



รายงานวิจัยฉบับสมบูรณ์

โครงการ เกจซุเปอร์กราวิตี้จากทฤษฎีสตริงและทฤษฎีเอ็มกับหลักไฮโลกราฟิค AdS/CFT

โดย รองศาสตราจารย์ ดร. ปริญญา การดำริห์

มีนาคม 2562

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รองศาสตราจารย์ ดร. ปริญญ์ การดำรง จุฬาลงกรณ์มหาวิทยาลัย

สนับสนุนโดยสำนักงานกองทุนสนับสนุนการวิจัย

และจุฬาลงกรณ์มหาวิทยาลัย

(ความเห็นในรายงานนี้เป็นของผู้วิจัยสกว. และจุฬาลงกรณ์มหาวิทยาลัยไม่จำเป็นต้องเห็นด้วยเสมอไป)

กิตติกรรมประกาศ

ตลอดระยะเวลา 3 ปีของการทำงานวิจัยในโครงการนี้ หัวหน้าโครงการวิจัยขอแสดงความขอบคุณสำนักงานกองทุนสนับสนุนการวิจัย (สกว.) และจุฬาลงกรณ์มหาวิทยาลัยที่ให้การสนับสนุนงานวิจัยมาตลอดระยะเวลาในการทำวิจัย ขอขอบคุณ Professor Jan Louis, University of Hamburg ที่ให้โอกาสหัวหน้าโครงการไปร่วมทำวิจัยด้วย ณ ประเทศเยอรมนี และขอขอบคุณ International Centre for Theoretical Physics (ICTP) ที่มอบทุนให้ไปทำงานวิจัย ณ ประเทศอิตาลี นอกจากนี้ หัวหน้าโครงการขอขอบคุณนายเชม อุปถัมภ์กุล นายภัทรดนัย นุชจิโน และ Ms. H. L. Dao ผู้ร่วมงานขอบคุณคณะกรรมการบริหารภาควิชาฟิสิกส์และคณะวิทยาศาสตร์สำหรับบรรยากาศการทำงานที่เอื้อต่อการจัดสรรเวลาในการทำงานวิจัยรวมทั้งเจ้าหน้าที่ของสกว. ทุกท่านทุกฝ่ายที่ช่วยให้การบริหารและการดำเนินงานต่างๆ ของโครงการวิจัยเป็นไปด้วยความเรียบร้อย และสุดท้ายนี้ โครงการวิจัยนี้คงไม่สามารถประสบความสำเร็จและเกิดผลลัพธ์ดังปรากฏอยู่ในรายงานฉบับนี้ได้หากไม่ได้รับการสนับสนุน กำลังใจ และความช่วยเหลือจากครอบครัวของหัวหน้าโครงการทั้งภรรยา ลูกชายตัวน้อยและบิดามารดาที่คอยรับฟังและให้การสนับสนุนตลอดเวลาที่งานวิจัยประสบปัญหาติดขัดหรือเกิดข้อผิดพลาด รวมไปถึงผู้มีส่วนเกี่ยวข้องอีกจำนวนมากที่ไม่สามารถระบุชื่อได้หมดในที่นี้ หัวหน้าโครงการวิจัยขอขอบคุณไว้ ณ ที่นี้โดยทั่วกัน

บทคัดย่อ

รหัสโครงการ: **RSA5980037**

ชื่อโครงการ: เกจชูเปอร์กราวิตี้จากทฤษฎีสตริงและทฤษฎีเอ็มกับหลักโฮโลกราฟี AdS/CFT

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ระยะเวลาโครงการ: 3 ปี

โครงการวิจัยนี้เป็นการศึกษาคำตอบโฮโลกราฟีภายในขอบเขตของทฤษฎีเกจชูเปอร์กราวิตี้ในกาลอวกาศสี่ ห้า หก และเจ็ดมิติ โดยเกจชูเปอร์กราวิตี้ที่พิจารณาในโครงการนี้คือทฤษฎีเกจชูเปอร์กราวิตี้สี่มิติที่มีจำนวนซูเปอร์ซิมเมทรี $N=3,4$ และเกจชูเปอร์กราวิตี้ที่คู่ควบกับสนามสสารในห้า หก และเจ็ดมิติ โดยมีซูเปอร์ซิมเมทรีครึ่งหนึ่งของค่าสูงสุด ในสี่มิติ นอกจากค้นพบสัญญาณเอกแอนติ-เดอ ซิตเตอร์ (AdS) แบบใหม่แล้ว ยังค้นพบคำตอบของการโฟลว์ RG โฮโลกราฟีและคำตอบเงินสุญญากาศใหม่อีกด้วย คำตอบที่ได้บางแบบมีต้นกำเนิดในมิติที่สูงกว่าจึงสามารถยกระดับขึ้นไปยังทฤษฎีสตริงและทฤษฎีเอ็มได้ ผลลัพธ์ที่ได้จะช่วยให้เกิดความเข้าใจพลวัตของทฤษฎีสตริงซูเปอร์คอนฟอร์มมอล (SCFTs) ในสามมิติในลิมิตที่มีการคู่ควบรุนแรงได้ ทฤษฎีสตริงเหล่านี้อธิบายพลวัตที่พลังงานต่ำของ M2-เบรนซึ่งเป็นองค์ประกอบพื้นฐานของทฤษฎีเอ็มและมีบทบาทสำคัญต่อการศึกษาหลุมดำเชิงจุลภาคภายในอวกาศ AdS_4 ที่เป็นหัวข้อวิจัยหลักในปัจจุบัน คำตอบรูปแบบนี้รวมทั้งคำตอบที่อธิบายหลุมดำสถิตและสตริงในอวกาศ AdS_5 ก็ได้ทำการศึกษาโดยใช้เกจชูเปอร์กราวิตี้ $N=4$ ในห้ามิติ ส่วนในหกมิติ ได้ทำการศึกษาเงื่อนไขของการมีสัญญาณ AdS₆ ที่รักษาซูเปอร์ซิมเมทรีและยังแสดงให้เห็นว่าสัญญาณดังกล่าวไม่มีสนามมอดูโลที่ไม่ทำลายซูเปอร์ซิมเมทรี และในเจ็ดมิติ ได้ศึกษาคำตอบโดเมนวอลล์ที่มีสนามสาม-ฟอร์มเพิ่มขึ้นมา คำตอบที่ได้อธิบายความบกพร่องคอนฟอร์มมอลสองมิติภายในทฤษฎี $N=(1,0)$ SCFTs ในหกมิติ รวมทั้งคำตอบที่ได้บางแบบยังสามารถยกระดับขึ้นไปยังทฤษฎีเอ็มได้อีกด้วย

คำหลัก: **Gauged supergravity, String/M-theory, AdS/CFT correspondence, Gauge/gravity correspondence**

Abstract

Project Code: RSA5980037

Project Title: Gauged Supergravity from String/M-theory and AdS/CFT holography

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Project Period: 3 years

The project concerns with the studies of holographic solutions within the framework of gauged supergravities in four, five, six and seven dimensions. The gauged supergravities under consideration are four-dimensional theories with $N=3,4$ supersymmetries and matter-coupled half-maximal gauged supergravities in five, six and seven dimensions. In four dimensions, in addition to new supersymmetric anti-de Sitter (AdS) vacuum, novel classes of holographic RG flows and Janus solutions are found. Some of these solutions have known higher-dimensional origins and can accordingly be uplifted to string/M-theory. The uplifted solutions would give more insight to dynamics of the dual superconformal field theories (SCFTs) in three dimensions at strongly coupled limits. These theories describe low energy dynamics of M2-brane, a fundamental ingredient of M-theory, and recently play an important role in microscopic study of black hole in AdS_4 space. Similar solutions including static black holes and strings in AdS_5 space are obtained within $N=4$ gauged supergravity in five dimensions. In six dimensions, general conditions for the existence of supersymmetric AdS_6 vacua are derived. Furthermore, it is shown that these vacua have no moduli preserving all supersymmetry. Finally, in seven dimensions, domain wall solutions with a non-vanishing three-form field are given. These solutions describe surface defects in $N=(1,0)$ SCFTs in six dimensions, and some of them can be uplifted to M-theory.

Keywords: Gauged supergravity, String/M-theory, AdS/CFT correspondence,
Gauge/gravity correspondence

1. Introduction to the research problem and its significance

It is widely accepted that superstring theory, or string theory for short, is the best candidate for a theory of quantum gravity. In the past fifty years, string theory has provided many insightful explanations to a number of unsolved problems in theoretical high energy physics such as black hole entropy and the Big Bang, the origin of our universe. At the beginning, there are five seemingly different superstring theories. But it has been shown that all of the five superstring theories are related to each other and to a unified eleven-dimensional theory, called M-theory, by a number of dualities.

At low energy, an energy scale much lower than the Planck scale at which the strength of gravity is comparable to other interactions, both string and M-theories can be successfully described by supergravity theories. Supergravity is an extension of Einstein general relativity by incorporating supersymmetry, the symmetry of exchanging bosons and fermions. It is presently known that string/M-theory has ten and eleven-dimensional supergravities as effective theories at low energy. Furthermore, in order to obtain a sensible physical theory in four-dimensional space-time, six or seven extra dimensions need to be compactified in a tiny volume. This will result in an effective lower dimensional theory. It has been shown that these effective theories take the form of supergravity and gauged supergravity theories. The latter is a gauged version of the former in the sense that some part of the global symmetry of the supergravity theory is promoted to a local or gauge symmetry. This procedure introduces non-abelian gauge symmetry to a supergravity theory. The resulting gauged theories possess many new features including the possibility of vacua different from Minkowski space-time.

An extremely important consequence of string theory is the so-called AdS/CFT correspondence, a duality relation between string theory on a particular space containing anti-de Sitter (AdS) space and a conformal field theory (CFT) on the boundary of the AdS space. At low energy, this reduces to a correspondence between a supergravity theory and a conformal field theory. This is a kind of strong-weak duality in which a weakly coupled gravity theory is dual to a strongly coupled field theory which is usually a gauge theory and vice versa. The correspondence is a direct consequence from the study of D-brane dynamics. In some sense, this is a correspondence between closed and open strings. Therefore, the AdS/CFT correspondence is expected to give some insight to the strong coupling limit of many interesting

gauge theories and the structure behind string/M-theory. In this respect, AdS spaces in different space-time dimensions are essential. It turns out that many gauged supergravities can accommodate vacua precisely of this type.

The original AdS/CFT correspondence has been extended through a number of different directions. One of these is the extension to non-conformal field theories. This is of particular interest since all of the gauge theories in the standard model of particle physics are not conformally symmetric. The result is called non-AdS/non-CFT or in a more common term gauge/gravity correspondence. Gravity backgrounds which are dual to this type of gauge theories are domain walls. Another interesting class of solutions is Janus solutions dual to interface CFTs which are useful in condensed matter physics as well. Solutions of these types also arise naturally in various types of gauged supergravities in the form of half-maximally supersymmetric vacuum solutions.

All of these facts express clearly that gauged supergravities play a very important role in understanding string/M-theory as well as gauge theories. It turns out that working in lower dimensions is much more convenient. Therefore, many interesting results are firstly obtained in lower dimensional gauged supergravities. To interpret the results in string/M-theory context, an embedding of these results is needed. This can be done by using a consistent reduction ansatz from ten or eleven dimensions to lower dimensions. Constructing the reduction ansatz is normally a highly non-trivial task, and presently only some particular cases of maximal gauged supergravities are known. In the case of half-maximal gauged supergravities, very little is known about their solutions and their embedding in higher dimensions. A reduced number of supersymmetry makes it more difficult to explicitly find the corresponding reduction ansatz since in this case supersymmetry allows for the coupling to an arbitrary number of matter fields.

2. Objectives

1. To study gauge/gravity correspondence within half-maximal gauged supergravities in 4, 5, 6 and 7 dimensions and to identify possible higher dimensional origins within string/M-theory.
2. To apply the results in extracting useful consequences in dual field theories and give some interpretations of the results in terms of brane configurations in string/M-theory.

3. Methodology

The procedure for finding holographic solutions is generally given by the following steps.

1. Analyze the structure of gauged supergravity under consideration.
2. Compute the scalar potential and look for possible AdS critical points.
3. Set up the corresponding BPS equations by setting fermionic supersymmetry transformations to zero. This step in general involves imposing certain projection conditions on the Killing spinors. These conditions in effect reduce the original number of supersymmetry.
4. Solve the resulting BPS equations to obtain the solutions of interest such as holographic RG flows and Janus solutions dual respectively to RG flows and interfaces in the dual field theories.
5. Check whether the BPS solutions satisfy the second-order field equations.
6. If the gauged supergravity has known higher dimensional origin, apply the reduction ansatz to uplift the BPS solutions to string/M-theory.
7. Look for implications of the holographic solutions in the dual SCFT.

4. Outcomes

A large number of new results have been obtained from this project. It is more convenient to report on these results separately in various space-time dimensions.

1. Four-dimensional gauged supergravities:

In N=3 gauged supergravity with $SO(3) \times SU(3)$ gauge group, the first holographic study of this gauged supergravity has been performed in this project. Two AdS_4 critical points preserving the full N=3 supersymmetry have been identified. One of these critical points is the expected trivial critical point with all scalars vanishing and $SO(3) \times SU(3)$ symmetry. The other one is a new non-trivial critical point with only $SO(3) \times U(1)$ symmetry. The full scalar masses and holographic RG flows interpolating between these critical points have been given. These solutions describe RG flows between N=3 three-dimensional SCFTs dual to the aforementioned AdS_4 critical points. A number of supersymmetric $AdS_2 \times \Sigma_2$ solutions have also been found. These describe RG flows across dimensions from N=3 SCFTs in three dimensions to one-dimensional conformal field theory or conformal quantum mechanics. The solutions also have an interpretation in terms of supersymmetric black holes in AdS_4 space.

In $N=4$ gauged supergravity, various gauge groups have been considered. The first gauge group is $SO(3) \times (T^3, T^3)$ arising from a truncation of eleven-dimensional supergravity on a tri-sasakian manifold. The resulting gauged supergravity admits two supersymmetric AdS_4 critical points with $N=3$ and $N=1$ supersymmetries. Holographic RG flows from these critical points to non-conformal field theories in the IR have been studied. In addition, a class of supersymmetric $N=1$ Janus solutions have also been given. All of these solutions can be uplifted to M-theory. The corresponding solutions in eleven dimensions have also been considered. The study has continued to find $AdS_2 \times \Sigma_2$ solutions arising as near horizon geometries of AdS_4 back holes. These provide a new class of $AdS_2 \times \Sigma_2$ solutions with known M-theory origin and should be useful in the context of black hole physics.

For $N=4$ gauged supergravities arising from type II string theories, $ISO(3) \times ISO(3)$ and $ISO(3) \times U(1)^6$ gauge groups have been considered. These gauge groups are obtained respectively from type IIB and type IIA string theories compactified on an orbifold $T^6/Z_2 \times Z_2$. RG flows and Janus solutions preserving different numbers of supersymmetries have been found within these gauged supergravities.

However, solutions in all these non-semisimple gauge groups have a limited structure in a sense that there are only a few supersymmetric AdS_4 critical points. The research project then moved on to consider semisimple gauge groups $SO(4) \times SO(4)$, $SO(4), SO(3,1)$ and $SO(3,1) \times SO(3,1)$. For these gauge groups, there are many supersymmetric $N=4$ AdS_4 critical points with different residual symmetries and various possibilities of RG flows between them. In addition, many of these solutions can be obtained analytically and hence very useful in all interesting applications.

2. Five-dimensional gauged supergravity:

A similar study has been performed in $N=4$ gauged supergravity in five dimensions coupled to five vector multiplets with gauge groups $U(1) \times SU(2) \times SU(2)$, $U(1) \times SO(3,1)$ and $U(1) \times SL(3, R)$. This gauged supergravity was previously less studied in the holographic context. Holographic RG flows and a large class of half-supersymmetric black strings in AdS_5 with $AdS_3 \times \Sigma_2$ horizons have been found. The study has also been extended to more general solutions with $1/4$ -supersymmetry

including static black holes in AdS_5 . All of these solutions have not appeared in previous literatures.

3. Six-dimensional gauged supergravity:

Supersymmetric AdS_6 vacua are rare. The only possibility is AdS_6 vacua preserving sixteen supercharges. These can be found using half-maximal $\mathcal{N}=(1,1)$ gauged supergravity, known as $F(4)$ gauged supergravity. In this research project, general conditions for the existence of supersymmetric AdS_6 vacua with all supersymmetry unbroken have been derived including the general form of gauge groups admitting these AdS_6 critical points. These gauge groups take the form of $G_1 \times G_2$ subgroup of the global symmetry group $\text{SO}(4,n)$ with G_1 and G_2 being subgroups of $\text{SO}(3,m)$ and $\text{SO}(1,m-n)$, respectively. It has also been shown that these AdS_6 vacua do not have moduli, deformations by massless scalars preserving all supersymmetry. These vacua are then isolated points in the scalar field space. This implies that the dual six-dimensional SCFTs do not have marginal deformations preserving all supersymmetry in agreement with the field theory results.

4. Seven-dimensional gauged supergravity:

The last part of the results in this project arises from a study of matter-coupled $\mathcal{N}=2$ gauged supergravity in seven dimensions. The study extends usual domain wall solutions, with only scalar fields non-vanishing, to include the three-form field in the $\mathcal{N}=2$ supergravity multiplet. An analytic solution of this type has been obtained, and for particular values of gauge coupling constants, the solution can be uplifted to eleven-dimensions. This solution takes the form of an AdS_3 -sliced domain wall and describes a conformal surface defect in the dual $\mathcal{N}=(1,0)$ SCFT. It has also been shown that supersymmetric solutions with non-vanishing gauge fields and a three-form field do not exist.

5. Conclusions and discussions

A number of new holographic solutions of various types such as domain walls, RG flows and Janus solutions in different space-time dimensions are discovered in this research project. Many of these solutions have known higher dimensional origins and can be uplifted or embedded in string/M-theory. The results of this project enlarge

known holographic solutions to a large extent. In addition, some gauged supergravities considered in this project have never been considered before namely, $N=3,4$ gauged supergravities in four dimensions and $N=4$ gauged supergravity in five dimensions coupled to vector multiplets.

Although many results have been discovered, there are many research directions to pursue. Examples of these are finding $AdS_2 \times \Sigma_2$ solutions in $N=4$ gauged supergravity in four dimensions with both electric and magnetic vector fields turned on. These will give a new class of supersymmetric dyonic AdS_4 black holes. There are some gauged supergravity theories that have not been investigated in the context of holography such as $N=5,6$ gauged supergravity in four dimensions and $N=6$ gauged supergravity in five dimensions. Maximal gauged supergravity in seven dimensions and various gauged supergravities in three dimensions also deserve further study. Hopefully, all of these issues and the related ones will be addressed in another research project.

6. Output จากโครงการวิจัยที่ได้รับทุนจากสกว.

1. ผลงานตีพิมพ์ในวารสารวิชาการนานาชาติ

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- 1.2 H. L. Dao and Parinya Karndumri, *Holographic RG flows and AdS_5 black strings from 5D half-maximal gauged supergravity*, Eur. Phys. J. **C79** (2019) 137
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- 1.6 Parinya Karndumri and Khem Upathambhakul, *Supersymmetric RG flows and Janus from type II orbifold compactification*, Eur. Phys. J. C (2017) **77**: 445
- 1.7 Parinya Karndumri, *Supersymmetric deformations of 3D SCFTs from tri-Sasakian truncation*, Eur. Phys. J. C (2017) **77**: 130
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2. การนำผลงานวิจัยไปใช้ประโยชน์

ในเชิงวิชาการ มีการนำผลงานวิจัยจากโครงการนี้ไปใช้ในการทำงานวิจัยต่อยอดออกไปจากผลลัพธ์ที่เกิดขึ้นภายในโครงการวิจัยนี้ตามที่ได้เผยแพร่ออกไปในวารสารทางวิชาการระดับนานาชาติ โดยจะเห็นได้จากการอ้างอิงผลงานวิจัยทั้ง 9 ชิ้นมีอยู่เป็นจำนวน 38 ครั้งภายในเวลา 3 ปี (การอ้างอิงสืบค้นจาก INSPIRE database ซึ่งเป็นฐานข้อมูลหลักของงานวิจัยในสาขาฟิสิกส์พลังงานสูง) นอกจากนี้ ยังได้สร้างนักวิจัยใหม่คือ นายเชม อุปถัมภากุล และนายภัทรดนัย นุชจิโนซึ่งขณะนี้เป็นนิสิตปริญญาเอกที่ภาควิชาฟิสิกส์ คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย โดยคาดว่าทั้งสองคนจะจบการศึกษาในปีพ.ศ. 2563

3. อื่นๆ (เช่น หนังสือ การจดสิทธิบัตร)

หนังสือตำราวิชาการเรื่อง “เกจซูปเปอร์กราวิตีกับโฮโลกราฟี AdS/CFT” ISBN: 9786164555464 ตีพิมพ์ในปีพ.ศ. 2561

ภาคผนวก

ตลอดระยะเวลา 3 ปีที่ดำเนินโครงการวิจัยนี้ มีผลการวิจัยเกิดขึ้นหลายเรื่องด้วยกัน มีบทความวิจัยที่ตีพิมพ์เผยแพร่ในวารสารทางวิชาการระดับนานาชาติทั้งสิ้น 9 ฉบับ โดยได้รวบรวมสำเนาของบทความวิจัยเหล่านี้ไว้ในภาคผนวกนี้แล้ว บทความวิจัยทั้งหมดได้รับการตีพิมพ์ในวารสาร Journal of High Energy Physics, The European Physical Journal C และ Physical Review D ซึ่งเป็นวารสารทางวิชาการที่มีคุณภาพสูงในฐานข้อมูล ISI และ Scopus โดยมีค่า impact factor ในปี 2017 เท่ากับ 5.541, 5.172 และ 4.394 ตามลำดับ ทุกวารสารเป็นวารสารที่อยู่ใน Q1 ทั้งสิ้น

ในด้านการร่วมมือกับต่างประเทศ ระหว่างดำเนินงานโครงการวิจัยนี้ หัวหน้าโครงการวิจัยได้ทุนจากมหาวิทยาลัย Hamburg เพื่อไปทำงานวิจัยร่วมกับกลุ่มวิจัยของ Professor Jan Louis ซึ่งเป็นผู้เชี่ยวชาญด้านทฤษฎีสตริงและซูปเปอร์กราวิตีที่มีชื่อเสียง โดยมีผลงานวิจัยตีพิมพ์ร่วมกัน 1 ฉบับ และได้รับทุนจาก International Centre for Theoretical Physics ไปทำงานวิจัย ณ ประเทศอิตาลี นอกจากนี้ ยังได้ร่วมทำงานวิจัยกับนักศึกษาปริญญาเอกจาก National University of Singapore อีกด้วย

Holographic RG flows in $N = 4$ SCFTs from half-maximal gauged supergravity

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Abstract We study four-dimensional $N = 4$ gauged supergravity coupled to six vector multiplets with semisimple gauge groups $SO(4) \times SO(4)$, $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$. All of these gauge groups are dyonically embedded in the global symmetry group $SO(6, 6)$ via its maximal subgroup $SO(3, 3) \times SO(3, 3)$. For $SO(4) \times SO(4)$ gauge group, there are four $N = 4$ supersymmetric AdS_4 vacua with $SO(4) \times SO(4)$, $SO(4) \times SO(3)$, $SO(3) \times SO(4)$ and $SO(3) \times SO(3)$ symmetries, respectively. These AdS_4 vacua correspond to $N = 4$ SCFTs in three dimensions with $SO(4)$ R-symmetry and different flavor symmetries. We explicitly compute the full scalar mass spectra at all these vacua. Holographic RG flows interpolating between these conformal fixed points are also given. The solutions describe supersymmetric deformations of $N = 4$ SCFTs by relevant operators of dimensions $\Delta = 1, 2$. A number of these solutions can be found analytically although some of them can only be obtained numerically. These results provide a rich and novel class of $N = 4$ fixed points in three-dimensional Chern–Simons–Matter theories and possible RG flows between them in the framework of $N = 4$ gauged supergravity in four dimensions. Similar studies are carried out for non-compact gauge groups, but the $SO(4) \times SO(4)$ gauge group exhibits a much richer structure.

1 Introduction

The study of holographic RG flows is one of the most interesting results from the celebrated AdS/CFT correspondence since its original proposal in [1]. The solutions take the form of domain walls interpolating between AdS vacua, for RG flows between two conformal fixed points, or between an AdS vacuum and a singular geometry, for RG flows from a

conformal fixed point to a non-conformal field theory. Many of these fixed points are described by superconformal field theories (SCFTs) which are also believed to give some insight into the dynamics of various branes in string/M-theory.

Rather than finding holographic RG flow solutions directly in string/M-theory, a convenient and effective way to find these solutions is to look for domain wall solutions in lower dimensional gauged supergravities. In some cases, the resulting solutions can be uplifted to interesting brane configurations within string/M-theory, see for example [2–4]. Apart from rendering the computation simpler, working in lower dimensional gauged supergravities also has an advantage of being independent of higher dimensional embedding. Results obtained within this framework are applicable in any models described by the same effective gauged supergravity regardless of their higher dimensional origins.

Many results along this direction have been found in maximally gauged supergravities, see for examples [5–13]. A number of RG flow solutions in half-maximally gauged supergravities in various dimensions have, on the other hand, been studied only recently in [14–21], see also [22, 23] for earlier results. In this paper, we will give holographic RG flow solutions within $N = 4$ gauged supergravity in four dimensions. Solutions in the case of non-semisimple gauge groups with known higher dimensional origins have already been considered in [19, 20]. This non-semisimple gauging, however, turns out to have a very restricted number of supersymmetric AdS_4 vacua. In this work, we will consider semisimple gauging of $N = 4$ supergravity similar to the study in other dimensions. Although some general properties of AdS_4 vacua and RG flows have been pointed out recently in [18], to the best of our knowledge, a detailed analysis and explicit RG flow solutions in $N = 4$ gauged supergravity have not previously appeared.

Gaugings of $N = 4$ supergravity coupled to an arbitrary number n of vector multiplets have been studied and classi-

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fied for a long time [24–26], and the embedding tensor formalism which includes all possible deformations of $N = 4$ supergravity has been given in [27]. For the case of $n < 6$, the relation between the resulting $N = 4$ supergravity and ten-dimensional supergravity is not known. Therefore, we will consider only the case of $n \geq 6$ which is capable of embedding in ten dimensions. Furthermore, we are particularly interested in $N = 4$ gauged supergravity coupled to six vector multiplets to simplify the computation. In this case, possible gauge groups are embedded in the global symmetry group $SL(2, \mathbb{R}) \times SO(6, 6)$. From a general result of [28], the existence of $N = 4$ supersymmetric AdS_4 vacua requires that the gauge group is purely embedded in the $SO(6, 6)$ factor. Furthermore, the gauge group must contain an $SO(3) \times SO(3)$ subgroup with one of the $SO(3)$ factors embedded electrically and the other one embedded magnetically.

We will consider gauge groups in the form of a simple product $G_1 \times G_2$ in which one of the two factors is embedded electrically in $SO(3, 3) \subset SO(6, 6)$ while the other is embedded magnetically in the other $SO(3, 3)$ subgroup of $SO(6, 6)$. Taking the above criteria for having supersymmetric AdS_4 vacua into account, we will study the case of $G_1, G_2 = SO(4)$ and $SO(3, 1)$. There are then three different product gauge groups to be considered, $SO(4) \times SO(4)$, $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$. We will identify possible supersymmetric AdS_4 vacua and supersymmetric RG flows interpolating between these vacua. These solutions should describe RG flows in the dual $N = 4$ Chern–Simons–Matter (CSM) theories driven by relevant operators dual to the scalar fields of the $N = 4$ gauged supergravity. As shown in [29], some of the $N = 4$ CSM theories can be obtained from a non-chiral orbifold of the ABJM theory [30]. Other classes of $N = 4$ CSM theories are also known, see [31, 32] for example. These theories play an important role in describing the dynamics of M2-branes on various backgrounds. The solutions obtained in this paper should also be useful in this context via the AdS/CFT correspondence. It should also be emphasized that all gauge groups considered here are currently not known to have higher dimensional origins. Therefore, the corresponding holographic duality in this case is still not firmly established.

The paper is organized as follows. In Sect. 2, we review $N = 4$ gauged supergravity coupled to vector multiplets in the embedding tensor formalism. This sets up the framework we will use throughout the paper and collects relevant formulae and notations used in subsequent sections. In Sect. 3, $N = 4$ gauged supergravity with $SO(4) \times SO(4)$ gauge group is constructed, and the scalar potential for scalars which are singlets under $SO(4)_{\text{inv}} \subset SO(4) \times SO(4)$ is computed. We will identify possible supersymmetric AdS_4 vacua and compute the full scalar mass spectra at these vacua. The section ends with supersymmetric RG flow solutions interpo-

lating between AdS_4 vacua and RG flows to non-conformal field theories. A similar study is performed in Sects. 4 and 5 for non-compact $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$ gauge groups. Conclusions and comments on the results will be given in Sect. 6. An appendix containing the convention on 't Hooft matrices is included at the end of the paper.

2 $N = 4$ gauged supergravity coupled to vector multiplets

To set up our framework, we give a brief review of four-dimensional $N = 4$ gauged supergravity. We mainly give relevant information and necessary formulae to find supersymmetric AdS_4 vacua and domain wall solutions. More details on the construction can be found in [27].

$N = 4$ supergravity can couple to an arbitrary number n of vector multiplets. The supergravity multiplet consists of the graviton $e_{\mu}^{\hat{\mu}}$, four gravitini ψ_{μ}^i , six vectors A_{μ}^m , four spin- $\frac{1}{2}$ fields χ^i and one complex scalar τ containing the dilaton ϕ and the axion χ . The complex scalar can be parametrized by $SL(2, \mathbb{R})/SO(2)$ coset. Each vector multiplet contains a vector field A_{μ} , four gaugini λ^i and six scalars ϕ^m . Similar to the dilaton and the axion in the gravity multiplet, the $6n$ scalar fields can be parametrized by $SO(6, n)/SO(6) \times SO(n)$ coset.

Throughout the paper, space-time and tangent space indices are denoted respectively by $\mu, \nu, \dots = 0, 1, 2, 3$ and $\hat{\mu}, \hat{\nu}, \dots = 0, 1, 2, 3$. The $SO(6) \sim SU(4)$ R-symmetry indices will be described by $m, n = 1, \dots, 6$ for the $SO(6)$ vector representation and $i, j = 1, 2, 3, 4$ for the $SO(6)$ spinor or $SU(4)$ fundamental representation. The n vector multiplets will be labeled by indices $a, b = 1, \dots, n$. All fields in the vector multiplets accordingly carry an additional index in the form of $(A_{\mu}^a, \lambda^{ia}, \phi^{ma})$.

