รายงานวิจัยฉบับสมบูรณ์

โครงการ การออกแบบระบบรองรับหัวอ่านฮาร์ดดิสก์แบบหลายฟังก์ชันเป้าหมายโดยใช้วิธีการ OMPBIL และวิธีแผ่นพื้นหลายระดับ

โดย ผศ.ดร. สุวิน สลีสองสม

พฤษภาคม 2563

สัญญาเลขที่ MRG6080148

รายงานวิจัยฉบับสมบูรณ์

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ผู้วิจัย

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สนับสนุนโดยสำนักงานคณะกรรมการการอุดมศึกษา และสำนักงานกองทุนสนับสนุนการวิจัย

กิตติกรรมประกาศ

ผู้วิจัยขอขอบคุณ สำนักงานคณะกรรมการการอุดมศึกษาและสำนักงานกองทุนสนับสนุนการวิจัย สำหรับทุนสนับสนุนการวิจัย ขอขอบพระคุณ ศาสตราจารย์ ดร. สุจินต์ บุรีรัตน์ ที่ได้ช่วยสละเวลาอันมีค่าเพื่อเป็นนักวิจัยที่ปรึกษา พร้อมทั้งให้คำปรึกษาที่เป็นประโยชน์ต่อการดำเนินงานวิจัย สุดท้ายนี้ขอขอบคุณครอบครัวที่คอยสนับสนุนและให้กำลังใจ

สารบัญ

บทคัดย่อ

รหัสโครงการ: MRG6080148

ชื่อโครงการ: การออกแบบระบบรองรับหัวอ่านฮาร์ดดิสก์แบบหลายฟังก์ชันเป้าหมายโดยใช้วิธีการ **OMPBIL** และวิธีแผ่นพื้นหลายระดับ

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บทคัดย่อ:

งานวิจัยนี้มีวัตถุประสงค์สามประการดังนี้ วัตถุประสงค์ที่หนึ่งคือต้องการนำเสนอวิธีการกริดหลายระดับสำหรับการหาโทโปโลยีเหมาะสมสุดของโครงสร้าง และขยายการศึกษาไปยังการหาโทโปโลยีเหมาะสมสุดแบบหลายฟังก์ชันเป้าหมายที่น่าเชื่อถือ และวัตถุประสงค์สุดท้ายนำเสนอการประยุกต์ใช้วิธีการที่ได้พัฒนาขึ้นกับปัญหาการหาโทโปโลยีเหมาะสมสุดของระบบรองรับหัวอ่านฮาร์ดดิสก์แบบหลายฟังก์ชันเป้าหมายจากการศึกษาพบว่าการใช้วิธีการกริดหลายระดับร่วมกับการหาโทโปโลยีเหมาะสมสุดของโครงสร้างให้ผลดีกว่าเมื่อเทียบกับวิธีการโทโปลีเหมาะสมสุดที่ใช้กริดระดับเดียว ยิ่งไปกว่านั้นวิธีการที่พัฒนาขึ้นที่เรียกว่าวิธีการหาโทโปโลยีเหมาะสมสุดแบบหลายฟังก์ชันเป้าหมายที่น่าเชื่อถือร่วมกับแบบจำลองแบบฟัสซี่ยังให้ผลการออกแบบที่มีความปลอดภัยมากกว่าวิธีการดั้งเดิม สุดท้ายผลการประยุกต์ใช้วิธีการ OMPBIL ร่วมกับวิธีการกริดหลายระดับกับปัญหาการสังเคราะห์โครงสร้างเหมาะสมสุดของระบบรองรับหัวอ่านฮาร์ดดิสก์ก็ให้ผลดีเช่นเดียวกัน

คำหลัก : การหาโทโปโลยีเหมาะสมสุด, ขั้นตอนวิธีวิวัฒนาการแบบหลายฟังก์ชันเป้าหมาย, วิธีการกริดหลายระดับ, ความน่าเชื่อถือ, ความไม่แน่นอน, ฟัสซี่เซต, ระบบรองรับหัวอ่านฮาร์ดดิสก์

Abstract

Project Code : MRG6080148

Project Title : Multiobjective optimization of a HDD suspension system using opposition-based population-based incremental learning and a Multi-grid approach

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Abstract:

This research has three objectives. Firstly, a multi-grid design approach for optimization of structural topology optimization is proposed. This idea proposed for solving a problem about grid or ground element resolution in structural topology optimization, which can help the designer to choose the best grid resolution at same time with finding the topology optimization. The design processes can be fulfilled by using multiple resolutions of ground elements, which is called a multi-grid approach. Secondly, the propose technique is extended to a multi-objective reliability-based topology optimization (MORBTO) for structural design, which considers uncertain structural parameters based on a fuzzy set model. The new technique is established in the form of multi-objective optimization where the equivalent possibilistic safety index (EPSI) is included as one of the objective functions along with mass, and compliance. This technique can reduce complexity due to a double-loop nest problem used previously due to performing single objective optimization. Finally, the propose approach is used to design a hard disk drive suspension (HDD), which also gives very good results.

Keywords : Topology optimization; multi-objective evolutionary algorithm; multi-grid design approach; reliability; uncertainty; fuzzy set; hard disk drive suspension

Chapter I
Executive Summary

1.1 Rationale of the study

This research has an extension from the previous wok by the authors and Thailand Research Fund (TRG5780123) to proposed the Opposite-based multiobjective population-based incremental learning (OMPBIL) and a Multi-grid approach for solving the structural topology optimization problems as a first aim. The second aim is developed the previous technique by combining a reliability analysis into topology optimization using a fuzzy set model. The new technique can reduce the complexity in analysis of triple-loop nest problem of the reliability-based topology. The new technique is established in the form of multi-objective optimization where the equivalent possibilistic safety index (EPSI) is included as one of the objective functions along with mass, and compliance. The third aim is an attention to use the method for design a topological design of a hard disk drive (HDD) suspension. For the first aim causes from the pre-process stage, when using a ground element approach, is what the best ground element resolution for a design problem should be. As a result, using several sets of ground segments at the same time when performing optimization is investigated and it is termed a multi-grid design approach. The propose technique has an expected to increase the performance in design topology of structures when it combines with the opposite-based concept. For the second aim, it has been found in multidisciplinary design optimization their design results are never used in practical due to the uncertainties of mechanical property of the material that used in the optimization design. The well-known technique that considers a reliability in the design optimization problem, which is called reliability-based topology optimization (RBTO). In the past is known that such problem is very complex in analysis due to it is triple-loops nest problem. Then this aim of this research is to reduce the complication of the double-loop nest problem in RBTO using multi-objective optimization technique with fuzzy uncertainties. From the fist and the second aim are reason of the third aim to integrate the propose techniques for designing a topology optimization of a hard disk drive (HDD) suspension. This problem is a multiobjective optimization problem, which has an objective are the maximizing the first sway mode natural frequency and minimizing bending stiffness of the suspension. Design constraints include the first torsion and bending modes frequencies of a structure. Structural analysis is carried out by using a finite element procedure.

1.2 Objective of the research

1.2.1 To apply the opposite-based multiobjective population-based incremental learning in structural design

1.2.2 To propose the new technique in reliability-based topology optimization.

1.2.3 To apply the propose technique in design of a hard disk drive suspension.

1.3 Scope of the research

1.3.1 Algorithm will be coded by the MATLAB program.

1.3.2 Structural design cases are coded in two-dimension.

1.3.3 Only a HDD suspension model is coded in three-dimension.

1.3.4 Optimizer is used in this research is in group of multiobjective evolutionary algorithm (MOEA).

Chapter II

Topology Optimisation Using MPBILs and Multi-Grid Ground Element

2.1 Introduction

The first question that always arises at pre-process stage, when using a ground element approach for topology optimization, is: What the best ground element resolution for a design problem should be? As a result, we investigate using several sets of ground elements when performing optimization, which we term the multi-grid design approach (MG). The MG approach is an extension of ground segment strategy, which has been proposed to solve a truss structural optimization problem [1,2] and morphing wing structural optimization problem [3].

The second question arises due to an opposition-based concept that could potentially improve the search performance of the evolutionary algorithm (EA) [4–7]; the multi-objective population-based incremental learning (MPBIL) was the best optimizer [8]. Additionally, it has been demonstrated that the opposition-based concept could improve population-based incremental learning (PBIL) performance for a single objective, which is called the opposition-based concept PBIL(OPBIL) [9], whereas the multi-objective optimization is called opposite-based, multi-objective, population-based incremental learning (OMPBIL) [3]. PBIL is categorized as an estimation distribution algorithm (EDA), which is still in the spotlight of many researchers due to this kind of algorithm being simple to adapt and apply for a single- and multi-objective optimization problem [10–13]. From our previous work, OMPBIL with a multi-grid approach has been used to solve partial topology optimization of morphing aircraft wings, and it promotes better results than the original multi-objective population-based incremental learning (MPBIL) with a single grid element. Moreover, the work reveals that the opposition concept could improve the search performance of MPBIL. The question remains whether the performance of OMPBIL can benefit from the opposite concept or two learning rates. To make it be clearer, we compare the performance of OMPBIL and the performance of MPBIL with multi-learning rate. If the former technique can achieve better results, it means that the opposition concept significantly improves the performance of MPBIL. Therefore, this question will be addressed in this study. Furthermore, it has been found [14] that learning rate was the most affective with search performance of PBIL. Another way to improve the search performance of MPBIL is to use an adaptive learning rate method [15]. This method is categorized as self-learning adaptations, so the effectiveness of this technique needs to be addressed in this research.

Therefore, in this chapter, the first objective is to apply the multi-grid approach (MG) approach to solve structural topology optimization problems, whereas the second objective is comparative performance of the three variants of MPBIL. The performance improvements are based on an opposite-based concept, a multi-learning rate, and an adaptive learning rate, respectively. This research expects to improve the performance of the proposed MPBIL and MG approaches that lead to the obtaining of better design results than the original MPBIL with a single grid. The rest of this paper is organized as follows. Section 2.2 promotes the details of topology with single-ground and multi-ground design approaches for structural topology optimization. We introduce some novel methods for enhancing the performance of multi-objective, population-based incremental learning in Section 2.3. The performance index and statistical testing are given in the same section. Numerical experiments and the design results are proposed in Section 2.4; moreover, the design results and discussion are in Section 2.5. Finally, the conclusions of the study are in Section 2.6.

2.2 Topological Designs with Single-and Multi-Ground Design Approaches

2.2.1. Topological Designs with Ground Element Filtering

Topology optimization is one mathematical tool used in the conceptual design stage of engineering systems for finding the best structural layout from a given design domain. Topological design can perform using an optimization method and finite element analysis. This technique is started by defining design domain represented as the discrete structural members such as panels, truss, and frame as shown in Figure 2.1. The optimization method can be performed by varying the width or thickness of each element in the design domain between zero and the maximum value. All elements were discarded, if the element width/thickness value was zero. Otherwise, the element was retained. With this concept, optimization of the structural layout and component sizes is performed. Two popular, well known topological methods are the solid isotropic material with penalization (SIMP) approach and the homogenization method, which use gradient-based optimizers. Later, an alternative optimizer is evolutionary algorithms due to the fact they are robust, simple to use, derivative-free, and free from intermediate pseudo densities [8]. Complicated problems, such as partial topology, simultaneous topology, shape, and sizing optimization, can be performed within one optimization run [3,8,16,17] by using such algorithms. In this paper [8], they presented the comparative performance of multi-objective evolutionary algorithms (MOEAs) for solving structural topology optimization test problems based on ground element filtering technique. It has been found that MPBIL is the best optimizer in their study, which outperforms other MOEAs [8], so MPBIL is the only MOEA selected to improve its search performance in this research. Furthermore, the ground element filtering technique is also used in this study. The ground element filtering technique (GEF technique) is a simple numerical scheme that can apply to all kinds of optimizers, which can prevent the checkerboard pattern problem and at the same time decrease the number of design variables [8,18,19]. The idea uses two mesh grids of design domain with different resolutions. The lower resolution grid is provided for design variables, whereas the higher resolution is used as a finite element grid. The conversion between two grids relates to threshold value (ε) that is defined at the first time before optimization run. Therefore, this technique has been proved to be an efficient technique to suppress the checkerboard problem. Next, the details of GEF technique are seen in [8,18,19]. Later, a method for solving checkerboard pattern was presented by Guirguis and Aly [20]. They proposed that derivative-free level-set method for solving structural topology can solve the checkerboard problem. This new technique can avoid the main limitations of non-gradient methods: dependence on the objective value. Moreover, the boundaries of structure are smooth, but it does not directly depend on the decision variables. A very recent work in multi-objective topology optimization has been proposed to address the limitations of generating infeasible structures and expensive computational cost by using the technique called "graphics processing unit (GPU)" [21]. On the contrary, this technique has been commented on usefulness in the case of truss-like structures and the solved examples are simple, and obtained results are sub-optimal solutions [22]. Recent applications of topology optimization appeared in design of composite molding processes [23]. More recently, applications of topology optimization appeared in many fields, e.g., composite molding processes [23], optimal design of piezoelectric [24], phononic crystals design [25] and stator configurations [26].

2.2.2. Single-and Multi-Grid Ground Elements

The MG approach for topology optimization is an extension of MG strategy, which proposes to solve a truss structural optimization [1] and morphing wing structures [3]. At the present, we propose to apply this technique to a structural topology optimization problem.

This technique has an improvement in both using the several ground resolutions. In this research, a ground structure has four sets of ground elements with different grid resolutions and the threshold value ε. The threshold value ε must be specified at the first stage before performing the optimization run. A special encoding and decoding scheme slightly changes from the previous work [3], but it is very important to the quality of final result. Especially, the threshold values are different in each grid resolution to prevent the checkerboard problem, which can occur in each grid. At the first stage, this scheme starts with defining the number of elements and the threshold values. The first set of ground elements has N_{11}elements, and the threshold value is set to be $\varepsilon 1$. Therefore, an example of a ground element set used in this study is the lowest resolution as number of elements N_{11}= 48 and $\varepsilon 1$= 0.07 as shown in Figure 2.1. The second set has N_{21} = 75 elements and the threshold value is $\varepsilon 2$ = 0.2. Then, the third set has N_{31} = 108 segments and the threshold value is $\varepsilon 3$ = 0.3, whereas the last set has the numbers of ground elements and the threshold value isN_{41}= 147 segments and$\varepsilon 4$ = 0.35, respectively. As a result, $N_{41} \geq N_{31} \geq N_{21} \geq N_{11}$ and $\varepsilon 4 \geq \varepsilon 3 \geq \varepsilon 2 \geq \varepsilon 1$, respectively. Therefore, the variables and the threshold values for encoding/decoding scheme for the MG approach, which is improved from previous algorithm, can be detailed as shown in Algorithm 2.1. For using this algorithm, the MPBIL and its improved versions perform with binary design variables, whereas it needs the conversion of binary string to become a real design vector x before entering into this algorithm. Furthermore, the ground element set with its ε used in this research for multi-objective optimization problem (MOP) MOP1, MOP3 and MOP4 is shown in Figure 2.1. For the design problem MOP2, the design domain is different from the other problems. The details of the ground element sets and the threshold values are presented in Section 4.

Algorithm 2.1. Encoding and decoding scheme for a MG approach.

Initialization: Generate four sets of ground elements and define the threshold value of ε for each set.

Inputx sized (N_{41} + 1) × 1.

Output: Thicknesses of ground elements.

Encoding

$x_1 \in$ [1, 4] is used for selecting a set of ground elements.

x_2 to $x_{N_{41}+1}$ are used for element thicknesses.

Decoding

1: Find $n = round(x_1)$ where $round(.)$ is a round-off operator.

2: If n = 1: x_2 to $x_{N_{41}+1}$ are set as N_{11} element thicknesses and ε=ε1.

3: If n = 2: x_2 to $x_{N_{41}+1}$ are set as N_{21} element thicknesses and ε = ε2.

4: If n = 3: x_2 to $x_{N_{41}+1}$ are set as N_{31} element thicknesses and ε = ε3.

5: If n = 4: x_2 to $x_{N_{41}+1}$ are set as N_{41} element thicknesses and ε = ε4.

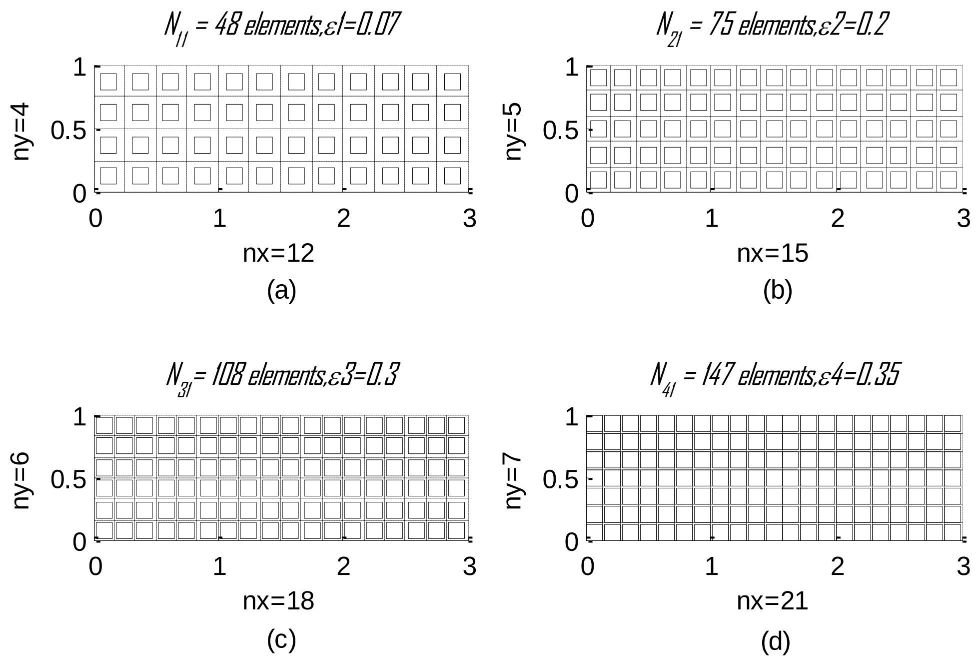


Figure 2.1.A ground elements set for MOP1, MOP3, and MOP4 (a) for n=1, (b) for n=2, (c) for n=3, and (d) for n = 4.

2.3. Performance Enhancements of Multi-Objective, Population-Based Incremental Learning

This section briefly details the concept of MPBIL and its three variants.

2.3.1. Multi-Objective, Population-Based Incremental learning

MPBIL is an extension of PBIL for solving a multi-objective optimization problem. This problem has more than one objective function, which promotes several solutions for this kind of problem, and it is called a Pareto solution set or a Pareto frontier. Rather than using a single probability vector, several probability vectors are used, so it is called a probability matrix. The matrix is used to maintain diversity of a binary population. At an initial step, the probability matrix has elements full of “0.5”. Each row of the probability matrix or probability vector is updated by Hebb’s rule [27]as follows

$$P_{ij}^{\text{new}} = P_{ij}^{\text{old}}(1 - LR) + b_j L_R \tag{1}$$

in which L_R is a PBIL learning rate, a small value usually recommends for the conventional operating [28], and b_j is the mean value of *j*th column of several binary solutions randomly selected from a current Pareto front. It is also useful to apply a mutation to probability matrix at some predefined probability as

$$P_{ij}^{new} = P_{ij}^{old}(1 - ms) + rand(0\,or\,1) \cdot ms \tag{2}$$

in which *ms* is mutation shift, and the default value is usually 0.2. For more details of MPBIL procedure, see [3].

2.3.2. Opposite-Based MPBIL

OMPBIL has been developed as an improved version of MPBIL [3]. Due to L_R affecting MPBIL performance, the issue is how to select a proper value of L_R for a general problem. It is expected to accelerate the convergence rate to find solution, as well as provide population diversity. Our previous work proposed the opposition-based concept embedded into MPBIL, which is an efficient technique that can upgrade MPBIL’s performance. Therefore, the outline of OMPBIL algorithm includes the opposition-based concept, which is not included in this paper. More details can be found in [3].

2.3.3. Multi-Learning Rate

The second approach to enhance the performance of MPBIL is the use of multi-learning rate. This question arises from the previous method, when it is using two learning rates that are of an opposite quantity. The question remains whether the performance of OMPBIL can benefit from the opposite concept or by using two learning rates. MPBIL with multi-learning rate (MPBILMLR) is proposed to solve topological optimization and to compare with the opposition-based concept. This algorithm differs from the traditional MPBIL by using three learning rates (L_R=0.25, 0.5, 0.75). The procedure of MPBILMLR algorithm is slightly different from OMPBIL. Therefore, the procedure of MPBIL with multi-learning rate algorithm is shown in Algorithm 2.2.

Algorithm 2.2. MPBIL with multi-learning rate.

Initialization Probability matrix **P**= $[\mathbf{0.5}]_{l\times nb}$, Probability matrix **P*i***= $[\mathbf{0.5}]_{l/M\times nb}$ where ***i*** = 1, ..., M = 3, external Pareto archive **Pareto**= {}.

1: Generate a binary population **B** from **P**.

2: Decode the binary population to be $\mathbf{x}_{n\times Np}$ and find the objective values $\mathbf{f}_{m\times Np}$.

3: Update **Pareto**by replacing it with non-dominated solutions of union set **Pareto** $\cup$ **x**.

4: If the number of members in **Pareto**exceeds the predefined archive size N_A, remove some of them by using an archiving technique.

5: If the termination criterion is fulfilled, stop the procedure. Otherwise, go to step 6:

6: Update **P** and create a binary population

6.1: Set a binary population **B** = {}.

6.2: For *i*= 1 to *l*/M.

6.2.1: Select n_0 binary solutions from **Pareto**randomly.

6.2.2: Use L_{Rk} = 0.25, 0.5, 0.75, for each *k* = 1, ..., M. (For this research M = 3)

6.2.3: Update the *i*th row of **P** by using (1).

