รายงานวิจัยฉบับสมบูรณ์

โครงการ การศึกษาการระบาดของแมลงศัตรูพืชและโรคพืชของข้าว มันสำปะหลัง อ้อย และข้าวโพด: การศึกษาโดยการสร้างแบบจำลอง

โดย รองศาสตราจารย์ ดร. ชนม์ทิตา รัตนกุล

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สัญญาเลขที่ RSA5880004

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รองศาสตราจารย์ ดร. ชนม์ทิตา รัตนกุล มหาวิทยาลัยมหิดล

สนับสนุนโดยสำนักงานกองทุนสนับสนุนการวิจัย
และมหาวิทยาลัยมหิดล

(ความเห็นในรายงานนี้เป็นของผู้วิจัย สกว.และมหาวิทยาลัยมหิดล
ไม่จำเป็นต้องเห็นด้วยเสมอไป)

สารบัญ

บทคัดย่อ

รหัสโครงการ: RSA5880004

ชื่อโครงการ: การศึกษาการระบาดของแมลงศัตรูพืชและโรคพืชของข้าว มันสำปะหลัง อ้อย และข้าวโพด: การศึกษาโดยการสร้างแบบจำลอง

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ข้าว มันสำปะหลัง อ้อย และข้าวโพดเป็นพืชเศรษฐกิจที่สำคัญของประเทศไทย โดยวัชพืชและแมลงศัตรูพืชเป็นปัจจัยที่มีผลต่อผลผลิตเป็นอย่างมาก การควบคุมการแพร่ระบาดของแมลงศัตรูพืชที่มีประสิทธิภาพจึงมีความสำคัญและจำเป็นอย่างยิ่งในการลดต้นทุนการผลิตซึ่งมีผลให้เกษตรกรได้ผลกำไรสูงสุด ในงานวิจัยชิ้นนี้ เราจึงศึกษาการแพร่ระบาดของแมลงศัตรูพืชโดยสร้างแบบจำลองทางคณิตศาสตร์และแบบจำลองเซลลูลาร์ออโตมาต้า โดยค่าพารามิเตอร์ต่างๆ ในแบบจำลองอ้างอิงมาจากข้อมูลที่มีการรายงานไว้ก่อนหน้า เราวิเคราะห์แบบจำลองที่สร้างขึ้นทั้งเชิงทฤษฎีและเชิงตัวเลข ทั้งนี้ เราได้ศึกษาการควบคุมการแพร่ระบาดของแมลงศัตรูพืชในหลากหลายวิธีที่แตกต่างกันเพื่อได้มาซึ่งการควบคุมการแพร่ระบาดของแมลงศัตรูพืชที่เหมาะสม

คำหลัก: แบบจำลองทางคณิตศาสตร์, แบบจำลองเซลลูลาร์ออโตมาต้า, การควบคุมแมลงศัตรูพืช

Abstract

Project Code: RSA5880004

Project Title: Investigating the outbreak of insect pests and plant diseases of rice, cassava, sugarcane and maize: A modeling approach

Investigator: Assoc. Prof. Dr. Chontita Rattanakul, Mahidol University

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Project Period: July 1, 2015 – September 30, 2017

Rice, cassava, sugarcane and maize are important agricultural products of Thailand. Weeds, animal pests and pathogens are regular concerns of economic importance. In order to maximize the agricultural products, the efficient strategies to control the outbreak of insect pests are necessary. In this project, we investigate the outbreak of insect pests by developing mathematical models and constructing cellular automata models. The available reported field data are utilized to estimate parameters in our models. The models are analyzed theoretically/ numerically. The different manners of the control for insect pests are investigated so that the appropriate controls are obtained.

Keywords: mathematical model, cellular automata model, pest control

Executive Summary

1. Rationale:

Agriculture has been the backbone of Thailand's economy for several decades. Cassava is considered to be one of the major agriculture crops of Thailand. Although Thailand is not a major consumer of cassava, it is the world's biggest exporter of cassava with the world's market share of 60.72% in 2011 according to the office of agricultural economics, ministry of agriculture and cooperatives, Thailand. Even though cassava can survive both hot and dry conditions, an increase in insect pests might easily cause a major loss in crop yield. Mealybugs (Hemiptera: Pseudococcidae) constitute a major family of insect pests of cassava. In Thailand, there are four species of mealybugs found in cassava fields which are striped mealybugs, Madeira mealybugs, pink mealybugs (cassava mealybug) and Jack-Beardsley mealybugs. In 2008, cassava mealybugs were first identified in Thailand and has spread aggressively throughout cassava's planting area in Thailand. In 2010, there was an outbreak of cassava mealybugs in Thailand resulting in a major loss in cassava yield. The total cassava yield reduced from 30 million tons per year to 22 tons per year according to the information from the Office of Agricultural Economics, Thailand.

There are various practices to control the spread of mealybugs in cassava fields in Thailand. Farmers might use biological controls, insecticides or a mixture of biological controls and insecticides. With biological controls, various practices have been recommended by the Thai Tapioca Development Institute and the Department of Agriculture, Ministry of Agriculture and Cooperatives, Thailand. The suggestions on the number of natural enemies to be released in a field and the period between each natural enemies released are diverse and also depend on the type of the natural enemies to be released. Natural enemies that have been used popularly to control the spread of cassava mealybugs are *Anagyrus lopezi* and green lacewings.

2. Objectives:

The objectives of this project are as follows:

2.1 To study the outbreak of insect pests of cassava.

2.2 To obtain mathematical models for studying the spreading of insect pests.

2.3 To obtain the efficient controls for the outbreak of insect pests in terms of how often/ how much farmers should release natural enemies of insect pests in order to control the outbreak of insect pests and maximize profit.

3. Methodology:

3.1 Gather information about rice, cassava, sugarcane, maize and the outbreak of their plant diseases and insect pests.

3.2 Develop mathematical models/ cellular automata models to study the outbreak of plant diseases and insect pests of rice, cassava, sugarcane and maize.

3.3 Use the available reported field data to estimate some parameters in the developed mathematical models.

3.4 Analyze the developed mathematical models theoretically/ numerically to obtain the appropriate controls of plant diseases and insect pests of rice, cassava, sugarcane and maize by using Hopf Bifurcation theorem, singular perturbation technique, Runge Kutta method or cellular automata/ Monte Carlo simulation.

4. Results:

4.1 Cellular automata Model

In this part, cellular automata together with Monte Carlo simulation are employed to investigate the spread of mealybugs in a cassava field with the usual practices of biological control in Thailand when green lacewings are used as the biological control agent. The effect of increased global temperatures, the frequency of the release of green lacewings as well as the stage of green lacewings released in the field are investigated.

A cellular automaton with Moore's neighborhood of a square lattice with the size $L \times L$ represents a cassava field. The states of every cell in the lattice will be updated in parallel at each time step (1 time step Δt = 1 day). Each cell in the lattice represents a state of cassava planted in the cell which will be updated at each time step according to given rules. The updating cell will be indexed by (i, j) with its immediate neighborhood, distant neighborhood and far distant neighborhood represented as the light grey, grey and dark grey areas, respectively. The possible states of each cell in the lattice are

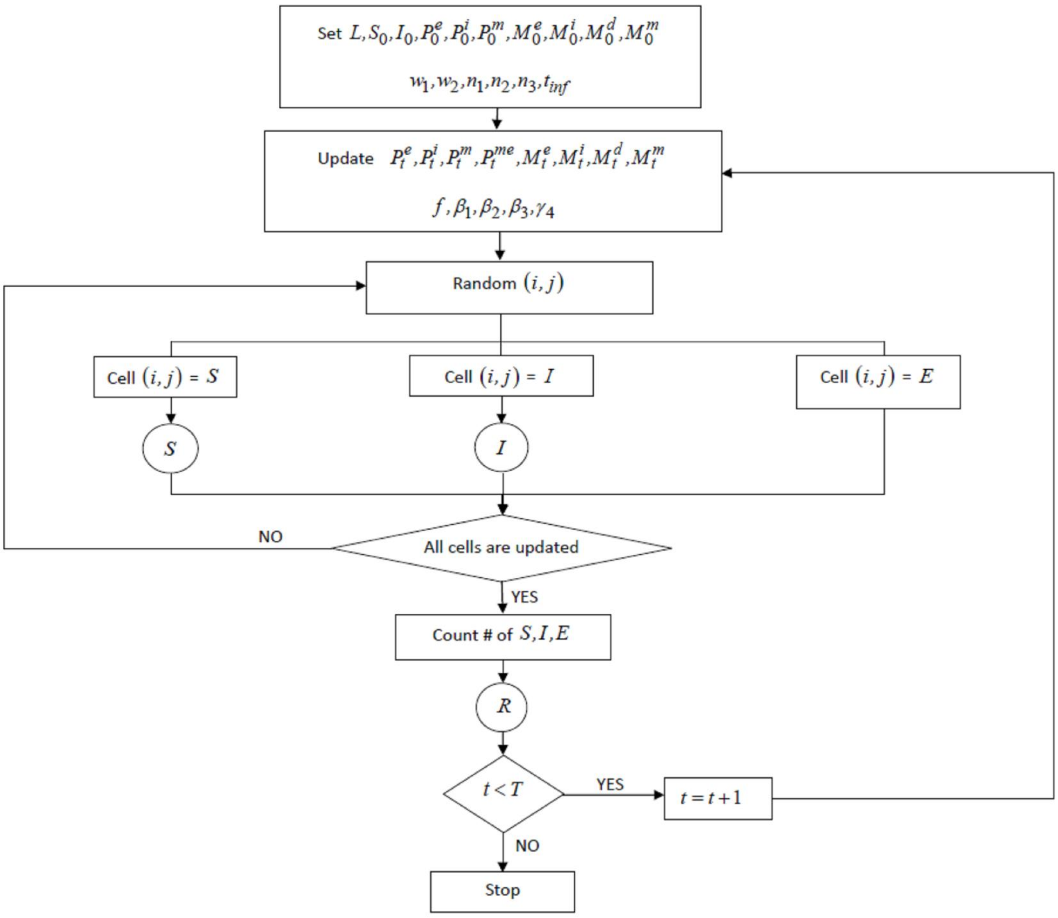
- susceptible cassava (*S*) which indicates that the cassava plant in that cell is free from mealybugs, or
- infested cassava (*I*) which indicates that the cassava plant in that cell has mealybugs on it, or
- empty cell (*E*) which indicates that the cassava plant in that cell was removed from the field.

(i-3,j-3)	(i-3,j-2)	(i-3,j-1)	(i-3,j)	(i-3,j+1)	(i-3,j+2)	(i-3,j+3)
(i-2,j-3)	(i-2,j-2)	(i-2,j-1)	(i-2,j)	(i-2,j+1)	(i-2,j+2)	(i-2,j+3)
(i-1,j-3)	(i-1,j-2)	(i-1,j-1)	(i-1,j)	(i-1,j+1)	(i-1,j+2)	(i-1,j+3)
(i,j-3)	(i,j-2)	(i,j-1)	(i,j)	(i,j+1)	(i,j+2)	(i,j+3)
(i+1,j-3)	(i+1,j-2)	(i+1,j-1)	(i+1,j)	(i+1,j+1)	(i+1,j+2)	(i+1,j+3)
(i+2,j-3)	(i+2,j-2)	(i+2,j-1)	(i+2,j)	(i+2,j+1)	(i+2,j+2)	(i+2,j+3)
(i+3,j-3)	(i+3,j-2)	(i+3,j-1)	(i+3,j)	(i+3,j+1)	(i+3,j+2)	(i+3,j+3)

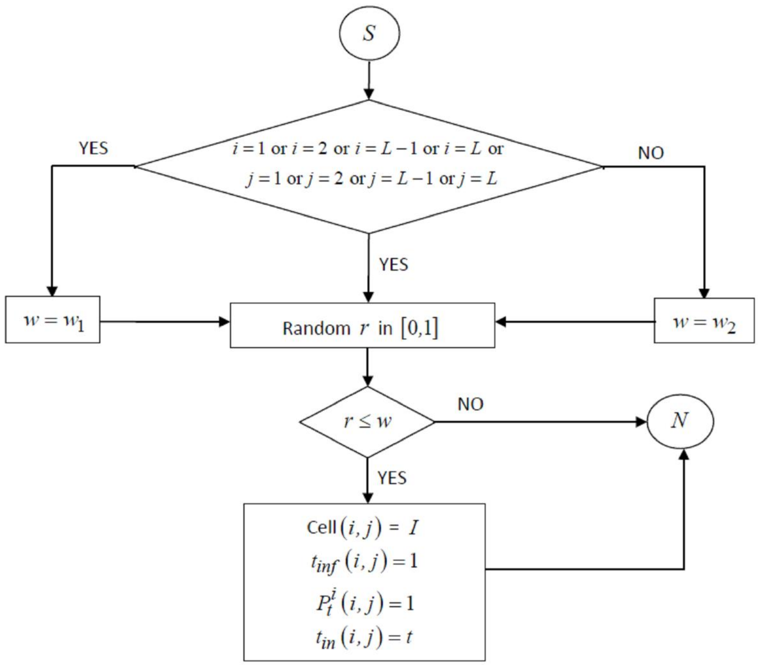
The light grey, grey and dark grey areas represent immediate neighborhood, distant neighborhood and far distant neighborhood, respectively.

Firstly, we investigate the use of green lacewings as a biological control agent. We let P_t^i, P_t^m and P_t^e denote the numbers of mealybugs in the instar state, adult state, egg state, respectively, at the time step t. M_t^i, M_t^d, M_t^m and M_t^e denote the numbers of green lacewings in the larva state, pupa stage, adult state and egg state, respectively, at the time step t. P_t and M_t denote the total numbers of mealybugs and green lacewings, respectively, at the time step t.

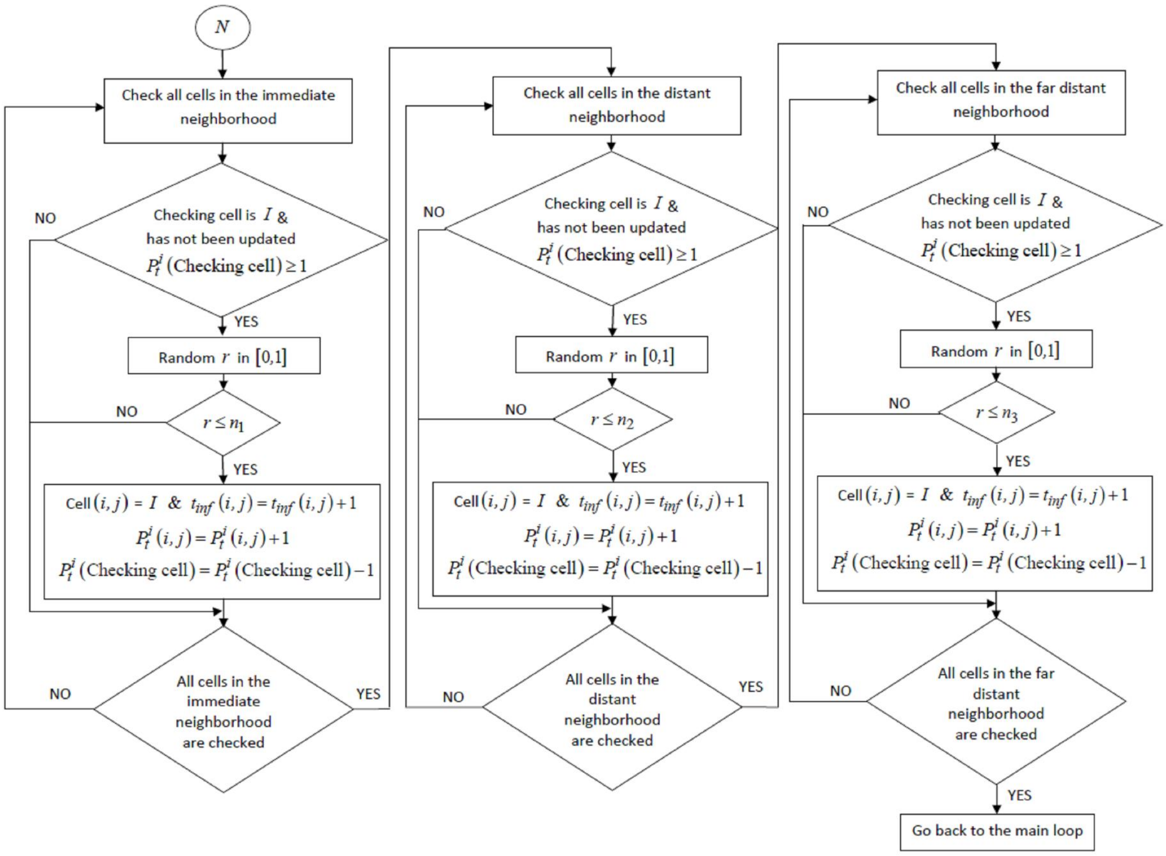
At each time step, a number $r, 0 \le r \le 1$ is randomized and each cell will be updated at random as shown in the following flowcharts.



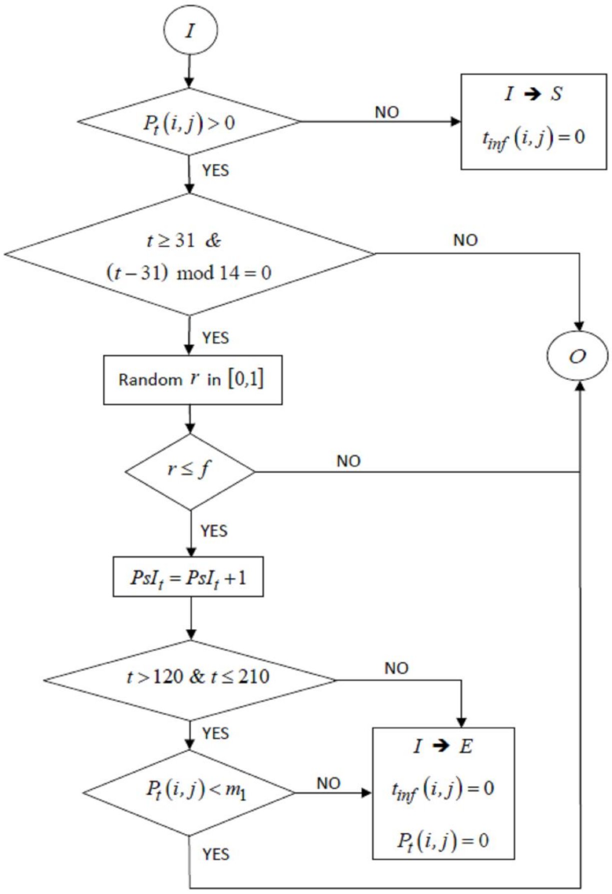
The main loop of CA



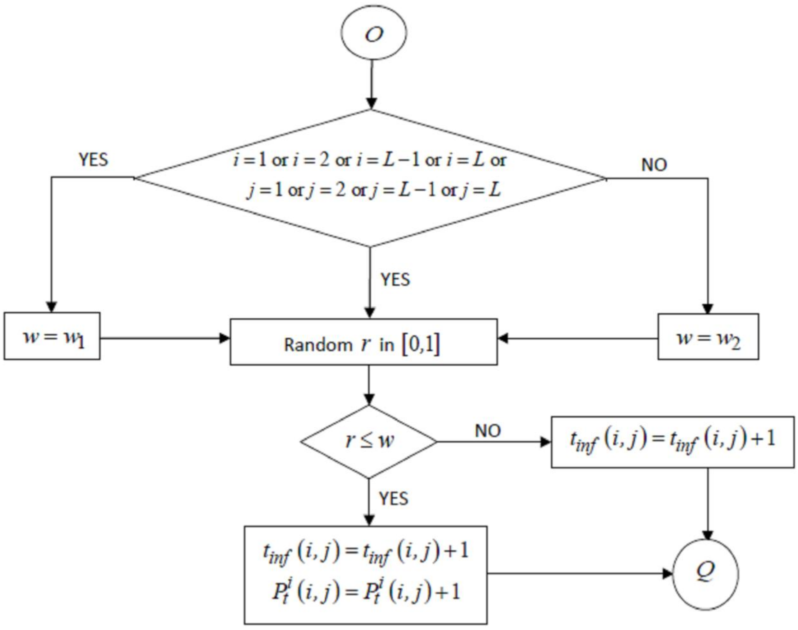
The main susceptible cassava updating loop



The neighbourhood checking loop of the susceptible cassava updating loop

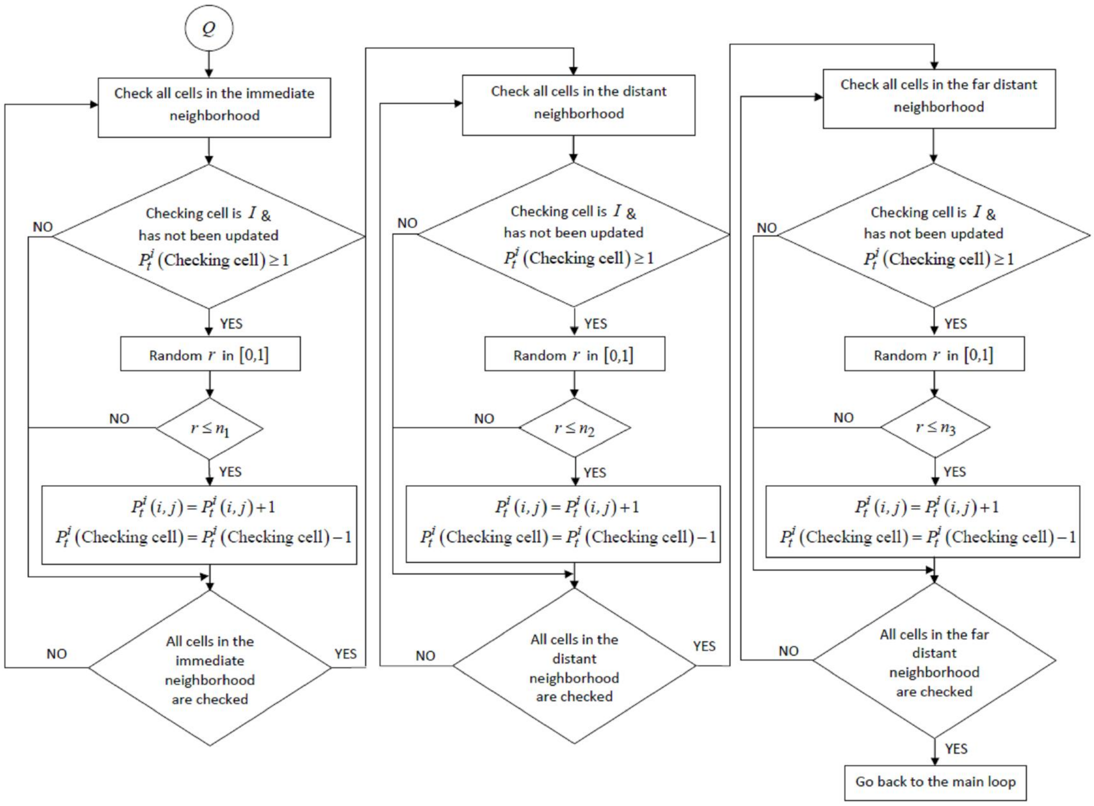


The main infected cassava updating loop

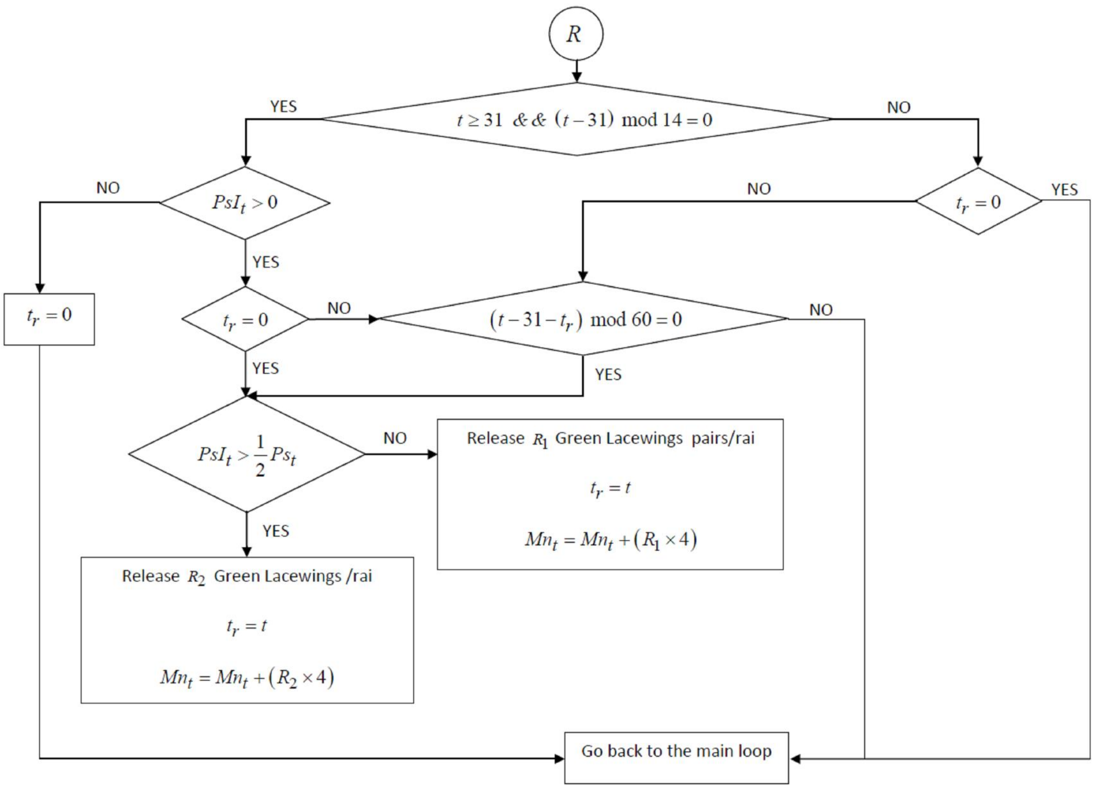


The number of mealybugs updating loop (incorporating wind effect) of the

infected cassava



The neighbourhood checking loop of the number of mealybugs updating loop (incorporating wind effect) of the infected cassava



The green lacewing releasing loop

Apart from the effect of the wind, the effects of the life cycles of mealybugs and green lacewings are also taken into account and the number of mealybugs and green lacewings at each stage are updated on each cell according to the system of difference equations. Parameters in the model are calculated from literatures. The simulation results are as follows.

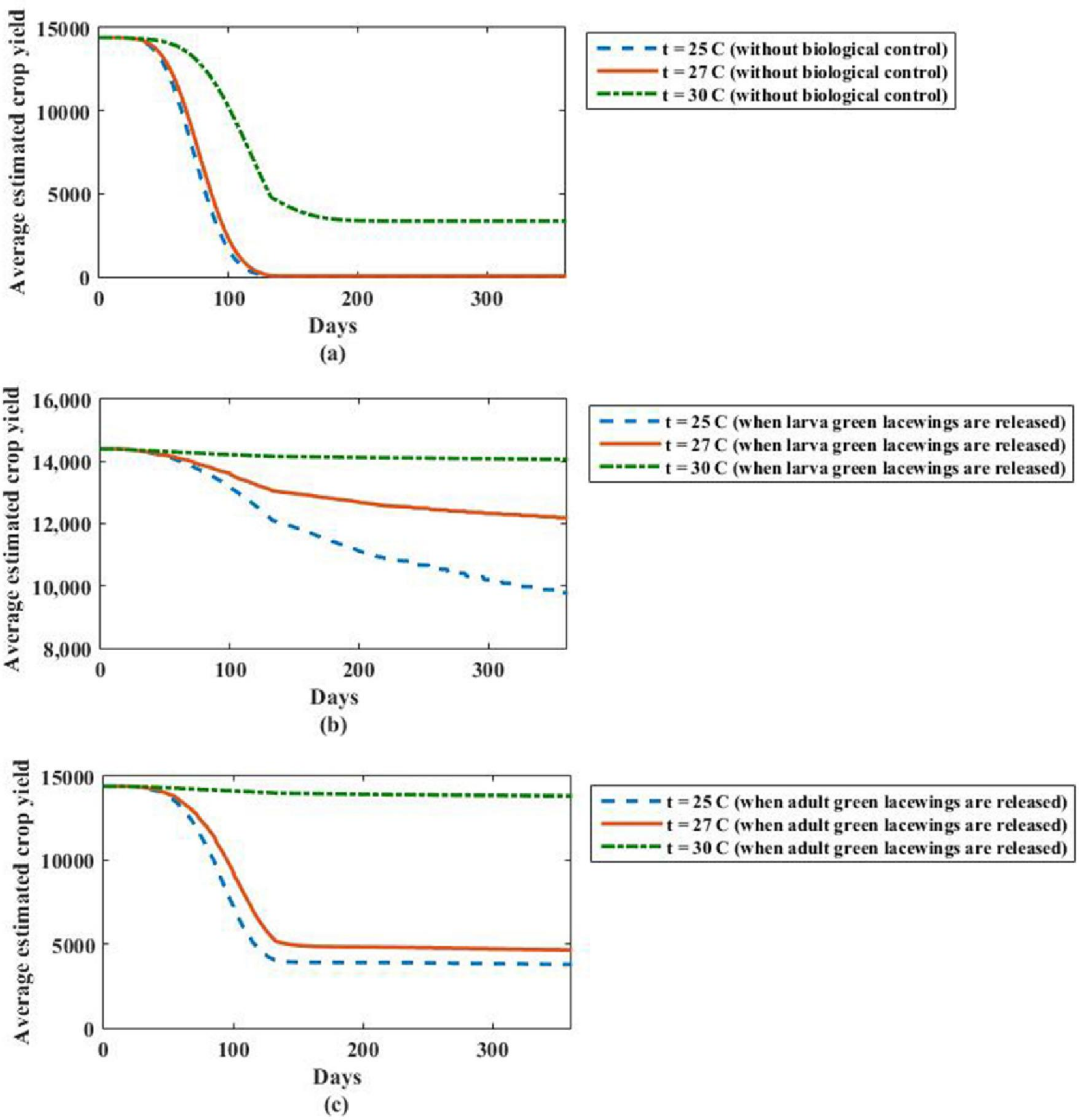


Figure 1 The average estimated cassava's crop yield of the 100 runs using MATLAB software.

The increase in global temperatures affects the sex ratio, survival rate, reproduction rate and life cycle of both mealybugs and green lacewings. Without biological control, the estimated crop yield decreases dramatically and tends to zero at 25°C, and at 27°C approximately 4 months after planting. At 30°C, the estimated crop yield decreases and tends to a constant level which is lower than 30% of the maximum estimated crop yield.

With biological control, green lacewings at the larva stage or adult stage may be released in the cassava field to control the spread of mealybugs. Hence, we study both manners of biological control. We can see that the number of infested cassava plants decreases whereas the number of susceptible cassava plants increases when the temperature increases which might be the results of shorter life cycle, lower survival rate, lower fecundity and shorter adult longevity of mealybugs. We can also see that the release of green lacewing larva gives a better result when there is a spread of mealybugs even though the lower amount of larva green lacewing is released compared to adult green lacewings. The reasons for this might be the shorter life span, lower survival rate, lower fecundity or shorter adult longevity of green lacewings because only green lacewings at the larva stage behave like a predator of mealybugs and if we release adult green lacewings it will take a period of time before they will lay eggs which develop into green lacewing larva, finally behaving like a predator of mealybugs.

With the increase of temperature, the survival rate and the fecundity rate are even lower and hence the greater amount of adult green lacewings should be released in the cassava field to control the spread of mealybugs. On the other hand, the estimated crop yield also increases when the temperature increases with the same level of released green lacewings. This implies that if farmers are satisfy with the estimated crop yield at the end of planting period when the temperature is 25°C, they might reduce the number of green lacewings released in the cassava field so that the cost for biological control will be decreased and the farmers then earn more profit. On the other hand, if the farmers would like to gain more estimated crop yield at the end of planting period, they might keep the released amount of green lacewings at the same level as they use when the temperature is 25°C. However, the cost for a green lacewings is approximately 0.50 baht (0.015 USD) while the selling price for cassava is quite low, approximately 2.50 baht (0.072 USD) per kilogram. Hence, the cost of biological control and the increase in crop yield should be calculated in order to obtain the most efficient biological control that maximizes profit.

Next, the cellular automata model is then modified to investigate the effect of the release frequency of green lacewings. The simulations of the spread of mealybugs are carried out by using parametric values that are estimated from the available reported data at the temperature of 30°C. The averaged values of the 10 runs using MATLAB software. The results are as follows.

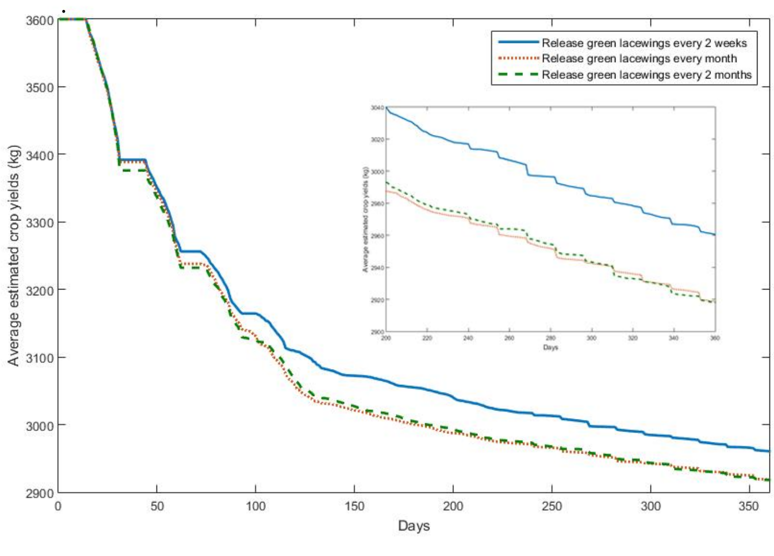


Figure 2 Average estimated crop yields.

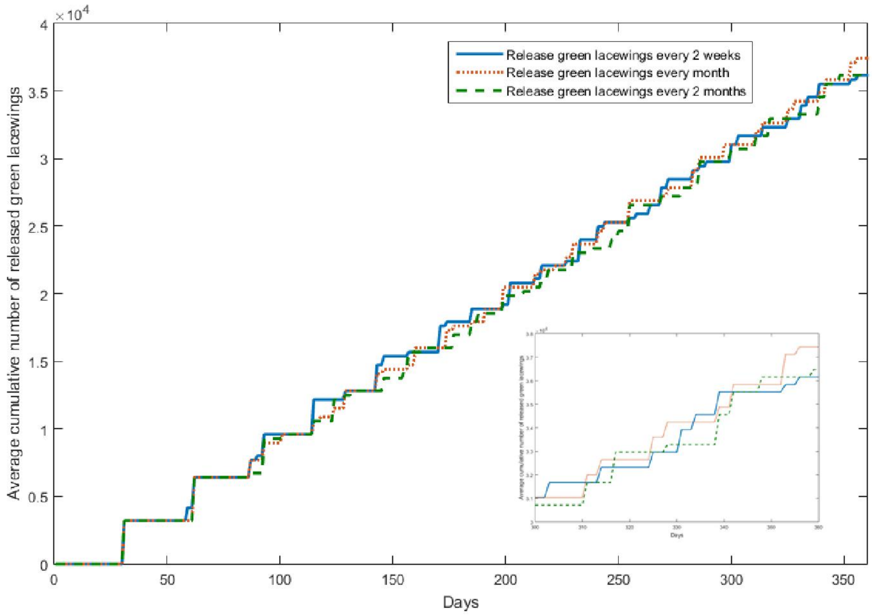


Figure 3 Average cumulative number of released green lacewings.

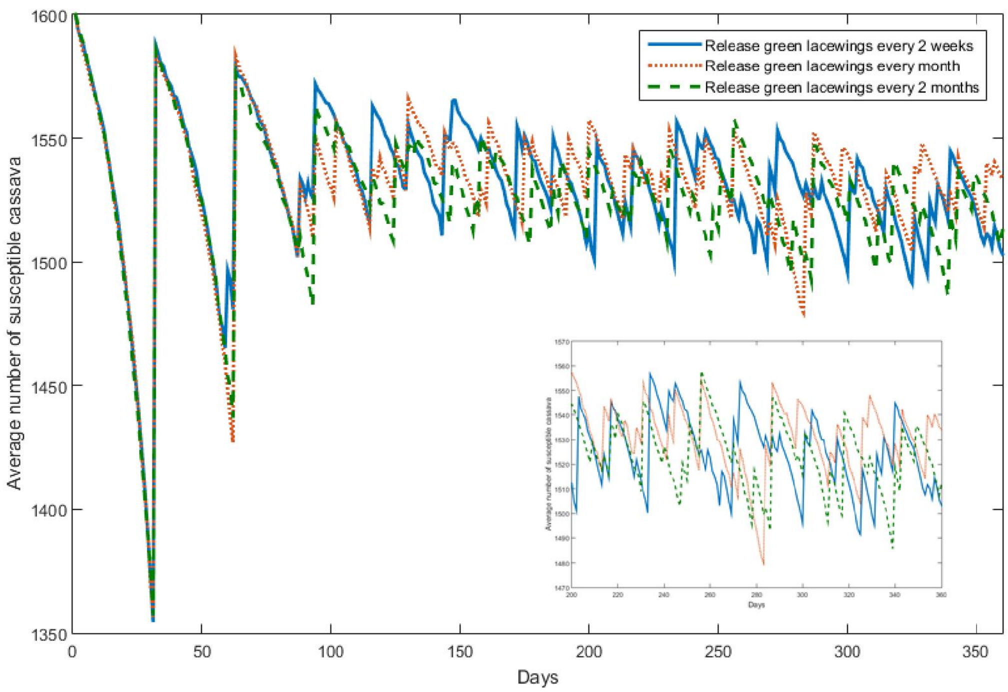


Figure 4 Average number of susceptible cassava.

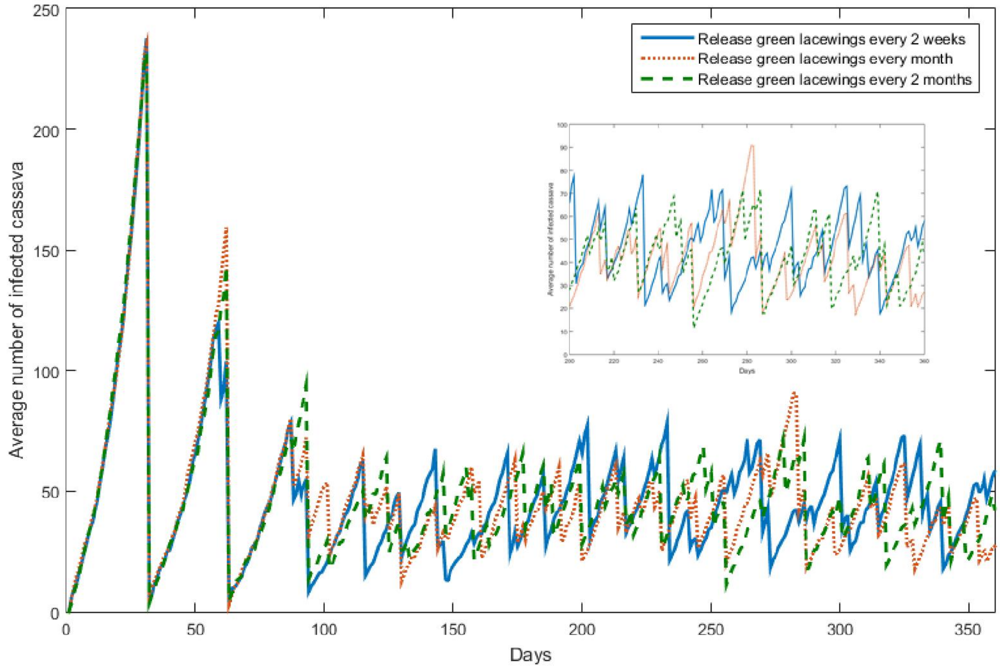


Figure 5 Average number of infected cassava.

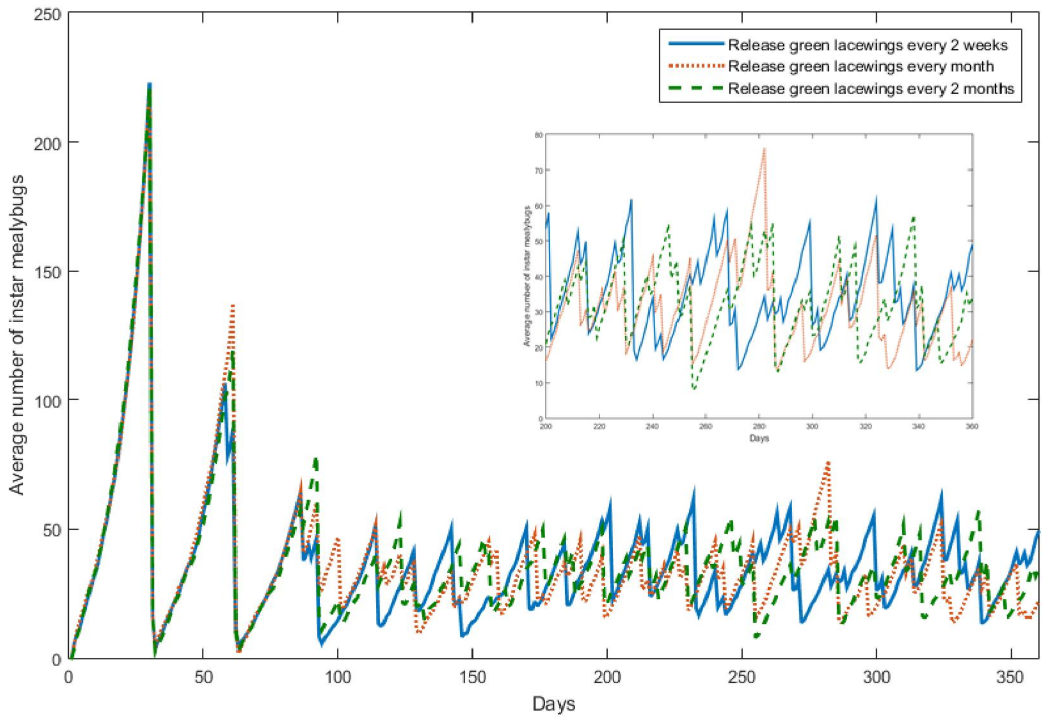


Figure 6 Average number of instar mealybugs.

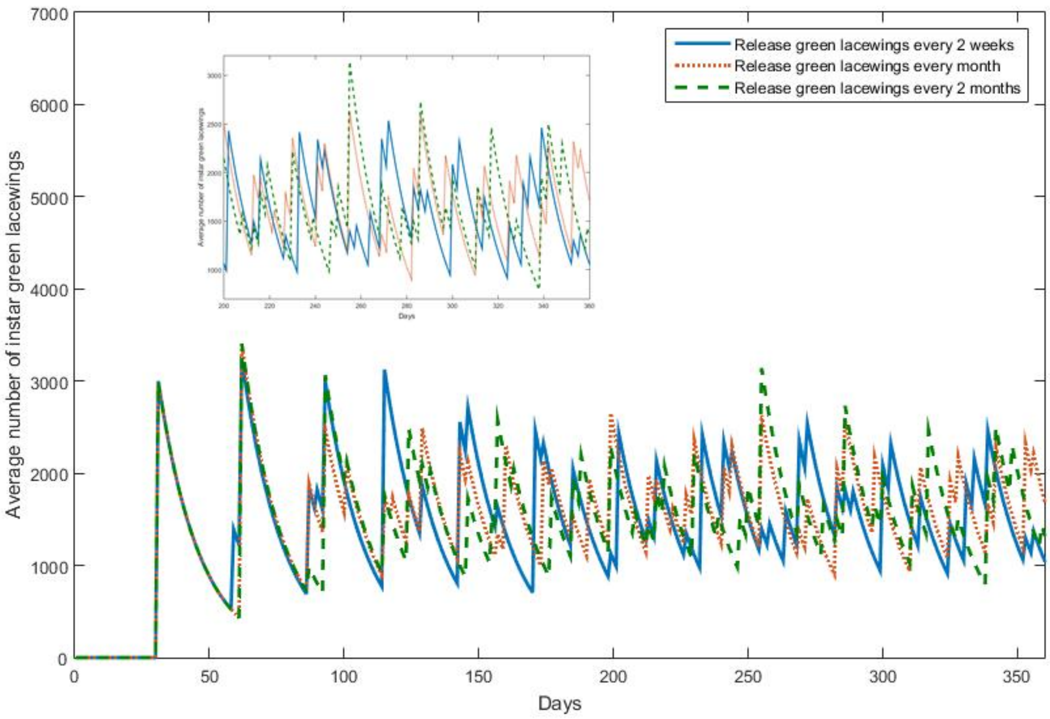


Figure 7 Average number of larva green lacewings.

From the simulation results shown in Figures 2–7, we can see that the release frequency of 2 weeks gives the better result as the estimated crop yields in this case is higher than in the other cases. However, the cumulative numbers of released green lacewings every 2 weeks and every 2 months are at about the same level while it is a little bit higher when they are released every month. Moreover, the spread of mealybugs seems to be controllable in all three cases.

Since the cumulative numbers of released green lacewings every 2 weeks and every 2 months are at about the same level, the costs for the release of green lacewings every 2 weeks and every 2 months are then different only with the wage costs. The wages in the case of 2 weeks release frequency will be four times those of the case of 2 months release frequency. In Figure 2, the estimated crop yields for the release frequency of 2 weeks is approximately 50 kg higher than the release frequency of 2 months at the end of the planting period. Suppose that the market price of cassava is 3 baht per kilogram, the release frequency of 2 weeks will give 150 baht or 1.67% more on the total crop sale income and hence, the increase in wages will not be covered by the increased income in this case. Therefore, the release frequency of 2 months seems to be the better option. However, in this study the cassava field of interest is just 1 rai (0.16 ha) and hence the infected probability through the wind might be higher compared to the larger field whereas more labor force may be necessary. It also depends on how high the wages are for such labor and how long it takes to finish the task. On the other hand, the increased income from the yields would be higher for a larger field. Therefore, further study is needed before any general conclusion can be drawn.

Next, we then modify the cellular automata model to investigate the use of *Anagyrus lopezi* as a biological control agent for controlling the spread of cassava mealybugs. Here, starting from the second month of planting, the survey for cassava mealybugs will be carried out every two weeks. If cassava mealybug is found when the survey is conducted during the 5th and the 7th month of planting, *Anagyrus lopezi* will be released in the field once or every three weeks for three times with the amount of 50-100 pairs per rai, 200 pairs per rai or 400 pairs per rai.

We then investigate six different tactics of releasing *Anagyrus lopezi* in a cassava field when the spread of cassava mealybugs is detected. The six tactics are listed as follows.

I: Release *Anagyrus lopezi* only once when the spread of cassava mealybug is first detected in the field at the amount of 50-100 pairs per rai.

II: Release *Anagyrus lopezi* only once when the spread of cassava mealybug is first detected in the field at the amount of 200 pairs per rai.

III: Release *Anagyrus lopezi* only once when the spread of cassava mealybug is first detected in the field at the amount of 400 pairs per rai.

IV: Release *Anagyrus lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field at the amount of 50-100 pairs per rai.

V: Release *Anagyrus lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field at the amount of 200 pairs per rai.

VI: Release *Anagyrus lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field at the amount of 400 pairs per rai.

Computer simulations of the six tactics are carried out by MATLAB software. The average of the 100 runs on the estimated crop yield of cassava at the end of planting period and the average of the 100 runs on the total number of *Anagyrus lopezi* released in the field are as follows

Tactic	Average estimated crop yield of cassava (kgs)	Average total number of wasps *Anagyrus lopezi* released in the cassava field (pairs)
I	14,277.96	60
II	14,234.04	160
III	14,252.09	320
IV	14,298.75	440
V	14,289.57	1,760
VI	14,301.05	3,520

The results indicate that the tactic VI (Release *Anagyrus lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field with the amount of 400 pairs per rai) gives the highest average estimated crop yield of cassava with the lowest number of infested cassava plants compared to the other five tactics. Even though the results indicate that the tactic VI is the best option for the control of the spread of cassava mealybugs and gives the highest average estimated cassava's yield at the end of planting period, the cassava's selling price is approximately 2.50 baht (0.072 USD) per kilogram and the cost for the biological control agent *Anagyrus lopezi* is approximately 4.50 baht (0.13 USD) per pair. In order that the most efficient biological control in terms of maximum profit for farmers may be obtained. The average estimated cost of *Anagyrus lopezi* released in the field, the average estimated income from selling cassava's yields and the average estimated (income – cost of biological control agents) at the end of planting period for each tactic is also provided here

Tactic	Average estimated cost of wasps textitAnagyrus lopezi released in the field (baht)	Average estimated income from selling cassava's crop yields (baht)	Average estimated (income − cost of wasps *Anagyrus lopezi*) at the end of planting period (baht)
I	270	35,694.90	35,424.90
II	720	35,585.10	34,865.10
III	1,440	35,630.23	34,190.25
IV	1,980	35,746.88	33,766.88
V	7,920	35,723.93	27,803.93
VI	15,840	35,752.63	19,912.63

We can see that even though the tactic VI give the highest average estimated cassava's crop yield, the tactic that gives the maximum profit is the tactic I (releasing *Anagyrus lopezi* only once when the spread of cassava mealybug is first detected in the field at the amount of 50-100 pairs per rai). Note that the planting area that we considered here is just 4 rai (0.64 ha). When the planting area is a large-scale cassava farm the results might not be the same as what we have found here. One reason is that the spread of cassava mealybugs might not be detected in the large-scale cassava farm as fast as in a small-scale cassava farm. Hence, further investigations are needed for a large-scale cassava farm.

Next, we then modify the cellular automata model to investigate the use of both green lacewings and *Anagyrus lopezi* as a biological control agent for controlling the spread of cassava mealybugs. Here, starting from the second month of planting, the survey for cassava mealybugs will be carried out every two weeks. If cassava mealybug is found when the survey is conducted during the 5th and the 7th month of planting, green lacewings and *Anagyrus lopezi* will be released in the field.

Here, the total of 54 manners of biological controls with *Anagyrus lopezi* and green lacewings are investigated. Computer simulations of 54 cases are carried out using MATLAB software. The results are as follows.