Fermionic fields and the supersymmetry parameters transforming in the fundamental representation of $SU(4)_R \sim SO(6)_R$ R-symmetry are subject to the chirality projections

$$\gamma_5 \psi_{\mu}^i = \psi_{\mu}^i, \quad \gamma_5 \chi^i = -\chi^i, \quad \gamma_5 \lambda^i = \lambda^i \quad (1)$$

while the fields transforming in the anti-fundamental representation of $SU(4)_R$ satisfy

$$\gamma_5 \psi_{\mu i} = -\psi_{\mu i}, \quad \gamma_5 \chi_i = \chi_i, \quad \gamma_5 \lambda_i = -\lambda_i. \quad (2)$$

Gaugings of the matter-coupled $N = 4$ supergravity can be described by two components of the embedding tensor $\xi^{\alpha M}$ and $f_{\alpha MNP}$ with $\alpha = (+, -)$ and $M, N = (m, a) = 1, \dots, n + 6$ denoting fundamental representations of $SL(2, \mathbb{R})$ and $SO(6, n)$, respectively. Under the full $SL(2, \mathbb{R}) \times SO(6, n)$ duality symmetry, the electric vector fields $A^{+M} = (A_{\mu}^m, A_{\mu}^a)$, appearing in the ungauged

Lagrangian, together with their magnetic dual A^{-M} form a doublet under $SL(2, \mathbb{R})$ denoted by $A^{\alpha M}$. A general gauge group is embedded in both $SL(2, \mathbb{R})$ and $SO(6, n)$, and the magnetic vector fields can also participate in the gauging. However, each magnetic vector field must be accompanied by an auxiliary two-form field in order to remove the extra degrees of freedom.

From the analysis of supersymmetric AdS_4 vacua in [28], see also [25, 26], purely electric gaugings do not admit AdS_4 vacua unless an $SL(2, \mathbb{R})$ phase is included [25]. The latter is however incorporated in the magnetic component f_{-MNP} [27]. Therefore, only gaugings involving both electric and magnetic vector fields, or dyonic gaugings, lead to AdS_4 vacua. Furthermore, the existence of maximally supersymmetric AdS_4 vacua requires $\xi^{\alpha M} = 0$. Accordingly, we will from now on restrict ourselves to the case of dyonic gaugings and $\xi^{\alpha M} = 0$.

With $\xi^{\alpha M} = 0$, the gauge covariant derivative can be written as

$$D_\mu = \nabla_\mu - g A_\mu^{\alpha M} f_{\alpha M}{}^{NP} t_{NP} \quad (3)$$

where ∇_μ is the usual space-time covariant derivative including the spin connection. t_{MN} are $SO(6, n)$ generators in the fundamental representation and can be chosen as

$$(t_{MN})_P{}^Q = 2\delta_{[M}^Q \eta_{N]P}. \quad (4)$$

$\eta_{MN} = \text{diag}(-1, -1, -1, -1, -1, -1, 1, \dots, 1)$ is the $SO(6, n)$ invariant tensor, and g is the gauge coupling constant that can be absorbed in the embedding tensor $f_{\alpha MNP}$. For a product gauge group consisting of many simple subgroups, there can be as many independent coupling constants as the simple groups within the product. Note also that with the component $\xi^{\alpha M} = 0$, the gauge group is embedded solely in $SO(6, n)$.

To define a consistent gauging, the embedding tensor has to satisfy a set of quadratic constraints

$$f_{\alpha R[MN} f_{\beta PQ]}{}^R = 0, \quad \epsilon^{\alpha\beta} f_{\alpha MNR} f_{\beta PQ}{}^R = 0. \quad (5)$$

as well as the linear or representation constraint $f_{\alpha MNP} = f_{\alpha[MNP]}$.

The scalar coset $SL(2, \mathbb{R})/SO(2)$ can be described by the coset representative $\mathcal{V}_\alpha$. We will choose the explicit form of $\mathcal{V}_\alpha$ as follow

$$\mathcal{V}_\alpha = e^{\frac{\phi}{2}} \begin{pmatrix} \chi - i e^{-\phi} \\ 1 \end{pmatrix}. \quad (6)$$

For the $SO(6, n)/SO(6) \times SO(n)$ coset, we introduce a coset representative $\mathcal{V}_M^A$ transforming under global

$SO(6, n)$ and local $SO(6) \times SO(n)$ by left and right multiplications, respectively. By splitting the index $A = (m, a)$, we can write the coset representative as

$$\mathcal{V}_M^A = (\mathcal{V}_M^m, \mathcal{V}_M^a). \quad (7)$$

Being an element of $SO(6, n)$, the matrix $\mathcal{V}_M^A$ satisfies the relation

$$\eta_{MN} = -\mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \quad (8)$$

In addition, we can parametrize the $SO(6, n)/SO(6) \times SO(n)$ coset in term of a symmetric matrix

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a \quad (9)$$

which is manifestly $SO(6) \times SO(n)$ invariant.

In this paper, we are mainly interested in supersymmetric solutions with only the metric and scalars non-vanishing. The bosonic Lagrangian with vector and auxiliary two-form fields vanishing can be written as

$$e^{-1} \mathcal{L} = \frac{1}{2} R + \frac{1}{16} \partial_\mu M_{MN} \partial^\mu M^{MN} - \frac{1}{4(\text{Im}\tau)^2} \partial_\mu \tau \partial^\mu \tau^* - V \quad (10)$$

where e is the vielbein determinant. The scalar potential is given in terms of the scalar coset representative and the embedding tensor by

$$V = \frac{g^2}{16} \left[f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta} \left[\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left(\frac{2}{3} \eta^{MQ} - M^{MQ} \right) \eta^{NR} \eta^{PS} \right] - \frac{4}{9} f_{\alpha MNP} f_{\beta QRS} \epsilon^{\alpha\beta} M^{MNPQRS} \right] \quad (11)$$

where M^{MN} is the inverse of M_{MN} , and M^{MNPQRS} is obtained from

$$M^{MNPQRS} = \epsilon_{mnpqrs} \mathcal{V}_M^m \mathcal{V}_N^n \mathcal{V}_P^p \mathcal{V}_Q^q \mathcal{V}_R^r \mathcal{V}_S^s \quad (12)$$

with indices raised by η^{MN} . Similar to M^{MN} , $M^{\alpha\beta}$ is the inverse of a symmetric 2×2 matrix $M_{\alpha\beta}$ defined by

$$M_{\alpha\beta} = \text{Re}(\mathcal{V}_\alpha \mathcal{V}_\beta^*). \quad (13)$$

Fermionic supersymmetry transformations are given by

$$\delta \psi_\mu^i = 2 D_\mu \epsilon^i - \frac{2}{3} g A_1^{ij} \gamma_\mu \epsilon_j, \quad (14)$$

$$\delta \chi^i = i \epsilon^{\alpha\beta} \mathcal{V}_\alpha D_\mu \mathcal{V}_\beta \gamma^\mu \epsilon^i - \frac{4}{3} i g A_2^{ij} \epsilon_j, \quad (15)$$

$$\delta \lambda_a^i = 2 i \mathcal{V}_a^M D_\mu \mathcal{V}_M^{ij} \gamma^\mu \epsilon_j + 2 i g A_{2aj}^i \epsilon^j. \quad (16)$$

The fermion shift matrices, also appearing in fermionic mass-like terms of the gauged Lagrangian, are given by

$$\begin{aligned} A_1^{ij} &= \epsilon^{\alpha\beta} (\mathcal{V}_\alpha)^* \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_2^{ij} &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_{2ai}^j &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_a^M \mathcal{V}_{ik}^N \mathcal{V}_P^{jk} f_{\beta MN}^P \end{aligned} \quad (17)$$

where $\mathcal{V}_M^{ij}$ is defined in terms of the 't Hooft symbols G_m^{ij} and $\mathcal{V}_M^m$ as

$$\mathcal{V}_M^{ij} = \frac{1}{2} \mathcal{V}_M^m G_m^{ij}. \quad (18)$$

Similarly, the inverse elements $\mathcal{V}_{ij}^M$ can be written as

$$\mathcal{V}_{ij}^M = -\frac{1}{2} \mathcal{V}_m^M (G_m^{ij})^*. \quad (19)$$

G_m^{ij} convert an index m in the vector representation of $SO(6)$ into an anti-symmetric pair of indices $[ij]$ in the $SU(4)$ fundamental representation. They satisfy the relations

$$G_{mij} = -(G_m^{ij})^* = -\frac{1}{2} \epsilon_{ijkl} G_m^{kl}. \quad (20)$$

The explicit form of these matrices can be found in the appendix.

The scalar potential can also be written in terms of the fermion shift matrices A_1 and A_2 as

$$V = -\frac{1}{3} A_1^{ij} A_{1ij} + \frac{1}{9} A_2^{ij} A_{2ij} + \frac{1}{2} A_{2ai}^j A_{2a}^i. \quad (21)$$

Together with the fermionic supersymmetry transformations, it then follows that unbroken supersymmetry corresponds to an eigenvalue of A_1^{ij} , α , satisfying $V_0 = -\frac{\alpha^2}{3}$ where V_0 is the value of the scalar potential at the vacuum or the cosmological constant.

3 Supersymmetric AdS_4 vacua and holographic RG flows in $SO(4) \times SO(4)$ gauged supergravity

We are interested in gauge groups that can be embedded in $SO(3, 3) \times SO(3, 3) \subset SO(6, 6)$. These gauge groups take the form of a product $G_1 \times G_2$ with $G_1, G_2 \subset SO(3, 3)$ being six-dimensional. Semisimple groups of dimension six that can be embedded in $SO(3, 3)$ are $SO(4)$, $SO(3, 1)$ and $SO(2, 2)$. The embedding tensors for these gauge groups are given in [33]. Since gauge groups involving $SO(2, 2)$ factors do not give rise to AdS_4 vacua, we will not consider these gauge groups in this paper. In this section, we will study $N = 4$ gauged supergravity with compact $SO(4) \times SO(4) \sim SO(3) \times SO(3) \times SO(3) \times SO(3)$ gauge group.

3.1 AdS_4 vacua

Non-vanishing components of the embedding tensor for $SO(4) \times SO(4)$ gauge group are given by

$$\begin{aligned} f_{+\hat{m}\hat{n}\hat{p}} &= \sqrt{2}(g_1 - \tilde{g}_1) \epsilon_{\hat{m}\hat{n}\hat{p}}, & f_{+\hat{a}\hat{b}\hat{c}} &= \sqrt{2}(g_1 + \tilde{g}_1) \epsilon_{\hat{a}\hat{b}\hat{c}}, \\ f_{-\tilde{m}\tilde{n}\tilde{p}} &= \sqrt{2}(g_2 - \tilde{g}_2) \epsilon_{\tilde{m}\tilde{n}\tilde{p}}, & f_{-\tilde{a}\tilde{b}\tilde{c}} &= \sqrt{2}(g_2 + \tilde{g}_2) \epsilon_{\tilde{a}\tilde{b}\tilde{c}}, \end{aligned} \quad (22)$$

where we have used the indices $M = (m, a) = (\hat{m}, \tilde{m}, \hat{a}, \tilde{a})$ with $\hat{m} = 1, 2, 3$, $\tilde{m} = 4, 5, 6$, $\hat{a} = 7, 8, 9$ and $\tilde{a} = 10, 11, 12$. As mentioned before, the first and second $SO(4)$ factors are embedded electrically and magnetically, respectively.

Non-compact generators of $SO(6, 6)$ are given by

$$Y_{ma} = e_{m,a+6} + e_{a+6,m} \quad (23)$$

in which the 12×12 matrices e_{MN} are defined by

$$(e_{MN})_{PQ} = \delta_{MP} \delta_{NQ}. \quad (24)$$

The 36 scalars in $SO(6, 6)/SO(6) \times SO(6)$ transform as $(\mathbf{6}, \mathbf{6})$ under the compact group $SO(6) \times SO(6)$. Under the gauge group $SO(4)_+ \times SO(4)_- \sim SO(3)_+^1 \times SO(3)_+^2 \times SO(3)_-^1 \times SO(3)_-^2$, these scalars transform as

$$(\mathbf{6}, \mathbf{6}) \rightarrow (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{3}). \quad (25)$$

We will consider scalars which are singlets under the diagonal subgroup $SO(4)_{\text{inv}} \sim [SO(3)_+^1 \times SO(3)_+^2]_D \times [SO(3)_-^1 \times SO(3)_-^2]_D$. Under $SO(4)_{\text{inv}}$, the scalars transform as

$$2(\mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{5}) + (\mathbf{5}, \mathbf{1}) + 2(\mathbf{3}, \mathbf{3}). \quad (26)$$

The two singlets correspond to the following non-compact generators

$$\hat{Y}_1 = Y_{11} + Y_{22} + Y_{33}, \quad \hat{Y}_2 = Y_{44} + Y_{55} + Y_{66} \quad (27)$$

in terms of which the coset representative is given by

$$L = e^{\phi_1 \hat{Y}_1} e^{\phi_2 \hat{Y}_2}. \quad (28)$$

Together with the $SL(2, \mathbb{R})/SO(2)$ scalars which are $SO(4) \times SO(4)$ singlets, there are four $SO(4)_{\text{inv}}$ singlet scalars. The scalar potential for these singlets can be computed to

$$\begin{aligned}
V = & \frac{1}{8} e^{-\phi-6\phi_1-6\phi_2} [e^{\phi+3\phi_2} [e^{\phi+3\phi_2} g_1^2 + e^{\phi+12\phi_1+3\phi_2} \tilde{g}_1^2 \\
& - 3e^{\phi+4\phi_1+3\phi_2} (2g_1^2 + \tilde{g}_1^2) - 3e^{\phi+8\phi_1+3\phi_2} (g_1^2 + 2\tilde{g}_1^2) \\
& + 2e^{3\phi_1} g_1 [g_2(1+3e^{4\phi_2}) - e^{2\phi_2} \tilde{g}_2(3+e^{4\phi_2})] \\
& + 6e^{7\phi_1} g_1 [g_2(1+3e^{4\phi_2}) - e^{2\phi_2} \tilde{g}_2(3+e^{4\phi_2})] \\
& - 6e^{5\phi_1} \tilde{g}_1 [g_2(1+3e^{4\phi_2}) - e^{2\phi_2} \tilde{g}_2(3+e^{4\phi_2})] \\
& - 2e^{9\phi_1} \tilde{g}_1 [g_2(1+3e^{4\phi_2}) - e^{2\phi_2} \tilde{g}_2(3+e^{4\phi_2})]] \\
& + e^{6\phi_1} [[1-3e^{4\phi_2}(2+e^{4\phi_2})](1+e^{2\phi} \chi^2) g_2^2 \\
& + 16g_2 \tilde{g}_2 e^{6\phi_2} (1+e^{2\phi} \chi^2) + e^{4\phi_2} [e^{2\phi} (16e^{2\phi_2} g_1 \tilde{g}_1 \\
& - 3\tilde{g}_2^2 \chi^2 - 6\tilde{g}_2^2 e^{4\phi_2} \chi^2 + e^{8\phi_2} \chi^2 \tilde{g}_2^2) \\
& + (e^{8\phi_2} - 6e^{4\phi_2} - 3)\tilde{g}_2^2]]]. \quad (29)
\end{aligned}$$

This potential admits a maximally supersymmetric AdS_4 critical point with $SO(4) \times SO(4)$ symmetry at $\chi = \phi_1 = \phi_2 = 0$ and

$$\phi = \ln \left| \frac{g_2 - \tilde{g}_2}{g_1 - \tilde{g}_1} \right|. \quad (30)$$

For convenience, we will denote this AdS_4 vacuum with $SO(4) \times SO(4)$ symmetry by critical point I.

Without loss of generality, we can shift the dilaton such that this critical point occurs at $\phi = 0$. For definiteness, we will choose

$$\tilde{g}_2 = g_1 + g_2 - \tilde{g}_1. \quad (31)$$

At this critical point, we find the value of the cosmological constant and the AdS_4 radius

$$V_0 = -6(g_1 - \tilde{g}_1)^2 \quad \text{and} \quad L = \frac{1}{\sqrt{2}(\tilde{g}_1 - g_1)} \quad (32)$$

where we have assumed that $\tilde{g}_1 > g_1$. All scalars have masses $m^2 L^2 = -2$. In general, using the relation $m^2 L^2 = \Delta(\Delta - 3)$, we find that these scalars can be dual to operators of dimensions $\Delta = 1$ or $\Delta = 2$. These correspond to mass terms of scalars ($\Delta = 1$) or fermions ($\Delta = 2$) in the dual three-dimensional SCFTs. Usually, the correct choice is fixed by supersymmetry as in the case of ABJM theory. However, in the present case, the identification is not so clear.

Furthermore, the scalar potential in (29) also admits additional three supersymmetric AdS_4 vacua:

- II. This critical point has $SO(3)_+ \times SO(4)_-$ symmetry with

$$\phi = \ln \left[\frac{2\sqrt{g_1 \tilde{g}_1}}{g_1 + \tilde{g}_1} \right], \quad \phi_1 = \frac{1}{2} \ln \left[\frac{g_1}{\tilde{g}_1} \right], \quad \phi_2 = 0,$$

$$\begin{aligned}
V_0 = & -\frac{3(g_1 + \tilde{g}_1)(g_1 - \tilde{g}_1)^2}{\sqrt{g_1 \tilde{g}_1}}, \\
L = & \frac{(g_1 \tilde{g}_1)^{\frac{1}{4}}}{(\tilde{g}_1 - g_1)\sqrt{g_1 + \tilde{g}_1}}. \quad (33)
\end{aligned}$$

- III. This critical point has $SO(4)_+ \times SO(3)_-$ symmetry with

$$\begin{aligned}
\phi = & -\ln \left[\frac{2\sqrt{g_2 \tilde{g}_2}}{g_2 + \tilde{g}_2} \right], \quad \phi_2 = \frac{1}{2} \ln \left[\frac{g_2}{\tilde{g}_2} \right], \quad \phi_1 = 0, \\
V_0 = & -\frac{3(g_2 + \tilde{g}_2)(g_1 - \tilde{g}_1)^2}{\sqrt{g_2 \tilde{g}_2}}, \\
L = & \frac{(g_2 \tilde{g}_2)^{\frac{1}{4}}}{(\tilde{g}_1 - g_1)\sqrt{g_2 + \tilde{g}_2}}. \quad (34)
\end{aligned}$$

- IV. This critical point is invariant under a smaller symmetry $SO(4)_{\text{inv}}$ with

$$\begin{aligned}
\phi = & \ln \left[\sqrt{\frac{g_1 \tilde{g}_1}{g_2 \tilde{g}_2}} \frac{g_2 + \tilde{g}_2}{g_1 + \tilde{g}_1} \right], \quad \phi_1 = \frac{1}{2} \ln \left[\frac{g_1}{\tilde{g}_1} \right], \\
\phi_2 = & \frac{1}{2} \ln \left[\frac{g_2}{\tilde{g}_2} \right], \\
V_0 = & -\frac{3(g_2 + \tilde{g}_2)^2 (g_1 - \tilde{g}_1)^2}{2\sqrt{g_1 \tilde{g}_1 g_2 \tilde{g}_2}}, \\
L = & \frac{\sqrt{2}(g_1 \tilde{g}_1 g_2 \tilde{g}_2)^{\frac{1}{4}}}{(\tilde{g}_1 - g_1)\sqrt{(g_1 + \tilde{g}_1)(g_2 + \tilde{g}_2)}}. \quad (35)
\end{aligned}$$

We have written the above equations in term of $\tilde{g}_2$ for brevity. All of these critical points preserve the full $N = 4$ supersymmetry and correspond to $N = 4$ SCFTs in three dimensions. Scalar masses at these critical points are given in Tables 1, 2 and 3.

It should also be noted that three massless scalars at critical points II and III are Goldstone bosons corresponding to the symmetry breaking $SO(4)_+ \times SO(4)_- \rightarrow SO(3)_+ \times SO(4)_-$ and $SO(4)_+ \times SO(4)_- \rightarrow SO(4)_+ \times SO(3)_-$.

Table 1 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(3)_+ \times SO(4)_-$ symmetry and the corresponding dimensions of the dual operators

Scalar field representations	$m^2 L^2$	Δ
(1, 1, 1)	$-2_{\times 2}$	1, 2
(1, 1, 1)	4	4
(3, 1, 1)	$0_{\times 3}$	3
(1, 3, 3)	$0_{\times 9}$	3
(5, 1, 1)	$-2_{\times 5}$	1, 2
(3, 1, 3) + (3, 3, 1)	$-2_{\times 18}$	1, 2

Table 2 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(4)_+ \times SO(3)_-$ symmetry and the corresponding dimensions of the dual operators

Scalar field representations	$m^2 L^2$	Δ
(1, 1, 1)	$-2_{\times 2}$	1, 2
(1, 1, 1)	4	4
(1, 1, 3)	$0_{\times 3}$	3
(3, 3, 1)	$0_{\times 9}$	3
(1, 1, 5)	$-2_{\times 5}$	1, 2
(1, 3, 3) + (3, 1, 3)	$-2_{\times 18}$	1, 2

Table 3 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(3) \times SO(3) \sim SO(4)_{\text{inv}}$ symmetry and the corresponding dimensions of the dual operators

Scalar field representations	$m^2 L^2$	Δ
(1, 1)	$-2_{\times 2}$	1, 2
(1, 1)	$4_{\times 2}$	4
(1, 5) + (5, 1)	$-2_{\times 10}$	1, 2
(1, 3) + (3, 1)	$0_{\times 6}$	3
(3, 3)	$0_{\times 18}$	3

These scalars live in representations (3, 1, 1) and (1, 1, 3), respectively. Similarly, for critical point IV, six of the massless scalars in the representation (1, 3) + (3, 1) are Goldstone bosons of the symmetry breaking $SO(4)_+ \times SO(4)_- \rightarrow SO(4)_{\text{inv}}$. The remaining massless scalars correspond to marginal deformations in the SCFTs dual to these AdS_4 vacua. These deformations necessarily break some amount of supersymmetry since the $N = 4$ AdS_4 vacua have no moduli preserving $N = 4$ supersymmetry [28]. It should also be noted that the vacuum structure of this gauged supergravity is very similar to two copies of $SO(3) \times SO(3) \sim SO(4)$ $N = 3$ gauged supergravity considered in [34].

3.2 Holographic RG flows between $N = 4$ SCFTs

We now consider holographic RG flow solutions interpolating between supersymmetric AdS_4 vacua previously identified. To find supersymmetric flow solutions, we begin with the metric ansatz

$$ds^2 = e^{2A(r)} dx_{1,2}^2 + dr^2 \quad (36)$$

where $dx_{1,2}^2$ is the flat Minkowski metric in three dimensions.

For spinor conventions, we will use the Majorana representation with all γ^μ real and γ_5 purely imaginary. This choice implies that ϵ_i is a complex conjugate of ϵ^i . All scalar fields will be functions of only the radial coordinate r in order to preserve Poincaré symmetry in three dimensions. The BPS

conditions coming from setting $\delta\chi^i = 0$ and $\delta\lambda_a^i = 0$ require the following projection

$$\gamma_{\hat{r}} \epsilon^i = e^{i\Lambda} \epsilon_i. \quad (37)$$

It follows from the $\delta\psi_{\mu i} = 0$ conditions for $\mu = 0, 1, 2$, that

$$A' = \pm W, \quad e^{i\Lambda} = \pm \frac{\mathcal{W}}{W} \quad (38)$$

where $W = |\mathcal{W}|$, and $'$ denotes the r -derivative. The superpotential $\mathcal{W}$ is defined by

$$\mathcal{W} = \frac{2}{3} \alpha \quad (39)$$

where α is the eigenvalue of A_1^{ij} corresponding to the unbroken supersymmetry. The detailed analysis leading to Eq. (38) can be found, for example, in [34].

For $SO(4)_{\text{inv}}$ singlet scalars, the tensor A_1^{ij} takes the form of a diagonal matrix

$$A_1^{ij} = \frac{3}{2} \mathcal{W} \delta^{ij} \quad (40)$$

with the superpotential given by

$$\begin{aligned} \mathcal{W} = \frac{1}{4\sqrt{2}} e^{-\frac{\phi}{2} - 3\phi_1 - 3\phi_2} & \left[3i \tilde{g}_1 e^{\phi + 2\phi_1 + 3\phi_2} + i \tilde{g}_1 e^{\phi + 6\phi_1 + 3\phi_2} \right. \\ & + e^{3\phi_1} (i + e^\phi \chi) \left[g_2 (1 + 3e^{4\phi}) - \tilde{g}_2 e^{2\phi_2} (3 + e^{4\phi_2}) \right] \\ & \left. - i g_1 e^{\phi + 3\phi_2} - 3i g_1 e^{\phi + 4\phi_1 + 3\phi_2} \right]. \end{aligned} \quad (41)$$

The variation of λ_a^i leads to the following BPS equations

$$\phi'_1 = -\frac{i}{2\sqrt{2}} e^{i\Lambda} e^{\frac{\phi}{2} - 3\phi_1} (e^{4\phi_1} - 1) (e^{2\phi_1} \tilde{g}_1 - g_1), \quad (42)$$

$$\phi'_2 = -\frac{1}{2\sqrt{2}} e^{i\Lambda} e^{-\frac{\phi}{2} - 3\phi_2} (e^{4\phi_2} - 1) (e^{2\phi_2} \tilde{g}_2 - g_2) (e^\phi \chi - i). \quad (43)$$

Consistency of the first condition implies that the phase $e^{i\Lambda}$ is purely imaginary, $e^{i\Lambda} = \pm i$. With this choice, the second condition requires that $\chi = 0$. This is also consistent with the variation of the dilatini χ^i . Furthermore, with $\chi = 0$, the superpotential (41) is purely imaginary in agreement with the spinor phase $e^{i\Lambda}$ in Eq. (38).

We will choose a definite sign in order to identify the $SO(4) \times SO(4)$ critical point with the limit $r \rightarrow \infty$. The BPS equations for ϕ , ϕ_1 and ϕ_2 can then be written as

$$\phi' = -4 \frac{\partial \mathcal{W}}{\partial \phi}, \quad \phi'_1 = -\frac{2}{3} \frac{\partial \mathcal{W}}{\partial \phi_1}, \quad \phi'_2 = -\frac{2}{3} \frac{\partial \mathcal{W}}{\partial \phi_2} \quad (44)$$

together with the A' equation

$$A' = W. \quad (45)$$

Explicitly, these equations read

$$\phi'_1 = -\frac{1}{2\sqrt{2}}e^{\frac{\phi}{2}-3\phi_1}(e^{4\phi_1}-1)(e^{2\phi_1}\tilde{g}_1-g_1), \quad (46)$$

$$\phi'_2 = \frac{1}{2\sqrt{2}}e^{-\frac{\phi}{2}-3\phi_2}(e^{4\phi_2}-1)(e^{2\phi_2}\tilde{g}_2-g_2), \quad (47)$$

$$\begin{aligned} \phi' = & -\frac{1}{2\sqrt{2}}e^{-\frac{\phi}{2}-3\phi_1-3\phi_2} \\ & \times \left[3\tilde{g}_1e^{\phi+2\phi_1+3\phi_2} - g_1e^{\phi+3\phi_2} - 3g_1e^{\phi+4\phi_1+3\phi_2} \right. \\ & + \tilde{g}_1e^{\phi+6\phi_1+3\phi_2} + e^{3\phi_1} \\ & \left. \times \left[\tilde{g}_2e^{2\phi_2}(3+e^{4\phi_2}) - g_2(1+3e^{4\phi_2}) \right] \right], \quad (48) \end{aligned}$$

$$\begin{aligned} A' = & \frac{1}{4\sqrt{2}}e^{-\frac{\phi}{2}-3\phi_1-3\phi_2} \\ & \times \left[3\tilde{g}_1e^{\phi+2\phi_1+3\phi_2} - g_1e^{\phi+3\phi_2} - 3g_1e^{\phi+4\phi_1+3\phi_2} \right. \\ & + \tilde{g}_1e^{\phi+6\phi_1+3\phi_2} + e^{3\phi_1} \\ & \left. \times \left[g_2(1+3e^{4\phi_2}) - \tilde{g}_2e^{2\phi_2}(3+e^{4\phi_2}) \right] \right] \quad (49) \end{aligned}$$

The scalar potential can be written in term of the real superpotential W as

$$V = 4 \left(\frac{\partial W}{\partial \phi} \right)^2 + \frac{2}{3} \left(\frac{\partial W}{\partial \phi_1} \right)^2 + \frac{2}{3} \left(\frac{\partial W}{\partial \phi_2} \right)^2 - 3W^2. \quad (50)$$

It can be verified that the above BPS equations are compatible with the second-order field equations. It should be noted that the consistency between BPS equations and field equations also requires $\chi = 0$.

We now consider various possible RG flows interpolating between the $N = 4$ supersymmetric fixed points. Some of these flows can be obtained analytically, but the others require some sort of numerical analysis. Near the $SO(4) \times SO(4)$ critical point as $r \rightarrow \infty$, the BPS equations give

$$\phi, \phi_1, \phi_2 \sim e^{-\frac{r}{L_I}} \quad (51)$$

in agreement with the fact that all these scalars are dual to operators of dimensions $\Delta = 1, 2$. L_I is the AdS_4 radius at critical point I.

We begin with the flow between critical points I and II. In this case, we can consistently set $\phi_2 = 0$. By considering ϕ and A as functions of ϕ_1 , we can combine the above BPS equations into

$$\frac{d\phi}{d\phi_1} = -\frac{g_1(1+3e^{4\phi_1}) + e^{2\phi_1}[4(g_2-\tilde{g}_2)e^{\phi_1-\phi} - \tilde{g}_1(e^{4\phi_1}+3)]}{(e^{4\phi_1}-1)(\tilde{g}_1e^{2\phi_1}-g_1)} \quad (52)$$

$$\frac{dA}{d\phi_1} = \frac{g_1(1+3e^{4\phi_1}) - e^{2\phi_1}[4(g_2-\tilde{g}_2)e^{\phi_1-\phi} + \tilde{g}_1(3+e^{4\phi_1})]}{2(e^{4\phi_1}-1)(\tilde{g}_1e^{2\phi_1}-g_1)}. \quad (53)$$

The first equation can be solved by

$$\phi = \ln \left[\frac{g_2 - \tilde{g}_2 + C_1(e^{4\phi_1} - 1)}{\tilde{g}_1e^{3\phi_1} - g_1e^{\phi_1}} \right]. \quad (54)$$

The integration constant C_1 will be chosen such that the solution interpolates between $\phi = 0$ at the $SO(4) \times SO(4)$ critical point and $\phi = \ln \left[\frac{\sqrt{g_1\tilde{g}_1}}{g_1+\tilde{g}_1} \right]$ at the $SO(3)_+ \times SO(4)_-$ critical point. This is achieved by choosing $C_1 = \frac{\tilde{g}_1^2(g_2-\tilde{g}_2)}{\tilde{g}_1^2-g_1^2}$, and the solution for ϕ is given by

$$\phi = \ln \left[\frac{(g_2 - \tilde{g}_2)(g_1 + \tilde{g}_1e^{2\phi_1})e^{-\phi_1}}{\tilde{g}_1^2 - g_1^2} \right]. \quad (55)$$

With this solution, Eq. (53) can be solved by

$$A = \frac{\phi_1}{2} - \ln(1-e^{4\phi_1}) + \ln(g_1 - \tilde{g}_1e^{2\phi_1}) + \frac{1}{2} \ln(g_1 + \tilde{g}_1e^{2\phi_1}) \quad (56)$$

where an irrelevant additive integration constant has been removed.

