สกว
TRF

รายงานวิจัยฉบับสมบูรณ์

การออกแบบโครงข่ายสื่อสารโดยการพิจารณาโครงข่ายที่ใช้งานอยู่แล้ว

Communication Network Design with the Consideration of Existing Network

โดย สุวรรณ รุ่งกีรติกุล

เมษายน 2544

Scan

สัญญาเลขที่ PDF/33/2541

รายงานวิจัยฉบับสมบูรณ์

การออกแบบโครงข่ายสื่อสารโดยการพิจารณาโครงข่ายที่ใช้งานอยู่แล้ว

Communication Network Design with the Consideration of Existing Network

สุวรรณ รุ่งกีรติกุล
สาขาวิชาวิศวกรรมไฟฟ้า สถาบันเทคโนโลยีนานาชาติสิรินธร
มหาวิทยาลัยธรรมศาสตร์

สนับสนุนโดย สำนักงานกองทุนสนับสนุนการวิจัย

บทคัดย่อ

Project Code: PDF/33/2541

Project Title: การออกแบบโครงข่ายสื่อสารโดยการพิจารณาโครงข่ายที่ใช้งานอยู่แล้ว

Investigator: ผู้ช่วยศาสตราจารย์ ดร. สุวรรณ รุ่งกีรติกุล
สาขาวิชาวิศวกรรมไฟฟ้า สถาบันเทคโนโลยีนานาชาติสิรินธร มหาวิทยาลัยธรรมศาสตร์
Email Address: suwan@siit.tu.ac.th

Project Period: 2 ปี 10 เดือน (1 กรกฎาคม 2541 – 30 เมษายน 2544)

Objectives: วัตถุประสงค์หลักของงานวิจัยนี้มุ่งเน้นไปที่การศึกษาปัญหาการออกแบบโครงข่ายสื่อสาร 2 ประเภทคือ โครงข่ายการสวิตช์กลุ่มข้อมูล และโครงข่าย ATM โดยให้มีการพิจารณาผลกระทบของโครงข่ายที่ใช้งานอยู่แล้ว

Methodology: ปัญหาการออกแบบโครงข่ายสื่อสารข้างต้นจะถูกแปลงให้อยู่ในรูปของปัญหาการทำให้เหมาะที่สุด และจากการวิเคราะห์จะทำให้ทราบคุณสมบัติของคำตอบที่เหมาะที่สุดของปัญหาเหล่านี้ ซึ่งจะนำไปสู่อัลกอริทึมสำหรับการแก้ปัญหาการออกแบบโครงข่าย

Results: อัลกอริทึมสำหรับการแก้ปัญหาการออกแบบโครงข่ายการสวิตช์กลุ่มข้อมูล และโครงข่าย ATM เมื่อพิจารณาโครงข่ายที่ใช้งานอยู่แล้ว

Discussion Conclusion: อัลกอริทึมที่ได้จากการศึกษามีคุณสมบัติที่ดีมากทั้งในแง่ของคุณภาพของคำตอบและระยะเวลาในการคำนวณ ดังนั้นจึงเหมาะสำหรับใช้ในการแก้ปัญหาการออกแบบโครงข่ายจริง

Suggestions: ควรมีการศึกษาเปรียบเทียบคุณสมบัติของอัลกอริทึมที่ได้จากการศึกษานี้ กับวิธีอื่นๆ เช่นวิธีทาง metaheuristics

Keywords: การออกแบบโครงข่ายสื่อสาร, โครงข่ายที่ใช้งานอยู่แล้ว, โครงข่ายการสวิตช์กลุ่มข้อมูล, โครงข่าย ATM, อัลกอริทึม

Abstract

Project Code: PDF/33/2541

Project Title: Communication Network Design with the Consideration of Existing Network

Investigator: Asst.Prof.Dr.Suwan Runggeratigul, Electrical Engineering Program, Sirindhorn International Institute of Technology, Thammasat University

Email Address: suwan@siit.tu.ac.th

Project Period: 2 years 10 months (1 July 1998 – 30 April 2001)

Objectives: The main objective of this project is to study the communication network design problems under the consideration of existing network facilities in packet-switched networks and ATM networks.

Methodology: The network design problems are formulated as optimization problems, which are analyzed to obtain the characteristics of optimal solutions. Based on these solution characteristics, heuristic algorithms are proposed to solve the problems.

Results: Two heuristic design algorithms are obtained for solving communication network design problems taking into consideration existing network facilities, where each of which is for packet-switched networks and ATM networks, respectively.

Discussion Conclusion: The proposed algorithms have very good performances in both the quality of solution and the computation time, and thus they are applicable to practical network design problems.

Suggestions: The performances of the proposed design should be compared with other design algorithms, e.g., algorithms based on metaheuristics.

Keywords: communication network design, existing network, packet-switched networks, ATM networks, algorithms.

Final Report

Communication Network Design with the Consideration of Existing Network

by Suwan Runggeratigul

April 2001

Contents

List of Figures

List of Tables

Acknowledgements

The author wishes to express his deepest appreciation to his mentor Prof. Dr. Sawasd Tantaratana for his illuminating guidance and invaluable discussions when conducting this research.

The author wishes to thank the Thailand Research Fund for financial support, and Sirindhorn International Institute of Technology, Thammasat University for the support of research facilities throughout this project.

At last, the author wishes to thank his wife Kanok and his son Parinthorn (μ) for their patience and support over the period of this project.

Suwan Runggeratigul
30 April 2001

Executive Summary

In the conventional communication network design, the concept of existing network is rarely taken into account. The main reason is that existing network facilities are treated as parts of network that are ready to be used in the construction of new network. Almost of the previous research works considered that the existing parts of network can be used without any cost. However, it is clear that the above idea is not applicable in practical cases. For example, although they are promptly to be used, there is cost of using the existing network facilities such as operating or maintenance cost.

In this research, we investigate the effect of existing network facilities to the design of communication network by introducing the cost difference between the existing network facilities and newly installed facilities. We focus on two communication network design problems, namely the link capacity assignment problem (CA problem) in packet-switched networks, and the virtual path routing problem (VPR problem) in asynchronous transfer mode networks (ATM networks).

For the CA problem in packet-switched networks, we investigate the case of long-term design by applying a piecewise linear concave link cost function, where the per-unit cost of existing link capacity is greater than that of newly installed capacity. By mathematical analysis, it is shown that there is no link whose capacity is equal to its existing capacity in the optimal solution of the problem. Based on this characteristic, the non-differentiable cost function can be treated as a differentiable function, and a design algorithm derived from the Lagrange multiplier method is then proposed.

For the VPR problem in ATM networks, we consider the cases of both short-term and long-term design by considering piecewise linear convex and concave link cost function, respectively. It is shown that the VPR problem has similar properties to shortest path problems. Accordingly, we propose a design algorithm derived from shortest path algorithm to solve the problem.

By numerical results, it is shown that the proposed design algorithms have very good performance in both the view of quality of solutions and computation time. This means that they are applicable to solve practical communication network design problems, especially for the case of large-scale networks.

The numerical results also show that how existing network affects the design of new network. In the case of short-term design, it is necessary to utilize the existing network facilities in a full range before installing new facilities to the network. This reflects the properties of existing network facilities in the short-term design. That is the existing parts of the network are promptly to be used in the construction of new network, and cannot be changed within a short period of time. Consequently, using existing facilities effectively before introducing new facilities is a very important design concept in this case.

On the other hand, it is shown in the case of long-term design that the above concept can be neglected when designing a new network. This result obviously reflects the properties of economy of scale in communication network resource for the case of long-term design. Hence, it is possible to augment new network facilities to the existing network where there are needs, while some parts of existing facilities can be left unused.

Chapter 1

Introduction

Many people stated that the current age is the information age [9][58][77][96]. With the advance of information-oriented society, communication networks play a key role as major means for transferring information between humans. Thus, networks that provide communication services to users efficiently, reliably, and economically are essentially required [102]. To construct such kind of networks, high-performance design method that considers several design factors is needed, where the method must be capable to give optimal solution (or near-optimal solution with high accuracy) in an acceptable running time.

The work in this research project is related to the study on communication network design that focuses on how the existing network facilities affects the design of a new network. This chapter briefly describes the statement of the problem, and gives the scope and objectives of the study.

1.1 General

Let's consider the graph given in Figure 1.1. This graph is a mathematical representation of a communication network, where each vertex or node represents a communication point, and each edge or link is equivalent to a communication channel connecting two communication points [2][10][50].

As an example of network design problem, we focus on the following case. When design parameters including the user traffic demand (forecasted value) are given, the capacity of each link in the network is determined such that all conditions of the design constraint are satisfied. One feasible solution to this problem is that we can set each link capacity to infinity. However, it is obvious that this is not an effective solution, i.e., the cost of implementing the network is also infinite. Consequently, we need one more requirement in the design, that is the cost of network implementation must be minimized. With this requirement, each link capacity will then be set to an optimal value. We call this design problem as the capacity assignment problem [10][52].

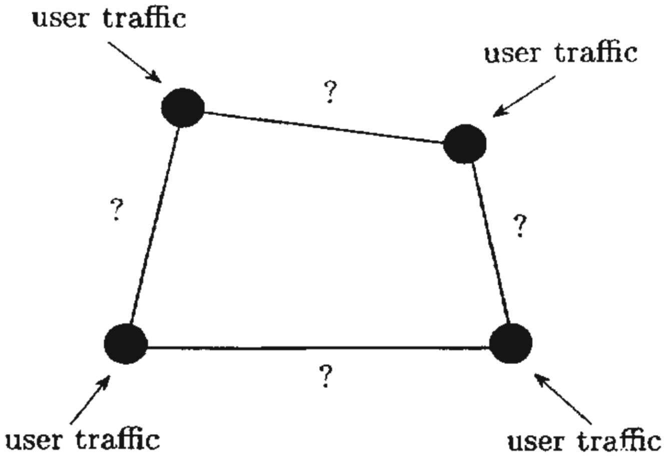


Figure 1.1: Design of a communication network.

The phase after the network design is the network implementation. After that, it will be the phase of network operation, where actual user traffic demand is applied to the network. However, after a period of running the network, the need of network redesign occurs [10][103]. The reasons that a network is needed to be rebuilt or redesigned can be listed as follows [103].

- There is a substantial increase in the number of users in the network.
- A substantial change occurs in the computation power of users' workstation machines.
- New applications emerge and demand more or different network services.

Clearly, the events in the above reasons result in the change of the user traffic demand to the network. With this new traffic demand, it is necessary to redesign the existing network as in Figure 1.2.

Besides the new user traffic demand, many design parameters and factors must be considered in the network redesign problem. New design constraint such as better quality of service (QoS) may be needed according to the new class of services introduced to the network. However, one of the most important design factors is the network facilities that are already implemented in the network. Although there are a lot of works that study the communication design/redesign problems in the literature [10][50][104], the concept of existing network, however, is rarely taken into account.

The main reason that the effect of existing network is usually neglected in the conventional study on communication network design is that existing network facilities are assumed as parts of network that are ready to be used in the construction of new network. Almost of the previous research works consider that the existing parts of network can be used without any cost, or can be treated in the same way as new facilities needed to be installed to the network [10][104]. However, it is clear that this idea is not generally true in practical cases. For example, although the existing network facilities are promptly to be used, there is cost of using them such as operating or maintenance cost. Moreover,

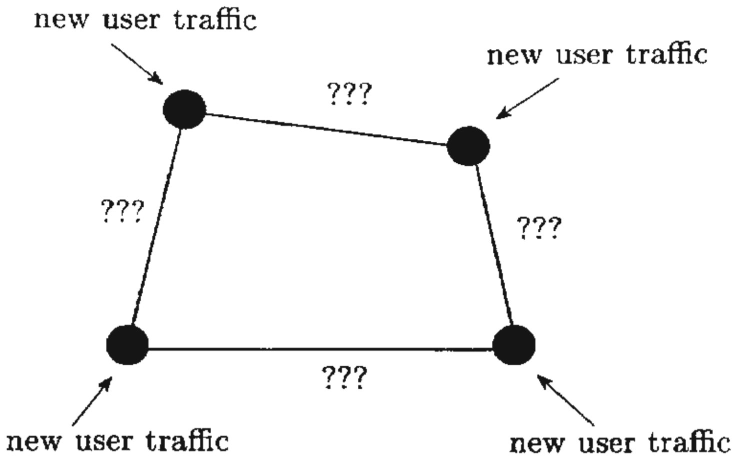


Figure 1.2: Redesign of a communication network.

the cost of using the existing network facilities needs not be the same as that of newly installed facilities.

Although most of the studies on communication network design/redesign do not include the concept of existing network in the design of new network, there are a few research works that study this major design factor as well as its effect to network design problem. These works are briefly described below.

Firstly, Shinohara studies the circuit dimensioning problem in circuit-switched networks in [92][93], where the design problem is formulated and solved under the condition that the number of circuits in the existing network is not zero. To include this design condition in the problem, a link cost function shown in Figure 1.3 is introduced to be applied to the design problem.

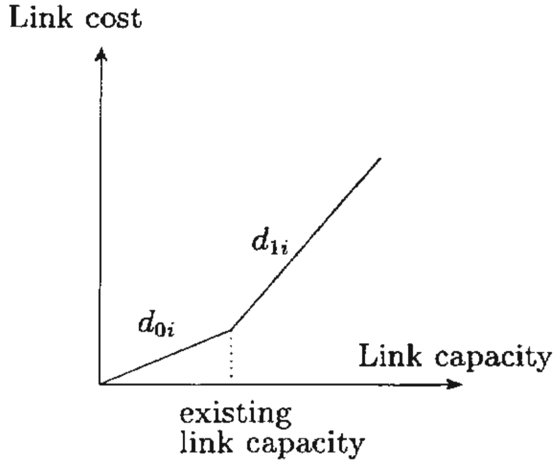


Figure 1.3: Link cost function taking into consideration an existing network: a piecewise linear convex function.

The cost function in Figure 1.3 is a piecewise linear convex function, where d_{0i} and d_{1i} are the per-unit cost of existing capacity and newly installed capacity on link i, respectively. The value of d_{0i} and d_{1i} are both non-negative. Shinohara examines the case that

$$d_{0i} \leq d_{1i}, \tag{1.1}$$

that is the per-unit cost of existing capacity is less than that of the newly installed capacity. This case is generally valid in network construction, especially in the case of short-term design. By taking the cost difference between the two types of link capacity (i.e., the existing part and the newly implemented part), the circuit dimensioning problem is solved to determine the number of circuits in all network links such that the total network cost is minimum, and the effect of the existing network to the network design can be investigated.

The other research work in the literature that takes into account the concept of existing network in network design problem is the work by Runggeratigul et. al. [80][81], where the link capacity assignment problem in packet-switched networks (short-term case) is studied. This work also applies the idea of the cost difference between existing capacity and new capacity to the packet-switched network design problem, and the capacity cost function as shown in Figure 1.3 is adopted. The main result of this study is that, the proposed design algorithm can give a solution in the way that the existing network facilities are utilized effectively, and new facilities are augmented to the existing network only in the places that there are needs.

Clearly, the cost function of Figure 1.3 used in the conventional research works is very useful in communication network design when the effect of existing network facilities is needed to consider. Moreover, this type of cost function is also applicable to other works on network design where the concept of existing network is omitted.

Firstly, when the cost of using existing network facilities is zero, we can set the value of d_{0i} as

$$d_{0i} = 0, \tag{1.2}$$

and the cost function as shown in Figure 1.4 is obtained.

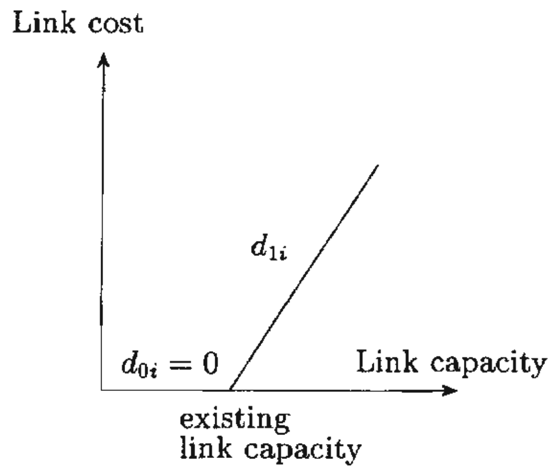


Figure 1.4: Cost function when there is no cost in using existing network facilities.

Secondly, when existing and new network facilities are treated in the same way, we can set the values of d_{0i} and d_{1i} as follows.

$$d_{0i} = d_{1i}, \tag{1.3}$$

Obviously, the cases in Eq.(1.2) and Eq.(1.3) are special cases both included in Eq.(1.1). This means that the research works using cost function of Eq.(1.1) is a more general case of the previous works.

Although there are already a few studies on network design with the consideration of existing network, this kind of study, however, is needed to be extended. The reasons can be given as follows.

- There are questions that the concept of cost difference between existing facilities and newly installed facilities is applicable or not to the design/redesign of other types of communication network besides the circuit-switched and packet-switched networks.
- Since the conventional works only study network design problem in the short-term case, it is interesting to investigate also the effect of existing network facilities in the case of long-term design.

The work in this research project is devoted to answer the above questions on network design with the consideration of existing network.

1.2 Scope of the Study

To extend the study in conventional research works, we will focus on the design of the following two types of communication networks.

1. Packet-switched networks.
2. Asynchronous transfer mode (ATM) based broadband integrated networks (ATM networks).

There are reasons why this project studies the above networks. Packet-switched network is the typical form of the Internet, which is the interconnection of communication networks around the world [74][96]. Nowadays, there are a large number of Internet users in almost of the parts of the earth, and the number increases gradually in a very high rate [14][60].

On the other hand, ATM is an advanced multiplexing and switching technique, which is recently accepted as the "target transfer mode" for the implementation of future broadband integrated networks [55][59][74]. Here, the term *broadband* means that ATM network can support high-speed communications, while *integrated* means it can support communications of all media of communication information including voice, image, video, and computer data.

Hence, the study in this research project has applications to the design of present and future communication networks, where we concentrate the study on the following network design problems.

1. Link capacity assignment problem (CA problem) in packet-switched networks (the long-term case).

2. Virtual path routing problem (VPR problem) in ATM networks (both the short-term and long-term cases).

To study the network design problems in the long-term case, it is proper to consider link cost function as a concave function [50][52]. This is because the economy of scale is often present in communication network resources. For example, we can obtain a cost function from the envelope of several cost functions due to the development in communication transmission technologies [30][107], the development in switching technologies [99], etc. To obtain a cost function for this case, we still use the concept of cost difference between existing and new network facilities. However, the relationship of d_{0i} and d_{1i} is now changed to

$$d_{0i} > d_{1i}. \tag{1.4}$$

This results in the cost function depicted as in Figure 1.5

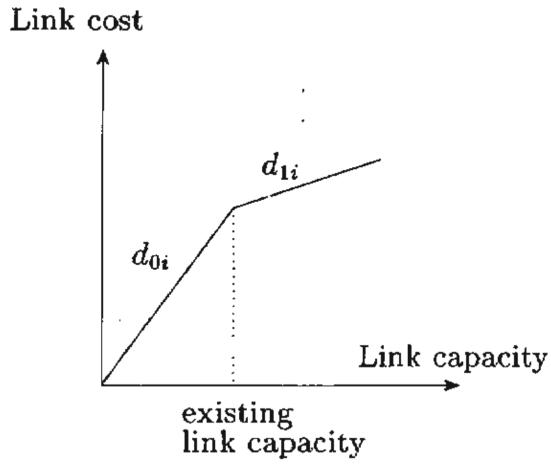


Figure 1.5: Link cost function taking into consideration an existing network: a piecewise linear concave function.

In this project, the above two network design problems are studied in the same manner. That is each network design problem is solved to obtain an optimal solution based on the information given, e.g., the amount of existing network facilities, the new user traffic demand, etc.

1.3 Objectives of the Research

As mentioned earlier, this research project is devoted to the study of communication network design taking into consideration an existing network. The study has the following objectives and goals.

1. To study the CA problem in packet-switched networks and the VPR problem in ATM networks under the condition that the effect of existing network is taken into account.
2. To analyze the characteristics of optimal solution in the above network design problems.
3. To construct, for each problem, a high-performance design algorithm derived from the characteristics of optimal solution.
4. To conduct experiments based on the algorithms obtained above.
5. To analyze the experimental results and examine the impact of existing network on each network design problem.

In this project, many efforts are made such that high-performance design algorithms can be obtained, where these algorithms must be able to give optimal solution or near-optimal solution within small computation time. It should be noted that the small computation time is very important, since this will make the algorithms applicable to solving network design problems when networks are of large size.

1.4 Organization of the Report

This research report begins in Chapter 1 with the general introduction of communication network design as well as the statement of problem studied in this project. After that, the scope and objectives of the study are given.

Chapter 2 provides the literature review for the study. It gives the overview of communication network design and a few examples of research works related to the work in this project.

Chapter 3 is the first part of the study, where the CA problem for packet-switched networks in the case of long-term design is examined. In this case, a piecewise linear concave cost function is considered in the design problem, and the characteristic of optimal solution is derived. After that, a design algorithm is proposed, and some numerical results are given.

Chapter 4, the second part of this study, deals with the VPR problem in ATM networks. The design problem is studied in this chapter in both short-term and long-term cases. A design algorithm is proposed for the VPR problem. Some experimental results are provided to verify the performance of the proposed algorithm as well as the effect of existing network to the network design.

Chapter 5 gives the summary of results of this research project, conclusions, as well as some interesting topics for further study.

Chapter 2

Literature Review

In this chapter, the overview of communication network design is discussed. After that, the detail of research works related to the study in this project is given.

2.1 Overview of Network Design

The life cycle of a communication network is the same as other systems and can be depicted as illustrated in Figure 2.1 [49]. The four phases called planning, design, implementation and operation are conducted sequentially and repeatedly in the cycle.

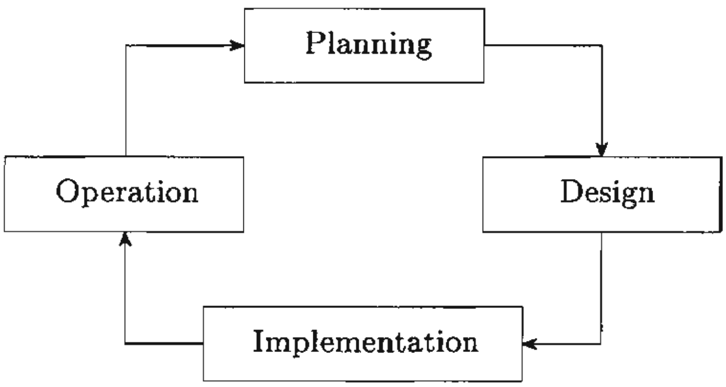


Figure 2.1: Life cycle of a communication network.

Generally, a communication network is needed to be planned and designed in a long time frame. This is due to not only the tremendous capital investment in the network implementation and operation, but also the necessity in the consideration of various factors and conditions such as future traffic demand, type of services, communication technology trends, etc.

Planning and design method for communication networks can be classified into two categories: dynamic planning method and static design method [49]. It should be noted that the term "planning" and "design" are sometimes used in the same meaning. That is the solving of a mathematical problem formulated from a realistic network with the consideration of factors in network implementation and network operation. However, the difference between *planning* and *design* can be made clear by the terms *dynamic planning* and *static design* as follows.

To plan or design a communication network in a long period of time, a tool called dynamic planning method is used. The planning tool will give solution of time, place, and the amount of facilities to be augmented to the network after considering factors affecting the network characteristics, e.g., traffic demand prediction, technology trends, rate of capital interest, etc.

One example of dynamic planning in the case of single facility is given in Figure 2.2 [49]. It can be seen from this figure that points of time (t_1 and t_2) and the amount of facilities that is necessarily augmented at each time point are determined.

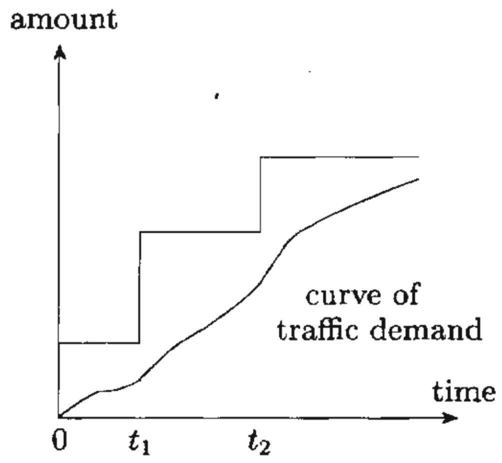


Figure 2.2: Dynamic planning in the case of single facility.

On the other hand, static design method deals with communication network in a comparatively short period of time, when the amount of traffic demand (forecasted value) in that period is given. The solution obtained from static design is concerned with the place and the amount of facilities to be installed in the network. Note that, in this case, there is no solution about the time for facility installation since the time period considered in the design is short, and facility installation is automatically conducted at the end of the period.

Additionally, static design method is also used for solving subproblems in the dynamic communication network planning. Their relationship can be shown in Figure 2.3, where a period of one year is taken as the time period of static design [49].

In dynamic planning, when a communication network is implemented and operated after a network design phase, the next time of network design can then be realized as the redesign phase of the network. It has been mentioned in Chapter 1 that, the existing network facilities are very important design factor and cannot be neglected in the redesign of a network. The study in this project is then focused on the static network design (re-

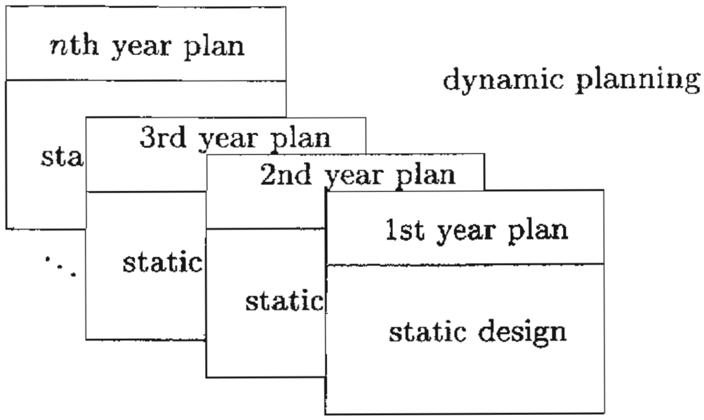


Figure 2.3: Relationship between dynamic planning and static design.

design) method for packet-switched and ATM networks, when the important information such as the amount of existing network facilities, new network traffic demand, is given.

2.2 Network Design Issues

In network design, communication network is generally defined as a set of communication points (routing/switching function facilities), and transmission lines (transmission function facilities) connecting the points. Mathematically, the network is usually modeled as a connected graph, containing a set of vertices and a set of edges [2][10][50]. A vertex and an edge are equivalent to each communication point and each transmission line in the network, respectively. Sometimes, vertex is also called *node*, and edge is referred to as *link*.

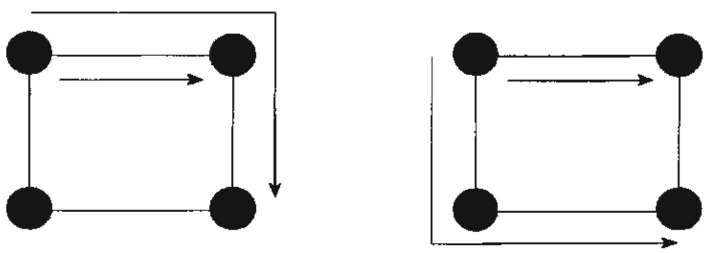
When traffic demand between each node-pair in the network is given, network design is concerned with the way to construct (or re-construct) the network with minimal cost while conditions of design constraint such as quality of service (QoS), network reliability are satisfied. It should be noted that we can also consider another version of problem, i.e., design a network with the best QoS or highest network reliability, while keeping network cost not exceeding the given amount of budget. However, QoS and network reliability are essential network parameters and are always requested by users, e.g., nobody wants to use a network with low QoS. Accordingly, network design problem should consider QoS or network reliability as design constraint, while network cost is treated as optimization (minimization) objective.

To deal with the real-world communication networks, there are a number of design issues that are necessarily considered in the network design problem. However, the following three items are those always be examined in the research works of this field [2][52].

- Network topology.
- Facility capacity.
- Routing.

(a) Network topology design.

(b) Facility capacity design.



(c) Routing design.

Figure 2.4: Network design issues.

Network topology design is concerned with the determination of the overall shape of the network, by giving a specific pattern of placing links between nodes such that the total network cost is minimized. Figure 2.4 (a) shows two possible topologies of a network, where there is one link different between each other, i.e., the link connecting the top-right node and the bottom-right node in the left figure, and the link between the top-left node and the bottom-right node in the right figure.

Facility capacity assignment deals with facility sizing, i.e., the determination of the amount of each facility in the network, e.g., speed of each transmission line, the bandwidth of switching equipment in each node, etc. Figure 2.4 (b) illustrates two different link capacity assignments of a network. In this figure, the thick and thin lines represent high speed and low speed links, respectively. The difference between the two capacity assignments in Figure 2.4 (b) is that a high speed link is used at the top-left to top-right link for the left figure, while a high speed link is used at the bottom-left to bottom-right link for the right figure.

Routing design involves selecting path (pattern of routes) for allocating traffic requirement between each node-pair. Figure 2.4 (c) gives two examples of routing pattern of a network. At the left of this figure, the traffic requirement between top-left and bottom-right node-pair is routed via the top-right node, while the same traffic is routed via the bottom-left node at the right of this figure.

The above three design issues can be solved simultaneously or separately. If the separated design manner is chosen, it is assumed that the solutions of some design issues are already known or given, and the task is concerned with the determination of the solution to the issues that are left unsolved. Several network design problems can be listed as follows [52].

- Capacity assignment (CA) problem

Given:	Flows and network topology
Minimize:	Network cost
Design parameters:	Facility capacities
Subject to:	Design constraint

- Flow assignment (FA) problem

Given:	Traffic demand, facility capacities and network topology
Optimize:	Parameters related to design constraint
Design parameters:	Flows
Subject to:	Design constraint

- Capacity and flow assignment (CFA) problem

Given:	Traffic demand and network topology
Minimize:	Network cost
Design parameters:	Capacities and flows
Subject to:	Design constraint

- Topology, capacity and flow assignment (TCFA) problem

Given:	Traffic demand
Minimize:	Network cost
Design parameters:	Topology, capacities, and flows
Subject to:	Design constraint

"Flow" in the above list is the sum of the amount of traffic requirements that are allocated in each network facility. "Design constraint" is control conditions of design problem and must be satisfied by the final design solution. A solution that does not satisfy all conditions of the design constraint of the problem is referred to as a non-feasible solution. Some good examples of constraint in the case of packet-switched network design are the upper limits of average packet delay, path delay, packet loss probability, etc.

It will be shown later in this report that the network design problems studied in this research project also belong to the problems listed above. The capacity assignment problem (CA problem) in packet-switched networks is just the member of the above CA problem. On the other hand, the virtual path routing problem (VPR problem) in ATM networks falls under the CFA problem.

To solve a network design problem, it is clear that solution obtained from simultaneous determination is better than that from separated solving manner. However, solving a design problem simultaneously is a very complicated task, especially in the case of large-scale network.

When solving a design problem of large network, one big problem is generally decomposed into several subproblems, which are later solved separately. For example, the above TCFA problem can be separated into two subproblems, namely topology design problem and the CFA problem. By the same idea, the CFA problem can be decomposed into two subproblems, i.e., the CA and FA problem.

The design method that decomposes one big problem into several subproblems and solves each subproblem separately is *heuristic* contrary to the optimum design method that takes all parameters into account simultaneously. Heuristic design method is usually adopted in solving network design problems [27][29][86][101]. Although the final solution obtained in the heuristic way might not be an optimal design solution, it generally has good quality and is acceptable in the sense that the computation time needed is relatively short.

2.3 Network Models in Network Design

In this section, network models that are adopted in the research works in the area of communication network design are discussed and a number of examples will be given.

In the earlier works of communication network design, communication network model is usually simplified by considering only major constraint and parameters. The main reason is that the optimization problem formulated from the simplified network model is easily to be solved. Although the model itself is easy, the solution obtained is, however, considered to be feasible, and can be used in realistic network design/implementation. The reason is straightforward that the main part of design components are not omitted in the model.

After the earlier steps, network model is revised to a more complex model by including more design constraint and parameters. This makes the model be able to deal with communication networks in the real-world. New design method according to the complex model is then studied. However, the method is usually developed based on the method or result obtained from the simplified model in preceding works.

Some examples on the change of network models in the study of communication network design are given below. Although all of these examples are concerned with the design of packet-switched networks, the same behavior can also be observed in other kind of communication networks.

- **Facility capacity model:** The model with discrete value of capacity is examined instead of model with continuous value [45][64]. This reflects the capacity characteristics of network facility in actual network construction, i.e., facility capacity cannot be set arbitrarily and must be chosen from a given set of values.

- **Facility cost model:** At first, the cost of network facility is considered to be linear with respect to the capacity. Then, non-linear cost functions including exponential and logarithmic are studied [52]. The discrete facility cost function is also adopted [45][64]. Clearly, discrete cost function comes from the model of discrete facility capacity.

- **Design constraint:** Firstly, the average packet delay throughout the network is considered as design constraint [52]. After that this is changed to average delay between each path (node-pair) [57][86]. Design method where the design constraint is packet loss probability can also be found in the literature [95].

- **Class of service:** Network model taking into consideration several classes of service is introduced to deal with multimedia network [95].

2.4 Conventional Research Works

As mentioned earlier, communication network is planned and designed in a long period of time according to the change and variation of several design parameters, including the change in traffic demand, that affect network characteristics. The network design methods proposed in the literature almost deal with network design problem where the network is in zero state, i.e., their is no link exists in the network, or capacities of all facilities are taken to be zero. Thus, the concept of **existing network** is rarely included in these design methods. Although some research works on network design/redesign tools inform that existing network is considered in the network design phase, no design method or algorithm is given explicitly [46][73].

