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รายงานวิจัยฉบับสมบูรณ์

การออกแบบโครงข่ายสื่อสารโดยการพิจารณาโครงข่ายที่ใช้งานอยู่แล้ว

Communication Network Design with the Consideration of Existing Network

โดย สุวรรณ รุ่งกีรติกุล

เมษายน 2544

สัญญาเลขที่ PDF/33/2541

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สุวรรณ รุ่งกีรติกุล
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สนับสนุนโดย สำนักงานกองทุนสนับสนุนการวิจัย

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Project Period: 2 ปี 10 เดือน (1 กรกฎาคม 2541 – 30 เมษายน 2544)

Objectives: วัตถุประสงค์หลักของงานวิจัยนี้มุ่งเน้นไปที่การศึกษาปัญหาการออกแบบโครงข่ายสื่อสาร 2 ประเภทคือ โครงข่ายการสวิตช์กลุ่มข้อมูล และโครงข่าย ATM โดยให้มีการพิจารณาผลกระทบของโครงข่ายที่ใช้งานอยู่แล้ว

Methodology: ปัญหาการออกแบบโครงข่ายสื่อสารข้างต้นจะถูกแปลงให้อยู่ในรูปของปัญหาการทำให้เหมาะที่สุด และจากการวิเคราะห์จะทำให้ทราบคุณสมบัติของคำตอบที่เหมาะที่สุดของปัญหาเหล่านี้ ซึ่งจะนำไปสู่อัลกอริทึมสำหรับการแก้ปัญหาการออกแบบโครงข่าย

Results: อัลกอริทึมสำหรับการแก้ปัญหาการออกแบบโครงข่ายการสวิตช์กลุ่มข้อมูล และโครงข่าย ATM เมื่อพิจารณาโครงข่ายที่ใช้งานอยู่แล้ว

Discussion Conclusion: อัลกอริทึมที่ได้จากการศึกษามีคุณสมบัติที่ดีมากทั้งในแง่ของคุณภาพของคำตอบและระยะเวลาในการคำนวณ ดังนั้นจึงเหมาะสำหรับใช้ในการแก้ปัญหาการออกแบบโครงข่ายจริง

Suggestions: ควรมีการศึกษาเปรียบเทียบคุณสมบัติของอัลกอริทึมที่ได้จากการศึกษานี้ กับวิธีอื่นๆ เช่นวิธีทาง metaheuristics

Keywords: การออกแบบโครงข่ายสื่อสาร, โครงข่ายที่ใช้งานอยู่แล้ว, โครงข่ายการสวิตช์กลุ่มข้อมูล, โครงข่าย ATM, อัลกอริทึม

Abstract

Project Code: PDF/33/2541

Project Title: Communication Network Design with the Consideration of Existing Network

Investigator: Asst.Prof.Dr.Suwan Runggeratigul, Electrical Engineering Program, Sirindhorn International Institute of Technology, Thammasat University

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Project Period: 2 years 10 months (1 July 1998 – 30 April 2001)

Objectives: The main objective of this project is to study the communication network design problems under the consideration of existing network facilities in packet-switched networks and ATM networks.

Methodology: The network design problems are formulated as optimization problems, which are analyzed to obtain the characteristics of optimal solutions. Based on these solution characteristics, heuristic algorithms are proposed to solve the problems.

Results: Two heuristic design algorithms are obtained for solving communication network design problems taking into consideration existing network facilities, where each of which is for packet-switched networks and ATM networks, respectively.

Discussion Conclusion: The proposed algorithms have very good performances in both the quality of solution and the computation time, and thus they are applicable to practical network design problems.

Suggestions: The performances of the proposed design should be compared with other design algorithms, e.g., algorithms based on metaheuristics.

Keywords: communication network design, existing network, packet-switched networks, ATM networks, algorithms.

Final Report

Communication Network Design with the Consideration of Existing Network

by Suwan Runggeratigul

April 2001

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Suwan Runggeratigul
30 April 2001

Executive Summary

In the conventional communication network design, the concept of existing network is rarely taken into account. The main reason is that existing network facilities are treated as parts of network that are ready to be used in the construction of new network. Almost of the previous research works considered that the existing parts of network can be used without any cost. However, it is clear that the above idea is not applicable in practical cases. For example, although they are promptly to be used, there is cost of using the existing network facilities such as operating or maintenance cost.

In this research, we investigate the effect of existing network facilities to the design of communication network by introducing the cost difference between the existing network facilities and newly installed facilities. We focus on two communication network design problems, namely the link capacity assignment problem (CA problem) in packet-switched networks, and the virtual path routing problem (VPR problem) in asynchronous transfer mode networks (ATM networks).

For the CA problem in packet-switched networks, we investigate the case of long-term design by applying a piecewise linear concave link cost function, where the per-unit cost of existing link capacity is greater than that of newly installed capacity. By mathematical analysis, it is shown that there is no link whose capacity is equal to its existing capacity in the optimal solution of the problem. Based on this characteristic, the non-differentiable cost function can be treated as a differentiable function, and a design algorithm derived from the Lagrange multiplier method is then proposed.

For the VPR problem in ATM networks, we consider the cases of both short-term and long-term design by considering piecewise linear convex and concave link cost function, respectively. It is shown that the VPR problem has similar properties to shortest path problems. Accordingly, we propose a design algorithm derived from shortest path algorithm to solve the problem.

By numerical results, it is shown that the proposed design algorithms have very good performance in both the view of quality of solutions and computation time. This means that they are applicable to solve practical communication network design problems, especially for the case of large-scale networks.

The numerical results also show that how existing network affects the design of new network. In the case of short-term design, it is necessary to utilize the existing network facilities in a full range before installing new facilities to the network. This reflects the properties of existing network facilities in the short-term design. That is the existing parts of the network are promptly to be used in the construction of new network, and cannot be changed within a short period of time. Consequently, using existing facilities effectively before introducing new facilities is a very important design concept in this case.

On the other hand, it is shown in the case of long-term design that the above concept can be neglected when designing a new network. This result obviously reflects the properties of economy of scale in communication network resource for the case of long-term design. Hence, it is possible to augment new network facilities to the existing network where there are needs, while some parts of existing facilities can be left unused.

Chapter 1

Introduction

Many people stated that the current age is the information age [9][58][77][96]. With the advance of information-oriented society, communication networks play a key role as major means for transferring information between humans. Thus, networks that provide communication services to users efficiently, reliably, and economically are essentially required [102]. To construct such kind of networks, high-performance design method that considers several design factors is needed, where the method must be capable to give optimal solution (or near-optimal solution with high accuracy) in an acceptable running time.

The work in this research project is related to the study on communication network design that focuses on how the existing network facilities affects the design of a new network. This chapter briefly describes the statement of the problem, and gives the scope and objectives of the study.

1.1 General

Let's consider the graph given in Figure 1.1. This graph is a mathematical representation of a communication network, where each vertex or node represents a communication point, and each edge or link is equivalent to a communication channel connecting two communication points [2][10][50].

As an example of network design problem, we focus on the following case. When design parameters including the user traffic demand (forecasted value) are given, the capacity of each link in the network is determined such that all conditions of the design constraint are satisfied. One feasible solution to this problem is that we can set each link capacity to infinity. However, it is obvious that this is not an effective solution, i.e., the cost of implementing the network is also infinite. Consequently, we need one more requirement in the design, that is the cost of network implementation must be minimized. With this requirement, each link capacity will then be set to an optimal value. We call this design problem as the capacity assignment problem [10][52].

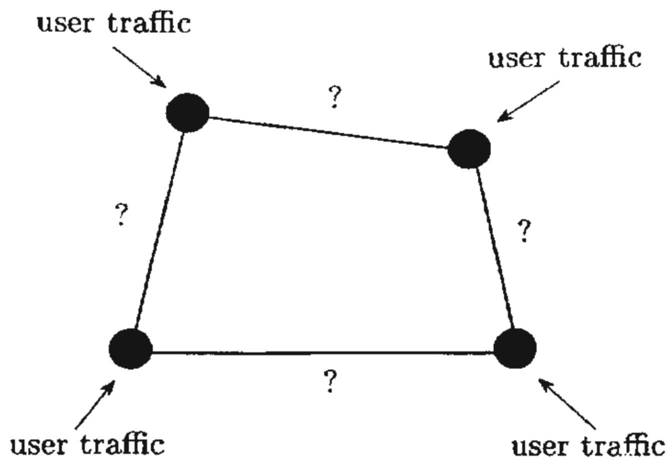


Figure 1.1: Design of a communication network.

The phase after the network design is the network implementation. After that, it will be the phase of network operation, where actual user traffic demand is applied to the network. However, after a period of running the network, the need of network redesign occurs [10][103]. The reasons that a network is needed to be rebuilt or redesigned can be listed as follows [103].

- There is a substantial increase in the number of users in the network.
- A substantial change occurs in the computation power of users' workstation machines.
- New applications emerge and demand more or different network services.

Clearly, the events in the above reasons result in the change of the user traffic demand to the network. With this new traffic demand, it is necessary to redesign the existing network as in Figure 1.2.

Besides the new user traffic demand, many design parameters and factors must be considered in the network redesign problem. New design constraint such as better quality of service (QoS) may be needed according to the new class of services introduced to the network. However, one of the most important design factors is the network facilities that are already implemented in the network. Although there are a lot of works that study the communication design/redesign problems in the literature [10][50][104], the concept of existing network, however, is rarely taken into account.

The main reason that the effect of existing network is usually neglected in the conventional study on communication network design is that existing network facilities are assumed as parts of network that are ready to be used in the construction of new network. Almost of the previous research works consider that the existing parts of network can be used without any cost, or can be treated in the same way as new facilities needed to be installed to the network [10][104]. However, it is clear that this idea is not generally true in practical cases. For example, although the existing network facilities are promptly to be used, there is cost of using them such as operating or maintenance cost. Moreover,

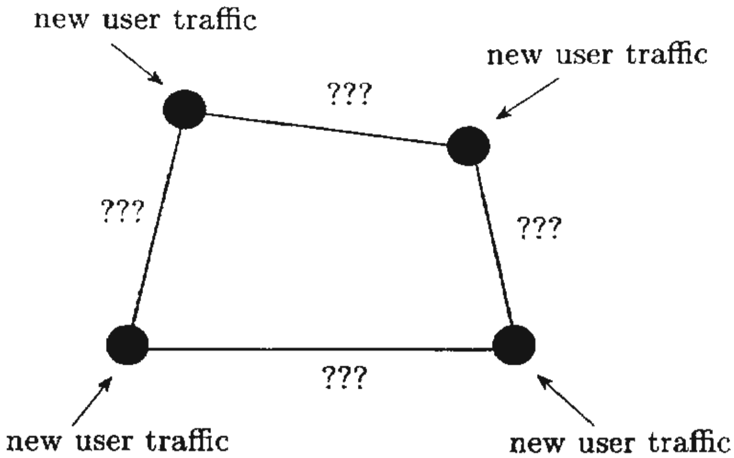


Figure 1.2: Redesign of a communication network.

the cost of using the existing network facilities needs not be the same as that of newly installed facilities.

Although most of the studies on communication network design/redesign do not include the concept of existing network in the design of new network, there are a few research works that study this major design factor as well as its effect to network design problem. These works are briefly described below.

Firstly, Shinohara studies the circuit dimensioning problem in circuit-switched networks in [92][93], where the design problem is formulated and solved under the condition that the number of circuits in the existing network is not zero. To include this design condition in the problem, a link cost function shown in Figure 1.3 is introduced to be applied to the design problem.

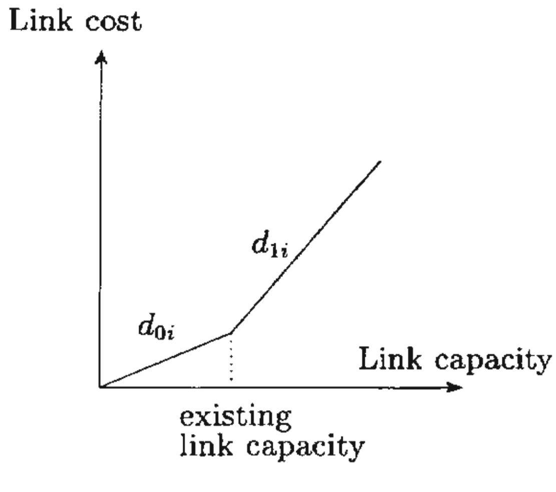


Figure 1.3: Link cost function taking into consideration an existing network: a piecewise linear convex function.

The cost function in Figure 1.3 is a piecewise linear convex function, where d_{0i} and d_{1i} are the per-unit cost of existing capacity and newly installed capacity on link i, respectively. The value of d_{0i} and d_{1i} are both non-negative. Shinohara examines the case that

$$d_{0i} \leq d_{1i}, \tag{1.1}$$

that is the per-unit cost of existing capacity is less than that of the newly installed capacity. This case is generally valid in network construction, especially in the case of short-term design. By taking the cost difference between the two types of link capacity (i.e., the existing part and the newly implemented part), the circuit dimensioning problem is solved to determine the number of circuits in all network links such that the total network cost is minimum, and the effect of the existing network to the network design can be investigated.

