Z-chart*.

4. An investigation examined the *EWMA* statistics with a control limit for each chart to find the run length (*RL*). The *EWMA$_\lambda$-chart* examined Z_i value with $H_\square$ value. The *EWMA$_T$-chart* examined δ_i value with H_T value. The *EWMA$_G$-chart* examined E_i value with H_G value, while the *c_Z-chart* examined y_i value with H_C value. Consider examining the Z_i, δ_i, E_i and y_i as out-of-control points. If there are points that fall outside the control limit, then they will be stored in the observations before a point indicated as out-of-control for run length (*RL*) calculation. If they are at *i* statistics indicating out-of-control, then $RL = i$ -1.

5. Steps 3 to 4 were repeated in 100,000 replications to compute the average run length (*ARL*) for each of the charts.

6. Comparison of the performance of control charts gives a low *ARL*, meaning that the control charts are efficient.

7. Changing during parameters value in the study to completely.

4. Results

In Table I, we defines the levels for both of the individual and simultaneous parameters shifts when $\lambda_0 = 1$, $\varphi_0 = 1.1$ and $\omega_0 = 0.1$.

In Table II, we show the upper control limit of *EWMA$_G$-chart* (H_G), *EWMA$_\lambda$-chart* $(H_\square)$ and *EWMA$_T$-chart* (H_T) were matching with the desired in-control performance 370 based on $\square_1 = \square_0 + 1$. We are chosen to be in-control performance 370, because proper control limits can be found for *EWMA* scheme. It can be seen that as all values of λ_0 and $\varphi_0 = 1.1$, when $\square = 0.1$, the values of the H_G (*EWMA$_G$-chart*) is lower when the values of ω_0 are higher. For the *EWMA$_\lambda$-chart* and *EWMA$_T$-chart*, when the $\square$ is lower, the values of $H_\square$, H_T and ω_0 are higher. The results we found were due to the *c_Z-chart* returning unsuitable values of the in-control performance 370, therefore we are not shown.

Table I defines the levels for both of the individual and simultaneous parameters shifts when $\lambda_0 = 1$, $\varphi_0 = 1.1$ and $\omega_0 = 0.1$

Levels of Shifts	The only shift in λ	The simultaneous shifts in λ, φ and ω
	$\lambda = \lambda_0 + \rho$	$\lambda = \lambda_0 + \rho, \varphi = \varphi_0 + \theta$ and $\omega = \omega_0 + \Omega$
1	$\lambda = 1.1$	$\lambda = 1.1$, $\varphi = 1.2$ and $\omega = 0.11$
2	$\lambda = 1.3$	$\lambda = 1.3$, $\varphi = 1.3$ and $\omega = 0.12$
3	$\lambda = 1.5$	$\lambda = 1.5$, $\varphi = 1.4$ and $\omega = 0.13$
4	$\lambda = 1.7$	$\lambda = 1.7$, $\varphi = 1.5$ and $\omega = 0.14$
5	$\lambda = 1.9$	$\lambda = 1.9$, $\varphi = 1.6$ and $\omega = 0.15$
6	$\lambda = 2.1$	$\lambda = 2.1$, $\varphi = 1.7$ and $\omega = 0.16$
7	$\lambda = 2.3$	$\lambda = 2.3$, $\varphi = 1.8$ and $\omega = 0.17$

8	$\lambda = 2.5$	$\lambda = 2.5$, $\varphi = 1.9$ and $\omega = 0.18$

Table II the upper control limit H_G , $H_□$ and H_T were matching with the desired in-control performance 370 for monitoring for shift in only parameter □and simultaneous shifts in λ , φ and ω

$□_0$	$□_0$	$□_0$	$EWMA_G$		$EWMA_\lambda$		$EWMA_T$	
			□	H_G	□	$H_□$	□	H_T
1	1.1	0.1	0.1000	1.6080	0.0440	0.9000	0.0670	0.9000
		0.2	0.1000	1.5000	0.0400	1.1000	0.0570	0.9400
		0.3	0.1000	1.3950	0.0380	1.2075	0.0540	1.0700
		0.4	0.1000	1.2880	0.0368	1.2780	0.0510	1.0980
2	1.1	0.1	0.1000	2.7900	0.0700	2.0784	0.0700	1.8700
		0.2	0.1000	2.6100	0.0680	2.0785	0.0680	1.8800
		0.3	0.1000	2.4300	0.0679	2.0786	0.0670	1.8900
		0.4	0.1000	2.2450	0.0678	2.0790	0.0660	1.9000