6.2.4: Generate the *i*th row of probability matrix **P*i*** using (2) and each L_{Rk}.

6.2.5: Generate *rand* $\in$ [0,1] a uniform random number.

6.2.6: If *rand* < the predefined mutation probability, update the *i*th row of **P1**, **P2** and**P3**using similar equation in [3].

6.2.7: Generate binary subpopulations **SB1**, **SB2** and **SB3** from the *i*th row of **P1**, **P2** and **P3,** respectively.

6.2.8: Set **B** = **B**$\cup$**SB1**$\cup$**SB2**$\cup$**SB3**

6.3: Next *i*.

7: Go to step 2.

2.3.4. Adaptive Learning Rate

The last method for MPBIL performance enhancement is using an adaptive learning rate, which proposes to modify the learning rate during the entire process [28]. A small value of learning rate is usually recommended for conventional PBIL to keep the algorithm reliable, but it usually causes low convergence rate. To balance the reliability and speed of convergence in all iterations, the learning rate needs to adapt. A model of adaptive learning rate has been proposed by Yang et al. [15] that satisfies the previous conditions. That model is shown as follows

$$L_R = L_{R0} + (L_{RM} - L_{R0})e^{-(\frac{SI}{NT})} \quad (3)$$

in which *SI* is the successive iterations with improvements in the objective function in the most recent *NT* iterations. L_{R0} and L_{RM} are the minimum and maximum learning rates that the designer defines before an optimization run. The learning rate depends on the ratio of *SI/NT*. Additionally, the high value of this ratio means that it is possible to locate better solutions using its current probability matrix, and consequently the learning rate should be small. In contrast, a low value of this ratio means the current probability matrix which is insufficiency, so the learning rate should be increased. Moreover, the outline of multi-objective, population-based incremental learning with adaptive learning rate (MPBILADLR) is slightly different from the traditional MPBIL, which uses Equation (3) to replace the original equation for finding L_R. This algorithm is shown as follow.

Algorithm 2.3.MPBIL with adaptive learning rate.

Initialization probability matrix **P**= [**0.5**]$_{l \times nb}$, external Pareto archive **Pareto**= {}.

1: Generate a binary population **B**from **P.**

2: Decode the binary population to be $\mathbf{x}_{n \times Np}$ and find the objective values $\mathbf{f}_{m \times Np}$.

3: Update **Pareto**by replacing it with non-dominated solutions of union set **Pareto**$\cup$**x**.

4: If the number of members in **Pareto**exceeds the predefined archive size N_A, remove some of them by using an archiving technique.

5: If the termination criterion is fulfilled, stop the procedure. Otherwise, go to step 6:

6: Update **P**.

6.1: For *i*= 1 to *l*.

6.1.1: Select n_0 binary solutions from **Pareto**randomly.

6.1.2: Generate L_R using (3).

6.1.3: Update the *i*th row of **P**by using (1).

6.1.4: Generate *rand* $\in$ [0,1] a uniform random number.

6.1.5: If *rand* <the predefined mutation probability, update the *i*th row of **P**using similar equation in [3].

6.2: Next *i*.

7: Go to step 1.

2.3.5. The Performance Index and Non-Parametric Statistical Test

MPBIL and its enhanced versions are classified as MOEAs, while the obtained results are classified as approximate Pareto optimal frontiers. In comparing the searching performance of MOEAs, the methods are employed to solve design optimization problems with equivalent total number of function of evaluations for number of attempts. The approximate Pareto frontiers obtained from various MOEAs are then compared using a performance indicator, which is called a hyper-volume (HV) [29] indicator. This indicator represents the hyper-area above a Pareto frontier for bi-objective optimization problem as shown in Figure 2.2, whereas it is called hyper-volume for three objective functions and more. Therefore, HV sums up all discrete areas v_i or volumes of hyper-areas or hyper-volumes with respect to a given referent point, respectively.

A technique for comparing the performance of each MOEA in this research is a non-parametric statistical test, which is called the Friedman test. This technique has been used by Sleesongsom and Bureerat [30] for studying the performance of meta-heuristics (MHs) in solving the four-bar linkage path generation problems. The Friedman test is suitable for comparing more classifiers over multiple data sets.

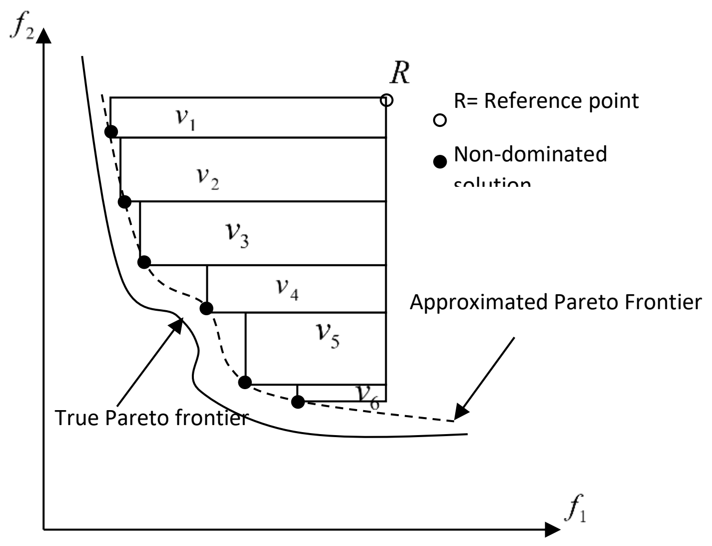


Figure 2.2. Hypervolume sums up all areas covered by the non-dominated solutions and a reference point.

2.4. Numerical Experiment

As mention earlier, the purpose of this research is to study the comparative performance of original MPBIL and its three variants with (WMG) and without the MG (WOMG) approach. Four design problems are used for testing performance of the proposed methods. The original MPBIL and three performance enhancements of MPBIL (OMPBIL, MPBILMLR, and MPBILADLR) are employed to solve multi-objective topology optimization problems that have been detailed in the previous section. Each algorithm is used to solve an optimization problem for 25 runs to measure its performance and consistency. For all design problems, all the algorithms are used with a population size of 35 and an iteration number of 400 whereas the external Pareto archive size is set to be 35 Non-dominated solutions obtained, so at the last iteration approximates the Pareto solutions. Therefore, four multi-objective problems are used for testing performance of MPBIL and performance enhancements of MPBIL, which has been proposed to study the comparative performance of some established multi-objective evolutionary algorithms (MOEAs) [8].The problems are structural topology optimization problem. The design problems are as follows:

MOP1: The topological design domain and loads are shown in Figure 2.3a. The structure is made of material with Young's modulus $E = 200\times10^9$ N/m^2, Poisson's ratio $\boldsymbol{\nu} = 0.3$, and tensile yield strength $\boldsymbol{\sigma}_{yt} = 200 \times 10^6$ N/m^2. The multi-objective design problem is set to minimize structural compliance and normalized mass as:

$$\min_{\rho^{GEF}} \{c, r\} \tag{5}$$

subject to

$$c \le 5c_{\min}$$

$$0.2 \le r \le 0.8$$

$$\boldsymbol{\rho}_i \in \{0.0001, 1\}$$

where $\boldsymbol{\rho}_i^{GEF}$ is the value of ith design variable; $\boldsymbol{\rho}_i$ is the thickness of i th finite element; m is the structural mass; $r = m(\boldsymbol{\rho})/m(\boldsymbol{\rho}^u)$ is the normalized mass or ratio of structural mass to maximum mass; c is the structural compliance; and $c_{\min} = c(\boldsymbol{\rho}^u)$. The first constraint is added to prevent topologies with a low global stiffness (or highly compliant structures) being included in the Pareto archive. The bound constraints are set as $\boldsymbol{\rho}_i \in \{0.0001, 1\}$. The parameter $\boldsymbol{\varepsilon}$ is set 0.3

and $[0.08, 0.1, 0.25, 0.3]^T$ for all MPBILs with WOMG and WMG design approach, respectively. The number of elements for single grid is set as highest resolution. A set of MG elements is use for this problem and show in Figure 2.1. The mean hypervolumes of the fronts of MOP1 for all optimization runs are given in Table 2.1, where the referent point for computing hypervolumes is set to be $\{2.5 \text{ kNm}, 2.5\}^T$.

MOP2: The second design problem promotes three objective functions, where the design domain and load illustrate in Figure 2.3b. The structure makes up the same material as MOP1. The multi-objective design problem can be written as:

$$\min_{\rho^{GEF}} \{c1, c2, r\} \tag{6}$$

subject to

$$c1 \leq 5c_{1,\min}$$

$$c2 \leq 5c_{2,\min}$$

$$0.2 \leq r \leq 0.8$$

$$\boldsymbol{\rho}_i \in \{0.0001, 1\}$$

where c_1 is the structural compliance due to the first load case and c_2 is the structural compliance due to the second load case, $c_{1,\min} = c_1(\boldsymbol{\rho}^u)$, and $c_{2,\min} = c_2(\boldsymbol{\rho}^u)$. A number of ground elements set, which uses in this study is [48,63,108,130], while the number of element for single grid is set as the highest resolution. The threshold parameter $\boldsymbol{\varepsilon}$ is set to be 0.35 and $[0.07, 0.2, 0.3, 0.35]^T$ for all MPBILs with WOMG and WMG approach, respectively. There are different from other problem due to the difference of design domain. The mean hypervolumes of the fronts of MOP2 for all optimization runs are given in Table 2.1, in which the referent point for computing the hypervolumes is set to be $\{1.5 \text{ kNm}, 1.5 \text{ kNm}, 1.5\}^T$.

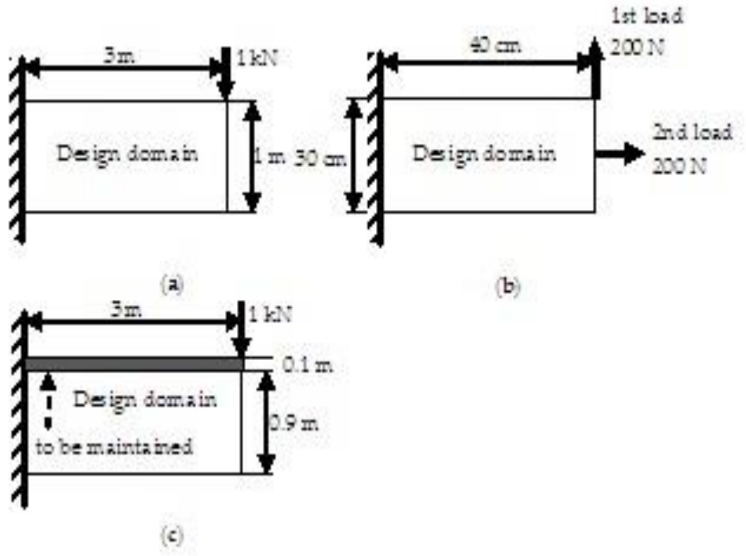


Figure 2.3. Structural design domains:(**a**) design domain of MOP1 & MOP3, (**b**) design domain of MOP2, (**c**) design domain of MOP 4 [8].

MOP3: The design problem has the same design conditions as set for MOP1 with the exception of the range of design variables. Addition constraints are $\sigma_{\max}^{eqv}$ ≤ **σ**yt and $\boldsymbol{\rho}_i \in$ {0.000001m, 0.01m}, in which $\sigma_{\max}^{eqv}$ is the maximum value of Von Mises stress (equivalent stress) on the ground elements. A set of MG elements is shown in Figure 2.1, and the number of elements for single grid is set as the highest resolution. In addition, the threshold parameter **ε** is set to be 0.3 and $[0.08, 0.1, 0.25, 0.3]^T$ for all MPBILs with WOMG and WMG design approach, respectively. The mean hypervolumes of the fronts of MOP3 for all optimization runs are given in Table 2.1, in which the reference point for computing the hypervolumes is set to be $\{3.5$ kNm, $3.5\}^T$.

MOP4: The design conditions of MOP4 are similar to MOP3, except in this design problem the top row finite elements are not assigned as design variables (unchanged) as displayed in Figure 2.3c, and the first objective of this problem changes to maximizing the first mode eigenvalue of structure ($\boldsymbol{\lambda}_1$). Note that all of design problems use a membrane finite element formulation for structural analysis. The number of ground elements and the parameter $\boldsymbol{\varepsilon}$ of MOP4 are similar to MOP3. The mean hypervolumes of the fronts of MOP4 for all optimization runs are given in Table 2.1, in which the referent point for computing the hypervolumes is set to be $\{1.0$ rad^2/s^2, $2.0\}^T$.

2.5. Design Results

The comparative performance of original MPBIL and the performance enhancements of MPBILs with MG and without MG approach for solving the design problems of MOP1–4 are given in Table 2.1, which compare based on HV indicator. It should be noted that all of the approximate Pareto fronts of the four design problems obtained from using the proposed MPBILs are normalized before calculating HV, as shown in Table 2.1. The highest mean of HV for each design problem is highlighted with grey color. The table shows OMPBIL promotes almost the best results for MOP1–4 except in case MOP4-WMG. Therefore, it is believed that the performance of OMPBIL is better result than the original MPBIL and their enhancements. In this study, the Friedman test and the Tukey–Kramer test are used for a statistical test to prove the significance of proposed algorithm. These tools are built-in functions in MATLAB/Octave. From our testing, the Friedman test gives OMPBIL has 1st rank, whereas the second rank is MPBIL at p-value (0.0002) $<\alpha(0.05)$ as shown in Table 2.2. It can be summarized that OMPBIL is the best performing algorithm for solving problem case MOP1–4. For multiple comparisons, we used the Tukey-Kramer test. The mean column ranks of OMPBIL are significantly different from MPBILMLR. The second best optimizer is MPBIL, whereas the third best is MPBILADLR. In addition, the worst optimizer for this design case is MPBILMLR. No questionable opposition concept is beneficial to improving the performance of MPBIL.

The average HV for all optimizers of each problem with MG and WOMG approach is shown in Table 2.3, which is summed along each column from Table 2.1. This table shows that the design problems with MG approach give higher HV than the design problem without MG approach in all design problems. Friedman test of average result in Table 2.3 can prove that the design problem with MG technique significantly outperforms WOMG technique at p-value (0.0455) $<\alpha(0.05)$. Furthermore, the best HVs of all cases give higher hypervolume than the previous work by [8] in all design cases, so OMPBIL with MG can improve the design results.

Table 2.1. Performance comparison based on hypervolume(HV)[1].

	MOP1		MOP2		MOP3		MOP4	
	WMG	WOMG	WMG	WOMG	WMG	WOMG	WMG	WOMG
MPBIL	0.8553	0.8255	0.7255	0.6420	0.7951	0.7229	0.7195	0.6219
OMPBIL	0.8556	0.8426	0.7259	0.6430	0.7968	0.7438	0.6723	0.6403
MPBILMLR	0.8115	0.7739	0.7212	0.6285	0.7016	0.5976	0.6167	0.5651
MPBILADLR	0.8543	0.8371	0.7240	0.6385	0.7954	0.7407	0.6404	0.6292

[1]WMG , with multi-grid approach; WOMG, without multi-grid approach; MPBIL, multi-objective population-based incremental learning; OMPBIL, opposite-based, multi-objective, population-based incremental learning; MPBILMLR , MPBIL with multi-learning rate;MPBILADLR, multi-objective, population-based incremental learning with adaptive learning rate.

Table 2.2. Average ranking and *p*-value of MPBIL, and enhanced performance of MPBIL achieved by Friedman test.

Average Ranking of Each Algorithm Friedman				*p*-Value
MPBIL	OMPBIL	MPBILMLR	MPBILADLR	
2.6250 (2)	3.8750 (1)	1 (4)	2.5000 (3)	0.0002

Table 2.3. Performance comparison of each MOP with and without MG for all algorithms.

Design Problems	Average Hypervolume	
	WMG	WOMG
MOP1	0.8442	0.8198
MOP2	0.7242	0.6380
MOP3	0.7722	0.7013
MOP4	0.6622	0.6141

Average Ranking (p-value = 0.0455)	2 (1)	1 (2)

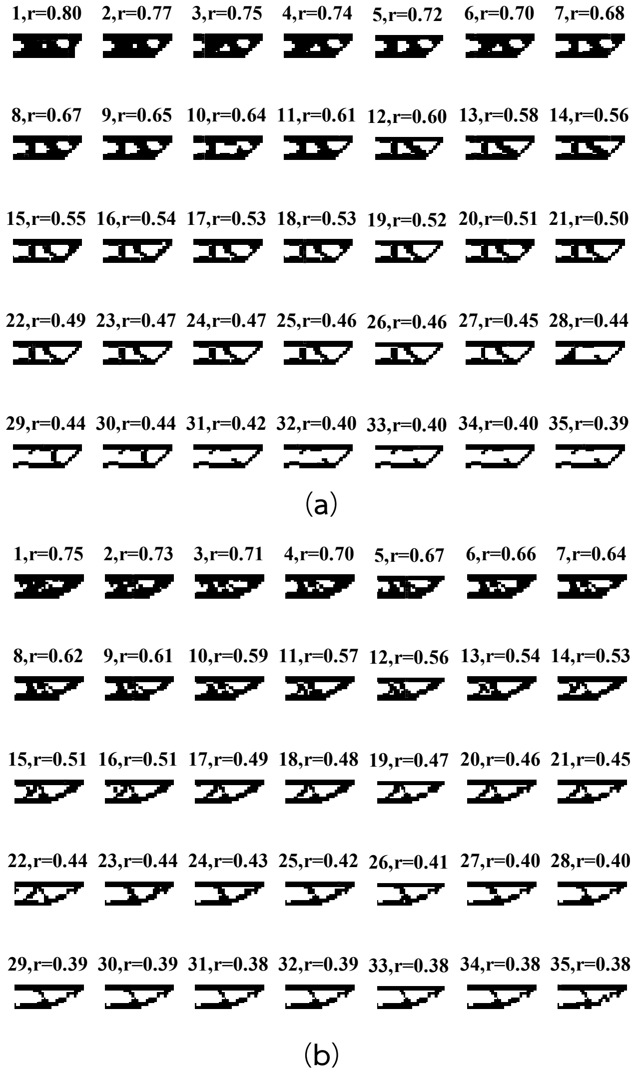
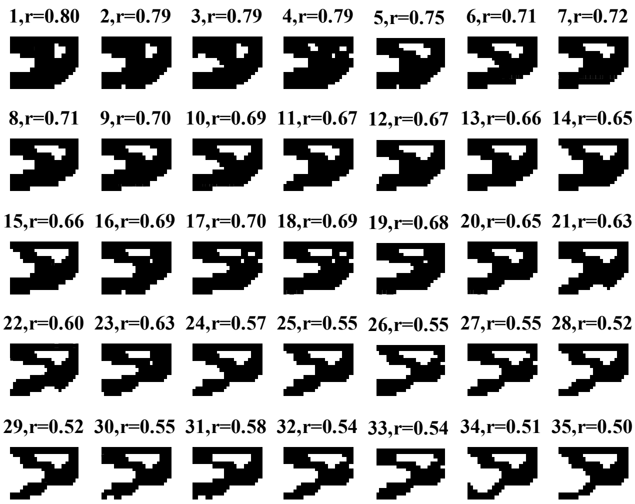
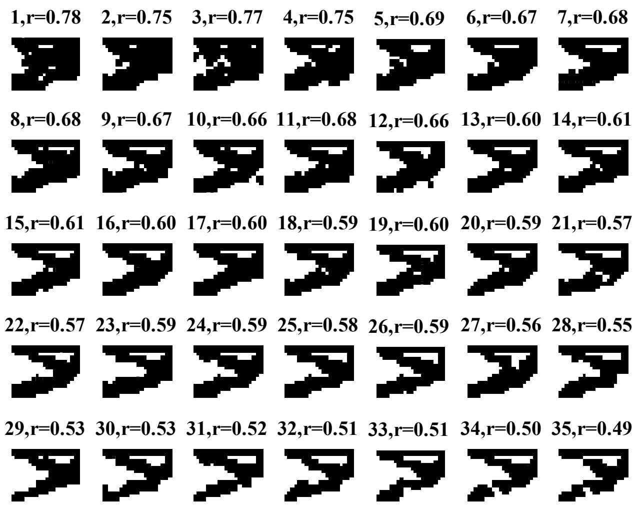


Figure 2.4. Topologies of MOP1:(**a**) OMPBIL with multi-grid, and (**b**) MPBIL without multi-grid.

Figures 2.4–2.7 shows some optimum topologies. The topologies in all figures are captioned with (a), which obtains from the best run of OMPBIL with multi-grid when solving each MOP with various r values. All figures are captioned with (b); they display the optimum topologies that are obtained from optimizing the design problem MOP1-4 with various *r* values by using MPBIL without multi-grid. These topologies are represented by the same technique from the previous work [8]. This shows that the topologies from OMPBIL with MG are better than the MPBIL technique without MG, and they can compare with the previous work using binary population-based incremental learning (BPBIL) and optimality criteria method (OCM) technique [8]. The optimum topologies are mostly from the ground elements with medium (MOP1 and MOP2) and low (MOP3, MOP4) resolutions. Therefore, the topology with the highest resolution is lower than the previous work by [8]. The lower resolution means lower computational time consumption. The use of highest ground element resolution is not the best selection for all design problems. However, in practice, a designer never knows which resolution is the most suitable for design problem, and employing the multi-grid approach is an advantage.



(a)



(b)

Figure 2.5. Topologies of MOP2:(**a**) OMPBIL with multi-grid, and (**b**) MPBIL without multi-grid.