The other research work in the literature that takes into account the concept of existing network in network design problem is the work by Runggeratigul et. al. [80][81], where the link capacity assignment problem in packet-switched networks (short-term case) is studied. This work also applies the idea of the cost difference between existing capacity and new capacity to the packet-switched network design problem, and the capacity cost function as shown in Figure 1.3 is adopted. The main result of this study is that, the proposed design algorithm can give a solution in the way that the existing network facilities are utilized effectively, and new facilities are augmented to the existing network only in the places that there are needs.

Clearly, the cost function of Figure 1.3 used in the conventional research works is very useful in communication network design when the effect of existing network facilities is needed to consider. Moreover, this type of cost function is also applicable to other works on network design where the concept of existing network is omitted.

Firstly, when the cost of using existing network facilities is zero, we can set the value of d_{0i} as

$$d_{0i} = 0, \tag{1.2}$$

and the cost function as shown in Figure 1.4 is obtained.

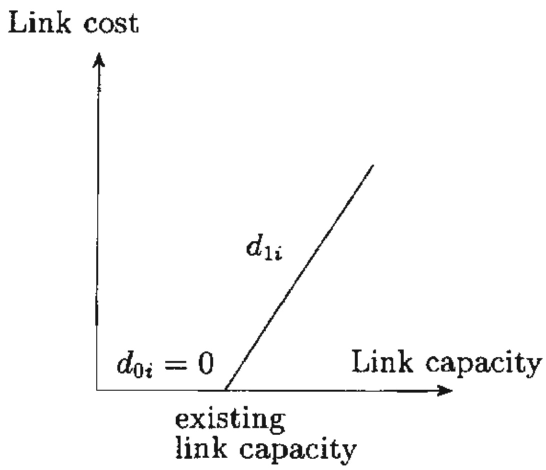


Figure 1.4: Cost function when there is no cost in using existing network facilities.

Secondly, when existing and new network facilities are treated in the same way, we can set the values of d_{0i} and d_{1i} as follows.

$$d_{0i} = d_{1i}, \tag{1.3}$$

Obviously, the cases in Eq.(1.2) and Eq.(1.3) are special cases both included in Eq.(1.1). This means that the research works using cost function of Eq.(1.1) is a more general case of the previous works.

Although there are already a few studies on network design with the consideration of existing network, this kind of study, however, is needed to be extended. The reasons can be given as follows.

- There are questions that the concept of cost difference between existing facilities and newly installed facilities is applicable or not to the design/redesign of other types of communication network besides the circuit-switched and packet-switched networks.
- Since the conventional works only study network design problem in the short-term case, it is interesting to investigate also the effect of existing network facilities in the case of long-term design.

The work in this research project is devoted to answer the above questions on network design with the consideration of existing network.

1.2 Scope of the Study

To extend the study in conventional research works, we will focus on the design of the following two types of communication networks.

1. Packet-switched networks.
2. Asynchronous transfer mode (ATM) based broadband integrated networks (ATM networks).

There are reasons why this project studies the above networks. Packet-switched network is the typical form of the Internet, which is the interconnection of communication networks around the world [74][96]. Nowadays, there are a large number of Internet users in almost of the parts of the earth, and the number increases gradually in a very high rate [14][60].

On the other hand, ATM is an advanced multiplexing and switching technique, which is recently accepted as the "target transfer mode" for the implementation of future broadband integrated networks [55][59][74]. Here, the term *broadband* means that ATM network can support high-speed communications, while *integrated* means it can support communications of all media of communication information including voice, image, video, and computer data.

Hence, the study in this research project has applications to the design of present and future communication networks, where we concentrate the study on the following network design problems.

1. Link capacity assignment problem (CA problem) in packet-switched networks (the long-term case).
2. Virtual path routing problem (VPR problem) in ATM networks (both the short-term and long-term cases).

To study the network design problems in the long-term case, it is proper to consider link cost function as a concave function [50][52]. This is because the economy of scale is often present in communication network resources. For example, we can obtain a cost function from the envelope of several cost functions due to the development in communication transmission technologies [30][107], the development in switching technologies [99], etc. To obtain a cost function for this case, we still use the concept of cost difference between existing and new network facilities. However, the relationship of d_{0i} and d_{1i} is now changed to

$$d_{0i} > d_{1i}. \tag{1.4}$$

This results in the cost function depicted as in Figure 1.5

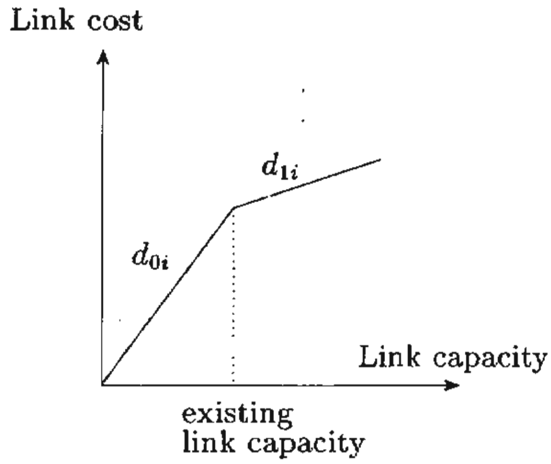


Figure 1.5: Link cost function taking into consideration an existing network: a piecewise linear concave function.

In this project, the above two network design problems are studied in the same manner. That is each network design problem is solved to obtain an optimal solution based on the information given, e.g., the amount of existing network facilities, the new user traffic demand, etc.

1.3 Objectives of the Research

As mentioned earlier, this research project is devoted to the study of communication network design taking into consideration an existing network. The study has the following objectives and goals.

1. To study the CA problem in packet-switched networks and the VPR problem in ATM networks under the condition that the effect of existing network is taken into account.
2. To analyze the characteristics of optimal solution in the above network design problems.
3. To construct, for each problem, a high-performance design algorithm derived from the characteristics of optimal solution.
4. To conduct experiments based on the algorithms obtained above.
5. To analyze the experimental results and examine the impact of existing network on each network design problem.

In this project, many efforts are made such that high-performance design algorithms can be obtained, where these algorithms must be able to give optimal solution or near-optimal solution within small computation time. It should be noted that the small computation time is very important, since this will make the algorithms applicable to solving network design problems when networks are of large size.

1.4 Organization of the Report

This research report begins in Chapter 1 with the general introduction of communication network design as well as the statement of problem studied in this project. After that, the scope and objectives of the study are given.

Chapter 2 provides the literature review for the study. It gives the overview of communication network design and a few examples of research works related to the work in this project.

Chapter 3 is the first part of the study, where the CA problem for packet-switched networks in the case of long-term design is examined. In this case, a piecewise linear concave cost function is considered in the design problem, and the characteristic of optimal solution is derived. After that, a design algorithm is proposed, and some numerical results are given.

Chapter 4, the second part of this study, deals with the VPR problem in ATM networks. The design problem is studied in this chapter in both short-term and long-term cases. A design algorithm is proposed for the VPR problem. Some experimental results are provided to verify the performance of the proposed algorithm as well as the effect of existing network to the network design.

Chapter 5 gives the summary of results of this research project, conclusions, as well as some interesting topics for further study.

Chapter 2

Literature Review

In this chapter, the overview of communication network design is discussed. After that, the detail of research works related to the study in this project is given.

2.1 Overview of Network Design

The life cycle of a communication network is the same as other systems and can be depicted as illustrated in Figure 2.1 [49]. The four phases called planning, design, implementation and operation are conducted sequentially and repeatedly in the cycle.

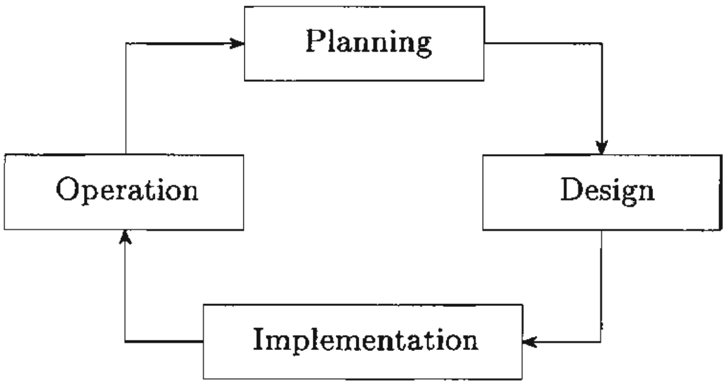


Figure 2.1: Life cycle of a communication network.

Generally, a communication network is needed to be planned and designed in a long time frame. This is due to not only the tremendous capital investment in the network implementation and operation, but also the necessity in the consideration of various factors and conditions such as future traffic demand, type of services, communication technology trends, etc.

Planning and design method for communication networks can be classified into two categories: dynamic planning method and static design method [49]. It should be noted that the term "planning" and "design" are sometimes used in the same meaning. That is the solving of a mathematical problem formulated from a realistic network with the consideration of factors in network implementation and network operation. However, the difference between *planning* and *design* can be made clear by the terms *dynamic planning* and *static design* as follows.

To plan or design a communication network in a long period of time, a tool called dynamic planning method is used. The planning tool will give solution of time, place, and the amount of facilities to be augmented to the network after considering factors affecting the network characteristics, e.g., traffic demand prediction, technology trends, rate of capital interest, etc.

One example of dynamic planning in the case of single facility is given in Figure 2.2 [49]. It can be seen from this figure that points of time (t_1 and t_2) and the amount of facilities that is necessarily augmented at each time point are determined.

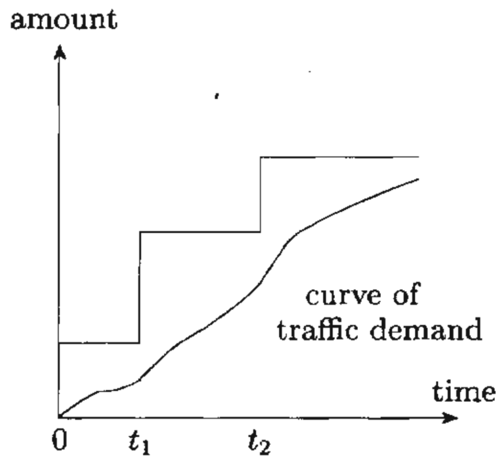


Figure 2.2: Dynamic planning in the case of single facility.

On the other hand, static design method deals with communication network in a comparatively short period of time, when the amount of traffic demand (forecasted value) in that period is given. The solution obtained from static design is concerned with the place and the amount of facilities to be installed in the network. Note that, in this case, there is no solution about the time for facility installation since the time period considered in the design is short, and facility installation is automatically conducted at the end of the period.

Additionally, static design method is also used for solving subproblems in the dynamic communication network planning. Their relationship can be shown in Figure 2.3, where a period of one year is taken as the time period of static design [49].

In dynamic planning, when a communication network is implemented and operated after a network design phase, the next time of network design can then be realized as the redesign phase of the network. It has been mentioned in Chapter 1 that, the existing network facilities are very important design factor and cannot be neglected in the redesign of a network. The study in this project is then focused on the static network design (re-

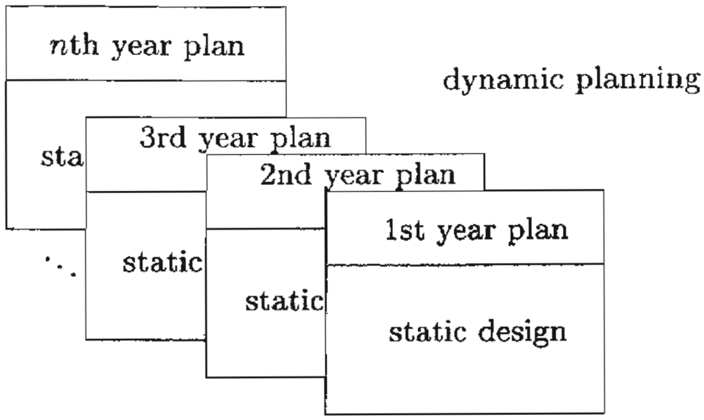


Figure 2.3: Relationship between dynamic planning and static design.

design) method for packet-switched and ATM networks, when the important information such as the amount of existing network facilities, new network traffic demand, is given.