Comparison of the ARL_1 values for the $EWMA_\lambda$*-chart* and $EWMA_G$*-chart* are presented in Table III, where $□_1 = □_0+1$. It is shown that if there are only shifts in λ for $□_0$ =1, $□_0$ =1.1 and the levels of shifts equal 1, 6, 7 and 8, the $EWMA_\lambda$*-chart* and $EWMA_G$*-chart* have similar faster detections. However, the $EWMA_\lambda$*-chart* is a little bit better for detecting changes than the $EWMA_G$*-chart*. It is found that the $EWMA_\lambda$*-chart* has better performance than $EWMA_G$*-chart* when the levels of shifts equal 2, 3, 4 and 5. When $□_0$ =2, $□_0$ =1.1 and all levels of shifts, the $EWMA_\lambda$*-chart* and $EWMA_G$*-chart* have similar faster detections. However, the $EWMA_\lambda$*-chart* returns lowest values of ARL_1 than the $EWMA_G$*-chart*. It can be seen that similar results can be observed as from figure I (The result of $□_0$ =1, $□_0$ =1.1 and $□_0$ = 0.3, 0.4 are similar to those we found for $□_0$ = 0.1, 0.2 and the result of $□_0$ =2, $□_0$ =1.1 when $□_0$ = 0.1, 0.2 are similar to at $□_0$ = 0.3, 0.4 (not shown here)). More results for evaluating when there is the $□_0$ increases for all $□_0$ and the levels of shifts, both the $EWMA_\lambda$*-chart* and $EWMA_G$*-chart* have slower detections. Sine, the performance of the $EWMA_\lambda$*-chart* and $EWMA_G$*-chart* are reduced detections by increases in $□_0$.

In table IV, the ARL_1 of the $EWMA_T$*-chart* and $EWMA_G$*-chart* for simultaneous shifts in λ , φ and ω based on $□_1 = □_0 + 1$, $□_1 = □_0 + 0.1$ and $□_1 = □_0 + 0.01$ are provided. The results can be observed in this table as there are simultaneous shifts in λ , φ and ω for all $□_0$, $□_0$ =1.1 and all levels of shifts, the $EWMA_T$*-chart* has better performance than the $EWMA_G$*-chart* when all parameters to increase simultaneously. In this situation, it is better to use the $EWMA_T$*-chart* based on Equation (6), which it is particularly designed. The similar results of the performances of the $EWMA_T$*-chart* and $EWMA_G$*-chart* for simultaneous shifts in λ , φ and ω in table IV are shown in figure II. However, the similar results for $□_0$ =1, $□_0$ =1.1 and $□_0$ = 0.3,
0.4 and $□_0$ =2, $□_0$ =1.1 and $□_0$ = 0.1, 0.2 are not shown here.

Table III the ARL_1 of the $EWMA_{\lambda}$-*chart* and $EWMA_G$-*chart* for shift in only parameter λ ($\lambda_1 = \lambda_0 + 1$)

λ_0	φ_0	λ	$EWMA_{\lambda}$-*chart*				$EWMA_G$-*chart*			
			ω_0=0.1	ω_0=0.2	ω_0=0.3	ω_0=0.4	ω_0=0.1	ω_0=0.2	ω_0=0.3	ω_0=0.4
1	1.1	1.1	173.58	182.18	184.40	187.89	194.09	196.56	204.55	219.11
		1.3	28.28	33.72	40.76	46.62	64.60	69.25	73.50	82.01
		1.5	9.32	11.51	15.00	18.75	31.73	32.95	36.11	40.09
		1.7	5.78	5.92	7.93	9.90	18.74	19.86	21.12	23.25
		1.9	2.46	3.48	4.75	6.39	13.05	13.78	14.64	15.35
		2.1	1.72	2.41	3.42	4.56	9.83	10.03	10.65	10.98
		2.3	1.23	1.77	2.55	3.50	7.75	7.92	8.16	8.51
		2.5	1.16	1.40	2.04	2.75	5.83	6.60	6.66	6.72
2	1.1	1.1	209.69	221.88	230.61	240.59	223.90	232.21	240.87	256.61
		1.3	67.46	73.97	87.27	97.15	84.86	97.27	102.48	113.17
		1.5	24.51	29.45	35.51	44.42	41.03	50.35	54.12	58.32
		1.7	11.26	14.44	18.04	21.74	24.23	30.62	32.99	35.86
		1.9	6.45	8.04	10.21	14.15	16.09	20.71	21.80	23.47
		2.1	4.17	5.01	6.62	9.25	11.25	15.37	15.94	16.96
		2.3	2.80	3.81	4.76	6.31	8.73	11.95	12.39	12.95
		2.5	1.94	2.71	3.53	4.69	6.84	9.56	9.83	10.14