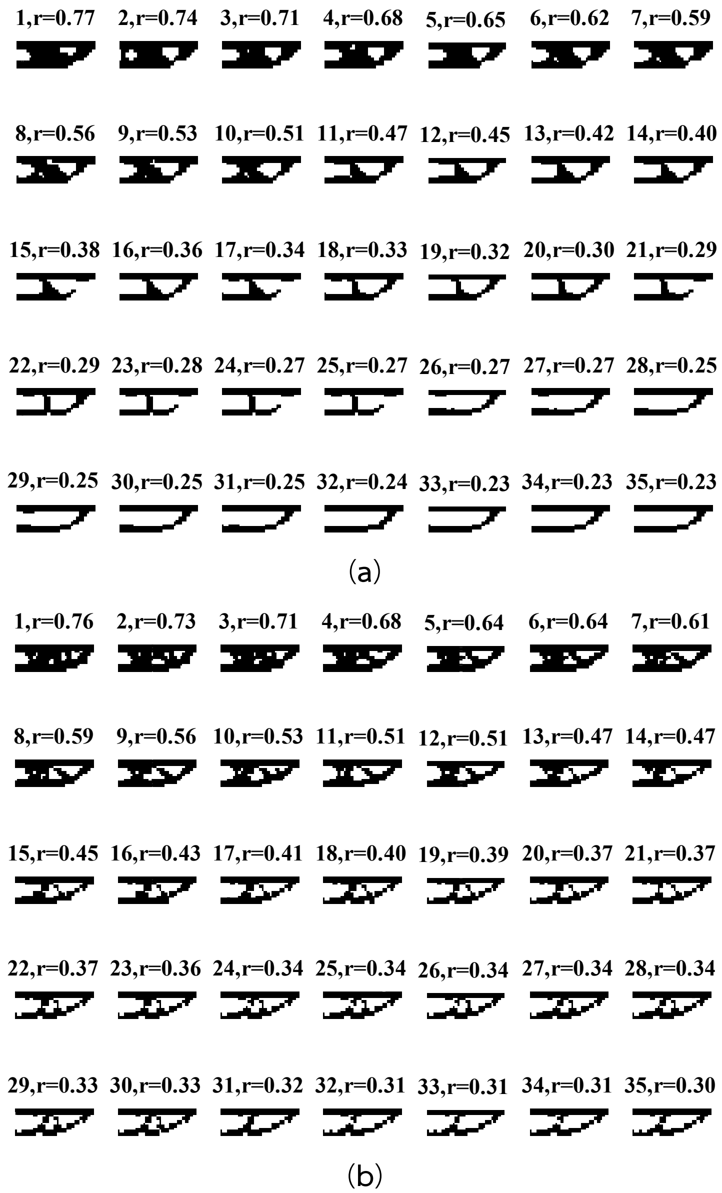
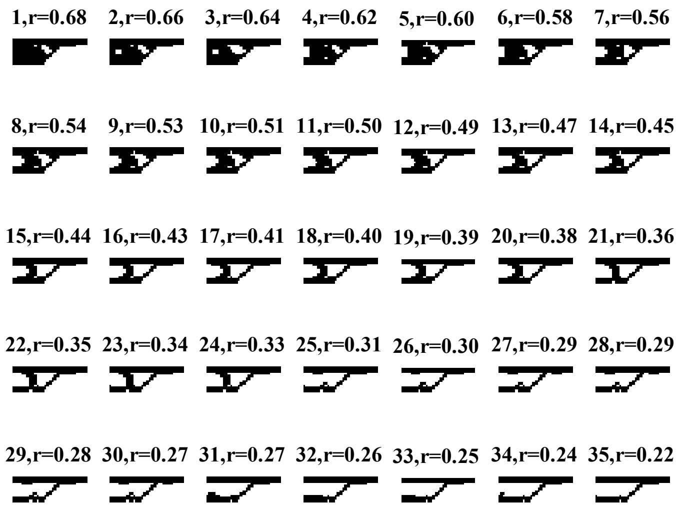
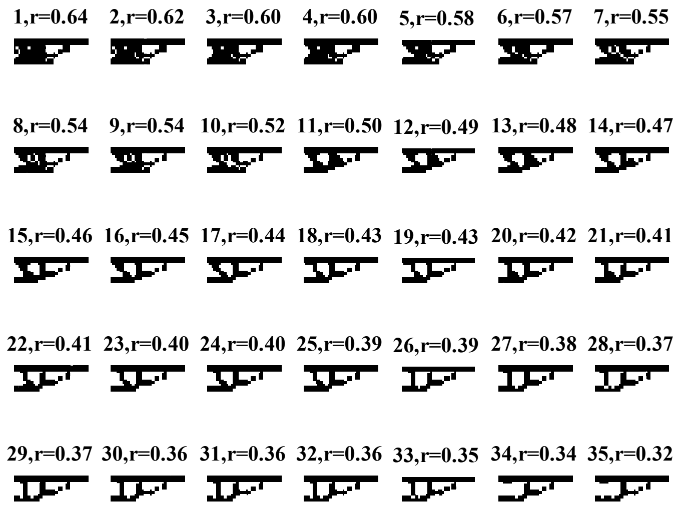


Figure 2.6. Topologies of MOP3:(**a**) OMPBIL with multi-grid, and (**b**) MPBIL without multi-grid.



(a)



(b)

Figure 2.7. Topologies of MOP4:(**a**)OMPBIL with multi-grid, and (**b**) MPBIL without multi-grid.

2.6. Conclusions

The purposes of this work are the demonstration of the performance comparison of an original MPBIL and their performance enhancement, and the MG approach for multi-objective structural topology optimization problems, respectively. Among the performance enhancements of MPBIL, OMPBIL outperforms other techniques. It promotes the opposition-based concept, which can improve the search performance of MPBIL. The use of MPBILs in combination with the MG approach is well capable of solving multi-objective structural topology optimization. The resulting topologies obtained from using OMPBIL are close to those obtained from the classical gradient-based approach. The new design strategy is a procedure for structural topology optimization, which uses multiple ground element resolutions, so the MG approach is more efficient than using single-resolution ground elements in the sense that the suitable grid resolution is automatically detected and used in one optimization run. These conclusions are very similar those obtained in our previous work [3]. In addition, the use of the MG approach combined with ground element filtering for alleviating checkerboards is effective. In future work, the proposed method is extended to solve topology optimization with uncertainty.

Chapter III

Multi-objective Reliability-based Topology Optimization of Structures Using a Fuzzy Set Model

3.1 Introduction

Design processes for derivative-free topology optimization has been developed [20, 31] though it is still far from using in reality. The design process always depends on material properties, external loading and other conditions. If uncertainties of such parameters take place, obtained deterministic optimum design results may be less reliable [32]. To address such a problem, there are two main strategies to account for uncertainties in topology optimization, robust topology optimization (RTO) [33] and reliability-based topology optimization (RBTO) [34]. The first technique is to optimize the expectation and variability of system performance with respect to uncertainties simultaneously, through which the robustness of system performance can be improved, while the second is concerned with failure probability constraints when optimizing the system performance, through which reliable optimization design can be achieved. Both methods are based on probabilistic [35] or non-probabilistic models [32, 36, 38]. The first model is the most popular due to its progress, but this technique requires precision on the statistical distribution of uncertainties. A good distribution of uncertainties usually leads to large amount of objective information, which spends more time costly in a practical conceptual design stage. In opposition to the first model, it is called non-probabilistic models where some well-known techniques of this type are anti-optimization [36] and a fuzzy set method [38]. In practical reliability-based design, there is both random uncertainty and fuzzy uncertainty [39, 40]. The fuzzy set model is an alternative technique due to it provides moderate conservative results. It is the best choice to collect the uncertainties into RBTO by using a level set to soft separation between the members and non-members of the set. It makes the model get an acceptable solution. However, disadvantage of the present RBTO is still complexity in analysis due to the combination of fuzzy set into the topology optimization problem is a triple-loop nest problem including the double loop nest in finding possibilistic safety index (PSI) and topology optimization. Later, it has been solved by using the target performance-based design approach resulting the triple loop being reduced to the double-loop nested problem [32, 41]. The target performance-based approach changes the PSI into the target performance of the i-th constraint where it is called the equivalent possibilistic safety index (EPSI) by minimizing the constraint at some level cut. This technique can add an experience of the expert opinion to select the level cut or membership level into RBTO.

The aim of this research is to reduce the complication of the double-loop nest problem in RBTO using multi-objective optimization technique with fuzzy uncertainties.

3.2. Topology optimization

The topology optimization is a mathematical problem, which aims to seek the optimal structural layout within a pre-specified design domain. The single-objective or multiobjective topology optimization problem can be expressed as

$$\text{Min } f_i(\boldsymbol{\rho}) \qquad i=1,.., M \tag{1}$$

Subject to

$$g_j(\boldsymbol{\rho}) \geq 0 \quad j = 1,2,..., N$$

$$0 < \boldsymbol{\rho}^l \leq \boldsymbol{\rho} \leq \boldsymbol{\rho}^u$$

where i = 1 is for single objective design, i > 1 is for a multiobjective problem, and $\boldsymbol{\rho}$ is the thickness of finite elements ranged between the lower limit ($\boldsymbol{\rho}^l$) and the upper limit ($\boldsymbol{\rho}^u$). For the topology optimization problem in this work, the constraint is expressed in a different form from a traditional optimization problem so as to make it more compatible with the derivation of a possibility safety index in the next section.

3.3. Fuzzy set and formulation of MORBTO

The fuzzy set theory becomes popular for optimization design of structures because its capability can describe uncertainties with helping by expert opinions. The fuzzy set can be used for describing uncertain parameters and extended to the possibility concept as detailed in [34, 41]. For a fuzzy variable with a membership function $\mu(a)$, the corresponding fuzzy set model can be expressed as

$$\Lambda = \{(a,\mu(a)) \mid a \in \Omega,\ \mu(a) \in [0,1]\} \quad (2)$$

where Ω is universal set, a is fuzzy variable. Each fuzzy variable a can be decomposed into a series of interval variables by using fuzzy set in accordance with degree of membership.

To construct the possibility of safety index, α-cut is used in this research ($\alpha \in [0,1]$) while a is in the interval $a^{\alpha} \in [a^{-\alpha}, a^{+\alpha}]$. The possibility that the fuzzy variable a is greater than s crisp number can be expressed as Pos($a \geq s$), so

$$\mathrm{Pos}(a \geq s) = \sup_{z>s} \mu(z) \quad (3)$$

where "sup" is the supremum.
From (3) and inequality constraint in (1) can write as

$$\mathrm{Pos}(g_j(\boldsymbol{\rho},\mathbf{a}) \leq 0) = \sup_{z\leq 0} \mu_j(z) \quad (4)$$

The topology optimization problem can be rearranged to be a RBTO problem based on the possibility safety index and the fuzzy set model:

$$\mathrm{Min}\ f_i(\boldsymbol{\rho}) \quad i=1,..,M \quad (5)$$

Subject to

$$\mathrm{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \leq 0) \leq \pi_{fi}^{max} \quad j = 1,2,\ldots, N$$

$$0 < \boldsymbol{\rho}^l \leq \boldsymbol{\rho} \leq \boldsymbol{\rho}^u$$

Let $\mathbf{a} = (a_1, a_2, \ldots, a_l)$ be fuzzy variables, π_{fj}^{max} is an allowable possibility index, π_f is a possibilistic safety index, and $g_j(\boldsymbol{\rho},\boldsymbol{a})$ becomes fuzzy rather than crisp. In comparing an inequality constraint in (1) and (5) where the equations are respectively represented the deterministic topology optimization and the RBTO, respectively, the constraint in the traditional problem is used to control the value of limit-state function to strictly higher than zero while the constraint in RBTO is used to control the possibility safety index value that is lower than zero and π_{fj}^{max}.

The possibility safety index can be applied to the topology optimization problem, which incorporates with the fuzzy set method to deal with the uncertainties as shown;

$$\text{Min } \{f_i(\boldsymbol{\rho}),\ \pi_f^{max}\}\ i=1,\ldots,M \tag{6}$$

Subject to

$$\pi_f^{max} = \max((\text{Pos}(g_j(\boldsymbol{\rho},a) \leq 0)) \leq 1)\quad j = 1,2,\ldots, N$$

$$0 < \boldsymbol{\rho}^l \leq \boldsymbol{\rho} \leq \boldsymbol{\rho}^u$$

New multi-objective reliability topology optimization problem is established in the possibility context.

To evaluate new objective function, the possibility safety index is derived in (4) and Figure 3.1, it is found: (1) if $g_j^{-0} \geq 0$, then $\text{Pos}(g_j(\boldsymbol{\rho},a) \leq 0) = 0$; (2) if $g_j^{-0} \leq 0 \leq g_j^{-1}$, then $\text{Pos}(g_j(\boldsymbol{\rho},a) \leq 0) = \alpha$, where $g_j^{-\alpha} = 0$; (3) if $g_j^{-1} < 0$, then $\text{Pos}(g_j(\boldsymbol{\rho},a) \leq 0) = 1$. Eq. (4) can rewrite as Eq. (7).

Its original shape is a trapezoidal, which is called a trapezoidal-shaped fuzzy set. The trapezoidal shape can degenerate its form to other shapes, such as, a triangular shape [41]. In this research, we still use the original form as shown in Figure 3.1 and the membership function of constraint function can be formulated in the following equation.

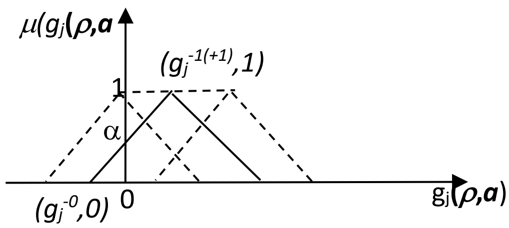


Figure 3.1. Membership function $\mu(g_i(\boldsymbol{\rho},a))$

$$\text{Pos}\ (g_j(\boldsymbol{\rho},a) \le 0) = \begin{cases} 0, & \\ \alpha,\ , \text{ where} & g_j^{-0} < 0 < g_j^{-1} \\ 1, & g_j^{-1} < 0 \end{cases} \quad g_j^{-0} \ge 0 \tag{7}$$

The solution of this equation can be calculated if g_j^{-0} and $g_j^{-1}$is known. If $g_j^{-0} \le 0$, we can obtain that Pos($g_j(\boldsymbol{\rho},a) \leq 0$) = 0 or 1, and the solution procedure can be terminated; else if $g_j^{-0} \leq 0 \leq g_j^{-1}$, the equation $g_j^{-\alpha} = 0$ should be solved, and its solution α will be the value of Pos($g_j(\boldsymbol{\rho},a) \leq 0$). Theoretically, the bi-section method uses to compute the value of Pos($g_j(\boldsymbol{\rho},a) \leq 0$), a procedure can be summarized as:

Step1: Initialization – let $\alpha_1^0 = 0$, $\alpha_2^0 = 1$, and specify the termination value as $\varepsilon = 1 \times 10^{-8}$;

Step2: Iteration 1 – Calculate $g_j^{-\alpha_1^0}$ and $g_j^{-\alpha_2^0}$, and if $g_j^{-\alpha_1^0} \ge 0$ or $g_j^{-\alpha_2^0} \le 0$ holds, we can obtain Pos($g_j(\boldsymbol{\rho},a) \leq 0$) = 0 or 1 and terminate the iterative procedure. Otherwise, to calculate $g_j^{-(\alpha_1^0+\alpha_2^0)/2}$ and go to step 3.

Step3: Iteration k (k ≥1) – if

$g_j^{-\alpha_1^{k-1}} \times g_k^{-(\alpha_1^{k-1}+\alpha_2^{k-1})/2} > 0$ holds, then let $\alpha_1^k = (\alpha_1^{k-1}+\alpha_2^{k-1})/2$ and $\alpha_2^k = \alpha_1^{k-1}$; if $g_j^{-\alpha_2^{k-1}} \times g_j^{-(\alpha_1^{k-1}+\alpha_2^{k-1})/2} > 1$ holds, then let $\alpha_2^k = (\alpha_1^{k-1}+\alpha_2^{k-1})/2$ and $\alpha_1^k = \alpha_1^{k-1}$. Go to step 4.

Step4: Termination – calculate the absolute value $\left|\alpha_2^{k-1} - \alpha_1^{k-1}\right|$, and if the termination condition $\left|\alpha_2^{k-1} - \alpha_1^{k-1}\right| \le \varepsilon$ holds, stop the iterative procedure, and estimate Pos($g_j(\boldsymbol{\rho},\mathbf{a}) \leq 0$) by Pos($g_j(\boldsymbol{\rho},\mathbf{a}) \leq 0$) = $(\alpha_1^{k-1}+\alpha_2^{k-1})/2$; otherwise, return to step 3 and continue the procedure till the termination condition $\left|\alpha_2^{k-1} - \alpha_1^{k-1}\right| \le \varepsilon$ is met.

For solving the multi-objective topology optimization problem in (6) is triple-loop nested problem, which is computational burden. The problem can be reduced to the double-loop problem by using the target performance-based approach [41] that has been proved the equivalent of the original failure possibility and the new one is described as follows.

$$\text{Pos}(g_j(\boldsymbol{\rho}, a) \leq 0)) \leq \pi_{fj}^{max} \approx \min(g_j(\boldsymbol{\rho}, a^{\pi_{fj}})) \geq 0 \tag{8}$$

where j = 1,2,..., N , and min($g_j(\boldsymbol{\rho}, a^{\pi_{fj}})$) is called the target performance of the constraint.

The equivalent of the previous topology optimization problem (6) can be changed to

$$\text{Min } \{f(\boldsymbol{\rho}), \text{EPSI}\} \tag{9}$$

Subject to

$$\text{EPSI}=\max(g_i(\boldsymbol{\rho}, a^{\pi_{fj}})) \geq 0,\ \boldsymbol{\pi}_{fj} \in [0,1],\ j = 1,2,\ldots, N$$

$$0 < \boldsymbol{\rho}^l \leq \boldsymbol{\rho} \leq \boldsymbol{\rho}^u$$

where EPSI is equivalent possibilistic safety index or target performance.

3.4. Design Examples

Two design examples demonstrate the proposed technique with the objective functions being volume fraction (r) or mass ratio and compliance (c). The objectives are conflicted as reducing mass affects to reduce the strength of structure. The difficulty of our objective is to find minimum mass ratio and compliance at the same time. The multi-objective optimization problem can be formulated and termed MOP1 and MOP2. The second problem (MOP2) is MOP1 with a stress constraint being added.

MOP1:

$$\min_{\rho^{GEF}} \{c, r\} \tag{10}$$

Subject to

$$5c_{\min} - c \geq 0$$

$$0.8 \geq r \geq 0.2$$

$$\boldsymbol{\rho}_i \in \{0.0001\text{m},\ 1\text{m}\}$$

MOP2:

$$\min_{\rho^{GEF}} \{c, r\} \tag{11}$$

Subject to

$5c_{\min} - c \geq 0$

$0.8 \geq r \geq 0.2$

$\boldsymbol{\sigma}_{yt} - \boldsymbol{\sigma}_{\max}^{eqv} \geq 0$

$\boldsymbol{\rho}_i \in \{0.000001\text{m}, 0.01\text{m}\}$

where $\boldsymbol{\rho}_i^{GEF}$ is the value of *i*th design variable; $\boldsymbol{\rho}_i$ is the thickness of *i*th finite element; *m* is the structural mass; $r = m(\boldsymbol{\rho})/m(\boldsymbol{\rho}^u)$ is the normalized mass or ratio of structural mass to maximum mass; *c* is the structural compliance; and $c_{\min} = c(\boldsymbol{\rho}^u)$. $\boldsymbol{\sigma}_{\max}^{eqv}$ is the maximum value of Von Mises stress of the ground element. The last constraint in the design problem is bound constraints.

The traditional RBTO combined with the fuzzy set model can be formulated as in Eq. (5). It can be expressed as follows.

$$\min_{\rho^{GEF}} \{c, r\} \tag{12}$$

Subject to

$\text{Pos}(g_j(\boldsymbol{\rho}, a) \leq 0) \leq \pi_{fi}^{max} \in [0,1] \quad j = 1,2,\ldots, 3$

$g_1 = 5c_{\min} - c \geq 0$

$g_2 = 0.8 \geq r \geq 0.2$

$g_3 = \boldsymbol{\sigma}_{yt} - \boldsymbol{\sigma}_{\max}^{eqv} \geq 0$ for MOP2

$\boldsymbol{\rho}_i \in \{0.0001\text{m}, 1\text{m}\}$ for MOP1

$\boldsymbol{\rho}_i \in \{0.000001\text{m}, 0.01\text{m}\}$ for MOP2

The proposed MORBTO incorporates the fuzzy set model from the previous section into the topology optimization to deal with the uncertainties in real situation can be formulated as:

$$\min_{\rho^{GEF}} \{c, r, \mathrm{EPSI}\} \quad (13)$$

Subject to

EPSI= $\max(g_j(\boldsymbol{\rho}, \mathbf{a}^{\pi_{fj}})) \geq 0$, $\boldsymbol{\pi}_{fj} \in [0,1]$, $j = 1,2,\ldots, 3$

$g_1 = 5c_{\min} - c \geq 0$

$g_2 = 0.8 \geq r \geq 0.2$

$g_3 = \sigma_{yt} - \sigma_{\max}^{eqv} \geq 0$ for MOP2

$\boldsymbol{\rho}_i \in \{0.0001\text{m}, 1\text{m}\}$ for MOP1

$\boldsymbol{\rho}_i \in \{0.000001\text{m}, 0.01\text{m}\}$ for MOP2

where EPSI is the equivalent possibilistic safety index, $\boldsymbol{x}$ are ρ^{GEF}, and $\boldsymbol{\pi}_{fj}$ = PSI and $\boldsymbol{a}$ is vector of fuzzy variable (E, ν, and F).

