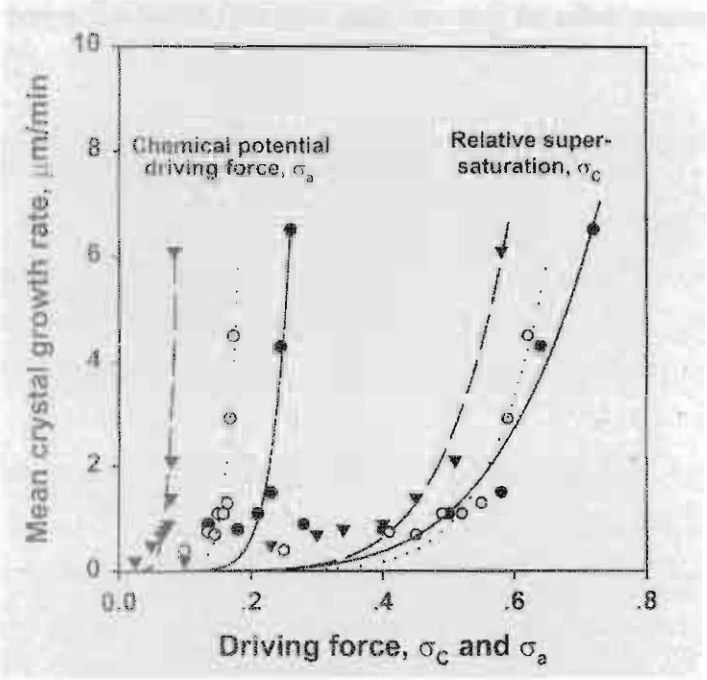

Figure 9.6 Comparing the commonly used relative supersaturation driving force with the analytical driving force for crystal growth of D-fructose from aqueous – ethanolic solutions.

9.4 Conclusions

The approximate driving forces (in particular the relative supersaturation) for fructose crystal growth from aqueous and aqueous-ethanol solutions is significantly different to the chemical potential driving force; in the worst cases there is an order of magnitude difference between the two measures. In aqueous-ethanol solutions, this is due to the system having low solubility, large driving forces (due to a large metastable limit), and a strong dependence between the activity coefficient and solute concentration. In aqueous solutions the conversion between relative supersaturation on a molar basis and a mass basis is responsible for most of the error. This suggests that the mass basis activity coefficients vary reasonably strongly with concentration at the experimental concentrations.

There is a linear relationship between the approximate and true driving forces in crystal growth from aqueous solutions, which leads to a difference in only the crystal growth rate constant in the power law models, not the exponent. This suggests that relative supersaturation is still a suitable driving force for crystal growth of D-fructose from aqueous solutions. In aqueous ethanol solutions the relationship between the approximate and true driving forces is strongly non-linear, suggesting that the relative supersaturation is not an appropriate driving force for crystal growth of D-fructose from aqueous ethanol solutions. Despite the conclusions here, a change to a true driving force does not help to explain the unusual growth kinetics in the aqueous ethanol system. The original

conclusion that the crystals are poisoned by an impurity formed in the solution are probably correct, as evidenced by no power law model fitting the data very well for either measure of the driving force.

10. Primary Conclusions for the Project

The following properties and mechanisms have been studied in this project in order to fully understand crystal growth of fructose and glucose by aqueous ethanolic crystallization of high fructose syrups.

1. Physical properties of mixtures of High Fructose Syrups with ethanol.
2. Physical properties of mixtures of aqueous glucose and fructose syrups with ethanol. This was performed because the High Fructose Syrups were very unstable to microbial action over time, and the four component mixture fructose + glucose + ethanol + water was an excellent model for aqueous ethanolic crystallization of the high fructose syrups. The two mixtures are more than 99 % similar in terms of components for solvent compositions used in crystallizations.
3. Solubility of fructose and glucose the system fructose + glucose + ethanol + water. This study is critical to determine the viability of ethanolic crystallization, however the information was not published until this study.
4. Yields of fructose and glucose that can be expected from aqueous ethanolic crystallization of high fructose syrups. This information was determined from analysis of the data in part 3.
5. Thermodynamic modeling of the system fructose + glucose + ethanol + water. Because the solubility data in limited to several temperatures it was decided to attempt to model the data using several variants of the UNIQUAC equation. It was evident that although the

This information is used to determine the feasibility of aqueous ethanolic crystallizations of high fructose syrups to replace the currently used chromatographic separation, which has proved to be very expensive in Thailand due to lack of production of the chromatographic resin. It was found that a co-crystallization of glucose and fructose was possible, although one component (typically glucose because of the position of the invariant point on the phase diagram) needed to be seeded, and the second component was allowed to nucleate. The two forms of crystal could then be separated by size separation. It is recommended to use three size fractions. The lowest size fraction will contain predominantly the nucleated solute, and the largest size fraction the seeded solute. There may be a size range in the centre which contains both solutes in significant amounts: the crystal in this size range should be re-dissolved and recycled back to the feed of the crystallizer to maintain good yields.

The main difficulty with the operation is the very low growth rate of both crystal forms, of the order of 0.1 micrometer/minute for both fructose and glucose. This makes the batch time very long, with two days being a suitable time based on the information in the current study. More research on this point is needed. (i) What is the mechanism that causes the growth rates to be so low: is it the poisoning of the solute by the second solute (i.e. fructose poisoning glucose) or is it the solvent (ethanol) poisoning the crystallization of the two solutes? (ii) Can the crystal growth rates be optimized at any point of the phase diagram, i.e. is there an optimum value of the addition of ethanol that will maximize the growth rates, or is it possible to get higher growth rates at elevated or

depressed temperatures? These questions are a large research project in their own right, and not enough time was available in the current project to solve them.

11. Outcomes

The research has shown that it is possible to replace the chromatographic separation of fructose and glucose prior to production of crystalline fructose with a single crystallization step involving both sugars. The crystallization step involves use of fructose seed crystals to produce large sized crystals of fructose, while glucose is simultaneously crystallized, but by requiring glucose crystals to nucleate. The nucleation of glucose crystals, combined with the low growth rates of both types of crystals means that the two species in the crystal phase occur at very different sizes, allowing a reasonable separation to be carried out by size separation of the crystal product. The crystallization process is advantages when aqueous ethanolic solutions are used because of the significantly lower solubilities in these solutions, and because the lower viscosities result in easier crystallizations.

The study of the phase diagram shows that it is not possible to separate fructose and glucose with a good yield using crystallization unless the method above is utilized. The solubilities of the two sugars in aqueous and aqueous ethanolic solutions are sufficiently close that both sugars will crystallize if a reasonable yield is desired, in either type of solution. Even at low yields the above method is not suitable, because glucose has the lower solubility unless the solution concentration of fructose is much higher than the solution concentration of glucose. In traditional High Fructose Syrup mixtures from starch conversion the concentration of glucose is slightly higher than the concentration of fructose.

The research has also studied the most important characteristics affecting the crystallization, including the

1. Phase diagram and potential yields.
2. Characteristics of the aqueous ethanolic mixtures of high fructose syrups.
3. Thermodynamic properties of the fructose – glucose solutions, including the ability to model solid-liquid equilibrium for sugars in aqueous solution, and a proof that these models do not extrapolate well over temperature, and perform particularly badly in mixed solute systems such as this one!
4. The effect of thermodynamic properties on crystal growth models (the effect of sugar activity). In Particular it has been shown that the commonly used supersaturation driving force is a poor representation of the true driving force, particularly for aqueous – ethanolic crystallizations.
5. The rate and equilibrium of the mutarotation reaction, which is likely to be a rate determining step for seeded crystallization processes in aqueous ethanolic solutions (see for example Flood et al. 2000).

The research has produced an international journal publication in an American Chemical Society Journal,

1. A. E. Flood and S. Puagsa, "Refractive Index, Viscosity, and Solubility at 30 °C, and Density at 25 °C for the System Fructose + Glucose + Ethanol + Water" *J. Chem. Eng. Data*, **45** (2000) 902 – 907.

with two other manuscripts intended for international publication being rearranged for resubmission;

1. A. E. Flood and Y. Hua, "Inhibition of the Mutarotation of D-(+)-Glucose by D-(-)-Fructose" submitted to the Journal of Carbohydrate Chemistry. This paper will be resubmitted with some extra data recorded by an RGJ PhD student, Sukanya Srisagna, and we expect it will be accepted in the next submission.

2. A. E. Flood, "UNIQUAC Modeling of Solid-Liquid Equilibrium in the System Fructose + Ethanol + Water" Submitted to the journal Fluid Phase Equilibria. I am currently working on the problem again with Dr. Juergen Rarey, and hope to make the paper suitable for publication.

The research has also produced several publications in the proceedings and presentations at international conferences.

1. A. E. Flood, P. Pantaraks, W. Monkaew, and Y. Hua, "A study of the mutarotation reaction in solutions of glucose and fructose" Proceedings of the Regional Symposium on Chemical Engineering, November 22-24, 1999, Songkhla, Thailand. Pages A11-1 – A11-7.

2 .A. E . Flood, "Measurement and modeling of solid liquid equilibrium in the system fructose + glucose + ethanol + water" Proceedings of the Regional Symposium on Chemical Engineering, December 11-13, 2000, Singapore. Pages ET1-1 – ET1-7

3. A. E. Flood, "Comparing Analytical and Approximate Expressions for the Driving Force in Crystal Growth of D-Fructose", Presented at the International Conference on Crystal Growth, ICCG-13, 30 July – 4 August, 2001, Kyoto, Japan.

Copies of the published journal and conference papers are given at the end of the report

The research funding has also been used to train staff and students in research, including two Research Assistants:

1. Miss Nichapat Toomwan.
2. Miss Srisuda Puagsa.

Miss Puagsa has continued to pursue a research career, and will graduate with a Master's degree in Chemical Engineering in 2546.

The research also partly supported six undergraduate research projects involving ten undergraduate students.

References

Addai-Mensah, J. "Crystallization of D-fructose from Aqueous Ethanolic Solutions" PhD Diss., Univ. of Queensland, Australia (1992).

Bates. F. J., "Polarimetry, Saccharimetry and the Sugars" Circular C-440, National Bureau of Standarsd, U.S. Department of Commerce (1942).

Bock, K., and Pedersen, C., *Adv. Carbohydr. Chem. Biochem.*,**21**, 27 (1983).

Bockstanz, G. L., Buffa, M., and Lira, C. T., *J. Chem. Eng. Data*,**34,** 426 (1989).

Catte, M., Dussap, C., Achard, C., and Gros, J., *Fluid Phase Equilibria*, **96**, 33 (1994).

Chu,Y. D., Shiau, L. D., and Berglund, K. A. *J. Crystal Growth*, **97**, 689 (1989).

Cockman, M., Kubler, D. G., Oswald, A. S., and Wilson, L., *J. Carbohydr. Chem.*, **6**, 181 (1997).

Edwards, L. W. "Process for continuous crystallization of alpha monohydrate dextrose utilizing high agitation" US Patent 4,357,172 (1982).

Elankovan, P. and Berglund, K. A., *AIChE J.* **33**, 1844 (1987).

Flood, A. E., Addai-Mensah, J., Johns, M. R., White, E. T., *J. Chem. Eng. Data,* **31,** 418 (1996a).

Flood, A. E., Johns, M. R., White, E. T., *Carbohydr. Res.*, **288,** 45 (1996b).

Flood, A. E., "Crystallization of D-fructose from Aqueous Ethanolic Solutions" PhD Diss., Univ. of Queensland, Australia (1996c).

Flood, A. E., Johns, M. R., White, E. T., *AIChE J.*, **46**, 239 (2000).

Funcke, W., von Sonntag, C., and Triantaphylides, C., *Carbohydr. Res.*, **75**, 305 (1979).

Gabas, N., and Laguerie, C., *Chem. Eng. Sci.*, **47**, 3148 (1992).

Goldberg, R. N. and Tiware, Y. B., *J. Phys. Chem. Ref. Data*, **18**, 809 (1989).

Jackson, R. F. and Silsbee, C. G., "The solubility of dextrose in water" Science Paper 437, U.S. Bureau of Standards (1922).

Kelly, F. H. C., *J. Appl. Chem.*, **4**, 409 (1954).

Kraus, J. and Nyvlt, J., *Zuckerind.* **118**, 219 (1993).

Kraus, J. and Nyvlt, J., *Zuckerind.* **119**, 24 (1994).

Kraus, J. and Nyvlt, J., *Zuckerind.* **119**, 299 (1994).

Kraus, J. and Nyvlt, J., *Zuckerind.* **119**, 407 (1994).

Mulvihill, P. J., "Crystalline and Liquid Dextrose Products" in Schenck, F. W. and Hebeda, R. E. "Starch Hydrolysis Products" VCH Publishers, New York (1992).

Osberger, T. F., "Crystalline Fructose" in O'Brien Nabors, L., and Gelardi, R. C. "Alternative Sweeteners" 2nd edition, Marcel Dekker Inc., New York (1991).

Peres, A. M. and Macedo, E. A., *Fluid Phase Equilibria*, **123,** 71 (1996).

Peres, A. M. and Macedo, E. A., *Carbohydr. Res.*, **303,** 135 (1997).

Perry, R. H., and Green, D., "Perry's Chemical Engineers' Handbook" 6th edition, McGraw-Hill, New York (1984).

Prausnitz, J. M., Lichtenthaler, R. N., and de Azevedom, E. G., "Molecular Thermodynamics of Fluid Phase Equilibria" 3rd edition, Prentice-Hall, Upper Saddle River, U.S. (1999).

Raemy, A., and Schweizer, T. F., *J. Therm Anal.*, **28**, 95 (1983).

Roos, Y., *Carbohydr. Res.*, **238**, 39 (1993).

Shiau, L. D., and Berglund, K. A., *AIChE J.*, **33**, 1028 (1997).

White, J. S. "Fructose Syrup: Production, Properties and Applications" in Schenck, F. W. and Hebeda, R. E. "Starch Hydrolysis Products" VCH Publishers, New York (1992).

Williams, C., and Allerhand, A., *Carbohydr. Res.*, **56**, 173 (1977).

Young, F. E., Jones, F. T., and Lewis, H. J., *J. Phys. Chem.*, **56**, 1093 (1952).

Young, F. E, *J. Phys. Chem.*, **61**, 616 (1957).

Appendix

Publications derived from the grant.

Refractive Index, Viscosity, and Solubility at 30 °C, and Density at 25 °C for the System Fructose + Glucose + Ethanol + Water

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Commercial crystalline fructose is currently prepared by chromatographic separation of aqueous solutions of fructose and glucose followed by crystallization in either aqueous or aqueous−ethanol solutions. It may be possible to use one or more crystallization steps instead of the chromatographic process, thus producing crystalline product more directly. In this study the solubility, refractive index, and viscosity of solutions of fructose + glucose + ethanol + water were measured at 30 °C, and the densities of solutions with solvent concentrations of 40, 60, and 80 mass % ethanol were measured at 25 °C. These properties will be useful for crystallization studies for this system.

Introduction

D-Fructose, $C_6H_{12}O_6$, is a monosaccharide widely used as a sweetener, largely due to its high sweetness value, although other physical and chemical properties also make it suitable for a number of products. Fructose is typically produced by hydrolysis of starch into glucose followed by isomerization to an aqueous solution of glucose and fructose. The product of this is a high-fructose syrup (HFS) that is (on a dry mass basis) approximately 42% fructose and 53% glucose with some residual higher carbohydrates (HFS-42). Higher purity syrups (such as HFS-90, which is 90% fructose on a dry mass basis) may be produced by chromatographic separation, while crystalline fructose is currently produced only from the high-purity syrups. Using the sweetness of sucrose as a basis (value 100), the sweetness of the crystalline form of fructose (β-D-fructopyranose) is approximately 180, while that of HFS-90 is only 106.[1] The difference is due to the noncrystallizing tautomers of fructose, which comprise approximately 30% of the fructose in solution, having lower sweetness than β-D-fructopyranose. HFS-42 has a sweetness of 92, which is lower than that of HFS-90, since glucose has low sweetness, approximately 65.

