Finally, we handle the effect of system boundary as follows: for i =1,2,...,k (dimensions) ; if (a_i=1 or b_i=n_i) then increment system boundary (SB) by 1, where at beginning the value of SB is set to zero. Then, we add the effect of combining with the system boundary (k - SB) into the CF(α) that is CF(α) = CF(α) + (k - SB). Since combinable buddies of α (or C) = 2, k, or 1 for three partitioning and combining methods (see Algorithm A.3-2), then their time complexities are O(k), O(k^2), and O(k), respectively.

ALGORITHM A.3-2: "Combinable nodes (C)" for each partitioning and combining method is identified based upon its corresponding partitioning and combining method, as follows:

- For the partitioning and combining by network degree, time complexity of the CF(α) is O(1) and the number of combinable buddies (C) are C = 1 (if n_i = 2 i.e., a hypercube network) or C = 2 (if n_i > 2) and their IDs are defined as follows: See some corresponding examples in Figure 50 and 51.

  If n_i = 2 and if ID of α = id, then C = 1 (β) and ID of β_i = 1 (if id = 2) or 2 (if id = 1).
  If n_i > 2 and if ID of α = id, then C = 2 (β_1, β_2) and

  $$\text{ID of } \beta_{i1} = \begin{cases} \text{id -1} & \text{if id = 2, 3, ..., } n_i \text{ for both networks and} \\ \text{0 (SB)} & \text{if id = 1 for a non-wraparound network or} \\ n_i & \text{if id = 1 for a wraparound network.} \end{cases}$$

  $$\text{and ID of } \beta_{i2} = \begin{cases} \text{id+1} & \text{if id+1} \le n_i\text{; otherwise 0 (SB) for a non-wraparound network (a buddy).} \\ \text{id+c} & \text{if id+c} \le n_i\text{; otherwise 0 (SB) for a non-wraparound network (a combine).} \\ \text{(id+1) mod } n_i & \text{for a wraparound network (a buddy).} \\ \text{(id+c) mod } n_i & \text{for a wraparound network (a combine).} \end{cases}$$

  where SB = System Boundary, c = #combined nodes.

- For the partitioning and combining by network size, the number of combinable buddies (C = k) and time complexity of the CF(α) is O(k^2) and each combinable node j at level L – i (β_{ij}, where i, j = 1, 2, ..., k) is defined by using the k-bit-map as follows: See also some corresponding examples in Figure 52 and 53.

  $\beta_{ij} = (b_{k-1} \ldots \underline{b_j} \ldots b_1 b_0)$ (or negate the j^{th} bit of α = $(b_{k-1} \ldots b_1 b_0)$; b_j = 0 or 1, for a buddy.
  $\beta_{ij} = (t_{k-1} \ldots \underline{t_j} \ldots t_1 t_0)$ (or negate the j^{th} bit (non*) of α=$(t_{k-1} \ldots t_1 t_0)$; t_j = 0,1,or * for a combine.

- For the partitioning and combining by network degree and size, there is only one combinable buddy (C = 1) at each level and time complexity of the CF(α) is O(k) and the combinable node at level L – i (β_i, where i = 1, 2, ..., k) is defined by negating ID of α (id), then ID of β_i = 1 (if id = 2) or 2 (if id = 1). See also a corresponding example in Figure 54.

Some examples of the combinable nodes for the partitioning and combining by network degree (method 1) are illustrated in Figure 50 for a 2-D system (i.e., mesh (non-wraparound network) or torus (wraparound network).) First, Figure 50.a illustrates two combinable buddies of a buddy (α = 2) for a 2-D mesh, which are β_{1j} = 1, 3 (indicated in dash blocks) for j = 1, 2, respectively. Figure 50.b illustrates two combinable buddies of a buddy (α = n_2) for a 2-D torus, which are β_{1j} = n_2–1, 1. Figure 50.c shows only one combinable buddy of a combine node (α = 1-2) for another 2-D mesh, which are β_1 = 3. Finally, Figure 50.d illustrates two combinable buddies of a combine node (α = $n_{k-1} - n_k$) for another 2-D torus, which are β_{1j} = n_2–2, 1 for j = 1, 2, respectively.

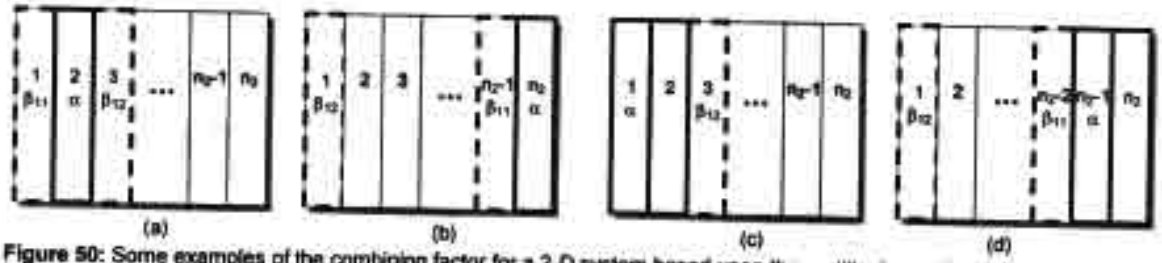


Figure 50: Some examples of the combining factor for a 2-D system based upon the partitioning and combining by network degree: a) a buddy 2-D mesh; b) a buddy 2-D torus; c) a combine 2-D mesh; and d) a combine 2-D torus.

Figure 51 illustrates the practical example of applying the partitioning and combining by network degree. Given a 2-D mesh (or non-wraparound network) and a considering node α at level 3 (L). The combinable β_{11} and β_{12} (at level 3) and β_{22} (at level 2) are illustrated in the k-Tree and the system status. Note: at level 2 (L-1), there is not β_{21} since it is a boundary of the system. Then, the combining factor (CF) of α (where k = 2) in Figure 51 is computed as follows:

$$CF(\alpha) = CF_1(\alpha_1 = \alpha, \beta_{1j}) + CF_2(\alpha_2 = R(\alpha_1), \beta_{2j})$$

$$= [PC_1(\alpha_1, \beta_{11}) + PC_2(\alpha_1, \beta_{12})] + [PC_1(\alpha_2, \beta_{21} + PC_2(\alpha_2, \beta_{22}]$$

$$= [1 + \tfrac{1}{4}] + [0 + 1] = 2\tfrac{1}{4}.$$

Finally, when considering the system boundary effect (where k = 2 and SB = 1), then CF(α) is 2¼ + (k - SB) = 3¼ since one size of this sub-system is a system boundary.

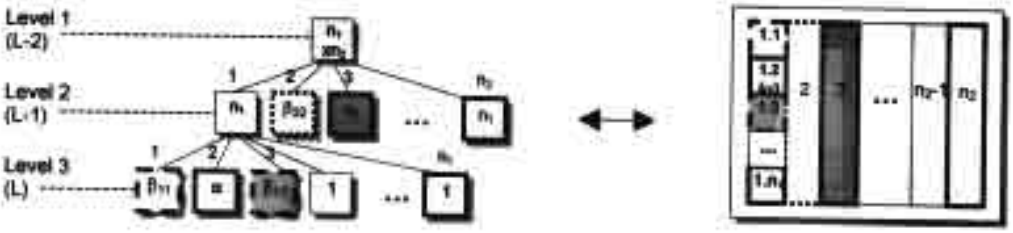


Figure 51: An example of computing CF(α) for the partitioning and combining by network degree.

Some examples of the combinable nodes for the partitioning and combining by network size are also illustrated in Figure 52 for a 2-D mesh system. Figure 52.a illustrates two combinable buddies of a buddy (α), where ID of α = 1 (00), 2 (01), 3 (10), or 4 (11), respectively. In the first figure, where ID of α = 1 (or 00), then IDs of two combinable nodes are $\beta_{11} = 0\underline{1}$ (or ID = 2) and $\beta_{12} = \underline{1}0$ (or ID = 3). Figure 52.b illustrates two combinable buddies of a combine node (α = *0 or {00, 10}). Therefore, the two combinable nodes are $\beta_{11} = 0\underline{1}$ (or ID = 2) and $\beta_{12} = 1\underline{1}$ (or ID = 4) respectively.

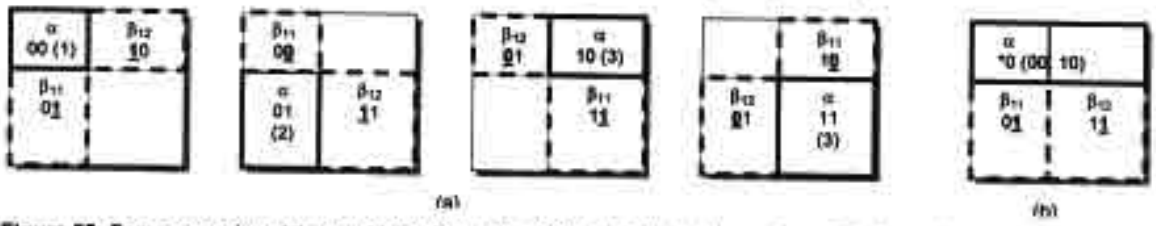


Figure 52: Some examples of the combining factor for a 2-D system based upon the partitioning and combining by network size: a) 4 possible case of a buddy and b) a combine node.

Figure 53 illustrates another practical example of applying the partitioning and combining by network size. Given a 2-D mesh (N = 64 x 64). Let α (or α_1) = b_1b_0 = 11 (or ID = 4), residing at level 3 (L) and 2 combinable buddies of α are $\beta_{11} = 1\underline{0}$ (or ID = 3) ; $\beta_{12} = \underline{0}1$ (or ID = 2). Assume the new considering node at level 2 (L-1) is the root of node α_1 at level 2 is $\alpha_2 = R(\alpha_1) = 10$ (or ID = 3) and then 2 adjacent nodes of α are $\beta_{21} = 1\underline{1}$ (or ID = 4) and $\beta_{22} = \underline{0}0$ (or ID = 1).

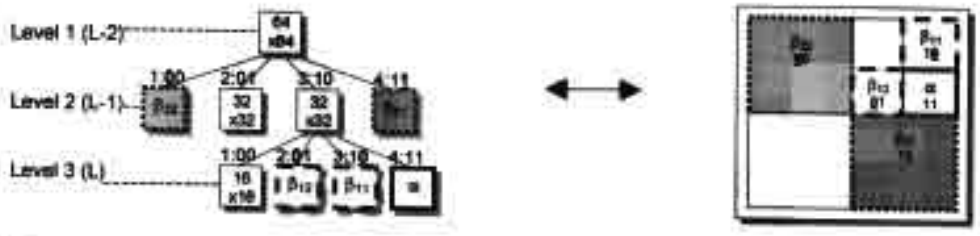


Figure 53: An example of computing CF(α) for the partitioning and combining by network size.

Then, the combining factor (CF) of α (where k = 2) in Figure 53 is computed as follows:

$$CF(\alpha) = CF_1(\alpha_1 = \alpha, \beta_{1j}) + CF_2(\alpha_2 = R(\alpha_1), \beta_{2j})$$

$$= [PC_1(\alpha_1, \beta_{11}) + PC_2(\alpha_1, \beta_{12})] + [PC_1(\alpha_2, \beta_{21} + PC_2(\alpha_2, \beta_{22}]$$

$$= [1 + \frac{1}{4}] + [\frac{1}{4} + \frac{1}{4}] = 1\frac{3}{4}.$$

Finally, when considering the system boundary effect (where k = 2 and SB = 2), then CF(α) is $1\frac{3}{4} + (k - SB) = 1\frac{3}{4}$

Figure 54 illustrates the practical example of applying the partitioning and combining by network degree and size. Given a 2-D mesh (N = 64x64). Let α is a Buddy#2 at level 5. A combinable buddy of α is β_1 = Buddy#1 (at the same level). For another level, β_2 = Buddy#1 (at level 4) since the root of α is Buddy#2 at level 4. Then, the combining factor (CF) of α (where k = 2) in Figure 54 is computed as follows:

$$CF(\alpha) = CF_1(\alpha_1 = \alpha, \beta_1) + CF_2(\alpha_2 = R(\alpha_1), \beta_2) = \frac{1}{4} + 1 = 1\frac{1}{4}.$$

Finally, when considering the system boundary effect (where k = 2 and SB = 1), then CF(α) is $1\frac{1}{4} + (k - SB) = 2\frac{1}{4}$.

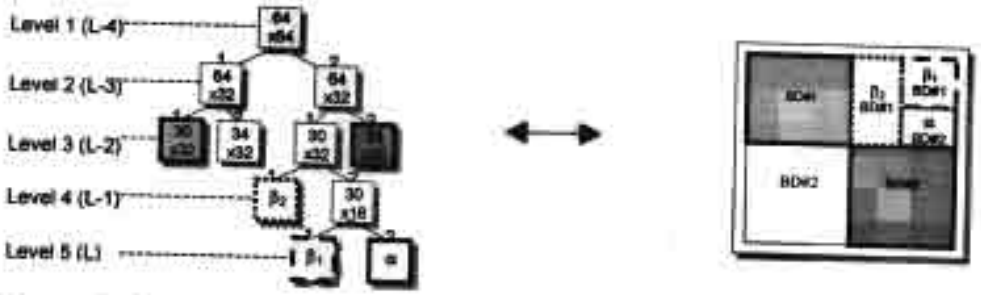


Figure 54: An example of computing CF(α) for the partitioning and combining by network degree and size.

3.4.1.4 Algorithm for Criterion 4 (Best Buddy Location after Partitioning)

After searching to visit all nodes in the tree, we obtain the best sub-system, whose size may equal or larger than the requested task. If the sub-system size already fits to the task, we do not have to apply the partitioning process and this criterion. Otherwise, we apply the partitioning process and Algorithm A.4 for each buddy node to find the most fit one (or the like best buddy node among B nodes, where the number of buddies B = n_k, 2^k, or 2 for three partitioning and combining methods in $O(kn_k)$, $O(k2^k)$, O(k), respectively.

ALGORITHM A.4: "The Best Buddy (or Best Sub-partition)" is applied for partitioning process (Step 4 in the best-fit heuristic). In this case, assume the best free sub-system from Steps 1-3 is the node represented as S, whose size is larger than the requested task. Then, the node S will be partitioned into B buddies and the best sub-partition (or one of B buddies will be allocated to the request), where B = n_k, 2^k, or 2 for the three partitioning and combining methods, respectively. In this case, we apply an approximate probability of combining (PC = 0, 1/4 , 1/2 , or 1) similar to that of the criterion 3.

However, the combining factor for the k-1 adjacent levels is similar for all possible B buddies since they always have the same root. Therefore, first we just apply the system boundary effect for each buddy, as follows: for i = 1, 2, ..., k ; if (a_i = 1 or b_i = n_i) then increment system boundary (SB) by 1, where at beginning the value of SB is set to zero. Then, the effect of combining with the system boundary is defined in terms of the probability of combining (PC = k – SB), which is computed for all buddy nodes in O(kn), $O(k2^k)$, O(k) time for three partitioning and combining methods, respectively. The buddy (α) that yields the minimum probability of combining (min PC), corresponding to the system boundary is selected.

If all buddy nodes provide the same PC, then we justify our decision in order to select the likely best buddy (α), based upon the local combining capability in terms of the probability of combining (PC), which is defined as follows:

- For the partitioning and combining by network degree, the best buddy (one of all n nodes) will be identified as follows: for j = 1, 2, ..., n; PC_j = PC (sub-buddies j) of the left combinable buddy (β_1) (or PC (β_1) if its sub-buddies do not exist) + PC (sub-buddies j) of the right combinable buddy (β_2) (or PC (β_2) if its sub-buddies do not exist). Then, the ID of α is set equal to the ID of the sub-buddy or j (if it yields the minimum PC). This process is computed in O(n) time since we have n buddy nodes and only two combinable nodes for each buddy. See some corresponding examples in Figure 55 and 56.

- For the partitioning and combining by network size, the best buddy (one of all 2^k nodes) can be identified directly from the ID of the best sub-system S (from step 1-3 or the one that needs to be partitioned). That is the ID of α is set equal to the ID of S (1, 2, ..., or 2^k). This can be computed in $O(2^k)$ time since we have 2^k buddy nodes. See some corresponding examples in Figure 57 and 58.

- For the partitioning and combining by network degree and size, the best buddy (one of two nodes) will be identified as follows: for j = 1, 2; PC_j = PC (sub-buddies j) of the combinable buddy (β) (or PC (β) if its sub-buddies do not exist). Then, the ID of α is set equal to the ID of the sub-buddy j (if it yields the minimum PC). This process is computed in O(1) time since we have only two buddy nodes. See some corresponding examples in Figure 59 and 60.

Some examples of identifying the best buddy nodes for the partitioning and combining by network degree are illustrated in Figure 55. Figure 55.a shows the computing of PC_j (1¼), which is equal for j = 1, 2, 3, and 4. Therefore, any buddy can be selected. Then, in this case the minimum address is selected. Figure 55.b illustrates another different result of PC_j (1¼ or ½) which is equal for all js, except PC_2 = ½.

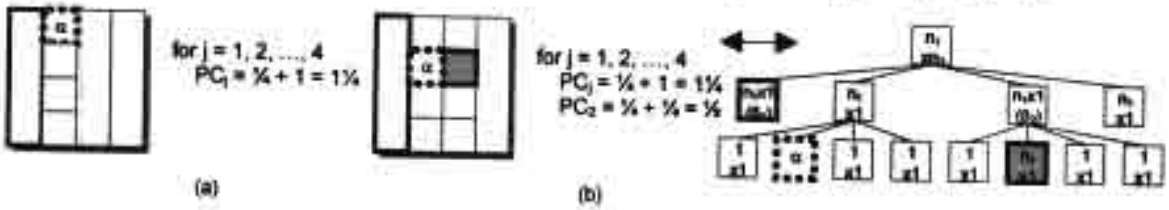


Figure 55: Some examples of the best buddy for the partitioning and combining by network degree: a) any of n buddies can be the best node; so the first one is selected and b) the second buddy is selected since it yields the minimum PC.

Figure 56 illustrates the practical example of applying the partitioning and combining by network degree. Given a 5-cube (or hypercube) system and four tasks (two 3-cubes, one 2-cube, and one 1-cubes) allocated. In this case, the 3-cube node is partitioned into two buddies. The PC of Buddy#1 is ¼ since its corresponding sub-buddy at the same level is busy. The PC of Buddy#2 is ½ since its corresponding sub-buddy at the same level is partially free. Then the Buddy#1 yielding the minimum PC is selected.

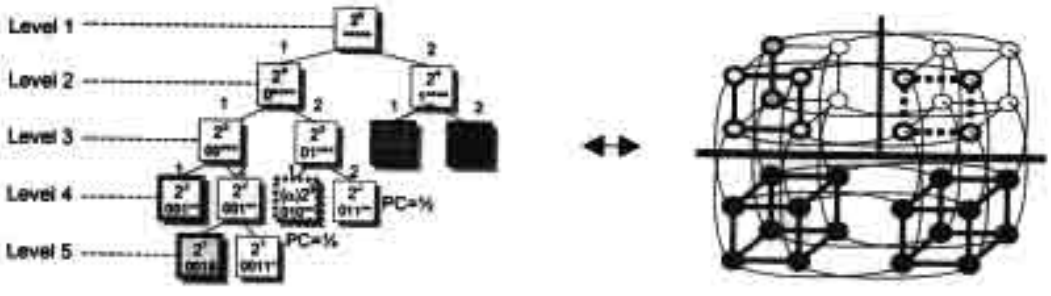


Figure 56: An example of identifying the best buddy for the partitioning and combining by network degree.

Some examples of identifying the best buddy nodes for the partitioning and combining by network size are illustrated in Figure 57. In the first figure, the ID α (or the selected buddy) is 1, which is the same as the ID of S. If the ID of buddy is 4 (the last figure), the ID of α (or the selected buddy) is Buddy#4.

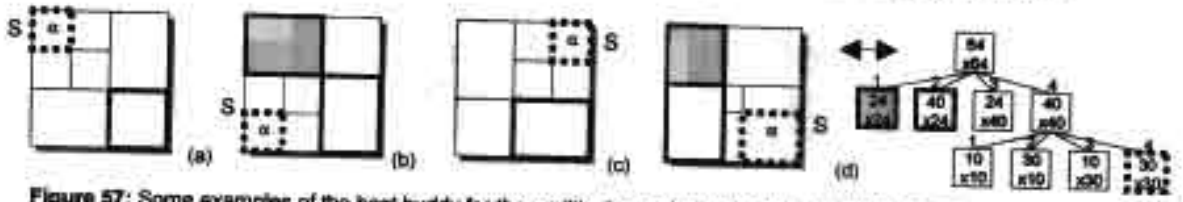


Figure 57: Some examples of the best buddy for the partitioning and combining by network degree.

Figure 58 illustrates the practical example of applying the partitioning and combining by network size. Given a 2-D mesh system (N = 8x10) and two tasks (4x4 and 2x4) allocated. For the next incoming task (3x3), suppose after applying the best fit heuristic, the best free node is Buddy#2 (4x4) at level 2 is selected. The PC of buddy 1, 2, 3, and 4 are 1, 0, 2, 1, respectively. Therefore, the sub-Buddy#2 is select since it yields the minimum PC = 0 (i.e., k = 2, SB = 2).

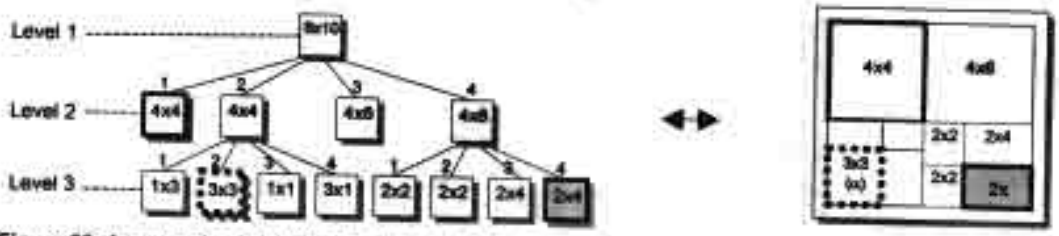


Figure 58: An example of identifying the best buddy for the partitioning and combining by network size.

Some examples of identifying the best buddy nodes for the partitioning and combining by network degree and size are illustrated in Figure 59. Figure 59.a shows the same PC of the first and second buddy, which is 0 (since k = 2 and SB = 2). Then the local combining factor is applied. Now the PC of the first buddy is ¼ and the PC of the second buddy is 1 and hence the first buddy is selected as the best buddy. Similar result is illustrated in Figure 59.b.

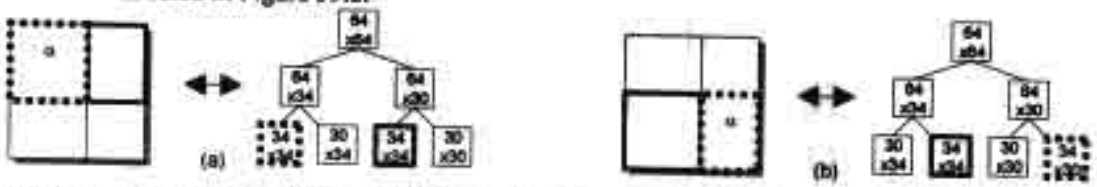


Figure 59: Some examples of the best buddy for the partitioning and combining by network degree and size: a) the first buddy is the best node and b) the second buddy is selected.

Figure 60 illustrates the practical example of applying the partitioning and combining by network degree and size. Given a 2-D mesh system (N = 64x64) and three tasks (20x20, 22x15, and 22x5) allocated. For the next incoming task (10x15), the best sub-system after step 1-3 is the second buddy (22x15) at level 5. After partitioning, the second buddy that yields the minimum PC (=0) is selected.

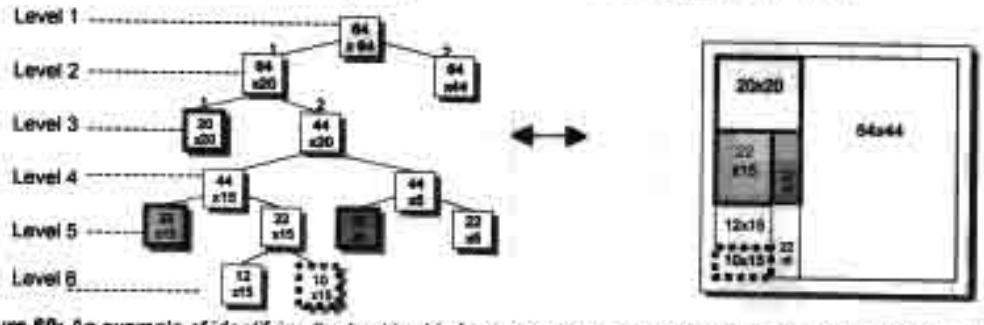


Figure 60: An example of identifying the best buddy for the partitioning and combining by network degree and size.

3.4.2 Best-Fit Heuristic for CU Allocation

In the reconfigurable SPP MSIMD/MIMD system design [2] that we use as our system-base for the resource (CU/PE) allocation, all processors are specially designed, called CPEs (control processor elements) since their roles (CU or PE) are assigned at run time. Therefore, in our study the CU for the selected sub-system (S) can be any CPE that is directly connected to that sub-system. For the CU allocation decision, first we introduce the general CU searching (in Section 3.4.2.1) and then the application of the general CU searching to our tree-based CU allocation method (in Section 3.4.2.2).

3.4.2.1 The General CU Scanning Methods

In general, we introduce two main strategies to find the appropriate CU for a selected sub-system, namely: 1) the processor-bit CU-scanning strategy in O(kN) time and 2) the k-sub-system CU-scanning strategy in $O(k^2)$ time, where N represents the system size ($N = n_1 \times n_2 \times \ldots \times n_k$).

3.4.2.1.1 The Processor-Bit CU-Scanning Strategy

Given a selected sub-system (S) of size $N' = m_1 \times m_2 \times \ldots \times m_k$ at address $\langle(a_1, a_2, \ldots, a_k), (b_1, b_2, \ldots, b_k)\rangle$, where the first k-coordinate $(a_1, a_2, \ldots, a_k)$ represents the base address and the second k-coordinate $(b_1, b_2, \ldots, b_k)$ represents the last cover address of the sub-system. Suppose the system size is $N = n_1 \times n_2 \times \ldots \times n_k$ and $m_k \le n_k$. First, we illustrate all possible processors (or CPEs); each of which can be assigned the CU role for the sub-system, by using two simple examples for 2-D and 3-D mesh systems (see Figure 61).

For the 2-D system (see Figure 61.a), it is not too difficult to find all possible processors being around the sub-system (S). Next in order to find an appropriate free CU at boundary of S, we can start scanning from the minimum address processor and go through all of them (one by one) from left to right and also from top to bottom. In the scanning from left to right (along dimension 1), we have to do the scanning twice for the top and the bottom. In the scanning from top to bottom (along dimension 2), we also have to do the scanning twice for the left and the right. However, it is more difficult for the 3-D system (see Figure 61.b) to perform such scanning for all possible CUs, and hence it is the most difficult for any k-D system.