The main reason that the concept of existing network is rarely considered in the previous works of communication network design is that existing network facilities are assumed as parts of network that are ready to be used in the construction of new network. Almost of the conventional research works assumed that the existing parts of network can be used with zero cost, or can be treated in the same way as new facilities needed to be installed to the network [10][104]. However, it is clear that this idea is generally not true in practical cases of network design. For example, although the existing network facilities are promptly to be used, there is cost concerning the use of these facilities such as operating or maintenance cost. Moreover, it is not necessary that the cost of using existing network facilities be the same as that of newly installed facilities.

There is one more reason that the existing network is not considered in the design of new network. It is that the growth of traffic demand applied to a network is often assumed to increase time-by-time. Based on this assumption, the capacity of each existing facility will not decrease lower than its current value after the new traffic pattern is applied. When the network is designed according to the new traffic, the amount of capacity needed to be augmented to each facility in the existing network can be determined and the network is then re-constructed.

However, in the realistic network operation, the amount of traffic applied to the network can be lower than its former value. Clearly, short-term cases such as daily or monthly network design fall in this case. In daily network design, user traffic demand between some node-pairs may decrease at night time comparing to that in the daytime or vice versa [65]. When the amount of traffic decreases, the value of facility capacity obtained from conventional design method will be less than the current value, and waste portion of facility will occur in the network. Moreover, when the traffic increases and new routing pattern is applied, it is also possible that waste of facilities will exist in the network if the network is not well designed. Obviously, this kind of waste is undesirable in the operation of communication network, but the way to deal with this important problem is rarely considered in the conventional research works. Furthermore, when the traffic increases more than the former value, some network facilities may not have enough capacity to cope with this change. Unfortunately, there is no good solution for dealing with this kind of problems [23].

From the above points of view, the research in this project devotes to the communication network design taking into consideration the effect of existing network facilities. Strictly speaking, the research focuses on both the short-term and long-term network design where the effect of existing network is examined in the design. It can be pointed out that the consideration of existing network in designing a new network is very important and cannot be neglected in the network design. The necessity of considering existing network can be listed as follows.

1. Communication network is planned and designed in a long time frame. In the design of each time section, i.e., in a short-term design, it is necessary to use the existing facilities in the full range before installing new facilities in the places where they are required.

2. There is difference between existing and newly installed facilities. For example, existing facilities are ready to be used in the network and there is cost associated in using them, e.g., maintenance cost. Moreover, existing facilities cannot be changed easily, especially in the short period of time. On the other hand, the cost concerning installation of new facilities needs not be the same as the cost of using the existing ones.

3. It is clear that utilizing existing facilities effectively is always important and cannot be disregarded during the network design phase.

2.5 Network Design taking into Consideration an Existing Network

Although the concept of existing network is rarely adopted in the design of new network as mentioned above, there are some recent research works on the study of communication network design that consider the effect and impact of existing network facilities. In this section, two examples of network design method that takes account of existing network will be described. They are the works conducted by Shinohara and by Runggeratigul as shortly described in Chapter 1.

2.5.1 Shinohara's Work

As mentioned briefly in Chapter 1, the work by Shinohara is concerned with the circuit dimensioning problem (CD problem) in circuit-switched networks [92][93]. The problem is formulated and solved under the condition that the number of circuits in the existing network are not zero. The circuit-switched network model used in the study is a multi-stage alternate routing network model, which composes of two kinds of links, namely high-usage links and final links. A simple three-node network is shown in Figure 2.5.

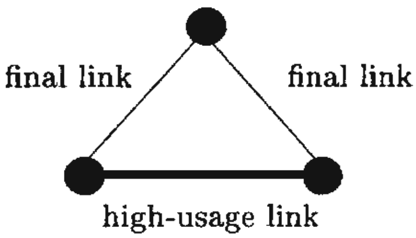


Figure 2.5: Triangular circuit-switched network.

In the figure, the thick line is a high-usage link assigned for the two nodes at the bottom, while the two thin lines are final links for the two nodes.

A high-usage link in the network is a circuit group from which overflow is permitted, and a final link is a circuit group from which overflow is not permitted [31]. Call, the traffic requirement between a node-pair, is firstly routed through a high-usage link assigned for the node-pair. However, if there is no empty channel left in the high-usage link, the call is treated as an overflow call and an alternate route selection procedure is activated to find a final link which has sufficient empty channel for the overflow call. If there is such kind of link exists, the call is routed through it. On the other hand, if there is no final link which has sufficient channel, the overflow call will be blocked, and no circuit is allocated for that call.

The CD problem is formulated as follows.

CD Problem

Given: $\boldsymbol{A}, b_j, \quad \forall j \in F$

Minimize: $Q = \sum_E q_i(x_i)$

Design variables: $x_i, \quad \forall i \in E$

Subject to: $B(x_j, a_j) \leq b_j, \quad \forall j \in F,$

$a_i = r_i(\boldsymbol{A}, \{x_i\}), \quad \forall i \in E.$

The meaning of notations in the above problem can be given as follows.

$\boldsymbol{A}$ · matrix of traffic (calls) applied to the network

E	set of all links in the network
F	set of final links
Q	total network cost (defined as the sum of all link costs)
$B(x_j, a_j)$	function giving the value of blocking probability on final link j
a_i	traffic offered to link i
b_j	upper limit of blocking probability on final link j
r_i	function that related to the flow allocation of link i
$q_i(x_i)$	cost of link i which is the function of x_i
x_i	the number of circuits in link i

The aim of this problem is to determine the number of circuits in all network links such that total network cost, defined as the sum of all link costs, is minimized. The conditions of design constraint are the upper limit of blocking probability on all final links and the route selection order according to the traffic requirement to the network. Note that, in the CD problem, there is no constraint of blocking probability on high-usage links since overflow is permitted in this kind of links.

In the study of the CD problem, the effect of the circuits in existing network facilities is included in the design by introducing the concept of cost difference between existing circuit and newly installed circuit. This results in a piecewise linear link cost function shown in Figure 2.6.

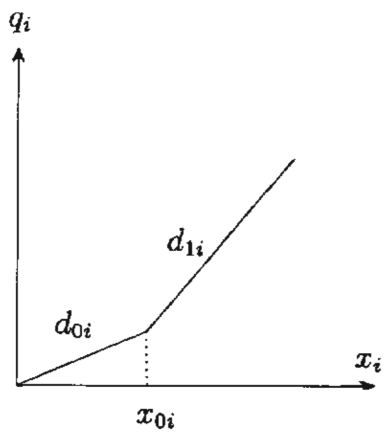


Figure 2.6: Link cost function in Shinohara's work.

The link cost q_i can be written as in the following equation.

$$q_i = \begin{cases} d_{0i}x_i & , x_i \le x_{0i}, \\ d_{1i}x_i - (d_{1i} - d_{0i})x_{0i} & , x_i > x_{0i}, \end{cases} \tag{2.1}$$

where d_{0i} and d_{1i} are the per-unit cost of existing circuit and newly installed circuit of link i, respectively, and x_{0i} is the number of existing circuits in link i.

The value of d_{0i} and d_{1i} are both non-negative, i.e.,

$$d_{0i} \geq 0, \tag{2.2}$$

$$d_{1i} \geq 0. \tag{2.3}$$

In his work, Shinohara examines the case that

$$d_{0i} \leq d_{1i}. \tag{2.4}$$

In other words, the per-unit cost of existing circuit is less than or equal to that of the newly installed circuit. The cost function in Figure 2.6 is generally valid in network construction, especially in the case of short-term design.

By applying the link cost function in Figure 2.6, the CD problem is solved to get the number of circuits in all links, $\{x_i\}$. The CD problem is a non-linear programming problem, and is solvable by the well-known Lagrange multiplier method. However, since the link cost function $q_i(x_i)$ is non-differentiable with respect to x_i at $x_i = x_{0i}$, the Lagrange multiplier method cannot be applied directly. In the study, the original non-differentiable function is approximated by another differentiable function as can be seen in Figure 2.7 (dotted line).

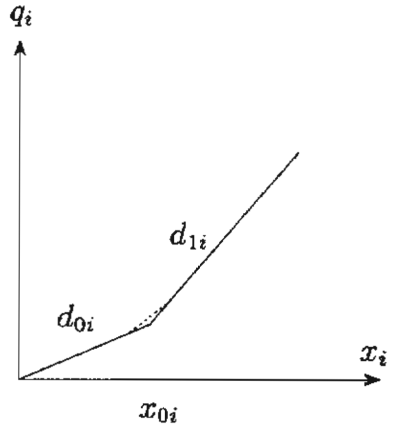


Figure 2.7: Approximation of non-differentiable cost function.

It is reported in the literature that a third order function is used for the approximation in this case [92]. By using the approximated function, $\frac{dq_i}{dx_i}$ becomes continuous and Lagrange multiplier method can be applied to solve the CD problem.

2.5.2 Runggeratigul's Work

Runggeratigul et. al. study the link capacity assignment problem (CA problem) in packet-switched networks with the consideration of existing network [80][81]. The design problem focuses on the case of short-term design. The fundamental idea of this work is to apply the concept of cost difference between the existing and new network facilities, that was proposed by Shinohara for circuit-switched networks.

The CA problem in packet-switched networks is a well-known communication network design problem, and has been studied widely since the early days of ARPANET's appearance [29][52]. This problem is concerned with means to determine capacity of links that minimize network cost, subject to some constraint, such as the upper limit of average packet delay. The CA problem can be solved when network information including network topology, routing pattern, etc., is given.

The CA problem can be formulated as follows.

CA Problem

Given: $\{f_i\}$ and $\{C_{0i}\}$

Minimize: $D = \sum_L D_i$

Design variables: $\{C_i\}$

Subject to:

$$T = \frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} \leq T_{\max}$$

$$C_i - f_i > 0, \quad \forall i \in L$$

$$C_i > 0, \quad \forall i \in L$$

The symbols used in the above problem are summarized as follows.

L	set of links in the network
f_i	traffic flow on link i
C_{0i}	existing value of capacity of link i
γ	overall traffic in the network
T	average packet delay
$T_{\max}$	maximum allowable average packet delay in the network
C_i	capacity of link i after assignment
D	total network cost
D_i	cost of link i

In the CA problem, $\{f_i\}$ is a new traffic pattern applied to the existing network where existing link capacity is $\{C_{0i}\}$. Capacity of all links $\{C_i\}$ is determined such that the total network cost D is minimized, where the network cost is defined as the sum of all link costs. Conditions of design constraint are the upper limit of average packet delay,

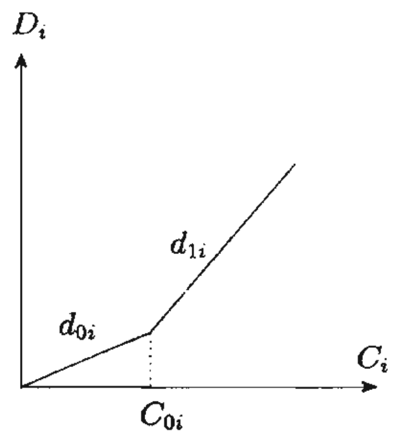


Figure 2.8: Link cost function in Runggeratigul's work.

and the relationship between link flow and link capacity. Obviously, link capacity must be greater than link flow so that packet delay will not grow indefinitely.

To include the existing network facilities in the CA problem, the concept of link cost difference is adopted, and a cost function shown in Figure 2.8 is applied.

From the figure, we can express link cost D_i as in Eq.(2.5).

$$D_i = \begin{cases} d_{0i}C_i & , C_i \leq C_{0i}, \\ d_{1i}C_i - (d_{1i} - d_{0i})C_{0i} & , C_i > C_{0i}, \end{cases} \tag{2.5}$$

where d_{0i} and d_{1i} are respectively the per-unit cost of existing and newly installed link capacity.

The values of d_{0i} and d_{1i} considered in the work are in the following range,

$$d_{0i} < d_{1i}, \tag{2.6}$$
$$d_{0i} \geq 0, \tag{2.7}$$
$$d_{1i} > 0. \tag{2.8}$$

This makes the cost function be a piecewise linear convex function.

It should be noted that the case $d_{0i} = d_{1i}$ is excluded in the study. The reason is straightforward that this is the case of linear cost function, and the CA problem with this simple cost function has been already studied [52]. Furthermore, the case $d_{1i} = 0$ is also excluded according to its triviality, i.e., the capacity of all links can be set as infinity, without making the total network cost be infinity (since d_{0i} and d_{1i} are both 0 in this case, the network cost is 0 !!!).

Since the link cost function is non-differentiable, the idea of using approximation function as in Shinohara's work is applicable. However, Runggeratigul et. al. propose a new procedure to solve the CA problem without using any approximation. The reasons are

- The selection of approximated function may affect the accuracy of the solution.

- Lengthy computation time is required to obtain approximated functions of all cost functions.

The procedure proposed in the study is referred to as the method of link set assignment [82], and its details can be described as follows.

After examining D_i in Eq.(2.5), it is clear that C_i belongs to one of the following three cases.

- $C_i < C_{0i}$,
- $C_i > C_{0i}$,
- $C_i = C_{0i}$.

Based on these cases, we can define the following three sets, L_0, L_1 and L_2, where

$$\begin{aligned} L_0 &= \{\, i \mid C_i < C_{0i}\}, &(2.9)\\ L_1 &= \{\, i \mid C_i > C_{0i}\}, &(2.10)\\ L_2 &= \{\, i \mid C_i = C_{0i}\}. &(2.11) \end{aligned}$$

Consequently, if each link i in the network is assigned to be a member of one of the above three sets, we will automatically have that D_i can be regarded as a differentiable function.

This is because

$$\frac{dD_i}{dC_i} = \begin{cases} d_{0i} & ,\forall i \in L_0, \\ d_{1i} & ,\forall i \in L_1. \end{cases} \qquad (2.12)$$

Obviously, there is no need to consider link i that belongs to L_2 since the link can be excluded from the CA problem and its capacity C_i can be set as a constant value of C_{0i}.

With the above procedure, we can derive the expression of $\{C_i\}$ as

$$C_i = f_i + \frac{\sum\limits_{L_0 \cup L_1} \sqrt{f_j d_j}}{\left(\gamma\, T_{\max} - \sum\limits_{L_2} \dfrac{f_j}{C_{0j} - f_j}\right)} \sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L_0 \cup L_1, \qquad (2.13)$$

where

$$d_i = \begin{cases} d_{0i} & \text{, when } i \in L_0, \\ d_{1i} & \text{, when } i \in L_1. \end{cases} \qquad (2.14)$$

The value of C_i calculated from Eq.(2.13) is optimum for the CA problem under the relevant link set assigned at the beginning. However, the solution has to be checked whether it contradicts its assigned link set or not, e.g., for $i \in L_0$, C_i that is greater than C_{0i} is thought to be a contradiction. If any contradiction occurs, the solution obtained will not be feasible. A design algorithm for the CA problem is then proposed based on the following idea [81].

Firstly, let's consider the case that all links are assigned to L_0. Then Eq.(2.13) is used to determine $\{C_i\}$, and the value is checked whether there is link i that $C_i > C_{0i}$ or not.

If such kind of links exist, we will have that these links cannot belong to L_0. Let's define this set of links as

$$L_{0x} = \{i \mid C_i > C_{0i}, \forall i \in L_0\}. \tag{2.15}$$

Secondly, we assign all links to be in L_1 and checking contradiction as in the above manner, and define

$$L_{1x} = \{i \mid C_i < C_{0i}, \forall i \in L_1\}. \tag{2.16}$$

We can also say that links in L_{1x} cannot belong to L_1 anymore.

From L_{0x} and L_{1x}, it is possible to conclude that

$$L_2 = \{i \mid \forall i \in L_{0x} \cap L_{1x}\}. \tag{2.17}$$

The proposed algorithm is the steps that examine links that are in L_2 repeatedly as in the above manner, and terminates when all links are assigned to proper sets without contradiction.

The main result of the study on the CA problem with the consideration of existing network (short-term case) is that, the design algorithm gives a solution in the way that the existing network facilities are utilized effectively, and new facilities are augmented to the existing network only in the places that they are required.

2.6 Summary

Although the concept of existing network is usually neglected in conventional research works on communication network design, there are a few studies that recently include this major design factor in the network design problems. These design problems are the circuit dimensioning problem (CD problem) in circuit-switched networks, and the link capacity assignment problem (CA problem) in packet-switched networks.

However, since the above research works consider the network design only in the short-term case, the following questions are needed to be answered.

- The question of how to study the case of long-term network design under the effect of existing network facilities.
- The question that the concept of cost difference between existing and newly installed facilities is applicable or not to the design of other types cf communication network besides the circuit-switched and packet-switched networks.

The work in this research project is devoted to give answers for the above questions. In Chapter 3, we will study the CA problem in packet-switched networks for the case of long-term design by applying a piecewise linear concave function. In Chapter 4, the virtual path routing problem (VPR problem) in ATM networks will be investigated for both short-term and long-term design.

Chapter 3

Link Capacity Assignment in Packet-Switched Networks

In this chapter, we discuss the first part of the work in this project. It is the study on the link capacity assignment problem in packet-switched networks with the consideration of existing network. However, since the case of short-term design has been already studied in the literature, this project then focuses the link capacity assignment problem on the case of long-term design.

The content of this chapter is organized as follows. Firstly, the outlines of the link capacity assignment problem in packet-switched networks will be given. Then the network model used in the study will be discussed, and a piecewise linear concave function is introduced as the network link cost function. After that, the capacity assignment problem is formulated as a non-linear programming problem. By analyzing the characteristic and property of the optimal solutions of the capacity assignment problem, we can propose a heuristic design algorithm derived from the well-known Lagrange multiplier method. At last, some numerical results are given to investigate the performance of the proposed algorithm, as well as to show the effects of existing network facilities to the link capacity assignment problem.

3.1 Introduction

Link capacity assignment problem (CA problem) in packet-switched networks has been studied widely since the early days of ARPANET's appearance in the late 1960s [29][52]. This network design problem is concerned with means to determine the capacity of links in the network such that the network cost is minimum, subject to some design constraint such as the upper limit of average packet delay. The CA problem can be solved when network information, including network topology, routing pattern, etc., is given. The problem has been studied for many kinds of network model. Some of the examples are given below.

- A model with several classes of packet is considered for dealing with multimedia networks [95].
- Besides the average packet delay throughout the network, path average delay [57][86] as well as packet loss probability [95] are defined as design constraint for more reasonable quality of service in a network.
- Discrete values of link capacity are introduced for dealing with links in an actual network construction [45][64].

Furthermore, many types of link cost function have been considered in the CA problem. For example,

- some fundamental mathematical functions including linear, exponential, logarithmic, are examined to see the behavior of link capacity assignment due to the effect of these cost functions [52],
- a piecewise linear convex cost function has also been considered as a link cost model in the case that the CA problem is solved with the consideration of existing network, where the per-unit cost of using existing capacity is less than that of installing new capacity [80][81] (this is the case of short-term network design already mentioned in Chapter 2).

The work in this chapter can be realized as an extension of the work by Runggeratigul et. al. in [80][81], where we extend the work in the way that the case of long-term network design can be studied.

It has been already known that utilizing existing network facilities in the construction of a new communication network is very important in the case of short-term network design [49]. The reason is straightforward, that is that existing network facilities are promptly to be used and cannot be changed easily in a short period of time. Thus, it is appropriate to use a convex link cost function in the case of short-term design as done in the previous works [80][81].

However, in other cases of network design such as long-term design, link cost function is not necessary convex. In many cases, it is proper to regard the function as a concave function [50][52]. This is because the economy of scale is often present in communication resources. For example, we can obtain a cost function from the envelope of several cost functions due to the development in communication transmission technologies [30][107], the development in switching technologies [99], etc.

In this chapter, we study the CA problem in packet-switched networks taking into consideration an existing network where the link cost function is piecewise linear concave. This is the case when the per-unit cost of using existing capacity is greater than that of installing new capacity. It is obvious that this type of link cost function is non-differentiable. To solve network design problems with this type of cost function, the method of approximating the non-differentiable cost function by a differentiable function has been proposed [93]. Although this method yields a solution within approximation error, there is no guarantee that the solution obtained from this method is optimum when link cost function is not convex. Moreover, lengthy time is needed to determine the approximation function for the non-differentiable cost function.

To study the CA problem in this research project, we then try to clarify the characteristic of optimal solution of the CA problem. The main procedure is to apply the method of link set assignment proposed for the piecewise linear convex function in [81]. This leads us to the analysis of the essential characteristic of the optimal solution to the CA problem. Based on this characteristic, there is no need to conduct approximation on the non-differentiable link cost function. Furthermore, it is possible to show that the non-differentiable cost function can be treated as a differentiable function. As a result, conventional methods such as Lagrange multiplier method can be applied to the solving of the CA problem.

3.2 Network Model

In the first part of this section, we discuss the model of packet-switched network considered in this study. After that in the second part, we introduce a link cost function with the consideration of cost difference between existing capacity and newly installed capacity for the case of long-term design.

3.2.1 Model Description

The symbols used in this chapter are summarized as follows.

L set of links in the network

f_i traffic flow on link i (in bits/second)

C_{0i} existing value of capacity of link i (in bits/second)

γ overall traffic in the network (in packets/second)

T average packet delay

$T_{\max}$ upper limit of the average packet delay T

C_i capacity of link i after assignment

D total network cost

D_i cost of link i

The packet-switched network model considered in this research project is the same as the one used in the early work of packet-switched network design [29][52][81]. The characteristics of this network model are given below.

Arrival process of packets transmitted on each link in the network is assumed to follow Poisson process, and packet length is assumed to be negative exponentially distributed. From this assumption, we can model each network link as an $M/M/1$ queueing system with infinite buffer. Then, the average packet delay throughout the network T can be given as in Eq.(3.1).

$$T = \frac{1}{\gamma} \sum_{L} \frac{f_i}{C_i - f_i}. \tag{3.1}$$

Based on the user traffic demand and the routing scheme applied to the network, it is possible to calculate the amount of traffic flow f_i on each link in the network. In this network model, the traffic flow is assumed to be a positive value,

$$f_i > 0. \tag{3.2}$$

Obviously, negative value of f_i has no meaning. In addition, a zero value of f_i is also trivial. If there is a link in the network with $f_i = 0$, we then have that there is no need to consider this link in the design problem anymore, and the link can be consequently removed from the network without any effect to other links in the network.

The value of link capacity C_i is assumed to be continuous, and can be set as an arbitrary positive value,

$$C_i > 0. \tag{3.3}$$

From Eq.(3.1), it is clear that the capacity of each link must be greater than the traffic flow on that link, i.e.,

$$C_i > f_i. \tag{3.4}$$

This is a necessary condition to make the value of packet delay on each link not grow indefinitely.

For simplicity, this network model does not take into account the node cost, e.g., cost of switching facilities, etc. Only link cost is considered, and the total network cost D is defined as the sum of all link costs. Then it yields

$$D = \sum_{L} D_i. \tag{3.5}$$

3.2.2 Link Cost Function

For each link, link cost D_i is a function of link capacity C_i as

$$D_i = f(C_i), \tag{3.6}$$

where $f(.)$ is an arbitrary function. Clearly, the function can be either a simplest case of linear function or a more realistic case of non-linear function.

Since the main objective of this research project is to investigate the network design problem under the impact of the consideration of existing network, we then define a link cost function that is possible to deal with the existing link capacity. Accordingly, we rewrite Eq.(3.6) as in the following equation.

$$D_i = f(C_i, C_{0i}). \tag{3.7}$$

Eq.(3.7) shows that the link cost function consists of two portions of link cost, i.e., the cost of using existing capacity and the cost concerned with the installation of new capacity.

Next, we apply the idea of cost difference between the existing capacity and the newly installed capacity [81][93] to the cost function in Eq.(3.7). We also assume that the link cost of the two parts are both linear with respect to link capacity. As a result, we obtain the following link cost function.

$$D_i = \begin{cases} d_{0i}C_i & ,C_i \leq C_{0i}, \\ d_{1i}C_i - (d_{1i} - d_{0i})C_{0i} & ,C_i > C_{0i}, \end{cases} \tag{3.8}$$

where d_{0i} and d_{1i} are the per-unit cost of existing capacity and newly installed capacity of link i, respectively. In general cases, the value of d_{0i} and d_{1i} are both non-negative. That is

$$d_{0i} \geq 0, \tag{3.9}$$

$$d_{1i} \geq 0. \tag{3.10}$$

For the case of short-term network design, we have considered the case of piecewise linear convex function, i.e.,

$$d_{0i} < d_{1i}, \tag{3.11}$$

$$d_{0i} \geq 0, \tag{3.12}$$

$$d_{1i} > 0. \tag{3.13}$$

However, we will have an opposite case for the long-term network design in our study. As mentioned above, it is appropriate to adopt a concave function for the case of long-term design. Hence, we focus on the following range of d_{0i} and d_{1i}.

$$d_{0i} > d_{1i}, \tag{3.14}$$

$$d_{0i} > 0, \tag{3.15}$$

$$d_{1i} > 0. \tag{3.16}$$

The above range of d_{0i} and d_{1i} results in a piecewise linear concave function as shown in Figure 3.1.

Note that the case $d_{0i} = d_{1i}$ is excluded from the range of d_{0i} and d_{1i} given above. The reason is that this case gives a linear link cost function, which has been already studied for the CA problem [52]. The case $d_{1i} = 0$ is also excluded due to its triviality. This is because the capacity of all links in this case can be set to infinity, while the total network cost is finite. In other words, there is no much need to solve the CA problem of this case anymore.

The cost function given in Figure 3.1 arises in many communication network design problems. One of the example is the link type selection problem where the economy of scale is present in communication resources. A practical example with two alternatives of link types is given in Figure 3.2. It can be seen from this example that the overall link cost function is a piecewise linear concave function, which is the envelope of cost functions of the two link types.

The concave link cost function in Figure 3.1 will be applied to the CA problem in packet-switched networks, where the problem is formulated as a mathematical programming problem in the next section.

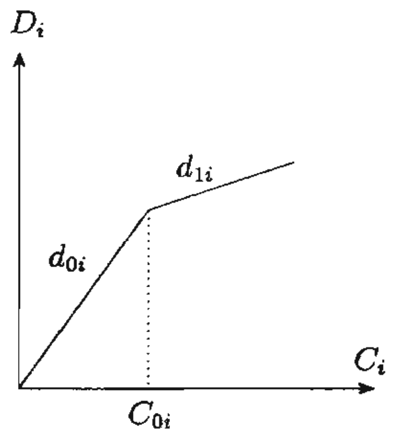


Figure 3.1: A piecewise linear concave link cost function.

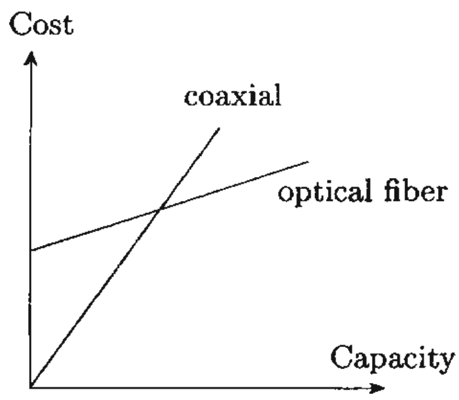


Figure 3.2: A practical example of link type selection.

3.3 Link Capacity Assignment Problem (CA Problem)

In this research project, we study the link capacity assignment in packet-switched networks in the case that new user traffic requirement is applied to an existing network, and then the capacity of each link is adjusted to cope with this new traffic requirement such that a network with minimal total cost is obtained under the constraint of packet delay.

Consequently, we formulate the CA problem as follows.

CA Problem

Given: $\{f_i\}$, $\{C_{0i}\}$, $\{d_{0i}\}$ and $\{d_{1i}\}$

Minimize: $D = \sum_L D_i$

Design variables: $\{C_i\}$

Subject to: $T = \frac{1}{\gamma}\sum_L \frac{f_i}{C_i - f_i} \leq T_{\max}$

$C_i - f_i > 0, \quad \forall i \in L$

$C_i > 0, \quad \forall i \in L$

In the above CA problem, $\{f_i\}$ is the new user traffic requirement applied to an existing packet-switched network where the existing link capacity is $\{C_{0i}\}$. Capacity of all links $\{C_i\}$ is determined so that the total network cost D is minimized. According to the network model described in the previous section, the network cost is defined as the sum of all link costs where each link cost D_i is given in Eq.(3.8).

The conditions of design constraint are the upper limit of average packet delay and the relationship between link flow and link capacity. The quality of service in our network model is indicated by the average packet delay throughout the network, T, where the network must be designed in the way that T does not exceed an upper limit $T_{\max}$. In the design result, link capacity must be greater than link flow such that the average packet delay on each link is finite as mentioned above.

Note that if link cost D_i is a linear function with respect to link capacity, i.e.,

$$D_i = c_i C_i, \tag{3.17}$$

where c_i is a positive value that denotes the per-unit cost of link capacity, we will have that C_i can be determined by the following equation [52].

$$C_i = f_i + \frac{\sum_L \sqrt{f_j c_j}}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{c_i}}, \qquad \forall i \in L. \tag{3.18}$$

From the above solution, the total network cost can then be calculated as in the following equation.

$$D = \sum_L f_i c_i + \frac{\left(\sum_L \sqrt{f_j c_j}\right)^2}{\gamma\, T_{\max}}. \tag{3.19}$$

3.4 The Method of Link Set Assignment

The CA problem formulated above is a non-linear programming problem. Many conventional methods such as Lagrange multiplier method are known as effective methods for solving non-linear programming problems, e.g., the CA problem with linear cost function given in the previous section.

However, since the link cost function D_i given in Eq.(3.8) is not differentiable with respect to C_i, Lagrange multiplier method cannot be applied to the CA problem directly. As a method to alleviate this type of network design problem, the non-differentiable function is approximated by a differentiable function [93] as described in Chapter 2. Figure 3.3 shows the idea of this method, where the dotted line is the part of approximated function.

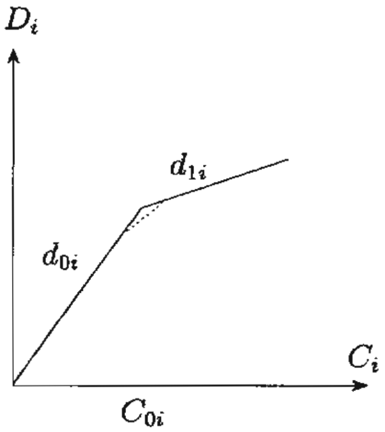


Figure 3.3: Approximation of non-differentiable function.

By using an approximated differentiable cost function, we can apply the Lagrange multiplier method to the CA problem. Although, this provides a solution to the problem within approximation error, there is no guarantee that the solution obtained is an optimal solution when the link cost is not convex. Moreover, lengthy time is needed for calculating approximation function of all link cost functions.

When the cost function is piecewise linear convex, a method of link set assignment has been proposed to solve the CA problem to obtain an optimal solution [81]. In this study, we extend this method such that it is applicable to the CA problem with piecewise linear concave cost function. The concept of the original version of the method can be described as follows.

D_i given in Eq.(3.8) can be regarded as a differentiable function, if we assign each link i to be a member of one of the following three sets,

$$L_0 = \{ i \mid C_i < C_{0i} \}, \tag{3.20}$$
$$L_1 = \{ i \mid C_i > C_{0i} \}, \tag{3.21}$$
$$L_2 = \{ i \mid C_i = C_{0i} \}. \tag{3.22}$$

This is because

$$\frac{dD_i}{dC_i} = \begin{cases} d_{0i} & ,\forall i \in L_0, \\ d_{1i} & ,\forall i \in L_1. \end{cases} \tag{3.23}$$

Note that link i that belongs to L_2 is excluded from the CA problem since its capacity C_i is set to a constant value of C_{0i}.

With the above link set assignment, Lagrange multiplier method can be applied to the CA problem, and Lagrangian G is constructed as in the following equation.

$$G = \sum_L D_i + \lambda \left(\frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} - T_{\max} \right), \tag{3.24}$$

where λ is a Lagrange multiplier.

Since $C_i = C_{0i}, \forall i \in L_2$, Eq.(3.24) can be rewritten as

$$G = \sum_{L_0 \cup L_1} D_i + \sum_{L_2} d_{0i} C_{0i} + \lambda \left(\frac{1}{\gamma} \sum_{L_0 \cup L_1} \frac{f_i}{C_i - f_i} + \frac{1}{\gamma} \sum_{L_2} \frac{f_i}{C_{0i} - f_i} - T_{\max} \right). \tag{3.25}$$

The constraint region of the CA problem can be described by the following three conditions.

$$\begin{aligned} \frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} &\leq T_{\max}, \\ C_i - f_i &> 0, \qquad \forall i \in L, \\ C_i &> 0, \qquad \forall i \in L, \end{aligned}$$

It can be shown that the constraint region of the problem mentioned above is convex. Since cost function of all links in the network can now be considered as linear function and the constraint region of the problem is convex, we consequently have that the CA problem is a convex programming problem.