Green Lacewings (GL) / ***Anagyrus Lopezi* (AL)**		**Release GL every 2 weeks**			**Release GL every month**			**Release GL every 2 months**		
		200 per rai	800 - 1,000 per rai	2,000 per rai	200 per rai	800 - 1,000 per rai	2,000 per rai	200 per rai	800 - 1,000 per rai	2,000 per rai
Release AL only 1 time	50-100 pairs per rai	1	2	3	4	5	6	7	8	9
	200 pairs per rai	10	11	12	13	14	15	16	17	18
	400 pairs per rai	19	20	21	22	23	24	25	26	27
Release AL 3 times every 3 weeks	50-100 pairs per rai	28	29	30	31	32	33	34	35	36
	200 pairs per rai	37	38	39	40	41	42	43	44	45
	400 pairs per rai	46	47	48	49	50	51	52	53	54

Average estimated cassava's crop yields at the end of planting period (kgs)

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	12,585.32	12,869.36	12,687.20	12,096.44	12,753.40	12,818.47	12,466.70	12,325.67	12,400.82
	200 pairs per rai	12,645.89	12,274.28	12,798.67	12,684.55	12,353.08	12,784.36	12,531.10	12,616.24	12,743.27
	400 pairs per rai	12,883.40	13,001.48	12,685.94	12,985.06	12,899.29	12,832.78	12,283.33	12,448.12	12,491.41
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	13,093.96	13,013.14	12,963.50	13,130.09	12,946.81	13,173.83	12,451.63	12,995.18	13,002.16
	200 pairs per rai	13,193.32	13,061.92	13,179.23	12,879.94	13,393.21	13,479.34	13,166.41	13,127.44	13,308.79
	400 pairs per rai	13,312.70	13,283.81	13,370.84	13,024.61	13,570.01	13,565.65	13,522.94	13,646.20	13,425.16

Average total numbers of AL and GW released in the field at the end of planting period

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	AL = 200 pairs GL = 10,400	AL = 200 pairs GL = 44,800	AL = 200 pairs GL = 120,000	AL = 200 pairs GL = 6,400	AL = 200 pairs GL = 25,600	AL = 200 pairs GL = 64,000	AL = 200 pairs GL = 3,200	AL = 200 pairs GL = 12,800	AL = 200 pairs GL = 32,000
	200 pairs per rai	AL = 800 pairs GL = 11,200	AL = 800 pairs GL = 35,200	AL = 800 pairs GL = 112,000	AL = 800 pairs GL = 6,400	AL = 800 pairs GL = 22,400	AL = 800 pairs GL = 48,000	AL = 800 pairs GL = 3,200	AL = 800 pairs GL = 12,800	AL = 800 pairs GL = 32,000
	400 pairs per rai	AL = 1,600 pairs GL = 11,200	AL = 1,600 pairs GL = 44,800	AL = 1,600 pairs GL = 96,000	AL = 1,600 pairs GL = 6,400	AL = 1,600 pairs GL = 16,000	AL = 1,600 pairs GL = 64,000	AL = 1,600 pairs GL = 3,200	AL = 1,600 pairs GL = 12,800	AL = 1,600 pairs GL = 32,000
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	AL = 600 pairs GL = 10,400	AL = 600 pairs GL = 51,200	AL = 600 pairs GL = 128,000	AL = 600 pairs GL = 6,400	AL = 600 pairs GL = 25,600	AL = 600 pairs GL = 48,000	AL = 600 pairs GL = 3,200	AL = 600 pairs GL = 12,800	AL = 600 pairs GL = 24,000
	200 pairs per rai	AL = 2,400 pairs GL = 9,600	AL = 2,400 pairs GL = 44,800	AL = 2,400 pairs GL = 80,000	AL = 2,400 pairs GL = 5,600	AL = 2,400 pairs GL = 25,600	AL = 2,400 pairs GL = 64,000	AL = 2,400 pairs GL = 3,200	AL = 2,400 pairs GL = 9,600	AL = 2,400 pairs GL = 32,000
	400 pairs per rai	AL = 4,800 pairs GL = 8,800	AL = 4,800 pairs GL = 25,600	AL = 4,800 pairs GL = 96,000	AL = 4,800 pairs GL = 6,400	AL = 4,800 pairs GL = 25,600	AL = 4,800 pairs GL = 56,000	AL = 4,800 pairs GL = 3,200	AL = 4,800 pairs GL = 12,800	AL = 4,800 pairs GL = 24,000

Average estimated cost of AL and GW released in the field at the end of planting period

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	6,100.00	23,300.00	60,900.00	4,100.00	13,700.00	32,900.00	2,500.00	7,300.00	16,900.00
	200 pairs per rai	9,200.00	21,200.00	59,600.00	6,800.00	14,800.00	27,600.00	14,800.00	58,000.00	144,400.00
	400 pairs per rai	12,800.00	29,600.00	55,200.00	10,400.00	15,200.00	39,200.00	5,200.00	10,000.00	19,600.00
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	7,900.00	28,300.00	66,700.00	5,900.00	15,500.00	26,700.00	15,200.00	58,400.00	144,800.00
	200 pairs per rai	15,600.00	33,200.00	50,800.00	13,600.00	23,600.00	42,800.00	8,800.00	13,600.00	23,200.00
	400 pairs per rai	26,000.00	34,400.00	69,600.00	24,800.00	34,400.00	49,600.00	14,700.00	57,900.00	144,300.00

1 pairs of AL = 4.50 baht

1 green lacewings = 0.50 baht

Average estimated income from selling cassava's crop yields at the end of planting period

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	35,404.33	35,452.70	34,804.13	33,917.08	35,100.00	34,748.45	31,860.00	35,016.20	34,375.50
	200 pairs per rai	35,528.63	35,620.88	35,337.95	33,837.75	35,191.70	35,240.63	32,646.95	35,233.88	35,458.33
	400 pairs per rai	35,390.25	35,334.00	35,395.33	35,346.95	35,155.13	35,182.13	33,777.63	32,940.63	34,855.33
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	35,502.20	35,319.38	35,286.20	35,185.50	35,235.58	35,380.70	31,642.33	34,986.95	35,623.70
	200 pairs per rai	35,546.63	35,275.50	35,521.88	35,367.75	35,397.58	35,047.70	34,851.38	34,891.88	35,058.95
	400 pairs per rai	35,644.50	35,669.83	35,542.13	35,398.13	35,210.83	34,998.20	34,740.00	34,939.70	35,291.25

1 kg. = 2.50 baht

Average estimated (income – cost of biological control agents) at the end of planting period

Anagyrus Lopezi (AL) \ Green Lacewings (GL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	29,304.33	12,152.70	-26,095.88	29,817.08	21,400.00	1,848.45	29,360.00	27,716.20	17,475.50
	200 pairs per rai	26,328.63	14,420.88	-24,262.05	27,037.75	20,391.70	7,640.63	17,846.95	-22,766.13	-108,941.68
	400 pairs per rai	22,590.25	5,734.00	-19,804.68	24,946.95	19,955.13	-4,017.88	28,577.63	22,940.63	15,255.33
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	27,602.20	7,019.38	-31,413.80	29,285.50	19,735.58	8,680.70	16,442.33	-23,413.05	-109,176.30
	200 pairs per rai	19,946.63	2,075.50	-15,278.13	21,767.75	11,797.58	-7,752.30	26,051.38	21,291.88	11,858.95
	400 pairs per rai	9,644.50	1,269.83	-34,057.88	10,598.13	810.82	-14,601.80	20,040.00	-22,960.30	-109,008.75

The results indicate that the method 53 gives the maximized estimated cassava's crop yield at the end of planting period and should be the most efficient biological control of the spread of cassava mealybugs. However, the numbers of *Anagyrus lopezi* and green lacewings released in Method 53 is quite high (4,800 pairs of *Anagyrus lopezi* and 12,800 green lacewings) whereas the selling price of cassava is approximately 2.50 baht (0.072 USD) per kilogram. The average estimated cost of *Anagyrus lopezi* and green lacewings released in the field, the average estimated income from selling cassava's crop yields at the end of planting period and the average estimated (income – cost of biological control agents) at the end of planting period, respectively, are then given here to compare the profit for the 54 biological control methods. We can see that although the method 53 gives the maximum estimated cassava's crop yield at the end of planting period, the cost for biological control is overcome the income from selling cassava's crop yield. On the other hand, the method 4 gives the maximum profit even though it gives the minimum estimated cassava's crop yield at the end of planting period. Therefore, to maximize the estimated cassava's crop yield at the end of planting period, the method 53

(Releasing 800-1,000 green lacewings per rai every 2 months together with 400 pairs of *Anagyrus lopezi* per rai every three weeks for three times after cassava mealybugs were first detected) is the most efficient biological control and to maximize profit, the method 4 (Releasing 200 green lacewings per rai every month together with releasing 50-100 pairs of *Anagyrus lopezi* per rai once after cassava mealybugs were first detected) is the most efficient control.

4.2 A Predator-Prey Model with Age Structure

In this part, a predator-prey model with age structure are developed in order to study the control of cassava's insect pest. The Sharpe-Lotka-McKendrick equation is extended and combined with an integro-differential equation to study population dynamics of mealybugs (prey) and released green lacewings (predator). Here, an age dependent formula is employed for mealybug population. The model is as follows

$$\frac{\partial P(a,t)}{\partial a}+\frac{\partial P(a,t)}{\partial t}=-\mu M(t)P(a,t)$$
$$\frac{dM(t)}{dt}=\mu\left(\int_0^\infty P(a,t)\,da\right)M(t)-\delta M(t)+g$$

with the following initial and boundary conditions

$$P(0^+,t)=\lim_{a\to 0^+}P(a,t)=b\int_0^\infty P(a,t)\,da,$$
$$P(a,0)=p_0(a),$$
$$M(0)=c,\quad c>0$$

where all variables and parameters are as in the following Table

Variable	Symbol	Unit
Prey age population density	$P(a,t)$	$BM\cdot T^{-1}$
Predator population	$M(t)$	BM
Death rate of prey	μ	$BM^{-1}\cdot T^{-1}$
Death rate of predator	δ	T^{-1}
Introduced predator rate	g	$BM\cdot T^{-1}$
Total prey (all ages)	$K(t)$	BM
Renewal rate of prey	b	T^{-1}

The solutions and their stability of the system are considered. The steady age distributions and their bifurcation diagrams are as follows

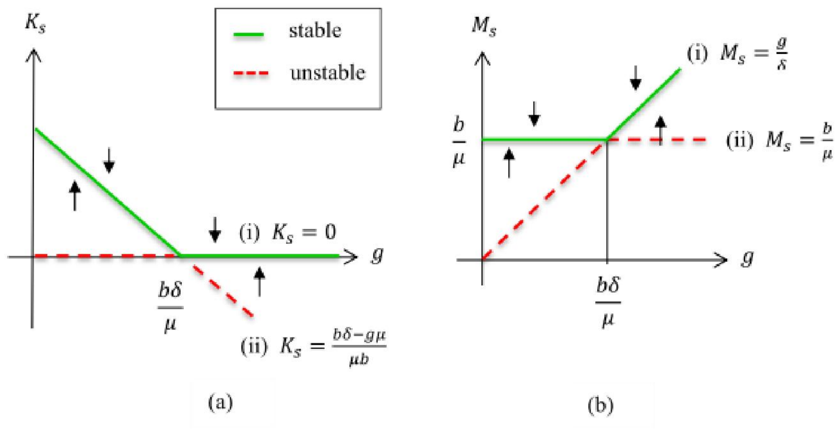


FIGURE 1. Bifurcation diagrams for (a) prey and (b) predator

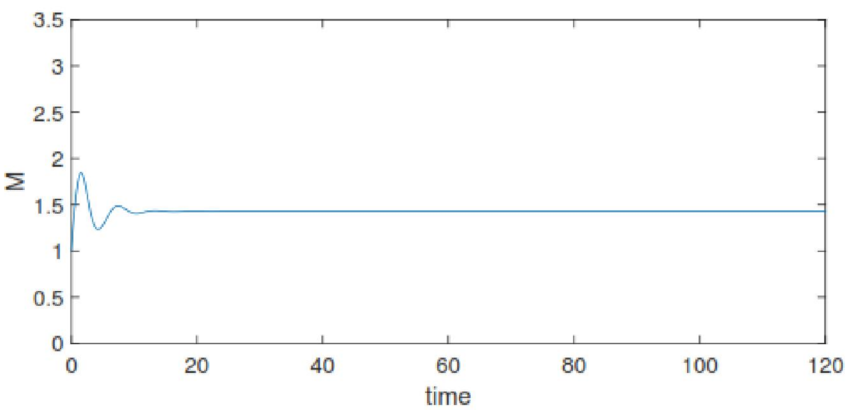


FIGURE 2. Population of predator for $b = 0.5$, $\mu = 1.6$, $\delta = 0.7$, $g = 1$

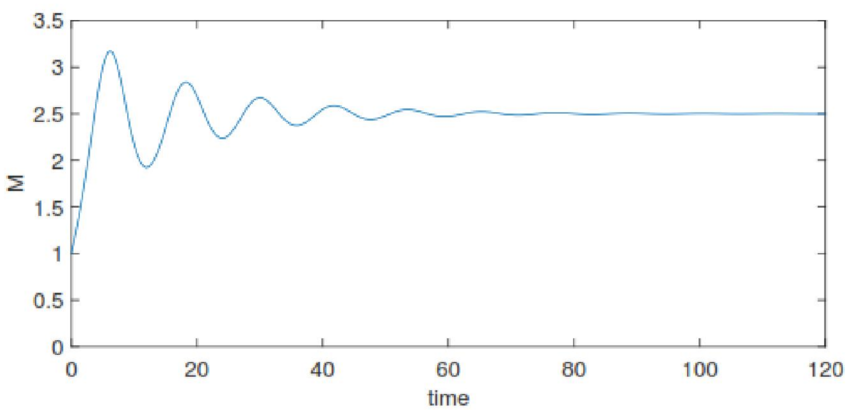


FIGURE 3. Population of predator for $b = 0.5$, $\mu = 0.2$, $\delta = 0.7$, $g = 0.3$

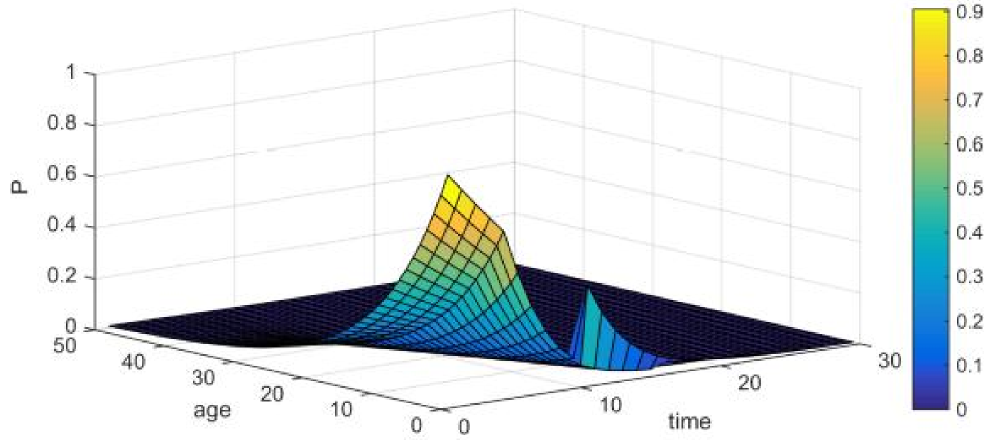


FIGURE 4. Population of prey for $b = 0.5$, $\mu = 1.6$, $\delta = 0.7$, $g = 1$

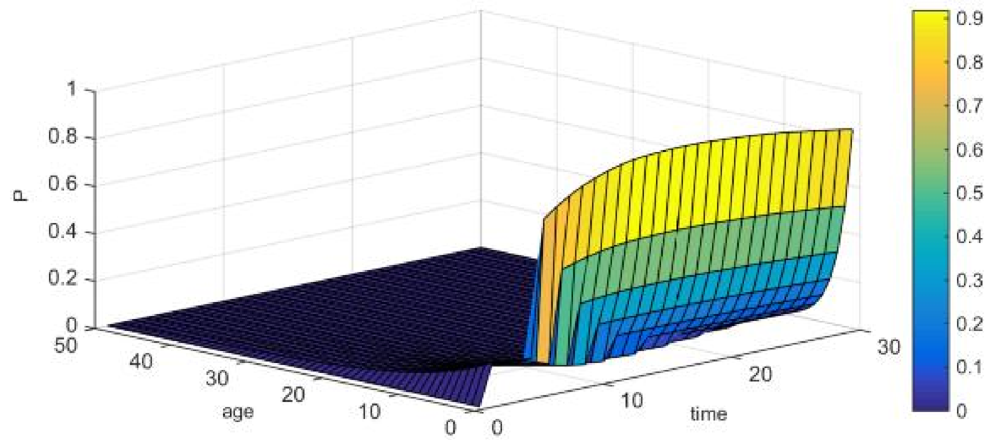


FIGURE 5. Population of prey for $b = 0.5$, $\mu = 0.2$, $\delta = 0.7$, $g = 0.3$

By employing the steady age distribution, two steady states of the system are obtained: one is mono-species and another one is coexisting species. Numerical results with biological meaning are provided which is very useful to visualize the mealybug controlling problem. Here we have shown that this useful hybrid model, with one age-structured compartment coupled to an unstructured compartment has exactly one asymptotically globally stable steady state. The solution of the transient model is obtained analytically, albeit implicitly, thus providing a check on computational solutions in more complex situations. This parallels the outcome in systems which are not age or spatially structured. To prevent recurring outbreaks requires that this predator release rate should ideally be maintained. It is expected that a similar outcome will apply when the parameters are functions of time and/or age.

เนื้อหางานวิจัย

1. Rationale:

Agriculture has been the backbone of Thailand's economy for several decades. Cassava is considered to be one of the major agriculture crops of Thailand. Although Thailand is not a major consumer of cassava, it is the world's biggest exporter of cassava with the world's market share of 60.72% in 2011 according to the office of agricultural economics, ministry of agriculture and cooperatives, Thailand. Even though cassava can survive both hot and dry conditions, an increase in insect pests might easily cause a major loss in crop yield. Mealybugs (Hemiptera: Pseudococcidae) constitute a major family of insect pests of cassava. In Thailand, there are four species of mealybugs found in cassava fields which are striped mealybugs, Madeira mealybugs, pink mealybugs (cassava mealybug) and Jack-Beardsley mealybugs. In 2008, cassava mealybugs were first identified in Thailand and has spread aggressively throughout cassava's planting area in Thailand. In 2010, there was an outbreak of cassava mealybugs in Thailand resulting in a major loss in cassava yield. The total cassava yield reduced from 30 million tons per year to 22 tons per year according to the information from the Office of Agricultural Economics, Thailand.

There are various practices to control the spread of mealybugs in cassava fields in Thailand. Farmers might use biological controls, insecticides or a mixture of biological controls and insecticides. With biological controls, various practices have been recommended by the Thai Tapioca Development Institute and the Department of Agriculture, Ministry of Agriculture and Cooperatives, Thailand. The suggestions on the number of natural enemies to be released in a field and the period between each natural enemies released are diverse and also depend on the type of the natural enemies to be released. Natural enemies that have been used popularly to control the spread of cassava mealybugs are *Anagyrus lopezi* and green lacewings.

2. Objectives:

The objectives of this project are as follows:

2.1 To study the outbreak of insect pests of cassava.

2.2 To obtain mathematical models for studying the spreading of insect pests.

2.3 To obtain the efficient controls for the outbreak of insect pests in terms of how often/ how much farmers should release natural enemies of insect pests in order to control the outbreak of insect pests and maximize profit.

3. Methodology:

3.1 Gather information about rice, cassava, sugarcane, maize and the outbreak of their plant diseases and insect pests.

3.2 Develop mathematical models/ cellular automata models to study the outbreak of plant diseases and insect pests of rice, cassava, sugarcane and maize.

3.3 Use the available reported field data to estimate some parameters in the developed mathematical models.

3.4 Analyze the developed mathematical models theoretically/ numerically to obtain the appropriate controls of plant diseases and insect pests of rice, cassava, sugarcane and maize by using Hopf Bifurcation theorem, singular perturbation technique, Runge Kutta method or cellular automata/ Monte Carlo simulation.

4. Results:

4.1 Cellular automata Model

In this part, cellular automata together with Monte Carlo simulation are employed to investigate the spread of mealybugs in a cassava field with the usual practices of biological control in Thailand.

A cellular automaton with Moore's neighborhood of a square lattice with the size $L \times L$ represents a cassava field. The states of every cell in the lattice will be updated in parallel at each time step (1 time step Δt = 1 day). Each cell in the lattice represents a state of cassava planted in the cell which will be updated at each time step according to given rules. The updating cell will be indexed by (i, j) with its immediate neighborhood, distant neighborhood and far distant neighborhood represented as the light grey, grey and dark grey areas, respectively, in Figure 1. The possible states of each cell in the lattice are

- susceptible cassava (*S*) which indicates that the cassava plant in that cell is free from mealybugs, or
- infested cassava (*I*) which indicates that the cassava plant in that cell has mealybugs on it, or
- empty cell (*E*) which indicates that the cassava plant in that cell was removed from the field.

(i-3,j-3)	(i-3,j-2)	(i-3,j-1)	(i-3,j)	(i-3,j+1)	(i-3,j+2)	(i-3,j+3)
(i-2,j-3)	(i-2,j-2)	(i-2,j-1)	(i-2,j)	(i-2,j+1)	(i-2,j+2)	(i-2,j+3)
(i-1,j-3)	(i-1,j-2)	(i-1,j-1)	(i-1,j)	(i-1,j+1)	(i-1,j+2)	(i-1,j+3)
(i,j-3)	(i,j-2)	(i,j-1)	(i,j)	(i,j+1)	(i,j+2)	(i,j+3)
(i+1,j-3)	(i+1,j-2)	(i+1,j-1)	(i+1,j)	(i+1,j+1)	(i+1,j+2)	(i+1,j+3)
(i+2,j-3)	(i+2,j-2)	(i+2,j-1)	(i+2,j)	(i+2,j+1)	(i+2,j+2)	(i+2,j+3)
(i+3,j-3)	(i+3,j-2)	(i+3,j-1)	(i+3,j)	(i+3,j+1)	(i+3,j+2)	(i+3,j+3)

Figure1 The light grey, grey and dark grey areas represent immediate neighborhood, distant neighborhood and far distant neighborhood, respectively.

4.1.1 Using green lacewings as a biological control agent

We let P_t^i, P_t^m and P_t^e denote the numbers of mealybugs in the instar state, adult state, egg state, respectively, at the time step t. M_t^i, M_t^d, M_t^m and M_t^e denote the numbers of green lacewings in the larva state, pupa stage, adult state and egg state, respectively, at the time step t. P_t and M_t denote the total numbers of mealybugs and green lacewings, respectively, at the time step t.

At each time step, a number $r, 0 \le r \le 1$ is randomized and each cell will be updated at random as shown in the flowcharts in Figure 2 – Figure 8.

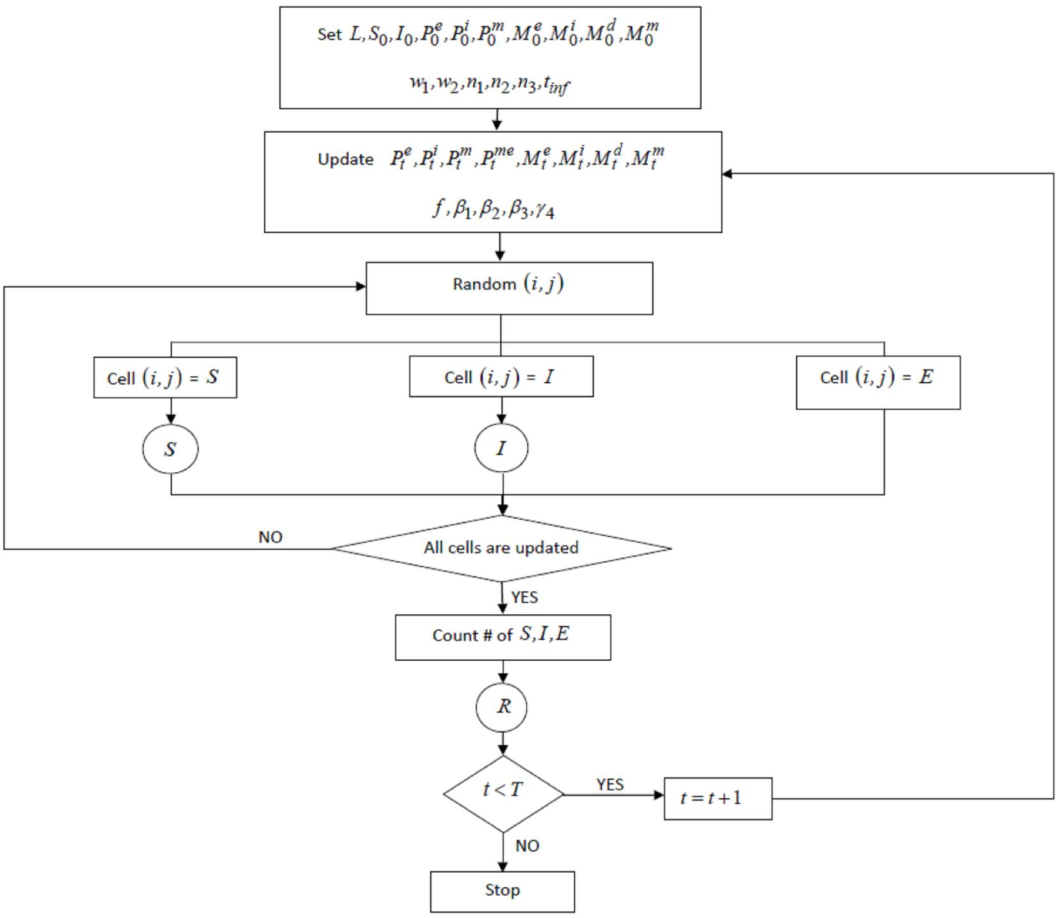


Figure 2 The main loop of CA.

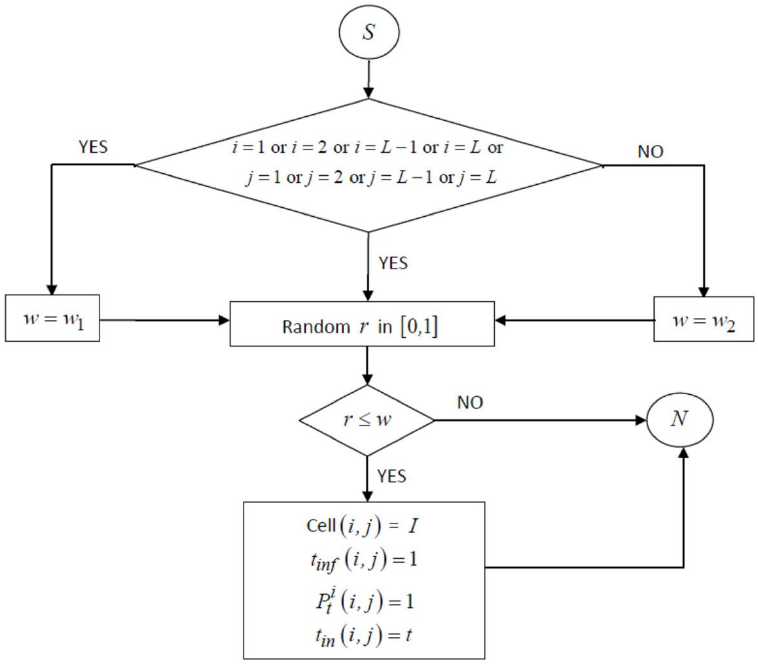


Figure 3 The main susceptible cassava updating loop.

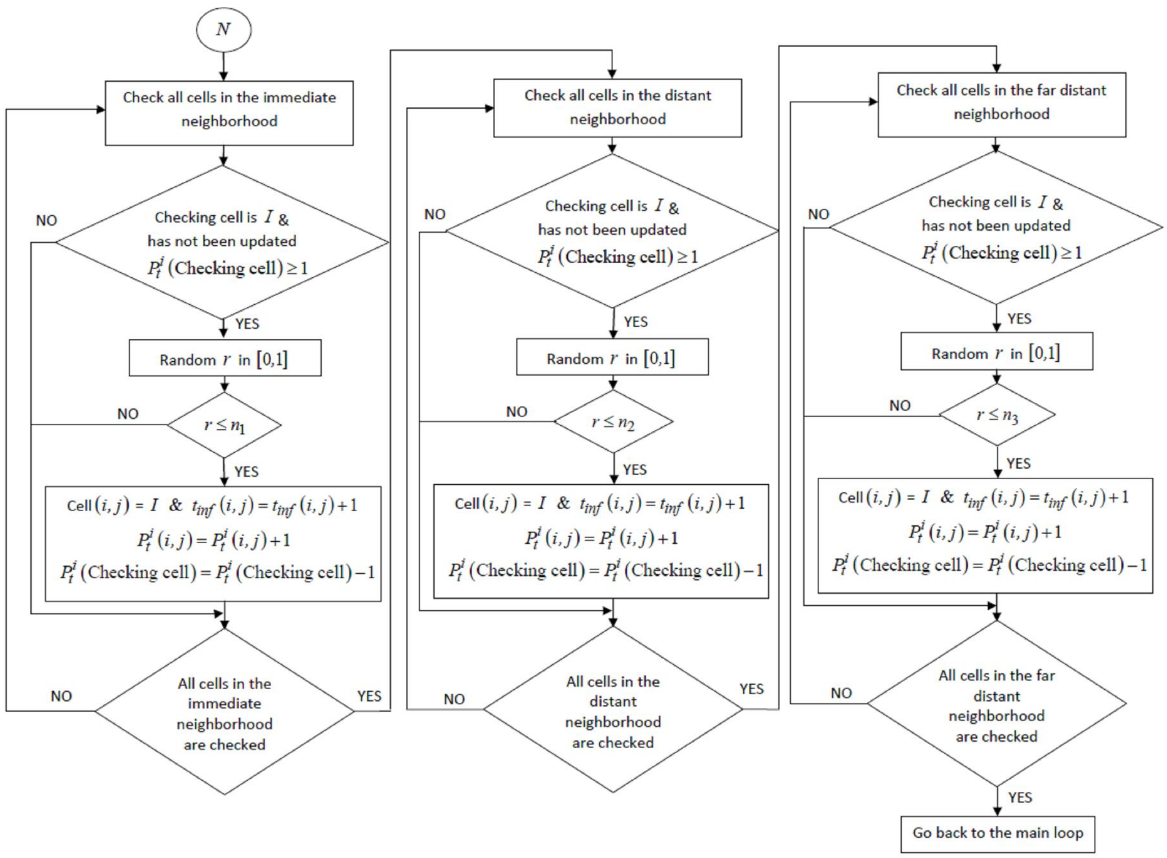


Figure 4 The neighbourhood checking loop of the susceptible cassava updating loop.

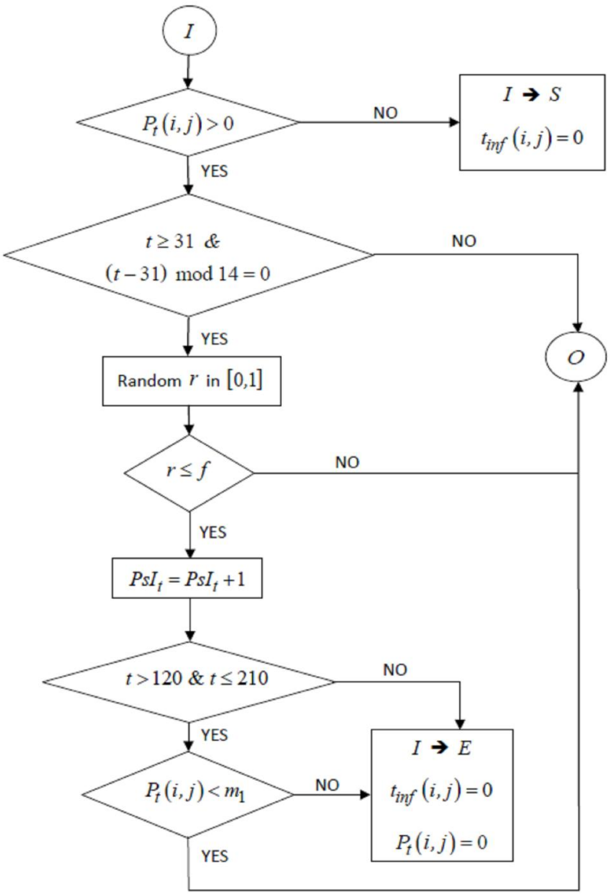


Figure 5 The main infected cassava updating loop.

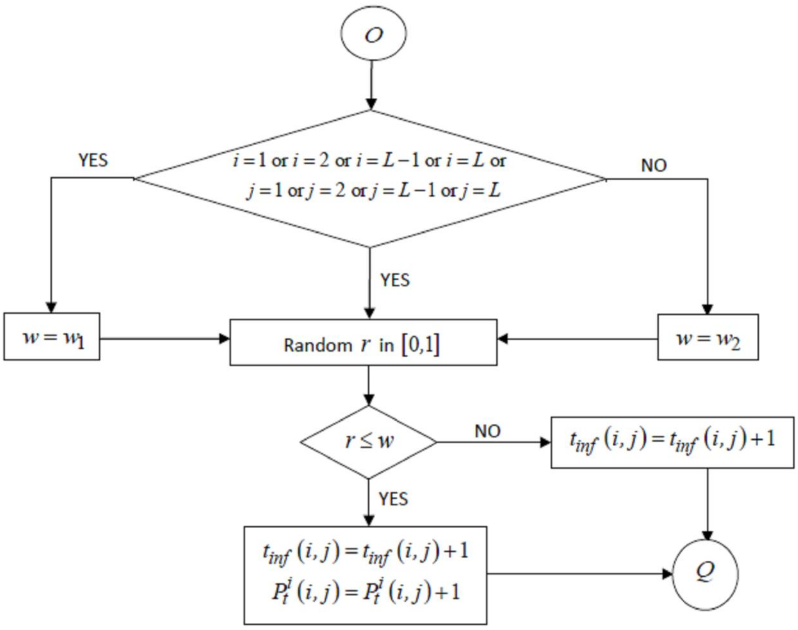


Figure 6 The number of mealybugs updating loop (incorporating wind effect) of the infected cassava.

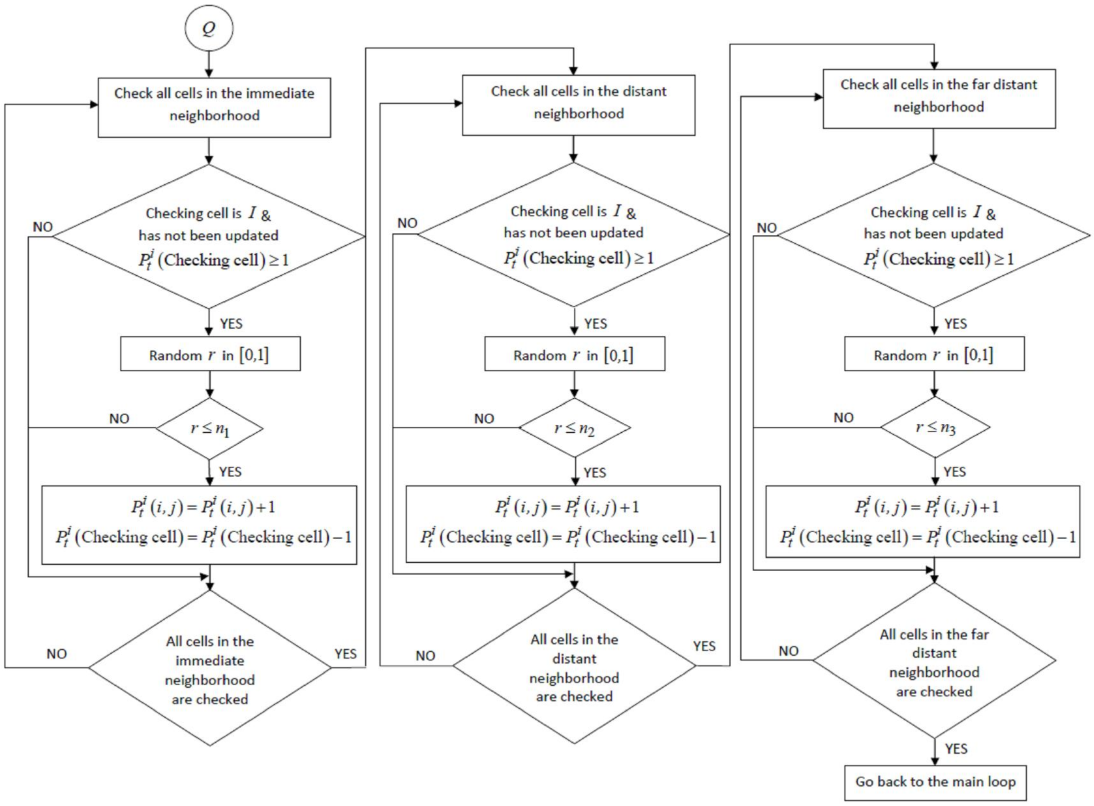


Figure7 The neighbourhood checking loop of the number of mealybugs updating loop (incorporating wind effect) of the infected cassava.

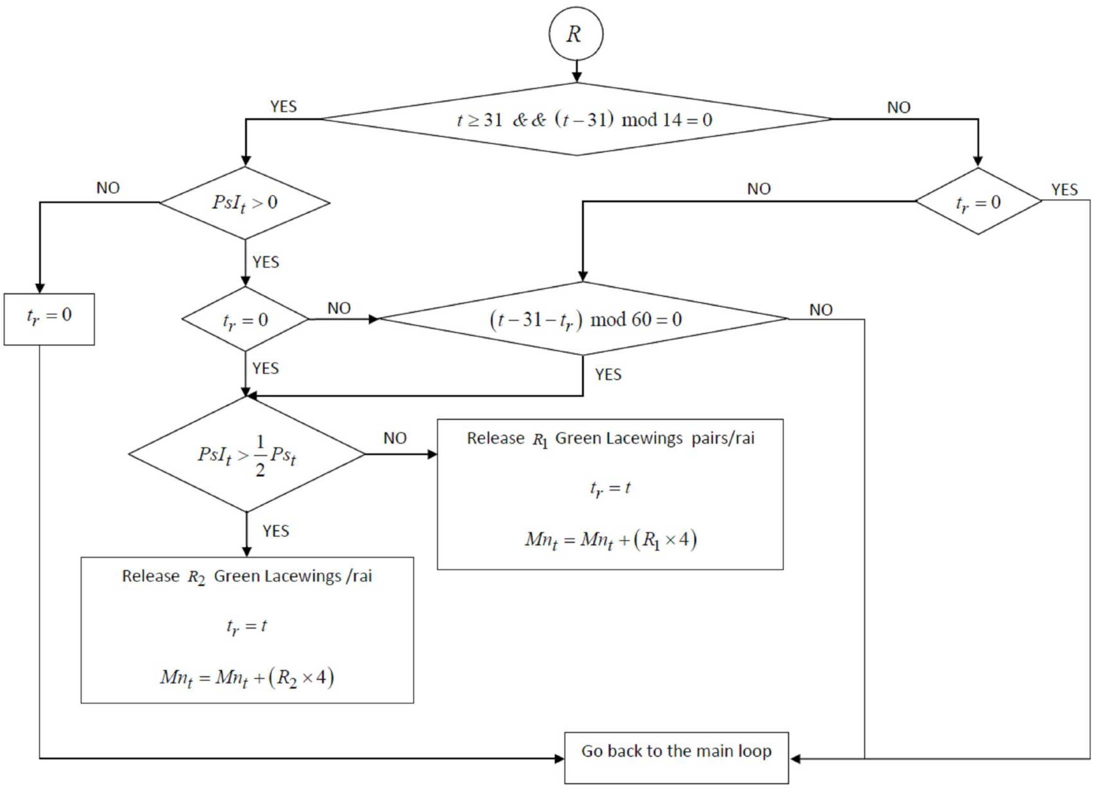


Figure 8 The green lacewing releasing loop.

Moreover, the populations of mealybugs and green lacewings in each cell at every stage are also monitored. We also assumed that only mealybugs of the instar stage may be blown from one infected cassava to other cassava plants in the immediate/ distant/ far distant neighborhood in the cassava field with the probabilities n_1, n_2 and n_3, respectively ($0 < n_3 < n_2 < n_1 \le 1$).

When a month has passed after cassava planting, if there is an infected cassava plant among the surveyed cassava plants, the green lacewings at the larva stage or the adult stage are to be released every two months if there still are mealybugs on the surveyed cassava plants in the field (Boonseng 2009). If the number of surveyed infected cassava plants is less than a half of the total number of surveyed cassava plants in the field, the number of green lacewings to be released in the field is R_1 per rai or else the number of green lacewings to be released in the field is R_2 per rai.

Despite the effect of the wind, the numbers of mealybugs at all stages on the cassava plant in each cell of the lattice are also updated according to the mealybug life-cycle as follows. For each cell $(i,j), i = 1,2,\ldots,L$ and $j = 1,2,\ldots,L$, using the following difference equations.

Mealybugs at the instar stage:

$$P_{t+\Delta t}^{i} = P_t^i + r_1\alpha_1 P_t^e - \alpha_2 P_t^i - \beta_1\left(P_t^i, M_t^i\right)M_t^i$$

Mealybugs at the adult stage:

$$P_{t+\Delta t}^{m} = P_t^m + r_2\alpha_2 P_t^i - \alpha_3 P_t^m - \beta_2\left(P_t^m, M_t^i\right)M_t^i$$

Mealybugs at the egg stage:

$$P_{t+\Delta t}^{e} = P_t^e + r_3\alpha_4 v_1 P_t^m - \alpha_1 P_t^e - \beta_3\left(P_t^e, M_t^i\right)M_t^i$$

where α_1 and α_2 are the fractions of mealybug's eggs and instar mealybugs that develop into instar mealybugs and adult mealybugs, respectively, in one time step.

r_1 and r_2 are the survival rates of mealybug's eggs and instar mealybugs that develop into instar mealybugs and adult mealybugs, respectively. α_3 is the natural death rate of adult mealybugs. r_3 is the fraction of female adult mealybugs. α_4 is the fraction of female adult mealybugs in the reproductive period. v_1 is the average number of eggs laid by a female adult mealybug in one time step. $\beta_1\left(P_t^i, M_t^i\right), \beta_2\left(P_t^m, M_t^i\right)$ and $\beta_3\left(P_t^e, M_t^i\right)$ are the average numbers of instar mealybugs, adult mealybugs and mealybug's eggs eaten by green lacewings of the larva stage in one time step.

Apart from the release of green lacewings as a biological control agent of mealybugs, the number of green lacewings at every stage on the cassava plant in each cell of the lattice is also updated according to green lacewing life-cycle as follows:

Green lacewings at the larva stage: (Only green lacewings in larva stage behave like a predator on mealybugs)

$$M_{t+\Delta t}^i = M_t^i + s_1\gamma_1 M_t^e - \gamma_2 M_t^i$$

Green lacewings at the pupa stage:

$$M_{t+\Delta t}^d = M_t^d + s_2\gamma_2\delta_1\left(P_t^i, P_t^m, P_t^e, M_t^i\right)M_t^i - \gamma_3 M_t^d$$

Green lacewings at the adult stage:

$$M_{t+\Delta t}^m = M_t^m + s_3\gamma_3 M_t^d - \delta_2 M_t^m$$

Green lacewings at the egg stage:

$$M_{t+\Delta t}^e = M_t^e + s_4 v_2 M_t^m - \gamma_1 M_t^e$$

where γ_1, γ_2 and γ_3 are the fractions of green lacewing's eggs, larva green lacewings and pupa green lacewings that develop into green lacewing larva, green lacewing pupa and adult green lacewings, respectively, in one time step. s_1, s_2 and s_3 are the survival rates of green lacewing's eggs, green lacewing larva and green lacewing pupa that develop into green lacewing larva, green lacewing pupa and adult green lacewings, respectively. $\delta_1\left(P_t^i, P_t^m, P_t^e, M_t^i\right)$ is the efficiency of converting green lacewing larva to green lacewing pupa. δ_2 is the natural death rate of adult green lacewings. s_4 is the fraction of female adult green lacewings. v_2 is the average number of eggs laid by a female adult green lacewings in one time step.

Furthermore, the approximated total crop yield is also monitored. We also assume that the estimated crop yield is a kilograms per cassava plant if there is no mealybug in the cassava field. The estimated crop yield will be reduced by 100%, 30% and 10%, approximately, if mealybugs spread on the cassava plants during the first 4 months, during the 5th and the 7th month, and during the 8th and the 12th month, respectively, according to the surveys of the Thai Tapioca Development Institute in 2007-2010. We then assume that the estimated crop yield will be reduced by 100%, 30% and 10%, approximately, if mealybugs spread on the cassava plants during the first 4 months, during the 5th and the 7th month, and during the 8th and the 12th month, respectively, if the infection period is more than two weeks. Hence, the estimated crop yield at each time step, $Y(t)$, is

$$Y(t) = a \cdot CS(t) + (0.9 \times a) \cdot CI_1(t) + (0.7 \times a) \cdot CI_2(t)$$

where $CS(t)$ represents the total number of susceptible cassava at the time step t, $CI_1(t)$ represents the total number of cassava infected by mealybugs during the 8th and the 12th month at the time step t and $CI_2(t)$ represents the total number of cassava infected by mealybugs during the 5th and the 7th month at the time step t.

4.1.1.1 Effect of different temperatures

Parameters in the model are then estimated from the available reported data the values are as shown in Table 1.

Parameter	Temperature		
	25°C	27°C	30°C
α_1	0.1075	0.1163	0.1493
α_2	0.0435	0.0482	0.0388
α_3	0.0612	0.0665	0.0596
α_4	0.4591	0.4991	0.4468
r_1	0.89	0.87	0.912
r_2	0.7737	0.6079	0.6589
r_3	0.74	0.72	0.85
v_1	40	36.53	2.9126
γ_1	0.1639	0.2222	0.2703
γ_2	0.0521	0.0585	0.0625
γ_3	0.0714	0.0885	0.1053
s_1	0.804	0.821	0.817
s_2	0.9191	0.9629	0.7938
s_3	0.9586	0.9614	0.7402
s_4	0.54	0.55	0.485
v_2	7.2789	5.8237	2.3876
δ_2	0.0227	0.0175	0.0206

Table 1. The estimation of the parameters in CA model.

The simulation of the spread of mealybugs in a cassava field at 25°C, 27°C and 30°C are as shown in Figures 9-13 (the averaged values of the 100 runs using MATLAB software).

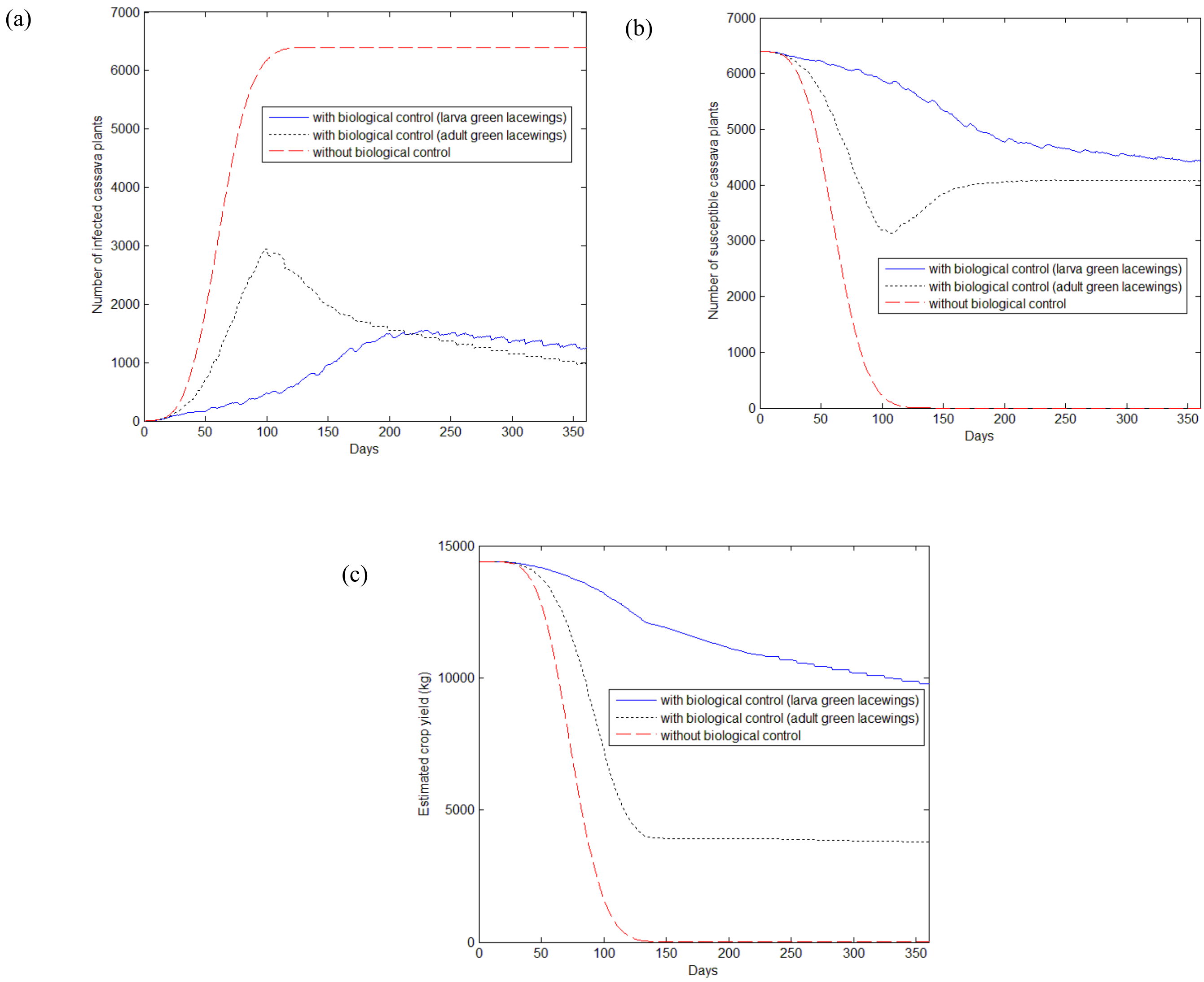


Figure 9 A simulation of the spread of mealybugs in a cassava field at 25°C.(a) the number of infected cassava (*I*). (b) the number of susceptible cassava (*S*). (c) the estimated crop yield (*Y*).

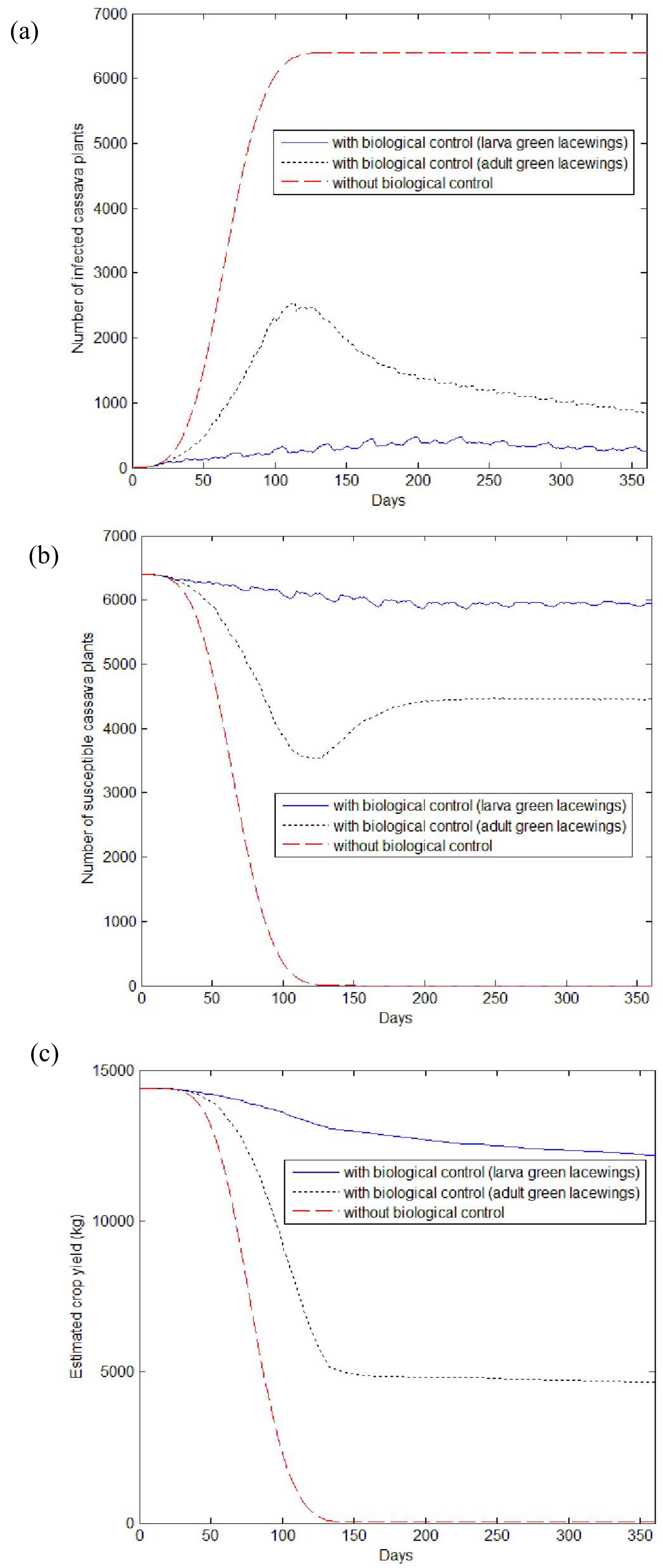


Figure 10 A simulation of the spread of mealybugs in a cassava field at 27°C. (a) the number of infected cassava (*I*). (b) the number of susceptible cassava (*S*). (c) the estimated crop yield (*Y*).