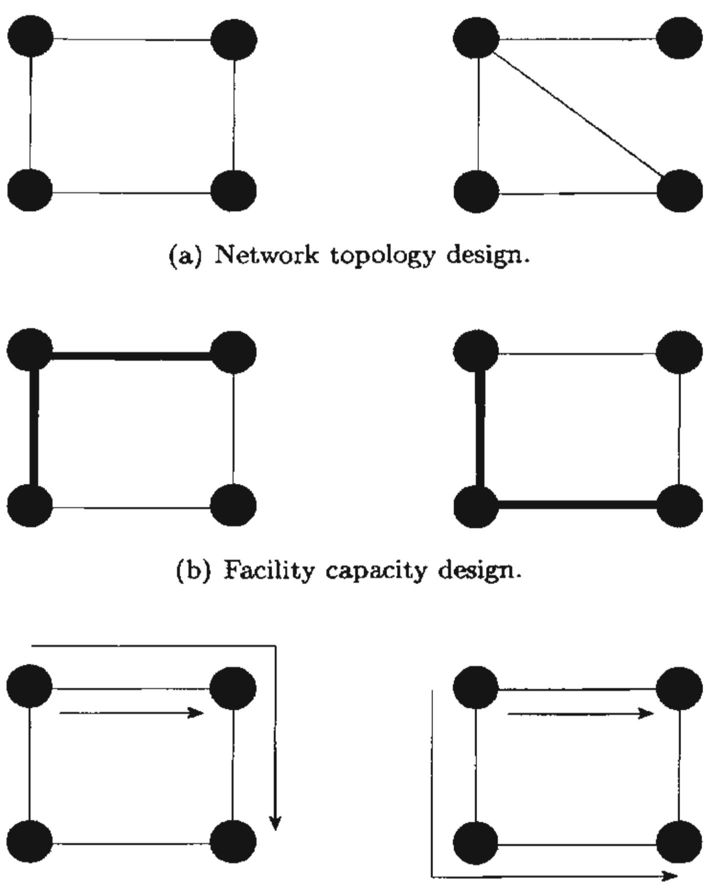
2.2 Network Design Issues

In network design, communication network is generally defined as a set of communication points (routing/switching function facilities), and transmission lines (transmission function facilities) connecting the points. Mathematically, the network is usually modeled as a connected graph, containing a set of vertices and a set of edges [2][10][50]. A vertex and an edge are equivalent to each communication point and each transmission line in the network, respectively. Sometimes, vertex is also called *node*, and edge is referred to as *link*.

When traffic demand between each node-pair in the network is given, network design is concerned with the way to construct (or re-construct) the network with minimal cost while conditions of design constraint such as quality of service (QoS), network reliability are satisfied. It should be noted that we can also consider another version of problem, i.e., design a network with the best QoS or highest network reliability, while keeping network cost not exceeding the given amount of budget. However, QoS and network reliability are essential network parameters and are always requested by users, e.g., nobody wants to use a network with low QoS. Accordingly, network design problem should consider QoS or network reliability as design constraint, while network cost is treated as optimization (minimization) objective.

To deal with the real-world communication networks, there are a number of design issues that are necessarily considered in the network design problem. However, the following three items are those always be examined in the research works of this field [2][52].

- Network topology.
- Facility capacity.
- Routing.



(a) Network topology design.

(b) Facility capacity design.

(c) Routing design.

Figure 2.4: Network design issues.

Network topology design is concerned with the determination of the overall shape of the network, by giving a specific pattern of placing links between nodes such that the total network cost is minimized. Figure 2.4 (a) shows two possible topologies of a network, where there is one link different between each other, i.e., the link connecting the top-right node and the bottom-right node in the left figure, and the link between the top-left node and the bottom-right node in the right figure.

Facility capacity assignment deals with facility sizing, i.e., the determination of the amount of each facility in the network, e.g., speed of each transmission line, the bandwidth of switching equipment in each node, etc. Figure 2.4 (b) illustrates two different link capacity assignments of a network. In this figure, the thick and thin lines represent high speed and low speed links, respectively. The difference between the two capacity assignments in Figure 2.4 (b) is that a high speed link is used at the top-left to top-right link for the left figure, while a high speed link is used at the bottom-left to bottom-right link for the right figure.

Routing design involves selecting path (pattern of routes) for allocating traffic requirement between each node-pair. Figure 2.4 (c) gives two examples of routing pattern of a network. At the left of this figure, the traffic requirement between top-left and bottom-right node-pair is routed via the top-right node, while the same traffic is routed via the bottom-left node at the right of this figure.

The above three design issues can be solved simultaneously or separately. If the separated design manner is chosen, it is assumed that the solutions of some design issues are already known or given, and the task is concerned with the determination of the solution to the issues that are left unsolved. Several network design problems can be listed as follows [52].

- Capacity assignment (CA) problem

Given:	Flows and network topology
Minimize:	Network cost
Design parameters:	Facility capacities
Subject to:	Design constraint

- Flow assignment (FA) problem

Given:	Traffic demand, facility capacities and network topology
Optimize:	Parameters related to design constraint
Design parameters:	Flows
Subject to:	Design constraint

- Capacity and flow assignment (CFA) problem

Given:	Traffic demand and network topology
Minimize:	Network cost
Design parameters:	Capacities and flows
Subject to:	Design constraint

- Topology, capacity and flow assignment (TCFA) problem

Given:	Traffic demand
Minimize:	Network cost
Design parameters:	Topology, capacities, and flows
Subject to:	Design constraint

"Flow" in the above list is the sum of the amount of traffic requirements that are allocated in each network facility. "Design constraint" is control conditions of design problem and must be satisfied by the final design solution. A solution that does not satisfy all conditions of the design constraint of the problem is referred to as a non-feasible solution. Some good examples of constraint in the case of packet-switched network design are the upper limits of average packet delay, path delay, packet loss probability, etc.

It will be shown later in this report that the network design problems studied in this research project also belong to the problems listed above. The capacity assignment problem (CA problem) in packet-switched networks is just the member of the above CA problem. On the other hand, the virtual path routing problem (VPR problem) in ATM networks falls under the CFA problem.

To solve a network design problem, it is clear that solution obtained from simultaneous determination is better than that from separated solving manner. However, solving a design problem simultaneously is a very complicated task, especially in the case of large-scale network.

When solving a design problem of large network, one big problem is generally decomposed into several subproblems, which are later solved separately. For example, the above TCFA problem can be separated into two subproblems, namely topology design problem and the CFA problem. By the same idea, the CFA problem can be decomposed into two subproblems, i.e., the CA and FA problem.

The design method that decomposes one big problem into several subproblems and solves each subproblem separately is *heuristic* contrary to the optimum design method that takes all parameters into account simultaneously. Heuristic design method is usually adopted in solving network design problems [27][29][86][101]. Although the final solution obtained in the heuristic way might not be an optimal design solution, it generally has good quality and is acceptable in the sense that the computation time needed is relatively short.

2.3 Network Models in Network Design

In this section, network models that are adopted in the research works in the area of communication network design are discussed and a number of examples will be given.

In the earlier works of communication network design, communication network model is usually simplified by considering only major constraint and parameters. The main reason is that the optimization problem formulated from the simplified network model is easily to be solved. Although the model itself is easy, the solution obtained is, however, considered to be feasible, and can be used in realistic network design/implementation. The reason is straightforward that the main part of design components are not omitted in the model.

After the earlier steps, network model is revised to a more complex model by including more design constraint and parameters. This makes the model be able to deal with communication networks in the real-world. New design method according to the complex model is then studied. However, the method is usually developed based on the method or result obtained from the simplified model in preceding works.

Some examples on the change of network models in the study of communication network design are given below. Although all of these examples are concerned with the design of packet-switched networks, the same behavior can also be observed in other kind of communication networks.

- **Facility capacity model:** The model with discrete value of capacity is examined instead of model with continuous value [45][64]. This reflects the capacity characteristics of network facility in actual network construction, i.e., facility capacity cannot be set arbitrarily and must be chosen from a given set of values.

- **Facility cost model:** At first, the cost of network facility is considered to be linear with respect to the capacity. Then, non-linear cost functions including exponential and logarithmic are studied [52]. The discrete facility cost function is also adopted [45][64]. Clearly, discrete cost function comes from the model of discrete facility capacity.

- **Design constraint:** Firstly, the average packet delay throughout the network is considered as design constraint [52]. After that this is changed to average delay between each path (node-pair) [57][86]. Design method where the design constraint is packet loss probability can also be found in the literature [95].

- **Class of service:** Network model taking into consideration several classes of service is introduced to deal with multimedia network [95].

2.4 Conventional Research Works

As mentioned earlier, communication network is planned and designed in a long period of time according to the change and variation of several design parameters, including the change in traffic demand, that affect network characteristics. The network design methods proposed in the literature almost deal with network design problem where the network is in zero state, i.e., their is no link exists in the network, or capacities of all facilities are taken to be zero. Thus, the concept of **existing network** is rarely included in these design methods. Although some research works on network design/redesign tools inform that existing network is considered in the network design phase, no design method or algorithm is given explicitly [46][73].

The main reason that the concept of existing network is rarely considered in the previous works of communication network design is that existing network facilities are assumed as parts of network that are ready to be used in the construction of new network. Almost of the conventional research works assumed that the existing parts of network can be used with zero cost, or can be treated in the same way as new facilities needed to be installed to the network [10][104]. However, it is clear that this idea is generally not true in practical cases of network design. For example, although the existing network facilities are promptly to be used, there is cost concerning the use of these facilities such as operating or maintenance cost. Moreover, it is not necessary that the cost of using existing network facilities be the same as that of newly installed facilities.

There is one more reason that the existing network is not considered in the design of new network. It is that the growth of traffic demand applied to a network is often assumed to increase time-by-time. Based on this assumption, the capacity of each existing facility will not decrease lower than its current value after the new traffic pattern is applied. When the network is designed according to the new traffic, the amount of capacity needed to be augmented to each facility in the existing network can be determined and the network is then re-constructed.

However, in the realistic network operation, the amount of traffic applied to the network can be lower than its former value. Clearly, short-term cases such as daily or monthly network design fall in this case. In daily network design, user traffic demand between some node-pairs may decrease at night time comparing to that in the daytime or vice versa [65]. When the amount of traffic decreases, the value of facility capacity obtained from conventional design method will be less than the current value, and waste portion of facility will occur in the network. Moreover, when the traffic increases and new routing pattern is applied, it is also possible that waste of facilities will exist in the network if the network is not well designed. Obviously, this kind of waste is undesirable in the operation of communication network, but the way to deal with this important problem is rarely considered in the conventional research works. Furthermore, when the traffic increases more than the former value, some network facilities may not have enough capacity to cope with this change. Unfortunately, there is no good solution for dealing with this kind of problems [23].

From the above points of view, the research in this project devotes to the communication network design taking into consideration the effect of existing network facilities. Strictly speaking, the research focuses on both the short-term and long-term network design where the effect of existing network is examined in the design. It can be pointed out that the consideration of existing network in designing a new network is very important and cannot be neglected in the network design. The necessity of considering existing network can be listed as follows.

1. Communication network is planned and designed in a long time frame. In the design of each time section, i.e., in a short-term design, it is necessary to use the existing facilities in the full range before installing new facilities in the places where they are required.

2. There is difference between existing and newly installed facilities. For example, existing facilities are ready to be used in the network and there is cost associated in using them, e.g., maintenance cost. Moreover, existing facilities cannot be changed easily, especially in the short period of time. On the other hand, the cost concerning installation of new facilities needs not be the same as the cost of using the existing ones.

3. It is clear that utilizing existing facilities effectively is always important and cannot be disregarded during the network design phase.

2.5 Network Design taking into Consideration an Existing Network

Although the concept of existing network is rarely adopted in the design of new network as mentioned above, there are some recent research works on the study of communication network design that consider the effect and impact of existing network facilities. In this section, two examples of network design method that takes account of existing network will be described. They are the works conducted by Shinohara and by Runggeratigul as shortly described in Chapter 1.

2.5.1 Shinohara's Work

As mentioned briefly in Chapter 1, the work by Shinohara is concerned with the circuit dimensioning problem (CD problem) in circuit-switched networks [92][93]. The problem is formulated and solved under the condition that the number of circuits in the existing network are not zero. The circuit-switched network model used in the study is a multi-stage alternate routing network model, which composes of two kinds of links, namely high-usage links and final links. A simple three-node network is shown in Figure 2.5.

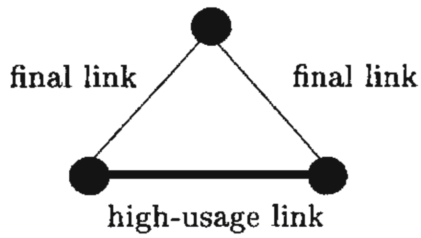


Figure 2.5: Triangular circuit-switched network.

In the figure, the thick line is a high-usage link assigned for the two nodes at the bottom, while the two thin lines are final links for the two nodes.

A high-usage link in the network is a circuit group from which overflow is permitted, and a final link is a circuit group from which overflow is not permitted [31]. Call, the traffic requirement between a node-pair, is firstly routed through a high-usage link assigned for the node-pair. However, if there is no empty channel left in the high-usage link, the call is treated as an overflow call and an alternate route selection procedure is activated to find a final link which has sufficient empty channel for the overflow call. If there is such kind of link exists, the call is routed through it. On the other hand, if there is no final link which has sufficient channel, the overflow call will be blocked, and no circuit is allocated for that call.