By using the Lagrangian G, the necessary and sufficient conditions for minimal solution of the CA problem are given by the following Kuhn-Tucker conditions [39][69].

$$\frac{\partial G}{\partial C_i} = 0, \qquad \forall i \in L_0 \cup L_1, \tag{3.26}$$
$$\lambda \geq 0, \tag{3.27}$$
$$\frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} - T_{\max} \leq 0, \tag{3.28}$$
$$\lambda . \left(\frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} - T_{\max} \right) = 0. \tag{3.29}$$

It is clear that $T = \dfrac{1}{\gamma}\sum_{L}\dfrac{f_i}{C_i - f_i}$ is a decreasing function of all C_i's. Therefore, we can set the constraint of packet delay time as

$$T = \frac{1}{\gamma}\sum_{L}\frac{f_i}{C_i - f_i} = T_{\max}. \tag{3.30}$$

As a result, conditions (3.28) and (3.29) are both satisfied simultaneously. Next, we take $\dfrac{\partial G}{\partial C_i} = 0$, $\forall i \in L_0 \cup L_1$ such that condition (3.26) is satisfied. From Eq.(3.25), we obtain the following equation.

$$d_i - \lambda.\frac{f_i}{\gamma(C_i - f_i)^2} = 0, \qquad \forall i \in L_0 \cup L_1, \tag{3.31}$$

where

$$d_i = \frac{dD_i}{dC_i} = \begin{cases} d_{0i} & ,\forall i \in L_0, \\ d_{1i} & ,\forall i \in L_1. \end{cases} \tag{3.32}$$

After mathematical manipulation on Eq.(3.31), we get

$$\lambda = \frac{\gamma d_i (C_i - f_i)^2}{f_i}, \qquad \forall i \in L_0 \cup L_1. \tag{3.33}$$

Since the values of d_{0i} and d_{1i} are non-negative, it is easy to show that $\lambda \geq 0$. This means that the condition (3.27) is satisfied. Thus, the solution from Lagrange multiplier method is guaranteed to be the minimal solution of the CA problem, where we can derive the solution as in the following steps.

From Eq.(3.30) and Eq.(3.33), we respectively have

$$\frac{1}{\gamma}\sum_{L_0 \cup L_1}\frac{f_i}{C_i - f_i} = T_{\max} - \frac{1}{\gamma}\sum_{L_2}\frac{f_i}{C_{0i} - f_i}, \tag{3.34}$$

$$C_i = f_i + \sqrt{\frac{\lambda f_i}{\gamma d_i}}, \qquad \forall i \in L_0 \cup L_1. \tag{3.35}$$

Replacing Eq.(3.35) into Eq.(3.34), we have

$$\sqrt{\lambda} = \frac{\displaystyle\sum_{L_0 \cup L_1}\sqrt{\frac{f_i d_i}{\gamma}}}{T_{\max} - \dfrac{1}{\gamma}\displaystyle\sum_{L_2}\frac{f_i}{C_{0i} - f_i}}. \tag{3.36}$$

Again, substituting Eq.(3.36) into Eq.(3.35), and after some mathematical manipulations, we obtain the solution of $\{C_i\}$ as in Eq.(3.37) [81].

$$C_i = f_i + \frac{\displaystyle\sum_{L_0 \cup L_1}\sqrt{f_j d_j}}{\left(\gamma\, T_{\max} - \displaystyle\sum_{L_2}\frac{f_j}{C_{0j} - f_j}\right)}\sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L_0 \cup L_1, \tag{3.37}$$

where

$$d_i = \begin{cases} d_{0i} & ,\forall i \in L_0, \\ \\ d_{1i} & ,\forall i \in L_1. \end{cases} \tag{3.38}$$

The value of C_i calculated from Eq.(3.37) is optimum for the CA problem under the relevant link set assigned at the beginning. However, the value has to be checked whether it contradicts its assigned link set or not, e.g., for $i \in L_0$, C_i that is greater than C_{0i} is thought to be a contradiction. If any contradiction occurs, the solution obtained will not be feasible and other combinations of link set have to be examined until a feasible solution with minimum network cost is found.

By the original version of the method of link set assignment, we can obtain an algorithm to the CA problem based on the concept of complete enumeration (CE). The algorithm is referred to as ALGORITHM CA_CE as follows.

ALGORITHM CA_CE

Step 1. Let network cost $D = \infty$.

Step 2. For $\forall i \in L$, put i into one of the three sets, L_0, L_1 or L_2.

Step 3. Set $d_i = d_{0i}$, $\forall i \in L_0$.

Set $d_i = d_{1i}$, $\forall i \in L_1$.

Reset $err = 0$.

Step 4. Determine C_i, $\forall i \in L_0 \cup L_1$ by Eq.(3.37).

Set $C_i = C_{0i}$, $\forall i \in L_2$.

Step 5. If $C_i \geq C_{0i}$, $\exists i \in L_0$, then set $err = 1$.

If $C_i \leq C_{0i}$, $\exists i \in L_1$, then set $err = 1$.

If $C_i \leq f_i$, $\exists i \in L$, then set $err = 1$.

If $T = \dfrac{1}{\gamma} \sum_L \dfrac{f_i}{C_i - f_i} > T_{\max}$, then set $err = 1$.

Step 6. Calculate network cost $D' = \sum_L D_i$ from the value of C_i's.

If $D' < D$ and $err \neq 1$, then set $D = D'$ and keep $\{C_i\}$ as the design solution.

Step 7. If all combinations of link set are completely examined, STOP and the final solution of the CA problem is $\{C_i\}$.

Step 8. Select another combination of link set (e.g., by swapping the members among L_0, L_1 and L_2) which has not been examined yet, and go to Step 3.

The parameter err in the above algorithm denotes whether the link set combination under consideration gives a feasible solution to the CA problem or not, where

$$err = \begin{cases} 0 & \text{, when the solution is feasible,} \\ 1 & \text{, otherwise.} \end{cases}$$

Clearly, the above ALGORITHM CA_CE can give global optimal solution to the CA problem with piecewise linear concave cost function in our study. However, the algorithm is obviously very time-consuming according to its nature of complete enumeration, that is we have to examine all of the link set combinations. Thus, a more efficient algorithm is required.

3.5 The Optimal Solution of the CA Problem

To find a way to solve the CA problem, we firstly review the modified version of the method of link set assignment proposed in [81]. Note that the basic concept of this idea has been already briefly described in Chapter 2. After that, we derive an important characteristic of the optimal solution of the CA problem.

3.5.1 Basic Concept

Since each link in the network certainly belongs to L_0, L_1, or L_2, we can select one of the three sets for each link and assign the link to that set. After that, we can check whether there is a contradiction between the assigned link set and the value of link capacity calculated from Eq.(3.37). The checking of contradiction can give us a way to decide that the link can belong to the set that it was assigned or not.

Before going further, it is important to note that there is a special kind of links whose link set must be fixed to L_1. Link i is a member of this special kind when its traffic flow and existing link capacity satisfy the following condition.

$$f_i \geq C_{0i}.$$

From the condition (3.4), it is easy to show that

$$C_i > C_{0i}.$$

The above implies that link i must be a member of only the set L_1. Hence, we will exclude this special kind of links from the CA problem, and concentrate on links with the following properties.

$$f_i < C_{0i} \qquad \text{or} \qquad C_{0i} - f_i > 0. \tag{3.39}$$

In the case of convex cost function, i.e., $d_{0i} < d_{1i}$, $\forall i \in L$, each link is firstly assigned to L_0 and if contradiction occurs, it is concluded that the link with contradiction cannot belong to L_0. In the next step, each link is set to L_1 and if contradiction occurs, we can say that the link with contradiction cannot belong to L_1. In the case that a link cannot belong to neither L_0 nor L_1, it must belong to L_2. By repeatedly examining links for the set L_2 until no more link has to be assigned to it, each link in the network will belong to a proper set, and link capacity can be computed by using Eq.(3.37). It is proven that the final solution by the above revised version of the method of link set assignment is a global optimal solution for the case of convex cost function [81].

Next, we investigate that how the above method can deal with the CA problem with piecewise linear concave cost function, i.e., $d_{0i} > d_{1i}$, $\forall i \in L$. Let's introduce two parameters for link i: e_{0i} and e_{1i}. The values of the two parameters are set as follows.

- e_{0i} will be 0 if link i is assigned to L_0 and no contradiction occurs, and be 1 if contradiction exists.
- e_{1i} will be 0 if link i is assigned to L_1 and no contradiction occurs, and be 1 if contradiction exists.

Then, we assign each link to L_0 and use Eq.(3.37) to determine $\{C_i\}$. After that, we check whether there is any contradiction or not, i.e., there is link i that $C_i \geq C_{0i}$ or not. In other words, we examine e_{0i} of link i whether the value is 0 or 1. Unlike the case of convex cost function, we focus on the links without contradiction, namely link i with $e_{0i} = 0$. This is because we can show that this kind of links can belong to L_0 without any contradiction although link set combination is changed. The proof related to this case is given below in Theorem 1.

Theorem 1 *When link i is assigned to belong to L_0 and $e_{0i} = 0$, the link i can be assigned to L_0 without any contradiction even some of other links are changed to belong to L_1.*

Proof: Firstly, we define the following sets and notations.

$$\begin{aligned} L_0' &= \{j \mid j \text{ is the link already assigned to } L_0\}, \\ L_1' &= \{j \mid j \text{ is the link already assigned to } L_1\}, \end{aligned}$$

$$\begin{aligned} b_0 &= \sum_{L_0'} \sqrt{f_j d_{0j}}, \\ b_0' &= \sum_{L-(L_0' \cup L_1')} \sqrt{f_j d_{0j}}, \\ b_1 &= \sum_{L_1'} \sqrt{f_j d_{1j}}, \\ b_1' &= \sum_{L-(L_0' \cup L_1')} \sqrt{f_j d_{1j}}. \end{aligned}$$

Next, we assume that there is a link i with $e_{0i} = 0$. Since there is no contradiction for this link, we have

$$C_i = f_i + \frac{b_0 + b_1 + b_0'}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{0i}}} < C_{0i}, \tag{3.40}$$

or

$$\frac{1}{\sqrt{d_{0i}}} < \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b_0')}. \tag{3.41}$$

Currently, all links in $L - (L_0' \cup L_1')$ are assigned to L_0 for the checking of link set contradiction. Let L_1'' be a set of links in $L - (L_0' \cup L_1')$ that are changed to belong to L_1. After the change, we have b_0' becomes b^* as

$$b^* = \sum_{L_0''} \sqrt{f_j d_{0j}} + \sum_{L_1''} \sqrt{f_k d_{1k}}, \tag{3.42}$$

where

$$L_0'' = L - (L_0' \cup L_1' \cup L_1'').$$

Note that the link i under consideration must be in L_0'', since we are checking link set contradiction when this link is assigned to L_0.

For the case of concave function, we have $d_{0i} > d_{1i}$, $\forall i \in L$. This leads to

$$b_0' = \sum_{L_0'' \cup L_1''} \sqrt{f_j d_{0j}} > b^*. \tag{3.43}$$

The above and (3.39) make

$$\frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b_0')} < \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b^*)}. \tag{3.44}$$

From (3.41) and (3.44), we have

$$\frac{1}{\sqrt{d_{0i}}} < \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b^*)}, \tag{3.45}$$

or

$$C_i = f_i + \frac{b_0 + b_1 + b^*}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{0i}}} < C_{0i}. \tag{3.46}$$

The last statement implies that e_{0i} is still equal to 0 as before, although link set combination is changed. In other words, link i with $e_{0i} = 0$ can belong to L_0 without any contradiction. □

By the same idea, when each link is set to L_1, we can say that the links with no contradiction can belong to L_1. This can be shown by Theorem 2.

Theorem 2 *When link i is assigned to belong to L_1 and $e_{1i} = 0$, the link i can be assigned to L_1 without any contradiction even some of other links are changed to belong to L_0.*

Proof: The proof can be done in almost the same way as in Theorem 1. Firstly, we assume that all of the links in $L - (L_0' \cup L_1')$ are currently assigned to L_1, and link set contradiction is checked. Assume again that there is a link i with $e_{1i} = 0$.

Since there is no contradiction for link i, we then have

$$C_i = f_i + \frac{b_0 + b_1 + b_1'}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{1i}}} > C_{0i}, \tag{3.47}$$

or

$$\frac{1}{\sqrt{d_{1i}}} > \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b_1')}. \tag{3.48}$$

Next, we try to change the set of some links. Let L_0'' be a set of links in $L - (L_0' \cup L_1')$ that are changed to belong to L_0. With this change, we have b_1' becomes b^{**} as

$$b^{**} = \sum_{L_0''} \sqrt{f_j d_{0j}} + \sum_{L_1''} \sqrt{f_k d_{1k}}, \tag{3.49}$$

where

$$L_1'' = L - (L_0' \cup L_1' \cup L_0'').$$

Note that the link i under consideration must be a member of L_1'', since we are checking link set contradiction when this link is assigned to L_1.

For the case of concave function, we have $d_{0i} > d_{1i}$, $\forall i \in L$. This leads to

$$b_1' = \sum_{L_0'' \cup L_1''} \sqrt{f_j d_{1j}} < b^{**}. \tag{3.50}$$

The above relation and (3.39) gives

$$\frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b_1')} > \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b^{**})}. \tag{3.51}$$

From (3.48) and (3.51), we have

$$\frac{1}{\sqrt{d_{1i}}} > \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b^{**})}, \tag{3.52}$$

or

$$C_i = f_i + \frac{b_0 + b_1 + b^{**}}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{1i}}} > C_{0i}. \tag{3.53}$$

This implies that the value of e_{1i} remains unchanged and is equal to 0 as before, although link set combination is changed. In other words, link i with $e_{1i} = 0$ can belong to L_1 without any contradiction. □

3.5.2 Characteristic of the Optimal Solution

By the above concept and theorems, we can construct the following procedure for assigning a proper set to each link. There are four possible cases related to the values of e_{0i} and e_{1i}. They are

1. $e_{0i} = 0$ and $e_{1i} = 1$,
2. $e_{0i} = 1$ and $e_{1i} = 0$,
3. $e_{0i} = 0$ and $e_{1i} = 0$,
4. $e_{0i} = 1$ and $e_{1i} = 1$.

In case 1, we can let link i belong to L_0 since there is no contradiction as mentioned in Theorem 1 above. Although link i may not belong to L_0 in the global optimal solution of the problem, we deal with the link in a greedy manner to obtain a feasible solution. The reason is that it has been guaranteed that there will be no contradiction even the sets of other links are changed. Moreover, due to the non-convexity of the CA problem, its global optimum cannot be determined easily, so we have to try to find its local optimum instead.

As same as case 1, link i that falls into case 2 is then assigned to L_1 as already shown in Theorem 2.

In case 3, since link i can belong to either L_0 or L_1 without any contradiction, we can apply any method to select the set for this type of links. For example, the set can be randomly chosen between L_0 and L_1.

Finally, for links of case 4, we do not assign this type of links to any set, but repeatedly do the above procedure again until all links have their proper set.

Obviously, there will be no problem if links of case 4 do not exist at the final stage of the examining, and each link in the network will then be assigned to either L_0 or L_1. However, if this kind of links exist, the question that they can be assigned to the set L_2 or not will arise. To answer this question, we introduce the following theorem.

Theorem 3 *In the optimal solution of the CA problem,*

$$L_2 = \emptyset.$$

Proof: We assume that there exists a link i such that $e_{0i} = 1$ and $e_{1i} = 1$. From this, we have

$$C_i = f_i + \frac{b_0 + b_1 + b'_0}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{0i}}} \;\geq\; C_{0i}, \tag{3.54}$$

$$C_i = f_i + \frac{b_0 + b_1 + b'_1}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{1i}}} \;\leq\; C_{0i}. \tag{3.55}$$

From (3.39), manipulating (3.54) and (3.55) yields

$$\alpha_i\,(b_0 + b_1 + b'_0) \;\geq\; \sqrt{f_i d_{0i}}, \tag{3.56}$$

$$\alpha_i\,(b_0 + b_1 + b'_1) \;\leq\; \sqrt{f_i d_{1i}}, \tag{3.57}$$

where

$$\alpha_i = \frac{f_i}{\gamma\, T_{\max}\,(C_{0i} - f_i)}.$$

At the final stage of the examining, (3.56) and (3.57) are valid for $\forall i \in L - (L'_0 \cup L'_1)$. Consequently, we can take the summation on both (3.56) and (3.57) over the set $L - (L'_0 \cup L'_1)$. This gives

$$\sum_{L-(L'_0 \cup L'_1)} \alpha_i\,(b_0 + b_1 + b'_0) \;\geq\; b'_0, \tag{3.58}$$

$$\sum_{L-(L'_0 \cup L'_1)} \alpha_i\,(b_0 + b_1 + b'_1) \;\leq\; b'_1. \tag{3.59}$$

From (3.58) and (3.59), we obtain

$$\frac{b_0 + b_1 + b'_0}{b_0 + b_1 + b'_1} \geq \frac{b'_0}{b'_1}. \tag{3.60}$$

After manipulating (3.60), we get

$$b'_1 \geq b'_0. \tag{3.61}$$

However, this contradicts to the fact that

$$d_{0i} > d_{1i}, \qquad \forall i \in L.$$

Thus, there cannot be any link i whose e_{0i} and e_{1i} are both equal to 1 simultaneously at the final stage of examining, and all links in the network will belong to either L_0 or L_1. This result implies that

$$L_2 = \emptyset. \tag{3.62}$$

This completes the proof. □

From Theorem 3, it has been shown that there is no link in the network that belongs to the set L_2 in the optimal solution of the CA problem. With this essential characteristic, we can obtain a heuristic design algorithm for the CA problem as proposed in the next section.

3.6 Algorithm for the CA Problem

Based on the fact that $L_2 = \emptyset$ in the optimal solution of the CA problem with piecewise linear concave cost function, we can exclude the point $C_i = C_{0i}$ from the cost function of all links in the network. Consequently, the link cost function can then be regarded as a differentiable function, where its differentiation $\frac{dD_i}{dC_i}$ can be expressed as in the following equation.

$$\frac{dD_i}{dC_i} = \begin{cases} d_{0i} & , \forall i \in L_0, \\ d_{1i} & , \forall i \in L_1. \end{cases} \tag{3.63}$$

Clearly, we have no need to approximate the non-differentiable function by a differentiable function as that was done in [93]. As a result, the Lagrange multiplier method can be directly applied to the CA problem as in the following algorithm.

ALGORITHM CA_LM

Step 1 Initialize d_i to either d_{0i} or d_{1i} by any method, $\forall i \in L$.

Set ϵ as a small positive value for using as algorithm termination parameter.

Determine the initial value of Lagrange multiplier β' by

$$\beta' = \left(\frac{\sum_L \sqrt{f_i d_i/\gamma}}{T_{\max}} \right)^2 .$$

Step 2 Determine C_i by

$$C_i = f_i + \sqrt{\frac{\beta' f_i}{\gamma\, d_i}}, \qquad \forall i \in L.$$

Step 3 If $C_i < C_{0i}$, then set $d_i = d_{0i}$, else set $d_i = d_{1i}$, $\forall i \in L$.

Step 4 Determine the Lagrange multiplier β by

$$\beta = \left(\frac{\sum_L \sqrt{f_i d_i/\gamma}}{T_{\max}} \right)^2 .$$

Step 5 If $|\beta - \beta'| > \epsilon$, then set $\beta' = \beta$ and go to Step 2, else STOP and determine C_i by

$$C_i = f_i + \frac{\sum_L \sqrt{f_j d_j}}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L.$$

Note that there are many ways to set the initial value of d_i at Step 1 of the algorithm. Some examples are given in the following section.

3.7 Numerical Results and Discussions

This section gives some numerical results on the performance of the proposed algorithm, ALGORITHM CA_LM, in the view of its optimality and computation amount, and some experimental results on the effect of existing network to the link capacity assignment in packet-switched networks for the case of long-term design.

3.7.1 Performance of the Proposed Algorithm

Firstly, we examine the optimality of ALGORITHM CA_LM by comparing its results with global optimal solutions. Note that we can determine the global optimal solutions by using ALGORITHM CA_CE, since this algorithm examines all link set combinations, i.e., it runs in a complete enumeration manner. However, this is possible only for the case of small-sized networks in practical applications.

In the numerical results below, we consider packet-switched networks with the following characteristics.

1. The number of nodes in the network, N, is equal to 3, 4, 5, 6, 7, and 8.

2. The network topology is fully connected. This means there is a direct link connecting every node-pair. The number of links in the network, n, is then equal to

$$n = \frac{N(N-1)}{2}.$$

3. The traffic flow on each link in existing network is randomly set in the range (0, 80] kbps following the uniform distribution.

4. The link cost function in the existing network is assumed to be linear with per-unit cost of c_i, which is randomly set over (0, 2]. Then the link capacity in existing network can be calculated by Eq.(3.18), where $T_{\max} = 20$ ms.

5. To design a new network, we assume that the new traffic flow on each link is uniformly randomized over the range (0, 80] kbps. Therefore, the new traffic flow can be greater than, equal to, or even less than the previous value of traffic flow in the existing network.

6. Link cost function in designing a new network is piecewise linear concave with

$$d_{0i} > d_{1i},$$

where d_{0i} and d_{1i} are both chosen between (0, 2] randomly.

For ALGORITHM CA_LM, we consider the following two methods for setting the initial value of d_i at Step 1 of the algorithm.

method A d_i is set according to the relationship between link flow f_i and the existing link capacity C_{0i} as follows.

$$d_i = \begin{cases} d_{0i} & \text{if } f_i < C_{0i}, \\ d_{1i} & \text{if } f_i \geq C_{0i}. \end{cases}$$

method B d_i is set randomly between d_{0i} and d_{1i}.

To obtain a good solution for a CA problem, we apply the proposed algorithm in the following five cases.

- case 1: method A
- case 2: method A and 10 times of method B
- case 3: method A and 20 times of method B
- case 4: method A and 50 times of method B
- case 5: method A and 100 times of method B

In the above cases, when there are many solutions obtained for a specific problem, we select the best solution, namely the solution with the smallest total network cost, as a final solution.

The relationships between the number of links in the network (n) and the percentage that each of the above five cases yields global optimal solution are shown in Table 3.1 and 3.2, where the number of random traffic patterns applied to the network is one hundred and five thousand, respectively.

Table 3.1: Percentage of yielding global optimum of ALGORITHM CA_LM: 100 traffic patterns.

n	case 1	case 2	case 3	case 4	case 5
3	90	95	96	96	100
6	90	96	97	97	100
10	83	97	98	99	100
15	77	98	98	98	100
21	75	98	100	100	100
28	61	94	97	99	100

Table 3.2: Percentage of yielding global optimum of ALGORITHM CA_LM: 5000 traffic patterns.

n	case 1	case 2	case 3	case 4	case 5
3	96.02	98.80	99.08	99.08	100.00
6	91.16	98.74	98.90	98.96	100.00
10	83.90	98.18	98.46	98.78	100.00
15	77.10	97.82	98.26	98.72	100.00
21	70.66	96.64	97.88	98.56	99.96
28	61.88	94.06	97.02	98.34	99.84

Next, we observe the difference between the solution obtained from each case and the global optimum. Table 3.3 and 3.4 show the average value of the ratio between the result obtained from the proposed algorithm and the global optimum.

From Table 3.1–3.4, it can be seen that the proposed algorithm has a lower percentage of yielding global optimum when the number of links in the network increases. However, the percentage is equal to 100 % for almost of the cases in case 5. Moreover, the solution by the proposed algorithm is very close to the global optimum as can be seen from the average ratio between the solution and the global optimum. Thus, we can say that the proposed algorithm, ALGORITHM CA_LM, solves the CA problem very efficiently.

Table 3.3: Average ratio of solution by ALGORITHM CA_LM and global optimum: 100 traffic patterns.

n	case 1	case 2	case 3	case 4	case 5
3	1.00080	1.00045	1.00030	1.00030	1.00000
6	1.00047	1.00020	1.00014	1.00014	1.00000
10	1.00078	1.00005	1.00005	1.00005	1.00000
15	1.00058	1.00002	1.00002	1.00002	1.00000
21	1.00051	1.00003	1.00000	1.00000	1.00000
28	1.00044	1.00005	1.00005	1.00000	1.00000

Table 3.4: Average ratio of solution by ALGORITHM CA_LM and global optimum: 5000 traffic patterns.

n	case 1	case 2	case 3	case 4	case 5
3	1.00034	1.00012	1.00009	1.00009	1.00000
6	1.00043	1.00004	1.00004	1.00004	1.00000
10	1.00055	1.00005	1.00004	1.00004	1.00000
15	1.00059	1.00004	1.00003	1.00002	1.00000
21	1.00061	1.00004	1.00003	1.00002	1.00000
28	1.00064	1.00004	1.00002	1.00001	1.00000

Next, we investigate the computation amount of the proposed algorithm by showing its computation time using actual computation results. Again, a network with fully connected topology is considered, and five thousand random traffic patterns are applied to the network. This time, the number of nodes in the network takes the values of 10, 20, 30, 40, ..., 120, 130, 140, and 150.

The relationships between the number of links in the network (n) and average, maximum, and variance of computation time are shown in Table 3.5, where the values of computation time are measured in second for one running time of the proposed algorithm. The machine used in the measurement in this experiment is a personal computer with CPU Pentium III 450 MHz and RAM 256 MBytes.

Table 3.5: Computation time of ALGORITHM CA_LM.

n	average	maximum	variance
45	0.003	0.020	0.00002084
190	0.007	0.030	0.00002001
435	0.015	0.030	0.00002739
780	0.025	0.040	0.00003034
1225	0.039	0.050	0.00001967
1770	0.056	0.070	0.00003407
2415	0.076	0.090	0.00003744
3160	0.099	0.110	0.00003712
4005	0.125	0.140	0.00005419
4950	0.154	0.170	0.00006902
5995	0.186	0.200	0.00006480
7140	0.222	0.250	0.00008756
8385	0.260	0.280	0.00012096
9730	0.302	0.320	0.00013495
11175	0.347	0.370	0.00014052

Results in Table 3.5 are depicted again in Figures 3.4 and 3.5.

Figure 3.4 shows that the average and maximum computation time of ALGORITHM CA_LM are approximately linear with respect to the number of links in the network. By linear regression, the values of linear correlation coefficients are 0.9999971 and 0.9993178, respectively.

Figure 3.5 shows that the variance of computation time of the proposed algorithm is very small. Hence, we can conclude that ALGORITHM CA_LM is very practical for applying to the CA problem of a large-scale network.

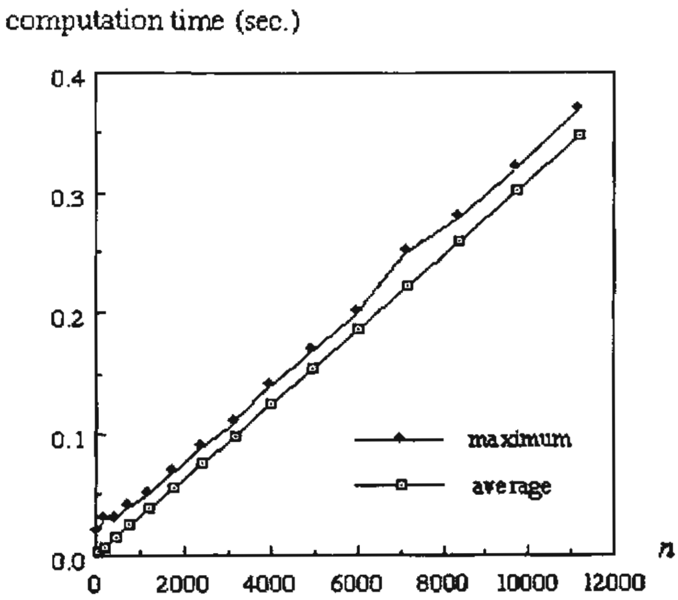


Figure 3.4: Average and maximum computation time of ALGORITHM CA_LM.

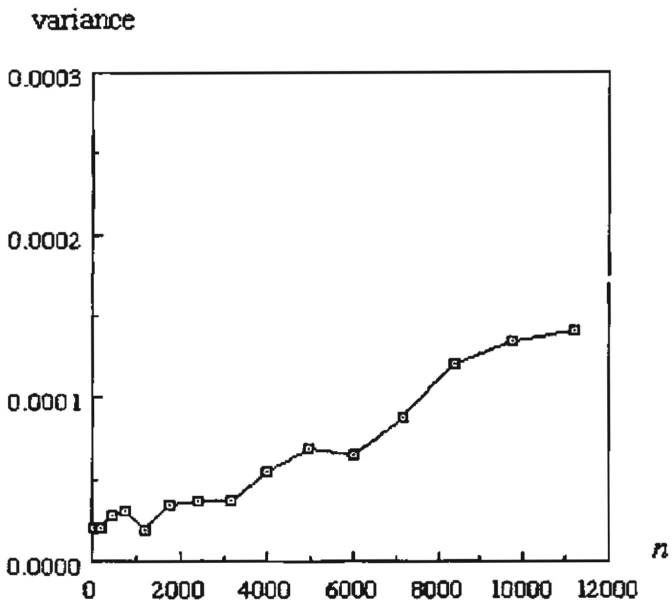


Figure 3.5: Variance of computation time of ALGORITHM CA_LM.

3.7.2 Effect of Existing Network to Network Design

Here, some numerical results will show that how the existing network facilities affect the CA problem in the case of long-term design, where the link cost is of a piecewise linear concave function. We consider a simple 3-node network as shown in Figure 3.6, where the network has three links: 1, 2, and 3.

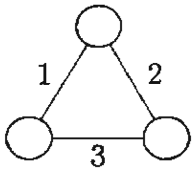


Figure 3.6: A 3-node packet-switched network.

Network parameters are set as follows.

- C_{0i} = 52 kbps for i = 1, 2, 3.
- The cost functions of link 1 and 2 are both linear with respect to link capacity, and the cost coefficients (per-unit cost) are equal to 1.
- $f_1 = f_2 = 40$ kbps.
- $T_{\max} = 20$ ms., and the mean value of packet length is 400 bits/packet.

By varying f_3 and applying several pairs of (d_{03}, d_{13}), we obtain the results as shown in Figure 3.7. It can be seen from the figure that the capacity of link 3 (C_3) is never equal to its existing link capacity (52 kbps) for all over the entire range of f_3. This confirms the characteristic of the optimal solution of the CA problem, namely $L_2 = \emptyset$.

It can also be observed from the result that there are some values of f_3 that make the curve of link capacity value to be discontinuous. These approximated values are summarized as follows.

$$\begin{array}{lclcl}
(d_{03}, d_{13}) & = & (1,0.2) & \rightarrow & f_3 = 29.32 \text{ kbps}, \\
(d_{03}, d_{13}) & = & (1,0.5) & \rightarrow & f_3 = 31.44 \text{ kbps}, \\
(d_{03}, d_{13}) & = & (5,1) & \rightarrow & f_3 = 37.47 \text{ kbps}, \\
(d_{03}, d_{13}) & = & (10,1) & \rightarrow & f_3 = 39.06 \text{ kbps}.
\end{array}$$

At these values of f_3, the total network cost is the same for both the cases when link 3 belongs to L_0, and when it belongs to L_1. It means that we can have two optimal solutions with the same network cost.

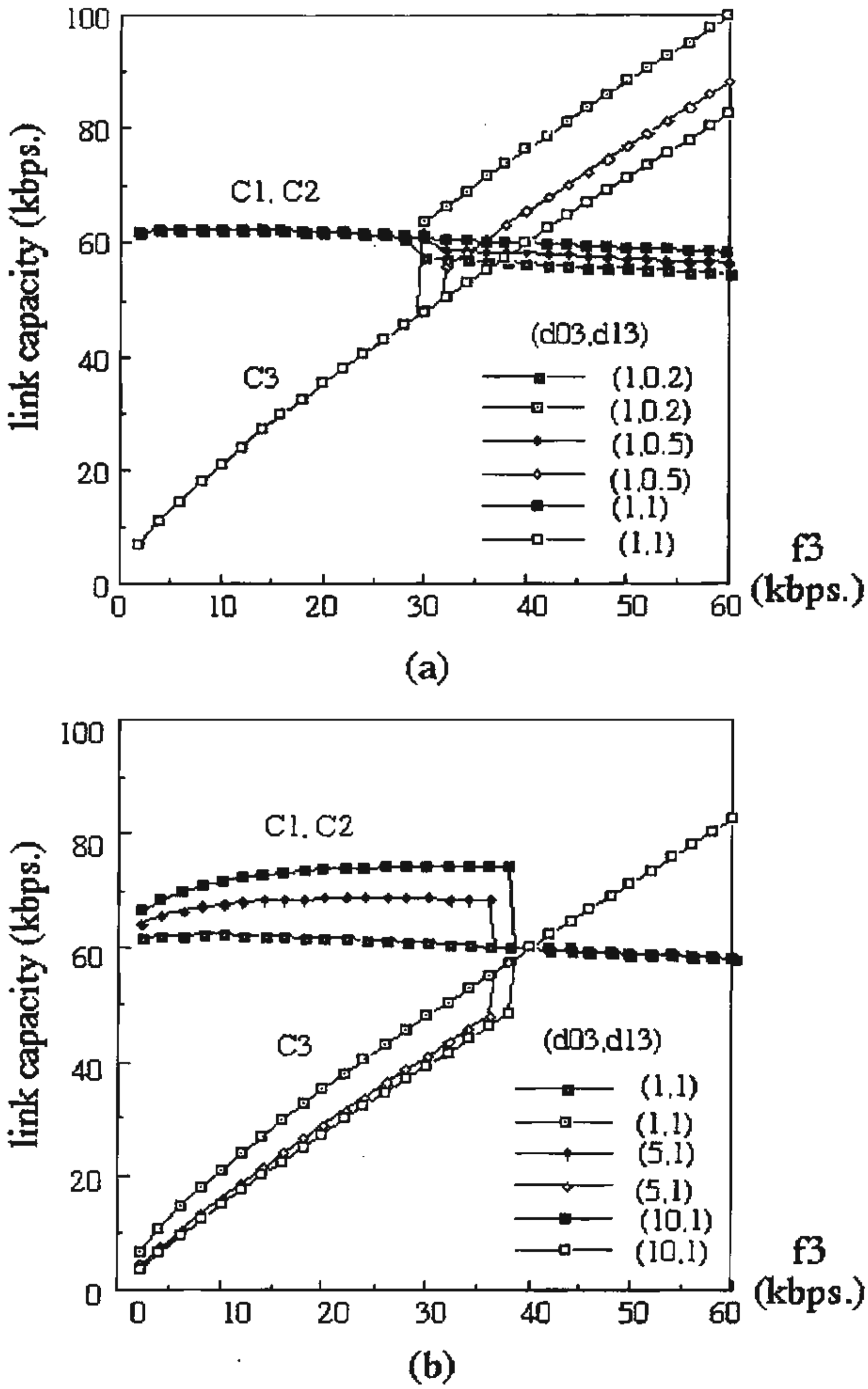


Figure 3.7: Results of the 3-node packet-switched network.