Opposite-based multiobjective population-based incremental learning (OMPBIL) is used for solving the optimization problem in this research due to its good performance as demonstrated in our previous study [20]. The improvement used the opposition-based concept embedded into MPBIL, which is found that it can upgrade MPBIL's performance. The parameter of the optimiser is set according to our previous study and the details of OMPBIL can see in [20]. The population size is 35, the total number of iterations is 600, and the external Pareto archive size is 35. The learning rate (LR) is generated randomly in the interval [0.4, 0.6]. The mutation probability and mutation shift are 0.1 and 0.2, respectively.

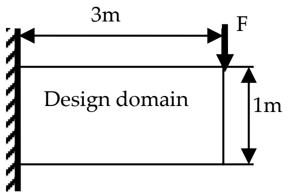


Figure. 3.2. Structural design domains

The topological design domain and loads are shown in Figure 3.2. The uncertainties are Young's modulus E, Poisson's ratio ν, and load F, which is assumed to be fuzzy variables. The membership function of the variables is triangular shaped with values are E = (190, 200, 210) $\times 10^9$ N/m^2, ν = (0.25, 0.3, 0.35), and F = (0.9, 1, 1.5) kN. The mechanical property is not considered to be fuzzy variable, which is the tensile yield strength σ_{yt} = 200 × 10^6 N/m^2. The traditional triple-loop nest multi-objective topology optimization problem is set to minimize structural compliance and normalized mass as shown in Eq. (12), while the proposed technique is presented in Eq. (13). The first constraint in every problem is added to prevent topologies with low global stiffness (or highly compliant structures) is included in the Pareto archive. The external force is applied at the right upper corner of the design domain. The traditional problem is used for comparing the time consuming with newly proposed technique.

The adaptation of a topology technique used in accomplishing the problems is from our previous proposed technique in [31], which is called the multi-grid ground element technique (MG). The proposed technique has been proved to be an efficient technique when combining with the OMPBIL. The Encoding and decoding scheme for the MG approach with PSI and fuzzy variables is needed as shown in the following algorithm 1.

Algorithm 1. Encoding and decoding scheme for a MG approach with PSI and fuzzy variables

Initialization: Generate four sets of ground elements and define the threshold value of ε for each set.

Inputx sized (N_{41} + 6) × 1.

Output: Thicknesses of ground elements.

Encoding

$x_1 \in$ [1, 4] is used for selecting a set of ground elements.

x_2= PSI is defined by the designer

x_3=is used for fuzzy variable F

x_4=is used for fuzzy variable E

x_5=is used for fuzzy variable ν

x_6 to $x_{N_{41}+6}$ are used for element thicknesses.

Decoding

1: Find n = *round*(x_1) where *round*(.) is a round-off operator.

2: If n = 1: x_6 to $x_{N_{41}+6}$ are set as N_{11} element thicknesses and ε=ε1.

3: If n = 2: x_6 to $x_{N_{41}+6}$ are set as N_{21} element thicknesses and ε = ε2.

4: If n = 3: x_6 to $x_{N_{41}+6}$ are set as N_{31} element thicknesses and ε = ε3.

5: If n = 4: x_6 to $x_{N_{41}+6}$ are set as N_{41} element thicknesses and ε = ε4.

The flow diagram of MORBTO is shown in Figure 3.3.

All computations are conducted using MATLAB and a personal computer with specifications being Intel(R) Core™ i5-3210M CPU @ 2.5 GHz, 4.00 RAM, and 64-bit Windows 10 operating system.

3.5. Design Results

The optimal topologies obtained from using the proposed MORBTO for MOP1 and MOP2 are shown in Figures 5 and 7, respectively. As a comparison, the optimal topology design obtained from the deterministic topology optimization for MOP1 and MOP2 is also presented in Figures 3.4 and 3.6, respectively. Some selected optimal topology designs obtained from Figures 3.4 and 3.5 are shown in Figures 3.8(a) and 3.8(b), respectively. From comparison, the optimal topologies obtained from the deterministic topology optimization also present in Figures 3.8(a) and 3.9(a). It can see in Figures 3.8(b) and 3.9(b) that the result from MORBTO is under the EPSI yields optimal topology design different from those yielded by the deterministic topology optimization (Figures 3.8(a) and 3.9(a)).

This can conclude that the proposed MORBTO approach presents a strategy that generate safer topology designs satisfying different failure possibility requirements, which causes topology changing follows the failure possibility change. As well as the proposed MORBTO yields optimal topology designs clearly that is different from the deterministic topology optimization

indicating that the fuzzy uncertainties remarkably influence the topology design. Furthermore, the time consumption comparison between the traditional triple-loop nest problem and the proposed technique for all cases are shown in Table 3.1. The results show that the time spent by the proposed technique is lower than the triple-loop nest problem in all the failure possibility constraints. It can be clearly stated that the proposed technique can reduce the complexity of the traditional technique.

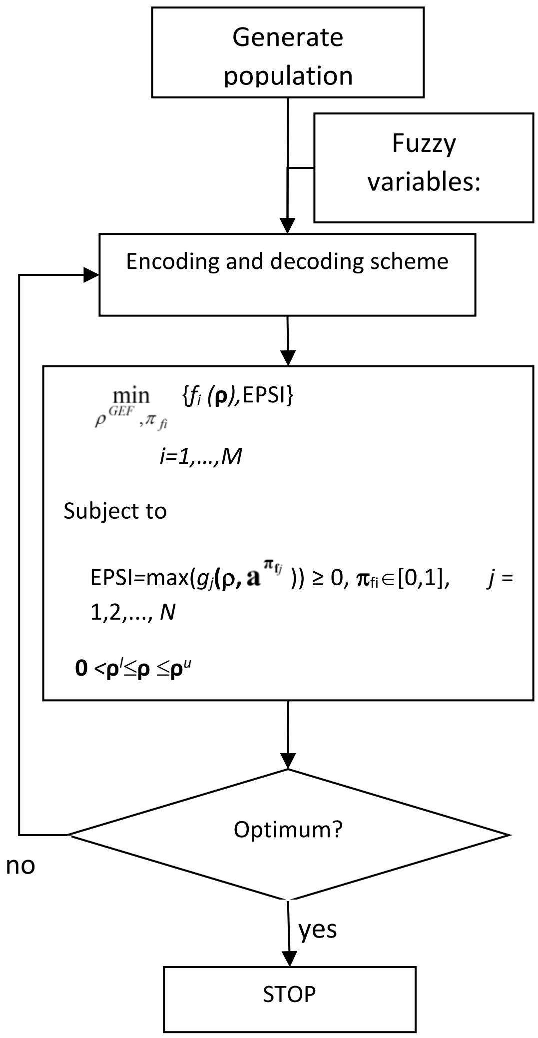


Figure 3.3. The flow diagram of MORBTO.

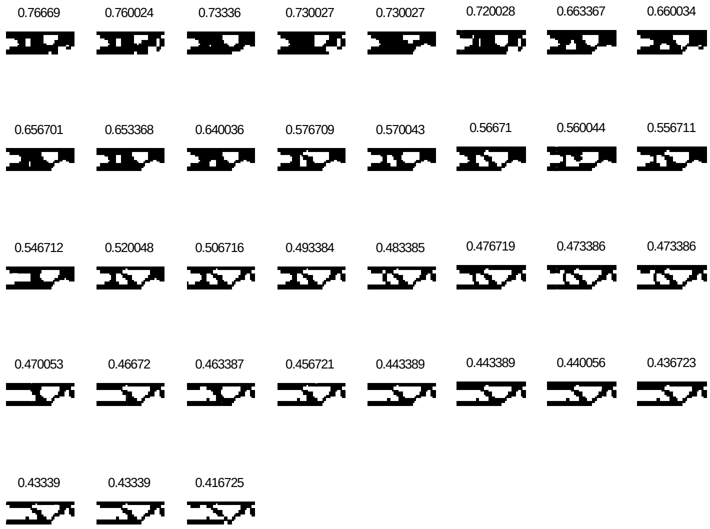


Figure 3.4. The deterministic optimal topology design of MOP1 under failure possibility constraint $\boldsymbol{\pi}_{fj}$= 1with various r.

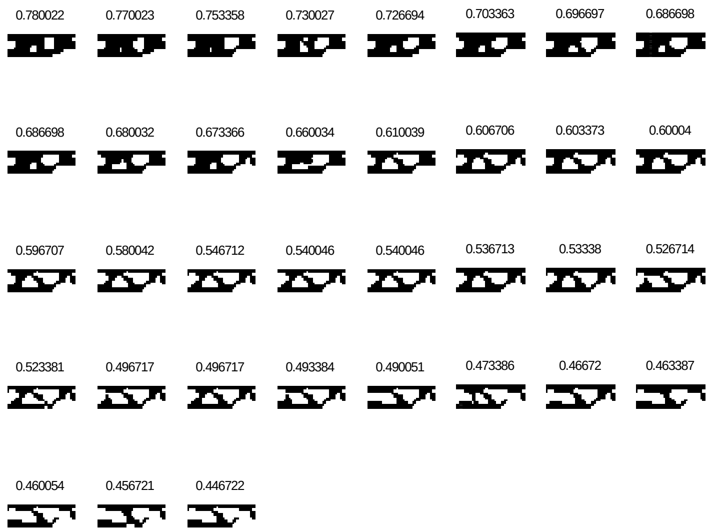


Figure 3.5. The optimal topology design of MOP1 under failure possibility constraint $\boldsymbol{\pi}_{fj}$= 0.001 with various *r*.

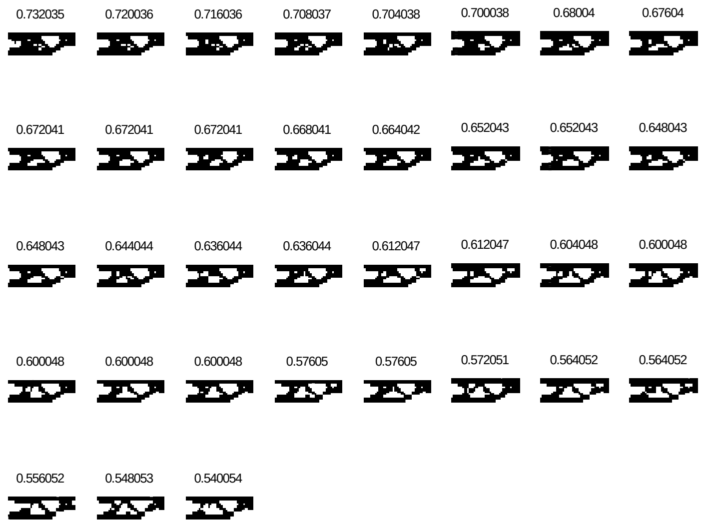


Figure 3.6. The deterministic optimal topology design of MOP2 under failure possibility constraint π_{fj}= 1 with various r.

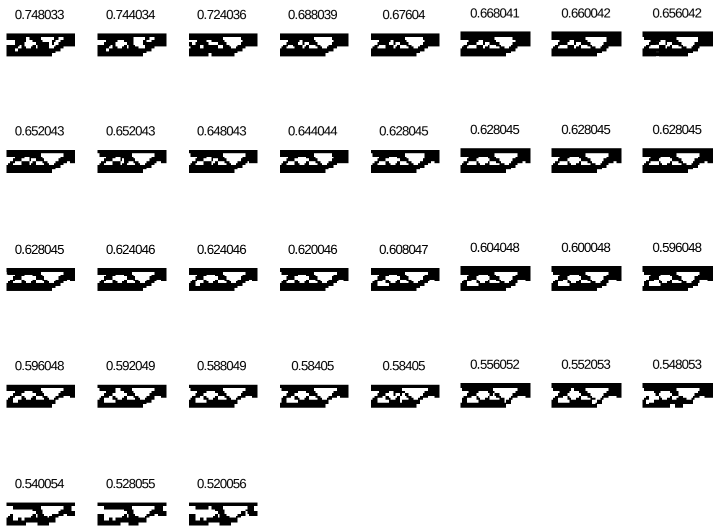
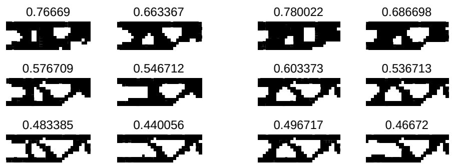
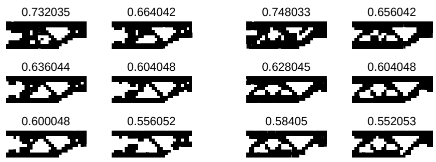


Figure 3.7. The optimal topology design of MOP2 under failure possibility constraint π_{fj}= 0.001 with various r.



(a) (b)

Figure 3.8. The selected optimal topologies of MOP1 (a) π_{fj} = 1 (b) π_{fj} = 0.001.



(a) (b)

Figure 3.9. The selected optimal topologies of MOP2 (a) π_{fj} = 1, (b) π_{fj} = 0.001.

Table 3.1. The time consuming of RBTO and MORBTO for MOP1 and MOP2

MOP1	RBTO		MORBTO	
π_{fj}	1	0.001	1	0.001
Time consuming (s)	5143.0	5168.7	966.1	973.3

MOP**2**	RBTO		MORBTO	
π_{fj}	1	0.001	1	0.001
Time consuming (s)	6755.8	9265.6	1093.9	1141.7

3.6. Conclusions and Discussions

The optimum structure from topology optimization problem may be less realisable due to inevitable uncertainties from various sources, thus, these uncertainties should take into account in design. A satisfactory design should take these uncertainties into consideration. This paper proposes new MORBTO approach which uses the fuzzy set model to describe the uncertainties. This technique can be applied for the multi-objective topology optimization problem, which has conflicting objectives. The proposed MORBTO is formulated as a problem of minimizing mass, compliance and equivalent possibilistic safety index with constraints being accomplished in the possibility context. Furthermore, this paper proposes to deal with the complex reliability of the structure using fuzzy that it is usually a double-looped problem by using multi-objective optimization reducing to single-loop optimization. The performance of the proposed technique is studied by two numerical examples. From the numerical results, it is shown that the proposed MORBTO can generate conservative optimal topology designs with various values of EPSIs, which is different from those obtained from the previous RBTO. These results indicate that the proposed MORBTO is an effective tool to deal with the uncertain topology optimization with considering the expert opinion. Furthermore, the proposed technique can reduce the complexity of the traditional technique and reducing time consumption in solving the optimization problem.

For future study, the reliability-based topology optimization approach will extend to handle uncertainties that take place in design of an aircraft structure.

Chapter 4

Topological Design of a Hard Disk Drive suspension Using OMPBIL

4.1 Introduction

A hard disk drive (HDD) suspension is a cantilever beam extended from an E-block to a sliding head of the HDD, which is used to protect the harmful vibration from the sliding head [42]. The suspension is one kind of compliance mechanism, which limits its motion by means of structural flexibility. It needs sufficiently low vertical bending stiffness, while the air bearing to suspension stiffness ratio is maintained at the proper range [43]. To project the off-tract phenomena and increase the servo bandwidth, the in-plane dynamic stiffness should be as high as possible [42- 43]. These reasons cause the conflict design objectives that we need to minimize in both the suspension vertical stiffness and maximize the natural frequencies associated with the sway and torsion modes [43].

The previous suspension system composes of three main components i.e. baseplate, load beam and gimbal (flexure), later it is added with a hinge to the system as shown in figure 1. The hinge is introduced to enable pitching and rolling movement of the load beam. The load beam is often stiffened by adding to its edges a couple of stiffeners, usually called rails [1]. The baseplate is an attachment of the E-block part. The sliding head is attached at the tip of the beam by a spherical joint called a dimmer that enables the head to move without a severe contact between itself and the platter's flexible shape. The complex of the system and conflict of objectives is caused the design of HDD suspension is more difficult problem. In the past, a trial-and-error approach is used in design of HDD suspension. However, it has been found recently that the topology optimization [31] and HDD suspension design [44] are more efficient approach. Some research work of topology optimization for HDD suspension design has been made [42-43, 44-47]. The most objective of the design problems are the maximization of sway or torsion mode natural frequencies whereas the mass and vertical stiffness are constrained [43]. The optimizer has been proved is an efficient technique for solving topology optimization problem without finding gradient of the objective is in group of evolutionary algorithms. Only one technique that the author proposes to solve this problem is called the opposite-based multi-objective population-based incremental learning (OMPBIL) [31, 48-50].

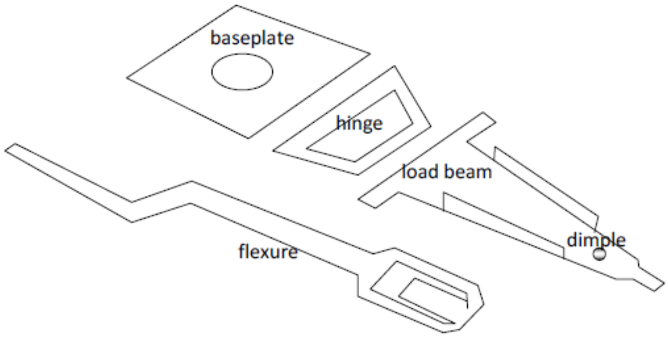


Fig. 4.1. Component of HDD suspension

The objective of this paper proposes a multiobjective evolutionary approach to deal with HDD suspension design. The design problem is expected to find a set of optimum suspension topologies, which is maximization of the first sway mode frequency and minimization of the suspension bending stiffness using OMPBIL optimizer. Design constrains include the first torsion and bending modes frequencies of a structure.

4.2 Hard disk suspension design

The recent design of HDD suspension is incorporated of topological optimization and finite element analysis [42-43, 44-46]. The final structure achieved by topology optimization technique is said to be the best structural layout accorded with a predefined objective function and design constraints. For a general process of structural topology design is added voids and unchangeable areas to the system, which has been defined the predefined design domain, boundary conditions and applied loads. In practical design process, the design domain is discretized into a few finite elements that called ground elements. Topological design variables are formed from the thicknesses of the elements. The elements with nearly zero thicknesses represent holes or voids on the structure whereas other elements indicate the existence of the structural material.

The well-known and the most efficient method for topology optimization is the solid isotropic material with penalization (SIMP) [19]. This technique is the gradient-based optimizers that well established in consistency of finding solution. The contrary behavior to the previous technique is called evolutionary algorithms (EAs) is questionable due to their low convergence rate and completely lack in consistency. The lacks have been the attention of many researchers to propose several attempts to enhance the searching performance of EAs for structural topological design [19]. The efficient numerical scheme to increase the performance of EAs is called the ground element filtering technique (GEF technique). This technique can solve the checkerboard pattern problem and decreases design variables by using two difference mesh grids resolution rather than a single grid one [19]. The lower resolution is used for a design variable, while the higher resolution is used for finite element grid. The results obtained from this technique can upgraded the EAs topologies is comparable with the gradient-based counterparts. Later, this technique has been extended to multi-grid ground elements technique. This technique can increase the performance of previous technique by increasing the number of grid resolution set [31]. Using EAs for topological design with acceptable searching performance is attractive especial for the multi-objective optimization problems due to the capability in finding Pareto solution set in one optimization run. The multiobjective evolutionary algorithms (MOEAs) require no function derivative, the methods can deal with almost all kinds of optimum design problem.

Multiobjective Population-Based Incremental Learning PBIL (MPBIL) is one kind of the estimation of distribution algorithm (EDA) that has been proved is one of efficient technique for solving practical problem in comparative performance testing of single and multi-objective design optimization problems [51]. The MPBIL has been proved can increase its performance by adding the opposite concept to the learning rule that governs the updating process. This technique is called the opposite-based multi-objective population-based incremental learning (OMPBIL) [48-50]. This technique is choose for solving the HDD suspension design problem. The details of OMPBIL, see [48-50].

4.3 **HDD suspension design optimisation problem and OMPBIL**

4.3.1 Optimisation problem

The multi-objective optimization problem is assigned to maximize the first sway mode frequency and minimize the suspension bending stiffness is shown as:

$$\text{Minimize: } K_{BS} \text{ \& max: } \omega_{1SW} \tag{1}$$

subject to

$$\omega_{1B} \geq 3.5 \text{ kHz}$$

$$\omega_{1T} \geq 8.0 \text{ kHz}$$

where K_{BS} is a bending stiffness, ω_{1SW} is the first frequency of sway mode, ω_{1B} is the first frequency of bending mode, ω_{1T} is the first frequency of torsion mode.

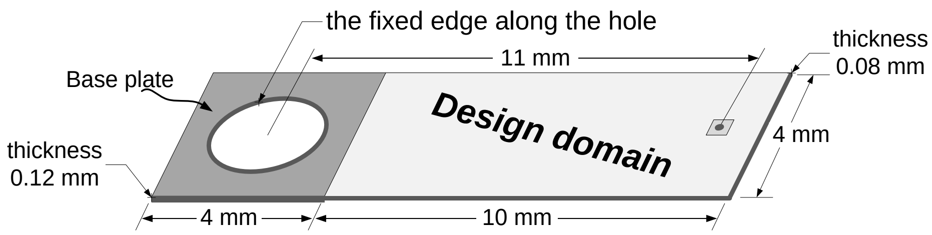


Figure. 4.2. The initial suspension design domain

The suspension system is modeled by one piece includes the baseplate, hinge, and load beam as shown in Figure 4.2. The design domain shape is a rectangular plate (14×4 mm) with thickness is 0.08 mm. The base plate (4×4×0.12 mm) with a hole radius is 1.333 mm, which has fixed constraint along the edge of the hole. At the right-hand side of the plate is a placement of a femto slider. During the optimization process the baseplate is unchangeable area due to it is an attachment of the slider. The material properties of the suspension are the Young's modulus of 193×109 N/m^2, 0.3 Poisson's ratio, and 8030 kg/m^3 density.

The design topology of the suspension is performed with the ground element filtering technique (GEF) [8] to reduce the number of design variables, and alleviate checkerboard patterns in topologies. The design domain is meshed with 50×20 shell elements, while the GEF is meshed with 25×10 elements. The lower and upper bounds of the shell element thickness is interval 0.00008 mm and 0.08 mm, respectively.