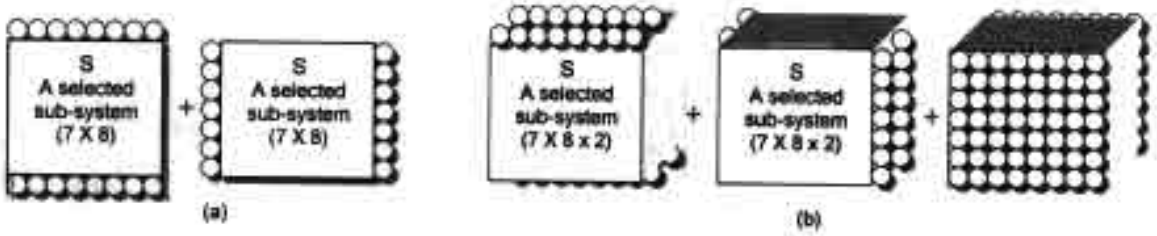


Figure 61: Some examples of a selected sub-system (S) and all of its corresponding CUs at boundary of S: a) all CUs for a 2-D mesh sub-system ($N' = 7 \times 8$) and b) all CUs for a 3-D mesh sub-system ($N' = 7 \times 8 \times 2$).

In general for a k-D system, a number of all possible CPEs that can be the CU (or candidate CUs) of the subsystem S are scanned from the first dimension to the k^{th} dimension and are identified as follows:

A number of all candidate CUs of S

$$= 2\sum_{i=1}^{k} m_1 \times m_2 \times \ldots \times m_{i-1} \times 1 \times m_{i+1} \times \ldots \times m_k$$

$$= 2[(1 \times m_2 \times \ldots \times m_{k-1} \times m_k) + (m_1 \times 1 \times \ldots \times m_{k-1} \times m_k) + \ldots + (m_1 \times m_2 \times \ldots \times 1 \times m_k) + (m_1 \times m_2 \times \ldots \times m_{k-1} \times 1)]$$

For example (see Figure 61 for the 2-D and 3-D systems), if $k = 2$ and the size of a considered sub-system (S) is $N' = m_1 \times m_2$ (7 x 8), all candidate CUs for this sub-system is $2(1 \times 8 + 7 \times 1) = 30$ processors (Figure 61.a). For $k = 3$ and the size of another sub-system (S) is $N'' = m_1 \times m_2 \times m_3$ (7 x 8 x 2), all candidate CUs for this sub-system is $2(1 \times 8 \times 2 + 7 \times 1 \times 2 + 7 \times 8 \times 1) = 172$ processors (Figure 61.b)

Next, we have to find the address of each of those candidate CUs and indicate its status (i.e., free (0) or busy (1)). In the k-D system, a processor address is represented in terms of a k-coordinate such as the base address $(a_1, a_2, \ldots, a_k)$, the cover address $(b_1, b_2, \ldots, b_k)$, etc. For the sub-system (S) of size $(N' = m_1 \times m_2 \times \ldots \times m_k)$ at address $\langle (a_1, a_2, \ldots, a_k), (b_1, b_2, \ldots, b_k) \rangle$, the minimum address of the first possible CU is $(a_1\text{-}1, a_2, \ldots, a_k)$. The scanning process starts from the first dimension to the k^{th} dimension. For each dimension i (i = 1, 2, ..., k), there are two consecutive groups of $(m_1 \times m_2 \times \ldots \times m_{i-1} \times 1 \times m_{i+1} \times \ldots \times m_k)$ processors, where each of their addresses is identified as follows:

CUs at dimension i = 1, 2, ... k: addressing of $2(m_1 \times m_2 \times \ldots \times m_{i-1} \times 1 \times m_{i+1} \times \ldots \times m_{k-1} \times m_k)$ CUs (where $m_i = 1$) are

Initialize Group 1 $(a_1, a_2, \ldots, a_i\text{-}1, \ldots, a_k)$

Group 2 $(a_1, a_2, \ldots, b_i\text{+}1, \ldots, a_k)$, where $b_i = a_i + m_i - 1$

Computation all dimensions, except i (since $m_i = 1$)

for $d_1 = 0, 1, 2, \ldots, m_1\text{-}1$ a_1+d_1

for $d_2 = 0, 1, 2, \ldots, m_2\text{-}1$ a_2+d_2

...

for $d_k = 0, 1, 2, \ldots, m_k\text{-}1$ a_k+d_k

Note: for each dimension i if $(a_i\text{-}1 < 0)$, it is a system boundary and no CUs in group 1 of that dimension.

For example (see Figure 62), if k = 2 and the size of a selected sub-system (S) is $N' = 7 \times 8$, addressing of all 30 candidate CUs, where $m_1 \times m_2 = 7 \times 8$ at $\langle (a_1, a_2), (b_1, b_2) \rangle = \langle (5, 5), (11, 12) \rangle$ are

- Candidate CUs at dimension 1: for $d_2 = 0, 1, 2, \ldots, 7$

  Group 1 (8 PEs): $(\mathbf{a_1\text{-}1}, a_2+d_2) = (\mathbf{4}, 5), (\mathbf{4}, 6), \ldots, (\mathbf{4}, 12)$

  Group 2 (8 PEs): $(\mathbf{b_1+1}, a_2+d_2) = (\mathbf{12}, 5), (\mathbf{12}, 6), \ldots, (\mathbf{12}, 12)$

- Candidate CUs at dimension 2: for $d_1 = 0, 1, 2, \ldots, 6$

  Group 1 (7 PEs): $(a_1+d_1, \mathbf{a_2\text{-}1}) = (5, \mathbf{4}), (6, \mathbf{4}), \ldots, (11, \mathbf{4})$

  Group 2 (7 PEs): $(a_1+d_1, \mathbf{b_2+1}) = (5, \mathbf{13}), (6, \mathbf{13}), \ldots (11, \mathbf{13})$

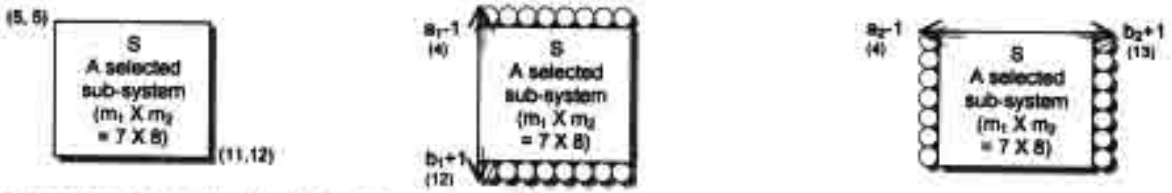


Figure 62: An example of a selected 2-D sub-system (S): all corresponding candidate CUs and their addressing.

In Figure 63, if k = 3 and the size of a selected sub-system (S) is $N' = 7 \times 8 \times 2$, addressing of all 172 candidate CUs, where $m_1 \times m_2 \times m_3 = 7 \times 8 \times 2$ at $\langle (a_1, a_2, a_3), (b_1, b_2, b_3) \rangle = \langle (5,5,5), (11,12,6) \rangle$ are

- Candidate CUs at dimension 1: $d_2 = 0, 1, 2, \ldots, 7$ and $d_3 = 0, 1$

  Group 1 (16 PEs): $(\mathbf{a_1\text{-}1}, a_2+d_2, a_3+d_3) = (\mathbf{4}, 5, 5), (\mathbf{4}, 5, 6), \ldots, (\mathbf{4}, 12, 6)$

  Group 2 (16 PEs): $(\mathbf{b_1+1}, a_2+d_2, a_3+d_3) = (\mathbf{12}, 5, 5), (\mathbf{12}, 5, 6), \ldots, (\mathbf{12}, 12, 6)$

- Candidate CUs at dimension 2: for $d_1 = 0, 1, 2, \ldots, 6$ and $d_3 = 0, 1$

  Group 1 (14 PEs): $(a_1+d_1, \mathbf{a_2\text{-}1}, a_3+d_3) = (5, \mathbf{4}, 5), (5, \mathbf{4}, 6), \ldots, (11, \mathbf{4}, 6)$

  Group 2 (14 PEs): $(a_1+d_1, \mathbf{b_2+1}, a_3+d_3) = (5, \mathbf{13}, 5), (5, \mathbf{13}, 6), \ldots, (11, \mathbf{13}, 6)$

- Candidate CUs at dimension 3: for $d_1 = 0, 1, 2, \ldots, 6$ and $d_2 = 0, 1, 2, \ldots, 7$

  Group 1 (56 PEs): $(a_1+d_1, a_2+d_2, \mathbf{a_3\text{-}1}) = (5, 5, \mathbf{4}), (5, 6, \mathbf{4}), \ldots, (11, 12, \mathbf{4})$

  Group 2 (56 PEs): $(a_1+d_1, a_2+d_2, \mathbf{b_3+1}) = (5, 5, 7), (5, 6, 7), \ldots, (11, 12, 7)$

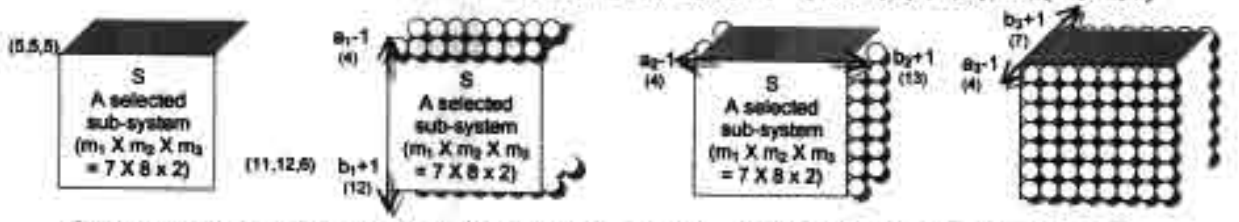


Figure 63: An example of a selected 3-D sub-system (S): all corresponding candidate CUs and their addressing.

3.4.2.1.2 The k-Sub-System CU-Scanning Strategy

Scanning for all possible CUs by using the processor-bit CU scanning strategy takes O(kN) time for any free sub-system (S), which is very time consuming. Then, the k-sub-system CU-scanning strategy is introduced to improve that time complexity. Given a selected sub-system (S) of size $N' = m_1 \times m_2 \times \ldots \times m_k$ at address $\langle(a_1, a_2, \ldots, a_k), (b_1, b_2, \ldots, b_k)\rangle$. In this case, for k dimensions there are 2k (non overlap) sub-systems of all possible candidate CUs (or CUSs), whose addressing are defined in $O(k^2)$ as follows:

> For dimension i =1, 2,..., k, addressing of two CUSs (each of size = $m_1 \times m_2 \times \ldots \times m_{i-1} \times 1 \times \ldots \times m_k$, where $m_i = 1$) are
>
> Min CUS address = $\langle(a_1, a_2, \ldots, a_i\text{-}1, \ldots, a_k), (b_1, b_2, \ldots, a_i\text{-}1, \ldots, b_k)\rangle$
> Max CUS address = $\langle(a_1, a_2, \ldots, b_i\text{+}1, \ldots, a_k), (b_1, b_2, \ldots, b_i\text{+}1, \ldots, b_k)\rangle$

For example (see Figure 62), if k = 2 and the size of a selected sub-system (S) is N'= 7 x 8, addressing of 4 CUSs (of 30 candidate CUs), where $m_1 \times m_2 = 7 \times 8$ at $\langle(a_1, a_2), (b_1, b_2)\rangle = \langle(5, 5), (11, 12)\rangle$ are

- At dimension 1:
  min CUS (8 PEs): $\langle(a_1\text{-}1, a_2), (a_1\text{-}1, b_2)\rangle = \langle(4, 5), (4, 12)\rangle$
  max CUS (8 PEs): $\langle(b_1\text{+}1, a_2), (b_1\text{+}1, b_2)\rangle = \langle(12, 5), (12, 12)\rangle$
- At dimension 2:
  min CUS (7 PEs): $\langle(a_1, a_2\text{-}1), (b_1, a_2\text{-}1)\rangle = \langle(5, 4), (11, 4)\rangle$
  max CUS (7 PEs): $\langle(a_1, b_2\text{+}1), (b_1, b_2\text{+}1)\rangle = \langle(5, 13), (11, 13)\rangle$

In another example (see Figure 63), if k = 3 and the size of a selected sub-system (S) is N" = 7 x 8 x 2, addressing of 6 CUSs (of 172 candidate CUs), where $m_1 \times m_2 \times m_3 = 7 \times 8 \times 2$ at $\langle(a_1, a_2, a_3), (b_1, b_2, b_3)\rangle = \langle(5, 5, 5), (11, 12, 6)\rangle$ are

- At dimension 1:
  min CUS (16 PEs): $\langle(a_1\text{-}1, a_2, a_3), (a_1\text{-}1, b_2, b_3)\rangle = \langle(4, 5, 5), (4, 12, 6)\rangle$
  max CUS (16 PEs): $\langle(b_1\text{+}1, a_2, a_3), (b_1\text{+}1, b_2, b_3)\rangle = \langle(12, 5, 5), (12, 12, 6)\rangle$
- At dimension 2:
  min CUS (14 PEs): $\langle(a_1, a_2\text{-}1, a_3), (b_1, a_2\text{-}1, b_3)\rangle = \langle(5, 4, 5), (11, 4, 6)\rangle$
  max CUS (14 PEs): $\langle(a_1, b_2\text{+}1, a_3), (b_1, b_2\text{+}1, b_3)\rangle = \langle(5, 13, 5), (11, 13, 6)\rangle$
- At dimension 3:
  min CUS (56 PEs): $\langle(a_1, a_2, a_3\text{-}1), (b_1, b_2, a_3\text{-}1)\rangle = \langle(5, 5, 4), (11, 12, 4)\rangle$
  max CUS (56 PEs): $\langle(a_1, a_2, b_3\text{+}1), (b_1, b_2, b_3\text{+}1)\rangle = \langle(5, 5, 7), (11, 12, 7)\rangle$

3.4.2.2 The Tree-Based CU Searching Methods

For the tree-based CU allocation approach, we introduce three best-fit heuristics to find the appropriate tree-node containing some candidate CUs for a selected sub-system in different time complexity: 1) the CU depth first search (CU-DFS) strategy ($O(N_A + kN_F + k^2)$); 2) the CU adjacent search (CU-AS) strategy ($O(k^2)$); and 3) the CU inside search (CU-IS) strategy (O(1)) for all partitioning and combining methods.

3.4.2.2.1 The CU Depth First Search (CU-DFS) Strategy

The CU depth first search (CU-DFS) strategy is used to find any free node (represented as R) in the tree that is adjacent to the selected sub-system (S). The searching starts from the root and goes to the left most (leaf) node, which is the first node R. If that node R is free, then it will check whether R is adjacent to S or not (see Definition 2), identified in O(k) time. If so, the best-fit value of that node R is computed for the candidate CU of size one processor (p = 1x1x...x1). During the DFS search, we also apply the best-fit value in Section 3.4.1 (Step 1 - 3 without task rotation) to the adjacent node only in O(k) time. Then, the new adjacent node R will be updated if it yields the better the best-fit value than the current adjacent node R in the record. The searching process is repeated for the next free node (if there exists) until all nodes in the tree are visited. So far, time complexity of the CU-DFS searching to select the best node R for the sub-system S is $O(N_A + kN_F + k^2)$ since there are N_A busy nodes (just visiting), N_F free nodes (visiting with adjacency checking), and k adjacent nodes (visiting with best-fit computing). Finally, for the best adjacent node R after complete the DFS search, first we identified all 2k CUSs of S (in $O(k^2)$ time) and select the best CUS and hence the best CU corresponding to the best adjacent node R (in $O(k^2)$ time). Therefore, total time complexity is $O(N_A + kN_F + k^2 + k^2 + k^2)$ or $O(N_A + kN_F + k^2)$. This CU-DFS strategy can be applied directly to all three partitioning and combining methods in $O(N_A + kN_F + k^2)$ time.

DEFINITION 2: Any two sub-systems (S and R) are adjacent along the dimension i if either $|a_{si} - b_{ri}| = 1$ or $|b_{si} - a_{ri}| = 1$, where $i = 1, 2, \ldots, k$; $(\alpha_s, \beta_s) = \langle(a_{s1}, a_{s2}, \ldots, a_{sk}), (b_{s1}, b_{s2}, \ldots, b_{sk})\rangle$ represents the address of S; and $(\alpha_r, \beta_r) = \langle(a_{r1}, a_{r2}, \ldots, a_{rk}), (b_{r1}, b_{r2}, \ldots, b_{rk})\rangle$ represents the address of R.

For example (see Figure 64), suppose $k = 2$ and a selected sub-system is S (of size $N' = m_1 \times m_2$ at the address $\langle(a_{s1}, a_{s2}), (b_{s1}, b_{s2})\rangle$) and a considering free node is R (of size $N'' = m'_1 \times m'_2$ at the address $\langle(a_{r1}, a_{r2}), (b_{r1}, b_{r2})\rangle$)). Figure 64.a illustrates that R and S are not adjacent since $|a_{si} - b_{ri}| \neq 1$ and $|b_{si} - a_{ri}| \neq 1$ for all $i = 1, 2$. Figure 64.b illustrates that R is adjacent to S since for dim $i = 2$, $|b_{s2} - a_{r2}| = 1$. Figure 64.c also illustrates that R is adjacent to S since for dim $i = 1$, $|a_{s1} - b_{r1}| = 1$, respectively.

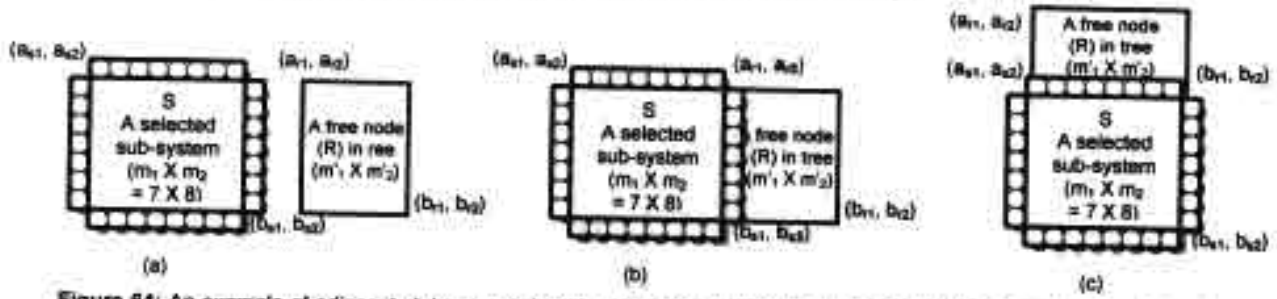


Figure 64: An example of adjacent statuses a selected sub-system S and a free node R: a) Non-adjacent status; b) and c) Two adjacent statuses.

3.4.2.2.2 The CU Adjacent Search (CU-AS) Strategy

Searching for all candidate CUs for S by using the CU-DFS strategy takes $O(N_A + kN_F + k^2)$ time for a current selected sub-system (S) and hence it is very time consuming for all candidate sub-systems (S). Then, the CU adjacent search (CU-AS) strategy is introduced to improve that time complexity.

The CU-AS strategy is introduced in order to find the free nodes R that are directly adjacent to the selected sub-system S. The searching starts from the selected S and goes to the first or next adjacent buddy node (R) in O(1) time. If that adjacent node R is free, its best-fit value (see Section 3.4.1 (only Step 1 - 3 without task rotation) is applied for the CU of size one processor ($p = 1x1x\ldots x1$) in O(k) time. Then, the new node R will be updated if it yields the better the best-fit value than the current node R in the record. This searching process is repeated for the next adjacent node for at most k nodes in $O(k^2)$ time including the best-fit computing. Note that the CU-AS strategy yields the similar system performance to that of the CU-DFS strategy (see Section 5), although it does not perform searching for all candidate CUs as the CU-DFS strategy do. Time complexity of the CU-AS strategy is only $O(k^2)$, described for each method as follows:

- For the partitioning and combining by network degree (Method 1), there are at most two adjacent buddy nodes (in the left and right) identified in O(1) time, where all processors in those two buddy nodes can be candidate CUs if they are free. Searching may need k levels for the minimum free node's size as must as possible, and hence time complexity of the CU-AS strategy for this partitioning and combining method is $O(k^2)$ including the O(k) time for best-fit computing. See the corresponding example in Figure 65.
- For the partitioning and combining by network size (Method 2), the searching starts from S and its adjacent free nodes (at most k nodes) can be identified directly (see Algorithm CU.1 and the corresponding example) in O(k) time. Searching for all k adjacent nodes of S and computing their best-fit values is $O(k^2)$ time. Therefore, time complexity of the CU-AS strategy is $O(k^2)$ time.
- For the partitioning and combining by network degree and size (Method 3), the searching starts from S. The adjacent node of S is directly identified in O(1) and its best-fit value can be computed in O(k) time since there is only one adjacent buddy (see Algorithm CU.2 and the corresponding example). Therefore, time complexity of the CU-AS is O(k) the next level searching and hence $O(k^2)$ for the k levels searching for the minimum free node's size as much as possible.

Note: In this CU-AS searching strategy, we illustrate time complexity based upon the idea of the combining and expanding, which is stored in the expanded node size (see more detail in Section 3.3 and Section 3.5).

Figure 65 illustrates the practical example of applying the CU-AS strategy for the partitioning and combining by network degree. Suppose the selected sub-system (S) is the buddy #2 at level 2 and hence its two adjacent buddy nodes are #1 and #3 (at level 2), respectively. All free nodes of the buddy #1 and 3 can be CUs for S, but the one providing best-fit value (i.e., the sub-buddy #1 at level 4) is selected.

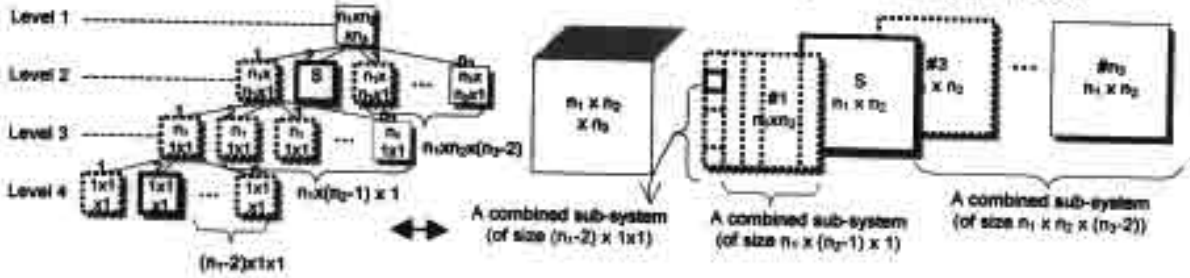
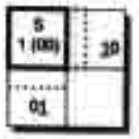


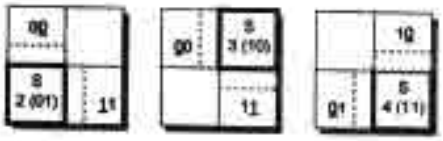
Figure 65: An example of applying the CU-AS for the partitioning and combining by network degree.

ALGORITHM CU.1: "Adjacent buddy nodes of a selected sub-system S" for the partitioning and combining by network size. There are three possible cases, concerning S as a buddy node or a combined sub-system (see Section 3.3.2):

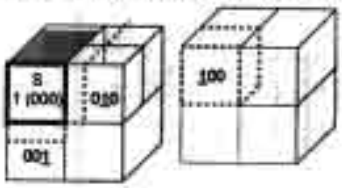
Case 1: if S is any buddy node (ID = 1, 2, ..., or 2^k in an integer format or (b_{k-1} ... b_1 b_0) in a conversion sub-system bit-map format), there are k adjacent buddy nodes and we compute the best-fit value (Step 1-3 without task rotation) of each adjacent node in O(k) time. For the same level as S, there exist k adjacent buddies, which are identified by negating b_j for one dimension at a time, where j = 1, 2, ..., k. Therefore, time complexity for finding k buddies including best-fit value is $O(k^2)$. For example, if k = 2, k adjacent buddies of any S (where ID = 1, 2, 3, or 4) are



2 adjacent buddies of S (b_1b_0 = 00) are 01 and 10.



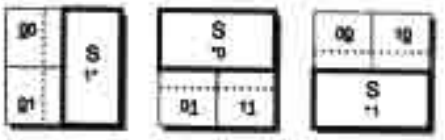
If k = 3, k adjacent buddies and k adjacent sub-buddies of an S where ID = 1 is



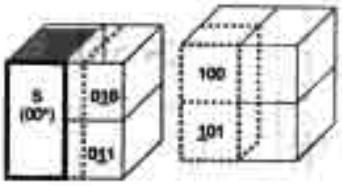
3 adjacent buddies of S ($b_2b_1b_0$ = 000) are 001, 010, and 100.

Case 2: if S is any combined node from 2^j buddies, represented in a ternary string format (t_{k-1} ... t_1 t_0) of S and stored in a special expand node size, there exist js' * and k-j bits (b_i) in that string, where j = 1, 2, ..., k-1. For each adjacent buddy, we compute the best-fit value (Step 1-3 without task rotation) in O(k) time. For the same level as S, there exist k adjacent buddies, which are identified by negating a non* (b_i) for one dimension at a time and then expanding all*s, where i = 1, 2, ..., k. Therefore, time complexity for finding k buddies including best-fit value is $O(k^2)$. For example, if k = 2, k adjacent buddies and k adjacent sub-buddies of any S (where j = 1 and a ternary string is 0*, 1*, *0, or *1) are

2 adjacent buddies of S (t_1t_0 = 0*) are 1* (10 and 11).



If k = 3, k adjacent buddies of an S where ID = 1 and either j = 1 or 2 (since only the maximum combined node at any level is stored in the expanded node) are

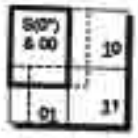


← For j = 1,
3 adjacent buddies of S ($b_2t_1t_0$ = 00*) are 01* (010, 011) and 10* (100,101).

→ or For j = 2,
3 adjacent buddies of S ($b_2t_1t_0$ = 00*) are 1** (100, 101, 110, 111).

Case 3: if S is any combined 1-buddy and 2^{k-1}-sub-buddies node, represented in a ternary string (t_{k-1} ... t_i t_0) and a binary string (b_{k-1} ... b_i b_0) of S, all adjacent buddies can be identified by applying Case 2. Note: there exists one * and k-1 bits (b_i) in that ternary string. For each adjacent buddy, we compute the best-fit value (Step 1-3 without task rotation) in O(k) time. For the same level as S, there exist 2(k-1) adjacent buddies, which are identified by negating a non* (b_i) for one dimension at a time and then expanding *, where i = 1, 2, ..., k. Hence, time complexity for finding 2(k-1) buddies including best-fit value is $O(k^2)$.