3.8 Summary

In this chapter, we have studied the link capacity assignment problem (CA problem) in packet-switched networks, where link cost function is piecewise linear concave. This type of cost function is used in order to investigate the CA problem taking into consideration an existing network in the long-term design.

After formulating the CA problem, the method of link set assignment is applied, and the characteristic of the optimal solution of the problem is derived. Based on this characteristic, it is shown that the non-differentiable link cost function can be treated as a differentiable function. As a result, the well-known Lagrange multiplier method can be directly applied to solve the problem, and a heuristic algorithm, ALGORITHM CA_LM, derived from the Lagrange multiplier method is proposed.

Due to the non-convexity of the CA problem studied in this chapter, it is hard to determine a global optimal solution to the problem. However, by investigating the numerical results, it has been shown that the proposed algorithm has very good performance in solving the CA problem, both from the view of optimality and computation time.

The work and results in this chapter are summarized and published in [82]. In addition, this publication also includes an extension of the study in this chapter by examining the CA problem with general piecewise linear concave link cost functions as illustrated in Figure 3.8. The main result in [82] is that, in the optimal solution of the CA problem, there is no link whose capacity is equal to the capacity values at the breakpoints of the link cost function, namely C_{0i}, C_{1i}, ..., $C_{(p-3)i}$, $C_{(p-2)i}$, where p is an index related to the number of breakpoints in the cost function.

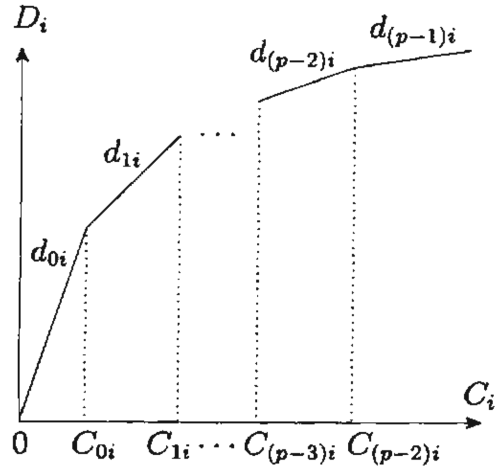


Figure 3.8: A general piecewise linear concave link cost function.

Chapter 4

Virtual Path Routing in ATM Networks

In this chapter, we discuss the second part of the work in this research project, where we study an important design problem in broadband integrated services digital networks (B-ISDN) using the technology of asynchronous transfer mode (ATM) as transmission and switching scheme. This kind of communication networks is generally referred to as ATM networks in the literature.

The network design problem studied in this chapter is the virtual path routing problem in ATM networks, where we consider the effect of existing network facilities in the design of a new network. The basic idea of cost difference between existing and new network facilities is again used in this chapter. The cases of short-term and long-term design are both considered in our study.

The content of this chapter is organized as follows. Firstly, the outlines of ATM networks will be given, where the concept of virtual channel and virtual path are described. It is shown later that the implementation of virtual channel and virtual path introduce a very high level of flexibility to the control and operation of ATM networks as well as the increasing in the network performance.

After that, we discuss the ATM network design problems, including the virtual path routing problem. Then the ATM network model used in this research work will be given, and piecewise linear functions are introduced as the network link cost functions. We consider a piecewise linear convex cost function for the case of short-term design, while a piecewise linear concave function is for the case of long-term design. Next, the virtual path routing problem is formulated as a non-linear programming problem, and analyzed to obtain some important characteristics and properties. Based on the problem analysis, we can show that the virtual path routing is in the class of shortest path problems. We then propose a heuristic design algorithm where its essential part is the well-known shortest path algorithm. Finally, some experimental results are given to investigate the performance of the proposed algorithm, as well as to show the effects of existing network facilities to the design of ATM networks.

4.1 Introduction

Recently, there has been considerable interest in integrated networks. In comparison to having several separate dedicated communication networks, integrating services and networks offers major advantages in planning, implementation, operation and maintenance. While dedicated networks require several distinct costly subscriber access lines and equipments, the integrated network access can be realized by a single high speed optical fiber for each user. An effort on integrating services and networks can be seen in the standardization of narrowband integrated services digital networks (N-ISDN or ISDN in short) [8][17][40][96].

More recently, there is a tendency that telecommunications will experience a dramatic change as broadband integrated services digital networks (B-ISDN) capabilities are introduced to communication networks. B-ISDN technology supports new user applications, including video and high speed computer data transfer, as well as existing services such as voice, facsimile, low speed data transfer, etc. [55][74][88][94]. Some important characteristics of B-ISDN can be listed as follows.

- To support high speed communications and applications.
- To support connectionless data communications to meet the needs of the interconnecting local area networks and high speed workstations.
- To support multi-point applications with a wide variety of media, including voice, video, image and computer data.
- To unify the service mechanism in order to achieve cost saving for the network provider in the transport, switching, control and operation.

4.1.1 Asynchronous Transfer Mode

There were several technologies proposed as transmission and switching scheme for broadband integrated networks. Some examples can be found in the literature, e.g., burst switching [3][42][61][63], hybrid switching [25][106][110][111].

Among these advanced technologies, ATM is recently selected by the International Telecommunications Union (ITU) as the target transfer mode for B-ISDN [15][55]. ATM does not stand for the well-known *automatic teller machine* in banking systems. Here, in the area of high speed telecommunications networks, ATM comes from "asynchronous transfer mode," which is an advanced multiplexing and switching technology proposed for the implementation of future high speed telecommunications networks. ATM is previously known as "fast packet switching" [15][70].

In ATM networks, all types of communication information including voice, video and computer data are fragmented into small fixed-length packet called "cell." This is contrary to the variable-length packet in conventional packet-switched networks (ones studied in the previous chapter). In the standardization of ATM and B-ISDN, ATM cell is 53 bytes (octets) in length, which is partitioned into two portions, namely 48 bytes for information payload field, and 5 bytes for header field [12][66]. The header consists of explicit label for channel identification, which simplifies the multiplexing process.

The main characteristics of ATM are to provide the following features for communication networks.

unity: Transmission of information from all kinds and classes of traffic source in unique small fixed-length packets.

flexibility: Capability of dealing with multimedia with a broad range of qualities of service (QoS).

high speed: Providing transmission bandwidth of up to the scale of Gbps.

Traffic Sources

Sources in ATM networks are usually considered to have bursty characteristic. It is that sources will generate cells whenever there is information to transmit, and on the other hand, no cell will be generated if there is no information. Some examples of bursty sources can be given as follows.

- Voice source with speech activity detector, which generates cells during a talkspurt and no cell during a silence period [48][109].
- Video source with frame-to-frame coding scheme such as moving picture and video conference, which generates cells based on the interframe difference information [72] where a large amount of cells are generated during the scene changing frames.

Traffic Multiplexing Level

The multiplexing method in an ATM network is a statistical one, i.e., there is no exclusive transmission bandwidth assigned to any specific source multiplexed in the system. However, all traffic sources share the same amount of bandwidth together. This is different to circuit-switched networks, where bandwidth is divided into several portions each of which is dedicated to a specific source (call).

When cells are generated and arrive at an ATM multiplexer, they will be multiplexed and transmitted as shown in Figure 4.1 [76]. As can be seen, the bandwidth in an ATM network is assigned to a source if and only if the source has information (cells) to transmit. This results in the statistical multiplexing of information from several sources.

Although statistical multiplexing is a very powerful method for handling cells from multiple sources, the QoS control is realized as a very complicated task. Furthermore, when there are a large number of classes of sources in the network, the level of multiplexing strongly affects the network performance and network characteristics. There are several ways to multiplex traffics of the same and different classes of sources. Some of them are illustrated in Figure 4.2 [98].

From the figure, the *channel independent* multiplexing level can be regarded as a method used in circuit-switched networks since each channel (source) has its own assigned

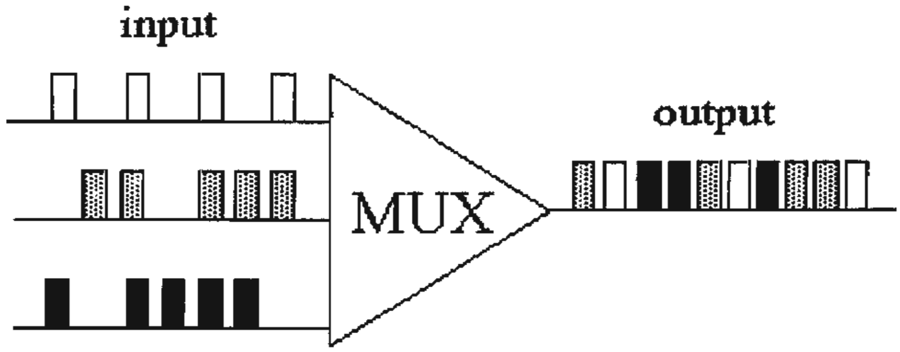


Figure 4.1: Cell multiplexing in ATM network.

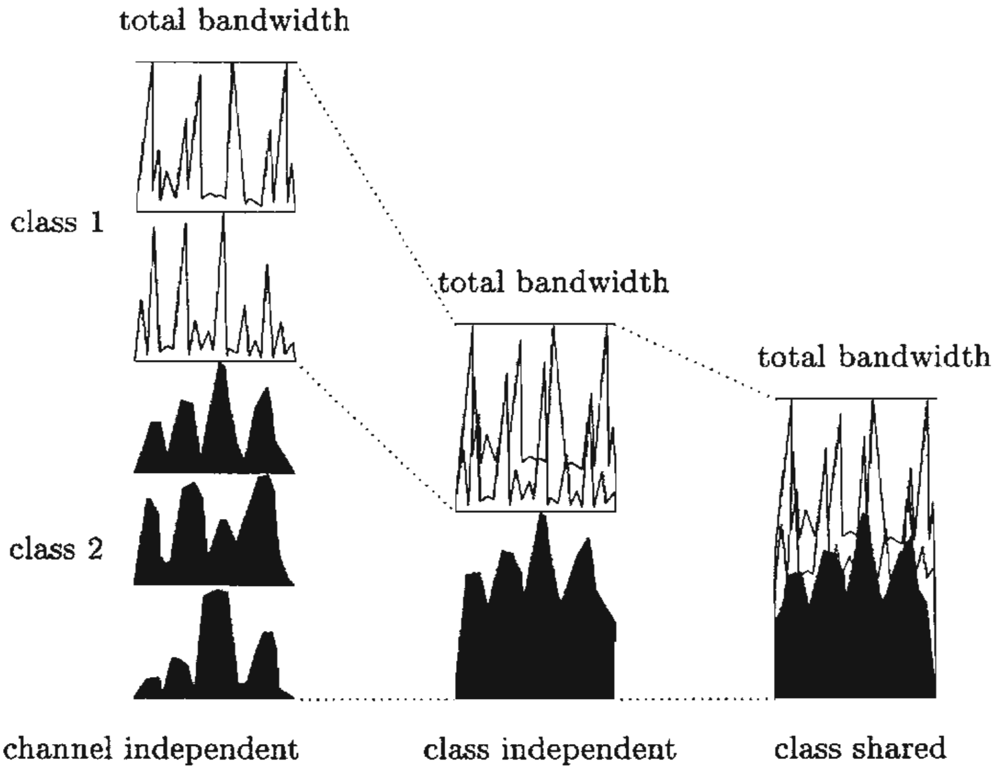


Figure 4.2: Levels of traffic multiplexing.

bandwidth. Therefore, no statistical multiplexing effect is obtained in this case. Clearly, the case of channel independent is not so effective, especially when traffic sources have high degree of burstiness.

In *class independent*, traffics from the same class of sources are multiplexed and share the same portion of bandwidth assigned to the class. Statistical multiplexing effect is obtained in this case as the total required bandwidth is obviously less than that in the case of channel independent. Although this results in a more complicated QoS control in each class, the QoS control among classes is simple since traffics from different classes do not share the same bandwidth in this case.

In the case of *class shared*, very high bandwidth utilization is obtained as can be seen from the figure. However, this leads to a very high complexity of traffic control as well as bandwidth assignment problem.

To determine whether what level of traffic multiplexing should be used in a network, it is necessary to take several parameters into account. They are, for example, the number of classes of sources in the network, the number of channels in each class, the QoS requested from each source, etc. It is realized that the complexity of traffic control in an ATM network grows dramatically when there are a large number of sources from multiple classes share a single transmission resource. Thus, some effective traffic control schemes are needed to cope with the difficulty in guaranteeing QoS of all traffic sources. Two main functions defined for traffic control in ATM networks are call admission control (CAC) and usage parameter control (UPC) [78][100].

4.1.2 The Concept of Virtual Path

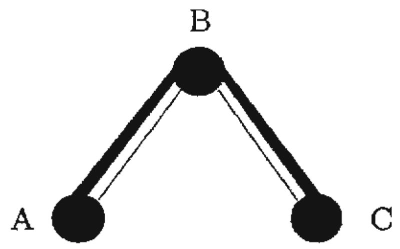
In an ATM network, traffics from multiple sources are multiplexed and transmitted via a shared bandwidth as described above. We can regard each individual traffic source as a communication channel that transmits cells. Since a communication channel in ATM networks is not an exclusive one, it is referred to in ATM standards as "virtual channel" (VC).

On the other hand, the shared bandwidth among sources can be treated as a transmission link that conveys multiplexed cells from multiple sources. Consequently, the bandwidth or capacity of the link is equal to the total bandwidth given in Figure 4.2. The value of link bandwidth deeply depends on the level of traffic multiplexing used in the system.

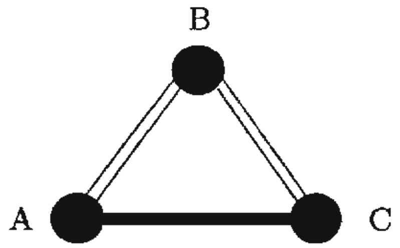
In the case of channel independent, although rarely used in ATM networks for bursty sources, link bandwidth is the same as the maximum speed that a source generates cells. Contrary to this, the link bandwidth in the cases of class independent and class shared can be determined by the traffic analysis and statistical multiplexing behavior of cells.

A sequence of links forms a *path*, which is dedicated to a pair of nodes where the origin and destination of the path are the two nodes. Actually, a path in an ATM network is a virtual one. Thus, it is referred to as "virtual path" (VP) in ATM terminology. A VP is a logical direct link connecting a node-pair even there is no physical direct link connecting the two nodes as illustrated in Figure 4.3.

The ATM network in Figure 4.3 has only two physical links, i.e., the link connecting A-B and B-C as in Figure 4.3 (a). Thus, when node A and C need to install a VP connecting



(a) Physical network.



(b) Logical network.

Figure 4.3: Virtual path (VP) in physical and logical network.

between them, the VP must be constructed and allocated over the two physical links via node B (the part of black line). In the view of logical network, there will be a direct link which is a VP connecting between A-C as shown in Figure 4.3 (b).

Next, the mechanism of traffic multiplexing in a VP will be discussed. When a source (call) has information to transmit, it generates a call-setup signal to the ATM node that it is connecting with. The node will then search for an appropriate VP for the call. If no such VP exists, the call will be rejected. Otherwise, the call will be accepted and allocated to the proper VP. The node will also assign the following two parameters to the call.

- Virtual channel identifier (VCI).
- Virtual path identifier (VPI).

VCI and VPI are both filled in the header field of all cells generated by the call, and will be used as reference at all nodes in the VP (origin, intermediate and destination nodes). The value of both VCI and VPI can be either fixed or changed at the intermediate nodes along the path from source to destination [21].

There are two methods of VPI assignment proposed in the literature, namely local VPI number assignment and global VPI number assignment [87]. However, by the performance comparison of the two methods, it can be concluded that the local VPI number assignment is appropriate for the implementation in ATM networks.

The accommodation of VP to a physical link as well as that of VC to a VP can be depicted as in Figure 4.4.

As a summary to this subsection, the implementation of VPs in ATM networks results in the following characteristics.

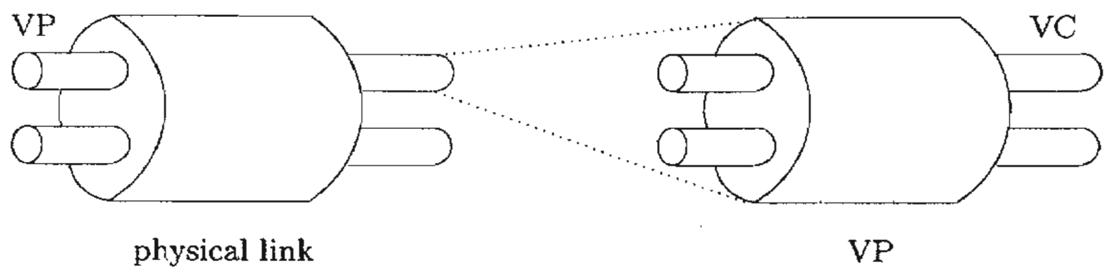


Figure 4.4: The relationship of physical link, virtual path and virtual channel.

- Flexibility in VP bandwidth control

  When a cell is generated and transmitted into a VP dedicated to the source and destination of the cell, it will be continuously transferred at intermediate nodes via the same VP until it reaches the destination node where it will be switched to the end-user. Consequently, VP bandwidth is then automatically depended on the number of cells fed into the VP at the origin node. Controlling the number of cells at the origin node can then alter the bandwidth of the VP, and this can be done without any change at intermediate nodes. Accordingly, flexibility is obtained.

- Flexibility in routing control

  Since a cell is transferred in a VP by assigning a logical path number (LPN) at intermediate node, renewing the pairs of LPNs in routing table at only origin and destination node can easily change the route of VP. Thus, flexibility in VP routing control is obtained.

- Improvement in network performance

  Based on the results of the above items of flexibility, network performance can then be improved. Firstly, the amount of processing overhead at intermediate nodes can be decreased. Secondly, the adaptation to the change in user traffic requirement can be conducted very simply. Finally, the adaptation to the failure of physical network facilities can also be conducted easily by rerouting the the group of VPs affected by the failure to other paths.

There are many research works on the topic of ATM network reconfiguration based on the use of flexibility mentioned above. When there are traffic load variations, physical facility failures, and traffic forecasting errors occur in a network, the network is reconfigured by changing VP bandwidth as well as VP routing to maintain the QoS of traffic at an acceptable level. This kind of research works can be found in the literature, e.g., [13][36][37][90][91].

4.1.3 ATM Network Design

As mentioned above, cells generated from sources (call) are multiplexed and transmitted via a VP assigned between each node-pair. Consequently, the bandwidth of VP will then be depended upon the number of VCs in the VP as well as traffic characteristics of each VC and its QoS requirement. Once the VP bandwidth is determined, it is necessary to accommodate the VP into physical facilities (links).

As a consequence, we have the following three design problems in ATM networks [41][48][98][108].

1. **Call Level Design Problem**

   In this problem, the dimensioning of the number of VCs multiplexed in each VP is conducted by considering the traffic demand, traffic characteristics in the call level and the QoS requirement of calls, e.g., call blocking probability.

2. **Cell Level Design Problem**

   The dimensioning of VP bandwidth is performed in this problem by taking into account the traffic characteristics in the cell level, QoS of cells, as well as the number of VCs multiplexed in the VP obtained in the call level design problem. There are many QoS requirements in the cell level, e.g., cell loss probability, cell delay time, cell delay jitter, etc.

3. **Virtual Path Routing Problem**

   This problem is also known as VP accommodation problem [108]. The objective of the virtual path routing problem is to solve for an *optimal route* (a sequence of physical links) for a VP by considering physical link cost function. Therefore, the optimal route is the route which results in the minimization of the total network cost when the VP is accommodated into the route.

For the traffic characteristics of calls, we have that the call arrival process and the probabilistic distribution of call duration (holding) time are necessary considered in the call level design problem. Actually, the call level design in ATM networks is similar to the circuit dimensioning in circuit-switched networks [31][47][75], since the two design problems can be formulated and solved almost in the same way.

In the cell level design, there is a very important item in this design problem. It is the cell arrival process, which indicates the degree of burstiness of cell arrival. As a VP in ATM network can be regarded as a queueing system, the cell level design is, therefore, equivalent to the analysis of the queue where cell arrival process is considered as the input to the system. The stochastic cell arrival process can be also referred to as *traffic source model*. In the literature, there are several traffic source models proposed for actual ATM traffic sources. For example,

- ON-OFF model for voice source [38][54][68][71], as well as for general traffic source [6][26][41][44][108],
- Interrupted Poisson Process (IPP) model [56] for single video (moving picture) source [4],
- Markov Modulated Poisson Process (MMPP) for video source [54][85],
- Auto Regressive (AR) model for video source using an interframe coding scheme [48][62].

As mentioned above, the objective of the virtual path routing problem (VPR problem) is to minimize the network cost by selecting an optimal route to accommodate each VP to physical network. Since this network design problem is directly concerned with physical link cost function, this research project then focuses on the VPR problem in ATM networks, by including the effect of existing network in the design of a new network. We study the VPR problem in this chapter by applying link cost functions that are capable to express the difference between the cost of existing and newly installed facility.

4.2 Network Model

In the first part of this section, we discuss the model of ATM network considered in our study. After that in the second part, we introduce link cost functions with the consideration of cost difference between existing facility and newly installed facility for both the cases of short-term and long-term design.

4.2.1 Model Description

The symbols used in this chapter are summarized as follows.

L set of virtual paths (VPs) in the network

P set of physical links

V_i bandwidth of VP i

Y_j capacity of physical link j

Y_{0j} existing value of capacity of physical link j

D total network cost

D_j cost of physical link j

α_{ij} VP accommodation parameter of VP i and physical link j, defined by

$$\alpha_{ij} = \begin{cases} 1, & \text{when VP } i \text{ is accommodated in physical link } j, \\ 0, & \text{otherwise.} \end{cases} \tag{4.1}$$

As a set of input parameters to the VPR problem, we assume that all of the VP bandwidths are obtained from the results of the network design in the level of call and cell. We also have the following condition for all VPs in the set L.

$$V_i > 0. \tag{4.2}$$

This means that each VP bandwidth takes only positive value. Clearly, negative and zero value of V_i have no significant meaning, and we can exclude this kind of VPs from the VPR problem.

Note that the number of VPs considered in the VPR problem can be an arbitrary positive integer. For a special case that there exists a single VP dedicated for each node-pair, we have

$$|L| = \frac{N(N-1)}{2}, \tag{4.3}$$

where $|L|$ is the number of members of set L, i.e., the number of VPs in the ATM network, and N is the number of ATM nodes in the network.

In the case that there are p VPs installed between each node-pair where each of the VP is allocated to each service class (so p is the number of service classes), we then have

$$|L| = p\,\frac{N(N-1)}{2}. \tag{4.4}$$

The relationship between the bandwidth of VP i, V_i, and the capacity of physical link j, Y_j, can be expressed by using the VP accommodation parameter α_{ij} as in the following equation.

$$Y_j = \sum_L \alpha_{ij} V_i. \tag{4.5}$$

In other words, the capacity of a physical link is equal to the sum of bandwidth of all VPs accommodated on the link.

For simplicity, our ATM network model defines the total network cost as the sum of all physical link costs. Then we have

$$D = \sum_P D_j. \tag{4.6}$$

4.2.2 Link Cost Function

For each physical link, link cost D_j is a function of link capacity Y_j as

$$D_j = g(Y_j), \tag{4.7}$$

where $g(.)$ is an arbitrary mathematical function, which can be a simplest case of linear function or a more realistic case of non-linear function.

Since the main objective of this research project is to study the communication network design problem under the effect of the consideration of existing network facilities, we then define a link cost function that is capable to deal with the existing link capacity. Consequently, Eq.(4.7) is rewritten as in the following equation.

$$D_j = g(Y_j, Y_{0j}). \tag{4.8}$$

Eq.(4.8) shows that the physical link cost function consists of two parts of link cost. They are the cost of using existing capacity and the cost concerned with the installation of new capacity.

Next, we apply the idea of cost difference between the existing capacity and the newly installed capacity [81][93] to the cost function in Eq.(4.8), as we have done in the preceding chapter. We also assume that the link cost of the two parts are both linear with respect to link capacity. As a consequence, we obtain the physical link cost function as in the following equation.

$$D_j = \begin{cases} d_{0j}Y_j & ,Y_j \le Y_{0j}, \\ d_{1j}Y_j - (d_{1j} - d_{0j})Y_{0j} & ,Y_j > Y_{0j}, \end{cases} \tag{4.9}$$

where d_{0j} and d_{1j} are the per-unit cost of existing capacity and newly installed capacity of physical link j, respectively. Generally, the value of d_{0j} and d_{1j} are both non-negative. That is

$$d_{0j} \ge 0, \tag{4.10}$$
$$d_{1j} \ge 0. \tag{4.11}$$

In this chapter, we study the VPR problem in ATM networks for two cases, namely the case of short-term network design and long-term network design.

Firstly, in the case of short-term network design, we can have that the cost function is a piecewise linear convex function [81][93], i.e.,

$$d_{0j} < d_{1j}, \tag{4.12}$$
$$d_{0j} \ge 0, \tag{4.13}$$
$$d_{1j} > 0. \tag{4.14}$$

The reason is straightforward that the existing capacity is promptly to be used in the construction of a new network, and it cannot be changed easily in a short period of time. Therefore, the per-unit cost of existing capacity can be regarded to be less than that of the new capacity. The above d_{0j} and d_{1j} results in a link cost function as shown in Figure 4.5.

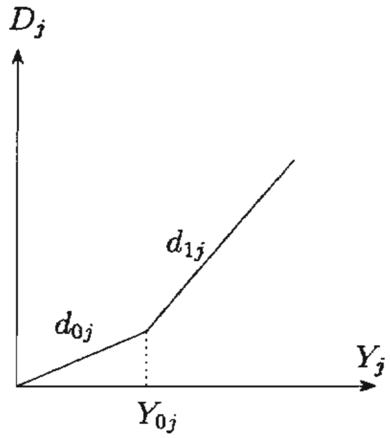


Figure 4.5: A link cost function for the case of short-term design.

On the other hand, we have an opposite case for the long-term network design. As mentioned many times in previous chapters, it is appropriate to apply a concave function in the case of long-term design. Hence, we focus on the following range of d_{0j} and d_{1j}.

$$d_{0j} > d_{1j}, \tag{4.15}$$
$$d_{0j} > 0, \tag{4.16}$$
$$d_{1j} > 0. \tag{4.17}$$

The above range of d_{0j} and d_{1j} results in a piecewise linear concave function as shown in Figure 4.6.

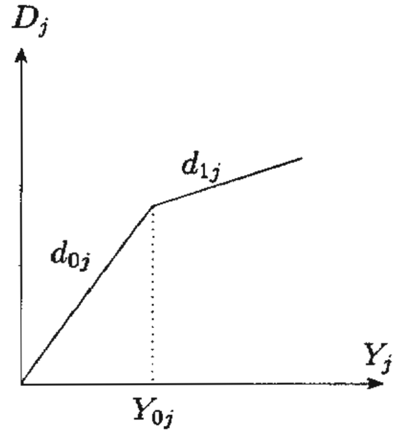


Figure 4.6: A link cost function for the case of long-term design.

It is important to note that the case of $d_{0j} = d_{1j}$ is excluded from the range of d_{0j} and d_{1j} given above, in both the cases of short-term and long-term design. The reason is that this case gives a simple case of linear link cost function to the VPR problem. However, we will examine the case of linear cost function as a special case in the analysis given later in this chapter, since it can lead us to an important property of the VPR problem, despite of its simplicity.

Furthermore, the case $d_{1j} = 0$ is also excluded due to its triviality. This is because the capacity of all physical links can be simply set to infinity, while the total network cost is maintained to be finite.

The convex link cost function in Figure 4.5 as well as the concave link cost function in Figure 4.6 will be applied to the VPR problem in ATM networks, where the problem can be formulated as an optimization problem in the next section.

4.3 Virtual Path Routing Problem (VPR Problem)

For the work in this chapter, we study the virtual path routing in ATM networks in the case that new user traffic requirement is applied to an existing network. It should be noted that the new set of traffic requirement results in the change of VP bandwidths between node-pairs. As a consequence, the routing pattern of VPs must be adjusted to cope with the change of VP bandwidths such that the total network cost is minimized.

Thus, we formulate the VPR problem as follows.

VPR Problem

Given: $\{V_i\}$, $\{Y_{0j}\}$, $\{d_{0j}\}$ and $\{d_{1j}\}$

Minimize: $D = \sum_P D_j$

Design variables: $\{\alpha_{ij}\}$

Subject to: $\alpha_{ij} \in \{0, 1\}, \quad \forall i \in L$ and $\forall j \in P$

$$Y_j = \sum_L \alpha_{ij} V_i, \quad \forall j \in P$$

In the above VPR problem, $\{V_i\}$ is the set of VP bandwidths resulted from the new user traffic requirement applied to an existing ATM network where the existing physical link capacity is $\{Y_{0j}\}$. The VP accommodation parameter $\{\alpha_{ij}\}$ is determined such that the total network cost D is minimized. Based on the network model described in the previous section, the network cost is defined as the sum of all link costs where each link cost D_j is given in Eq.(4.9).

The conditions of design constraint in the VPR problem are the value of α_{ij} and the relationship between VP bandwidths and physical link capacity. It is obvious from the definition in Eq.(4.1) that the value of α_{ij} can have only two possibilities, that is either 0 or 1.

The set $\{\alpha_{ij}\}$ forms the set of VP routing pattern, where we need to determine the optimal routing pattern in the way that it gives a network with minimal cost.

It is very important to note that, although we have two types of link cost function (convex function and concave function), the VPR problem can be formulated in a same way for each type of cost function. Furthermore, it will be shown later in this chapter that heuristic design algorithm for the VPR problem is common and applicable to solve the two types of cost function.

4.4 VPR Problem Analysis

In this section, we analyze the VPR problem formulated in the preceding chapter. Firstly, we try to find a method to solve the VPR problem such that its global optimal solution can be obtained. It can be seen later that the method solves the problem in a complete enumeration manner. Hence, the method is not applicable to practical network design.

Based on the above situation, we then analyze the VPR problem to find a new way to construct a heuristic method for solving the problem. We then investigate the simple case of the VPR problem where the link cost function is linear. From the analysis, it can be shown that the VPR problem is a member in the class of shortest path problems.

4.4.1 Global Optimal Solution

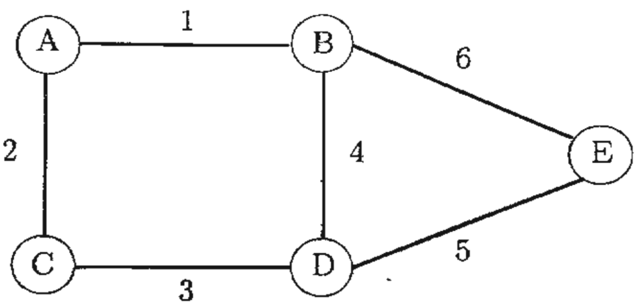


Figure 4.7: A 5-node 6-link ATM network.

Let's consider an ATM network with 5 nodes and 6 physical links as shown in Figure 4.7.

Assume that there are a VP installed exclusively for each node-pair in the network. Therefore, the overall number of VPs is equal to $5(5-1)/2 = 10$. They are the VPs connecting between A-B, A-C, A-D, A-E, B-C, B-D, B-E, C-D, C-E, and D-E.

Let's focus on the VP for the node-pair A-B. From Figure 4.7, we can derive that there are three possibilities to accommodate or route this VP in the physical network. Using the sequence of physical links, we have that the three possible routes are $\{1\}$, $\{2,3,4\}$, and $\{2,3,5,6\}$. By the same idea, there are also three possible routes for the VP of D-E node-pair, i.e., $\{5\}$, $\{4,6\}$, and $\{3,2,1,6\}$.

Consequently, we can determine all possible routes for accommodating each VP to the physical network. Assume that the number of possible routes for VP i is equal to r_i, the total number of routing patterns in the network, R, is then equal to

$$R = \prod_{L} r_i. \tag{4.18}$$

Obviously, examining all of the possible routes leads to the global optimal solution to the VPR problem.

From the above analysis, we can construct an algorithm, ALGORITHM VPR_CE, based on the complete enumeration (CE) as follows.

ALGORITHM VPR_CE

Step 1. Let network the cost $D = \infty$.

Step 2. For $\forall i \in L$, select a route for VP i from its r_i possible routes.

Step 3. Determine $\{\alpha_{ij}\}$ from the routing pattern of all VPs.

Step 4. Calculate the capacity of physical link j, $\forall j \in P$, by

$$Y_j = \sum_{L} \alpha_{ij} V_i.$$

Step 5. Calculate physical link cost D_j, $\forall j \in P$, and also the total network cost $D' = \sum_{P} D_j$.

Step 6. If $D' < D$, then set $D = D'$ and keep $\{\alpha_{ij}\}$ as the design solution.

Step 7. If all possible routing patterns are completely examined, STOP and the final solution of the VPR problem is $\{\alpha_{ij}\}$.

Step 8. Select another routing pattern (e.g., by selecting another possible route of each VP) which has not been examined yet, and go to Step 3.

From the selected route of a VP, the parameter α_{ij} can be determined as in the following example. Assume that the VP for node-pair A-B is VP 1 (i.e., $i = 1$), and the route for this VP is chosen to be $\{2, 3, 4\}$. We then have

$$\begin{aligned}
\alpha_{11} &= 0, \\
\alpha_{12} &= 1, \\
\alpha_{13} &= 1, \\
\alpha_{14} &= 1, \\
\alpha_{15} &= 0, \\
\alpha_{16} &= 0.
\end{aligned}$$

The above ALGORITHM VPR_CE can give global optimal solution to the VPR problem with piecewise linear cost functions in our study. However, it is clear that the algorithm is very time-consuming according to its characteristic of complete enumeration where we have to examine all of the possible VP routing patterns. Thus, a more efficient design algorithm is needed.