Crystalline fructose is currently prepared using either aqueous or aqueous−ethanolic crystallization of high-purity (90−95%) fructose syrups. Aqueous crystallization is made difficult by the high solubility of fructose in water (approximately 4.3 g of fructose/g of water at 30 °C), which not only affects the yield but also produces very highly viscous solutions. The fructose−water phase diagram is well-known,[2] and property data suitable for use in crystallization of aqueous fructose solutions by the addition of ethanol have also been determined for the system fructose + ethanol + water.[3] Suitable processes for crystallizing fructose using ethanol as a nonsolvent are described in patents,[4−6] and crystallization data have also been published.[7,8] Processes crystallizing fructose or glucose directly from lower purity high-fructose syrups (HFS-42 for instance) are not currently used.

Published data on the solubility of sugars in solvents containing alcohols is limited (for example sucrose in ethanol−water mixtures,[9] xylose and mannose in ethanol−water mixtures,[10] and glucose in ethanol−water mixtures[11]). More interest has been shown recently, partly due to an interest in thermodynamic modeling of these systems particularly by the group of Macedo.[12−14] There are solubility data for a very limited number of multiple-sugar solute systems (fructose and glucose in water[15] and xylose and mannose in water[16] are examples) and essentially no data for the solubility of multiple-sugar solutes in mixed solvents.

Experimental Section

Chemicals. D-(−)-Fructose, D-(+)-glucose anhydrous (both ACS, for analysis), and ethanol anhydrous (99.9% v/v, for analysis) were obtained from Carlo Erba Reagenti (Milan) and used without further purification.

Experimental Procedures. The solubility of fructose and glucose in solutions of ethanol and water was measured at 30 °C, with the solutions maintained at the desired temperature with an uncertainty of ±0.2 °C. All determinations were made in sealed glass Schott bottles into which a known quantity of ethanol + water (of desired concentration) and a known quantity of anhydrous fructose were added. The ethanol concentration in the ethanol + water solution was known to an accuracy of 0.1 mg/g of solution. An amount of fructose was dissolved in the bottles, with the exact amount varying between bottles such that the experiments covered a range of points between the previously published systems glucose + ethanol + water[11,12] and fructose + ethanol + water.[3] An amount of crystalline anhydrous glucose sufficient to achieve at least 50% excess of glucose over the amount needed for saturation was added to each bottle, and the bottles were then shaken in an orbital shaking bath at 100 rpm and 30 °C until saturation was reached. After 24 h, the refractive index of the liquid was determined every 6 h to determine if saturation was complete. Saturation was complete within 7−10 days for all determinations.

This system proved difficult for accurate measurement of fructose and glucose concentrations. In most cases with sugars it has been preferable to determine concentrations using a gravimetric method, such as the total solids

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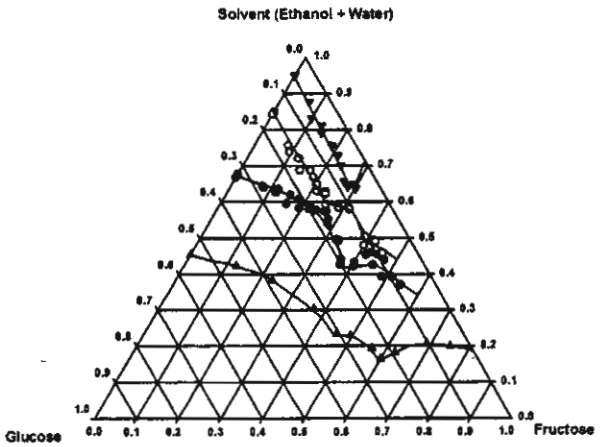

Figure 1. Solubility for the system fructose + glucose + ethanol + water at 30 °C. Solvent compositions: ▼, 80 mass % ethanol; ○, 60 mass % ethanol; ●, 40 mass % ethanol; ▲, 0 mass % ethanol.[15]

determination[17] or the method of Peres and Macedo,[12] as these methods have excellent reproducibility. This type of method was not used in the current study, since an accurate determination of two solutes was required, and hence a HPLC method was used. It was also found that if samples containing high concentrations of ethanol but low concentrations of sugar were left to stand over several days, then a detectable (by HPLC) amount of an unknown reaction product formed, whereas this product did not form if the ethanol was removed from solution. The reaction product is not known, although the reaction may involve sugar dehydration. For this reason, saturated liquid samples of approximately 1 mL were taken from the Schott bottles, mass was determined to ±0.1 mg in sealed weighing bottles, the samples were then partially dried at room temperature (approximately 30 °C) for 17 h to remove the bulk of the ethanol from the sample, and mass was again determined to ±0.1 mg. Drying at high temperatures was not used because fructose tends to degrade at temperatures higher than 65 °C. After the drying step, the samples were diluted to approximately 1 g of solids/100 mL of solution by the addition of a known amount (±0.1 mg) of distilled water, which was a suitable concentration for the HPLC method used. It should be noted that the drying process was not used to totally dry the sample but only to remove most of the ethanol so that the reaction between the ethanol and the sugars did not occur. After this sample preparation was carried out, the peak indicating the sugar−ethanol reaction product was not detected for any sample.

The diluted samples were filtered through a 0.45 μm filter, and then a 2 μL sample was injected onto a 250 mm × 4 mm Aminex HPX-87C (Biorad, Bangkok, Thailand) column using a water mobile phase at a flow rate of 0.3 mL/min. The column temperature was 80 °C. Detection was with a diode array detector measuring UV at 192 nm. The uncertainty (95% probable error) in the concentration determinations, including the dilution and HPLC, was 0.002 g of glucose/g of solution and 0.003 g of fructose/g of solution. Duplicate solubility determinations showed that the uncertainty (95% probable error) in the solubility measurements was 0.005 g of sugar/g of solution for both fructose and glucose. Uncertainties in other variables, such as bath temperature, solvent composition, or saturation point, may be responsible for the duplicate bottles having larger uncertainties than were seen in the concentration measurement alone.

Table 1. Solubilities of D-(−)-Fructose and D-(+)-Glucose in Ethanol + Water at 30 °C

solvent comp (mass % ethanol)	solubility (g of sugar/g of solution)		
	glucose	fructose	total
40.0	0.332	0	0.332
	0.323	0	0.323
	0.280	0.076	0.356
	0.279	0.081	0.360
	0.247	0.118	0.365
	0.257	0.117	0.364
	0.247	0.158	0.405
	0.224	0.157	0.381
	0.210	0.182	0.392
	0.222	0.194	0.416
	0.193	0.217	0.410
	0.200	0.219	0.419
	0.178	0.326	0.504
	0.174	0.332	0.506
	0.176	0.284	0.460
	0.168	0.279	0.447
	0.168	0.259	0.427
	0.162	0.263	0.425
	0.191	0.233	0.424
	0.185	0.237	0.422
	0.203	0.370	0.573
	0.194	0.364	0.558
	0.173	0.404	0.577
	0.165	0.401	0.566
	0.128	0.418	0.546
	0.114	0.426	0.540
	0.124	0.450	0.574
	0.099	0.446	0.545
	0.120	0.486	0.606
	0.091	0.467	0.558
	0.095	0.512	0.607
	0.086	0.543	0.629
	0.000	0.718[a]	0.718
60.0	0.150	0	0.150
	0.157	0	0.157
	0.161	0.081	0.242
	0.168	0.094	0.262
	0.155	0.124	0.279
	0.169	0.143	0.312
	0.147	0.185	0.332
	0.144	0.169	0.313
	0.149	0.203	0.352
	0.156	0.214	0.370
	0.138	0.234	0.372
	0.141	0.237	0.378
	0.154	0.254	0.408
	0.154	0.252	0.406
	0.126	0.281	0.407
	0.128	0.286	0.414
	0.103	0.312	0.415
	0.105	0.316	0.421
	0.119	0.400	0.519
	0.104	0.391	0.495
	0.090	0.416	0.506
	0.086	0.422	0.508
	0.087	0.453	0.540
	0.084	0.456	0.540
	0	0.603[a]	0.603
80.0	0.049	0	0.049
	0.050	0	0.050
	0.053	0.071	0.124
	0.070	0.099	0.169
	0.066	0.143	0.209
	0.056	0.133	0.189
	0.060	0.186	0.246
	0.060	0.182	0.242
	0.059	0.214	0.273
	0.081	0.263	0.344
	0.066	0.235	0.301
	0.078	0.281	0.359
	0.061	0.302	0.363
	0.059	0.291	0.350
	0.000	0.266[a]	0.266

[a] From ref 3.

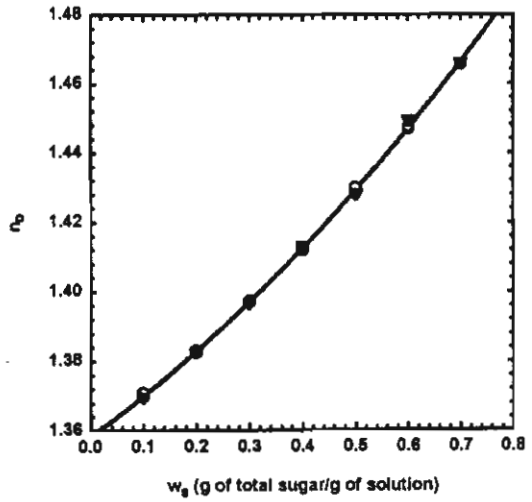

Figure 2. Refractive index for solutions of glucose and fructose in ethanol–water mixtures of 40 mass % ethanol. Ratio of glucose–fructose: ●, 1:1; ○, 2:1; ▼, 1:2.

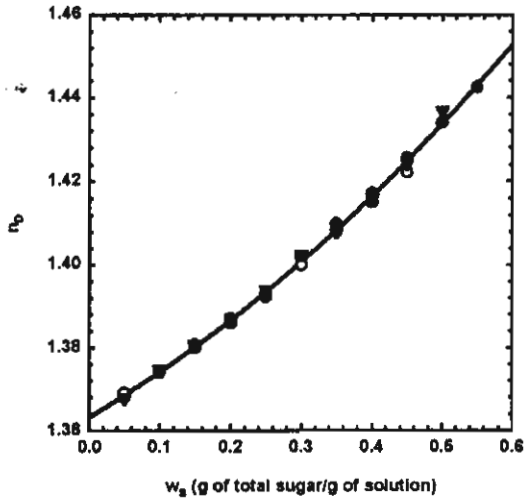

Figure 3. Refractive index for solutions of glucose and fructose in ethanol–water mixtures of 60 mass % ethanol. Ratio of glucose–fructose: ●, 1:1; ○, 2:1; ▼, 1:2.

Refractive index was measured for solvent compositions of 40, 60, and 80 mass % ethanol, and solute compositions of glucose–fructose equal to 1:1, 1:2, and 2:1. The total solute concentrations were chosen so that a range of data points up to approximately the saturation condition were measured. Duplicate refractive index determinations were made on an Abbe refractometer with temperature control to within ±0.1 °C. The precision of the refractive index was ±0.0005 refractive index unit.

Solution viscosity was measured in duplicate in a falling ball viscometer (Haake) with the sample viscosity determining which ball was used in the determination. The viscometer was jacketed, and water from a constant-temperature bath kept the viscometer temperature constant to within ±0.1 °C. The solutions studied had viscosities in the range 1 to 1000 mPa·s, and hence balls 1 (2.4 g·cm^{-3}, 15.81 mm) and 4 (8.13 g·cm^{-3}, 15.2 mm) were used. These balls were calibrated against sugar solutions of known viscosity. The time period used in the viscosity determination was the average of eight measurements of the time required for the ball to travel the required

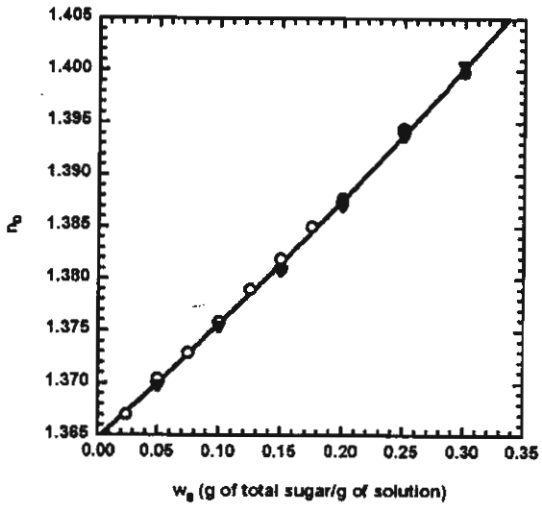

Figure 4. Refractive index for solutions of glucose and fructose in ethanol–water mixtures of 80 mass % ethanol. Ratio of glucose–fructose: ●, 1:1; ○, 2:1; ▼, 1:2.

distance. The error of the viscosity measurements is expected to be within 3%.

Solution density was measured in triplicate at 25 °C using 10 cm^3 density determination bottles weighed to ±0.1 mg. The uncertainty (95% probable error) of the density measurement was 0.0003 g·cm^{-3}.

Results and Discussion

The solubilities of fructose and glucose in ethanol + water are plotted as a ternary diagram in Figure 1. Since the system contains four components, it is not easy to illustrate the data on a two-dimensional plot. The illustration is simplified by having one axis as total solvent (ethanol + water), with the lines on the plot depicting constant solvent composition (in the case of this study 40, 60, and 80 mass % ethanol). The data for the system fructose + glucose + water (equivalent to a 0 mass % ethanol line), which also appear on the plot, were taken from an earlier study at 30 °C.[15] The three data points for pure fructose in ethanol + water (on the glucose axis) have been taken from a recent study by the same author.[3] The solubility data from the present study are shown in Table 1.

Two studies[11,12] have investigated the solubility of glucose in ethanol + water solutions; however, these studies were conducted at different temperatures, 35 °C for the former and 40 and 60 °C for the latter. For this reason, the solubility of glucose in ethanol + water was measured directly in this study. The study of Peres and Macedo[14] gave interaction parameters for a modified UNIQUAC model (optimized using their own experimental results) which could be used to predict these solubility values. At 30 °C, this model predicts a solubility of 0.064 g of glucose/g of solution at 80 mass % ethanol, 0.270 g of glucose/g of solution at 60 mass % ethanol, and 0.459 g of glucose/g of solution at 40 mass % ethanol. These values are significantly higher than the experimental values in this work; however, it should be noted that the temperature used in this study is outside the range of temperatures on which the model is based (40 °C and 60 °C).

The solubilities of both glucose and fructose are decreased as the concentration of ethanol in the solvent is increased over the range of concentrations investigated in the study. The solubilities of glucose and fructose in ethanol

Table 2. Refractive Indexes (n_D) of D-(−)-Fructose + D-(+)-Glucose + Ethanol + Water Solutions at 30 °C

solvent comp[a]	tot sugar conc[b]	n_D at these ratios of glucose–fructose 1:1	2:1	1:2
40.0	0.100	1.3699	1.3709	1.3699
	0.200	1.3824	1.3830	1.3830
	0.300	1.3968	1.3973	1.3970
	0.400	1.4119	1.4124	1.4131
	0.500	1.4293	1.4298	1.4283
	0.600	1.4470	1.4474	1.4495
	0.700	1.4655		1.4660
60.0	0.050	1.3681	1.3691	1.3680
	0.100	1.3740	1.3741	1.3748
	0.150	1.3800	1.3807	1.3808
	0.200	1.3860	1.3870	1.3871
	0.250	1.3893	1.3932	1.3942
	0.300	1.4016	1.4000	1.4025
	0.350	1.4099	1.4080	1.4080
	0.400	1.4170	1.4151	1.4161
	0.450	1.4255	1.4221	1.4243
	0.500	1.4340		1.4369
	0.550	1.4424		
80.0	0.025		1.3670	
	0.050	1.3701	1.3704	1.3699
	0.075		1.3729	
	0.100	1.3758	1.3758	1.3755
	0.125		1.3790	
	0.150	1.3810	1.3819	1.3809
	0.175		1.3850	
	0.200	1.3877	1.3872	1.3871
	0.250	1.3944		1.3938
	0.300	1.3998		1.4005

[a] mass % ethanol. [b] g of sugar/g of solution.