By changing to a new radial coordinate $\tilde{r}$ defined by $\frac{d\tilde{r}}{dr} = e^{\frac{\phi}{2}}$, Eq. (46) becomes

$$\frac{d\phi_1}{d\tilde{r}} = -\frac{1}{2\sqrt{2}}e^{-3\phi_1}(e^{4\phi_1}-1)(\tilde{g}_1e^{2\phi_1}-g_1) \quad (57)$$

whose solution is given by

$$\begin{aligned} \frac{(g_1^2 - \tilde{g}_1^2)\tilde{r}}{\sqrt{2}} = & (g_1 - \tilde{g}_1) \tan^{-1} e^{\phi_1} - (g_1 + \tilde{g}_1) \tanh^{-1} e^{\phi_1} \\ & + 2\sqrt{g_1\tilde{g}_1} \tanh^{-1} \left[\sqrt{\frac{\tilde{g}_1}{g_1}} e^{\phi_1} \right]. \quad (58) \end{aligned}$$

Near critical point II, the operator dual to ϕ_1 becomes irrelevant with dimension $\Delta = 4$, but the operator dual to the dilaton ϕ is still relevant with dimensions $\Delta = 1, 2$. This can be seen by looking at the behavior of scalars near critical point II

$$\phi \sim e^{-\frac{r}{L_{II}}} \quad \text{and} \quad \phi_1 \sim e^{\frac{r}{L_{II}}} \quad (59)$$

as $r \rightarrow -\infty$.

We can then consider a flow from critical point II to critical point IV. Along this flow, we have $\phi_1 = \frac{1}{2} \ln \frac{g_1}{\tilde{g}_1}$, and a similar analysis gives the solution

$$\phi = \ln \left[\frac{2\sqrt{g_1\tilde{g}_1}(g_2 + \tilde{g}_2)e^{2\phi_2}}{(g_1 + \tilde{g}_1)(g_2 + \tilde{g}_2e^{2\phi_2})} \right], \quad (60)$$

$$A = \frac{\phi_2}{2} - \ln(1 - e^{4\phi_2}) + \ln(\tilde{g}_2e^{2\phi_2} - g_2) + \frac{1}{2} \ln(g_2 + \tilde{g}_2e^{2\phi_2}). \quad (61)$$

Along this flow, the running of ϕ_2 is described by

$$\begin{aligned} & \frac{(g_1 - \tilde{g}_1)(g_2 + \tilde{g}_2)}{\sqrt{2}} \bar{r} \\ &= (\tilde{g}_1 - g_1) \tan^{-1} e^{\phi_2} - (g_2 + \tilde{g}_2) \tanh^{-1} e^{\phi_2} \\ &+ 2\sqrt{g_2\tilde{g}_2} \tanh^{-1} \left[e^{\phi_2} \sqrt{\frac{\tilde{g}_2}{g_2}} \right] \end{aligned} \quad (62)$$

with $\bar{r}$ defined by $\frac{d\bar{r}}{dr} = e^{-\frac{\phi}{2}}$.

Similarly, the flows between critical points I and III and between critical points III and IV are given respectively by

$$\phi_1 = 0, \quad (63)$$

$$\phi = \ln \left[\frac{e^{\phi_2}(g_2 + \tilde{g}_2)}{g_2 + e^{2\phi_2}\tilde{g}_2} \right], \quad (64)$$

$$A = \frac{\phi_2}{2} - \ln(1 - e^{4\phi_2}) + \ln(e^{2\phi_2}\tilde{g}_2 - g_2) + \frac{1}{2} \ln(g_2 + \tilde{g}_2e^{2\phi_2}) \quad (65)$$

and

$$\phi_2 = \frac{1}{2} \ln \left[\frac{g_2}{\tilde{g}_2} \right], \quad (66)$$

$$\phi = \ln \left[\frac{e^{-\phi_1}(g_2 + \tilde{g}_2)(g_1 + \tilde{g}_1e^{2\phi_1})}{2(g_1 + \tilde{g}_1)\sqrt{g_2\tilde{g}_2}} \right], \quad (67)$$

$$A = \frac{\phi_1}{2} - \ln(1 - e^{4\phi_1}) + \ln(g_1 - e^{2\phi_1}\tilde{g}_1) + \frac{1}{2} \ln(g_1 + \tilde{g}_1e^{2\phi_1}) \quad (68)$$

In these cases, the r -dependent of ϕ_1 and ϕ_2 can be obtained in the same way as Eqs. (58) and (62).

For a direct flow from critical point I to critical point IV, a numerical solution is needed. This solution is given in Fig. 1. More generally, a flow from critical point I to critical point II and finally to critical point IV can also be found. This solution is given in Fig. 2 and describes a cascade of RG flows with smaller flavor symmetry along the flow.

3.3 RG flows to $N = 4$ non-conformal theory

A consistent truncation of the above $N = 4$ $SO(4) \times SO(4)$ gauged supergravity is obtained by setting $\phi_1 = \phi_2 = 0$.

In this case, only scalars in the gravity multiplet are present. As previously mentioned, the axion χ cannot be turned on simultaneously with ϕ_1 and ϕ_2 .

For $\phi_1 = \phi_2 = 0$, the superpotential is complex and given by

$$\mathcal{W} = \frac{1}{\sqrt{2}} e^{-\frac{\phi}{2}} [(g_2 - \tilde{g}_2)\chi e^{\phi} - i(\tilde{g}_2 - g_2 + e^{\phi}(g_1 - \tilde{g}_1))]. \quad (69)$$

With $\tilde{g}_2 = g_1 + g_2 - \tilde{g}_1$, the scalar potential takes a simpler form

$$\begin{aligned} V &= 4 \left(\frac{\partial W}{\partial \phi} \right)^2 + 4e^{-2\phi} \left(\frac{\partial W}{\partial \chi} \right)^2 - 3W^2 \\ &= -(g_1 - \tilde{g}_1)^2 e^{-\phi} [1 + 4e^{\phi} + e^{2\phi}(1 + \chi^2)] \end{aligned} \quad (70)$$

which has only one AdS_4 critical point at $\phi = \chi = 0$. This is critical point I of the previous subsection.

The BPS equations in this truncation are given by

$$\phi' = -4 \frac{\partial W}{\partial \phi} = -\frac{\sqrt{2}(\tilde{g}_1 - g_1)[e^{2\phi}(1 + \chi^2) - 1]}{\sqrt{(1 + e^{\phi})^2 + e^{2\phi}\chi^2}}, \quad (71)$$

$$\chi' = -4e^{-2\phi} \frac{\partial W}{\partial \chi} = -\frac{2\sqrt{2}(\tilde{g}_1 - g_1)e^{-\frac{\phi}{2}}\chi}{\sqrt{(1 + e^{\phi})^2 + e^{2\phi}\chi^2}}, \quad (72)$$

$$A' = W = \frac{1}{\sqrt{2}}(\tilde{g}_1 - g_1)e^{-\frac{\phi}{2}}\sqrt{(1 + e^{\phi})^2 + e^{2\phi}\chi^2}. \quad (73)$$

Near the AdS_4 critical point, we find

$$\phi \sim \chi \sim e^{-\frac{r}{L_1}} \quad (74)$$

implying that ϕ and χ correspond to relevant operators of dimensions $\Delta = 1, 2$.

By considering ϕ and A as functions of χ , we can combine the BPS equations into

$$\frac{d\phi}{d\chi} = \frac{e^{2\phi}(1 + \chi^2) - 1}{2\chi}, \quad (75)$$

$$\frac{dA}{d\chi} = -\frac{1 + 2e^{\phi} + e^{2\phi}(1 + \chi^2)}{4\chi} \quad (76)$$

which can be solved by

$$\phi = -\frac{1}{2} \ln(1 - 2C\chi - \chi^2), \quad (77)$$

$$\begin{aligned} A &= -\ln \chi + \frac{1}{2} \ln[1 - C\chi + \sqrt{1 - 2C\chi - \chi^2}] \\ &+ \frac{1}{4} \ln(1 - 2C\chi - \chi^2) \end{aligned} \quad (78)$$

in which an additive integration constant for A has been neglected. It should also be noted that we must keep the constant $C \neq 0$ in order to obtain the correct behavior near the AdS_4 critical point as given in Eq. (74).

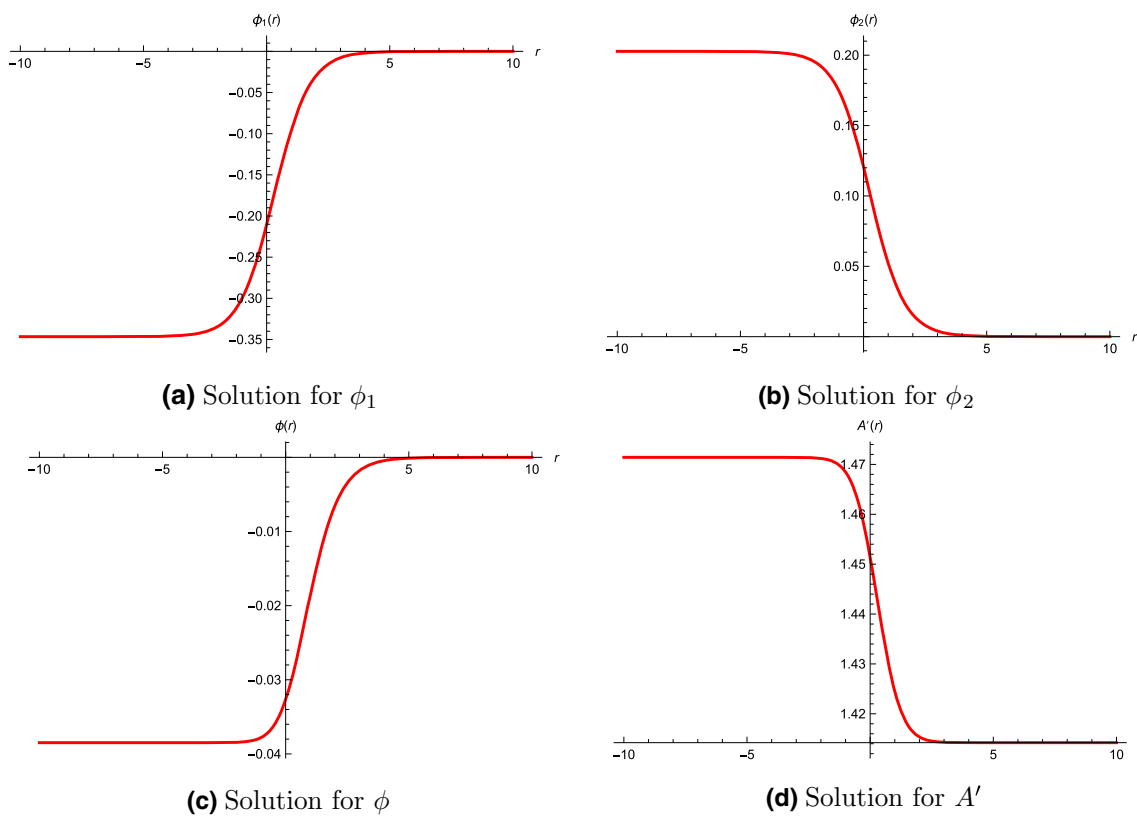


Fig. 1 An RG flow between critical points I and IV with $g_1 = 1$, $\tilde{g}_1 = \tilde{g}_2 = 2$ and $g_2 = 3$

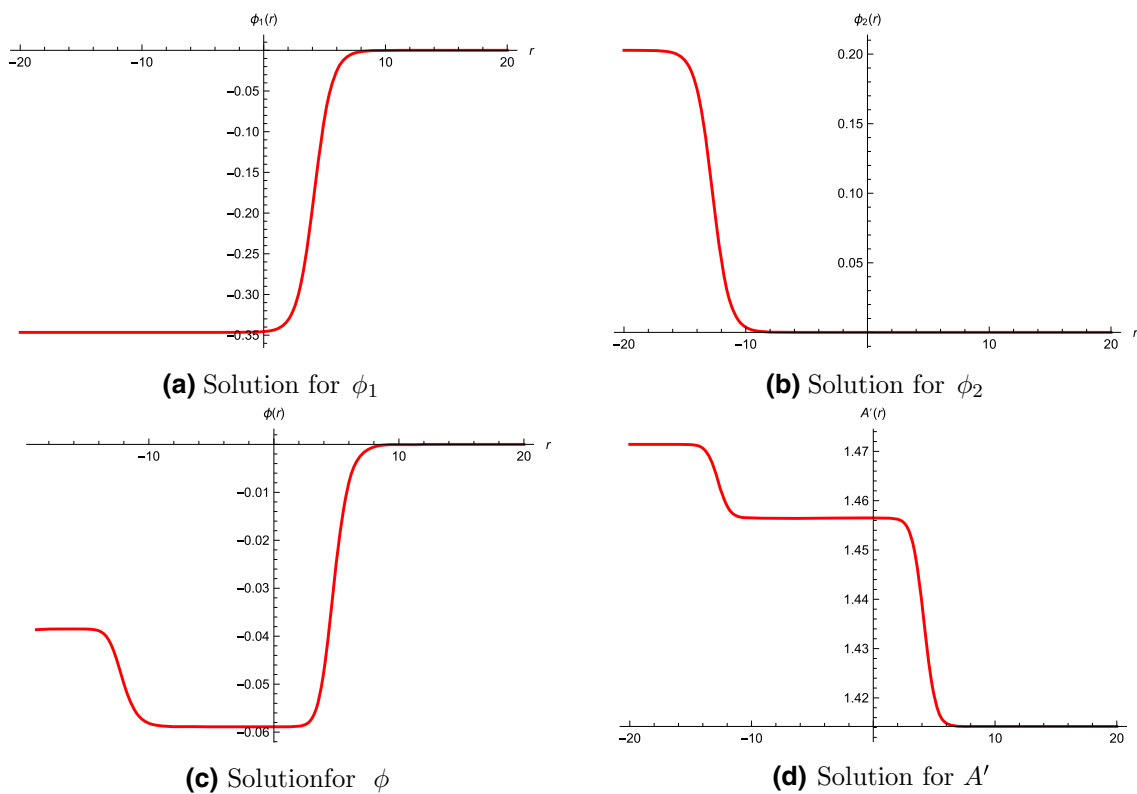


Fig. 2 An RG flow from critical point I to critical point II and continue to critical point IV with $g_1 = 1$, $\tilde{g}_1 = \tilde{g}_2 = 2$ and $g_2 = 3$

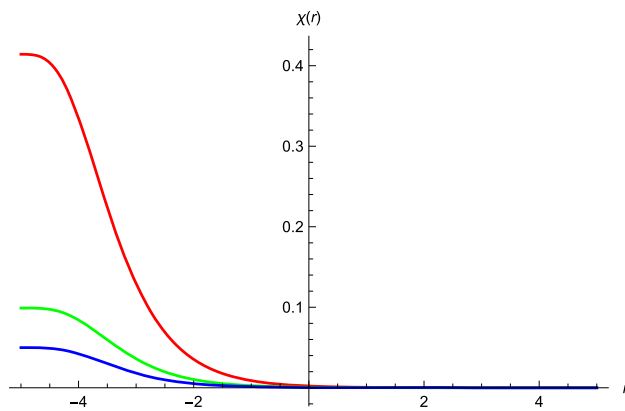


Fig. 3 Solutions for χ with $g_1 = 1$, $\tilde{g}_1 = 2$ and $\chi_0 = \sqrt{1 + C^2} - C$ for $C = 1$ (red), $C = 5$ (green) and $C = 10$ (blue)

Finally, we can substitute the ϕ solution in (77) in Eq. (72) and in principle solve for χ as a function of r . However, we are not able to solve for χ analytically. We then look for numerical solutions. From Eq. (77), we see that $\phi \rightarrow 0$ as $\chi \rightarrow 0$. This limit, as usual, corresponds to the AdS_4 critical point. We can also see that ϕ is singular at χ_0 for which $1 - 2C\chi_0 - \chi_0^2 = 0$ or

$$\chi_0 = -C \pm \sqrt{1 + C^2}. \quad (79)$$

This implies that ϕ flows from the value $\phi = 0$ at the critical point to a singular value $\phi \rightarrow \infty$ while χ flows between the values $\chi = 0$ and $\chi = \chi_0$. Examples of solutions for χ is shown in Fig. 3.

Near the singularity $\phi \rightarrow \infty$ and $\chi \rightarrow \chi_0$, we find that

$$\chi - \chi_0 \sim r^4, \quad \phi \sim -\ln r^2, \quad A \sim \ln r. \quad (80)$$

This gives the metric

$$ds^2 = r^2 dx_{1,2}^2 + dr^2. \quad (81)$$

From the scalar potential (70), we find $V \rightarrow -\infty$ for any value of χ_0 . Therefore, the singularity is physical according to the criterion of [35]. We then conclude that the solution describes an RG flow from the $N = 4$ SCFT in the UV to a non-conformal field theory in the IR corresponding to the above singularity. The deformations break conformal symmetry but preserve the $SO(4)$ flavor symmetry and $N = 4$ Poincaré supersymmetry in three dimensions.

4 $N = 4$ $SO(3, 1) \times SO(3, 1)$ gauged supergravity

In this section, we consider non-compact gauge group $SO(3, 1) \times SO(3, 1)$ with the embedding tensor

$$\begin{aligned} f_{+123} &= f_{+189} = f_{+729} = -f_{+783} = \frac{1}{\sqrt{2}}(g_1 - \tilde{g}_1), \\ f_{+789} &= f_{+183} = f_{+723} = -f_{+129} = \frac{1}{\sqrt{2}}(g_1 + \tilde{g}_1), \\ f_{-456} &= f_{-4,11,12} = f_{-10,5,12} = -f_{-10,11,6} \\ &= \frac{1}{\sqrt{2}}(g_2 - \tilde{g}_2), \\ f_{-10,11,12} &= f_{-4,11,6} = f_{-10,5,6} = -f_{-45,12} \\ &= \frac{1}{\sqrt{2}}(g_2 + \tilde{g}_2). \end{aligned} \quad (82)$$

We now repeat the analysis performed in the previous section.

4.1 Supersymmetric AdS_4 vacuum

We will parametrize the $SO(6, 6)/SO(6) \times SO(6)$ coset by using scalars that are $SO(3) \times SO(3) \subset SO(3, 1) \times SO(3, 1)$ invariant. From the embedding of $SO(3, 1)$ in $SO(3, 3)$, there are two $SO(3) \times SO(3)$ singlets corresponding to the non-compact generators

$$\tilde{Y}_1 = Y_{11} + Y_{22} - Y_{33}, \quad \tilde{Y}_2 = Y_{44} + Y_{55} - Y_{66}. \quad (83)$$

The coset representative can be parametrized as

$$L = e^{\phi_1 \tilde{Y}_1} e^{\phi_2 \tilde{Y}_2}. \quad (84)$$

In this case, the scalar potential is given by

$$\begin{aligned} V &= \frac{1}{8} e^{-\phi - 6\phi_1 - 6\phi_2} \left[2g_2 e^{\phi + 3\phi_1 + 9\phi_2} \right. \\ &\quad \times (e^{6\phi_1} g_1 - 3g_1 e^{2\phi_1} - \tilde{g}_1 + 3\tilde{g}_1 e^{4\phi_1}) \\ &\quad - 6g_2 e^{\phi + 3\phi_1 + 5\phi_2} (g_1 e^{6\phi_1} - 3g_1 e^{2\phi_1} - \tilde{g}_1 + 3\tilde{g}_1 e^{4\phi_1}) \\ &\quad + 6\tilde{g}_2 e^{\phi + 3\phi_1 + 7\phi_2} (3\tilde{g}_1 e^{4\phi_1} - 3g_1 e^{2\phi_1} + g_1 e^{6\phi_1} - \tilde{g}_1) \\ &\quad - 2\tilde{g}_2 e^{\phi + 3\phi_1 + 3\phi_2} (g_1 e^{6\phi_1} - 3g_1 e^{2\phi_1} - \tilde{g}_1 + 3\tilde{g}_1 e^{4\phi_1}) \\ &\quad + 3e^{6\phi_1 + 4\phi_2} [e^{4\phi_2} (2g_2^2 - \tilde{g}_2^2)(1 + \chi^2 e^{2\phi}) \\ &\quad - 3(g_2^2 - 2\tilde{g}_2^2)(1 + \chi^2 e^{2\phi})] \\ &\quad + g_2^2 e^{6\phi_1 + 6\phi_2} (1 + \chi^2 e^{2\phi}) + \tilde{g}_2^2 e^{6\phi_1} (1 + \chi^2 e^{2\phi}) \\ &\quad + e^{6\phi_2} \left[3(2g_1^2 - \tilde{g}_1^2) e^{2\phi + 8\phi_1} \right. \\ &\quad + 16e^{6\phi_1} [g_2 \tilde{g}_2 + e^{2\phi} (g_1 \tilde{g}_1 + g_2 \tilde{g}_2 \chi^2)] \\ &\quad \left. + g_1^2 e^{2\phi + 12\phi_1} \tilde{g}_1^2 e^{2\phi} - 3(g_1^2 - 2\tilde{g}_1^2) e^{\phi + 4\phi_1} \right] \left. \right]. \end{aligned} \quad (85)$$

This potential admits only one supersymmetric AdS_4 critical point at

$$\begin{aligned} \phi &= \frac{1}{2} \ln \left[\frac{g_1 \tilde{g}_1 (g_2^2 + \tilde{g}_2^2)^2 (g_1^2 + \tilde{g}_1^2)^2}{g_2 \tilde{g}_2} \right], \quad \chi = 0, \\ \phi_1 &= \frac{1}{2} \ln \left[-\frac{\tilde{g}_1}{g_1} \right], \quad \phi_2 = \frac{1}{2} \ln \left[-\frac{\tilde{g}_2}{g_2} \right]. \end{aligned} \quad (86)$$

Table 4 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(3) \times SO(3)$ symmetry and the corresponding dimensions of the dual operators for $SO(3, 1) \times SO(3, 1)$ gauge group

Scalar field representations	$m^2 L^2$	Δ
$(\mathbf{1}, \mathbf{1})$	-2×2	1, 2
$(\mathbf{1}, \mathbf{1})$	4×2	4
$(\mathbf{1}, \mathbf{5}) + (\mathbf{5}, \mathbf{1})$	-2×10	1, 2
$(\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{1})$	0×6	3
$(\mathbf{3}, \mathbf{3})$	0×18	3

This critical point preserves $N = 4$ supersymmetry and $SO(3) \times SO(3)$ symmetry. The latter is the maximal compact subgroup of $SO(3, 1) \times SO(3, 1)$ gauge group. Without loss of generality, we can shift the scalars such that the critical point occurs at $\phi = \phi_1 = \phi_2 = 0$. This can be achieved by setting

$$\tilde{g}_1 = -g_1, \quad \tilde{g}_2 = -g_2, \quad g_2 = -g_1. \quad (87)$$

With these values, the cosmological constant and AdS_4 radius are given by

$$V_0 = -6g_1^2 \quad \text{and} \quad L^2 = \frac{1}{2g_1^2}. \quad (88)$$

It should be noted that the choice $\tilde{g}_1 = -g_1$, $\tilde{g}_2 = -g_2$ and $g_2 = g_1$ makes the critical point at $\phi = \phi_1 = \phi_2 = \chi = 0$ a dS_4 with $V_0 = 2g_1^2$.

At the $N = 4$ AdS_4 critical point, the gauge group $SO(3, 1) \times SO(3, 1)$ is broken down to its maximal compact subgroup $SO(3) \times SO(3)$. All scalar masses at this critical point are given in Table 4. The two singlet representations $(\mathbf{1}, \mathbf{1})$ corresponding to ϕ_1 and ϕ_2 are dual to irrelevant operators of dimensions $\Delta = 4$, and six massless scalars in representation $(\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{1})$ are Goldstone bosons.

4.2 RG flows without vector multiplet scalars

Since there is only one supersymmetric AdS_4 critical point, there is no supersymmetric RG flow between the dual SCFTs. In this case, we instead consider RG flows from the SCFT dual to the $N = 4$ AdS_4 vacuum with $SO(3) \times SO(3)$ symmetry. We begin with a simple truncation to scalar fields in the supergravity multiplet obtained by setting $\phi_1 = \phi_2 = 0$. Within this truncation, the superpotential is given by

$$\mathcal{W} = \frac{3i}{2\sqrt{2}} g_1 e^{-\frac{\phi}{2}} [1 + e^{\phi} (1 - i\chi)] \quad (89)$$

in term of which the scalar potential can be written as

$$V = \frac{16}{9} \left(\frac{\partial W}{\partial \phi} \right)^2 + \frac{16}{9} e^{-2\phi} \left(\frac{\partial W}{\partial \chi} \right)^2 - \frac{4}{3} W^2 \\ = -g_1^2 e^{-\phi} [1 + 4e^{\phi} + e^{2\phi} (1 + \chi^2)]. \quad (90)$$

The flow equations obtained from $\delta\chi^i = 0$ conditions are given by

$$\phi' = -\frac{8}{3} \frac{\partial W}{\partial \phi} = -\frac{\sqrt{2} g_1 e^{-\frac{\phi}{2}} [e^{2\phi} (1 + \chi^2) - 1]}{\sqrt{(1 + e^{\phi})^2 + e^{2\phi} \chi^2}}, \quad (91)$$

$$\chi' = -\frac{8}{3} e^{-2\phi} \frac{\partial W}{\partial \chi} = -\frac{2\sqrt{2} g_1 e^{-\frac{\phi}{2}} \chi}{\sqrt{(1 + e^{\phi})^2 + e^{2\phi} \chi^2}}. \quad (92)$$

The BPS conditions from $\delta\lambda_a^i = 0$ are, of course, identically satisfied by setting $\phi_1 = \phi_2 = 0$.

The flow equation for the metric function is simply given by

$$A' = W = \frac{3}{2\sqrt{2}} g_1 e^{-\frac{\phi}{2}} \sqrt{(1 + e^{\phi})^2 + e^{2\phi} \chi^2}. \quad (93)$$

Near the AdS_4 critical point, we find

$$\phi \sim \chi \sim e^{-\sqrt{2} g_1 r} \sim e^{-\frac{r}{L}} \quad (94)$$

as expected for the dual operators of dimensions $\Delta = 1, 2$.

Apart from some numerical factors involving gauge coupling constants, the structure of the resulting BPS equations are very similar to the $SO(4) \times SO(4)$ case. We therefore only give the solution without going into any details here

$$\phi = -\frac{1}{2} \ln(1 - \chi^2 - 2C\chi), \quad (95)$$

$$A = -\frac{3}{2} \ln \chi + \frac{3}{8} \ln(1 - 2C\chi - \chi^2) \\ + \frac{3}{4} \ln(1 - C\chi + \sqrt{1 - 2C\chi - \chi^2}). \quad (96)$$

As in the $SO(4) \times SO(4)$ case, we are able to solve for χ only numerically. An example of solutions for χ is shown in Fig. 4. From the figure, it can be readily seen that, along the flow, χ interpolates between $\chi = 0$ and $\chi_0 = -C \pm \sqrt{1 + C^2}$. The ϕ solution, on the other hand, interpolates between $\phi = 0$ and $\phi \rightarrow \infty$ as can be seen from the solution (95). The singularity $\phi \rightarrow \infty$ also gives rise to $V \rightarrow -\infty$ for any value of χ_0 . Therefore, the singularity is physical, and the solution describes an RG flow from the $N = 4$ SCFT in the UV with $SO(3) \times SO(3)$ symmetry to a non-conformal field theory in the IR corresponding to this singularity.

4.3 RG flows with vector multiplet scalars

We now consider solutions with non-vanishing vector multiplet scalars. In this case, we need to set $\chi = 0$ in order to

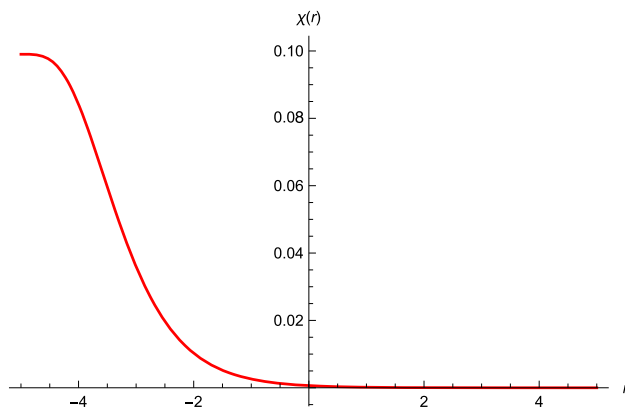


Fig. 4 Solution for χ with $\chi_0 = \sqrt{1 + C^2} - C$ for $C = 5$ and $g_1 = 1$ in $SO(3, 1) \times SO(3, 1)$ gauging

make the solutions of the BPS equations solve the second-order field equations as in the case of $SO(4) \times SO(4)$ gauging. The corresponding BPS equations are given by

$$\phi'_1 = \sqrt{2}g_1 e^{\frac{\phi}{2}} \cosh(2\phi_1) \sinh \phi_1, \quad (97)$$

$$\phi'_2 = \sqrt{2}g_1 e^{-\frac{\phi}{2}} \cosh(2\phi_2) \sinh \phi_2, \quad (98)$$

$$\phi' = \sqrt{2}g_1 e^{-\frac{\phi}{2}} [e^{\phi} \cosh \phi_1 (\cosh(2\phi_1) - 2) - \cosh \phi_2 (\cosh(2\phi_2) - 2)], \quad (99)$$

$$A' = \frac{1}{\sqrt{2}} g_1 e^{-\frac{\phi}{2}} [e^{\phi} \cosh \phi_1 (\cosh(2\phi_1) - 2) + \cosh \phi_2 (\cosh(2\phi_2) - 2)]. \quad (100)$$

With suitable boundary conditions, these equations can be solved numerically as in the previous cases. We will, however, look at particular truncations for which analytic solutions can be found. These solutions should be more interesting and more useful than the numerical ones in many aspects.