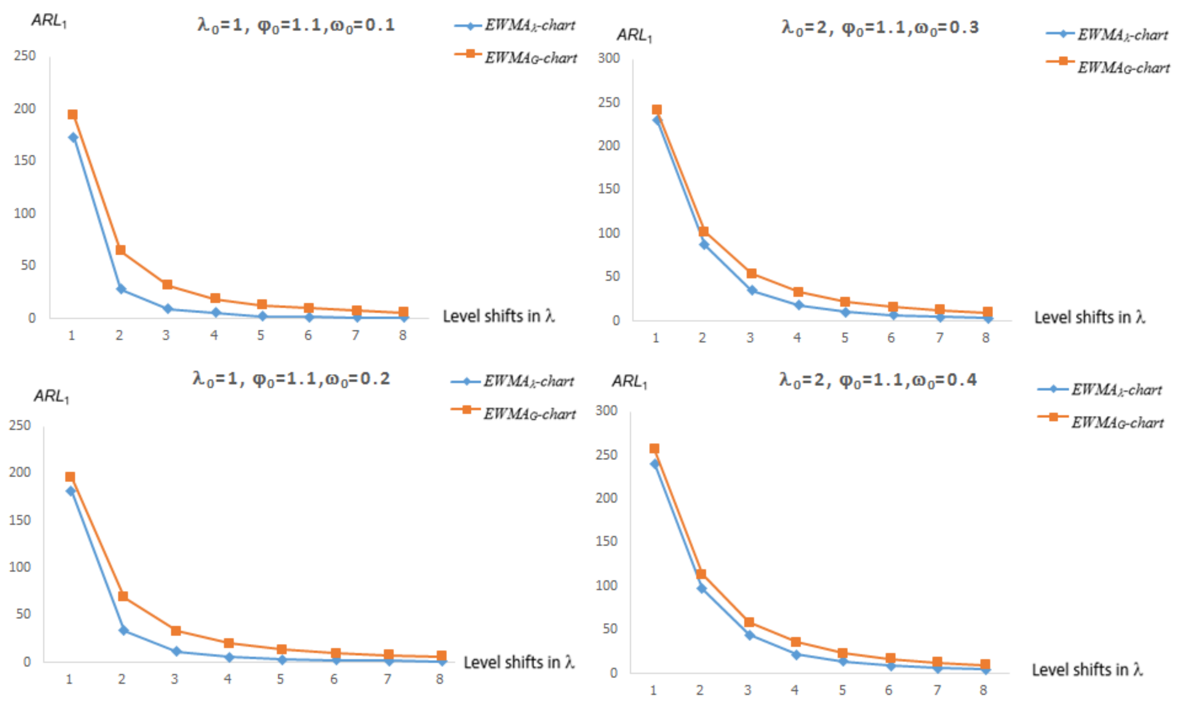

Fig. I the ARL_1 of the $EWMA_{\lambda}$-*chart* and $EWMA_G$-*chart* for the shifts in only parameter λ

Table IV the ARL_1 of the *EWMA$_T$-chart* and *EWMA$_G$-chart* for simultaneous shifts in λ, φ and ω ($\square_1 = \square_0 + 1$, $\square_1 = \square_0 + 0.1$ and $\square_1 = \square_0 + 0.01$)