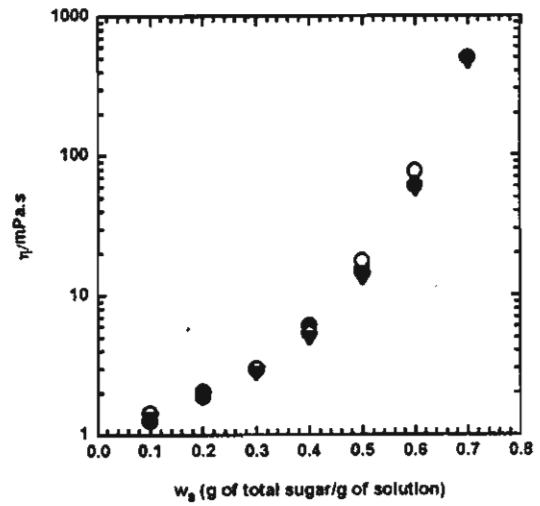

Figure 5. Viscosity for solutions of glucose and fructose in ethanol–water mixtures of 40 mass % ethanol. Ratio of glucose–fructose: ●, 1:1; ○, 2:1; ▼, 1:2.

at 30 °C are 0.0036 g of glucose/g of solution (extrapolated from the data at 40 and 60 °C using the modified UNIQUAC model of Peres and Macedo[14]) and 0.035 g of fructose/g of solution,[3] respectively, so it is expected that the solubility is monotonically decreasing as the ethanol content of the solvent increases.

When the solvent is 80 mass % ethanol, there is a "salting in" effect whereby the total sugar concentration is higher when both glucose and fructose occur together in solution compared to where only one solute appears. This maximum sugar concentration is 0.36 g of sugar/g of solution (compared to 0.050 g of glucose/g of solution or 0.266 g of fructose/g of solution at saturation for the one-

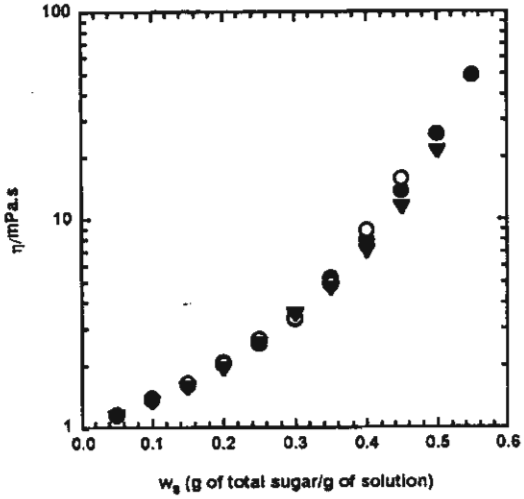

Figure 6. Viscosity for solutions of glucose and fructose in ethanol–water mixtures of 60 mass % ethanol. Ratio of glucose–fructose: ●, 1:1; ○, 2:1; ▼, 1:2.

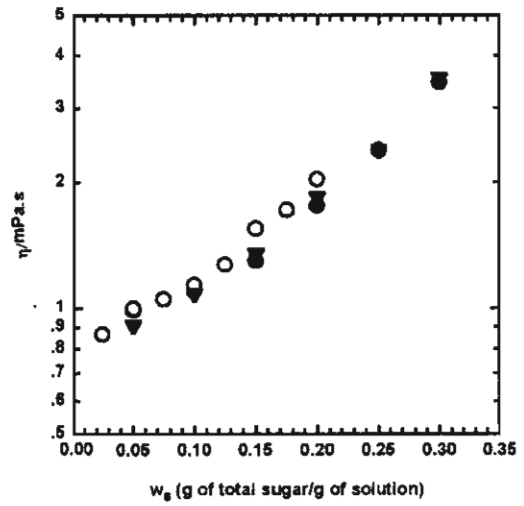

Figure 7. Viscosity for solutions of glucose and fructose in ethanol–water mixtures of 80 mass % ethanol. Ratio of glucose–fructose: ●, 1:1; ○, 2:1; ▼, 1:2.

solute mixtures). For both 40 and 60 mass % ethanol, the minimum total sugar concentration is when glucose is the only solute (0.33 and 0.15 g of glucose/g of solution, respectively), while the maximum occurred when fructose was the only solute (0.718 and 0.603 g of fructose/g of solution, respectively). The solubility lines for 60 and 80 mass % ethanol show similar behavior when glucose is the crystallizing form but markedly different behavior when fructose is the crystallizing form. It is noticeable that the solubility of glucose decreases most rapidly between 40 and 60 mass % ethanol, while the solubility of fructose decreases most rapidly between 60 and 80 mass % ethanol. The solubility curve for the system for 40 mass % ethanol is most similar to the behavior of the ternary system fructose + glucose + water.

The solubility curve for the system D-(−)-fructose + D-(+)-glucose + water[15] shows two distinct eutectics: one where the crystal form of glucose changes from glucose monohydrate to anhydrous glucose and one where fructose becomes the preferred crystalline phase. The first of these points is not clearly evident in the four-component system, although it may still exist. The second eutectic point is

Table 3. Viscosities of D-(−)-Fructose + D-(+)-Glucose + Ethanol + Water Solutions at 30 °C

solvent comp[a]	tot sugar conc[b]	η/(mPa·s) at these ratios of glucose−fructose 1:1	2:1	1:2
40.0	0.100	1.26	1.43	1.35
	0.200	2.04	1.90	1.92
	0.300	2.92	3.00	2.89
	0.400	6.12	5.37	5.12
	0.500	15.4	17.6	13.7
	0.600	60.7	77.2	59.6
	0.700	502		485
60.0	0.050	1.16	1.15	1.16
	0.100	1.39	1.36	1.37
	0.150	1.61	1.64	1.60
	0.200	2.06	2.02	1.98
	0.250	2.54	2.66	2.58
	0.300	3.33	3.36	3.59
	0.350	5.24	4.89	4.79
	0.400	8.00	8.85	7.18
	0.450	13.6	15.7	11.7
	0.500	25.6		21.7
	0.550	49.2		
80.0	0.025		0.86	
	0.050	0.99	1.00	0.91
	0.075		1.05	
	0.100	1.10	1.14	1.08
	0.125		1.27	
	0.150	1.30	1.56	1.36
	0.175		1.72	
	0.200	1.76	2.03	1.85
	0.250	2.38		2.39
	0.300	3.46		3.55

[a] mass % ethanol. [b] g of sugar/g of solution.

clearly evident. Work is in progress to determine the preferred crystalline phase for a range of temperatures and concentrations in this system.

In general, it is preferable to measure solubility through both dissolution and crystallization experiments, which will bracket the solubility by approach from both above and below. In the present study, measurement through crystallization was not attempted, since it is possible that glucose monohydrate would crystallize under certain conditions and thus the water content of the solvent would be reduced as crystallization progresses. The present study uses only dissolution of anhydrous sugars, which will not alter the solvent composition.

The refractive indexes for solutions of D-(−)-fructose + D-(+)-glucose + ethanol + water with solvent compositions of 40, 60, and 80 mass % ethanol are shown in Figures 2, 3, and 4, respectively. It is clear from these diagrams that the proportion of glucose to fructose in the solution does not have a significant effect on the refractive index at any of the solvent compositions studied. This is significant in that it shows that refractive index will give no information on the solute ratio in solution, although it is still useful as a measure of total solute for this system. As the solvent ratio increases, the refractive index of infinitely dilute solutions increases slightly, probably as a result of the differences in the refractive indexes of ethanol (n_D = 1.3594 at 25 °C) and water (n_D = 1.3325 at 25 °C);[18] however, the change in refractive index due to changes in solvent composition is not as significant at higher sugar concentrations. The refractive index data for the system are shown in Table 2.

Viscosities for the system D-(−)-fructose + D-(+)-glucose + ethanol + water for solvent compositions of 40, 60, and 80 mass % ethanol are shown in Figures 5, 6, and 7, respectively. The viscosity for the systems is strongly dependent on the total sugar content, with increasing

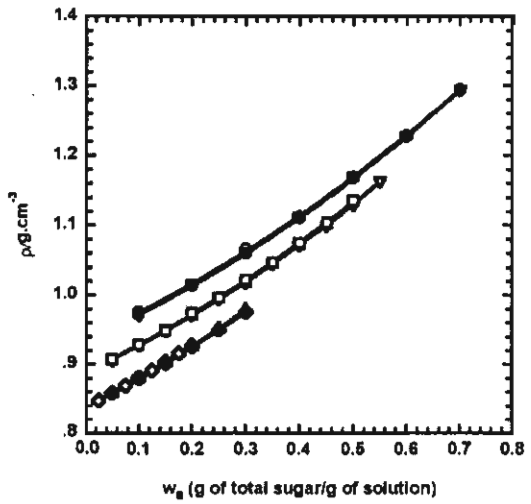

Figure 8. Density for solutions of fructose and glucose in ethanol−water mixtures. Curves are 40 mass % ethanol with the following ratios of glucose−fructose (●, 1:1; ○, 2:1; ▼, 1:2), 60 mass % ethanol with the following ratios of glucose−fructose (▽, 1:1; ■, 2:1; □, 1:2), and 80 mass % ethanol with the following ratios of glucose−fructose (◆, 1:1; ◇, 2:1; ▲, 1:2).

Table 4. Densities of D-(−)-Fructose + D-(+)-Glucose + Ethanol + Water Solutions at 25 °C

solvent comp[a]	tot sugar conc[b]	ρ/(g·cm^{-3}) at these ratios of glucose−fructose 1:1	2:1	1:2
40.0	0.100	0.9726	0.9757	0.9724
	0.200	1.0140	1.0143	1.0162
	0.300	1.0604	1.0661	1.0625
	0.400	1.1115	1.1123	1.1141
	0.500	1.1691	1.1686	1.1700
	0.600	1.2285	1.2280	1.2287
	0.700	1.2944		1.2948
60.0	0.050	0.9070	0.9071	0.9074
	0.100	0.9285	0.9282	0.9283
	0.150	0.9497	0.9502	0.9489
	0.200	0.9722	0.9717	0.9740
	0.250	0.9957	0.9963	0.9964
	0.300	1.0198	1.0200	1.0221
	0.350	1.0459	1.0466	1.0470
	0.400	1.0724	1.0739	1.0755
	0.450	1.1006	1.1030	1.1044
	0.500	1.1310		1.1356
	0.550	1.1640		
80.0	0.025		0.8473	
	0.050	0.8583	0.8587	0.8573
	0.075		0.8694	
	0.100	0.8804	0.8806	0.8793
	0.125		0.8918	
	0.150	0.9019	0.9038	0.9011
	0.175		0.9158	
	0.200	0.9259	0.9280	0.9250
	0.250	0.9503		0.9520
	0.300	0.9763		0.9793

[a] mass % ethanol. [b] g of sugar/g of solution.

solute concentration giving very strongly increasing viscosity. The highest viscosities recorded (around 500 mPa·s) were for 40 mass % ethanol, where the high solubility allows for high solute concentrations. The solutions measured were all undersaturated: since sugar solutions may be held at solute concentrations substantially higher than saturation without crystallization, the viscosities for the system are potentially very high. The viscosity is weakly dependent on the ethanol content of the solvent, with higher ethanol contents giving slightly lower viscosities,

although this dependence is much weaker than the dependence on solute concentration. The viscosity of pure ethanol at 30 °C (0.964 mPa·s) is greater than that of water (0.815 mPa·s),[18] so the behavior in this system is unusual, although it is probably due to differences in the solution structures. The solute (glucose−fructose) ratio has no significant effect on the viscosity over the range of values in this study. Data for the viscosity of the system are given in Table 3.

The densities of solutions of D-(−)-fructose + D-(+)-glucose + ethanol + water are shown in Figure 8. The density of the solution is not significantly affected by the solute (glucose−fructose) ratio in solution, although this is likely due to the two solutes having very similar densities. The densities of solid anhydrous glucose and fructose are 1.562 g·cm^{-3} (at 18 °C) and 1.600 g·cm^{-3} (at 20 °C), respectively.[19] The densities of the solutions are strongly (and nonlinearly) dependent on the total sugar content, with the density increasing with increasing sugar content. If a power law was fitted to the data, the exponent would be of the order 1.15 to 1.30 for all solvent compositions studied, with higher ethanol contents giving smaller exponents. The ethanol content of the solvent has a significant effect on the density, with solutions having higher ethanol content displaying lower density for the same sugar content, as would be expected from the differences in density between the two solvents. The densities of pure ethanol and water at 30 °C are 0.783 g·cm^{-3} and 1.023 g·cm^{-3}, respectively.[18] Density data for the system are given in Table 4.

Literature Cited

(1) White, J. S. Fructose Syrups: Production, Properties, and Applications. In *Starch Hydrolysis Products: Worldwide Technology, Production and Applications*; Schenck, F. W., Hebeda, R. E., Eds.; VCH Publishers: New York, 1992.

(2) Young, F. E.; Jones, F. T.; Lewis, H. J. D-Fructose−Water Phase Diagram. *J. Phys. Chem.* **1952**, *56*, 1093−1096.

(3) Flood, A. E.; Addai-Mensah, J.; Johns, M. R.; White, E. T. Refractive Index, Viscosity, Density, and Solubility in the System Fructose + Ethanol + Water at 30, 40, and 50 °C. *J. Chem. Eng. Data* **1996**, *41*, 418−421.

(4) Mahoney, J. C. US Patent 2 357 838, 1940.

(5) Dwivedi, B. K.; Subodh, K. R. US Patent 4 199 374, 1980.

(6) Bateman, J. R.; Goodacre, B. C.; Smithson, A. UK Patent 3 133 796 A, 1984.

(7) Flood, A. E.; Johns, M. R.; White, E. T. Mutarotation of D-fructose in aqueous−ethanolic solutions and its influence on crystallization. *Carbohydr. Res.* **1996**, *288*, 45−56.

(8) Flood, A. E.; Johns, M. R.; White, E. T. Crystal Growth Rates and Dispersion for D-Fructose from Aqueous Ethanol. *AIChE J.* **2000**, *46* (2), 239−246.

(9) Reber, L. A. The Solubility of Sucrose in Hydroalcoholic Solutions. *J. Am. Pharm. Assoc., Sci. Ed.* **1953**, *42* (4), 192−193.

(10) Gabas, N.; Carillon, T.; Hiquily, N. Solubilities of D-Xylose and D-Mannose in Water−Ethanol Mixtures at 25 °C. *J. Chem. Eng. Data* **1988**, *33*, 128−130.

(11) Bockstanz, G. L.; Buffa, M.; Lira, C. T. Solubilities of α-Anhydrous Glucose in Ethanol/Water Mixtures. *J. Chem. Eng. Data* **1989**, *34*, 426−429.

(12) Peres, A. M.; Macedo, E. A. Solid−Liquid Equilibrium of Sugars in Mixed Solvents. *Entropie* **1997**, *202/203*, 1−5.

(13) Peres, A. M.; Macedo, E. A. Measurement and Modeling of Solubilities of D-Glucose in Water/Alcohol and Alcohol/Alcohol Systems. *Ind. Eng. Chem. Res.* **1997**, *36*, 2816−2820.