The first truncation is obtained by setting $\phi_2 = \phi_1$ and $\phi = 0$. It can be easily verified that this is a consistent truncation. The relevant BPS equations read

$$\phi'_1 = \sqrt{2}g_1 \cosh(2\phi_1) \sinh \phi_1, \quad (101)$$

$$A' = \sqrt{2}g_1 \cosh \phi_1 [\cosh(2\phi_1) - 2] \quad (102)$$

which have a solution

$$2g_1 r = \ln \left[\frac{1 - \sqrt{2} \cosh \phi_1}{1 + \sqrt{2} \cosh \phi_1} \right] - 2\sqrt{2} \tanh^{-1} e^{\phi_1}, \quad (103)$$

$$A = \ln(1 + e^{4\phi_1}) - \ln(1 - e^{2\phi_1}) - \phi_1. \quad (104)$$

The solution for ϕ_1 is clearly seen to be singular at a finite value of r .

Recall that ϕ_1 and ϕ_2 are dual to irrelevant operators, we expect that in this case, the $N = 4$ SCFT should appear in the IR. Near the singularity, we find

$$\phi_1 \sim \pm \frac{1}{3} \ln \left[C - \frac{3g_1 r}{2\sqrt{2}} \right] \quad \text{and} \quad A \sim -\frac{1}{3} \ln \left[C - \frac{3g_1 r}{2\sqrt{2}} \right] \quad (105)$$

for a constant C . It can be verified that, in this limit, the scalar potential blows up as $V \rightarrow \infty$. Therefore, the singularity is unphysical.

Another truncation is obtained by setting $\phi_2 = 0$ which gives rise to the BPS equations

$$\phi'_1 = \sqrt{2}g_1 e^{\frac{\phi}{2}} \cosh(2\phi_1) \sinh \phi_1, \quad (106)$$

$$\phi' = \sqrt{2}g_1 e^{-\frac{\phi}{2}} (1 + e^{\phi} \cosh \phi_1) [\cosh(2\phi_1) - 2], \quad (107)$$

$$A' = \frac{1}{\sqrt{2}} g_1 e^{-\frac{\phi}{2}} [e^{\phi} \cosh \phi_1 (\cosh(2\phi_1) - 2) - 1]. \quad (108)$$

An analytic solution to these equations is given by

$$\phi = \ln \left[\cosh \phi_1 - \frac{1}{2} C \cosh(2\phi_1) \operatorname{csch} \phi_1 \right], \quad (109)$$

$$\sqrt{2}g_1 \tilde{r} = \ln[C - \tanh(2\phi_1)], \quad (110)$$

$$A = \ln[\cosh(2\phi_1)] - \frac{1}{2} \ln(\sinh \phi_1) - \frac{1}{2} \ln[C \cosh(2\phi_1) - \sinh(2\phi_1)] \quad (111)$$

where the coordinate $\tilde{r}$ is defined via $\frac{d\tilde{r}}{dr} = e^{-\frac{\phi}{2}}$. It should be noted that to give the correct behavior for ϕ and ϕ_1 near the AdS_4 critical point, we need $C \neq 0$.

The solution is singular at a finite value of $\tilde{r}$. Near this singularity, we find

$$\phi_1 \sim \pm \frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right| \quad (112)$$

where $\tilde{C}$ is a constant. The behavior of ϕ and A depends on the value of C .

We begin with the case $\phi_1 \rightarrow \infty$. For $C = 2$, we find from the explicit solution that

$$\phi \sim -\phi_1 \sim \frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|, \quad A \sim \phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|. \quad (113)$$

For $C \neq 2$, we find

$$\phi \sim \phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|, \quad A \sim \phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|. \quad (114)$$

Both of these singularities lead to $V \rightarrow \infty$ and hence are unphysical.

We now move to another possibility with $\phi_1 \rightarrow -\infty$. In this case, we find

$$\phi \sim \phi_1 \sim \frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|,$$

$$A \sim -\phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2} g_1 \tilde{r} - \tilde{C} \right| \quad (115)$$

for $C = -2$ and

$$\begin{aligned} \phi &\sim -\phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2} g_1 \tilde{r} - \tilde{C} \right|, \\ A &\sim -\phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2} g_1 \tilde{r} - \tilde{C} \right| \end{aligned} \quad (116)$$

for $C \neq -2$. These behaviors also give $V \rightarrow \infty$. Therefore, we conclude that the solutions in this particular truncation do not holographically describe RG flows from $N = 4$ SCFT.

A similar analysis shows that the truncation with $\phi_1 = 0$ also leads to unphysical singularities. It would be interesting to uplift these solutions to ten or eleven dimensions and determine whether these singularities are resolved.

5 $N = 4$ $SO(4) \times SO(3, 1)$ gauged supergravity

In this section, we consider a gauge group with one compact and one non-compact factors of the form $SO(4) \times SO(3, 1)$. All the procedures are essentially the same, so we will not present much detail here. The $SO(4)$ and $SO(3, 1)$ are electrically and magnetically embedded in $SO(3, 3) \times SO(3, 3)$, respectively. The corresponding embedding tensor is given by

$$\begin{aligned} f_{+123} &= \sqrt{2}(g_1 - \tilde{g}_1), \quad f_{+789} = \sqrt{2}(g_1 + \tilde{g}_1), \\ f_{-456} &= f_{-4,11,12} = f_{-10,5,12} = -f_{-10,11,6} \\ &= \frac{1}{\sqrt{2}}(g_2 - \tilde{g}_2), \\ f_{-10,11,12} &= f_{-4,11,6} = f_{-10,5,6} = -f_{-45,12} \\ &= \frac{1}{\sqrt{2}}(g_2 + \tilde{g}_2). \end{aligned} \quad (117)$$

5.1 Supersymmetric AdS_4 vacua

We consider scalar fields invariant under $SO(4)_{\text{inv}} \subset SO(4) \times SO(3) \subset SO(4) \times SO(3, 1)$. The corresponding coset representative for the $SO(6, 6)/SO(6) \times SO(6)$ coset is now given by

$$L = e^{\phi_1 \hat{Y}} e^{\phi_2 \tilde{Y}_2} \quad (118)$$

where $\hat{Y}_1$ and $\tilde{Y}_2$ are defined in (27) and (83), respectively.

The scalar potential turns out to be

$$\begin{aligned} V &= \frac{1}{8} e^{-\phi - 6\phi_1 - 6\phi_2} [(g_1 + g_2)^2 e^{2\phi + 12\phi_1 + 6\phi_2} \\ &\quad - 3(3g_1^2 + 2g_1 g_2 + g_2^2) e^{2\phi + 4\phi_1 + 6\phi_2} \\ &\quad + e^{6\phi_1} [g_2^2 (1 + e^{2\phi} \chi^2) (1 + e^{4\phi_2})^3 \\ &\quad + 16e^{6\phi_2} [e^{2\phi} (g_1^2 + g_1 g_2 - g_2^2 \chi^2) - g_2^2]] \\ &\quad + 8g_2 e^{\phi + 3\phi_1 + 6\phi_2} [g_1 (e^{2\phi_1} - 1)^3 \end{aligned}$$

Table 5 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(4) \times SO(3)$ symmetry and the corresponding dimensions of the dual operators for $SO(4) \times SO(3, 1)$ gauge group

Scalar field representations	$m^2 L^2$	Δ
$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$-2_{\times 2}$	1, 2
$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	4	4
$(\mathbf{1}, \mathbf{1}, \mathbf{3})$	$0_{\times 3}$	3
$(\mathbf{3}, \mathbf{3}, \mathbf{1})$	$0_{\times 9}$	3
$(\mathbf{1}, \mathbf{1}, \mathbf{5})$	$-2_{\times 5}$	1, 2
$(\mathbf{1}, \mathbf{3}, \mathbf{3}) + (\mathbf{3}, \mathbf{1}, \mathbf{3})$	$-2_{\times 18}$	1, 2

$$\begin{aligned} &+ g_2 e^{2\phi_1} (3 + e^{4\phi_1})] \cosh \phi_2 \\ &\times [\cosh(2\phi_2) - 2] - 3(3g_1^2 + 4g_1 g_2 + 2g_2^2) \\ &\times e^{2\phi + 8\phi_1 + 6\phi_2} + g_1^2 e^{2\phi + 6\phi_2} \end{aligned} \quad (119)$$

where we have imposed the following relations

$$\tilde{g}_1 = g_1 + g_2 \quad \text{and} \quad \tilde{g}_2 = -g_2 \quad (120)$$

in order to have an $N = 4$ supersymmetric AdS_4 critical point with $SO(4) \times SO(3)$ symmetry at $\phi_1 = \phi_2 = \phi = \chi = 0$.

There are two supersymmetric AdS_4 vacua with $N = 4$ supersymmetry:

- The first critical point is a trivial one with $SO(4) \times SO(3)$ symmetry at

$$\phi = \chi = \phi_1 = \phi_2 = 0, \quad V_0 = -6g_2^2. \quad (121)$$

- A non-trivial supersymmetric critical point is given by

$$\begin{aligned} \phi_2 = \chi = 0, \quad \phi_1 &= \frac{1}{2} \ln \left[\frac{g_1}{g_1 + g_2} \right], \\ \phi &= \frac{1}{2} \ln \left[\frac{4g_1(g_1 + g_2)}{(2g_1 + g_2)^2} \right], \quad V_0 = -\frac{3g_2^2(2g_1 + g_2)}{\sqrt{g_1(g_1 + g_2)}}. \end{aligned} \quad (122)$$

This critical point is invariant under a smaller symmetry $SO(3) \times SO(3)$.

Scalar masses at these two critical points are given in Tables 5 and 6. It can be seen that the mass spectra are very similar to critical points III and IV in the case of $SO(4) \times SO(4)$ gauge group.

5.2 Holographic RG flow

In this section, we will give a supersymmetric RG flow solution interpolating between the two AdS_4 vacua identified

Table 6 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(3) \times SO(3)$ symmetry and the corresponding dimensions of the dual operators for $SO(4) \times SO(3, 1)$ gauge group

Scalar field representations	$m^2 L^2$	Δ
(1, 1)	-2×2	1, 2
(1, 1)	4×2	4
(1, 5) + (5, 1)	-2×10	1, 2
(1, 3) + (3, 1)	0×6	3
(3, 3)	0×18	3

above. As in the previous cases, turning on vector multiplet scalars requires the vanishing of the axion χ . Since we are only interested in the solution interpolating between two AdS_4 vacua, we will accordingly set $\chi = 0$ from now on.

With $\chi = 0$, the superpotential is given by

$$\mathcal{W} = \frac{i}{4\sqrt{2}} e^{-\frac{\phi}{2} - 3\phi_1 - 3\phi_2} \left[g_1 e^{\phi + 3\phi_2} + 3g_1 e^{\phi + 4\phi_1 + 3\phi_2} - 3(g_1 + g_2) e^{\phi + 2\phi_1 + 3\phi_2} + g_2 e^{3\phi_1} (1 + e^{2\phi_2}) (1 - 4e^{2\phi_2} + e^{4\phi_2}) - (g_1 + g_2) e^{\phi + 6\phi_1 + 3\phi_2} \right] \quad (123)$$

in term of which the scalar potential can be written as

$$V = 4 \left(\frac{\partial W}{\partial \phi} \right)^2 + \frac{2}{3} \left(\frac{\partial W}{\partial \phi_1} \right)^2 + \frac{2}{3} \left(\frac{\partial W}{\partial \phi_2} \right)^2 - 3W^2. \quad (124)$$

The BPS equations read

$$\phi'_1 = -\frac{2}{3} \frac{\partial W}{\partial \phi_1} = -\frac{1}{2\sqrt{2}} e^{\frac{\phi}{2} - 3\phi_1} (e^{4\phi_1} - 1) (e^{2\phi_1} (g_1 + g_2) - g_1), \quad (125)$$

$$\phi'_2 = -\frac{2}{3} \frac{\partial W}{\partial \phi_2} = \frac{1}{2\sqrt{2}} g_2 e^{-\frac{\phi}{2} - 3\phi_1} (e^{2\phi_2} - 1) (e^{4\phi_1} + 1), \quad (126)$$

$$\phi' = -4 \frac{\partial W}{\partial \phi} = -\frac{1}{2\sqrt{2}} e^{-\frac{\phi}{2} - 3\phi_1} \left[4g_2 e^{3\phi_1} \cosh \phi_2 [\cosh(2\phi_2) - 2] + e^{\phi} [(e^{2\phi_1} - 1)^3 g_1 + e^{2\phi_1} (3 + e^{4\phi_1}) g_2] \right], \quad (127)$$

$$A' = \frac{1}{4} \sqrt{2} e^{-\frac{\phi}{2} - 3\phi_1} \left[e^{\phi} [(e^{2\phi_1} - 1)^3 g_1 + e^{2\phi_1} (3 + e^{4\phi_1}) g_2] - 4g_2 e^{3\phi_1} \cosh \phi_2 [\cosh(2\phi_2) - 2] \right]. \quad (128)$$

Since $\phi_2 = 0$ at both critical points, we can consistently truncate ϕ_2 out. Note also that ϕ_2 is dual to an irrelevant operator of dimension $\Delta = 4$ as can be seen from the linearized BPS equations which give

$$\phi \sim \phi_1 \sim e^{-\frac{r}{L}}, \quad \phi_2 \sim e^{\frac{r}{L}}. \quad (129)$$

With $\phi_2 = 0$, we find an RG flow solution driven by ϕ and ϕ_1 as follow

$$g_2(2g_1 + g_2)\tilde{r} = \sqrt{2}g_2 \tan^{-1} e^{\phi_1} + \sqrt{2}(2g_1 + g_2) \tanh^{-1} e^{\phi_1} - 2\sqrt{2g_1(g_1 + g_2)} \tanh^{-1} \left[e^{\phi_1} \sqrt{\frac{g_1 + g_2}{g_1}} \right], \quad (130)$$

$$\phi = \ln \left[\frac{e^{-\phi_1} g_1 + e^{\phi_1} (g_1 + g_2)}{2g_1 + g_2} \right], \quad (131)$$

$$A = \frac{1}{2} \phi_1 - \ln(1 - e^{4\phi_1}) + \ln[(e^{2\phi_1} - 1)g_1 + e^{2\phi_1} g_2] + \frac{1}{2} \ln[g_1 + (g_1 + g_2)e^{2\phi_1}] \quad (132)$$

where the coordinate $\tilde{r}$ is related to r by the relation $\frac{d\tilde{r}}{dr} = e^{\frac{\phi}{2}}$.

This solution preserves $N = 4$ supersymmetry in three dimensions and describes an RG flow from $N = 4$ SCFT in the UV with $SO(4) \times SO(3)$ symmetry to another $N = 4$ SCFT in the IR with $SO(3) \times SO(3)$ symmetry at which the operator dual to ϕ_1 is irrelevant. Although the number of supersymmetry is unchanged, the flavor symmetry $SO(3)$ in the UV is broken by the operator dual to ϕ_1 . We can also truncate out the vector multiplet scalars and study supersymmetric RG flows to non-conformal field theories as in the previous cases. However, we will not consider this truncation since it leads to similar structure as in the previous two gauge groups.

6 Conclusions and discussions

We have studied dyonic gaugings of $N = 4$ supergravity coupled to six vector multiplets with compact and non-compact gauge groups $SO(4) \times SO(4)$, $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$. We have identified a number of supersymmetric $N = 4$ AdS_4 vacua within these gauged supergravities and studied several RG flows interpolating between these vacua. The solutions describe supersymmetric deformations of the dual $N = 4$ SCFTs with different flavor symmetries in three dimensions. These deformations are driven by relevant operators of dimensions $\Delta = 1, 2$ which deform the UV SCFTs to other SCFTs or to non-conformal field theories in the IR.

For $SO(4) \times SO(4)$ gauge group, there are four supersymmetric AdS_4 vacua with $SO(4) \times SO(4)$, $SO(4) \times SO(3)$, $SO(3) \times SO(4)$ and $SO(4)$ symmetries. These vacua should correspond to $N = 4$ conformal fixed points of $N = 4$ CSM theories with $SO(4)$, $SO(3)$ and no flavor symmetries, respectively. We have found various RG flows interpolating between these critical points including RG flows connecting three critical points or a cascade of RG flows. These should be useful in holographic studies of $N = 4$ CSM theories.

In the case of non-compact $SO(3, 1) \times SO(3, 1)$ gauge group, we have found only one supersymmetric AdS_4 vacuum with $SO(3) \times SO(3)$ symmetry. We have given a number of RG flow solutions describing supersymmetric deformations of the dual $N = 4$ SCFT to $N = 4$ non-conformal field theories. The solutions with only scalar fields from the gravity multiplet non-vanishing give rise to physical singularities. Flows with vector multiplet scalars turned on, however, lead to physically unacceptable singularities. The mixed gauge group $SO(4) \times SO(3, 1)$ also exhibits similar structure of vacua and RG flows with two supersymmetric AdS_4 critical points.

Given our solutions, it is interesting to identify their higher dimensional origins in ten or eleven dimensions. Along this line, the result of [36, 37] on $S^3 \times S^3$ compactifications could be a useful starting point for the $SO(4) \times SO(4)$ gauge group. The uplifted solutions would be desirable for a full holographic study of $N = 4$ CSM theories. This should provide an analogue of the recent uplifts of the GPPZ flow describing a massive deformation of $N = 4$ SYM [38, 39]. The embedding of the non-compact gauge groups $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$ would also be worth considering.

Another direction is to find interpretations of the solutions given here in the dual $N = 4$ CSM theories with different flavor symmetries similar to the recent study in [18] for AdS_5/CFT_4 correspondence. The results found here is also in line with [18]. In particular, scalars in the gravity multiplet are dual to relevant operators at all critical points. These operators are in the same multiplet as the energy-momentum tensor. Another result is the exclusion between the operators dual to the axion and vector multiplet scalars which cannot be turned on simultaneously as required by supersymmetry in the gravity solutions. It would be interesting to find an analogous result on the field theory side.

A generalization of the present results to include more active scalars with smaller residual symmetries could provide more general holographic RG flow solutions in particular flows that break some amount of supersymmetry. We have indeed performed a partial analysis for $SO(3)_{\text{inv}}$ scalars. In this case, there are six singlets. It seems to be possible to have solutions that break supersymmetry from $N = 4$ to $N = 1$, but the scalar potential takes a highly complicated form. Therefore, we refrain from presenting it here. Solutions from other gauge groups more general than those considered here also deserve investigations. Finally, finding other types of solutions such as supersymmetric Janus and flows across dimensions to $AdS_2 \times \Sigma_2$, with Σ_2 being a Riemann surface, would also be useful in the holographic study of defect SCFTs and black hole physics. Recent works along this line include [20, 40–45].

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Appendix A: Useful formulae

To convert an $SO(6)$ vector index m to a pair of anti-symmetric $SU(4)$ fundamental indices $[ij]$, we use the following 't Hooft symbols

$$\begin{aligned} G_1^{ij} &= \begin{bmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix}, & G_2^{ij} &= \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix}, \\ G_3^{ij} &= \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, & G_4^{ij} &= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \\ G_5^{ij} &= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, & G_6^{ij} &= \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (133)$$

These matrices satisfy the relation

$$G_{mij} = -\frac{1}{2}\epsilon_{ijkl}G_m^{kl} = -(G_m^{ij})^*. \quad (134)$$

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Supersymmetric deformations of 3D SCFTs from tri-Sasakian truncation

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Abstract We holographically study supersymmetric deformations of $N = 3$ and $N = 1$ superconformal field theories in three dimensions using four-dimensional $N = 4$ gauged supergravity coupled to three-vector multiplets with non-semisimple $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ gauge group. This gauged supergravity can be obtained from a truncation of 11-dimensional supergravity on a tri-Sasakian manifold and admits both $N = 1, 3$ supersymmetric and stable non-supersymmetric AdS_4 critical points. We analyze the BPS equations for $SO(3)$ singlet scalars in detail and study possible supersymmetric solutions. A number of RG flows to non-conformal field theories and half-supersymmetric domain walls are found, and many of them can be given analytically. Apart from these “flat” domain walls, we also consider AdS_3 -sliced domain wall solutions describing two-dimensional conformal defects with $N = (1, 0)$ supersymmetry within the dual $N = 1$ field theory while this type of solutions does not exist in the $N = 3$ case.

1 Introduction

In recent years, superconformal field theories (SCFTs) in three dimensions have attracted much attention in the context of the AdS/CFT correspondence [1]. Apart from being effective world-volume theories of M2-branes [2, 3], three-dimensional gauge theories and their conformal fixed points have also interesting applications in condensed matter physics [4–6].

Along this line, four-dimensional gauged supergravities have been a very useful tool in various holographic studies including the holographic Renormalization Group (RG) flows and conformal defects of co-dimension one. The former can be described holographically by domain walls interpolating between two AdS vacua or between an AdS vacuum in one limit and a domain wall in the other limit; see for

example [7–9]. These two classes of solutions correspond, respectively, to RG flows between conformal fixed points and flows to non-conformal field theories. These solutions are called “flat” or Minkowski-sliced domain walls. The conformal defects on the other hand can be described in the holographic context by AdS-sliced domain walls [10–16].

A number of holographic RG flows within four-dimensional gauged supergravities have been studied; see for example [17–21] and [22–24] for more recent results. Some of these solutions can be uplifted to 11 dimensions resulting in many interesting geometric interpretations such as a polarization of M2-branes into M5-branes in [25]. On the other hand, supersymmetric Janus solutions in four dimensions have been studied recently in the maximal $N = 8$, $SO(8)$ gauged supergravity in [26]. Some of these solutions have been uplifted to 11 dimensions via a consistent S^7 reduction in [25]. In the context of lower supersymmetry, a number of supersymmetric Janus solutions within $N = 3$, $SU(2) \times SU(3)$ gauged supergravity have been explored in [27]. This gauged supergravity is expected to describe the lowest Kaluza–Klein modes of a compactification of M-theory on a tri-Sasakian manifold N^{010} [28]. The gauge group $SU(2) \times SU(3)$ is an isometry of N^{010} , and the two factors are identified with the $N = 3$ $SO(3)_R$ R-symmetry and $SU(3)$ flavor symmetry in the dual SCFT, respectively.

The complete spectrum of this compactification has been carried out in [29], and the structure of the supermultiplets has been given in [30]. Furthermore, the dual SCFT to this compactification has been proposed in [31]. It has also been discovered in [31] and further investigated in [32] that all compactifications of M-theory giving rise to $N = 3$ supersymmetric AdS_4 backgrounds contain a universal massive spin- $\frac{3}{2}$ multiplet. All components of this multiplet arise only from constant harmonics. The truncation keeping only the lowest Kaluza–Klein modes and this massive multiplet is

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accordingly expected to be consistent. The resulting theory is expected to be $N = 4$ gauged supergravity with $N = 4$ supersymmetry broken to $N = 3$ at the vacuum. The dual composite operators to this long, massive, gravitino multiplet have also been proposed in [31, 32].

Up to now, only the complete truncation of 11-dimensional supergravity on a generic tri-Sasakian manifold has been carried out in [33] in which all the fields which are singlet under the flavor $SU(3)$ symmetry have been kept. The enhancement by the Betti vector multiplet, which is also an $SU(3)$ singlet, in the compactification on N^{010} has also been pointed out. This is due to a non-trivial cohomology of degree two in N^{010} giving rise to an additional massless vector multiplet.

This tri-Sasakian truncation results in $N = 4$ gauged supergravity coupled to three-vector multiplets. The theory admits two supersymmetric AdS_4 solutions with unbroken $SO(3)_R$ R-symmetry and $N = 3, 1$ supersymmetries. These solutions correspond to compactifications on N^{010} and its squashed version, respectively. A possible candidate for the $N = 3$ SCFT dual to the $N = 3$ solution is given in [31], but there is a puzzle with this SCFT as regards the baryonic spectrum; see the discussion in [34] and [35]. For the $N = 1$ case, the situation is less clear. In particular, the $N = 1$ SCFT dual to the squashed $N = 1$, $AdS_4 \times N^{010}$ solution has not previously appeared although the $N = 1$ SCFT dual to the squashed S^7 compactification has been given in [36]. In this paper, we will analyze the BPS equations for $SO(3)_R$ invariant scalar fields and investigate possible deformations of the dual $N = 3$ and $N = 1$ SCFTs within the framework of four-dimensional gauged supergravity.

We will mainly consider supersymmetric deformations in the forms of RG flows to non-conformal field theories and two-dimensional defects described by Janus solutions. Regarding to the N^{010} compactification, a number of holographic RG flows and Janus solutions within the framework of $N = 3$ gauged supergravity have already been studied in [27, 37], but these solutions currently cannot be uplifted to 11 dimensions due to the lack of the complete consistent truncation keeping all lowest Kaluza–Klein modes including the $SU(3)$ non-singlet ones.

The paper is organized as follows. In Sect. 2, we review $N = 4$ gauged supergravity coupled to three-vector multiplets and the tri-Sasakian truncation of 11-dimensional supergravity to this $N = 4$ gauged supergravity. The analysis of BPS equations for $SO(3)_R$ singlet scalars will also be carried out. These are relevant for finding supersymmetric RG flow and Janus solutions in Sects. 3 and 4. We will also explicitly give the uplift of some solutions to 11 dimensions and finally give some conclusions and comments on the results in Sect. 5. In the two appendices, we give an explicit form of the relevant field equations and some of the complicated BPS equations.

2 $N = 4$ gauged supergravity and tri-Sasakian truncation of 11-dimensional supergravity

In this section, we briefly review $N = 4$ gauged supergravity in the embedding tensor formalism to set up the framework for finding supersymmetric solutions. Further details of the construction can be found in [38] on which this review is mainly based. We will also give basic information and relevant formulas of the tri-Sasakian truncation of 11-dimensional supergravity to $N = 4$ gauged supergravity with $SO(3) \times (T^3, \hat{T}^3)$ gauge group. This is the strategy we will follow in order to uplift four-dimensional solutions to 11 dimensions.

2.1 $N = 4$ gauged supergravity coupled to three-vector multiplets

We now consider the half-maximal $N = 4$ supergravity in four dimensions. The supergravity multiplet consists of the graviton $e_{\mu}^{\hat{\mu}}$, four gravitini ψ_{μ}^i , six vectors A_{μ}^m , four spin- $\frac{1}{2}$ fields χ^i and one complex scalar τ . The complex scalar, or equivalently two real scalars, can be parametrized by the $SL(2, \mathbb{R})/SO(2)$ coset.

In this half-maximal supersymmetry, the supergravity multiplet can couple to an arbitrary number n of vector multiplets although we will later set $n = 3$. Each multiplet contains a vector field A_{μ} , four gaugini λ^i and six scalars ϕ^m . The scalar fields can be parametrized by the $SO(6, n)/SO(6) \times SO(n)$ coset. Before moving to possible gaugings of this matter-coupled supergravity, we will first give some details as regards various indices used throughout this paper.

Space-time and tangent space indices are denoted respectively by $\mu, \nu, \dots = 0, 1, 2, 3$ and $\hat{\mu}, \hat{\nu}, \dots = 0, 1, 2, 3$. The $SO(6) \sim SU(4)$ R-symmetry indices will be described by $m, n = 1, \dots, 6$ for the $SO(6)$ vector representation and $i, j = 1, 2, 3, 4$ for the $SO(6)$ spinor or $SU(4)$ fundamental representations. The n vector multiplets will be labeled by indices $a, b = 1, \dots, n$. Therefore, all the fields in the vector multiplets will carry an additional index in the form of $(A_{\mu}^a, \lambda^{ia}, \phi^{ma})$. All fermionic fields and the supersymmetry parameters transform in the fundamental representation of $SU(4)_R \sim SO(6)_R$ R-symmetry and are subject to the chirality projections

$$\gamma_5 \psi_{\mu}^i = \psi_{\mu}^i, \quad \gamma_5 \chi^i = -\chi^i, \quad \gamma_5 \lambda^i = \lambda^i. \quad (1)$$

Similarly, for the corresponding fields transforming in the anti-fundamental representation of $SU(4)_R$, we have

$$\gamma_5 \psi_{\mu i} = -\psi_{\mu i}, \quad \gamma_5 \chi_i = \chi_i, \quad \gamma_5 \lambda_i = -\lambda_i. \quad (2)$$

Gaugings of the matter-coupled $N = 4$ supergravity can be efficiently described by using the embedding tensor Θ . This constant tensor encodes the information as regards the embedding of any gauge group G_0 in the global or duality symmetry $SL(2, \mathbb{R}) \times SO(6, n)$ in a covariant way. It has been shown in [38] that there are two components of the embedding tensor $\xi^{\alpha M}$ and $f_{\alpha MNP}$ with $\alpha = (+, -)$ and $M, N = (m, a) = 1, \dots, n+6$ denoting fundamental representations of $SL(2, \mathbb{R})$ and $SO(6, n)$, respectively. The electric vector fields $A^{+M} = (A_\mu^m, A_\mu^a)$, appearing in the ungauged Lagrangian, and their magnetic dual A^{-M} form a doublet under $SL(2, \mathbb{R})$ denoted by $A^{\alpha M}$.

In general, a subgroup of both $SL(2, \mathbb{R})$ and $SO(6, n)$ can be gauged, and the magnetic vector fields can also participate in the gauging. In particular, it has been shown in [39], see also [40], that purely electric gaugings do not admit AdS_4 vacua. In this paper, we will only consider gaugings involving both electric and magnetic vector fields in order to obtain AdS_4 vacua relevant for applications in the AdS/CFT correspondence.

The full covariant derivative can be written as

$$D_\mu = \nabla_\mu - g A_\mu^{\alpha M} \Theta_{\alpha M}^{NP} t_{NP} + g A_\mu^{M(\alpha} \epsilon^{\beta)\gamma} \xi_{\gamma M} t_{\alpha\beta} \quad (3)$$

where ∇_μ is the usual space-time covariant derivative. t_{MN} and $t_{\alpha\beta}$ are $SO(6, n)$ and $SL(2, \mathbb{R})$ generators which can be chosen as

$$(t_{MN})_P^Q = 2\delta_{[M}^Q \eta_{N]P}, \quad (t_{\alpha\beta})_\gamma^\delta = 2\delta_{(\alpha}^\delta \epsilon_{\beta)\gamma} \quad (4)$$

with $\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}$ and $\epsilon^{+-} = 1$. $\eta_{MN} = \text{diag}(-1, -1, -1, -1, -1, -1, 1, \dots, 1)$ is the $SO(6, n)$ invariant tensor, and g is the gauge coupling constant that can be absorbed in the embedding tensor Θ . The embedding tensor appearing in the above equation can be written in terms of $\xi^{\alpha M}$ and $f_{\alpha MNP}$ as

$$\theta_{\alpha MNP} = f_{\alpha MNP} - \xi_{\alpha[N} \eta_{P]M}. \quad (5)$$

In the following discussions, we will only consider solutions with only the metric and scalars non-vanishing. Therefore, we will set all of the vector fields to zero from now on.