$\square_0$	$\square_0$	$\square$	*EWMA$_T$-chart*				*EWMA$_G$-chart*			
			$\square_0$=0.1	$\square_0$=0.2	$\square_0$=0.3	$\square_0$=0.4	$\square_0$=0.1	$\square_0$=0.2	$\square_0$=0.3	$\square_0$=0.4
1	1.1	1.1	116.87	128.82	134.74	147.67	148.85	152.74	161.47	171.97
		1.3	28.30	31.02	35.85	38.12	52.93	56.33	61.07	66.86
		1.5	12.38	14.44	16.73	18.75	29.53	31.33	33.49	36.81
		1.7	7.17	8.29	10.35	12.43	19.14	20.41	21.85	24.72
		1.9	5.57	6.14	7.80	9.32	14.61	15.66	16.66	18.43
		2.1	4.37	4.92	6.46	7.60	11.83	12.68	13.47	14.88
		2.3	3.61	4.17	5.36	6.50	9.98	10.77	11.26	12.56
		2.5	3.32	3.54	4.71	6.07	8.51	9.20	9.88	11.28
2	1.1	1.1	121.77	131.73	142.26	158.55	178.80	183.68	197.73	212.16
		1.3	36.74	38.35	42.84	51.84	73.94	78.92	86.26	93.42
		1.5	15.83	17.57	20.96	25.00	42.41	45.79	49.69	52.75
		1.7	9.68	11.07	12.89	16.02	28.42	31.31	32.80	36.78
		1.9	6.47	7.90	9.44	12.03	21.43	22.74	24.74	26.38
		2.1	5.55	6.22	7.56	9.61	16.63	18.12	19.74	22.28
		2.3	4.81	5.30	6.37	7.85	14.23	15.17	16.62	18.31
		2.5	4.12	4.55	5.78	7.19	12.34	13.27	14.37	15.67