(14) Peres, A. M.; Macedo, E. A. Phase equilibria of D-glucose and sucrose in mixed solvent mixtures: Comparison of UNIQUAC-based models. *Carbohydr. Res.* **1997**, *303*, 135−151.

(15) Kelly, F. H. C. The solubility of sucrose in impure solutions. In *Principles of Sugar Technology*; Honig, P., Ed.; Elsevier: Amsterdam, 1959; Vol. II.

(16) Gabas, N.; Laguérie, C. Solubilitié du Système ternaire eau-D-xylose-D-mannose. Modélisation de l'équilibre liquide-solide à l'aide de la méthode UNIFAC. *Bull. Soc. Chim. Fr.* **1990**, *127*, 391−395.

(17) BSES. Total Solids (Dry Substance) Determination in Mill Products. *Standard Laboratory Manual for Australian Sugar Mills*; Method 19; BSES: Brisbane, Australia, 1991; Vol. 2.

(18) Yaws, C. L. *Chemical Property Handbook*; McGraw-Hill: New York, 1999.

(19) *Handbook of Chemistry and Physics*, 76th ed.; Lide, D. R., Ed.; Chemical Rubber Publishing Company: Boca Raton, FL, 1995.

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A Study of the Mutarotation Reaction in Solutions of Glucose and Fructose

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Abstract

Mutarotation reactions have been shown to be industrially significant due to their effect on the crystallization of common sugars. In cases where the mutarotation rate is small and the crystal growth rate large, the rate of mutarotation will be the rate determining step for the process. The mutarotation reaction was studied using both F.T.-I.R. and ^{13}C N.M.R. techniques. It was found that the F.T.-I.R. technique gave accurate results for the mutarotation rate, and also could be used to determine the mutarotational equilibria if all tautomers/anomers in solution were also available as pure compounds. This is the case for glucose (where β-D-glucopyranose and α-D-glucopyranose are both available in crystalline form), but is not the case for fructose (where β-D-fructopyranose is the only tautomer available in pure form). It was also noted that F.T.-I.R is less convenient where glucose and fructose appear together in solution due to constructive interference in the 1060 - 1100 wavenumber range. The ^{13}C N.M.R. studies were better able to measure the mutarotational equilibria of the sugars, although a drawback of N.M.R. is its greater cost and difficulty. The most interesting result of the studies is that where glucose and fructose occurred together in solution, the mutarotation reactions for the two sugars appeared to be competitive. A possible explanation for this behavior is that the two sugars compete for the H^+ and OH^- ions necessary for ring opening and closing in the mutarotation reaction. The current study of the mutarotation rate of aqueous solutions of glucose and fructose is part of a larger project studying crystallization of High Fructose Syrups, which are aqueous syrups containing approximately equal concentrations of glucose and fructose.

Key words: fructose, glucose, mutarotation.

Introduction

Most naturally occurring sugars occur in two or more distinct forms. Glucose occurs as α-D-glucopyranose and β-D-glucopyranose: these forms are identical except for the configuration around the C-1 carbon, and these types of sterioisomers are known as anomers. Fructose occurs in five forms which are detectable in solution; two are six-membered rings (α-D-fructopyranose and β-D-fructopyranose), two are five-membered rings (α-D-fructofuranose and β-D-fructofuranose), and a straight chain form. These five forms interconvert rapidly, and are known as tautomers. The equilibrium reactions between sugar anomers or tautomers are known as mutarotation reactions.

Mutarotation of sugars has received considerable attention from researchers using a variety of techniques. The references in the following discussion are a small sample characteristic of a much wider range of studies. Polarimetry was used in most of the early studies, (Barry and Honeyman, 1952; Hyvönen et al., 1977), while other techniques have been used more frequently in recent studies. The newer techniques include gas chromatography of the trimethylsilyl derivatives (Cockman et al., 1987; Nikolov and Reilly, 1983; Flood et al., 1996), analysis of ^{1}H N.M.R. spectra (Coxon, 1972), which has been used particularly on aldoses, and ^{13}C N.M.R. (Angyal and Bethell, 1976; Wilbur et al., 1977), which has mainly been used on ketoses. Details of the ^{13}C N.M.R. spectra of most common monosaccharides and some of their derivatives are given in Bock and Pedersen (1983). F.T.-I.R. has also been used to study mutarotation, particularly by the groups of Yaylayan (Yaylayan and Ismail, 1992; Yaylayan et al., 1993) and Polavarapu (Back et al., 1984; Back and Polavarapu, 1987). The studies nearly all concentrate on measurement of the mutarotational equilibrium, with only a few, notably those of Cockman et al., (1987), Back and Polavarapu (1987), and Flood et al., (1996) determining rate constants to any accuracy.

Experimental Procedure

A. Chemical

Analytical grade sugars were purchased from a number of manufacturers. The same sugar from different origins displayed no significant difference in mutarotation rate, mutarotational equilibria, ^{13}C N.M.R. spectra, or F.T.-I.R. spectra. The analytical grade sugars were used without further purification. Details of the reagents are:

D(-)fructose (β-D-fructopyranose): 1. Carlo Erba Reagenti, reagent analytical grade. 2. E. Merck, pure. D(+)glucose anhydrous (α-D-glucopyranose): 1. Carlo Erba Reagenti, reagent analytical grade. 2. Riedel-de Haën, analytical reagent. D(+)glucose monohydrate (α-D-glucopyranose.H_2O): Carlo Erba, reagent pharmaceutical grade. β-D(+)glucose anhydrous (β-D-glucopyranose): Sigma-Aldrich, 97+% (with up to 3 % α-D-glucopyranose).

B. Procedure

F.T.-I.R. Procedure: Samples of sugars in aqueous solutions were placed into an ATR cell at room temperature immediately after being dissolved, and the ATR cell was placed into the Fourier Transform Infrared Spectrometer, Model FTS 175 C. Samples containing only fructose were scanned every one minute for the first ten minutes, and thereafter every ten minutes up to a total of 150 minutes. Each set of data corresponded to 16 scans. Samples containing glucose, or mixtures of glucose and fructose, were scanned every one minute for ten minutes, then every ten minutes up to a total of 200 minutes. The longer scan time for samples with glucose was chosen due to the slower mutarotation rates expected for glucose. The spectra for the initial set of scans were subtracted from each subsequent set of scans to provide a display of how different parts of the spectra changed with time. Room temperature was controlled at 22.5 °C, and all samples were analyzed at this temperature.

^{13}C N.M.R. Procedure: Samples of glucose and/or fructose were dissolved in either D_2O or water (depending on the experiment performed). Proton-decoupled ^{13}C N.M.R. spectra were obtained at 75.4 MHz using 5 mm spinning tubes in a Varian Anova 300 MHz machine. Proton decoupling was at 300 MHz. Fourier transforms were achieved using 1.0 Hz line broadening.

Chemical shifts were determined relative to one drop of D_2O used as an internal standard for the aqueous samples. Initially data were taken at 77 s intervals, with 16 scans used at each interval. This procedure allowed enough time between scans for the sample to become fully relaxed. After 50 of these samples (more than 1 hr) the time between samples was increased to 377 s, as the solutions were approaching equilibrium. When the solutions reached equilibrium (around 4 hr after dissolution) one set of 1000 scans was performed to obtain an accurate measurement of the mutarotational equilibrium. Spectra were taken at 22.5 °C, room temperature. The peak heights of the C-3 and C-5 carbons were used (for all tautomers in the solution) to determine the relative proportions of each tautomer. These were chosen of representative of other peaks in the spectra. The C-1 (anomeric) carbon could not be used because these atoms are quaternary, and hence will always give smaller peaks than the others. The C-4 carbon could not be used as the C-4 chemical shift for both glucose anomers is the same, and hence they are not resolved.

Data Fitting Procedure: In each case it was assumed that the increase (or decrease) in the proportion of each tautomer in the mixture could be fitted by a first order reaction. Since there may be some small time delay either in getting the sample into the equipment (resulting in the reaction beginning before the apparent start of the experiment), or delay due to slow dissolution (resulting in the reaction beginning after the apparent start of the experiment) the experimental data was fitted to a first order equation with a variable time delay. The form of the equation for tautomers which increase in concentration is:

$$\text{for } t \le t_d\text{: } y = 0$$

$$\text{for } t > t_d\text{: } y = A\exp(-k(t - t_d))$$

Equivalent equations may be written for those forms which decrease in concentration. The variables are the time delay (t_d), the rate constant (k), and the equilibrium value (A). The dependent variable (y) may be the relative proportion of the tautomer (if known) or some other value such as a peak height on a spectrum. The addition of the time delay term is important to get a good fit for the data, especially in cases (such as fructose mutarotation) where the time constant for the mutarotation is of a similar magnitude to the possible time delays in sample preparation. This equation was fitted to the experimental data using the regression feature of the program SigmaPlot 5.0 (SPSS Inc.) using a tolerance of 1×10^{-4}.

Results and Discussion

F.T.-I.R.

F.T.-I.R. spectra were measured between approximately 1000 and 4000 cm^{-1}. The main regions of interest for glucose are 950 - 1050 cm^{-1} and 2700 - 3000 cm^{-1}. The second of these regions is particularly important for the study of mutarotation: when α-D-glucopyranose transforms to β-D-glucopyranose there is an increase in the intensity of absorption at 2880 cm^{-1} and a decrease in the intensity at 2940 cm^{-1}. This result is in agreement with Back and Polavarapu (1987), who suggested this change is due to C-H stretching vibrations during α-β isomerization. The rate of change in intensity in these regions was used to determine the mutarotation rate. It is easier to measure this change if the first spectra is subtracted from each of the later spectra, showing increasing and decreasing peaks at these points. Obviously if the mutarotation is measured starting from the β-form, then the peak at 2940 cm^{-1} will increase, and the peak at 2880 cm^{-1} will decrease. A similar change can be seen in the region 1040 - 1100 cm^{-1}, where the α-β transformation causes an intensity increase at 1080 cm^{-1} and an intensity decrease at 1040 cm^{-1} (with the opposite occurring for the β-α transformation). An example time dependent spectra (without subtraction) for the transformation β-glucose - α-glucose is shown in Figure 1a. An example spectra (with subtraction) for the transformation β-glucose - α-glucose is shown in Figure 1b. If mutarotation experiments are begun from both the α and β forms then the mutarotation equilibrium may be determined from the relative changes in intensity between the pure forms and the equilibrium.

The main spectral changes which occur during fructose mutarotation (which is mainly a conversion between the β-pyranose form and the two furanose forms) are a increase in intensity at

1044 cm^{-1} and decrease at 1085 cm^{-1}. Again this information can be used to determine the mutarotation rate, however since only the β-D-fructopyranose form is available as a pure compound it is not possible to determine the mutarotational equilibrium.

F.T.I.R. experiments were used to measure the mutarotation rates of solutions of 20, 30 and 40 g/100 ml fructose and glucose in water. The mutarotational equilibrium of glucose was also measured at 20 g/100 ml by measuring mutarotation from both α-glucose and β-glucose at this concentration. An attempt was also made to study the mutarotation of both glucose and fructose when they appear together in the same solution. For this, mutarotation was monitored in solutions of 10 g/100 ml fructose + 10 g/100 ml glucose, and 20 g/100 ml fructose + 20 g/100 ml glucose. This study was made difficult by the interference between glucose and fructose in the 1040 - 1100 cm^{-1} range, however it was possible to determine probable rate constants by fitting two constructive first order reactions to the data. It was not possible to completely determine which sugar had the faster rate, although it is very probably that glucose, which has a slower rate in pure solutions, should also have a slower rate in the mixed solution. This was confirmed by the later ^{13}C N.M.R. experiments. Results of the mutarotation rates found from the time dependent F.T.-I.R. spectra are shown in Table 1. No accuracy is given on the experiments measuring mutarotation rates in solutions containing both fructose and glucose: because of the constructive interference in the significant parts of the spectra the accuracy is likely to be poor.

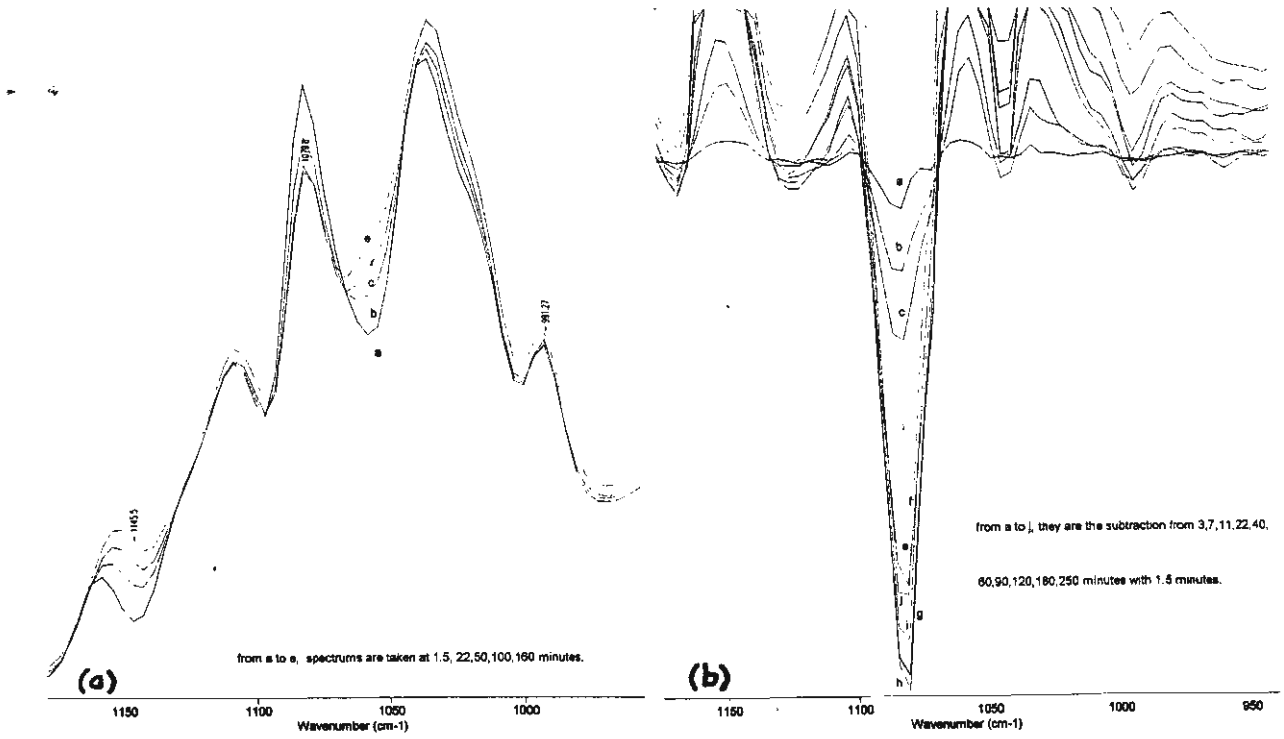

Figure 1: Spectra for the transformation of 20 g/100 ml β-D-glucopyranose in water at 22.5 °C. (a) Time dependent spectra (no subtraction) in the 950 - 1200 cm^{-1} range. Starting from a, the spectra are at 1.5, 22, 50, 100, and 160 min. (b) Time dependent spectra (with subtraction of 1.5 min spectra) in the 950 - 1200 cm^{-1} range. Starting from a, the spectra are 3, 7, 11, 22, 40, 60, 90, 120, 180, 250 min.

The mutarotational equilibrium was determined for glucose at the concentration of 20 g glucose/100 ml water through analysis of the results for both α- and β-glucose. It was found that the equilibrium was 60 % β-D-glucopyranose and 40 % α-D-glucopyranose. This result is not expected to be accurate to greater than 5 % because of the variation in the subtracted spectra's baseline. The result does display good agreement with the value of 62 % β-D-glucopyranose at 25 °C determined by Lee et al., (1969).