The CD problem is formulated as follows.

CD Problem

Given: $\boldsymbol{A}, b_j, \quad \forall j \in F$

Minimize: $Q = \sum_{E} q_i(x_i)$

Design variables: $x_i, \quad \forall i \in E$

Subject to: $B(x_j, a_j) \leq b_j, \quad \forall j \in F,$

$a_i = r_i(\boldsymbol{A}, \{x_i\}), \quad \forall i \in E.$

The meaning of notations in the above problem can be given as follows.

$\boldsymbol{A}$ matrix of traffic (calls) applied to the network

E	set of all links in the network
F	set of final links
Q	total network cost (defined as the sum of all link costs)
$B(x_j, a_j)$	function giving the value of blocking probability on final link j
a_i	traffic offered to link i
b_j	upper limit of blocking probability on final link j
r_i	function that related to the flow allocation of link i
$q_i(x_i)$	cost of link i which is the function of x_i
x_i	the number of circuits in link i

The aim of this problem is to determine the number of circuits in all network links such that total network cost, defined as the sum of all link costs, is minimized. The conditions of design constraint are the upper limit of blocking probability on all final links and the route selection order according to the traffic requirement to the network. Note that, in the CD problem, there is no constraint of blocking probability on high-usage links since overflow is permitted in this kind of links.

In the study of the CD problem, the effect of the circuits in existing network facilities is included in the design by introducing the concept of cost difference between existing circuit and newly installed circuit. This results in a piecewise linear link cost function shown in Figure 2.6.

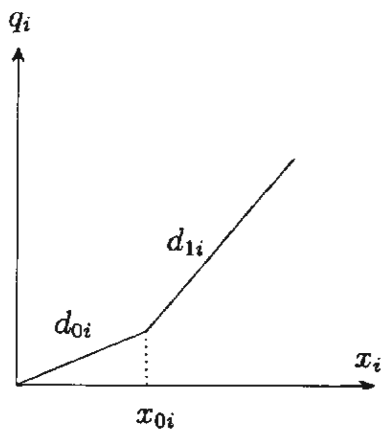


Figure 2.6: Link cost function in Shinohara's work.

The link cost q_i can be written as in the following equation.

$$q_i = \begin{cases} d_{0i}x_i & , x_i \leq x_{0i}, \\ d_{1i}x_i - (d_{1i} - d_{0i})x_{0i} & , x_i > x_{0i}, \end{cases} \tag{2.1}$$

where d_{0i} and d_{1i} are the per-unit cost of existing circuit and newly installed circuit of link i, respectively, and x_{0i} is the number of existing circuits in link i.

The value of d_{0i} and d_{1i} are both non-negative, i.e.,

$$d_{0i} \geq 0, \tag{2.2}$$
$$d_{1i} \geq 0. \tag{2.3}$$

In his work, Shinohara examines the case that

$$d_{0i} \leq d_{1i}. \tag{2.4}$$

In other words, the per-unit cost of existing circuit is less than or equal to that of the newly installed circuit. The cost function in Figure 2.6 is generally valid in network construction, especially in the case of short-term design.

By applying the link cost function in Figure 2.6, the CD problem is solved to get the number of circuits in all links, $\{x_i\}$. The CD problem is a non-linear programming problem, and is solvable by the well-known Lagrange multiplier method. However, since the link cost function $q_i(x_i)$ is non-differentiable with respect to x_i at $x_i = x_{0i}$, the Lagrange multiplier method cannot be applied directly. In the study, the original non-differentiable function is approximated by another differentiable function as can be seen in Figure 2.7 (dotted line).

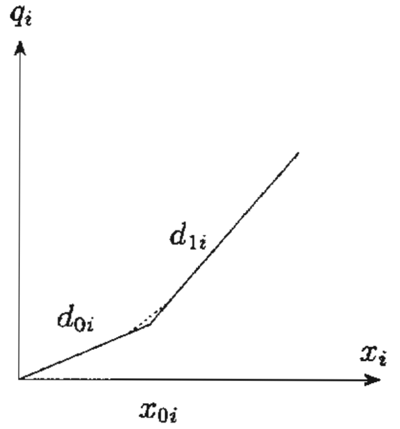


Figure 2.7: Approximation of non-differentiable cost function.

It is reported in the literature that a third order function is used for the approximation in this case [92]. By using the approximated function, $\dfrac{dq_i}{dx_i}$ becomes continuous and Lagrange multiplier method can be applied to solve the CD problem.

2.5.2 Runggeratigul's Work

Runggeratigul et. al. study the link capacity assignment problem (CA problem) in packet-switched networks with the consideration of existing network [80][81]. The design problem focuses on the case of short-term design. The fundamental idea of this work is to apply the concept of cost difference between the existing and new network facilities, that was proposed by Shinohara for circuit-switched networks.

The CA problem in packet-switched networks is a well-known communication network design problem, and has been studied widely since the early days of ARPANET's appearance [29][52]. This problem is concerned with means to determine capacity of links that minimize network cost, subject to some constraint, such as the upper limit of average packet delay. The CA problem can be solved when network information including network topology, routing pattern, etc., is given.

The CA problem can be formulated as follows.

CA Problem

Given: $\{f_i\}$ and $\{C_{0i}\}$

Minimize: $D = \sum_L D_i$

Design variables: $\{C_i\}$

Subject to:

$$T = \frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} \leq T_{\max}$$

$$C_i - f_i > 0, \quad \forall i \in L$$

$$C_i > 0, \quad \forall i \in L$$

The symbols used in the above problem are summarized as follows.

L	set of links in the network
f_i	traffic flow on link i
C_{0i}	existing value of capacity of link i
γ	overall traffic in the network
T	average packet delay
$T_{\max}$	maximum allowable average packet delay in the network
C_i	capacity of link i after assignment
D	total network cost
D_i	cost of link i

In the CA problem, $\{f_i\}$ is a new traffic pattern applied to the existing network where existing link capacity is $\{C_{0i}\}$. Capacity of all links $\{C_i\}$ is determined such that the total network cost D is minimized, where the network cost is defined as the sum of all link costs. Conditions of design constraint are the upper limit of average packet delay,

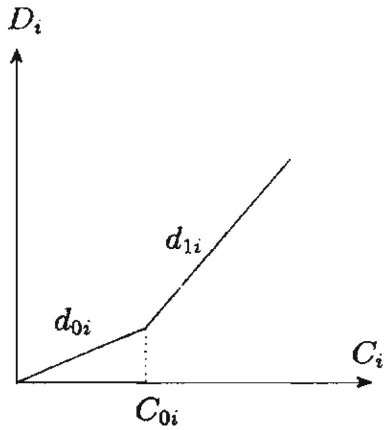


Figure 2.8: Link cost function in Runggeratigul's work.

and the relationship between link flow and link capacity. Obviously, link capacity must be greater than link flow so that packet delay will not grow indefinitely.

To include the existing network facilities in the CA problem, the concept of link cost difference is adopted, and a cost function shown in Figure 2.8 is applied.

From the figure, we can express link cost D_i as in Eq.(2.5).

$$D_i = \begin{cases} d_{0i}C_i & , C_i \leq C_{0i}, \\ d_{1i}C_i - (d_{1i} - d_{0i})C_{0i} & , C_i > C_{0i}, \end{cases} \tag{2.5}$$

where d_{0i} and d_{1i} are respectively the per-unit cost of existing and newly installed link capacity.

The values of d_{0i} and d_{1i} considered in the work are in the following range,

$$d_{0i} < d_{1i}, \tag{2.6}$$
$$d_{0i} \geq 0, \tag{2.7}$$
$$d_{1i} > 0. \tag{2.8}$$

This makes the cost function be a piecewise linear convex function.

It should be noted that the case $d_{0i} = d_{1i}$ is excluded in the study. The reason is straightforward that this is the case of linear cost function, and the CA problem with this simple cost function has been already studied [52]. Furthermore, the case $d_{1i} = 0$ is also excluded according to its triviality, i.e., the capacity of all links can be set as infinity, without making the total network cost be infinity (since d_{0i} and d_{1i} are both 0 in this case, the network cost is 0 !!!).

Since the link cost function is non-differentiable, the idea of using approximation function as in Shinohara's work is applicable. However, Runggeratigul et. al. propose a new procedure to solve the CA problem without using any approximation. The reasons are

- The selection of approximated function may affect the accuracy of the solution.

- Lengthy computation time is required to obtain approximated functions of all cost functions.

The procedure proposed in the study is referred to as the method of link set assignment [82], and its details can be described as follows.

After examining D_i in Eq.(2.5), it is clear that C_i belongs to one of the following three cases.

- $C_i < C_{0i}$,
- $C_i > C_{0i}$,
- $C_i = C_{0i}$.

Based on these cases, we can define the following three sets, L_0, L_1 and L_2, where

$$\begin{aligned} L_0 &= \{\, i \mid C_i < C_{0i}\}, && (2.9)\\ L_1 &= \{\, i \mid C_i > C_{0i}\}, && (2.10)\\ L_2 &= \{\, i \mid C_i = C_{0i}\}. && (2.11)\end{aligned}$$

Consequently, if each link i in the network is assigned to be a member of one of the above three sets, we will automatically have that D_i can be regarded as a differentiable function.

This is because

$$\frac{dD_i}{dC_i} = \begin{cases} d_{0i} & ,\forall i \in L_0, \\ d_{1i} & ,\forall i \in L_1. \end{cases} \tag{2.12}$$

Obviously, there is no need to consider link i that belongs to L_2 since the link can be excluded from the CA problem and its capacity C_i can be set as a constant value of C_{0i}.

With the above procedure, we can derive the expression of $\{C_i\}$ as

$$C_i = f_i + \frac{\sum\limits_{L_0 \cup L_1} \sqrt{f_j d_j}}{\left(\gamma\, T_{\max} - \sum\limits_{L_2} \dfrac{f_j}{C_{0j} - f_j}\right)} \sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L_0 \cup L_1, \tag{2.13}$$

where

$$d_i = \begin{cases} d_{0i} & \text{, when } i \in L_0, \\ d_{1i} & \text{, when } i \in L_1. \end{cases} \tag{2.14}$$

The value of C_i calculated from Eq.(2.13) is optimum for the CA problem under the relevant link set assigned at the beginning. However, the solution has to be checked whether it contradicts its assigned link set or not, e.g., for $i \in L_0$, C_i that is greater than C_{0i} is thought to be a contradiction. If any contradiction occurs, the solution obtained will not be feasible. A design algorithm for the CA problem is then proposed based on the following idea [81].

Firstly, let's consider the case that all links are assigned to L_0. Then Eq.(2.13) is used to determine $\{C_i\}$, and the value is checked whether there is link i that $C_i > C_{0i}$ or not.

If such kind of links exist, we will have that these links cannot belong to L_0. Let's define this set of links as

$$L_{0x} = \{i \mid C_i > C_{0i}, \forall i \in L_0\}. \tag{2.15}$$

Secondly, we assign all links to be in L_1 and checking contradiction as in the above manner, and define

$$L_{1x} = \{i \mid C_i < C_{0i}, \forall i \in L_1\}. \tag{2.16}$$

We can also say that links in L_{1x} cannot belong to L_1 anymore.

From L_{0x} and L_{1x}, it is possible to conclude that

$$L_2 = \{i \mid \forall i \in L_{0x} \cap L_{1x}\}. \tag{2.17}$$

The proposed algorithm is the steps that examine links that are in L_2 repeatedly as in the above manner, and terminates when all links are assigned to proper sets without contradiction.

The main result of the study on the CA problem with the consideration of existing network (short-term case) is that, the design algorithm gives a solution in the way that the existing network facilities are utilized effectively, and new facilities are augmented to the existing network only in the places that they are required.

2.6 Summary

Although the concept of existing network is usually neglected in conventional research works on communication network design, there are a few studies that recently include this major design factor in the network design problems. These design problems are the circuit dimensioning problem (CD problem) in circuit-switched networks, and the link capacity assignment problem (CA problem) in packet-switched networks.

However, since the above research works consider the network design only in the short-term case, the following questions are needed to be answered.

- The question of how to study the case of long-term network design under the effect of existing network facilities.
- The question that the concept of cost difference between existing and newly installed facilities is applicable or not to the design of other types of communication network besides the circuit-switched and packet-switched networks.

The work in this research project is devoted to give answers for the above questions. In Chapter 3, we will study the CA problem in packet-switched networks for the case of long-term design by applying a piecewise linear concave function. In Chapter 4, the virtual path routing problem (VPR problem) in ATM networks will be investigated for both short-term and long-term design.

Chapter 3

Link Capacity Assignment in Packet-Switched Networks

In this chapter, we discuss the first part of the work in this project. It is the study on the link capacity assignment problem in packet-switched networks with the consideration of existing network. However, since the case of short-term design has been already studied in the literature, this project then focuses the link capacity assignment problem on the case of long-term design.