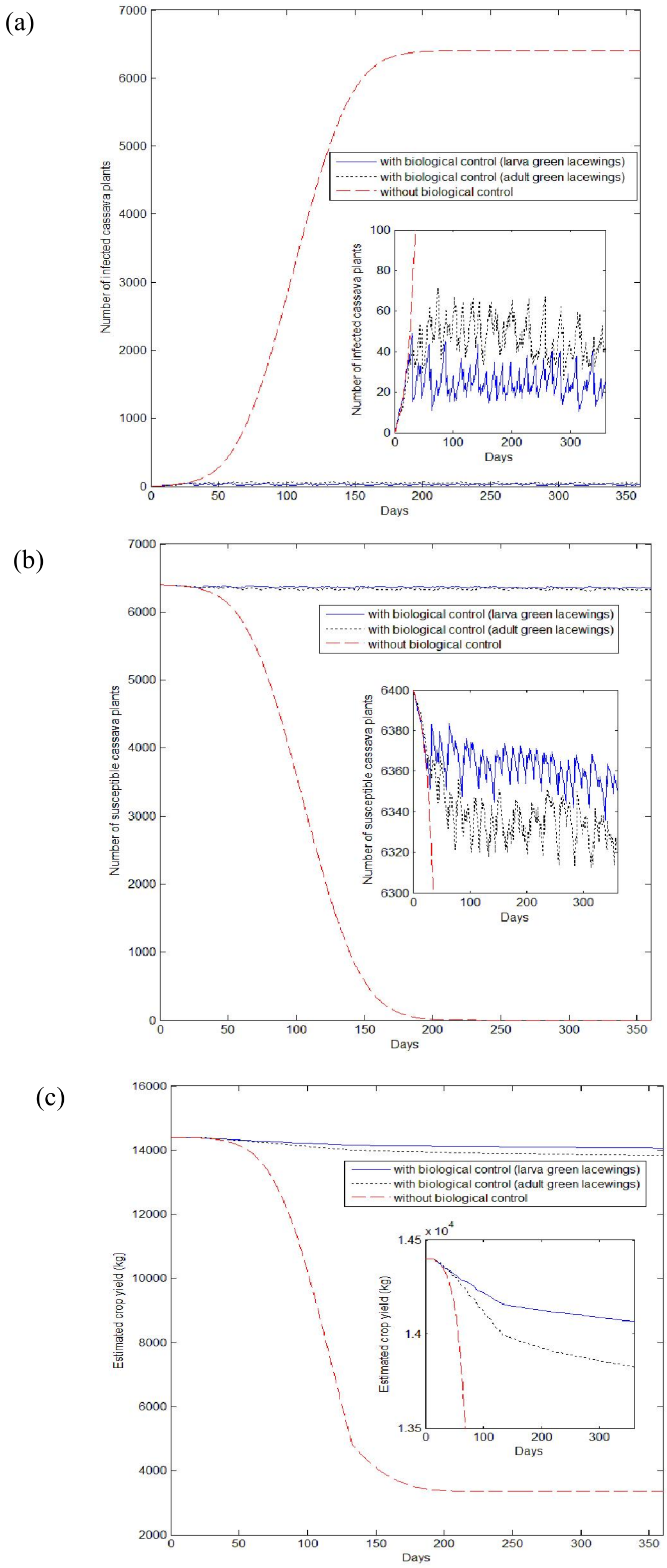


Figure 11 A simulation of the spread of mealybugs in a cassava field at 30°C. (a) the number of infected cassava (I). (b) the number of susceptible cassava (S). (c) the estimated crop yield (Y).

(a)

(b)

(c)

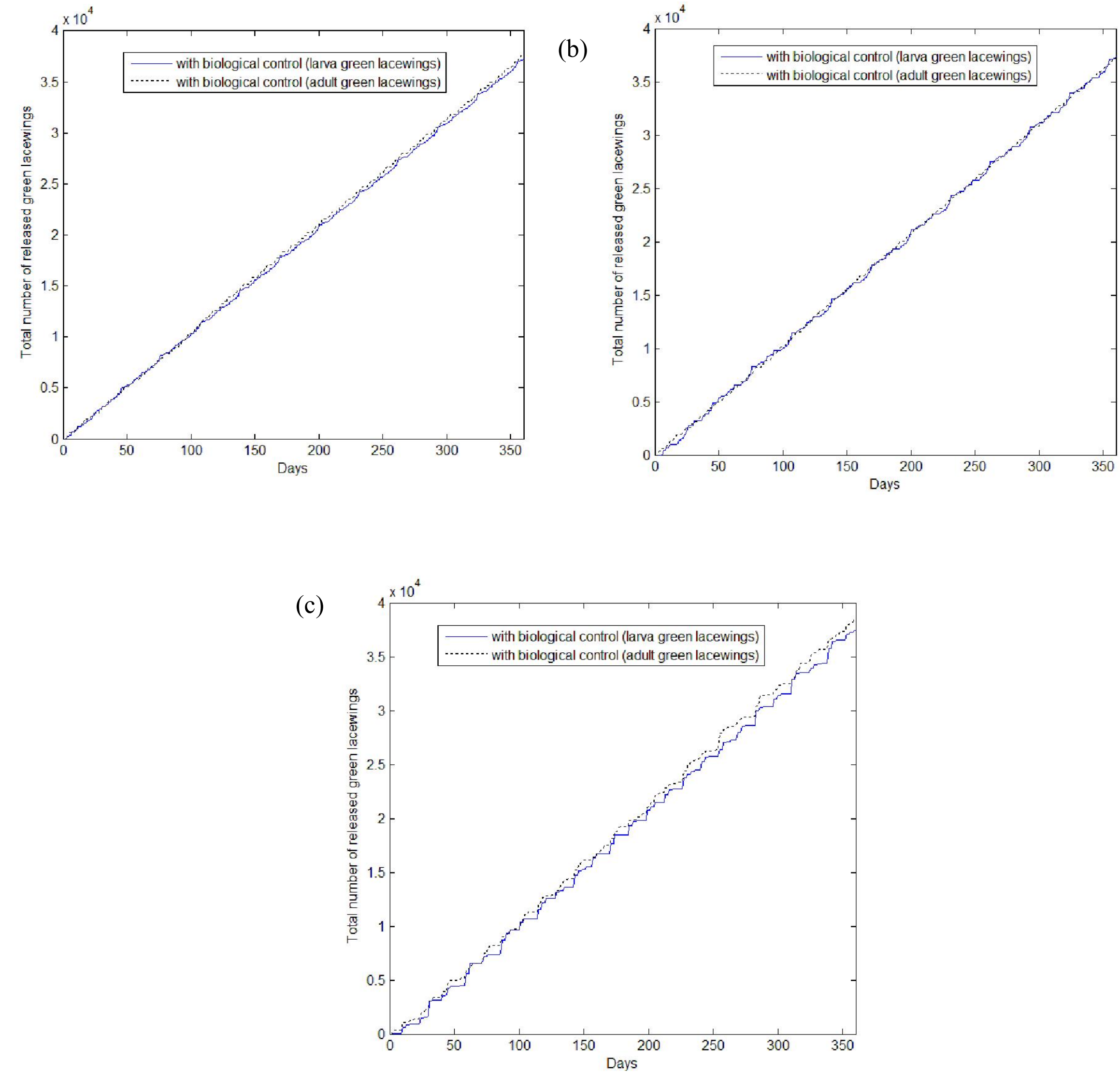


Figure 12 A simulation of the spread of mealybugs in a cassava field. (a) the total number of released green lacewings in the cassava field at 25°C. (b) the total number of released green lacewings in the cassava field at 27°C. (c) the total number of released green lacewings in the cassava field at 30°C.

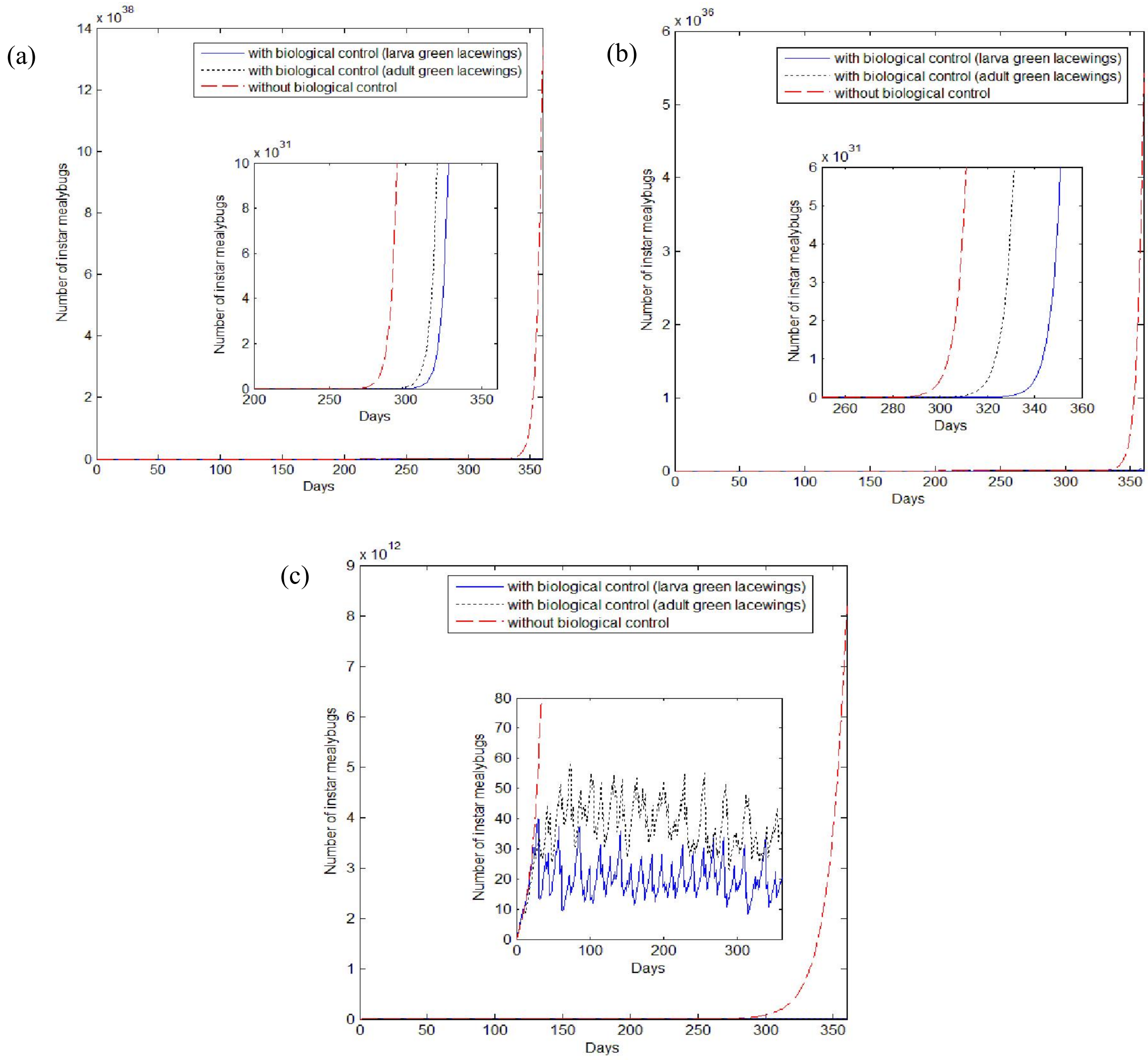


Figure 13 A simulation of the spread of mealybugs in a cassava field. (a) number of instar mealybugs in the cassava field at 25°C. (b) number of instar mealybugs in the cassava field at 27°C. (c) number of instar mealybugs in the cassava field at 30°C.

The simulation results indicated that

- Without biological control, the estimated crop yield decreases dramatically and tends to zero at 25°C, and at 27°C approximately 4 months after planting. At 30°C, the estimated crop yield decreases and tends to a constant level which is lower than 30% of the maximum estimated crop yield.

- With biological control, the number of infected cassava plants decreases whereas the number of susceptible cassava plants increases when the temperature increases which might be the results of shorter life cycle, lower survival rate, lower fecundity and shorter adult longevity of mealybugs. We can also see that the release of green lacewing larva gives a better result when there is a spread of mealybugs even though the lower amount of green lacewing larva is released compared to adult green lacewings. The reasons for this might be the shorter life span, lower survival rate, lower fecundity or shorter adult longevity of green lacewings because only green lacewings at the larva stage behave like a predator of mealybugs and if we release adult green lacewings it will take a period of time before they will lay eggs which develop into green lacewing larva, finally behaving like a predator of mealybugs. With the increase of temperature, the survival rate and the fecundity rate are even lower and hence the greater amount of adult green lacewings should be released in the cassava field to control the spread of mealybugs. On the other hand, the estimated crop yield also increases when the temperature increases, however, the number of green lacewings released in the cassava field is also increased. Hence, the cost of biological control and the increase in crop yield should be calculated in order to obtain the most efficient biological control that maximizes profit.

4.1.1.2 Effect of different release frequencies of green lacewings

In addition, the simulation of the spread of mealybugs in a cassava field at 30°C with different frequencies of the release of green lacewings are as shown in

Figures 14-16 (the averaged values of the 10 runs using MATLAB software). In the simulations, we assume that

$$L = 40, a = 3.601, R_1 = 800, R_2 = 1{,}000, n_1 = 0.05, n_2 = 0.005, n_3 = 0.0005,$$

$$w_1 = 0.01, w_2 = 0.001$$

(a)

(b)

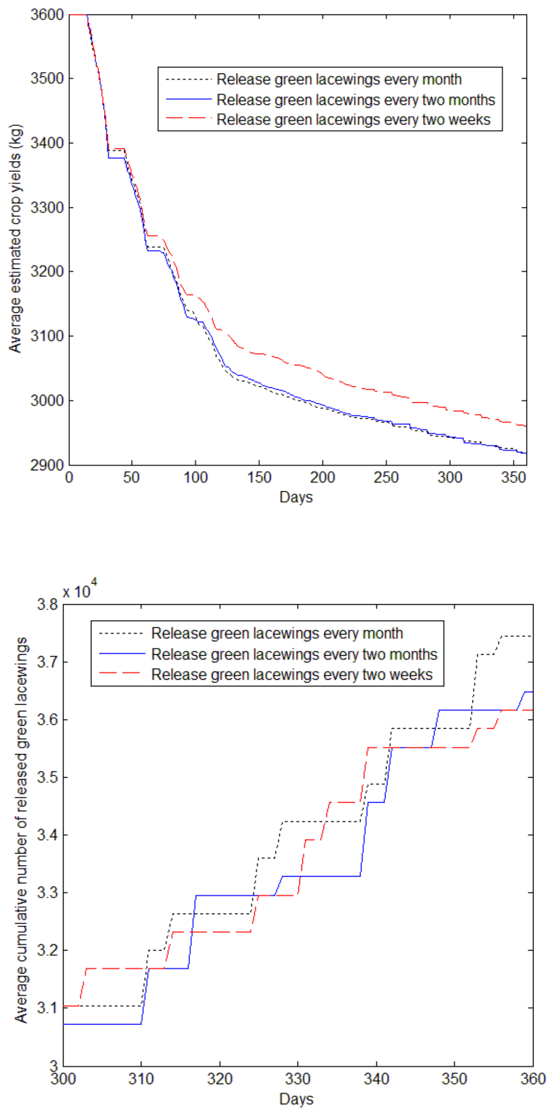


Figure 14 Simulation results of the spread of mealybugs in a cassava field. (a) The estimated crop yields. (b) The cumulative number of released green lacewings.

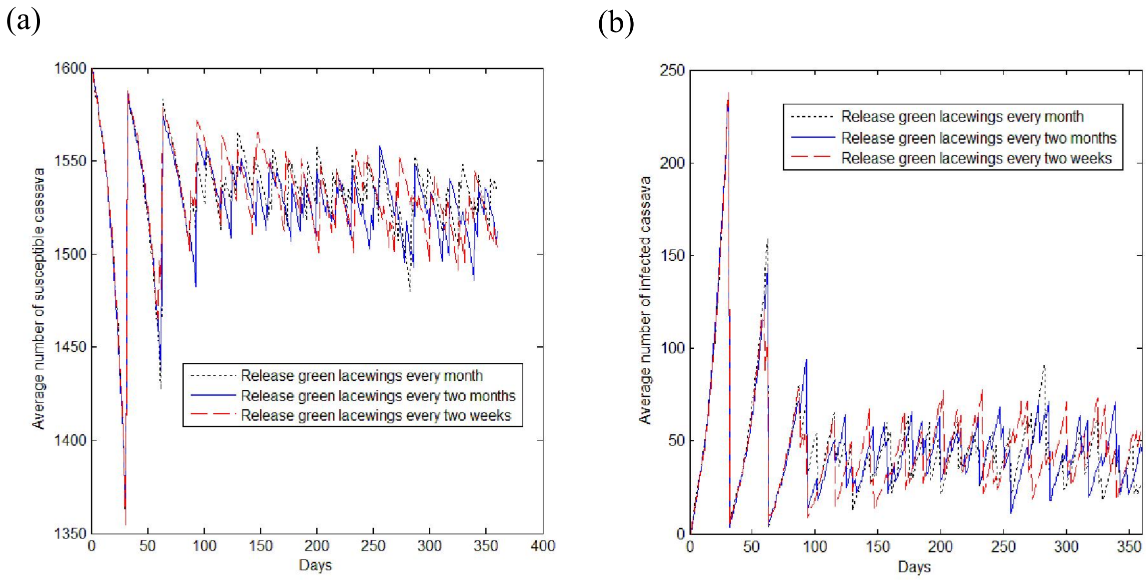


Figure 15 Simulation results of the spread of mealybugs in a cassava field. (a) The number of susceptible cassava. (b) The number of infected cassava.

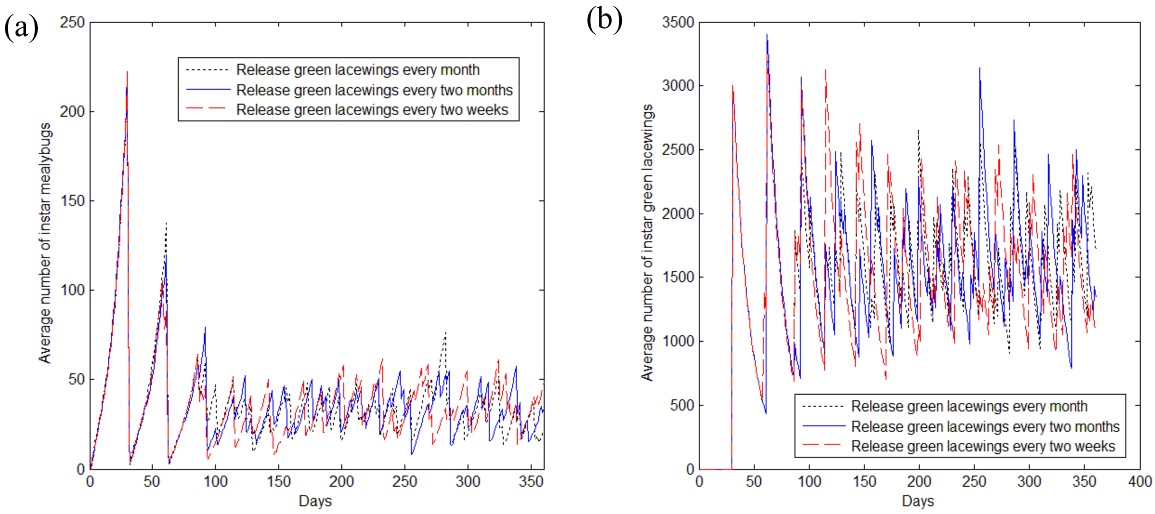


Figure 16 Simulation results of the spread of mealybugs in a cassava field. (a) The number of instar mealybugs. (b) The number of larva green lacewings.

From the simulation results shown in Figures 14 – 16, we can see that the release frequency of 2 weeks gives the better result as the estimated crop yields in this case is higher than in the other cases. However, the cumulative numbers of released green lacewings every 2 weeks and every 2 months are at about the same level while it is a little bit higher when they are released every month. Moreover, the spread of mealybugs seems to be controllable in all three cases.

Since the cumulative numbers of released green lacewings every 2 weeks and every 2 months are at about the same level, the costs for the release of green lacewings every 2 weeks and every 2 months are then different only at the wages. The wages in the case of 2 weeks release frequency will be four times those of the case of 2 months release frequency. In Figure 14 (a), the estimated crop yields for the release frequency of 2 weeks is approximately 50 kg higher than the release frequency of 2 months at the end of the planting period. Suppose that the market price of cassava is 3 baht per kilogram, the release frequency of 2 weeks will give 150 baht or 1.67% more on the total crop sale income and hence, the increase in wages will not be covered by the increased income in this case. Therefore, the release frequency of 2 months seems to be the better option. However, in this study the cassava field of interest is just 1 rai (0.16 ha) and hence the infected probability through the wind might be higher compared to the larger field whereas more labor force may be necessary. It also depends on how high the wages are for such labor and how long it takes to finish the task. On the other hand, the increased income from the yields would be higher for a larger field. Therefore, the further study is needed before any general conclusion can be drawn.

4.1.2 Using *Anagyrus Lopezi* as a biological control agent

According to the recommended cassava's planting instructions of the Department of Agricultural Extension, Ministry of Agriculture and Cooperatives, Thailand, the appropriate planting distance between two cassava plants is 1 metre. Suppose that the cassava's planting area is 4 rai (0.64 ha) and the total number of cassava plants in this field is then 6,400 plants initially. Cellular automata and Monte Carlo simulation technique are then employ. A 80 × 80 lattice will be used to represent the cassava field while each cell in the lattice represents a cassava plant. There are three possible states for each cell. Susceptible cell (S) represents the cassava plant which is not infested by cassava mealybug. Infested cell (I) represents the cassava plant which is infested by cassava mealybug. Empty cell (E) is the cell that the cassava plant is removed from the field because it was infested during the first 4 months or the last 5 months of planting period (the planting period of cassava is about a year) and will be removed immediately when it is surveyed (this is the usual practice recommended by the Department of Agricultural Extension, Ministry of Agriculture and Cooperatives, Thailand).

Initially, every cell in the lattice will be assumed to be susceptible cells. After that, at each time step (1 time step (Δt) = 1 day), the states of every cell in the lattice will be updated at random order according to the following rules where a number $r, 0 \le r \le 1$ will be randomized.

Rules for updating susceptible cell (S)

If the randomized cell is a susceptible cell, the following rules are used.

(i) The randomized cell might become an infested cell due to the infestation of instar cassava mealybugs through wind transfer with the probability v. Note that the probability that the cells belonged to the first two rows next to each of the four borders of the lattice might become an infested cell $(w = w_1)$ is higher than the probability that the other cells in the lattice might become an infested cell $(w = w_2)$ that is $0 \le w_2 < w_1 \le 1$.

(ii) The randomized cell might become an infested cell if at least one of the cells in the immediate neighborhood, distant neighborhood and far distant neighborhood of the randomized cell is an infested cell with the probabilities n_1, n_2, n_3, respectively where $0 \le n_3 < n_2 < n_1 \le 1$ (The immediate neighborhood, distant neighborhood and far distant neighbood are as shown in Figure 1).

Rules for updating infested cell (I)

If the randomized cell is an infestation cell, the following rules are used.

(i) The randomized cell might become a susceptible cell if the wasp Anagyrus lopezi successfully feed/parasitism on cassava mealybugs so that there is no cassava mealybugs on the cassava plant in the randomized cell.

(ii) After a month of planting, each of the cassava plants might be surveyed with the probability s every two weeks.

(a) If the randomized cell is surveyed during the first 4 months or the last 5 months after planting then the cell will become an empty cell.

(b) If the randomized cell is surveyed during the 5th month and the 7th month of planting and the number of cassava mealybugs on the cassava plant in the cell is greater than M_1, then the cell will become an empty cell.

Rule for updating empty cell (E)

If the randomized cell is an empty cell, then no change occurs.

Starting from the 2nd month of cassava planting, the cassava field will be surveyed every 2 weeks. If cassava mealybugs are found on the surveyed cassava plants, the wasp *Anagyrus lopezi* will be released randomly on the infested cassava plants. The number of the wasp *Anagyrus lopezi* to be released in the cassava field depends on the severity of the spread of cassava mealybugs. We then assume that if over 50% of surveyed cassava plants are infested then the number of *Anagyrus lopezi* to be released is A_1 or else the number of *Anagyrus lopezi* to be released is A_2.

In addition to the wind effect, the numbers of cassava mealybug and *Anagyrus lopezi* at all stages on each cassava plant are also updated based upon their life-cycle as follows

Mealybugs at the instar stage:

$$C^i_{t+\Delta t} = C^i_t + r_1\alpha_1 C^e_t - \alpha_2 C^i_t - \beta_1\left(C^i_t, W^m_t\right)W^m_t$$

Mealybugs at the adult stage:

$$C^m_{t+\Delta t} = C^m_t + r_2\alpha_2 C^i_t - \alpha_3 C^m_t - \beta_2\left(C^m_t, W^m_t\right)W^m_t$$

Mealybugs at the egg stage:

$$C^e_{t+\Delta t} = C^e_t + r_3\alpha_4 v_1 C^m_t - \alpha_1 C^e_t - \beta_3\left(C^e_t, W^m_t\right)W^m_t$$

Anagyrus lopezi at the larva stage:

$$W^i_{t+\Delta t} = W^i_t + s_1\gamma_1 W^e_t - \gamma_2 W^i_t$$

Anagyrus lopezi at the pupa stage:

$$W^d_{t+\Delta t} = W^d_t + s_2\gamma_2 W^i_t - \gamma_3 W^d_t$$

Anagyrus lopezi at the adult stage:

$$W^m_{t+\Delta t} = W^m_t + s_3\gamma_3 W^d_t - \delta_1 W^m_t$$

Anagyrus lopezi at the egg stage:

$$W^e_{t+\Delta t} = W^e_t + s_4\delta_2\left(C^i_t, C^m_t, C^e_t, W^m_t\right)W^m_t - \gamma_1 W^e_t$$

where C_t^i, C_t^m and C_t^e represent the number of cassava mealybugs in instar stage, adult stage and egg stage, respectively, at the time step t; W_t^i, W_t^d, W_t^m and W_t^e represent the numbers of *Anagyrus lopezi* in larva stage, pupa stage, adult stage and egg stage, respectively, at the time step t. α_1 and α_2 are the fractions of eggs and instar cassava mealybugs that develop into instar cassava mealybugs and adult cassava mealybugs, respectively, per time step. r_1 and r_2 are the survival probabilities that cassava mealybugs of egg and instar stage in this time step develop to cassava mealybugs of the instar and adult stage in the next time step, respectively. α_3 is the natural death rate of adult cassava mealybugs per time step. r_3 is the fraction of adult female cassava mealybugs. α_4 is the fraction of adult female cassava mealybug in the reproductive period. v_1 is the average number of eggs that laid by an adult female cassava mealybug in the reproductive period per time step. β_1, β_2 and β_3 are the average numbers of instar cassava mealybugs, adult cassava mealybugs and cassava mealybug's eggs, respectively, eaten by an adult *Anagyrus lopezi* per time step depending on the number of cassava mealybugs at the instar and adult stage and the number of *Anagyrus lopezi* on each cassava plant. γ_1, γ_2 and γ_3 are the fractions of *Anagyrus lopezi* at the egg, larva and pupa stage in this step develop into the larva, pupa and adult stage in the next step, respectively, with the corresponding survival probabilities s_1, s_2 and s_3. δ_1 is the natural death rate of the adult *Anagyrus lopezi* per time step. s_4 is the fraction of adult female *Anagyrus lopezi*. $\delta_2\left(C_t^i, C_t^m, C_t^e, W_t^i\right)$ is the efficiency of laying eggs of an adult female *Anagyrus lopezi* per time step depending on the number of consumed cassava mealybugs.

In addition, the estimated crop yield $y(t)$ at the end of the planting period is also monitored at each time step t. Assuming that the estimated crop yield is e

kilograms per a cassava plant if the plant has never been infested with cassava mealybugs longer than 2 weeks during the planting period, whereas the estimated crop yields will be reduced by 100%, 30% and 10% if the cassava plant is infested with cassava mealybugs longer than 2 weeks during the first 4 months, during the period between the 5th and the 7th months, and during the period between the 8th and the 12th months, respectively, the estimated cassava's crop yield at each time step can be calculated as follows

$$y(t) = e \cdot y_S(t) + (0.9 \times e) \cdot y_{I_1}(t) + (0.7 \times e) \cdot y_{I_2}(t)$$

where $y_S(t)$ represents the total number of cassava plants that have never been infested with cassava mealybugs at the time step t. $y_{I_1}(t)$ represents the total number of cassava plants that have been infested with cassava mealybugs longer than 2 weeks during the period between the 8th and the 12th months of planting at the time step t. $y_{I_2}(t)$ represents the total number of cassava plants that have been infested with cassava mealybugs longer than 2 weeks during the period between the 5th and the 7th months of planting at the time step t.

4.1.2.1 Effect of different temperatures

Parameters in the model are then estimated from the available reported data the values are as shown in Table 2.

Parameter	Temperature		
	20°C	25°C	30°C
α_1	0.0613	0.1075	0.1493
α_2	0.0199	0.0435	0.0388
α_3	0.0440	0.0612	0.0596
α_4	0.4836	0.4591	0.4468
r_1	0.9100	0.8900	0.9120
r_2	0.5602	0.7737	0.6589
r_3	0.7800	0.7400	0.8500
v_1	23.6364	40.0000	2.9126

Parameter	Temperature		
	20°C	25°C	30°C
γ_1	0.6885	0.6849	1.0000
γ_2	0.1780	0.2346	0.3385
γ_3	0.1065	0.1659	0.2087
s_1	0.9710	0.9390	0.9700
s_2	0.8856	0.8789	0.7828
s_3	0.8856	0.8789	0.7828
s_4	0.5405	0.6579	0.5405
δ_1	0.0223	0.0541	0.1099

Table 2. The estimation of the parameters in CA model.

The simulation of the spread of mealybugs in a cassava field at 20°C, 25°C and 30°C are as shown in Figures 17 (using MATLAB software).

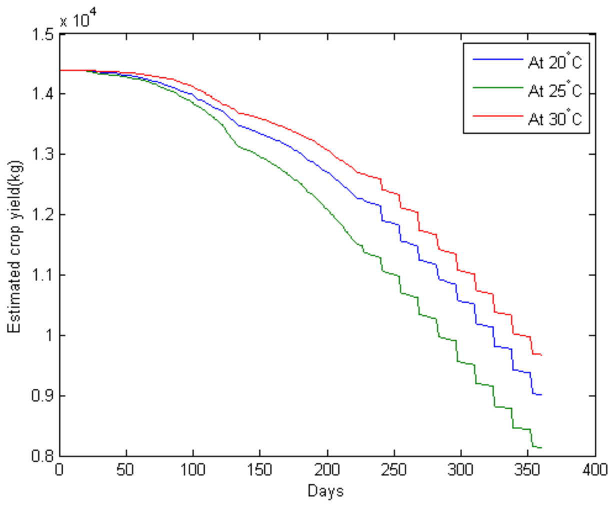


Figure 17 Simulation results of the spread of mealybugs in a cassava field at 20°C, 25°C and 30°C when *Anagyrus lopezi* are released once cassava mealybugs are detected.

4.1.2.2 Effect of different release frequencies and different number of released *Anagyrus Lopezi*

There are various recommended practices for farmers when an outbreak of cassava mealybug occurs in Thailand. One suggestion is to release *Anagyrus lopezi* once when cassava mealybugs are detected whereas the other suggestion is to release *Anagyrus lopezi* every three weeks for three times since cassava mealybugs are detected. Here, the simulation of the spread of mealybugs in a cassava field at 25°C with different frequencies of the release of *Anagyrus lopezi* and different number of released *Anagyrus Lopezi* are as shown in Figures 18 (the averaged values of the 10 runs using MATLAB software). Here, we investigate six different manners of releasing of *Anagyrus Lopezi* in a cassava field when the spread of cassava mealybugs is detected. The six manners are listed as follows.

I: Release *Anagyrus Lopezi* only once when the spread of cassava mealybug is first detected in the field with the amount of 50-100 pairs per rai.

II: Release *Anagyrus Lopezi* only once when the spread of cassava mealybug is first detected in the field with the amount of 200 pairs per rai.

III: Release *Anagyrus Lopezi* only once when the spread of cassava mealybug is first detected in the field with the amount of 400 pairs per rai.

IV: Release *Anagyrus Lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field with the amount of 50-100 pairs per rai.

V: Release *Anagyrus Lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field with the amount of 200 pairs per rai.

VI: Release *Anagyrus Lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field with the amount of 400 pairs per rai.

Computer simulations of the six manners are carried out using MATLAB software. In the simulations, the parameters in the system of difference equations are as in Table 2 at 25°C where

$L = 80,\ n_1 = 0.001, n_2 = 0.0001, n_3 = 0.00001, w_1 = 0.0001, w_2 = 0.00001$ and $a = 2.25$.

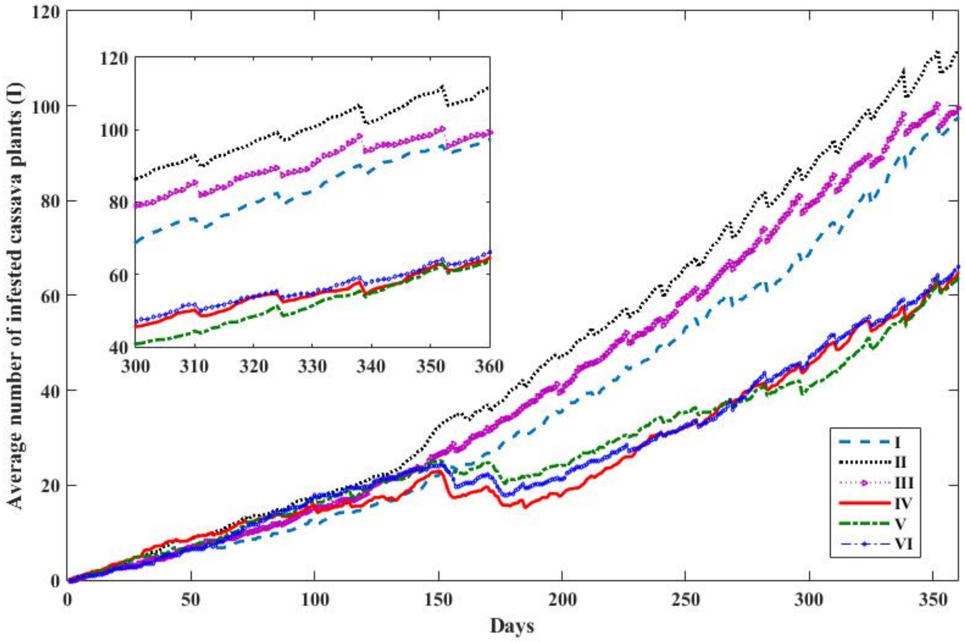


Figure 18 Simulation results of the spread of mealybugs in a cassava field at 25°C when *Anagyrus lopezi* are released in the six different manners.

4.1.3 Using a mixed of green lacewings and *Anagyrus Lopezi* as biological control agents

Based upon the recommended instructions of the Department of Agricultural Extension, Ministry of Agriculture and Cooperatives, Thailand, we then make the following assumptions. Initially, all cassava plants in the field are free of cassava mealybugs. However, the wind might blow instar cassava mealybugs into the field and some cassava plants might be infested. Note that only cassava mealybug at the instar stage can be blown with the wind.

Starting on the 2nd month of cassava's planting the surveyed for cassava mealybugs will be carried out every two weeks by collecting the numbers of mealybugs at all stages on the cassava plants that are not planted on the two rows next to the four borders of the cassava field. The survey will be conducted on every two rows of plants, and every eleven plants as instructed by the Department of Agricultural Extension, Ministry of Agriculture and Cooperatives, Thailand.

If the plant infested with cassava mealybugs is subjected to be surveyed and the day that the survey is conducted is during the first 4 months or the last 5 months of planting period, the plant will then be removed from the field. On the other hand, if cassava mealybug is found on a surveyed plant during the 5th - 8th month of planting, *Anagyrus lopezi* and green lacewings will be released in the field in order to control the spread of cassava mealybugs in the field. Note that the planting period of cassava is assumed to be one year.

We then assume further that if a cassava plant is infested with cassava mealybugs longer than two weeks, the estimated crop yields at the end of planting period will decrease. Crop yields will be reduced by 100\%, 30\% and 10\% if the cassava plant is infested with cassava mealybugs during the first 4 months, during the period between the 5th and the 7th months, and during the last 5 months, respectively, according to the surveys of the Thai Tapioca Development Institute in 2007-2010.

Here, a square lattice of the size $L \times L$ lattice will be used to represent a cassava field while each cell in the lattice represents a cassava plant. The possible states for each cell in the lattice are susceptible cell (*S*) representing the cassava plant which is not infested by cassava mealybug, infested cell (*I*) representing the cassava plant which is infested by cassava mealybug and empty cell (*E*) representing the cell that the cassava plant was removed from the field. At first, every cell in the lattice will be assumed to be susceptible cells. After that, at each time step (1 time step) = 1 day), the states of every cell (i, j) in the lattice will be updated at random order according to the following rules where a number $r, 0 \le r \le 1$ will be randomized.

Rules for updating susceptible cell (*S*)

If the randomized cell is a susceptible cell, the following rules are used.

(i) The randomized cell might become an infested cell due to the infestation of instar cassava mealybugs through wind transfer with the probability v. Note that the probability that the cells belonged to the first two rows next to each of the four borders of the lattice might become an infested cell $(w = w_1)$ is higher than the probability that the other cells in the lattice might become an infested cell $(w = w_2)$ that is $0 \le w_2 < w_1 \le 1$.

(ii) The randomized cell might become an infested cell if at least one of the cells in the immediate neighborhood, distant neighborhood and far distant neighborhood of the randomized cell is an infested cell with the probabilities n_1, n_2, n_3, respectively where $0 \le n_3 < n_2 < n_1 \le 1$ (The immediate neighborhood, distant neighborhood and far distant neighbood are as shown in Figure 1).

Rules for updating infested cell (*I*)

If the randomized cell is an infestation cell, the following rules are used.

(i) The randomized cell might become a susceptible cell if the wasp Anagyrus lopezi successfully feed/parasitism on cassava mealybugs so that there is no cassava mealybugs on the cassava plant in the randomized cell.

(ii) After a month of planting, each of the cassava plants might be surveyed with the probability s every two weeks.

(a) If the randomized cell is surveyed during the first 4 months or the last 5 months after planting then the cell will become an empty cell.

(b) If the randomized cell is surveyed during the 5th month and the 7th month of planting and the number of cassava mealybugs on the cassava plant in the cell is greater than M_1, then the cell will become an empty cell.

Rule for updating empty cell (*E*)

If the randomized cell is an empty cell, then no change occurs.

Starting on the 2nd month of cassava's planting the surveyed for cassava mealybugs will be conducted every two weeks.

(i) During the first 4 months and the last 5 months of cassava's planting period: if the infested plant is surveyed, it will be removed from the cassava field.

(ii) During the 5th and the 7th month of cassava's planting period: if the infested plant is surveyed, *Anagyrus lopezi* and green lacewings will be released in the cassava field in the following manners

***Anagyrus lopezi*:**

Anagyrus lopezi of the adult stage (the only stage that feed on/parasitism cassava mealybugs) will be released only once or every 3 weeks for 3 times. The number of *Anagyrus lopezi* to be released in the field is assumed to be R_3 pairs per rai.

Green lacewings:

Green lacewings of the larva stage (the only stage that feed on cassava mealybugs) will be released every 2 weeks, every month or every two months until there is no cassava mealybugs found on the surveyed cassava plant. The number of larva green lacewings to be released in the cassava field depends on the severity of the spread of cassava mealybugs. If the number of surveyed infested cassava plants is less than a half of the total number of surveyed cassava plants in the field, the number of green lacewings to be released in the field is R_1 per rai or else the number of green lacewings to be released in the field is R_2 where $0 < R_1 < R_2$.

In this study, we also monitor the number of cassava mealybugs, *Anagyrus lopezi* and green lacewings at each stage on each cell of the lattice.

Apart from the wind effects the number of cassava mealybugs on each cassava plant is also subjected to change according to its life cycle. Similarly, the life cycles of *Anagyrus lopezi* and green lacewings will also effects the numbers of *Anagyrus lopezi* and green lacewings, respectively, on each cassava plant in the field apart from the numbers of *Anagyrus lopezi* and green lacewings released in the field to control the spread of cassava mealybugs. Hence, we also assume that the following systems of difference equations represent the change in the numbers of cassava mealybugs, *Anagyrus lopezi* and green lacewings at each stage on each cell (i, j) of the lattice according to their life cycles.

Cassava mealybug

$$\begin{aligned}
\textit{Instar Stage:} \quad C^i_{t+\Delta t} &= C^i_t + r_1\alpha_1 C^e_t - \alpha_2 C^i_t - \beta_{11}(C^i_t, A^m_t, G^i_t)A^m_t \\
&\quad -\beta_{12}(C^i_t, A^m_t, G^i_t)G^i_t \\
\textit{Adult Stage:} \quad C^m_{t+\Delta t} &= C^m_t + r_2\alpha_2 C^i_t - \alpha_3 C^m_t - \beta_{21}(C^m_t, A^m_t, G^i_t)A^m_t \\
&\quad -\beta_{22}(C^m_t, A^m_t, G^i_t)G^i_t \\
\textit{Egg Stage:} \quad C^e_{t+\Delta t} &= C^e_t + r_3\alpha_4 v_1 C^m_t - \alpha_1 C^e_t - \beta_{31}(C^e_t, A^m_t, G^i_t)A \\
&\quad -\beta_{32}(C^e_t, A^m_t, G^i_t)G^i_t
\end{aligned}$$

Anagyrus lopezi

$$\begin{aligned}
\textit{Larva stage:} \quad A^i_{t+\Delta t} &= A^i_t + s_{11}\gamma_{11}A^e_t - \gamma_{21}A^i_t \\
\textit{Pupa stage:} \quad A^d_{t+\Delta t} &= A^d_t + s_{21}\gamma_{21}A^i_t - \gamma_{31}A^d_t \\
\textit{Adult stage:} \quad A^m_{t+\Delta t} &= A^m_t + s_{31}\gamma_{31}A^d_t - \delta_{11}A^m_t \\
\textit{Egg stage:} \quad A^e_{t+\Delta t} &= A^e_t + s_{41}\delta_{21}(C^i_t, C^m_t, C^e_t, A^m_t)A^m_t - \gamma_{11}A^e_t
\end{aligned}$$

Green lacewings

$$
\begin{aligned}
\textit{Larva stage:} \quad G^i_{t+\Delta t} &= G^i_t + s_{12}\gamma_{12}G^e_t - \gamma_{22}G^i_t \\
\textit{Pupa stage:} \quad G^d_{t+\Delta t} &= G^d_t + s_{22}\gamma_{22}\delta_{12}(C^i_t, C^m_t, C^e_t, G^i_t)G^i_t - \gamma_{32}G^d_t \\
\textit{Adult stage:} \quad G^m_{t+\Delta t} &= G^m_t + s_{32}\gamma_{32}G^d_t - \delta_{22}G^m_t \\
\textit{Egg stage:} \quad G^e_{t+\Delta t} &= G^e_t + s_{42}v_{22}G^m_t - \gamma_{12}G^e_t
\end{aligned}
$$

Parameter	Definition	Value
• Cassava Mealybug		
α_1	the fraction of cassava mealybug's eggs that develop into instar cassava mealybugs in one time step	0.1299
α_2	the fraction of instar cassava mealybugs that develop into adult cassava mealybugs in one time step	0.0571
α_3	the natural death rate of adult cassava mealybugs	0.0483
α_4	the fraction of survived female adult cassava mealybugs in the reproductive period	0.5334
r_1	the survival rate of cassava mealybug's eggs that develop into instar cassava mealybugs	0.9575
r_2	the survival rate of instar cassava mealybugs that develop into adult cassava mealybugs	0.9666
r_3	the fraction of female adult cassava mealybugs	1.0000
v_1	the average number of eggs laid by a female adult cassava mealybug in one time step	16.9250
• *Anagyrus lopezi*		
γ_{11}	the fraction of *Anagyrus lopezi*'s eggs that develop into larva *Anagyrus lopezi* in one time step	0.5000
γ_{21}	the fraction of larva *Anagyrus lopezi* that develop into pupa *Anagyrus lopezi* in one time step	0.1000
γ_{31}	the fraction of pupa *Anagyrus lopezi* that develop into adult *Anagyrus lopezi* in one time step	0.1667
δ_{11}	the natural death rate of adult *Anagyrus lopezi* per a time step	0.0855
s_{11}	the survival rate of *Anagyrus lopezi*'s eggs that develop into larva *Anagyrus lopezi*	0.9000
s_{21}	the survival rate of larva *Anagyrus lopezi* that develop into pupa *Anagyrus lopezi*	0.9000
s_{31}	the survival rate of pupa *Anagyrus lopezi* that develop into adult *Anagyrus lopezi*	0.9000
s_{41}	the fraction of female adult *Anagyrus lopezi*	0.5290
• Green lacewings		
γ_{12}	the fraction of green lacewing's eggs that develop into larva green lacewing in one time step	0.1639
γ_{22}	the fraction of larva green lacewings that develop into pupa green lacewing in one time step	0.0521
γ_{32}	the fraction of pupa green lacewings that develop into adult green lacewings in one time step	0.0714
δ_{22}	the natural death rate of adult green lacewings	0.0227
s_{12}	the survival rate of green lacewing's eggs that develop into larva green lacewing	0.8040
s_{22}	the survival rate of larva green lacewing that develop into pupa green lacewing	0.9191
s_{32}	the survival rate of pupa green lacewing that develop into adult green lacewings	0.9586
s_{42}	the fraction of female adult green lacewings	0.5400
v_2	the average number of eggs laid by a female adult green lacewings in one time step	7.2789

Here, the total of 54 manners of biological controls with *Anagyrus lopezi* and green lacewings are investigated. Computer simulations of 54 cases are carried out using MATLAB software. The results are as follows.

Anagyrus Lopezi (AL) \ Green Lacewings (GL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 - 1,000 per rai	2,000 per rai	200 per rai	800 - 1,000 per rai	2,000 per rai	200 per rai	800 - 1,000 per rai	2,000 per rai
Release AL only 1 time	50-100 pairs per rai	1	2	3	4	5	6	7	8	9
	200 pairs per rai	10	11	12	13	14	15	16	17	18
	400 pairs per rai	19	20	21	22	23	24	25	26	27
Release AL 3 times every 3 weeks	50-100 pairs per rai	28	29	30	31	32	33	34	35	36
	200 pairs per rai	37	38	39	40	41	42	43	44	45
	400 pairs per rai	46	47	48	49	50	51	52	53	54

Average estimated cassava's crop yields at the end of planting period (kgs)

Anagyrus Lopezi (AL) \ Green Lacewings (GL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	12,585.32	12,869.36	12,687.20	12,096.44	12,753.40	12,818.47	12,466.70	12,325.67	12,400.82
	200 pairs per rai	12,645.89	12,274.28	12,798.67	12,684.55	12,353.08	12,784.36	12,531.10	12,616.24	12,743.27
	400 pairs per rai	12,883.40	13,001.48	12,685.94	12,985.06	12,899.29	12,832.78	12,283.33	12,448.12	12,491.41
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	13,093.96	13,013.14	12,963.50	13,130.09	12,946.81	13,173.83	12,451.63	12,995.18	13,002.16
	200 pairs per rai	13,193.32	13,061.92	13,179.23	12,879.94	13,393.21	13,479.34	13,166.41	13,127.44	13,308.79
	400 pairs per rai	13,312.70	13,283.81	13,370.84	13,024.61	13,570.01	13,565.65	13,522.94	13,646.20	13,425.16

Average total numbers of AL and GW released in the field at the end of planting period

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	AL = 200 pairs GL = 10,400	AL = 200 pairs GL = 44,800	AL = 200 pairs GL = 120,000	AL = 200 pairs GL = 6,400	AL = 200 pairs GL = 25,600	AL = 200 pairs GL = 64,000	AL = 200 pairs GL = 3,200	AL = 200 pairs GL = 12,800	AL = 200 pairs GL = 32,000
	200 pairs per rai	AL = 800 pairs GL = 11,200	AL = 800 pairs GL = 35,200	AL = 800 pairs GL = 112,000	AL = 800 pairs GL = 6,400	AL = 800 pairs GL = 22,400	AL = 800 pairs GL = 48,000	AL = 800 pairs GL = 3,200	AL = 800 pairs GL = 12,800	AL = 800 pairs GL = 32,000
	400 pairs per rai	AL = 1,600 pairs GL = 11,200	AL = 1,600 pairs GL = 44,800	AL = 1,600 pairs GL = 96,000	AL = 1,600 pairs GL = 6,400	AL = 1,600 pairs GL = 16,000	AL = 1,600 pairs GL = 64,000	AL = 1,600 pairs GL = 3,200	AL = 1,600 pairs GL = 12,800	AL = 1,600 pairs GL = 32,000
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	AL = 600 pairs GL = 10,400	AL = 600 pairs GL = 51,200	AL = 600 pairs GL = 128,000	AL = 600 pairs GL = 6,400	AL = 600 pairs GL = 25,600	AL = 600 pairs GL = 48,000	AL = 600 pairs GL = 3,200	AL = 600 pairs GL = 12,800	AL = 600 pairs GL = 24,000
	200 pairs per rai	AL = 2,400 pairs GL = 9,600	AL = 2,400 pairs GL = 44,800	AL = 2,400 pairs GL = 80,000	AL = 2,400 pairs GL = 5,600	AL = 2,400 pairs GL = 25,600	AL = 2,400 pairs GL = 64,000	AL = 2,400 pairs GL = 3,200	AL = 2,400 pairs GL = 9,600	AL = 2,400 pairs GL = 32,000
	400 pairs per rai	AL = 4,800 pairs GL = 8,800	AL = 4,800 pairs GL = 25,600	AL = 4,800 pairs GL = 96,000	AL = 4,800 pairs GL = 6,400	AL = 4,800 pairs GL = 25,600	AL = 4,800 pairs GL = 56,000	AL = 4,800 pairs GL = 3,200	AL = 4,800 pairs GL = 12,800	AL = 4,800 pairs GL = 24,000

Average estimated cost of AL and GW released in the field at the end of planting period

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	6,100.00	23,300.00	60,900.00	4,100.00	13,700.00	32,900.00	2,500.00	7,300.00	16,900.00
	200 pairs per rai	9,200.00	21,200.00	59,600.00	6,800.00	14,800.00	27,600.00	14,800.00	58,000.00	144,400.00
	400 pairs per rai	12,800.00	29,600.00	55,200.00	10,400.00	15,200.00	39,200.00	5,200.00	10,000.00	19,600.00
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	7,900.00	28,300.00	66,700.00	5,900.00	15,500.00	26,700.00	15,200.00	58,400.00	144,800.00
	200 pairs per rai	15,600.00	33,200.00	50,800.00	13,600.00	23,600.00	42,800.00	8,800.00	13,600.00	23,200.00
	400 pairs per rai	26,000.00	34,400.00	69,600.00	24,800.00	34,400.00	49,600.00	14,700.00	57,900.00	144,300.00

1 pairs of AL = 4.50 baht

1 green lacewings = 0.50 baht

Average estimated income from selling cassava's crop yields at the end of planting period

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	35,404.33	35,452.70	34,804.13	33,917.08	35,100.00	34,748.45	31,860.00	35,016.20	34,375.50
	200 pairs per rai	35,528.63	35,620.88	35,337.95	33,837.75	35,191.70	35,240.63	32,646.95	35,233.88	35,458.33
	400 pairs per rai	35,390.25	35,334.00	35,395.33	35,346.95	35,155.13	35,182.13	33,777.63	32,940.63	34,855.33
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	35,502.20	35,319.38	35,286.20	35,185.50	35,235.58	35,380.70	31,642.33	34,986.95	35,623.70
	200 pairs per rai	35,546.63	35,275.50	35,521.88	35,367.75	35,397.58	35,047.70	34,851.38	34,891.88	35,058.95
	400 pairs per rai	35,644.50	35,669.83	35,542.13	35,398.13	35,210.83	34,998.20	34,740.00	34,939.70	35,291.25

1 kg. = 2.50 baht

Average estimated (income – cost of biological control agents) at the end of planting period

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	29,304.33	12,152.70	-26,095.88	29,817.08	21,400.00	1,848.45	29,360.00	27,716.20	17,475.50
	200 pairs per rai	26,328.63	14,420.88	-24,262.05	27,037.75	20,391.70	7,640.63	17,846.95	-22,766.13	-108,941.68
	400 pairs per rai	22,590.25	5,734.00	-19,804.68	24,946.95	19,955.13	-4,017.88	28,577.63	22,940.63	15,255.33
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	27,602.20	7,019.38	-31,413.80	29,285.50	19,735.58	8,680.70	16,442.33	-23,413.05	-109,176.30
	200 pairs per rai	19,946.63	2,075.50	-15,278.13	21,767.75	11,797.58	-7,752.30	26,051.38	21,291.88	11,858.95
	400 pairs per rai	9,644.50	1,269.83	-34,057.88	10,598.13	810.82	-14,601.80	20,040.00	-22,960.30	-109,008.75

The results indicate that the method 53 gives the maximized estimated cassava's crop yield at the end of planting period and should be the most efficient biological control of the spread of cassava mealybugs. However, the numbers of *Anagyrus lopezi* and green lacewings released in Method 53 is quite high (4,800 pairs of *Anagyrus lopezi* and 12,800 green lacewings) whereas the selling price of cassava is approximately 2.50 baht (0.072 USD) per kilogram. Figures 8, 9 and 10 showing the average estimated cost of *Anagyrus lopezi* and green lacewings released in the field, the average estimated income from selling cassava's crop yields at the end of planting period and the average estimated (income – cost of biological control agents) at the end of planting period, respectively, are then given here to compare the profit for the 54 biological control methods. We can see that although the method 53 gives the maximum estimated cassava's crop yield at the end of planting period, the cost for biological control is overcome the income from selling cassava's crop yield. On the other hand, the method 4 gives the maximum profit even though it gives the minimum estimated cassava's crop yield at the end of planting period. Therefore, to maximize the estimated cassava's crop yield at the end of planting period, the method 53 (Releasing 800-1,000 green lacewings per rai every 2 months together with 400 pairs of *Anagyrus lopezi* per rai every three weeks for three times after cassava mealybugs were first detected) is the most efficient biological control and to maximize profit, the method 4 (Releasing 200 green lacewings per rai every month together with releasing 50-100 pairs of *Anagyrus lopezi* per rai once after cassava mealybugs were first detected) is the most efficient control.