For example, if k = 2, 2(k-1) adjacent buddies of any S (where a ternary string is 0* and 00 or 01, *0 and 00 or 01, 1* and 10 or 11, or *1 and 10 or 11) are



2 (2(k-1)) adjacent buddies of S (t_1t_0=0*, b_1b_0=00) are 10, 11.

2 (2(k-1)) adjacent buddies of S (t_1t_0=*0, b_1b_0=00) are 01, 11.

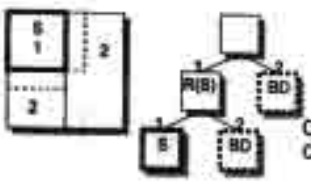
ALGORITHM CU.2: "Adjacent buddy of a selected sub-system S" for the partitioning and combining by network degree and size.

If S is any buddy node (ID = 1, 2), there is only one adjacent buddy nodes and we compute the best-fit value (Step 1-3 without task rotation) of each adjacent node in O(k) time. In order to find CUs from k sides as identified by the partitioning and combining by network size, we consider the following two cases:

Case 1: Beginning at the same level as S. For the same level as S, there exists only one adjacent buddy, which is identified by negating the ID of S (i.e. if ID of S = 1 it is 2 and if ID of S = 2 it is 1). Therefore, time complexity for finding a buddy including best-fit value is O(k).

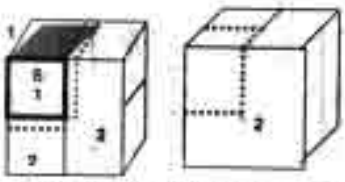
Case 2: Beginning at the upper level or the sub-sequence root(s) of S (R(S)) up to k-1 levels. In each upper level, find an adjacent buddy of the R(S), which is identified by negating the ID of the R(S). Time complexity for finding a buddy including best-fit value is O(k) and $O(k^2)$ for at most k levels

For example, if k = 2, an adjacent buddy of any S and an adjacent buddy of S's root (where ID = 1 or 2) are



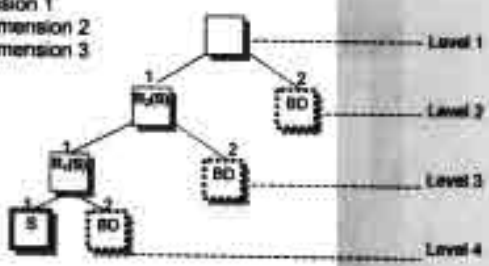
Case 1: 1 adjacent buddy of S (ID = 1) is 2.
Case 2: 1 adjacent buddy of R(S) (ID = 1) is 2.

If k = 3, an adjacent buddy of S (where ID = 1) and adjacent buddies of (k-1)-level S's roots are



Case 1: 1 adjacent buddy of S (ID = 1) is 2 at level 4
Case 2: For $R_1(S)$,
1 adjacent buddy of $R_1(S)$ (ID = 1) is 2 at level 3
For $R_2(S)$,
1 adjacent buddy of $R_2(S)$ (ID = 1) is 2 at level 2

Note:
S is partitioned along dimension 1
$R_1(S)$ is partitioned along dimension 2
$R_2(S)$ is partitioned along dimension 3



After DFS searching of both searching methods (the CU-DFS strategy and the CU-AS strategy) to visit all nodes in the tree for the SIMD task, we obtain the best sub-system S and the best node for CU, whose size may equal or larger than the requested task. The final step (applied only once) for both strategies is selecting the best-fit PE and CU after partitioning. Let's consider the following criterion:

- If the sub-system size already fits to the task and the CU node size is equal to one PE, we do not have to apply the partitioning method.
- Otherwise, we apply Algorithm A.4 for each buddy node to find the most fit one (or the like best buddy node among B nodes, where B = n_k, 2^k, or 2 for three partitioning and combining methods in $O(kn_k)$, $O(k2^k)$, O(k) time, respectively. Note that
  - After partitioning, we will have more candidate CUs inside S, where adjacent sub-buddy nodes = 2, k, or 1 for Method 1, 2, or 3, respectively. Then, we have to compute the best-fit value (Step 4 in Section 3.4.1) for each adjacent node of the best buddy (from S) in O(k), $O(k^2)$, O(k) time, for Method 1, 2, 3.
  - Then, between the best adjacent node inside S and the best adjacent node outside S, the one yields the better best-fit value is selected.
  - Finally, if the CU node size is equal to 1, we do not have to perform the partitioning. Otherwise, we partition that CU node and select the best CU for that best buddy for the sub-system (PE) for the requested tasks.

3.4.2.2.3 The CU Inside Search (CU-IS) Strategy

Searching for all possible candidate CUs by using the CU-DFS strategy takes $O(N_A + kN_P + k^2)$ time and searching for some candidate CUs by using the CU-AS needs $O(k^2)$ time. The later strategy improves time complexity over the previous one for a current sub-system (S). However, it is still time consuming for all candidate sub-systems. Then, the CU inside search (CU-IS) strategy is introduced to improve that time complexity, which is O(1) time for a current sub-system (S). In this case, the searching is similar to that for the sub-system in PE allocation, except now we are always looking for the sub-system that is larger than the requested and hence it always includes CUs inside that sub-system. Thus, we do not need extra time to search for CUs outside the sub-system, as the above two strategies. Although this strategy does not search for outside CUs as those two strategies do, they yield the comparable system performance as those of two previous strategies (see Section 5). Therefore, time complexity of the CU-IS strategy is O(1).

3.5 Searching for Allocation/Deallocation

This section integrates all computing functions (in Section 3.2 – 3.4) to form the searching for resource (CU/PE) allocation/deallocation decision. As an introduction in Section 3.1, Figure 2.2 illustrates the diagram of the dynamic tree-based resource (CU/PE) allocation/deallocation computing flow for a reconfigurable and partitionable MSIMD/MIMD parallel system.

When there is an incoming task, the dynamic resource allocation process will check in the waiting queue first. If the wait priority of the first task in the waiting queue is more than the threshold value, that task will be put in the waiting queue. Otherwise, the processor "allocation" procedure will find an appropriate free sub-system for that task by searching into the tree. If there is a free sub-system, the requested task will be allocated on the system. If no available sub-system, the request will be put in the waiting queue with FCFS scheduling. In particular, in the tree-based resource (CU/PE) allocation procedure, searching starts from the root and performing depth first search (DFS) to visit all free nodes in the tree by visiting the left most (leaf) node first. If that node is free and its size can accommodate the request, its best-fit value (see Section 3.4.1) is computed. For an SIMD task, the CU-searching strategy is also applied (see Section 3.4.2.2). Then, the best (SIMD/MIMD) sub-system (S) is updated if the new free S yields the better best-fit value. The above process is repeated for the next free leaf node in the tree. After all nodes are visited, the final process is applied, which is either to 1) allocate the best sub-system directly to the request (if its size is equal to that of the request) or 2) partition (see Section 3.2 and 3.4) to find the best sub-partition of the corresponding node for the request (if its size is larger than that of the request). Next step is applying the sub-system combining process (see Section 3.3) to recombine sub-systems by starting from the

partitioned node. Note that only the maximum combined size among all possible combinations is stored in the tree since we always keep the non-overlap sub-systems in the tree. Then the expanded node size (see an example in Figure 66) is stored in a selected node in the combined group and updated as free and other corresponding nodes are updated as busy. Finally, the maximum free size (applied in the best-fit heuristic) is updated.

Table 1 summarizes main functions of our resource (CU/PE) allocation model for the three partitioning and combining methods.

Table 1. Main functions of the universal resource (CU/PE) allocation process.

Main Functions	Method 1	Method 2	Method 3
(1) Leaf node operation (for N_f free nodes by using DFS) - Compute best-fit value for each node (for SIMD/MIMD) - Compute best-fit value for CUs of each node (for SIMD)	Section 3.4.1 Section 3.4.2	Section 3.4.1 Section 3.4.2	Section 3.4.1 Section 3.4.2
(2) Partitioning after finding the best free node - Best sub-partition - for PE (SIMD/MIMD) - for CU (SIMD) - Network partitioning - Allocate (update status of corresponding nodes)	Section 3.4.1.4 Section 3.4.2.2 Section 3.2.1	Section 3.4.1.4 Section 3.4.2.2 Section 3.2.2	Section 3.4.1.4 Section 3.4.2.2 Section 3.2.3
(3) Combining after allocation the best sub-partition - Sub-system combining (Algorithm C.2-C.3) - Expand size & update status of corresponding nodes	Section 3.3.1	Section 3.3.2	Section 3.3.3
(4) Update the maximum free size	Section 3.4.1	Section 3.4.1	Section 3.4.1

Note: Method 1 is based upon the partitioning and combining by network degree. Method 2 is based upon the partitioning and combining by network size. Method 3 is based upon the partitioning and combining by network degree and size.

When a task is finished, the processor "deallocation" procedure will find the allocated position of that finished task by using sub-set path searching into the tree. After finishing free (or deallocate) the node that stores information, the recombining process is applied to combine all corresponding (free) sub-partitions (or Buddy nodes) as soon as they become available. The recombining process starts form the new free node (of the completed task) to the root of the tree. This process will stop when there is at least one buddy of a corresponding node (along the combining path) is not available. Finally, the maximum free size (applied in the best-fit heuristic) is updated. At this time, if there are task(s) in the waiting queue, the priority FCFS scheduling will be applied to perform scheduling and allocation for these waiting tasks.

Table 2 summarizes main functions of our resource (CU/PE) deallocation model for three partitioning and combining methods.

Table 2. Main functions of the universal resource (CU/PE) deallocation process.

Main Functions	Method 1	Method 2	Method 3
(1) Sub-set-path searching to the finished node			
(2) Deallocation status - Deallocate (update status of corresponding nodes) - Unexpanded size & update status of corresponding nodes			
(3) Combining after deallocation - Sub-system combining (Algorithm C.1) up to root	Section 3.3.1	Section 3.3.2	Section 3.3.3
(4) Update the maximum free size	Section 3.4.1	Section 3.4.1	Section 3.4.1

Note that in the searching for allocation/deallocation algorithm, especially in the implementation part, we introduce the "expanded free node size" function (in step 3) for a combined sub-system, which is selected as the best free sub-system for the request. This idea is so important for the general tree-based allocation for the partitionable k-D systems in order to limit the number of nodes in the tree and provide the same methodology to update and partition as a regular (free) leaf node. For any combined sub-system, a sub-buddy node (such as the one with either the minimum buddy-ID or the largest buddy size) is selected to be the expanded node. That node is used to store the combined information (i.e., a combined size, a new base address, a free status), then other nodes in the combined sub-system are updated as busy. These busy nodes will be free whenever the expanded node is free (in step 2 of the deallocation process).

Figure 66 illustrates an example of the combining and expanding for given a 2-D mesh system with three tasks allocated (buddy#1 and #2 at level 2, buddy#1 at level 3: see Figure 66.a). Suppose that the last allocated node is the buddy 1 at level 3. Then, the expanded node processing of a combined (6x6) sub-system is illustrated in Figure 66.b. The expanded node's information (i.e. expand status, new size, new base address, old size, old base address) will be stored on a selected node (buddy#4 at level 2).

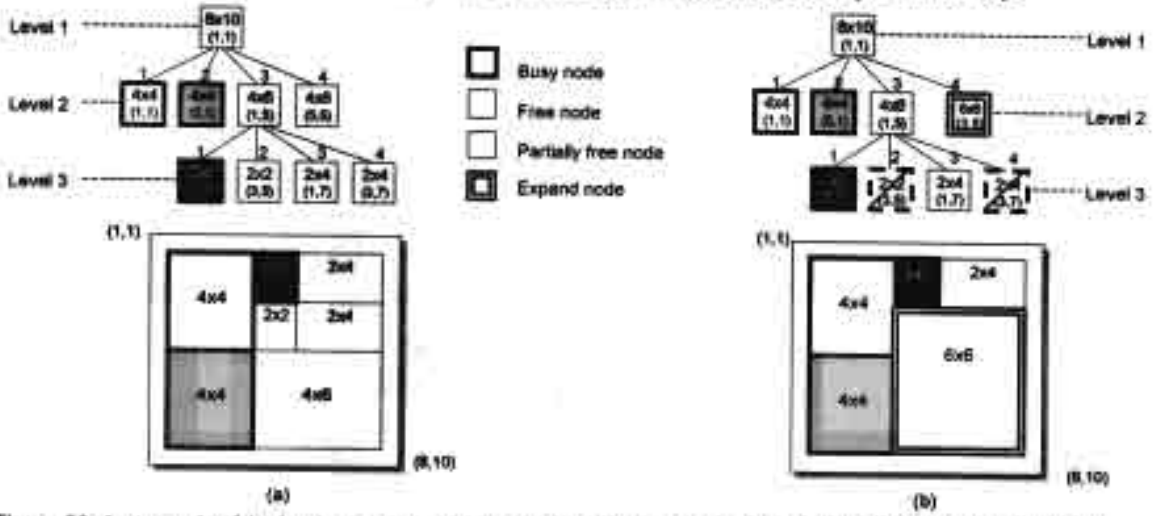


Figure 66: An example of the "node size expanding": a) the current tree with 3 task allocated and b) the corresponding tree with "side expanding".

3.6 Time Complexity of the Universal Resource (CU/PE) Allocation Model

Before we start deriving the total time complexity of the universal resource allocation/deallocation model, we show the summary of time complexity for all major processes (in Section 3.2 – Section 3.4) of the allocation model in Table 3.

Table 3. The number of buddies and time complexity of each process in the universal resource (CU/PE) allocation model.

Functions	Method 1	Method 2	Method 3
Number of Buddies	n	2^k	2
	Time Complexity		
Network Partitioning	$O(k^2n)$	$O(k2^k)$	$O(k^2)$
Sub-system Combining - in Allocation process - in Deallocation process	$O(kn^{k+1})$ $O(n)$	$O(k^2 2^{2k})$ $O(2^k)$	$O(k^4)$ $O(k)$
Best-Fit Heuristic for PE Allocation (for any SIMD / MIMD task) - for N_F external nodes	$O(k^2)$		
Best-Fit Heuristic for CU Allocation (for any SIMD task) - CU-DFS for any S - CU-AS for any S - CU-IS for any S	$O(N_A + kN_F + k^2)$ $O(k^2)$ $O(1)$		

Note: Method 1 is based upon the partitioning and combining by network degree. Method 2 is based upon the partitioning and combining by network size. Method 3 is based upon the partitioning and combining by network degree and size.

3.6.1 Time Complexity of Searching for Allocation/Deallocation for any MIMD task

Let N be the system size ($N = n_1 \times n_2 \times \ldots \times n_k$),
N_A be the maximum number of allocated tasks ($N_A \le N$),
N_F be the corresponding number of free nodes in the k-Tree ($N_A + N_F \le N$),
M be the maximum number of nodes in the k-Tree, where
M = external (leaf) nodes + internal (non-leaf) nodes < 2N) and $n = \max(n_1, n_2, \ldots, n_k)$.
$(\le (N_A + N_F) + (N_A + N_F - 1) / (b - 1)$, where b = n, 2^k, or 2 for Method 1, 2, or 3.)

Table 4 illustrates the summary of each function time complexity and total time complexity of the universal PE (or sub-system) allocation for any MIMD task.

Table 4. Time complexity of main functions of the universal resource (CU/PE) allocation process for any MIMD task.

Main Functions	Method 1	Method 2	Method 3
(1) Leaf node operation (for N_F nodes by DFS) Compute best-fit value for each node	$O(N_A + k^2N_F)$	$O(N_A + k^2N_F)$	$O(N_A + k^2N_F)$
(2) Partitioning after visit all nodes - Best sub-partition - Allocate (update status)	$O(kn)$ $O(k)$	$O(k2^k)$ $O(k)$	$O(k)$ $O(k)$
(3) Combining after allocation - Combining (Algorithm C.2-C.3) - Expand size & update status - Update the maximum free size	$O(kn^{k+1})$ $O(k)$ $O(N_A + kN_F)$	$O(k^2 2^{2k})$ $O(k)$ $O(N_A + kN_F)$	$O(k^4)$ $O(k)$ $O(N_A + kN_F)$
Total time complexity = (1) + (2) + (3)	$O(N_A + k^2N_F + kn^{k+1})$	$O(N_A + k^2N_F + k^2 2^{2k})$	$O(N_A + k^2N_F + k^4)$

Note: Method 1 is based upon the partitioning and combining by network degree. Method 2 is based upon the partitioning and combining by network size. Method 3 is based upon the partitioning and combining by network degree and size.

Table 5. Time complexity of main functions of the universal resource (CU/PE) deallocation process for any MIMD task.

Main Functions	Method 1	Method 2	Method 3
(1) Subset-path searching (from the root to the finished node)	$O(kn)$	$O(n2^k)$	$O(kn)$
(2) Deallocation status - Deallocate (update status) - Unexpanded size & update status	$O(1)$ $O(kn)$	$O(1)$ $O(k^2 2^k)$	$O(1)$ $O(k^4)$
(3) Combining after deallocation - Combining (Algorithm C.1) up to root - Update the maximum free size	$O(kn)$ $O(N_A + kN_F)$	$O(n2^k)$ $O(N_A + kN_F)$	$O(kn)$ $O(N_A + kN_F)$
Total time complexity = (1) + (2) + (3)	$O(N_A + kN_F + kn)$	$O(N_A + kN_F + n2^k)$	$O(N_A + kN_F + kn)$

In order to simplify the time complexity analysis, we assume that for a particular system network, requested tasks always require sub-systems that have the same interconnection network as that provided by the system. Therefore, we derive total time complexity for each partitioning and combining method in the following sub-section.

3.6.1.1 Time Complexity when applying the partitioning and combining by network degree

THEOREM 1: Time complexity of the tree-based allocation to find the best free sub-system for each incoming task on a product network-based systems (of size $N = n_1 \times n_2 \times \ldots \times n_k$) that partitioning and combining by network degree is $O(N_A + k^2N_F + kn^{k+1})$.

PROOF: In the allocation algorithm, a number of recursive iterations of the DFS (depth first search) are at most a number of nodes in the tree and only nodes whose sizes are larger than (or equal to) the request are visited. In this (sub-system bit-map) approach, the number of nodes in the tree is proportional to the number of busy nodes (N_A) and the number of free nodes (N_F), where $N_A + N_F \leq N$ and $N_F \leq (n-1)N_A$. Since the number of external (or leaf) nodes are at most $N_A + N_F \leq N$ and the number of internal nodes are at most (#leaf nodes-1) divided by (n-1). Therefore, the total number of nodes in the tree is at most M nodes, where $M = (N_A + N_F) + (N_A + N_F - 1) / (n-1)$. For each (free) leaf node (of N_F nodes), the best-fit value is computed in $O(k^2)$ time and hence $O(k^2N_F)$ for all leaf nodes. After finding the best free sub-system (Step 3), if its size is equal to the request, then it is directly allocated to the request. Otherwise, the network partitioning and the best sub-partition will be applied, which can be computed in O(kn) time. And the corresponding node(s) in the tree is updated in O(k) time. Next, the combining process is applied in $O(kn^{k+1})$ time. Then, the expanding size and update status takes O(k) time. Finally, the maximum free size is updated by applying the DFS in $O(N_A + kN_F)$. Thus, total time complexity to visit all nodes in the tree is approximately $O(N_A + k^2N_F + kn^{k+1})$. In addition, this method when applied to the hypercube (or k-cube) provides time complexity $O(N_A + k^2N_F + k2^{k+1})$ since $n = 2$.

THEOREM 2: Time complexity of the tree-based deallocation to free the particular tree node that stores the finished task and to combine the free buddy nodes of the root sub-tree to the root of the tree on the partitionable product network-based systems ($N = n_1 \times n_2 \times \ldots \times n_k$) that partitioning and combining by network degree is $O(N_A + kN_F + kn)$.

PROOF: In the deallocation procedure, searching for the location of a finished sub-system from the root is at most kn steps. Combining all n buddy nodes from the finished sub-system to the root (if possible) takes another n(kn) steps. Then, the resume-node-size process of the expand-node-size process (if any) may be required in O(kn) time. Finally, the maximum free size is updated by applying the DFS in $O(N_A + kN_F)$. Therefore, total time complexity of the tree-based deallocation is $O(N_A + kN_F + kn)$. Note: Our model, when applied to the hypercube (or k-cube) systems, provides time complexity $O(N_A + kN_F)$ since n = 2.

3.6.1.2 Time Complexity when applying the partitioning and combining by network size

THEOREM 3: Time complexity of the tree-based allocation to find the best free sub-system (PE) for each incoming task on a product network-based systems (of size $N = n_1 \times n_2 \times \ldots \times n_k$) that partitioning and combining by network size is $O(N_A + k^2N_F + k^2 2^k)$.

PROOF: In the allocation algorithm, a number of recursive iterations of the DFS (depth first search) are at most a number of nodes in the tree and only nodes whose sizes are larger than (or equal to) the request are visited. In this (sub-system bit-map) approach, the number of nodes in the tree is proportional to the number of allocated tasks or busy nodes (N_A) and the number of free nodes (N_F) in the tree, where $N_A + N_F \leq N$ and $N_F \leq (2^k - 1)N_A$. Since the number of external (or leaf) nodes are at most $N_A + N_F \leq N$ and the number of internal nodes are at most (#leaf nodes-1) divided by $(2^k - 1)$. Therefore, the total number of nodes in is at most M nodes, where $M = (N_A + N_F) + (N_A + N_F - 1) / (2^k - 1)$. For each (free) leaf node (of N_F nodes), the best-fit value is computed in $O(k^2)$ time and hence $O(k^2N_F)$ for all leaf nodes. After finding the best free sub-system (Step 3), if its size is equal to the request, then it is directly allocated to the request. Otherwise, the network partitioning and the best sub-partition will be applied, which can be computed in $O(k2^k)$ time. And, the corresponding node(s) in the tree is updated in O(k) time. Next, the combining process is applied in $O(k^2 2^{2k})$ time. Then, the expanding size and update status takes O(k) time. Finally, the maximum free size is updated by applying the DFS in $O(N_A + kN_F)$. Thus, total time complexity to visit all nodes in the tree is approximately $O(N_A + k^2N_F + k^2 2^k)$. In addition, this tree-based model, when applied to the 2-D/3-D mesh or 2-D/3-D torus systems, provides a linear time complexity $O(N_A + N_F)$ since k = 2 for the 2-D systems and k = 3 for the 3-D systems.

THEOREM 4: Time complexity of the tree-based deallocation to free the particular tree node that stores the finished task and to combine the free buddy nodes of the root sub-tree to the root of the tree on the partitionable product network-based systems ($N = n_1 \times n_2 \times \ldots \times n_k$) that partitioning and combining by network size is $O(N_A + kN_F + nk^2)$.

PROOF: In the deallocation procedure, searching for the location of a finished sub-system from the root is at most $n(2^k)$ steps. Combining all 2^k buddy nodes from the finished sub-system to the root (if possible) takes another $n(2^k)$ steps. Then, the resume-node-size process of the expand-node-size process (if any) may be required in $O(k^2 2^k)$ time. Finally, the maximum free size is updated by applying the DFS in $O(N_A + kN_F)$. Therefore, total time complexity of the tree-based deallocation is $O(N_A + kN_F + n2^k)$. Note: Our best-fit tree-based model, when applied to the 2-D/3-D mesh or torus systems, provides a linear time complexity $(N_A + N_F + n)$ since since k = 2 for the 2-D systems and k = 3 for the 3-D systems.

3.6.1.3 Time Complexity when applying the partitioning and combining by network degree and size

THEOREM 5: Time complexity of the tree-based allocation to find the best free sub-system (PE) for each incoming task on a product network-based systems (of size $N = n_1 \times n_2 \times \ldots \times n_k$) that partitioning and combining by network degree and size is $O(N_A + k^2N_F + k^4)$.

PROOF: In the allocation algorithm, a number of recursive iterations of the DFS (depth first search) are at most a number of nodes in the tree and only nodes whose sizes are larger than (or equal to) the request are visited. In this (sub-system bit-map) approach, the number of nodes in the tree is proportional to the number of allocated tasks or busy nodes (N_A) and the number of free nodes (N_F) in the tree, where $N_A + N_F \leq N$ and $N_F \leq N_A$. Since the number of external (or leaf) nodes are at most $N_A + N_F \leq N$ and the number of internal nodes are at most (#leaf nodes-1) divided by (2 – 1). Therefore, the total number of nodes in the tree is at most M nodes, where $M = (N_A + N_F) + (N_A + N_F - 1) / (2 - 1)$. For each (free) leaf node (of N_F

nodes), the best-fit value is computed in $O(k^2)$ time and hence $O(k^2N_F)$ for all leaf nodes. After finding the best free sub-system (Step 3), if its size is equal to the request, then it is directly allocated to the request. Otherwise, the network partitioning and the best sub-partition will be applied, which can be computed in O(k) time. And, the corresponding node(s) in the tree is updated in O(k) time. Next, the combining process is applied in $O(k^4)$ time. Then, the expanding size and update status takes O(k) time. Finally, the maximum free size is updated by applying the DFS in $O(N_A + kN_F)$. Thus, total time complexity to visit all nodes in the tree is approximately $O(N_A + k^2N_F + k^4)$. In addition, our tree-based model, when applied to the 2-D/3-D mesh or 2-D/3-D torus systems, provides a linear time complexity $O(N_A + N_F)$ since $k = 2$ for the 2-D systems and $k = 3$ for the 3-D systems.

THEOREM 6: Time complexity of the tree-based deallocation to free the particular tree node that stores the finished task and to combine the free buddy nodes of the root sub-tree to the root of the tree on the partitionable product network-based systems ($N = n_1 \times n_2 \times \ldots \times n_k$) that partitioning and combining by network degree and size is $O(N_A + kN_F + kn)$.

PROOF: In the deallocation procedure, searching for the location of a finished sub-system from the root is at most kn steps. Then, combining all 2 buddy nodes from the finished sub-system to the root (if it is possible) takes another kn steps. Then, the resume-node-size process of the expand-node-size process (if any) may be required in $O(k^4)$ time. Finally, the maximum free size is updated by applying the DFS in $O(N_A + k N_F)$. Therefore, total time complexity of the tree-based deallocation is $O(N_A + kN_F + kn)$. Note: Our best-fit tree-based model, when applied to the 2-D/3-D mesh or torus systems, provides a linear time complexity $O(N_A + N_F + n)$ since $k = 2$ for the 2-D systems and $k = 3$ for the 3-D systems.

3.6.2 Time Complexity of Searching for Allocation/Deallocation for any SIMD task

Time complexity of searching for allocation for any SIMD task is similar to that for the MIMD task, except we have to add the CU allocation for each node (or each selected sub-system) in both Step 1 and 2.

3.6.2.1 Time Complexity when applying the partitioning and combining by network degree

Table 6 illustrates each function time complexity and total time complexity of the universal CU/PE allocation approach for any SIMD task, based upon the partitioning and combining by network degree.