The content of this chapter is organized as follows. Firstly, the outlines of the link capacity assignment problem in packet-switched networks will be given. Then the network model used in the study will be discussed, and a piecewise linear concave function is introduced as the network link cost function. After that, the capacity assignment problem is formulated as a non-linear programming problem. By analyzing the characteristic and property of the optimal solutions of the capacity assignment problem, we can propose a heuristic design algorithm derived from the well-known Lagrange multiplier method. At last, some numerical results are given to investigate the performance of the proposed algorithm, as well as to show the effects of existing network facilities to the link capacity assignment problem.

3.1 Introduction

Link capacity assignment problem (CA problem) in packet-switched networks has been studied widely since the early days of ARPANET's appearance in the late 1960s [29][52]. This network design problem is concerned with means to determine the capacity of links in the network such that the network cost is minimum, subject to some design constraint such as the upper limit of average packet delay. The CA problem can be solved when network information, including network topology, routing pattern, etc., is given. The problem has been studied for many kinds of network model. Some of the examples are given below.

- A model with several classes of packet is considered for dealing with multimedia networks [95].
- Besides the average packet delay throughout the network, path average delay [57][86] as well as packet loss probability [95] are defined as design constraint for more reasonable quality of service in a network.
- Discrete values of link capacity are introduced for dealing with links in an actual network construction [45][64].

Furthermore, many types of link cost function have been considered in the CA problem. For example,

- some fundamental mathematical functions including linear, exponential, logarithmic, are examined to see the behavior of link capacity assignment due to the effect of these cost functions [52],
- a piecewise linear convex cost function has also been considered as a link cost model in the case that the CA problem is solved with the consideration of existing network, where the per-unit cost of using existing capacity is less than that of installing new capacity [80][81] (this is the case of short-term network design already mentioned in Chapter 2).

The work in this chapter can be realized as an extension of the work by Runggeratigul et. al. in [80][81], where we extend the work in the way that the case of long-term network design can be studied.

It has been already known that utilizing existing network facilities in the construction of a new communication network is very important in the case of short-term network design [49]. The reason is straightforward, that is that existing network facilities are promptly to be used and cannot be changed easily in a short period of time. Thus, it is appropriate to use a convex link cost function in the case of short-term design as done in the previous works [80][81].

However, in other cases of network design such as long-term design, link cost function is not necessary convex. In many cases, it is proper to regard the function as a concave function [50][52]. This is because the economy of scale is often present in communication resources. For example, we can obtain a cost function from the envelope of several cost functions due to the development in communication transmission technologies [30][107], the development in switching technologies [99], etc.

In this chapter, we study the CA problem in packet-switched networks taking into consideration an existing network where the link cost function is piecewise linear concave. This is the case when the per-unit cost of using existing capacity is greater than that of installing new capacity. It is obvious that this type of link cost function is non-differentiable. To solve network design problems with this type of cost function, the method of approximating the non-differentiable cost function by a differentiable function has been proposed [93]. Although this method yields a solution within approximation error, there is no guarantee that the solution obtained from this method is optimum when link cost function is not convex. Moreover, lengthy time is needed to determine the approximation function for the non-differentiable cost function.

To study the CA problem in this research project, we then try to clarify the characteristic of optimal solution of the CA problem. The main procedure is to apply the method of link set assignment proposed for the piecewise linear convex function in [81]. This leads us to the analysis of the essential characteristic of the optimal solution to the CA problem. Based on this characteristic, there is no need to conduct approximation on the non-differentiable link cost function. Furthermore, it is possible to show that the non-differentiable cost function can be treated as a differentiable function. As a result, conventional methods such as Lagrange multiplier method can be applied to the solving of the CA problem.

3.2 Network Model

In the first part of this section, we discuss the model of packet-switched network considered in this study. After that in the second part, we introduce a link cost function with the consideration of cost difference between existing capacity and newly installed capacity for the case of long-term design.

3.2.1 Model Description

The symbols used in this chapter are summarized as follows.

L set of links in the network

f_i traffic flow on link i (in bits/second)

C_{0i} existing value of capacity of link i (in bits/second)

γ overall traffic in the network (in packets/second)

T average packet delay

$T_{\max}$ upper limit of the average packet delay T

C_i capacity of link i after assignment

D total network cost

D_i cost of link i

The packet-switched network model considered in this research project is the same as the one used in the early work of packet-switched network design [29][52][81]. The characteristics of this network model are given below.

Arrival process of packets transmitted on each link in the network is assumed to follow Poisson process, and packet length is assumed to be negative exponentially distributed. From this assumption, we can model each network link as an $M/M/1$ queueing system with infinite buffer. Then, the average packet delay throughout the network T can be given as in Eq.(3.1).

$$T = \frac{1}{\gamma} \sum_{L} \frac{f_i}{C_i - f_i}. \quad (3.1)$$

Based on the user traffic demand and the routing scheme applied to the network, it is possible to calculate the amount of traffic flow f_i on each link in the network. In this network model, the traffic flow is assumed to be a positive value,

$$f_i > 0. \quad (3.2)$$

Obviously, negative value of f_i has no meaning. In addition, a zero value of f_i is also trivial. If there is a link in the network with $f_i = 0$, we then have that there is no need to consider this link in the design problem anymore, and the link can be consequently removed from the network without any effect to other links in the network.

The value of link capacity C_i is assumed to be continuous, and can be set as an arbitrary positive value,

$$C_i > 0. \quad (3.3)$$

From Eq.(3.1), it is clear that the capacity of each link must be greater than the traffic flow on that link, i.e.,

$$C_i > f_i. \quad (3.4)$$

This is a necessary condition to make the value of packet delay on each link not grow indefinitely.

For simplicity, this network model does not take into account the node cost, e.g., cost of switching facilities, etc. Only link cost is considered, and the total network cost D is defined as the sum of all link costs. Then it yields

$$D = \sum_{L} D_i. \quad (3.5)$$

3.2.2 Link Cost Function

For each link, link cost D_i is a function of link capacity C_i as

$$D_i = f(C_i), \quad (3.6)$$

where $f(.)$ is an arbitrary function. Clearly, the function can be either a simplest case of linear function or a more realistic case of non-linear function.

Since the main objective of this research project is to investigate the network design problem under the impact of the consideration of existing network, we then define a link cost function that is possible to deal with the existing link capacity. Accordingly, we rewrite Eq.(3.6) as in the following equation.

$$D_i = f(C_i, C_{0i}). \quad (3.7)$$

Eq.(3.7) shows that the link cost function consists of two portions of link cost, i.e., the cost of using existing capacity and the cost concerned with the installation of new capacity.

Next, we apply the idea of cost difference between the existing capacity and the newly installed capacity [81][93] to the cost function in Eq.(3.7). We also assume that the link cost of the two parts are both linear with respect to link capacity. As a result, we obtain the following link cost function.

$$D_i = \begin{cases} d_{0i}C_i & ,C_i \le C_{0i}, \\ d_{1i}C_i - (d_{1i} - d_{0i})C_{0i} & ,C_i > C_{0i}, \end{cases} \tag{3.8}$$

where d_{0i} and d_{1i} are the per-unit cost of existing capacity and newly installed capacity of link i, respectively. In general cases, the value of d_{0i} and d_{1i} are both non-negative. That is

$$d_{0i} \ge 0, \tag{3.9}$$
$$d_{1i} \ge 0. \tag{3.10}$$

For the case of short-term network design, we have considered the case of piecewise linear convex function, i.e.,

$$d_{0i} < d_{1i}, \tag{3.11}$$
$$d_{0i} \ge 0, \tag{3.12}$$
$$d_{1i} > 0. \tag{3.13}$$

However, we will have an opposite case for the long-term network design in our study. As mentioned above, it is appropriate to adopt a concave function for the case of long-term design. Hence, we focus on the following range of d_{0i} and d_{1i}.

$$d_{0i} > d_{1i}, \tag{3.14}$$
$$d_{0i} > 0, \tag{3.15}$$
$$d_{1i} > 0. \tag{3.16}$$

The above range of d_{0i} and d_{1i} results in a piecewise linear concave function as shown in Figure 3.1.

Note that the case $d_{0i} = d_{1i}$ is excluded from the range of d_{0i} and d_{1i} given above. The reason is that this case gives a linear link cost function, which has been already studied for the CA problem [52]. The case $d_{1i} = 0$ is also excluded due to its triviality. This is because the capacity of all links in this case can be set to infinity, while the total network cost is finite. In other words, there is no much need to solve the CA problem of this case anymore.

The cost function given in Figure 3.1 arises in many communication network design problems. One of the example is the link type selection problem where the economy of scale is present in communication resources. A practical example with two alternatives of link types is given in Figure 3.2. It can be seen from this example that the overall link cost function is a piecewise linear concave function, which is the envelope of cost functions of the two link types.

The concave link cost function in Figure 3.1 will be applied to the CA problem in packet-switched networks, where the problem is formulated as a mathematical programming problem in the next section.

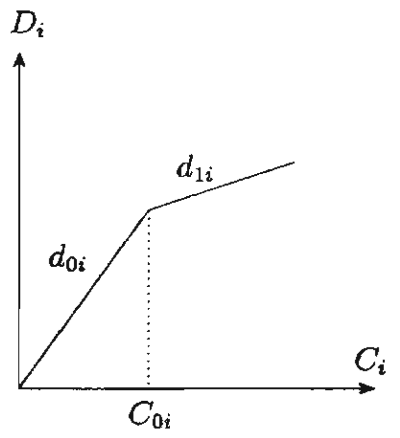


Figure 3.1: A piecewise linear concave link cost function.

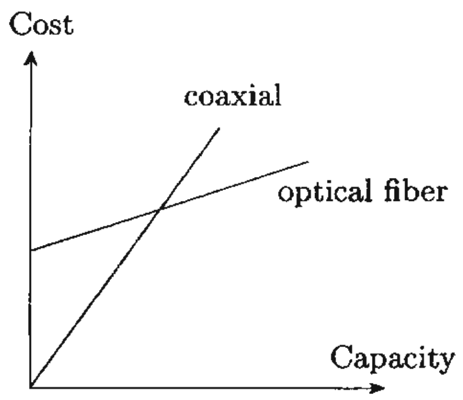


Figure 3.2: A practical example of link type selection.

3.3 Link Capacity Assignment Problem (CA Problem)

In this research project, we study the link capacity assignment in packet-switched networks in the case that new user traffic requirement is applied to an existing network, and then the capacity of each link is adjusted to cope with this new traffic requirement such that a network with minimal total cost is obtained under the constraint of packet delay.

Consequently, we formulate the CA problem as follows.

CA Problem

Given: $\{f_i\}$, $\{C_{0i}\}$, $\{d_{0i}\}$ and $\{d_{1i}\}$

Minimize: $D = \sum_L D_i$

Design variables: $\{C_i\}$

Subject to:
$$T = \frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} \leq T_{\max}$$
$$C_i - f_i > 0, \quad \forall i \in L$$
$$C_i > 0, \quad \forall i \in L$$

In the above CA problem, $\{f_i\}$ is the new user traffic requirement applied to an existing packet-switched network where the existing link capacity is $\{C_{0i}\}$. Capacity of all links $\{C_i\}$ is determined so that the total network cost D is minimized. According to the network model described in the previous section, the network cost is defined as the sum of all link costs where each link cost D_i is given in Eq.(3.8).

The conditions of design constraint are the upper limit of average packet delay and the relationship between link flow and link capacity. The quality of service in our network model is indicated by the average packet delay throughout the network, T, where the network must be designed in the way that T does not exceed an upper limit $T_{\max}$. In the design result, link capacity must be greater than link flow such that the average packet delay on each link is finite as mentioned above.

Note that if link cost D_i is a linear function with respect to link capacity, i.e.,

$$D_i = c_i C_i, \tag{3.17}$$

where c_i is a positive value that denotes the per-unit cost of link capacity, we will have that C_i can be determined by the following equation [52].

$$C_i = f_i + \frac{\sum_L \sqrt{f_j c_j}}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{c_i}}, \qquad \forall i \in L. \tag{3.18}$$

From the above solution, the total network cost can then be calculated as in the following equation.

$$D = \sum_L f_i c_i + \frac{\left(\sum_L \sqrt{f_j c_j}\right)^2}{\gamma\ T_{\max}}. \tag{3.19}$$

3.4 The Method of Link Set Assignment

The CA problem formulated above is a non-linear programming problem. Many conventional methods such as Lagrange multiplier method are known as effective methods for solving non-linear programming problems, e.g., the CA problem with linear cost function given in the previous section.

However, since the link cost function D_i given in Eq.(3.8) is not differentiable with respect to C_i, Lagrange multiplier method cannot be applied to the CA problem directly. As a method to alleviate this type of network design problem, the non-differentiable function is approximated by a differentiable function [93] as described in Chapter 2. Figure 3.3 shows the idea of this method, where the dotted line is the part of approximated function.