4.2 A Predator-Prey Model with Age Structure

In this part, a predator-prey model with age structure are developed in order to study the control of cassava's insect pest. The Sharpe-Lotka-McKendrick equation is extended and combined with an integro-differential equation to study population dynamics of mealybugs (prey) and released green lacewings (predator). Here, an age dependent formula is employed for mealybug population. The model is as follows

$$\frac{\partial P(a,t)}{\partial a}+\frac{\partial P(a,t)}{\partial t}=-\mu M(t)P(a,t)$$
$$\frac{dM(t)}{dt}=\mu\left(\int_0^\infty P(a,t)\,da\right)M(t)-\delta M(t)+g$$

with the following initial and boundary conditions

$$P(0^+,t)=\lim_{a\to 0^+}P(a,t)=b\int_0^\infty P(a,t)\,da,$$
$$P(a,0)=p_0(a),$$
$$M(0)=c,\quad c>0$$

where all variables and parameters are as in the following Table

Variable	Symbol	Unit
Prey age population density	$P(a,t)$	$BM\cdot T^{-1}$
Predator population	$M(t)$	BM
Death rate of prey	μ	$BM^{-1}\cdot T^{-1}$
Death rate of predator	δ	T^{-1}
Introduced predator rate	g	$BM\cdot T^{-1}$
Total prey (all ages)	$K(t)$	BM
Renewal rate of prey	b	T^{-1}

The solutions and their stability of the system are considered. The steady age distributions and their bifurcation diagrams are as follows

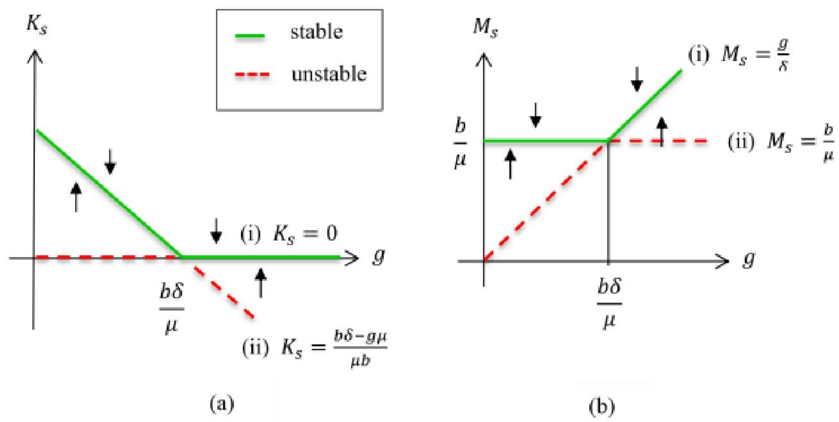


FIGURE 1. Bifurcation diagrams for (a) prey and (b) predator

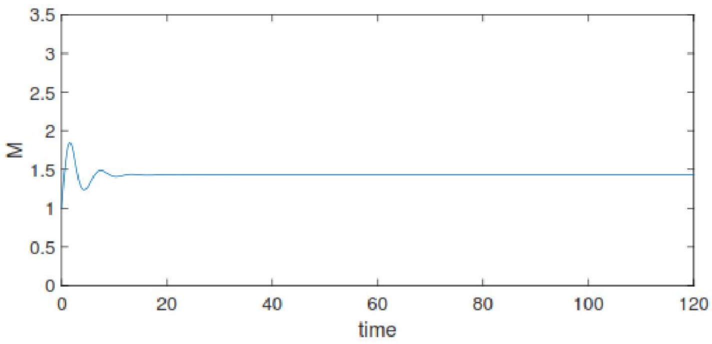


FIGURE 2. Population of predator for $b = 0.5$, $\mu = 1.6$, $\delta = 0.7$, $g = 1$

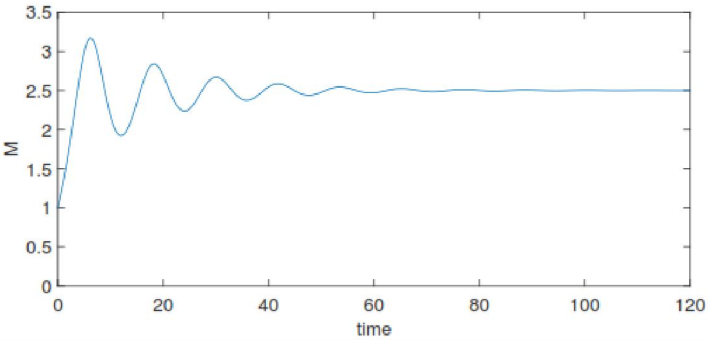


FIGURE 3. Population of predator for $b = 0.5$, $\mu = 0.2$, $\delta = 0.7$, $g = 0.3$

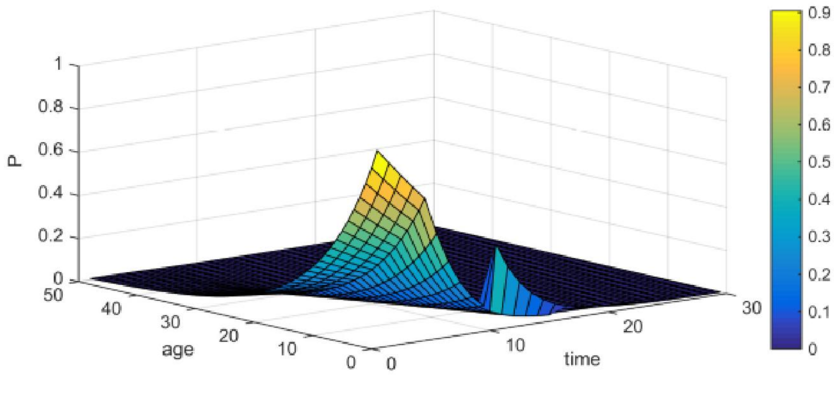


FIGURE 4. Population of prey for $b = 0.5$, $\mu = 1.6$, $\delta = 0.7$, $g = 1$

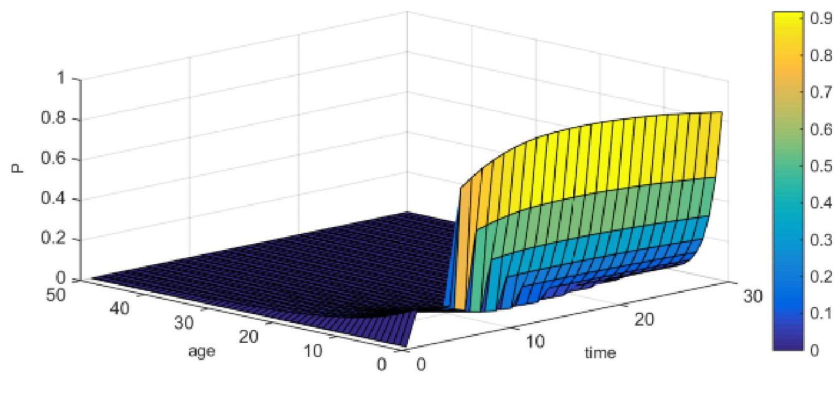


FIGURE 5. Population of prey for $b = 0.5$, $\mu = 0.2$, $\delta = 0.7$, $g = 0.3$

By employing the steady age distribution, two steady states of the system are obtained: one is mono-species and another one is coexisting species. Numerical results with biological meaning are provided which is very useful to visualize the mealybug controlling problem. Here we have shown that this useful hybrid model, with one age-structured compartment coupled to an unstructured compartment has exactly one asymptotically globally stable steady state. The solution of the transient model is obtained analytically, albeit implicitly, thus providing a check on computational solutions in more complex situations. This parallels the outcome in systems which are not age or spatially structured. To prevent recurring outbreaks requires that this predator release rate should ideally be maintained. It is expected that a similar outcome will apply when the parameters are functions of time and/or age.

Output ที่ได้จากโครงการวิจัย

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1. ผลงานวิจัยที่ตีพิมพ์ในวารสารวิชาการระดับนานาชาติ

1.1 Promrak J., Rattanakul C.* Effect of Increased Global Temperatures on a Biological Control of Green Lacewings on the Spread of Mealybugs in a Cassava Field: A Simulation Study. *Advances in Difference Equations* (2017) 2017:161 (doi: 10.1186/s13662-017-1218-y).

- JCR (ISI-Web of Science) with IF (2016) = 0.335 (Q4)
- SJR (Scopus) (Q2).

1.2 Promrak J., Wake G., Rattanakul C. A Predator-Prey Model with Age Structure. *ANZIAM Journal* (Accepted).

- JCR (ISI-Web of Science) with IF (2016) = 0.898 (Q3)
- SJR (Scopus) (Q4).

1.3 Promrak J., Rattanakul C.* A Simulation Study of the Spread of Mealybugs in a Cassava Field: Effect of Release Frequency of a Biological Control Agent. *Kasetsart Journal: Natural Science* 49 (2015): 963-970.

- SJR (Scopus) (Q4).

1.4 Promrak J., Wake G., Rattanakul C. Modified Predator-Prey Model for Mealybug Population with Biological Control. *Journal of Mathematics and System Science* 6 (2016): 180-193.

1.5 Rattanakul C. A cellular automata model of a biological control of the spread of cassava mealybugs in a cassava field using *Anagyrus Lopezi* as a biological control agent. *Applied Mathematics and Computation* (Manuscript).

- JCR (ISI-Web of Science) with IF (2016) = 1.738 (Q1)

1.6 Rattanakul C. A cellular automata model of a biological control of the spread of cassava mealybugs in a cassava field using *Anagyrus Lopezi* and green lacewings as biological control agents. *Pest Management Science* (Manuscript).

- JCR (ISI-Web of Science) with IF (2016) = 3.253 (Q1)

2. กิจกรรมอื่นๆ ที่เกี่ยวข้อง

2.1 ผลงานอื่นๆ เช่น การไปเสนอผลงาน การได้รับเชิญไปเป็นวิทยากร

2.1.1 **Chontita Rattanakul**, Monte Carlo Simulation of the Spread of Mealybugs in a Cassava Field, *International Symposium on Fundamental and Applied Sciences*, March 29-31, 2016, Kyoto, Japan.

2.1.2 **Chontita Rattanakul**, A Biological Control of the Spread of Cassava Mealybugs in a Cassava Field Using a Mix of *Anagyrus lopezi* and Green Lacewings: A Simulation Study, International Conference on Mathematics and Applications (ICMA-MU 2016), December 17-19, 2016, Bangkok, Thailand. (Invited Speaker)

2.2 ความเชื่อมโยงทางวิชาการกับนักวิชาการอื่นๆ ทั้งในและต่างประเทศ

2.2.1 Prof. Graeme Wake, Institute of Natural and Mathematical Sciences, Massey University, Auckland, New Zealand.

ภาคผนวก

1.1 Promrak J., Rattanakul C.* Effect of Increased Global Temperatures on a Biological Control of Green Lacewings on the Spread of Mealybugs in a Cassava Field: A Simulation Study. *Advances in Difference Equations* (2017) 2017:161 (doi: 10.1186/s13662-017-1218-y).

- JCR (ISI-Web of Science) with IF (2016) = 0.335 (Q4)
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Advances in Difference Equations

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2016	849	0.335	0.330	0.455	0.071	326	3.8	7.1	0.00233	0.113	99.39	0.26726	7.336
2015	626	0.297	0.283	0.403	0.047	362	3.7	7.8	0.00239	0.134	99.72	0.27277	5.439
2014	785	0.640	0.615	0.694	0.086	327	2.9	7.3	0.00221	0.150	99.69	0.24753	42.985
2013	627	0.634	0.562	0.754	0.263	372	2.9	6.9	0.00207	0.210	99.19	0.22838	47.017
2012	382	0.760	0.715	0.774	0.124	129	2.9	7.8	0.00158	0.233	98.45	Not A...	62.583
2011	338	0.845	0.730	0.790	0.256	90	2.7	7.5	0.00163	0.270	97.78	Not A...	69.728
2010	253	0.891	0.504	0.826	0.271	118	2.7	6.6	0.00083	0.186	99.15	Not A...	70.163
2009	161	0.892	0.430	Not A...	0.130	77	3.3	6.7	0.00091	Not A...	100.00	Not A...	64.877
2008	105	0.494	0.379	Not A...	0.211	38	3.1	6.9	0.00065	Not A...	97.37	Not A...	27.721
2007	61	0.437	0.310	Not A...	0	23	Not A...	6.3	0.00076	Not A...	100.00	Not A...	28.994

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2015	246/254	Q4	3.346	289/312	Q4	7.532
2014	177/257	Q3	31.323	142/312	Q2	54.647
2013	163/251	Q3	35.259	125/302	Q2	58.775
2012	119/247	Q2	52.024	80/296	Q2	73.142
2011	95/245	Q2	61.429	64/289	Q1	78.028
2010	91/236	Q2	61.653	60/279	Q1	78.674
2009	93/204	Q2	54.657	64/255	Q2	75.098
2008	144/175	Q4	18.000	135/215	Q3	37.442
2007	130/165	Q4	21.515	132/207	Q3	36.473

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JCR Year	MATHEMATICS
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2015	254/488-Q3
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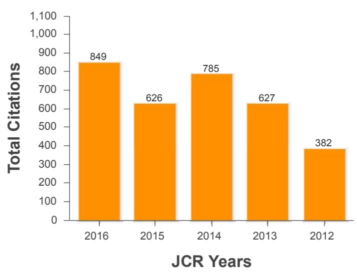

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Advances in Difference Equations

Country	Egypt
Subject Area and Category	Mathematics Algebra and Number Theory Analysis Applied Mathematics
Publisher	Hindawi Publishing Corporation
Publication type	Journals
ISSN	16871847, 16871839
Coverage	2004-ongoing
Scope	The theory of difference equations, the methods used, and their wide applications have advanced beyond their adolescent stage to occupy a central position in applicable analysis. In fact, in the last 12 years, the proliferation of the subject has been witnessed by hundreds of research articles, several monographs, many international conferences, and numerous special sessions. The theory of differential and difference equations forms two extreme representations of real world problems. For example, a simple population model when represented as a differential equation shows the good behavior of solutions whereas the corresponding discrete analogue shows the chaotic behavior. (source)

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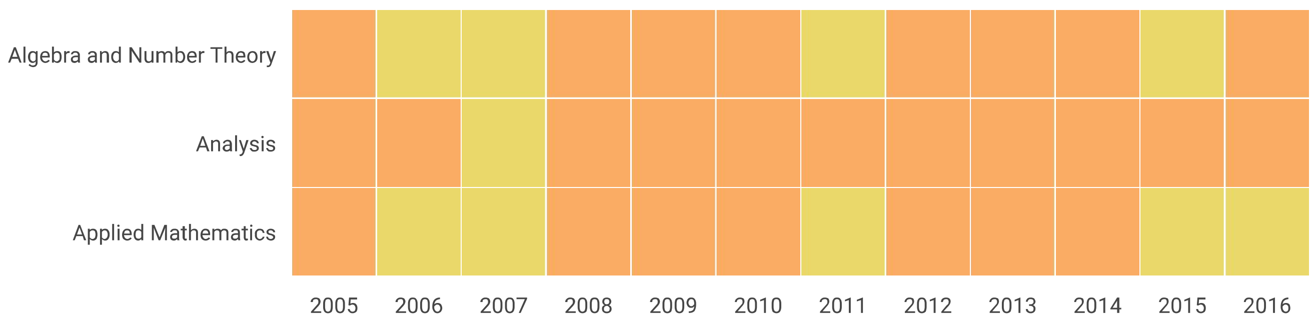

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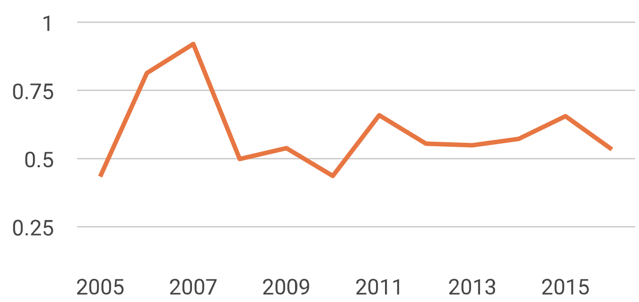

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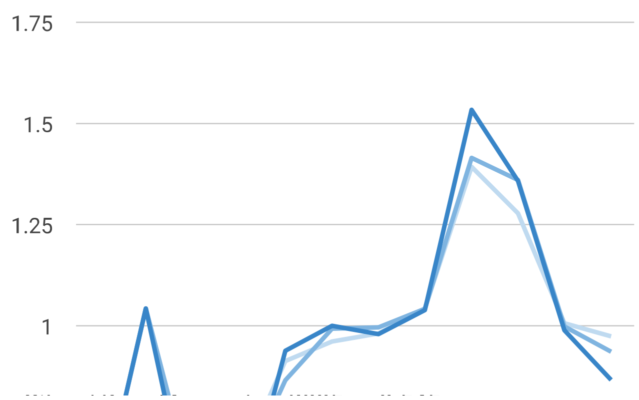

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RESEARCH

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Effect of increased global temperatures on biological control of green lacewings on the spread of mealybugs in a cassava field: a simulation study

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Abstract

Even though the effect of release frequency of green lacewings in controlling the spread of mealybugs in a cassava field was investigated by Promrak and Rattanakul in 2015, the effect of increased global temperature was not taken into account. In this work, cellular automata and Monte Carlo simulation are employed in order to study the effect of an increased global temperature on the life cycles of mealybugs and green lacewings which in turn effects the efficacy of the biological control of the spread of mealybugs. Computer simulations are carried out at different temperatures so that an efficient biological control of the spread of mealybugs in a cassava field is obtained.

Keywords: biological control; cassava; cellular automata; green lacewings; mealybugs; Monte Carlo simulation

1 Introduction

The term 'global warming' refers to an increase in the average temperature of Earth's atmosphere and oceans. According to the report from Goddard Institute for Space Studies (GISS) and Climatic Research Unit (CRU), the temperature has been increasing almost every year since 1880 and has climbed up fast in the last few decades. The Intergovernmental Panel on Climate Change (IPCC) forecasts that, for the next two decades, a further warming trend will occur at the rate 0.1-0.2°C per decade. Global warming may cause several severe problems such as the increase in the spread of insect-borne diseases, heavier rainfall and flooding, food and water deficiency, season changing and migration [1]. On crops and insect pests, the increase in the global temperature may lead to the change in the life cycle of insect pests at any stage and the rate of pest development might be higher while the host plants are more attractive to insect pests in drought areas [2, 3]. As a result, the loss of crop yields will then increases.

Under the global increased temperature condition, crops which can thrive in hot and dry climates such as cassava is considered as a key for food security. Cassava (*Manihot esculenta* Crantz) is also known as yuca, manioc, tapioca, mandioc, etc. Its root and tuber are the main source of food for Africans and are popular in the tropics [4]. Since it requires little skill to cultivate with moderate soil nutrient and water, this crop is very attractive to agriculturists worldwide [5]. In 2005, the top importers of cassava are East Asia (US$460,070,000), EU25 (US$59,534,000), and NAFTA (US$48,725,000) while the primary exporters are South East Asia (US$533,926,000), Central America (US$66,173,000), and EU25 (US$ 7,853,000) [6]. For Thailand, the cassava export of the year 2015 has been reported as 79.56% of the global market share [7]. Therefore, cassava is considered as an economic plant of Thailand.

The major crop loss of cassava is due to its insect pests, especially cassava mealybug (*Phenacoccus manihoti* Matile-Ferrero) which was first detected in Zaire and Congo in the early 1970s and quickly became the most severe pest on cassava [8]. To control the spread of mealybugs, biological control using their natural enemies has proved experimentally to be successful [9–14]. One of the natural enemies of mealybugs that has been used popularly is green lacewings. It has been used in a mealybug controlling project in Thailand [15, 16] as well. However, various instructions are recommended to farmers in Thailand when the spread of mealybugs is detected. Moreover, the effect of an increased global temperature on the life cycles of both mealybugs and green lacewings has not been taken into account yet.

In our previous work [17], a cellular automata (CA) model together with the Monte Carlo simulation technique has been employed to study the effect of the release frequency of green lacewings in controlling the spread of mealybugs in a cassava field [17]. Since the effect of an increased global temperature on the life cycles of mealybugs and green lacewings should be investigated so that we can appropriately modify the usual practices in the control of the spread of mealybugs in response to those changes, we investigate the effect of an increased temperature on the controlling of the spread of mealybugs in a cassava field using green lacewings in this paper.

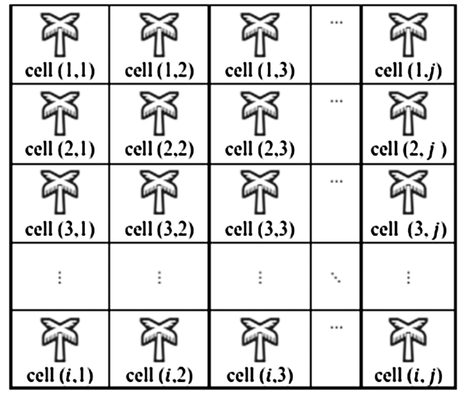
2 Cellular automata model

We assume that cassava is planted in the field based on the recommended instructions of the Department of Agricultural Extension, Ministry of Agriculture and Cooperatives, Thailand. Cassava is then planted at the beginning of the rainy season and the stem cuttings are soaked by the recommended chemical reactants before they are planted in the cassava field. Hence, the major factor of the spread of mealybugs in the field that we will take into account is the wind. Note that only mealybugs of the instar stage can be blown by the wind. Moreover, the recommended planting distance between two cassava plants is 1 m and the planting period of cassava is 1 year. We also assume further that the survey for the spread of mealybugs will be conducted every 2 weeks after a month of planting in the recommended manner by many agricultural technical officers from the Thai Tapioca Development Institute and the Department of Agriculture, Ministry of Agriculture and Cooperatives, Thailand [18].

A cellular automaton with Moore's neighborhood of a square lattice with the size 80×80 is used to represent a cassava field of the area 4 rai (or 0.64 ha) as shown in Figure 1. The possible states of each cell in the lattice are susceptible cassava (S), infested cassava (I) and removed cassava (E) representing a cassava plant that is free from mealybugs, a cassava plant that is infested with mealybugs and a cassava plant that is removed from the field, respectively.

At each time step (1 time step $\Delta t = 1$ day), a number r, $0 \leq r \leq 1$ is randomized and each cell will be updated at random according to the following rules:

Figure 1 A lattice representing a cassava field.



(i-3,j-3)	(i-3,j-2)	(i-3,j-1)	(i-3,j)	(i-3,j+1)	(i-3,j+2)	(i-3,j+3)
(i-2,j-3)	(i-2,j-2)	(i-2,j-1)	(i-2,j)	(i-2,j+1)	(i-2,j+2)	(i-2,j+3)
(i-1,j-3)	(i-1,j-2)	(i-1,j-1)	(i-1,j)	(i-1,j+1)	(i-1,j+2)	(i-1,j+3)
(i,j-3)	(i,j-2)	(i,j-1)	(i,j)	(i,j+1)	(i,j+2)	(i,j+3)
(i+1,j-3)	(i+1,j-2)	(i+1,j-1)	(i+1,j)	(i+1,j+1)	(i+1,j+2)	(i+1,j+3)
(i+2,j-3)	(i+2,j-2)	(i+2,j-1)	(i+2,j)	(i+2,j+1)	(i+2,j+2)	(i+2,j+3)
(i+3,j-3)	(i+3,j-2)	(i+3,j-1)	(i+3,j)	(i+3,j+1)	(i+3,j+2)	(i+3,j+3)

Figure 2 Neighborhoods of the updating cell (i,j). The light gray, gray and dark gray areas represent immediate neighborhood, distant neighborhood and far distant neighborhood, respectively.

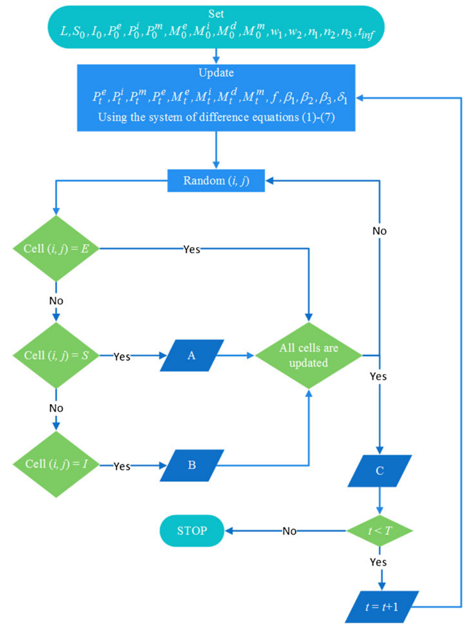
(a) If the randomized cell is a removed cassava (E), then it remains the removed cassava.

(b) If the randomized cell is a susceptible cassava (S), then there are possibilities that the randomized cell may become an infested cassava (I) due to the following reasons:

  (i) Mealybugs of the instar stage from outside of the cassava field might be blown through the wind to the randomized cell. If the randomized cell belongs to the first two rows next to each of the four borders of the lattice then the cell may become an infested cell with the probability w_1, $0 \leq w_1 \leq 1$ or else the randomized cell may become an infested cell with the probability w_2, $0 \leq w_2 < w_1 \leq 1$.

  (ii) Mealybugs of the instar stage from the neighborhood of the randomized might be blown through the wind to the randomized cells. Here, we consider only three levels of neighborhood of the randomized cell which are the immediate neighborhood, the distant neighborhood and the far distant neighborhood as shown in Figure 2. The probabilities that mealybugs of the instar stage from the immediate neighborhood, distant neighborhood and far distant neighborhood might be blown through the wind to the randomized cells are n_1, n_2 and n_3, respectively, where $0 \leq n_3 < n_2 < n_1 \leq 1$.

Figure 3 The main loop of CA.



(c) If the randomized cell is an infested cassava (I), then the following rules will be used.
   (i) It may become a removed cassava (E) if it is subjected to a survey during the first 4 months or the last 5 months of planting or the number of mealybugs on the randomized cell is greater than m_1.
   (ii) It may become a susceptible cassava (S) if green lacewings feed on mealybugs on the cassava plant in the randomized cell successfully and there is no mealybug on the cassava plant in the randomized cell.

When a month has passed after cassava planting, if there is an infested cassava plant among the surveyed cassava plants, green lacewings are to be released every 2 months if there still are mealybugs on the surveyed cassava plants in the field. If the number of surveyed infested cassava plants is less than a half of the total number of surveyed cassava plants in the field, the number of green lacewings to be released in the field is R_1 per rai (or $R_1/0.16$ per ha) or else the number of green lacewings to be released in the field is R_2 per rai (or $R_2/0.16$ per ha).

In what follows, we let P_t^i, P_t^m, P_t^e be the number of instar mealybugs, adult mealybugs and mealybug's eggs, respectively, at time t. Let M_t^i, M_t^d, M_t^m and M_t^e be the number of larva green lacewings, pupa green lacewings, adult green lacewings and green lacewings' egg, respectively, at time t. The numbers of mealybugs and green lacewings at each stage on the cassava plant in each cell of the lattice are also updated according to the life cycles of mealybugs and green lacewings using a system of difference equations as follows.

Instar mealybug:

$$P_{t+\Delta t}^i = P_t^i + r_1\alpha_1 P_t^e - \alpha_2 P_t^i - \beta_1\left(P_t^i, M_t^i\right)M_t^i. \tag{1}$$

Equation (1) represents the number of instar mealybugs at the time step $t + \Delta t$. The first term on the right hand side represents the number of instar mealybugs at the time

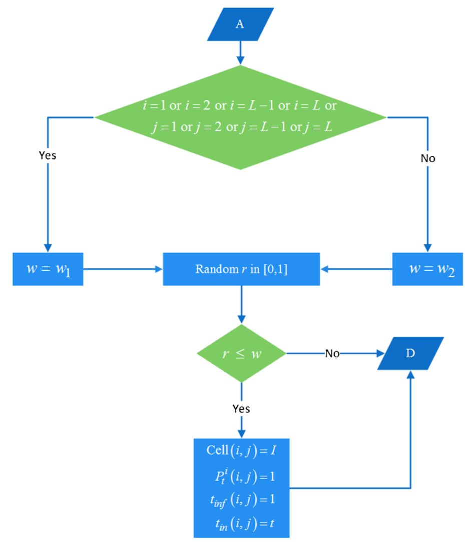


Figure 4 The main susceptible cassava updating loop.

step t. The second term on the right hand side represents the number of instar mealybugs developed from mealybug's egg of the time step t. The third term on the right hand side represents the number of instar mealybugs of the time step t that develop into adult mealybugs in the time step $t + \Delta t$. The last term on the right hand side represents the number of instar mealybugs eaten by green lacewings of the larva stage in the time step t.

Adult mealybug:

$$P^m_{t+\Delta t} = P^m_t + r_2\alpha_2 P^i_t - \alpha_3 P^m_t - \beta_2\left(P^m_t, M^i_t\right)M^i_t. \tag{2}$$

Equation (2) represents the number of adult mealybugs at the time step $t + \Delta t$. The first term on right hand side represents the number of adult mealybugs at the time step t. The second term on the right hand side represents the number of adult mealybugs developed from instar mealybug of the time step t. The third term on the right hand side represents the number of adult mealybugs of the time step t that die in the time step $t + 1$ due to mealybug's life cycle. The last term on the right hand side represents the number of adult mealybugs eaten by green lacewings of the larva stage in the time step t.

Mealybug's egg:

$$P^e_{t+\Delta t} = P^e_t + r_3\alpha_4 v_1 P^m_t - \alpha_1 P^e_t - \beta_3\left(P^e_t, M^i_t\right)M^i_t. \tag{3}$$

Equation (3) represents the number of mealybug's eggs at the time step $t + \Delta t$. The first term on the right hand side represents the number of mealybug's eggs at the time step t. The second term on the right hand side represents the number of mealybug's eggs laid by adult mealybugs of the time step t. The third term on the right hand side represents the number of mealybug's eggs in the time step t that develop into instar mealybugs in the time step $t + \Delta t$. The last term on the right hand side represents the number of mealybug's eggs eaten by green lacewings of the larva stage in the time step t.

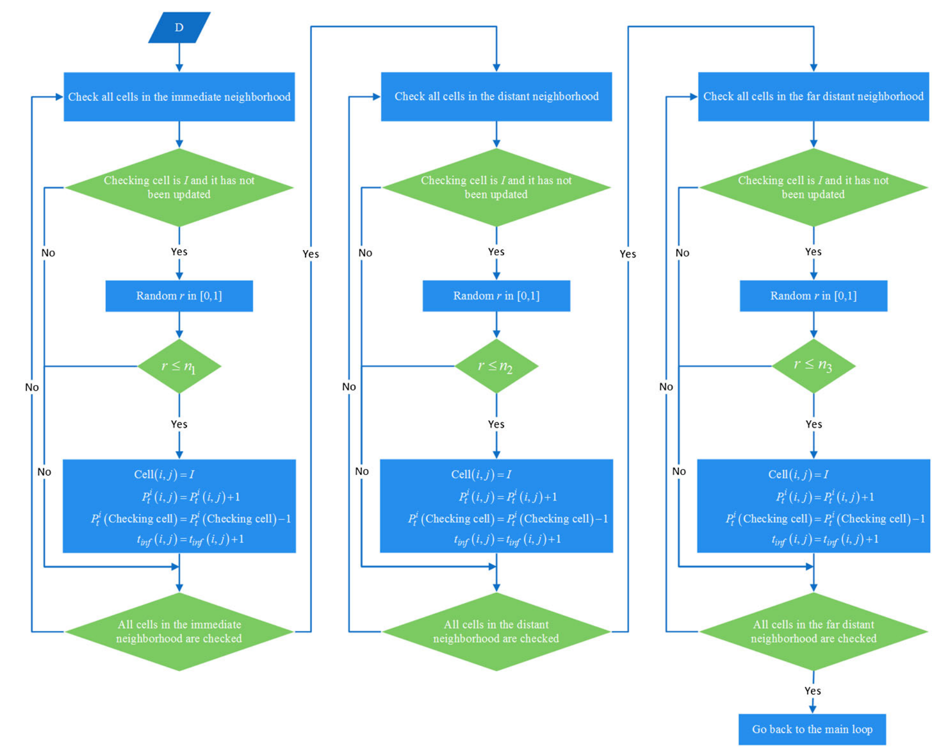


Figure 5 The neighborhood checking loop of the susceptible cassava updating loop.

Larva green lacewing:

$$M^i_{t+\Delta t} = M^i_t + s_1\gamma_1 M^e_t - \gamma_2 M^i_t. \tag{4}$$

Equation (4) represents the number of larva green lacewings at the time step $t + \Delta t$. The first term on the right hand side represents the number of larva green lacewings at the time step t. The second term on the right hand side represents the number of larva green lacewings developed from green lacewing's eggs of the time step t. The last term on the right hand side represents the number of larva green lacewings in the time step t that develop into pupa green lacewings in the time step $t + \Delta t$.

Pupa green lacewing:

$$M^d_{t+\Delta t} = M^d_t + s_2\gamma_2\delta_1\left(P^i_t, P^m_t, P^e_t, M^i_t\right) M^i_t - \gamma_3 M^d_t. \tag{5}$$

Equation (5) represents the number of pupa green lacewings at the time step $t + \Delta t$. The first term on the right hand side represents the number of pupa green lacewings at the time step t. The second term on the right hand side represents the number of pupa green lacewings developed from larva green lacewings of the time step t depending on the number of consumed mealybugs. The last term on the right hand side represents the number of pupa green lacewings in the time step t that develop into adult green lacewings in the time step $t + \Delta t$.

Figure 6 The main infested cassava updating loop.

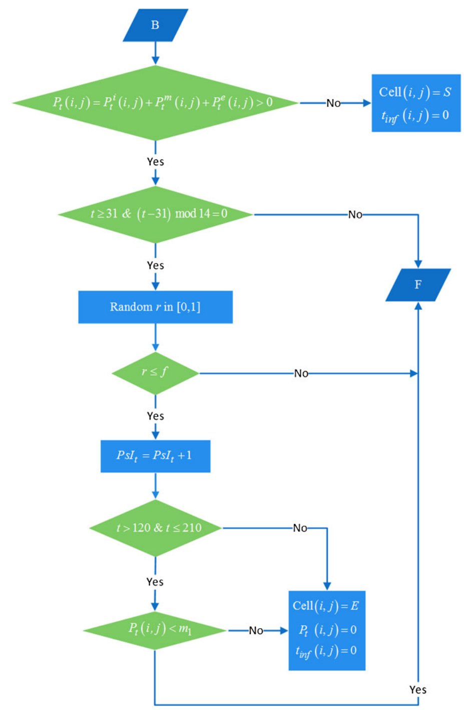
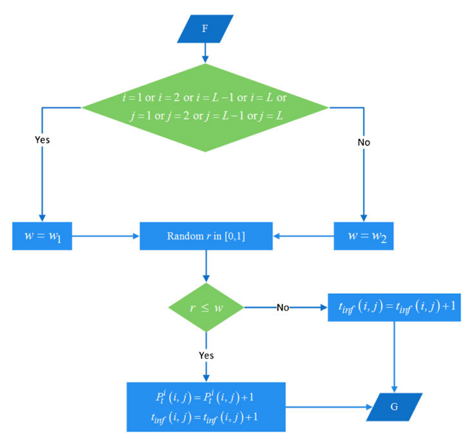


Figure 7 The number of mealybugs updating loop (incorporating wind effect) of infested cassava.



Adult green lacewing:

$$M_{t+\Delta t}^{m}=M_{t}^{m}+s_3\gamma_3M_t^d-\delta_2M_t^m. \tag{6}$$

Equation (6) represents the number of adult green lacewings at the time step $t+\Delta t$. The first term on the right hand side represents the number of adult green lacewings at the time step t. The second term on the right hand side represents the number of adult green lacewings developed from pupa green lacewings of the time step t. The last term on the right hand side represents the number of adult green lacewings of the time step t that die in the time step $t+\Delta t$ due to green lacewing's life cycle.

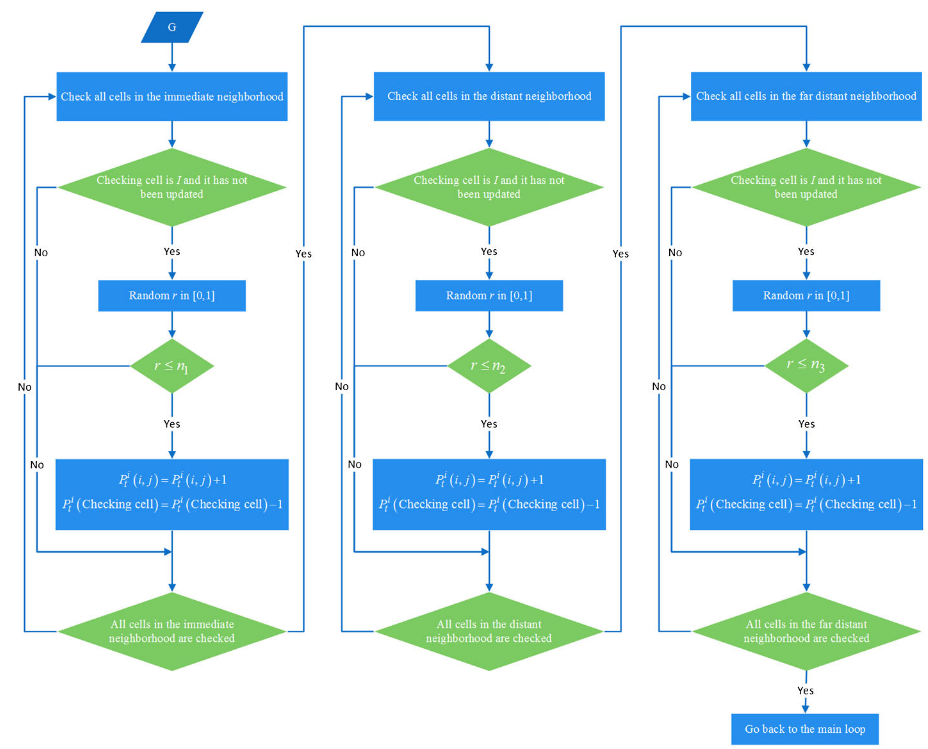


Figure 8 The neighborhood checking loop of the number of mealybugs updating loop (incorporating wind effect) of the infested cassava.

Green lacewing's eggs:

$$M^e_{t+\Delta t} = M^e_t + s_4 v_2 M^m_t - \gamma_1 M^e_t. \tag{7}$$

Equation (7) represents the number of green lacewing's eggs at the time step $t + \Delta t$. The first term on the right hand side represents the number of green lacewing's eggs at the time step t. The second term on the right hand side represents the number of green lacewing's eggs laid by adult green lacewings of the time step t. The last term on the right hand side represents the number of green lacewing's eggs in the time step t that develop into larva green lacewings in the time step $t + \Delta t$.

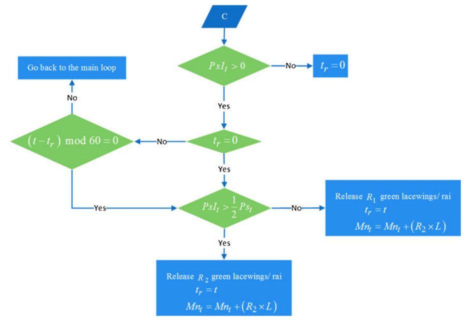
$\beta_1(P^i_t, M^i_t)$, $\beta_2(P^m_t, M^i_t)$ and $\beta_3(P^e_t, M^i_t)$ are the numbers of instar mealybugs, adult mealybugs and mealybug's eggs eaten by green lacewings of the larva stage, respectively, in one time step and $\delta_1(P^i_t, P^m_t, P^e_t, M^i_t)$ is the efficiency of converting larva green lacewing to pupa green lacewing. The definitions of other parameters in the model are given in Table 1 together with their approximated values calculated from the literature [19–21] at three different temperatures.

Furthermore, the approximated total crop yield is also monitored. We also assume that the estimated crop yield is a kilograms per cassava plant if there is no mealybug in the cassava field. The estimated crop yield will be reduced by 100%, 30% and 10%, approximately, if mealybugs spread on the cassava plants during the first 4 months, during the 5th and the 7th month, and during the 8th and the 12th month, respectively, according to the surveys of the Thai Tapioca Development Institute in 2007-2010. Hence, the estimated crop yield

Table 1 Parameters in the system of difference equations (1)-(7)

Parameter	Definition	$t = 25°C$	$t = 27°C$	$t = 30°C$
• Mealybugs				
α_1	The fraction of mealybug's eggs that develop into instar mealybugs in one time step	0.1075	0.1163	0.1493
α_2	The fraction of instar mealybugs that develop into adult mealybugs in one time step	0.0435	0.0482	0.0388
α_3	The natural death rate of adult mealybugs	0.0612	0.0665	0.0596
α_4	The fraction of female adult mealybugs in the reproductive period	0.4591	0.4991	0.4468
r_1	The survival rate of mealybug's eggs that develop into instar mealybugs	0.8900	0.8700	0.9120
r_2	The survival rate of instar mealybugs that develop into adult mealybugs	0.7737	0.6079	0.6589
r_3	The fraction of female adult mealybugs	0.7400	0.7200	0.8500
v_1	The average number of eggs laid by a female adult mealybug in one time step	40.0000	36.5300	2.9126
• Green lacewings				
γ_1	The fraction of green lacewing's eggs that develop into larva green lacewing in one time step	0.1639	0.2222	0.2703
γ_2	The fraction of larva green lacewings that develop into pupa green lacewing in one time step	0.0521	0.0585	0.00625
γ_3	The fraction of pupa green lacewings that develop into adult green lacewings in one time step	0.0714	0.0885	0.1053
δ_2	The natural death rate of adult green lacewings	0.0227	0.0175	0.0206
s_1	The survival rate of green lacewing's eggs that develop into larva green lacewing	0.8040	0.8210	0.8170
s_2	The survival rate of larva green lacewing that develop into pupa green lacewing	0.9191	0.9629	0.7938
s_3	The survival rate of pupa green lacewing that develop into adult green lacewings	0.9586	0.9614	0.7402
s_4	The fraction of female adult green lacewings	0.5400	0.5500	0.4850
v_2	The average number of eggs laid by a female adult green lacewings in one time step	7.2789	5.8237	2.3876

Figure 9 The green lacewings releasing loop.



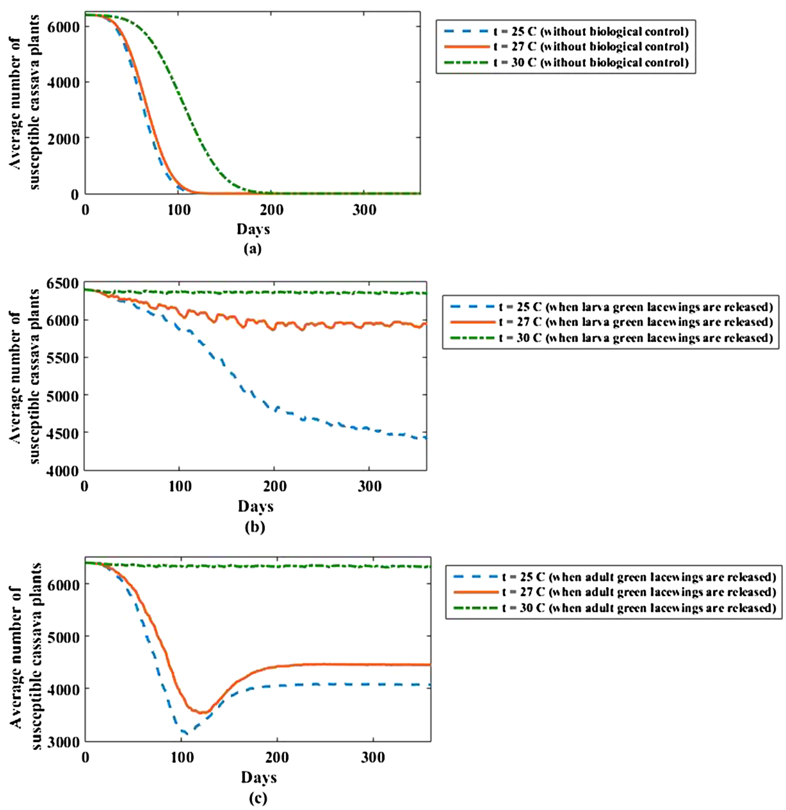


Figure 10 The average number of susceptible cassava at 25°C, 27°C and 30°C. (a) No biological control is applied, **(b)** larva green lacewings are released in the cassava field, **(c)** adult green lacewings are released in the cassava field.

at each time step, $Y(t)$, is then assumed to be represented by the following equation:

$$Y(t) = a \cdot C_1 + (0.9 \times a) \cdot C_2 + (0.7 \times a) \cdot C_3, \tag{8}$$

where C_1 is the total number of susceptible cassava at the time step t, C_2 is the total number of cassava infested by mealybugs during the 8th and the 12th month at the time step t and C_3 is the total number of cassava infested by mealybugs during the 5th and the 7th month at the time step t.

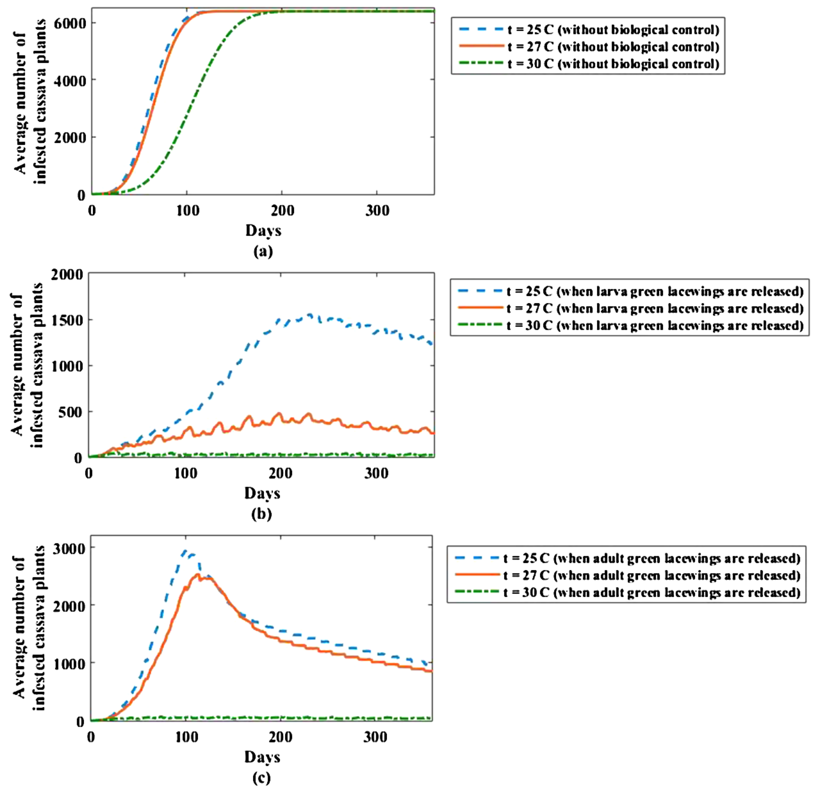


Figure 11 The average number of infested cassava at 25°C, 27°C and 30°C. (a) No biological control is applied, **(b)** larva green lacewings are released in the cassava field, **(c)** adult green lacewings are released in the cassava field.

3 Simulation results

In this section, numerical simulations at the three different temperatures 25°C, 27°C and 30°C are carried out in order to investigate the effect of increased temperatures on the control of mealybugs.