4.4.2 VPR Problem with Linear Cost Function

Now, let's consider a simple case of the VPR problem. It is the case of linear physical link cost function. That is

$$D_j = d_j Y_j, \tag{4.19}$$

where d_j is the per-unit capacity cost of physical link j. In general cases, we have

$$d_j > 0.$$

By this type of link cost function, we have

$$\begin{aligned}
D &= \sum_P D_j, \\
&= \sum_P d_j Y_j, \\
&= \sum_P d_j \cdot \left(\sum_L \alpha_{ij} V_i \right), \\
&= \sum_P \sum_L d_j \alpha_{ij} V_i, \\
&= \sum_L \sum_P d_j \alpha_{ij} V_i, \\
&= \sum_L V_i \cdot \left(\sum_P d_j \alpha_{ij} \right).
\end{aligned}$$

Thus, minimizing D is equal to

$$\min D = \min \left(\sum_L V_i \cdot \left(\sum_P d_j \alpha_{ij}\right)\right), \tag{4.20}$$

$$= \sum_L V_i \cdot \min \left(\sum_P d_j \alpha_{ij}\right). \tag{4.21}$$

Let's consider a general shortest path problem for a network graph. We assume that F is the minimized objective function of the problem, and z_j is the flow allocated on link j of the graph. To solve the shortest path problem, we firstly assign a number denoted the link length l_j to each link j in a network graph, where l_j is defined by

$$l_j = \frac{\partial F}{\partial z_j}. \tag{4.22}$$

As a result, the shortest path between a node-pair is the path that is connecting the two nodes and the sum of link lengths along the path is minimum [2][7].

From the above property of shortest path problem, we can see that Eq.(4.21) implies that, in the case of linear link cost function, the optimal result can be obtained by routing each VP to a sequence of physical links that forms a shortest path between the two nodes connecting by the VP, where the link length in this case of VPR problem is

$$l_j = d_j. \tag{4.23}$$

By matching the VPR problem and the general shortest path problem mentioned above, it yields

$$F = D, \tag{4.24}$$

$$z_j = Y_j. \tag{4.25}$$

Therefore, from Eq.(4.22), we have

$$\begin{aligned} l_j &= \frac{\partial F}{\partial z_j}, \\ &= \frac{\partial D}{\partial Y_j}, \\ &= \frac{dD_j}{dY_j}, \\ &= d_j. \end{aligned}$$

Obviously, the last equation is the same as Eq.(4.23).

From the above analysis, we can conclude that the VPR problem is a member in the class of shortest path problems and can be solved by using any shortest path algorithm when link length l_j associated with each physical link j is determined by

$$l_j = \frac{dD_j}{dY_j}. \tag{4.26}$$

4.5 Algorithm for the VPR Problem

Based on the analysis of the VPR problem in the previous section, the problem can be solved similarly to a shortest path problem. Hence, we can propose the following heuristic algorithm based on shortest path (SP) algorithm. The proposed algorithm is referred to as ALGORITHM VPR_SP.

ALGORITHM VPR_SP

Step 1. Initialize the algorithm by feasible link length $\{l_j\}$ and let $D = \infty$.

Step 2. Use a shortest path algorithm to determine the route of each VP.

Step 3. Determine $\{\alpha_{ij}\}$ from the routing pattern of all VPs.

Step 4. Calculate the capacity of physical link j, $\forall j \in P$, by

$$Y_j = \sum_L \alpha_{ij} V_i.$$

Step 5. Calculate physical link cost D_j, $\forall j \in P$, and also the total network cost $D' = \sum_P D_j$.

Step 6. If the network cost is not improved, i.e., $D' \geq D$, then STOP with $\{\alpha_{ij}\}$ as the final design solution.

Otherwise set $D = D'$ and keep $\{\alpha_{ij}\}$ as the design solution.

Step 7. Compute $\{l_j\}$ by Eq.(4.26).

Go to Step 2.

The shortest path algorithm used in the above ALGORITHM VPR_SP can be any algorithm proposed in the literature, e.g., the shortest path algorithm by Dijkstra, by Dantzig, by Floyd, etc. [2][89].

To determine the value of link length l_j at Step 7. in the proposed algorithm, we have that

$$l_j = \frac{dD_j}{dY_j}.$$

For the physical link cost functions D_j in our study (those shown in Figure 4.5 and 4.6), we obtain

$$l_j = \begin{cases} d_{0j}, & \text{when } Y_j < Y_{0j}, \\ d_{1j}, & \text{when } Y_j > Y_{0j}. \end{cases} \tag{4.27}$$

However, it is clear that the link cost functions in our study are not differentiable with respect to physical link capacity Y_j at the point $Y_j = Y_{0j}$. To find the link length in this case, we examine Eq.(4.27) again. It is found that

- in the case of short-term design, i.e., $d_{0j} < d_{1j}$,

$$d_{0j} \leq l_j \leq d_{1j}, \quad \text{when } Y_j = Y_{0j}. \tag{4.28}$$

- in the case of long-term design, i.e., $d_{0j} > d_{1j}$,

$$d_{1j} \leq l_j \leq d_{0j}, \quad \text{when } Y_j = Y_{0j}. \tag{4.29}$$

In other words, the link length l_j, although the exact value is not known, falls in the range between d_{0j} and d_{1j}. Thus, we can have

$$l_j|_{Y_j=Y_{0j}} = (1-\theta).d_{0j} + \theta.d_{1j}, \tag{4.30}$$

where $0 \leq \theta \leq 1$.

Note that the value of θ can be fixed, varied, or even randomly set. In our algorithm, we consider a general case to choose the value randomly over the range [0, 1].

4.6 Numerical Results and Discussions

This section gives some numerical results on the performance of the proposed algorithm, ALGORITHM VPR_SP, in the view of its optimality and computation amount, and some experimental results on the effect of existing network to the virtual path routing in ATM networks.

4.6.1 Performance of the Proposed Algorithm

Firstly, we examine the optimality of ALGORITHM VPR_SP by comparing its results with global optimal solutions. Note that the global optimal solutions can be obtained by using ALGORITHM VPR_CE given in previous section. However, the ALGORITHM VPR_CE can be used only in the case of small-sized networks according to the fact that it runs in a complete enumeration manner.

In the numerical results given below, we consider ATM networks with the following characteristics.

1. The number of nodes in the network, N, is equal to 3, 4, 5, 6, 7, 8, and 9.
2. The physical network topology is randomly selected, where the probability of having a physical link between each node-pair is equal to 0.5. The topology of the physical network must be connected [2][53], i.e., for any node-pair in the network, there must be at least one path connecting the two nodes.
3. There is one VP installed for each node-pair. Consequently, the total number of VPs in the network is
$$|L| = \frac{N(N-1)}{2}.$$
This also results in a fully connected logical network topology. The value of each VP bandwidth, V_i, is randomly set in the range (0, 20] Mbps following the uniform distribution.

4. The existing capacity of each physical link, Y_{0j}, is uniformly randomized over the range (0, 40] Mbps.

5. The physical link cost function in designing a new network is piecewise linear with

$$d_{0j} < d_{1j}, \quad \text{convex function for the case of short-term design,}$$

or

$$d_{0j} > d_{1j}, \quad \text{concave function for the case of long-term design,}$$

where d_{0j} and d_{1j} are both chosen between (0, 2] randomly.

For ALGORITHM VPR_SP, we consider the following two methods for the initialization of link length l_j at Step 1 of the algorithm.

method A l_j is set equally to be 1 for every physical link. This means the shortest path between a node-pair is equivalent to the path with minimal hop-count.

method B l_j is set randomly between d_{0j} and d_{1j}.

To obtain a good solution for a VPR problem, we apply the proposed algorithm in the following five cases.

- case 1: method A
- case 2: method A and 10 times of method B
- case 3: method A and 20 times of method B
- case 4: method A and 50 times of method B
- case 5: method A and 100 times of method B

In the above cases, when there are many solutions obtained for a specific problem, we select the best solution, that is the solution with the smallest total network cost, as a final solution.

The relationships between the number of VPs in the network ($|L|$) and the percentage that each of the above five cases yields global optimal solution when applying one hundred random VP bandwidth patterns to the network are shown in Table 4.1 and 4.2 for the case of convex and concave cost function, respectively.

We next examine the difference between the solution obtained from each case and the global optimum. Table 4.3 and 4.4 show the average value of the ratio between the result obtained from the proposed algorithm and the global optimum.

From Table 4.1−4.4, we can see that the proposed algorithm has a lower percentage of yielding global optimum when the number of VPs in the network increases. However, the solution by the proposed algorithm is very close to the global optimum as can be seen from the average ratio between the solution and the global optimum. Thus, it is possible to conclude that the proposed algorithm, ALGORITHM VPR_SP, efficiently solves the VPR problem.

Table 4.1: Percentage of yielding global optimum of ALGORITHM VPR_SP: convex cost function.

$\|L\|$	case 1	case 2	case 3	case 4	case 5
3	100	100	100	100	100
6	97	99	100	100	100
10	85	97	98	98	98
15	66	83	87	90	96
21	42	61	74	82	88
28	30	45	49	56	62
36	10	20	27	32	43

Table 4.2: Percentage of yielding global optimum of ALGORITHM VPR_SP: concave cost function.

$\|L\|$	case 1	case 2	case 3	case 4	case 5
3	96	100	100	100	100
6	87	100	100	100	100
10	73	97	100	100	100
15	58	94	97	99	100
21	32	90	94	97	97
28	15	74	81	90	97
36	14	64	76	89	95

Table 4.3: Average ratio of solution by ALGORITHM VPR_SP and global optimum: convex cost function.

$\|L\|$	case 1	case 2	case 3	case 4	case 5
3	1.00000	1.00000	1.00000	1.00000	1.00000
6	1.00263	1.00038	1.00000	1.00000	1.00000
10	1.01143	1.00005	1.00000	1.00000	1.00000
15	1.02214	1.00631	1.00344	1.00309	1.00084
21	1.03710	1.01043	1.00568	1.00295	1.00129
28	1.04976	1.01572	1.01137	1.00805	1.00464
36	1.05748	1.02460	1.01902	1.01339	1.01026

Table 4.4: Average ratio of solution by ALGORITHM VPR_SP and global optimum: concave cost function.

$\|L\|$	case 1	case 2	case 3	case 4	case 5
3	1.00270	1.00000	1.00000	1.00000	1.00000
6	1.01588	1.00000	1.00000	1.00000	1.00000
10	1.03792	1.00063	1.00000	1.00000	1.00000
15	1.05143	1.00132	1.00050	1.00015	1.00000
21	1.07451	1.00284	1.00231	1.00112	1.00054
28	1.08498	1.00897	1.00372	1.00110	1.00050
36	1.09165	1.01171	1.00593	1.00172	1.00095

Next, we investigate the computation amount of the proposed algorithm by showing its computation time using actual computation results. Again, a network with fully connected logical topology is considered, and one hundred random VP bandwidth patterns are applied to the network. This time, the number of nodes in the network takes the values of 10, 20, 30, ..., 80, 90, and 100.

The relationships between the number of VPs in the network ($|L|$) and average, maximum, and variance of computation time are shown in Table 4.5 and 4.6 for the case of convex and concave cost function, respectively. The value of computation time indicates one running time of the proposed algorithm in second. Again, the machine used in the experiment is a personal computer with CPU Pentium III 450 MHz and RAM 256 MBytes.

Table 4.5: Computation time of ALGORITHM VPR_SP: convex cost function.

$\|L\|$	average	maximum	variance
45	0.002	0.003	0.00000004
190	0.016	0.020	0.00000185
435	0.054	0.066	0.00002203
780	0.118	0.159	0.00008462
1225	0.221	0.273	0.00015363
1770	0.364	0.417	0.00011367
2415	0.568	0.616	0.00015055
3160	0.833	0.894	0.00013897
4005	1.177	1.238	0.00011064
4950	1.608	1.688	0.00031559

Results in Table 4.5 and 4.6 are also depicted in Figures 4.8–4.11.

Figure 4.8 and 4.10 show that the average and maximum computation time of the proposed algorithm, ALGORITHM VPR_SP, are approximately linear with respect to the number of VPs in the network. By linear regression, the values of linear correlation coefficients are

Table 4.6: Computation time of ALGORITHM VPR_SP: concave cost function.

$\|L\|$	average	maximum	variance
45	0.002	0.003	0.00000004
190	0.016	0.020	0.00000186
435	0.054	0.067	0.00002048
780	0.118	0.159	0.00008415
1225	0.221	0.274	0.00016059
1770	0.364	0.415	0.00011095
2415	0.568	0.615	0.00014500
3160	0.833	0.894	0.00012900
4005	1.178	1.239	0.00011115
4950	1.608	1.689	0.00031068

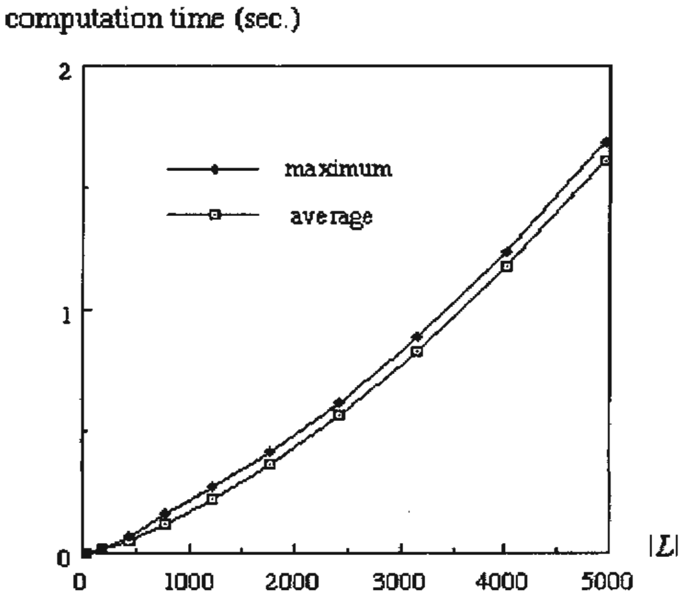


Figure 4.8: Average and maximum computation time of ALGORITHM VPR_SP: convex cost function.

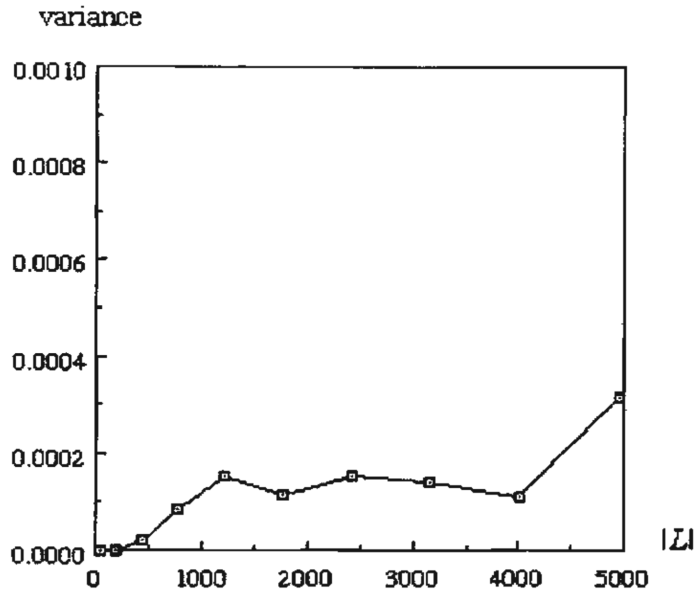


Figure 4.9: Variance of computation time of ALGORITHM VPR_SP: convex cost function.

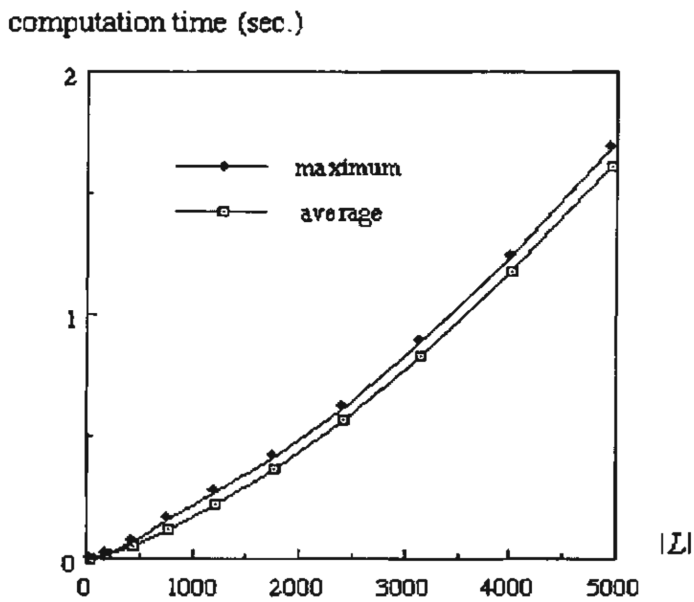


Figure 4.10: Average and maximum computation time of ALGORITHM VPR_SP: concave cost function.

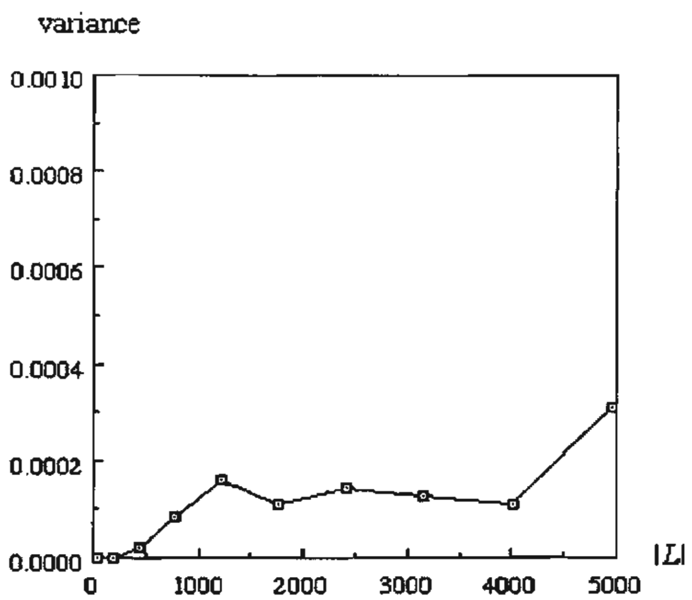


Figure 4.11: Variance of computation time of ALGORITHM VPR_SP: concave cost function.

- the case of convex cost function: 0.9911706 and 0.9884848, respectively,
- the case of concave cost function: 0.9911623 and 0.9886147, respectively.

Figure 4.9 and 4.11 show that the variance of computation time of the proposed algorithm is very small. Hence, we can conclude that ALGORITHM VPR_SP is very practical for applying to the VPR problem of a large-scale ATM network.

4.6.2 Effect of Existing Network to Network Design

Some numerical results will be given in this subsection to show that how the existing network facilities affect the solution of the VPR problem in the case of convex and concave link cost function. We consider a simple 4-node 4-link ATM network as shown in Figure 4.12.

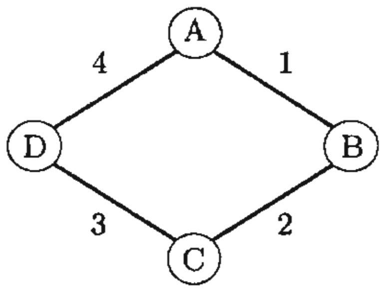


Figure 4.12: A 4-node 4-link ATM network.

We use the following network parameters.

- $Y_{0j} = 4$ Mbps for $j = 1, 2, 3$, and 4.
- There is one VP dedicated for every node-pair. This makes the number of VPs be 6. All VP bandwidths are 2 Mbps, except the only one between node-pair A-B, which is varied and denoted by V_x.

For the network in Figure 4.12, we can show that VP assigned for a node-pair will be routed via a direct physical link if there is a direct physical link connecting the node-pair. Clearly, there are four node-pairs with direct physical link in this network. Accordingly, we have the optimal route of the four VPs as follows.

- node-pair A-B: {1}
- node-pair B-C: {2}
- node-pair C-D: {3}
- node-pair A-D: {4}

For the other two VPs of node-pair A-C and B-D, each of them has 2 possible routes as follows.

- node-pair A-C: {1, 2} or {4, 3}
- node-pair B-D: {1, 4} or {2, 3}

Consequently, the optimal solutions of the VPR problem for this network must belong to the four routing patterns, W, X, Y, and Z in Table 4.7.

Table 4.7: VP routing patterns.

node-pair	W	X	Y	Z
A-B	{1}	{1}	{1}	{1}
A-C	{1, 2}	{1, 2}	{4, 3}	{4, 3}
A-D	{4}	{4}	{4}	{4}
B-C	{2}	{2}	{2}	{2}
B-D	{1, 4}	{2, 3}	{1, 4}	{2, 3}
C-D	{3}	{3}	{3}	{3}

By applying each of the above VP routing patterns to the network in Figure 4.12, we can calculate the capacity of each physical link in the network. The results are summarized in Table 4.8. It can be seen that, although each routing pattern produces different link capacity in each physical link, the total physical link capacity in the network, i.e., $\sum_P Y_j$, is surprisingly the same. That is,

$$\sum_P Y_j = V_x + 14.$$

In the next step, we examine the VPR problem for both the cases of convex and concave link cost function.

Table 4.8: Physical link capacity by each VP routing pattern.

physical link	W	X	Y	Z
1	$V_x + 4$	$V_x + 2$	$V_x + 2$	V_x
2	4	6	2	4
3	2	4	4	6
4	4	2	6	4

The Case of Convex Cost Function

In this case, we set $d_{0j} = 0.5$ and $d_{1j} = 1.0$ for all physical links to satisfy the condition of convex cost function,

$$d_{0j} < d_{1j}.$$

After applying ALGORITHM VPR_SP, we obtain the following results.

- When $V_x < 2$ Mbps, the optimal routing pattern is W. From this result, we have that the capacity of physical links are { V_x + 4, 4, 2, 4 } (in the sequence of physical link 1, 2, 3, 4).
- When $V_x = 2$ Mbps, all possible routing patterns, W, X, Y, and Z, can be an optimal pattern since all of them give the same network cost.
- When $V_x > 2$ Mbps, the optimal routing pattern is Z, where the capacity of physical links are { V_x, 4, 6, 4 }.

From the above results, it is clear that the convex cost function affects the selection of optimal VP routing in the way that the network tries to utilize existing physical link capacity in a full range before implementing new parts of capacity. In other words, we always have at least two physical links with final capacity equal to their existing capacity, i.e., 4 Mbps. Consequently, the optimum must be either the routing pattern W or Z. On the other hand, the routing pattern X and Y cannot be an optimal result, except in the case $V_x = 2$ Mbps where all four patterns give the same result. The reason is straightforward that the two patterns X and Y do not use existing capacity effectively (there is at least one link that the capacity is 2 Mbps) while new portion of capacity is added to one link in the network (there is a link with capacity of 6 Mbps).

The above results strongly support the characteristics of existing network that the network facilities already installed in the network are promptly to be used and cannot be changed easily in a short period of time. Accordingly, utilizing existing network in a full range before augmenting new network facilities is a very important concept in the case of short-term network design.

The Case of Concave Cost Function

In this case, we set $d_{0j} = 1.0$ and $d_{1j} = 0.5$ for all physical links to satisfy the following condition for concave cost function.

$$d_{0j} > d_{1j}.$$

By ALGORITHM VPR_SP, we obtain the results of VP routing in this case as follows.

- When $V_x < 2$ Mbps, the optimal routing pattern can be X, Y, or Z.
- When $V_x = 2$ Mbps, all possible routing patterns, W, X, Y, and Z, can be an optimal pattern.
- When $V_x > 2$ Mbps, the optimal routing pattern can be W, X, or Y.

It can be seen from the results in this case that optimal solution needs not be fixed to any routing pattern, but there can be multiple alternatives each of which gives the same result, i.e., the same total network cost. Furthermore, unlike the case of convex cost function, the restriction of utilizing existing network effectively before installing new facilities does not affect the VP routing in this case. It can be considered that the main reason comes from the fact that economy of scale is present in the link capacity in this case of long-term network design. Consequently, new network facilities can be added to the network in the places that there are needs, while some parts of the existing facilities are left unused. This means that the VP routing in the case of long-term design can be conducted more freely than that in the case of short-term design.

4.7 Summary

We have studied in this chapter the virtual path routing problem (VPR problem) in ATM networks taking into consideration an existing network, where we examine the cases of both short-term and long-term design. Piecewise linear cost functions are used to represent the cost difference between existing and new network facilities. We use a piecewise linear convex cost function in the case of short-term design, while a piecewise linear concave function is used in the case of long-term design.

After the VPR problem is formulated as a non-linear programming problem, it is analyzed to obtain some important characteristics. By focusing on the special case of linear link cost function, we can show that the VPR problem is a member in the class of shortest path problems. As a result, we, therefore, propose a heuristic design algorithm, ALGORITHM VPR_SP, based on shortest path algorithm.

In the first part of the numerical examples, it is shown that the proposed algorithm has good performance in solving the VPR problem, both from its optimality and computation amount. The impact of existing network to the VPR problem is investigated in the second part of the numerical results. It can be shown that, in the case of short-term design, the concept of using existing network facilities effectively before installing new facilities is very important. On the other hand, the above concept does not affect the results of the VPR problem in the case of long-term design according to the economy of scale in the network facilities, and new facilities can be installed if needed while some existing facilities can be left unused.

Chapter 5

Conclusions

This is the last chapter of this research report. It gives firstly the summary of results obtained from the study in this project. After that, some important topics for further study are given.

5.1 Summary of Results

The study in this research project is concerned with communication network design with the consideration of existing network facilities. We study two major network design problems in this project. They are

- the link capacity assignment problem (CA problem) in packet-switched networks,
- the virtual path routing problem (VPR problem) in ATM networks.

We focus on the case of long-term design for the CA problem, and the cases of both short-term and long-term design for the VPR problem. In our study, we use piecewise linear cost functions to represent the cost difference between existing network facilities and newly installed facilities. In the case of short-term network design, the cost function is piecewise linear convex where the per-unit cost of existing network facilities is smaller than that of new facilities. On the other hand, a piecewise linear concave cost function is used in the case of long-term design for dealing with the economy of scale present in communication network facilities.

After the CA problem and the VPR problem are formulated as mathematical programming problems, they are analyzed to find some important characteristics and properties of the problems.

From the study in this research project, we obtain the following results.

1. The CA problem: It has been proven that there is no link in the network whose capacity is equal to the existing capacity. Based on this characteristic, the non-differentiable link cost function can be treated as a differentiable function, and

conventional method such as the well-known Lagrange multiplier method can be applied to solve the CA problem. This results in a heuristic algorithm referred to as ALGORITHM CA_LM in Chapter 3 of this report.

2. The VPR problem: It has been shown that the problem is a member in the class of shortest path problems. With this property, we can propose a heuristic design algorithm, ALGORITHM VPR_SP, based on conventional shortest path algorithms in Chapter 4 of this report.

3. Numerical results show that the above two proposed heuristic algorithms have very good performance in solving the network design problems. Moreover, their computation amount is very small, which means that they are applicable to solve practical design problems of large-scale network.

4. The effect of existing network to the design of a new network has been investigated by numerical examples. It can be shown that the concept of using existing network facilities before augmenting new facilities is very important in the case of short-term network design. This reflects the characteristics of existing network facilities in short-term design, that is that the existing facilities are ready to be used in the construction of new network, and they cannot be changed easily in a short period of time.

   In the case of long-term design, it has been shown that the above concept of using existing network facilities can be neglected. In this case, new part of network facilities can be introduced and installed to the existing network if they are required, while some existing facilities can be left unused. This result reflects the property of economy of scale in communication network facilities for the case of long-term design.

From the results listed above, we can conclude that the objectives of this research project to study the effect of existing network facilities in communication network design has been fulfilled.

5.2 Topics for Further Work

The following research topics can be considered as the study in further work.

1. It is interesting to investigate whether the design concepts and the proposed algorithms in this project are applicable or not to the control and operation of practical communication networks, especially in the case of VP routing in ATM networks. Although there are many research works in the literature on the study of dynamic VP routing and assignment in ATM networks [5][24][23][105], almost of these works do not take into account the effect of existing network to the setting of new network, namely the relationship between existing and new VP routing and assignment. Thus, including the consideration of existing network parameter setting in the phase of network control can lead to the increasing of the network performance.

2. It is also interesting to examine the capability of the concepts used in this study to the design of other kinds of communication networks in the real world, e.g., local area networks, optical networks, wireless communication networks, etc. If the concepts can be applied to the design of these networks, it is very important to verify whether similar results are obtained or not. Since there is nothing guaranteeing that the same kind of results will be obtained when networks have different characteristics. This, therefore, becomes a very challenging problem.

3. It is worth trying to improve the quality of solutions for the CA and VPR problems. According to the heuristic nature of the proposed algorithms, they have high chance to get stuck to local optimum of the design problems [79]. For the methods to escape from local optimum, there is a class of methods in the literature known as *metaheuristics* proposed to solve a broad range of optimization and design problems. Some of the well-known and frequently used techniques are ant colony system [18][19][20], genetic algorithm [28][35], memetic algorithm [43][67], simulated annealing [1][51], and tabu search [32][33][34]. These methods are also applied to the solving of communication network design problems [11][16][22][97].

   As related works to the study in this project, we also apply genetic algorithm and memetic algorithm to solve the CA problem [83][84]. It is shown in [83] that ALGORITHM CA_LM proposed in this research project has high chance to yield local optimum, especially in the case of large networks. However, this does not mean that ALGORITHM CA_LM has no use. Actually, to obtain good quality of solution for the CA problem, ALGORITHM CA_LM can be used as an operator to generate good initial population for genetic algorithm [83], as well as be used as a local search operator in memetic algorithm [84].

   Thus, the application of metaheuristics to network design problems and the hybrid version of the proposed algorithm in this project with other metaheristics are interesting topics for further study.

References

[1] Aarts, E. and Korst, J. (1989) *Simulated Annealing and Boltzmann Machines*, John Wiley & Sons.

[2] Ahuja, V. (1985). *Design and Analysis of Computer Communication Networks*, Mc Graw-Hill, Inc., Singapore.

[3] Amstutz, S.R. (1989). "Burst Switching—An Update," *IEEE Communications Magazine*, vol. 27, no. 9, Sept. 1989, pp. 50–57.

[4] Arai, H., Kawarasaki, M. and Nogami, S. (1989). "An Analysis of Call Admission Control in the ATM Network," *IECEJ*, vol. J72-B-I, no. 11, Sept. 1989, pp. 1000–1007. (in Japanese)

[5] Aydemir, M. and Viniotis, Y. (1996). "Virtual Path Assignment in ATM Networks," In *Data Communication and their Performance*, Edited by Fdida, S. and Onvural, R.O., pp. 382–400.

[6] Baiocchi, A., Melazzi, N.B., Listanti, M., Roveri, A. and Winkler, R. (1991). "Loss Performance Analysis of an ATM Multiplexer Loaded with High-Speed ON-OFF Sources," *IEEE J. Select. Areas Commun.*, vol. 9, no. 3, Apr. 1991, pp. 388–393.

[7] Bertsekas, D. and Gallager, R. (1992). *Data Networks*, 2nd Edition, Prentice Hall.

[8] Bhatnagar, P.K. (1997). *Engineering Networks for Synchronization, CCS 7, and ISDN*, IEEE Press.

[9] Busis, N.A. (1999). "Neurology in the Electronic Information Age," *European Journal of Neurology*, vol. 6, pp. 385–414. The document is available on-line at **http://www.neuroguide.com/ejnreview.html**.

[10] Cahn, R.S. (1998). *Wide Area Network Design*, Morgan Kaufmann Publishers, Inc., San Francisco, California.

[11] Chakraborty, M. and Chakraborty, U.K. (1997). "Applying Genetic Algorithm and Simulated Annealing to a Combinatorial Optimization Problem," In *Proceedings of International Conference on Information, Communication and Signal Processing (ICICS'97)*, 2C2.6, pp. 929–933.

[12] Chen, T.M. and Liu, S.S. (1995). *ATM Switching Systems*, Artech House, Inc.

[13] Coan, B.A., Vecchi, M.P. and Wu, L.T. (1990). "A Distributed Protocol to Improve the Survivability of Trunk Networks," *Proc. XIII ISS*, Stockholm, vol. 4, pp. 173–179.

[14] Comer, D. (1995). *Internetworking with TCP/IP Volume I: Principles, Protocols, and Architecture*, 3rd Edition, Prentice-Hall Inc., Englewood Cliffs.

[15] Crutcher, L. (1991). "Transport protocols for ATM," IEE Colloquium on Fast Packet Switching, pp. 7/1–7/4.

[16] Dengiz, B., Altiparmak, F. and Smith, A.E. (1997). "Local Search Genetic Algorithm for Optimal Design of Reliable Networks," *IEEE Trans Evol Comp*, vol. 1, pp. 179–188.

[17] Dodd, A.Z. (1999). *The Essential Guide to Telecommunications*, Prentice-Hall.

[18] Dorigo, M. and Di Caro, G. (1999). "The Ant Colony Optimization Meta-Heuristic," In *New Ideas in Optimization*, Edited by Corne, D., Dorigo, M. and Glover, F., pp. 11–32, McGraw-Hill, London.

[19] Dorigo, M. and Gambardella, L.M. (1997). "Ant Colonies for the Traveling Salesman Problem," *BioSystems*, vol. 43, pp. 73–81.

[20] Dorigo, M. and Gambardella, L.M. (1997). "Ant Colony System: A Cooperative Learning Approach to the Traveling Salesman Problem," *IEEE Trans Evol Comp*, vol. 1, pp. 53–66.

[21] Dutton, H.J.R. and Lenhard, P. (1995). *Asynchronous Transfer Mode (ATM): Technical Overview*, 2nd Edition, Prentice Hall.