The optimizer uses for solving this problem in Eq. (1) is OMPBIL. The number of iterations is 250 and the population size of 50. Each elements thickness in the design domain represent with binary, which have the values of “1” if it is set to have 0.08 mm element thickness, in contrast the values is “0” if it is 0.00008 mm element thickness.

4.4 Design Results

The best approximate Pareto fronts of the design problem (1) obtained from using OMPBIL is displayed in Figure 4.3. The HDD suspension topologies corresponding to the selected solutions in Figure 4.3 are illustrated in Figure 4.4. It is shown that various suspension layouts are obtained within one optimization run. All optimum topology fulfils with the design constraints, the designer can choose it later. A selected topology of the HDD suspension is shown in Figure 4.5 for looking the detail inside.

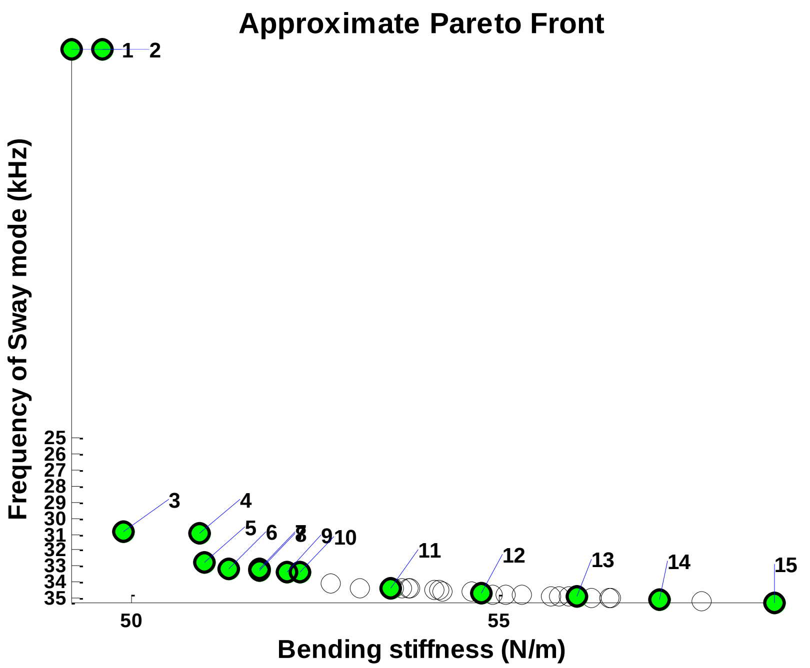


Figure 4.3. The best non-dominated Pareto front

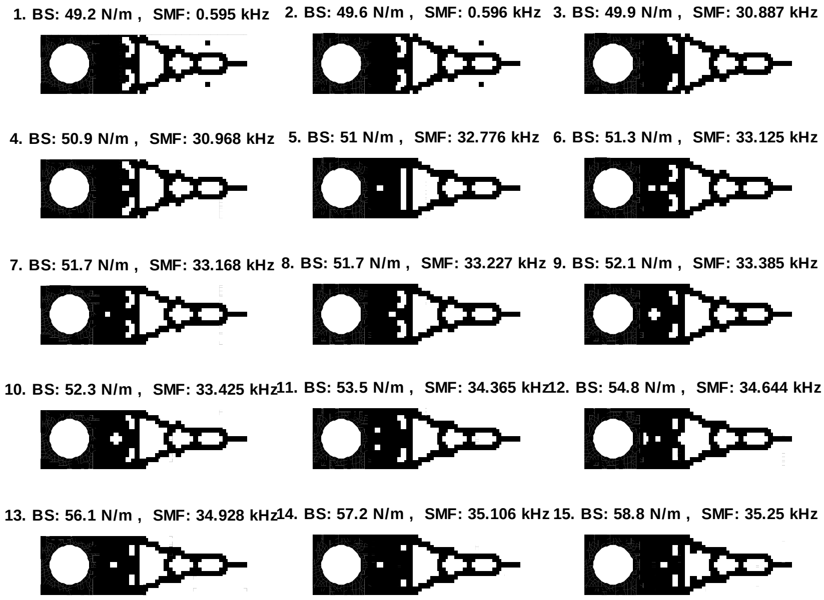


Figure 4.4. Selected suspension accord with the Pareto front

15. BS: 58.8 N/m , SMF: 35.25 kHz

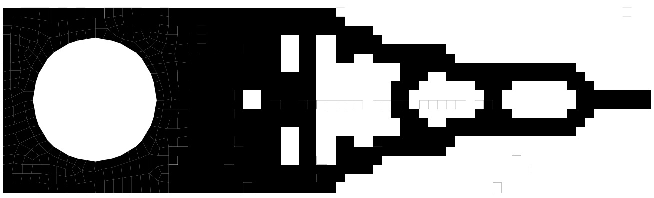


Figure 4.5. Selected suspension accord with the Pareto front

4.5 Conclusions

From the design results it can be said that the opposite-based multiobjective population-based incremental learning is a powerful optimizer in design of a HDD suspension system. By the present technique, the topology of HDD suspension can be achieved within one optimization run. The present study is only preliminary study of the HDD suspension design, which still needs the advance technique in chapter 2 (multi-grid ground elements technique) and the MORBTO in chapter 3 in case of considering of the uncertainties in the design formulate. The future work will be considered such techniques in design of the HDD suspension.

Chapter V
Conclusions and Future work

This research has 3 aims are developed the technique for topology optimization of structures with/without reliability analysis, and HDD suspension design. The first purposes of this work are the demonstration of the performance comparison of an original MPBIL and their performance enhancement, and the MG approach for multi-objective structural topology optimization problems, respectively. Among the performance enhancements of MPBIL, OMPBIL outperforms other techniques. It promotes the opposition-based concept, which can improve the search performance of MPBIL. The use of MPBILs in combination with the MG approach is well capable of solving multi-objective structural topology optimization. The resulting topologies obtained from using OMPBIL are close to those obtained from the classical gradient-based approach. The new design strategy is a procedure for structural topology optimization, which uses multiple ground element resolutions, so the MG approach is more efficient than using single-resolution ground elements in the sense that the suitable grid resolution is automatically detected and used in one optimization run. In addition, the use of the MG approach combined with ground element filtering for alleviating checkerboards is effective. In future work, the proposed method is extended to solve topology optimization with uncertainty.

The second aim is to consider uncertainties into topology design. A satisfactory design should take these uncertainties into consideration. The new MORBTO approach is proposed, which uses the fuzzy set model to describe the uncertainties. This technique can be applied for the multi-objective topology optimization problem, which has conflicting objectives. The proposed MORBTO is formulated as a problem of minimizing mass, compliance and equivalent possibilistic safety index with constraints being accomplished in the possibility context. Furthermore, this research proposes to deal with the complex reliability of the structure using fuzzy that it is usually a double-looped problem by using multi-objective optimization reducing to single-loop optimization. The performance of the proposed technique is studied by two numerical examples. From the numerical results, it is shown that the proposed MORBTO can generate conservative optimal topology designs with various values of EPSIs, which is different from those obtained from the previous RBTO. These results indicate that the proposed MORBTO is an effective tool to deal with the uncertain topology optimization with considering the expert opinion. Furthermore, the proposed technique can reduce the complexity of the traditional technique and reducing time consumption in solving the optimization problem. For future study, the reliability-based topology optimization approach will extend to handle uncertainties that take place in design of HDD suspension.

The third aim is said is an only preliminary study of the HDD suspension design, which still needs the advance technique in chapter 2 (multi-grid ground elements technique) and the MORBTO in chapter 3 in case of considering of the uncertainties in the design formulate. From the design results it can be said that the opposite-based multiobjective population-based incremental learning is a powerful optimizer in design of a HDD suspension system. By the present technique, the topology of HDD suspension can be achieved within one optimization run. The future work will be considered such techniques in design of the HDD suspension.

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APPENDIX A

LIST OF PUBLICATIONS

Output จากโครงการวิจัยที่ได้รับทุนจาก สกว.

1. ผลงานตีพิมพ์ในวารสารวิชาการนานาชาติ

   Sleesongsom, S.; Bureerat, S. Topology Optimization Using MPBILs and Multi-Grid Ground Element, *Applied Sciences*. **2018**, 8, 271.

   Sleesongsom, S.; Bureerat, S. Multi-objective Reliability-based Topology Optimization of Structures Using a Fuzzy Set Model, *Journal of Mechanical Science and Technology*. **2020**, (accepted).

2. การนำผลงานวิจัยไปใช้ประโยชน์

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3. อื่นๆ (เช่น ผลงานตีพิมพ์ในวารสารวิชาการในประเทศ การเสนอผลงานในที่ประชุมวิชาการ หนังสือ การจดสิทธิบัตร)

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Article

Topology Optimisation Using MPBILs and Multi-Grid Ground Element

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Abstract: This paper aims to study the comparative performance of original multi-objective population-based incremental learning (MPBIL) and three improvements of MPBIL. The first improvement of original MPBIL is an opposite-based concept, whereas the second and third method enhance the performance of MPBIL using the multi and adaptive learning rate, respectively. Four classic multi-objective structural topology optimization problems are used for testing the performance. Furthermore, these topology optimization problems are improved by the method of multiple resolutions of ground elements, which is called a multi-grid approach (MG). Multi-objective design problems with MG design variables are then posed and tackled by the traditional MPBIL and its improved variants. The results show that using MPBIL with opposite-based concept and MG approach can outperform other MPBIL versions.

Keywords: topology optimization; multi-objective optimization; opposite-based evolutionary algorithm; population-based incremental learning; adaptive learning rate

1. Introduction

The first question that always arises at pre-process stage, when using a ground element approach for topology optimization, is:What the best ground element resolution for a design problem should be? As a result, we investigate using several sets of ground elements when performing optimization, which we term the multi-grid design approach (MG). The MG approach is an extension of ground segment strategy, which has been proposed to solve a truss structural optimization problem [1,2] and morphing wing structural optimization problem [3].

The second question arises due to an opposition-based concept that could potentially improve the search performance of the evolutionary algorithm (EA) [4–7]; the multi-objective population-based incremental learning (MPBIL) was the best optimizer [8]. Additionally, it has been demonstrated that the opposition-based concept could improve population-based incremental learning (PBIL) performance for a single objective, which is called the opposition-based concept PBIL(OPBIL) [9], whereas the multi-objective optimization is called opposite-based, multi-objective, population-based incremental learning (OMPBIL) [3]. PBIL is categorized as an estimation distribution algorithm (EDA), which is still in the spotlight of many researchers due to this kind of algorithm being simple to adapt and apply for a single- and multi-objective optimization problem [10–13]. From our previous work, OMPBIL with a multi-grid approach has been used to solve partial topology optimization of morphing aircraft wings, and it promotes better results than the original multi-objective population-based incremental learning (MPBIL) with a single grid element. Moreover, the work reveals that the opposition concept could improve the search performance of MPBIL. The question remains whether the performance of OMPBIL can benefit from the opposite concept or two learning rates. To make it be clearer, we compare the performance of OMPBIL and the performance of MPBIL with multi-learning rate. If the former technique can achieve better results, it means that the opposition concept significantly improves the performance of MPBIL. Therefore, this question will be addressed in this study. Furthermore, it has been found [14] that learning rate was the most affective with search performance of PBIL. Another way to improve the search performance of MPBIL is to use an adaptive learning rate method [15]. This method is categorized as self-learning adaptations, so the effectiveness of this technique needs to be addressed in this research.

Therefore, in this paper, the first objective is to apply the multi-grid approach (MG) approach to solve structural topology optimisation problems, whereas the second objective is comparative performance of the three variants of MPBIL. The performance improvements are based on an opposite-based concept, a multi-learning rate, and an adaptive learning rate, respectively. This research expects to improve the performance of the proposed MPBIL and MG approaches that lead to the obtaining of better design results than the original MPBIL with a single grid. The rest of this paper is organized as follows. Section 2 promotes the details of topology with single-ground and multi-ground design approaches for structural topology optimization. We introduce some novel methods for enhancing the performance of multi-objective, population-based incremental learning in Section 3. The performance index and statistical testing are given in the same section. Numerical experiments and the design results are proposed in Section 4; moreover, the design results and discussion are in Section 5. Finally, the conclusions of the study are in Section 6.

2. Topological Designs with Single-and Multi-Ground Design Approaches

2.1. Topological Designs with Ground Element Filtering

Topology optimization is one mathematical tool used in the conceptual design stage of engineering systems for finding the best structural layout from a given design domain. Topological design can perform using an optimization method and finite element analysis. This technique is started by defining design domain represented as the discrete structural members such as panels, truss, and frame as shown in Figure 1. The optimization method can be performed by varying the width or thickness of each element in the design domain between zero and the maximum value. All elements were discarded, if the element width/thickness value was zero. Otherwise, the element was retained. With this concept, optimization of the structural layout and component sizes is performed. Two popular, well known topological methods are the solid isotropic material with penalization (SIMP) approach and the homogenization method, which use gradient-based optimizers. Later, an alternative optimizer is evolutionary algorithms due to the fact they are robust, simple to use, derivative-free, and free from intermediate pseudo densities [8]. Complicated problems, such as partial topology, simultaneous topology, shape, and sizing optimization, can be performed within one optimization run [3,8,16,17] by using such algorithms. In this paper [8], they presented the comparative performance of multi-objective evolutionary algorithms (MOEAs) for solving structural topology optimization test problems based on ground element filtering technique. It has been found that MPBIL is the best optimizer in their study, which outperforms other MOEAs [8], so MPBIL is the only MOEA selected to improve its search performance in this research. Furthermore, the ground element filtering technique is also used in this study. The ground element filtering technique (GEF technique) is a simple numerical scheme that can apply to all kinds of optimizers, which can prevent the checkerboard pattern problem and at the same time decrease the number of design variables [8,18,19]. The idea uses two mesh grids of design domain with different resolutions. The lower resolution grid is provided for design variables, whereas the higher resolution is used as a finite element grid. The conversion between two grids relates to threshold value (ε) that is defined at the first time before optimization run. Therefore, this technique has been proved to be an efficient technique to suppress the checkerboard problem. Next, the details of GEF technique are seen in [8,18,19]. Later, a method for solving checkerboard pattern was presented by Guirguis and Aly [20]. They proposed that derivative-free level-set method for solving structural topology can solve the checkerboard problem. This new technique can avoid the main limitations of non-gradient methods: dependence on the objective value. Moreover, the boundaries of structure are smooth, but it does not directly depend on the decision variables. A very recent work in multi-objective topology optimization has been proposed to address the limitations of generating infeasible structures and expensive computational cost by using the technique called "graphics processing unit (GPU)" [21]. On the contrary, this technique has been commented on usefulness in the case of truss-like structures and the solved examples are simple, and obtained results are sub-optimal solutions [22]. Recent applications of topology optimization appeared in design of composite molding processes [23]. More recently, applications of topology optimization appeared in many fields, e.g., composite molding processes [23], optimal design of piezoelectric [24], phononic crystals design [25] and stator configurations [26].

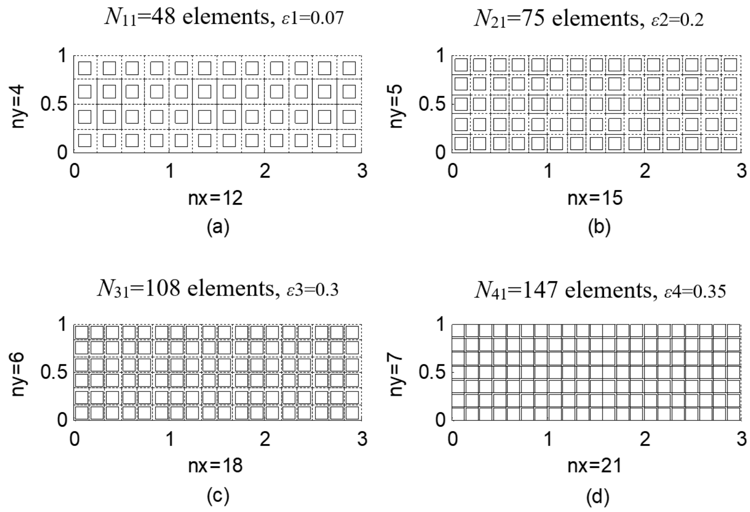


Figure 1. A ground elements set for MOP1, MOP3, and MOP4 (**a**) for $n = 1$; (**b**) for $n = 2$; (**c**) for $n = 3$, and (**d**) for $n = 4$.

2.2. Single-and Multi-Grid Ground Elements

The MG approach for topology optimization is an extension of MG strategy, which proposes to solve a truss structural optimization [1] and morphing wing structures [3]. At the present, we propose to apply this technique to a structural topology optimization problem. This technique has an improvement in both using the several ground resolutions. In this research, a ground structure has four sets of ground elements with different grid resolutions and the threshold value ε. The threshold value ε must be specified at the first stage before performing the optimization run. A special encoding and decoding scheme slightly changes from the previous work [3], but it is very important to the quality of final result. Especially, the threshold values are different in each grid resolution to prevent the checkerboard problem, which can occur in each grid. At the first stage, this scheme starts with defining the number of elements and the threshold values. The first set of ground elements has N_{11} elements, and the threshold value is set to be $\varepsilon 1$. Therefore, an example of a ground element set used in this study is the lowest resolution as number of elements $N_{11} = 48$ and $\varepsilon 1 = 0.07$ as shown in Figure 1. The second set has $N_{21} = 75$ elements and the threshold value is $\varepsilon 2 = 0.2$. Then, the third set has $N_{31} = 108$ segments and the threshold value is $\varepsilon 3 = 0.3$, whereas the last set has the numbers of ground elements and the threshold value is $N_{41} = 147$ segments and $\varepsilon 4 = 0.35$, respectively. As a result, $N_{41} \geq N_{31} \geq N_{21} \geq N_{11}$ and $\varepsilon 4 \geq \varepsilon 3 \geq \varepsilon 2 \geq \varepsilon 1$, respectively. Therefore, the variables and the threshold values for encoding/decoding scheme for the MG approach, which is improved from previous algorithm, can be detailed as shown in Algorithm 1. For using this algorithm, the MPBIL and its improved versions perform with binary design variables, whereas it needs the conversion of binary string to become a real design vector x before entering into this algorithm. Furthermore, the ground element set with its ε used in this research for multi-objective optimization problem (MOP) MOP1, MOP3 and MOP4 is shown in Figure 1. For the design problem MOP2, the design domain is different from the other problems. The details of the ground element sets and the threshold values are presented in Section 4.

Algorithm 1. Encoding and decoding scheme for a MG approach.

Initialization: Generate four sets of ground elements and define the threshold value of ε for each set.
Inputx sized (N_{41} + 1) × 1.
Output: Thicknesses of ground elements.
Encoding
$x_1 \in [1, 4]$ is used for selecting a set of ground elements.
x_2 to $x_{N_{41}+1}$ are used for element thicknesses.
Decoding
1: Find n = $round(x_1)$ where $round(.)$ is a round-off operator.
2: If n = 1: x_2 to $x_{N_{41}+1}$ are set as N_{11} element thicknesses and $\varepsilon = \varepsilon 1$.
3: If n = 2: x_2 to $x_{N_{41}+1}$ are set as N_{21} element thicknesses and $\varepsilon = \varepsilon 2$.
4: If n = 3: x_2 to $x_{N_{41}+1}$ are set as N_{31} element thicknesses and $\varepsilon = \varepsilon 3$.
5: If n = 4: x_2 to $x_{N_{41}+1}$ are set as N_{41} element thicknesses and $\varepsilon = \varepsilon 4$.

3. Performance Enhancements of Multi-Objective, Population-Based Incremental Learning

This section briefly details the concept of MPBIL and its three variants.

3.1. Multi-Objective, Population-Based Incremental Learning

MPBIL is an extension of PBIL for solving a multi-objective optimization problem. This problem has more than one objective function, which promotes several solutions for this kind of problem, and it is called a Pareto solution set or a Pareto frontier. Rather than using a single probability vector, several probability vectors are used, so it is called a probability matrix. The matrix is used to maintain diversity of a binary population. At an initial step, the probability matrix has elements full of "0.5". Each row of the probability matrix or probability vector is updated by Hebb's rule [27] as follows

$$P_{ij}^{\text{new}} = P_{ij}^{\text{old}}(1 - LR) + b_j L_R \tag{1}$$

in which L_R is a PBIL learning rate, a small value usually recommends for the conventional operating [28], and b_j is the mean value of jth column of several binary solutions randomly selected from a current Pareto front. It is also useful to apply a mutation to probability matrix at some predefined probability as

$$P_{ij}^{new} = P_{ij}^{old}(1 - ms) + rand(0\,or\,1) \cdot ms \tag{2}$$

in which ms is mutation shift, and the default value is usually 0.2. For more details of MPBIL procedure, see [3].

3.2. Opposite-Based MPBIL

OMPBIL has been developed as an improved version of MPBIL [3]. Due to L_R affecting MPBIL performance, the issue is how to select a proper value of L_R for a general problem. It is expected to accelerate the convergence rate to find solution, as well as provide population diversity. Our previous work proposed the opposition-based concept embedded into MPBIL, which is an efficient technique that can upgrade MPBIL's performance. Therefore, the outline of OMPBIL algorithm includes the opposition-based concept, which is not included in this paper. More details can be found in [3].

3.3. Multi-Learning Rate

The second approach to enhance the performance of MPBIL is the use of multi-learning rate. This question arises from the previous method, when it is using two learning rates that are of an opposite quantity. The question remains whether the performance of OMPBIL can benefit from the opposite concept or by using two learning rates. MPBIL with multi-learning rate (MPBILMLR) is proposed to solve topological optimization and to compare with the opposition-based concept. This algorithm differs from the traditional MPBIL by using three learning rates (L_R = 0.25, 0.5, 0.75). The procedure of MPBILMLR algorithm is slightly different from OMPBIL. Therefore, the procedure of MPBIL with multi-learning rate algorithm is shown in Algorithm 2.