Table 1: Mutarotation in solutions of fructose and glucose determined by F.T.-I.R. spectroscopy.

Initial mixture	Fructose mutarotation rate (min^{-1})	Glucose mutarotation rate (min^{-1})
20 % (g/ml) β-D-fructopyranose in H_2O	0.45 ± 0.05	-
30 % (g/ml) β-D-fructopyranose in H_2O	0.34 ± 0.03	-
40 % (g/ml) β-D-fructopyranose in H_2O	0.30 ± 0.03	-
20 % (g/ml) β-D-glucopyranose in H_2O	-	0.039 ± 0.005
20 % (g/ml) α-D-glucopyranose in H_2O	-	0.034 ± 0.005
30 % (g/ml) α-D-glucopyranose in H_2O	-	0.017 ± 0.005
40 % (g/ml) α-D-glucopyranose in H_2O	-	0.017 ± 0.005
10 % β-D-fp + 10 % α-D-gp in H_2O	0.24	0.03
20 % β-D-fp + 20 % α-D-gp in H_2O	0.04	$< 1 \times 10^{-4}$

^{13}C N.M.R.

Two ^{13}C N.M.R. experiments were performed in order to confirm and extend the results of the F.T.-I.R. work. The first experiment used 50 mg α-**D**-glucopyranose in 0.7 ml of D_2O, and took scans at 4.5, 12, 40, and 1230 minutes, the last of these being to measure the equilibrium. This small number of data points gives the mutarotation rate constant as 0.0047 min^{-1}, which is in reasonably good agreement with the values determined by F.T.-I.R. when it is considered that the solution is lower concentration (approximately 7 g/100 ml) than the earlier solutions. The equilibrium was determined to be 60.6 % β-**D**-glucopyranose and 39.4 % α-**D**-glucopyranose, which is similar to the value in H_2O for both the literature and the F.T.-I.R. results. This value is expected to be accurate to with 1 %.

The second experiment uses the method given in the procedure section earlier, and measured the mutarotational rates and equilibria of both fructose and glucose in an aqueous solution of 10 g/100 ml fructose and 10 g/100 ml glucose. Because of the large number of time dependent data, and the large number of scans at equilibrium, the rate constant of glucose, and the equilibrium of both fructose and glucose could be determined very accurately. Unfortunately there is a time delay of approximately 2 minutes before the sample could start to be scanned, and a further 77 s before the end of the first set of scans: by the time the first set of scans was completed, the fructose mixture appears to be already at equilibrium. This means that the mutarotation rate of fructose cannot be determined accurately, although it is certainly possible to put a lower limit on it. An example of the ^{13}C N.M.R. spectra of the aqueous solution of glucose and fructose is shown in Figure 2. Each peak in the spectra may be assigned to a particular carbon in a particular tautomer easily by using the reference data of Bock and Pedersen (1983), although each peak in this spectra is shifted upfield by about 0.8 ppm. (The chemical shift measured is quite instrument dependent, and can vary 1 or more ppm on different machines). A plot of the mutarotation of fructose and glucose in the sample is given in Figure 3. It is easy to see that fructose is at equilibrium before the first sample, while glucose is not yet at equilibrium at 6000 s.

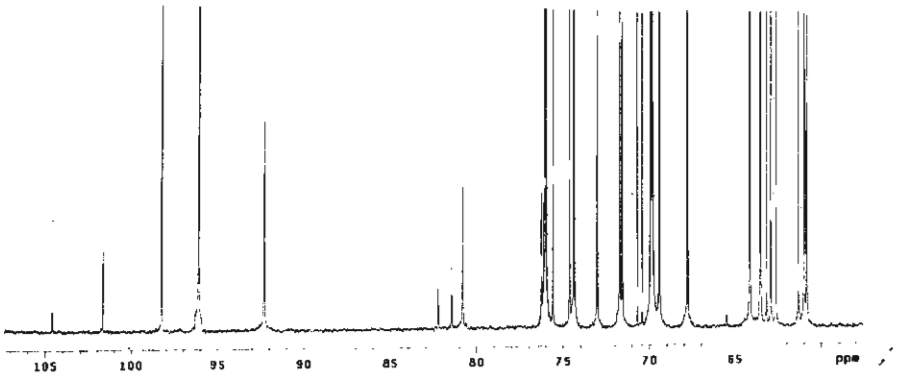

Figure 2: Spectra of glucose and fructose at equilibrium for 10 % (w/v) glucose and fructose.

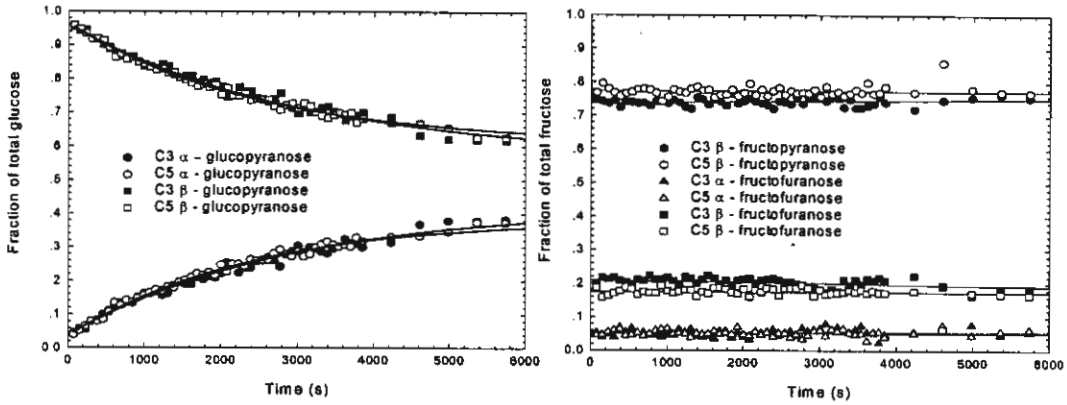

Figure 3: Mutarotation of glucose and fructose in an aqueous solution of 10 % (w/v) glucose and 10 % (w/v) fructose.

From this result it may be seen that the equilibrium for glucose is 40.4 % α-**D**-glucopyranose and 59.6 % β-**D**-glucopyranose, and the equilibrium for fructose is 5.5 % α-**D**-fructofuranose, 19.0 % β-**D**-fructofuranose, and 75.5 % β-**D**-fructopyranose. The values for both glucose and fructose agree very well with values in pure solutions, so it appears that fructose has no effect on the glucose equilibrium, and vice versa. The mutarotation rate of fructose in this experiment must be at least 0.5 min^{-1} in order to be at equilibrium with the first 3 minutes. This figure agrees reasonably well with the 0.45 min^{-1} found by the F.T.-I.R. experiments, remembering that the total sugar concentration is 20 g/100 ml in both cases. The mutarotation rate of glucose is determined to be 0.023 ± 0.002 min^{-1}. This result is interesting, in that it is much slower than the 20 g/100 ml glucose sample determined by F.T.-I.R. It appears that the mutarotation reactions of fructose and glucose are competitive, likely due to competition of H^+ needed for ring opening in the reaction, or OH^- required for ring closure. The reaction for fructose, which is faster than that of glucose and therefore probably more competitive for the H^+ and OH^-, is not slowed by the addition of glucose, however the glucose reaction does appear significantly slowed by the addition of fructose.

Conclusions

This study has shown that both F.T.-I.R. and ^{13}C N.M.R. can be used to some extent for monitoring rates and equilibrium of mutarotation reactions. The F.T.-I.R. technique is most suitable for reactions involving only two anomers, and even then only when both anomers are available in the pure form. If only one anomer or tautomer is available in pure form it is still possible to measure the mutarotation rate, but not the equilibrium. Glucose is a good example where F.T.-I.R. is a good technique for studying mutarotation, and fructose is a good example where it is not a good technique. In this case the F.T.-I.R. technique has the same drawbacks as polarimetry. ^{13}C N.M.R. is suitable for measuring both the rate and the equilibrium of any sugar, because the fraction of each tautomer may be measured directly from peak heights or integrals. Usually all, or nearly all carbon atoms are able to be resolved for each tautomer in the mixture, and given proper technique (for instance consideration of relaxation times and suitable numbers of scans) the results will be very accurate. The main difficulty is in getting the sample into the equipment and starting scans before the reaction has progressed too far.

The results of the experiments show that the mutarotation rate of fructose is much higher (about 1 order of magnitude) than that of glucose. It can also be seen that the mutarotation rate is dependent on the concentration of the sugar in the solution, with higher concentrations giving lower rates. This may be due to a lowering of the proportion of H^+ and OH^- to sugar molecules, which

slows the ring opening and closing required for the reaction. The equilibrium has been shown in this study (and earlier studies) to be largely independent of the sugar concentration.

The most interesting result in the current study relates to the effect sugars have on the mutarotation rates of other sugars. It appears the mutarotation reaction is competitive, whereby the mutarotation rate of glucose in solutions containing fructose is much lower than that of pure glucose solutions. This is not just a concentration dependence because it also occurs in solutions of equal total sugar concentration. This slowing of the mutarotation rate is extremely significant for industrial crystallizations, where the crystallization rate may be controlled by the rate of mutarotation (Flood et al., 1996; Flood et al., 1998). The current study was started to determine mutarotation rates likely to be involved in crystallizations of High Fructose Syrups, which are aqueous mixtures of glucose and fructose. It appears as if the crystallization of glucose from such mixtures will be extremely slow due to the low rates of glucose mutarotation which could be expected. More work is progressing in this lab on competitive mutarotation reactions, and their effect on crystallization.

Acknowledgments

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References

Angyal, S.J. and Bethell, G.S. 1976. Conformational Analysis in Carbohydrate Chemistry. III. The ^{13}C N.M.R. Spectra of the Hexuloses. Aust. J. Chem., 29: 1249-1265.

Back, D.M., Michalska, D.F. and Polavarapu, P.L. 1984. Fourier Transform Spectroscopy as a Powerful Tool for the Study of carbohydrates in Aqueous Solutions. Appl. Spectrosc., 38(2): 173-180.

Back, D.M. and Polavarapu, P.L. 1987. Vibrational assignments and isomerization rate-constants from time-dependent F.T.-I.R. spectra of sugars. Carbohydr. Res., 165: 173-187.

Barry, C.P. and Honeyman, J. 1952. Fructose and its derivatives. Adv Carbohydr. Chem., 7: 53-85.

Bock, K. and Pedersen, C. 1983. Carbon-13 magnetic resonance spectroscopy of monosaccharides. Adv. Carbohydr. Chem. Biochem., 41: 27-66.

Cockman, M., Kubler, D.G., Oswald, A.S., and Wilson, L. 1987. The mutarotation of fructose and the invertase hydrolysis of sucrose. J. Carbohydr. Chem., 6(2): 181-201.

Coxon, B. 1972. Proton magnetic Resonance Spectroscopy of Carbohydrates. Adv. Carbohydr. Chem. Biochem., 27: 7-83.

Flood, A.E., Johns, M.R., and White, E.T. 1996. Mutarotation of D-fructose in aqueous-ethanolic solutions and its influence on crystallization. Carbohydr. Res., 288: 45 - 56.

Flood, A.E., Johns, M.R., and White, E.T. 1999. Crystal Growth Rates and Dispersion for D-fructose from Aqueous Ethanol. J. A.I.Ch.E. (accepted for publication).

Hyvönen, L., Varo, P. and Koivistoinen, P. 1977. Tautomeric equilibria of D-glucose and D-fructose. J. Food Sci., 42(3): 652-659.

Nikolov, Z.L. and Reilly, P.J. 1983. Isothermal capillary column gas chromatography of trimethylsilyl disaccharides. J. Chromatogr., 254: 157-162.

Wilbur, D.J., Williams, C. and Allerhand, A. 1977. Detection of Furanose Anomers of D-Mannose in Aqueous Solution. Application of Carbon-13 Nuclear Magnetic Resonance Spectroscopy at 68 MHz. J. Am. Chem. Soc., 99: 5450-5452.

Yaylayan, V.A. and Ismail, A.A. 1992. Determination of the effect of temperature on the concentration of *keto* form of D-fructose by FT-IR spectroscopy. J. Carbohydr. Chem., 11(2): 149-158.

Yaylayan, V.A., Ismail, A.A., and Mandeville, S. 1993. Quantitative determination of the effect of pH and temperature on the *keto* form of D-fructose by FT IR spectroscopy. Carbohydr. Res., 248: 355-360.

Measurement and modeling of solid - liquid equilibrium in the system fructose + glucose + ethanol + water.

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ABSTRACT

Measurement and modeling of the solubility, and determination of preferred crystalline phases under different solvent and temperature conditions is essential information required at the beginning of crystallizer design. However this information is rarely available in the scientific literature, particularly for mixed solute and mixed solvent solutions. This paper describes methods and results for measuring and modeling solid-liquid equilibrium in the quaternary system fructose + glucose + ethanol + water at 30 and 40 °C. Experiments covered a range of solvent (ethanol + water) compositions from 40 weight percent ethanol to 80 weight percent ethanol. This range is suitable for determining solubilities under conditions likely to be found in alcoholic crystallization from high fructose syrups (HFS). The solubility results showed that addition of ethanol greatly reduces the solubility of both sugars over the entire range of compositions, as would be expected from the ternary systems glucose + ethanol + water and fructose + ethanol + water. The solubility also increases significantly with increasing temperature for all solvent compositions. The fructose + glucose + water system displays two invariant points, and thus three crystalline phases (glucose monohydrate, anhydrous glucose, and anhydrous fructose) over the range of solute compositions; however the system fructose + glucose + ethanol + water displays only one invariant point, where the preferred crystalline phase changes from anhydrous glucose to anhydrous fructose. It is believed that the ethanol in solution stabilizes the presence of water in the liquid phase with respect to the solid phase. The activity coefficient of the sugars in the solid phase could be determined from rigorous thermodynamic methods for both hydrated and anhydrous forms. The availability of solid-phase activity coefficients and solubility measurements at two temperatures allowed the system to be modeled using a UNIQUAC-type model.

1. INTRODUCTION

Fructose is a monosaccharide sugar that is widely used as a sweetener and food additive. The sweetness of its crystalline form is 1.8 times that of sucrose, although the sweetness of its non-crystallizing forms is not as high. Fructose is mostly sold as high fructose syrups (HFS) which are produced from starch. The starch is converted to glucose using alpha - amylase and glucoamylase, and the glucose syrup further reacted to a mixture of glucose and fructose using glucose isomerase. The equilibrium conversion of the last reaction is approximately 50 %, so the high fructose syrups have substantial quantities of glucose.

Crystalline fructose is currently produced from HFS by chromatographic separation to produce a 95 % (dry basis) fructose syrup, followed by either aqueous or aqueous – ethanolic crystallization. Fructose has extremely high solubility in aqueous solutions, and thus ethanol may be used to reduce the solubility of fructose, thus increasing yields and reducing solution viscosity in the crystallizer. Crystallization of these relatively pure fructose syrups is well understood, with significant research on the phase equilibrium and crystallization for aqueous solutions (Young *et al.*; 1952: Shiau and Berglund; 1987: Chu *et al.*; 1989), and aqueous ethanolic solutions (Flood *et al.*; 1996a: Flood *et al.*; 1996b: Flood *et al.*; 2000) already completed. Recently an investigation has begun which aims to crystallize and separate fructose and glucose directly from lower purity high fructose syrups: this research has required significant measurement and modeling of the solid liquid equilibrium in the system fructose + glucose + ethanol + water, which has been unavailable in the scientific literature.