Table 6. Time complexity of main functions of the universal resource (CU/PE) allocation process (by Method 1) for any SIMD task.

Main Functions	Method 1 (partitioning and combining by network degree)		
	CU-DFS	CU-AS	CU-IS
(1) Leaf node operation (for N_F nodes)	$O(N_A+(k^2+N_A+kN_F+k^2)N_F)$	$O(N_A+(k^2+k^2)N_F)$	$O(N_A+(k^2+1)N_F)$
Compute best-fit value			
- for each free node (SIMD/MIMD)	$O(k^2)$	$O(k^2)$	$O(k^2)$
- for CUs of each selected node (SIMD)	$O(N_A + kN_F + k^2)$	$O(k^2)$	$O(1)$
(2) Partitioning after visit all nodes			
- Best sub-partition			
- for PE (SIMD/MIMD)		$O(kn)$	
- for CU (SIMD)		$O(kn)$	
- Allocate (update status)		$O(k)$	
(3) Combining after allocation			
- Combining (Algorithm C.2-C.3)		$O(kn^{k+1})$	
- Expand size & update status		$O(k)$	
- Update the maximum free size		$O(N_A + kN_F)$	
Total time complexity	$O(k^2N_F + N_AN_F + k(N_F)^2 + kn^{k+1})$	$O(N_A + k^2N_F + kn^{k+1})$	$O(N_A + k^2N_F + kn^{k+1})$

THEOREM 7: Time complexity of the tree-based allocation approach including the CU-DFS strategy to find the best free sub-system and the best CU for each incoming task on a product network-based systems that partitioning and combining by network degree is $O(k^2N_F + N_AN_F + k(N_F)^2 + kn^{k+1})$.

PROOF: Similar to Theorem 1, except that for each (free) leaf node (of N_F nodes), the best-fit value is computed in $O(k^2)$ time for finding PE (or sub-system) and $O(N_A+kN_F+k^2)$ time for finding CU by applying the CU-DFS strategy. Therefore, it takes $O(k^2 + N_A + kN_F + k^2)$ or $O(k^2 + N_A + kN_F)$ time in finding both

PE and CU for each node and hence $O(N_A + (k^2 + N_A + kN_F) N_F)$ for all leaf nodes. Thus, total time complexity to visit all nodes in the tree is approximately $O(k^2N_F + N_AN_F + k(N_F)^2 + kn^{k+1})$, where $O(kn^{k+1})$ is time complexity of the combining process by network degree.

THEOREM 8: Time complexity of the tree-based allocation approach including the CU-AS strategy to find the best free sub-system and the best CU for each incoming task on a product network-based systems that partitioning and combining by network degree is $O(N_A + k^2N_F + kn^{k+1})$. [*PROOF:* Similar to that illustrated in Theorem 7, except time complexity of the CU-AS strategy is $O(k^2)$.]

THEOREM 9: Time complexity of the tree-based allocation approach including the CU-IS strategy to find the best free sub-system and the best CU for each incoming task on a product network-based systems that partitioning and combining by network degree is $O(N_A+k^2N_F+kn^{k+1})$. [*PROOF:* Similar to that illustrated in Theorem 7, except time complexity of the CU-IS strategy is $O(1)$.]

THEOREM 10: Time complexity of the tree-based deallocation approach to free the particular tree PE node and CU node that stores the finished task and to combine the free buddy nodes of the root sub-tree to the root of the tree on the partitionable product network-based systems that partitioning and combining by network degree is $O(N_A + kN_F + kn)$. [*PROOF:* Similar to that illustrated in Theorem 2.]

3.6.2.2 Time Complexity when applying the partitioning and combining by network size

Table 7 illustrates each function time complexity and total time complexity of the universal resource (CU/PE) allocation for any SIMD task, based upon the partitioning and combining by network size.

Table 7. Time complexity of main functions of the universal resource (CU/PE) allocation process (by Method 2) for any SIMD task.

Main Functions	Method 2 (partitioning and combining by network size)		
	CU-DFS	CU-AS	CU-IS
(1) Leaf node operation (for N_F nodes) Compute best-fit value - for each free node (SIMD/MIMD) - for CUs of each selected node (SIMD)	$O(N_A+(k^2+N_A+kN_F+k^2)N_F)$ $O(k^2)$ $O(N_A + kN_F + k^2)$	$O(N_A+(k^2+k^2)N_F)$ $O(k^2)$ $O(k^2)$	$O(N_A+(k^2+1)N_F)$ $O(k^2)$ $O(1)$
(2) Partitioning after visit all nodes - Best sub-partition - for PE (SIMD/MIMD) - for CU (SIMD) - Allocate (update status)	$O(k2^k)$ $O(k2^k)$ $O(k)$		
(3) Combining after allocation - Combining (Algorithm C.2-C.3) - Expand size & update status - Update the maximum free size	$O(k^22^{2k})$ $O(k)$ $O(N_A + kN_F)$		
Total time complexity	$O(k^2N_F + N_AN_F + k(N_F)^2 + k^22^{2k})$	$O(N_A + k^2N_F + k^22^{2k})$	$O(N_A + k^2N_F + k^22^{2k})$

THEOREM 11: Time complexity of the tree-based allocation approach including the CU-DFS strategy to find the best free sub-system (PE) for each incoming task on a product network-based systems that partitioning and combining by network size is $O(k^2N_F + N_AN_F$ or $k(N_F)^2 + k^22^{2k})$.

PROOF: Similar to Theorem 3, except that for each (free) leaf node (of N_F nodes), the best-fit value is computed in $O(k^2)$ time for finding PE (or sub-system) and $O(N_A+kN_F+k^2)$ time for finding CU by applying the CU-DFS strategy. Therefore, it takes $O(k^2+N_A+kN_F+ k^2)$ or $O(k^2+N_A+kN_F)$ time in finding both PE and CU for each node and hence $O(N_A + (k^2 + N_A + kN_F) N_F)$ for all leaf nodes. Thus, total time complexity to visit all nodes in the tree is approximately $O(k^2N_F + N_AN_F + k(N_F)^2 + k^22^{2k})$, where $O(k^22^{2k})$ is time complexity of the combining process by network size.

THEOREM 12: Time complexity of the tree-based allocation approach including the CU-AS strategy to find the best free sub-system (PE) for each incoming task on a product network-based systems that partitioning and combining by network size is $O(N_A + k^2N_F + k^22^{2k})$. [*PROOF:* Similar to that illustrated in Theorem 11, except time complexity of the CU-AS strategy is $O(k^2)$.]

THEOREM 13: Time complexity of the tree-based allocation approach including the CU-IS strategy to find the best free sub-system (PE) for each incoming task on a product network-based systems that partitioning and combining by network size is $O(N_A + k^2N_F + k^2 2^{2k})$. [*PROOF:* Similar to that illustrated in Theorem 11, except time complexity of the CU-IS strategy is O(1).]

THEOREM 14: Time complexity of the tree-based deallocation approach to free the particular tree PE node and CU node that stores the finished task and to combine the free buddy nodes of the root sub-tree to the root of the tree on the partitionable product network-based systems that partitioning and combining by network size is $O(N_A + kN_F + k^2 2^{2k})$. [*PROOF:* Similar to that illustrated in Theorem 4.]

3.6.2.3 Time Complexity when applying the partitioning and combining by network degree and size

Table 8 illustrates each function time complexity and total time complexity of the universal CU/PE allocation approach for any SIMD task, based on the partitioning and combining by network degree & size.

Table 8. Time complexity of main functions of the universal resource (CU/PE) allocation process (by Method 3) for any SIMD task.

Main Functions	Method 3 (partitioning and combining by network degree and size)		
	CU-DFS	CU-AS	CU-IS
(1) Leaf node operation (for N_F nodes) Compute best-fit value - for each free node (SIMD/MIMD) - for CUs of each selected node (SIMD)	$O(N_A+(k^2+N_A+kN_F+k^2)N_F)$ $O(k^2)$ $O(N_A + kN_F + k^2)$	$O(N_A+(k^2+k^2)N_F)$ $O(k^2)$ $O(k^2)$	$O(N_A+(k^2+1)N_F)$ $O(k^2)$ $O(1)$
(2) Partitioning after visit all nodes - Best sub-partition - for PE (SIMD/MIMD) - for CU (SIMD) - Allocate (update status)	$O(k)$ $O(k)$ $O(k)$		
(3) Combining after allocation - Combining (Algorithm C.2-C.3) - Expand size & update status - Update the maximum free size	$O(k^4)$ $O(k)$ $O(N_A + kN_F)$		
Total time complexity	$O(k^2N_F + N_AN_F + k(N_F)^2 + k^4)$	$O(N_A + k^2N_F + k^4)$	$O(N_A + k^2N_F + k^4)$

THEOREM 15: Time complexity of the tree-based allocation approach including the CU-DFS strategy to find the best free sub-system (PE) for each incoming task on a product network-based systems that partitioning and combining by network degree and size is $O(k^2N_F + N_AN_F + k(N_F)^2 + k^4)$

PROOF: Similar to Theorem 5, except that for each (free) leaf node (of N_F nodes), the best-fit value is computed in $O(k^2)$ time for finding PE (or sub-system) and $O(N_A + kN_F + k^2)$ time for finding CU by applying the CU-DFS strategy. Therefore, it takes $O(k^2 + N_A + kN_F + k^2)$ or $O(k^2+N_A+kN_F)$ time in finding both PE and CU for each node and hence $O(N_A + (k^2 + N_A + kN_F) N_F)$ for all leaf nodes. Thus, total time complexity to visit all nodes in the tree is approximately $O(k^2N_F + N_AN_F + k(N_F)^2 + k^4)$, where $O(k^4)$ is time complexity of the combining process by network degree and size.

THEOREM 16: Time complexity of the tree-based allocation approach including the CU-AS strategy to find the best free sub-system (PE) for each incoming task on a product network-based systems that partitioning and combining by network degree and size is $O(N_A + k^2N_F + k^4)$. [*PROOF:* Similar to that illustrate in Theorem 15, except time complexity of the CU-AS strategy is $O(k^2)$.]

THEOREM 17: Time complexity of the tree-based allocation approach including the CU-IS strategy to find the best free sub-system (PE) for each incoming task on a product network-based systems that partitioning and combining by network degree and size is $O(N_A + k^2N_F + k^4)$. [*PROOF:* Similar to that illustrate in Theorem 15, except time complexity of the CU-IS strategy is O(1).]

THEOREM 18: Time complexity of the tree-based deallocation to free the particular tree node that stores the finished task and to combine the free buddy nodes of the root sub-tree to the root of the tree on the partitionable product network-based systems that partitioning and combining by network degree and size is $O(N_A + kN_F + kn)$ [*PROOF:* Similar to that illustrated in Theorem 6.]

4. APPLICATION OF THE UNIVERSAL CU/PE ALLOCATION MODEL

The universal tree-based resource (CU/PE) allocation model can be applied to all interconnection networks that belong to the product network class such as multi-dimensional (k-D) meshes, multi-dimensional (k-D) tori, n-ary k-cubes, hypercubes, hypercycles, etc. In this section, we show some applications of the universal resource allocation model to two popular interconnection networks, which are the 2-D mesh networks and the hypercube (or k-cube) networks.

4.1 The Universal Resource (CU/PE) Allocation Model for 2-D Meshes

In the reconfigurable MSIMD/MIMD system, there are two different modes providing for incoming tasks: SIMD (single instruction, multiple data) and MIMD (multiple instructions, multiple data). The MIMD task requires only a free sub-system (or partition), consisting of processing elements (PEs) for distributed computing of many instructions and data. The SIMD task needs both a free partition (PEs) and a control unit (CU) for parallel computing of a single instruction with multiple data. Next, in order to simplify our explanation, we present the application on the 2-D mesh for all MIMD tasks first (in Section 4.1.1) and then the application on the 2-D mesh for all SIMD tasks (in Section 4.1.2). However in practical (see Section 5), mixing modes are utilized for parallel and distributed computing in the reconfigurable and partitionable MSIMD/MIMD parallel system.

4.1.1 Sub-system (PEs) Allocation for MIMD Tasks

Suppose we have a 2-D mesh system of size 16x16 and a sequence of 5 incoming tasks (4x7, 2x2, 3x4, 8x8, and 3x3), which come in one at a time. Before we apply the universal resource (CU/PE) allocation model on this system, let's show how the product network ($G = G_1 \times G_2$) of the 2-D mesh-connected system of size $N = n_1 \times n_2$ (16x16) is constructed. Figure 67 illustrates the product network G, a product of two basic networks (or linear arrays) G_i of size $n_i = 16$, where $k = 2$ and $i = 1, 2$.

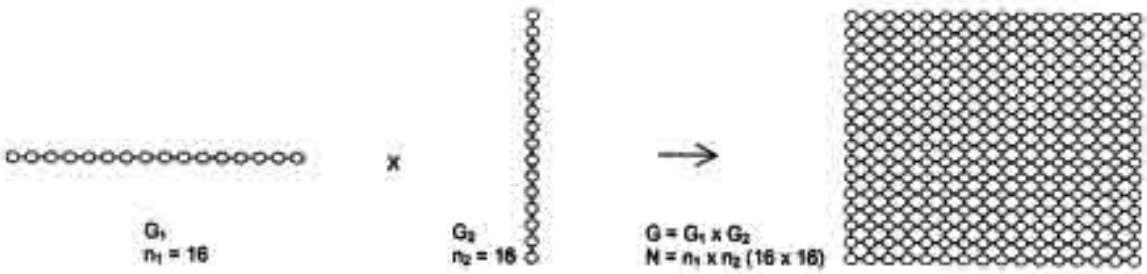


Figure 67: An example of a 2-D mesh-connected system, a product network of two linear array networks.

For the 2-D mesh-connected system ($k = 2$), the value of k is very small and hence we can apply either Method 2 (the partitioning and combining by network size) or Method 3 (the partitioning and combining by network degree and size) of the universal resource (CU/PE) allocation model.

4.1.1.1 Apply Method 2: the Partitioning and Combining by Network Size

Figure 68 illustrates the system status and the corresponding k-Tree that shows the allocation of the first incoming task (4x7). For this task, the root node (or the first node) of the k-Tree is created (starting at level 1) to store the system information (i.e., size = 16x16, based address = <1, 1>, status = 0). For the initial system, we have only one free node in the tree and hence it is the best one (when applying the best-fit heuristic (Step 1 - 3)). For the final step (Step 4), the partitioning process to select the best buddy node for allocating to the task is applied. Usually for the first task, we select the first buddy node (see Figure 68.a). After the allocation, we apply the combining process to the corresponding nodes of the current partitioning. Now, we have two possible combined sub-systems of sizes 12x16 and 16x9. Then, the expand-node size is applied to the larger size (12x16), the combining of the buddy#2 and the byddy#4 (at level 2). We store the new expand size into the free buddy#2 and mark the buddy#4 as a busy node (see Figure 68.b). Then the maximum free size is updated, which is the buddy#2 (12x16) at level 2.

Figure 69 illustrates the system status and the corresponding k-Tree that shows the allocation of the second task (2x2) and the third task (3x4), respectively. For the task (2x2), the searching starts from the root and goes to the left most free node (see Figure 68.b), which is the buddy#2 (12x16) at level 2. Then its best-fit value (Step 1-3) is computed (i.e., preserve maxFS = F, diffSF = 2, size = 192, CF = 2¼) and it is recorded as the first best node. The searching then visits the next free node, which is the buddy#3 (4x9) at level 2. The best-fit value of this node (Step 1-3) is computed (i.e., preserve maxFS = T, diffSF = 2, size = 36, CF = 3¼). Since this node (4x9) performs the better best-fit value (i.e. it can preserve the maximum free size (maxFS), the first criterion in the best-fit heuristic), it is updated as the current best node. At this time, all free nodes are visited and then Step 4 of the best-fit heuristic is applied to the best node (from Step 1-3). After the partitioning process, the best sub-partition for the second task (2x2) is the buddy#3 at level 3 (see Figure 69.a). Finally after the combining process of the current partitioning, there are two possible combined sub-systems of sizes 4x7 and 2x9. Then, the expand-node size is applied to the larger size (4x7), the combining of the buddy#1 and the byddy#2 (at level 3). We store the new expand size into the free buddy#1 and mark the buddy#2 as a busy node. Next since the maximum free size (12x16), the buddy#2 at level 2, is not partitioned, it is not necessary to be updated at this time.

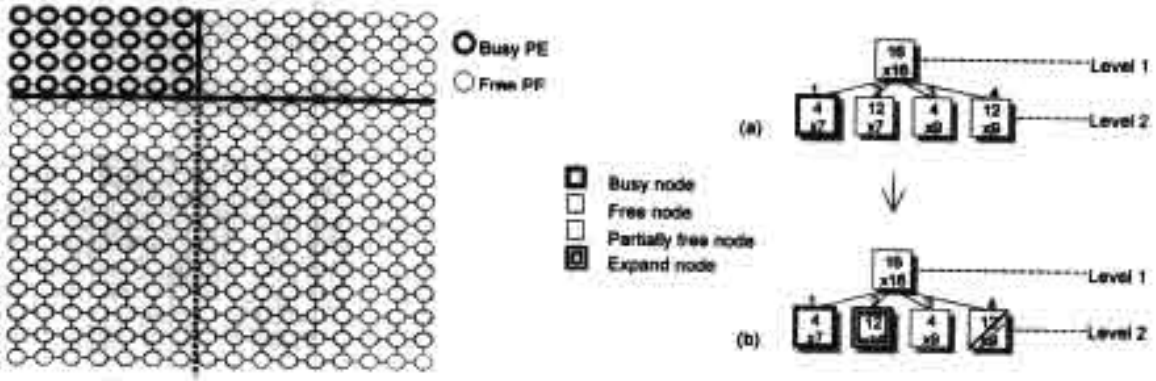


Figure 68: The system status and the corresponding k-Tree of the allocation of the first task (4x7), based on the partitioning and combining by network size, for a 2-D mesh.

For the third task (3x4), the first free node (see Figure 69.a) is the buddy#2 (12x16) at level 2 with computed best-fit value (i.e., preserve maxFS = F, diffSF = 2, size = 192, CF = 2¼). Then, it is recorded as the first best node. The next free node is the buddy#1 (4x7) at level 3 with the computed best-fit value (i.e., preserve maxFS = T, diffSF = 1 (for the rotated size 4x3), size = 28, CF = 5½) and hence it is updated as the new best node. Next, there is the last free node (the buddy#4 at level 3) to visit but it size (2x2) can not accommodate to the request (3x4). Now, all free nodes are visited and then Step 4 is applied to the best node (from Step 1-3) in order to select the best sub-partition. After the partitioning process, the buddy#1 at level 4 is selected for the task (3x4) with the rotated size 4x3 (see Figure 69.b) since it yields the better best-fit value. Finally for the maximum free size (12x16), in this case it is not necessary to be updated.

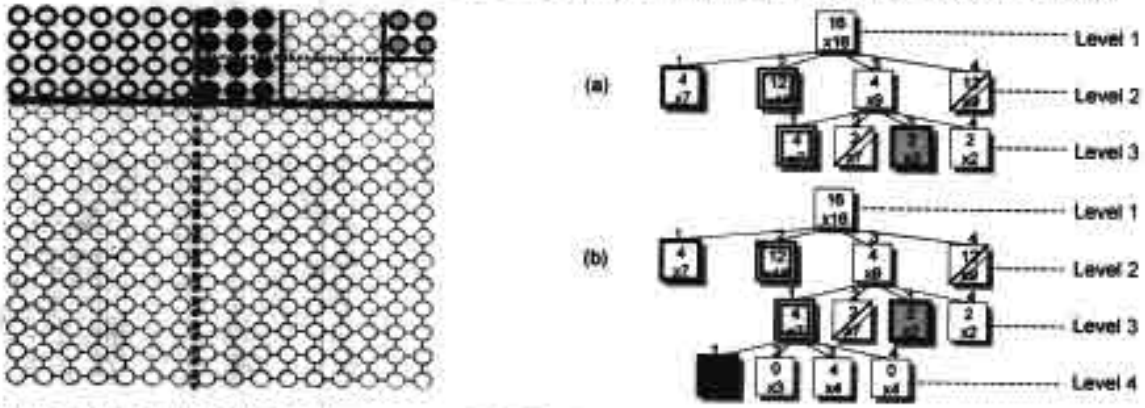


Figure 69: The system status and the corresponding k-Tree of the allocation of a) the second task (2x2) and b) the third task (3x4), based on the partitioning and combining by network size, for a 2-D mesh.

Figure 70 illustrates the system status and the corresponding k-Tree of the allocation of the fourth task (8x8) and the fifth task (3x3), respectively. For the task (8x8), the first free node (see Figure 69.b) is the buddy#2 (12x16) at level 2 with the computed best-fit value (i.e., preserve maxFS = F, diffSF = 2, size = 192, CF = 2¼) and it is recorded as the first best node. Next, two sub-sequence free nodes are the buddy#3 (4x4) at level 4 and the buddy#4 (2x2) at level 3 but their sizes can not accommodate to the request (8x8). So far, all free nodes are visited and then Step 4 of the best-fit heuristic is applied to partition the best node (from Step 1-3), in order to select the best sub-partition (the buddy#2 (8x8) at level 3) for the fourth task (see Figure 70.a). After combining the corresponding nodes of the current partitioning, we have two possible combined sub-systems (4x16 and 12x8). The larger size (12x8), the combining of the buddy#3 and the byddy#4 (at level 3), is selected. We store the expand size in the free buddy#3 and mark the buddy#4 as a busy node. Finally, since the buddy#2 at level 2, the maximum free size (12x16), is partitioned, we have to compute the new maximum free size. At beginning, we select the new expanded node (12x8), the buddy#3 at level 3, as the temporary maximum free size. Then, we have to update the maximum free size by performing DFS to visit all free nodes in the tree if the larger node exists.

For the last incoming task (3x3), the first free node (see Figure 70.a) is the buddy#1 (4x8) at level 3 with the computed best-fit value (i.e., preserve maxFS = T, diffSF = 2, size = 32, CF = 5) and it is recorded as the first best node. The next free node is the buddy#3 (12x8) at level 3, but it cannot preserve the maximum free size, the first best-fit criterion. Then, the searching goes to the next free node, the buddy#3 (4x4) at level 4. Its best-fit value is computed (i.e., preserve maxFS = T, diffSF = 2, size = 16, CF = 5½) and updated as the current best node since it performs the better best-fit value, according to the criterion 3 (the smaller size). The last free node is the buddy#4 at level 3, but its size (2x2) can not accommodate to the request (3x3). Now all free nodes are visited and then Step 4 of the best-fit heuristic is applied to the best node (from Step 1-3) in order to select the best sub-partition. After the partitioning process, the buddy#3 at level 5 is selected for the fifth task (see Figure 70.b). After the combining process of the current partitioning, we have two possible combined sub-systems (4x1 and 1x4). The expand-node size is applied to the first size (4x1), the combining of the buddy#1 and the byddy#2 (at level 5). The new expand size is stored into the free buddy#1 and the buddy#2 is marked as a busy node. For the maximum free size computing, since the current maximum free size (12x8), is not partitioned, it is not necessary to be updated.

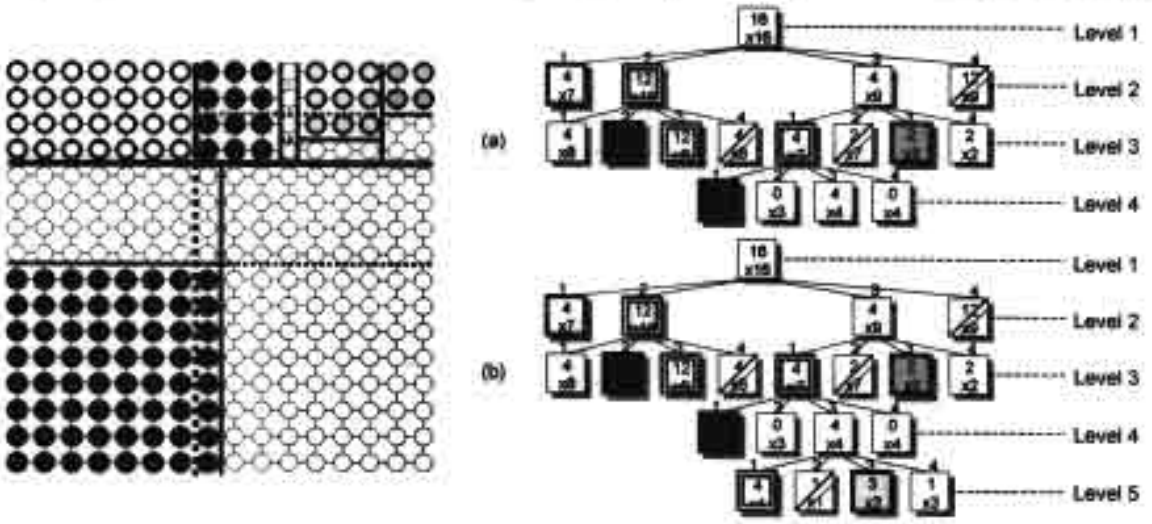


Figure 70: The system status and the corresponding k-Tree of the allocation of a) the fourth task (8x8) and b) the fifth task (3x3), based on the partitioning and combining by network size, for a 2-D mesh.

4.1.1.2 Apply Method 3: the Partitioning and Combining by Network Degree and Size

In order to see the compared results to the previous application, we use the same 2-D mesh system and incoming tasks, defined in Section 4.1.1.1. Figure 71 illustrates the system status and the corresponding binary-Tree that illustrates the allocation of the first incoming task (4x7). For this task, the root of the tree is created (at level 1) to store the system information (i.e., size = 16x16, based address = <1, 1>, status = 0). Initially, we have only one free node, which is the best one (when applying the best-fit heuristic (Step 1-3)).

The final step (Step 4), the partitioning process to select the best buddy node for allocating to the task, is applied. Usually for the first task, we select the first buddy node. Finally, the maximum free size is updated, which is the buddy#2 (12x16) at level 2.