Algorithm 2. MPBIL with multi-learning rate.

Initialization Probability matrix **P** = $[\mathbf{0.5}]_{l\times nb}$, Probability matrix **P***i* = $[\mathbf{0.5}]_{l/\mathrm{M}\times nb}$ where $i = 1, \ldots, \mathrm{M} = 3$, external Pareto archive **Pareto** = {}.
1: Generate a binary population **B** from **P**.
2: Decode the binary population to be $\mathbf{x}_{n\times Np}$ and find the objective values $\mathbf{f}_{m\times Np}$.
3: Update **Pareto** by replacing it with non-dominated solutions of union set **Pareto** ∪ **x**.
4: If the number of members in **Pareto** exceeds the predefined archive size N_A, remove some of them by using an archiving technique.
5: If the termination criterion is fulfilled, stop the procedure. Otherwise, go to step 6:
6: Update **P** and create a binary population
6.1: Set a binary population **B** = {}.
6.2: For i = 1 to l/M.
  6.2.1: Select n_0 binary solutions from **Pareto** randomly.
  6.2.2: Use $L_{R\mathrm{k}}$ = 0.25, 0.5, 0.75, for each $k = 1, \ldots, \mathrm{M}$. (For this research M = 3)
  6.2.3: Update the ith row of **P** by using (1).
  6.2.4: Generate the ith row of probability matrix **P***i* using (2) and each $L_{R\mathrm{k}}$.
  6.2.5: Generate $rand \in [0, 1]$ a uniform random number.
  6.2.6: If $rand$ < the predefined mutation probability, update the ith row of **P1**, **P2** and **P3** using similar equation in [3].
  6.2.7: Generate binary subpopulations **SB1**, **SB2** and **SB3** from the ith row of **P1**, **P2** and **P3**, respectively.
  6.2.8: Set **B** = **B**∪**SB1**∪**SB2**∪**SB3**
6.3: Next i.
7: Go to step 2.

3.4. Adaptive Learning Rate

The last method for MPBIL performance enhancement is using an adaptive learning rate, which proposes to modifythe learning rate during the entire process [28]. A small value of learning rate is usually recommended for conventional PBIL to keep the algorithm reliable, but it usually causes low convergence rate. To balance the reliability and speed of convergence in all iterations, the learning rate needs to adapt. A model of adaptive learning rate has been proposed by Yang et al. [15] that satisfies the previous conditions. That model is shown as follows

$$L_R = L_{R0} + (L_{RM} - L_{R0})e^{-\left(\frac{SI}{NT}\right)} \tag{3}$$

in which SI is the successive iterations with improvements in the objective function in the most recent NT iterations. L_{R0} and L_{RM} are the minimum and maximum learning rates that the designer defines before an optimization run. The learning rate depends on the ratio of SI/NT. Additionally, the high value of this ratio means that it is possible to locate better solutions using its current probability matrix, and consequently the learning rate should be small. In contrast, a low value of this ratio means the current probability matrix which is insufficiency, so the learning rate should be increased. Moreover, the outline of multi-objective, population-based incremental learning with adaptive learning rate

(MPBILADLR) is slightly different from the traditional MPBIL, which uses Equation (3) to replace the original equation for finding L_R. This algorithm is shown as follow.

Algorithm 3. MPBIL with adaptive learning rate.

Initialization probability matrix $\mathbf{P} = [\mathbf{0.5}]_{l\times nb}$, external Pareto archive **Pareto** = {}.
1: Generate a binary population **B** from **P**.
2: Decode the binary population to be $\mathbf{x}_{n\times Np}$ and find the objective values $\mathbf{f}_{m\times Np}$.
3: Update **Pareto** by replacing it with non-dominated solutions of union set **Pareto**∪**x**.
4: If the number of members in **Pareto** exceeds the predefined archive size N_A, remove some of them by using an archiving technique.
5: If the termination criterion is fulfilled, stop the procedure. Otherwise, go to step 6:
6: Update **P**.
    6.1: For i = 1 to l.
    6.1.1: Select n_0 binary solutions from **Pareto** randomly.
    6.1.2: Generate L_R using (3).
    6.1.3: Update the ith row of **P** by using (1).
    6.1.4: Generate *rand* ∈ [0, 1] a uniform random number.
    6.1.5: If *rand* <the predefined mutation probability, update the ith row of P using similar equation in [3].
6.2: Next i.
7: Go to step 1.

3.5. The Performance Index and Non-Parametric Statistical Test

MPBIL and its enhanced versions are classified as MOEAs, while the obtained results are classified as approximate Pareto optimal frontiers. In comparing the searching performance of MOEAs, the methods are employed to solve design optimization problems with equivalent total number of function of evaluations for number of attempts. The approximate Pareto frontiers obtained from various MOEAs are then compared using a performance indicator, which is called a hyper-volume (HV) [29] indicator. This indicator represents the hyper-area above a Pareto frontier for bi-objective optimization problem as shown in Figure 2, whereas it is called hyper-volume for three objective functions and more. Therefore, HV sums up all discrete areas v_i or volumes of hyper-areas or hyper-volumes with respect to a given referent point, respectively.

A technique for comparing the performance of each MOEA in this research is a non-parametric statistical test, which is called the Friedman test. This technique has been used by Sleesongsom and Bureerat [30] for studying the performance of meta-heuristics (MHs) in solving the four-bar linkage path generation problems. The Friedman test is suitable for comparing more classifiers over multiple data sets.

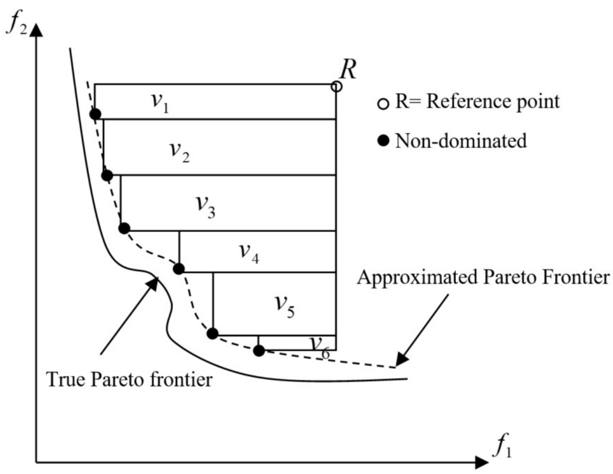


Figure 2. Hyper volume sums up all areas covered by the non-dominated solutions and a reference point.

4. Numerical Experiment

As mention earlier, the purpose of this research is to study the comparative performance of original MPBIL and its three variants with (WMG) and without the MG (WOMG) approach. Four design problems are used for testing performance of the proposed methods. The original MPBIL and three performance enhancements of MPBIL (OMPBIL, MPBILMLR, and MPBILADLR) are employed to solve multi-objective topology optimisation problems that have been detailed in the previous section. Each algorithm is used to solve an optimisation problem for 25 runs to measure its performance and consistency. For all design problems, all the algorithms are used with a population size of 35 and an iteration number of 400 whereas the external Pareto archive size is set to be 35 Non-dominated solutions obtained, so at the last iteration approximates the Pareto solutions. Therefore, four multi-objective problems are used for testing performance of MPBIL and performance enhancements of MPBIL, which has been proposed to study the comparative performance of some established multi-objective evolutionary algorithms (MOEAs) [8]. The problems are structural topology optimisation problem. The design problems are as follows:

MOP1: The topological design domain and loads are shown in Figure 3a. The structure is made of material with Young's modulus $E = 200 \times 10^9$ N/m^2, Poisson's ratio $\nu = 0.3$, and tensile yield strength $\sigma_{yt} = 200 \times 10^6$ N/m^2. The multi-objective design problem is set to minimize structural compliance and normalized mass as:

$$\min_{\rho^{GEF}}\{c,\ r\} \tag{5}$$

subject to

$$c \leq 5c_{\min}$$
$$0.2 \leq r \leq 0.8$$
$$\rho_i \in \{0.0001, 1\}$$

where ${\rho_i}^{GEF}$ is the value of ith design variable; ρ_i is the thickness of ith finite element; m is the structural mass; $r = m(\boldsymbol{\rho})/m(\boldsymbol{\rho}^u)$ is the normalized mass or ratio of structural mass to maximum mass; c is the structural compliance; and $c_{\min} = c(\boldsymbol{\rho}^u)$. The first constraint is added to prevent topologies with a low global stiffness (or highly compliant structures) being included in the Pareto archive. The bound constraints are set as $\rho_i \in \{0.0001, 1\}$. The parameter ε is set 0.3 and $[0.08, 0.1, 0.25, 0.3]^T$ for all MPBILs with WOMG and WMG design approach, respectively. The number of element for single grid is set as highest resolution. A set of MG elements is use for this problem and show in Figure 1. The mean hypervolumes of the fronts of MOP1 for all optimisation runs are given in Table 1, where the referent point for computing hypervolumes is set to be $\{2.5\text{ kNm}, 2.5\}^{\mathrm{T}}$.

MOP2: The second design problem promotes three objective functions, where the design domain and load illustrate in Figure 3b. The structure makes up the same material as MOP1. The multi-objective design problem can be written as:

$$\min_{\rho^{GEF}}\{c1,\ c2,\ r\} \tag{6}$$

subject to

$$c1 \leq 5c_{1,\min}$$
$$c2 \leq 5c_{2,\min}$$
$$0.2 \leq r \leq 0.8$$
$$\rho_{\mathrm{i}} \in \{0.0001, 1\}$$

where c_1 is the structural compliance due to the first load case and c_2 is the structural compliance due to the second load case, $c_{1,\min} = c_1(\boldsymbol{\rho}^u)$, and $c_{2,\min} = c_2(\boldsymbol{\rho}^u)$. A number of ground elements set, which uses in this study is [48,63,108,130], while the number of element for single grid is set as the highest resolution. The threshold parameter ε is set to be 0.35 and $[0.07, 0.2, 0.3, 0.35]^T$ for all MPBILs with WOMG and WMG approach, respectively. There are different from other problem due to the difference of design domain. The mean hypervolumes of the fronts of MOP2 for all optimization runs are given in Table 1, in whichthe referent point for computing the hypervolumes is set to be {1.5 kNm, 1.5 kNm, 1.5}^{T}.

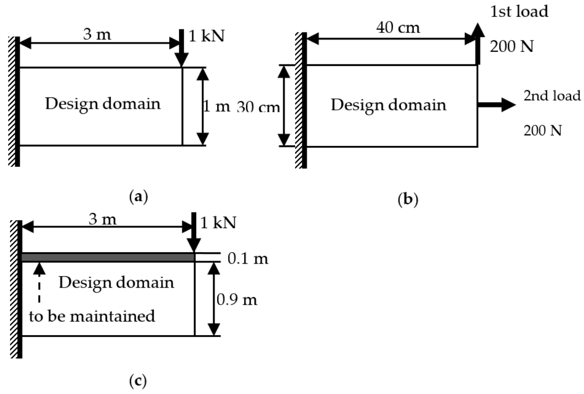


Figure 3. Structural design domains: (**a**) design domain of MOP1 & MOP3; (**b**) design domain of MOP2; (**c**) design domain of MOP 4. Reproduce with permission from [8], Taylor & Francis, 2011.

MOP3: The design problem has the same design conditions as set for MOP1 with the exception of the range of design variables. Addition constraints are $\sigma_{\max}^{eqv} \leq \sigma$yt and $\rho_i \in$ {0.000001 m, 0.01 m}, in which $\sigma_{\max}^{eqv}$ is the maximum value of Von Mises stress (equivalent stress) on the ground elements. A set of MG elements is shown in Figure 1, and the number of elements for single grid is set as the highest resolution. In addition, the threshold parameter ε is set to be 0.3 and $[0.08, 0.1, 0.25, 0.3]^T$ for all MPBILs with WOMG and WMG design approach, respectively. The mean hypervolumes of the fronts of MOP3 for all optimization runs are given in Table 1, in which the reference point for computing the hypervolumes is set to be {3.5 kNm, 3.5}^{T}.

MOP4: The design conditions of MOP4 are similar to MOP3, except in this design problem the top row finite elements are not assigned as design variables (unchanged) as displayed in Figure 3c, and the first objective of this problem changes to maximizing the first mode eigenvalue of structure (λ_1). Note that all of design problems use a membrane finite element formulation for structural analysis. The number of ground elements and the parameter ε of MOP4 are similar to MOP3. The mean hypervolumes of the fronts of MOP4 for all optimization runs are given in Table 1, in which the referent point for computing the hypervolumes is set to be {1.0 rad^2/s^2, 2.0}^{T}.

Table 1. Performance comparison based on hypervolume (HV) [1].

	MOP1		MOP2		MOP3		MOP4	
	WMG	**WOMG**	**WMG**	**WOMG**	**WMG**	**WOMG**	**WMG**	**WOMG**
MPBIL	0.8553	0.8255	0.7255	0.6420	0.7951	0.7229	0.7195	0.6219
OMPBIL	0.8556	0.8426	0.7259	0.6430	0.7968	0.7438	0.6723	0.6403
MPBILMLR	0.8115	0.7739	0.7212	0.6285	0.7016	0.5976	0.6167	0.5651
MPBILADLR	0.8543	0.8371	0.7240	0.6385	0.7954	0.7407	0.6404	0.6292

[1] WMG, with multi-grid approach; WOMG, without multi-grid approach; MPBIL, multi-objective population-based incremental learning; OMPBIL, opposite-based, multi-objective, population-based incremental learning; MPBIL MLR, MPBIL with multi-learning rate; MPBIL ADLR, multi-objective, population-based incremental learning with adaptive learning rate.

5. Design Results

The comparative performance of original MPBIL and the performance enhancements of MPBILs with MG and without MG approach for solving the design problems of MOP1–4 are given in Table 1, which compare based on HV indicator. It should be noted that all of the approximate Pareto fronts of the four design problems obtained from using the proposed MPBILs are normalized before calculating HV, as shown in Table 1. The highest mean of HV for each design problem is highlighted with grey color. The table shows OMPBIL promotes almost the best results for MOP1–4 except in case MOP4-WMG. Therefore, it is believed that the performance of OMPBIL is better result than the original MPBIL and their enhancements. In this study, the Friedman test and the Tukey–Kramer test are used for a statistical test to prove the significance of proposed algorithm. These tools are built-in functions in MATLAB/Octave. From our testing, the Friedman test gives OMPBIL has 1st rank, whereas the second rank is MPBIL at p-value (0.0002) < α(0.05) as shown in Table 2. It can be summarized that OMPBIL is the best performing algorithm for solving problem case MOP1–4. For multiple comparisons, we used the Tukey-Kramer test. The mean column ranks of OMPBIL are significantly different from MPBILMLR. The second best optimizer is MPBIL, whereas the third best is MPBILADLR. In addition, the worst optimizer for this design case is MPBILMLR. No questionable opposition concept is beneficial to improving the performance of MPBIL.

Table 2. Average ranking and p-value of MPBIL, and enhanced performance of MPBIL achieved by Friedman test.

Average Ranking of Each AlgorithmFriedman				p-Value
MPBIL	OMPBIL	MPBILMLR	MPBILADLR	
2.6250 (2)	3.8750 (1)	1 (4)	2.5000 (3)	0.0002

The average HV for all optimizers of each problem with MG and WOMG approach is shown in Table 3, which is summed along each column from Table 1. This table shows that the design problems with MG approach give higher HV than the design problem without MG approach in all design problems. Friedman test of average result in Table 3 can prove that the design problem with MG technique significantly outperforms WOMG technique at p-value (0.0455) < α(0.05). Furthermore, the best HVs of all cases give higher hypervolume than the previous work by [8] in all design cases, so OMPBIL with MG can improve the design results.

Table 3. Performance comparison of each MOP with and without MG for all algorithms.

Design Problems	Average Hypervolume	
	WMG	WOMG
MOP1	0.8442	0.8198
MOP2	0.7242	0.6380
MOP3	0.7722	0.7013
MOP4	0.6622	0.6141
Average Ranking (p-value = 0.0455)	2 (1)	1 (2)

Figures 4–7 shows some optimum topologies. The topologies in all figures are captioned with (a), which obtains from the best run of OMPBIL with multi-grid when solving each MOP with various r values. All figures are captioned with (b); they display the optimum topologies that are obtained from optimizing the design problem MOP1–4 with various r values by using MPBIL without multi-grid. These topologies are represented by the same technique from the previous work [8]. This shows that the topologies from OMPBIL with MG are better than the MPBIL technique without MG, and they can compare with the previous work using binary population-based incremental learning (BPBIL) and optimality criteria method (OCM) technique [8]. The optimum topologies are mostly from the ground elements with medium (MOP1 and MOP2) and low (MOP3, MOP4) resolutions. Therefore, the topology with the highest resolution is lower than the previous work by [8]. The lower resolution means lower computational time consumption. The use of highest ground element resolution is not the

best selection for all design problems. However, in practice, a designer never knows which resolution is the most suitable for design problem, and employing the multi-grid approach is an advantage.

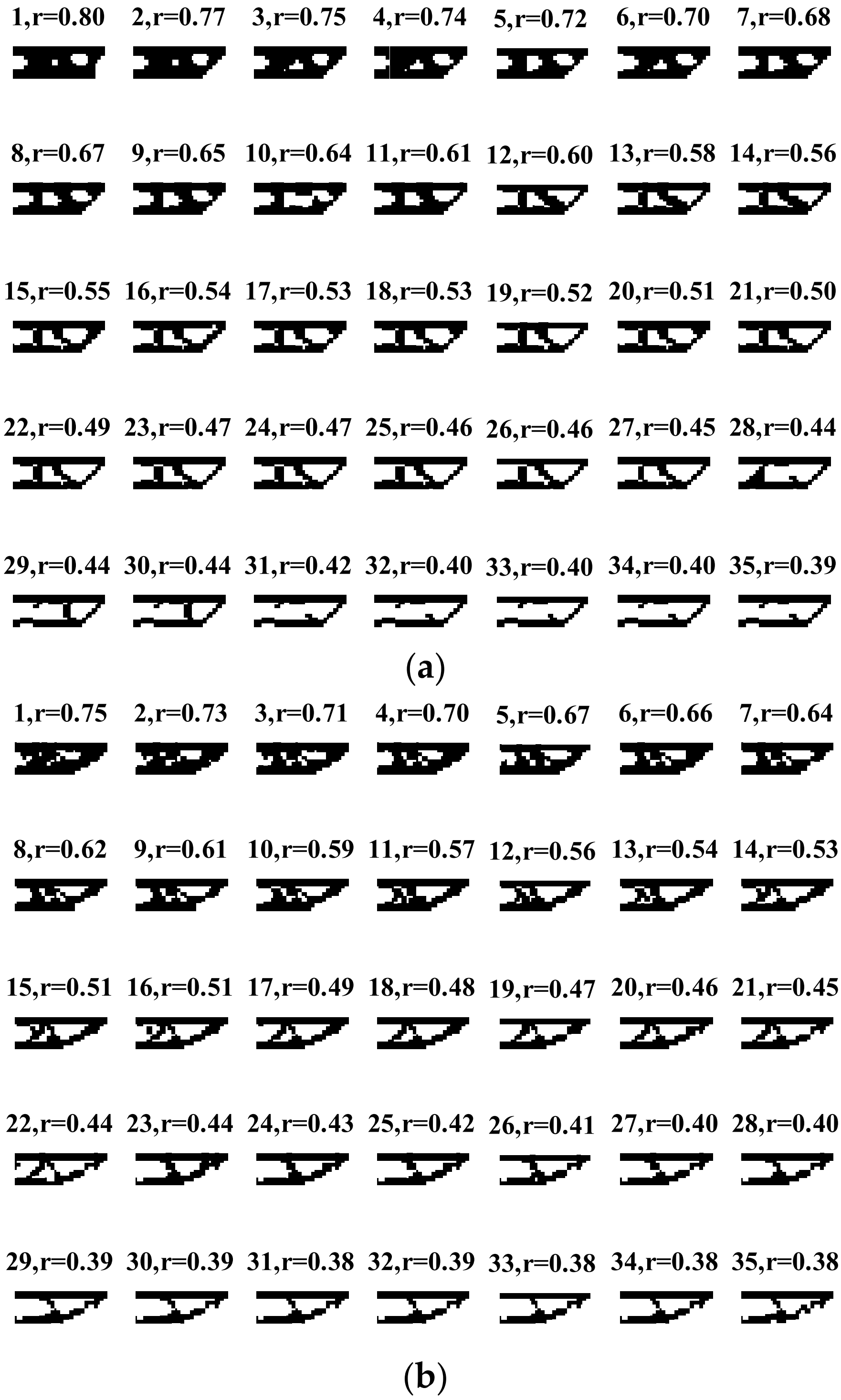
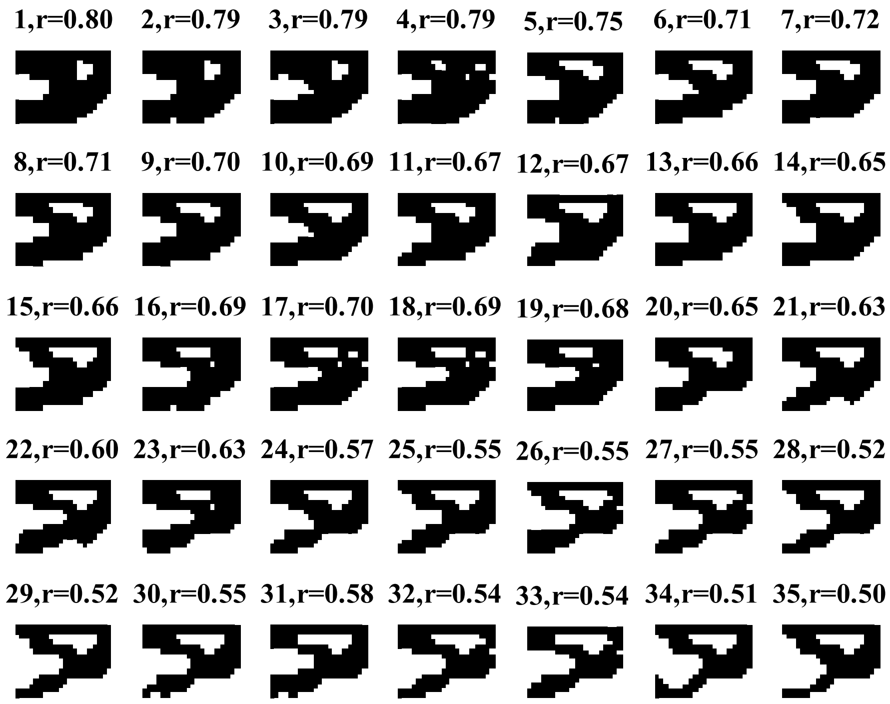
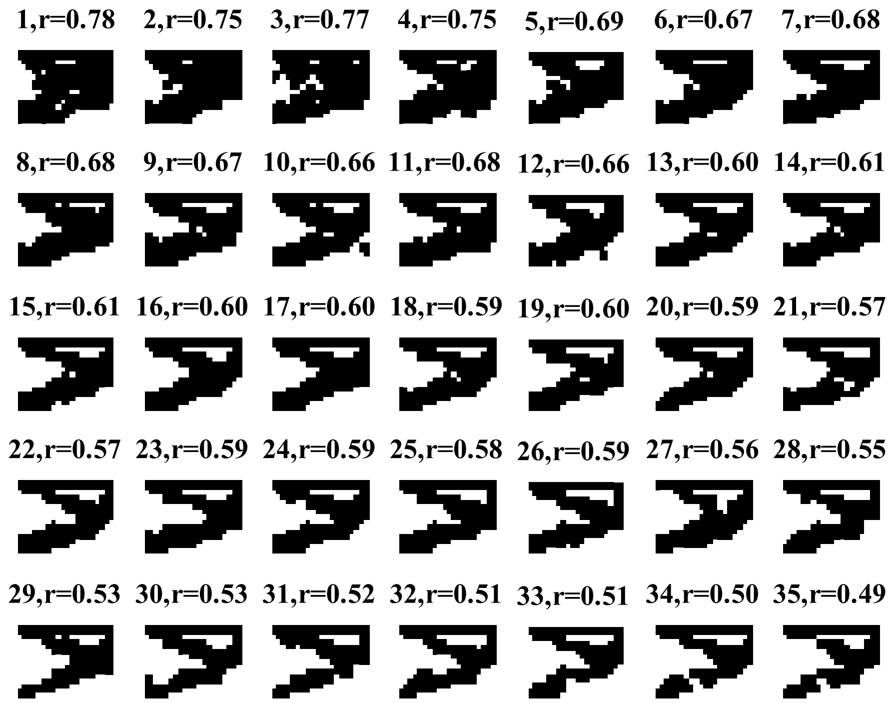


Figure 4. Topologies of MOP1: (**a**) OMPBIL with multi-grid; and (**b**) MPBIL without multi-grid.