Thermodynamic modeling of solid-liquid equilibrium has focused on two areas; the determination of activity coefficients at the solubility limit, and modeling of the liquid phase activity coefficients using available thermodynamic models. The first area has been solved for anhydrous crystalline forms for any solvent, and gives the well known equation:

$$\ln(\gamma_{sug} x_{sug}) = \left[\frac{-\Delta H}{R} + \frac{\Delta A - \Delta B T^0}{R} T_m + \frac{\Delta B}{2R} T_m^2\right]\left(\frac{1}{T} - \frac{1}{T_m}\right) + \frac{\Delta A - \Delta B T^0}{R} \ln\left(\frac{T}{T_m}\right) + \frac{\Delta B}{2R}(T - T_m) \quad (1)$$

The ΔA and ΔB terms are related to the difference in the heat capacity of the pure liquid and the pure solid, via $\Delta C_p = \Delta A + \Delta B \times (T - T^0)$.

For hydrated crystalline forms, Catté *at al.* (1994) have developed a suitable equation based on the dilution enthalpy of the sugar: unfortunately their equation is only useful in binary sugar – water systems, and is not suitable for use in this study. Glucose monohydrate was not found in solutions containing ethanol, and hence their equation was not required.

Most current research involving modeling of solid-liquid equilibrium of sugars involves the UNIQUAC method, or modifications of this method (Catté *et al.*; 1994: Peres and Macedo; 1996, 1997a,b,c), although a UNIFAC method has also been attempted (Catté *et al.*; 1995). The modified UNIQUAC model of Peres and Macedo (1997a) has proved to be successful in the modeling of sugar solubility in mixed solvents (Peres and Macedo; 1997a,b: Flood; 2000). The UNIQUAC models break up the activity coefficient into a combinatorial part and a residual part;

$$\ln(\gamma_i) = \ln(\gamma_i^C) + \ln(\gamma_i^R) \quad (2)$$

This study uses the modified UNIQUAC model proposed by the group of Macedo. The combinatorial part is:

$$\ln(\gamma_i^C) = \ln\frac{\varphi_i}{x_i} + 1 - \frac{\varphi_i}{x_i} \quad \text{with } \varphi_i \text{ defined as } \varphi_i = \frac{x_i Z_i}{\sum_j x_j Z_j}, \quad Z_i = (R_i)^{2/3} \quad (3)$$

The UNIQUAC equation for the residual part is given by:

$$\ln(\gamma_i^R) = Q_i\left[1 - \ln\left(\sum_j \theta_j \tau_{ji}\right) - \sum_j \frac{\theta_j \tau_{ij}}{\sum_k \theta_k \tau_{kj}}\right] \quad \text{with } \theta_i = \frac{x_i Q_i}{\sum_j x_j Q_j} \quad (4)$$

The parameters τ_{ij} (commonly known as Boltzmann factors) are given in the modified UNIQUAC method as:

$$\tau_{ij} = \exp\left(-\frac{a_{ij}}{T}\right) \quad \text{where the } a_{ij} \text{ are fitting parameters, } a_{ij} = a_{ij}^0 + a_{ij}^T (T - T^0) \quad (5)$$

Structural parameters for the molecules involved, and physical properties required for the solubility equation are given in Table 1.

Table 1. UNIQUAC structure parameters and physical properties for fructose, glucose, ethanol, and water.

Species	R_i	Q_i	Melting Temp. (K)	Enthalpy of Fusion (J/mol)	ΔA	ΔB
D-fructose	8.1529	8.004	378.15	33,000	320.0	0
D-glucose	8.1528	7.920	421.15	32,000	140.0	0
Ethanol	2.5755	2.588	a	a	a	a
Water	0.9200	1.400	a	a	a	a

[a] Not required for the present study.

2. SOLUBILITY

Determining the solubility of solutes in solution is vitally important for crystallization design, however it is rare that accurate solubility data for industrially important systems is published. This is partly because the solubility is a function of many variables including temperature, solvent, co-solutes, and impurities. This section describes experimental methods to determine solubility in the mixed solute – mixed solvent system fructose + glucose + ethanol + water at 30 and 40 °C.

2.1 Solubility: Methods

The method used in the current study is the same as used in a previous study (Flood and Puagsa; 2000), of which a short discussion will be given here. The chemicals, D-(−)-fructose, D-(+)-glucose anhydrous (both ACS grade), and ethanol anhydrous (99.9 % v/v, for analysis) were obtained from Carlo Erba Reagenti (Milan) and were used without further purification. Solutions were made by dissolving a quantity of fructose (below the solubility limit) in a known solution of ethanol + water. Glucose was added in at least 50 % excess of that needed for saturation. A range of experiments was performed to determine solubility between the limits given by the systems fructose + ethanol + water (see Flood *et al.*; 1996) and glucose + ethanol + water (Bockstanz *et al.*; 1989, and Peres and Macedo; 1997a). Solubility was approached using sealed glass Schott bottles held in a constant temperature (30 or 40 ± 0.2 °C) orbital shaking bath operating at 100 rpm (200 stroke). After 24 h the refractive index was measured every 6 h to test for equilibrium. Equilibrium was achieved within 7 days for all determinations.

The fructose and glucose concentration in equilibrated samples was determined by a HPLC method. In general it is preferable to measure solute concentrations in sugar systems using a gravimetric method such as the dry substance determination procedure (BSES; 1991), however in this study the determination of two solutes was required, and hence a separative method was preferred. Removal of the ethanol in solution was performed since it could interfere with the sugar peaks on the chromatogram and also partially react with the sugars during storage. Samples of approximately 1 mL were weighed, partially dried at room temperature for 17 h to remove the bulk of the ethanol, and then re-weighed. After drying the samples were diluted to approximately 1 % (w/v) by addition of a known amount of distilled water. All weights were determined to ± 0.1 mg. Diluted samples were filtered through a 0.45 μm syringe filter and then injected into a 250 mm × 4 mm Aminex HPX-87C (Biorad, Bangkok) column using a water mobile phase at a flow rate of 0.3 mL/min. The column temperature was 80 °C, and detection was with a diode array detector measuring UV at 192 nm.

2.2 Solubility: Results and Conclusions

Results for the solubility of fructose and glucose in solutions of ethanol and water at 30 and 40 °C are given as ternary diagrams in Figure 1. Two axes give concentrations of glucose and fructose, and the third gives the concentration of the solvent, which is a known mixture of ethanol and water. The lines on the ternary diagram represent lines of constant solvent composition, in this case 0, 40, 60, and 80 weight % ethanol.

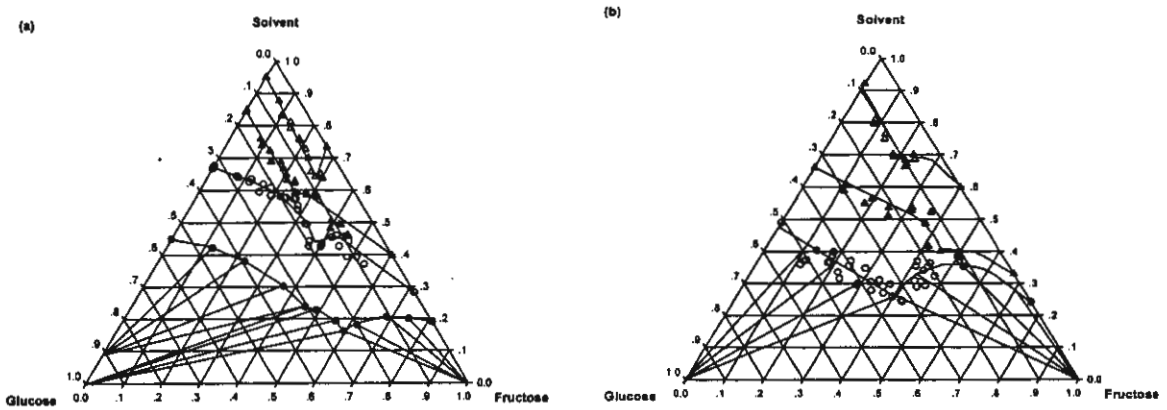

Figure 1. *Solubility and preferred crystal phase in the system fructose + glucose + ethanol + water at 30 °C (a) (see Flood and Puagsa; 2000 for tabulated solubility data at 30 °C), and 40 °C (b). Solvent compositions:* ● 0 weight % ethanol (aqueous, from Kelly; 1954); ○ 40 weight percent ethanol; ▲ 60 weight % ethanol; △ 80 weight % ethanol. Phases are indicated at 0 weight % ethanol at 30 °C (glucose monohydrate is ~ 9.1 % water) and 40 weight percent ethanol at 40 °C.

The solubility results show that the solubilities of fructose and glucose are both strong functions of temperature and solvent composition. The solubility of both sugars increases significantly over the 10 °C temperature range used in this study, and decrease markedly with increasing ethanol content in the solvent. In general, the minimum total sugar content at a particular temperature and solvent composition is where glucose appears as the

only solute, and the maximum is where fructose appears as the only solute. An exception to this is at 80 % ethanol, 30 °C, where there is a noticeable "salting in" effect: the maximum sugar content of 0.36 g sugar/g of solution is higher than the solubility of fructose in ethanol + water (0.266 g of fructose/g of solution) and that of glucose (0.050 g of glucose/g of solution).

It is also noticeable that while the fructose + glucose + water system has two invariant points (at 30 °C), the fructose + glucose + ethanol + water system appears to have only one invariant point for all solvent compositions studied. This will be discussed in greater detail in the crystal phase determination section.

3. CRYSTAL PHASE DETERMINATION

Crystal phase determination at different points of the phase diagram is important for industrial crystallizer design, as it enables the designer to choose a set of conditions resulting in the preferred product. The crystalline phase of the product will determine product purity (such as if the solvent is part of the crystal structure), crystal shape (which will influence the crystal – liquor separation process), and decomposition and melting temperatures. Sugar solutes may crystallize in a range of hydrate forms: the most likely forms for fructose and glucose are α-D-glucose anhydrous, α-D-glucose monohydrate, β-D-glucose anhydrous, D-fructose anhydrous (β-D-fructopyranose), D-fructose dihydrate, and D-fructose hemihydrate. Crystal phases in the system fructose + glucose + water were determined by Kelly (1954), and these phases are shown in Figure 1(a). β-D-fructopyranose is the preferred crystalline phase in the system fructose + ethanol + water between (at least) 30 to 50 °C.

3.1 Crystal Phase Determination: Method

Supersaturated solutions were made based on the results of the solubility work. (The supersaturated region for a particular solvent composition is the region below the solubility line in this figure). Points were chosen on either side of possible invariant points. Solutions were produced at 60 °C to dissolve the solute completely. After dissolution the samples were cooled to 40 °C, and seeded with a small amount of all likely preferred crystalline phases. Seeding was performed because sugar solutions have very large metastable regions and primary nucleation was unlikely at 20 °C subcooling. Seeding also resulted in large sized crystals, suitable for easy separation from solution.

Product crystals were vacuum filtered and then dried at 70 °C for 24 h, and then stored over silica gel. Higher temperatures were not used due to the melting point of glucose monohydrate (83 °C) and the decomposition temperature of fructose (~ 75 °C). It is believed that drying at this temperature could not alter the crystal phase from glucose monohydrate to anhydrous glucose. After drying the crystal phase was determined using X-ray diffraction (XRD). XRD was performed on a Bruker D5005 diffractometer using a copper anode. The 2θ range was 5 to 60°, using a step of 0.020°, and a step time of 0.6 s. In all cases there was excellent agreement between the intensity spectrum of the unknown and the intensity spectrum of the related compound in the XRD library.

3.2 Crystal Phase Determination: Results and Conclusions

The study of Kelly (1954) determined three phases in the system fructose + glucose + water at 30 °C. α-D-glucose monohydrate crystallizes at low fructose contents, up to the first invariant point. This crystalline form of glucose is approximately 9.1 % water, as shown on Figure 1(a). At high fructose concentrations (between the second invariant point and the glucose axis) anhydrous fructose is the preferred crystalline phase, and anhydrous glucose is the preferred crystalline phase between the two invariant points. The phases are shown on Figure 1(a).

In the system fructose + glucose + ethanol + water there is only one obvious invariant point, which suggests that only two distinct crystalline phases will be apparent. However, it was possible that a second invariant point still exists but was not evident, and hence phase determination was performed. At 40 °C, with a solvent composition of 40 % ethanol, it was determined that anhydrous glucose was the only crystalline phase to the left of the invariant point. To the right of the invariant point only anhydrous fructose crystallizes. This shows that (at least under these solvent conditions) there is only one invariant point in the system fructose + glucose + ethanol + water. Preferred crystalline phases are shown for 40 °C, 40 % ethanol in Figure 1(b). It appears that ethanol lowers the phase transition temperature (glucose monohydrate to anhydrous glucose) which is around 90 °C in aqueous solutions. This is probably due to ethanol increasing the affinity of water in the liquid phase.

4. THERMODYNAMIC MODELING

The modified UNIQUAC model was used to model solubility in the quaternary system fructose + glucose + ethanol + water. Two related ternary systems, fructose + ethanol + water and glucose + ethanol + water, have already been completed (Peres and Macedo, 1997b: Flood, 2000). The glucose-water, water-glucose, fructose-water, water-fructose, ethanol-water, and water-ethanol interaction parameters may be taken from these studies. The solubility of the third relevant ternary system, fructose + glucose + water, has not been studied since there is data only at 30 °C for this system. This is not significant; the fructose-glucose and glucose-fructose interaction parameters are set to zero in these studies. The relevant interaction parameters (from the earlier studies) are given in Table 2.

Table 2. Modified UNIQUAC interaction parameters for the system fructose + glucose + ethanol + water. (The first row is a_{ij}^0 and the second is a_{ij}^T. Only sugar-water parameters assume linear temperature dependence).

i / j	D-fructose	D-glucose	Ethanol	Water
D-fructose	0	0	-8.5681[b]	58.5738[b]
	0	0	0	0.7329[b]
D-glucose	0	0	53.5398[a]	-68.6157[a]
	0	0	0	-0.0690[a]
Ethanol	159.6180[b]	136.2574[a]	0	207.4055[a]
	0	0	0	0
Water	97.3045[b]	96.5267[a]	-78.5272[a]	0
	0.6761[b]	0.2770[a]	0	0

[a] From Peres and Macedo (1997b). [b] From Flood (2000).

This model fits the experimental data very well along the glucose axis (the ternary system fructose + ethanol + water) and along the fructose axis (the ternary system glucose + ethanol + water) as the model parameters were optimized using this experimental data in earlier studies. It is also likely that the model must fit the data reasonably well within the vicinity of these axes, where the second solute may be considered as only a low concentration impurity. Therefore it was decided a suitable (and quick) test of the model is whether the activity coefficients given by the model agree with the activity coefficients given by equation (1) at the invariant point for different temperatures and solvent compositions, where the error is likely to be close to the maximum error. There is one invariant point at each solvent composition (40, 60, and 80 weight percent ethanol) for each temperature (30 and 40 °C), and hence 6 points were tested. The results are given in Table 3.

Table 3. Comparison between the Modified UNIQUAC activity coefficient, and the experimental activity coefficient (determined from equation (1)) for the invariant points.

Temperature (K)	Solvent Composition (weight % ethanol)	Activity coefficients from experiment		Activity coefficients from Modified UNIQUAC	
		D-Fructose	D-Glucose	D-Fructose	D-Glucose
303.15	40.0	2.101	1.705	1.721	0.461
303.15	60.0	3.413	2.050	2.547	0.865
303.15	60.0	2.672	3.596	3.009	1.999
313.15	40.0	2.007	0.804	2.208	0.603
313.15	60.0	2.069	2.028	2.858	1.035
313.15	80.0	5.136	3.691	2.629	1.855

It can be noted that the Modified UNIQUAC model is quite poor at estimating the activity coefficients at the invariant points. This is particularly so for glucose, where the activity coefficient is significantly underestimated at all data points. This is despite the model predicting activity coefficients in the ternary sugar + ethanol + water systems very well. It is also clear that better predictions could be produced if the Modified UNIQUAC model was parameterized using the results of the current study. Further work is in progress to optimize the model in terms of the current quaternary solubility data, and to determine whether the re-optimized model still fits the ternary systems well. This work does call into question the ability of UNIQUAC type models to extrapolate from simple solutions to more complex ones. It is the author's own experience that UNIQUAC models do not extrapolate to temperatures outside the range of data used to optimize the model particularly well.