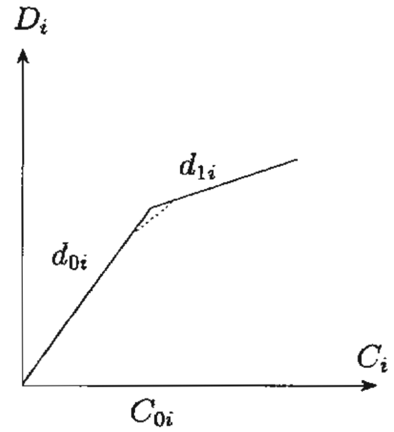


Figure 3.3: Approximation of non-differentiable function.

By using an approximated differentiable cost function, we can apply the Lagrange multiplier method to the CA problem. Although, this provides a solution to the problem within approximation error, there is no guarantee that the solution obtained is an optimal solution when the link cost is not convex. Moreover, lengthy time is needed for calculating approximation function of all link cost functions.

When the cost function is piecewise linear convex, a method of link set assignment has been proposed to solve the CA problem to obtain an optimal solution [81]. In this study, we extend this method such that it is applicable to the CA problem with piecewise linear concave cost function. The concept of the original version of the method can be described as follows.

D_i given in Eq.(3.8) can be regarded as a differentiable function, if we assign each link i to be a member of one of the following three sets,

$$L_0 = \{ i \mid C_i < C_{0i}\}, \tag{3.20}$$
$$L_1 = \{ i \mid C_i > C_{0i}\}, \tag{3.21}$$
$$L_2 = \{ i \mid C_i = C_{0i}\}. \tag{3.22}$$

This is because

$$\frac{dD_i}{dC_i} = \begin{cases} d_{0i} & ,\forall i \in L_0, \\ d_{1i} & ,\forall i \in L_1. \end{cases} \tag{3.23}$$

Note that link i that belongs to L_2 is excluded from the CA problem since its capacity C_i is set to a constant value of C_{0i}.

With the above link set assignment, Lagrange multiplier method can be applied to the CA problem, and Lagrangian G is constructed as in the following equation.

$$G = \sum_L D_i + \lambda \left(\frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} - T_{\max} \right), \tag{3.24}$$

where λ is a Lagrange multiplier.

Since $C_i = C_{0i}, \forall i \in L_2$, Eq.(3.24) can be rewritten as

$$G = \sum_{L_0 \cup L_1} D_i + \sum_{L_2} d_{0i} C_{0i} + \lambda \left(\frac{1}{\gamma} \sum_{L_0 \cup L_1} \frac{f_i}{C_i - f_i} + \frac{1}{\gamma} \sum_{L_2} \frac{f_i}{C_{0i} - f_i} - T_{\max} \right). \tag{3.25}$$

The constraint region of the CA problem can be described by the following three conditions.

$$\begin{aligned} \frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} &\leq T_{\max}, \\ C_i - f_i &> 0, \qquad \forall i \in L, \\ C_i &> 0, \qquad \forall i \in L, \end{aligned}$$

It can be shown that the constraint region of the problem mentioned above is convex. Since cost function of all links in the network can now be considered as linear function and the constraint region of the problem is convex, we consequently have that the CA problem is a convex programming problem.

By using the Lagrangian G, the necessary and sufficient conditions for minimal solution of the CA problem are given by the following Kuhn-Tucker conditions [39][69].

$$\frac{\partial G}{\partial C_i} = 0, \qquad \forall i \in L_0 \cup L_1, \tag{3.26}$$
$$\lambda \geq 0, \tag{3.27}$$
$$\frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} - T_{\max} \leq 0, \tag{3.28}$$
$$\lambda . \left(\frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} - T_{\max} \right) = 0. \tag{3.29}$$

It is clear that $T = \frac{1}{\gamma}\sum_L \frac{f_i}{C_i - f_i}$ is a decreasing function of all C_i's. Therefore, we can set the constraint of packet delay time as

$$T = \frac{1}{\gamma}\sum_L \frac{f_i}{C_i - f_i} = T_{\max}. \tag{3.30}$$

As a result, conditions (3.28) and (3.29) are both satisfied simultaneously. Next, we take $\frac{\partial G}{\partial C_i} = 0$, $\forall i \in L_0 \cup L_1$ such that condition (3.26) is satisfied. From Eq.(3.25), we obtain the following equation.

$$d_i - \lambda . \frac{f_i}{\gamma(C_i - f_i)^2} = 0, \qquad \forall i \in L_0 \cup L_1, \tag{3.31}$$

where

$$d_i = \frac{dD_i}{dC_i} = \begin{cases} d_{0i} & , \forall i \in L_0, \\ d_{1i} & , \forall i \in L_1. \end{cases} \tag{3.32}$$

After mathematical manipulation on Eq.(3.31), we get

$$\lambda = \frac{\gamma d_i (C_i - f_i)^2}{f_i}, \qquad \forall i \in L_0 \cup L_1. \tag{3.33}$$

Since the values of d_{0i} and d_{1i} are non-negative, it is easy to show that $\lambda \geq 0$. This means that the condition (3.27) is satisfied. Thus, the solution from Lagrange multiplier method is guaranteed to be the minimal solution of the CA problem, where we can derive the solution as in the following steps.

From Eq.(3.30) and Eq.(3.33), we respectively have

$$\frac{1}{\gamma}\sum_{L_0 \cup L_1} \frac{f_i}{C_i - f_i} = T_{\max} - \frac{1}{\gamma}\sum_{L_2} \frac{f_i}{C_{0i} - f_i}, \tag{3.34}$$

$$C_i = f_i + \sqrt{\frac{\lambda f_i}{\gamma d_i}}, \qquad \forall i \in L_0 \cup L_1. \tag{3.35}$$

Replacing Eq.(3.35) into Eq.(3.34), we have

$$\sqrt{\lambda} = \frac{\sum_{L_0 \cup L_1} \sqrt{\frac{f_i d_i}{\gamma}}}{T_{\max} - \frac{1}{\gamma}\sum_{L_2} \frac{f_i}{C_{0i} - f_i}}. \tag{3.36}$$

Again, substituting Eq.(3.36) into Eq.(3.35), and after some mathematical manipulations, we obtain the solution of $\{C_i\}$ as in Eq.(3.37) [81].

$$C_i = f_i + \frac{\sum_{L_0 \cup L_1} \sqrt{f_j d_j}}{\left(\gamma\, T_{\max} - \sum_{L_2} \frac{f_j}{C_{0j} - f_j}\right)} \sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L_0 \cup L_1, \tag{3.37}$$

where

$$d_i = \begin{cases} d_{0i} & ,\forall i \in L_0, \\ d_{1i} & ,\forall i \in L_1. \end{cases} \tag{3.38}$$

The value of C_i calculated from Eq.(3.37) is optimum for the CA problem under the relevant link set assigned at the beginning. However, the value has to be checked whether it contradicts its assigned link set or not, e.g., for $i \in L_0$, C_i that is greater than C_{0i} is thought to be a contradiction. If any contradiction occurs, the solution obtained will not be feasible and other combinations of link set have to be examined until a feasible solution with minimum network cost is found.

By the original version of the method of link set assignment, we can obtain an algorithm to the CA problem based on the concept of complete enumeration (CE). The algorithm is referred to as ALGORITHM CA_CE as follows.

ALGORITHM CA_CE

Step 1. Let network cost $D = \infty$.

Step 2. For $\forall i \in L$, put i into one of the three sets, L_0, L_1 or L_2.

Step 3. Set $d_i = d_{0i}$, $\forall i \in L_0$.

Set $d_i = d_{1i}$, $\forall i \in L_1$.

Reset $err = 0$.

Step 4. Determine C_i, $\forall i \in L_0 \cup L_1$ by Eq.(3.37).

Set $C_i = C_{0i}$, $\forall i \in L_2$.

Step 5. If $C_i \geq C_{0i}$, $\exists i \in L_0$, then set $err = 1$.

If $C_i \leq C_{0i}$, $\exists i \in L_1$, then set $err = 1$.

If $C_i \leq f_i$, $\exists i \in L$, then set $err = 1$.

If $T = \frac{1}{\gamma} \sum_L \frac{f_i}{C_i - f_i} > T_{\max}$, then set $err = 1$.

Step 6. Calculate network cost $D' = \sum_L D_i$ from the value of C_i's.

If $D' < D$ and $err \neq 1$, then set $D = D'$ and keep $\{C_i\}$ as the design solution.

Step 7. If all combinations of link set are completely examined, STOP and the final solution of the CA problem is $\{C_i\}$.

Step 8. Select another combination of link set (e.g., by swapping the members among L_0, L_1 and L_2) which has not been examined yet, and go to Step 3.

The parameter err in the above algorithm denotes whether the link set combination under consideration gives a feasible solution to the CA problem or not, where

$$err = \begin{cases} 0 & \text{, when the solution is feasible,} \\ 1 & \text{, otherwise.} \end{cases}$$

Clearly, the above ALGORITHM CA_CE can give global optimal solution to the CA problem with piecewise linear concave cost function in our study. However, the algorithm is obviously very time-consuming according to its nature of complete enumeration, that is we have to examine all of the link set combinations. Thus, a more efficient algorithm is required.

3.5 The Optimal Solution of the CA Problem

To find a way to solve the CA problem, we firstly review the modified version of the method of link set assignment proposed in [81]. Note that the basic concept of this idea has been already briefly described in Chapter 2. After that, we derive an important characteristic of the optimal solution of the CA problem.

3.5.1 Basic Concept

Since each link in the network certainly belongs to L_0, L_1, or L_2, we can select one of the three sets for each link and assign the link to that set. After that, we can check whether there is a contradiction between the assigned link set and the value of link capacity calculated from Eq.(3.37). The checking of contradiction can give us a way to decide that the link can belong to the set that it was assigned or not.

Before going further, it is important to note that there is a special kind of links whose link set must be fixed to L_1. Link i is a member of this special kind when its traffic flow and existing link capacity satisfy the following condition.

$$f_i \geq C_{0i}.$$

From the condition (3.4), it is easy to show that

$$C_i > C_{0i}.$$

The above implies that link i must be a member of only the set L_1. Hence, we will exclude this special kind of links from the CA problem, and concentrate on links with the following properties.

$$f_i < C_{0i} \qquad \text{or} \qquad C_{0i} - f_i > 0. \tag{3.39}$$

In the case of convex cost function, i.e., $d_{0i} < d_{1i}$, $\forall i \in L$, each link is firstly assigned to L_0 and if contradiction occurs, it is concluded that the link with contradiction cannot belong to L_0. In the next step, each link is set to L_1 and if contradiction occurs, we can say that the link with contradiction cannot belong to L_1. In the case that a link cannot belong to neither L_0 nor L_1, it must belong to L_2. By repeatedly examining links for the set L_2 until no more link has to be assigned to it, each link in the network will belong to a proper set, and link capacity can be computed by using Eq.(3.37). It is proven that the final solution by the above revised version of the method of link set assignment is a global optimal solution for the case of convex cost function [81].

Next, we investigate that how the above method can deal with the CA problem with piecewise linear concave cost function, i.e., $d_{0i} > d_{1i}$, $\forall i \in L$. Let's introduce two parameters for link i: e_{0i} and e_{1i}. The values of the two parameters are set as follows.

- e_{0i} will be 0 if link i is assigned to L_0 and no contradiction occurs, and be 1 if contradiction exists.
- e_{1i} will be 0 if link i is assigned to L_1 and no contradiction occurs, and be 1 if contradiction exists.

Then, we assign each link to L_0 and use Eq.(3.37) to determine $\{C_i\}$. After that, we check whether there is any contradiction or not, i.e., there is link i that $C_i \geq C_{0i}$ or not. In other words, we examine e_{0i} of link i whether the value is 0 or 1. Unlike the case of convex cost function, we focus on the links without contradiction, namely link i with $e_{0i} = 0$. This is because we can show that this kind of links can belong to L_0 without any contradiction although link set combination is changed. The proof related to this case is given below in Theorem 1.

Theorem 1 *When link i is assigned to belong to L_0 and $e_{0i} = 0$, the link i can be assigned to L_0 without any contradiction even some of other links are changed to belong to L_1.*

Proof: Firstly, we define the following sets and notations.

$$\begin{aligned} L_0' &= \{j \mid j \text{ is the link already assigned to } L_0\}, \\ L_1' &= \{j \mid j \text{ is the link already assigned to } L_1\}, \end{aligned}$$

$$\begin{aligned} b_0 &= \sum_{L_0'} \sqrt{f_j d_{0j}}, \\ b_0' &= \sum_{L-(L_0' \cup L_1')} \sqrt{f_j d_{0j}}, \\ b_1 &= \sum_{L_1'} \sqrt{f_j d_{1j}}, \\ b_1' &= \sum_{L-(L_0' \cup L_1')} \sqrt{f_j d_{1j}}. \end{aligned}$$

Next, we assume that there is a link i with $e_{0i} = 0$. Since there is no contradiction for this link, we have

$$C_i = f_i + \frac{b_0 + b_1 + b_0'}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{0i}}} < C_{0i}, \tag{3.40}$$

or

$$\frac{1}{\sqrt{d_{0i}}} < \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b_0')}. \tag{3.41}$$

Currently, all links in $L - (L'_0 \cup L'_1)$ are assigned to L_0 for the checking of link set contradiction. Let L''_1 be a set of links in $L - (L'_0 \cup L'_1)$ that are changed to belong to L_1. After the change, we have b'_0 becomes b^* as

$$b^* = \sum_{L''_0} \sqrt{f_j d_{0j}} + \sum_{L''_1} \sqrt{f_k d_{1k}}, \tag{3.42}$$

where

$$L''_0 = L - (L'_0 \cup L'_1 \cup L''_1).$$

Note that the link i under consideration must be in L''_0, since we are checking link set contradiction when this link is assigned to L_0.