We now consider explicit parametrization of the scalar coset manifold $SL(2, \mathbb{R})/SO(2) \times SO(6, n)/SO(6) \times SO(n)$. The first factor can be described by a coset representative

$$\mathcal{V}_\alpha = \frac{1}{\sqrt{\text{Im}\tau}} \begin{pmatrix} \tau \\ 1 \end{pmatrix} \quad (6)$$

or equivalently by a symmetric matrix

$$M_{\alpha\beta} = \text{Re}(\mathcal{V}_\alpha \mathcal{V}_\beta^*) = \frac{1}{\text{Im}\tau} \begin{pmatrix} |\tau|^2 & \text{Re}\tau \\ \text{Re}\tau & 1 \end{pmatrix}. \quad (7)$$

Note that $\text{Im}(\mathcal{V}_\alpha \mathcal{V}_\beta^*) = \epsilon_{\alpha\beta}$. The complex scalar τ can in turn be written in terms of the dilaton ϕ and the axion χ as

$$\tau = \chi + ie^\phi. \quad (8)$$

For the $SO(6, n)/SO(6) \times SO(n)$ factor, we introduce the coset representative $\mathcal{V}_M^A$ transforming by a left and right multiplication under $SO(6, n)$ and $SO(6) \times SO(n)$, respectively. We will split the $SO(6) \times SO(n)$ index $A = (m, a)$ and write the coset representative as $\mathcal{V}_M^A = (\mathcal{V}_M^m, \mathcal{V}_M^a)$. Being an element of $SO(6, n)$, the matrix $\mathcal{V}_M^A$ satisfies the relation

$$\eta_{MN} = -\mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \quad (9)$$

As in the $SL(2, \mathbb{R})/SO(2)$ factor, we can parametrize the $SO(6, n)/SO(6) \times SO(n)$ coset in terms of a symmetric matrix

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \quad (10)$$

We are now in a position to give the bosonic Lagrangian with the vector fields and auxiliary two-form fields vanishing,

$$e^{-1} \mathcal{L} = \frac{1}{2} R + \frac{1}{16} \partial_\mu M_{MN} \partial^\mu M^{MN} - \frac{1}{4(\text{Im}\tau)^2} \partial_\mu \tau \partial^\mu \tau^* - V \quad (11)$$

where e is the vielbein determinant. The scalar potential is given by

$$V = \frac{g^2}{16} \left[f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta} \left[\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left(\frac{2}{3} \eta^{MQ} - M^{MQ} \right) \eta^{NR} \eta^{PS} \right] - \frac{4}{9} f_{\alpha MNP} f_{\beta QRS} \epsilon^{\alpha\beta} M^{MNPQRS} + 3 \xi_\alpha^M \xi_\beta^N M^{\alpha\beta} M_{MN} \right] \quad (12)$$

where M^{MN} is the inverse of M_{MN} , and M^{MNPQRS} is defined by

$$M_{MNPQRS} = \epsilon_{mnpqrs} \mathcal{V}_M^m \mathcal{V}_N^n \mathcal{V}_P^p \mathcal{V}_Q^q \mathcal{V}_R^r \mathcal{V}_S^s \quad (13)$$

with indices raised by η^{MN} .

The gauge group we will consider here is a non-semisimple group $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3) \subset SO(6, 3)$ described by the non-vanishing component $f_{\alpha MNP}$ of the embedding tensor. We will then set $\xi^{\alpha M} = 0$ in the following discussion. The embedding of this $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ gauge group is described by the following components of the embedding tensor:

$$\begin{aligned}
f_{+IJ,K+6} &= -f_{+I+3,J+6,K+6} \\
&= -2\sqrt{2}\epsilon_{IJK}, \quad I, J, K = 1, 2, 3, \\
f_{+I+6,J+6,K+6} &= 6\sqrt{2}k\epsilon_{IJK}, \quad f_{-I,J+6,K+6} = -4\epsilon_{IJK}.
\end{aligned} \quad (14)$$

The constant k is related to the four-form flux along the four-dimensional space-time; see Eq. (30) below. This gauge group arises from a truncation of 11-dimensional supergravity on a tri-Sasakian manifold [33]. It should be noted that both electric and magnetic components participate in the gauging, $f_{\pm MNP} \neq 0$, since purely electric gaugings do not lead to AdS_4 vacua as mentioned above.

We should also remark that the identification of this gauge group and other computations in [33] have been done in the off-diagonal η_{MN}

$$\eta_{MN} = \begin{pmatrix} -\mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 \\ \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_3 \end{pmatrix} \quad (15)$$

where $\mathbf{0}_3$ and $\mathbf{I}_3$ denote 3×3 zero and identity matrices, respectively. Accordingly, in computing M_{MNPQRS} in (13) and some parts of the supersymmetry transformations given below, $\mathcal{V}_M^m$ and $\mathcal{V}_M^a$ must be projected to the negative and positive eigenvalue subspaces of η_{MN} , respectively.

By transforming to a purely electric frame, the gauge algebra will be more transparent. We will not explicitly give this transformation here since we will mainly work in the above electric-magnetic frame. However, for completeness, we will discuss the structure of the gauge algebra here; see [33] for more details. The $SO(3)$ part is the diagonal subgroup of $SO(3) \times SO(3) \times SO(3) \subset SO(6) \times SO(3) \subset SO(6, 3)$. The six generators of $\mathbf{T}^3$ and $\hat{\mathbf{T}}^3$ transform as $\mathbf{3} + \mathbf{3}$ under $SO(3)$. $\mathbf{T}^3$ generators commute with each other while $\hat{\mathbf{T}}^3$ generators close on to $\mathbf{T}^3$ generators.

We now turn to another important ingredient of the $N = 4$ gauged supergravity namely the supersymmetry transformations of fermionic fields. These are given by

$$\delta\psi_\mu^i = 2D_\mu\epsilon^i - \frac{2}{3}gA_1^{ij}\gamma_\mu\epsilon_j, \quad (16)$$

$$\delta\chi^i = i\epsilon^{\alpha\beta}\mathcal{V}_\alpha D_\mu\mathcal{V}_\beta\gamma^\mu\epsilon^i - \frac{4}{3}igA_2^{ij}\epsilon_j, \quad (17)$$

$$\delta\lambda_a^i = 2i\mathcal{V}_a^M D_\mu\mathcal{V}_M^{ij}\gamma^\mu\epsilon_j + 2igA_{2aj}^i\epsilon^j. \quad (18)$$

The fermion shift matrices are defined by

$$\begin{aligned}
A_1^{ij} &= \epsilon^{\alpha\beta}(\mathcal{V}_\alpha)^*\mathcal{V}_{kl}^M\mathcal{V}_N^{ik}\mathcal{V}_P^{jl}f_{\beta M}^{NP}, \\
A_2^{ij} &= \epsilon^{\alpha\beta}\mathcal{V}_\alpha\mathcal{V}_{kl}^M\mathcal{V}_N^{ik}\mathcal{V}_P^{jl}f_{\beta M}^{NP}, \\
A_{2ai}^j &= \epsilon^{\alpha\beta}\mathcal{V}_\alpha\mathcal{V}_a^M\mathcal{V}_{ik}^N\mathcal{V}_P^{jk}f_{\beta MN}^P
\end{aligned} \quad (19)$$

where $\mathcal{V}_M^{ij}$ is defined in terms of the 't Hooft symbols G_m^{ij} and $\mathcal{V}_M^m$ as

$$\mathcal{V}_M^{ij} = \frac{1}{2}\mathcal{V}_M^m G_m^{ij}, \quad (20)$$

and similarly for its inverse,

$$\mathcal{V}_{ij}^M = -\frac{1}{2}\mathcal{V}_M^m (G_m^{ij})^*. \quad (21)$$

The G_m^{ij} satisfy the relations

$$G_{mij} = (G_m^{ij})^* = \frac{1}{2}\epsilon_{ijkl}G_m^{kl}. \quad (22)$$

The explicit form of these matrices can be found for example in [39]. Note that we use the convention about the (anti) self-duality of G_{mij} opposite to that of [39]. It should also be noted that the scalar potential can be written in terms of A_1 and A_2 tensors as

$$V = -\frac{1}{3}A_1^{ij}A_{1ij} + \frac{1}{9}A_2^{ij}A_{2ij} + \frac{1}{2}A_{2ai}^jA_{2a}^i{}_j. \quad (23)$$

2.2 $N = 4$ gauged supergravity from 11 dimensions

Four-dimensional $N = 4$ gauged supergravity coupled to three-vector multiplets with $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ gauge group has been obtained from a truncation of 11-dimensional supergravity on a generic tri-Sasakian manifold in [33]. In this section, we review the relevant formulas involving the reduction ansatz which will be useful for uplifting four-dimensional solutions in the next sections. In particular, we will set all of the vector fields to zero as well as the auxiliary two-form and magnetic vector fields.

The 11-dimensional metric can be written as

$$ds_{11}^2 = e^{2\varphi}ds_4^2 + e^{2U}ds^2(B_{\text{QK}}) + g_{IJ}\eta^I\eta^J. \quad (24)$$

The three-dimensional internal metric g_{IJ} can be written in terms of the vielbein as

$$g = Q^T Q. \quad (25)$$

For convenience, as in [33], we will parametrize the matrix Q in terms of a product of a diagonal matrix V and an $SO(3)$ matrix O as

$$Q = VO, \quad V = \text{diag}(e^{V_1}, e^{V_2}, e^{V_3}). \quad (26)$$

The scalar φ is chosen in such a way that the four-dimensional Einstein–Hilbert term is obtained

$$\varphi = -\frac{1}{2}(4U + V_1 + V_2 + V_3). \quad (27)$$

Finally, B_{QK} denotes a four-dimensional quaternionic Kahler manifold.

The three-form field and its four-form field strength are given, respectively, by

$$C_3 = c_3 + c_{IJ}\eta^I \wedge J^I + \frac{1}{6}\chi\epsilon_{IJK}\eta^I \wedge \eta^J \wedge \eta^K \quad (28)$$

and

$$G_4 = H_4 + 4\text{Trc vol}(\text{QK}) + \frac{1}{6}\epsilon_{IJK}\text{d}\chi \wedge \eta^I \eta^J \eta^K + \text{d}c_{IJ} \wedge \eta^I \wedge J^J \epsilon_{IJL}[(\chi + \text{Trc})\delta_{LK} - 2c_{(LK)}] \times \eta^I \wedge \eta^J \wedge J^K \quad (29)$$

where $H_4 = \text{d}c_3$, c_{IJ} is a 3×3 matrix and $\text{Trc} = \delta^{IJ}c_{IJ}$. In the present case, the H_4 will be given by

$$H_4 = -6ke^{4\varphi-V_1-V_2-V_3-4U}\text{vol}_4 \quad (30)$$

where vol_4 is the volume form of the four-dimensional metric ds_4^2 . The volume form of B_{QK} , $\text{vol}(\text{QK})$, can be written in terms of the two-forms J^I as

$$\text{vol}(\text{QK}) = \frac{1}{6}J^I \wedge J^I. \quad (31)$$

For the N^{010} tri-Sasakian manifold, we can take a simple description in terms of a coset manifold $SU(3)/U(1)$. This is enough for our propose although the full $SU(3) \times SU(2)$ isometry is not manifest; see [41] for another description. Using the standard Gell-Mann matrices, we can choose the $SU(3)$ generators to be $-\frac{i}{2}\lambda_\alpha$, $\alpha = 1, \dots, 8$. The coset and $U(1)$ generators can be chosen to be

$$K_i = -\frac{i}{2}(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7), \quad H = -\frac{i\sqrt{3}}{2}\lambda_8. \quad (32)$$

The vielbein on N^{010} can eventually be obtained from the decomposition of the Maurer–Cartan one-form

$$L^{-1}\text{d}L = e^i K_i + \omega H \quad (33)$$

where L is the coset representative for $SU(3)/U(1)$. ω is the corresponding $U(1)$ connection.

Following [33], we will use the tri-Sasakian structures of the form

$$\eta^I = \frac{1}{2}(e^1, e^2, e^3), \\ J^I = \frac{1}{8}(e^4 \wedge e^5 - e^3 \wedge e^6, -e^3 \wedge e^5 - e^4 \wedge e^6, e^5 \wedge e^6 - e^3 \wedge e^4). \quad (34)$$

From these, we find the metric on B_{QK} to be

$$\text{d}s^2(B_{\text{QK}}) = \frac{1}{256}[(e^3)^2 + (e^4)^2 + (e^5)^2 + (e^6)^2] \quad (35)$$

with the volume form given by

$$\text{vol}(\text{QK}) = \frac{1}{6}J^I \wedge J^I = -\frac{1}{64}e^3 \wedge e^4 \wedge e^5 \wedge e^6. \quad (36)$$

In the remaining parts of this paper, we will not need the explicit form of $\text{d}s^2(B_{\text{QK}})$ and η^I 's since we will not consider the deformations of these metrics. Therefore, we will leave these as generic expressions.

2.3 BPS equations for $SO(3)$ invariant scalars

We now give an explicit parametrization of the $SL(2, \mathbb{R})/SO(2) \times SO(6, 3)/SO(6) \times SO(3)$ coset and relevant information for setting up the BPS equations corresponding to $SO(3)$ singlet scalars.

Since we will study both RG flows and Janus solutions, and the former can formally be obtained as a limit of the latter, we will first construct the BPS equations for finding supersymmetric Janus solutions and take an appropriate limit to find the BPS equations for RG flow solutions. The metric ansatz takes the form of an AdS_3 -sliced domain wall,

$$\text{d}s^2 = e^{2A(r)}(e^{\frac{2\xi}{\ell}}\text{d}x_{1,1}^2 + \text{d}\xi^2) + \text{d}r^2. \quad (37)$$

As can be clearly seen, this metric becomes a flat domain wall used in the study of holographic RG flows in the limit $\ell \rightarrow \infty$. The vielbein components can be chosen to be

$$e^{\hat{\mu}} = e^{A+\frac{\xi}{\ell}}\text{d}x^\mu, \quad e^{\hat{\xi}} = e^A\text{d}\xi, \quad e^{\hat{r}} = \text{d}r. \quad (38)$$

The non-vanishing spin connections of this metric are then given by

$$\omega_{\hat{r}}^{\hat{\xi}} = A'e^{\hat{\xi}}, \quad \omega_{\hat{\xi}}^{\hat{\mu}} = \frac{1}{\ell}e^{-A}e^{\hat{\mu}}, \quad \omega_{\hat{r}}^{\hat{\mu}} = A'e^{\hat{\mu}} \quad (39)$$

where $'$ denotes the r -derivative. For the moment, indices μ, ν will take values 0, 1, and hatted indices are the tangent space indices.

In this paper, we are only interested in $SO(3)$ singlet scalars. These scalar fields depend only on the radial coordinate r . There are four $SO(3)$ singlets corresponding to two scalars from $SL(2, \mathbb{R})/SO(2)$ and another two from $SO(6, 3)/SO(6) \times SO(3)$ according to the branching of $SO(6, 3) \rightarrow SO(3) \times SO(3) \times SO(3) \rightarrow SO(3)_{\text{diag}}$

$$(6, 3) \rightarrow (3, 1, 3) + (1, 3, 3) \rightarrow 2 \times (1 + 3 + 5). \quad (40)$$

Following [33], we parametrize the $SO(6, 3)/SO(6) \times SO(3)$ coset representative by

$$\mathcal{V} = \exp \begin{pmatrix} \mathbf{0}_3 & \sqrt{2}Z\mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \sqrt{2}Z\mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \end{pmatrix} \times \begin{pmatrix} \mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 e^{-2U-V_1}\mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & e^{2U+V_1}\mathbf{I}_3 \end{pmatrix}. \quad (41)$$

Note that $SO(3)$ invariance requires c_{IJ} to be proportional to the identity, $c_{IJ} = \sqrt{2}Z\delta_{IJ}$, and $V_1 = V_2 = V_3$.

The $SL(2, \mathbb{R})/SO(2)$ scalars are given by

$$\tau = \chi + ie^{3V_1}. \quad (42)$$

For convenience, we will define another scalar,

$$U_1 = 2U + V_1. \quad (43)$$

This also gives a diagonal scalar kinetic term

$$\begin{aligned} & \frac{1}{16} \partial_\mu M_{MN} \partial^\mu M^{MN} - \frac{1}{4(\text{Im}\tau)^2} \partial_\mu \tau \partial^\mu \tau^* \\ &= -\frac{3}{2} U_1'^2 - \frac{9}{4} V_1'^2 - \frac{1}{4} e^{-6V_1} \chi'^2 - \frac{3}{2} e^{-2U_1} Z'^2. \end{aligned} \quad (44)$$

In order to setup the BPS equations corresponding to $\delta\chi^i = 0$ and $\delta\lambda_a^i = 0$, a projector involving γ_r is needed. Since the procedure is essentially the same as in [26, 27], we will only repeat the relevant formulas. Following [26], we will use Majorana representation in which all gamma matrices γ_μ are real, and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ is purely imaginary. In the chiral notation, we have, for example,

$$\epsilon^i = \frac{1}{2}(1 + \gamma_5)\epsilon_M^i, \quad \epsilon_i = \frac{1}{2}(1 - \gamma_5)\epsilon_M^i \quad (45)$$

where ϵ_M is a four-component Majorana spinor. From all this, it follows that $\epsilon_i = (\epsilon^i)^*$.

Accordingly, the γ_r -projector can be written as

$$\gamma^{\hat{r}} \epsilon^i = e^{i\Lambda} \epsilon_i \quad (46)$$

or equivalently

$$\gamma^{\hat{r}} \epsilon_i = e^{-i\Lambda} \epsilon^i. \quad (47)$$

The analysis of $\delta\psi_\mu^i = 0$ equations leads to the following $\gamma_{\hat{\xi}}$ projection:

$$\gamma_{\hat{\xi}} \epsilon_i = i\kappa e^{i\Lambda} \epsilon^i \quad (48)$$

see [26] for more detail. The constant κ satisfying $\kappa^2 = 1$ determines the chirality of the unbroken supercharges on the

two-dimensional defect. Up to a phase, the full Killing spinor can be written as

$$\epsilon^i = e^{\frac{A}{2} + \frac{\xi}{2\ell} + i\frac{\Lambda}{2}} \epsilon^{(0)i} \quad (49)$$

with the constant spinors $\epsilon^{(0)i}$ satisfying

$$\gamma_{\hat{r}} \epsilon^{(0)i} = \epsilon_i^{(0)} \quad \text{and} \quad \gamma_{\hat{\xi}} \epsilon_i^{(0)} = i\kappa \epsilon^{(0)i}. \quad (50)$$

The integrability conditions of $\delta\psi_{\hat{0},\hat{1}}^i = 0$ equations give

$$A'^2 + \frac{1}{\ell^2} e^{-2A} = |\mathcal{W}|^2 \quad (51)$$

where $\mathcal{W}$ is the “superpotential” given by the eigenvalue α of the A_1^{ij} tensor corresponding to the unbroken supersymmetry

$$\mathcal{W} = \frac{2}{3}\alpha. \quad (52)$$

The cosmological constant at AdS_4 critical points is given in terms of α by the relation $V_0 = -\frac{4}{3}\alpha^2$.

Finally, we note the expression for the phase $e^{i\Lambda}$ in terms of $\mathcal{W}$

$$e^{i\Lambda} = \frac{A'}{W} + \frac{i\kappa}{\ell} \frac{e^{-A}}{W}. \quad (53)$$

$$\text{and} \quad e^{i\Lambda} = \frac{\mathcal{W}}{A' + \frac{i\kappa}{\ell} e^{-A}}. \quad (54)$$

for real and complex $\mathcal{W}$, respectively. These relations can be obtained by considering the gravitino variations in each case; see [27] for more detail.

For the RG flows, the corresponding BPS equations can be found by formally taking the limit $\ell \rightarrow \infty$. We simply find

$$A' = \pm W \quad \text{and} \quad e^{i\Lambda} = \frac{\mathcal{W}}{W}. \quad (55)$$

where $W = |\mathcal{W}|$ is call the “real superpotential”. The $\gamma_{\hat{\xi}}$ projector drops out, and there is no chirality restriction on the preserved supercharges.

We now give the scalar potential for $SO(3)$ singlet scalars

$$\begin{aligned} V = 3e^{-6U_1-3V_1} [2e^{2U_1+6V_1} + 12e^{6V_1} Z^2 - e^{4U_1} - 8e^{3(U_1+V_1)} \\ + 2e^{2U_1} (\chi + Z)^2 + 3(k - 2\chi Z - Z^2)^2]. \end{aligned} \quad (56)$$

As pointed out in [33], this is the scalar potential of the truncated $N = 1$ supergravity in which only $SO(3)$ singlet fields are retained.

The scalar field equations can be obtained by using this potential in the effective Lagrangian

$$\mathcal{L}_{\text{scalar}} = e^{3A} \left[\frac{1}{16} \partial_\mu M_{MN} \partial^\mu M^{MN} - \frac{1}{4(\text{Im}\tau)^2} \partial_\mu \tau \partial^\mu \tau^* - V \right]. \quad (57)$$

Note that the scalar field equations are the same for both the RG flows and the Janus solutions since scalars do not depend on the ξ coordinate. This is the reason we can take $\sqrt{-g}$ to be just e^{3A} not $e^{3A+2\frac{\xi}{\ell}}$. The explicit form of these equations and Einstein equations will be given in Appendix A.

As shown in [33], the above potential admits a number of AdS_4 critical points both supersymmetric and non-supersymmetric. In this paper, we will only consider the following supersymmetric AdS_4 vacua:

$$\text{I: } U_1 = 3V_1 = \frac{1}{2} \ln |k|, \quad V_0 = -12|k|^{-\frac{3}{2}} \quad (58)$$

$$\text{II: } U_1 = \ln 5 + \frac{1}{2} \ln \frac{|k|}{15}, \quad V_1 = \frac{1}{6} \ln \frac{|k|}{15},$$

$$V_0 = -12|k|^{-\frac{3}{2}} \sqrt{\frac{37}{55}} \quad (59)$$

with $\chi = Z = 0$. The cosmological constant V_0 is related to the AdS_4 radius by

$$L^2 = -\frac{3}{V_0}. \quad (60)$$

Within the $N = 4$ gauged supergravity, critical point I with $k > 0$ gives $N = 3$ supersymmetric AdS_4 vacuum while $k < 0$ solution gives a non-supersymmetric skew-whiffle solution as will be shown in the next section. Similarly, critical point II with $k < 0$ and $k > 0$ corresponds, respectively, to weak G_2 $N = 1$ AdS_4 and non-supersymmetric skew-whiffle solutions. In particular, the $N = 1$ critical point corresponds to a squashed version of N^{010} manifold. It is also useful to note the two metrics here

$$N = 3: \quad ds_{11}^2 = |k|^{-\frac{7}{6}} \left(e^{\frac{2r}{L_3}} dx_{1,2}^2 + dr^2 \right) + |k|^{\frac{1}{3}} \left[ds^2(B_{\text{QK}}) + \eta^I \eta^I \right], \quad (61)$$

$$N = 1: \quad ds_{11}^2 = \frac{1}{25} \left(\frac{|k|}{15} \right)^{-\frac{7}{6}} \left(e^{\frac{2r}{L_1}} dx_{1,2}^2 + dr^2 \right) + 5 \left(\frac{|k|}{15} \right)^{\frac{1}{3}} \left[ds^2(B_{\text{QK}}) + \frac{1}{5} \eta^I \eta^I \right] \quad (62)$$

where the AdS_4 radii are given by $L_3 = \frac{1}{2}|k|^{\frac{3}{4}}$ and $L_1 = \frac{5^{\frac{5}{4}}}{2(3)^{\frac{7}{4}}}|k|^{\frac{3}{4}}$.

Before carrying out the analysis of BPS equations, we briefly discuss the dual SCFTs to these critical points. The SCFT dual to the $N = 3$ critical point has been proposed in [31]. At low energy, this is an $SU(N) \times SU(N)$ gauge theory of interacting three hypermultiplets transforming in a triplet of the $SU(3)$ flavor symmetry. Each hypermultiplet transforms as a bifundamental under the $SU(N) \times SU(N)$ gauge group and as a doublet of the $SU(2)_R \sim SO(3)_R$ R-symmetry. In terms of the $N = 2$ superfields, these hypermultiplets can be written as

$$U_\alpha^i = (u^i, -\bar{v}^i) \quad \text{and} \quad V_{i\alpha} = -\epsilon_{\alpha\beta} \bar{U}_i^\beta = (v_i, \bar{u}_i) \quad (63)$$

where $i = 1, 2, 3$ and $\alpha = 1, 2$.

From the Kaluza–Klein spectrum given in [29,30], the massless graviton multiplet corresponds to the usual stress-energy tensor multiplet, including the $SO(3)_R$ R-symmetry current, in the dual $N = 3$ SCFT. There are also nine massless vector multiplets transforming in the adjoint and singlet (Betti multiplet) representations of $SU(3)$. These correspond to the following operator:

$$\Sigma_j^i = \frac{1}{\sqrt{2}} \text{Tr}(U^i \bar{U}_j + \bar{V}^i V_j) - \frac{1}{3\sqrt{2}} \delta_j^i \text{Tr}(U^k \bar{U}_k + \bar{V}^k V_k), \quad (64)$$

$$\Sigma = \frac{1}{\sqrt{2}} \text{Tr}(U^i \bar{U}_i + \bar{V}^i V_i) \quad (65)$$

which are the conserved currents of the flavor $SU(3)$ and the baryonic $U(1)$ global symmetries, respectively.

In [31]; see also [32], the operator dual to the massive gravitino multiplet, which is of particular interest in the present work, has also been proposed. The corresponding operator is given by the $SO(3)_R$ singlet composite superfield

$$\mathcal{SH} = \text{Tr}(\Theta_\Sigma^+ \Theta_\Sigma^0 \Theta_\Sigma^-) \quad (66)$$

where Θ_Σ is the field strength superfield. The components $(\Theta_\Sigma^+, \Theta_\Sigma^0, \Theta_\Sigma^-)$ are denoted in the $N = 2$ language by $(Y, \Sigma, -Y^\dagger)$ together with derivative terms. The explicit form of these can be found in [31]. Upon expanding in powers of the superspace coordinates $(\theta^\pm, \theta^0)$, we obtain the composite operators dual to the various component fields within the massive gravitino multiplet. For example, the scalar operator of dimension 6 corresponding to the breathing mode of the N^{010} manifold is given by the $N = 3$ supersymmetrization of the operator

$$\epsilon^{\lambda\mu\nu} \epsilon^{\rho\sigma\tau} F_{\lambda\mu} F_{\nu\rho} F_{\sigma\tau}. \quad (67)$$

It should be noted that this operator is the highest component of the supermultiplet with six factors of the $(\theta^\pm, \theta^0)$ coordinates. The deformation corresponding to this operator is then expected to preserve supersymmetry.

It has been pointed out in [33] that the SCFT dual to the $N = 1$ critical point on the other hand should be identified with the $N = 1$ SCFT arising from the squashed seven-sphere given in [36]. This is due to a similar spectrum within the truncation of [33] and that of the squashed seven-sphere. However, very little is known about $N = 1$ SCFT in three dimensions apart from holographic descriptions.

3 $N = 3$ supersymmetric solutions

We now look at the resulting BPS equations and their solutions. By using the coset representative (41), we find that A_1^{ij} tensor is diagonal

$$A_1^{ij} = \text{diag}(\alpha_1, \alpha_3, \alpha_3, \alpha_3). \quad (68)$$

The two eigenvalues α_1 and α_3 correspond to Killing spinors ϵ^1 and $\epsilon^{2,3,4}$ and give rise to the superpotentials

$$\mathcal{W}_1 = \frac{3}{2}e^{-\frac{3}{2}(2U_1+V_1)}[e^{2U_1} + 2e^{U_1+3V_1} + k - 2\chi Z - Z^2 + 2i[e^{U_1}\chi + (e^{U_1} + e^{3V_1})Z]], \quad (69)$$

$$\mathcal{W}_3 = -\frac{1}{2}e^{-\frac{3}{2}(2U_1+V_1)}[5e^{2U_1} + 2e^{U_1+3V_1} - 3k + 6\chi Z + 3Z^2 + 2i[e^{U_1}(\chi + Z) - 3e^{3V_1}Z]]. \quad (70)$$

In this section, we will consider only $\mathcal{W}_3$ corresponding to unbroken $N = 3$ supersymmetry and leave the analysis of $\mathcal{W}_1$ to the next section.

3.1 Flow to $N = 3$ non-conformal field theory

The analysis of $\delta\lambda_a^i = 0$ equations along $\epsilon^{2,3,4}$ requires $U_1 = 3V_1$ and $\chi = 2Z$. However, we need to further set $\chi = Z = 0$ in the BPS equations in order to satisfy the field equations. With all these requirements, we end up with the $N = 3$ BPS equations

$$V_1' = e^{-\frac{21}{2}V_1}(e^{6V_1} - k), \quad (71)$$

$$A' = \frac{1}{2}e^{-\frac{21}{2}V_1}(7e^{6V_1} - 3k). \quad (72)$$

From these equations, we find an $N = 3$ AdS_4 critical point

$$V_1 = \frac{1}{6} \ln k, \quad A' = \frac{2}{k^{\frac{3}{4}}} = \frac{1}{L_3}. \quad (73)$$

We also see that there is no critical point for $k < 0$. This is in agreement with the fact that the solutions with $k < 0$ break all supersymmetry as mentioned before. In Eqs. (71) and (72), we have chosen a definite sign choice to obtain the correct behavior near the critical point

$$V_1 \sim e^{\frac{3r}{L_3}}. \quad (74)$$

This is consistent with the fact that V_1 is dual to an irrelevant operator of dimension six. We then see that the dual $N = 3$ SCFT appears in the IR.

It can be checked that these equations satisfy the scalar field equations and Einstein equations. In this case, the superpotential is real

$$\mathcal{W}_3 = W_3 = \frac{1}{2}e^{-\frac{21}{2}V_1}(7e^{6V_1} - 3k), \quad (75)$$

and the scalar potential can be written as

$$V = \frac{4}{189} \left(\frac{\partial W_3}{\partial V_1} \right) - 3W_3^2, \\ = 9k^2 e^{-21V_1} - 21e^{-9V_1}. \quad (76)$$

For non-vanishing pseudoscalars χ and Z and $U_1 \neq 3V_1$, $N = 3$ supersymmetry is broken, and the scalar potential cannot be written in terms of the real superpotential W_3 . It should also be noted that the vanishing of χ and Z rules out any supersymmetric Janus solutions since the corresponding BPS equations cannot be consistent for finite ℓ . This is similar to the results of [26] and [27] in which pseudoscalars are required for supersymmetric Janus solutions to exist.