[22] Elbaum, R. and Sidi, M. (1996). "Topological Design of Local-Area Networks using Genetic Algorithms," *IEEE/ACM Trans Netw*, vol. 4, pp. 766–778.

[23] Feng, G. and Liu, Z. (1998). "Virtual Path (VP) Topology Optimization Using a Neural Network Approach in Multistage VP Control," *IEICE Trans. Commun.*, vol. E81-B, no. 6, June 1998, pp. 1139–1151.

[24] Feng, G. and Kheong, D.S.C. (2000). "Efficient Support for Multicast Applications over VP-Based ATM Networks," *IEICE Trans. Commun.*, vol. E83-B, no. 12, Dec. 2000, pp. 2661–2674.

[25] Fischer, M.J. and Harris, T.C. (1976). "A Model for Evaluating the Performance of an Integrated Circuit and Packet-Switched Multiplex Structure," *IEEE Trans. Commun.*, vol. COM-24, no. 2, Feb. 1976, pp. 195–202.

[26] Gallassi, G., Rigolio, G. and Fratta, L. (1989). "ATM: Bandwidth Assignment and Bandwidth Enforcement Policies," *Proc. GLOBECOM'89*, pp. 49.6.1–6.

[27] Gavish, B. and Neuman, I. (1989). "A System for Routing and Capacity Assignment in Computer Communication Network," *IEEE Trans. Commun.*, vol. 37, no. 4, Apr. 1989, pp. 360–366.

[28] Gen, M. and Cheng, R. (1997). *Genetic Algorithms and Engineering Design*, John Wiley & Sons, New York.

[29] Gerla, M. and Kleinrock, L. (1977). "On the Topological Design of Distributed Computer Networks," *IEEE Trans. Commun.*, vol. COM-25, no. 1, Jan. 1977, pp. 48–60.

[30] Gersht, A. and Weihmayer, R. (1990) "Joint Optimization of Data Network Design and Facility Selection," *IEEE J. Select. Area Commun.*, vol. 8, no. 9, Dec. 1990, pp. 1667–1681.

[31] Girard, A. (1990). *Routing and Dimensioning in Circuit-Switched Networks*, Addison-Wesley Publishing Company.

[32] Glover, F. (1989). "Tabu Search – Part I," ORSA Journal on Computing, vol. 1, pp. 190–206.

[33] Glover, F. (1990). "Tabu Search – Part II," ORSA Journal on Computing, vol. 2, pp. 4–32.

[34] Glover, F. and Laguna, M. (1997). *Tabu Search*, Kluwer Academic Publishers.

[35] Goldberg, D.E. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison Wesley Longman, Inc.

[36] Gopal, G., Kim, C.K. and Weinrib, A. (1991). "Algorithms for Reconfigurable Networks," *Proc. of ITC-13*, pp. 341–347.

[37] Hadama, H. and Ohta, S. (1989). "Routing Control of Virtual Paths in Large-Scale ATM-Based Transport Network," *IECEJ*, vol. J72-B-I, no. 11, Nov. 1989, pp. 970–978. (in Japanese)

[38] Heffes, H. and Lucantoni, D.M. (1986). "A Markov Modulated Characterization of Packetized Voice and Data Traffic and Related Statistical Multiplexer Performance," *IEEE J. Select. Areas Commun.*, vol. SAC-4, no. 6, Sept. 1986, pp. 856–868.

[39] Hillier, F.S. and Lieberman, G.J. (1995). *Introduction to Operations Research*, 6th Edition, McGraw-Hill, Inc.

[40] Hopkins, G.L. (1995). *The ISDN Literacy Book*, Addison-Wesley Publishing.

[41] Horigome, H. and Uose, H. (1992). "Considerations on Cost-Efficiency of ATM Network," *Proc. 5th International Network Planning Symposium*, Kobe.

[42] Hseih, J. and Ucci, D.R. (1989). "Model and Performance Study of Burst Switching for Integrated Voice and Data Services," *Electronics Letters*, vol. 25 25, 7 Dec. 1989, pp. 1721–1722.

[43] Ishibuchi, H., Murata, T. and Tomioka, S. (1997). "Effectiveness of Genetic Local Search Algorithms," In *Proc. 7-th Int. Conf. Genetic Algorithms*, pp. 505–512.

[44] Jacobsen, S.B., Moth, K. and Dittmann, L. (1990). "Load Control in ATM Networks," *Proc. XIII ISS*, Stockholm, vol. 5, pp. 131–138.

[45] Kamimura, K. (1987). "Packet Switched Network Design with Constraints on the Circuit Discreteness," *IECEJ*, vol. J70-B, no. 9, Sep. 1987, pp. 999−1008. (in Japanese)

[46] Kamimura, K., Takai, A. and Kawakita, Y. (1991). "CONPASS, Expert System Supporting Private Network Design," *NTT R&D*, vol. 40, no. 1, pp. 99−108.

[47] Kaufman, J.S. (1981). "Blocking in a Shared Resource Environment," *IEEE Trans. Commun.*, vol. COM-29, no. 10, pp. 1474−1481.

[48] Kawashima, K. and Saito, H. (1990). "Teletraffic Issues in ATM Networks," *Computer Networks and ISDN Systems*, 20, pp. 369−375.

[49] Kawashima, K. and Kobayashi, H. (1993). "Network Planning Tools," *Jour. of IEICE*, vol. 76, no. 2, Feb. 1993, pp. 141−148. (in Japanese)

[50] Kershenbaum, A. (1993). *Telecommunications Network Design Algorithms*, McGraw-Hill, Inc., Singapore.

[51] Kirkpatrick, S., Gelatt, C.D. and Vecchi, M.P. (1983). "Optimization by Simulated Annealing," *Science*, vol. 220, no. 4598, pp. 671−680.

[52] Kleinrock, L. (1976). *Queueing Systems Volume II, Computer Applications*, John Wiley & Sons, New York, pp. 314−360.

[53] Knuth, D.E. (1997). *The Art of Computer Programming, Volume 1: Fundamental Algorithms*, 3rd Edition, Addison-Wesley Longman.

[54] Komine, T. and Yamada, H. (1992). "An Analysis on Statistical Multiplexing Characteristics of ATM Cells with Voice and Video Inputs," *IECEJ*, vol. J75-B-I, no. 8, Aug. 1992, pp. 509−516. (in Japanese)

[55] Kumar, B. (1996). *Broadband Communications*, McGraw-Hill, Inc., Singapore.

[56] Kuczura, A. (1973). "The Interrupted Poisson Process as an Overflow Process," *Bell Syst. Tech. J.*, vol. 52, no. 3, Mar. 1973, pp. 437−448.

[57] Liu, X., Igarashi, J. and Miyamoto, H. (1990). "A Packet-Switch Networks Optimization Method with a Different Kind of Channel Rates for Each Link and the Fairness on Grades of Service," *IECEJ*, vol. J73-B-I, no. 4, Apr. 1990, pp. 267−275. (in Japanese)

[58] Library and Information Commission. (1999). "Keystone for the Information Age: a National Information Policy for the UK." The document is available on-line at **http://www.lic.gov.uk/publications/policyreports/keystone.html**.

[59] Lloyd-Evans, R. (1996). *Wide Area Network Performance and Optimization*, Addison-Wesley Longman, England.

[60] Lynch, D.C. and Rose, M.T. (1993). *Internet System Handbook*, Addison-Wesley Publishing Company, Inc.

[61] Ma, B.N.W. and Mark, J.W. (1988). "Performance Analysis of Burst Switching for Integrated Voice/Data Services," *IEEE Trans. Commun.*, vol. 36, no. 3, March 1988, pp. 282–297.

[62] Maglaris, B., Anastassiou, D., Sen, P., Karlsson, G. and Robbins, J.D. (1988). "Performance Models of Statistical Multiplexing in Packet Video Communications," *IEEE Trans. Commun.*, vol. 36, no. 7, July 1988, pp. 834–844.

[63] Maglaris, B., Boorstyn, R.R., Panwar, S., Spirtos, T., O'Reilly, P., and Jack, C. (1990). "Routing of Voice and Data in Burst-Switched Networks," *IEEE Trans. Commun.*, vol. 38, no. 6, June 1990, pp. 889–897.

[64] Maruyama, K. and Tang, D.T. (1976). "Discrete Link Capacity Assignment in Communication Network," *Proc. 3rd ICCC*, Toronto, pp. 92–97.

[65] Mase, K. and Runggeratigul, S. (1997). "Planning and Design Issues for Information Delivery Using Idle Periods in Telecommunication Networks," *IEICE Trans. Commun.*, vol. E80-B, no. 2, Feb. 1997, pp. 230–239.

[66] Minoli, D. and Golway, T. (1997). *Planning and Managing ATM Networks*, Manning Publications Company.

[67] Moscato, P. (1999). "Memetic Algorithms: A Short Introduction," In *New Ideas in Optimization*, Edited by Corne, D., Dorigo, M. and Glover, F., pp. 219–234, McGraw-Hill, London.

[68] Nagarajan, R., Kurose, J.F. and Towsley, D. (1991). "Approximation Techniques for Computing Packet Loss in Finite-Buffered Voice Multiplexers," *IEEE J. Select. Areas Commun.*, vol. 9, no. 3, Apr. 1991, pp. 368–377.

[69] Nash, S.G. and Sofer, A. (1996). *Linear and Nonlinear Programming*, McGraw-Hill, Inc.

[70] Newman, P. (1989). "Fast Packet Switching for Broadband ISDN," Second IEE National Conference on Telecommunications, pp. 391–396.

[71] Nogami, S. (1990). "Quality Control at the Cell Level and Its Characteristics," *IECEJ*, vol. J73-B-I, no. 9, Sept. 1990, pp. 671–680. (in Japanese)

[72] Nomura, M., Fujii, T. and Ohta, N. (1989). "Basic Characteristics of Variable Rate Video Coding in ATM Environment," *IEEE J. Select. Areas Commun.*, vol. 7, no. 5, June 1989, pp. 752–760.

[73] Okazaki, H. (1987). "A Study on Planning Packet Switched Networks," *IEICE Tech. Rep.*, SE-87-38.

[74] Partridge, C. (1994). *Gigabit Networking*, Addison-Wesley Publishing Company.

[75] Pióro, M., Lubacz, J. and Körner, U. (1990). "Traffic Engineering Problems in Multiservice Circuit Switched Networks," *Computer Networks and ISDN Systems*, 20, pp. 127–136.

[76] Pitts, J.M. and Schormans, J.A. (1996). *Introduction to ATM Design and Performance*, John Wiley & Sons.

[77] Potts, C.H. (1999). "The Digital Divide: Social Justice in the Information Age," *Issues in Education & Technology*, no. 1. The document is available on-line at **http://www.centerx.gseis.ucla.edu/x/projects/xtech/dd.htm**.

[78] Prycker, M. (1995). *Asynchronous Transfer Mode: Solution for Broadband ISDN*, 3rd Edition, Prentice Hall.

[79] Reeves, C.R. (1996). "Modern Heuristic Techniques," In *Modern Heuristic Search Methods*, Edited by Rayward-Smith, V.J., Osman, I.H., Reeves, C.R. and Smith, G.D., pp. 17–23, John Wiley & Sons, Chichester.

[80] Runggeratigul, S., Zhao, W. and Asano, S. (1994). "Packet-Switched Network Design Method with Consideration of Existing Network," 1994 IEICE Fall Conference, Japan, B-659, Sep. 1994.

[81] Runggeratigul, S., Zhao, W., Ji, Y., Aizawa, A. and Asano, S. (1995). "Link Capacity Assignment in Packet-Switched Network with Existing Network Consideration," *IEICE Trans. Commun.*, vol. E78-B, no. 5, May 1995, pp. 709–719.

[82] Runggeratigul, S. and Tantaratana, S. (1999). "Link Capacity Assignment in Packet-Switched Networks: The Case of Piecewise Linear Concave Cost Function," *IEICE Trans. Commun.*, vol. E82-B, no. 10, Oct. 1999, pp. 1566–1576.

[83] Runggeratigul, S. (2000). "A Genetic Algorithms Approach to Communication Network Design taking into Consideration an Existing Network", *ScienceAsia*: Journal of the Science Society of Thailand, vol. 26, no. 3, pp. 181–186.

[84] Runggeratigul, S. (2001). "A Memetic Algorithm to Communication Network Design taking into Consideration an Existing Network", accepted by the 4th Metaheuristics International Conference.

[85] Saito, H., Kawarasaki, M. and Yamada, H. (1991). "An Analysis of Statistical Multiplexing in an ATM Transport Network," *IEEE J. Select. Areas Commun.*, vol. 9, no. 3, Apr. 1991, pp. 359–367.

[86] Saksena, V.R. (1989). "Topological Analysis of Packet Networks," *IEEE J. Select. Areas Commun.*, vol. 7, no. 8, Oct. 1989, pp. 1243–1252.

[87] Sato, K., Ohta, S. and Tokizawa, I. (1989). "Broadband Transport Network Architecture Based on Virtual Path Concept," *IECEJ*, vol. J72-B-I, no. 11, Nov. 1989, pp. 904–916. (in Japanese)

[88] Schwartz, M. (1996). *Broadband Integrated Networks*, Prentice Hall.

[89] Sedgewick, R. (1988). *Algorithms*, 2nd Edition, Addison-Wesley Publishing Company.

[90] Shioda, S. and Uose, H. (1991). "Virtual Path Bandwidth Control Method for ATM Networks: Successive Modification Method," *IEICE Trans.*, vol. E74, no. 12, Dec. 1991, pp. 4061–4068.

[91] Shioda, S. and Uose, H. (1992). "Virtual Path Bandwidth Control Method for ATM Networks: Batch Modification Method," *IECEJ*, vol. J75-B-I, no. 5, May 1992, pp. 333–342. (in Japanese)

[92] Shinohara, M. (1985). "Optimum Dimensioning of Switching Network by Newton Method," 1985 IEICE National Conference, Japan, 436. (in Japanese)

[93] Shinohara, M. (1990). "Generalization of Pratt's Formula for Dimensioning Circuit-Switched Network," *Computer Networks and ISDN Systems*, 20, pp. 115–126.

[94] Sterbenz, J.P.G. and Touch, J.D. (2001). *High-Speed Networking: A Systematic Approach to High-Bandwidth Low-Latency Communication*, John Wiley & Sons.

[95] Sugano, M., Murata, M. and Miyahara, H. (1993). "Capacity Assignment Algorithm for Packet Switching Network with Different Grades of Service", *IECEJ*, vol. J76-B-I, no. 8, Aug. 1993, pp. 581–589. (in Japanese)

[96] Tanenbaum, A.S. (1989). *Computer Networks*, 2nd Edition, Prentice-Hall Inc., Englewood Cliffs.

[97] Tang, K., Ko, K., Man, K.F. and Kwong, S. (1998). "Topology Design and Bandwidth Allocation of Embedded ATM Networks using Genetic Algorithm," *IEEE Comm Letters*, vol. 2, pp. 171–173.

[98] Uose, H., Shioda, S., Horigome, H. and Yamamoto, H. (1990). "Design and Control Aspects of ATM Transit Networks," *IEICE Tech. Rep.*, IN-90-96. (in Japanese)

[99] Verma, P.K. (1989). *Performance Estimation of Computer Communication Networks*, Computer Science Press.

[100] Vickers, B.J., Kim, J.B., Suda, T. and Hong, D.P. (1993). "Congestion Control and Resource Management in Diverse ATM Environments," *IECEJ*, vol. J76-B-I, no. 11, Nov. 1993, pp. 759–774.

[101] Watanabe, A. and Ohmae, Y. (1991). "Topological Design of Corporate Communication Networks," *IECEJ*, vol. J74-B-I, no. 2, Feb. 1991, pp. 107–115.

[102] Watanabe, Y. and Kashper, A. (1991) "Communications Network Design and Planning," Proceedings of Global Telecommunications Conference 1991 (GLOBECOM '91), pp. 1336–1340, vol. 2.

[103] Wittmann, A. (1995). "Designing-And Redesigning-Today's Local Area Network," Network Computing. The document is available on-line at **http://www.networkcomputing.com/netdesign/lanindex.html.**

[104] Wobus, J. (1997). "Upgrading Your Network Backbone," Network Computing. The document is available on-line at **http://www.networkcomputing.com/netdesign/bone1.html.**

[105] Wong, E.W.M., Chan, A.K.M., Chan, S. and Ko, K.T. (2000). "Bandwidth Allocation for Virtual Paths in ATM Networks with Dynamic Routing," *IEICE Trans. Commun.*, vol. E83-B, no. 3, Mar. 2000, pp. 626–637.

[106] Wu, J.Y. and Roux, R. (1990). "Some Policies for Circuit Allocation and their Packet Capacity in Hybrid Switching Systems like FDDI-II," 1990 Global Telecommunications Conference and Exhibition. GLOBECOM '90., vol. 3, pp. 1763–1767.

[107] Yaged, B. (1971) "Minimum Cost Routing for Static Network Models," *Networks*, vol. 1, pp. 139–172.

[108] Yoshida, M. and Okazaki, H. (1993). "A Study on ATM Network Planning Based on Evaluation of Design Items," *IEICE Trans. Commun.*, vol. E76-B, no. 11, Nov. 1993.

[109] Yuan, C. and Silvester, J.A. (1989). "Queueing Analysis of Delay Constrained Voice Traffic in a Packet Switching System," *IEEE J. Select. Areas Commun.*, vol. 7, no. 5, June 1989, pp. 729–738.

[110] Zukerman, M. (1988). "Circuit Allocation and Overload Control in a Hybrid Switching System," *Computer Networks and ISDN Systems*, 16, pp. 281–298.

[111] Zukerman, M. (1989). "Bandwidth Allocation for Bursty Isochronous Traffic in a Hybrid Switching System *IEEE Trans. Commun.*, vol. 37, no. 12, Dec. 1989, pp. 1367–1371.

Outputs

The result of this research project has been published in the following international journal paper.

- Runggeratigul, S. and Tantaratana, S. (1999). "Link Capacity Assignment in Packet-Switched Networks: The Case of Piecewise Linear Concave Cost Function," *IEICE Trans. Commun.*, vol. E82-B, no. 10, Oct. 1999, pp. 1566–1576. **http://search.ieice.or.jp/1999/files/e000b10.htm#e82-b,10,1566**

In addition, the results of related work to this project are published in international journal and international conference papers as follows.

- Runggeratigul, S. (2000). "A Genetic Algorithms Approach to Communication Network Design taking into Consideration an Existing Network", *ScienceAsia*: Journal of the Science Society of Thailand, vol. 26, no. 3, pp. 181–186.
- Runggeratigul, S. (2001). "A Memetic Algorithm to Communication Network Design taking into Consideration an Existing Network", accepted by the 4th Metaheuristics International Conference.

The above papers are attached in the appendix of this report.

Appendix A

Reprint of
Runggeratigul, S. and Tantaratana, S. (1999). "Link Capacity Assignment in Packet-Switched Networks: The Case of Piecewise Linear Concave Cost Function," *IEICE Trans. Commun.*, vol. E82-B, no. 10, Oct. 1999, pp. 1566–1576.

IEICE TRANS. COMMUN., VOL E82 B, NO.10 OCTOBER 1999

PAPER

Link Capacity Assignment in Packet-Switched Networks: The Case of Piecewise Linear Concave Cost Function

Suwan RUNGGERATIGUL†, *Member and* Sawasd TANTARATANA†, *Nonmember*

SUMMARY In this paper, we study the link capacity assignment problem in packet-switched networks (CA problem) focusing on the case where link cost function is a piecewise linear concave function. This type of cost function arises in many communication network design problems such as those arising from developments in communication transmission technologies. It is already known that the method of link set assignment is applicable for solving the CA problem with piecewise linear convex cost function. That is, each link in the network is assigned to one of a group of specific sets, and checked for link set contradiction. By extending the method of link set assignment to the case of piecewise linear concave cost function, an important characteristic of the optimal solution of the CA problem is derived. Based on this characteristic, the non-differentiable link cost function can be treated as a differentiable function, and a heuristic algorithm derived from the Lagrange multiplier method is then proposed. Although it is difficult to determine the global optimum of the CA problem due to its non-convexity, it is shown by numerical results that the solution obtained from the proposed algorithm is very close to the global optimum. Moreover, the computation time is linearly dependent on the number of links in the problem. These performances show that the proposed algorithm is very efficient in solving the CA problem, even in the case of large-scale networks.

key words: communication networks and services, packet-switched networks, network design, link capacity assignment, non-linear programming, algorithm

1. Introduction

Link capacity assignment problem in packet-switched networks (CA problem) has been studied widely since the early days of ARPANET's appearance [1], [2]. This problem is concerned with means to determine capacity of links that minimize network cost, subject to some constraints, such as the upper limit of average packet delay. The CA problem can be solved when network information, including network topology, routing pattern, etc., is given. The problem has been studied for many kinds of network model, e.g., network with single class or multi-class packets, network with several kinds of design parameter: packet delay or packet loss rate or both of them, network with continuous or discrete link capacity model, etc.

Many types of link cost function have been considered in the CA problem. For example, some fundamental mathematical functions including linear, exponential, logarithmic, are examined to see the behavior of link capacity assignment due to the effect of these cost functions [2]. A piecewise linear convex cost function has also been considered as a link cost model in the case that the CA problem is solved with the consideration of existing network, where the per-unit cost of using existing capacity is less than that of installing new capacity [3].

In general cases of communication network design, it is proper to consider link cost function as a concave function [2], [4]. This is because the economy of scale is often present in communication resources. For example, we can obtain a cost function from the envelope of several cost functions due to the development in communication transmission technologies [5], [6], the development in switching technologies [7], etc.

In this paper, we study the CA problem in packet-switched networks where link cost function is piecewise linear concave. This is the case when each link in the network is implemented by selecting a link type from several alternatives whose cost function is linear. Clearly, this type of link cost function is non-differentiable. To solve problems with this type of cost function, the method of approximating the non-differentiable cost function by a differentiable function has been proposed [8]. Although this method yields a solution within approximation error, there is no guarantee that the solution obtained from this method is optimum when link cost function is not convex. Moreover, lengthy time is needed to determine the approximation function.

The main objective of this paper is to clarify the characteristic of optimal solution of the CA problem. By applying the method of link set assignment [3], we can derive an important characteristic of the optimal solution. Based on this characteristic, there is no need to perform approximation on the non-differentiable link cost function as in the above method, and the cost function can be treated as a differentiable function. As a result, conventional methods such as Lagrange multiplier method can be applied to the problem.

It should be noted that the CA problem taking into consideration an existing network in the case of long-term design (the per-unit cost of installing new capacity is less than that of using existing capacity) is a special case of the CA problem studied in this paper.

The rest of this paper is organized as follows. First,

Manuscript received January 8, 1999.
Manuscript revised March 31, 1999.

†The authors are with Sirindhorn International Institute of Technology, Thammasat University, Thailand.

a packet-switched network model used in this paper and the detail of link cost function are discussed in Sect. 2. The CA problem is then formulated in Sect. 3, where the concept of link set assignment for solving the CA problem is given. In Sect. 4, an important characteristic of the optimal solution of the CA problem is derived. Based on this characteristic, a heuristic algorithm derived from the Lagrange multiplier method is presented in Sect. 5. Some numerical results are given in Sect. 6, and conclusions are given in Sect. 7.

2. Network Model

In this section, the model of packet-switched networks considered in this paper is discussed. Then a piecewise linear concave link cost function is introduced.

2.1 Model Description

The packet-switched network model used in this paper is the same as the one used in the early work of packet-switched network design [1]–[3]. Packet arrival process on each link is assumed to be Poisson, and packet length is assumed to be negative exponentially distributed. From this assumption, we can model each link as an $M/M/1$ system with infinite buffer, and the average packet delay throughout the network T can be given as in Eq. (1).

$$T = \frac{1}{\gamma}\sum_{L}\frac{f_i}{C_i - f_i}, \tag{1}$$

where L is the set of links in the network, f_i is the traffic flow on link i (in bits/second), C_i is the capacity of link i (in bits/second), and γ is the overall traffic in the network (in packets/second). The value of link capacity C_i is assumed to be continuous, and can be set as an arbitrary positive value. For simplicity, node cost (e.g., cost of switching facility, etc.) is not taken into account in this model. Only link cost is considered, and the total network cost D is defined as the sum of all link costs, i.e.,

$$D = \sum_{L} D_i, \tag{2}$$

where D_i is the cost of link i.

2.2 Link Cost Function

We assume that there are p alternatives of link types that can be selected to implement each link in the network, where $p > 1$ and the cost function of each link type is linear. Consequently, the cost function of link type k for link i can be given as

$$D_{ki} = d_{ki}C_i + r_{ki}, \qquad 0 \le k \le p-1, \tag{3}$$

where d_{ki} and r_{ki} are respectively the per-unit cost and

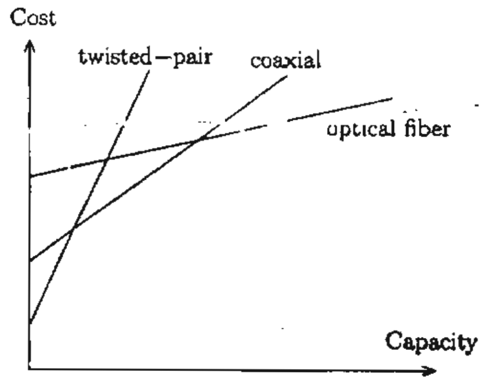


Fig. 1 An example of practical link type selection.

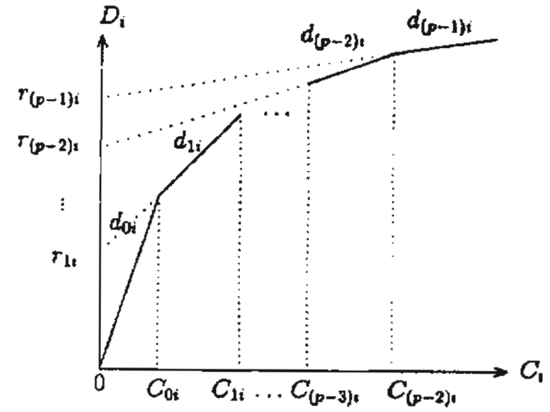


Fig. 2 A piecewise linear concave link cost function.

the start-up cost of link type k for link i.

Hence, the link cost function is the envelope of cost functions of all link types, which is a piecewise linear function of link capacity as

$$D_i = \min_k(d_{ki}C_i + r_{ki}), \qquad 0 \le k \le p-1. \tag{4}$$

Since we are going to investigate the CA problem with piecewise linear concave cost function, we consider the following case:

$$d_{ui} > d_{vi}, \qquad r_{ui} < r_{vi}, \tag{5}$$

when $0 \le u, v \le p-1$ and $u < v$. Without loss of generality, we can assume that $r_{0i} = 0, \forall i \in L$.

A practical example with three alternatives of link types is given in Fig. 1.

As a whole, the expression of the link cost function can be given as in Eq. (6), which is depicted in Fig. 2.

$$D_i = \begin{cases} d_{0i}C_i & , \quad C_i \le C_{0i}, \\ d_{1i}C_i + r_{1i} & , \quad C_{0i} \le C_i \le C_{1i}, \\ \vdots & \\ d_{(p-1)i}C_i + r_{(p-1)i} & , \quad C_i \ge C_{(p-2)i}. \end{cases} \tag{6}$$

3. Link Capacity Assignment Problem

In this section, the CA problem for packet-switched networks is formulated as a non-linear programming problem, and then the concept of link set assignment for solving the problem is discussed.

3.1 Problem Formulation

We formulate the CA problem as follows:

CA Problem

Given: $\{f_i\}$

Minimize: $D = \sum_L D_i$

Design variables: $\{C_i\}$

Subject to:
$$T = \frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} \leq T_{\max},$$
$$C_i - f_i > 0, \quad \forall i \in L$$

where $T_{\max}$ is the constraint value of T allowed in the network.

In the above CA problem, $\{f_i\}$ is the traffic pattern in the network, and $f_i > 0, \forall i \in L$. Link capacity $\{C_i\}$ is determined so that the total network cost D is minimized. Design constraints are the upper limit of packet delay, and the relationship between link flow and link capacity. Obviously, link capacity must be greater than link flow so that packet delay will not grow indefinitely.

3.2 The Method of Link Set Assignment

The CA problem formulated above is a non-linear programming problem. Many conventional methods such as Lagrange multiplier method are known as efficient methods for solving non-linear programming problems, e.g., the CA problem studied in the earlier work [2].

However, since the link cost function D_i given in Eq. (6) is not differentiable with respect to C_i, Lagrange multiplier method cannot be applied to the CA problem. As a method to alleviate this type of network design problem, the non-differentiable function is approximated by a differentiable function as shown in Fig. 3 [8]. By using an approximated differentiable link cost function, Lagrange multiplier method can be applied to the CA problem. Although, this provides a solution to the problem within approximation error, there is no guarantee that the solution obtained is an optimal solution when the link cost is not convex. Moreover, lengthy time is needed for calculating approximation function of all cost functions.

When the cost function is piecewise linear convex,

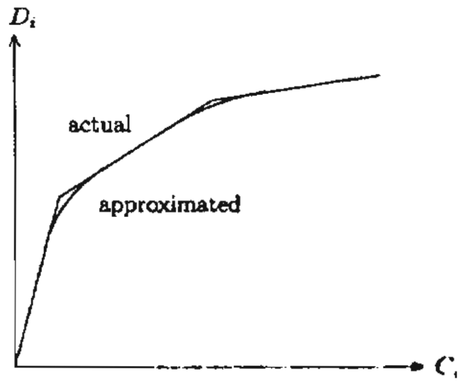


Fig. 3 Approximation of non-differentiable function

a method of link set assignment has been proposed to solve the CA problem to obtain an optimal solution [3]. In this paper, we extend this method so that it is applicable to the CA problem with piecewise linear concave cost function. The concept of the method is as follows.

D_i given in Eq. (6) can be considered as a differentiable function, if we assign each link i to be a member of one of the following $2p-1$ sets:

$$\begin{aligned}
L_0 &= \{\, i \mid C_i < C_{0i} \},\\
L_1 &= \{\, i \mid C_i = C_{0i} \},\\
L_2 &= \{\, i \mid C_{0i} < C_i < C_{1i} \},\\
L_3 &= \{\, i \mid C_i = C_{1i} \},\\
&\vdots\\
L_{2p-4} &= \{\, i \mid C_{(p-3)i} < C_i < C_{(p-2)i} \},\\
L_{2p-3} &= \{\, i \mid C_i = C_{(p-2)i} \},\\
L_{2p-2} &= \{\, i \mid C_i > C_{(p-2)i} \}.
\end{aligned}$$

This is because

$$\frac{dD_i}{dC_i} = \begin{cases} d_{0i} & , \quad i \in L_0, \\ d_{1i} & , \quad i \in L_2, \\ & \vdots \\ d_{(p-2)i} & , \quad i \in L_{2p-4}, \\ d_{(p-1)i} & , \quad i \in L_{2p-2}. \end{cases} \tag{7}$$

Note that link i that belongs to L_{2k+1} $(0 \leq k \leq p-2)$ is excluded from the CA problem since its capacity C_i is set to be a constant value of C_{ki}.

With the above link set assignment, Lagrange multiplier method can be applied to the CA problem, and the solution of $\{C_i\}$ can be given as in Eq. (8) [3].

$$C_i = f_i + \frac{\sum_{L_E} \sqrt{f_j d_j}}{\left(\gamma\, T_{\max} - \sum_{L_O} \frac{f_j}{X_j - f_j}\right)} \sqrt{\frac{f_i}{d_i}}, \quad \forall i \in L_E, \tag{8}$$

where

$$L_E = L_0 \cup L_2 \cup \cdots \cup L_{2p-4} \cup L_{2p-2},$$
$$L_O = L_1 \cup L_3 \cup \cdots \cup L_{2p-5} \cup L_{2p-3},$$
$$X_j = C_{kj}, \text{ when } j \in L_{2k+1}, \quad 0 \le k \le p-2,$$

and d_i is equal to $\frac{dD_i}{dC_i}$ given in Eq. (7).

The value of C_i calculated from Eq. (8) is optimum for the CA problem under the relevant link set assigned at the beginning. However, the value has to be checked whether it contradicts its assigned link set or not, e.g., for $i \in L_0$, C_i that is greater than C_{0i} is thought to be a contradiction. If any contradiction occurs, the solution obtained will not be feasible and other combinations of link set have to be examined until a feasible solution with minimum network cost is found.

Although the above method gives global optimal solution of the CA problem, it is very time-consuming since we have to examine all link set combinations. Thus, a more efficient algorithm is required.

4. Characteristic of the Optimal Solution of the CA Problem

To find a way to solve the CA problem, we first review the modified method of link set assignment given in Ref. [3]. For simplicity, we first discuss the case $p = 2$, and then extend the discussion to the case $p > 2$.

4.1 The Case $p = 2$

For this case, we have

$$L_0 = \{\, i \mid C_i < C_{0i}\},$$
$$L_1 = \{\, i \mid C_i = C_{0i}\},$$
$$L_2 = \{\, i \mid C_i > C_{0i}\},$$

and

$$C_i = f_i + \frac{\sum_{L_0 \cup L_2} \sqrt{f_j d_j}}{\left(\gamma\, T_{\max} - \sum_{L_1} \frac{f_j}{C_{0j} - f_j}\right)} \sqrt{\frac{f_i}{d_i}}, \quad \forall i \in L_0 \cup L_2, \tag{9}$$

where $d_i = d_{0i}$ if $i \in L_0$ and $d_i = d_{1i}$ if $i \in L_2$.

Since each link in the network certainly belongs to L_0, L_1, or L_2, we can select one of the three sets for each link and assign the link to that set. Then checking the contradiction between the assigned link set and the calculated link capacity will give us a way to decide that the link can belong to the set that it was assigned or not.