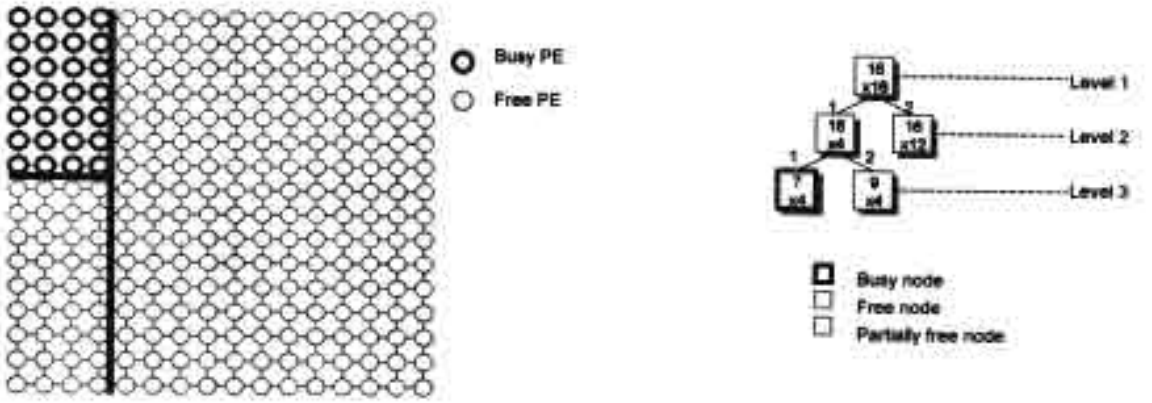


Figure 71: The system status and the corresponding binary-Tree of the allocation of the first task (4x7), based on the partitioning and combining by network degree and size, for a 2-D mesh.

Figure 72 illustrates the system status and the corresponding binary-Tree that shows the allocation of the second task (2x2) and the third task (3x4), respectively. For the task (2x2), the searching starts from the root and the first free node is the buddy#2 (9x4) at level 3 (see Figure 71). Then its best-fit value is computed (i.e., preserve maxFS = T, diffSF = 2, size = 36, CF = 3¼) and it is recorded as the first best node. The next free node is the buddy#2 (16x12) at level 2. According to the best-fit criterion 1, it cannot preserve the maximum free size, which is not better than the current best-fit node. Then, searching goes to the next free node but now all free nodes are visited. Then, Step 4 of the best-fit heuristic is applied to the best node (from Step 1-3). After applying the partitioning process twice, the best sub-partition for the second task (2x2) is the buddy#2 (2x4) at level 4, partitioned along the 1th dimension, and then the buddy#1 (2x2) at level, partitioned alone the 2nd dimension (see Figure 72.a). Next since the maximum free size (16x12), the buddy#2 at level 2, is not partitioned, it is not necessary to be updated at this time.

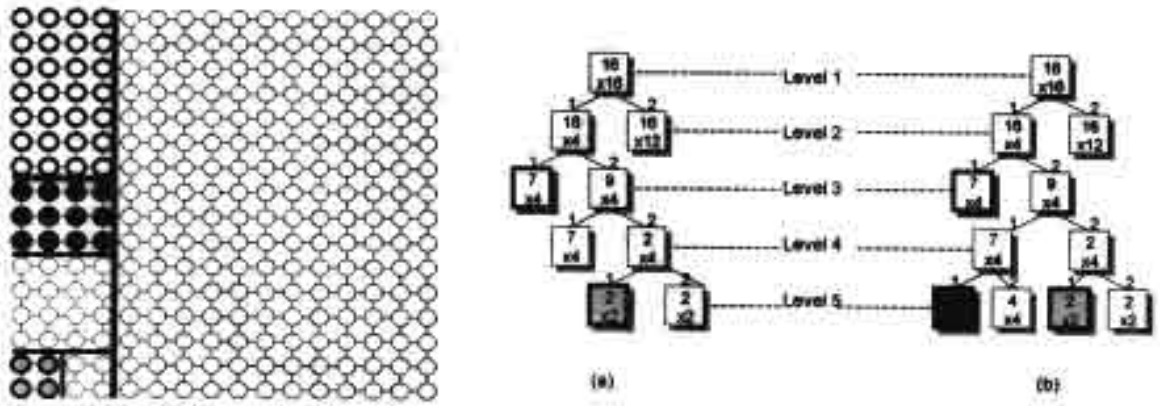


Figure 72: The system status and the corresponding binary-Tree of the allocation of a) the second task (2x2) and b) the third task (3x4), based on the partitioning and combining by network degree and size, for a 2-D mesh.

For the third task (3x4), the first free node (see Figure 72.a) is the buddy#1 (7x4) at level 4 with computed best-fit value (i.e., preserve maxFS = T, diffSF = 1, size = 28, CF = 3¼) and it is the first best node. The next free node is the buddy#2 (2x2) at level 5 but it size (2x2) can not accommodate to the request (3x4). The last free node is the buddy#2 (16x12) at level 2 but it cannot preserve the maximum free size, the criterion 1 of the best-fit heuristic. Then the partitioning process (Step 4) is applied to the best node (from Step 1-3) in order to select the best sub-partition, the buddy#1 (3x4) at level 5 for the fifth task (see Figure 72.b). Finally for the maximum free size (16x12), in this case it is not necessary to be updated.

Figure 73 illustrates the system status and the corresponding binary-Tree of the allocation of the fourth task (8x8) and the fifth task (3x3). For the task (8x8), the first free node (see Figure 72.b) is the buddy#2 (4x4) at level 5 but its size is less than the task's size. The next free node is the buddy#2 (2x2) at level 5 but its size is too small for the task. The last free node is the buddy#2 (16x12) at level 2 with the computed best-fit value (i.e., preserve maxFS = F, diffSF = 2, size = 192, CF = 2¼) and it is recorded as the first free node. So far, all free nodes are visited and then Step 4 of the best-fit heuristic is applied to partition the best node (from Step 1-3), in order to select the best sub-partition for the fourth task (see Figure 73.a). In this case, the partitioning process is applied twice. According to the criterion 4 of the best-fit heuristic, we select the buddy#1 (8x12) at level 3, partitioned along the 1st dimension, and then the buddy#2 (8x8) at level 4, partitioned along the 2nd dimension. Finally, since the buddy#2 at level 2, the maximum free size (16x12), is partitioned, we have to compute the new maximum free size. At beginning, we select the new maximum node (the buddy#2 (8x12) at level 3) after the current partitioning. Then, we update the maximum free size by performing DFS to visit all free nodes in the tree if the larger node exists.

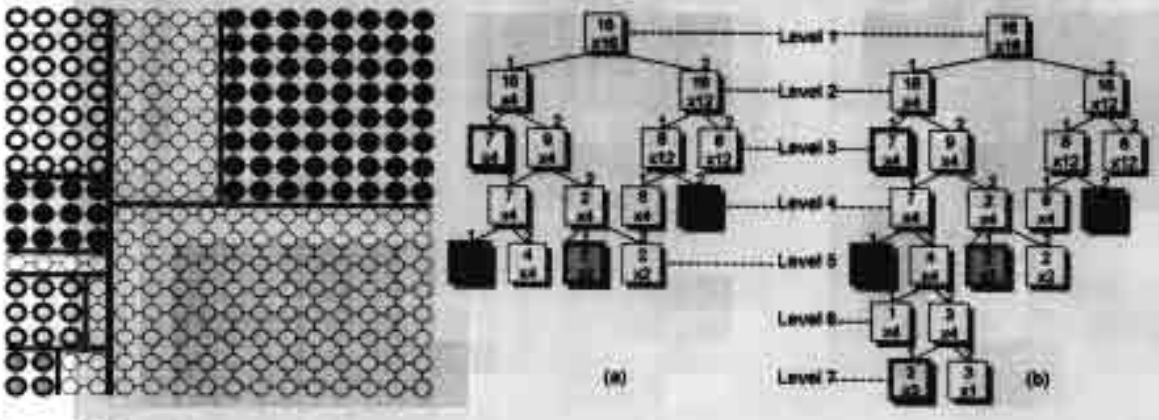


Figure 73: The system status and the corresponding binary-Tree of the allocation of a) the fourth task (8x8) and b) the fifth task (3x4), based on the partitioning and combining by network degree and size, for a 2-D mesh.

For the last incoming task (3x3), the first free node (see Figure 73.a) is the buddy#2 (4x4) at level 5 with the computed best-fit value (i.e., preserve maxFS = T, diffSF = 2, size = 16, CF = 3¾) and it is recorded as the first best node. The next free node is the buddy#2 (2x2) at level 5, but it cannot accommodate to the task (3x3). Then, the searching goes to the next free node, the buddy#1 (8x4) at level 4 with the computed best-fit value (preserve maxFS = T, diffSF = 2, size = 32). This node does not perform the better best-fit value, according to the criterion 3 of the best-fit heuristic. The last free node is the buddy#2 (8x12) at level 3. However, it cannot preserve the maximum free size. Now all free nodes are visited and then Step 4 of the best-fit heuristic is applied to the best node (from Step 1-3) in order to select the best sub-partition. The partitioning process is applied twice. According to the criterion 4 of the best-fit heuristic, we select the buddy#2 (3x4) at level 6, partitioned along the 1st dimension and then the buddy#1 (3x3) at level 7, partitioned along the 2nd dimension for the fifth task (see Figure 73.b). In this case, we do not have to update the maximum free size since the current maximum free size (8x12) is not partitioned.

4.1.2 Sub-System (PEs) and Control Unit (CU) Allocation for SIMD tasks

Suppose we have a 2-D mesh system of size 16x16 and a sequence of 5 incoming SIMD tasks (4x7, 2x2, 3x4, 8x8, and 3x3), which come in one at a time. For the 2-D mesh system (k = 2), the value of k is very small and thus we can apply either Method 2 (the partitioning and combining by network size) or Method 3 (the partitioning and combining by network degree and size) of the universal CU/PE allocation model.

4.1.2.1 Apply Method 2: the Partitioning and Combining by Network Size

Figure 74 illustrates the system status and the corresponding k-Tree that shows the allocation of the first incoming task (4x7). The sub-system (or PEs) allocation is similar to the allocation illustrated in previous section (see Figure 68) for the MIMD task. For the SIMD task in this section, we add the allocation for the

corresponding CU. Usually for the first task, we select the first buddy node for the PE allocation and perform another partitioning process to the smallest adjacent sub-partition for the CU allocation. After the CU/PE allocation, we apply the combining process to the corresponding nodes of the partitioning for PEs and the partitioning for CU. At this time, the maximum free size is the buddy#2 (12x16) at level 2.

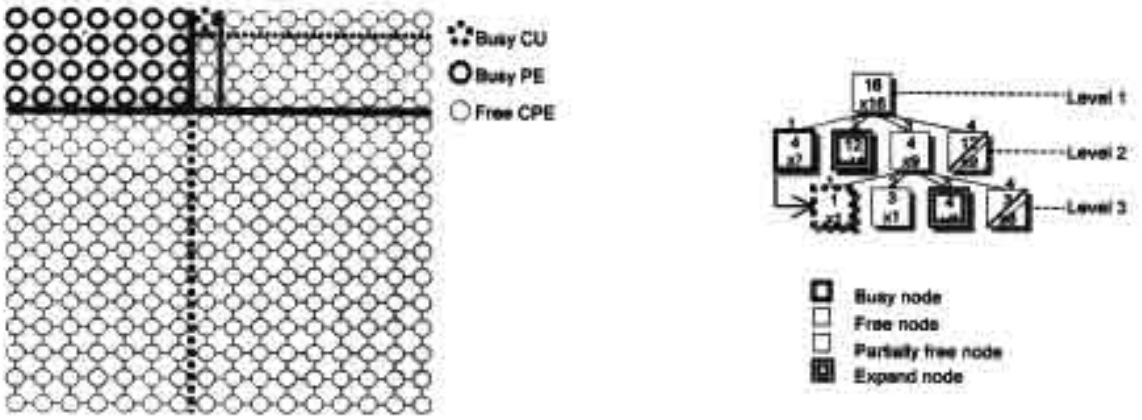


Figure 74: The system status and the corresponding k-Tree of the allocation of the first SIMD task (4x7), based on the partitioning and combining by network size, for a 2-D mesh.

Figure 75 illustrates the system status and the corresponding k-Tree that shows the allocation of the second task (2x2). The sub-system (or PEs) allocation is similar to the allocation, illustrated in previous section (Figure 69.a) for the MIMD task. For the SIMD task, we have to apply the CU searching (i.e., the CU-DFS, CU-AS, or CU-IS strategy) to find the adjacent node (containing some CUs) for each visiting node. For example, the last free node, the buddy#3 (4x8) at level 3 (see Figure 74) is updated as the best node since it yields the better best-fit value (i.e., preserve maxFS = T, diffSF = 2, size = 32, CF = 4½). For CU searching of this node, the best adjacent node is the buddy#2 at level 3 if we apply the CU-DFS or CU-AS strategy. If we apply the CU-IS strategy, we do not have to find adjacent node because the corresponding CU can be inside the best sub-system, the buddy#3 at level 3. After all free nodes are visited, Step 4 of the best-fit heuristic (the partitioning process) is applied to the best node for PEs and its adjacent node for CU (from Step 1-3). After the partitioning process, the best sub-partition for the second task (2x2) is the buddy#1 at level 4 and the corresponding CU is selected from the outside node (from applying the CU-DFS or CU-AS), the buddy#2 (3x1) at level 2.

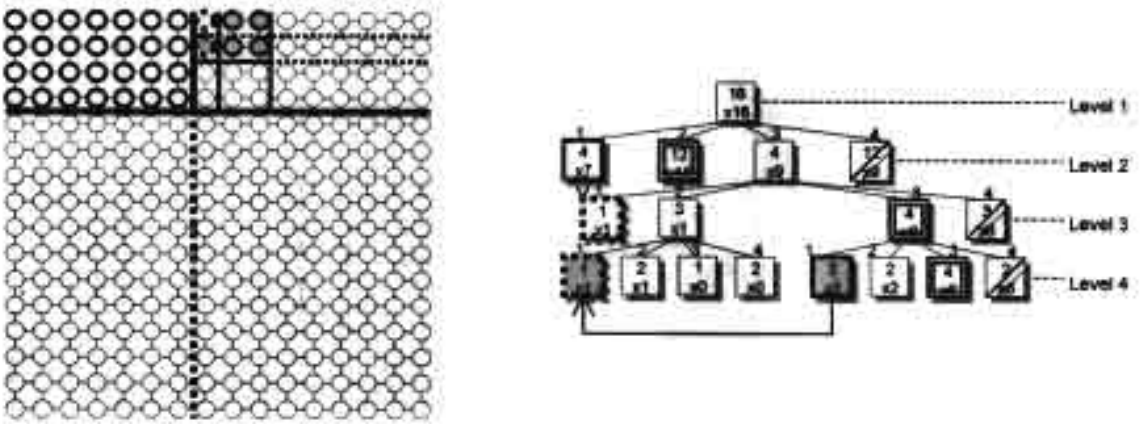


Figure 75: The system status and the corresponding k-Tree of the allocation of the second SIMD task (2x2), based on the partitioning and combining by network size, for a 2-D mesh.

Figure 76 illustrates the system status and the corresponding k-Tree that shows the allocation of the third task (3x4), the fourth task (8x8), and the fifth task (3x3), respectively. The searching for the best free node for the sub-system (PEs) allocation and the corresponding CU for the CU allocation for each of these tasks is similar to that applied for the second task.

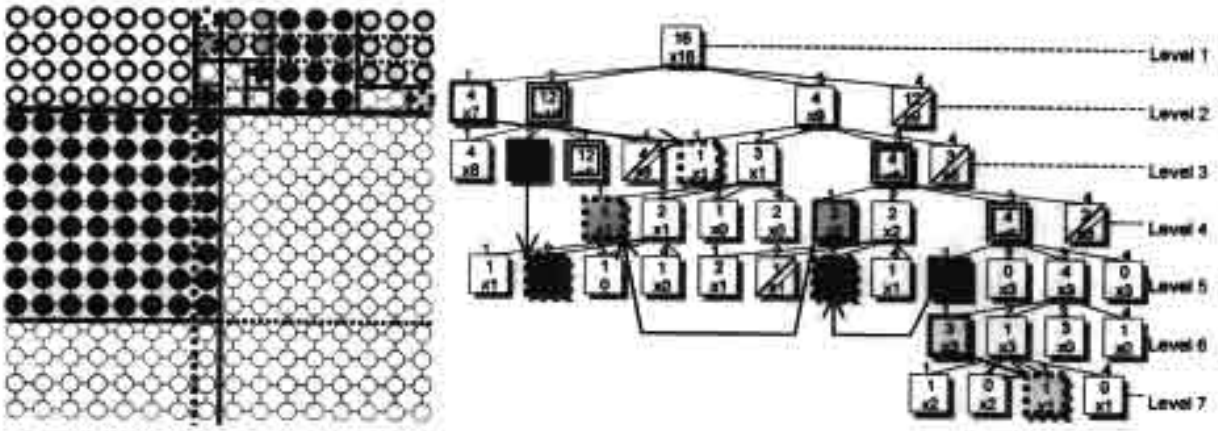


Figure 76: The system status and the corresponding k-Tree of the allocation of the 3rd, 4th, and 5th SIMD tasks (3x4, 8x8, 3x3), based on the partitioning and combining by network size, for a 2-D mesh.

4.1.2.2 Apply Method 3: the Partitioning and Combining by Network Degree and Size

Figure 77 illustrates the system status and the corresponding binary-Tree that shows the allocation of the first incoming task (4x7). The sub-system (or PEs) allocation is similar to the allocation illustrated in previous section (Figure 71) for the MIMD task. Then, for the SIMD task we have to add the allocation for the corresponding CU. Usually for the first task, we select the first buddy node for the PE allocation and perform another partitioning process to the smallest adjacent sub-partition for the CU allocation. After the CU/PE allocation, we apply the combining process to the corresponding nodes of the partitioning for PEs and the partitioning for CU. The first maximum free size is the buddy#2 (12x16) at level 2.

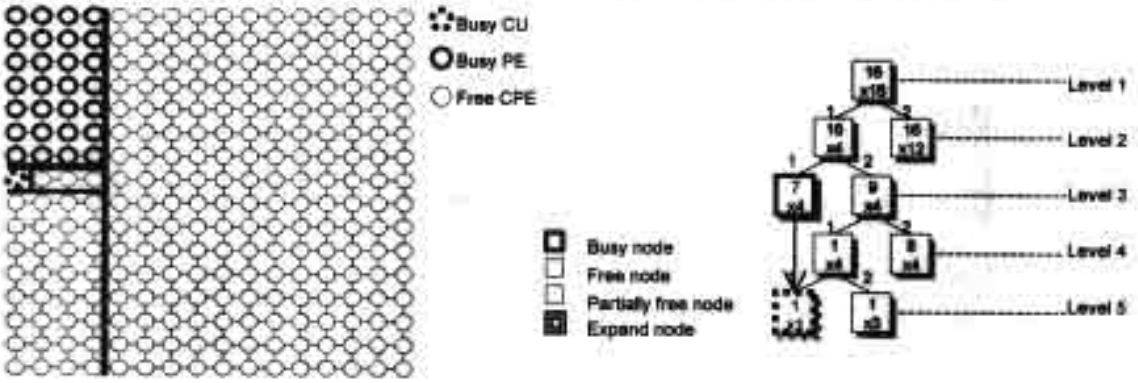


Figure 77: The system status and the corresponding k-Tree of the allocation of the first SIMD task (4x7), based on the partitioning and combining by network degree and size, for a 2-D mesh.

Figure 78 illustrates the system status and the corresponding binary-Tree that shows the allocation of the second task (2x2). The sub-system (or PEs) allocation is similar to the allocation illustrated in previous section (see Figure 72.a) for the MIMD task. For the SIMD task we have to apply the CU searching (i.e., the CU-DFS, CU-AS, or CU-IS strategy) to find the adjacent node (containing some CUs) for each visiting node. After visiting all free nodes, the best node (from Step 1-3) is the buddy#2 (8x4) at level 4 and its adjacent node is the buddy#2 (1x3) at level 5. After the partitioning process, the best sub-partition for the second task (2x2) is the buddy#1 at level 6 and the corresponding CU is selected from the outside node (from applying the CU-DFS or CU-AS strategy), the buddy#2 (1x3) at level 5.

Figure 79 illustrates the system status and the corresponding k-Tree that shows the allocation of the third task (3x4), the fourth task (8x8), and the fifth task (3x3), respectively. The searching for the best free node for the sub-system (PEs) allocation and the corresponding CU for the CU allocation for each of these tasks is similar to that applied for the second task.

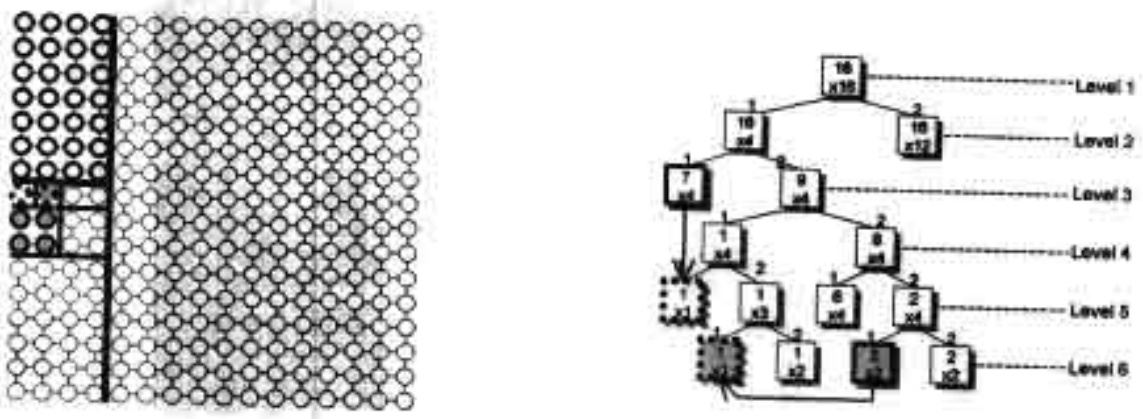


Figure 78: The system status and the corresponding k-Tree of the allocation of the second SIMD task (2x2), based on the partitioning and combining by network degree and size, for a 2-D mesh.

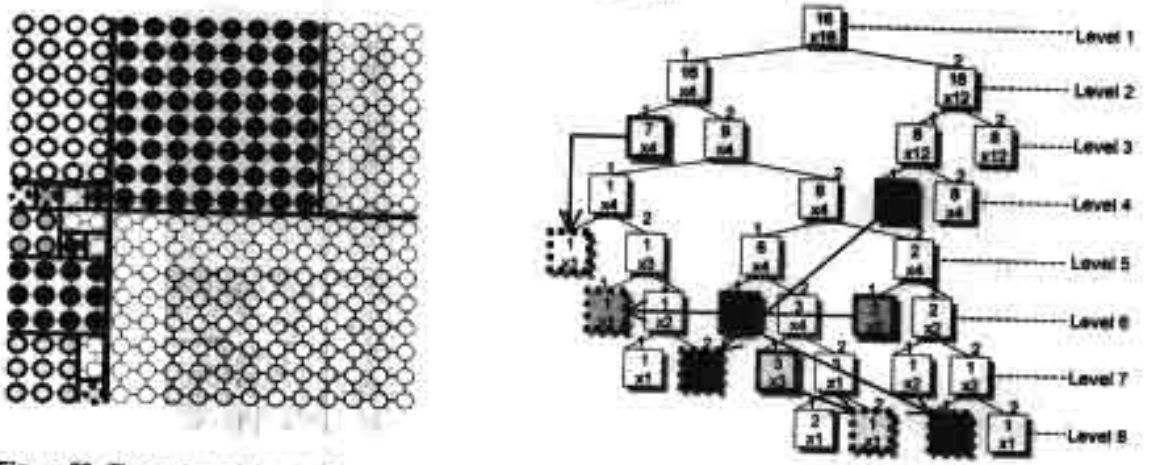


Figure 79: The system status and the corresponding k-Tree of the allocation of the 3rd, 4th, and 5th SIMD tasks (3x4, 8x8, 3x3), based on the partitioning and combining by network degree and size, for a 2-D mesh.

4.2 The Universal Resource (CU/PE) Allocation Model for Hypercubes

For the reconfigurable MSIMD/MIMD system with the hypercube network, two different modes providing for incoming tasks: SIMD (single instruction, multiple data) and MIMD (multiple instructions, multiple data). The MIMD task requires only a free sub-system (or partition), consisting of processing elements (PEs) for distributed computing of many instructions and data. The SIMD task needs both a free partition (PEs) and a control unit (CU) for parallel computing of a single instruction with multiple data. Next, in order to simplify our explanation, we present the application on the hypercube for all MIMD tasks first (in Section 4.2.1) and then the application on the hypercube for all SIMD tasks (in Section 4.2.2). However in practical (see Section 5), mixing modes are allowed for parallel and distributed computing in the reconfigurable and partitionable MSIMD/MIMD parallel system.

4.2.1 Sub-system (PEs) Allocation for MIMD Tasks

Suppose we have a hypercube (or 5-cube) system of size $N = 2^5$ and a sequence of three incoming tasks (3-cube, 4-cube, 2-cube), coming in one at a time. Before we apply the universal resource (CU/PE) allocation model on this system, let's show how the product network ($G = G_1xG_2xG_3xG_4xG_5$) of the 5-cube-connected system of size $N = n_1xn_2xn_3xn_4xn_5$ (2^5) is constructed.

Figure 80 illustrates the product network G, a product of five basic networks (or linear arrays) G_i of size $n_i = 2$, where $k = 5$ and $i = 1, 2, 3, 4, 5$. For the hypercube-connected system, the value of n is 2 and hence we apply the partitioning and combining by network degree (Method 1).

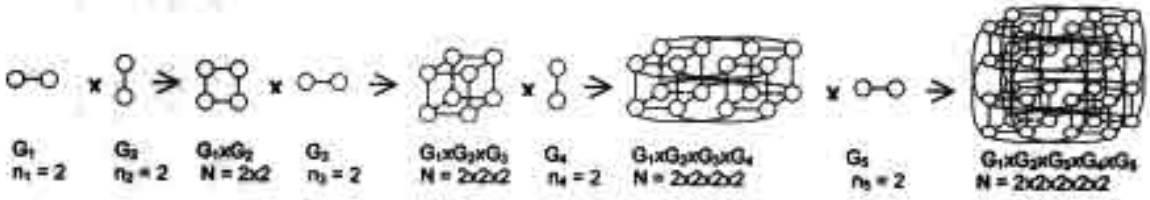


Figure 80: An example of a hypercube (5-cube)-connected system, a product network of five linear array networks.

Figure 81 illustrates the system status and the corresponding binary tree that shows the allocation of the first incoming task (3-cube). For this task, the root node (or the first node) of the tree is created (starting at level 1) to store the system information (i.e., size = 2^5, based address = <1,1,1,1,1>, status = 0). For the initial system, we have only one free node in the tree and hence it is the best one (when applying the best-fit heuristic (Step 1 - 3)). For the final step (Step 4), the partitioning process is applied (twice for this task size) to select the best buddy node for allocating to the task. Usually for the first task, we always select the first buddy node. After the allocation, we will apply the combining process (i.e., combining for a 4-cube) to the corresponding nodes of the sub-sequence partitioning if the buddy of its root is partially free (or some of its buddy nodes are busy). In this case, we do not have to apply the combining process since the buddy#2 (4-cube) is free. Then the first maximum free size is the buddy#2 (4-cube) at level 2.