We now return to a supersymmetric RG flow solution. The BPS equations given above have a simple solution

$$A = \frac{3}{2}V_1 + \frac{1}{3} \ln(e^{6V_1} - k), \quad (77)$$

$$V_1 = -\frac{1}{6} \ln \left[\frac{1 - e^{6k\tilde{r}+C}}{k} \right] \quad (78)$$

where the new radial coordinate $\tilde{r}$ is related to r by $\frac{d\tilde{r}}{dr} = e^{-\frac{21}{2}V_1}$. As $\tilde{r} \sim r \rightarrow -\infty$, we find $V_1 \sim e^{6k\tilde{r}} \sim e^{\frac{3r}{L_3}}$. As usual in flows to non-conformal field theories, there is a singularity at $\tilde{r} \sim -\frac{C}{6k}$ which gives $V_1 \rightarrow \infty$. Near this singularity, we find

$$V_1 \sim -\frac{1}{6} \ln(6k\tilde{r}+C) \quad \text{and} \quad A \sim \frac{7}{2}V_1 \sim -\frac{7}{12} \ln(6k\tilde{r}+C). \quad (79)$$

In this limit, the scalar potential vanishes. This implies that the singularity is physical according to the criterion of [42].

We can also see this by looking at the 11-dimensional metric and considering the criterion of [43]. In the present case, we have $U = V_1$ and

$$ds_{11}^2 = e^{-7V_1} ds_4^2 + e^{2V_1} (ds^2(B_{\text{QK}}) + \eta^I \eta^I) \\ = dx_{1,2}^2 + (6k\tilde{r} + C)^{-\frac{7}{3}} d\tilde{r}^2 \\ + (6k\tilde{r} + C)^{-\frac{1}{3}} [ds^2(B_{\text{QK}}) + \eta^I \eta^I], \\ G_4 = -6k dx^0 \wedge dx^1 \wedge dx^2 \wedge d\tilde{r}. \quad (80)$$

By changing to a new coordinate R via the relation $dR = (6k\tilde{r} + C)^{-\frac{7}{6}} d\tilde{r}$, we can write the metric as

$$ds_{11}^2 = dx_{1,2}^2 + dR^2 + (kR)^2 [ds^2(B_{\text{QK}}) + \eta^I \eta^I] \quad (81)$$

Near the singularity, we then see that the metric component g_{00} is bounded, $g_{00}^{(11)} = -e^{2A-7V_1} \rightarrow -1$. Therefore, the singularity is also physical by the criterion of [43]. This solution should be identified with the flow from $E^{1,2} \times HK$, HK being a Hyper-Kähler manifold, to $AdS_4 \times N^{010}$ studied in [44] by using another approach.

It should also be noted that when $k = 0$, AdS_4 critical points do not exist. In this case, the gauged supergravity, however, admits an $N = 3$ supersymmetric domain wall vacuum. This solution preserves only six supercharges due to the γ_r projection and accordingly is a half-BPS solution. By setting $k = 0$ in the BPS equations, we can find a simple domain wall solution

$$V_1 = \frac{2}{9} \ln \frac{9r}{2}, \quad A = \frac{7}{9} \ln \frac{9r}{2} \quad (82)$$

where, for convenience, we have set the associated integration constants to zero by shifting the coordinates. This solution can be readily lifted to 11 dimensions in which the metric is given by

$$ds_{11}^2 = dx_{1,2}^2 + \left(\frac{9r}{2}\right)^{-\frac{14}{9}} dr^2 + \left(\frac{9r}{2}\right)^{\frac{4}{9}} ds^2(B_{QK}) + \eta^I \eta^I, \quad (83)$$

$$= dx_{1,2}^2 + dR^2 + R^2(ds^2(B_{QK}) + \eta^I \eta^I) \quad (84)$$

where we have defined a new coordinate $R = \left(\frac{9r}{2}\right)^{\frac{2}{9}}$. In this case, the four-form field vanishes.

As a final comment on the $N = 3$ solution, we can also give a geometric interpretation of the condition $U_1 = 3V_1$. Recall that $U_1 = 3V_1$ means $U = V_1$, we find that only the breathing mode is consistent with $N = 3$ supersymmetry. As mentioned previously, the breathing mode corresponds to an operator which is the highest component of the supermultiplet and hence does not break supersymmetry. On the other hand, the squashing mode corresponding to the scalar $V_1 - U$, dual to a dimension-4 operator, breaks all of the supersymmetry. Non-supersymmetric RG flows between $N = (3, 0)$ and $N = (0, 1)$ supersymmetric AdS_4 critical points driven by this scalar have been studied in [45]; see also [46]. The dual operator driving the flow has also been proposed in [45].

4 $N = 1$ supersymmetric solutions

In this section, we will carry out a similar analysis for the case of unbroken $N = 1$ supersymmetry corresponding to the Killing spinor ϵ^1 . The real superpotential is given by

$$W_1 = \frac{3}{2} e^{-3U_1 - \frac{3}{2}V_1} \times \sqrt{[2\chi e^{U_1} + 2Z(e^{U_1} + e^{3V_1})]^2 + [e^{2U_1} + 2e^{U_1+3V_1} + k - 2\chi Z - Z^2]^2} \quad (85)$$

in terms of which the scalar potential can be written as

$$V = -2G^{\alpha\beta} \frac{\partial W_1}{\partial \phi^\alpha} \frac{\partial W_1}{\partial \phi^\beta} - 3W_1^2 \quad (86)$$

where $\phi^\alpha = (U_1, V_1, Z, \chi)$ and $G^{\alpha\beta}$ is the inverse of the metric in the scalar kinetic terms given in (44). We now look at the BPS equations and possible supersymmetric solutions.

4.1 RG flow solutions

We begin with an RG flow solution with only U_1 and V_1 scalars non-vanishing. These correspond to the breathing and squashing modes of N^{010} . It can be checked that keeping only U_1 and V_1 is consistent with the BPS equations and the corresponding field equations. From 11-dimensional point of view, this corresponds to pure metric modes since the pseudoscalars Z and χ appear in the internal components of the four-form field strength. A non-supersymmetric flow between this $N = 1$ AdS_4 and the skew-whiffle $N = 3$ AdS_4 has already been studied in [45, 46].

In this work, we will study a supersymmetric flow to a non-conformal field theory. The BPS equations in this case are given by

$$U'_1 = e^{-\frac{3}{2}(2U_1+V_1)}(e^{2U_1} + 4e^{U_1+3V_1} + 3k), \quad (87)$$

$$V'_1 = e^{-\frac{3}{2}(2U_1+V_1)}(e^{2U_1} - 2e^{U_1+3V_1} + k), \quad (88)$$

$$A' = \frac{3}{2} e^{-\frac{3}{2}(2U_1+V_1)}(e^{2U_1} + 2e^{U_1+3V_1} + k). \quad (89)$$

From these equations, we clearly see that there is only one AdS_4 critical point given by the $N = 1$ critical point II in Sect. 2, and there exists a critical point only for $k < 0$ as previously remarked.

Near this $N = 1$ critical point, we find an asymptotic behavior

$$3V_1 - U_1 \sim e^{-\frac{5r}{3L}}, \quad 2U_1 + V_1 \sim e^{\frac{3r}{L}} \quad (90)$$

corresponding to relevant and irrelevant operators of dimensions $\Delta = \frac{5}{3}, \frac{4}{3}$ and $\Delta = 6$, respectively.

We begin with a simple case in which the relevant deformation is further truncated out. This can be achieved by setting $V_1 = \frac{U_1}{3} - \frac{1}{3} \ln 5$. By taking appropriate combinations, we find new BPS equations

$$3V'_1 - U'_1 = 2e^{-2U_1 - \frac{3}{2}V_1}(e^{U_1} - 5e^{3V_1}), \quad (91)$$

$$2U'_1 + V'_1 = e^{-\frac{3}{2}V_1 - 3U_1}(3e^{2U_1} + 6e^{U_1+3V_1} + 7k) \quad (92)$$

from which we immediately see that the above truncation is consistent. Under this truncation, the remaining BPS equations become

$$U'_1 = \frac{3}{\sqrt{5}} e^{-\frac{7}{2}U_1}(3e^{2U_1} + 5k), \quad (93)$$

$$A' = \frac{3}{2\sqrt{5}} e^{-\frac{7}{2}U_1}(7e^{2U_1} + 5k). \quad (94)$$

By changing to a new radial coordinate $\tilde{r}$, defined by $\frac{d\tilde{r}}{dr} = e^{-\frac{7}{2}U_1}$, as in the $N = 3$ case, we obtain the solution

$$U_1 = -\frac{1}{2} \ln \left[\frac{e^{-6\sqrt{5}k\tilde{r}} - 3}{5k} \right],$$

$$A = \frac{1}{2}U_1 + \frac{1}{3} \ln(6e^{2U_1} + 10k), \quad (95)$$

where we have absorbed all integration constants by shifting $\tilde{r}$ and rescaling $dx_{1,2}^2$ coordinates. It should also be remembered that in this case $k < 0$. The singularity at $6\sqrt{5}k\tilde{r} \rightarrow -\ln 3$ is physical by the criteria of both [42] and [43]. In this case, we find, as $6\sqrt{5}k\tilde{r} \rightarrow -\ln 3$,

$$V \rightarrow 0, \quad g_{00}^{(11)} \rightarrow -5^{\frac{1}{3}}. \quad (96)$$

We identify this solution with the flow from $E^{1,2} \times Spin(7)$ to $AdS_4 \times \tilde{S}^7$ where $\tilde{S}^7$ is the squashed seven-sphere with a weak G_2 holonomy.

To solve Eqs. (91) and (92) in the presence of both types of deformations, we introduce new scalar fields defined by

$$\tilde{V} = 3V_1 - U_1 \quad \text{and} \quad \tilde{U} = 2U_1 + V_1 \quad (97)$$

in terms of which the BPS equations become

$$\tilde{V}' = e^{-\frac{2}{7}\tilde{V} - \frac{9}{14}\tilde{U}} (2 - 10e^{\tilde{V}}), \quad (98)$$

$$\tilde{U}' = e^{-\frac{3}{2}\tilde{U}} (3e^{-\frac{2}{7}(\tilde{V}-2\tilde{U})} + 6e^{\frac{5}{7}\tilde{V} + \frac{6}{7}\tilde{U}} + 7k), \quad (99)$$

$$A' = \frac{3}{2}e^{-\frac{2}{7}\tilde{V} - \frac{3}{2}\tilde{U}} (2e^{\frac{6}{7}\tilde{V} + \frac{6}{7}\tilde{U}} + e^{\frac{6}{7}\tilde{U}} + ke^{\frac{2}{7}\tilde{V}}). \quad (100)$$

We then define a new coordinate ρ via the relation

$$\frac{d\rho}{dr} = e^{-\frac{2}{7}\tilde{V} - \frac{9}{14}\tilde{U}}. \quad (101)$$

An analytic solution to the above equations can subsequently be obtained

$$\tilde{V} = -\ln(5 + C_1 e^{2\rho}),$$

$$\tilde{U} = \frac{7}{3}\rho + \frac{7}{6} \ln \left[-3k(5 + C_1 e^{-2\rho})^{\frac{5}{7}} \right.$$

$$\left. \times (5 + C_1 e^{-2\rho})^{\frac{18}{35}} \left[3k(5)^{\frac{1}{5}} {}_2F_1 \left(\frac{4}{5}, \frac{4}{5}, \frac{9}{5}, -\frac{C_1}{5} e^{-2\rho} \right) \right. \right.$$

$$\left. \left. - \frac{162}{7} (3^{\frac{3}{5}}) C_1 C_2 e^{\frac{8}{5}\rho} \right] \right],$$

$$A = \frac{6}{7}\rho + \frac{3}{14}\tilde{U} + \frac{6}{35} \ln[8C_1 + 40e^{2\rho}] \quad (102)$$

where ${}_2F_1$ is the hypergeometric function.

We now consider the asymptotic behavior of this RG flow to $N = 1$ non-conformal field theories. Near the singularity at $\rho = \frac{1}{2} \ln \left(-\frac{C_1}{5} \right)$, we find that

$$\tilde{V} = -\ln(5 + C_1 e^{-2\rho}) \rightarrow \infty,$$

$$\tilde{U} \sim \frac{3}{5} \ln(C_1 + 5e^{2\rho}) \rightarrow -\infty,$$

$$A \sim \frac{3}{10} \ln(C_1 + 5e^{2\rho}). \quad (103)$$

Although the scalar potential diverges near this singularity, the 11-dimensional metric gives $g_{00}^{(11)} \sim \text{constant}$. The singularity is then physical and the solution describes an RG flow between the dual $N = 1$ SCFT and a non-conformal $N = 1$ field theory. Near this singularity, the corresponding 11-dimensional solution is given by

$$ds_{11}^2 = dx_{1,2}^2 + \frac{d\rho^2}{(C_1 + 5e^{2\rho})^{\frac{2}{5}}} + (C_1 + 5e^{2\rho})^{\frac{3}{5}} ds^2(B_{\text{QK}}) + (C_1 + 5e^{2\rho})^{-\frac{2}{5}} \eta^I \eta^I, \quad (104)$$

$$G_4 = -6k(C_1 + 5e^{2\rho})^{-\frac{8}{35}} dx^0 \wedge dx^1 \wedge dx^2 \wedge \rho \quad (105)$$

where we have absorbed a constant in the $dx_{1,2}^2$ coordinates.

We then move to more complicated RG flows involving the $SO(3)_R$ singlet pseudoscalars. In this case, the flows will involve the internal components of the four-form field strength. Before considering possible solutions, we give an explicit form of the uplift formulas for the metric and the four-form with non-vanishing Z and χ :

$$ds_{11}^2 = e^{-(2U_1+V_1)} ds_4^2 + e^{U_1-V_1} ds^2(B_{\text{QK}}) + e^{2V_1} \eta^I \eta^I,$$

$$G_4 = -6ke^{-6U_1-3V_1} \text{vol}_4 + 12Z \text{vol}(\text{QK}) + \chi' dr \wedge \eta^1 \wedge \eta^2 \wedge \eta^3$$

$$+ (\chi + Z) \epsilon_{IJK} \eta^I \wedge \eta^J \wedge J^K + Z' dr \wedge \eta^I \wedge J^I. \quad (106)$$

The $N = 1$ BPS equations with four non-vanishing scalars are given by

$$U_1' = -\frac{2}{3} \frac{\partial W_1}{\partial U_1}, \quad V_1' = -\frac{4}{9} \frac{\partial W_1}{\partial V_1},$$

$$Z' = -\frac{2}{3} e^{2U_1} \frac{\partial W_1}{\partial Z}, \quad \chi' = -4e^{6V_1} \frac{\partial W_1}{\partial \chi}, \quad A' = W_1 \quad (107)$$

where the superpotential W_1 is given in (85). The explicit form of these equations can be found in Appendix B.

We will begin with the solutions near the $N = 1$ AdS_4 critical point. Near this critical point with $r \rightarrow \infty$, we find that

$$3V_1 - U_1 \sim e^{-\frac{5r}{3L_1}}, \quad 2U_1 + V_1 \sim e^{\frac{3r}{L_1}},$$

$$\chi + \frac{6}{5}Z \sim e^{-\frac{5r}{L_1}}, \quad \chi - \frac{Z}{5} \sim e^{-\frac{r}{3L_1}}. \quad (108)$$

From these, we see that U_1 and V_1 are combinations of a relevant and an irrelevant operators of dimensions $\Delta = \frac{5}{3}, \frac{4}{3}$ and $\Delta = 6$ as in the previous case while Z and χ are combinations of a relevant and an irrelevant operators of dimensions

$\Delta = \frac{8}{3}$ and $\Delta = 5$, respectively. These are consistent with the scalar masses given in [33].

Even with pseudoscalars turned on, there is a consistent truncation keeping only irrelevant scalars. This truncation is given by

$$V_1 = \frac{1}{3}U_1 - \frac{1}{3}\ln 5 \quad \text{and} \quad Z = 5\chi. \quad (109)$$

Within this truncation, the BPS equations become

$$U_1' = \frac{e^{-\frac{7}{2}U_1}}{\sqrt{5}\tilde{W}} [63e^{4U_1} + 150ke^{2U_1} + 75k^2 + 5250\chi^2(e^{2U_1} - k) + 91875\chi^4], \quad (110)$$

$$\chi' = -\frac{12\chi e^{-\frac{3}{2}U_1}(7e^{2U_1} - 5k + 175\chi^2)}{\sqrt{5}\tilde{W}}, \quad (111)$$

$$A' = \frac{3e^{-\frac{7}{2}U_1}}{2\sqrt{5}}\tilde{W} \quad (112)$$

where

$$\tilde{W} = \sqrt{(7e^{2U_1} + 5k)^2 + 350\chi^2(7e^{2U_1} - 5k) + 30625\chi^4}. \quad (113)$$

In this case, the BPS equations cannot be completely solved analytically. However, the solution can be implicitly given by defining a new scalar field F via the relation $F = e^{2U_1}$ in terms of which the BPS equations read

$$\frac{d\chi}{dr} = -\frac{12\chi(175\chi^2 + 7F - 5k)}{\sqrt{5}F^{\frac{3}{4}}\sqrt{49F^2 + 25(k - 35\chi^2)^2 + 70F(k + 35\chi^2)}}, \quad (114)$$

$$\frac{dF}{d\chi} = -\frac{21F^2 + 50F(k + 35\chi^2) + 25(k - 35\chi^2)^2}{2\chi(7F + 175\chi^2 - 5k)}, \quad (115)$$

$$\frac{dA}{d\chi} = -\frac{49F^2 + 70F(k + 35\chi^2) + 25(k - 35\chi^2)^2}{8\chi F(7F + 175\chi^2 - 5k)} \quad (116)$$

where in the last two equations we have taken χ as an independent variable by combining F' and A' equations with χ' equation, respectively. By solving Eq. (115), we can determine $F(\chi)$ implicitly from the following solution:

$$C\chi = 2^{\frac{2}{3}}7^{\frac{1}{3}}[5(k + 35\chi^2) + 7F]_2F_1\left(\frac{1}{2}, \frac{4}{5}, \frac{3}{2}, \frac{[5(k + 35\chi^2) + 7F]^2}{3500k\chi^2}\right) - 175(5^{\frac{2}{3}})\chi^2\left[\frac{49F^2 + 70F(k + 35\chi^2) + 25(k - 35\chi^2)^2}{k\chi^2}\right]^{\frac{1}{2}}. \quad (117)$$

In principle, $F(\chi)$ can be substituted in Eqs. (114) and (116) to determine $\chi(r)$ and $A(\chi)$.

We now look for asymptotic behavior for large values of scalar fields. At large χ , we find that

$$U_1 = \frac{1}{2}\ln F \sim \frac{1}{2}\ln \frac{C}{\chi^{\frac{3}{2}}}, \quad \chi \sim C'r^{-\frac{8}{9}} \quad (118)$$

where for convenience we have shifted the coordinate r such that the singularity is present at $r = 0$. In general, $\chi(r \rightarrow 0)$ can be ∞ or $-\infty$ depending on the sign of the constant C' . For definiteness, we will take $C' > 0$ in the present discussion. This behavior gives the metric warped factor

$$A \sim \frac{7}{9}\ln r. \quad (119)$$

Near the singularity, we find that the scalar potential diverges but $g_{00}^{(11)}$ becomes constant. We then conclude that the singularity is physical by the criterion of [43]. For completeness, we give an example of numerical solutions with $k = -1$ in Fig. 1.

Note that we have identified the $N = 1$ AdS_4 critical point at $\chi = 0$ and $U_1 = 0.22541$, for $k = -1$, with the IR SCFT at $r = -\infty$. The numerical solution also gives a singularity consistent with the above analysis namely the divergence of scalars and the potential as well as the constancy of $g_{00}^{(11)}$. The 11-dimensional solution near the singularity can be obtained as follows:

$$\begin{aligned} ds_{11}^2 &= dx_{1,2}^2 + dR^2 + \left(\frac{2R}{3}\right)^2 \left[ds^2(B_{\text{QK}}) + \frac{1}{5}\eta^I \eta^I \right], \\ G_4 &= -6k \left(\frac{2R}{9}\right)^{\frac{7}{2}} dx^0 \wedge dx^1 \wedge dx^2 \wedge dr \\ &\quad + 60 \left(\frac{2R}{9}\right)^{-4} \text{vol}(B_{\text{QK}}) \\ &\quad + 6 \left(\frac{2R}{9}\right)^{-4} \epsilon_{IJK} \eta^I \wedge \eta^J \wedge \eta^K - \left(\frac{2}{9}\right)^{-\frac{17}{2}} \\ &\quad \times R^{-5} dR \wedge \eta^1 \wedge \eta^2 \wedge \eta^3 \\ &\quad - 5 \left(\frac{2}{9}\right)^{-\frac{17}{2}} R^{-5} dR \wedge \eta^I \wedge \eta^I \end{aligned} \quad (120)$$

where we have defined a new coordinate $R = \frac{9}{2}r^{\frac{2}{9}}$.

We now look at the most general flow solution with all four scalars turned on. The BPS equations are too complicated to be solved analytically. In any case, numerical solutions can be obtained by suitable boundary conditions similar to the previous case. From the asymptotic behavior of these scalars given in (108), there could be many possible singularities at the end of the flows due to the presence of various vacuum expectation values and operator deformations as in the solutions studied in [24]. We will only give an example of these solutions. This is shown in Fig. 2 in which we take $k = -1$, and the $N = 1$ critical point corresponds to the values of the scalar fields

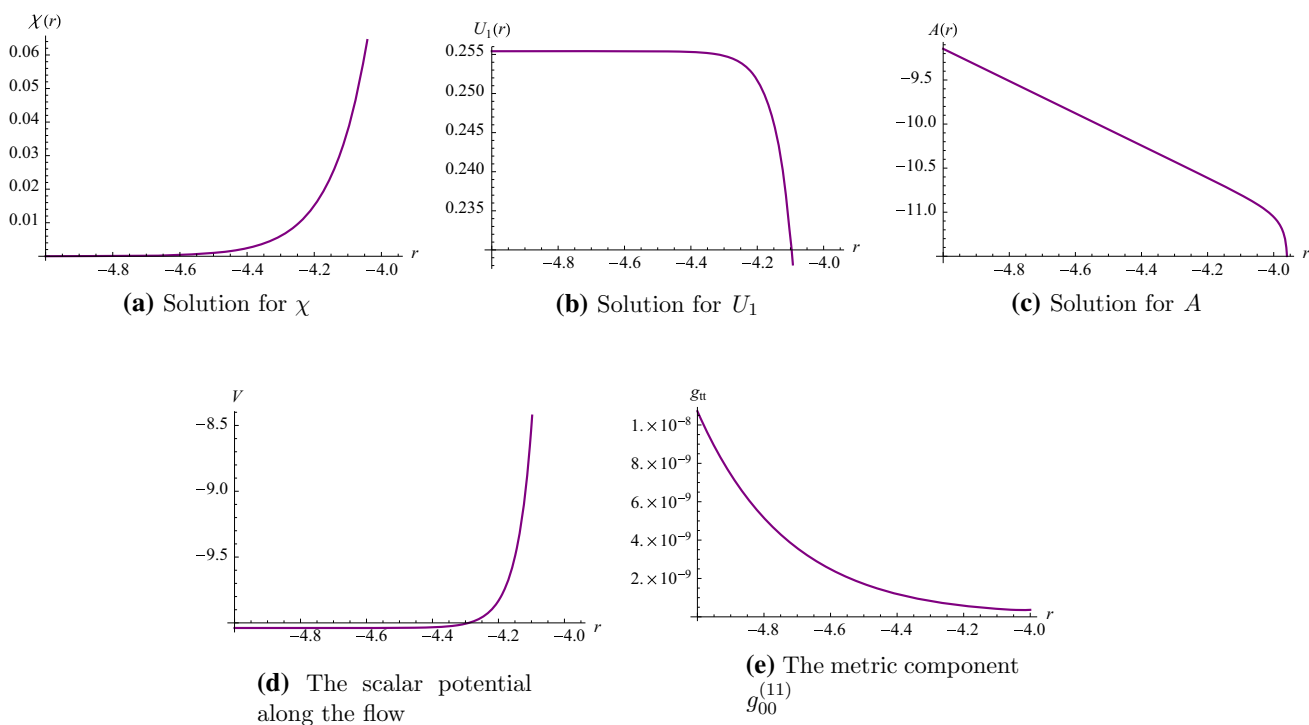


Fig. 1 An RG flow solution from $N = 1$ non-conformal field theory to $N = 1$ SCFT with one scalar and one pseudoscalar and $k = -1$

$$U_1 = 0.25541, \quad V_1 = -0.45134, \quad Z = \chi = 0. \quad (121)$$

Near the singularity, we can see that $\chi \sim Z \rightarrow \infty$ and $U_1 \sim V_1 \rightarrow -\infty$. From the BPS equations, we can make an analysis near this limit resulting in the asymptotic behavior

$$U_1 \sim 3V_1 \sim \frac{1}{2} \ln \chi^{-\frac{3}{2}}, \quad Z \sim 5\chi \sim r^{-\frac{8}{9}}. \quad (122)$$

Using these expressions or the numerical analysis in Fig. 2, we can see that the singularity is physical due to the constancy of $g_{00}^{(11)}$ although the scalar potential becomes infinite. The uplift of this solution can be obtained along the same line as in the previous case.

4.2 Domain wall solutions

Similar to the $N = 3$ case, we will consider $N = 1$ domain wall solutions to the BPS equations with $k = 0$. All of the relevant BPS equations can be obtained from those given above by setting $k = 0$, so we will not repeat them here.

In the case of vanishing pseudoscalars, we find a domain wall solution to Eqs. (98), (99), and (100) with $k = 0$

$$\begin{aligned} \tilde{U} &= \frac{3}{2} \tilde{V} - \frac{21}{20} \ln[105e^{\tilde{V}} - 21], \quad A = \frac{1}{2} \tilde{U}, \\ r &= \frac{2e^{\frac{1}{4}\tilde{V}} \left[2 + 25e^{\tilde{V}} - 2(1 - 5e^{\tilde{V}})^{\frac{27}{20}} {}_2F_1\left(\frac{1}{4}, \frac{7}{20}, \frac{5}{4}, 5e^{\tilde{V}}\right) \right]}{3969(21)^{\frac{7}{20}} (5e^{\tilde{V}} - 1)^{\frac{27}{20}}}. \end{aligned} \quad (123)$$

The last equation implicitly gives the scalar $\tilde{V}(r)$.

With non-vanishing pseudoscalars, we find an analytic solution only for the subtruncation to irrelevant scalars, $V_1 = \frac{1}{3}U_1 - \frac{1}{3}\ln 5$ and $Z = 5\chi$. The solution to Eqs. (110), (111), and (112) with $k = 0$ is given by

$$\begin{aligned} U_1 &= \frac{1}{2} \ln \left[\frac{C_1}{2\chi^{\frac{3}{2}}} - 25\chi^2 \right], \\ A &= \frac{1}{4} \left(50\chi^{\frac{7}{2}} - C_1 \right) - \frac{7}{8} \ln \chi, \\ r &= \frac{2\sqrt{52}^{\frac{1}{4}} (C_1 - 50\chi^{\frac{7}{2}}) + 525\chi^{\frac{13}{8}} \left(50\chi^{\frac{7}{2}} - C_1 \right)^{\frac{1}{4}} {}_2F_1\left(-\frac{1}{7}, \frac{1}{4}, \frac{6}{7}, \frac{C_1}{50\chi^{\frac{7}{2}}}\right)}{54\chi^{\frac{9}{8}} (C_1 - 50\chi^{\frac{7}{2}})^{\frac{1}{4}}}. \end{aligned} \quad (124)$$

When uplifted to 11 dimensions, these solutions will provide domain walls with internal four-form fluxes. All of these solutions should describe non-conformal $N = 1$ field theories in three dimensions according to the DW/QFT correspondence [47, 48].