When biological control is applied, we assume that the survey for the spread of mealybugs will be done every two weeks beginning one month after planting as recommended by many agricultural technical officers from the Thai Tapioca Development Institute and the Department of Agriculture, Ministry of Agriculture and Cooperatives, Thailand. According to the recommendation of the Department of Agricultural Extension, Ministry of Agriculture and Cooperatives, Thailand, the cassava field will be surveyed by collecting the numbers of mealybugs at all stages on the cassava plants that are not planted on the two rows next to the four borders of the cassava field. The survey will be conducted on every two rows of plants, and every 11 plants. On the cassava field of the size 80 m × 80 m with 1 m between two cassava plants, a cassava plant might be surveyed with the probability

$$
\begin{aligned}
f &= \text{the number of surveyed cassava plants in the field} \\
&\quad \div \text{the total number of cassava plants that have not been removed} \\
&\quad \text{from the cassava field.}
\end{aligned}
$$

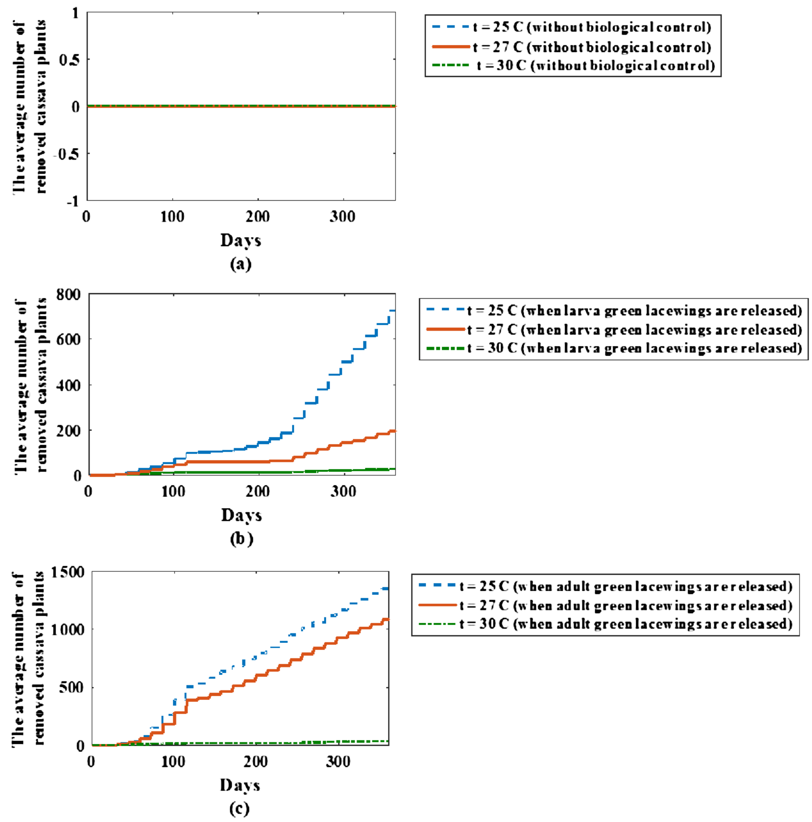


Figure 12 The average number of removed cassava at 25°C, 27°C and 30°C. (a) No biological control is applied, **(b)** larva green lacewings are released in the cassava field, **(c)** adult green lacewings are released in the cassava field.

Flow charts of the simulations are given in Figures 3-9. The results shown in Figures 10-15 are the averaged values of the 100 runs using MATLAB software. In the simulations, the parameters in the system of difference equations (1)-(7) are as in Table 1 where $n_1 = 0.05$, $n_2 = 0.005$, $n_3 = 0.0005$, $w_1 = 0.001$, $w_2 = 0.0001$, $a = 2.25$, $R_1 = 800$, $R_2 = 1{,}000$ and

$$\beta_1(P_t^i, M_t^i) = \min\left\{20, \frac{P_t^i(i,j)}{M_t^i(i,j)}\right\},$$

$$\beta_2(P_t^m, M_t^i) = \min\left\{20, \frac{P_t^m(i,j)}{M_t^i(i,j)}\right\},$$

$$\beta_3(P_t^e, M_t^i) = \min\left\{20, \frac{P_t^e(i,j)}{M_t^i(i,j)}\right\},$$

$$\delta_1(P_t^i, P_t^m, P_t^e, M_t^i) = 0.0521 \times \min\left\{1, \frac{P_t^i(i,j) + P_t^m(i,j) + P_t^e(i,j)}{M_t^i(i,j)} \div 60\right\}.$$

First, we investigate the spread of mealybugs when there is no biological control, when larva green lacewings are released to control the spread of mealybugs and when adult green lacewings are released to control the spread of mealybugs. The number of susceptible cassava plants, the number of infested cassava plants, the number of removed cassava plants and the estimated crop yield at 25°C, 27°C and 30°C are as shown in Figures 10, 11, 12 and 13.

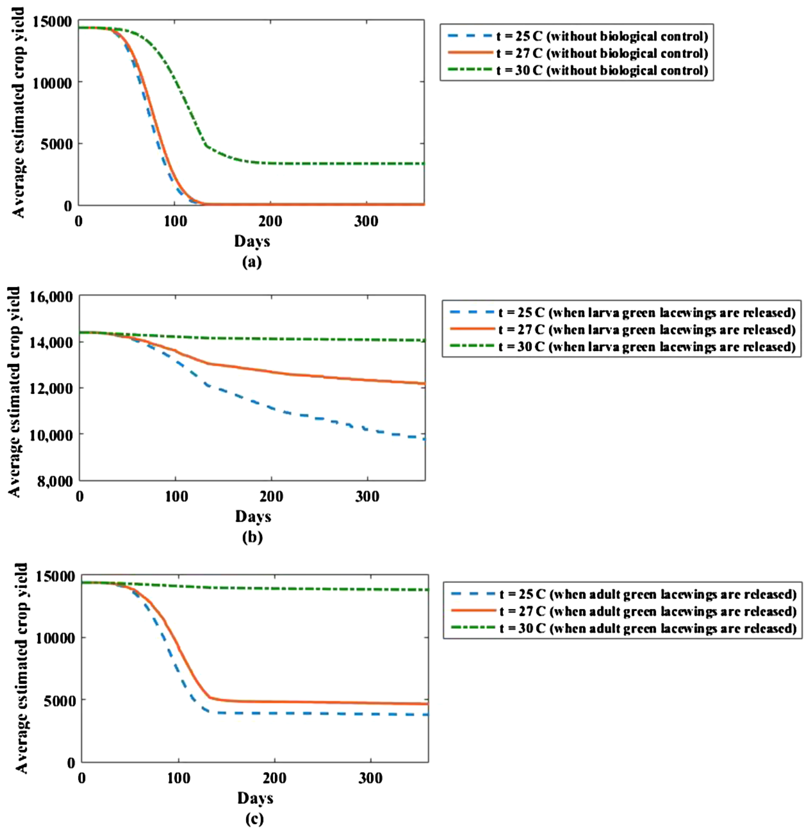
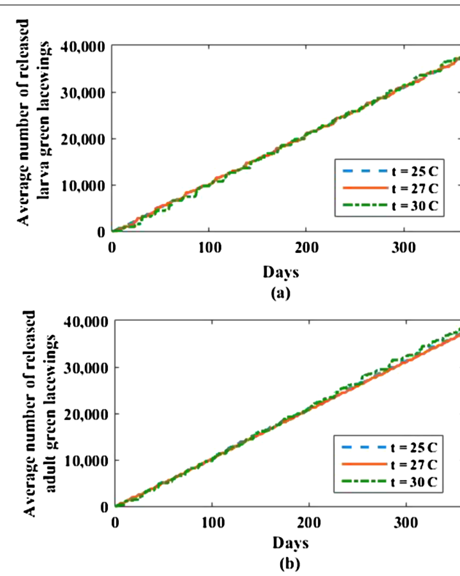


Figure 13 The average estimated cassava's crop yield at 25°C, 27°C and 30°C. (a) No biological control is applied, **(b)** when larva green lacewings are released in the cassava field, **(c)** adult green lacewings are released in the cassava field.

Figure 14 The total numbers of green lacewings that are released to control the spread of mealybugs in a cassava field at 25°C, 27°C and 30°C. (a) Larva green lacewings are released in the cassava field, **(b)** adult green lacewings are released in the cassava field.



Next, the total numbers of larva and adult green lacewings that are released to control the spread of mealybugs in a cassava field at 25°C, 27°C and 30°C are presented in com-

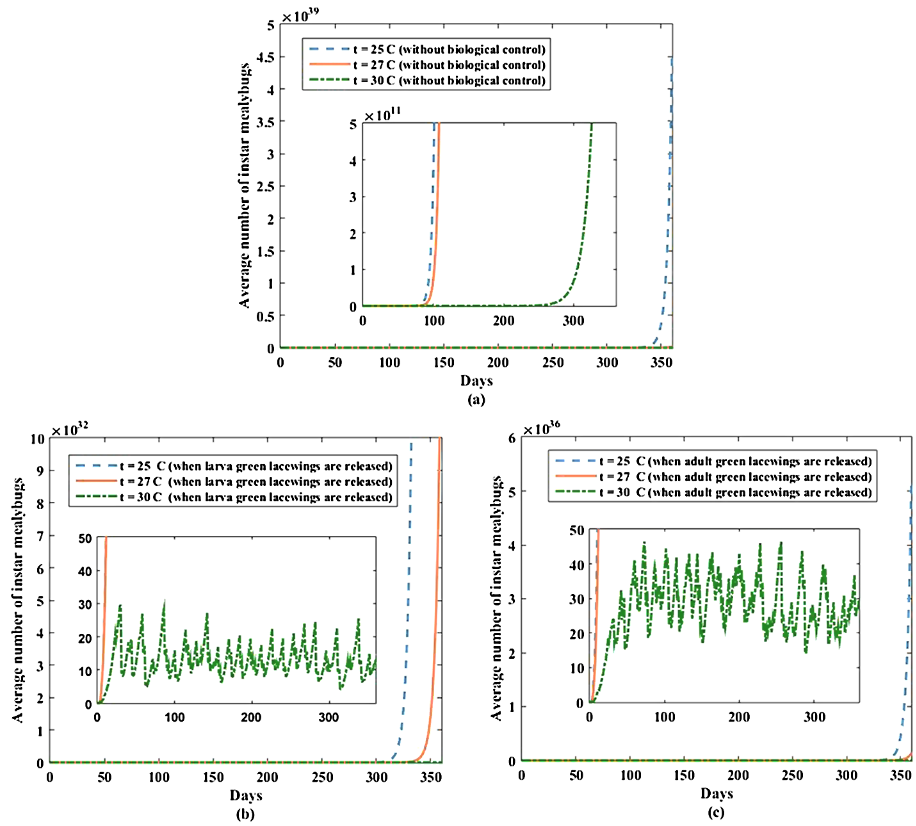


Figure 15 The average numbers of instar mealybugs at 25°C, 27°C and 30°C. (a) No biological control is applied, **(b)** larva green lacewings are released in the cassava field, **(c)** adult green lacewings are released in the cassava field.

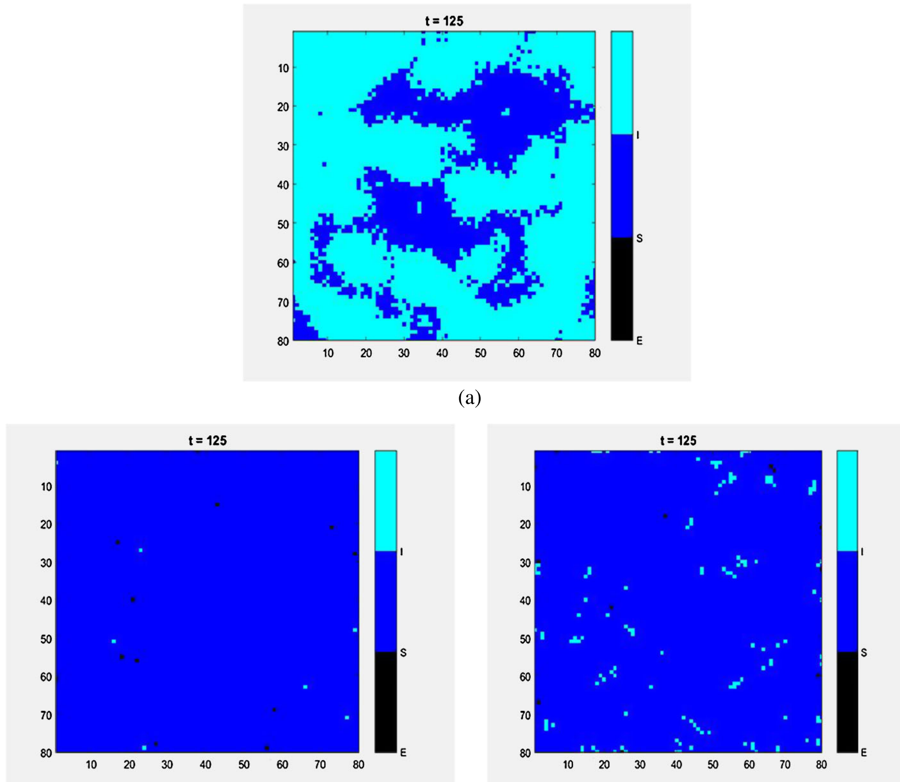
parison as shown in Figure 14. Moreover, the number of instar mealybugs when there is no biological control, when larva green lacewings are released to control the spread of mealybugs and when adult green lacewings are released to control the spread of mealybugs at 25°C, 27°C and 30°C are also shown in Figure 15.

We can see from the simulation results that even though the average numbers of instar mealybugs is very high when a biological control is applied, the number of the infested cassava plants is not that high. This means that the spread of mealybugs can be controlled to be located on only a small number of cassava plants although the number of mealybugs might be high on those cassava plants. Here, snapshots showing the distribution of susceptible cassava, infested cassava and removed cassava in the cassava field of a simulation at 30°C are also given in Figures 16 and 17 for a better understanding.

4 Discussion and conclusion

The increase in global temperatures affects the sex ratio, survival rate, reproduction rate and life cycle of both mealybugs and green lacewings. Simulations of the spread of mealybugs in a cassava field at 25°C, 27°C and 30°C have been carried out.

Without biological control, the estimated crop yield decreases dramatically and tends to zero at 25°C, and at 27°C approximately 4 months after planting. At 30°C, the estimated crop yield decreases and tends to a constant level which is lower than 30% of the maximum estimated crop yield.



(a)

(b)

(c)

Figure 16 Snapshots showing the distribution of susceptible cassava, infested cassava and removed cassava in the cassava field of a simulation at 30°C on the 125th day of planting. (a) No biological control is applied, **(b)** larva green lacewings are released in the cassava field, **(c)** adult green lacewings are released in the cassava field.

With biological control, green lacewings at the larva stage or adult stage may be released in the cassava field to control the spread of mealybugs. Hence, we study both manners of biological control. We can see that the number of infested cassava plants decreases whereas the number of susceptible cassava plants increases when the temperature increases which might be the results of shorter life cycle, lower survival rate, lower fecundity and shorter adult longevity of mealybugs. We can also see that the release of green lacewing larva gives a better result when there is a spread of mealybugs even though the lower amount of larva green lacewing is released compared to adult green lacewings. The reasons for this might be the shorter life span, lower survival rate, lower fecundity or shorter adult longevity of green lacewings because only green lacewings at the larva stage behave like a predator of mealybugs and if we release adult green lacewings it will take a period of time before they will lay eggs which develop into green lacewing larva, finally behaving like a predator of mealybugs. With the increase of temperature, the survival rate and the fecundity rate are even lower and hence the greater amount of adult green lacewings should be released in the cassava field to control the spread of mealybugs. On the other hand, the estimated crop yield also increases when the temperature increases with the same level of released green lacewings. This implies that if farmers are satisfied with the estimated crop yield at the end of planting period when the temperature is 25°C, they might reduce the number of green lacewings released in the cassava field

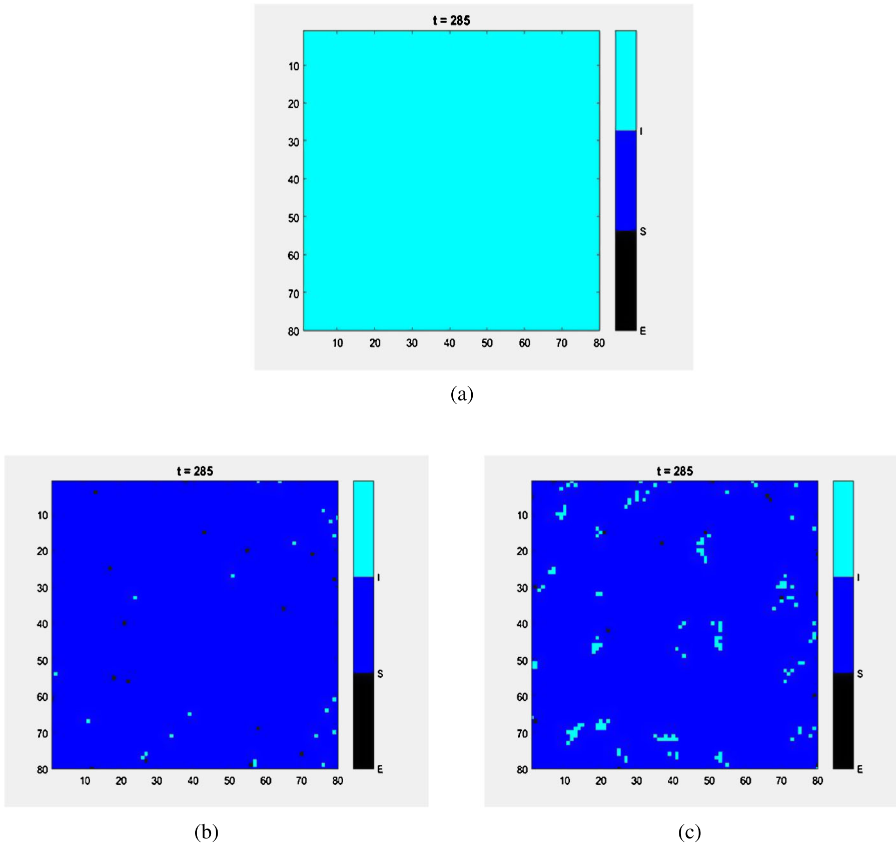


Figure 17 Snapshots showing the distribution of susceptible cassava, infested cassava and removed cassava in the cassava field of a simulation at 30°C on the 285th day of planting. (a) No biological control is applied, **(b)** larva green lacewings are released in the cassava field, **(c)** adult green lacewings are released in the cassava field.

so that the cost for biological control will be decreased and the farmers then earn more profit. On the other hand, if the farmers would like to gain more estimated crop yield at the end of planting period, they might keep the released amount of green lacewings at the same level as they use when the temperature is 25°C. However, the cost for a green lacewings is approximately 0.50 baht (US$0.015) while the selling price for cassava is quite low, approximately 2.50 baht (US$0.072) per kilogram. Hence, the cost of biological control and the increase in crop yield should be calculated in order to obtain the most efficient biological control that maximizes profit.

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The first author carried out a numerical investigation of the developed model and the second author developed the model and carried out numerical investigations. All authors read and approved the final manuscript.

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1.2 Promrak J., Wake G., Rattanakul C. A Predator-Prey Model with Age Structure. *ANZIAM Journal* (Accepted).

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2015	346	0.957	0.914	1.191	0.083	24	8.0	>10.0	0.00102	0.615	100.00	0.11657	60.827
2014	344	1.025	0.950	0.896	0.125	24	7.4	>10.0	0.00095	0.479	100.00	0.10655	67.899
2013	328	0.829	0.829	0.917	0.176	17	7.5	10.0	0.00098	0.444	100.00	0.10851	57.570
2012	244	1.043	1.043	0.806	0	4	5.9	>10.0	0.00115	0.425	100.00	Not A...	70.648
2011	266	0.254	0.238	0.712	0.167	6	6.2	>10.0	0.00118	0.364	100.00	Not A...	6.327
2010	220	0.414	0.371	0.694	0	13	5.5	9.1	0.00130	0.360	100.00	Not A...	15.890
2009	141	0.286	0.271	0.468	0.227	22	5.3	10.0	0.00091	0.233	100.00	Not A...	1.716
2008	117	0.380	0.380	0.490	0.050	20	3.8	>10.0	0.00122	0.272	100.00	Not A...	9.429
2007	99	0.455	0.428	0.361	0.036	28	Not A...	>10.0	0.00119	0.246	100.00	Not A...	22.727
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2014	83/257	Q2	67.899
2013	107/251	Q2	57.570
2012	73/247	Q2	70.648
2011	230/245	Q4	6.327
2010	199/236	Q4	15.890
2009	201/204	Q4	1.716
2008	159/175	Q4	9.429
2007	128/165	Q4	22.727
2006	135/150	Q4	10.333
2005	142/151	Q4	6.291
2004	159/162	Q4	2.160
2003	148/153	Q4	3.595
2002	148/156	Q4	5.449
2001	119/158	Q4	25.000

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2014	326/488-Q3
2013	317/473-Q3
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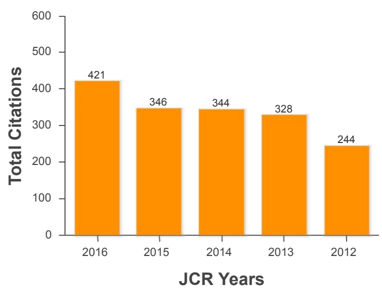

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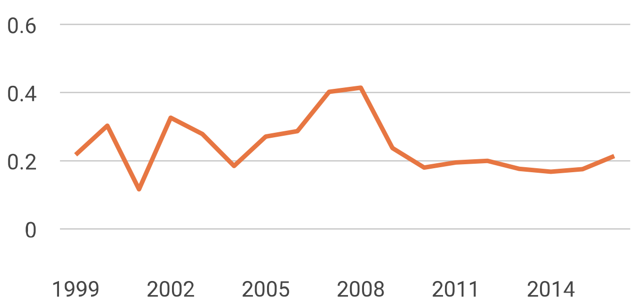

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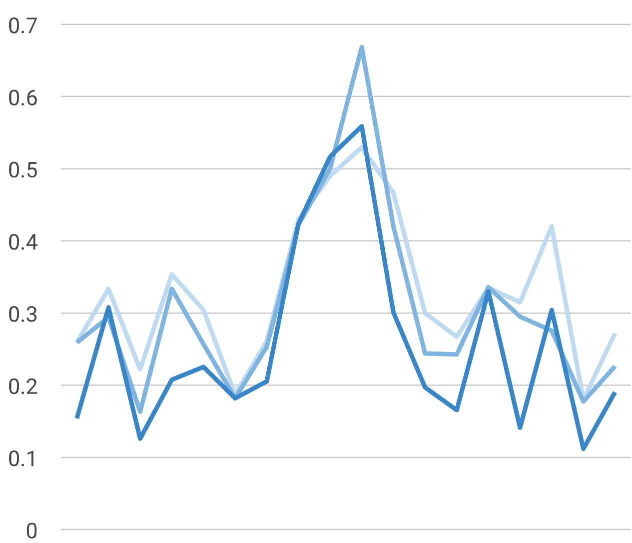

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To: "Jairaj Promrak (jpjairaj@gmail.com)" <jpjairaj@gmail.com>

Hullo Jairaj,

Here is the official acceptance, please inform Chonitta. Congratulations. I assume we will get asked for the LaTeX version quite soon, in the ANZIAM format.

Will you do this? If not forward it as is and I will get it done here..

I think it being in the ANZIAM journal is great...it represents your connection to New Zealand, and it was presented at the ANZIAM 2017 conference in 2017.

I am sorry that it took seven months.

Regards,

Graeme

Professor Emeritus Graeme Wake D Sc FRSNZ, Centre for Mathematics in Industry and Principal of Wakes' Scientific Consulting (Wakescience), Institute of Natural and Mathematical Sciences, Massey University at Albany, P.B 102904, North Shore MC, Auckland, New Zealand. Room 2.19, MS Building: Tel +64 (0) 9 2136602; Mobile +64 (0) 27 441-8247. E-mail g.c.wake@massey.ac.nz Web http://www.riddet.ac.nz/our- people/professor-graeme-wake

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From: e-Editor [mailto:editor@journal.austms. org.au]
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To: Wake, Graeme
Subject: [ANZIAMJ] Editor Decision

Graeme Wake:

I am pleased to advise you that your submission to ANZIAM Journal, "A Predator-Prey Model with Age Structure for Prey: Application to the Control of Mealybugs in Crops" is now accepted for publication. You should hear from our copyediting team in due course.

Thank you for your contribution to the ANZIAM Journal.

Nāku noa, nā

Dr Michael John Plank

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PREDATOR-PREY MODEL WITH AGE STRUCTURE

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Abstract

Mealybug is an important pest of cassava plant in Thailand and tropical countries leading to severe damage of crop yield. One of the most successful controls of mealybug spread is using its natural enemies such as green lacewings, where the development of mathematical models forecasting mealybug's population dynamics will improve implementation of biological control. In this work, the Sharpe-Lotka-McKendrick equation is extended and combined with an integro-differential equation to study population dynamics of mealybugs (prey) and released green lacewings (predator). Here, an age dependent formula is employed for mealybug population. The solutions and their stability of the system are considered. The steady age distributions and their bifurcation diagrams are presented. Finally, the threshold of the rate of released green lacewings for mealybug extermination is investigated.

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Keywords and phrases: Sharpe-Lotka-McKendrick equation, predator-prey model, steady age distribution, mealybugs.

1. Introduction

Mealybug is a serious pest of food crops, fruit trees and many other cultivated plants such as cassava, cotton, mango, grape vine and orchid [9]. It is in the *Pseudococcidae* family. The commonly found species are *Phenacoccus manihoti*, *Ferrisiana virgata (cockerell)*, *Phenacoccus*

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madeirensis, *Phenacoccus solenopis* and *Phenacoccus jackbeardsleyi*. Mealybug at any stages can cause damage in plants. It sucks sap from many parts of the plant leading to reduction of photosynthesis, distortion of leaf and stem and death [15]. In the absence of its natural enemies and other control managements, this damage can reduce yields by more than 80% [9].

To control the spread of mealybugs, the biological control by releasing their natural enemies has proved experimentally to be an effective method where one of the most powerful pradators is green lacewing [11, 12]. There are many sources of guidelines to deal with the rate of releasing green lacewings to control the spread of mealybugs. In 1995, Broadley and Thomas [6] employed green lacewing Malladasignata (Schneider) to control aphid populations with releasing rates of 500-1,000 larvae lacewings per hectare or 1-5 larvae lacewings per plant. Alternatively, pest management specialists in Australia offered that suitable releasing rate for biological control is about 2,000-10,000 green lacewings per hectare [7]. In January 2010, National Biological Control Research Center (NBCRC)-Central Regional Center suggested to release 10,000,000 eggs of green lacewings to control the spread of mealybugs in the infested areas of Kanchanaburi and Suphanburi provinces in Thailand [32]. To verify the optimal rate of releasing green lacewings in farm work, an in-depth understanding of the mealybug-green lacewing (predator-prey) interactions is necessary.

Traditionally, the population dynamics of two species have been described by Lotka and Volterra [25, 35] as follows:

$$\frac{dx}{dt} = cx - \alpha xy, \tag{1.1}$$

$$\frac{dy}{dt} = \gamma xy - dy, \tag{1.2}$$

where $x(t)$ and $y(t)$ denote the populations of the prey and predator species, respectively; c is natural growth rate of prey, d is natural death rate of predator, α is the death rate per encounter of prey due to predation, and γ is the efficiency of conversion eaten prey into predator. Over the past few decades, this model has been improved and applied by many researchers. Because of the difference of two behavioural time scales, Arditi and Ginzburg [4] suggested to include predator abundance term into the trophic function (the per capita rate of consumption) which led to ratio-dependent models. This new form of function is more appropriate for heterogeneous systems, and can solve the problems which may occur in the classical predator-prey model (1.1)-(1.2), including paradoxes of enrichment and biological control [3, 5]. Since the integer-order differential operator is local, the fractional-order differential operator is introduced to obtain non-local properties by paying attention to all preceding states to result in the next state [21].

A fractional Lotka-Volterra model was proposed by Das et al. [13]. Its solution was obtained through an analytical method, called the homotopy perturbation method. The influence of the fractional order on both predator and prey populations was discussed [14]. On the other hand, the approximate analytical solutions of fractional Lotka-Volterra model was derived by using hybrid approach which is the combination of homotopy analysis method, Laplace transform and homotopy polynomials [21]. To handle spatial and individual behaviours of heterogeneity, Hugo et al. proposed a population-driven, individual-based model where the individual scale was used only for the predation process [33]. Their work demonstrated the link between individual and population scales. Recently, the Lotka-Volterra model with the state-dependent riccati equation control technique was employed to study biological control of spider mite *Panonychus Ulmi*. The results indicated that the approaches are efficient to stabilize the system at the desired point, which can minimize economic damage [34].

Due to the widely recognized biological fact that age plays an important role in death and fecundity rates of a population, a partial differential equation in which time and age are independent was introduced. Let $u(a,t)$ be the population density (or age distribution) of individuals of age a at time t, $a \geq 0, t \geq 0$. A general age structured model for the evolution process was proposed by Sharpe et al. [26]:

$$\frac{\partial u(a,t)}{\partial a} + \frac{\partial u(a,t)}{\partial t} = -\mu(a,t)u(a,t), \tag{1.3}$$

where $\mu(a,t)$ is the mortality rates per capita. Equation (1.3) is also known as the Sharpe-Lotka-McKendrick equation used to describe the dynamic of population density of individuals.

The birth rate process is described by the renewal law:

$$u(a,t) = \int_0^\infty \beta(a,t)u(a,t)\, da, \tag{1.4}$$

where $\beta(a,t)$ is the renewal rate, and gives the proportion of newborn population at time t with parents of age a. The initial age distribution is given by

$$u(a,0) = u_0(a). \tag{1.5}$$

The derivation and properties of this age structure model are discussed (for example, see [2, 16, 27]). Equations (1.3)-(1.5) have been developed in many directions including partition into two subpopulations [18], spatial effect with diffusive process [8, 17, 22, 23], and age-sex structured population [31]. Different numerical methods for age-structured population were reviewed and their numerical solutions were also presented [1, 24, 28, 36]. Solutions of common age-structured models: the Leslie matrix, the difference equation, the integral equation were compared [20]. The approximation of equations (1.3)-(1.5) in various forms of mortality function was discussed [19]. Moreover, age-structured models have been tested with the real data. Sharpe and Lotka derived the expression of fixed age-distribution (independent of time), and applied this formula to calculate the population of England and Wales 1871-1880. Their results indicated the calculated values conform quite closely to the observed data [30]. Chiu [10] proposed some new algorithms used to estimate parameter functions in the models by practical data. With these algorithms and a numerical method, the human population can be predicted.

Unlike the previous research which considers predator-prey interaction regardless to age or focuses on the age structure model of a single population, in this work, we study population dynamics of two species with age-structure for prey by extending equation (1.3) and considering the equation for prey in the Lotka-Volterra model. We then also introduce released-predator term in the system as a biological control. Therefore, this model will be useful in the areas of pest management program.

In Section 2, the predator-prey model with age-dependent formula for prey is proposed. We initially derive the implicit solutions, one for predator and another for prey. To simplify the model, we analyze the steady age distributions (s.a.d.) in Section 3. Then, the system can be solved explicitly. After the steady distributions are found, we investigate their stabilities and obtain the bifurcation diagrams presented in Section 4. In Section 5, we focus on numerical examples for the mealybug problem controlled by green lacewings. Finally, in Section 6, we summarize the discussion of the results and draw some conclusions.

2. The predator-prey model with age-dependent formula for the prey

Let $P(a,t)$ be the population size density of prey over age a at time t, and $M(t)$ be the population size of predator at time t for $a, t \geq 0$. To investigate the population dynamics of prey, we extend the Sharpe-Lotka-McKendrick equation (1.3) by multiplying mortality rate with the population size of its predator. Combining this equation to integro-differential equation for predator leads to a population model with age structure for prey as follows:

$$\frac{\partial P(a,t)}{\partial a} + \frac{\partial P(a,t)}{\partial t} = -\mu M(t) P(a,t), \tag{2.1}$$

$$\frac{dM(t)}{dt} = \mu \left(\int_0^\infty P(a,t)\, da \right) M(t) - \delta M(t) + g \tag{2.2}$$

TABLE 1. Details of variables and parameters used

Variable	Symbol	Unit
Prey age population density	$P(a,t)$	$BM \cdot T^{-1}$
Predator population	$M(t)$	BM
Death rate of prey	μ	$BM^{-1} \cdot T^{-1}$
Death rate of predator	δ	T^{-1}
Introduced predator rate	g	$BM \cdot T^{-1}$
Total prey (all ages)	$K(t)$	BM
Renewal rate of prey	b	T^{-1}

with initial and boundary conditions

$$P(0^+,t) = \lim_{a \to 0^+} P(a,t) = b \int_0^\infty P(a,t)\,da, \tag{2.3}$$

$$P(a,0) = p_0(a), \tag{2.4}$$

$$M(0) = c, \quad c > 0. \tag{2.5}$$

We also define

$$K(t) = \int_0^\infty P(a,t)\,da. \tag{2.6}$$

Variables and their units are given in Table 2 where BM and T stand for biomass and time (usually in days), respectively. We have taken β in equation (1.4) as a constant and denoted it by b as shown in (2.3). Parameters μ, δ, and g are also constants for sampling where the rate of released predator g is used to control the growing population of prey (see more information in [29]).

2.1. Implicit solution of prey We solve the equation (2.1) using method of characteristics. Let $s = t - a$ and $\tau = t$. Then equation (2.1) becomes the partial differential equation

$$\frac{\partial P(s,\tau)}{\partial \tau} = -\mu(\tau) P(s,\tau).$$

We solve this equation by integrating by parts and obtain

$$P(s,\tau) = F(s) e^{-\mu \int_0^\tau M(\tau')d\tau'},$$

where F is an arbitrary function to be found. The solution of the original equation is, therefore,

$$P(a,t) = F(t-a) e^{\mu \int_0^t M(t')\,dt'}. \tag{2.7}$$

Case I: $t < a$
The initial condition at $t = 0$ gives $P(a, 0) = F(-a) = p_0(a)$, so $F(t - a) = p_0(a - t)$ and

$$P(a, t) = p_0(a - t)e^{-\mu \int_0^t M(t')\, dt'} \quad \text{for } t < a. \tag{2.8}$$

Case II: $t > a$
The renewal condition at $a = 0$: from equations (2.3) and (2.7), $P(0, t) = F(t)e^{-\mu \int_0^t M(t')\, dt'} = b \int_0^\infty P(a, t)\, da$. So,

$$\begin{aligned} F(t) &= be^{\mu \int_0^t M(t')\, dt'} \left(\int_0^t P(a, t)\, da + \int_t^\infty P(a, t)\, da \right) \\ &= be^{\mu \int_0^t M(t')\, dt'} \left(\int_0^t F(t - a)e^{-\mu \int_0^t M(t')dt'}\, da + \int_t^\infty p_0(a - t)e^{-\mu \int_0^t M(t')\, dt'} da \right) \\ &= b \int_0^t F(t - a)\, da + b \int_t^\infty p_0(a - t)\, da. \end{aligned} \tag{2.9}$$

Let $a'' = t - a$ and $a' = a - t$. Then equation (2.9) becomes

$$F(t) = b \int_0^t F(a'')\, da'' + b \int_0^\infty p_0(a')\, da',$$

which yields

$$\begin{aligned} F'(t) &= bF(t) \quad \text{and so} \\ F(t) &= Ce^{bt}. \end{aligned} \tag{2.10}$$

From equation (2.9)

$$F(0) = C = b \int_0^\infty p_0(a)\, da. \tag{2.11}$$

Substituting for C into (2.10)

$$F(t) = be^{bt} \int_0^\infty p_0(a)\, da \equiv be^{bt} K_0,$$

where $K_0 = \int_0^\infty P_0(a)\, da$. Thus,

$$P(a, t) = be^{b(t-a)} K_0 e^{-\mu \int_0^t M(t')\, dt'} \quad \text{for } t > a. \tag{2.12}$$

2.2. Implicit solution for the predator Solving equation (2.2) by using integrating factor $e^{-\mu \int_0^t K(z)\, dz + \delta t}$ and the definition of $K(t)$ in equation (2.3) yields

$$M(t) = ge^{\mu \int_0^t K(z)\, dz - \delta t} \int_0^t e^{-\mu \int_0^{t'} K(z)\, dz + \delta t'}\, dt'. \tag{2.13}$$

Later in Section 4, we find an equation for $M(t)$.

3. The steady age distribution of the system

Let $P_s(a)$ and M_s be the equilibrium solutions of our age structured population model (2.1-2.5) called the steady age distributions which are independent of time; we then obtain the corresponding system:

$$\frac{dP_s}{da} = -\mu M_s P_s, \tag{3.1}$$

$$\frac{dM_s}{dt} = \mu K_s M_s - \delta M_s + g = 0 \tag{3.2}$$

with renewal condition

$$P_s(0) = b \int_0^\infty P_s(a)\, da \equiv bK_s. \tag{3.3}$$

Here we define $K_s = \int_0^\infty P_s(a)\, da$. So either there is no prey (the monoculture solution), or

$$P_s(a) = bK_s e^{-\mu a M_s}. \tag{3.4}$$

From equations (2.2), (3.2)–(3.4), we obtain

$$M_s = \frac{b}{\mu}, \quad P_s(a) = \frac{b\delta - g\mu}{\mu} e^{-ab}, \quad \text{and} \quad K_s = \frac{b\delta - g\mu}{\mu b}.$$

Thus, the solutions of the system (3.1)–(3.3) are

1. $(P_s, M_s) = (0, g/\delta)$
2. $(P_s, M_s) = (e^{-ab}(b\delta - g\mu)/\mu, b/\mu)$ which is biologically feasible, provided that $P_s(a) \geq 0$ if and only if $g \leq b\delta/\mu \equiv \delta M_s$.

4. Stability

Recall from equation (2.2) that

$$\frac{dM(t)}{dt} = \mu \left(\int_0^\infty P(a,t)\, da \right) M(t) - \delta M(t) + g.$$

Substituting $P(a,t)$ from (2.8) and (2.12) into the following formula

$$\begin{aligned}\int_0^\infty P(a,t)da &= \int_0^t P(a,t)\, da + \int_t^\infty P(a,t)\, da \\ &= \left(\int_0^t be^{b(t-a)} K_0\, da + \int_t^\infty p_0(a-t)\, da \right) e^{-\mu \int_0^t M(t')\, dt'}.\end{aligned}$$

Let $a' = a - t$; then

$$\begin{aligned}\int_0^\infty P(a,t)\,da &= \left(K_0 e^{bt}(1-e^{-bt}) + \int_0^\infty p_0(a')\,da'\right) e^{-\mu\int_0^t M(t')\,dt'}\\ &= K_0 e^{bt-\mu\int_0^t M(t')\,dt'}\end{aligned}$$

leads to

$$\frac{dM(t)}{dt} = \mu K_0 e^{bt-\mu\int_0^t M(t')\,dt'} M(t) - \delta M(t) + g \tag{4.1}$$

with $M(0) = c$.

To obtain a local equation, we differentiate (4.1) with respect to t which gives

$$\frac{d^2M}{dt^2} = \left(\mu K_0 e^{bt-\mu\int_0^t M(t')\,dt'} - \delta\right)\frac{dM}{dt} + \mu K_0 M(b-\mu M) e^{bt-\mu\int_0^t M(t')\,dt'}. \tag{4.2}$$

From equation (4.1)

$$e^{bt-\mu\int_0^t M(t')\,dt'} = \frac{\delta}{\mu K_0} - \frac{g}{\mu K_0 M}.$$

Then, equation (4.2) becomes

$$\frac{d^2M}{dt^2} = -\frac{g}{M}\frac{dM}{dt} + (b-\mu M)(\delta M - g). \tag{4.3}$$

For stability of the steady-age distribution, we let solutions of model (4.3) be

$$M(t) = M_s + m(t),$$

where $|m(t)| \ll 1$, that is, $m(t)$ is a small disturbance from the fixed point. We get

$$\frac{d^2m}{dt^2} = -\frac{g}{M_s}\frac{dm}{dt} + (b-\mu(M_s+m))(\delta(M_s+m)-g).$$

From the previous section, we have two steady-state age-distributions, which are $(P_s, M_s) = (0, g/\delta)$ and $(P_s, M_s) = \left(e^{-ab}(b\delta - g\mu)/\mu, b/\mu\right)$.

For $M_s = g/\delta$, we have

$$\frac{d^2m}{dt^2} + \delta\frac{dm}{dt} + (\mu g - \delta b)m + \underbrace{\delta\mu m^2}_{\text{higher order term}} = 0.$$

We determine the stability of this system using the corresponding eigenvalues. We then obtain that $(P_s, M_s) = (0, g/\delta)$ is stable when $g > \delta b/\mu$, and is unstable elsewhere.

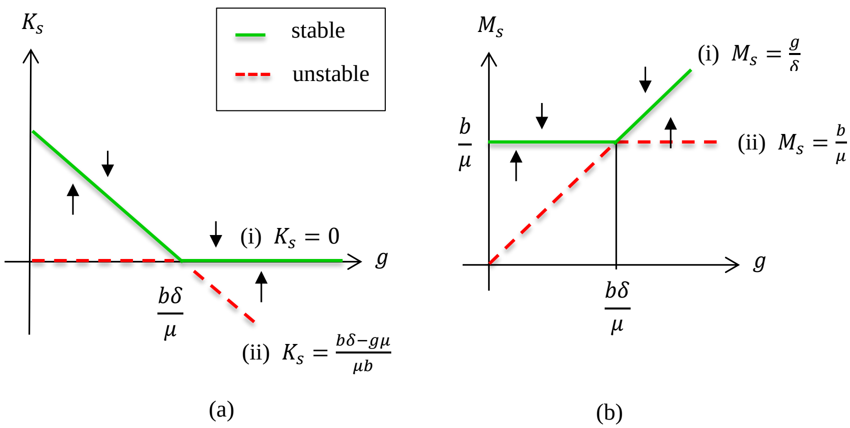

FIGURE 1. Bifurcation diagrams for (a) prey and (b) predator

For $M_s = b/\mu$,

$$\frac{d^2m}{dt^2} + \frac{g\mu}{b}\frac{dm}{dt} + (\delta b - \mu g)m + \underbrace{\delta\mu m^2}_{\text{higher order term}} = 0.$$

Again, the eigenvalue-technique is employed providing the stability condition for $(P_s, M_s) = \left(e^{-ab}(b\delta - g\mu)/\mu, b/\mu\right)$ which is $g < \delta b/\mu$. Therefore, the stabilities of this system are found and shown on the bifurcation diagrams in Figure 1.

5. Numerical results: application to the control of mealybugs in crops

In this section, specific examples are presented to verify the theoretical results divided into two cases based on the steady state where the threshold is $g = \delta b/\mu$. Let $b = 0.5, \mu = 1.6, \delta = 0.7, g = 1$ $(g > \delta b/\mu)$ for the first case, and $b = 0.5, \mu = 0.2, \delta = 0.7, g = 0.3$ for the second one. By breaking the second order equation (4.3) into two first order equations, the population dynamics of predator are carried out, as shown in Figures 2 and 3. The long-term solutions for the first and the second cases are $M_{s1} = g/\delta = 1.429$ and $M_s = b/\mu = 2.5$, respectively. Observe that the behaviour of predator population is very fluctuated, and it takes more time before stable period when g is smaller. In other words, the more predator is added, the faster the controlling of mealybugs is achieved. Moreover, there is no overwhelming population of predators which may cause another problem.

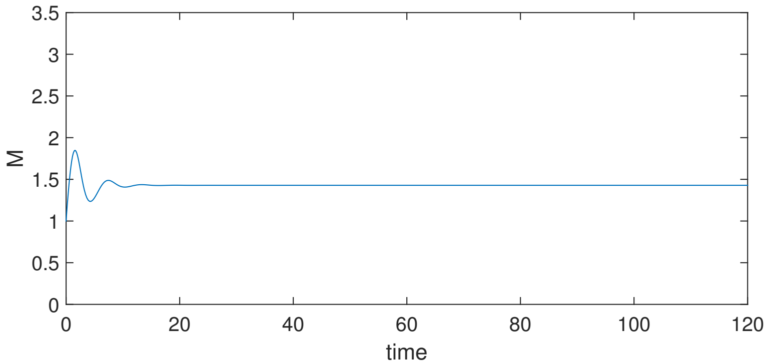

FIGURE 2. Population of predator for $b = 0.5$, $\mu = 1.6$, $\delta = 0.7$, $g = 1$

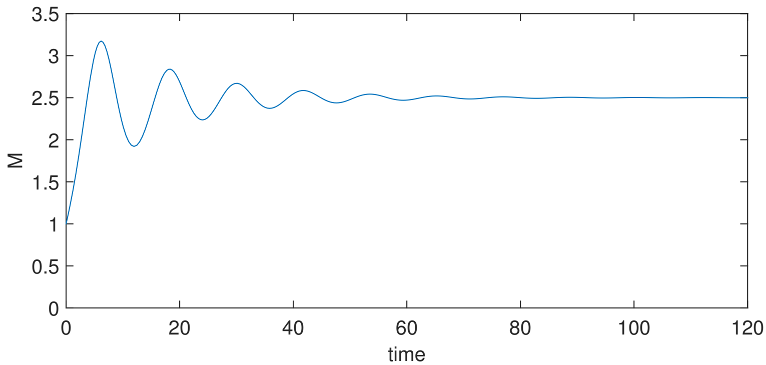

FIGURE 3. Population of predator for $b = 0.5$, $\mu = 0.2$, $\delta = 0.7$, $g = 0.3$

After obtaining the numerical results of M, we can simulate the prey density using equations (2.8) and (2.12). Define the initial condition (2.4) as $P(a,0) = p_0(a) = Ce^{ka}$ for some positive constants C and k. This function represents the population of mealybugs over age a at the first observed time. We then evaluate the population density of prey for corresponding cases. The results are illustrated in Figures 4 and 5.

In the case that $g > \delta b/\mu$, $P(a,t)$ tends to zero for large t, that is, no mealybug survives. Otherwise, $P(a,t)$ converges to $e^{-ab}(b\delta - g\mu)/\mu = 1.45e^{-0.5a}$.

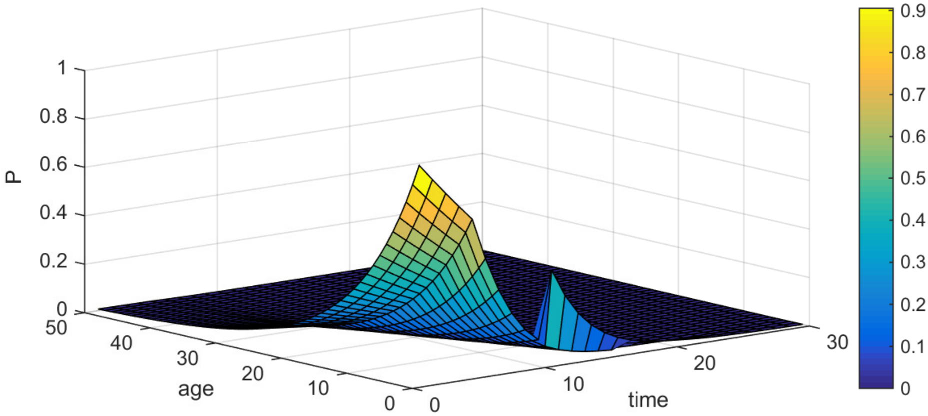

FIGURE 4. Population of prey for $b = 0.5$, $\mu = 1.6$, $\delta = 0.7$, $g = 1$

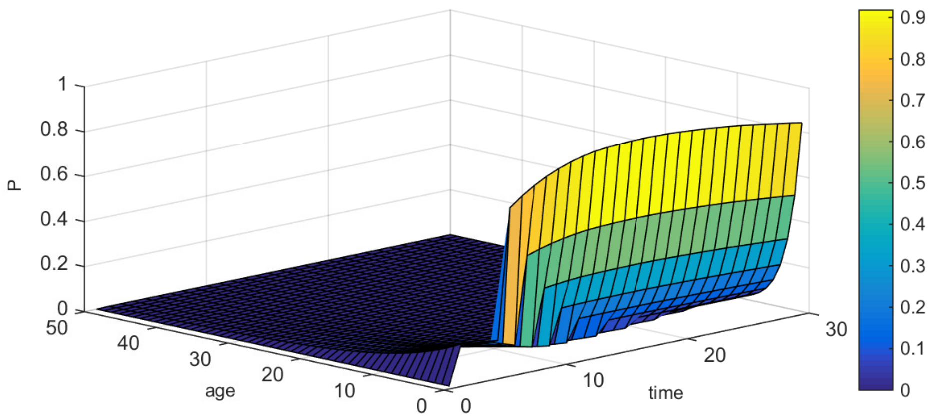

FIGURE 5. Population of prey for $b = 0.5$, $\mu = 0.2$, $\delta = 0.7$, $g = 0.3$

6. Discussion and conclusion

We have analyzed the predator-prey population model with age-dependent formula for the prey. The implicit solutions for both predator and prey are evaluated as equations (2.3), (2.12) and (2.13). By employing the steady age distribution, two steady states of the system are obtained: one is mono-species and another is coexisting species. Then, local stability of both steady states is explained. Furthermore, we get the threshold of the introduced predator level leading to mealybug extinction. Numerical results with biological meaning are provided in Section 5, which is very useful to visualize the mealybug controlling problem.

Here we have shown that this useful hybrid model, with one age-structured compartment coupled to an unstructured compartment, has exactly one asymptotically globally stable steady state The solution of the transient model is obtained analytically, albeit implicitly, thus providing a check on computational solutions in more complex situations. This parallels the outcome in systems which are not age or spatially structured. The threshold for extermination of the mealybugs ($g > \delta b/\mu$) will be useful for practical situations. To prevent recurring outbreaks, requires that this predator release rate should ideally be maintained. It is expected that a similar outcome will apply when the parameters are functions of time and/or age.

Acknowledgements

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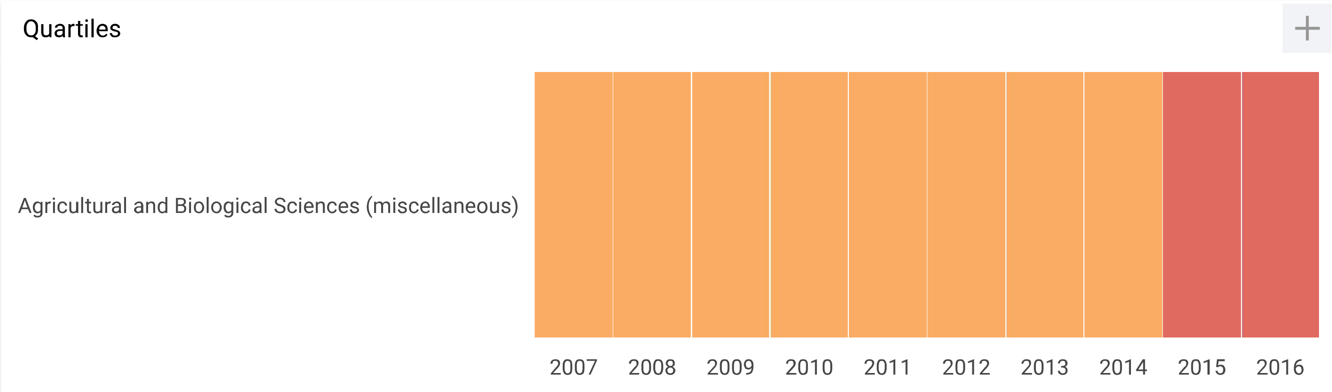

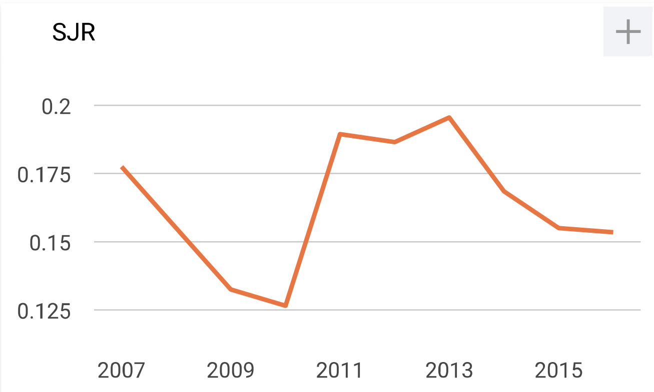

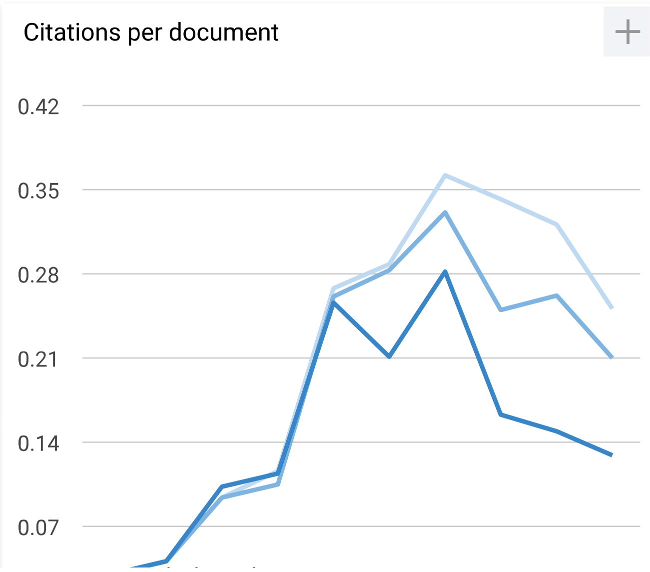

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Simulation Study of the Spread of Mealybugs in a Cassava Field: Effect of Release Frequency of a Biological Control Agent

Jairaj Promrak[1,2] and Chontita Rattanakul[1,2,*]

ABSTRACT

Cellular automata and Monte Carlo simulation techniques were employed to study the spread of mealybugs (a major cassava insect pest) in a cassava field. There are various recommended instructions on how often farmers should release green lacewings (a biological control agent) to control the spread of mealybugs. In this study, the effects of different release frequencies of green lacewings in controlling the spread of mealybugs were investigated.