In the case of convex cost function ($d_{0i} < d_{1i}$, $\forall i \in L$), each link is first assigned to L_0 and if contradiction occurs, it is concluded that the link with contradiction cannot belong to L_0. In the next step, each link is set to L_2 and if contradiction occurs, we can say that the link with contradiction cannot belong to L_2. In the case that a link cannot belong to neither L_0 nor L_2, it must belong to L_1. By repeatedly examining link for L_1 until no more link has to be assigned to it, each link in the network will belong to a proper set, and link capacity can be computed by using Eq. (9). It is proven that the final solution by the method of link set assignment is a global optimal solution for the case of convex cost function [3].

Next, we investigate that how the above method can deal with the CA problem with concave cost function ($d_{0i} > d_{1i}$, $\forall i \in L$). First, we assign each link to L_0 and use Eq. (9) to determine $\{C_i\}$, then check whether there is any contradiction or not, i.e., there is link i that $C_i \ge C_{0i}$ or not. Unlike the case of convex cost function, we focus on the links without contradiction. This is because we can show that this kind of links can belong to L_0 without any contradiction although link set combination is changed. See Appendix for the proof. Again when each link is set to L_2, we can say that the links with no contradiction can belong to L_2.

By the above concept, we have the following procedure for assigning a proper set to each link. Let's introduce two parameters for link i: e_{0i} and e_{2i}. The values of the two parameters are set as follows: e_{0i} will be 0 if link i is assigned to L_0 and no contradiction occurs, and be 1 if contradiction exists. e_{2i} will be 0 if link i is assigned to L_2 and no contradiction occurs, and be 1 if contradiction exists. Then we consider the following four cases:

1. $e_{0i} = 0$ and $e_{2i} = 1$,
2. $e_{0i} = 1$ and $e_{2i} = 0$,
3. $e_{0i} = 0$ and $e_{2i} = 0$,
4. $e_{0i} = 1$ and $e_{2i} = 1$.

In case 1, we can let link i belong to L_0 since there is no contradiction as mentioned above. Although in global optimal solution of the problem, link i may not belong to L_0, we deal with the link in a greedy manner to obtain a feasible solution since it is guaranteed that there will be no contradiction even the sets of other links are changed. Moreover, due to the non-convexity of the CA problem, its global optimum cannot be determined easily, so we have to try to find its local optimum instead. As same as case 1, link i that falls into case 2 is then assigned to L_2.

92 In case 3, since link i can belong to either L_0 or L_2

without any contradiction, we can apply any method to select the set for this type of links, e.g., L_0 and L_2 are randomly chosen.

Finally, for link of case 4, we do not assign this type of links to any set, but repeatedly do the above procedure again until all links have their proper set.

There will be no problem if links of case 4 do not exist at the final stage of the examining, and each link in the network will then be assigned to L_0 or L_2. However, if this kind of links exist, the question that they can be assigned to L_1 or not will arise. To answer this question we introduce the following theorem.

Theorem 1: In the optimal solution of the CA problem with $p = 2$,

$$L_1 = \emptyset.$$

Proof: We define the following sets and notations.

$$L'_0 = \{i \mid i \text{ is the link already assigned to } L_0\},$$
$$L'_2 = \{i \mid i \text{ is the link already assigned to } L_2\},$$

$$b_0 = \sum_{L'_0} \sqrt{f_j d_{0j}}, \quad b'_0 = \sum_{L-(L'_0 \cup L'_2)} \sqrt{f_j d_{0j}},$$
$$b_1 = \sum_{L'_2} \sqrt{f_j d_{1j}}, \quad b'_1 = \sum_{L-(L'_0 \cup L'_2)} \sqrt{f_j d_{1j}}.$$

Next, assume that there exists link i such that $e_{0i} = 1$ and $e_{2i} = 1$. Then we have

$$C_i = f_i + \frac{b_0 + b_1 + b'_0}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{0i}}} \geq C_{0i}, \tag{10}$$
$$C_i = f_i + \frac{b_0 + b_1 + b'_1}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{1i}}} \leq C_{0i}. \tag{11}$$

Manipulating (10) and (11) yields

$$\alpha_i\,(b_0 + b_1 + b'_0) \geq \sqrt{f_i d_{0i}}, \tag{12}$$
$$\alpha_i\,(b_0 + b_1 + b'_1) \leq \sqrt{f_i d_{1i}}, \tag{13}$$

where $\alpha_i = \dfrac{f_i}{\gamma\, T_{\max}\,(C_{0i} - f_i)}$.

At the final stage of the examining, (12) and (13) are valid for $\forall i \in L - (L'_0 \cup L'_2)$, so we can take the summation of (12) and (13) over the set $L - (L'_0 \cup L'_2)$. This gives

$$\sum_{L-(L'_0 \cup L'_2)} \alpha_i\,(b_0 + b_1 + b'_0) \geq b'_0, \tag{14}$$
$$\sum_{L-(L'_0 \cup L'_2)} \alpha_i\,(b_0 + b_1 + b'_1) \leq b'_1. \tag{15}$$

From (14) and (15), we obtain

$$\frac{b_0 + b_1 + b'_0}{b_0 + b_1 + b'_1} \geq \frac{b'_0}{b'_1}. \tag{16}$$

After manipulating (16), we get

$$b'_1 \geq b'_0. \tag{17}$$

However, this contradicts to the fact that

$$d_{0i} > d_{1i}, \qquad \forall i \in L.$$

Thus, there cannot be any link i that $e_{0i} = 1$ and $e_{2i} = 1$ at the final stage of examining, and all links in the network will belong to L_0 or L_2. This implies that

$$L_1 = \emptyset. \tag{18}$$

□

4.2 The Case $p > 2$

Next, the method of link set assignment and the above procedure are extended to the case $p > 2$.

First, we define E_i as the number of sets among $L_0, L_2, \ldots, L_{2p-4}, L_{2p-2}$ that make contradictions when link i is assigned to. Then, $E_i \neq p$ means there is at least one set among $L_0, L_2, \ldots, L_{2p-4}, L_{2p-2}$ that makes no contradiction with link i. Consequently, link i can be assigned to that set without any contradiction. If the number of this kind of sets is more than one, we can take any criteria to select one set from the possible sets, e.g., the set is chosen in random.

In the case that $E_i = p$, we do not assign link i to any set, but repeatedly do the procedure of link set assignment until each link in the network is assigned to a proper set.

Again, the above procedure will work successfully if there is no link i that $E_i = p$ at the final stage of examining, and each link in the network will be assigned to one of the following sets: $L_0, L_2, \ldots, L_{2p-4}$, and L_{2p-2}. Therefore we come to the question: Is there any link i that $E_i = p$ at the final stage of the examining? To answer this question, we first introduce the following lemma.

Lemma 1: If there is link i that $E_i = p$ at the final stage of examining, we cannot have the following cases at the same time.

1. $C_i \geq C_{mi}$ when $i \in L_{2m}$.
2. $C_i \leq C_{mi}$ when $i \in L_{2m+2}$.

where $0 \leq m \leq p-2$.

Proof: We define the following sets and notations where $0 \leq k \leq p-1$.

$$L'_{2k} = \{i \mid i \text{ is the link already assigned to } L_{2k}\},$$

$$b_k = \sum_{L'_{2k}} \sqrt{f_j d_{kj}}, \qquad b'_k = \sum_{L'} \sqrt{f_j d_{kj}},$$

$$B = b_0 + b_1 + b_2 + \cdots + b_{p-2} + b_{p-1},$$
$$L' = L - (L'_0 \cup L'_2 \cup L'_4 \cup \cdots \cup L'_{2p-4} \cup L'_{2p-2}).$$

When link i is assigned to set L_{2m} and L_{2m+2}, we

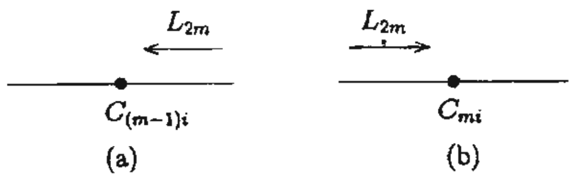


Fig. 4 Representation of link set contradictions.

assume that link set contradictions occur in the case that $C_i \geq C_{mi}$ when $i \in L_{2m}$, and $C_i \leq C_{mi}$ when $i \in L_{2m+2}$. Then we have

$$C_i = f_i + \frac{B + b'_m}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{mi}}} \geq C_{mi}, \tag{19}$$

$$C_i = f_i + \frac{B + b'_{m+1}}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{(m+1)i}}} \leq C_{mi}. \tag{20}$$

From (19) and (20), we get

$$\alpha_i\ (B + b'_m) \geq \sqrt{f_i d_{mi}}, \tag{21}$$

$$\alpha_i\ (B + b'_{m+1}) \leq \sqrt{f_i d_{(m+1)i}}, \tag{22}$$

where $\alpha_i = \dfrac{f_i}{\gamma\, T_{\max}\ (C_{mi} - f_i)}$.

At the final stage of the examining, (21) and (22) are valid for $\forall i \in L'$, so we can take the summation of (21) and (22) over the set L'. This yields

$$\sum_{L'} \alpha_i\ (B + b'_m) \geq b'_m, \tag{23}$$

$$\sum_{L'} \alpha_i\ (B + b'_{m+1}) \leq b'_{m+1}. \tag{24}$$

From (23) and (24), we obtain

$$b'_{m+1} \geq b'_m. \tag{25}$$

However, this contradicts to the fact that

$$d_{mi} > d_{(m+1)i}, \qquad \forall i \in L.$$

Hence, the link set contradiction at L_{2m} and L_{2m+2} cannot be $C_i \geq C_{mi}$ and $C_i \leq C_{mi}$, respectively, at the same time. □

When link i is assigned to L_{2m}, $1 \leq m \leq p-3$, and contradiction occurs, we can have two cases:

$$C_i \leq C_{(m-1)i} \text{ or } C_i \geq C_{mi}.$$

We represent the above link set contradictions by arrows pointing toward $C_{(m-1)i}$ and C_{mi}, respectively, as in Fig. 4.

From Lemma 1, we cannot have the contradictions at L_{2m} and L_{2m+2} as two arrows pointing toward $C_i = C_{mi}$ at the same time, i.e., the case that is shown in Fig. 5(a). As a consequence, if there are contradictions

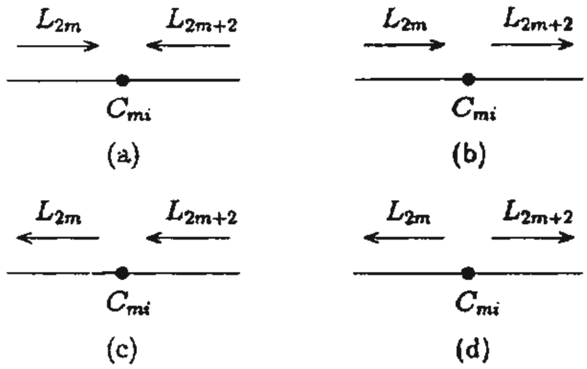


Fig. 5 Link set contradictions at L_{2m} and L_{2m+2}.

Fig. 6 Link set contradictions at L_0 and L_{2p-2}.

at both L_{2m} and L_{2m+2}, the possible cases must be those given in Fig. 5(b), (c), or (d).

From Lemma 1 and arrow representation of link set contradiction, we introduce the following theorem to give an answer for the question that there is any link i with $E_i = p$ at the final stage of link set examining or not.

Theorem 2: In the optimal solution of the CA problem with $p > 2$,

$$L_{2k+1} = \emptyset, \qquad 0 \leq k \leq p-2.$$

Proof: Assume that at the final stage of link set examining, there are links that have contradictions at all of the sets: $L_0, L_2, L_4, \ldots, L_{2p-4}$, and L_{2p-2}. In other words, there are links i that $E_i = p$.

Let's consider the contradictions at L_0 and L_{2p-2}. Clearly, we must have $C_i \geq C_{0i}$ when $i \in L_0$, and $C_i \leq C_{(p-2)i}$ when $i \in L_{2p-2}$. By arrow representation, these two cases can be depicted as in Fig. 6.

First, we investigate the link set contradiction at L_0. This contradiction can be represented by an arrow pointing to $C_i = C_{0i}$. To make link set contradictions occur at $L_2, L_4, \ldots, L_{2p-6}, L_{2p-4}$, we must have arrows pointing toward the points C_i equal to $C_{1i}, C_{2i}, \ldots, C_{(p-3)i}, C_{(p-2)i}$, respectively. However, the contradiction at L_{2p-2} is an arrow pointing toward $C_i = C_{(p-2)i}$ as shown in Fig. 7(a). By Lemma 1, this is impossible.

Next, we consider the contradiction at L_{2p-2}. This contradiction is represented by an arrow pointing to $C_i = C_{(p-2)i}$. To make the contradictions occur at $L_{2p-4}, L_{2p-6}, \ldots, L_4, L_2$, we must have arrows pointing toward the points C_i equal to

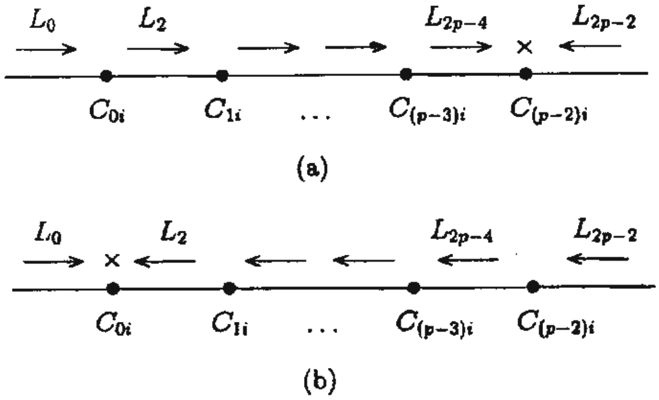


Fig. 7 The impossibility of link set contradictions.

$C_{(p-3)i}, C_{(p-4)i}, \ldots, C_{1i}, C_{0i}$, respectively. However, the contradiction at L_0 is an arrow pointing toward $C_i = C_{0i}$ as shown in Fig. 7(b). By Lemma 1, this is also impossible.

Thus, at the final stage, there is no link that has contradictions at $L_0, L_2, L_4, \ldots, L_{2p-4}$, and L_{2p-2}, i.e., there is no link i that $E_i = p$. This also means that each link in the network is assigned to one of the following sets: $L_0, L_2, L_4, \ldots, L_{2p-4}$, and L_{2p-2}.

Hence,

$$L_1 = L_3 = \cdots = L_{2p-5} = L_{2p-3} = \emptyset \tag{26}$$

or

$$L_{2k+1} = \emptyset, \qquad 0 \le k \le p-2. \tag{27}$$

□

5. Algorithm for the CA Problem

Based on the fact that $L_{2k+1} = \emptyset$, $0 \le k \le p-2$ in the optimal solution of the CA problem, we can exclude the point $C_i = C_{ki}$ from the cost function of all links in the network, and the link cost function can then be treated as a differentiable function, where $\frac{dD_i}{dC_i}$ can be expressed as in Eq. (7).

Clearly, we have no need to approximate the non-differentiable function by a differentiable function as that was done in Ref. [8]. As a result, the Lagrange multiplier method can be directly applied to the CA problem as in the following algorithm.

ALGORITHM

Step 1 Initialize d_i as d_{ki}, $0 \le k \le p-1$ by any method, $\forall i \in L$.

Set ϵ as a small positive value for using as algorithm termination parameter.

Determine the initial value of Lagrange multiplier β' by

$$\beta' = \left(\frac{\sum_L \sqrt{f_i d_i / \gamma}}{T_{\max}} \right)^2.$$

Step 2 Determine C_i by

$$C_i = f_i + \sqrt{\frac{\beta' f_i}{\gamma d_i}}, \qquad \forall i \in L.$$

Step 3 Set d_i, $\forall i \in L$ as follows:

$$\begin{aligned} d_i &= d_{0i} \text{ if } C_i < C_{0i}, \\ d_i &= d_{1i} \text{ if } C_{0i} < C_i < C_{1i}, \\ &\vdots \\ d_i &= d_{(p-1)i} \text{ if } C_i > C_{(p-2)i}. \end{aligned}$$

Step 4 Determine Lagrange multiplier β by

$$\beta = \left(\frac{\sum_L \sqrt{f_i d_i / \gamma}}{T_{\max}} \right)^2.$$

Step 5 If $|\beta - \beta'| > \epsilon$, then set $\beta' = \beta$ and go to Step 2, else STOP and determine C_i as

$$C_i = f_i + \frac{\sum_L \sqrt{f_j d_j}}{\gamma T_{\max}} \sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L.$$

Note that there are many ways that can be adopted for setting the initial value of d_i at Step 1. Some examples are given in the following section.

6. Numerical Results and Discussions

This section gives some numerical results on the performance of the proposed algorithm in the view of optimality and computation amount, and some results on the effect of concave cost function on the link capacity assignment in packet-switched networks.

6.1 Performance of the Proposed Algorithm

First, we examine the optimality of the proposed algorithm by comparing its results with global optimal solutions. For comparison, the results obtained from the method using approximation function [8] are also given. A network with fully connected topology is considered with five thousand random traffic patterns. For the method of setting the initial value of d_i, we consider the following two methods in this paper.

method A d_i is set according to the relationship between link flow f_i and link capacity at breakpoints of the cost function as follows:

Table 1 Percentage of yielding global optimum ($p = 2$).

n	case 1	case 2	case 3	case 1′	case 2′	case 3′
3	96.0	98.8	99.1	95.7	96.6	97.8
6	91.2	98.7	98.9	90.4	95.5	96.9
10	83.9	98.2	98.5	83.0	94.1	95.7
15	77.1	97.8	98.3	74.4	93.6	95.4
21	70.7	96.8	97.9	65.0	92.7	94.4
28	61.9	94.2	97.0	54.0	90.7	93.0

Table 2 Percentage of yielding global optimum ($p = 3$).

n	case 1	case 2	case 3	case 1′	case 2′	case 3′
3	92.2	97.1	97.5	85.1	94.9	95.9
6	83.5	92.3	94.2	77.0	86.7	89.0
10	74.5	88.9	90.9	66.6	78.4	85.8
15	65.4	83.9	86.8	53.7	58.0	75.4

Table 3 Percentage of yielding global optimum ($p = 4$).

n	case 1	case 2	case 3	case 1′	case 2′	case 3′
3	92.6	98.0	98.2	85.5	93.5	94.5
6	83.9	95.3	95.9	76.1	85.5	88.8
10	75.4	92.1	93.0	67.0	77.8	82.2

Table 4 Average ratio of solution and global optimum ($p = 2$).

n	case 1	case 2	case 3	case 1′	case 2′	case 3′
3	1.00034	1.00012	1.00009	1.00074	1.00033	1.00030
6	1.00043	1.00004	1.00004	1.00149	1.00031	1.00029
10	1.00055	1.00005	1.00004	1.00205	1.00030	1.00028
15	1.00059	1.00004	1.00003	1.00236	1.00028	1.00026
21	1.00061	1.00004	1.00003	1.00237	1.00027	1.00025
28	1.00064	1.00004	1.00002	1.00260	1.00027	1.00025

Table 5 Average ratio of solution and global optimum ($p = 3$).

n	case 1	case 2	case 3	case 1′	case 2′	case 3′
3	1.00047	1.00018	1.00017	1.00082	1.00030	1.00026
6	1.00064	1.00030	1.00019	1.00154	1.00059	1.00056
10	1.00067	1.00025	1.00019	1.00195	1.00054	1.00046
15	1.00066	1.00024	1.00018	1.00227	1.00052	1.00042

Table 6 Average ratio of solution and global optimum ($p = 4$).

n	case 1	case 2	case 3	case 1′	case 2′	case 3′
3	1.00027	1.00008	1.00007	1.00045	1.00024	1.00020
6	1.00040	1.00014	1.00011	1.00068	1.00027	1.00021
10	1.00039	1.00012	1.00011	1.00082	1.00035	1.00027

$$
\begin{aligned}
d_i &= d_{0i} \text{ if } f_i < C_{0i},\\
d_i &= d_{1i} \text{ if } C_{0i} \leq f_i < C_{1i},\\
&\vdots\\
d_i &= d_{(p-1)i} \text{ if } f_i \geq C_{(p-2)i}.
\end{aligned}
$$

method B d_i is set randomly among $d_{0i}, d_{1i}, \ldots$, and $d_{(p-1)i}$.

To obtain a good solution for a CA problem, we apply the proposed algorithm and the method using approximation function in the following three cases:

- case 1 (1′): method A
- case 2 (2′): method A and 10 times of method B
- case 3 (3′): method A and 20 times of method B

In the results given below, we denote cases 1, 2, 3 for the proposed algorithm, and cases 1′, 2′, 3′ for the method using approximation function. In the cases that there are many solutions obtained, we select the best solution, i.e., the solution with the smallest network cost, as a final solution.

The relationships between the number of links in the network n and the percentage that each method yields global optimal solution are shown in Tables 1–3. Note that global optimum can be obtained by examining all link set combinations.

Next, we observe the difference between the solution obtained from each method and global optimum. Tables 4–6 show the average value of the ratio of the solution obtained from the method and global optimum.

From Tables 1–6, it can be seen that although the proposed algorithm has a lower percentage of yielding global optimum when the number of links in the network increases, its solution is very close to the global

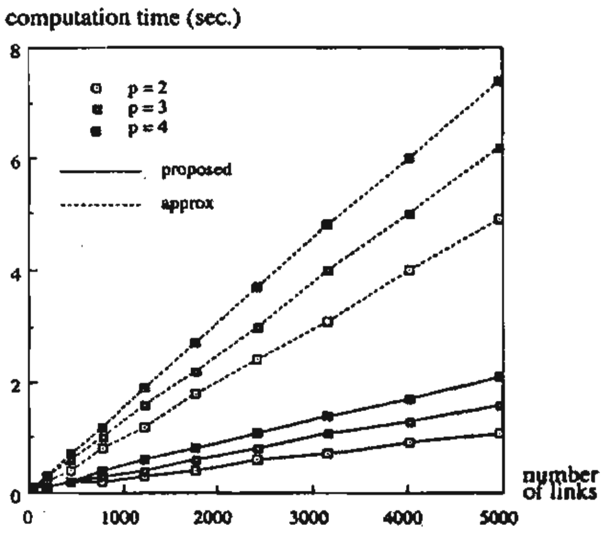


Fig. 8 Maximum computation time.

optimum. Moreover, the proposed algorithm has better performance than the method using approximation function. This means that the proposed algorithm solves the CA problem very efficiently.

Next, we investigate the computation amount of the algorithm. We then show the computation time of each method by using actual computation results. Again, a network with fully connected topology is considered, and five thousand of random traffic patterns are applied to the network. The relationships between the number of links in the network and maximum, average, and variance of computation time are shown in Figs. 8, 9, and 10, respectively (SPARC station 4 is used in determining numerical results). The values of computation time are measured for one executing time of

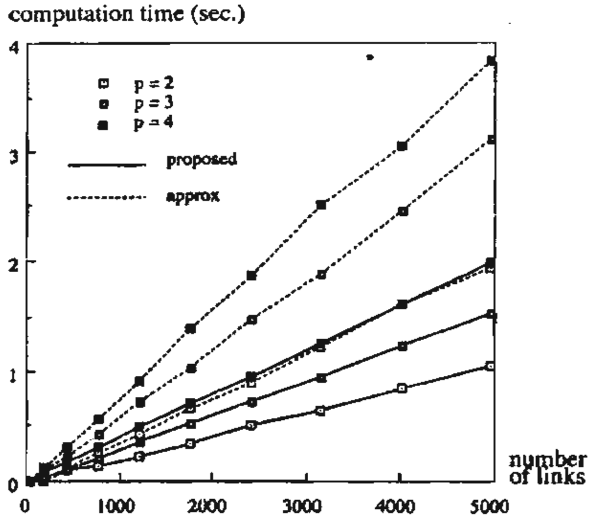


Fig. 9 Average computation time.

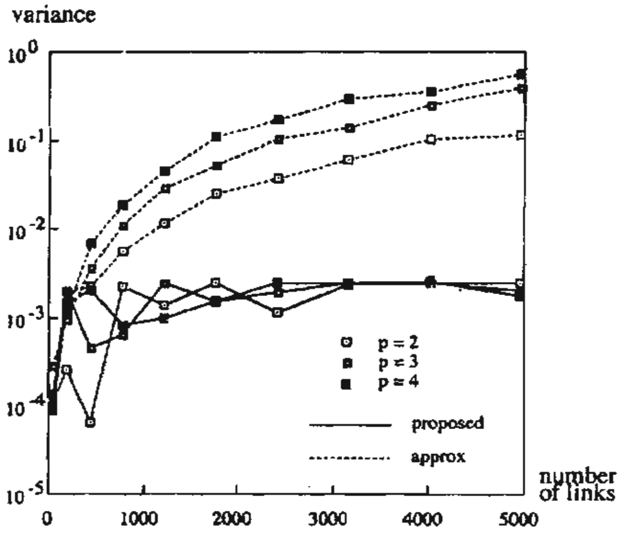


Fig. 10 Variance of computation time.

each method.

Figures 8 and 9 show that the maximum and average computation time of both methods are approximately linear with respect to the number of links in the network. (By linear regression, the values of linear correlation coefficients are very close to 1.) However, the method using approximation function needs longer computation time as can be seen from the figures.

Figure 10 shows that the variance of computation time of the proposed algorithm is very small. Hence, it can be concluded that the proposed algorithm is very practical for applying to the CA problem of a large-scale network.

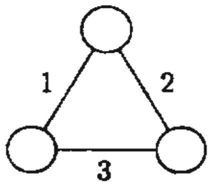


Fig. 11 A 3-node network.

6.2 Effect of Concave Link Cost Function on the Network Design

For simplicity, we focus on the case $p = 2$. A CA problem of this case is the same as the problem taking into consideration an existing network, where the per-unit cost of installing new capacity is less than that of using existing capacity. For link i, we let d_{0i} and d_{1i} be respectively the per-unit cost of existing and new capacity, and C_{0i} be the existing link capacity.

A 3-node network as shown in Fig. 11 is considered. Network parameters are set as follows: $C_{0i} = 52$ kbps for $i = 1, 2, 3$, cost function of link 1 and 2 are linear with respect to link capacity with cost coefficient (per-unit cost) equal to 1, $f_1 = f_2 = 40$ kbps, $T_{max} = 20$ ms., mean value of packet length is 400 bits/packet. By varying f_3 and applying several pairs of (d_{03}, d_{13}), the results as shown in Fig. 12 are obtained. From Fig. 12, we can see that the capacity of link 3 (C_3) is never equal to its own existing capacity (52 kbps) for all over the entire range of f_3. Moreover, there are some values of f_3 that make the curve of link capacity value to be discontinuous. These approximated values are summarized as follows:

$$
\begin{aligned}
(d_{03}, d_{13}) &= (1, 0.2) \rightarrow f_3 = 29.32 \text{ kbps.},\\
(d_{03}, d_{13}) &= (1, 0.5) \rightarrow f_3 = 31.44 \text{ kbps.},\\
(d_{03}, d_{13}) &= (5, 1) \rightarrow f_3 = 37.47 \text{ kbps.},\\
(d_{03}, d_{13}) &= (10, 1) \rightarrow f_3 = 39.06 \text{ kbps.}
\end{aligned}
$$

At these values of f_3, the total network cost is the same for both the cases when link 3 belongs to L_0, and when it belongs to L_2. It means that we can have two solutions with the same network cost.

7. Conclusions

In this paper, we have studied the link capacity assignment problem in packet-switched networks (CA problem), where link cost function is piecewise linear concave. This type of cost function exists in many design problems, e.g., a design problem taking into consideration an existing network in long-term design, a design problem with link type selection, etc.

After formulating the CA problem, the method of link set assignment is applied, and the characteristic of the optimal solution of the problem is derived.

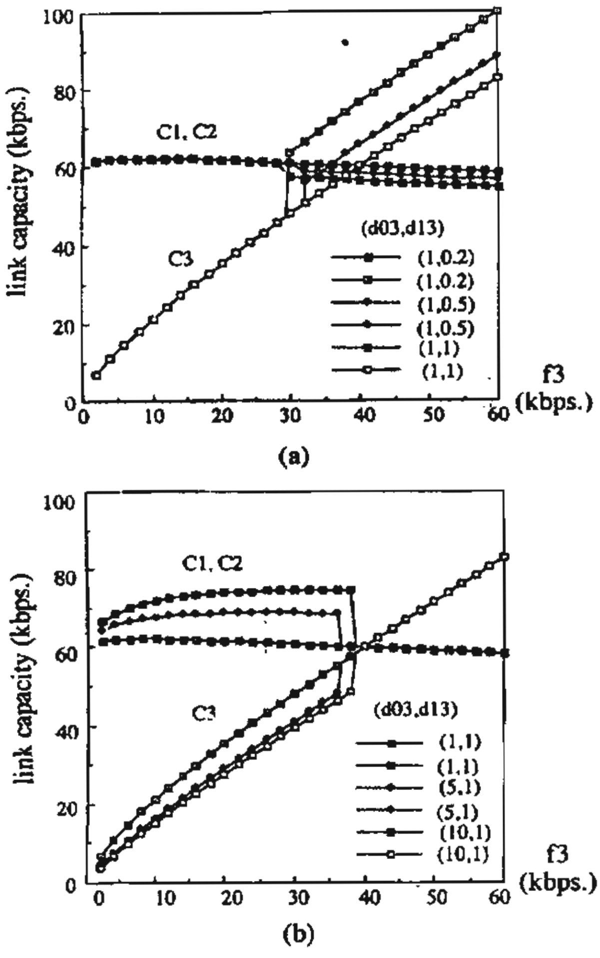


Fig. 12 Results of the 3-node network.

Based on this characteristic, it is shown that the non-differentiable link cost function can be treated as a differentiable function. As a result, the Lagrange multiplier method can be directly applied to solve the problem, and a heuristic algorithm derived from the Lagrange multiplier method is proposed.

Due to the non-convexity of the CA problem, it is hard to determine a global optimal solution. However, by investigating numerical results, it is shown that the proposed algorithm has very good performance for solving the CA problem, both from the view of optimality and computation time.

Acknowledgement

The work in this paper is supported by the Thailand Research Fund under the Post-Doctoral Research Fellowship.

References

[1] M. Gerla and L. Kleinrock, "On the topological design of distributed computer networks," IEEE Trans. Commun., vol.COM-25, no.1, pp.48–60, Jan. 1977.

[2] L. Kleinrock, Queueing Systems Volume II, Computer Applications, John Wiley & Sons, New York, 1976.

[3] S. Runggeratigul, W. Zhao, Y. Ji, A. Aizawa, and S. Asano, "Link capacity assignment in packet-switched network with existing network consideration," IEICE Trans. Commun., vol.E78-B, no.5, pp.709–719, May 1995.

[4] A. Kershenbaum, Telecommunications Network Design Algorithms, McGraw-Hill, 1993.

[5] A. Gersht and R. Weihmayer, "Joint optimization of data network design and facility selection," IEEE J. Select. Area Commun., vol.8, no.9, pp.1667–1681, Dec. 1990.

[6] B. Yaged, "Minimum cost routing for static network models," Networks, vol.1, pp.139–172, 1971.

[7] P.K. Verma, Performance Estimation of Computer Communication Networks, Computer Science Press, 1989.

[8] M. Shinohara, "Generalization of Pratt's formula for dimensioning circuit-switched network," Computer Networks and ISDN Systems, vol.20, pp.115–126, 1990.

Appendix

In this appendix, we will show that from e_{0i} defined in Sect. 4, we can conclude that if $e_{0i} = 0$, link i can be assigned to L_0 even if some of other links are changed to L_2. Also, the conclusion about e_{2i} is that, if $e_{2i} = 0$, link i can be assigned to L_2 even if some of other links are changed to L_0.

Proof: Assume that there is a link i with $e_{0i} = 0$. For this link, we have

$$C_i = f_i + \frac{b_0 + b_1 + b'_0}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{0i}}} < C_{0i}, \qquad \text{(A·1)}$$

or

$$\frac{1}{\sqrt{d_{0i}}} < \frac{\gamma\, T_{\max}\,(C_{0i} - f_i)}{\sqrt{f_i}\,(b_0 + b_1 + b'_0)}. \qquad \text{(A·2)}$$

The definitions of b_0, b_1, and b'_0 are given in Sect. 4.

Currently, all links in $L - (L'_0 \cup L'_2)$ are assigned to L_0 for checking link set contradiction. Let L''_2 be a set of links in $L - (L'_0 \cup L'_2)$ that are changed to L_2. After the change, we have b'_0 becomes b^* as

$$b^* = \sum_{L''_0} \sqrt{f_j d_{0j}} + \sum_{L''_2} \sqrt{f_k d_{1k}}, \qquad \text{(A·3)}$$

where $L''_0 = L - (L'_0 \cup L'_2 \cup L''_2)$. See the definition of L'_0 and L'_2 in Sect. 4.

Note that link i under consideration must be in L''_0, since we are checking link set contradiction when it is assigned to L_0.

From $d_{0i} > d_{1i}$, $\forall i \in L$, we have

$$b'_0 = \sum_{L''_0 \cup L''_2} \sqrt{f_j d_{0j}} > b^*. \qquad \text{(A·4)}$$

This makes

$$\frac{\gamma\, T_{\max}\,(C_{0i} - f_i)}{\sqrt{f_i}\,(b_0 + b_1 + b'_0)} < \frac{\gamma\, T_{\max}\,(C_{0i} - f_i)}{\sqrt{f_i}\,(b_0 + b_1 + b^*)}. \qquad \text{(A·5)}$$

From Eqs. (A·2) and (A·5),

$$\frac{1}{\sqrt{d_{0i}}} < \frac{\gamma\, T_{\max}\,(C_{0i} - f_i)}{\sqrt{f_i}\,(b_0 + b_1 + b^*)}, \qquad \text{(A·6)}$$

or

$$C_i = f_i + \frac{b_0 + b_1 + b^*}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{0i}}} < C_{0i}, \qquad \text{(A·7)}$$

which implies that e_{0i} is still equal to 0 although link set combination is changed. In other words, link i with $e_{0i} = 0$ can belong to L_0 without any contradiction.