5. NOMENCLATURE

a_{ij}	UNIQUAC interaction parameter (K)
C_p	Heat capacity (J/mol.K)
Q	UNIQUAC group area parameter
R	UNIQUAC group volume parameter
R	Universal gas constant (J/mol.K)
T	Absolute temperature (K)
T_m	Melting temperature (K)
$T°$	Reference temperature, set to 298.15 K
x	Mole fraction of component
Z	Parameter for UNIQUAC model

Greek

γ	Activity coefficient
ΔA, ΔB	Temperature dependencies
ΔC_p	Pure liquid heat capacity minus pure solid heat capacity (J/mol.K)
ΔH_f	Enthalpy of fusion (J/mol)
φ	Molecular volume fraction
θ	Molecular surface area fraction
τ	UNIQUAC parameter

Subscripts

i, j, k	Property of component *i, j, k*
sug	Property of a sugar

Superscripts

C	Combinatorial
R	Residual

6. REFERENCES

Bockstanz, G. L., Buffa, M. and Lira, C. T. (1989). *Solubilities of α-Anhydrous Glucose in Ethanol/Water Mixtures.* J. Chem. Eng. Data **34 (3)**, 426–429.

BSES (Bureau of Sugar Experiment Stations) (1991). *Total Solids (Dry Substance) Determination in Mill Products,* in *Standard Laboratory Manual for Australian Sugar Mills, Volume 2.* Method 19. BSES, Brisbane.

Catté, M., Dussap, C., Achard, C. and Gros. J. (1994). *Excess properties and solid-liquid equilibria for aqueous solutions of sugars using a UNIQUAC model.* Fluid Phase Equilibria **96,** 33–50.

Catté, M., Dussap, C. and Gros. J. (1995). *A physical Chemical UNIFAC model for aqueous solutions of sugars.* Fluid Phase Equilibria **105,** 1–25.

Chu, Y. D., Shiau, L. D. and Berglund, K. A. (1989). *Effects of Impurities on Crystal Growth in Fructose Crystallization.* J. Crystal Growth **97,** 689–696.

Flood, A. E., Addai-Mensah, J., Johns, M. R. and White, E. T. (1996a). *Refractive Index, Viscosity, Density, and Solubility in the System Fructose + Ethanol + Water at 30, 40, and 50 °C.* J. Chem. Eng. Data **41 (3),** 418–421.

Flood, A. E., Johns, M. R. and White, E. T. (1996b). *Mutarotation of D-fructose in aqueous–ethanolic solutions and its influence on crystallization.* Carbohydr. Res. **288,** 45–56.

Flood, A. E., Johns, M. R. and White, E. T. (2000). *Crystal Growth Rates and Dispersion for D-Fructose from Aqueous Ethanol.* AIChE. J. **46 (2)** 239–246.

Flood, A. E. and Puagsa, S. (2000). *Refractive Index, Viscosity, and Solubility at 30 °C, and Density at 25 °C for the System Fructose + Glucose + Ethanol + Water.* J. Chem. Eng. Data **45 (5),** in press.

Flood, A.E. (2000) *Modeling Solid-Liquid Equilibrium in the System Fructose + Ethanol + Water using the Modified UNIQUAC Model.* Fluid Phase Equilibria. [In preparation].

Kelly, F. H. C. (1954). *Phase Equilibria in Sugar Solutions. IV. Ternary System of Water - Glucose - Fructose.* J. Appl. Chem. (London) **4,** 409–410.

Peres, A. M. and Macedo, E. A. (1996). *Thermodynamic properties of sugars in aqueous solutions: correlation and prediction using a modified UNIQUAC model.* Fluid Phase Equilibria **123,** 71–95.

Peres, A. M. and Macedo, E. A. (1997a). *Measurement and modeling of Solubilities of D-Glucose in Water/Alcohol and Alcohol/Alcohol Systems.* Ind. Eng. Chem. Res. **36,** 2816–2820.

Peres, A. M. and Macedo, E. A. (1997b). *Phase Equilibria of D-glucose and sucrose in mixed solvent mixtures: Comparison of UNIQUAC-based models.* Carbohydr. Res. **303,** 135–151.

Peres, A. M. and Macedo, E. A. (1997c). *Solid-Liquid Equilibrium of Sugars in Mixed Solvents.* Entropie **202/203,** 1–5.

Shiau, L.D. and Berglund, K. A. (1987). *Growth Kinetics of Fructose Crystals formed by Contact Nucleation.* AIChE J., **33,** 1028–1032.

Young, F. E., Jones, F. T. and Lewis, H. J. (1952). *D-Fructose – Water Phase Diagram.* J. Physical Chem., **56,** 1093–1096.

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Comparing Analytical and Approximate Expressions for the Driving Force in Crystal Growth of D-Fructose.

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Abstract

The relative supersaturation driving force, $(c - c^*)/c^*$, and an approximation to the chemical potential driving force, $\ln(x/x^*)$, are compared to the chemical potential driving force for the crystal growth of D-fructose from aqueous and aqueous ethanol solutions. The activity coefficients required for the chemical potential driving force have been calculated from available UNIQUAC models of the systems. It is believed that these models can accurately predict activity coefficients as they are optimized using a wide range of experimental data. It can be shown that both the approximate driving forces are poor representations of the chemical potential driving force, largely because the solutions are non-ideal, and the activity coefficients vary strongly over the composition range of the supersaturated solutions. This does not significantly affect the order of the power law model relating the crystal growth rate to the driving force in aqueous solutions; the main result of the approximation is a change in the crystal growth rate constant, k_G. The conversion to the more accurate driving force for crystal growth of D-fructose from aqueous ethanol solutions changes both the crystal growth rate constant, k_G, and the exponent, n.

PACS: 82.60.L; 82.20; 81.10.A;

Keywords: A1. Driving force; Activity; Supersaturated solutions; A2. Growth from solutions; B1. Fructose

1. Introduction

It is well known that phase transition occurs when there is a difference in chemical potential between the phases, with the phase transition serving to equalize the chemical potentials, and thus bring the phases to equilibrium. The 'true' driving force for crystallization from solution is therefore the difference in chemical potential between the crystallizing molecule in the supersaturated solution and in

experiment, when the relative supersaturation was around 1.0. It is possible that this is due to inaccuracies in the relative supersaturation driving force. The aim of this study is to compare approximate measures of the driving force to the chemical potential driving force for the crystal growth of D-fructose from aqueous and aqueous ethanol solutions, and to determine whether inaccuracies in the driving force are responsible for unusual features of the system.

2. Experimental procedure

Crystal growth rate – supersaturation data for crystal growth of D-fructose from aqueous [7] and aqueous ethanol [8,9] solutions were taken from the literature. The concentration range of this data determined the range of comparison for the three driving forces in the study. The solubility data required for the calculations were determined from the studies of Young et al. [10] and Flood et al. [11].

The activity coefficients required for the study were determined from modified-UNIQUAC models [5] of the systems being investigated. The modified UNIQUAC model of the system D-fructose – water has been developed by Peres and Macedo [5], and is optimized based on a wide range of thermodynamics data, including water activity, osmotic coefficient, boiling point, freezing point, vapor pressure, and solubility. The modified UNIQUAC model of the ternary system D-fructose – ethanol – water [12] is based only on solubility (since other thermodynamic data on this system is not available), however appears consistent with other thermodynamic data available in the binary system. The solubility data used in the model covers a reasonably wide range of temperature (between 24 and 50 °C), and a wide range of D-fructose composition (mass fractions between 0.035 and 0.87).

Mole fraction units were used in the calculation of the chemical potential driving force, σ_a, as these units are required in the UNIQUAC models of the activity coefficients. The same units were used for the simplified driving force, σ_l, which assumes constant activity coefficients. In the two crystal growth studies investigated, the relative supersaturation is defined in terms of mass fraction concentrations, and this is how the relative supersaturation is defined in the current study.

3. Results and discussion

3.1. Driving force in the D-fructose – water system

the crystalline state [1,2]. In the chemical engineering literature the driving force for crystallization is usually presented as the supersaturation, ($c - c^*$), or more commonly the relative supersaturation, ($c - c^*)/c^*$. In some cases this simplification may be reasonably accurate, however in many cases, particularly those with high supersaturation and where the activity coefficient varies strongly with concentration, the approximation is poor.

The difficulty in using the chemical potential driving force has been that in many cases the activity coefficient of the solute was not known at the equilibrium point, or at the solution concentration. These difficulties are no longer a significant problem. The activity coefficients of the solute at solid-liquid equilibrium may be determined for anhydrous or hydrated crystals if a few simple thermodynamic variables are known. The activity coefficient in the supersaturated solutions may be predicted from thermodynamic models (UNIQUAC, for example) optimized using solubility and other thermodynamic data. Thermodynamic models are already available for a range of systems, including sugars [3-6].

The chemical potential driving force may be given by the equation

$$\sigma_a = \ln \frac{\gamma c}{\gamma^* c^*} \tag{1}$$

In the absence of activity coefficient data it is often assumed that the activity coefficient of the supersaturated solution is not significantly different to that of the saturated solution. This leads to the approximate driving force

$$\sigma_I = \ln \frac{c}{c^*} \tag{2}$$

The approximate driving force may be simplified to the relative supersaturation if the value of the driving force is very small. The relative supersaturation is defined as

$$\sigma_c = \frac{c - c^*}{c^*} \tag{3}$$

The concentrations represented by c and c^* may be in any consistent units, however they must agree with the concentrations used to determine the activity coefficients. A good discussion of the use of these equations is given in the review of Garside [2].

Crystal growth kinetics based on a relative supersaturation driving force have been published for crystal growth of fructose from aqueous [7] and aqueous-ethanol solutions between 24 and 50 °C [8,9]. Unusual growth kinetics were found in the fructose – ethanol – water system in the initial times of the

Activity coefficients, and activity coefficient ratios (γ_F/γ_F^*) for D-fructose are shown as functions of the mole fraction at 30, 40 and 50 °C in Fig. 1. The range of the data in Fig. 1 is between the solubility limit and the maximum supersaturation used by the study of Shanks and Berglund [7]. While the activity coefficients do vary in this concentration range, the activity coefficient is never more than about four percent larger than in the corresponding saturated solution.

Fig. 2 shows a comparison between the approximate driving forces, σ_I and σ_C, and the chemical potential driving force σ_a. It is clear that neither driving force is close to the analytical driving force (as shown by the 45 degree line), and that the relative supersaturation is significantly worse than the constant activity coefficient approximation. The inaccuracy in the relative supersaturation driving force is largely due to the conversion between mass fraction and mole fraction units. The relative change in concentration between saturated and supersaturated solutions is larger in mole fraction units, and it is also likely that the mass based activity coefficients may vary more strongly than the mole fraction based activity coefficients in this range.

Fortunately there is a linear relationship between the true and the approximate driving forces, and this relationship is a very weak function of temperature, especially for the constant activity coefficient approximation.

The original crystal growth data of Shiau and Berglund [7] plotted against both the relative supersaturation and the chemical potential driving force is shown in Fig. 3. The original data was fitted to a model

$$\overline{G} = A\exp(-E_A / RT)\sigma_C^{1.25} \tag{4}$$

where 1.25 was determined as the optimum growth rate exponent for the experimental data. This model has been plotted with appropriate pre-exponential and activation energy terms in Fig. 3. When the crystal growth rate data is re-plotted against the chemical potential driving force, the optimum growth rate exponent is reduced only slightly to 1.226. It is clear that the change in the driving force has not changed the growth rate exponent of the power law model ($G = k_G\sigma^n$), only the growth rate constant, k_G. The exponent is most significant in the power law model, as it is often used to distinguish between different

mechanisms, or models of crystal growth. Thus it seems reasonable to use either of the two approximations of the driving force in crystal growth studies of D-fructose from aqueous solutions.

3.2. Driving force in the D-fructose – ethanol – water system

The crystal growth rate studies in aqueous ethanol solutions covered a range of supersaturations (up to a relative supersaturation of one), ethanol/solvent ratios (75 percent ethanol – 90 percent ethanol) and temperatures (24 to 40 °C). This study will only consider a fraction of the results at ethanol/solvent ratio equal to 90 percent, as an example of the behavior of the larger system. Activity coefficients and activity coefficient ratios in supersaturated solutions for the system fructose – ethanol – water are shown in Fig. 4. The maximum supersaturation in these figures corresponds to a relative supersaturation of 1.0 at each temperature; this is the maximum supersaturation used in the earlier crystal growth study. Non-nucleating batch experiments can be performed at relatively high supersaturations in this system, since the fructose solubility is very low in aqueous ethanol solutions, and the metastable limit is large. The concentration ranges for the three temperatures overlap, due to the large range of supersaturation in these experiments.

It may be clearly seen that the activity coefficients in these solutions are strong (and non-linear) functions of the fructose mole fraction, but have weak dependence on temperature in this range. The infinite dilution activity coefficient for fructose in aqueous ethanol solutions is very large, around 20, and rapidly drops between infinite dilution and a mole fraction of ten percent. The activity coefficients reduce to between 47 and 62 percent of the saturation values at a relative supersaturation of one, suggesting that the approximate driving forces will be quite inaccurate.

Fig. 5 shows the relationship between the true and the approximate driving forces for crystal growth of D-fructose from aqueous ethanol solutions. In this system the approximate driving forces are significantly (potentially an order of magnitude) larger than the true driving force. The two approximate measures of the driving force are very similar for this system, unlike growth from aqueous systems where the constant activity coefficient driving force is significantly better than the relative supersaturation. The non-linear relation between the activity coefficients and the mole fraction also leads to a non-linear

relation between the approximate and analytical driving forces; thus there is no simple proportionality between the relative supersaturation and the true driving force.

Crystal growth rates are plotted against the true driving force and the relative supersaturation driving force in Fig. 6. The power law functions relating crystal growth to the relative supersaturation have exponents between 4.9 and 7.2; these exponents are highly unlikely based on accepted models of crystal growth. The explanation given in the original study is that the crystals are poisoned by an impurity in the solution, probably a reaction product of fructose and ethanol, and this causes the crystal growth rate to quickly decrease to a low value.

The true driving force in the experiments is significantly less than the relative supersaturation would suggest. The change to the true driving force increases the optimum power law exponent in these experiments, and it is clear that the unusual growth kinetics are not due to the inaccurate driving force; it is likely that the poisoning of the crystal surface is responsible.

4. Conclusions

The approximate driving forces (in particular the relative supersaturation) for fructose crystal growth from aqueous and aqueous-ethanol solutions is significantly different to the chemical potential driving force; in the worst cases there is an order of magnitude difference between the two measures. In aqueous-ethanol solutions, this is due to the system having low solubility, large driving forces (due to a large metastable limit), and a strong dependence between the activity coefficient and solute concentration. In aqueous solutions the conversion between relative supersaturation on a molar basis and a mass basis is responsible for most of the error. This suggests that the mass basis activity coefficients vary reasonably strongly with concentration at the experimental concentrations.

There is a linear relationship between the approximate and true driving forces in crystal growth from aqueous solutions, which leads to a difference in only the crystal growth rate constant in the power law models, not the exponent. This suggests that relative supersaturation is still a suitable driving force for crystal growth of D-fructose from aqueous solutions. In aqueous ethanol solutions the relationship between the approximate and true driving forces is strongly non-linear, suggesting that the relative supersaturation is not an appropriate driving force for crystal growth of D-fructose from aqueous ethanol

solutions. Despite the conclusions here, a change to a true driving force does not help to explain the unusual growth kinetics in the aqueous ethanol system. The original conclusion that the crystals are poisoned by an impurity formed in the solution are probably correct, as evidenced by no power law model fitting the data very well for either measure of the driving force.