(**a**)



(**b**)

Figure 5. Topologies of MOP2: (**a**) OMPBIL with multi-grid; and (**b**) MPBIL without multi-grid.

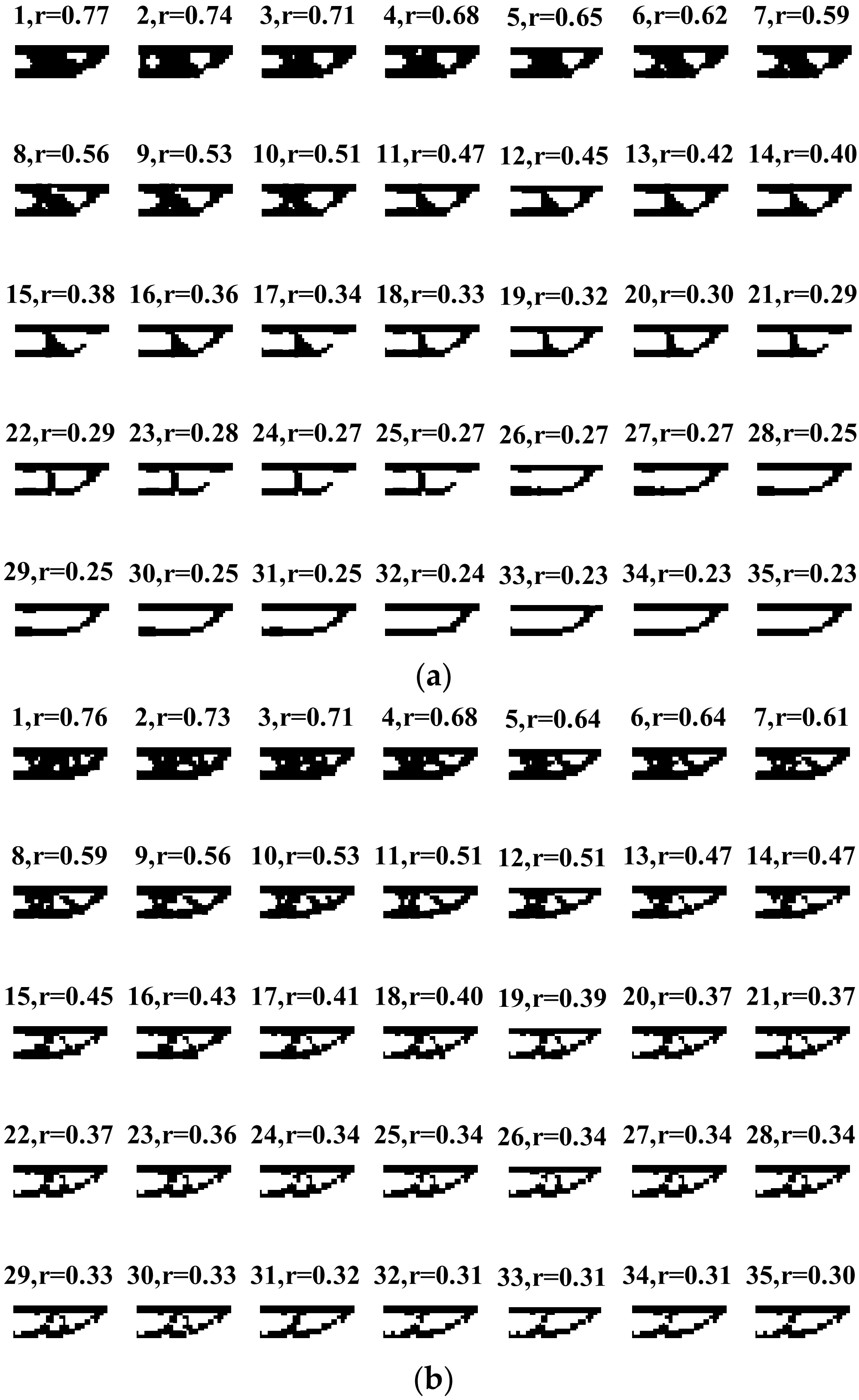


Figure 6. Topologies of MOP3: (**a**) OMPBIL with multi-grid; and (**b**) MPBIL without multi-grid.

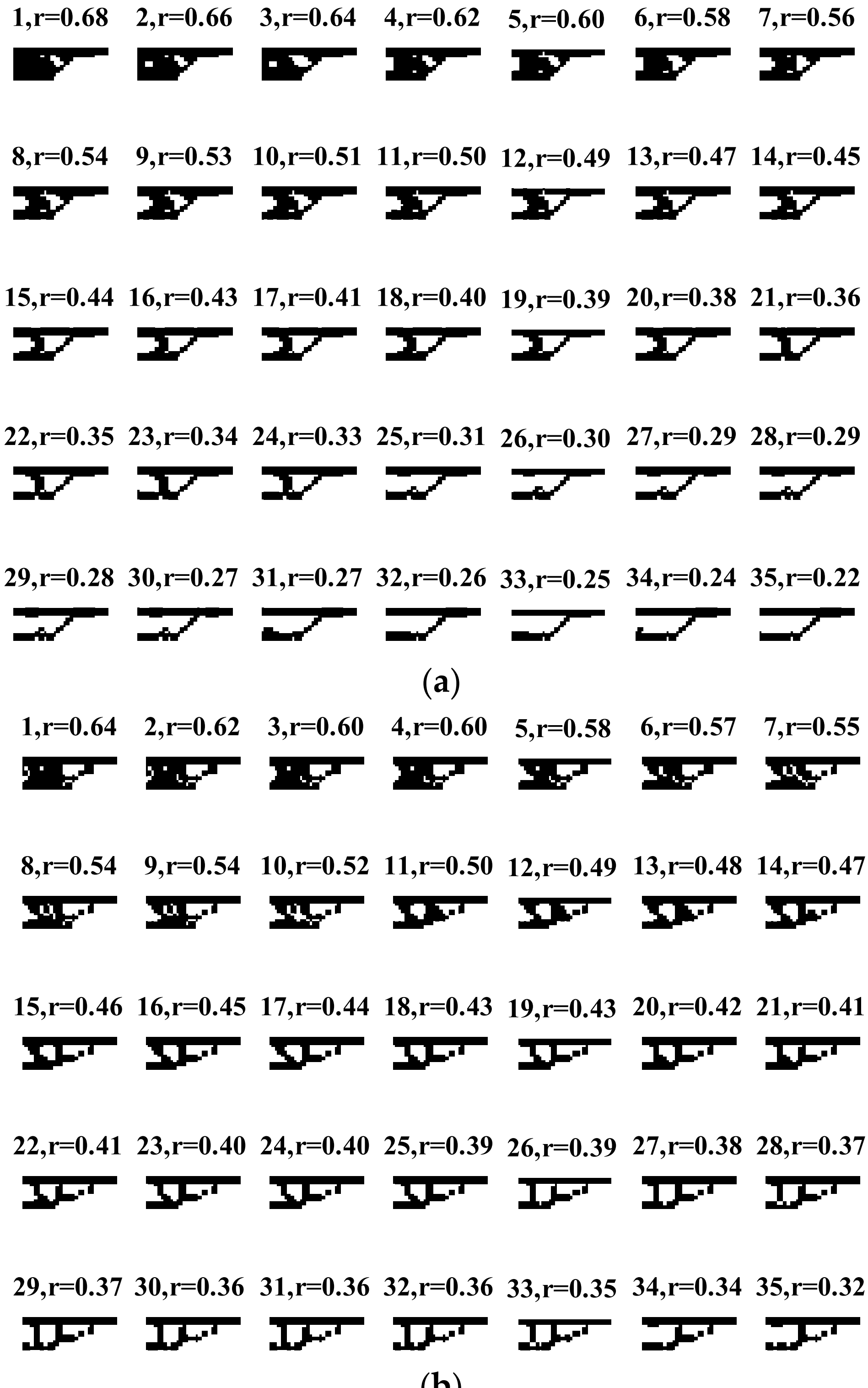


Figure 7. Topologies of MOP4: (**a**) OMPBIL with multi-grid; and (**b**) MPBIL without multi-grid.

6. Conclusions

The purposes of this work are the demonstration of the performance comparison of an original MPBIL and their performance enhancement, and the MG approach for multi-objective structural topology optimization problems, respectively. Among the performance enhancements of MPBIL, OMPBIL outperforms other techniques. It promotes the opposition-based concept, which can improve the search performance of MPBIL. The use of MPBILs in combination with the MG approach is well capable of solving multi-objective structural topology optimization. The resulting topologies obtained from using OMPBIL are close to those obtained from the classical gradient-based approach. The new design strategy is a procedure for structural topology optimization, which uses multiple ground element resolutions, so the MG approach is more efficient than using single-resolution ground elements in the sense that the suitable grid resolution is automatically detected and used in one optimization run. These conclusions are very similar those obtained in our previous work [3]. In addition, the use of the MG approach combined with ground element filtering for alleviating checkerboards is effective. In future work, the proposed method is extended to solve topology optimization with uncertainty.

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Conflicts of Interest: The authors declare no conflict of interest.

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Multi-objective Reliability-based Topology Optimization of Structures Using a Fuzzy Set Model

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Abstract This research proposes a multi-objective reliability-based topology optimization (MORBTO) for structural design, which considers uncertain structural parameters based on a fuzzy set model. The new technique is established in the form of multi-objective optimization where the equivalent possibilistic safety index (EPSI) is included as one of the objective functions along with mass, and compliance. This technique can reduce complexity due to a double-loop nest problem used previously due to performing single objective optimization. The present technique can accomplish within one optimization run using a multi-objective approach. Two design examples are used to demonstrate the present technique, which have the objectives as structural mass and compliance with the constraint of structural strength. The results show the proposed technique is effective and simple compared to previous techniques.

1. Introduction

Design processes for derivative-free topology optimization has been developed [1, 2] though it is still far from using in reality. The design process always depends on material properties, external loading and other conditions. If uncertainties of such parameters take place, obtained deterministic optimum design results may be less reliable [3]. To address such a problem, there are two main strategies to account for uncertainties in topology optimization, robust topology optimization (RTO) [4] and reliability-based topology optimization (RBTO) [5]. The first technique is to optimize the expectation and variability of system performance with respect to uncertainties simultaneously, through which the robustness of system performance can be improved, while the second is concerned with failure probability constraints when optimizing the system performance, through which reliable optimization design can be achieved. Both methods are based on probabilistic [6] or non-probabilistic models [3, 7, 9]. The first model is the most popular due to its progress, but this technique requires precision on the statistical distribution of uncertainties. A good distribution of uncertainties usually leads to large amount of objective information, which spends more time costly in a practical conceptual design stage.

In opposition to the first model, it is called non-probabilistic models where some well-known techniques of this type are anti-optimization [7] and a fuzzy set method [9]. The fuzzy set model is an alternative technique due to it gives moderate conservative results. It is the best choice to collect the uncertainties into RBTO by using a level set to soft separation between the members and non-members of the set. It makes the model get an acceptable solution. However, disadvantage of the present RBTO is still complexity in analysis due to the combination of fuzzy set into the topology optimization problem is a triple-loop nest problem including the double loop nest in finding possibilistic safety index (PSI) and topology optimization. Later, it has been solved by using the target performance-based design approach resulting the triple loop being reduced to the double-loop nested problem [3, 10]. The target performance-based approach changes the PSI into the target performance of the i-th constraint where it is called the equivalent possibilistic safety index (EPSI) by minimizing the constraint at some level cut. This technique can add an experience of the expert opinion to select the level cut or membership level into RBTO. The aim of this research is to reduce the complication of the double-loop nest problem in RBTO using multi-objective optimization technique with fuzzy uncertainties.

The rest of this paper is organised as follows; Topology optimization is presented in Section 2. Fuzzy set and MORBTO are proposed in Section 3. The numerical examples are given in Section 4. The conclusions of this study are detailed in Section 5, respectively.

2. Topology optimization

The topology optimization is a mathematical problem, which aims to seek the optimal structural layout within a pre-specified design domain. The single-objective or multiobjective topology optimization problem can be expressed as

$$\text{Min } f_i(\boldsymbol{\rho}) \quad i=1,..,M \tag{1}$$

Subject to

$$g_j(\boldsymbol{\rho}) \geq 0 \qquad j = 1,2,..., N$$

$$\mathbf{0} < \boldsymbol{\rho}^l \leq \boldsymbol{\rho} \leq \boldsymbol{\rho}^u$$

where $i = 1$ is for single objective design, $i > 1$ is for a multiobjective problem, and ρ is the thickness of finite elements ranged between the lower limit ($\boldsymbol{\rho}^l$) and the upper limit ($\boldsymbol{\rho}^u$). For the topology optimization problem in this work, the constraint is expressed in a different form from a traditional optimization problem so as to make it more compatible with the derivation of a possibility safety index in the next section.

3. Fuzzy set and formulation of MORBTO

The fuzzy set theory becomes popular for optimization design of structures because its capability can describe uncertainties with helping by expert opinions. The fuzzy set can be used for describing uncertain parameters and extended to the possibility concept as detailed in [5, 10]. For a fuzzy variable with a membership function $\mu(a)$, the corresponding fuzzy set model can be expressed as

$$\Lambda = \{(a,\mu(a)) \mid a \in \Omega, \quad \mu(a) \in [0,1]\} \tag{2}$$

where Ω is universal set, a is fuzzy variable. Each fuzzy variable a can be decomposed into a series of interval variables by using fuzzy set in accordance with degree of membership.

To construct the possibility of safety index, α-cut is used in this research ($\alpha \in [0,1]$) while a is in the interval $a^{\alpha} \in [a^{-\alpha}, a^{+\alpha}]$. The possibility that the fuzzy variable a is greater than s crisp number can be expressed as $\text{Pos}(a \geq s)$, so

$$\text{Pos}(a \geq s) = \sup_{z>s} \mu(z) \tag{3}$$

where "sup" is the supremum.

From (3) and inequality constraint in (1) can write as

$$\text{Pos}(g_j(\boldsymbol{\rho},\mathbf{a}) \leq 0) = \sup_{z \leq 0} \mu_j(z) \tag{4}$$

The topology optimization problem can be rearranged to be a RBTO problem based on the possibility safety index and the fuzzy set model:

$$\text{Min } f_i(\boldsymbol{\rho}) \quad i=1,..,M \tag{5}$$

Subject to

$$\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \leq 0) \leq \pi_{fj}^{max} \quad j = 1,2,..., N$$

$$\mathbf{0} < \boldsymbol{\rho}^l \leq \boldsymbol{\rho} \leq \boldsymbol{\rho}^u$$

Let $\mathbf{a} = (a_1, a_2, \ldots, a_l)$ be fuzzy variables, π_{fj}^{max} is an allowable possibility index, π_f is a possibilistic safety index, and $g_j(\boldsymbol{\rho},\boldsymbol{a})$ becomes fuzzy rather than crisp. In comparing an inequality constraint in (1) and (5) where the equations are respectively represented the deterministic topology optimization and the RBTO, respectively, the constraint in the traditional problem is used to control the value of limit-state function to strictly higher than zero while the constraint in RBTO is used to control the possibility safety index value that is lower than zero and π_{fj}^{max}.

The possibility safety index can be applied to the topology optimization problem, which incorporates with the fuzzy set method to deal with the uncertainties as shown;

$$\text{Min } \{f_i(\boldsymbol{\rho}), \pi_f^{max}\} \; i=1,...,M \tag{6}$$

Subject to

$$\pi_f^{max} = \max((\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0)) \le 1) \quad j = 1,2,..., N$$

$$\mathbf{0} < \boldsymbol{\rho}^l \le \boldsymbol{\rho} \le \boldsymbol{\rho}^u$$

New multi-objective reliability topology optimization problem is established in the possibility context.

To evaluate new objective function, the possibility safety index is derived in (4) and Fig. 1, it is found: (1) if $g_j^{-0} \ge 0$, then $\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0) = 0$; (2) if $g_j^{-0} \le 0 \le g_j^{-1}$, then $\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0) = \alpha$, where $g_j^{-\alpha} = 0$; (3) if $g_j^{-1} < 0$, then $\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0) = 1$. Eq. (4) can rewrite as Eq. (7).

Its original shape is a trapezoidal, which is called a trapezoidal-shaped fuzzy set. The trapezoidal shape can degenerate its form to other shapes, such as, a triangular shape [10]. In this research, we still use the original form as shown in Fig.1 and the membership function of constraint function can be formulated in the following equation.

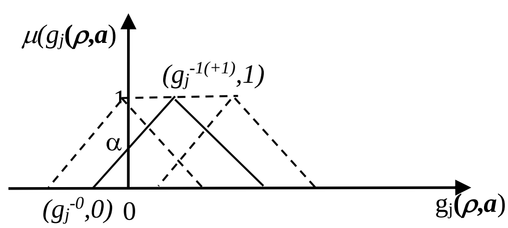


Fig. 1. Membership function $\mu(g_i(\boldsymbol{\rho},\boldsymbol{a}))$

$$\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0) = \begin{cases} 0, & g_j^{-0} \ge 0 \\ \alpha, & \text{where } g_j^{-0} < 0 < g_j^{-1} \\ 1, & g_j^{-1} < 0 \end{cases} \tag{7}$$

The solution of this equation can be calculated if g_j^{-0} and $g_j^{-1}$is known. If $g_j^{-0} \le 0$, we can obtain that $\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0) = 0$ or 1, and the solution procedure can be terminated; else if $g_j^{-0} \le 0 \le g_j^{-1}$, the equation $g_j^{-\alpha} = 0$ should be solved, and its solution α will be the value of $\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0)$. Theoretically, the bi-section method uses to compute the value of $\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0)$, a procedure can be summarized as:

Step1: Initialization – let $\alpha_1^0 = 0$, $\alpha_2^0 = 1$, and specify the termination value as $\varepsilon = 1 \times 10^{-8}$;

Step2: Iteration 1 – Calculate $g_j^{-\alpha_1^0}$ and $g_j^{-\alpha_2^0}$, and if $g_j^{-\alpha_1^0} \ge 0$ or $g_j^{-\alpha_2^0} \le 0$ holds, we can obtain $\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0) = 0$ or 1 and terminate the iterative procedure. Otherwise, to calculate $g_j^{-(\alpha_1^0+\alpha_2^0)/2}$ and go to step 3.

Step3: Iteration k (k ≥1) – if $g_j^{-\alpha_1^{k-1}} \times g_k^{-(\alpha_1^{k-1}+\alpha_2^{k-1})/2} > 0$ holds, then let $\alpha_1^k = (\alpha_1^{k-1}+\alpha_2^{k-1})/2$ and $\alpha_2^k = \alpha_1^{k-1}$; if $g_j^{-\alpha_2^{k-1}} \times g_j^{-(\alpha_1^{k-1}+\alpha_2^{k-1})/2} > 1$ holds, then let $\alpha_2^k = (\alpha_1^{k-1}+\alpha_2^{k-1})/2$ and $\alpha_1^k = \alpha_1^{k-1}$. Go to step 4.