In the same manner, we can also prove that link i with $e_{2i} = 0$ can be assigned to L_2 without any contradiction. □

Suwan Runggeratigul received the B.E. degree in electrical engineering from Chulalongkorn University, Thailand in 1989, and the M.E. and D.Eng. degrees in electronic engineering from the University of Tokyo, Japan in 1992, and 1995, respectively. He was a visiting researcher at the National Center for Science Information Systems, and Nippon Telegraph and Telephone Multimedia Networks Laboratory, Japan from 1995 to 1997. He is now with Sirindhorn International Institute of Technology, Thammasat University, Thailand. His research interests include analysis of communication systems and network design.

Sawasd Tantaratana was born in Thailand. He received his B.E.E. degree from the University of Minnesota in 1971, M.S. degree from Stanford University in 1972, and Ph.D. degree from Princeton University in 1977, all in electrical engineering. He has spent most of his career in teaching in both Thai and U.S. universities: King Mongkut's Institute of Technology, Thonburi, Thailand (1972–1974 and 1977–1980); Auburn University, Alabama, USA (1981–1984); University of Massachusetts at Amherst, Massachusetts, USA (1986–1996); and Sirindhorn International Institute of Technology, Thammasat University, Pathumthani, Thailand (since 1997), where he is a Professor of Electrical Engineering. He was a visiting faculty member at the University of Illinois at Urbana-Champaign, Illinois, USA, from 1980 to 1981, a member of the Technical Staff at AT&T Bell Laboratories in Holmdel, New Jersey, USA, from 1984 to 1986, and a senior researcher at the National Electronic and Computer Technology Center, Bangkok, Thailand, from 1996–1997. His research interests are signal processing and communications.

Appendix B

Reprint of
Runggeratigul, S. (2000). "A Genetic Algorithms Approach to Communication Network Design taking into Consideration an Existing Network", *ScienceAsia*: Journal of the Science Society of Thailand, vol. 26, no. 3, pp. 181–186.

ScienceAsia 26 (2000) : 181-186

A Genetic Algorithms Approach to Communication Network Design taking into Consideration an Existing Network

Suwan Runggeratigul
Department of Electrical Engineering, Sirindhorn International Institute of Technology, Thammasat University, Pathumthani 12121, Thailand.
Corresponding author, Email: suwan@siit.tu.ac.th

Received 31 Mar 2000
Accepted 3 Jul 2000

Abstract This paper proposes the application of genetic algorithms (GAs) to solve communication network design problem taking into consideration existing network facilities. We study the link capacity assignment problem for packet-switched networks (CA problem) as an example of network design problem. In the CA problem, we focus on the case where link cost function is a piecewise linear concave function in which the per-unit cost of newly-installed link capacity is smaller than that of the existing link capacity. This type of cost function arises in many communication network design problems such as those arising from developments in communication transmission technologies. Experimental results show that GAs solve the CA problem very efficiently, and the solutions obtained from GAs are better than those from existing design algorithm.

KEYWORDS: communication network design, packet-switched network, link capacity assignment, genetic algorithms.

Introduction

It is well-known that utilizing existing network facilities in the construction of new network is very important in the case of short-term network design.[1] This is because existing network facilities are ready to be used and cannot be changed easily in a short period of time. For the short-term network design, we have studied the link capacity assignment problem for packet-switched networks with convex piecewise linear cost function, where the per-unit cost of using existing capacity is less than that of installing new capacity.[2] However, in other cases of network design such as long-term design, link cost function is not convex. In many cases, it is proper to consider the function as a concave function. This is because the economy of scale is often present in communication resources. For example, the cost function obtained by enveloping cost functions due to the development in communication transmission technologies,[3,4] the development in switching technologies,[5] *etc.*

The link capacity assignment problem (CA problem) for packet-switched networks taking into consideration an existing network where the link cost function is piecewise linear concave has been studied, and a heuristic design algorithm has been proposed for solving the problem.[6,7] However, it can be shown that this heuristic algorithm gives local optimal solutions for the CA problem of large-scale networks. Consequently, we need other methods to solve the CA problem more efficiently. This paper applies genetic algorithms (GAs) to the CA problem. Although there are several evolutionary algorithms developed for optimization problems, we choose GAs in this paper because the representation of the CA problem to GAs is very straightforward, and can be done by simple coding technique: fixed-length binary coding. Experimental results show that GAs solve the CA problem very efficiently, and the GAs solutions are better than those by the existing heuristic algorithm, especially in the case of large-scale networks.

Problem Description

This section describes the CA problem considered in this paper.

Network Model

The model of packet-switched network used in this paper is the same as the one used in the early work of packet-switched network design.[2,8,9] In this model, the average packet delay throughout the network, T, can be given by Eq.(1).

$$T=\frac{1}{\gamma}\sum_{L}\frac{f_i}{C_i-f_i}, \tag{1}$$

where L is the set of links in the network f_i is the traffic flow on link i (in bits/second), C_i is the capacity of link i (in bits/second), and γ is the overall traffic in the network (in packets/second). We assume that C_i is continuous and can be set as an arbitrary positive value. For simplicity, the total network cost D is defined as the sum of all link costs. Then, we have

$$D=\sum_{L}D_i. \tag{2}$$

For link i, link cost D_i is a piecewise linear function of link capacity as follows.

$$D_i=\begin{cases} d_{oi}C_i & ,C_i \le C_{oi} \\ d_{li}C_i-(d_{li}-d_{oi})C_{oi} & ,C_i > C_{oi} \end{cases} \tag{3}$$

where d_{oi} and d_{li} are respectively the per-unit cost of existing and newly-installed capacity, and C_{oi} is the value of the existing capacity of link i. Equation (3) shows that the cost function consists of two portions of link cost, *ie*, cost of using existing capacity and cost concerned with the installation of new capacity. In this paper, we focus on the case of concave cost function, where $d_{oi} > d_{li}$, $d_{oi} > 0$, $d_{li} > 0$, $\forall i \in L$. The concave link cost function is depicted in Fig 1.

This type of cost function arises in many communication network design problems. One of the example is the link type selection problem where the economy of scale is present in communication resources. A practical example with two alternatives of link types is given in Fig 2. It can be seen from this example that the link cost function is a piecewise linear function of link capacity, which is the envelope of cost functions of the two link types.

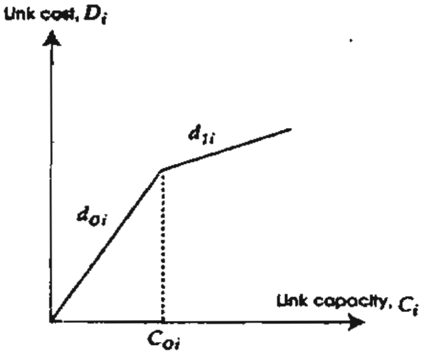


Fig 1. A piecewise linear concave link cost function.

Link Capacity Assignment Problem

In this paper, we investigate link capacity assignment for packet-switched networks in the case that new traffic pattern is applied to the existing network and then the capacity of each link is adjusted to cope with this new pattern, under the packet delay constraint and minimization of network cost. The CA problem can be formulated as follows.

CA Problem

Given: $\{f_i\}$

Minimize: $D=\sum_{L}D_i$

Variables: $\{C_i\}$

Subject to: $T=\frac{1}{\gamma}\sum_{L}\frac{f_i}{C_i-f_i}\le T_{max}$

$C_i-f_i>0,\ \forall i \in L$

$C_i>0,\ \forall i \in L$

where T_{max} is the maximum allowable packet delay in the network, and link cost function is that given in Eq (3).

In the above CA problem, $\{f_i\}$ is the traffic pattern applied to the network, and $f_i > 0$, $\forall i \in L$. Link capacity $\{C_i\}$ is determined so that the total network cost D is minimized. Design constraints are the upper limit of packet delay, and the relationship between link flow and link capacity. Obviously, link capacity must be greater than link flow so that packet delay will not grow indefinitely.

The optimal solution of the CA problem can be obtained by the following procedure.[2] First, we assign each link i in the network to be the member of one of the following three sets,

$L_0=\{i \mid C_i < C_{0i}\}$,
$L_1=\{i \mid C_i > C_{0i}\}$,
$L_2=\{i \mid C_i = C_{0i}\}$,

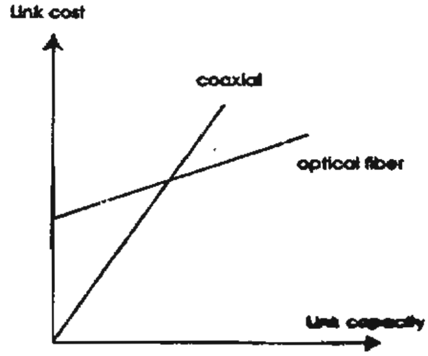


Fig 2. A practical example of link type selection.

then the solution of $\{C_i\}$ can be determined by Lagrange multiplier method as

$$C_i = f_i + \frac{\sum_{L_0 \cup L_1} \sqrt{f_j d_j}}{\left(\gamma T_{max} - \sum_{L_2} \frac{f_j}{C_{0j} - f_j}\right)} \sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L_0 \cup L_1 \quad (4)$$

where $d_i = d_{0i}$ if $i \in L_0$, and $d_i = d_{1i}$ if $i \in L_1$. Note that $C_i = C_{0i}$, $\forall i \in L_2$. With the assigned link set, we can determine the value of C_i in each link from Eq.(4). Then to find a global optimal solution, we have to examine all link set combinations such that the solution with minimum network cost is found.

For the case of concave cost function, *ie*, $d_{0i} > d_{1i}$, $\forall i \in L$, it has been proven mathematically that there is no link in the network whose capacity is equal to its existing capacity in the optimal solution.[6,7] In other words, $L_2 = \phi$. Based on this characteristic of the solution, we can exclude the point $C_i = C_{0i}$ from the cost function, which can then be treated as a differentiable function. Then a heuristic algorithm derived from Lagrange multiplier method can be proposed to solve the CA problem.[6,7]

The algorithm starts with initial value of d_i which is set as either d_{0i} or d_{1i} for each link i in the network. The Lagrange multiplier β is defined by

$$\beta = \left(\frac{\sum_{L} \sqrt{f_i d_i / \gamma}}{T_{max}}\right)^2. \qquad (5)$$

Link capacity C_i is then determined by

$$C_i = f_i + \sqrt{\frac{\beta f_i}{\gamma d_i}}, \qquad \forall i \in L. \qquad (6)$$

If the value of C_i calculated above is less than C_{0i}, d_i will be set as d_{0i}, otherwise it will be set as d_{1i}. The algorithm iteratively calculates the value of β, and terminates when β converges.

It has been reported that the above heuristic algorithm has good performance for problems of small networks. However, it will be shown later in this paper that the algorithm has a large chance to give local optima for large-scale networks. As a consequence, we need other methods that have better performance. In this paper, we propose the application of GAs for solving the CA problem.

GENETIC ALGORITHMS

Genetic algorithms (GAs) are heuristic search method derived from the mechanisms of natural selection and natural genetics.[10,11] GAs have been adopted to solve various engineering design problems including communication network design.[12,13,14] This section discusses the implementation of GAs for solving the CA problem.

Encoding

From the characteristic of the optimal solution of the CA problem, i.e., $L_2 = \phi$, we have that each link in the network belongs to either L_0 or L_1. Since there are only two possibilities for each link, it is clear that binary representation can be used for the encoding of the CA problem as follows. Each candidate solution in the search space is encoded as a binary string of length n, where n is the total number of links in the network. Here, bit i of a string is set to be 0 when link i belongs to L_0, and 1 when it belongs to L_1.

The fitness value of any string is defined as the network cost D corresponding to the link set assigned by bits in the string. Since $L_2 = \phi$, Eq.(4) becomes

$$C_i = f_i + \frac{\sum_{L} \sqrt{f_j d_j}}{\gamma T_{max}} \sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L, \qquad (7)$$

where

$$d_i = \begin{cases} d_{0i}, \text{ when bit } i \text{ is } 0, \\ d_{1i}, \text{ when bit } i \text{ is } 1. \end{cases} \qquad (8)$$

By using Eq.(7), we can determine the capacity C_i of link i for a given string. Link cost D_i can then be calculated by Eq.(3), and the sum of all link costs is the network cost D, which will be used as the fitness value of the string. Since the CA problem is a minimization problem, a string with low network cost is the one that has good fitness value.

Initial Population

The population size of 100 is used in this paper. We consider three variants of initial population as follows.

- ga0 — 100 strings are randomly generated as initial population
- ga1 — the heuristic algorithm described in the previous section is executed for 100 independent runs, and the solutions obtained are used as initial population

ga2 50 strings are generated at random, and the other 50 strings are from the heuristic algorithm

For the strings which are randomly generated, each bit in the string is set to be 0 or 1 with equal probability.

Parent Selection

This paper uses tournament selection[10,11] to select a pair of parent strings for the crossover process. First, we choose a pair of different strings randomly from the population, and compare their fitness values. The string with better fitness value (the one that gives lower network cost) is used as father string. Next, we randomly choose two more different strings, and evaluate their fitness values. The string with better fitness is then set as mother string. If the father and mother strings are the same string, we then repeatedly choose mother string until the different one is obtained. Each string in the population has equal probability to be chosen to enter the tournament comparison, i.e., 1/100. There are totally 100 pairs of parents chosen in this process.

Crossover

There are many types of crossover proposed in the literature, *eg*, one-point, two-point, multi-point, uniform crossover. Since uniform crossover is a general form of other types of crossover mentioned above,[15] we use uniform crossover[10,11,16] to create two child strings from two parent strings in this paper. The crossover mask for each pair of parents is generated as a binary string of length n, where each bit is randomly set to 0 or 1 with the same probability. From the crossover mask, the bit of the first (second) child string will be set to be the same as that of the father string if the mask bit is 0 (1), and as the bit of the mother string if the mask bit is 1 (0). One example of crossover process is given below with the case $n = 7$.

father string	1001100
mother string	0010111
crossover mask	0110011
child string #1	1011111
child string #2	0000100

At the end of the crossover process, the number of child strings is 200.

Mutation

The mutation operator is applied to all bits in all child strings with mutation probability of 0.01. If the probability test is passed, the bit will be mutated to its complement, *ie*, 0 to 1, or 1 to 0.

Population Replacement

To find the population of the next generation, we compare the fitness value of two child strings produced from the same pair of parents, and the one with better fitness value will be survived to the next generation. After that, elitism[11] is applied by discarding one string in the new population and replacing it with the best string in the former population.

Termination Criterion

This paper applies GAs to solve the CA problem for 1000 generations, and the best solution obtained so far is then used as the solution by GAs.

Numerical Results and Discussion

First, we examine the computation time of applying GAs to the CA problem. A network with fully connected topology is considered and the machine used in the experiment is a personal computer with CPU Pentium III 450 MHz. The result of computation time is given in Table 1, where it can be seen that the computation time of GAs is approximately linear with respect to the number of links in the network. (By linear regression, the value of linear correlation coefficient is very close to 1.)

Then we compare the performances of the existing heuristic algorithm and GAs with randomly generated initial population (ga0). To do this, we let the existing heuristic algorithm execute for a number

Table 1. Computation time of GAs when solving the CA problem.

n	computation time (sec.)
45	12.254
190	61.610
435	119.068
780	216.280
1225	342.084
1770	495.544
2415	676.684
3160	884.486
4005	1122.746
4950	1388.482

of independent runs, where the total execution time is kept to be the same as that of GAs, and the best solution obtained is used as the solution by the algorithm. A network with fully connected topology is considered again with one hundred random traffic patterns. The comparison of solutions by the two methods are given in Table 2, where the results are the number of cases that GAs give better, the same, or worse solutions when compared with the heuristic algorithm. We can see from the results that the solutions by GAs are never worse than those by the heuristic algorithm. Furthermore, GAs always give better solutions for large-scale networks. This means that GAs solve the CA problem more efficiently than the existing heuristic algorithm.

Next, we compare the performances of ga0, ga1, and ga2. The results are given in Table 3-5. For the CA problem of small networks, the performances of the three methods are almost the same. However, in the case of large-scale networks, ga1 and ga2 always perform better than ga0. For the performance comparison between ga1 and ga2, we have ga1 is better than ga2 in average. From these results, we can say that to obtain a good GAs solution for the CA problem, a good initial population should be used, and this can be generated by using the solutions by the existing heuristic algorithm.

Conclusions

In this paper, we have proposed the application of genetic algorithms (GAs) to the link capacity assignment problem (CA problem) for packet-switched networks with the consideration of existing facilities. The link cost function is considered to be piecewise linear-concave which is the case that can be found in long-term network design. It has been shown that GAs can be applied to the CA problem very easily where the solution candidates are represented by fixed-length binary coding. Experimental results show that GAs outperform the existing heuristic algorithm. It is also recommended

Table 2. Performance comparison between ga0 and the heuristic algorithm.

n	ga0 is better	same	ga0 is worse
45	0	100	0
190	11	89	0
435	99	1	0
780	100	0	0
1225	100	0	0
1770	100	0	0
2415	100	0	0
3160	100	0	0
4005	100	0	0
4950	100	0	0

Table 3. Performance comparison between ga0 and ga1.

n	ga0 is better	same	ga1 is better
45	0	100	0
190	3	97	0
435	14	33	53
780	2	0	98
1225	1	0	99
1770	0	0	100
2415	0	0	100
3160	0	0	100
4005	0	0	100
4950	0	0	100

Table 4. Performance comparison between ga0 and ga2.

n	ga0 is better	same	ga2 is better
45	0	100	0
190	1	99	0
435	9	37	54
780	2	0	98
1225	1	0	99
1770	0	0	100
2415	0	0	100
3160	0	0	100
4005	0	0	100
4950	0	0	100

Table 5. Performance comparison between ga1 and ga2.

n	ga1 is better	same	ga2 is better
45	0	100	0
190	0	[illegible]	[illegible]
435	0	92	[illegible]
780	[illegible]	[illegible]	[illegible]
1225	[illegible]	[illegible]	[illegible]
1770	40	[illegible]	[illegible]
2415	71	1	28
3160	57	0	43
4005	58	0	42
4950	65	0	35

that using good initial population leads to good GAs solution for the CA problem, where the good initial population can be obtained by the existing heuristic algorithm.

The following topics can be considered as future works.

- Performance comparison between GAs and other search methods, *eg*, random search, simulated annealing, tabu search, *etc*.
- Improvement of GAs solutions such as by using appropriate initial population obtained from some search methods mentioned above.

References

1. Kawashima K and Kobayashi H (1993) Network planning tools. *Jour of IEICE* 76, 141-8.
2. Runggeratigul S, Zhao W, Ji Y, Aizawa A and Asano S (1995) Link capacity assignment in packet-switched network with existing network consideration. *IEICE Trans Commun* E78-B, 709-19.
3. Yaged B Jr (1971) Minimum cost routing for static network models. *Networks* 1, 139-72.
4. Gersht A and Weihmayer R (1990) Joint optimization of data network design and facility selection. *IEEE J Select Area Commun* 8, 1667-81.
5. Verma PK (1989) *Performance estimation of computer communication networks*, pp.116-30. Computer Science Press, Rockville.
6. Runggeratigul S and Tantaratana S (1998) Communication network design taking into consideration an existing network: the case of concave cost function. In *Proceedings of The Second International Conference on Engineering Design and Automation (EDA'98)*, MS#79.
7. Runggeratigul S and Tantaratana S (1999) Link capacity assignment in packet-switched networks: the case of piecewise linear concave cost function. *IEICE Trans Commun* E82-B, 1566-76.
8. Kleinrock L (1976) *Queueing systems, volume II*, pp.314-60. John Wiley & Sons, New York.
9. Gerla M and Kleinrock L (1977) On the topological design of distributed computer networks. *IEEE Trans Commun* COM-25, 48-60.
10. Reeves CR (1996) Modern heuristic techniques. In *Modern heuristic search methods* (Edited by Rayward-Smith VJ, Osman IH, Reeves CR and Smith GD), pp.17-23. John Wiley & Sons, Chichester.
11. Gen M and Cheng R (1997) *Genetic algorithms and engineering design*, pp.1-34. John Wiley & Sons, New York.
12. Elbaum R and Sidi M (1996) Topological design of local-area networks using genetic algorithms. *IEEE/ACM Trans Netw* 4, 766-78.
13. Chakraborty M and Chakraborty UK (1997) Applying genetic algorithm and simulated annealing to a combinatorial optimization problem. *In Proceedings of International Conference on Information, Communication and Signal Processing (ICICS'97)*, 2C2.6, pp.929-33.
14. Tang K, Ko K, Man KF and Kwong S (1998) Topology design and bandwidth allocation of embedded ATM networks using genetic algorithm. *IEEE Comm Letters* 2, 171-3.
15. Michalewicz Z (1999) *Genetic algorithms + data structures = evolution programs*, pp.88-90. Springer-Verlag, New York.
16. Syswerda G (1989) Uniform crossover in genetic algorithms. In *Proceedings of the Third International Conference on Genetic Algorithms (ICGA'89)*, pp.2-9.

Appendix C

Manuscript of
Runggeratigul, S. (2001). "A Memetic Algorithm to Communication Network Design taking into Consideration an Existing Network", accepted by the 4th Metaheuristics International Conference.

A Memetic Algorithm to Communication Network Design taking into Consideration an Existing Network

Suwan Runggeratigul *

* Electrical Engineering Program, Sirindhorn International Institute of Technology
Thammasat University, Pathumthani 12121, Thailand.
Email: suwan@siit.tu.ac.th

1 Introduction

It is well-known that utilizing existing network facilities in the construction of new network is very important in the case of short-term network design [1]. The reason is that existing network facilities are ready to be used and cannot be changed easily in a short period of time. For the short-term network design, we have studied the link capacity assignment for packet-switched networks with convex piecewise linear cost function, where the per-unit cost of using existing capacity is less than that of installing new capacity [2]. However, in other cases of network design such as long-term design, it is proper to consider the function as a concave function since the economy of scale is often present in communication resources. Some good examples are the cost function obtained by enveloping cost functions due to the development in communication transmission/switching technologies [3][4][5].

The link capacity assignment problem (CA problem) for packet-switched networks taking into consideration an existing network where the link cost function is piecewise linear concave has been studied, and a heuristic design algorithm (we refer to it as HDA in this paper) has been proposed to solve the problem [6]. Genetic algorithms (GAs) are also applied to the CA problem, and it has been shown that GAs solve the problem more efficiently than HDA [7]. However, although GAs efficiently solve several optimization problems, it has been reported that the search ability of GAs to find optimal solutions in some problems is a bit inferior to local search techniques [8]. Consequently, there are many efforts to consider hybrid version of GAs by combining the evolutionary operators with local search operators. This new class of methods are known as memetic algorithms (MAs) [9][10]. Since MAs have been successfully applied to a wide range of problems, this paper then investigates the capability of applying MA to the CA problem. To design an MA, we consider HDA as local search operator and implement it after each evolutionary operator. Experimental results show that MA solves the CA problem very efficiently, and it outperforms GA, especially in the case of large-scale networks.

2 Problem Description

The packet-switched network model in this paper is the same as the one used in the early works of packet-switched network design [2][11][12]. In this model, the average packet delay throughout the network, T, can be given by Eq.(1).

$$T = \frac{1}{\gamma} \sum_{L} \frac{f_i}{C_i - f_i}, \tag{1}$$

where L is the set of links in the network, f_i is the traffic flow on link i (in bits/second), C_i is the capacity of link i (in bits/second), and γ is the overall traffic in the network (in packets/second). The total network cost D is defined as the sum of all link costs,

$$D = \sum_L D_i. \tag{2}$$

For link i, link cost D_i is a piecewise linear function of link capacity as follows.

$$D_i = \begin{cases} d_{0i}C_i & , \quad C_i \leq C_{0i}, \\ d_{1i}C_i - (d_{1i} - d_{0i})C_{0i} & , \quad C_i > C_{0i}, \end{cases} \tag{3}$$

where d_{0i} and d_{1i} are respectively the per-unit cost of existing and newly-installed capacity, and C_{0i} is the value of the existing capacity of link i. In this paper, we focus on the case of concave cost function, where $d_{0i} > d_{1i}$, $d_{0i} > 0$, $d_{1i} > 0$, $\forall i \in L$. The concave link cost function is depicted in Fig.1.

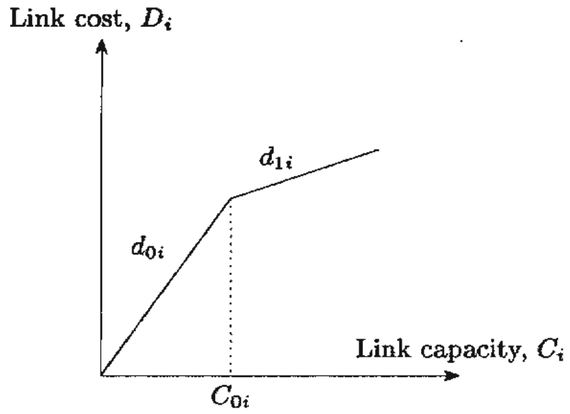


Figure 1: A piecewise linear concave link cost function.

This type of cost function arises in many communication network design problems. One example is the link type selection problem where the economy of scale is present in communication resources. We then formulate the CA problem as follows.

CA Problem

Given: $\{f_i\}$, $\{C_{0i}\}$, $\{d_{0i}\}$ and $\{d_{1i}\}$

Minimize: $D = \sum_L D_i$

Variables: $\{C_i\}$

Subject to: $T = \frac{1}{\gamma}\sum_L \frac{f_i}{C_i - f_i} \leq T_{\max}$

$C_i - f_i > 0, \quad C_i > 0, \quad \forall i \in L$

where $T_{\max}$ is the maximum allowable packet delay in the network. In the CA problem, $\{f_i\}$ is a new traffic pattern applied to an existing network. Link capacity $\{C_i\}$ is determined to cope with the new traffic pattern such that the total network cost D is minimized. Design constraints are the upper limit of packet delay, and the relationship between link flow and link capacity. Obviously, link capacity must be greater than link flow so that packet delay will not grow indefinitely. It can be shown that the complexity of the CA problem is $O(2^n)$, where n is the number of links in the network.

3 Memetic Algorithm

Memetic algorithms (MAs) are metaheuristics that take the advantage of the evolutionary operators in determining interesting regions of the search space, as well as the local search operator in rapidly finding good solutions in a small region of the search space [10]. In this paper, we propose the following memetic algorithm for the CA problem.

Memetic Algorithm (MA)

Step 1 Initialization: generate individuals for initial population.

Step 2 Local search: apply local search to each individual.

Step 3 Parent selection: select pairs of parents for crossover process.

Step 4 Crossover: produce two offsprings from each pair of parents.

Step 5 Local search: apply local search to each offspring.

Step 6 Mutation: perform mutation on each offspring.

Step 7 Local search: apply local search to each offspring after mutation.

Step 8 Population replacement: select offsprings for the population in the next generation.

Step 9 Termination: If the predefined number of generations have been completed, then STOP. Otherwise, go to Step 3.

The implementation of MA is described below in two parts: genetic algorithm and local search operator.

3.1 Genetic Algorithm

The part of genetic algorithm (GA) in the MA is the same as the one used in [7] for the CA problem. Each candidate solution of the CA problem in the search space is encoded as a binary string of length n, where n is the total number of links in the network. Here, bit i of a string is set to be 0 when $C_i < C_{0i}$, and 1 when $C_i > C_{0i}$ (it has been proven mathematically that there is no link i that $C_i = C_{0i}$ [6]). The fitness value of a string is defined as the network cost D corresponding to the bits in the string. To calculate the fitness of a string, we firstly determine the link capacity C_i by Eq.(4).

$$C_i = f_i + \frac{\sum_L \sqrt{f_j d_j}}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L, \tag{4}$$

where

$$d_i = \begin{cases} d_{0i}, & \text{when bit } i \text{ is } 0, \\ d_{1i}, & \text{when bit } i \text{ is } 1. \end{cases} \tag{5}$$

Link cost D_i can then be calculated by Eq.(3), and the sum of all link costs is the network cost D, which is used as the fitness value of the string.

The population size of 100 is used, and 100 strings are randomly generated as initial population, where each bit in the string is set to be 0 or 1 with equal probability.

Since the CA problem is a minimization problem, we avoid the problem of fitness value scaling by using tournament selection to select a pair of parents for the crossover process. Each string in the population has equal probability to be chosen to enter the tournament comparison. We use uniform crossover to create two offsprings from each pair of parents. The crossover mask is generated as a binary string of length n, where each bit is randomly set to 0 or 1 with the same probability. The mutation operator is applied to all bits in all offsprings with mutation probability of 0.01.

To determine the population of the next generation, we compare the fitness value of two offsprings produced from the same pair of parents, and the one with better fitness value will be survived to the next generation. After that, elitism is applied by randomly discarding one string in the new population and replacing it with the best string in the previous population.

3.2 Local Search Operator

To solve the CA problem with concave cost function, a heuristic design algorithm (HDA) derived from Lagrange multiplier method has been proposed [6]. The algorithm starts with initial value of d_i which is set as either d_{0i} or d_{1i} for each link i in the network. The Lagrange multiplier β is defined by

$$\beta = \left(\frac{\sum_L \sqrt{f_i d_i / \gamma}}{T_{\max}} \right)^2 . \tag{6}$$

Link capacity C_i is then determined by

$$C_i = f_i + \sqrt{\frac{\beta f_i}{\gamma\, d_i}}, \qquad \forall i \in L. \tag{7}$$

If the value of C_i calculated above is less than C_{0i}, d_i will be set as d_{0i}, otherwise it will be set as d_{1i}. The algorithm iteratively calculates the value of β, and terminates when β converges.

We consider HDA as a local search operator in our MA, by applying it to every string newly generated by GA (at Steps 2, 5 and 7). The initial value of d_i will be set as d_{0i} (d_{1i}) if bit i is 0 (1). HDA will iteratively search for a local optimum of a given string.

4 Numerical Results

In this section, we compare the performance of MA and GA (the MA without local search). For a fair comparison, we let the two methods solve a given problem instance in the same running time. This makes the number of generations of applying MA and GA to a problem be 1000 and 2500, respectively. The best solution obtained from each method is used as the final solution of that method.

A network with fully connected topology is considered with 100 random traffic patterns. The comparison of the two methods are given in Table 1. The results in the first part are the number of cases that MA gives better, the same, or worse solutions when compared with GA. The second part is results on the ratio between solutions of MA and GA (average, minimum, maximum). Note that GA is the current best known method for the CA problem [7]. The results show that the solutions by MA are never worse than those by GA. Furthermore, MA always gives better solutions for large-scale networks, and the rate of improvement in the quality of solution increases when the network size increases.

5 Conclusions

In this paper, we have proposed the application of memetic algorithm (MA) to the link capacity assignment problem (CA problem) for packet-switched networks with the consideration of existing

Table 1: Performance comparison between MA and GA.

n	MA is better	same	MA is worse	avg ratio	min ratio	max ratio
45	0	100	0	1.00000	1.00000	1.00000
190	30	70	0	0.99999	0.99982	1.00000
435	95	5	0	0.99997	0.99985	1.00000
780	100	0	0	0.99990	0.99972	0.99998
1225	100	0	0	0.99979	0.99944	0.99994
1770	100	0	0	0.99967	0.99929	0.99985
2415	100	0	0	0.99954	0.99917	0.99976
3160	100	0	0	0.99937	0.99907	0.99963
4005	100	0	0	0.99919	0.99892	0.99940
4950	100	0	0	0.99902	0.99870	0.99926

network facilities. The link cost function is considered to be piecewise linear concave. The MA proposed in this paper is designed by combining a local search operator to evolutionary operators of genetic algorithm (GA), where the local search is the heuristic algorithm previously proposed for the CA problem. Experimental results show that MA outperforms GA in all tested problems.

References

[1] Kawashima K and Kobayashi H (1993) Network planning tools. *Jour of IEICE* **76**, 141–148.

[2] Runggeratigul S, Zhao W, Ji Y, Aizawa A and Asano S (1995) Link capacity assignment in packet-switched network with existing network consideration. *IEICE Trans Commun* **E78-B**, 709–719.

[3] Yaged B Jr (1971) Minimum cost routing for static network models. *Networks* **1**, 139–172.

[4] Gersht A and Weihmayer R (1990) Joint optimization of data network design and facility selection. *IEEE J Select Area Commun* **8**, 1667–1681.

[5] Verma PK (1989) *Performance estimation of computer communication networks*, pp.116–130. Computer Science Press, Rockville.

[6] Runggeratigul S and Tantaratana S (1999) Link capacity assignment in packet-switched networks: the case of piecewise linear concave cost function. *IEICE Trans Commun* **E82-B**, 1566–1576.

[7] Runggeratigul S (2000) Application of genetic algorithms to communication network design taking into consideration an existing network. In *Proc. 2-nd Asia-Pacific Conference on Genetic Algorithms and Applications*, pp.323–331.

[8] Ishibuchi H, Murata T and Tomioka S (1997) Effectiveness of genetic local search algorithms. In *Proc. 7-th Int. Conf. Genetic Algorithms*, pp.505–512.

[9] Moscato P (1999) Memetic algorithms: a short introduction. In *New ideas in optimization* (Edited by Corne D, Dorigo M and Glover F), pp.219–234. McGraw-Hill, London.

[10] Merz P and Freisleben B (1999) A comparison of memetic algorithms, tabu search, and ant colonies for the quadratic assignment problem. In *Proc. 1999 Congress on Evolutionary Computation*, pp.2063–2070.

[11] Kleinrock L (1976) *Queueing systems, volume II*, pp.314–360. John Wiley & Sons, New York.

[12] Gerla M and Kleinrock L (1977) On the topological design of distributed computer networks. *IEEE Trans Commun* **COM-25**, 48–60.