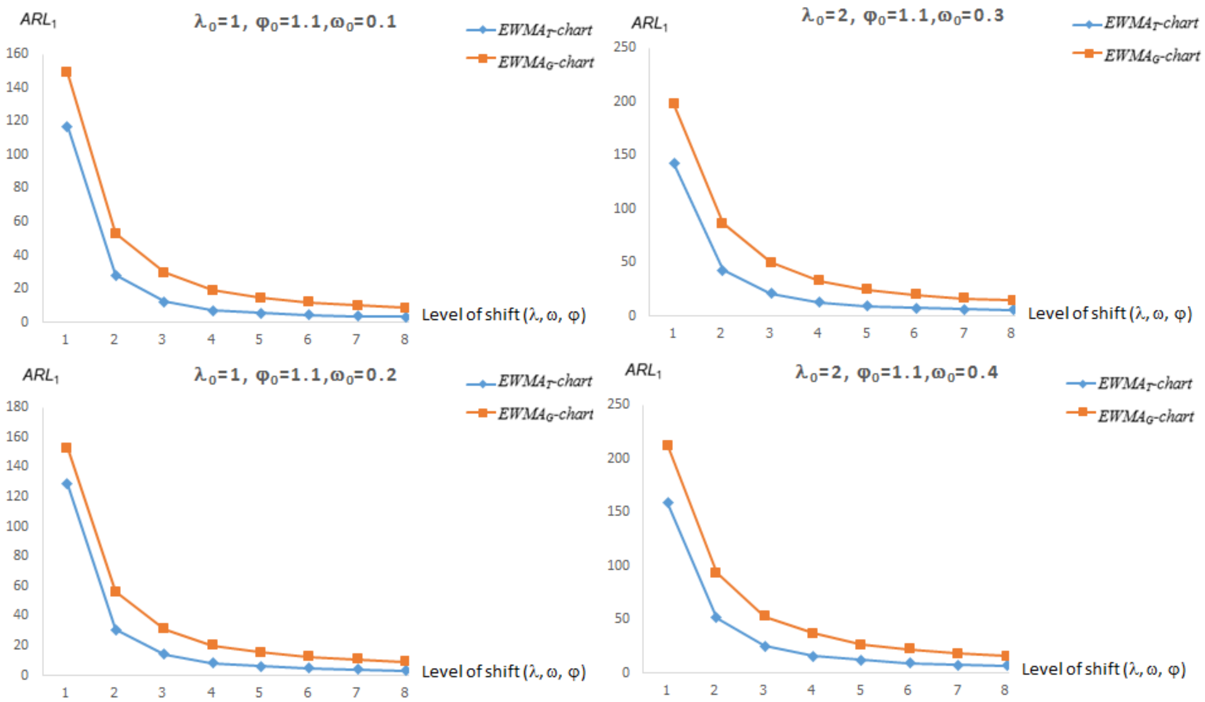

Fig. II the ARL_1 of the *EWMA$_T$-chart* and *EWMA$_G$-chart* for simultaneous shifts in λ, φ and ω

Another situation in both Tables V and VI, we evaluated the ARL_1 of the $EWMA_{\lambda}$-*chart* and the $EWMA_T$-*chart* based on $\square_1 = \square_0+1$ and $\square_1 = \square_0+2$.It is found that for $\square_0$, $\square_0$ =1.1 and levels of shifts equal 1-3, the $EWMA_{\lambda}$-*chart* ($\square_1 = \square_0+1$) has better performance than the $EWMA_{\lambda}$-*chart* ($\square_1 = \square_0+2$) when only shift in parameter □. Thus, if only shift in parameter □, it is better to use the $EWMA_{\lambda}$-*chart* based on $\square_1 = \square_0+1$. However, the $EWMA_{\lambda}$-*chart* ($\square_1 = \square_0+1$) is a little bit better for detecting changes than the $EWMA_{\lambda}$-*chart* ($\square_1 = \square_0+2$) when the levels of shifts equal 4, 5 and 6 (These results are similar to the $EWMA_T$-*chart* for simultaneous shifts in λ, φ and ω). In this situation, we would select using the $EWMA_{\lambda}$-*chart* and $EWMA_T$-*chart* based on $\square_1 = \square_0+1$ since it is better in the sense of the performance. It can be seen that similar results in tables V and VI can be observed as from figure III and IV respectively.

Table V the ARL_1 of the $EWMA_{\lambda}$-*chart* for shift in only parameter □ with $\square_0 = 1$ and $\square_0 = 1.1$

		$\square_1 = \square_0+1$				$\square_1 = \square_0+2$			
	□	$\square_0$=0.1	$\square_0$=0.2	$\square_0$=0.3	$\square_0$=0.4	$\square_0$=0.1	$\square_0$=0.2	$\square_0$=0.3	$\square_0$=0.4
$EWMA_{\lambda}$-*chart*	1.15	112.78	115.29	121.89	128.16	134.93	142.80	152.26	175.75
	1.35	19.56	24.83	29.40	36.46	36.55	43.23	47.66	59.44
	1.55	7.18	9.21	12.41	15.71	14.56	19.46	20.41	23.87
	1.75	2.24	4.91	6.62	9.22	7.29	10.73	11.29	13.50
	1.95	1.59	3.33	4.36	6.03	4.08	6.35	6.68	7.98
	2.15	1.16	2.32	3.18	4.19	2.81	4.24	4.61	5.32

Table VI the ARL_1 of the $EWMA_T$-*chart* for shift in simultaneous of λ, φ and ω with $\square_0 = 1$ and $\square_0 = 1.1$

		$\square_1 = \square_0+1$, $\square_1 = \square_0+0.1$ and $\square_1 = \square_0+0.01$				$\square_1 = \square_0+2$, $\square_1 = \square_0+0.2$ and $\square_1 = \square_0+0.02$			
	□	$\square_0$=0.1	$\square_0$=0.2	$\square_0$=0.3	$\square_0$=0.4	$\square_0$=0.1	$\square_0$=0.2	$\square_0$=0.3	$\square_0$=0.4
$EWMA_T$-*chart*	1.15	120.79	124.65	142.22	139.36	147.31	149.49	160.67	179.28
	1.35	27.05	29.05	37.02	40.99	40.44	44.08	52.41	59.92
	1.55	9.74	10.79	14.95	17.69	16.17	17.70	20.55	26.70
	1.75	4.53	5.53	7.92	9.95	7.78	9.36	11.06	14.17
	1.95	2.81	3.59	5.01	6.27	4.35	5.38	6.92	8.06
	2.15	1.86	2.48	3.50	4.45	2.98	3.48	4.25	5.63