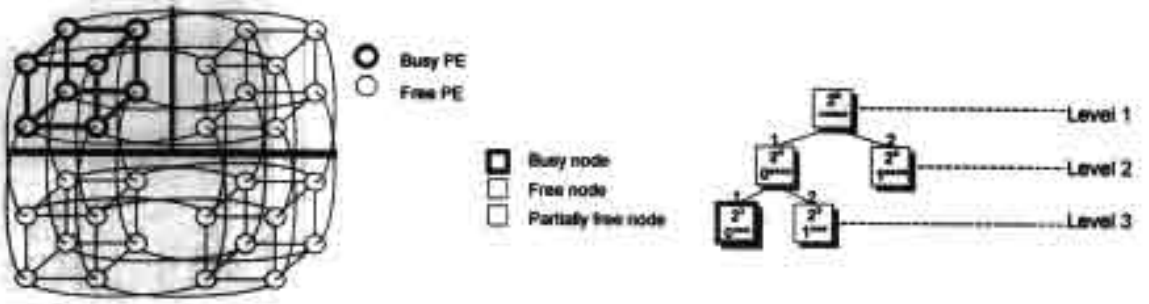


Figure 81: The system status and the corresponding binary-Tree of the allocation of the first task (3-cube), based on the partitioning and combining by network degree, for a hypercube.

Figure 82 illustrates the system status and the corresponding binary tree that shows the allocation of the second task (4-cube) and the third task (2-cube). For the task (4-cube), the searching starts from the root and goes to the left most free node (see Figure 81), which is the buddy#2 (3-cube) at level 3 but its size cannot accommodate to the task. The searching then visits the next free node, which is the buddy#2 (4-cube) at level 2. The best-fit value of this node (Step 1-3) is computed (i.e., preserve maxFS = F, diffSF = 0, size = 16, CF = ½) and it is recorded as the first best node. At this time, all free nodes are visited and then Step 4 of the best-fit heuristic is applied to the best node (from Step 1-3) if its size is larger than the task. In this case, we do not apply the partitioning process since the node's size is equal to the task's size (4-cube) and hence no need for the combining process (see Figure 82.a). Next since the maximum free size (4-cube), the buddy#2 at level 2, is allocated, we have to find the new maximum free size by using DFS to visit all free nodes in the tree. Now, the maximum free size is the buddy#2 (3-cube) at level 3.

For the third task (2-cube), the searching starts from the root and goes to the left most free node (see Figure 82.a), which is the buddy#2 (3-cube) at level 3. The best-fit value of this node (Step 1-3) is computed (i.e., preserve maxFS = F, diffSF = 0, size = 8, CF = ½) and it is recorded as the first best node. The searching is continuing to the next free node. So far, all free nodes are visited and then Step 4 of the best-fit heuristic is applied to the best node (from Step 1-3), the buddy#2 (3-cube) at level 3. After applying the partitioning process once, we allocate the buddy#1 (at level 4) for the third task (2-cube). After the allocation, we will apply the combining process (i.e., combining for a 4-cube) to the corresponding nodes of the sub-sequence partitioning if the buddy of its root is partially free (or some of its buddy nodes are busy). In this case, we do not have to apply the combining process since the buddy#2 (4-cube) is busy. Next since the maximum free size (3-cube), the buddy#2 at level 3, is partitioned, we have to find the new maximum free size. At beginning, we set the temporary maximum free size to the buddy#2 (2-cube) at level 4 and then using DFS to visit all free nodes in the tree to update if the larger node exists.

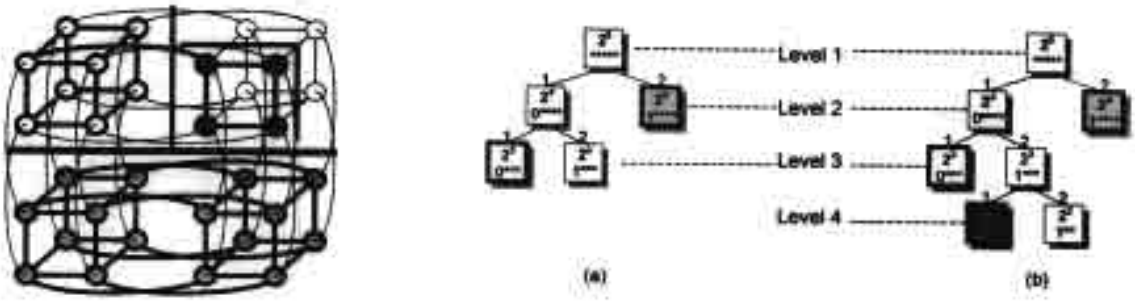


Figure 82: The system status and the corresponding binary-Tree of the allocation of (a) the second task (4-cube) and (b) the third task (2-cube), based on the partitioning and combining by network degree, for a hypercube.

4.2.2 Sub-system (PEs) Allocation and Control Unit (CU) for SIMD Tasks

For the application for SIMD tasks, suppose we have a hypercube (or 5-cube) system of size $N = 2^5$ and a sequence of three incoming SIMD tasks (3-cube, 4-cube, and 2-cube), which come in one at a time. For the hypercube-connected system, the value of n is 2 and thus we apply Method 1 (the partitioning and combining by network degree).

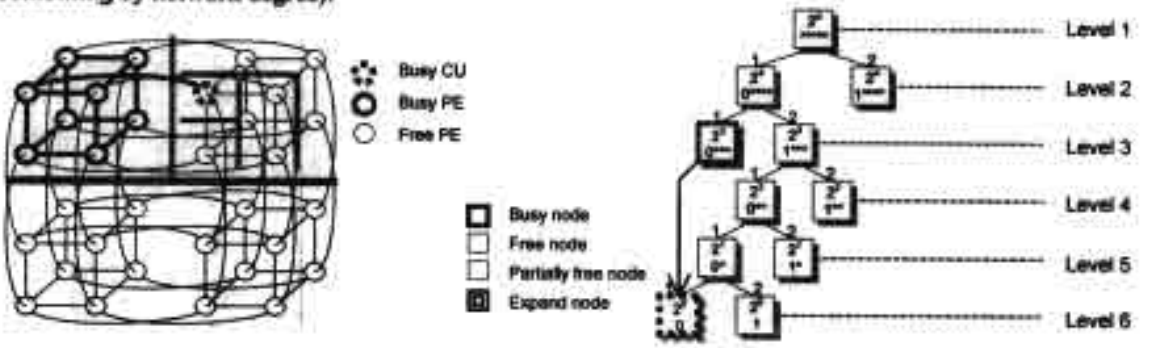


Figure 83: The system status and the corresponding binary-Tree of the allocation of the first SIMD task (3-cube), based on the partitioning and combining by network degree, for a hypercube.

Figure 83 illustrates the system status and the corresponding binary tree that shows the allocation of the first incoming SIMD task (3-cube). For this task, the root node of the tree is created (starting at level 1) to store the system information (i.e., size = 2^5, based address = <1,1,1,1,1>, status = 0). Initially, we have only one free node in the tree and hence it is the best one (from applying the best-fit heuristic (Step 1 - 3)). For the final step (Step 4), the partitioning process is applied (twice for the 3-cube) and for the first task, we always select the first buddy node (at level 3) for the sub-system (PEs) allocation. Then we perform another partition process on its buddy, the buddy#2 (at level 3), for the CU allocation. After the allocation, we will apply the combining process to the corresponding nodes of the sub-sequence partitioning if the buddy of its root is partially free (or some of its buddy nodes are busy). In this case, we do not have to apply the combining process since the buddy#2 is free. Finally, the first maximum free size is the buddy#2 (4-cube) at level 2.

Figure 84 illustrates the system status and the corresponding binary tree that shows the allocation of the second SIMD task (4-cube) and the third SIMD task (2-cube), respectively. For the task (4-cube), the searching starts from the root and goes to first free node (see Figure 83), which is the buddy#2 (1 PE) at level 6 but its size cannot accommodate to the task. The searching then visits the next free nodes, which are the buddy#2 (1-cube) at level 5, the buddy#2 (2-cube) at level 4, the buddy#2 (3-Cube) at level 5 but their sizes cannot accommodate to the task (4-cube). Then, the last free node is the buddy#2 at level 2. The best-fit value of this node (Step 1-3) is computed (i.e., preserve maxFS = F, diffSF = 0, size = 32, CF = ½) and it is recorded as the first best node. For CU searching for this node, the best adjacent node is the buddy#2 at level 6 if we apply the CU-DFS or the CU-AS strategy. After all free nodes are visited, then Step 4 of the best-fit heuristic is applied to the best node (from Step 1-3) if its size is larger than the task.

In this case, we do not apply the partitioning process since the node's size is equal to the task's size (4-cube) and hence no need for the combining process (see Figure 84.a). Next since the maximum free size (4-cube), the buddy#2 at level 2, is allocated, we have to find the new maximum free size by using DFS to visit all free nodes in the tree. Now, the maximum free size is the buddy#2 (2-cube) at level 4. For the third task (2-cube), the searching for the best node for the sub-system (PEs) allocation and the corresponding CU for the CU allocation is similar to that applied for the second task (see Figure 84.b).

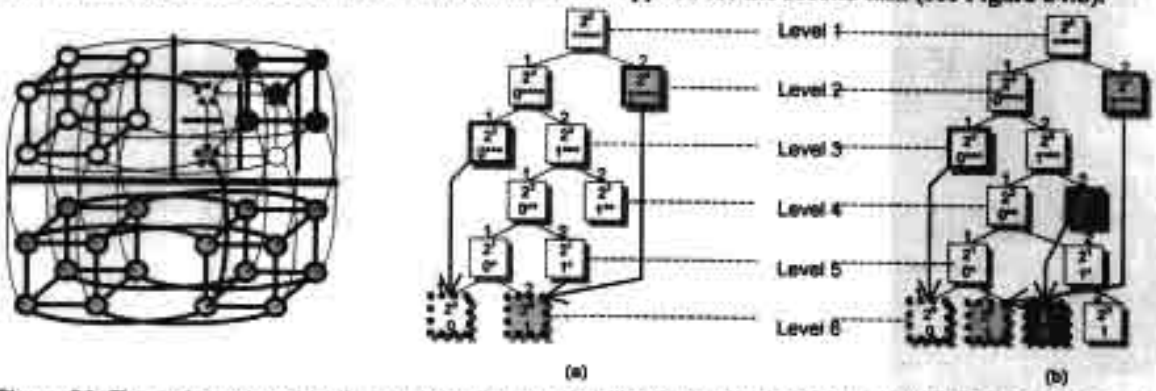


Figure 84: The system status and the corresponding binary-Tree of the allocation of a) the second SIMD task (4-cube) and b) the third SIMD task (2-cube), based on the partitioning and combining by network degree, for a hypercube.

5. PERFORMANCE EVALUATION

In order to evaluate the system performance, the universal tree-based resource (CU/PE) allocation approach was developed and applied for two network applications, which are the 2-D meshes and the 3-D meshes. These two interconnection networks are efficient for the reconfigurable and partitionable systems since they provide small node degrees (or links), low cost per node, and hence low system cost of links. Therefore, in the partitioning process we have not to cut many links in order to form a partition. For example, there are three values of the node degrees of the 2-D meshes: 1) node degree = 4 for each internal node, 2) node degree = 3 for each border node, and 3) node degree = 2 for each corner node (see Figure 85.a). For the 3-D meshes, there are four values of the node degrees, which are 6, 5, 4, and 3 for each inner node, each side node, each border node, and each corner node, respectively (see Figure 85.b).

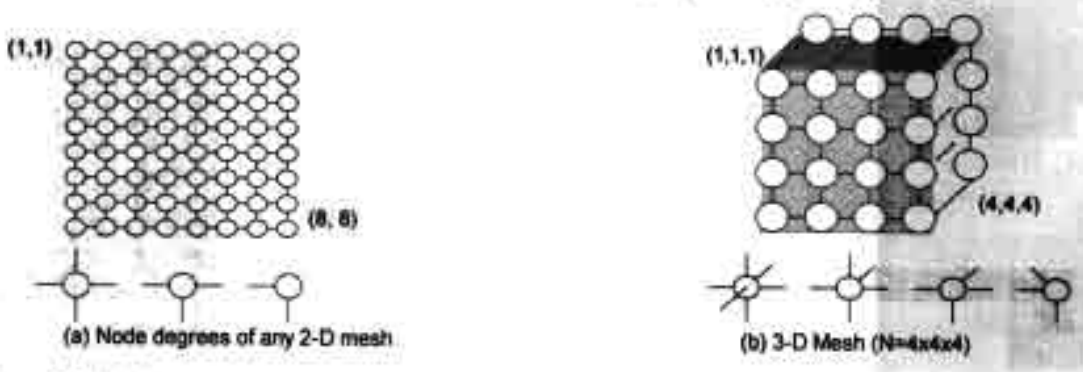


Figure 85: Node degrees: a) at most 4 for the 2-D meshes and b) at most 6 for the 3-D meshes.

By simulation study, a number of experiments are performed to investigate the effect of applying the "universal tree-based resource (CU/PE) allocation model" for performing processor allocation/deallocation for those reconfigurable and partitionable mesh-connected systems. The system performance is investigated in terms of system utilization, system fragmentation, etc. Note that the system utilization (U_{sys}) is measured as a summation of the busy processors (allocated for tasks) over the number of processors in the system, computed when the system reaches the steady state. Similarly, the system fragmentation (F_{sys}) is measured as a summation of the free processors over the total number of processor in the system, also computed when the system reaches the steady state, where $F_{sys} = 1 - U_{sys}$ (if there is no internal fragmentation.)

For those 2-D and 3-D mesh-connected systems (where k = 2 and 3) the value of k (or the number of dimensions) is very small and hence we can apply either Method 2 (the partitioning and combining by network size) and sometime later is called "the modified k-Tree-based resource allocation strategy", or Method 3 (the partitioning and combining by network degree and size), called "the b inary-tree-based resource allocation strategy". In particular, for the reconfigurable and partitionable 2-D and 3-D mesh-connected systems (for both MIMD and SIMD tasks), we introduces the comparison results between the modified k-Tree-based resource allocation strategy and the binary-tree-based resource allocation strategy. Also for the partitionable 2-D mesh-connected systems (for only MIMD tasks) we presents the comparison results of our universal tree-based resource (PE) allocation model (when applied to the 2-D meshes (in $O(N_A + N_F)$ time)) to recently 2-D mesh-based strategies, which are the BUSY LIST strategy ($O(N_s^3)$) [14], the FREE SUB-LIST strategy ($O(N_f^3)$) [26], and the QUICK ALLOCATION strategy ($O(N_s\sqrt{N})$) [50]. When considering time complexity, our tree-based approach perform the processor allocation decision in linear time (of the number of allocated tasks (N_A) and corresponding free nodes (N_F)), which is efficient, compared to those ($O(N_s^3)$, $O(N_f^3)$, and , $O(N_s\sqrt{N})$ of the above existing methods, where $N_A \approx N_s$ (see system performance results) and $N_F < N_f$ (since our model stores only non-overlap free nodes while N_f includes overlap free sub-systems in the free lists.)

For each experiment, a number of simulation time units are iterated around 5,000-50,000 time units and a number of incoming tasks are generated approximately 1,000-10,000 tasks, according to the setting of the system size parameter, the task size (i.e., row, column) parameter and the task size's distribution. For each evaluated result, a number of different data sets are generated and the algorithm is repeated until an average system performance does not change (or at least 100 iterations). Experimental results of applying the universal resource (CU/PE) allocation model are represented for the static system performance (with concerning processor allocation for incoming tasks (or jobs) only (or assumed that no task finishes during the considering time)) and the dynamic system performance (with taking into account of deallocation for some finished tasks). In this study, in order to set the same incoming tasks and environment to all strategies for the comparison purpose, the static system performance is concerned (i.e., when we measure the system utilization and system fragmentation); otherwise the dynamic system performance is concerned. In each experiment, two task-size distributions are considered: the Uniform distribution $U(\alpha, \beta)$ and the Normal distribution $N(\mu, \sigma)$. For each of these distributions, the system sizes ($N = n_1 \times n_2$) are varied and the task sizes [1x1 - $n_1 \times n_2$] are generated, where $\alpha = 1$, $\beta = \max(n_1, n_2)$ for the Uniform distribution $U(\alpha, \beta)$ and $\mu = \sigma = \max(n_1, n_2)/2$ for the Normal distribution $N(\mu, \sigma)$. Other parameters are fixed such as task arrival rate ~ Poisson (λ) (or inter-arrival time ~ Exp($1/\lambda$=5)), and service time ~ Exp(μ=10), etc.

5.1 System Performance Evaluation for the Partitionable 2-D Meshes (only MIMD Tasks)

In order to set the same incoming tasks and environment to all allocation strategies for the comparison purpose, we assumed that no task finishes during the considering time. Therefore, in this study N_A represents the maximum number of allocated tasks in the system and N_F represents the corresponding free nodes in the tree.

5.1.1 Investigate the Effect of System Sizes to the System Performance

In the first experiment, we investigated "the effect of system sizes (N) to the system utilization (U_{sys}) and the system fragmentation (F_{sys})" for the 2-D meshes, executing only MIMD tasks. In this experiment, the system sizes ($N = n_1 x n_2$) were varied and the task sizes (1x1 - $n_1 x n_2$) were generated and fixed by using the Uniform distribution (see Table 9 and Table 11) or the Normal distribution (see Table 10 and Table 12)).

Table 9: Effect of "the system sizes" to the system utilization (%) for the Uniform distribution.

System Sizes ($N = n_1 x n_2$)	Binary-Tree	k-Tree	Free Sub-List	Busy List	Quick Allocation
64x64	60.75	60.97	56.06	57.27	55.64
128x128	61.63	60.11	56.98	56.67	57.84
256x256	60.30	59.77	55.77	54.61	56.87
512x512	59.36	58.86	56.11	55.91	56.53

Table 9 illustrates the results of the system utilization for the Uniform distribution (see also Figure 86). For all test cases our binary-tree strategy performed approximately 60 - 61% system utilization which was comparable to those of the k-tree strategy and was improved over those of the recently 2-D mesh-based strategies, which were at most 57% system utilization.)

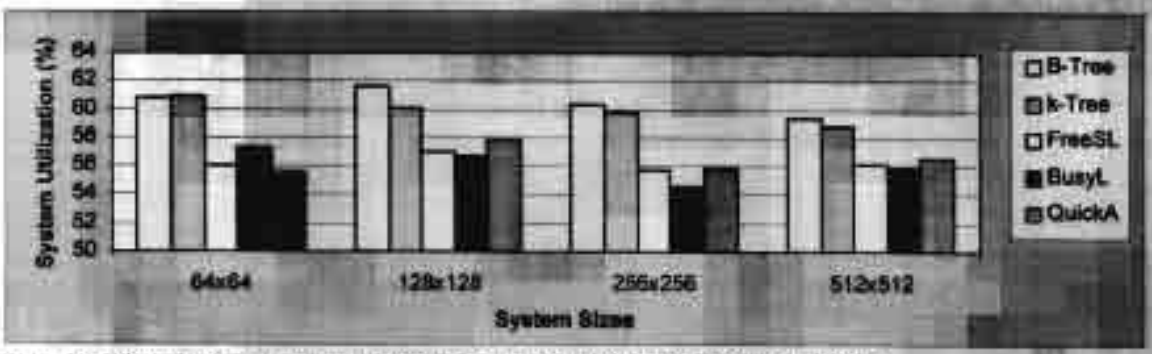


Figure 86: Effect of "the system sizes" to the system utilization (%) for the Uniform distribution.

Table 10 illustrates the results of the system utilization for the Normal distribution (see also Figure 87). For the system size N = 64x64, the binary-tree, the k-tree, and the free sub-list strategies yielded the comparable results (~61% system utilization) and were improved over those (~58% system utilization) of the busy list and the quick allocation strategies.

For the system size N = 512x512, our binary-tree strategy performed 62.2% system utilization which was improved over that (61.3%) of the k-tree strategy and those (57.0%, 57.2%, and 55.9%) of the recently 2-D mesh-based strategies (the free sub-list, the busy list, and the quick allocation strategies.)

For other test cases (N = 128x128 and 256x256), our binary-tree strategy performed approximately 59 - 60% system utilization, which was comparable to those of the k-tree strategy and improved over those of the recently 2-D mesh-based strategies, which were at most 58% system utilization.)

Table 10: Effect of "the system sizes" to the system utilization (%) for the Normal distribution.

System Sizes ($N = n_1 x n_2$)	Binary-Tree	k-Tree	Free Sub-List	Busy List	Quick Allocation
64x64	61.05	61.18	61.42	58.46	57.53
128x128	59.34	59.16	56.89	58.02	54.79
256x256	59.41	60.10	58.02	55.12	55.09
512x512	62.20	61.28	57.04	57.22	55.89

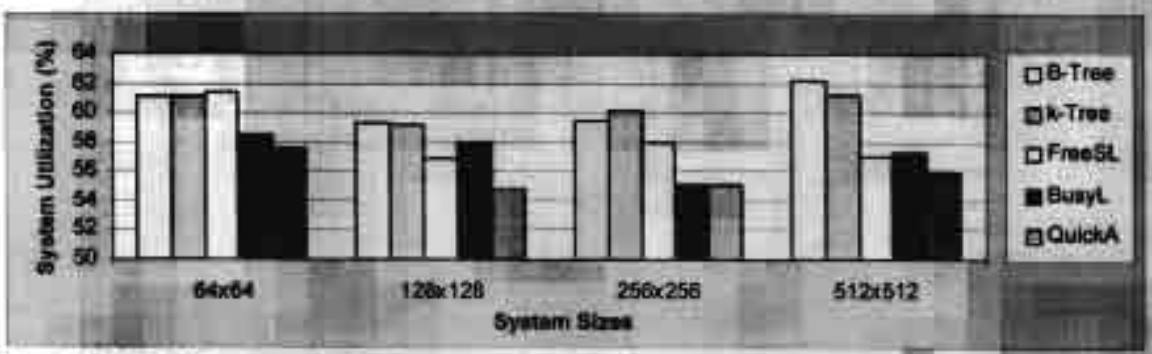


Figure 87: Effect of "the system sizes" to the system utilization (%) for the Normal distribution.

Table 11 and Figure 88 illustrate the results of the system fragmentation for the Uniform distribution. For all test cases our binary-tree strategy performed approximately 39 - 40% system fragmentation which was comparable to those of the k-tree strategy and was improved over those of the recently 2-D mesh-based strategies, which were approximately 42 - 44% system fragmentation.

Table 11: Effect of "the system sizes" to the system fragmentation (%) for the Uniform distribution.

System Sizes ($N = n_1 x n_2$)	Binary-Tree	k-Tree	Free Sub-List	Busy List	Quick Allocation
64x64	39.25	39.03	43.94	42.73	44.36
128x128	38.37	39.89	43.02	43.33	42.16
256x256	39.70	40.23	44.23	45.39	44.13
512x512	40.64	41.14	43.89	44.09	43.47

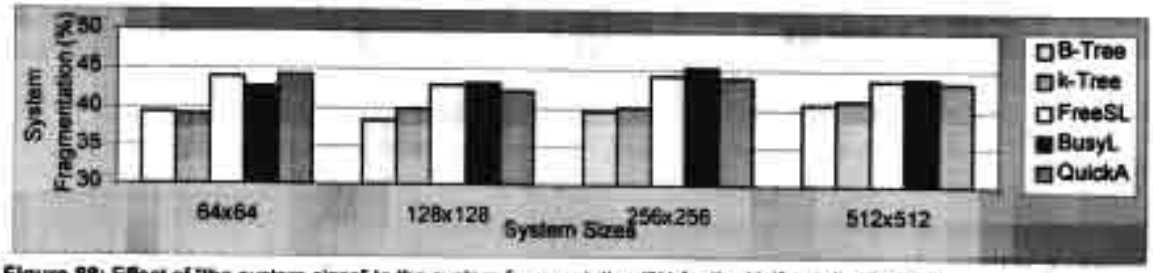


Figure 88: Effect of "the system sizes" to the system fragmentation (%) for the Uniform distribution.

Table 12 and Figure 89 illustrate the results of the system fragmentation for the Normal distribution. For N = 64x64, the binary-tree strategy performed approximately 38.9% system fragmentation, which was comparable to those (38.8% and 38.6%) of the k-tree and the free sub-list strategies and was improved over those (41.5% and 42.5%) of the busy list and the quick allocation strategies. For other test cases (N = 128x128 and 256x256), our binary-tree strategy performed approximately 40 - 41% system fragmentation, which was comparable to those of the k-tree strategy and improved over those (42 - 45%) of the recently 2-D mesh-based strategies (the free sub-list, the busy list, and the quick allocation strategies.)

Table 12: Effect of "the system sizes" to the system fragmentation (%) for the Normal distribution.

System Sizes ($N = n_1 x n_2$)	Binary-Tree	k-Tree	Free Sub-List	Busy List	Quick Allocation
64x64	38.95	38.82	38.58	41.54	42.47
128x128	40.66	40.84	43.11	41.98	45.21
256x256	40.59	39.90	41.98	44.86	44.91
512x512	37.80	38.72	42.96	42.78	44.11

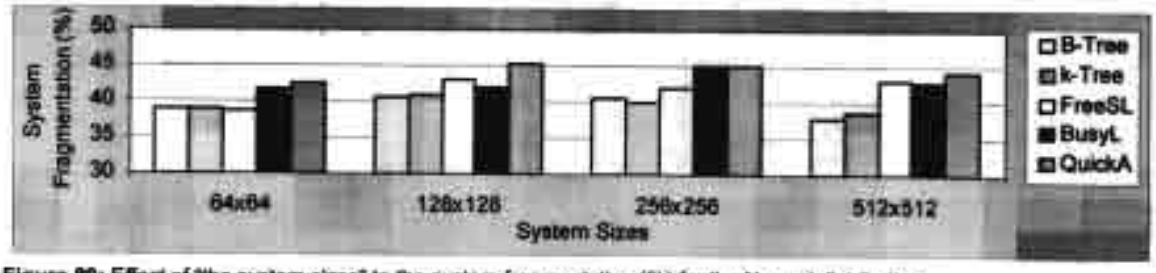


Figure 89: Effect of "the system sizes" to the system fragmentation (%) for the Normal distribution.

In summary, the varying the system sizes (and the generating the task sizes in $1x1 - n_1 \times n_2$) do not effect to the system performance (i.e., U_{sys} and F_{sys}) for all allocation methods since each strategy tends to yield the results, effected by the method itself for all system sizes. In addition, the effect of task sizes generated by using the Uniform or Normal distributions tends to be the same. Therefore, in the next investigation, we will show the results of the Uniform distribution only.