4.3 Janus solutions

In the case of $N = 1$ supersymmetry, it is possible to have a supersymmetric Janus solution describing a conformal interface within the three-dimensional $N = 1$ SCFT. The resulting BPS equations for an AdS_3 -sliced domain wall metric can be written as

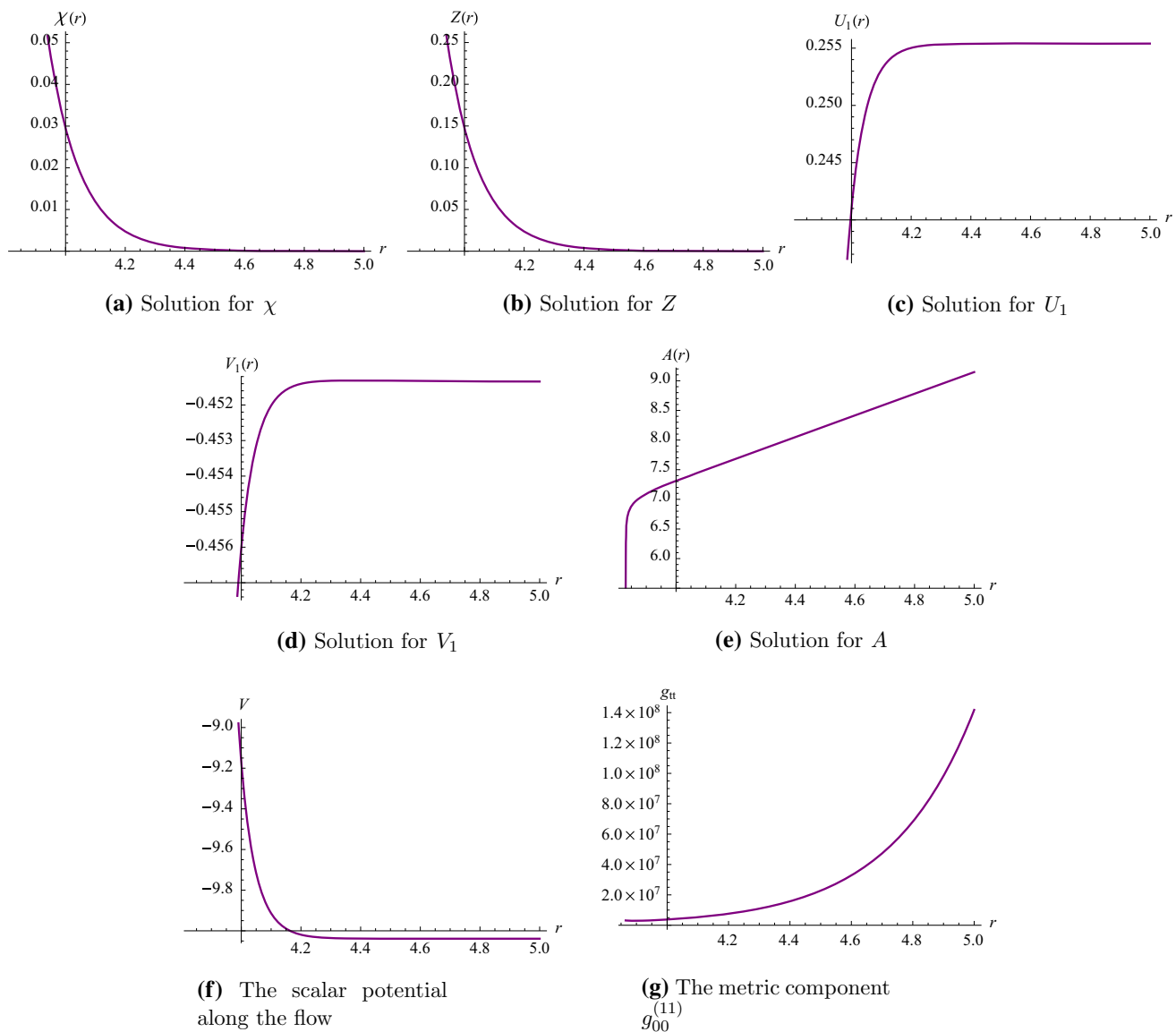


Fig. 2 An RG flow solution from $N = 1$ SCFT to $N = 1$ non-conformal field theory with two scalars and two pseudoscalars and $k = -1$

$$U_1' = -\frac{2}{3} \frac{A'}{W_1} \frac{\partial W_1}{\partial U_1} - \frac{2}{3} \kappa e^{U_1} \left(\frac{e^{-A}}{\ell W_1} \right) \frac{\partial W_1}{\partial Z}, \quad (125)$$

$$Z' = -\frac{2}{3} \frac{A'}{W_1} e^{2U_1} \frac{\partial W_1}{\partial Z} + \frac{2}{3} \kappa e^{U_1} \left(\frac{e^{-A}}{\ell W_1} \right) \frac{\partial W_1}{\partial U_1}, \quad (126)$$

$$V_1' = -\frac{4}{9} \frac{A'}{W_1} \frac{\partial W_1}{\partial V_1} - \frac{4}{3} \kappa e^{3V_1} \left(\frac{e^{-A}}{\ell W_1} \right) \frac{\partial W_1}{\partial \chi}, \quad (127)$$

$$\chi' = -4e^{6V_1} \frac{A'}{W_1} \frac{\partial W_1}{\partial \chi} + \frac{4}{3} \kappa e^{3V_1} \left(\frac{e^{-A}}{\ell W_1} \right) \frac{\partial W_1}{\partial V_1}, \quad (128)$$

$$A'^2 = W_1^2 - \frac{e^{-2A}}{\ell^2}. \quad (129)$$

These equations reduce to the RG flow equations in the limit $\ell \rightarrow \infty$, as expected. They take a very similar form to the equations studied within the $N = 8$ and $N = 3$ gauged supergravities in [26, 27]. All of these equations satisfy the corre-

sponding second-order field equations. We will not present the explicit form of these equations here due to their complexity. This can be obtained from the above equations by taking the superpotential W_1 from Eq. (85).

There is, however, a consistent truncation that can be performed by keeping only the irrelevant deformations. It can be straightforwardly checked that setting $V_1 = \frac{U_1}{3} - \frac{1}{3} \ln 5$ and $Z = 5\chi$ is a consistent truncation for both the above BPS equations and the corresponding field equations. The resulting equations are given by

$$U_1' = \frac{2A'}{\mathcal{Y}} [21e^{4U_1} + 50\kappa e^{2U_1} + 25k^2 + 1750\chi^2(e^{2U_1} - k) + 30,625\chi^4] - \frac{40\kappa\chi e^{-A+U_1}}{\ell\mathcal{Y}} (7e^{2U_1} + 175\chi^2 - 5k), \quad (130)$$

$$\chi' = -\frac{2\kappa e^{-A+U_1}}{5\ell\mathcal{Y}}[21e^{4U_1} + 50e^{2U_1} + 25k^2 + 1750\chi^2(e^{2U_1} - k) + 30,625\chi^4] - \frac{8A'\chi e^{2U_1}}{\mathcal{Y}}(7e^{2U_1} + 175\chi^2 - 5k), \quad (131)$$

$$A'^2 = \frac{9}{20}e^{-7U_1}[(7e^{U_1} + 5k^2)^2 + 350\chi^2(7e^{2U_1} - 5k) + 30,625\chi^4] - \frac{e^{-2A}}{\ell^2} \quad (132)$$

where $\mathcal{Y}$ is defined by

$$\mathcal{Y} = (7e^{U_1} + 5k^2)^2 + 350\chi^2(7e^{2U_1} - 5k) + 30,625\chi^4. \quad (133)$$

Even within this simpler truncation, it is not possible to find any analytic solutions.

We now return to the BPS equations for all $SO(3)$ singlet scalars. As in the $N = 8$ gauged supergravity case, these

BPS equations have a turning point at which $A' = 0$. Also, the regular Janus solution is required to approach the $N = 1$ AdS_4 critical point as $r \rightarrow \pm\infty$. As discussed in [26], for a given branch of A' near one of these limits, the first term in the scalar flow equations dominates. When the solution moves from the critical point, the second term will make the solution begin to loop around. At the point when $A' = 0$, the other branch of A' equation will bring the solution back to the AdS_4 critical point. The solution preserves $N = (1, 0)$ or $N = (0, 1)$ supersymmetry on the two-dimensional interface depending on the sign of κ .

However, from an intensive numerical search, we have not found this type of solutions even starting from $A'' > 0$ at the turning point. All of the solutions we obtain are singular on both sides of the turning point. Example of these solutions for the two-scalar truncation and all four scalars are shown, respectively, in Figs. 3 and 4. Note that the singulari-

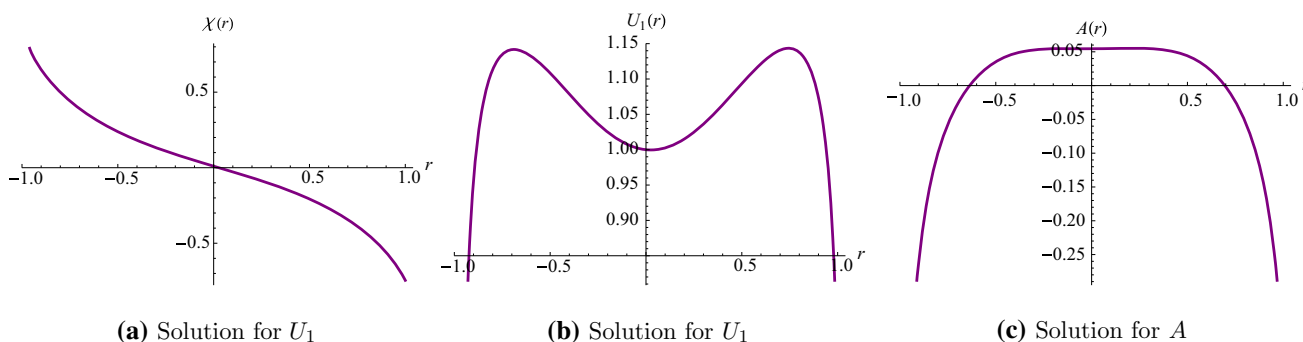


Fig. 3 $N = 1$ Janus solution within a truncation to two irrelevant scalars with $k = -1$, $\kappa = 1$ and $\ell = 1$

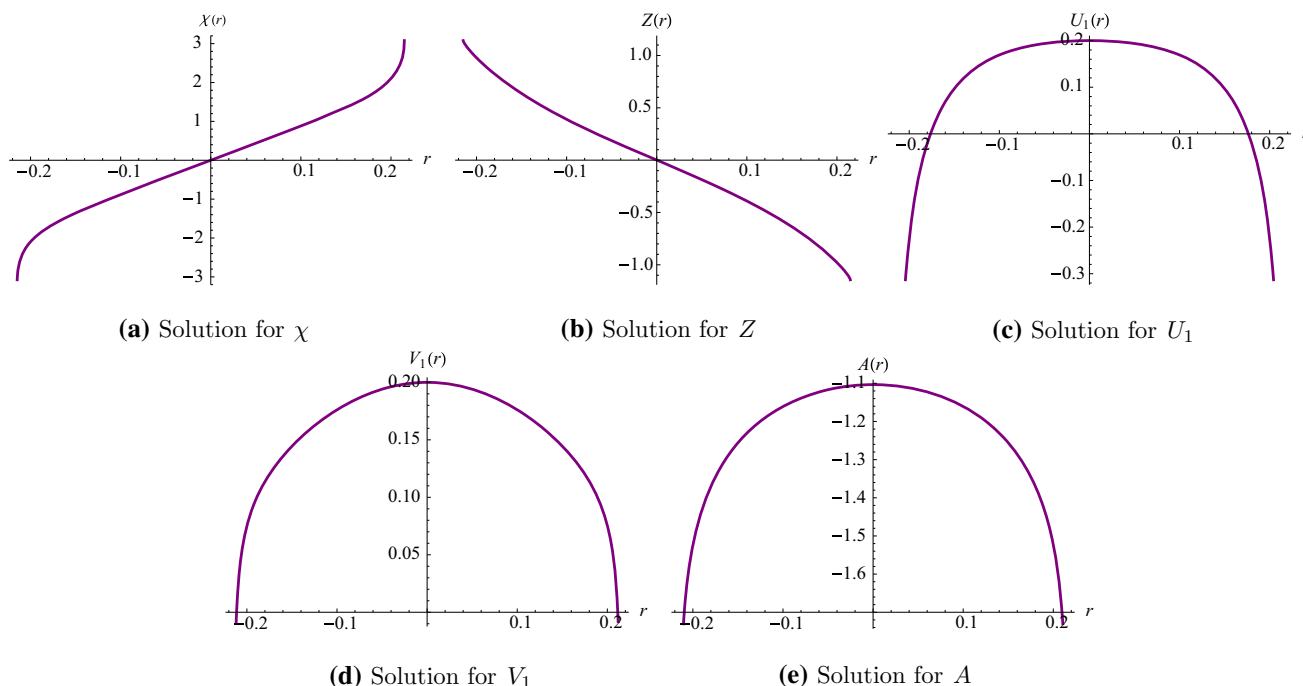


Fig. 4 $N = 1$ Janus solution with all $SO(3)$ singlet scalars and $k = -1$, $\kappa = 1$ and $\ell = 1$

ties appearing at both ends correspond to the non-conformal phases of the dual $N = 1$ SCFT studied in the previous section. These are also physical singularities according to the criterion of [43]. Therefore, we expect that these singular solutions might give some physical insight in the dual $N = 1$ field theories.

5 Conclusions

We have studied $N = 4$ gauged supergravity in four dimensions with $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ gauge group. This theory is a consistent truncation of 11-dimensional supergravity on a tri-Sasakian manifold including massive Kaluza–Klein modes. The theory admits two supersymmetric AdS_4 critical points with $N = 3$ and $N = 1$ supersymmetries and unbroken $SO(3)_R$ R-symmetry. We have fully analyzed the BPS equations for both cases and checked that they satisfy all the second-order field equations. This analysis has not been carried out in the truncation given in [33] in which only the structure of the supermultiplets has been discussed. The result obtained in this paper is consistent with all the expectations in [33] and in a sense could be viewed as an extension of the analysis in [33] to include the fermionic supersymmetry variations.

We have subsequently used these BPS equations to study possible supersymmetric deformations of the dual three-dimensional $N = 3$ and $N = 1$ SCFTs. These deformations correspond to turning on scalar composite operators dual to the massive gravitino multiplet of the gauged supergravity or their vacuum expectation values. We have studied a number of RG flows between these SCFTs and non-conformal field theories in three dimensions. Many of these deformations lead to various singularities corresponding to possible non-conformal phases of the dual SCFTs. We have also checked that all of the new $N = 1$ flow solutions presented here flow to physical singularities. Among the various solutions found in this paper, we have recovered the $N = 3$ flow from $E^{1,2} \times HK$ to $AdS_4 \times N^{010}$ and the $N = 1$ flow from $E^{1,2} \times Spin(7)$ to $AdS_4 \times \tilde{S}^7$ studied in [44].

The results given here provide additional gravity solutions to AdS_4/CFT_3 correspondence and might be useful in many studies along this line. In addition, we have found a number of supersymmetric domain wall solutions which might be useful in the context of DW/QFT correspondence. All of these solutions can be straightforwardly uplifted to 11 dimensions. The corresponding prescription of the uplift has also been given. It could be interesting to further study the implications of these solutions in the dual $N = 1$ SCFT and $N = 1$ gauge theory. The interpretation of these solutions in terms of M-brane geometries when uplifted to 11 dimensions also deserves further investigation.

Furthermore, we have looked at possible supersymmetric Janus solutions. In the $N = 3$ case, this type of solutions is not possible at least with unbroken $SO(3)_R$ symmetry. This is similar to the five-dimensional Janus solution with unbroken $SO(6)$ symmetry [12]. There could also be non-supersymmetric Janus solutions in this case as well. For the $N = 1$ case, the supersymmetric Janus solution is possible numerically. This solution corresponds to a two-dimensional conformal interface with $N = (1, 0)$ unbroken supersymmetry. We have given examples of numerical Janus solutions between $N = 1$ non-conformal phases of three-dimensional SCFTs. These solutions might be useful in the context of interfaced and boundary CFTs [49]. It would be interesting (if possible) to look for regular Janus solutions interpolating between $N = 1$ AdS_4 critical points which describe defect CFTs in three dimensions [50].

We end the paper by pointing out other possible future work. First of all, it is interesting to consider more general solutions with a residual symmetry less than $SO(3)_R$. From the $N = 1$ BPS equations studied here, it could be readily seen that this analysis would be very complicated. Alternatively, we could consider solutions with non-vanishing gauge fields that interpolate between $N = 1, 3$ AdS_4 solutions to $AdS_2 \times \Sigma_2$ in which Σ_2 is a Riemann surface. These solutions should correspond to twisted three-dimensional SCFTs and would be interesting in the study of black hole physics. Another issue, which should be of much interest, is to construct a more general and complete truncation of 11-dimensional supergravity on N^{010} . The truncation given in [33] has taken into account only $SU(3)$ singlet fields. This more general truncation could be used to uplift the RG flows and Janus solutions studied in [27, 37] resulting in new holographic solutions in 11 dimensions. Finally, by taking the Betti multiplet into account, it would be interesting to study baryon states corresponding to M5-branes wrapped on supersymmetric 5-cycles of N^{010} similar to the study of the four-dimensional gauge theory in [51].

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Appendix A: Field equations for $SO(3)_R$ singlet scalars

In this appendix, we explicitly give the field equations for all of the four $SO(3)_R$ singlet scalars and the corresponding Einstein equations. Since the equations in the RG flow case can be obtained from those of the Janus solutions, we will only give the equations for the Janus solutions.

The scalar equations are given by

$$U_1'' + 3A'U_1' - e^{-6U_1-3V_1}[2e^{4U_1} + 24e^{3U_1+3V_1} - 8e^{2U_1+6V_1} - 18k^2 - 4Z^2(2e^{2U_1} + 18e^{6V_1} - 9k) - 18Z^4 - 8\chi^2(e^{2U_1} + 9Z^2) - 8\chi Z(2e^{2U_1} - 9k + 9Z^2) - Z'^2e^{4U_1+3V_1}] = 0, \quad (134)$$

$$V_1'' + 3A'V_1' - \frac{1}{3}e^{-6U_1-6V_1}[6e^{4U_1+3V_1} + 12e^{2U_1+9V_1} - 18k^2e^{3V_1} + 12e^{3V_1}Z^2(6e^{6V_1} - e^{2U_1} + 3k) - 24\chi Ze^{3V_1}(e^{2U_1} - 3k + 3Z^2) - 18Z^4e^{3V_1} - 12\chi^2e^{3V_1}(e^{2U_1} + 6Z^2) - \chi'^2e^{6U_1}] = 0, \quad (135)$$

$$\chi'' + 3A'\chi' - 6\chi'V_1' - 6e^{-6U_1+3V_1}[4e^{2U_1}(\chi + Z) + 12Z(Z^2 + 2\chi Z - k)] = 0, \quad (136)$$

$$Z'' + 3A'Z' - 2U_1'Z' - 4e^{-4U_1-3V_1}[Z(e^{2U_1} + 6e^{6V_1} - 3k + 3Z^2) + 6\chi^2Z + \chi(e^{2U_1} - 3k + 9Z^2)] = 0. \quad (137)$$

With the metric ansatz (37), the Einstein equations give rise to the following (dependent) equations:

$$2A'' + 3A'^2 + \frac{e^{-2A}}{\ell^2} + \frac{3}{2}U_1'^2 + \frac{9}{4}V_1'^2 + \frac{1}{4}e^{-6V_1}\chi'^2 + \frac{3}{2}e^{-2U_1}Z'^2 + V = 0, \quad (138)$$

$$3A'^2 + \frac{3}{\ell^2}e^{-2A} - \frac{3}{2}U_1'^2 - \frac{9}{4}V_1'^2 - \frac{1}{4}e^{-6V_1}\chi'^2 - \frac{3}{2}e^{-2U_1}Z'^2 + V = 0 \quad (139)$$

where V is the scalar potential given in (56).

Appendix B: BPS equations for $N = 1$ supersymmetry

We give the BPS equations for the $N = 1$ RG flow solutions here. These equations are given by

$$U_1' = \frac{e^{-\frac{3}{2}(2U_1+V_1)}}{\mathcal{Q}}[(e^{2U_1} + 2e^{U_1+3V_1} + k) \times (e^{2U_1} + 4e^{U_1+3V_1} + 3k) + 2Z^2(2e^{2U_1} + 6e^{6V_1} + 5e^{U_1+3V_1} - 3k) + 4\chi^2(2e^{2U_1} + 3Z^2) + 3Z^4 + 4\chi Z(2e^{2U_1} + 3Z^2 - 3k)], \quad (140)$$

$$V_1' = \frac{e^{-\frac{3}{2}(2U_1+V_1)}}{\mathcal{Q}}[(e^{2U_1} + k)^2 - 4e^{2U_1+6V_1} + 2Z^2 \times (e^{2U_1} - 2e^{6V_1} - k)]$$

$$+ Z^4 + 4\chi^2(e^{2U_1} + Z^2) + 4\chi Z(e^{2U_1} + Z^2 - k)], \quad (141)$$

$$Z' = -\frac{2e^{-U_1-\frac{3}{2}V_1}}{\mathcal{Q}}[2Ze^{6V_1} + 2Ze^{U_1+3V_1} + (\chi + Z) \times (e^{2U_1} + 2\chi Z + Z^2 - k)], \quad (142)$$

$$\chi' = -\frac{12e^{-3U_1+\frac{9}{2}V_1}}{\mathcal{Q}}[(2\chi + Z)e^{2U_1} + Z(Z^2 + 2\chi Z - k)] \quad (143)$$

where

$$\mathcal{Q} = \sqrt{[2e^{3V_1}Z + 2(\chi + Z)e^{U_1}]^2 + [e^{2U_1} + 2e^{U_1+3V_1} + k - Z(2\chi + Z)]^2}. \quad (144)$$

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Supersymmetric $\text{AdS}_2 \times \Sigma_2$ solutions from tri-sasakian truncation

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Abstract A class of $\text{AdS}_2 \times \Sigma_2$, with Σ_2 being a two-sphere or a hyperbolic space, solutions within four-dimensional $N = 4$ gauged supergravity coupled to three-vector multiplets with dyonic gauging is identified. The gauged supergravity has a non-semisimple $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ gauge group and can be obtained from a consistent truncation of 11-dimensional supergravity on a tri-sasakian manifold. The maximally symmetric vacua contain AdS_4 geometries with $N = 1, 3$ supersymmetry corresponding to $N = 1$ and $N = 3$ superconformal field theories (SCFTs) in three dimensions. We find supersymmetric solutions of the form $\text{AdS}_2 \times \Sigma_2$ preserving two supercharges. These solutions describe twisted compactifications of the dual $N = 1$ and $N = 3$ SCFTs and should arise as near horizon geometries of dyonic black holes in asymptotically AdS_4 space-time. Most solutions have hyperbolic horizons although some of them exhibit spherical horizons. These provide a new class of $\text{AdS}_2 \times \Sigma_2$ geometries with known M-theory origin.

1 Introduction

Apart from giving deep insight to strongly coupled gauge theories and string/M-theory compactifications in various dimensions, the AdS/CFT correspondence has been recently used to explain the entropy of asymptotically AdS_4 black holes [1–3]. In this context, the black hole entropy is computed using topologically twisted index of three-dimensional superconformal field theories (SCFTs) compactified on a Riemann surface Σ_2 [4–8]. In the dual gravity solutions, the black holes interpolate between the asymptotically AdS_4 and the near horizon $\text{AdS}_2 \times \Sigma_2$ geometries. These can be interpreted as RG flows from three-dimensional SCFTs in the form of Chern–Simons–Matter (CSM) theories possibly with flavor matters to superconformal quantum mechanics corresponding to the AdS_2 geometry.

Along this line of research, BPS black hole solutions in four-dimensional gauged supergravity, in particular near horizon geometries, with known higher-dimensional origins are very useful. Most of the solutions have been studied within $N = 2$ gauged supergravities [9–15], for recent results; see [16, 17]. Many of these solutions can be uplifted to string/M-theory since these $N = 2$ gauged supergravities can be obtained either from truncations of the maximal $N = 8$ gauged supergravity, whose higher-dimensional origin is known, or direct truncations of M-theory on Sasaki–Einstein manifolds.

In this work, we give evidence for a new class of BPS black hole solutions in the half-maximal $N = 4$ gauged supergravity with known higher-dimensional origin by finding a number of new $\text{AdS}_2 \times \Sigma_2$ solutions. This gauged supergravity has $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ gauge group and can be obtained from a compactification of M-theory on a tri-sasakian manifold [18]. Holographic RG flows and supersymmetric Janus solutions, describing $(1+1)$ -dimensional interfaces in the dual SCFTs have recently appeared in [19]. In the present paper, we will look for supersymmetric solutions of the form $\text{AdS}_2 \times \Sigma_2$ within this tri-sasakian compactification.

Apart from giving this type of solutions in gauged supergravity with more supersymmetry, to the best of the authors' knowledge, the results are the first example of $\text{AdS}_2 \times \Sigma_2$ solutions from the truncation of M-theory on a tri-sasakian manifold. The truncation given in [18] gives a reduction ansatz for 11-dimensional supergravity on a generic tri-sasakian manifold including massive Kaluza–Klein modes. Among this type of manifolds, N^{010} with isometry $SU(2) \times SU(3)$ is of particular interest. In this case, there is a non-trivial two-form giving rise to an extra vector multiplet; see [20, 21] for the Kaluza–Klein spectrum of $\text{AdS}_4 \times N^{010}$. This background, discovered long ago in [22], preserves $N = 3$ out of the original $N = 4$ supersymmetry. There is another supersymmetric AdS_4 vacuum with $SO(3)$ symmetry and $N = 1$ supersymmetry, the one broken by $\text{AdS}_4 \times N^{010}$.

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This vacuum corresponds to $\text{AdS}_4 \times \tilde{N}^{010}$ geometry, with $\tilde{N}^{010}$ being a squashed version of N^{010} .

Not much is known about the dual $N = 1$ SCFT, but the dual $N = 3$ SCFT has been proposed in a number of previous works [23, 24]; see also [25, 26]. This SCFT takes the form of a CSM theory with $SU(N) \times SU(N)$ gauge group. It is a theory of interacting three hypermultiplets transforming in a triplet of the $SU(3)$ flavor symmetry, and each hypermultiplet transforms as a bifundamental under the $SU(N) \times SU(N)$ gauge group and as a doublet of the $SU(2)_R \sim SO(3)_R$ R-symmetry. There are also a number of previous works giving holographic studies of this theory both in 11-dimensional context and in the effective $N = 3$ and $N = 4$ gauged supergravities [19, 27–31]. Solutions given in these works are holographic RG flows, Janus solutions and supersymmetric $\text{AdS}_2 \times \Sigma_2$ solutions with magnetic charges.

In this work, we consider $N = 4$ gauged supergravity constructed in the embedding tensor formalism in [32]. This construction is the most general supersymmetric gaugings of $N = 4$ supergravity in which both the “electric” vector fields, appearing in the ungauged Lagrangian, and their magnetic duals can participate. Therefore, magnetic and dyonic gaugings are allowed in this formulation. Furthermore, this formulation contains the “purely electric” gauged $N = 4$ supergravity constructed long time ago in [33] and the non-trivial $SL(2, \mathbb{R})$ phases of [34, 35] as special cases. We will look for supersymmetric $\text{AdS}_2 \times \Sigma_2$ solutions in the $N = 4$ gauged supergravity with a dyonic gauging of the non-semisimple group $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$. The solutions are required to preserve $SO(2) \subset SO(3)_R$, so only a particular combination of vector fields corresponding to this $SO(2)$ residual gauge symmetry appears in the gauge covariant derivative. The strategy is essentially similar to the wrapped brane solutions of [36], implementing the twist by canceling the spin connections on Σ_2 by the $SO(2)$ gauge connection.

These $\text{AdS}_2 \times \Sigma_2$ solutions should appear as near horizon geometries of supersymmetric black holes in asymptotically AdS_4 space-time. Since the $N = 4$ gauged supergravity admits two supersymmetric AdS_4 vacua with unbroken $SO(3)_R$ symmetry and $N = 1, 3$ supersymmetries, the $\text{AdS}_2 \times \Sigma_2$ solutions should be RG fixed points in one dimension of the dual $N = 1, 3$ SCFTs. Although the structure of the dual $N = 1$ SCFT is presently not clear, we expect that there should be RG flows between these twisted $N = 1, 3$ SCFTs on Σ_2 to one-dimensional superconformal quantum mechanics dual to the $\text{AdS}_2 \times \Sigma_2$ solutions. In this sense, the entropy of these black holes would possibly be computed from the topologically twisted indices of the dual $N = 1, 3$ SCFTs. Furthermore, these solutions should provide a new class of AdS_2 geometries within M-theory.

The paper is organized as follows. In Sect. 2, we review $N = 4$ gauged supergravity coupled to vector multiplets and relevant formulas for uplifting the resulting solutions to 11

dimensions. The analysis of BPS equations for $SO(2) \subset SO(3)_R$ singlet scalars and Yang–Mills equations, for static black hole ansatz consistent with the symmetry of Σ_2 , will be carried out in Sect. 3. In Sect. 4, we will explicitly give $\text{AdS}_2 \times \Sigma_2$ solutions to the BPS flow equations. We separately consider the $N = 1$ and $N = 3$ cases and end the section with the uplift formulas for embedding the solutions in 11 dimensions. We finally give some conclusions and comments on the results in Sect. 5. In the appendix, we collect the convention regarding ‘t Hooft matrices and give the explicit form of the Yang–Mills and BPS equations.

2 $N = 4$ gauged supergravity with dyonic gauging

In this section, we review $N = 4$ gauged supergravity in the embedding tensor formalism following [32]. We mainly focus on the bosonic Lagrangian and supersymmetry transformations of fermions which provide basic ingredients for finding supersymmetric solutions. Since the gauged supergravity under consideration is known to arise from a trisaskian truncation of 11-dimensional supergravity, we will also give relevant formulas which are useful to uplift four-dimensional solutions to 11 dimensions. The full detail of this truncation can be found in [18].

2.1 $N = 4$ gauged supergravity coupled to vector multiplets

In the half-maximal $N = 4$ supergravity in four dimensions, the supergravity multiplet consists of the graviton $e_{\mu}^{\hat{\mu}}$, four gravitini ψ_{μ}^i , six vectors A_{μ}^m , four spin- $\frac{1}{2}$ fields χ^i and one complex scalar τ . The complex scalar can be parametrized by the $SL(2, \mathbb{R})/SO(2)$ coset. The supergravity multiplet can couple to an arbitrary number n of vector multiplets containing a vector field A_{μ} , four gaugini λ^i and six scalars ϕ^m . The scalar fields can be parametrized by the $SO(6, n)/SO(6) \times SO(n)$ coset.

Space-time and tangent space indices are denoted, respectively, by $\mu, \nu, \dots = 0, 1, 2, 3$ and $\hat{\mu}, \hat{\nu}, \dots = 0, 1, 2, 3$. Indices $m, n = 1, \dots, 6$ and $i, j = 1, 2, 3, 4$, respectively, describe the vector and spinor representations of the $SO(6)_R \sim SU(4)_R$ R-symmetry or equivalently a second-rank anti-symmetric tensor and fundamental representations of $SU(4)_R$. The n vector multiplets are labeled by indices $a, b = 1, \dots, n$. All the fields in the vector multiplets will accordingly carry an additional index in the form of $(A_{\mu}^a, \lambda^{ia}, \phi^{ma})$.

All fermionic fields and the supersymmetry parameters transform in the fundamental representation of $SU(4)_R$ R-symmetry with the chirality projections

$$\gamma_5 \psi_{\mu}^i = \psi_{\mu}^i, \quad \gamma_5 \chi^i = -\chi^i, \quad \gamma_5 \lambda^i = \lambda^i. \quad (1)$$

Similarly, for the fields transforming in the anti-fundamental representation of $SU(4)_R$, we have

$$\gamma_5 \psi_{\mu i} = -\psi_{\mu i}, \quad \gamma_5 \chi_i = \chi_i, \quad \gamma_5 \lambda_i = -\lambda_i. \quad (2)$$

General gaugings of the matter-coupled $N = 4$ supergravity can be efficiently described by the embedding tensor Θ , which encodes the information as regards the embedding of any gauge group G_0 in the global or duality symmetry $SL(2, \mathbb{R}) \times SO(6, n)$. There are two components of the embedding tensor $\xi^{\alpha M}$ and $f_{\alpha MNP}$ with $\alpha = (+, -)$ and $M, N = (m, a) = 1, \dots, n+6$ denoting fundamental representations of $SL(2, \mathbb{R})$ and $SO(6, n)$, respectively. The electric vector fields $A^{M+} = (A_\mu^m, A_\mu^a)$, appearing in the ungauged Lagrangian, together with their magnetic dual A^{M-} form a doublet under $SL(2, \mathbb{R})$. These are denoted collectively by $A^{M\alpha}$. In general, a subgroup of both $SL(2, \mathbb{R})$ and $SO(6, n)$ can be gauged, and the magnetic vector fields can also participate in the gauging. However, in this paper, we only consider gaugings with only $f_{\alpha MNP}$ non-vanishing. We then set $\xi^{\alpha M}$ to zero from now on. This also considerably simplifies many formulas given below.