Keyword: Cellular automata, cassava, mealybugs, green lacewings, biological control

INTRODUCTION

Cassava is one of Thailand's agriculture crops of importance and its share on the world's export market has been reported as approximately 61% (Office of Agricultural Economics, 2007). The spread of mealybugs in cassava fields in Thailand might cause a dramatic loss in crop yields as was recorded in 2009 when the total cassava yield was reduced from 30 million tonnes per year to 22 million tonnes per year (Boonseang, 2010). Therefore, the efficient control of the spread of mealybugs is essential.

The spread of mealybugs might be controlled by using insecticide, biological control or a mixed approach incorporating both insecticide and biological control. With biological control, green lacewing is considered as one of the major natural enemies of mealybugs and hence, it is often used as a biological control agent and there are various recommended release frequencies of green lacewings such as every two weeks, every month and every two months until there are no mealybugs on the surveyed cassava plants in the field (Centre for Pest Management, 2014). This study investigated the effect of different release frequencies on estimated crop yields.

CELLULAR AUTOMATA MODEL DEVELOPMENT

We assume that planting of cassava follows the recommended instructions of the Department of Agricultural Extension, Ministry of Agriculture and Cooperatives, Thailand (Field Crops Research Institute, 2014). In the simulations, a 40 × 40 lattice will be considered to represent a cassava field, so that the planting area is 0.16 ha (40 × 40 m). The planting distance between two cassava plants is 1 m and hence the total number of cassava plants in the field is 1,600 plants.

The states of every cell in the lattice will be updated in parallel at each time step (1 time step (Δt) = 1 day). Each cell in the lattice represents a

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state of the cassava planted in the cell which will be updated at each time step according to given rules. There are three possible states for each cell in the lattice. S, I and E representing susceptible cassava, infected cassava and removed cassava, respectively.

A number r, $0 \leq r \leq 1$ will be randomized at each time step and each cell will be updated at random according to the following rules:

Updating rule for removal cassava

If the randomized cell is a removed cassava (E), then no change occurs.

Updating rules for susceptible cassava

If the randomized cell is a susceptible cassava (S), the following rules will be applied.

(a) The cell might become an infected cassava with the probability w due to the infection of instar mealybugs from outside of the cassava field through wind transfer. Note that if the randomized cell belongs to the first two rows next to each of the four borders of the lattice then $w = w_1$, or else $w = w_2$, $0 \leq w_2 < w_1 \leq 1$.

(b) The cell might become an infected cassava with the probabilities n_1, $n_2$and n_3 if at least one of the cells in the immediate neighborhood, distant neighborhood and far distant neighborhood of the randomized cell is an infected cassava, respectively, where n_3, $0 \leq n_3 < n_2 < n_1 \leq 1$.

Updating rules for infected cassava

If the randomized cell is an infected cassava (I), the following rules will be applied.

(a) The cell might become a susceptible cassava if green lacewings successfully feed on mealybugs and there are no mealybugs on the cassava plant in the randomized cell.

(b) One month after cassava planting, a survey will be done every two weeks. We assume that each of the cassava plants might be surveyed with the probability f. We also assume further that the planting period is 12 months. If the cell is surveyed during the first 4 months or the last 5 months after planting then the cell will become a removed cassava. On the other hand, if the cell is surveyed during the 5th month and the 7th month and the number of mealybugs on the cassava plant in the cell is greater than m_1, then the cell will become a removed cassava.

The flowchart of the main loop is given in Figure 1.

The cassava field will be surveyed every 2 weeks starting from the 2nd month of planting. If mealybugs are found on the surveyed cassava plants, green lacewings at the larva stage will be released randomly on the infected cassava plants every two months until there are no mealybugs on the surveyed cassava plants. The number of green lacewings to be released depends on the severity of the spread of mealybugs. If over 50% of surveyed cassava plants are infected then the number of green lacewings to be released is G_1 or else the number of green lacewings to be released is G_2.

Apart from the wind effect, the numbers of mealybugs and green lacewings at all stages on the cassava plant in each cell of the lattice are also updated according to their life-cycle as given in Equations 1–7:

$$P_{t+\Delta t}^{i} = P_t^i + r_1\alpha_1 P_t^e - \alpha_2 P_t^i - \beta_1\left(P_t^i, M_t^i\right)M_t^i \quad (1)$$

$$P_{t+\Delta t}^{m} = P_t^m + r_2\alpha_2 P_t^i - \alpha_3 P_t^m - \beta_2\left(P_t^m, M_t^i\right)M_t^i \quad (2)$$

$$P_{t+\Delta t}^{e} = P_t^e + r_3\alpha_4 v_1 P_t^m - \alpha_1 P_t^e - \beta_3\left(P_t^e, M_t^i\right)M_t^i \quad (3)$$

$$M_{t+\Delta t}^{i} = M_t^i + s_1\gamma_1 M_t^e - \gamma_2 M_t^i \quad (4)$$

$$M_{t+\Delta t}^{d} = M_t^d + s_2\gamma_2\delta_1\left(P_t^i, P_t^m, P_t^e, M_t^i\right)M_t^i - \gamma_3 M_t^d \quad (5)$$

$$M_{t+\Delta t}^{m} = M_t^m + s_3\gamma_3 M_t^d - \delta_2 M_t^m \quad (6)$$

$$M_{t+\Delta t}^{e} = M_t^e + s_4 v_2 M_t^m - \gamma_1 M_t^e \quad (7)$$

where P_t^i, P_t^m and P_t^e represent the numbers of instar mealybugs, adult mealybugs and mealybug eggs, respectively, at the time step t; M_t^i, M_t^d, M_t^m and M_t^e represent the numbers of larva green lacewings, pupa green lacewings, adult green lacewings and green lacewing eggs, respectively, at the time step t ; α_1 and α_2 are the fractions of mealybug eggs and instar mealybugs that develop into instar mealybugs and adult mealybugs, respectively, in one time step; r_1 and r_2 are the probabilities that mealybug eggs and instar mealybugs survive this time step and develop into instar mealybugs and adult mealybugs, respectively, in the next time step; α_3 is the natural death rate of adult mealybugs in one time step; r_3 is the fraction of female adult mealybugs. α_4 is the fraction of female adult mealybugs in the reproductive period; ν_1 is the average number of eggs laid by a female adult mealybug in one time step; $\beta_1\left(P_t^i, M_t^i\right), \beta_2\left(P_t^m, M_t^i\right)$ and $\beta_3\left(P_t^e, M_t^i\right)$ are the average numbers of instar mealybugs, adult mealybugs and mealybug eggs, respectively, eaten by a green lacewings of the larva stage in one time step; γ_1, γ_2 and γ_3 are the fractions of green lacewing eggs, larva green lacewings and pupa green lacewings, respectively, that develop into green lacewing larva, green lacewing pupa and adult green lacewings, respectively, in one time

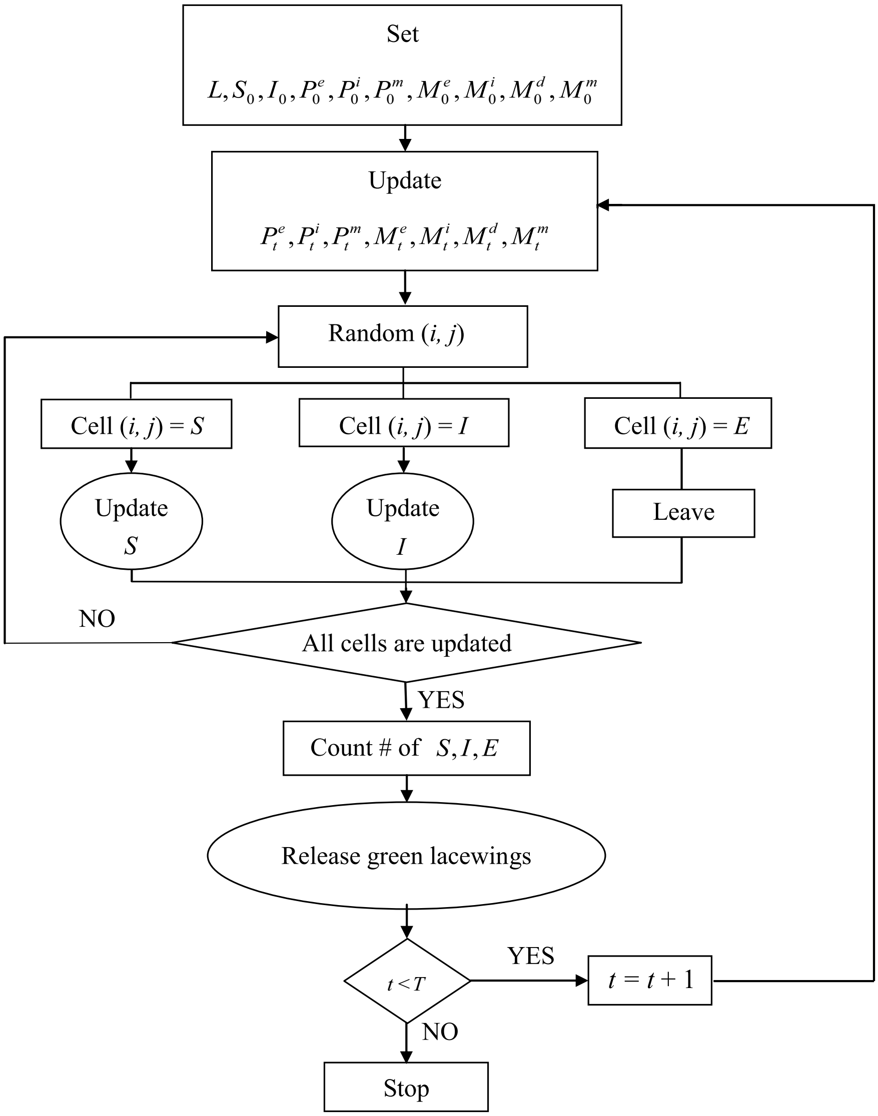

Figure 1 Main loop of Cellular Automata Model.

step; s_1, s_2 and s_3 are the probabilities of green lacewing eggs, green lacewing larva and green lacewing pupa that survive this time step and develop into green lacewing larva, green lacewing pupa and adult green lacewings, respectively, in the next time step; $\delta_1\left(P_t^i, P_t^m, P_t^e, M_t^i\right)$ is the efficiency of converting green lacewing larva to green lacewing pupa in one time step; δ_2 is the natural death rate of adult green lacewings in one time step; s_4 is the fraction of female adult green lacewings; and v_2 is the average number of eggs laid by a female adult green lacewings in one time step.

Moreover, the estimated crop yield at the end of the planting period is also monitored at each time step. By assuming that the estimated crop yield per cassava plant is c kilograms if the plant has never been infected with mealybugs longer than 2 weeks during the planting period, whereas the estimated crop yields will be reduced by 100%, 30% and 10% if the plant is infected for longer than 2 weeks during the first 4 months, during the period between the 5th and the 7th months, and during the period between the 8th and the 12th months, respectively, the estimated crop yields at each time step can be calculated using Equation 8:

$$P(t) = c \cdot C_S(t) + (0.9 \times c) \cdot C_{I_1}(t) + (0.7 \times c) \cdot C_{I_2}(t) \quad (8)$$

where $P(t)$ represents the estimated crop yields at the time step t; $C_S(t)$ represents the total number of cassava plants that have never been infected at the time step t; $C_{I_1}(t)$ represents the total number of cassava plants that have been infected longer than 2 weeks during the period between the 8th and the 12th months of planting at the time step; and $C_{I_2}(t)$ represents the total number of cassava plants that have been infected longer than 2 weeks during the period between the 5th and the 7th months of planting at the time step t.

SIMULATION RESULTS

The simulations of the spread of mealybugs were carried out using parametric values that were estimated from the available reported data at 30°C (Office of Agricultural Economics, 2007; Chong *et al*., 2008; Pappas *et al*., 2009; Pappas and Koveos, 2011).

The results shown in Figures 2–4 are the averaged values of the 10 runs using the MATLAB software (Version 7.11.0.584 (R2010b); MathWorks Inc.; Natick, MA, USA) with α_1 = 0.1493 per day, α_2 = 0.0388 per day, α_3 = 0.0596 per capita per day, α_4 = 0.4468 per day, r_1 = 0.9120, r_2 = 0.6589, r_3 = 0.8500 per day, γ_1 = 0.2703 per day, γ_2 = 0.0625 per day, γ_3 = 0.1053 per day, s_1 = 0.8170, s_2 = 0.7938, s_3 = 0.7402, s_4 = 0.4850, v_1 = 2.9126 per capita per day, v_2 = 2.3876 per capita per day, δ_2 = 0.0206 per capita per day, n_1 = 0.05, n_2 = 0.005, n_3 = 0.0005, w_1 = 0.01, w_2 = 0.001, c = 2.25 kg, G_1 = 1,000 green lacewings, G_2 = 800 green lacewings,

$$\beta_1\left(P_t^i, M_t^i\right) = \min\left\{20, \frac{P_t^i(i,j)}{M_t^i(i,j)}\right\}$$ mealybugs per larva green lacewings per day,

$$\beta_2\left(P_t^m, M_t^i\right) = \min\left\{20, \frac{P_t^m(i,j)}{M_t^i(i,j)}\right\}$$ mealybugs per larva green lacewings per day,

$$\beta_3\left(P_t^e, M_t^i\right) = \min\left\{20, \frac{P_t^e(i,j)}{M_t^i(i,j)}\right\}$$ mealybugs per larva green lacewings per day,

$$f = \left\{\frac{\text{the number of surveyed cassava plants in the field}}{\text{the total number of cassava plants that have not been removed from the cassava field}}\right\}$$

and $\delta_1 = 0.0521 \times \min\left\{1, \frac{P_t^i(i,j) + P_t^m(i,j) + P_t^e(i,j)}{M_t^i(i,j)} \div 60\right\}$

where min is the minimum and other terms are as previously defined.

DISCUSSION

From the simulation results shown in Figures 2–4, we can see that the release frequency of 2 weeks gives the best result as the estimated crop yield in this case is higher than in the other cases. However, the cumulative numbers of released green lacewings every 2 weeks and every 2 months are at about the same level while they is a little higher when they are released every month. Moreover, the spread of mealybugs seems to be controllable in all three cases.

Since the cumulative numbers of released green lacewings every 2 weeks and every 2 months are at about the same level, the costs for the release of green lacewings every 2 weeks and every 2 months are then different only with regard to the wage costs. The wages in the case of a 2 week release frequency will be four times those of the case of a 2 month release frequency. In Figure 2a, the estimated crop yields for the release frequency of 2 weeks is approximately 50 kg higher than the release frequency of 2 months at the end of

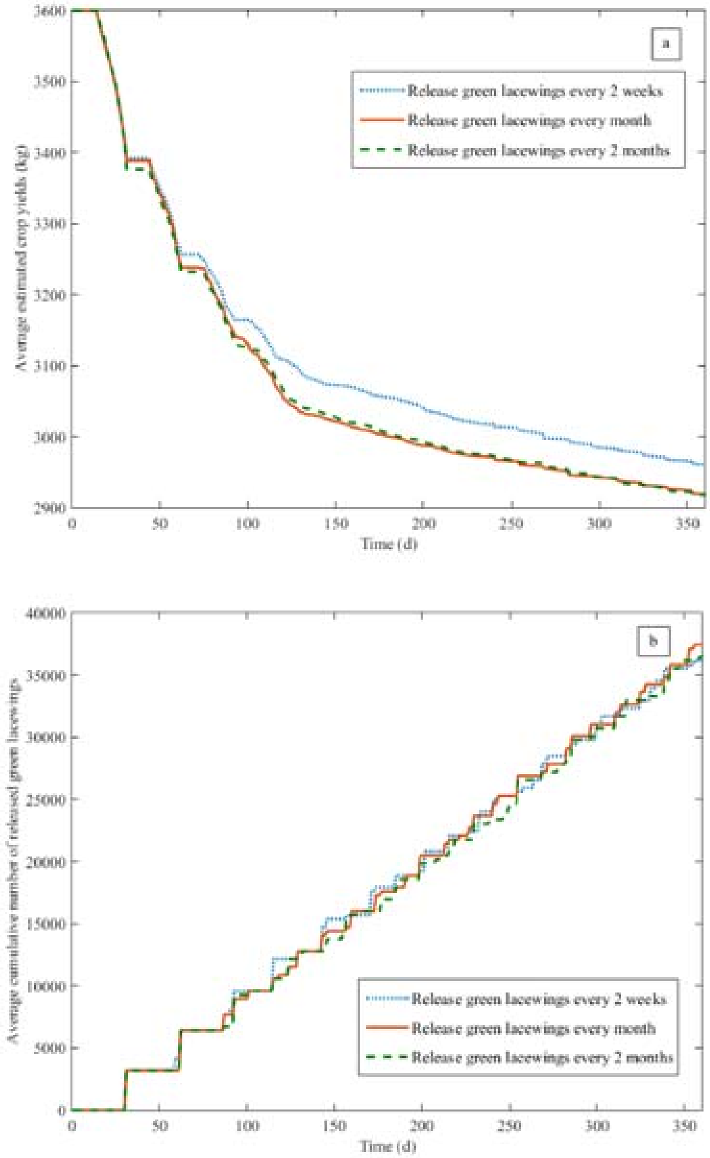

Figure 2 Simulation results of the spread of mealybugs in a cassava field: (a) Average estimated crop yields; (b) Average cumulative number of released green lacewings.

the planting period. Suppose that the market price of cassava is 0.08 USD per kilogram, the release frequency of 2 weeks will give 4.17 USD or 1.67% more on the total crop sale income and hence, the increase in wages will not be covered by the increased income in this case. Therefore, the release frequency of 2 months seems to be the better option. However, in this study, the cassava field of interest is just 0.16 ha and hence the infected probability through wind might be higher compared to a larger field whereas more labor may be necessary. It also depends on how high the wage level is and how long it takes to finish the task. On the other hand, the increased income from the yields would be higher for a larger field. Therefore, further study is needed before any general conclusion can be drawn.

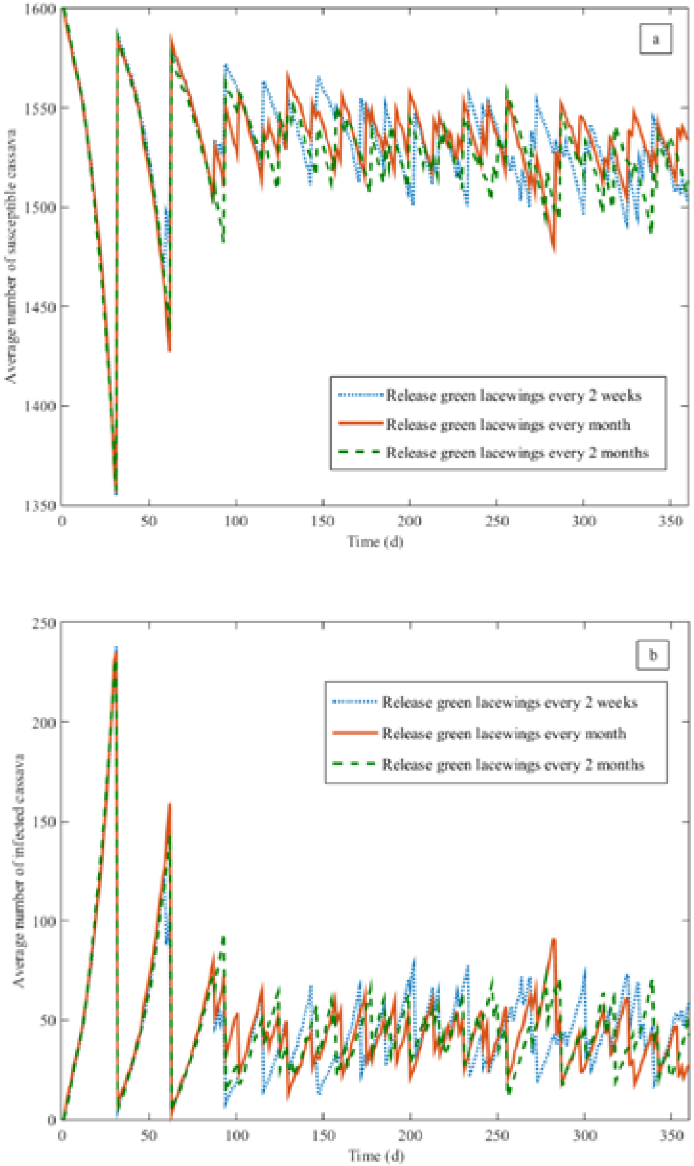

Figure 3 Simulation results of the spread of mealybugs in a cassava field: (a) Average number of susceptible cassava; (b) Average number of infected cassava.

CONCLUSION

The release frequency of green lacewings is one of the factors that must be taken into account to achieve the most efficient control of mealybugs. The cost of biological control for each of the different release frequencies of green lacewings such as wages and the reproduction cost of green lacewings, will be investigated further in comparison to the increased crop yields so that the optimal frequency that maximizes profit can be obtained.

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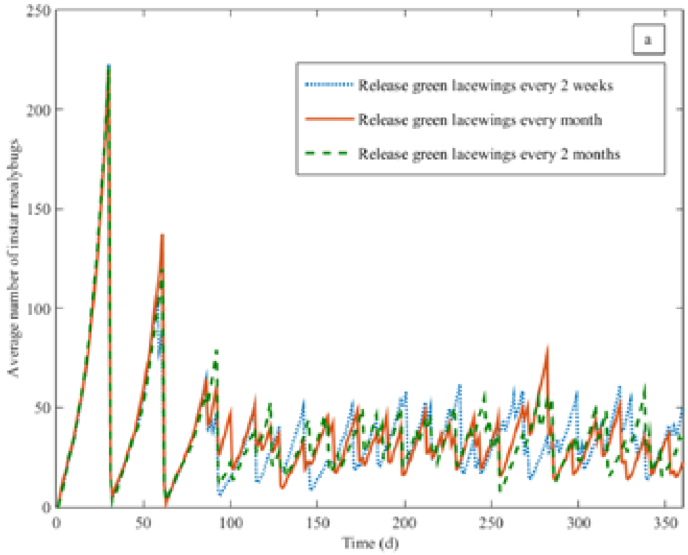

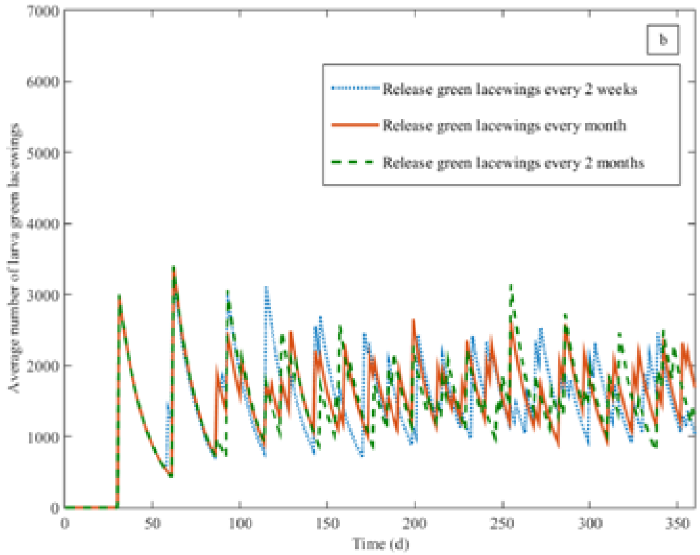

Figure 4 Simulation results of the spread of mealybugs in a cassava field: (a) Average number of instar mealybugs; (b) Average number of larva green lacewings.

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Modified Predator-Prey Model for Mealybug Population with Biological Control

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Abstract: Mealybugs are a major pest for many crops (such as the vegetable Cassava, in Thailand). An environmentally-friendly bio-control method is implemented using an introduced predator (green lacewings) of the mealybugs to mitigate plant damage. This is analyzed so as to devise and determine an optimal strategy for control of the mealybug population. A predator-prey model has been proposed and analyzed to study the effect of the biological control of the spread of the mealybugs in the plant field. The behaviour of the system in terms of stability, phase space and bifurcation diagrams are considered. The results obtained from different numbers of predators being released are compared. In particular we obtain thresholds of introduced-predator level above which the prey is driven to extinction. Future models will include age-structured multi-compartments for both the prey and predator populations.

Keywords: Predator-prey model, mealybug, biological control

1. Introduction

Mealybugs are a type of scale insect which is belongs to the family Pseudococcidae, order Homoptera (Johnson and Triplehorn, 2004). They are serious pests that infest a wide range of agricultural, horticultural and forest species including cassava, mango, tomato, peach, grape vine, redcurrant, cotton and orchid (Royal Horticultural Society, 2015). In 1980s, exotic mealybugs caused 50-90% loss of mango yields in West Africa (Moore, 2004). In 1999, the cost of Postharvest Management (PHM) of the pink hibiscus mealybug invading crops in the U.S. was estimated to be around \$750 million per year in the absence of control (Moffitt, 1999). During 2006-2007, the economic damage caused by mealybug in cotton area reached \$500,000 in north India while 0.2 million bales and 50,000 acres of cotton region were destroyed in Pakistan (Nagrare *et al.*, 2009; Institute of Science in Society, 2010). In 2008, the spread of cassava mealybug over 160,000 hectares led to a 8-10 million tonnes decrease in the cassava production in Thailand (Department of agriculture, 2008). Not only is this an enormous economic loss, but these pest infestations also cause social and cultural problems.

To control mealybug population, the biological method by releasing natural enemies has proved experimentally to be successful as shown in Table 1. Although the impact of the biological control of mealybugs has been widely tested, there are few established theoretical models to support such projects.

Mathematical modelling is an important tool to study the behaviour of prey and its predator populations. It allows us to determine the range of parameters required for a stable system and also provides a way of determining the effect when conditions are changed, especially where there is a distinct abrupt change in the long-term behaviour. Determination of these

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Table 1 Successful experiments in controlling mealybugs using its natural enemies.

Mealybug	Natural enemy	Plant	Area	Reference
Phenacoccus manihoti	Parasitoid: *Epidinocarsis lopezi*	Cassava	Africa	Herren *et al.*, 1991
Rastrococcus invadens Williams	Parasitoid: *Gyranusoidea tebygi* Noyes	Mango	Benin	Bokonon-ganta *et al.*,1995
Maconellicoccus hirsutus	*Encyrtid* wasps	-	-	Kairo *et al.,* 2000
	Coccinellid beetles	Sapota (India)	India, Caribbean, Egypt	Mani *et al.,* 2008 Baskaran *et al.,* 2007 Kairo *et al.,* 2000
	Parasitoid: *Anagyrus kamali* and *Achrysopophagus*sp.	-	Egypt	Bartlett, 1978
	Metarhizium anisopliae var. *acridum*	-	Laboratory	Ujjan *et al.,* 2007

thresholds is crucial for effective management of the situations.

The aim of this paper is to apply methods from the theory of dynamical systems to pest-control problem. We modify the predator-prey equations to analyze mealybug population with and without releasing its natural enemy. In the beginning, the population dynamics of two species using the predator-prey equations have been studied.

Let P and M be the population size of prey (mealybugs) and predator respectively. The first mathematical model is given by the following system of coupled differential equations:

$$\frac{dP}{dt} = aP\left(1-\frac{P}{K}\right) - bPM = f_1 \quad (1)$$

$$\frac{dM}{dt} = -cM + dPM = f_2 \quad (2)$$

$aP(1-P/K)$ and cM are the growth rate of prey and death rate of predator, respectively. Further, bPM represents the decreasing rate of prey caused by its predator whereas dPM is the increasing rate of predator growth depending on its prey. This model is corresponding to the following assumptions:

(i) Prey grows logistically.

(ii) Predator eats only the particular prey, under a mass-action law.

By solving equations (1) and (2), we can obtain the behaviour of the system in terms of steady states, phase planes and bifurcation diagrams which are the main focus of this work.

The outline of this paper is as follows. In section 2, the models and theoretical solutions are given. Numerical simulations and bifurcation diagrams are presented in section 3. Section 4 is devoted to discussion our results. Finally, in section 5, we draw conclusions and suggest the idea for future research.

2. Methods

In this work, three models are considered. The first model is the original predator-prey model with logistic growth for prey. The last two models are modified predator-prey models by adding the natural enemy continuously and periodically.

2.1 Predator-prey Model

Assume that $a, b, c, d,$ and K are positive parameters. A predator-prey model is shown in equations (1) and (2). To deal with a system of two first-order equations, the eigenvalue-eigenvector method is applied.

2.1.1 Steady-state solutions

A steady state (also called equilibrium point or fixed point) is a situation in which the system does not change [10]. Setting derivatives equal to zero; $dP/dt = 0$ and $dM/dt = 0$. Equations (1) and (2) become

$$aP\left(1-\frac{P}{K}\right) - bPM = 0, \quad (3)$$

$$-cM + dPM = 0. \quad (4)$$

From equations (3) and (4), we obtain three steady states which are $(P_{s1}, M_{s1}) = (0,0), (P_{s2}, M_{s2}) = (K, 0)$ and $(P_{s3}, M_{s3}) = \left(c/d, a/b(1 - c/dK)\right)$.
Get better setting out. Call them respectively "Extinction", "Monospecies" and "Coexisting species". The last one is feasible if and only if $c < dK$.

2.1.2 Stability

After finding all steady-states, the type of each point is specified to complete the solution's diagram. From (1) and (2), the Jacobian matrix of this system is

$$J_{(P_s,M_s)} = \begin{bmatrix} \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial M} \\ \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial M} \end{bmatrix}_{(P_s,M_s)}$$

$$= \begin{bmatrix} a - \frac{2aP}{K} - bM & -bP \\ dM & -c + dP \end{bmatrix}_{(P_s,M_s)}.$$

At $(P_{s1}, M_{s1}) = (0,0)$, "extinction"

$$J_{(0,0)} = \begin{bmatrix} a & 0 \\ 0 & -c \end{bmatrix} \rightarrow \lambda_1 = a,\ \lambda_2 = -c$$

where $a > 0$ and $c > 0$.
Since there is one eigenvalue which is positive and one which is negative, $(P_{s1}, M_{s1}) = (0,0)$ is an unstable saddle.

At $(P_{s2}, M_{s2}) = (K, 0)$, " monospecies"

$$J_{(K,0)} = \begin{bmatrix} -a & -bK \\ 0 & dK - c \end{bmatrix} \rightarrow \lambda_1 = -a,\ \lambda_2 = dK - c$$

where $a, c, d, K > 0$.
$(P_{s2}, M_{s2}) = (K, 0)$ is stable if both eigenvalues are negative, i.e., $dK - c < 0$.

At $(P_{s3}, M_{s3}) = \left(c/d, a/b(1 - c/dK)\right)$, "coexisting species"

$$J_{\left(\frac{c}{d},\frac{a}{b}\left(1-\frac{c}{dK}\right)\right)} = \begin{bmatrix} -\frac{ac}{dK} & -\frac{bc}{d} \\ \frac{a}{b}\left(d - \frac{c}{K}\right) & 0 \end{bmatrix}$$

$$\rightarrow \lambda = \frac{1}{2}\left(-\frac{ac}{dK} \pm \sqrt{\left(\frac{ac}{dK}\right)^2 - 4\mathrm{ac}\left(1 - \frac{c}{dK}\right)}\right)$$

Case 1: $dK < c$

$$\lambda_1 = \frac{1}{2}\left(-\frac{ac}{dK} + \sqrt{\left(\frac{ac}{dK}\right)^2 - 4\mathrm{ac}\left(1 - \frac{c}{dK}\right)}\right) > 0,$$

$$\lambda_2 = \frac{1}{2}\left(-\frac{ac}{dK} - \sqrt{\left(\frac{ac}{dK}\right)^2 - 4\mathrm{ac}\left(1 - \frac{c}{dK}\right)}\right) < 0$$

Since there is one eigenvalue which is positive and one which is negative, in this case, (P_{s3}, M_{s3}) is an unstable saddle.

Case 2: $dK > c$

$$\lambda_1 < 0, \lambda_2 < 0$$

Both eigenvalues have negative real parts so this point is stable (either a node or a spiral in the phase-plane).

To verify that the stability is defined within the appropriate area, we will show that $R = \{(P, M): 0 < K \le P \le c/d, 0 < M \le M_0\}$ is positively invariant. Let A and B be the upper bounds of P and M respectively. Consider the boundary region of four components:

Ω_1 - straight segment from $(A, 0)$ to (A, B),

Ω_2 - straight segment from (A, B) to $(0, B)$,

Ω_3 - straight segment from $(0, B)$ to $(0,0)$,

Ω_4 - straight segment from $(0,0)$ to $(A, 0)$, see Figure 1.

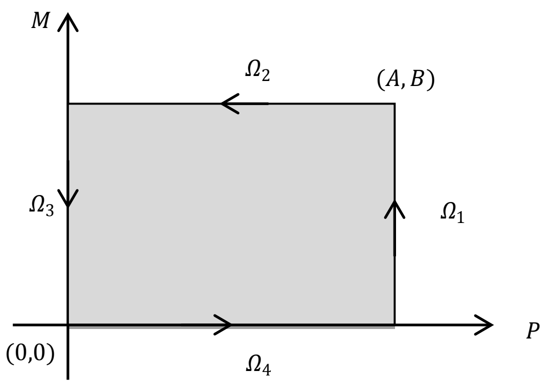


Fig. 1 A two-dimensional positively invariant region.

Choose the normal vectors $\overrightarrow{n_1} = (1,0)$ and $\overrightarrow{n_2} = (0,1)$ to point outside the region for segments Ω_1 and Ω_2 whereas apply the normal vectors $\overrightarrow{n_3} = \overrightarrow{n_1} = (1,0)$ and $\overrightarrow{n_4} = \overrightarrow{n_2} = (0,1)$ to point inside the area for segments Ω_3 and Ω_4 using dot product.

(a) Along Ω_1, $(\dot{P},\dot{M})\cdot(1{,}0)\le 0$;

$$(\dot{P},\dot{M})\cdot(1{,}0)=\dot{P}=\ aP\left(1-\frac{P}{K}\right)-bPM$$
$$=\ aA\left(1-\frac{A}{K}\right)-bAM\le 0.$$
$$aA\left(1-\frac{A}{K}\right)\le bAM\ \rightarrow\ \frac{a}{b}\left(1-\frac{A}{K}\right)\le M$$

Since $0\le M\le B,\ A\ge K$.

(b) Along Ω_2, $(\dot{P},\dot{M})\cdot(0{,}1)\le 0$;

$$(\dot{P},\dot{M})\cdot(0{,}1)=\dot{M}=-cM+dPM$$
$$=-cB+dPB\le 0.$$
$$dPB\le cB\ \rightarrow P\le c/d.$$

Since $0\le P\le A,\ A\le c/d$.

(c) Along Ω_3,

$$(\dot{P},\dot{M})\cdot(1{,}0)=\dot{P}=\ aP\left(1-\frac{P}{K}\right)-bPM=0.$$

That is, when $P=0$, M is on the y-axis with $\dot{M}<0$ (cannot cross Ω_3).

(d) Along Ω_4,

$$(\dot{P},\dot{M})\cdot(0{,}1)=\dot{M}=-cM+dPM=0.$$

That is, when $M=0$, P is on the x-axis with $\dot{P}>0$ (cannot cross Ω_4).

(e) Find $B=max\{M\}$.

$$\dot{M}=-cM+dPM\le -cB+dPB$$
$$\le -cB+d(c/d)B=0\ \text{ for }\ P\le c/d.$$
$$\dot{M}<0\ \rightarrow M(t)<M(0)=M_0.$$

Let $t\ge 0$. Suppose $P(0)\in R$ and $M(0)\in R$. We then obtain $P(t)\in R$ and $M(t)\in R$, i.e., $R=\{(P,M):0<K\le P\le c/d, 0<M\le M_0\}$ is a positively invariant set with respect to equations (1) and (2) for $K\le c/d$.

2.2 New Model for Biological Control with Continuous effect

Assume $P,M,a,b,c,d,$ and K are defined as in the previous model. We modify the predator-prey equations by adding a natural enemy of the mealybugs into the system at a positive constant rate g. Our new model is shown below.

$$\frac{dP}{dt}=aP\left(1-\frac{P}{K}\right)-bPM=f_1 \tag{5}$$

$$\frac{dM}{dt}=g-cM+dPM=f_3 \tag{6}$$

Then, we analyze this system in the same manner.

2.2.1 Steady-state solutions

Setting derivatives equal to zero; $dP/dt=0$ and $dM/dt=0$. Equations (5) and (6) become

$$aP\left(1-\frac{P}{K}\right)-bPM=0, \tag{7}$$

$$g-cM+dPM=0. \tag{8}$$

So, three steady states are $(P_{ss1},M_{ss1})=(0,g/c)$, $(P_{ss2},M_{ss2})=\left(P_{ss2},\frac{g}{c-dP_{ss2}}\right)$ and $(P_{ss3},M_{ss3})=\left(P_{ss3},\frac{g}{c-dP_{ss3}}\right)$ where P_{ss2} and P_{ss3} are defined in equations (9) and (10), respectively.

$$P_{ss2}=\frac{(adK+ca)+\sqrt{(adK+ca)^2-4ad\,(caK-gbK)}}{2ad}, \tag{9}$$

$$P_{ss3}=\frac{(adK+ca)-\sqrt{(adK+ca)^2-4ad\,(caK-gbK)}}{2ad}. \tag{10}$$

The first one only is a monospecies while the last two solutions are coexisting species.

2.2.2 Stability

The Jacobian matrix of (5) and (6) is

$$J_{(P_{ss},M_{ss})}=\begin{bmatrix}\frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial M}\\ \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial M}\end{bmatrix}_{(P_{ss},M_{ss})}$$
$$=\begin{bmatrix}a-\frac{2aP}{K}-bM & -bP\\ dM & -c+dP\end{bmatrix}_{(P_{ss},M_{ss})} \tag{11}$$

At $(P_{ss1},M_{ss1})=(0,g/c)$,

$$J_{(0,g/c)}=\begin{bmatrix}a-\frac{bg}{c} & 0\\ \frac{dg}{c} & -c\end{bmatrix}\rightarrow \lambda_1=a-\frac{bg}{c},\ \lambda_2=-c$$

where $a,b,c,g>0$.

We obtain that $(P_{ss1},M_{ss1})=(0,g/c)$ is stable when $g>ac/b$ (both eigenvalues are negative).

At $(P_{ss2},M_{ss2})=\left(P_{ss2},\frac{g}{c-dP_{ss2}}\right)$,

$$J_{\left(P_{ss2},\frac{g}{c-dP_{ss2}}\right)} = \begin{bmatrix} a-\frac{2aP_{ss2}}{K}-\frac{bg}{c-dP_{ss2}} & -bP_{ss2} \\ \frac{dg}{c-dP_{ss2}} & -c+dP_{ss2} \end{bmatrix}$$

$$\lambda^2-\left(a-\frac{2aP_{ss2}}{K}-\frac{bg}{c-dP_{ss2}}-c+dP_{ss2}\right)\lambda -\left(a-\frac{2aP_{ss2}}{K}-\frac{bg}{c-dP_{ss2}}\right)(c-dP_{ss2})+bP_{ss2}\left(\frac{dg}{c-dP_{ss2}}\right)=0 \quad (12)$$

The eigenvalue λ can be calculated from the quadratic equation (12). However, for convenience, particular λ for the corresponding parameters will be considered. In other words, the stability of equations (7) and (8) at (P_{ss2}, M_{ss2}) and (P_{ss3}, M_{ss3}) will be approached numerically by the following steps.

(i) Separate value of g.

(ii) Find the corresponding P_{ss2} and P_{ss3} from equations (9) and (10).

(iii) Apply equation (11) to obtain the Jacobian matrix.

(iv) Calculate the eigenvalues λ.

(v) Classify the stability.

By solving equations (7) and (8), we obtain

$$M=\frac{a}{b}\left(1-\frac{P}{K}\right), \quad (13)$$

$$M=\frac{g}{c-dP}. \quad (14)$$

It leads to the following formula:

$$g=\frac{adP^2-(adK+ca)P+caK}{bK} \quad (15)$$

Equation (15) is a parabola which has vertex at $(g^*,P^*)=\left(\frac{ac}{b}-\frac{(adK+ca)^2}{4adbK},\frac{K}{2}+\frac{c}{2d}\right)$ and passes through the points

$$(g,P)=(0,K),(0,c/d),(ac/b,0).$$

Suppose

$$K=1, a=1.3, b=0.5, c=0.7$$

and

$$d=1.6$$

in appropriate units. The graph of g is shown as Figure 2.

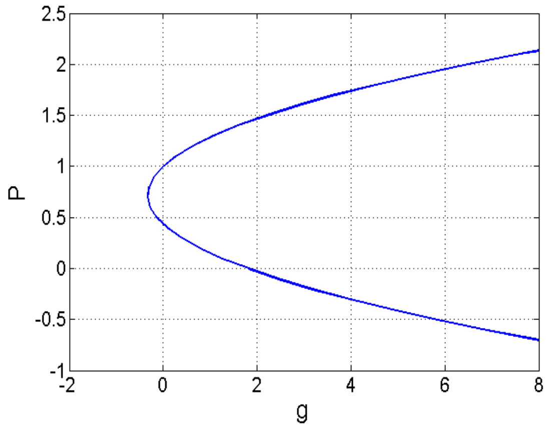


Fig. 2 Graph of g against P.

Remark: (a) P_{ss2} is the upper half of the parabola and P_{ss3} is the lower half of it.

(b) $g^*=\frac{ac}{b}-\frac{(adK+ca)^2}{4adbK}<0$.

Separate the graph into three regions based on the value of g as follows:

R1. $g^*<g<0$,

R2. $0<g<ac/b$,

R3. $g>ac/b$.

We pick up the value of g for each region and then calculate the eigenvalue by the step as we mention earlier. Finally, the stability can be specified shown in Table 2.

Moreover, we found that the positively invariant region of equations (5) and (6) is $R=\{(P,M):0<K\le P<c/d, M>0\}$ for $K\le c/d$.

2.3 New Model for Biological Control with Impulse Effect

In practice, predator will be released not continuously but periodically so modified predator-prey equations with impulse effect are considered as follows:

Table 2 Stability of modified predator-prey model at (P_{ss2}, M_{ss2}) and (P_{ss3}, M_{ss3}).

Region	g	P_s	Jacobian matrix	λ	Stability
P_{ss2}-R1	-0.2	0.89	$\begin{bmatrix}-1.16 & -0.45\\ 0.44 & 0.73\end{bmatrix}$	-1.05, 0.62	unstable
P_{ss2}-R2	0.2	1.08	$\begin{bmatrix}-1.40 & -0.54\\ -0.31 & 1.02\end{bmatrix}$	-1.47, 1.09	unstable
P_{ss2}-R3	2.0	1.47	$\begin{bmatrix}-1.91 & -0.73\\ -1.94 & 1.65\end{bmatrix}$	-2.27, 2.01	unstable
P_{ss3}-R1	-0.2	0.54	$\begin{bmatrix}-0.71 & -0.27\\ 1.90 & 0.17\end{bmatrix}$	-0.27±0.57i	stable
P_{ss3}-R2	0.2	0.36	$\begin{bmatrix}-0.47 & -0.18\\ 2.65 & -0.12\end{bmatrix}$	-0.30±0.67i	stable
P_{ss3}-R3	2.0	-0.03	$\begin{bmatrix}0.04 & 0.01\\ 4.28 & -0.75\end{bmatrix}$	0.11, -0.82	unstable

$$\frac{dP}{dt} = aP\left(1-\frac{P}{K}\right) - bPM \qquad (16)$$

$$\frac{dM}{dt} = -cM + dPM,\ T_i < t < T_{i+1} \qquad (17)$$

$$M(T_{i^+}) = M(T_{i^-}) + m, m = g * (\Delta t) \qquad (18)$$

where P, M, a, b, c, d, g and K are defined as section 2.2. Let m be the size of the added predator with period Δt. The relationship of parameters in equations (17) and (18) can be drawn in Figure 3.

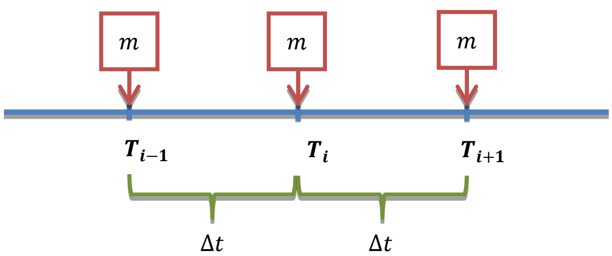


Fig. 3 Relationship of parameters for added predator.

The numerical results from this model will be shown and compared in section 3.

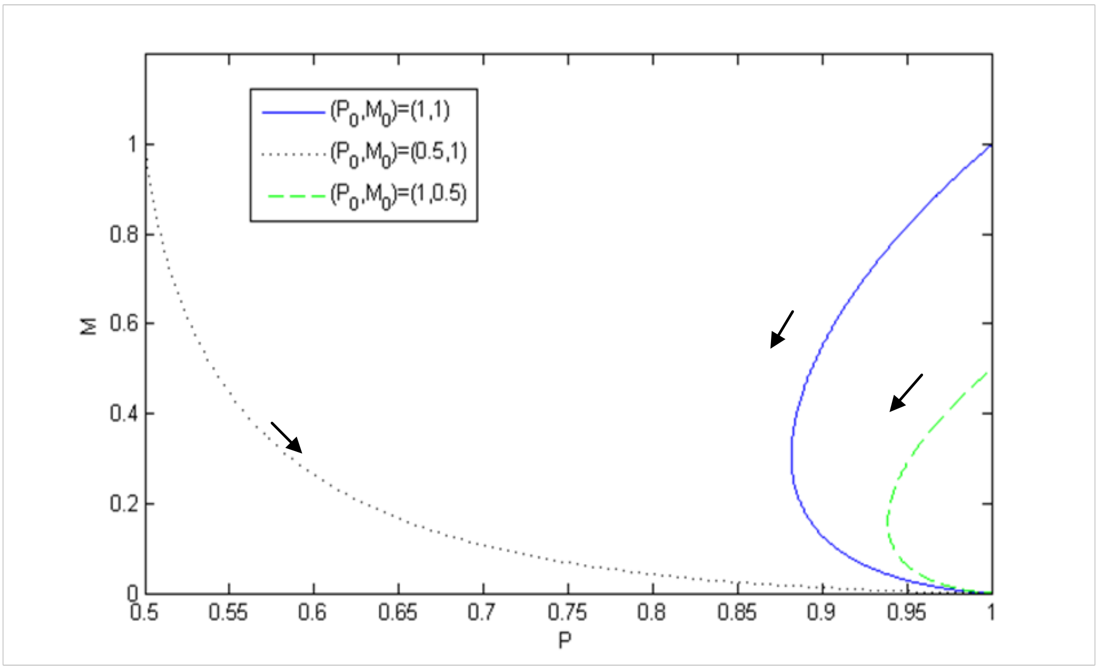
3. Simulation Results

The simulation of models leads to the following numerical results. Firstly, we show illustrative phase plane plots for the original model where the arrows represent temporal changes. From Figure 4, we obtain that the solutions tend to the monospecies and the coexisting species steady states when $c = 3.2$ and $c = 0.7$ consecutively.

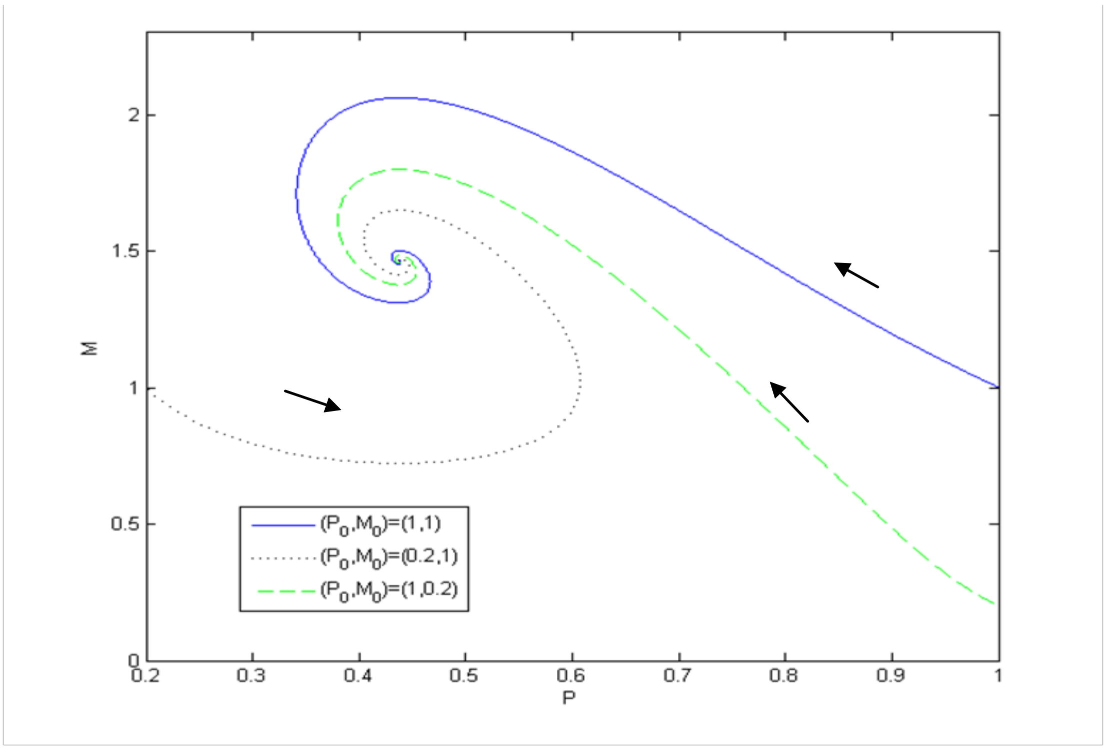
Secondly, the corresponding bifurcation diagrams of equations (1) and (2) are shown in Figure 5 where solid and dash lines represent stable and unstable states, respectively. We simplify the problem by assuming $b = d$. Then, long-term solutions depending on parameter b can be explained. In the case that $dK < c$, solutions will converge to monospecies $(P_{s2}, M_{s2}) = (K, 0)$. Otherwise, they will tend to the coexisting species $(P_{s3}, M_{s3}) = \left(\frac{c}{d}, \frac{a}{b}\left(1-\frac{c}{dK}\right)\right)$ where P_{s3} is strictly decreasing.

Next, we move to the bio-control model satisfying equations (5) and (6). Phase plane graphs with different rate of added predator are compared and shown in Figure 6. For $g = 1$, solutions meet coexisting species steady state. And we obtain monospecies as a long-term solution when $g = 3$.