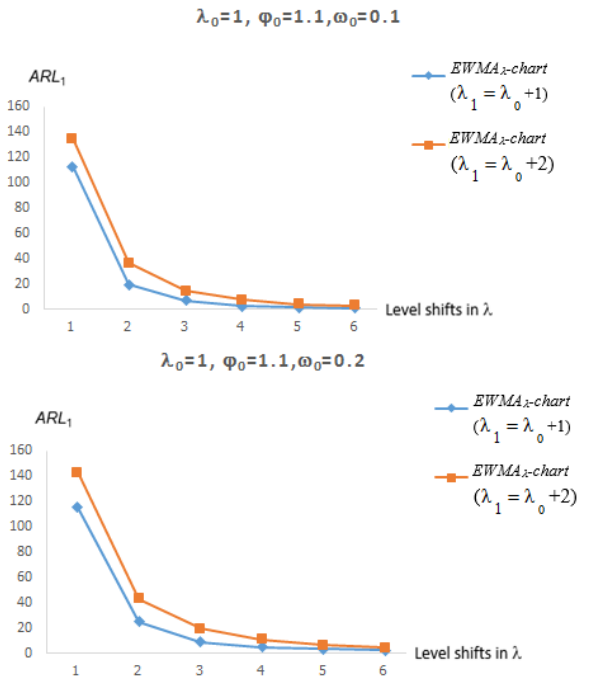

Fig. III the ARL_1 of the $EWMA_{\lambda}$*-chart* for the shifts in only parameter λ with $\square_1 = \square_0 + 1$ and $\square_1 = \square_0 + 2$

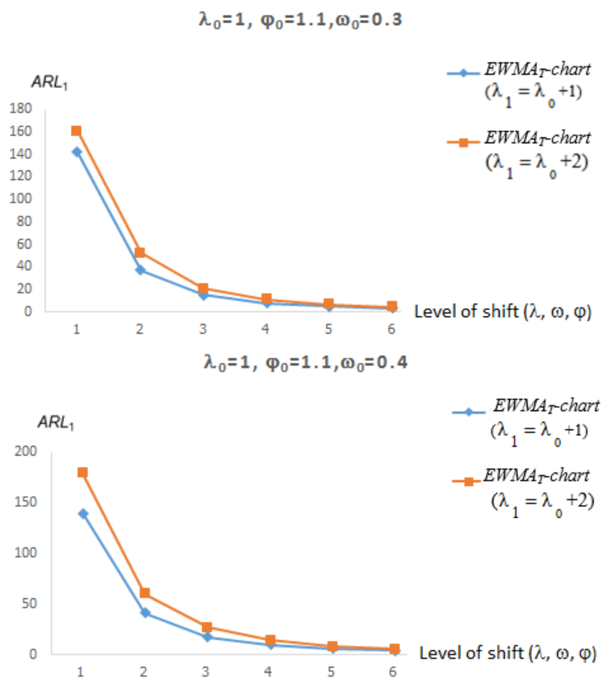

Fig. IV the ARL_1 of the $EWMA_T$*-chart* for simultaneous shifts in λ, φ and ω with $\square_1 = \square_0 + 1$ and $\square_1 = \square_0 + 2$

5. Case study

An example is used here for illustration. The data set used in Table VII is the number of defects in ceramics of a manufacturing process.