5. Notation

A	pre-exponential term in the growth rate model, m/s
c	solute concentration, mass fraction
c^*	equilibrium solute concentration, mass fraction
E_A	activation energy, J/mole
G	linear crystal growth rate, m/s
k_G	crystal growth rate constant, m/s
n	exponent in the power law model, dimensionless
R	universal gas constant, J/mole·K
T	temperature, K
x	mole fraction of solute, dimensionless
γ	activity coefficient of the solute, dimensionless
γ^*	activity coefficient of the solute at equilibrium, dimensionless
σ_c	relative supersaturation, dimensionless
σ_l	driving force assuming constant activity coefficients, dimensionless
σ_a	driving force based on difference in chemical potential, dimensionless

References

[1] A. van Hook, Ind. Eng. News 36 (1949) 1042.

[2] J. Garside, in G. R. Youngquist (Ed.). Advances in Crystallization from Solutions, AIChE Symp. Ser. 80 (1984) 23.

[3] M. Catté, C.-G. Dussap, C. Achard, J.-B. Gros, Fluid Phase Equilibria 96 (1994) 33.

[4] M. Catté, C.-G. Dussap, J.-B. Gros, Fluid Phase Equilibria 105 (1995) 1.

[5] A. M. Peres, E. A. Macedo, Fluid Phase Equilibria 123 (1996) 71.

[6] A. M. Peres, E. A. Macedo, Carbohydr. Res. 303 (1997) 135.

[7] L. D. Shiau, K. A. Berglund, AIChE J. 33 (1987) 1028.

[8] A. E. Flood, Thesis, U Queensland, 1996.

[9] A. E. Flood, M. R. Johns, E. T. White, AIChE J. 46 (2000) 239.

[10] F: E. Young, F. T. Jones, H. J. Lewis, J. Phys. Chem. 56 (1952) 1093.

[11] A. E. Flood, J. Addai-Mensah, M. R. Johns, E. T. White, J. Chem. Eng. Data 41 (1996) 418.

[12] A. E. Flood, Fluid Phase Equilibria (submitted for publication).

Figure captions.

Fig. 1. Dependence of the D-fructose activity coefficient, and activity coefficient ratio, on the mole fraction of D-fructose (in aqueous solution).

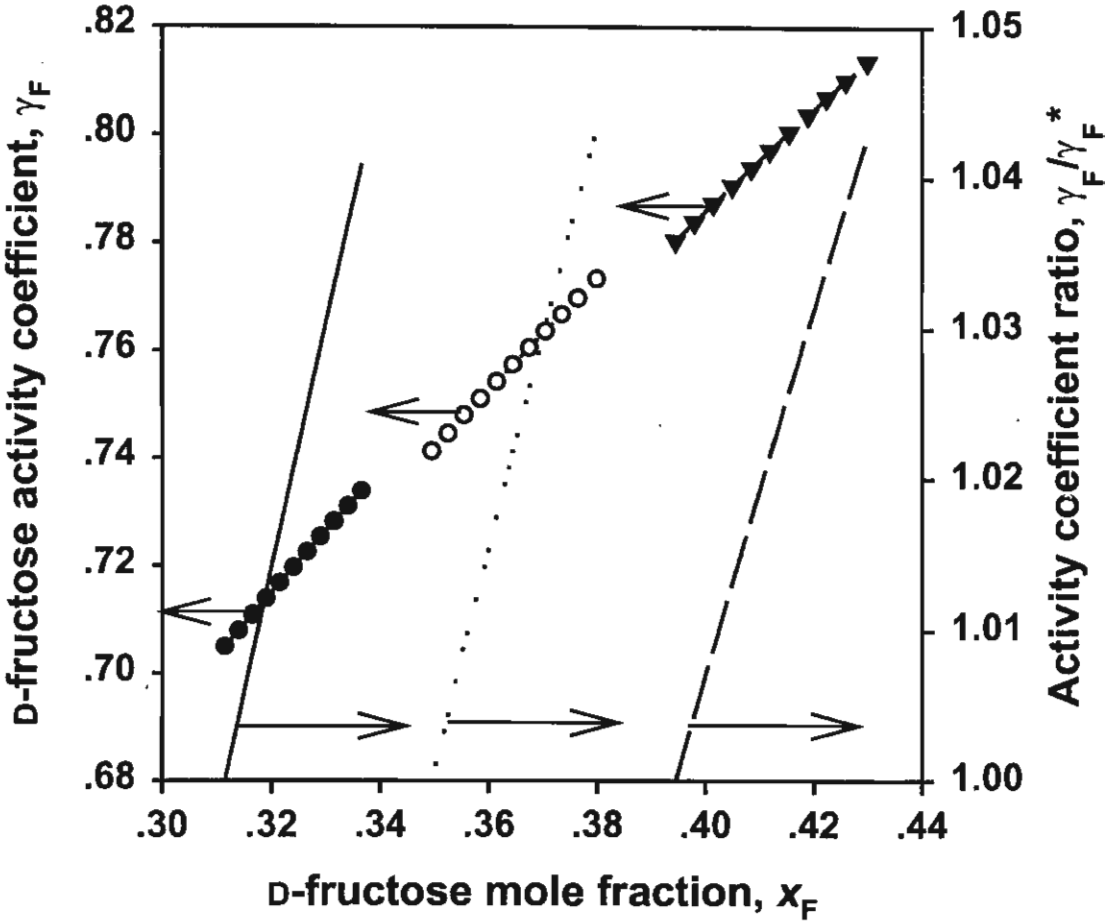
Fig. 2. Comparison of the approximate driving forces and true driving force of crystal growth of D-fructose from aqueous solution.

Fig. 3. Crystal growth rates of D-fructose from aqueous solution as a function of the relative supersaturation and the chemical potential driving force.

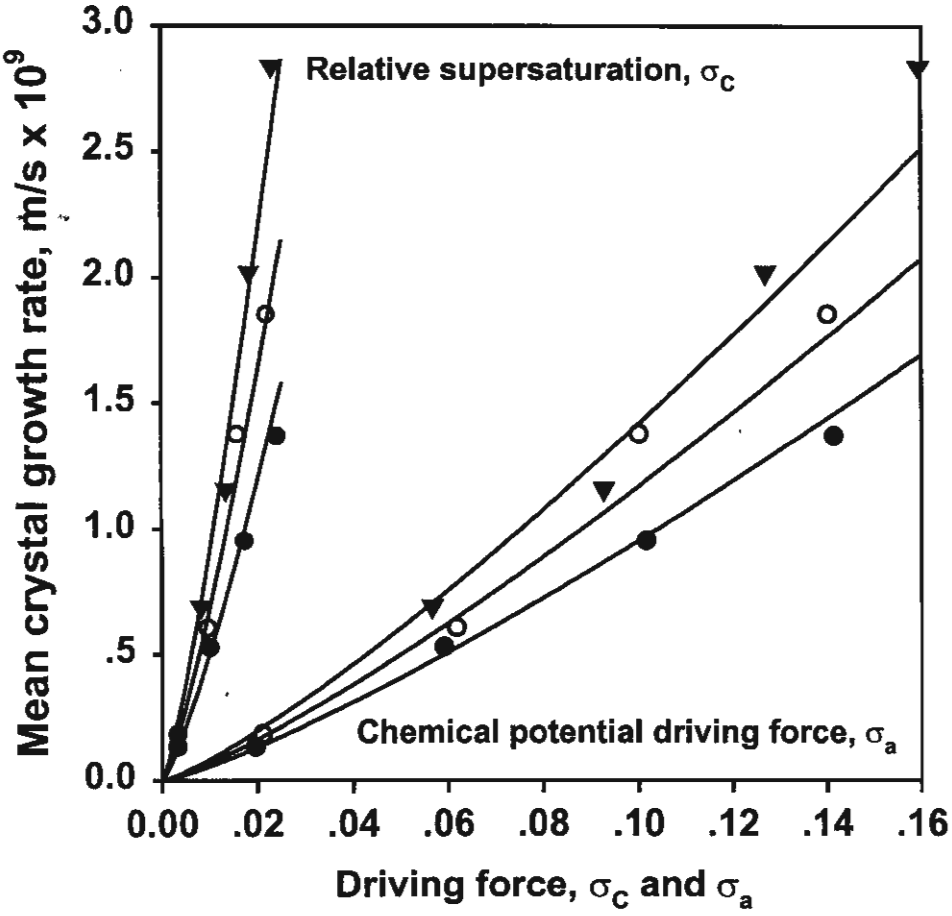
Fig. 4. Dependence of the D-fructose activity coefficient, and activity coefficient ratio, on the mole fraction of D-fructose (in aqueous ethanol solution).

Fig. 5. Comparison of the approximate driving forces and true driving force of crystal growth of D-fructose from aqueous ethanol solution.

Fig. 6. Crystal growth rates of D-fructose from aqueous ethanol solution as a function of the relative supersaturation and the chemical potential driving force.


.82
.80
.78
.76
.74
.72
.70
.68
1.05
1.04
1.03
1.02
1.01
1.00
.30
.32
.34
.36
.38
.40
.42
.44
D-fructose activity coefficient, γ_F
Activity coefficient ratio, γ_F/γ_F^*
D-fructose mole fraction, x_F

Approximate driving force, σ_I and σ_C
Fundamental driving force, σ_a
0.00
.02
.04
.06
.08
.10
.12
.14


Mean crystal growth rate, m/s x 10^9
3.0
2.5
2.0
1.5
1.0
.5
0.0
Relative supersaturation, σc
Chemical potential driving force, σa
0.00
.02
.04
.06
.08
.10
.12
.14
.16
Driving force, σc and σa

D-fructose activity coefficient, γ_F
D-fructose mole fraction, x_F
Activity coefficient ratio, γ_F/γ_F^*
18
16
14
12
10
8
6
4
2
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.04
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1.0
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.5
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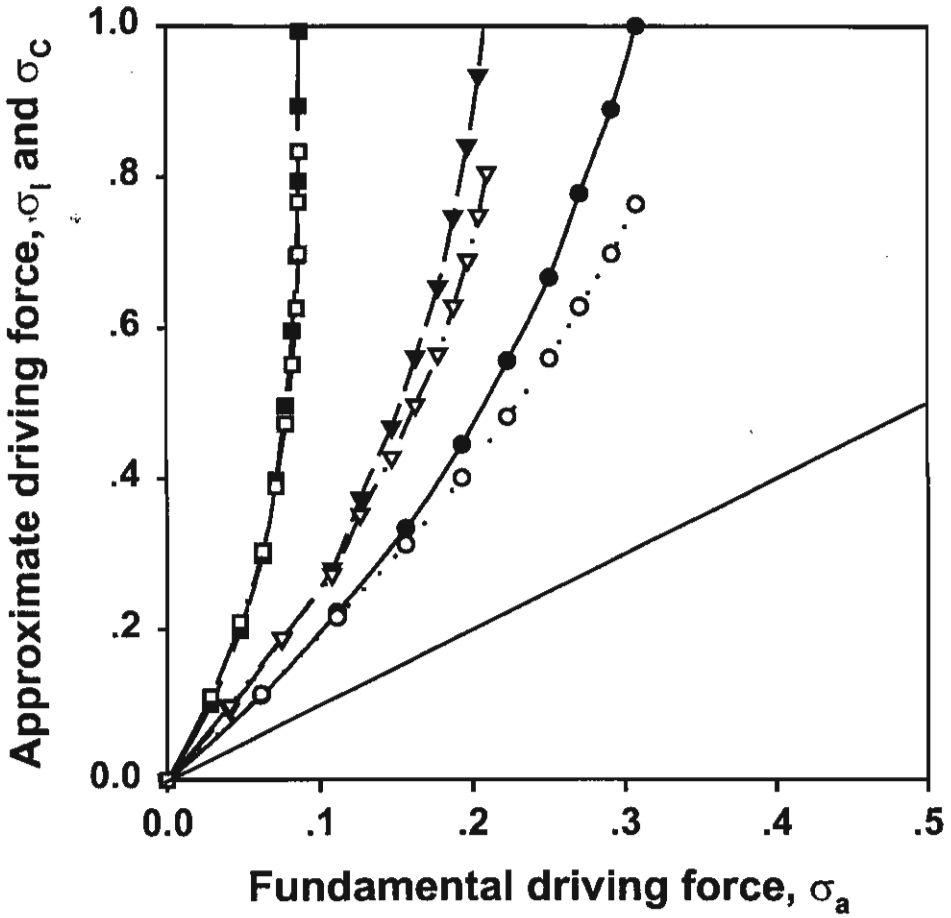
Approximate driving force, σ_l and σ_c
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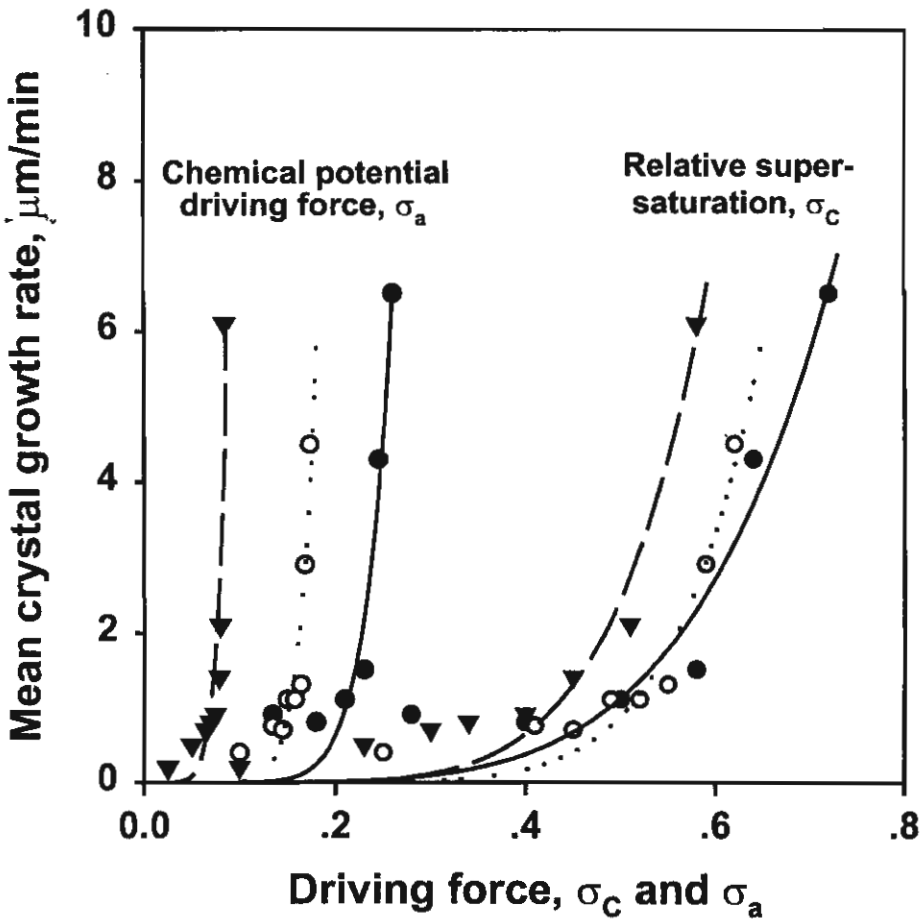
.2
0.0
0.0
.1
.2
.3
.4
.5
Fundamental driving force, σ_a


Mean crystal growth rate, μm/min
10
8
6
4
2
0
0.0
.2
.4
.6
.8
Chemical potential driving force, σa
Relative super-saturation, σc
Driving force, σc and σa