Again, bifurcation diagrams are employed to determine the stability of the system. Since we focus on the effect of added predator to mealybug population, our bifurcation diagrams are drawn with respect to g. Recall from Figure 1 that g can be negative (take some existing predators out of the system); however, it is out of our domain. From Figure (7a), we can see

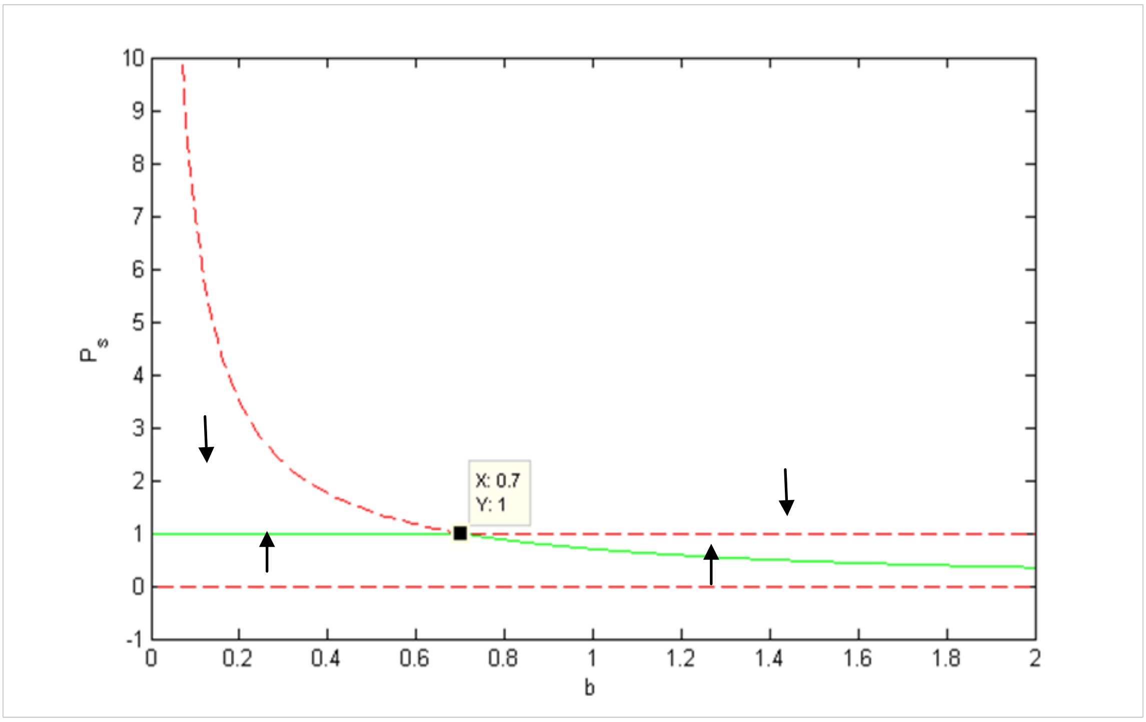


(a)

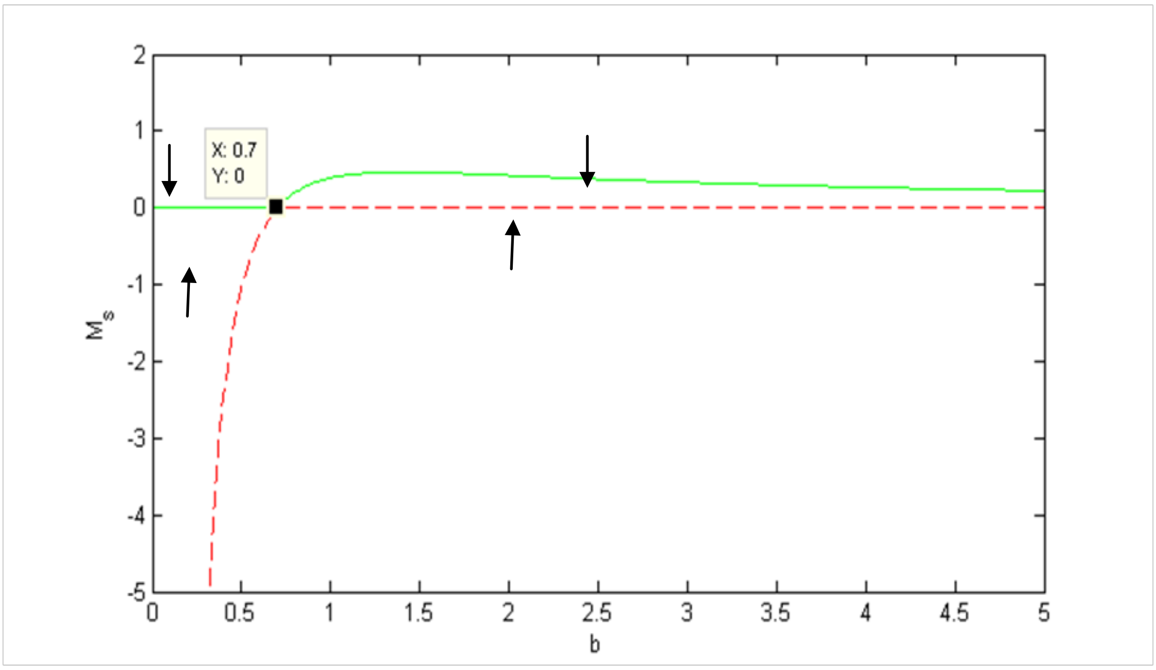


(b)

Fig. 4 Simulation results of mealybug and predator populations of the predator-prey model for $K = 1, a = 1.3, b = 0.5, d = 1.6$ (4a) $c = 3.2$ (4b) $c = 0.7$.

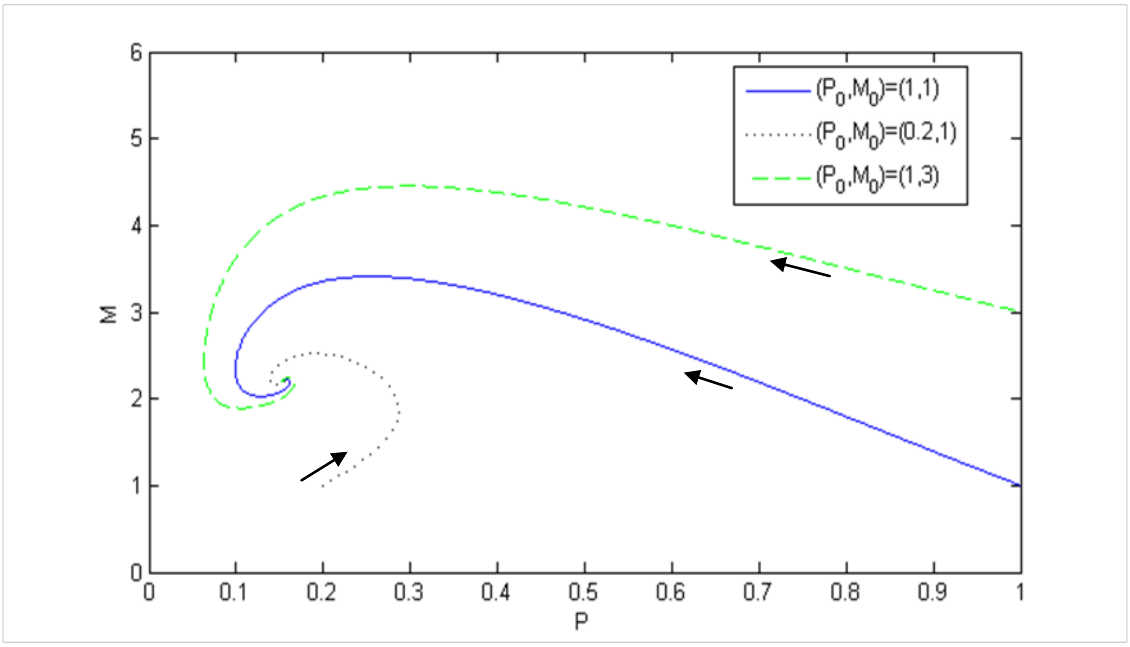


(a)

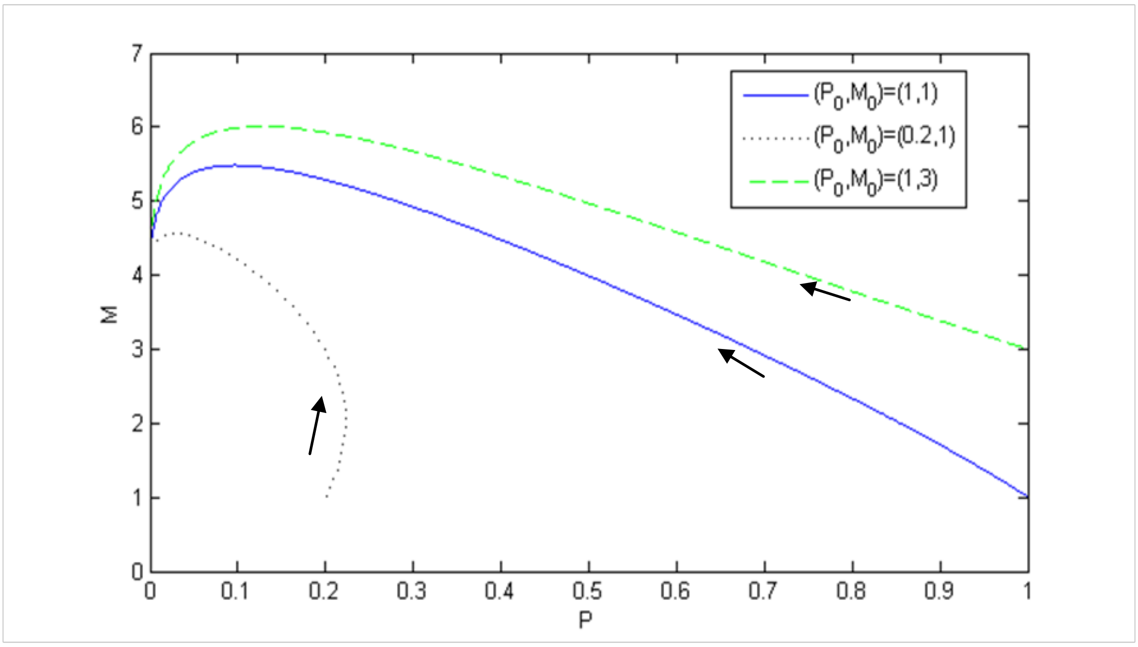


(b)

Fig. 5 Bifurcation diagrams of the predator-prey model for $K = 1, a = 1.3, c = 0.7$ (5a) mealybug population (5b) predator population.

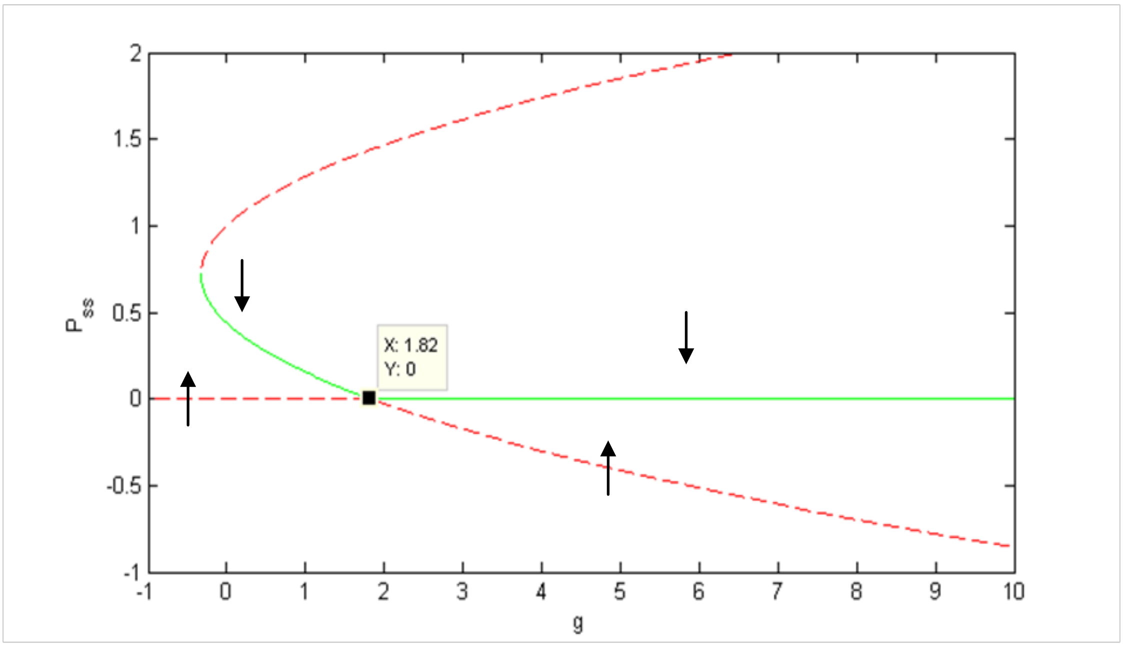


(a)

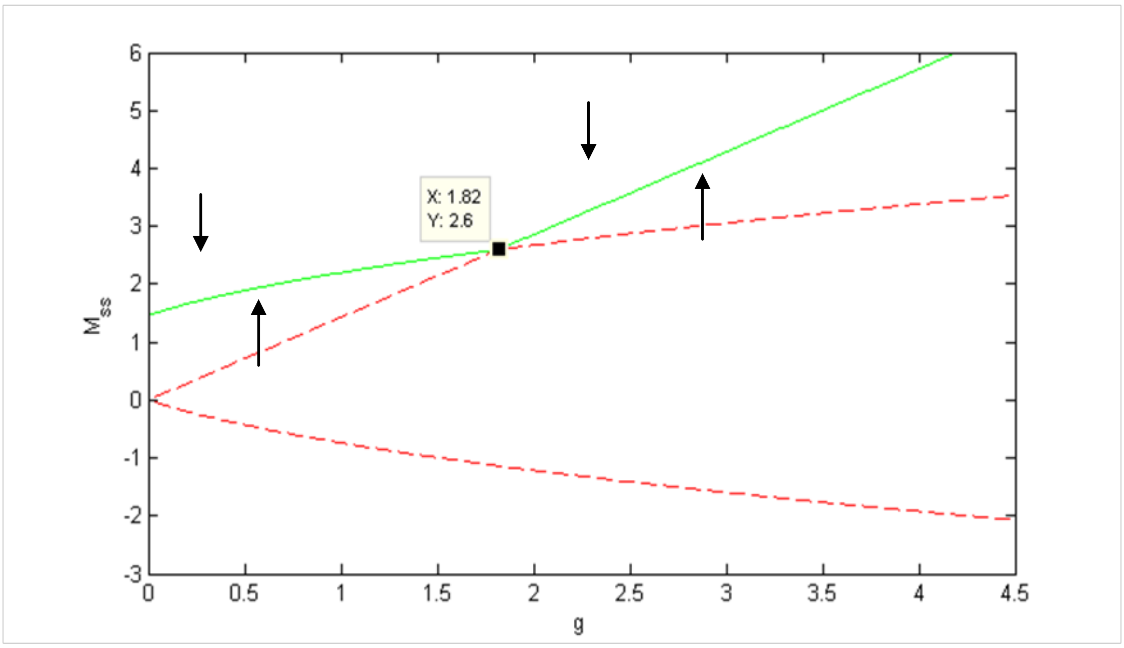


(b)

Fig. 6 Simulation results of mealybug population of modified predator-prey model for $K = 1, a = 1.3, b = 0.5, c = 0.7, d = 1.6$ (6a) $g = 1$ (6b) $g = 3$.



(a)



(b)

Fig. 7 Bifurcation diagrams of modified predator-prey model (7a) mealybug population (7b) predator population.

that population size of mealybug is dropped by increasing g and it becomes zero after $g = ac/b$. Behaviour of predator population is divided into two cases, see Figure (7b). The first one occurs when $g < ac/b$; solutions grow logistically while g is enlarged. The second type is for $g > ac/b$; solutions linearly increase corresponding to higher rate g.

Comparisons of the results of two models are shown in Figures (8) and (9) where figures on the left and the right hand sides represent the simulation results of biological control models of continuous and periodic predator-adding, respectively. Two values of g are studied: $g = 1$ $(g < ac/b)$ and $g = 3$ $(g > ac/b)$.

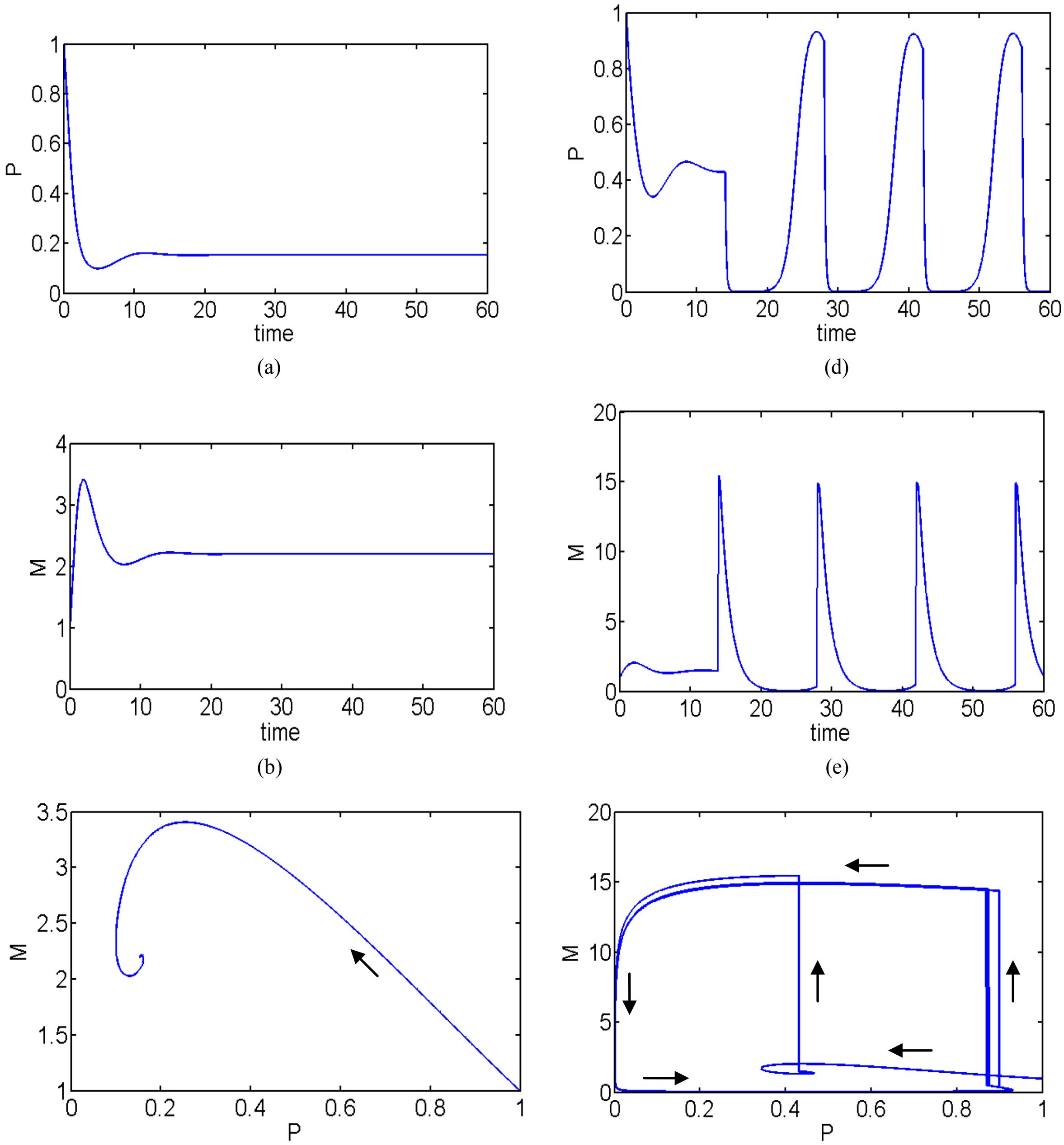


Fig. 8 Comparison of the results between models of equations (5-6) (left hand side) and equations (16-18) (right hand side) for $g = 1, K = 1, a = 1.3, b = 0.5, c = 0.7, d = 1.6, (P_0, M_0) = (1, 1), \Delta t = 14$.

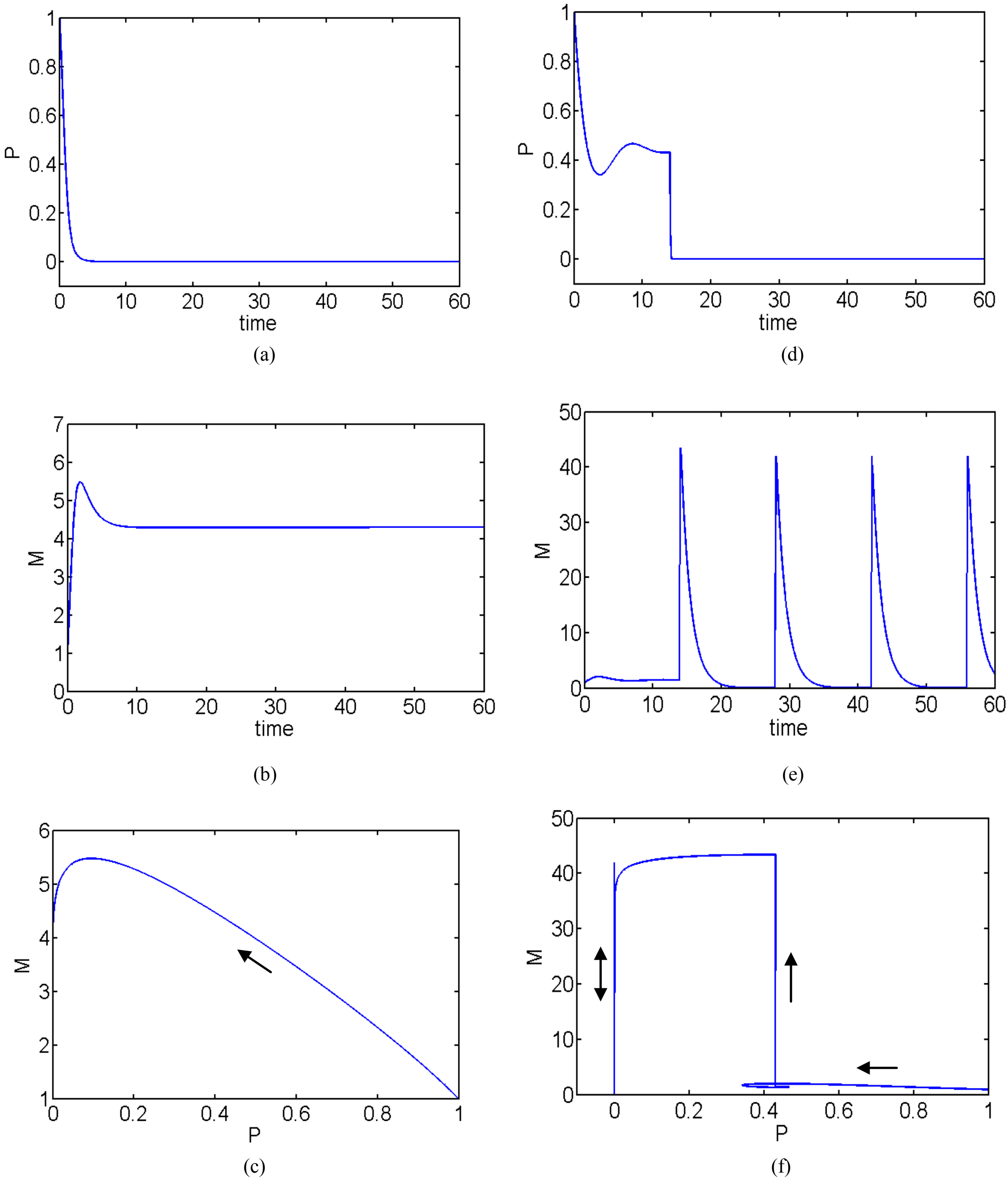


Fig. 9 Comparison of the results between models of equations (5-6) (left hand side) and equations (16-18) (right hand side) for $g = 3, K = 1, a = 1.3, b = 0.5, c = 0.7, d = 1.6, (P_0, M_0) = (1, 1), \Delta t = 14$.

From Figures (8f) and (9f) satisfying impulse effect model, we obtain the periodic solutions with coexisting-species for $g = 1$ and with mono-species for $g = 3$.

4. Discussion

From the simulation results shown in Figures 4, we verify that our solutions will converge to mono-population $(P, M) = (K, 0)$ if $dK < c$. Otherwise, the long term steady-state solution is $(P, M) = \left(\frac{c}{d}, \frac{a}{b}\left(1 - \frac{c}{dK}\right)\right)$. That is, for a non-control situation, some mealybugs survive finally. Moreover, from Figures (5a) and (5b), we can see that the bifurcation occurs at $b = d = c/K$.

After introducing its natural enemy, the size of the population of mealybugs is decreasing as desired. With the same conditions, the mealybug's level reduces from 0.45 (as a proportion) in Figure (4b) to 0.18 in Figure (6a). From Figures (6a) and (6b), we can see that if more predators are added, more mealybugs are eliminated. Furthermore, bifurcation diagram in Figure (7a) reveals that if added predator rate is more than ac/b, the mealybugs are driven to extinction.

Furthermore, our experiment provides the same trend for two models of biological control (continuous or impulse effect) that is if we add large enough amount of predator, mealybugs can be eliminated, see Figures (8) and (9).

In future work, the multistage physiological structures of the predator and prey will be taken into account before applying such model to the real experimental data.

5. Conclusion

This work presents mathematical models for a mealybug population with biological control. The stability of the system has been analyzed by the eigenvalue-eigenvector method. Some examples of relationship between the predator and its prey are given in terms of phase planes. The simulation results obtained from different parameters are compared. Finally, the bifurcation diagrams have been proposed in order to describe the overall behaviour of our models. In particular we discover thresholds of the predator release-rate which can eliminate mealybugs ultimately.

Acknowledgement

This work was supported by the Development and Promotion of Science and Technology Talents Project (DPST), Thailand, The Thailand Research Fund and the Centre of Excellence in Mathematics, Thailand.

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1.5 **Rattanakul C. A cellular automata model of a biological control of the spread of cassava mealybugs in a cassava field using *Anagyrus Lopezi* as a biological control agent. *Applied Mathematics and Computation* (Manuscript).**

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2015	17,704	1.345	1.086	1.436	0.305	1,372	6.1	9.0	0.04075	0.465	98.91	4.64415	78.937
2014	17,778	1.551	1.192	1.686	0.318	1,533	5.9	9.3	0.03785	0.469	98.37	4.23897	86.576
2013	16,463	1.600	1.248	1.672	0.298	1,089	5.7	9.2	0.04111	0.486	97.70	4.53078	88.247
2012	13,684	1.349	0.995	1.454	0.430	1,062	5.2	9.1	0.04132	0.437	97.83	Not A...	82.794
2011	11,905	1.317	0.997	1.338	0.212	1,102	4.9	9.5	0.04829	0.469	97.28	Not A...	82.245
2010	11,480	1.536	1.131	1.371	0.251	1,014	4.3	8.6	0.04274	0.390	98.13	Not A...	87.924
2009	9,741	1.124	0.873	1.236	0.214	812	4.1	8.7	0.04288	0.361	99.88	Not A...	70.343
2008	7,704	0.961	0.705	1.124	0.283	1,088	3.8	9.4	0.04102	0.357	100.00	Not A...	65.429
2007	5,549	0.821	0.476	1.007	0.093	1,403	3.6	>10.0	0.03481	0.362	99.93	Not A...	63.333
2006	3,518	0.816	0.446	Not A...	0.074	1,208	3.2	>10.0	Not A...	Not A...	99.92	Not A...	61.000
2005	2,211	0.688	0.427	Not A...	0.082	995	3.1	>10.0	Not A...	Not A...	99.90	Not A...	56.623
2004	1,419	0.567	0.315	Not A...	0.072	865	3.9	>10.0	Not A...	Not A...	99.88	Not A...	49.691
2003	999	0.426	0.224	Not A...	0.127	575	4.8	>10.0	Not A...	Not A...	99.83	Not A...	27.778
2002	841	0.436	0.298	Not A...	0.070	355	5.2	>10.0	Not A...	Not A...	99.44	Not A...	34.295
2001	623	0.359	0.223	Not A...	0.051	197	6.7	>10.0	Not A...	Not A...	99.49	Not A...	28.797

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2014	35/257	Q1	86.576
2013	30/251	Q1	88.247
2012	43/247	Q1	82.794
2011	44/245	Q1	82.245
2010	29/236	Q1	87.924
2009	61/204	Q2	70.343
2008	61/175	Q2	65.429
2007	61/165	Q2	63.333
2006	59/150	Q2	61.000
2005	66/151	Q2	56.623
2004	82/162	Q3	49.691
2003	111/153	Q3	27.778
2002	103/156	Q3	34.295
2001	113/158	Q3	28.797
2000	111/145	Q4	23.793

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2013	3/473-Q1
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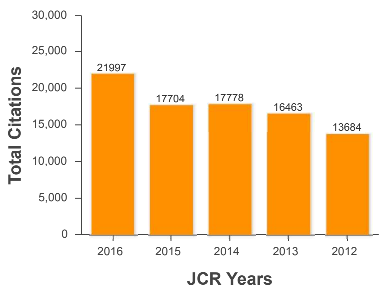

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RESEARCH

Investigating the use of wasps *Anagyrus Lopezi* for biological control of cassava mealybugs: A cellular automata model

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Abstract

In this paper, a cellular automata model and Monte Carlo simulation are utilized in order to investigate the control of cassava mealybugs in a cassava field when wasps *Anagyrus lopezi* are used as a biological control agent. The model is constructed based upon farmers' usual practices of cassava's planting in Thailand. The effects of life cycles of cassava mealybugs and wasps *Anagyrus lopezi* are also taken into account. Computer simulations of six different tactics of biological control are carried out. The results indicate that to maximize the profit farmers should release wasps *Anagyrus lopezi* only once when the spread of cassava mealybug is first detected in the field at the amount of 50-100 pairs per rai.

Keywords: cassava; cassava mealybug; cellular automata; wasps *Anagyrus lopezi*

1 Background

Nowadays, the crops that could survive hot and dry conditions such as cassava (*Manihot esculenta Crantz*) are getting more attention when the global temperatures increase every year. Cassava is a root crop with high starch content and can be used in many food and non-food industries such as pharmaceuticals, material, plywood, paper, textiles. It can also be used as biomass to produce ethanol fuel [1]. Cassava is considered to be one of the major agriculture crops of Thailand. However, a major loss in crop yield might occur if there is the outbreak of it's insect pest. In 2008, cassava mealybugs were first identified in Thailand as one of the most important cassava's insect pests. Since then, cassava mealybugs have spread throughout Thailand's cassava fields [3]. In 2010, there was an outbreak of cassava mealybugs in Thailand resulting in a major loss in cassava yield. The total cassava yield reduced from 30 million tons per year to 22 tons per year as recorded by the information from the Office of Agricultural Economics, Thailand.

The controls of the spread of mealybugs in Thailand are practiced in various ways. Farmers might use biological controls, insecticides or both of biological controls and insecticides. The Department of Agriculture, Ministry of Agriculture and Cooperatives, Thailand and the Thai Tapioca Development Institute suggest various practices for biological controls. The recommended instructions on the amount of natural enemies to be released and the period between each natural enemies released are diverse and depend on the type of the natural enemies to be released.

One of natural enemies that have been used frequently to control the spread of cassava mealybugs is wasps *Anagyrus lopezi* (*Apoanagyrus lopezi*) [2]. Wasps

Anagyrus lopezi was first imported to Thailand in the year 2009 in order to control the outbreak of cassava mealybugs in the infested cassava fields because it attacks cassava mealybugs specifically. However, there are various instructions on how to released wasps *Anagyrus lopezi* when a spread of cassava mealybugs occurs. Wasps *Anagyrus lopezi* is suggested to be released once or three times every three weeks when cassava mealybugs is first detected in a cassava field where the recommended number of wasps *Anagyrus lopezi* to be released in a cassava field are also varied such as 50-100 pairs per rai (0.16 ha), 200 pairs per rai (0.16 ha) and 400 pairs per rai (0.16 ha). In this paper, different tactics of biological control of cassava mealybugs in a cassava field are investigated.

2 Model development

According to the cassava's planting instructions recommended by the department of agricultural extension, ministry of agriculture and cooperatives, Thailand, the suggested distance for planting any two connected cassava plants is 1 metre. Suppose that the total area of cassava field is 4 rai (0.64 ha), the number of cassava plants on the first day of planting is 6,400 plants in total. A Cellular automata model is constructed as follows to investigate biological control of cassava mealybugs in the cassava field.

A 80×80 lattice will be used to stand for the cassava field. A cassava plant in the field is represented by a cell in the lattice. There are three possible states for each cell. Susceptible cell (S) refers to the non-infested cassava plant (the cassava plant that is free of cassava mealybug). Infested cell (I) refers to the infested cassava plant (the cassava plant that has cassava mealybug on the plant). Empty cell (E) refers to the removed cassava plant.

At first, every cell in the lattice will be assumed to be in the state S. In each time step (1 time step (Δt) = 1 day), the updates for the state of every cell in the lattice will be carried out at random order based upon the following rules.

Rule for updating E

If an empty cell E is randomized, it will remain at the state E.

Rules for updating S

If a susceptible cell S is randomized and it is located on the 1st or 2nd row next to the borders of the lattice, the state of the cell might become I because the cassava plant in the cell might be infested with cassava mealybugs that blown in by the wind from outside of the field with the probability $w = w_1$. On the other hand, if the randomized susceptible cell is not located on the 1st and 2nd row next to the borders of the lattice, it might become an infested cell I with the probability $w = w_2$ where $0 \leq w_2 < w_1 \leq 1$.

Moreover, the randomized susceptible cell might become I with the probability n because it is infested from cassava plants in its neighborhood shown in Figure 1. The probability that the cassava plant in the randomized cell will be infested from the cassava plants belonging to its immediate neighborhood is higher than from the distant neighborhood. The probability that the cassava plant in the randomized cell will be infested from the cassava plants belonging to its distant neighborhood is higher than from the far distant neighborhood.

Rules for updating I

If an infested cell I is randomized, it might return to the susceptible state S in the next time step if wasps *Anagyrus lopezi* successfully eliminates cassava mealybugs on the randomized cell.

On the other hand, when cassava has been planted in the field for a month, a surveyed for cassava mealybugs will be done every two weeks. If the randomized infested cell I is surveyed during the 1^{st}, 120^{th} or 211^{th}, 360^{th} day of planting, the state of the cell will then change to E.

Rule for release of wasps *Anagyrus lopezi*

Starting from the second month of planting, the survey for cassava mealybugs will be carried out every two weeks. If cassava mealybug is found when the survey is conducted during the 5th and the 7th month of planting, wasps *Anagyrus lopezi* will be released in the field once or every three weeks for three times with the amount of 50-100 pairs per rai, 200 pairs per rai or 400 pairs per rai.

Rule for updating numbers of cassava mealybug

In addition to the wind effect that might bring cassava mealybugs from inside or outside of the field so that the number of cassava mealybugs on each cassava plant might be changed, the effect of the life-cycle of cassava mealybug is also taken into account. Here, the difference equations (1)-(3) are employed to update the number of cassava mealybugs at each stage on each cell in the lattice due to the effect of the life-cycle of cassava mealybug where C_t^i,C_t^m and C_t^e are the numbers of cassava mealybugs of the instar stage, adult stage and egg stage, respectively, at the time step t.

$$\begin{aligned} \textit{Instar Stage:} \quad C_{t+\Delta t}^i &= C_t^i + r_1\alpha_1 C_t^e - \alpha_2 C_t^i - \beta_1(C_t^i, A_t^m)A_t^m & (1)\\ \textit{Adult Stage:} \quad C_{t+\Delta t}^m &= C_t^m + r_2\alpha_2 C_t^i - \alpha_3 C_t^m - \beta_2(C_t^m, A_t^m)A_t^m & (2)\\ \textit{Egg Stage:} \quad C_{t+\Delta t}^e &= C_t^e + r_3\alpha_4 v_1 C_t^m - \alpha_1 C_t^e - \beta_3(C_t^e, A_t^m)A_t^m & (3) \end{aligned}$$

where $\beta_1\left(C_t^i, A_t^m\right)$, $\beta_2\left(C_t^m, A_t^m\right)$ and $\beta_3\left(C_t^e, A_t^m\right)$ are the average numbers of instar cassava mealybugs, adult cassava mealybugs and cassava mealybug's eggs, respectively, killed by an adult wasps *Anagyrus lopezi* per time step. The definitions of other parameters in equations (1)-(3) are provided in Table 1 as well as their approximated values calculated based on the literatures [7]- [11] at $25 \pm 2°C$.

Rule for updating numbers of wasps *Anagyrus lopezi*

Apart from the increase in the number of wasps *Anagyrus lopezi* in the field due to the release of wasps *Anagyrus lopezi* when cassava mealybug is first detected, the effect of the life-cycle of wasps *Anagyrus lopezi* is also taken into account. Here, the difference equations (4)-(7) are employed to update the number of wasps *Anagyrus lopezi* at each stage on each cell in the lattice due to the effect of the life-cycle of wasps *Anagyrus lopezi* where A_t^i,A_t^d,A_t^m and A_t^e are the numbers of wasps *Anagyrus lopezi* of the larva stage, pupa stage, adult stage and egg stage, respectively, at the time step t..

$$\begin{aligned} \textit{Larva stage:} \quad A_{t+\Delta t}^i &= A_t^i + s_1\gamma_1 A_t^e - \gamma_2 A_t^i & (4)\\ \textit{Pupa stage:} \quad A_{t+\Delta t}^d &= A_t^d + s_2\gamma_2 A_t^i - \gamma_3 A_t^d & (5)\\ \textit{Adult stage:} \quad A_{t+\Delta t}^m &= A_t^m + s_3\gamma_3 A_t^d - \delta_1 A_t^m & (6)\\ \textit{Egg stage:} \quad A_{t+\Delta t}^e &= A_t^e + s_4\delta_2(C_t^i, C_t^m, C_t^e, A_t^m)A_t^m - \gamma_1 A_t^e & (7) \end{aligned}$$

where $\delta_2\left(C_t^i, C_t^m, C_t^e, A_t^m\right)$ is the efficiency of an adult female wasps *Anagyrus lopezi* on laying eggs per time step depending on the amount of consumed cassava mealybugs. The definitions of parameters in equations (4)-(7) are provided in Table 1 as well as their approximated values calculated from literatures [7]- [11] at $25 \pm 2°C$.

Note that, on each cassava plant, the number of cassava mealybugs at each stage and the number of wasps *Anagyrus Lopezi* at each stage are also monitored. In this study, wasps *Anagyrus Lopezi* at the adult stage on an infested cassava plant might fly to another infested cassava plant in their immediate, distant or far distant neighbourhood. Cassava mealybugs of the instar stage on an infested cassava plant might be blown by the wind to a cassava plant(infested or non-infested) in its immediate, distant or far distant neighbourhood.

In addition, we also monitor the approximated total crop yield. Here, the estimated crop yield is assumed to be α kilograms per cassava plant if the plant has not been infested by cassava mealybugs longer than two weeks. The crop yield of the plant will be damaged by 100%, 30% and 10%, approximately, if the cassava plant has been infested during the 1^{st}, 121^{st}, 210^{th} and 360^{th} day, respectively, by cassava mealybugs longer than two weeks. At each time step, the total estimated crop yield, $Z(t)$, can then be calculated by

$$Z(t) = \alpha \cdot Z_1 + (0.9 \times \alpha) \cdot Z_2 + (0.7 \times \alpha) \cdot Z_3 \tag{8}$$

where Z_1 is the number of cassava plants that have not been infested by cassava mealybugs longer than two weeks in total at the time step t, Z_2 is the number of cassava plants that have been infested by cassava mealybugs longer than two weeks in total during the 211^{st} and 360^{th} day at the time step t and Z_3 is the number of cassava plants that have been infested by cassava mealybugs longer than two weeks in total during the 121^{st} and 210^{th} day at the time step t.

Table 1 Definition and calculated value of parameters in equations (1)-(7)

Parameter	Definition	Value
• Cassava Mealybug		
α_1	the fraction of cassava mealybug of the egg stage that develop into cassava mealybug of the instar stage in one time step	0.12990
α_2	the fraction of cassava mealybug of the instar stage that develop into cassava mealybug of the adult stage in one time step	0.05710
α_3	the natural death rate of cassava mealybug of the adult stage	0.04810
α_4	the fraction of survived female cassava mealybugs of the adult stage in the reproductive period	0.63530
r_1	the survival rate of cassava mealybug of the egg stage that develop into cassava mealybug of the instar stage	0.95750
r_2	the survival rate of cassava mealybug of the instar stage that develop into cassava mealybug of the adult stage	0.96660
r_3	the fraction of female cassava mealybugs of the adult stage	0.97700
v_1	the average number of eggs that are laid by a female cassava mealybug of the adult stage in one time step	16.92500
• Wasps *Anagyrus lopezi*		
γ_1	the fraction of wasps *Anagyrus lopezi* of the egg stage that develop into *Anagyrus lopezi* of the larva stage in one time step	0.50000
γ_2	the fraction of wasps *Anagyrus lopezi* of the larva stage that develop into *Anagyrus lopezi* of the pupa stage in one time step	0.16670
γ_3	the fraction of wasps *Anagyrus lopezi* of the pupa stage that develop into *Anagyrus lopezi* of the adult stage in one time step	0.16670
δ_1	the natural death rate of wasps *Anagyrus lopezi* of the adult stage per a time step	0.06040
s_1	the survival rate of wasps *Anagyrus lopezi* of the egg stage that develop into *Anagyrus lopezi* of the larva stage	0.78325
s_2	the survival rate of wasps *Anagyrus lopezi* of the larva stage that develop into *Anagyrus lopezi* of the pupa stage	0.78325
s_3	the survival rate of wasps *Anagyrus lopezi* of the pupa stage that develop into *Anagyrus lopezi* of the adult stage	0.78325
s_4	the fraction of female adult wasps *Anagyrus lopezi*	0.39710

3 Numerical Simulations

In the simulations, the lattice is of the size 80×80 that is the area of cassava's planting is 4 rai (0.64 ha) while the distance between each of the two connected cassava plants is one metre and hence, the initial number of cassava plants in the field is 6,400. The planting period is one year. The simulations are carried out step by step as indicated in Figure 2.

Here, we investigate six different tactics of releasing wasps *Anagyrus lopezi* in a cassava field when the spread of cassava mealybugs is detected. The six tactics are listed as follows.

I: Release wasps *Anagyrus lopezi* only once when the spread of cassava mealybug is first detected in the field at the amount of 50-100 pairs per rai.

II: Release wasps *Anagyrus lopezi* only once when the spread of cassava mealybug is first detected in the field at the amount of 200 pairs per rai.

III: Release wasps *Anagyrus lopezi* only once when the spread of cassava mealybug is first detected in the field at the amount of 400 pairs per rai.

IV: Release wasps *Anagyrus lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field at the amount of 50-100 pairs per rai.

V: Release wasps *Anagyrus lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field at the amount of 200 pairs per rai.

VI: Release wasps *Anagyrus lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field at the amount of 400 pairs per rai.

Computer simulations of the six tactics are carried out by MATLAB software. In the simulations, $n_1 = 0.001$, $n_2 = 0.0001$, $n_3 = 0.00001$, $w_1 = 0.0001$, $w_2 = 0.00001$ and $\alpha = 2.25$. The averaged simulation result of the 100 runs are shown in Figures 3-8. The average of the 100 runs on the estimated crop yield of cassava at the end of planting period and the average of the 100 runs on the total number of wasps *Anagyrus lopezi* released in the field are also given in Table 2.

Table 2 The average estimated crop yield of cassava at the end of planting period and the average total number of wasps Anagyrus lopezi released in the field for each tactic.

Tactic	Average estimated crop yield of cassava (kgs)	Average total number of wasps *Anagyrus lopezi* released in the cassava field (pairs)
I	14,277.96	60
II	14,234.04	160
III	14,252.09	320
IV	14,298.75	440
V	14,289.57	1,760
VI	14,301.05	3,520

The results indicate that the tactic VI (Release wasps *Anagyrus lopezi* three times every three weeks when the spread of cassava mealybug is first detected in the field with the amount of 400 pairs per rai) gives the highest average estimated crop yield of cassava with the lowest number of infested cassava plants compared to the other five tactics.

4 Discussion and conclusion

We investigate the biological control of cassava mealybugs in a cassava field when wasps *Anagyrus lopezi* are used as a biological control agent. The six tactics of the control are considered. Even though the results indicate that the tactic VI is the best option for the control of the spread of cassava mealybugs and gives the highest average estimated cassava's yield at the end of planting period, the cassava's selling price is approximately 2.50 baht (0.072 USD) per kilogram and the cost for the biological control agent wasps *Anagyrus lopezi* is approximately 4.50 baht (0.13 USD) per pair. In order that the most efficient biological control in terms of maximum profit for farmers may be obtained. Table 3, showing the average estimated cost of wasps *Anagyrus lopezi* released in the field, the average estimated income from selling cassava's yields and the average estimated (income – cost of biological control agents) at the end of planting period for each tactic is also provided here.

Table 3 The average estimated cost of wasps Anagyrus lopezi released in the field, the average estimated income from selling cassava's crop yields and the average estimated (income – cost of biological control agents) at the end of planting period for each tactic.

Tactic	Average estimated cost of wasps textitAnagyrus lopezi released in the field (baht)	Average estimated income from selling cassava's crop yields (baht)	Average estimated (income – cost of wasps *Anagyrus lopezi*) at the end of planting period (baht)
I	270	35,694.90	35,424.90
II	720	35,585.10	34,865.10
III	1,440	35,630.23	34,190.25
IV	1,980	35,746.88	33,766.88
V	7,920	35,723.93	27,803.93
VI	15,840	35,752.63	19,912.63

In Table 3, we can see that even though the tactic VI give the highest average estimated cassava's crop yield, the tactic that gives the maximum profit is the tactic I (releasing wasps *Anagyrus lopezi* only once when the spread of cassava mealybug is first detected in the field at the amount of 50-100 pairs per rai). Note that the planting area that we considered here is just 4 rai (0.64 ha). When the planting area is a large-scale cassava farm the results might not be the same as what we have found here. One reason is that the spread of cassava mealybugs might not be detected in the large-scale cassava farm as fast as in a small-scale cassava farm. Hence, further investigations are needed for a large-scale cassava farm.

Declarations
Not applicable

List of abbreviations
Not applicable

Ethics approval and consent to participate
Not applicable

Consent for publication
Not applicable

Availability of data and materials
Parameters in equations (4)-(7) are provided in Table 1 as well as their approximated values calculated from literatures [7]- [11].

Competing interests
The authors declare that they have no competing interests.

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Author's contributions
Sole author.

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Authors' information
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Endnotes
Not applicable

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Figures

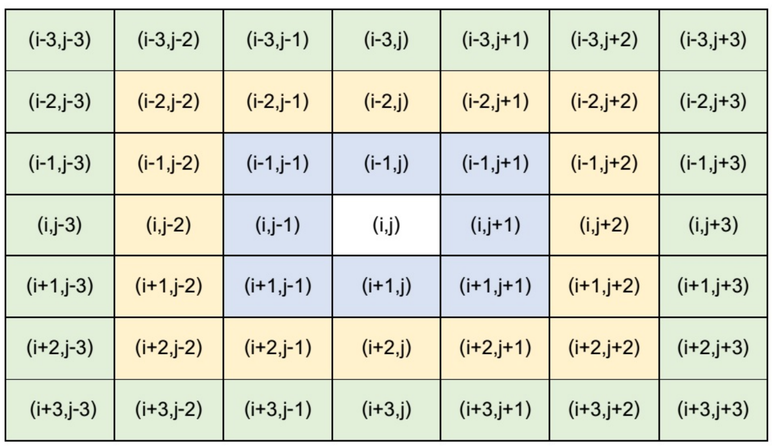

Figure 1 The blue, yellow and green areas represent immediate neighborhood, distant neighborhood and far distant neighborhood, respectively, of the cell (i, j).

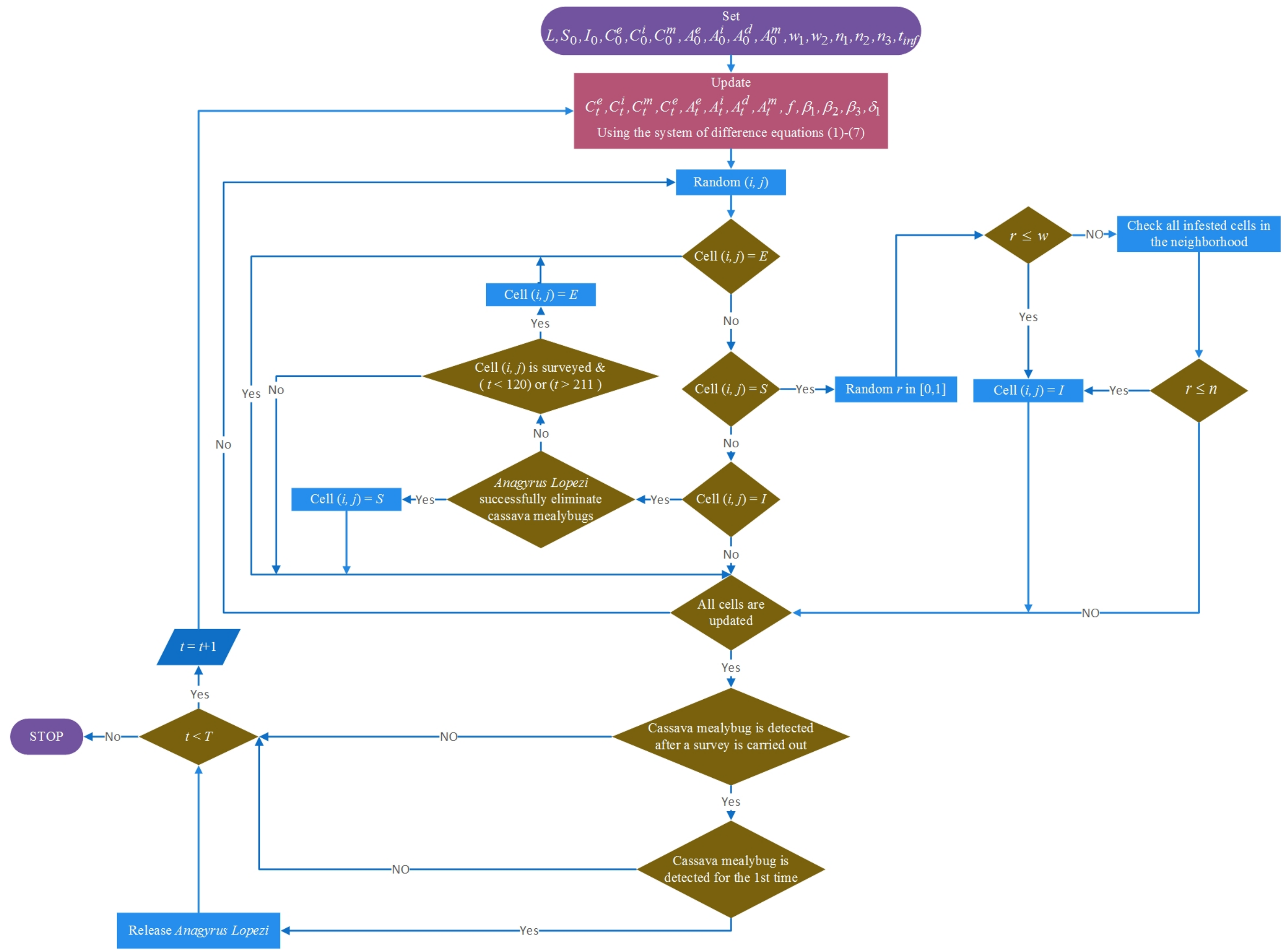

Figure 2 The main loop of cellular automata.