5.1.2 Investigate the Effect of Task Sizes to the System Performance

In the second experiment, we investigated "the effect of task sizes to the system utilization (see Table 13) and the system fragmentation (see Table 14)". In this experiment, the system size was fixed ($N = n_1 \times n_2 = 512x512$) and the task sizes were generated and varied (by using the Uniform distribution) in various ranges (i.e., the "large" range ($1x1 - n_1 \times n_2$), the "medium" range ($1x1 - {}^{n1}/_2 \times {}^{n2}/_2$), and the "small" range

(1x1 - $^{n1}/_4$ x $^{n2}/_4$)). Table 13 and Figure 90 illustrate the results of the system utilization. For the small range of task sizes (1x1 - 128x128 or 1x1 - $^{n1}/_4$ x $^{n2}/_4$), the binary-tree, the free sub-list, and the busy list allocation strategies performed the comparable system utilization, which were 81.1%, 82.6%, and 82.6%, respectively and were improved over those (79.1%, 80.1%) of the k-tree and the quick allocation strategies. For the medium range of task sizes (1x1 - 256x256 or 1x1 - $^{n1}/_2$ x $^{n2}/_2$), the top three strategies that performs the best system utilization were are the busy list (73.4%), free sub-list (71.1%), and the binary-tree (69.2%), which were improved over those (68.3% and 66.3%) of the k-tree and quick allocation strategies. For the large range of task sizes (1x1 - 512x512 or 1x1 - n_1 x n_2), the binary-tree and k-tree strategies yielded the best comparable system utilization (59.3% and 58.6%), which were improved over those (56.11%, 55.9%, and 56.5%) of the free sub-list, the busy list, and the quick allocation strategies, respectively.

Table 13: Effect of "the task sizes" to the system utilization (%) for the Uniform distribution.

Task Sizes (1x1 - rxc)	Binary-Tree	k-Tree	Free Sub-List	Busy List	Quick Allocation
1x1 - 128x128	81.08	79.12	82.57	82.62	80.15
1x1 - 256x256	69.20	66.29	71.11	73.38	66.31
1x1 - 512x512	59.36	58.86	58.11	55.91	56.53

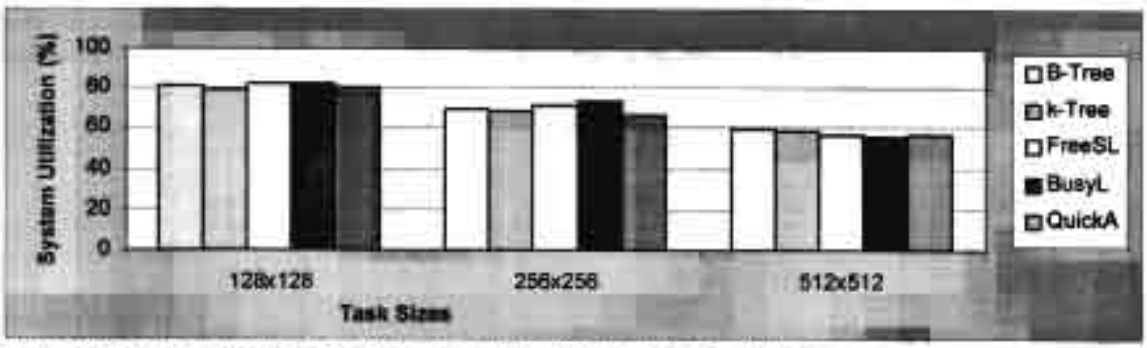


Figure 90: Effect of "the task sizes" to the system utilization (%) for the Uniform distribution.

Table 14 and Figure 91 illustrate the results of the system fragmentation, the tree-based approach and those existing 2-D mesh-based strategies also tended to perform the same effecting results similar to the system utilization since $U_{sys} = 1\text{-}F_{sys}$ (or since there was no internal system fragmentation).

Table 14: Effect of "the task sizes" to the system fragmentation (%) for the Uniform distribution.

Task Sizes (1x1 - rxc)	Binary-Tree	k-Tree	Free Sub-List	Busy List	Quick Allocation
1x1 - 128x128	18.92	20.88	17.43	17.38	19.85
1x1 - 256x256	30.80	31.71	28.89	26.62	33.69
1x1 – 512x512	40.64	41.14	43.89	44.09	43.37

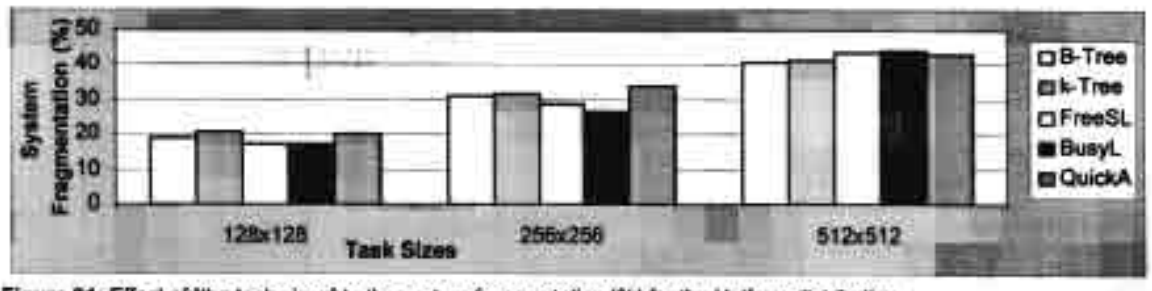


Figure 91: Effect of "the task sizes" to the system fragmentation (%) for the Uniform distribution.

In summary, the varying of task sizes on the fixed system size (N = n_1 x n_2) is effecting to the system performance (i.e., U_{sys} and F_{sys}). When generating range of task sizes increases (small - large), the system utilization increases while the system fragmentation decreases. For the small and medium ranges, the top ranking are the busy list, the free sub-list, and the binary-tree strategies respectively, whereas for the large range the top ranking are the binary-tree, the k-tree, and the busy list strategies, respectively.

5.2 System Performance Evaluation for the Reconfigurable and Partitionable 2-D Meshes and 3-D Meshes (both MIMD / SIMD Tasks)

In order to set the same incoming tasks and environment to the CU allocation strategies (the CU-DFS and CU-AS) for the comparison purpose, we assumed that no task finishes during the considering time.

In the last experiment, we investigated the effect of system sizes (N) to the system utilization (U_{sys}), where the system sizes ($N = n_1 \times n_2$) were varied and the task sizes ($1x1 - {}^{n1}/_2 \times {}^{n2}/_2$) were generated and fixed. For all test cases the CU-AS and CU-DFS strategies performed the same system utilization. The reason is that the system performance results of these two methods were different only when sub-system (S) and task (T) sizes were equal which rarely occurred.

Table 15 and Figure 92 illustrate the results of the system utilization of the modified k-tree strategy for the 2-D and 3-D mesh-connected systems when the percentage of the SIMD tasks were increased (such as 0%, 10%, 20%). The results showed that increasing the percentage of SIMD tasks did not effect the system utilization for the 2-D and the 3-D meshes, except when N = 64x64.

Table 15. Effect of the "system sizes" to the system utilization (%) for the 2-D and 3-D Meshes.

2-D Mesh				3-D Mesh			
% of SIMD Tasks mixed with MIMD Tasks							
System Sizes ($N = n_1 x n_2$)	0%	10%	20%	System Sizes ($N = n_1 x n_2 x n_3$)	0%	10%	20%
64x64	68.76	68.46	71.91	32x32x32	58.14	58.55	58.60
128x128	68.25	67.82	67.62	64x64x64	49.73	54.32	54.33
256x256	70.16	70.16	70.16	128x128x128	50.52	50.52	50.52

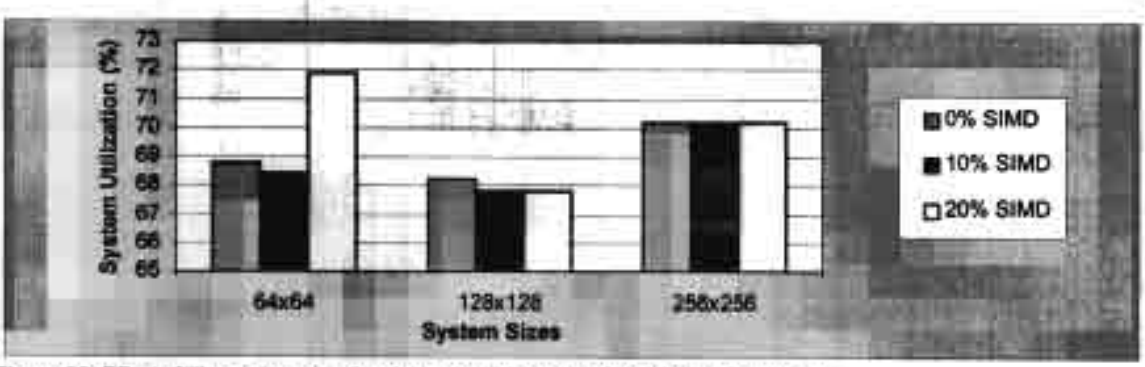


Figure 92: Effect of "the system sizes" to the system utilization (%): a) for the 2-D Meshes.

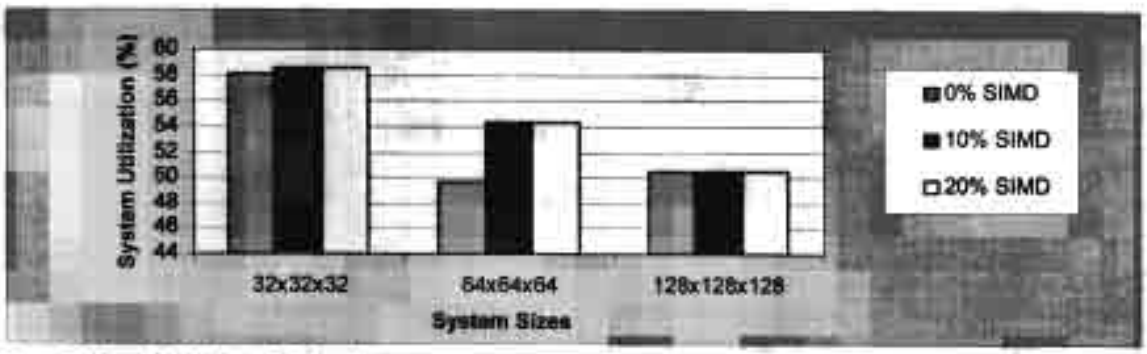


Figure 92 (cont.): b) for the 3-D Meshes.

Table 16 and Figure 93 illustrate the results of the system utilization of the binary-tree and the k-tree strategies for the 2-D and 3-D meshes. For the 2-D meshes, the results showed that the binary-tree strategy yielded the improved system utilization over that the k-tree strategy, except when N = 64x64. For the 3-D meshes, both binary-tree and k-tree strategies yielded the comparable system utilization in all test cases.

Table 16: Effect of "the system sizes" to the system utilization (%) for the 2-D and 3-D Meshes.

2-D Mesh			3-D Mesh		
System Sizes ($N = n_1 x n_2$)	Binary-Tree	k-Tree	System Sizes ($N = n_1 x n_2 x n_3$)	Binary-Tree	k-Tree
64x64	60.75	60.97	32x32x32	47.24	47.71
128x128	61.63	60.11	64x64x64	44.46	44.28
256x256	60.30	59.77	128x128x128	43.30	42.53

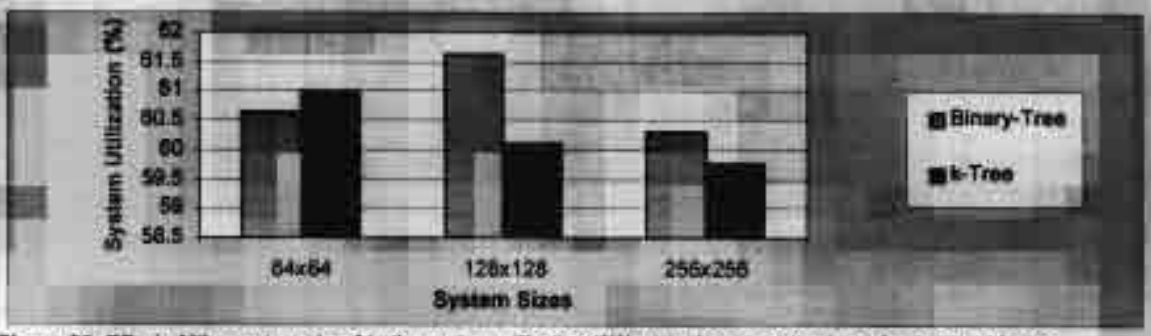


Figure 93: Effect of "the system sizes" to the system utilization (%)by tree-based methods: a) for the 2-D Meshes.

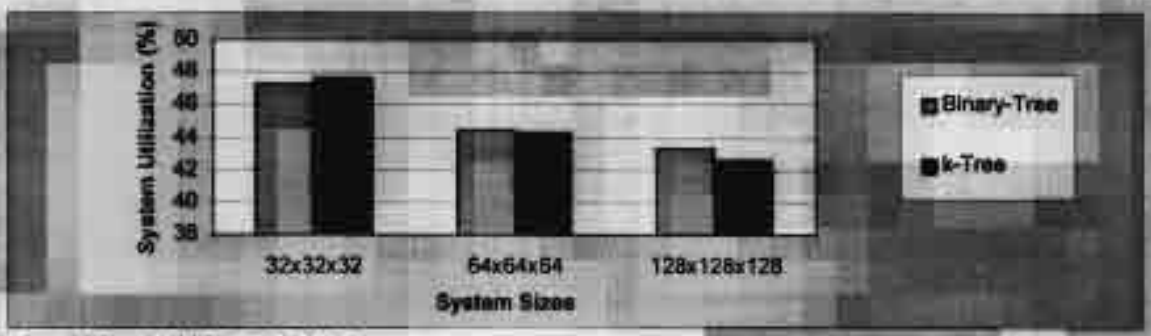


Figure 93 (cont.): b) for the 3-D Meshes.

Table 17 and Figure 94 illustrate the results of the system fragmentation of the modified k-tree strategy for the 2-D and 3-D meshes when the percentage of the SIMD tasks were increased (such as 0%, 10%, 20%). The results showed that increasing the percentage of having SIMD tasks in the system did not effect the system fragmentation for the 2-D and the 3-D meshes, except when N = 64x64. The results were similar to the system utilization since $U_{sys} = 1\text{-}F_{sys}$ (or since there was no internal system fragmentation).

Table 17. Effect of the "system sizes" to the system fragmentation (%) for the 2-D and 3-D Meshes.

2-D Mesh				3-D Mesh			
	% of SIMD Tasks mixed with MIMD Tasks						
System Sizes ($N = n_1 x n_2$)	0%	10%	20%	System Sizes ($N = n_1 x n_2 x n_3$)	0%	10%	20%
64x64	31.24	31.54	28.09	32x32x32	41.86	41.45	41.40
128x128	31.75	32.18	32.18	64x64x64	50.27	45.68	45.67
256x256	29.84	29.84	29.84	128x128x128	49.48	49.48	49.48

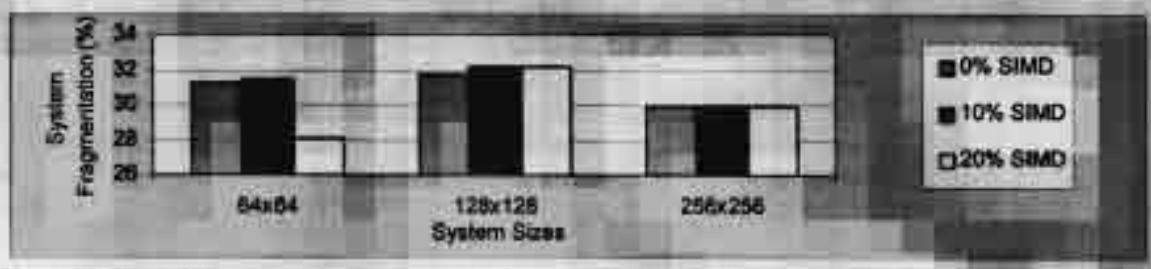


Figure 94: Effect of "the system sizes" to the system fragmentation (%): a) for the 2-D Meshes.

Figure 94 (cont.): b) for the 3-D Meshes.

Table 18 and Figure 95 illustrate the results of the system fragmentation of the binary-tree and the k-tree strategies for the 2-D and 3-D meshes. The results showed that the binary-tree strategy yielded the improved system utilization over that the k-tree strategy, except when N = 64x64 they are comparable.

Table 18: Effect of "the system sizes" to the system fragmentation (%) for the 2-D and 3-D Meshes.

2-D Mesh			3-D Mesh		
System Sizes ($N = n_1 x n_2$)	Binary-Tree	k-Tree	System Sizes ($N = n_1 x n_2 x n_3$)	Binary-Tree	k-Tree
64x64	39.25	39.03	32x32x32	52.76	52.39
128x128	38.37	39.89	64x64x64	55.54	55.72
256x256	39.70	40.23	128x128x128	56.70	57.47

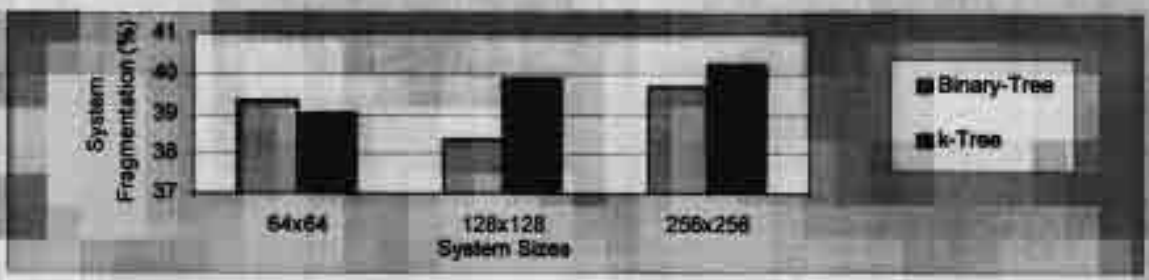


Figure 95: Effect of "the system sizes" to the system fragmentation (%) by tree-based methods: a) for the 2-D Meshes.

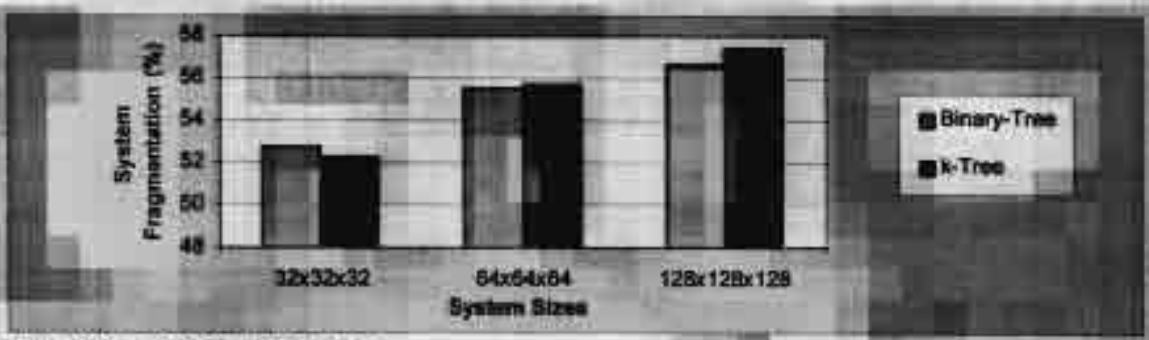


Figure 95 (cont.): b) for the 3-D Meshes.

6. CONCLUSION AND FUTURE STUDY

This study introduces "a universal tree-based model" to perform dynamic resource (CU/PE) allocation decision for the reconfigurable and partitionable MSIMD/MIMD parallel systems. Our universal resource (CU/PE) allocation model can be applied to all interconnection networks in the product-network class such as multi-dimensional meshes, multi-dimensional tori, hypercubes, n-ary k-cubes, etc. Since these reconfigurable systems can execute various dynamic tasks (with MIMD and SIMD modes) in different partitions during run time, we present the new binary-tree-based approach for MIMD and SIMD tasks in efficient time. Moreover, we provide the modified k-tree-based approach to be more useful by adding the CU allocation to cover SIMD partitions for the reconfigurable and partitionable MSIMD/MIMD parallel systems. Time complexity of the tree-based universal model (for MIMD tasks) is efficient for any k-D product-network systems, compared to the original k-tree-based approach and also efficient when applied to the 2-D mesh systems, compared to the recently 2-D mesh-based processor allocation strategies (i.e., the free sub-list strategy, the busy list strategy, and the quick allocation strategy). The total time complexity (for CU/PE allocation) depends on the time complexity of integrating the CU allocation method into the system. Therefore, we also introduces three best-fit heuristics for the tree-based CU allocation: 1) the CU-DFS strategy in $O(N_A + kN_F + k^2)$ time and 2) the CU-AS strategy in $O(k^2)$ time and 3) the CU-IS strategy in $O(1)$ time. Finally, we perform many experiments to investigate system performance of applying our new binary tree-based (CU/PE) allocation model for the reconfigurable and partitionable 2-D and 3-D meshes. By simulation study, the results showed that our binary-tree strategy yielded the comparable system utilization and system fragmentation to those by the k-tree strategy and improved over those by the k-tree strategy in some cases. For the 2-D partitionable meshes, our binary-tree-based results and modified k-tree-based results were also comparable to those of the recently 2-D mesh-based strategies (i.e., the free sub-list strategy, the busy list strategy, and the quick allocation strategy) and improved over those of the recently strategies for some test cases in more efficient time.

In the future study, we will apply our universal and general resource allocation model to some practical applications in parallel and distributed computing, high performance computing, and super computing.

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APPENDIX A

OUTPUT

1. ผลงานที่คาดว่าตีพิมพ์

1. J. Srisawat, W. Surakampontorn, and N.A. Alexandridis, **"A New Binary-Tree-Based Subsystem Allocation for Partitionable k-D Mesh Multicomputers,"** (expected to be published in Journal of Parallel and Distributed Computing in 2004 - 2005.)

2. J. Srisawat, W. Surakampontorn, and N.A. Alexandridis, **"A Universal CU and PE Allocation for Dynamic Reconfigurable MSIMD/MIMD Parallel Systems,"** (expected to be published in IEEE Transactions on Parallel and Distributed Systems in 2004 - 2005.)

2. การนำผลงานวิจัยไปใช้ประโยชน์

ผลงานวิจัยในขั้นนี้ สามารถนำไปใช้ประโยชน์ในเชิงวิชาการ ในด้านการพัฒนาการเรียนการสอนในระดับปริญญาโท และปริญญาโทเอก

3. การเสนอผลงานในที่ประชุมวิชาการ

J. Srisawat, W. Surakampontorn, and N.A. Alexandridis, **"A k-Tree-Based Resource (CU/PE) Allocation for Reconfigurable MSIMD/MIMD Multi-dimensional Mesh-connected Architectures,"** *in Proceeding of the 2002 International Technical Conference on circuits/systems, Computers and Communications*, v. 1, pp. 58-62, Phuket, Thailand, July 2002.

APPENDIX B

REPRINT PUBLICATION

Title: A k-Tree-Based Resource (CU/PE) Allocation for Reconfigurable MSIMD/MIMD Multi-dimensional Mesh-connected Architectures

Authors: Jeeraporn Srisawat, Wanlop Surakampontorn, and Nikitas A. Alexandridis

Proceeding: In Proceeding of the 2002 International Technical Conference on Circuits/Systems, Computers and Communications, v. 1, pp. 58-62, Phuket, Thailand, July 2002.

A k-Tree-Based Resource (CU/PE) Allocation for Reconfigurable MSIMD/MIMD Multi-Dimensional Mesh-Connected Architectures*

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Abstract: In this paper, we present a new generalized k-Tree-based (CU/PE) allocation model to perform dynamic resource (CU/PE) allocation/deallocation decision for the reconfigurable MSIMD/MIMD multi-dimensional (k-D) mesh-connected architectures. Those reconfigurable multi-SIMD/MIMD systems allow dynamic modes of executing tasks, which are SIMD and MIMD. The MIMD task requires only the free sub-system; however the SIMD task needs not only the free sub-system but also the corresponding free CU. In our new k-Tree-based (CU/PE) allocation model, we introduce two best-fit heuristics for the CU allocation decision: 1) the CU depth first search (CU-DFS) in $O(kN_F)$ time and 2) the CU adjacent search (CU-AS) in $O(k2^k)$ time. By the simulation study, the system performance of these two CU allocation strategies was also investigated. Our simulation results showed that the CU-AS and CU-DFS strategies performed the same system performance when applied for the reconfigurable MSIMD/MIMD 2-D and 3-D mesh-connected architectures.

1. Introduction

A partitionable multicomputer is a special type of parallel systems that provides (at run time) for executing various independent parallel/distributed applications (or tasks) on different sub-systems in parallel. For this system, each of these tasks requests an MIMD mode. The more flexible partitionable parallel system, called the reconfigurable multi-SIMD/MIMD system, provide independent sub-systems for the requested tasks, executing in the SIMD and MIMD modes. At run time some tasks may call the SIMD mode (which is good at synchronization and communication) whereas some tasks may need to execute independent branching or different instructions (which are suitable for the MIMD mode). Therefore, the dynamic reconfigurable MSIMD/MIMD parallel architecture has become increasingly important for the parallel and distributed computing environment. The SPP MSIMD/MIMD architecture [1] is the flexible design of reconfigurable MSIMD/MIMD systems since the roles of its processors, called CPEs (control processor elements), will be assigned to be either PE or CU at run time. This architecture performs dynamic reconfiguration at the network level (for any independent SIMD/MIMD task) not at the instruction level (that reconfiguring by altering the connections between the PEs in order to match the task graph or particular algorithm.)

In such a reconfigurable MSIMD/MIMD environment, a number of independent tasks (of the same or different applications) come in. Each of these tasks requires (at run time) a separate sub-system (or partition) to execute in either the SIMD mode or the MIMD mode. At the front-end computer, a special designed OS (known as the resource allocator and task scheduler) will provide appropriate free sub-systems for the new incoming task(s). In particular, that OS has to dynamically find the location of a free sub-system in order to allocate for each incoming task, as well as to deallocate a busy sub-system and recombine partitions as soon as they become available when a task completes. In the reconfigurable MSIMD/MIMD systems, the requested MIMD mode requires only the free sub-systems but the requested SIMD mode needs both the free sub-system and the corresponding free CU.

In the past five years, most existing processor (PE) allocation methods were introduced for partitionable multicomputers to allocate independent tasks, (executing in the MIMD mode) and for specific interconnection networks such as 2-D meshes. Those PE allocation strategies includes FRAME SLIDE [2], BUSY LIST with Scheduling [3], ADAPTIVE SCAN [4], FREE SUB-LIST [5], 2-D BUDDY [6], FREE LIST [7], BIT-MAP with Partition [8], QUAD TREE [9], QUICK ALLOCATION [11], and BIT MAP [12]). All of them were introduced (at the front-end computer) for the partitionable MIMD 2-D mesh-connected multicomputers. For the reconfigurable SPP MSIMD/MIMD architecture [1], the resource (CU/PE) allocation strategy, called bit-map BUDDY, for hypercube networks was also introduced. However, the bit-map BUDDY strategy was handled by the special OS at the back-end MSIMD/MIMD parallel system.