Step4: Termination – calculate the absolute value $\left|\alpha_2^{k-1} - \alpha_1^{k-1}\right|$, and if the termination condition $\left|\alpha_2^{k-1} - \alpha_1^{k-1}\right| \le \varepsilon$ holds, stop the iterative procedure, and estimate $\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0)$ by $\text{Pos}(g_j(\boldsymbol{\rho},\boldsymbol{a}) \le 0) = (\alpha_1^{k-1}+\alpha_2^{k-1})/2$; otherwise, return to step 3 and continue the procedure till the termination condition $\left|\alpha_2^{k-1} - \alpha_1^{k-1}\right| \le \varepsilon$ is met.

For solving the multi-objective topology optimization problem in (6) is triple-loop nested problem, which is computational burden. The problem can be reduced to the double-loop problem by using the target performance-based approach [10] that has been proved the equivalent of the original failure possibility and the new one is described as follows.

$$\text{Pos}(g_j(\boldsymbol{\rho}, \boldsymbol{a}) \le 0)) \le \pi_{fj}^{max} \approx \min(g_j(\boldsymbol{\rho}, \boldsymbol{a}^{\pi fj})) \ge 0 \tag{8}$$

where $j = 1,2,..., N$, and $\min(g_j(\boldsymbol{\rho},\boldsymbol{a}^{\pi fj}))$ is called the target performance of the constraint.

The equivalent of the previous topology optimization problem (6) can be changed to

$$\text{Min } \{f(\boldsymbol{\rho}), \text{EPSI}\} \tag{9}$$

Subject to

$$\text{EPSI} = \max(g_j(\boldsymbol{\rho},\boldsymbol{a}^{\pi fj})) \ge 0, \; \pi_{fj} \in [0,1], \; j = 1,2,..., N$$

$$\mathbf{0} < \boldsymbol{\rho}^l \le \boldsymbol{\rho} \le \boldsymbol{\rho}^u$$

where EPSI is equivalent possibilistic safety index or target performance.

4. Design Examples

Two design examples demonstrate the proposed technique with the objective functions being volume fraction (r) or mass ratio and compliance (c). The objectives are conflicted as reducing mass affects to reduce the strength of structure. The difficulty of our objective is to find minimum mass ratio and compliance at the same time. The multi-objective optimization problem can be formulated and termed MOP1 and MOP2. The second problem (MOP2) is MOP1 with a stress constraint being added.

MOP1:

$$\min_{\rho^{GEF}} \{c, r\} \quad (10)$$

Subject to

$5c_{\min} - c \geq 0$

$0.8 \geq r \geq 0.2$

$\rho_i \in \{0.0001\text{m}, 1\text{m}\}$

MOP2:

$$\min_{\rho^{GEF}} \{c, r\} \quad (11)$$

Subject to

$5c_{\min} - c \geq 0$

$0.8 \geq r \geq 0.2$

$\sigma_{yt} - \sigma_{max}^{eqv} \geq 0$

$\rho_i \in \{0.000001\text{m}, 0.01\text{m}\}$

where ρ_i^{GEF} is the value of ith design variable; ρ_i is the thickness of ith finite element; m is the structural mass; $r = m(\boldsymbol{\rho})/m(\boldsymbol{\rho}^u)$ is the normalized mass or ratio of structural mass to maximum mass; c is the structural compliance; and $c_{\min} = c(\boldsymbol{\rho}^u)$. σ_{max}^{eqv} is the maximum value of Von Mises stress of the ground element. The last constraint in the design problem is bound constraints.

The traditional RBTO combined with the fuzzy set model can be formulated as in Eq. (5). It can be expressed as follows.

$$\min_{\rho^{GEF}} \{c, r\} \quad (12)$$

Subject to

$\text{Pos}(g_j(\boldsymbol{\rho}, \boldsymbol{a}) \leq 0) \leq \pi_{fi}^{max} \in [0,1] \quad j = 1,2,..., 3$

$g_1 = 5c_{\min} - c \geq 0$

$g_2 = 0.8 \geq r \geq 0.2$

$g_3 = \sigma_{yt} - \sigma_{max}^{eqv} \geq 0$ for MOP2

$\rho_i \in \{0.0001\text{m}, 1\text{m}\}$ for MOP1

$\rho_i \in \{0.000001\text{m}, 0.01\text{m}\}$ for MOP2

The proposed MORBTO incorporates the fuzzy set model from the previous section into the topology optimization to deal with the uncertainties in real situation can be formulated as:

$$\min_{\rho^{GEF}} \{c, r, \text{EPSI}\} \quad (13)$$

Subject to

$\text{EPSI} = \max(g_j(\boldsymbol{\rho}, \mathbf{a}^{\pi fj})) \geq 0, \; \boldsymbol{\pi}_{fj} \in [0,1], \; j = 1,2,..., 3$

$g_1 = 5c_{\min} - c \geq 0$

$g_2 = 0.8 \geq r \geq 0.2$

$g_3 = \sigma_{yt} - \sigma_{max}^{eqv} \geq 0$ for MOP2

$\rho_i \in \{0.0001\text{m}, 1\text{m}\}$ for MOP1

$\rho_i \in \{0.000001\text{m}, 0.01\text{m}\}$ for MOP2

where EPSI is the equivalent possibilistic safety index, $\boldsymbol{x}$ are ρ^{GEF}, and $\boldsymbol{\pi}_{fj}$ = PSI and $\boldsymbol{a}$ is vector of fuzzy variable (E, ν, and F).

Opposite-based multiobjective population-based incremental learning (OMPBIL) is used for solving the optimization problem in this research due to its good performance as demonstrated in our previous study [2]. The improvement used the opposition-based concept embedded into MPBIL, which is found that it can upgrade MPBIL's performance. The parameter of the optimiser is set according to our previous study and the details of OMPBIL can see in [2]. The population size is 35, the total number of iterations is 600, and the external Pareto archive size is 35. The learning rate (LR) is generated randomly in the interval [0.4, 0.6]. The mutation probability and mutation shift are 0.1 and 0.2, respectively.

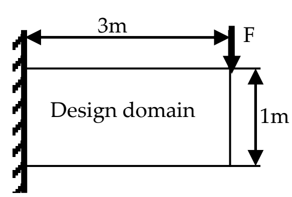


Fig. 2. Structural design domains

The topological design domain and loads are shown in Fig. 2. The uncertainties are Young's modulus E, Poisson's ratio ν, and load F, which is assumed to be fuzzy variables. The membership function of the variables is triangular shaped with values are E = (190, 200, 210) $\times 10^9$ N/m^2, ν = (0.25, 0.3, 0.35), and F = (0.9, 1, 1.5) kN. The mechanical property is not considered to be fuzzy variable, which is the tensile yield strength $\sigma_{yt} = 200 \times 10^6$ N/m^2. The traditional triple-loop nest multi-objective topology optimization problem is set to minimize structural compliance and normalized mass as shown in Eq. (12), while the proposed technique is presented in Eq. (13). The first constraint in every problem is added to prevent topologies with low global stiffness (or highly compliant structures) is included in the Pareto archive. The external force is applied at the right upper corner of the design domain. The traditional problem is used for comparing the time consuming with newly proposed technique.

The adaptation of a topology technique used in accomplishing the problems is from our previous proposed technique in [1], which is called the multi-grid ground element technique (MG). The proposed technique has been proved to be an efficient technique when combining with the OMPBIL. The Encoding and decoding scheme for the MG approach with PSI and fuzzy variables is needed as shown in the following algorithm 1.

Algorithm 1. Encoding and decoding scheme for a MG approach with PSI and fuzzy variables
Initialization: Generate four sets of ground elements and define the threshold value of ε for each set.
Inputx sized $(N_{41} + 6) \times 1$.
Output: Thicknesses of ground elements.
Encoding
$x_1 \in [1, 4]$ is used for selecting a set of ground elements.
x_2= PSI is defined by the designer
x_3=is used for fuzzy variable F
x_4=is used for fuzzy variable E
x_5=is used for fuzzy variable ν
x_6 to $x_{N_{41}+6}$ are used for element thicknesses.
Decoding
1: Find $n = round(x_1)$ where $round(.)$ is a round-off operator.
2: If $n = 1$: x_6 to $x_{N_{41}+6}$ are set as N_{11} element thicknesses and $\varepsilon=\varepsilon1$.
3: If $n = 2$: x_6 to $x_{N_{41}+6}$ are set as N_{21} element thicknesses and $\varepsilon = \varepsilon2$.
4: If $n = 3$: x_6 to $x_{N_{41}+6}$ are set as N_{31} element thicknesses and $\varepsilon = \varepsilon3$.
5: If $n = 4$: x_6 to $x_{N_{41}+6}$ are set as N_{41} element thicknesses and $\varepsilon = \varepsilon4$.

The flow diagram of MORBTO is shown in Fig. 3.

All computations are conducted using MATLAB and a personal computer with specifications being Intel(R) Core™ i5-3210M CPU @ 2.5 GHz, 4.00 RAM, and 64-bit Windows 10 operating system.

5. Design Results

The optimal topologies obtained from using the proposed MORBTO for MOP1 and MOP2 are shown in Figs. 5 and 7, respectively. As a comparison, the optimal topology design obtained from the deterministic topology optimization for MOP1 and MOP2 is also presented in Figs. 4 and 6, respectively. Some selected optimal topology designs obtained from Figs. 4 and 5 are shown in Figs. 8(a) and 8(b), respectively. From comparison, the optimal topologies obtained from the deterministic topology optimization also present in Figs. 8(a) and 9(a). It can see in Figs. 8(b) and 9(b) that the result from MORBTO is under the EPSI yields optimal topology design different from those yielded by the deterministic topology optimization (Figs. 8(a) and 9(a)).

This can conclude that the proposed MORBTO approach presents a strategy that generate safer topology designs satisfying different failure possibility requirements, which causes topology changing follows the failure possibility change. As well as the proposed MORBTO yields optimal topology designs clearly that is different from the deterministic topology optimization, indicating that the fuzzy uncertainties remarkably influence the topology design. Furthermore, the time consumption comparison between the traditional triple-loop nest problem and the proposed technique for all cases are shown in Table 1. The results show that the time spent by the proposed technique is lower than the triple-loop nest problem in all the failure possibility constraints. It can be clearly stated that the proposed technique can reduce the complexity of the traditional technique.

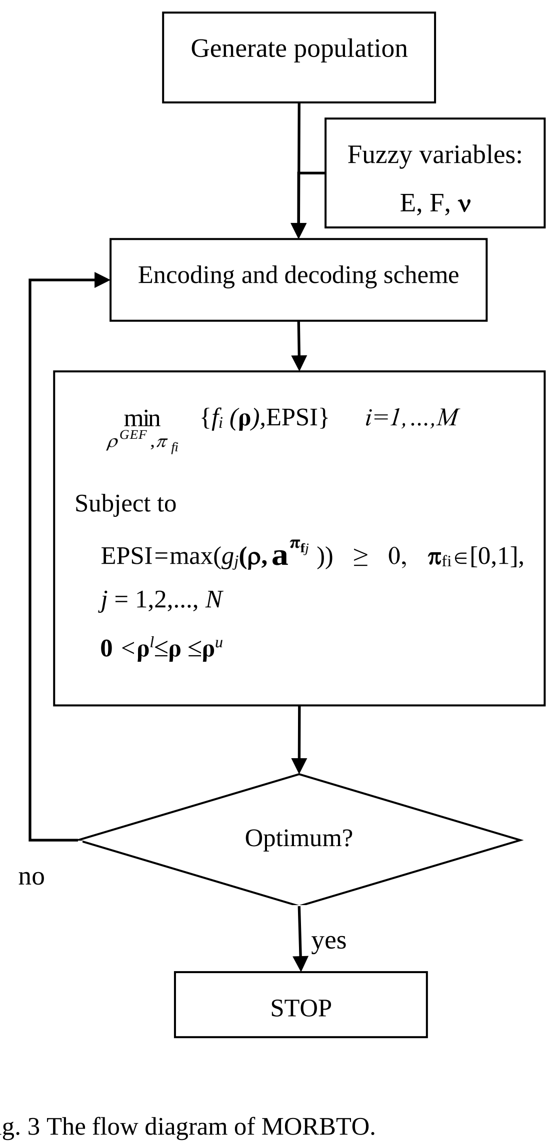


Fig. 3 The flow diagram of MORBTO.

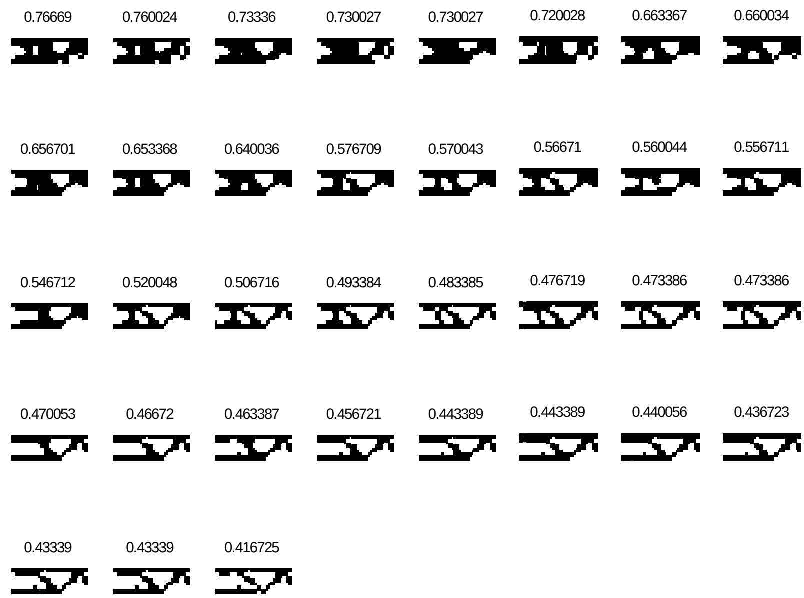


Fig. 4.The deterministic optimal topology design of MOP1 under failure possibility constraint π_{fj}= 1with various r.

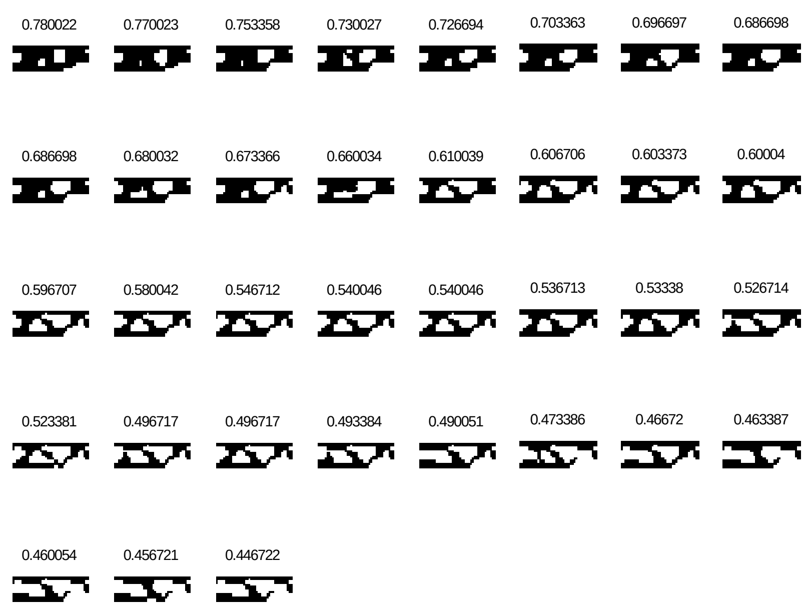


Fig. 5. The optimal topology design of MOP1 under failure possibility constraint π_{fj}= 0.001 with various *r*.

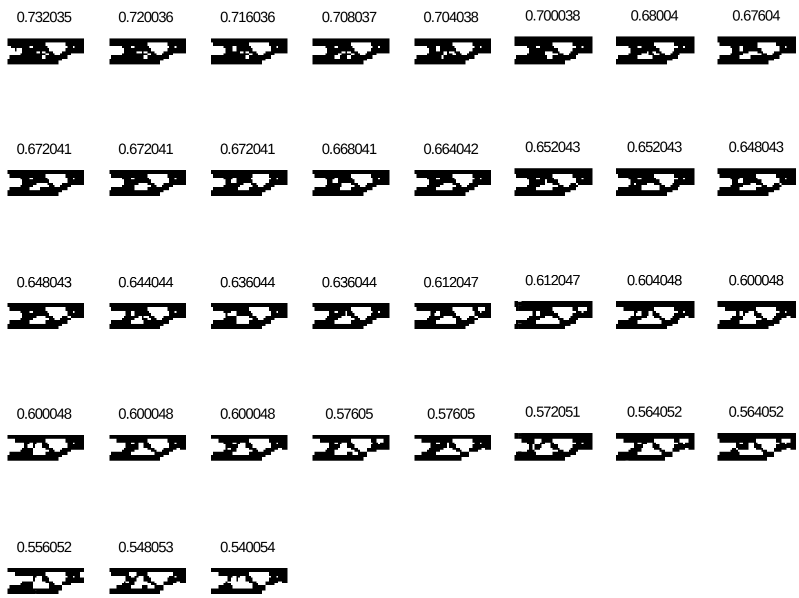


Fig. 6. The deterministic optimal topology design of MOP2 under failure possibility constraint π_{fj}= 1 with various r.

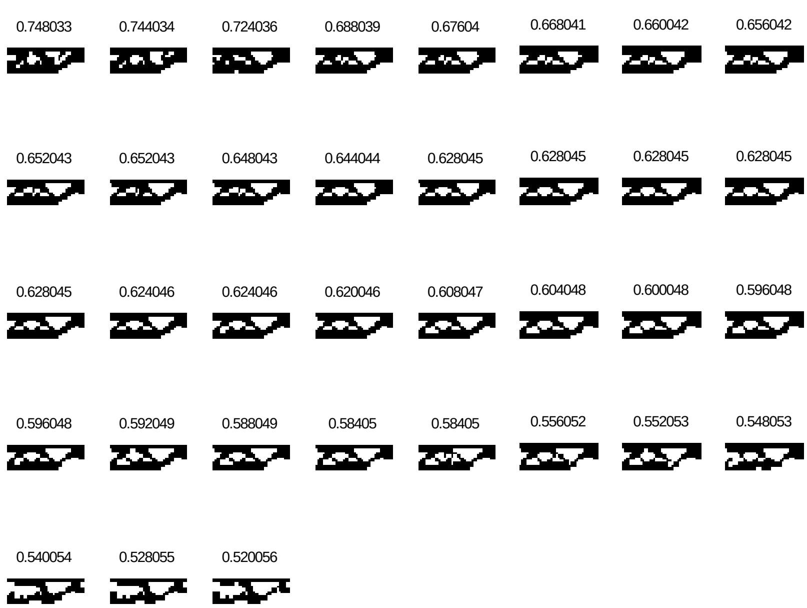
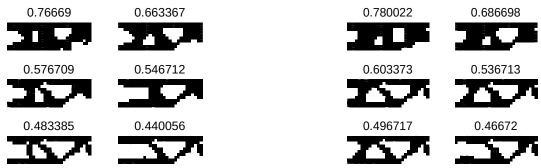
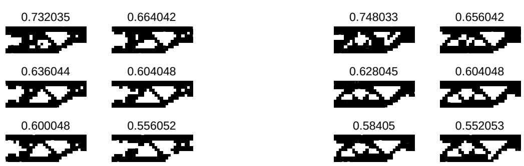


Fig. 7. The optimal topology design of MOP2 under failure possibility constraint π_{fj}= 0.001 with various r.



(a) (b)

Fig. 8. The selected optimal topologies of MOP1 (a) π_{fj} = 1 (b) π_{fj} = 0.001.



(a) (b)

Fig. 9. The selected optimal topologies of MOP2 (a) π_{fj} = 1, (b) π_{fj} = 0.001.

Table 1. The time consuming of RBTO and MORBTO for MOP1 and MOP2

MOP1	RBTO		MORBTO	
π_{fj}	1	0.001	1	0.001
Time consuming (s)	5143.0	5168.7	966.1	973.3

MOP**2**	RBTO		MORBTO	
π_{fj}	1	0.001	1	0.001
Time consuming (s)	6755.8	9265.6	1093.9	1141.7

6. Conclusions and Discussions

The optimum structure from topology optimization problem may be less realisable due to inevitable uncertainties from various sources, thus, these uncertainties should take into account in design. A satisfactory design should take these uncertainties into consideration. This paper proposes new MORBTO approach which uses the fuzzy set model to describe the uncertainties. This technique can be applied for the multi-objective topology optimization problem, which has conflicting objectives. The proposed MORBTO is formulated as a problem of minimizing mass, compliance and equivalent possibilistic safety index with constraints being accomplished in the possibility context. Furthermore, this paper proposes to deal with the complex reliability of the structure using fuzzy that it is usually a double-looped problem by using multi-objective optimization reducing to single-loop optimization. The performance of the proposed technique is studied by two numerical examples. From the numerical results, it is shown that the proposed MORBTO can generate conservative optimal topology designs with various values of EPSIs, which is different from those obtained from the pvious RBTO. These results indicate that the proposed MORBTO is an effective tool to deal with the uncertain topology optimization with considering the expert opinion. Furthermore, the proposed technique can reduce the complexity of the traditional technique and reducing time consumption in solving the optimization problem.

For future study, the reliability-based topology optimization approach will extend to handle uncertainties that take place in design of an aircraft structure.

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References (in font 9) should appear with items referred to by numerals in square brackets. All journal articles must include volume, number, and pages. The journal title, conference title, and book title must be in italic.

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