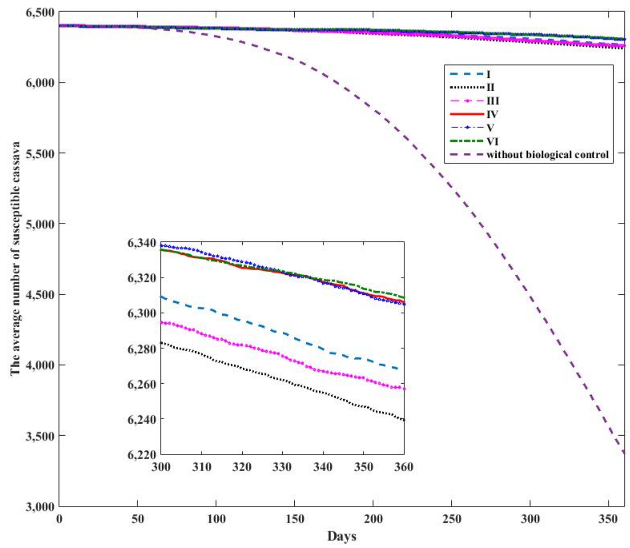

Figure 3 The average number of susceptible cassava plants.

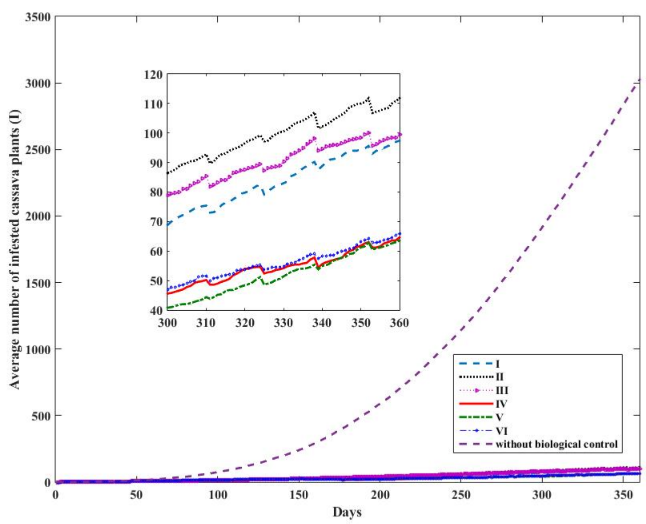

Figure 4 The average number of infested cassava plants.

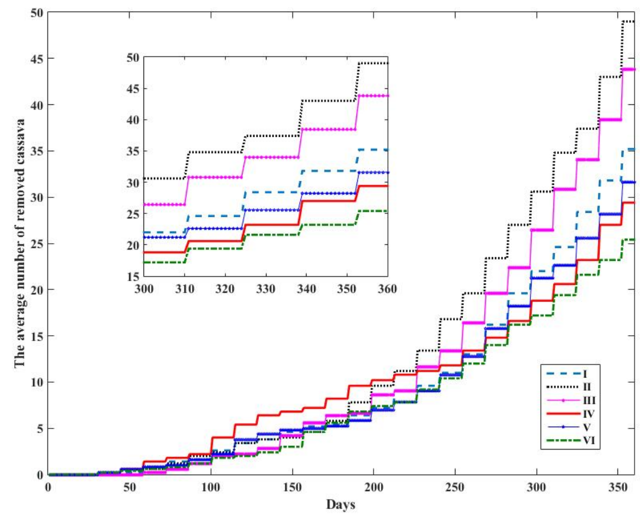

Figure 5 The average number of removed cassava plants.

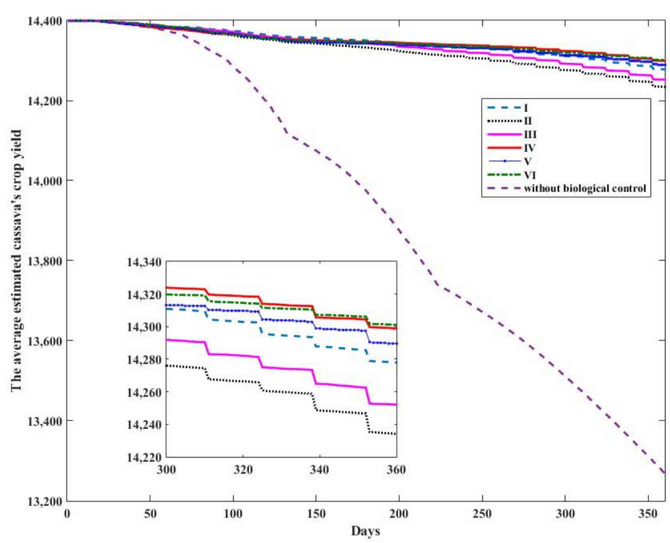

Figure 6 The average estimated cassava's crop yield.

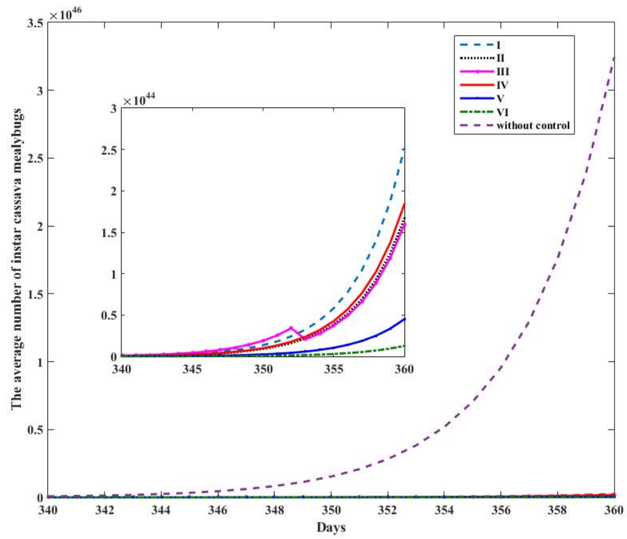

Figure 7 The average number of instar cassava mealybugs.

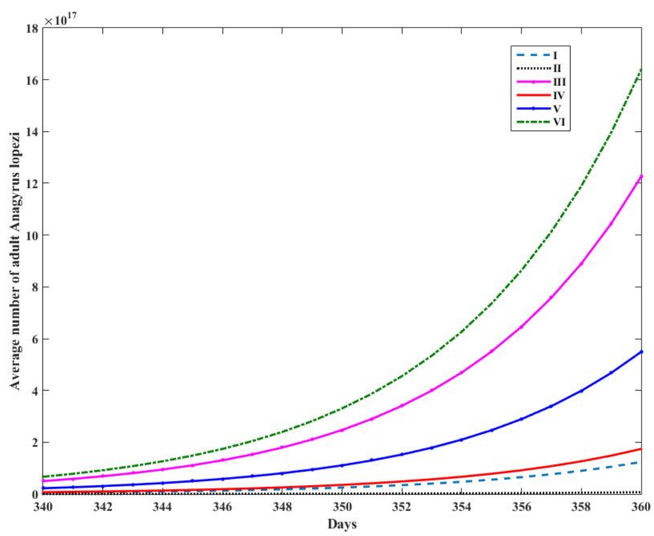

Figure 8 The average number of adult wasps *Anagyrus lopezi*.

1.6 Rattanakul C. A cellular automata model of a biological control of the spread of cassava mealybugs in a cassava field using *Anagyrus Lopezi* and green lacewings as biological control agents. *Pest Management Science* (Manuscript).

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2016	7,943	3.253	2.894	3.338	0.719	263	6.7	9.8	0.01288	0.868	93.54	1.47563	92.460
2015	7,172	2.811	2.565	3.116	0.818	198	6.5	9.6	0.01308	0.851	91.41	1.49104	90.217
2014	6,246	2.694	2.323	2.863	0.642	240	6.3	9.8	0.01227	0.799	86.67	1.37385	89.986
2013	5,500	2.743	2.457	2.889	0.525	183	5.9	9.3	0.01316	0.833	93.44	1.45064	91.410
2012	4,883	2.594	2.226	2.771	0.517	209	5.7	9.4	0.01254	0.806	94.74	Not A...	91.324
2011	4,101	2.251	1.938	2.545	0.500	200	5.5	8.3	0.01035	0.675	93.00	Not A...	87.202
2010	3,521	2.313	2.047	2.358	0.353	184	5.5	9.3	0.01078	0.688	95.65	Not A...	89.751
2009	2,990	2.190	1.930	2.079	0.534	191	5.1	8.7	0.01050	0.652	87.43	Not A...	87.002
2008	2,732	2.040	1.767	2.170	0.530	164	5.1	8.2	0.01118	0.653	92.07	Not A...	89.548
2007	2,175	1.867	1.617	1.892	0.278	151	4.7	8.5	0.01123	0.620	90.73	Not A...	89.996
2006	1,608	1.428	1.258	Not A...	0.185	146	4.2	9.0	Not A...	Not A...	96.58	Not A...	78.054
2005	1,269	1.175	1.068	Not A...	0.342	155	3.8	8.2	Not A...	Not A...	98.71	Not A...	69.650
2004	937	1.184	0.996	Not A...	0.215	158	3.2	9.8	Not A...	Not A...	98.10	Not A...	73.000
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2002	375	1.178	0.949	Not A...	0.133	158	2.1	7.9	Not A...	Not A...	97.47	Not A...	82.379
2001	111	0.642	0.496	Not A...	0.095	147	1.4	7.9	Not A...	Not A...	99.32	Not A...	60.095

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2016	8/83	Q1	90.964	6/91	Q1	93.956
2015	11/83	Q1	87.349	7/94	Q1	93.085
2014	11/81	Q1	87.037	7/92	Q1	92.935
2013	11/79	Q1	86.709	4/90	Q1	96.111
2012	10/78	Q1	87.821	5/87	Q1	94.828
2011	14/80	Q1	83.125	8/86	Q1	91.279
2010	10/75	Q1	87.333	7/83	Q1	92.169
2009	11/61	Q1	82.787	7/74	Q1	91.216
2008	7/49	Q1	86.735	6/72	Q1	92.361
2007	6/49	Q1	88.776	7/74	Q1	91.216
2006	11/49	Q1	78.571	16/69	Q1	77.536
2005	14/48	Q2	71.875	22/66	Q2	67.424
2004	15/50	Q2	71.000	17/66	Q2	75.000
2003	7/53	Q1	87.736	11/64	Q1	83.594
2002	10/55	Q1	82.727	12/64	Q1	82.031
2001	21/55	Q2	62.727	29/67	Q2	57.463

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2015	39/336-Q1
2014	42/325-Q1
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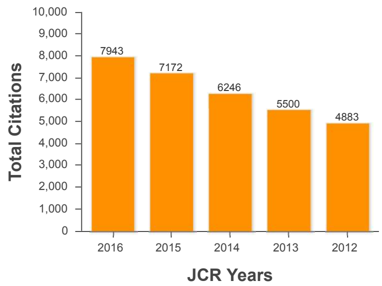
2012	45/327-Q1



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RESEARCH ARTICLE

A cellular automata model of biological control of cassava mealybug in a cassava field using *Anagyrus lopezi* and green lacewing as biological control agents

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ABSTRACT
Cassava is one of the most important agricultural products of Thailand. Weeds, animal pests and pathogens are regular concern of economic importance. In this paper, a cellular automata (CA) model is constructed to investigate the spread of cassava mealybugs in a cassava field with the usual practices of biological control in Thailand. The available reported data from many sources will be utilized so that parameter values in the model are obtained. To obtain an efficient control of the spread of cassava mealybugs in a cassava field, computer simulations are then carried out for 54 different manners of biological control when *Anagyrus lopezi* and green lacewings are used as biological control agents. The results indicate that the most efficient control yielding maximum profit is to release 200 green lacewings per rai every month together with 50-100 pairs of *Anagyrus lopezi* per rai once after cassava mealybugs is first detected.

KEYWORDS
Cassava; cassava mealybug; *Anagyrus Lopezi*; green lacewing; biological control; cellular automata; Monte Carlo simulation

1. Introduction

Agriculture has been the backbone of Thailand's economy for several decades. Cassava is considered to be one of the major agriculture crops of Thailand. Although Thailand is not a major consumer of cassava, it is the world's biggest exporter of cassava with the world's market share of 60.72% in 2011 according to the office of agricultural economics, ministry of agriculture and cooperatives, Thailand. Even though cassava can survive both hot and dry conditions, an increase in insect pests might easily cause a major loss in crop yield. Mealybugs (Hemiptera: Pseudococcidae) constitute a major family of insect pests of cassava. In Thailand, there are four species of mealybugs found in cassava fields which are striped mealybugs, Madeira mealybugs, pink mealybugs (cassava mealybug) and Jack-Beardsley mealybugs (Boonseng 2009). In 2008, cassava mealybugs were first identified in Thailand and has spread aggressively throughout cassava's planting area in Thailand (Winotai et al. 2010). In 2010, there was an outbreak of cassava mealybugs in Thailand resulting in a major loss in cassava yield.

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The total cassava yield reduced from 30 million tons per year to 22 tons per year according to the information from the Office of Agricultural Economics, Thailand.

There are various practices to control the spread of mealybugs in cassava fields in Thailand. Farmers might use biological controls, insecticides or a mixture of biological controls and insecticides. With biological controls, various practices have been recommended by the Thai Tapioca Development Institute and the Department of Agriculture, Ministry of Agriculture and Cooperatives, Thailand. The suggestions on the number of natural enemies to be released in a field and the period between each natural enemies released are diverse and also depend on the type of the natural enemies to be released. Natural enemies that have been used popularly to control the spread of cassava mealybugs are *Anagyrus lopezi* and green lacewings (Boonseng 2009).

In Thailand, different manners are recommended when an outbreak of cassava mealybug occurs. When *Anagyrus lopezi* is used as a biological control agent, one recommendation is to release *Anagyrus lopezi* once when cassava mealybugs are detected whereas the other suggestion is to release *Anagyrus lopezi* every three weeks for three times since cassava mealybugs are detected. The recommended number of *Anagyrus lopezi* to be released in the field is also various such as 50-100 pairs per rai (300-600 pairs per ha, approximately). On the other hand, when green lacewings is used as a biological control agent, the recommended release frequencies are various such as every two weeks, every month or every two months. The instructions on how many of green lacewings to be released in the field are also various such as 200 per rai (1,200 per ha), 800 green lacewings per rai (4,800 per ha) or 1,000 per rai (6,000 per ha).

The lack of the knowledge may cause a major loss in cassava crop yields. When an outbreak of cassava mealybugs occurs and farmers have to determine on how much/ how often the natural enemies of cassava mealybugs should be released to control the outbreak, might turn to use insecticide instead if they do not know which instructions should be applied. The outbreak might get worst depending on the type of the insecticide and the age of the plant when the insecticide is applied because the insect pest might resist to the insecticide while the natural enemies of the insect pest might be destroyed and the plant might be damaged.

In this paper, the 54 different manners of biological control using both *Anagyrus lopezi* and green lacewings will be investigated to get a better understanding on each manner. In order to do so, a cellular automata (CA) model will be constructed in the next section.

2. A Cellular Automata Model

Based upon the recommended instructions of the Department of Agricultural Extension, Ministry of Agriculture and Cooperatives, Thailand, we then make the following assumptions.

Initially, all cassava plants in the field are free of cassava mealybugs. However, the wind might blow instar cassava mealybugs into the field and some cassava plants might be infested. Note that only cassava mealybug at the instar stage can be blown with the wind.

Starting on the 2nd month of cassava's planting the surveyed for cassava mealybugs will be carried out every two weeks by collecting the numbers of mealybugs at all stages on the cassava plants that are not planted on the two rows next to the four borders of the cassava field. The survey will be conducted on every two rows of plants, and every eleven plants as instructed by the Department of Agricultural Extension, Ministry of Agriculture and Cooperatives, Thailand.

If the plant infested with cassava mealybugs is subjected to be surveyed and the day that the survey is conducted is during the first 4 months or the last 5 months of planting period, the plant will then be removed from the field. On the other hand, if cassava mealybug is found on a surveyed plant during the 5th - 8th month of planting, *Anagyrus lopezi* and green lacewings will be released in the field in order to control the spread of cassava mealybugs in the field. Note that the planting period of cassava is assumed to be one year.

We then assume further that if a cassava plant is infested with cassava mealybugs longer than two weeks, the estimated crop yields at the end of planting period will decrease. Crop yields will be reduced by 100%, 30% and 10% if the cassava plant is infested with cassava mealybugs during the first 4 months, during the period between the 5th and the 7th months, and during the last 5 months, respectively, according to the surveys of the Thai Tapioca Development Institute in 2007-2010.

Here, a square lattice of the size $L \times L$ lattice will be used to represent a cassava field while each cell in the lattice represents a cassava plant. The possible states for each cell in the lattice are susceptible cell (S) representing the cassava plant which is not infested by cassava mealybug, infested cell (I) representing the cassava plant which is infested by cassava mealybug and empty cell (E) representing the cell that the cassava plant was removed from the field.

At first, every cell in the lattice will be assumed to be susceptible cells. After that, at each time step (1 time step (Δt) = 1 day), the states of every cell $(i, j), 1 \leq i \leq L, 1 \leq j \leq L, i, j \in \mathbf{Z}$ in the lattice will be updated at random order according to the following rules where a number $r, 0 \leq r \leq 1$ will be randomized.

2.1 Rules for updating each cell in the lattice

2.1.1 Rules for updating a susceptible cell (S)

If the randomized cell (i, j) is a susceptible cell then the following rules are applied.

(i) If the randomized cell (i, j) is belonged to the first two rows next to each of the four borders of the lattice, the cell might becomes an infested cell because the cassava plant in that cell might be infested with cassava mealybugs blown with the wind from the outside of the field with a probability $w = w_1$, or else it might becomes an infested cell with a probability $w = w_2$ where $0 \leq w_2 < w_1 \leq 1$. Hence, if $r \leq w$ then the randomized cell changes the state from S to I.

(ii) The randomized cell (i, j) might become an infested cell if there is cassava mealybug in the cells belonging to it's neighborhoods with the probability n. Here, the neighborhoods of the randomized cell refer to the immediate neighborhood, distant neighborhood or far distant neighborhood as shown in Figure 1. The the randomized cell (i, j) might change the state from S to I if $r \leq n$ where $n = n_1, n = n_2$ and $n = n_3$ when there is cassava mealybug in the immediate neighborhood, distant neighborhood and far distant neighborhood, respectively where $0 \leq n_3 < n_2 < n_1 \leq 1$.

2.1.2 Rules for updating an infested cell (I)

If the randomized cell (i, j) is an infested cell then the following rules are applied.

(i) The randomized cell (i, j) might become a susceptible cell if green lacewings and *Anagyrus lopezi* successfully feed/parasitism on cassava mealybugs so that there is no cassava mealybugs on the cassava plant in the randomized cell. In this case, the state of the randomized cell (i, j) will change from I to S.

(ii) Starting from the 2nd month of planting, each of the cassava plants might be surveyed with the probability f every two weeks. Here, a cassava plant might be surveyed with the probability

f = the number of cassava plants to be surveyed in the field according to the above manner ÷ the total number of cassava plants that have not been removed from the cassava field

(a) If the randomized cell (i, j) is surveyed during the first 4 months or the last 5 months after planting then the cassava plant in the randomized cell will be removed and the state of the randomized cell will change from I to E.

(b) If the randomized cell (i, j) is surveyed during the 5th month and the 7th month of planting and the number of cassava mealybugs on the cassava plant in the cell is greater than m_1, then cassava plant in the randomized cell will be removed and the state of the randomized cell will change from I to E.

2.1.3 Rule for updating an empty cell (E)

If the randomized cell (i, j) is an empty cell then it remains an empty cell.

2.2 The release of *Anagyrus lopezi* and green lacewings

Starting on the 2nd month of cassava's planting the surveyed for cassava mealybugs will be conducted every two weeks.

(i) During the first 4 months and the last 5 months of cassava's planting period: if the infested plant is surveyed, it will be removed from the cassava field.

(ii) During the 5th and the 7th month of cassava's planting period: if the infested plant is surveyed, *Anagyrus lopezi* and green lacewings will be released in the cassava field in the following manners

Anagyrus lopezi: *Anagyrus lopezi* of the adult stage (the only stage that feed on/parasitism cassava mealybugs) will be released only once or every 3 weeks for 3 times. The number of *Anagyrus lopezi* to be released in the field is assumed to be R_3 pairs per rai (or $\frac{R_3}{0.16}$ pairs per ha).

Green lacewings: Green lacewings of the larva stage (the only stage that feed on cassava mealybugs) will be released every 2 weeks, every month or every two months until there is no cassava mealybugs found on the surveyed cassava plant. The number of larva green lacewings to be released in the cassava field depends on the severity of the spread of cassava mealybugs. If the number of surveyed infested cassava plants is less than a half of the total number of surveyed cassava plants in the field, the number of green lacewings to be released in the field is R_1 per rai (or $\frac{R_1}{0.16}$ per ha) or else the number of green lacewings to be released in the field is R_2 per rai (or $\frac{R_2}{0.16}$ per ha) where $0 < R_1 < R_2$.

2.3 Effects of life cycles of cassava mealybugs, *Anagyrus lopezi* and green lacewings

In this study, we also monitor the number of cassava mealybugs, *Anagyrus lopezi* and green lacewings at each stage on each cell of the lattice.

Apart from the wind effects the number of cassava mealybugs on each cassava plant is also subjected to change according to its life cycle. Similarly, the life cycles of *Anagyrus lopezi* and green lacewings will also effects the numbers of *Anagyrus lopezi* and green lacewings, respectively, on each cassava plant in the field apart from the numbers of *Anagyrus lopezi* and green lacewings released in the field to control the spread of cassava mealybugs. Hence, we also assume that the following systems of difference equations represent the change in the numbers of cassava mealybugs, *Anagyrus lopezi* and green lacewings at each stage on each cell (i, j) of the lattice

according to their life cycles.
Cassava mealybug: The life-cycle of cassava mealybugs consists of three stages which are eggs, instar and adult stages.
Instar Stage:

$$C^i_{t+\Delta t} = C^i_t + r_1\alpha_1 C^e_t - \alpha_2 C^i_t - \beta_{11}(C^i_t, A^m_t, G^i_t)A^m_t - \beta_{12}(C^i_t, A^m_t, G^i_t)G^i_t \qquad (1)$$

Equation (1) represents the number of instar cassava mealybugs at the time step $t + \Delta t$. The first term on the right hand side represents the number of instar cassava mealybugs at the time step t while the second term represents the number of instar cassava mealybugs developed from cassava mealybug's egg of the time step t. The third term on the right hand side represents the number of instar cassava mealybugs of the time step t that develop into adult cassava mealybugs in the time step $t + \Delta t$. The fourth term on the right hand side represents the number of instar cassava mealybugs killed by *Anagyrus Lopezi* of the adult stage in the time step t. The last term on the right hand side represents the number of instar cassava mealybugs eaten by green lacewings of the larva stage in the time step t.
Adult Stage:

$$C^m_{t+\Delta t} = C^m_t + r_2\alpha_2 C^i_t - \alpha_3 C^m_t - \beta_{21}(C^m_t, A^m_t, G^i_t)A^m_t - \beta_{22}(C^m_t, A^m_t, G^i_t)G^i_t \qquad (2)$$

Equation (2) represents the number of adult cassava mealybugs at the time step $t+\Delta t$. The first term on right hand side represents the number of adult cassava mealybugs at the time step t. The second term on the right hand side represents the number of adult cassava mealybugs developed from instar cassava mealybug of the time step t. The third term on the right hand side represents the number of adult cassava mealybugs of the time step t that die in the time step $t+1$ due to cassava mealybug's life cycle. The fourth term on the right hand side represents the number of adult cassava mealybugs killed by *Anagyrus Lopezi* of the adult stage in the time step t. The last term on the right hand side represents the number of adult cassava mealybugs eaten by green lacewings of the larva stage in the time step t.
Egg Stage:

$$C^e_{t+\Delta t} = C^e_t + r_3\alpha_4 v_1 C^m_t - \alpha_1 C^e_t - \beta_{31}(C^e_t, A^m_t, G^i_t)A^m_t - \beta_{32}(C^e_t, A^m_t, G^i_t)G^i_t \qquad (3)$$

Equation (3) represents the number of cassava mealybug's eggs at the time step $t+\Delta t$. The first term on the right hand side represents the number of cassava mealybug's eggs at the time step t. The second term on the right hand side represents the number of cassava mealybug's eggs laid by adult cassava mealybugs of the time step t. The third term on the right hand side represents the number of cassava mealybug's eggs in the time step t that develop into instar cassava mealybugs in the time step $t + \Delta t$. The fourth term on the right hand side represents the number of cassava mealybug's eggs killed by *Anagyrus Lopezi* of the adult stage in the time step t. The last term on the right hand side represents the number of cassava mealybug's eggs eaten by green lacewings of the larva stage in the time step t.

Anagyrus lopezi: The life-cycle of *Anagyrus lopezi* consists of four stages which are eggs, larva, pupa and adult stages.

Larva stage:

$$A^i_{t+\Delta t} = A^i_t + s_{11}\gamma_{11}A^e_t - \gamma_{21}A^i_t \tag{4}$$

Equation (4) represents the number of larva *Anagyrus lopezi* at the time step $t + \Delta t$. The first term on the right hand side represents the number of larva *Anagyrus lopezi* at the time step t. The second term on the right hand side represents the number of *Anagyrus lopezi* developed from *Anagyrus lopezi*'s eggs of the time step t. The last term on the right hand side represents the number of *Anagyrus lopezi* in the time step t that develop into pupa *Anagyrus lopezi* in the time step $t + \Delta t$.

Pupa stage:

$$A^d_{t+\Delta t} = A^d_t + s_{21}\gamma_{21}A^i_t - \gamma_{31}A^d_t \tag{5}$$

Equation (5) represents the number of pupa *Anagyrus lopezi* at the time step $t + \Delta t$. The first term on the right hand side represents the number of pupa *Anagyrus lopezi* at the time step t. The second term on the right hand side represents the number of pupa *Anagyrus lopezi* developed from larva *Anagyrus lopezi* of the time step t. The last term on the right hand side represents the number of pupa *Anagyrus lopezi* in the time step t that develop into adult *Anagyrus lopezi* in the time step $t + \Delta t$.

Adult stage:

$$A^m_{t+\Delta t} = A^m_t + s_{31}\gamma_{31}A^d_t - \delta_{11}A^m_t \tag{6}$$

Equation (6) represents the number of adult *Anagyrus lopezi* at the time step $t + \Delta t$. The first term on the right hand side represents the number of adult *Anagyrus lopezi* at the time step t. The second term on the right hand side represents the number of adult *Anagyrus lopezi* developed from pupa *Anagyrus lopezi* of the time step t. The last term on the right hand side represents the number of adult *Anagyrus lopezi* of the time step t that that die in the time step $t + \Delta t$ due to *Anagyrus lopezi*'s life cycle.

Egg stage:

$$A^e_{t+\Delta t} = A^e_t + s_{41}\delta_{21}(C^i_t, C^m_t, C^e_t, A^m_t)A^m_t - \gamma_{11}A^e_t \tag{7}$$

Equation (7) represents the number of *Anagyrus lopezi*'s eggs at the time step $t + \Delta t$. The first term on the right hand side represents the number of *Anagyrus lopezi*'s eggs at the time step t. The second term on the right hand side represents the number of *Anagyrus lopezi*'s eggs laid by adult *Anagyrus lopezi* of the time step t depending on the number of consumed cassava mealybugs. The last term on the right hand side represents the number of *Anagyrus lopezi*'s eggs in the time step t that develop into larva *Anagyrus lopezi* in the time step $t + \Delta t$.

Green lacewings: The life-cycle of *Anagyrus lopezi* consists of four stages which are eggs, larva, pupa and adult stages.

Larva stage:

$$G^i_{t+\Delta t} = G^i_t + s_{12}\gamma_{12}G^e_t - \gamma_{22}G^i_t \tag{8}$$

Equation (8) represents the number of larva green lacewings at the time step $t+\Delta t$. The first term on the right hand side represents the number of larva green lacewings at the time step t. The second term on the right hand side represents the number of larva green lacewings developed from green lacewing's eggs of the time step t. The last term on the right hand side represents the number of larva green lacewings in the time step t that develop into pupa green lacewings in the time step $t+\Delta t$.

Pupa stage:

$$G^d_{t+\Delta t} = G^d_t + s_{22}\gamma_{22}\delta_{12}(C^i_t, C^m_t, C^e_t, G^i_t)G^i_t - \gamma_{32}G^d_t \tag{9}$$

Equation (9) represents the number of pupa green lacewings at the time step $t+\Delta t$. The first term on the right hand side represents the number of pupa green lacewings at the time step t. The second term on the right hand side represents the number of pupa green lacewings developed from larva green lacewings of the time step t depending on the number of consumed mealybugs. The last term on the right hand side represents the number of pupa green lacewings in the time step t that develop into adult green lacewings in the time step $t+\Delta t$.

Adult stage:

$$G^m_{t+\Delta t} = G^m_t + s_{32}\gamma_{32}G^d_t - \delta_{22}G^m_t \tag{10}$$

Equation (10) represents the number of adult green lacewings at the time step $t+\Delta t$. The first term on the right hand side represents the number of adult green lacewings at the time step t. The second term on the right hand side represents the number of adult green lacewings developed from pupa green lacewings of the time step t. The last term on the right hand side represents the number of adult green lacewings of the time step t that that die in the time step $t+\Delta t$ due to green lacewing's life cycle.

Egg stage:

$$G^e_{t+\Delta t} = G^e_t + s_{42}\upsilon_{22}G^m_t - \gamma_{12}G^e_t \tag{11}$$

Equation (11) represents the number of green lacewing's eggs at the time step $t+\Delta t$. The first term on the right hand side represents the number of green lacewing's eggs at the time step t. The second term on the right hand side represents the number of green lacewing's eggs laid by adult green lacewings of the time step t. The last term on the right hand side represents the number of green lacewing's eggs in the time step t that develop into larva green lacewings in the time step $t+\Delta t$.
where $\beta_{11}\left(C^i_t, A^m_t, G^i_t\right)$, $\beta_{21}\left(C^m_t, A^m_t, G^i_t\right)$ and $\beta_{31}\left(C^e_t, A^m_t, G^i_t\right)$ are the average numbers of instar cassava mealybugs, adult cassava mealybugs and cassava mealybug's eggs, respectively, killed by an adult *Anagyrus lopezi* per a time step. $\beta_{12}\left(C^i_t, A^m_t, G^i_t\right)$, $\beta_{22}\left(C^m_t, A^m_t, G^i_t\right)$ and $\beta_{32}\left(C^e_t, A^m_t, G^i_t\right)$ are the average numbers of instar cassava mealybugs, adult cassava mealybugs and cassava mealybug's eggs, respectively, eaten by an instar green lacewings per a time step. $\delta_{12}\left(C^i_t, C^m_t, C^e_t, G^i_t\right)$ is the efficiency of converting larva green lacewings to pupa green lacewings depending on the number of consumed cassava mealybugs. $\delta_{21}\left(C^i_t, C^m_t, C^e_t, A^m_t\right)$ is the efficiency of laying eggs of an adult female *Anagyrus lopezi* per a time step depending on the number of consumed cassava mealybugs. The definitions of other parameters in the model are given in Table 1 together with their approximated values calculated from literatures (Chong JH, Roda AL, Mannion CM 2008)-(Souissi R, Rü BL 1997) at $25 \pm 2°C$.

Table 1. Parameters in the system of difference equations (1)-(11)

Parameter	Definition	Value
• Cassava Mealybug		
α_1	the fraction of cassava mealybug's eggs that develop into instar cassava mealybugs in one time step	0.1299
α_2	the fraction of instar cassava mealybugs that develop into adult cassava mealybugs in one time step	0.0571
α_3	the natural death rate of adult cassava mealybugs	0.0483
α_4	the fraction of survived female adult cassava mealybugs in the reproductive period	0.5334
r_1	the survival rate of cassava mealybug's eggs that develop into instar cassava mealybugs	0.9575
r_2	the survival rate of instar cassava mealybugs that develop into adult cassava mealybugs	0.9666
r_3	the fraction of female adult cassava mealybugs	1.0000
v_1	the average number of eggs laid by a female adult cassava mealybug in one time step	16.9250
• *Anagyrus lopezi*		
γ_{11}	the fraction of *Anagyrus lopezi*'s eggs that develop into larva *Anagyrus lopezi* in one time step	0.5000
γ_{21}	the fraction of larva *Anagyrus lopezi* that develop into pupa *Anagyrus lopezi* in one time step	0.1000
γ_{31}	the fraction of pupa *Anagyrus lopezi* that develop into adult *Anagyrus lopezi* in one time step	0.1667
δ_{11}	the natural death rate of adult *Anagyrus lopezi* per a time step	0.0855
s_{11}	the survival rate of *Anagyrus lopezi*'s eggs that develop into larva *Anagyrus lopezi*	0.9000
s_{21}	the survival rate of larva *Anagyrus lopezi* that develop into pupa *Anagyrus lopezi*	0.9000
s_{31}	the survival rate of pupa *Anagyrus lopezi* that develop into adult *Anagyrus lopezi*	0.9000
s_{41}	the fraction of female adult *Anagyrus lopezi*	0.5290
• Green lacewings		
γ_{12}	the fraction of green lacewing's eggs that develop into larva green lacewing in one time step	0.1639
γ_{22}	the fraction of larva green lacewings that develop into pupa green lacewing in one time step	0.0521
γ_{32}	the fraction of pupa green lacewings that develop into adult green lacewings in one time step	0.0714
δ_{22}	the natural death rate of adult green lacewings	0.0227
s_{12}	the survival rate of green lacewing's eggs that develop into larva green lacewing	0.8040
s_{22}	the survival rate of larva green lacewing that develop into pupa green lacewing	0.9191
s_{32}	the survival rate of pupa green lacewing that develop into adult green lacewings	0.9586
s_{42}	the fraction of female adult green lacewings	0.5400
v_2	the average number of eggs laid by a female adult green lacewings in one time step	7.2789

2.4 Estimate cassava's crop yields

The estimated total crop yields at the end of cassava planting period is also monitored. Assuming that the estimated crop yield is a kilograms per a cassava plant if the plant has never been infested with cassava mealybugs longer than 2 weeks during the planting period, whereas the estimated crop yields will be reduced by 100%, 30% and 10% if the cassava plant was infested with cassava mealybugs longer than 2 weeks during the first 4 months of planting, during the period between the 5th and the 7th months of planting, and during the period between the 8th and the 12th months of planting, respectively, the estimated cassava's crop yield at each time step will be calculated using the following equation:

$$Y(t) = a \cdot Y_1 + (0.9 \times a) \cdot Y_2 + (0.7 \times a) \cdot Y_3 \tag{12}$$

where Y_1 is the the total number of cassava plants that have never been infested with cassava mealybugs at the time step t, Y_2 is the total number of cassava plants that have been infested with cassava mealybugs longer than 2 weeks during the period between the 8th and the 12th months of planting at the time step t and Y_3 is the total number of cassava plants that have been infested with cassava mealybugs longer than 2 weeks during the period between the 5th and the 7th months of planting at the time step t.

3. Numerical Simulations

In the simulations, the lattice is of the size 80×80 that is the area of cassava's planting is 4 rai (0.64 ha) while the distant between two cassava plants is 1 metre and hence, the initial number of cassava plants in the field is 6,400. The planting period is one year. The total of 54 methods of biological controls with *Anagyrus lopezi* and green lacewings are investigated. The number of cassava mealybugs, the number of *Anagyrus Lopezi* and the number of green lacewings at each stage on each cassava plant are also monitored. Here, *Anagyrus Lopezi* and green lacewings of the adult stage on an infested cassava plant might fly to another infested cassava plant in their immediate, distant or far distant neighbourhood. Cassava mealybugs of the instar stage on an infested cassava plant might be blown by the wind to a cassava plant(infested or non-infested) in its immediate, distant or far distant neighbourhood as well.

Computer simulations of 54 cases are carried out using MATLAB software. The 54 cases are as listed in Figure 2. The parameters in the system of difference equations (1)-(11) are as in Table 1 where $n_1 = 0.005$, $n_2 = 0.0005$, $n_3 = 0.00005$, $w_1 = 0.0001$, $w_2 = 0.00001$ and $a = 2.25$. The results are as shown in Figures 2-5. The average estimated cassava's crop yield at the end of planting period and the average total number of *Anagyrus lopezi* and green lacewings released in the field are also given in Figure 6 and 7, respectively.

Figures 2-5 indicate that the method 53 (Releasing 800-1,000 green lacewings per rai every 2 months together with 400 pairs *Anagyrus lopezi* per rai every three weeks for three times after cassava mealybugs were first detected) gives the maximum number of susceptible cassava plants with the minimum numbers of infested cassava plant and removed cassava plants whereas the method 4 (Releasing 200 green lacewings per rai every month together with releasing 50-100 pairs of *Anagyrus lopezi* per rai once after cassava mealybugs were first detected) gives the minimum number of susceptible cassava plants with the maximum numbers of infested cassava plant and removed cassava plants. In addition, the estimated cassava's crop yield summarized in Figure

6 shows that the method 53 gives the maximum estimated cassava's crop yield at the end of planting period whereas the method 4 gives the minimum estimated cassava's crop yield. Hence, in order to maximized estimated cassava's crop yield at the end of planting period, farmers should apply method 53 for a biological control.

4. Discussion and conclusion

Cellular automata model and Monte Carlo simulation are utilized in this paper to investigate the 54 manners of biological control of the spread of cassava mealybugs in a cassava field with the planting area of 4 rai (0.64 ha) where *Anagyrus lopezi* and green lacewings are used as biological control agents. The results indicate that the method 53 gives the maximized estimated cassava's crop yield at the end of planting period and should be the most efficient biological control of the spread of cassava mealybugs. However, the numbers of *Anagyrus lopezi* and green lacewings released in Method 53 is quite high (4,800 pairs of *Anagyrus lopezi* and 12,800 green lacewings) whereas the selling price of cassava is approximately 2.50 baht (0.072 USD) per kilogram. Figures 8, 9 and 10 showing the average estimated cost of *Anagyrus lopezi* and green lacewings released in the field, the average estimated income from selling cassava's crop yields at the end of planting period and the average estimated (income – cost of biological control agents) at the end of planting period, respectively, are then given here to compare the profit for the 54 biological control methods. We can see that although the method 53 gives the maximum estimated cassava's crop yield at the end of planting period, the cost for biological control is overcome the income from selling cassava's crop yield. On the other hand, the method 4 gives the maximum profit even though it gives the minimum estimated cassava's crop yield at the end of planting period. Therefore, to maximize the estimated cassava's crop yield at the end of planting period, the method 53 (Releasing 800-1,000 green lacewings per rai every 2 months together with 400 pairs of *Anagyrus lopezi* per rai every three weeks for three times after cassava mealybugs were first detected) is the most efficient biological control and to maximize profit, the method 4 (Releasing 200 green lacewings per rai every month together with releasing 50-100 pairs of *Anagyrus lopezi* per rai once after cassava mealybugs were first detected) is the most efficient control.

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Figures

(i-3,j-3)	(i-3,j-2)	(i-3,j-1)	(i-3,j)	(i-3,j+1)	(i-3,j+2)	(i-3,j+3)
(i-2,j-3)	(i-2,j-2)	(i-2,j-1)	(i-2,j)	(i-2,j+1)	(i-2,j+2)	(i-2,j+3)
(i-1,j-3)	(i-1,j-2)	(i-1,j-1)	(i-1,j)	(i-1,j+1)	(i-1,j+2)	(i-1,j+3)
(i,j-3)	(i,j-2)	(i,j-1)	(i,j)	(i,j+1)	(i,j+2)	(i,j+3)
(i+1,j-3)	(i+1,j-2)	(i+1,j-1)	(i+1,j)	(i+1,j+1)	(i+1,j+2)	(i+1,j+3)
(i+2,j-3)	(i+2,j-2)	(i+2,j-1)	(i+2,j)	(i+2,j+1)	(i+2,j+2)	(i+2,j+3)
(i+3,j-3)	(i+3,j-2)	(i+3,j-1)	(i+3,j)	(i+3,j+1)	(i+3,j+2)	(i+3,j+3)

Figure 1. Neighborhoods of the updating cell (i, j). The blue, yellow and green areas represent immediate neighborhood, distant neighborhood and far distant neighborhood, respectively.

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 - 1,000 per rai	2,000 per rai	200 per rai	800 - 1,000 per rai	2,000 per rai	200 per rai	800 - 1,000 per rai	2,000 per rai
Release AL only 1 time	50-100 pairs per rai	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6	Method 7	Method 8	Method 9
	200 pairs per rai	Method 10	Method 11	Method 12	Method 13	Method 14	Method 15	Method 16	Method 17	Method 18
	400 pairs per rai	Method 19	Method 20	Method 21	Method 22	Method 23	Method 24	Method 25	Method 26	Method 27
Release AL 3 times every 3 weeks	50-100 pairs per rai	Method 28	Method 29	Method 30	Method 31	Method 32	Method 33	Method 34	Method 35	Method 36
	200 pairs per rai	Method 37	Method 38	Method 39	Method 40	Method 41	Method 42	Method 43	Method 44	Method 45
	400 pairs per rai	Method 46	Method 47	Method 48	Method 49	Method 50	Method 51	Method 52	Method 53	Method 54

Figure 2. Methods of biological controls with *Anagyrus lopezi* and green lacewings.

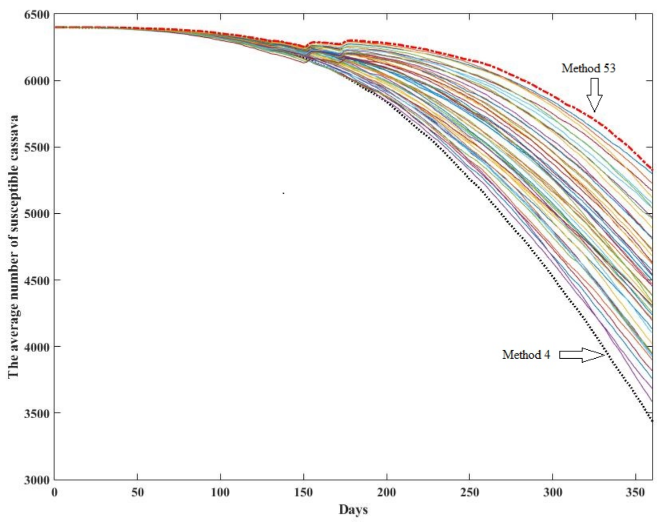

Figure 3. The average number of susceptible cassava at $25 \pm 2°C$ when *Anagyrus Lopezi* and green lacewings are released in the cassava field with the 54 different manners.

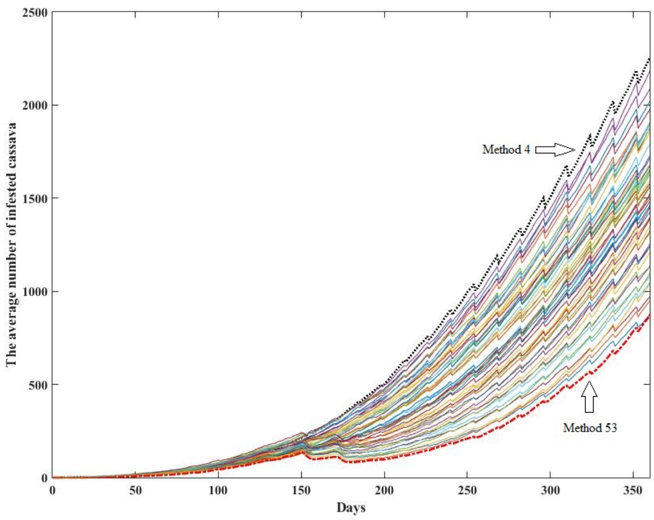

Figure 4. The average number of infested cassava at $25 \pm 2°C$ when *Anagyrus Lopezi* and green lacewings are released in the cassava field with the 54 different manners.

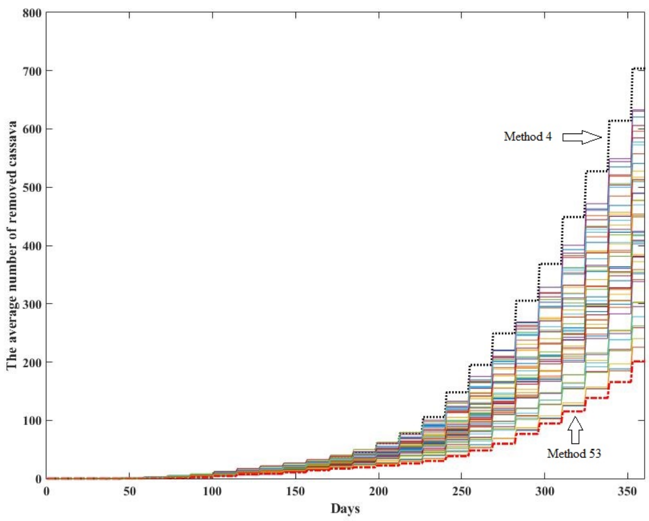

Figure 5. The average number of removed cassava at $25 \pm 2°C$ when *Anagyrus Lopezi* and green lacewings are released in the cassava field with the 54 different manners.

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	12,585.32	12,869.36	12,687.20	12,096.44	12,753.40	12,818.47	12,466.70	12,325.67	12,400.82
	200 pairs per rai	12,645.89	12,274.28	12,798.67	12,684.55	12,353.08	12,784.36	12,531.10	12,616.24	12,743.27
	400 pairs per rai	12,883.40	13,001.48	12,685.94	12,985.06	12,899.29	12,832.78	12,283.33	12,448.12	12,491.41
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	13,093.96	13,013.14	12,963.50	13,130.09	12,946.81	13,173.83	12,451.63	12,995.18	13,002.16
	200 pairs per rai	13,193.32	13,061.92	13,179.23	12,879.94	13,393.21	13,479.34	13,166.41	13,127.44	13,308.79
	400 pairs per rai	13,312.70	13,283.81	13,370.84	13,024.61	13,570.01	13,565.65	13,522.94	13,646.20	13,425.16

Figure 6. The average estimated cassava's crop yield at the end of planting period when *Anagyrus lopezi* and green lacewings are used as biological control agents (baht).

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	AL = 200 pairs GL = 10,400	AL = 200 pairs GL = 44,800	AL = 200 pairs GL = 120,000	AL = 200 pairs GL = 6,400	AL = 200 pairs GL = 25,600	AL = 200 pairs GL = 64,000	AL = 200 pairs GL = 3,200	AL = 200 pairs GL = 12,800	AL = 200 pairs GL = 32,000
	200 pairs per rai	AL = 800 pairs GL = 11,200	AL = 800 pairs GL = 35,200	AL = 800 pairs GL = 112,000	AL = 800 pairs GL = 6,400	AL = 800 pairs GL = 22,400	AL = 800 pairs GL = 48,000	AL = 800 pairs GL = 3,200	AL = 800 pairs GL = 12,800	AL = 800 pairs GL = 32,000
	400 pairs per rai	AL = 1,600 pairs GL = 11,200	AL = 1,600 pairs GL = 44,800	AL = 1,600 pairs GL = 96,000	AL = 1,600 pairs GL = 6,400	AL = 1,600 pairs GL = 16,000	AL = 1,600 pairs GL = 64,000	AL = 1,600 pairs GL = 3,200	AL = 1,600 pairs GL = 12,800	AL = 1,600 pairs GL = 32,000
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	AL = 600 pairs GL = 10,400	AL = 600 pairs GL = 51,200	AL = 600 pairs GL = 128,000	AL = 600 pairs GL = 6,400	AL = 600 pairs GL = 25,600	AL = 600 pairs GL = 48,000	AL = 600 pairs GL = 3,200	AL = 600 pairs GL = 12,800	AL = 600 pairs GL = 24,000
	200 pairs per rai	AL = 2,400 pairs GL = 9,600	AL = 2,400 pairs GL = 44,800	AL = 2,400 pairs GL = 80,000	AL = 2,400 pairs GL = 5,600	AL = 2,400 pairs GL = 25,600	AL = 2,400 pairs GL = 64,000	AL = 2,400 pairs GL = 3,200	AL = 2,400 pairs GL = 9,600	AL = 2,400 pairs GL = 32,000
	400 pairs per rai	AL = 4,800 pairs GL = 8,800	AL = 4,800 pairs GL = 25,600	AL = 4,800 pairs GL = 96,000	AL = 4,800 pairs GL = 6,400	AL = 4,800 pairs GL = 25,600	AL = 4,800 pairs GL = 56,000	AL = 4,800 pairs GL = 3,200	AL = 4,800 pairs GL = 12,800	AL = 4,800 pairs GL = 24,000

Figure 7. The average total number of *Anagyrus lopezi* and green lacewings released in the field.

Anagyrus Lopezi (AL) \ Green Lacewings (GL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	6,100.00	23,300.00	60,900.00	4,100.00	13,700.00	32,900.00	2,500.00	7,300.00	16,900.00
	200 pairs per rai	9,200.00	21,200.00	59,600.00	6,800.00	14,800.00	27,600.00	14,800.00	58,000.00	144,400.00
	400 pairs per rai	12,800.00	29,600.00	55,200.00	10,400.00	15,200.00	39,200.00	5,200.00	10,000.00	19,600.00
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	7,900.00	28,300.00	66,700.00	5,900.00	15,500.00	26,700.00	15,200.00	58,400.00	144,800.00
	200 pairs per rai	15,600.00	33,200.00	50,800.00	13,600.00	23,600.00	42,800.00	8,800.00	13,600.00	23,200.00
	400 pairs per rai	26,000.00	34,400.00	69,600.00	24,800.00	34,400.00	49,600.00	14,700.00	57,900.00	144,300.00

Figure 8. The average estimated cost of *Anagyrus lopezi* and green lacewings released in the field (baht).

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	35,404.33	35,452.70	34,804.13	33,917.08	35,100.00	34,748.45	31,860.00	35,016.20	34,375.50
	200 pairs per rai	35,528.63	35,620.88	35,337.95	33,837.75	35,191.70	35,240.63	32,646.95	35,233.88	35,458.33
	400 pairs per rai	35,390.25	35,334.00	35,395.33	35,346.95	35,155.13	35,182.13	33,777.63	32,940.63	34,855.33
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	35,502.20	35,319.38	35,286.20	35,185.50	35,235.58	35,380.70	31,642.33	34,986.95	35,623.70
	200 pairs per rai	35,546.63	35,275.50	35,521.88	35,367.75	35,397.58	35,047.70	34,851.38	34,891.88	35,058.95
	400 pairs per rai	35,644.50	35,669.83	35,542.13	35,398.13	35,210.83	34,998.20	34,740.00	34,939.70	35,291.25

Figure 9. The average estimated income from selling cassava's crop yields at the end of planting period (baht).

Green Lacewings (GL) / *Anagyrus Lopezi* (AL)		Release GL every 2 weeks			Release GL every month			Release GL every 2 months		
		200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai	200 per rai	800 or 1,000 per rai	2,000 per rai
Release AL only 1 time	50 or 100 pairs per rai	29,304.33	12,152.70	-26,095.88	29,817.08	21,400.00	1,848.45	29,360.00	27,716.20	17,475.50
	200 pairs per rai	26,328.63	14,420.88	-24,262.05	27,037.75	20,391.70	7,640.63	17,846.95	-22,766.13	-108,941.68
	400 pairs per rai	22,590.25	5,734.00	-19,804.68	24,946.95	19,955.13	-4,017.88	28,577.63	22,940.63	15,255.33
Release AL 3 times every 3 weeks	50 or 100 pairs per rai	27,602.20	7,019.38	-31,413.80	29,285.50	19,735.58	8,680.70	16,442.33	-23,413.05	-109,176.30
	200 pairs per rai	19,946.63	2,075.50	-15,278.13	21,767.75	11,797.58	-7,752.30	26,051.38	21,291.88	11,858.95
	400 pairs per rai	9,644.50	1,269.83	-34,057.88	10,598.13	810.82	-14,601.80	20,040.00	-22,960.30	-109,008.75

Figure 10. The average estimated (income – cost of biological control agents) at the end of planting period (baht).