In this paper, we present a new generalized k-Tree-based (CU/PE) allocation model to perform dynamic resource (CU/PE) allocation decision (at the front-end computer) for the reconfigurable MSIMD/MIMD parallel systems, which utilize the multi-dimensional (k-D) mesh interconnection networks. This new generalized k-Tree-based (CU/PE) allocation model is extended from our previous study [10]. That k-Tree-based model was introduced for performing (PE) allocation for the partitionable MIMD k-D mesh-connected systems. Our new model covers the resource (CU/PE) allocation for the reconfigurable MSIMD/MIMD k-D mesh-connected architectures, which allows independent tasks, executing in the MIMD and SIMD modes. In addition, in order to complete the SIMD partition, we introduce two best-fit heuristics for the CU allocation decision: 1) the CU depth first search (CU-DFS) strategy in $O(kN_F)$ time and 2) the CU adjacent search (CU-AS) strategy in $O(k2^k)$ time. With the CU-AS strategy, our k-Tree-based (CU/PE) allocation model yields the same time complexity as that of the MIMD sub-system (PE) allocation in our previous study (when applied to 2-D and 3-D meshes). Finally, the system performance of these two strategies was also investigated and compared (in terms of system utilization and system fragmentation) by the simulation study. In particular, the results of applying our model to the reconfigurable MSIMD/MIMD 2-D and 3-D mesh-connected systems are presented.

Next section illustrates our new generalized k-Tree-based (CU/PE) allocation model to perform the resource allocation/deallocation decision for the reconfigurable multi-SIMD/MIMD k-D mesh-connected architectures. Section 3 presents the evaluated system performance of applying the new k-Tree-based (CU/PE) allocation model for some interconnection networks such as 2-D meshes and 3-D meshes. Finally, conclusion and future study are discussed in Section 4.

* This research was supported by the Thailand Research Fund under Grant PDF44-Jeeraporn Srisawat.

2. k-Tree-Based (CU/PE) Allocation Model for Reconfigurable MSIMD/MIMD k-D Meshes

Our generalized k-Tree-based (CU/PE) allocation model includes a k-Tree system state representation (Section 2.1) and algorithms for network partitioning (Section 2.2), sub-system combining (Section 2.3), best-fit heuristic (Section 2.4), searching for allocation/deallocation decision (Section 2.5). This new generalized k-Tree-based (CU/PE) allocation model is extended from our previous study [10], applied only for the partitionable MIMD k-D meshes, to cover the reconfigurable multi-SIMD/MIMD k-D mesh-connected architectures. In particular, in this paper we introduce two best-fit heuristics for the CU allocation decision for the SIMD task (in section 2.4.2) to complete the SIMD partition in efficient time.

2.1 k-Tree System State Representation

We use a data structure, called a k-Tree to represent system states of the reconfigurable MSIMD/MIMD k-D mesh-connected system. In our k-Tree-based (CU/PE) allocation model, the number of nodes in the k-Tree are dynamic, corresponding to the number allocated tasks. At start, the k-Tree consists of only one (root) node, used to store the system information (i.e., a size, a base-address, a status, etc.) of the initialized system. During run time when many tasks are executing, each leaf node (or a sub-system) may be free (for incoming task(s)) or busy (for executing task(s)) and each internal node is partially available. In order to allocate an incoming task, each larger free node can be dynamically created and partitioned into 2^k buddies/node (see Figure 1). Note: in this new k-Tree-based model, any k-Tree node is modified to include a link to a CU for the SIMD task (see Figure 2).

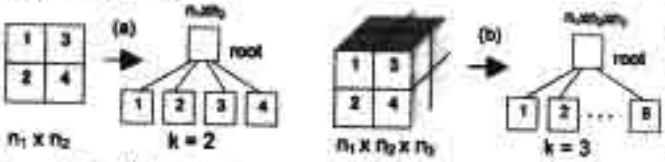


Figure 1. The 2^k buddies of the k-Trees: (a) 2-D mesh; and (b) 3-D mesh

Figure 2 illustrates an example of the system state representation of applying the k-Tree-based (CU/PE) allocation model for allocating three SIMD tasks (of sizes 2x3, 2x2, and 1x5) and two MIMD tasks (of sizes 4x4 and 3x6) on an 8x10-mesh system.

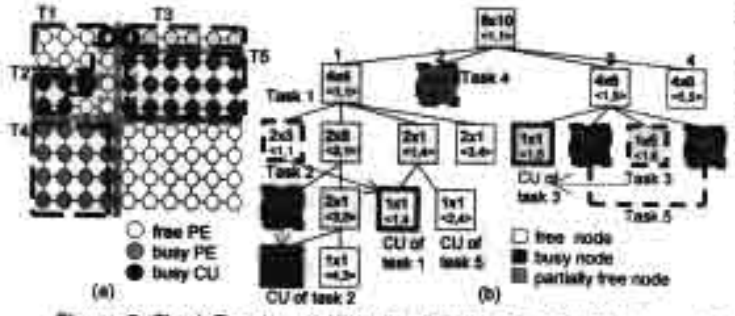


Figure 2. The k-Tree-based (CU/PE) allocation for 3 SIMD tasks and 2 MIMD tasks on an 8x10 mesh system: (a) the allocated system status and (b) the corresponding k-Tree system state representation.

2.2 Network Partitioning

The k-Tree-based network partitioning is the partitioning process that partitions all k dimensions of the k-D system ($N = n_1 x n_2 x \ldots x n_k$) into smaller 2^k sub-systems and allocates an appropriate one for the request (of size $p_1 \times p_2 \times \ldots \times p_k$, where $p_i \le n_i$, $i = 1, 2, \ldots, k$). In this paper, we utilize Buddy-ID-Address-Size-Conversion algorithm of our previous study. This network partitioning process (i.e., identifying #buddies = 2^k, base-addresses, and sizes) is computed in $O(k2^k)$ time (see more detail in [10]). Note: the network partitioning will be applied and modified later (in Section 2.4.2) in order to handle the CU partitioning for a selected sub-system.

2.3 Sub-System Combining

The sub-system combining is applied during processor allocation or deallocation. We also utilize the combinations of 2^j-Adjacent-Buddies algorithm of our previous study in order to combine 2^j buddies (where $j = 1, 2, \ldots, k-1$) in the larger free sub-systems. This algorithm is computed in $O(k2^{k-j})$ time for each j and hence $O(k2^k)$ time for all js (since $k2^1 + k2^2 + k2^3 + \ldots + k2^{k-1} = 2k[2^{k-1}-1]$). The k-Tree-based combining process is classified into four main groups: 1) Combining all buddies, 2) Combining some adjacent buddies, 3) Combining (adjacent) buddy(s) and corresponding adjacent sub-buddies, and 4) Combining some adjacent sub-buddies (see more details in [10]).

2.4 Best-Fit Heuristic

2.4.1 Best-fit Heuristic for PE Allocation

The best-fit heuristic is to find the likely best free sub-system for an incoming task. For PE allocation decision, we also utilize the generalized best-fit heuristic [10] for the partitionable k-D meshes.

Best-Fit Criteria:
1. Find all free Ss that can preserve the "max free size" [*O(k) time*].
2. If many Ss have property (1), S that gives "min different size factor (diffSF)" is selected [*$O(k^2)$ time*].
3. If many Ss have (1)&(2), the "smallest size" S yielding "min combining factor (CF)" is selected [*O(k) time*]. Otherwise, select by random.
4. After all nodes are visited,
   - If the best S = request, then it is directly allocated to the request.
   - Otherwise it is partitioned and one of its buddies that yields "min modified CF (MCF)" will be selected [*$O(k2^k)$ time*].

Note: Criteria 1-3 are applied for every free node and combined S. But criterion 4 is computed only once for the best free S of Steps 1-3.

2.4.2 Best-Fit Heuristic for CU Allocation

In the reconfigurable SPP MSIMD/MIMD design [1], CPEs (control processor elements) were added in the system and their roles (CU or PE) are assigned at run time. Therefore, a CU for a selected sub-system (S) can be any CPE that is directly connected to S. First, we introduce a generalized method to identify all possible CUs and their addressing. If the size of the selected sub-system (S) is $m_1 x\ m_2 x \ldots x\ m_k$ at address $\langle(a_1, a_2, \ldots, a_k), (b_1, b_2, \ldots, b_k)\rangle$, then the number of all possible CUs are $2 \sum_{i=1}^{k} m_1 \times m_2 \times \ldots \times m_{i-1} \times 1 \times m_{i+1} \times \ldots \times m_k$. For example, if k = 2 and S = 7 x 8, # possible CUs is 2(1x8 + 7x1) = 30.

In general, for k dimensions of S there are 2k (outside) sub-systems of CUs (CUSs). CUSs' addressing are defined as follows: for each dim i (of size $= m_1 x m_2 x \ldots x 1 x m_{i+1} x \ldots x m_k$), where i=1,2,...,k.
Min CUS address = $\langle(a_1, a_2, \ldots, a_i\text{-}1, \ldots, a_k), (b_1, b_2, \ldots, a_i\text{-}1, \ldots, b_k)\rangle$
Max CUS address = $\langle(a_1, a_2, \ldots, b_i\text{+}1, \ldots, a_k), (b_1, b_2, \ldots, b_i\text{+}1, \ldots, b_k)\rangle$
For example, if k = 2 and S = 7 x 8, addressing of all 30 CUs (or 4 CUSs) for a selected sub-system (S = $m_1 x\ m_2$ = 7x 8) (at $\langle(a_1, a_2), (b_1, b_2)\rangle = \langle(5, 5), (11, 12)\rangle$) are

- For a fixed dim 1,
  - a min CUS (8 PEs): $\langle(\mathbf{a_1\text{-}1}, a_2), (\mathbf{a_1\text{-}1}, b_2)\rangle = \langle(4, 5), (4, 12)\rangle$
  - a max CUS (8 PEs): $\langle(\mathbf{b_1\text{+}1}, a_2), (\mathbf{b_1\text{+}1}, b_2)\rangle = \langle(12,5), (12,12)\rangle$
- For a fixed dim 2,
  - a min CUS (7 PEs): $\langle(a_1, \mathbf{a_2\text{-}1}), (b_1, \mathbf{a_2\text{-}1})\rangle = \langle(5, 4), (11, 4)\rangle$
  - a max CUS (7 PEs): $\langle(a_1, \mathbf{b_2\text{+}1}), (b_1, \mathbf{b_2\text{+}1})\rangle = \langle(5,13), (11,13)\rangle$

For any free node R (of size $d_1 \times d_2 \times \ldots \times d_k$) in the k-Tree, there are 2k inside sub-systems at boundary (BSs). BSs' addressing are defined as follows: for each dimension i (each of size $[(d_1\text{-}2) x (d_2\text{-}2) x \ldots x (d_{i-1}\text{-}2) x 1 x d_{i+1} x \ldots x d_k]$), where i = 1, 2, ..., k.
Min BS addr=$\langle(a_1\text{+}1, a_2\text{+}1, \ldots, a_i, a_{i+1}, \ldots, a_k), (b_1\text{-}1, b_2\text{-}1, \ldots, a_i, b_{i+1}, \ldots, b_k)\rangle$
Max BS addr=$\langle(a_1\text{+}1, a_2\text{+}1, \ldots, b_i, a_{i+1}, \ldots, a_k), (b_1\text{-}1, b_2\text{-}1, \ldots, b_i, b_{i+1}, \ldots, b_k)\rangle$
For example, if k = 2 and R = 7 x 8, addressing of all 26 PEs (or 4 BSs) for a free node (R) of size $d_1 \times d_2$ = 7 x 8) (at $\langle(a_1, a_2), (b_1, b_2)\rangle = \langle(5, 5), (11, 12)\rangle$) are

- For a fixed dim 1,
  - a min BS (8 PEs): $\langle(a_1, a_2), (a_1, b_2)\rangle = \langle(5, 5), (5, 12)\rangle$
  - a max BS (8 PEs): $\langle(b_1, a_2), (b_1, b_2)\rangle = \langle(11\_5), (11,12)\rangle$
- For a fixed dim 2,
  - a min BS (5 PEs): $\langle(a_1\text{+}1, \mathbf{a_2}), (b_1\text{-}1, \mathbf{a_2})\rangle = \langle(6, 5), (10, 5)\rangle$
  - a max BS (5 PEs): $\langle(a_1\text{+}1, \mathbf{b_2}), (b_1\text{-}1, \mathbf{b_2})\rangle = \langle(6,12), (10,12)\rangle$

2.4.2.1 The CU Depth First Search (CU-DFS)

The CU depth first search (CU-DFS) is used to find any free node R that is adjacent to the selected sub-system S. The searching starts from the root and goes to the left most (leaf) node. If it is free, it then will be checked whether it is adjacent to S. If so, its best-fit value (Section 2.4.1) is computed. Then, new S and R will be updated if they yield the better best-fit value. The above process is repeated for the next free node (if there exists). Time complexity of the CU-DFS is $O(kN_F)$. In order to identify any boundary free PEs of a free node whether or not it is adjacent to the selected sub-system S, we define the adjacent status in O(k), as follows:

Let S_1 is an selected sub-system S with expanded boundary (Figure 3)
S_2 is a free node, $j = 1, 2$.
$|S_i|$ is $n_1 \times n_2 \times \ldots \times n_k$ at $<(a_1, a_2, \ldots, a_k), (b_1, b_2, \ldots, b_k)>$, $j=1,2, \ldots, k$.
Then, S_1 is adjacent to S_2 if they are not disjoint and are different only one bit. "Disjoint status" can be identified (O(k)) as for $\exists j$, $j = 1,2,\ldots, k$
S_1 and S_2 are disjoint either if $(a_1 + a_2 \geq n_2)$ or if $(a_2 + a_1 \geq n_1)$

For example, suppose k = 2 and a selected sub-system is S ($m_1 \times m_2$) and a free node is R ($d_1 \times d_2$). Figure 3.a illustrates the adjacent status of S (10) and R (11), which are satisfied both not-disjoint and one bit different. Figure 3.b shows the non-adjacent status of S (10) and R (01) since they are not disjoint but are different in 2 bits.

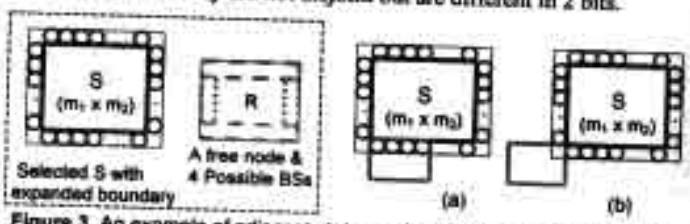


Figure 3. An example of adjacent statuses: (a) adjacent; (b) not adjacent.

2.4.2.2 The CU Adjacent Search (CU-AS)

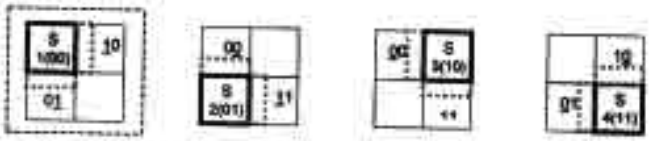
The CU adjacent search (CU-AS) is another approach used to find any free node R that is adjacent to the selected sub-system S. In this method, the searching starts from S and its adjacent nodes can be identified directly (see the following 4 cases). If the node R is free, its corresponding BSs are identified and its best-fit value (Section 2.4.1) is computed. New S and R will be updated if they have the better best-fit value. Time complexity of the CU-DFS is $O(k2^k)$.

In order to identify any boundary free PEs of a free node (R) whether or not it is adjacent to the selected sub-system S, we define the adjacent buddies in $O(k^2)$ for any non-combined sub-system (S) or $O(k2^k)$ for any combined sub-system (S).

Case 1: if S is any buddy node (1, 2, …, or 2^k), we compute its BS(s) in $O(k^2)$ time.

Let S is a selected sub-system ($S = m_1 \times m_2 \times \ldots \times m_k$)
I is a Buddy ID of S, where $I = 1, 2, \ldots,$ or 2^k.
Then k Adjacent Buddies can be identified by
1. Convert I to a binary strings $(b_{k-1} \ldots b_1 b_0)$
2. Compute k adjacent Buddies: for dim j, $j = 1, 2, \ldots, k$
negate b_{j-1} $(b_{k-1} \ldots \underline{b_{j-1}} \ldots b_1 b_0)$.
if $n_{j-1} \neq 1$, min BS $<(a_1, a_2, \ldots, a_j, \ldots, a_k), (b_1, b_2, \ldots, a_j, \ldots, b_k)>$
if $n_{j-1} = 0$, max BS $<(a_1, a_2, \ldots, b_j, \ldots, a_k), (b_1, b_2, \ldots, b_j, \ldots, b_k)>$

For example, if k = 2, 2 adjacent buddies of any node S (where I = 1, 2, 3, or 4) are

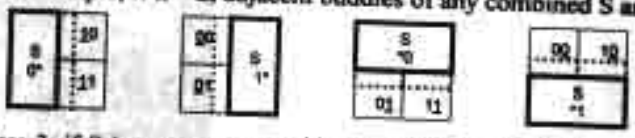


† - Negate dim 1 ($b_1\underline{b_0}$ => 01). Compute minBS $<(a_1, a_2), (a_1, b_2)>$
- Negate dim 2 ($\underline{b_1}b_0$ => 10). Compute minBS $<(a_1, a_2), (b_1, a_2)>$

Case 2: if S is any combined (2^{k-j}) buddy node (j = 1, 2, …, k-1), we compute its BS(s) in $O(k2^k)$.

Let S is a selected sub-system (or a combined S of 2^{k-j}, $j = 1, 2, \ldots, k-1$)
T is a ternary string of $S = (b_{k-1} \ldots b_j \ldots b_0)$
There exist ja' * in T [i.e., k Ts are $(b_{k-1} \ldots b_j \ldots *), (b_{k-1} \ldots b_{j-1} \ldots *b_0), \ldots,$ $(*b_{k-2} \ldots b_{j-1} \ldots *)$ and $(k-j)2^j$ adjacent Buddies of each T are identified by
- Negate a non* dim (from min to max Nos.), set each of "..*" = 0/1 &
compute minBS (n=1) or maxBS (n=0)
Example, for $j = 1$ & $T = (b_{k-1} \ldots b_2 b_1 *)$, $(k-1)2^1$ adjacent nodes are
negate b_1 $(b_{k-1} \ldots b_2 n_1 *)$, set dim 1 = 0/1,
if n=1, compute min BS $<(a_1, a_2, a_3, \ldots, a_k), (b_1, a_2, b_3, \ldots, b_k)>$
if n=0,compute max BS $<(a_1, b_2, a_3, \ldots, a_k), (b_1, b_2, b_3, \ldots, b_k)>$
, . . . , negate b_{k-1} $(n_{k-1} \ldots b_2 b_1 *)$, set dim 1 = 0/1,
if n=1, compute min BS $<(a_1, a_2, a_3, \ldots, a_k), (b_1, b_2, b_3, \ldots, a_k)>$
if n=0,compute max BS $<(a_1, a_2, a_3, \ldots, b_k), (b_1, b_2, b_3, \ldots, b_k)>$
for $j = k-1$ &T = $(b_{k-1} * \ldots **)$, 2^{k-1} adjacent nodes are
negate b_{k-1} $(n_{k-1} * \ldots **)$, set dim 1, 2, …, k-1= 0/1(min->max).

For example, if k = 2, adjacent buddies of any combined S are



Case 3: if S is any combined (2^{k-j}) buddy(s)&(2^j) sub-buddies node (j=1,2,…,k-1), all adjacent buddies can be identified by applying Case 1 and Case 2 in $O(k2^k)$. For example, if k=2, adjacent buddies of any combined S are

Case 4: if S is any combined ($k2^{k-1}$) sub-buddies node (j=1,2,…,k-1), all adjacent buddies can be identified by applying Case 1 and 2 in $O(k2^k)$. For example, if k=2, adjacent buddies of any combined S are

2.5 Searching for Allocation/Deallocation

In the k-Tree-based (CU/PE) allocation procedure, searching starts from the root and perform DFS (depth first search) by visiting the left most (leaf) node. If that node is free and its size can accommodate the request, its best-fit value is computed. For an SIMD task, either the CU-DFS or CU-AS strategy is applied. Then, the best (SIMD/MIMD) sub-system (S) is updated if the new free S yields the better best-fit value. The above process is repeated for the next node in the k-Tree including all external (leaf) nodes and internal nodes. After all nodes are visited, the final process is applied to either 1) allocate the best sub-system directly to the request (if its size is equal to that of the request) or 2) partition the corresponding node for the request (if its size is larger than that of the request).

Whenever a task is finished, the k-Tree-based (CU/PE) deallocation procedure is applied by searching for the location of the finished sub-system starts from the root and goes to the subset path until reaching the leaf node that stores information of the finished task. After finding that k-Tree's node of the finished task, its status is updated. Finally, the combining process is recursively applied (to remove free internal nodes(s)) from both PE and CU partitions to the root (if it is possible).

THEOREM 1: Time complexity of the k-Tree-based (CU/PE) allocation with applying the CU-AS strategy (to find the best free sub-system for each incoming task) on a k-D mesh is $O(k^3 2^{2k}(N_A+N_F))$. N_A is the max allocated tasks ($N_A \leq N$), N_F is the corresponding free nodes in k-Tree ($N_A+N_F \leq N$ and $N_F \leq (2^k-1)N_A$).

PROOF: Since #external (or leaf) nodes are at most $N_A + N_F \leq N$ and #internal nodes are at most (#leaf nodes-1) divided by (2^k-1),

therefore all nodes (M) = $(N_A+N_F) + (N_A+N_F-1) / (2^k-1) < 2N$. [For each (free) leaf node, the best-fit value is computed in $O(k^3)$ and $O(k^3N_F)$ for all free nodes. For each internal node, the best-fit value is computed in $O(k^32^{2k})$ for all combined sub-systems and $O(k^32^k(N_A+N_F))$ for all internal nodes.] Finally, after all nodes are visited, if the best S's size is equal to the request, then it is directly allocated to the request. Otherwise, the network partitioning and the best sub-partition will be applied in $O(k^32^{2k})$. Thus, total time complexity of the k-Tree-based (CU/PE) allocation with applying the CU-AS heuristic is $O(k^32^k(N_A+N_F))$, where $N_A+N_F \le N$ and hence $O(N_A+N_F)$ when applied to the 2-D/3-D meshes.

Note: time complexity of the k-Tree-based (CU/PE) allocation with applying the CU-DFS heuristic is $O(k^42^k(N_A+N_F)\ (kN_F))$ and hence $O(N_F(N_A+N_F))$ when applied to the 2-D/3-D meshes.

THEOREM 2: Time complexity of the k-Tree-based (CU/PE) deallocation (to free the particular k-Tree node that stores the finished task and to combine the free internal nodes to the root of the k-Tree) on a k-D mesh is $O(n2^k)$, where $n=\max(n_1,n_2,\ldots,n_k)$.

PROOF: Searching for the location of a finished sub-system from the root is at most $n(2^k)$ steps. Then, combining all 2^k buddy nodes from the finished sub-system to the root(if it is possible) takes another $n(2^k)$ steps. Therefore, total time complexity is $O(n2^k < M)$.

3. System Performance Evaluation

By simulation study, a number of experiments were performed to investigate the system performance effect (i.e., system utilization and fragmentation) of applying our k-tree-based (CU/PE) allocation model for the reconfigurable MSIMD/MIMD 2-D and 3-D meshes. For each experiment, (simulation) time units were iterated around 5,000-50,000 units and incoming tasks were generated around 1,000-10,000 tasks, according to the system parameter(s) setting. For each evaluated result, different data sets were generated and the algorithm was repeated until an average system performance does not change. The Uniform distribution $U(\alpha, \beta)$ was considered for the task-size distribution. Task arrival rate – Poisson(λ) (or inter-arrival time – Exp($1/\lambda$=5)), and service time – Exp(μ=10). Note: in order to set the same incoming tasks and environment to both CU allocation strategies for the comparison purpose, we assumed that no task finishes during the considering time.

In Experiment 1, we investigated the effect of system sizes to the system utilization (U_{sys}), where the system sizes ($N=n_1 x n_2$) were varied and the task sizes ($1x1 - n_1/2 \times n_2/2$) were generated and fixed. For all test cases the CU-AS and CU-DFS strategies performed the same system utilization (since these methods were different only when sub-system (S) and task (T) sizes were equal which hardly occurred.). Table 1 showed the results (%U_{sys}) of applying the k-Tree-based (CU/PE) allocation for 2-D and 3-D meshes, which yielded the same results when increasing percentage of SIMD tasks.

Table 1. Effect of the system sizes to the system utilization (%).

2-D Mesh				3-D Mesh			
$N=n_1xn_2$	0%	10%	20%	N	0%	10%	20%
64x64	68.76	68.46	71.91	n=32	58.14	58.55	59.80
128x128	66.25	67.62	67.62	n=64	49.73	54.32	54.33
256x256	70.16	70.16	70.16	n=128	50.52	50.52	50.52

Table 2. Effect of the task sizes to the system utilization (%).

Task size	0%	10%	20%	50%
1x1-64x64	81.403	78.565	78.268	78.200
1x1-128x128	70.158	70.160	70.162	69.007
1x1-256x256	59.995	59.995	59.995	59.997

In Experiment 2, we investigated the effect of task sizes to the system utilization, where the system size was fixed (N = 256x256) and the task sizes were varied. Table 2 showed that the system utilization increased when the maximum task-size parameter was reduced since a number of small tasks could be allocated. For the system utilization, these strategies performed the same results since $U_{sys} = 1 - F_{sys}$ (or no effect of internal system fragmentation).

4. Conclusion and Future Study

This paper introduces two best-fit heuristics for the k-Tree-based CU allocation: 1) the CU-DFS strategy in $O(kN_F)$ and 2) the CU-AS strategy in $O(k2^k)$. The CU allocation is added to complete the design of the new generalized k-tree-based (CU/PE) allocation model for the reconfigurable MSIMD/MIMD k-D mesh-connected architectures. By simulation study, a number of experiments were performed to investigate system performance of applying our new k-Tree-based (CU/PE) allocation model for reconfigurable 2-D and 3-D meshes. System performance results (i.e., system utilization & fragmentation) of applying our model with including the CU-AS strategy showed the same results to those of the CU-DFS strategy. However, for the 2-D or 3-D meshes the CU-AS strategy yields O(1) time which is better than $O(N_F)$ time performed by the CU-DFS strategy.

In the future study, we will modify and add the CU searching to some existing 2-D mesh-based PE allocation methods. Those modified strategies can support SIMD tasks for the reconfigurable MSIMD/MIMD 2-D meshes. Therefore, the system performance of those (CU/PE) allocation methods will be investigated and compared to our k-Tree-based (CU/PE) allocation approach.

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