For the case of concave function, we have $d_{0i} > d_{1i}$, $\forall i \in L$. This leads to

$$b'_0 = \sum_{L''_0 \cup L''_1} \sqrt{f_j d_{0j}} > b^*. \tag{3.43}$$

The above and (3.39) make

$$\frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b'_0)} < \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b^*)}. \tag{3.44}$$

From (3.41) and (3.44), we have

$$\frac{1}{\sqrt{d_{0i}}} < \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b^*)}, \tag{3.45}$$

or

$$C_i = f_i + \frac{b_0 + b_1 + b^*}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{0i}}} < C_{0i}. \tag{3.46}$$

The last statement implies that e_{0i} is still equal to 0 as before, although link set combination is changed. In other words, link i with $e_{0i} = 0$ can belong to L_0 without any contradiction. □

By the same idea, when each link is set to L_1, we can say that the links with no contradiction can belong to L_1. This can be shown by Theorem 2.

Theorem 2 *When link i is assigned to belong to L_1 and $e_{1i} = 0$, the link i can be assigned to L_1 without any contradiction even some of other links are changed to belong to L_0.*

Proof: The proof can be done in almost the same way as in Theorem 1. Firstly, we assume that all of the links in $L - (L'_0 \cup L'_1)$ are currently assigned to L_1, and link set contradiction is checked. Assume again that there is a link i with $e_{1i} = 0$.

Since there is no contradiction for link i, we then have

$$C_i = f_i + \frac{b_0 + b_1 + b'_1}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{1i}}} > C_{0i}, \tag{3.47}$$

or

$$\frac{1}{\sqrt{d_{1i}}} > \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b'_1)}. \tag{3.48}$$

Next, we try to change the set of some links. Let L_0'' be a set of links in $L - (L_0' \cup L_1')$ that are changed to belong to L_0. With this change, we have b_1' becomes b^{**} as

$$b^{**} = \sum_{L_0''} \sqrt{f_j d_{0j}} + \sum_{L_1''} \sqrt{f_k d_{1k}}, \tag{3.49}$$

where

$$L_1'' = L - (L_0' \cup L_1' \cup L_0'').$$

Note that the link i under consideration must be a member of L_1'', since we are checking link set contradiction when this link is assigned to L_1.

For the case of concave function, we have $d_{0i} > d_{1i},\ \forall i \in L$. This leads to

$$b_1' = \sum_{L_0'' \cup L_1''} \sqrt{f_j d_{1j}} < b^{**}. \tag{3.50}$$

The above relation and (3.39) gives

$$\frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b_1')} > \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b^{**})}. \tag{3.51}$$

From (3.48) and (3.51), we have

$$\frac{1}{\sqrt{d_{1i}}} > \frac{\gamma\, T_{\max}\, (C_{0i} - f_i)}{\sqrt{f_i}\, (b_0 + b_1 + b^{**})}, \tag{3.52}$$

or

$$C_i = f_i + \frac{b_0 + b_1 + b^{**}}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{1i}}} > C_{0i}. \tag{3.53}$$

This implies that the value of e_{1i} remains unchanged and is equal to 0 as before, although link set combination is changed. In other words, link i with $e_{1i} = 0$ can belong to L_1 without any contradiction. □

3.5.2 Characteristic of the Optimal Solution

By the above concept and theorems, we can construct the following procedure for assigning a proper set to each link. There are four possible cases related to the values of e_{0i} and e_{1i}. They are

1. $e_{0i} = 0$ and $e_{1i} = 1$,
2. $e_{0i} = 1$ and $e_{1i} = 0$,
3. $e_{0i} = 0$ and $e_{1i} = 0$,
4. $e_{0i} = 1$ and $e_{1i} = 1$.

In case 1, we can let link i belong to L_0 since there is no contradiction as mentioned in Theorem 1 above. Although link i may not belong to L_0 in the global optimal solution of the problem, we deal with the link in a greedy manner to obtain a feasible solution. The reason is that it has been guaranteed that there will be no contradiction even the sets of other links are changed. Moreover, due to the non-convexity of the CA problem, its global optimum cannot be determined easily, so we have to try to find its local optimum instead.

As same as case 1, link i that falls into case 2 is then assigned to L_1 as already shown in Theorem 2.

In case 3, since link i can belong to either L_0 or L_1 without any contradiction, we can apply any method to select the set for this type of links. For example, the set can be randomly chosen between L_0 and L_1.

Finally, for links of case 4, we do not assign this type of links to any set, but repeatedly do the above procedure again until all links have their proper set.

Obviously, there will be no problem if links of case 4 do not exist at the final stage of the examining, and each link in the network will then be assigned to either L_0 or L_1. However, if this kind of links exist, the question that they can be assigned to the set L_2 or not will arise. To answer this question, we introduce the following theorem.

Theorem 3 *In the optimal solution of the CA problem,*

$$L_2 = \emptyset.$$

Proof: We assume that there exists a link i such that $e_{0i} = 1$ and $e_{1i} = 1$. From this, we have

$$C_i = f_i + \frac{b_0 + b_1 + b_0'}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{0i}}} \;\geq\; C_{0i}, \tag{3.54}$$

$$C_i = f_i + \frac{b_0 + b_1 + b_1'}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_{1i}}} \;\leq\; C_{0i}. \tag{3.55}$$

From (3.39), manipulating (3.54) and (3.55) yields

$$\alpha_i\,(b_0 + b_1 + b_0') \;\geq\; \sqrt{f_i d_{0i}}, \tag{3.56}$$

$$\alpha_i\,(b_0 + b_1 + b_1') \;\leq\; \sqrt{f_i d_{1i}}, \tag{3.57}$$

where

$$\alpha_i = \frac{f_i}{\gamma\, T_{\max}\,(C_{0i} - f_i)}.$$

At the final stage of the examining, (3.56) and (3.57) are valid for $\forall i \in L - (L_0' \cup L_1')$. Consequently, we can take the summation on both (3.56) and (3.57) over the set $L - (L_0' \cup L_1')$. This gives

$$\sum_{L-(L_0'\cup L_1')} \alpha_i\,(b_0 + b_1 + b_0') \;\geq\; b_0', \tag{3.58}$$

$$\sum_{L-(L_0'\cup L_1')} \alpha_i\,(b_0 + b_1 + b_1') \;\leq\; b_1'. \tag{3.59}$$

From (3.58) and (3.59), we obtain

$$\frac{b_0 + b_1 + b'_0}{b_0 + b_1 + b'_1} \geq \frac{b'_0}{b'_1}. \tag{3.60}$$

After manipulating (3.60), we get

$$b'_1 \geq b'_0. \tag{3.61}$$

However, this contradicts to the fact that

$$d_{0i} > d_{1i}, \qquad \forall i \in L.$$

Thus, there cannot be any link i whose e_{0i} and e_{1i} are both equal to 1 simultaneously at the final stage of examining, and all links in the network will belong to either L_0 or L_1. This result implies that

$$L_2 = \emptyset. \tag{3.62}$$

This completes the proof. □

From Theorem 3, it has been shown that there is no link in the network that belongs to the set L_2 in the optimal solution of the CA problem. With this essential characteristic, we can obtain a heuristic design algorithm for the CA problem as proposed in the next section.

3.6 Algorithm for the CA Problem

Based on the fact that $L_2 = \emptyset$ in the optimal solution of the CA problem with piecewise linear concave cost function, we can exclude the point $C_i = C_{0i}$ from the cost function of all links in the network. Consequently, the link cost function can then be regarded as a differentiable function, where its differentiation $\frac{dD_i}{dC_i}$ can be expressed as in the following equation.

$$\frac{dD_i}{dC_i} = \begin{cases} d_{0i} & , \forall i \in L_0, \\ d_{1i} & , \forall i \in L_1. \end{cases} \tag{3.63}$$

Clearly, we have no need to approximate the non-differentiable function by a differentiable function as that was done in [93]. As a result, the Lagrange multiplier method can be directly applied to the CA problem as in the following algorithm.

ALGORITHM CA_LM

Step 1 Initialize d_i to either d_{0i} or d_{1i} by any method, $\forall i \in L$.

Set ϵ as a small positive value for using as algorithm termination parameter.

Determine the initial value of Lagrange multiplier β' by

$$\beta' = \left(\frac{\sum_L \sqrt{f_i d_i / \gamma}}{T_{\max}} \right)^2 .$$

Step 2 Determine C_i by

$$C_i = f_i + \sqrt{\frac{\beta' f_i}{\gamma\, d_i}}, \qquad \forall i \in L.$$

Step 3 If $C_i < C_{0i}$, then set $d_i = d_{0i}$, else set $d_i = d_{1i}$, $\forall i \in L$.

Step 4 Determine the Lagrange multiplier β by

$$\beta = \left(\frac{\sum_L \sqrt{f_i d_i / \gamma}}{T_{\max}} \right)^2 .$$

Step 5 If $|\beta - \beta'| > \epsilon$, then set $\beta' = \beta$ and go to Step 2, else STOP and determine C_i by

$$C_i = f_i + \frac{\sum_L \sqrt{f_j d_j}}{\gamma\, T_{\max}} \sqrt{\frac{f_i}{d_i}}, \qquad \forall i \in L.$$

Note that there are many ways to set the initial value of d_i at Step 1 of the algorithm. Some examples are given in the following section.

3.7 Numerical Results and Discussions

This section gives some numerical results on the performance of the proposed algorithm, ALGORITHM CA_LM, in the view of its optimality and computation amount, and some experimental results on the effect of existing network to the link capacity assignment in packet-switched networks for the case of long-term design.

3.7.1 Performance of the Proposed Algorithm

Firstly, we examine the optimality of ALGORITHM CA_LM by comparing its results with global optimal solutions. Note that we can determine the global optimal solutions by using ALGORITHM CA_CE, since this algorithm examines all link set combinations, i.e., it runs in a complete enumeration manner. However, this is possible only for the case of small-sized networks in practical applications.

In the numerical results below, we consider packet-switched networks with the following characteristics.

1. The number of nodes in the network, N, is equal to 3, 4, 5, 6, 7, and 8.
2. The network topology is fully connected. This means there is a direct link connecting every node-pair. The number of links in the network, n, is then equal to

$$n = \frac{N(N-1)}{2}.$$

3. The traffic flow on each link in existing network is randomly set in the range (0, 80] kbps following the uniform distribution.
4. The link cost function in the existing network is assumed to be linear with per-unit cost of c_i, which is randomly set over (0, 2]. Then the link capacity in existing network can be calculated by Eq.(3.18), where $T_{\max} = 20$ ms.
5. To design a new network, we assume that the new traffic flow on each link is uniformly randomized over the range (0, 80] kbps. Therefore, the new traffic flow can be greater than, equal to, or even less than the previous value of traffic flow in the existing network.
6. Link cost function in designing a new network is piecewise linear concave with

$$d_{0i} > d_{1i},$$

where d_{0i} and d_{1i} are both chosen between (0, 2] randomly.

For ALGORITHM CA_LM, we consider the following two methods for setting the initial value of d_i at Step 1 of the algorithm.

method A d_i is set according to the relationship between link flow f_i and the existing link capacity C_{0i} as follows.

$$d_i = \begin{cases} d_{0i} & \text{if } f_i < C_{0i}, \\ d_{1i} & \text{if } f_i \geq C_{0i}. \end{cases}$$

method B d_i is set randomly between d_{0i} and d_{1i}.

To obtain a good solution for a CA problem, we apply the proposed algorithm in the following five cases.

- case 1: method A
- case 2: method A and 10 times of method B
- case 3: method A and 20 times of method B
- case 4: method A and 50 times of method B
- case 5: method A and 100 times of method B

In the above cases, when there are many solutions obtained for a specific problem, we select the best solution, namely the solution with the smallest total network cost, as a final solution.

The relationships between the number of links in the network (n) and the percentage that each of the above five cases yields global optimal solution are shown in Table 3.1 and 3.2, where the number of random traffic patterns applied to the network is one hundred and five thousand, respectively.