Table VII a set of defect in ceramic from a manufacturing process

Phase I										**Phase II**									
1	0	2	1	0	1	1	0	1	3	0	1	3	4	5	5	6	7	1	0
0	0	1	0	1	0	2	0	1	1	4	0	1	0	3	1	0	1	0	2
0	3	0	2	1	3	0	1	0	1	1	1	2	3	2	2	0	2	0	1
3	1	1	1	1	0	2	1	0	0	1	0	2	1	1	1	1	0	1	0
1	1	0	2	3	2	1	1	3	0	2	0	2	1	0	0	3	0	3	1
1	0	1	1	0	1	0	1	0	2	1	0	3	0	1	0	1	0	1	0
2	1	0	1	1	0	3	0	1	0	1	1	0	4	0	1	0	1	1	2
1	3	1	1	2	1	1	0	1	2	2	0	3	0	1	1	0	1	1	1
0	2	0	1	0	3	0	2	1	0	2	0	1	0	3	0	1	2	0	1
1	0	1	1	0	2	4	0	1	0	0	1	3	0	2	1	0	5	0	1
1	0	1	1	1	0	2	2	3	4	0	1	0	1	1	2	1	0	2	0
2	0	1	1	1	0	2	1	1	1	2	0	2	2	0	3	1	1	2	0
1	1	0	1	2	1	0	1	0	1	3	1	2	0	2	0	1	0	2	0
0	1	0	0	0	2	0	3	0	**5**	2	1	0	3	0	2	0	1	2	0
0	2	1	0	2	1	2	0	3	0	1	1	0	2	0	1	1	2	0	1
1	0	0	3	0	2	1	0	1	0	2	1	0	1	0	1	1	2	3	1
2	0	0	1	0	4	0	1	0	1	0	2	1	1	0	3	0	1	2	1
1	2	0	1	2	0	1	2	0	1	0	1	2	3	1	1	0	1	1	1
1	1	0	1	2						1	0	0	0	1					

The *EWMA*$_\lambda$*-chart* was applied as a Phase I method to find for matching with the desired in-control performance. However, some data values (bolded) in Table VIII were cut off and still 184 counts were then used for construct the upper control limit in Phase I. The parameters were the parameters were calculated to be $\square_0$ = 1.089, $\square_0$ = 1.184 and $\square_0$ = 0.343 based on $\square_1 = \square_0 + 1$. Control limit of *EWMA*$_\lambda$*-chart* was selected based on methods discussed in section 4 (Table II), we found that $H_{\square}$ = 1.233. Figure V provides *EWMA*$_\lambda$*-chart* based on the data we obtained. It is shown that *EWMA*$_\lambda$*-chart* is signaled at the 188th sample.

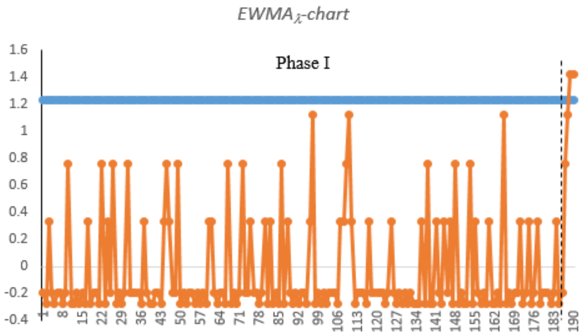

Fig. V the *EWMA*$_\lambda$*-chart* for the case study

6. Conclusion

This paper presented the new exponentially-weighted moving average (*EWMA-chart*) based on the Zero-Inflated Generalized Poisson (*ZIGP*) process when there is situation of over-dispersion. The two different methods resulting in two new *EWMA-charts* are called the *EWMA*$_\lambda$*-chart* and *EWMA*$_T$*-chart*. The new *EWMA-chart*s are sensitive to small parameter shifts, we presented a *EWMA* statistics based on log-likelihood ratio for plotting on the new *EWMA-chart*. The *EWMA*$_\lambda$*-chart* for detecting shift in only parameter $\square$, which is a case study of *EWMA*$_\lambda$*-chart* based on *ZIGP* process. We also presented the *EWMA*$_T$*-chart* for detecting shift in simultaneous of λ, φ and ω. The performance of *EWMA-charts* were considered from simulation of the average run length (*ARL*). We presented in-control parameter of $\square_0$ = 1 and 2, and $\square_0$ = 0.1, 0.2, 0.3 and 0.4, $\square_0$ = 1.1, and $\square_1 = \square_0 + 1$ to be detections shifts in λ is desired. The results showed that the Shewhart *c-chart* (c_Z*-chart*) is unsuitable in detecting small shift in λ. The performance of the *EWMA*$_\lambda$*-chart* is most suitable for detecting small changes in λ. Moreover, the *EWMA*$_T$*-chart* is the best for monitoring of simultaneous shifts in λ, φ and ω of *ZIGP* processes. Finally, we presented case study to illustrate for used in a manufacturing process.

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