Table 3.1: Percentage of yielding global optimum of ALGORITHM CA_LM: 100 traffic patterns.

n	case 1	case 2	case 3	case 4	case 5
3	90	95	96	96	100
6	90	96	97	97	100
10	83	97	98	99	100
15	77	98	98	98	100
21	75	98	100	100	100
28	61	94	97	99	100

Table 3.2: Percentage of yielding global optimum of ALGORITHM CA_LM: 5000 traffic patterns.

n	case 1	case 2	case 3	case 4	case 5
3	96.02	98.80	99.08	99.08	100.00
6	91.16	98.74	98.90	98.96	100.00
10	83.90	98.18	98.46	98.78	100.00
15	77.10	97.82	98.26	98.72	100.00
21	70.66	96.64	97.88	98.56	99.96
28	61.88	94.06	97.02	98.34	99.84

Next, we observe the difference between the solution obtained from each case and the global optimum. Table 3.3 and 3.4 show the average value of the ratio between the result obtained from the proposed algorithm and the global optimum.

From Table 3.1−3.4, it can be seen that the proposed algorithm has a lower percentage of yielding global optimum when the number of links in the network increases. However, the percentage is equal to 100 % for almost of the cases in case 5. Moreover, the solution by the proposed algorithm is very close to the global optimum as can be seen from the average ratio between the solution and the global optimum. Thus, we can say that the proposed algorithm, ALGORITHM CA_LM, solves the CA problem very efficiently.

Table 3.3: Average ratio of solution by ALGORITHM CA_LM and global optimum: 100 traffic patterns.

n	case 1	case 2	case 3	case 4	case 5
3	1.00080	1.00045	1.00030	1.00030	1.00000
6	1.00047	1.00020	1.00014	1.00014	1.00000
10	1.00078	1.00005	1.00005	1.00005	1.00000
15	1.00058	1.00002	1.00002	1.00002	1.00000
21	1.00051	1.00003	1.00000	1.00000	1.00000
28	1.00044	1.00005	1.00005	1.00000	1.00000

Table 3.4: Average ratio of solution by ALGORITHM CA_LM and global optimum: 5000 traffic patterns.

n	case 1	case 2	case 3	case 4	case 5
3	1.00034	1.00012	1.00009	1.00009	1.00000
6	1.00043	1.00004	1.00004	1.00004	1.00000
10	1.00055	1.00005	1.00004	1.00004	1.00000
15	1.00059	1.00004	1.00003	1.00002	1.00000
21	1.00061	1.00004	1.00003	1.00002	1.00000
28	1.00064	1.00004	1.00002	1.00001	1.00000

Next, we investigate the computation amount of the proposed algorithm by showing its computation time using actual computation results. Again, a network with fully connected topology is considered, and five thousand random traffic patterns are applied to the network. This time, the number of nodes in the network takes the values of 10, 20, 30, 40, ..., 120, 130, 140, and 150.

The relationships between the number of links in the network (n) and average, maximum, and variance of computation time are shown in Table 3.5, where the values of computation time are measured in second for one running time of the proposed algorithm. The machine used in the measurement in this experiment is a personal computer with CPU Pentium III 450 MHz and RAM 256 MBytes.

Table 3.5: Computation time of ALGORITHM CA_LM.

n	average	maximum	variance
45	0.003	0.020	0.00002084
190	0.007	0.030	0.00002001
435	0.015	0.030	0.00002739
780	0.025	0.040	0.00003034
1225	0.039	0.050	0.00001967
1770	0.056	0.070	0.00003407
2415	0.076	0.090	0.00003744
3160	0.099	0.110	0.00003712
4005	0.125	0.140	0.00005419
4950	0.154	0.170	0.00006902
5995	0.186	0.200	0.00006480
7140	0.222	0.250	0.00008756
8385	0.260	0.280	0.00012096
9730	0.302	0.320	0.00013495
11175	0.347	0.370	0.00014052

Results in Table 3.5 are depicted again in Figures 3.4 and 3.5.

Figure 3.4 shows that the average and maximum computation time of ALGORITHM CA_LM are approximately linear with respect to the number of links in the network. By linear regression, the values of linear correlation coefficients are 0.9999971 and 0.9993178, respectively.

Figure 3.5 shows that the variance of computation time of the proposed algorithm is very small. Hence, we can conclude that ALGORITHM CA_LM is very practical for applying to the CA problem of a large-scale network.

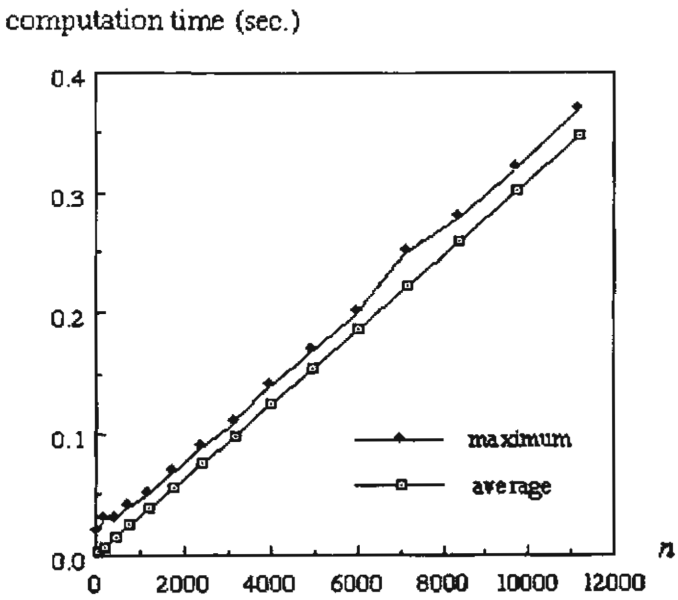


Figure 3.4: Average and maximum computation time of ALGORITHM CA_LM.

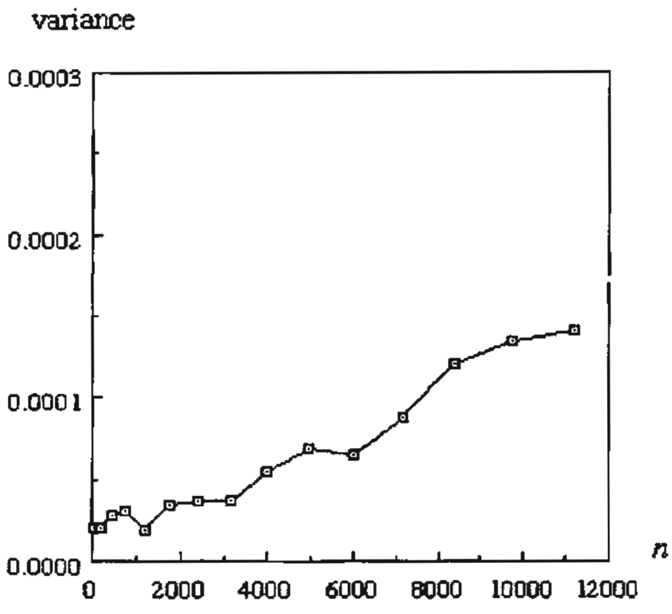


Figure 3.5: Variance of computation time of ALGORITHM CA_LM.

3.7.2 Effect of Existing Network to Network Design

Here, some numerical results will show that how the existing network facilities affect the CA problem in the case of long-term design, where the link cost is of a piecewise linear concave function. We consider a simple 3-node network as shown in Figure 3.6, where the network has three links: 1, 2, and 3.

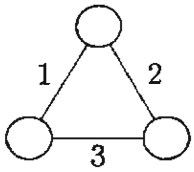


Figure 3.6: A 3-node packet-switched network.

Network parameters are set as follows.

- C_{0i} = 52 kbps for i = 1, 2, 3.
- The cost functions of link 1 and 2 are both linear with respect to link capacity, and the cost coefficients (per-unit cost) are equal to 1.
- $f_1 = f_2 = 40$ kbps.
- $T_{\max} = 20$ ms., and the mean value of packet length is 400 bits/packet.

By varying f_3 and applying several pairs of (d_{03}, d_{13}), we obtain the results as shown in Figure 3.7. It can be seen from the figure that the capacity of link 3 (C_3) is never equal to its existing link capacity (52 kbps) for all over the entire range of f_3. This confirms the characteristic of the optimal solution of the CA problem, namely $L_2 = \emptyset$.

It can also be observed from the result that there are some values of f_3 that make the curve of link capacity value to be discontinuous. These approximated values are summarized as follows.

$$\begin{aligned}
(d_{03}, d_{13}) &= (1,0.2) &\rightarrow\ f_3 &= 29.32 \text{ kbps}, \\
(d_{03}, d_{13}) &= (1,0.5) &\rightarrow\ f_3 &= 31.44 \text{ kbps}, \\
(d_{03}, d_{13}) &= (5,1) &\rightarrow\ f_3 &= 37.47 \text{ kbps}, \\
(d_{03}, d_{13}) &= (10,1) &\rightarrow\ f_3 &= 39.06 \text{ kbps}.
\end{aligned}$$

At these values of f_3, the total network cost is the same for both the cases when link 3 belongs to L_0, and when it belongs to L_1. It means that we can have two optimal solutions with the same network cost.

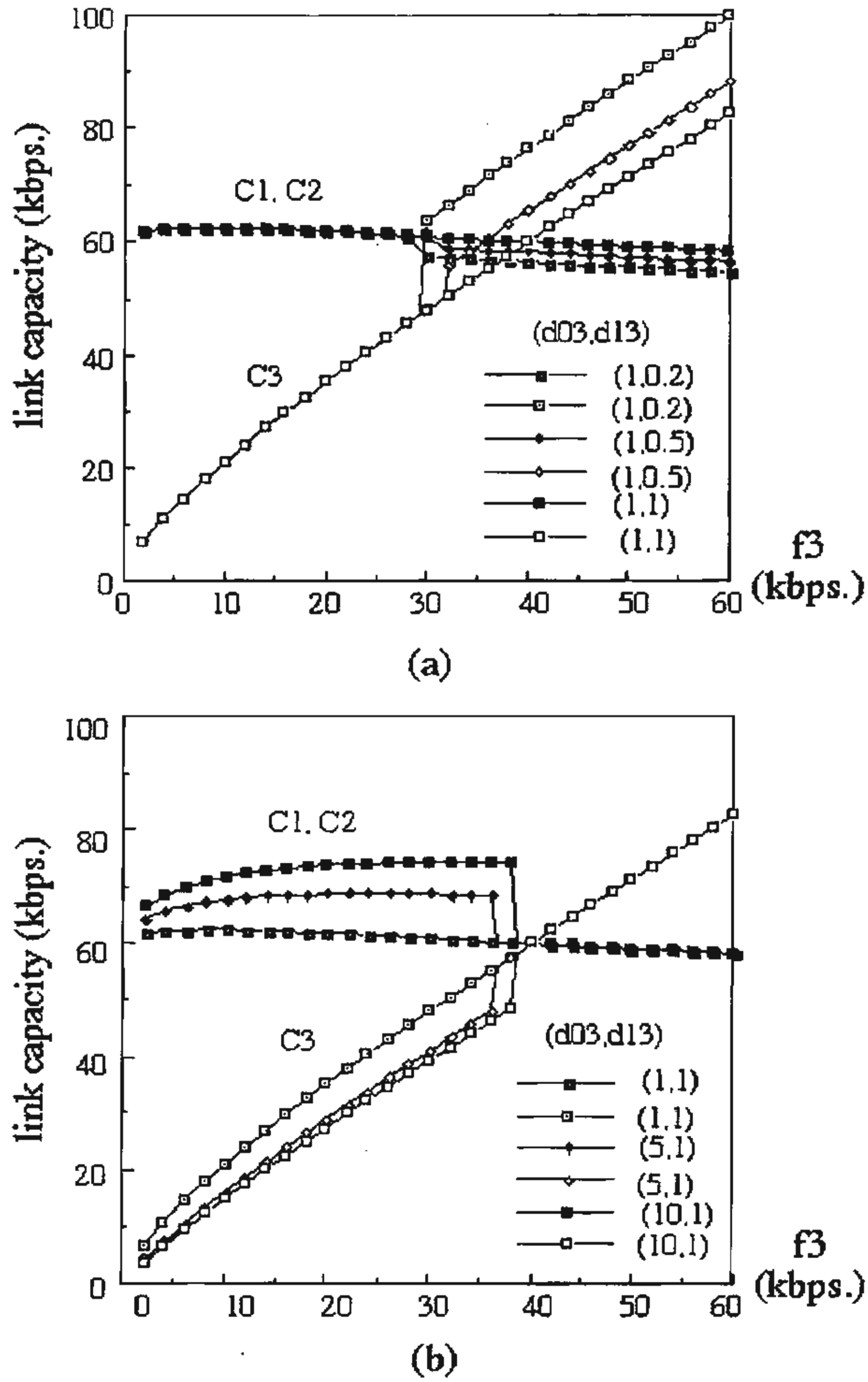


Figure 3.7: Results of the 3-node packet-switched network.