The full covariant derivative can be written as

$$D_\mu = \nabla_\mu - g A_\mu^{M\alpha} f_{\alpha M}^{NP} t_{NP} \quad (3)$$

where ∇_μ is the space-time covariant derivative including the spin connections. t_{MN} are $SO(6, n)$ generators which can be chosen as

$$(t_{MN})_P^Q = 2\delta_{[M}^Q \eta_{N]P}, \quad (4)$$

with η_{MN} being the $SO(6, n)$ invariant tensor. The gauge coupling constant g can be absorbed in the embedding tensor Θ . The original gauging considered in [33] only involves electric vector fields and is called electric gauging. In this case, only f_{+MNP} are non-vanishing. In the following discussions, we will consider dyonic gauging involving both electric and magnetic vector fields. In this case, both A^{M+} and A^{M-} enter the Lagrangian, and $f_{\alpha MNP}$ with $\alpha = \pm$ are non-vanishing. Consistency requires the presence of two-form fields when magnetic vector fields are included. In the present case with $\xi^{\alpha M} = 0$, these two-forms transform as an anti-symmetric tensor under $SO(6, n)$ and will be denoted by $B_{\mu\nu}^{MN} = B_{\mu\nu}^{[MN]}$. The two-forms modify the gauge field strengths to

$$\begin{aligned} \mathcal{H}^{M\pm} &= dA^{M\pm} - \frac{1}{2} \eta^{MQ} f_{\alpha QNP} A^{N\alpha} \wedge A^{P\pm} \\ &\quad \pm \frac{1}{2} \eta^{MQ} f_{\mp QNP} B^{NP}. \end{aligned} \quad (5)$$

Note that for non-vanishing f_{-MNP} the field strengths of electric vectors $\mathcal{H}^{M+}$ have a contribution from the two-form fields.

Before moving to the Lagrangian, we explicitly give the parametrization of the scalar coset manifold $SL(2, \mathbb{R})/SO(2)$

$\times SO(6, n)/SO(6) \times SO(n)$. The first factor can be described by a coset representative

$$\mathcal{V}_\alpha = \frac{1}{\sqrt{\text{Im}\tau}} \begin{pmatrix} \tau \\ 1 \end{pmatrix} \quad (6)$$

or equivalently by a symmetric matrix

$$M_{\alpha\beta} = \text{Re}(\mathcal{V}_\alpha \mathcal{V}_\beta^*) = \frac{1}{\text{Im}\tau} \begin{pmatrix} |\tau|^2 & \text{Re}\tau \\ \text{Re}\tau & 1 \end{pmatrix}. \quad (7)$$

It should also be noted that $\text{Im}(\mathcal{V}_\alpha \mathcal{V}_\beta^*) = \epsilon_{\alpha\beta}$. The complex scalar τ can also be written in terms of the dilaton ϕ and the axion χ as

$$\tau = \chi + i e^\phi. \quad (8)$$

For the $SO(6, n)/SO(6) \times SO(n)$ factor, we can introduce the coset representative $\mathcal{V}_M^A$ transforming by left and right multiplications under $SO(6, n)$ and $SO(6) \times SO(n)$, respectively. The $SO(6) \times SO(n)$ index will be split as $A = (m, a)$ according to which the coset representative can be written as $\mathcal{V}_M^A = (\mathcal{V}_M^m, \mathcal{V}_M^a)$. As an element of $SO(6, n)$, the matrix $\mathcal{V}_M^A$ also satisfies the relation

$$\eta_{MN} = -\mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \quad (9)$$

As in the $SL(2, \mathbb{R})/SO(2)$ factor, the $SO(6, n)/SO(6) \times SO(n)$ coset can also be parametrized in term of a symmetric matrix defined by

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \quad (10)$$

The bosonic Lagrangian of the $N = 4$ gauged supergravity is given by

$$\begin{aligned} e^{-1} \mathcal{L} &= \frac{1}{2} R + \frac{1}{16} \mathcal{D}_\mu M_{MN} \mathcal{D}^\mu M^{MN} \\ &\quad - \frac{1}{4(\text{Im}\tau)^2} \partial_\mu \tau \partial^\mu \tau^* - V \\ &\quad - \frac{1}{4} \text{Im} \tau M_{MN} \mathcal{H}_{\mu\nu}^{M+} \mathcal{H}^{N+\mu\nu} \\ &\quad - \frac{1}{8} \text{Re} \tau e^{-1} \epsilon^{\mu\nu\rho\sigma} \eta_{MN} \mathcal{H}_{\mu\nu}^{M+} \mathcal{H}_{\rho\sigma}^{N+} \\ &\quad - \frac{1}{2} e^{-1} \epsilon^{\mu\nu\rho\sigma} \left[f_{-MNP} A_\mu^{M-} A_\nu^{N+} \partial_\rho A_\sigma^{P-} \right. \\ &\quad \left. + \frac{1}{4} f_{\alpha MNR} f_{\beta PQS} \eta^{RS} A_\mu^{M\alpha} A_\nu^{N+} A_\rho^{P\beta} A_\sigma^{Q-} \right. \\ &\quad \left. - \frac{1}{4} f_{-MNP} B_{\mu\nu}^{NP} \right. \\ &\quad \left. \times \left(2\partial_\rho A_\sigma^{M-} - \frac{1}{2} \eta^{MS} f_{\alpha SQR} A_\rho^{Q\alpha} A_\sigma^{R-} \right) \right. \\ &\quad \left. - \frac{1}{16} f_{+MNR} f_{-PQS} \eta^{RS} B_{\mu\nu}^{MN} B_{\rho\sigma}^{PQ} \right] \end{aligned} \quad (11)$$

where e is the vielbein determinant. The scalar potential is given by

$$V = \frac{g^2}{16} \left[f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta} \times \left[\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left(\frac{2}{3} \eta^{MQ} - M^{MQ} \right) \eta^{NR} \eta^{PS} \right] - \frac{4}{9} f_{\alpha MNP} f_{\beta QRS} \epsilon^{\alpha\beta} M^{MNPQRS} \right] \quad (12)$$

where M^{MN} is the inverse of M_{MN} , and M^{MNPQRS} is defined by

$$M_{MNPQRS} = \epsilon_{mnpqrs} \mathcal{V}_M^m \mathcal{V}_N^n \mathcal{V}_P^p \mathcal{V}_Q^q \mathcal{V}_R^r \mathcal{V}_S^s \quad (13)$$

with indices raised by η^{MN} . The covariant derivative of M_{MN} is defined by

$$DM_{MN} = dM_{MN} + 2A^{P\alpha} \eta^{QR} f_{\alpha QP(M} M_{N)R}. \quad (14)$$

It should be pointed out here that the magnetic vectors and the two-forms do not have kinetic terms. They are auxiliary fields whose field equations give rise to the duality relation between two-forms and scalars and the electric-magnetic duality of A^{M+} and A^{M-} , respectively. Together with the Yang–Mills equations obtained from the variation with respect to A^{M+} , these equations are given by

$$\eta_{MN} * \mathcal{D}\mathcal{H}^{N-} = -\frac{1}{4} f_{+MP}^N M_{NQ} \mathcal{D}M^{QP}, \quad (15)$$

$$\eta_{MN} * \mathcal{D}\mathcal{H}^{N+} = \frac{1}{4} f_{-MP}^N M_{NQ} \mathcal{D}M^{QP}, \quad (16)$$

$$\mathcal{H}^{M-} = \text{Im } \tau M^{MN} \eta_{NP} * \mathcal{H}^{P+} - \text{Re } \tau \mathcal{H}^{M+}, \quad (17)$$

where we have used differential form language for later computational convenience. By substituting $\mathcal{H}^{M-}$ from (17) in (15), we obtain the usual Yang–Mills equations for $\mathcal{H}^{M+}$ while Eq. (16) simply gives the relation between the Hodge dual of the three-form field strength and the scalars due to the usual Bianchi identity of the gauge field strengths

$$\mathcal{F}^{M\pm} = dA^{M\pm} - \frac{1}{2} \eta^{MQ} f_{\alpha QNP} A^{N\alpha} \wedge A^{P\pm}. \quad (18)$$

In this paper, we are interested in $N = 4$ gauged supergravity coupled to three vector multiplets. The gauge group of interest here is a non-semisimple group $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3) \subset SO(6, 3)$ described by the following components of the embedding tensor:

$$\begin{aligned} f_{+IJ, K+6} &= -f_{+I+3, J+6, K+6} \\ &= -2\sqrt{2} \epsilon_{IJK}, \quad I, J, K = 1, 2, 3, \\ f_{+I+6, J+6, K+6} &= 6\sqrt{2} k \epsilon_{IJK}, \\ f_{-I, J+6, K+6} &= -4\epsilon_{IJK}. \end{aligned} \quad (19)$$

The constant k is related to the four-form flux along the four-dimensional space-time; see Eq. (122).

We should also remark that we follow the convention of [18] in all of the computations carried out here. In particular, the $SO(6, 3)$ tensor η_{MN} is off-diagonal

$$\eta_{MN} = \begin{pmatrix} -\mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 \\ \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_3 \end{pmatrix} \quad (20)$$

where $\mathbf{0}_3$ and $\mathbf{I}_3$ denote 3×3 zero and identity matrices, respectively. As a result, the computation of M_{MNPQRS} in (13) and parts of the supersymmetry transformations given below which involve $\mathcal{V}_M^m$ and $\mathcal{V}_M^a$ must be done with the projection to the negative and positive eigenvalues of η_{MN} , respectively. This can be achieved by using the projection matrix

$$P = \begin{pmatrix} \mathbf{0}_3 & \sqrt{2} \tilde{P}_3 & \mathbf{0}_3 \\ -\tilde{P}_3 & \mathbf{0}_3 & \tilde{P}_3 \\ \tilde{P}_3 & \mathbf{0}_3 & \tilde{P}_3 \end{pmatrix} \quad (21)$$

where the 3×3 matrix $\tilde{P}_3$ is given by

$$\tilde{P}_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (22)$$

We now turn to the supersymmetry transformations of fermionic fields. These are given by

$$\delta\psi_\mu^i = 2D_\mu \epsilon^i - \frac{2}{3} g A_1^{ij} \gamma_\mu \epsilon_j + \frac{i}{4} (\mathcal{V}_\alpha)^* \mathcal{V}_M^{ij} \mathcal{H}_{\nu\rho}^{M\alpha} \gamma^{\nu\rho} \gamma_\mu \epsilon_j, \quad (23)$$

$$\delta\chi^i = i\epsilon^{\alpha\beta} \mathcal{V}_\alpha D_\mu \mathcal{V}_\beta \gamma^\mu \epsilon^i - \frac{4}{3} i g A_2^{ij} \epsilon_j + \frac{i}{2} \mathcal{V}_\alpha \mathcal{V}_M^{ij} \mathcal{H}_{\mu\nu}^{M\alpha} \epsilon_j, \quad (24)$$

$$\begin{aligned} \delta\lambda_a^i &= 2i \mathcal{V}_a^M D_\mu \mathcal{V}_M^{ij} \gamma^\mu \epsilon_j + 2i g A_{2aj}^i \epsilon^j \\ &\quad - \frac{1}{4} \mathcal{V}_\alpha \mathcal{V}_{Ma} \mathcal{H}_{\mu\nu}^{M\alpha} \gamma^{\mu\nu} \epsilon^i. \end{aligned} \quad (25)$$

The fermion shift matrices are defined by

$$\begin{aligned} A_1^{ij} &= \epsilon^{\alpha\beta} (\mathcal{V}_\alpha)^* \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_2^{ij} &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_{2ai}{}^j &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_a^M \mathcal{V}_{ik}^N \mathcal{V}_P^{jk} f_{\beta MN}^P \end{aligned} \quad (26)$$

where $\mathcal{V}_M^{ij}$ is defined in terms of the 't Hooft matrices G_m^{ij} and $\mathcal{V}_M^m$ as

$$\mathcal{V}_M^{ij} = \frac{1}{2} \mathcal{V}_M^m G_m^{ij} \quad (27)$$

and similarly for its inverse

$$\mathcal{V}^M_{ij} = -\frac{1}{2} \mathcal{V}^M_m (G_m^{ij})^*. \quad (28)$$

G_m^{ij} satisfy the relations

$$G_{mij} = (G_m^{ij})^* = \frac{1}{2} \epsilon_{ijkl} G_m^{kl}. \quad (29)$$

The explicit form of these matrices is given in the appendix. It should also be noted that the scalar potential can be written in terms of A_1 and A_2 tensors as

$$V = -\frac{1}{3}A_1^{ij}A_{1ij} + \frac{1}{9}A_2^{ij}A_{2ij} + \frac{1}{2}A_{2ai}{}^jA_{2a}{}^i{}_j. \quad (30)$$

With the explicit form of $\mathcal{V}_\alpha$ given in (6) and Eq. (17), it is straightforward to derive the following identities:

$$\begin{aligned} i\mathcal{V}_\alpha\mathcal{V}_M^{ij}\mathcal{H}_{\mu\nu}^{M\alpha}\gamma^{\mu\nu} &= -(\mathcal{V}_-)^{-1}\mathcal{V}_M^{ij}\mathcal{H}_{\mu\nu}^{M+}\gamma^{\mu\nu}(1-\gamma_5), \quad (31) \\ i\mathcal{V}_\alpha\mathcal{V}_M^a\mathcal{H}_{\mu\nu}^{M\alpha}\gamma^{\mu\nu} &= -(\mathcal{V}_-)^{-1}\mathcal{V}_M^a\mathcal{H}_{\mu\nu}^{M+}\gamma^{\mu\nu}(1+\gamma_5), \quad (32) \\ i(\mathcal{V}_\alpha)^*\mathcal{V}_M^{ij}\mathcal{H}_{\mu\nu}^{M\alpha}\gamma^{\mu\nu}\gamma_\rho &= (\mathcal{V}_-)^{-1}\mathcal{V}_M^{ij}\mathcal{H}_{\mu\nu}^{M+}\gamma^{\mu\nu}\gamma_\rho(1-\gamma_5). \quad (33) \end{aligned}$$

In obtaining these results, we have used the following relations for the $SO(6, n)$ coset representative [33]:

$$\begin{aligned} \eta_{MN} &= -\frac{1}{2}\epsilon_{ijkl}\mathcal{V}_M^{ij}\mathcal{V}_N^{kl} + \mathcal{V}_M^a\mathcal{V}_N^a, \\ \mathcal{V}_M^a\mathcal{V}^M{}_{ij} &= 0, \\ \mathcal{V}_M^{ij}\mathcal{V}^M{}_{kl} &= -\frac{1}{2}(\delta_k^i\delta_l^j - \delta_l^i\delta_k^j), \quad \mathcal{V}_M^a\mathcal{V}^M{}_b = \delta_b^a. \quad (34) \end{aligned}$$

These relations are useful in simplifying the BPS equations resulting from the supersymmetry transformations. Note also that these relations are slightly different from those given in [32] due to a different convention on $\mathcal{V}_\alpha$ in terms of the scalar τ . In more detail, $\mathcal{V}_\alpha$ used in this paper and in [18] satisfies $\mathcal{V}_+/\mathcal{V}_- = \tau$ while $\mathcal{V}_\alpha$ used in [32] gives $\mathcal{V}_+/\mathcal{V}_- = \tau^*$. This results in some sign changes in the above equations compared to those of [32].

2.2 Uplift formulas to 11 dimensions

As mentioned above, four-dimensional $N = 4$ gauged supergravity coupled to three vector multiplets with $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ gauge group has been obtained from a truncation of 11-dimensional supergravity on a tri-sasakian manifold in [18]. We will briefly review the structure and relevant formulas focusing on the reduction ansatz which will be useful for embedding four-dimensional solutions. Essentially, we simply collect some formulas without giving detailed explanations for which we refer the interested reader to [18].

The 11-dimensional metric can be written as

$$ds_{11}^2 = e^{2\varphi}ds_4^2 + e^{2U}ds^2(B_{\text{QK}}) + g_{IJ}(\eta^I + A_1^I)(\eta^J + A_1^J). \quad (35)$$

The three-dimensional internal metric g_{IJ} can be written in terms of the vielbein as

$$g = Q^T Q. \quad (36)$$

Following [18], we will parametrize the matrix Q in terms of a product of a diagonal matrix V and an $SO(3)$ matrix O

$$Q = VO, \quad V = \text{diag}(e^{V_1}, e^{V_2}, e^{V_3}). \quad (37)$$

The scalar φ is chosen to be

$$\varphi = -\frac{1}{2}(4U + V_1 + V_2 + V_3) \quad (38)$$

in order to obtain the Einstein frame action in four dimensions. B_{QK} denotes a four-dimensional quaternionic Kähler manifold whose explicit metric is not needed in the following discussions.

The ansatz for the four-form field is given by

$$\begin{aligned} G_4 &= H_4 + H_{3I} \wedge (\eta + A_1)^I + \frac{1}{2}\epsilon_{IJK}\tilde{H}_2^I \wedge (\eta + A_1)^J \\ &\quad \wedge (\eta + A_1)^K + 4\text{Trc vol}(\text{QK})H_{1IJ} \\ &\quad \wedge (\eta + A_1)^I \wedge J^J + \frac{1}{6}\epsilon_{IJK} \\ &\quad \times d\chi \wedge (\eta + A_1)^I \wedge (\eta + A_1)^J \wedge (\eta + A_1)^K \\ &\quad + H_{2I} \wedge J^I + \epsilon_{IJL}[(\chi + \text{Trc})\delta_{LK} \\ &\quad - 2c_{(LK)}](\eta + A_1)^I \wedge (\eta + A_1)^J \wedge J^K. \quad (39) \end{aligned}$$

c_{IJ} is a 3×3 matrix and $\text{Trc} = \delta^{IJ}c_{IJ}$. The volume form of B_{QK} , $\text{vol}(\text{QK})$, can be written in terms of the two-forms J^I as

$$\text{vol}(\text{QK}) = \frac{1}{6}J^I \wedge J^I. \quad (40)$$

Various forms in the above equation are defined by

$$\begin{aligned} H_4 &= dc_3 + c_{2I} \wedge F_2^I, \\ H_{3I} &= Dc_{2I} + \epsilon_{IJK}F_2^J \wedge \tilde{c}_{1K}, \\ \tilde{H}_{2I} &= D\tilde{c}_{1I} - 2c_{2I} + \chi F_{2I}, \\ H_{2I} &= Dc_{1I} + 2c_{2I} + c_{JI}F_2^J, \\ H_{1IJ} &= Dc_{IJ} + 2\epsilon_{IJK}(c_{1K} + \tilde{c}_{1K}) \quad (41) \end{aligned}$$

with the $SO(3)$ covariant derivative

$$Dc_{I_1\dots I_n} = dc_{I_1\dots I_n} + 2\sum_{l=1}^n \epsilon_{JlK}A_1^J \wedge c_{I_1\dots K\dots I_n}. \quad (42)$$

The $SO(3)_R$ field strengths are defined by

$$F_2^I = dA_1^I - \epsilon_{IJK}A_1^J \wedge A_1^K. \quad (43)$$

It is useful to note here that the $SL(2, \mathbb{R})/SO(2)$ scalars are given by

$$\tau = \chi + ie^{V_1+V_2+V_3}. \quad (44)$$

Although we will not directly need the explicit form of $ds^2(B_{\text{QK}})$ and η^I 's in the remaining parts of this paper, it is useful to give some information on the N^{010} tri-sasakian manifold. N^{010} is a 7-manifold with $SU(2) \times SU(3)$ isometry. The $SU(2)$ is identified with the R-symmetry of the dual $N = 3$ SCFT while $SU(3)$ is the flavor symmetry. A

simple description of N^{010} can be obtained in terms of a coset manifold $SU(3)/U(1)$. With the standard Gell-Mann matrices, the $SU(3)$ generators can be chosen to be $-\frac{i}{2}\lambda_\alpha$, $\alpha = 1, \dots, 8$. The coset and $U(1)$ generators are accordingly identified as

$$K_i = -\frac{i}{2}(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7),$$

$$H = -\frac{i\sqrt{3}}{2}\lambda_8. \quad (45)$$

The vielbein on N^{010} can eventually be obtained from the decomposition of the Maurer–Cartan one-form

$$L^{-1}dL = e^i K_i + \omega H \quad (46)$$

where L is the coset representative for $SU(3)/U(1)$, and ω is the corresponding $U(1)$ connection.

Following [18], we can use the tri-sasakian structures of the form

$$\eta^I = \frac{1}{2}(e^1, e^2, e^7),$$

$$J^I = \frac{1}{8}(e^4 \wedge e^5 - e^3 \wedge e^6, -e^3 \wedge e^5 - e^4 \wedge e^6, e^5 \wedge e^6 - e^3 \wedge e^4). \quad (47)$$

From these, we find the metric on the quaternionic–Kähler base B_{QK} to be

$$ds^2(B_{\text{QK}}) = \frac{1}{256}[(e^3)^2 + (e^4)^2 + (e^5)^2 + (e^6)^2] \quad (48)$$

with the volume form given by

$$\text{vol}(\text{QK}) = \frac{1}{6}J^I \wedge J^I = -\frac{1}{64}e^3 \wedge e^4 \wedge e^5 \wedge e^6. \quad (49)$$

As mentioned before, all of the fields appearing in the reduction of [18] are $SU(3)$ singlets.

3 BPS flow equations

In this section, we perform the analysis of Yang–Mills equations and supersymmetry transformations in order to obtain BPS equations for the flows between AdS_4 vacua and possible $\text{AdS}_2 \times \Sigma_2$ geometries. We set all fermions to zero and truncate the bosonic fields to $SO(2) \subset SO(3)_R$ singlets. This $SO(2)$ is generated by

$$\hat{X} = X_{9+} + X_{6+} + X_{3-} \quad (50)$$

where the gauge generators are defined by

$$X_{M\alpha} = -f_{\alpha MNP} \hat{I}^{NP}. \quad (51)$$

We see that a combination of the electric vectors A^{9+} , A^{6+} and the magnetic vector A^{3-} becomes the corresponding $SO(2)$ gauge field.

We are interested in supersymmetric solutions of the form $\text{AdS}_2 \times \Sigma_2$ with $\Sigma_2 = S^2, H^2$. We will then take the ansatz for the four-dimensional metric to be

$$ds_4^2 = -e^{2f(r)}dt^2 + dr^2 + e^{2g(r)}(d\theta^2 + F(\theta)^2 d\phi^2) \quad (52)$$

with

$$F(\theta) = \sin \theta \quad \text{and} \quad F(\theta) = \sinh \theta \quad (53)$$

for the S^2 and H^2 , respectively. We will also use the parameter $\kappa = \pm 1$ to denote the S^2 and H^2 cases. The functions $f(r)$, $g(r)$ and all other fields only depend on the radial coordinate r for static solutions. With the obvious vielbein

$$e^{\hat{t}} = e^f dt, \quad e^{\hat{r}} = dr,$$

$$e^{\hat{\theta}} = e^g d\theta, \quad e^{\hat{\phi}} = e^g F d\phi, \quad (54)$$

it is now straightforward to compute the spin connections of the above metric

$$\omega^{\hat{t}\hat{r}} = f' e^{\hat{t}}, \quad \omega^{\hat{\theta}\hat{r}} = g' e^{\hat{\theta}},$$

$$\omega^{\hat{\phi}\hat{r}} = g' e^{\hat{\phi}}, \quad \omega^{\hat{\theta}\hat{\phi}} = \frac{F'(\theta)}{F(\theta)} e^{-g} e^{\hat{\phi}}. \quad (55)$$

In the above expressions, we have used the hat to denote “flat” indices while $'$ stands for the r -derivative with the only exception that $F'(\theta) = \frac{dF(\theta)}{d\theta}$. The ansatz for electric and magnetic vector fields are given by

$$A^{M+} = \mathcal{A}_r^M dt - p^M F'(\theta) d\phi, \quad (56)$$

$$A^{M-} = \tilde{\mathcal{A}}_r^M dt - e_M F'(\theta) d\phi \quad (57)$$

where we have chosen the gauge such that $A_r^{M\alpha} = 0$. p^M and e_M correspond to magnetic and electric charges, respectively. In the present case, only $A^{M\alpha}$ with $M = 3, 6, 9$ are relevant.

We finally give the explicit form of the scalar coset representative for $SO(6, 3)/SO(6) \times SO(3)$. The parametrization of [18] which is directly related to the higher-dimensional origin is given by

$$\mathcal{V} = \mathcal{C} \mathcal{Q} \quad (58)$$

where the matrices $\mathcal{Q}$ and $\mathcal{C}$ are defined by

$$\mathcal{Q} = \begin{pmatrix} \mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & e^{-2U} Q^{-1} & \mathbf{I}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & e^{2U} Q^T \end{pmatrix},$$

$$\mathcal{C} = \exp \begin{pmatrix} \mathbf{0}_3 & \sqrt{2}c^T & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \sqrt{2}c & a & \mathbf{0}_3 \end{pmatrix}. \quad (59)$$

For $SO(2)$ invariant scalars, the 3×3 matrices c and a are given by

$$c = \begin{pmatrix} Z_1 & Z_3 & 0 \\ -Z_3 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{pmatrix}, \quad a = \begin{pmatrix} 0 & \Phi & 0 \\ -\Phi & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (60)$$

while Q can be obtained from (37) with $V_2 = V_1$ and O being

$$O = \exp \begin{pmatrix} 0 & \beta & 0 \\ -\beta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (61)$$

This is a generalization of the coset representative of the $SO(3)_R$ singlet scalars used in [19] in which $\Phi = \beta = Z_3 = 0$, $Z_1 = Z_2$ and $V_1 = V_2 = V_3$. In the following, we will rename the scalars $V_3 \rightarrow V_2$ such that the complex scalar τ becomes

$$\tau = \chi + ie^{2V_1+V_2}. \quad (62)$$

We now give the scalar potential for $SO(2)$ singlet scalars,

$$\begin{aligned} V = e^{-3(4U+2V_1+V_2)} [e^{4(U+V_2)} (e^{4U} + 2e^{4V_1}) \\ + 9k^2 + 4\chi^2 e^{4U+2V_1} \\ - 4e^{6U+4V_1+2V_2} (6 + e^{2(U-V_1)} \\ - e^{-2(U-V_1)}) + 24k\chi Z_1 + 16\chi^2 Z_1^2 \\ + 8\chi Z_2 e^{4U+2V_1} - 12k\chi Z_2 \\ + (16\chi^2 - 24k)Z_1 Z_2 + 32\chi Z_1^2 Z_2 \\ + 4Z_2^2 e^{4U+2V_1} + 4\chi^2 Z_2^2 + 8\chi Z_1 Z_2^2 \\ + 16Z_1^2 Z_2^2 - 4\chi Z_2^3 - 8Z_1 Z_2^3 \\ + 6kZ_2^2 + Z_2^4 + 2e^{2V_2} \\ [e^{4U} (\chi + 2Z_1 - Z_2)^2 + 2e^{4V_1} (2Z_1 + Z_2)^2]]. \end{aligned} \quad (63)$$

The scalars β , Φ and Z_3 do not appear in the potential. It can also be checked that setting $\beta = \Phi = Z_3 = 0$ is a consistent truncation. In fact, β never appears in any equations, so we can set it to zero. On the other hand, the Yang–Mills equations, to be given later, demand that Φ and Z_3 must be constant. Since we are interested in the flow solutions interpolating between $\text{AdS}_2 \times \Sigma_2$ and AdS_4 vacua, and at supersymmetric AdS_4 critical points, both Φ and Z_3 vanish. We then choose $Z_3 = \Phi = 0$.

The kinetic terms for the remaining scalars read

$$\begin{aligned} \mathcal{L}_{\text{kin}} = -6U'^2 - 2U'(2V_1' + V_2') - 2V_1'^2 - V_1'V_2' \\ - \frac{1}{4}[3V_2'^2 + e^{-2(2V_1+V_2)} \chi'^2 + 4e^{-2(2U+V_1)} \\ \times Z_1'^2 + 2e^{-2(2U+V_2)} Z_2'^2]. \end{aligned} \quad (64)$$

We now redefine the scalars such that the kinetic terms are diagonal

$$\tilde{V} = 2V_1 + V_2, \quad \tilde{U}_1 = 2U + V_1, \quad \tilde{U}_2 = 2U + V_2, \quad (65)$$

in terms of which we find

$$\begin{aligned} \mathcal{L}_{\text{kin}} = -\frac{1}{4}(4\tilde{U}_1'^2 + 2\tilde{U}_2'^2 + \tilde{V}'^2 \\ + e^{-2\tilde{V}} \chi'^2 + 4e^{-2\tilde{U}_1} Z_1'^2 + 2e^{-2\tilde{U}_2} Z_2'^2). \end{aligned} \quad (66)$$

These new scalars will also be useful in the analysis of the BPS equations below.

The above scalar potential admits two supersymmetric AdS_4 vacua with $N = 1$ and $N = 3$ supersymmetries [18]. At these vacua the symmetry is enhanced from $SO(2)$ to $SO(3)$. For convenience, before carry out the analysis of the Yang–Mills and BPS equations, we review the $N = 3$ and $N = 1$ AdS_4 critical points in terms of the new scalars defined above:

$$N = 3 : \quad \tilde{V} = \tilde{U}_1 = \tilde{U}_2 = \frac{1}{2} \ln k, \quad (67)$$

$$V_0 = -12|k|^{-\frac{3}{2}}, \quad k > 0,$$

$$N = 1 : \quad \tilde{U}_1 = \tilde{U}_2 = \ln 5 + \frac{1}{2} \ln \left[-\frac{k}{15} \right],$$

$$\tilde{V} = \frac{1}{2} \ln \left[-\frac{k}{15} \right],$$

$$V_0 = -12|k|^{-\frac{3}{2}} \sqrt{\frac{37}{55}}, \quad k < 0. \quad (68)$$

V_0 is the cosmological constant related to the AdS_4 radius by

$$L^2 = -\frac{3}{V_0}. \quad (69)$$

3.1 The analysis of Yang–Mills equations

We now solve the equations of motion for the gauge fields given in (15)–(17). We should emphasize that, in the reduction of [18], the magnetic vectors A^{M-} with $M = 4, 5, 6$ do not appear in the reduction ansatz. These might arise from the reduction of the dual internal seven-dimensional metric. Furthermore, in this reduction, the two-form fields corresponding to these magnetic vectors do not appear.

Although the present analysis involves A^{6+} , we will truncate out the A^{6-} in order to use the reduction ansatz of [18] to uplift the resulting solutions to 11 dimensions. This amounts to setting e_6 and $\tilde{\mathcal{A}}_t^6$ in (57) to zero. It turns out that this truncation is consistent provided that the two-form fields are properly truncated. Therefore, we will set $e_6 = \tilde{\mathcal{A}}_t^6 = 0$ in the following analysis. Note also that the vanishing of A^{6-} does not mean the covariant field strength $\mathcal{H}^{6-}$ vanishes although the usual gauge field strength $\mathcal{F}^{6-}$ vanishes. This is due to the fact that $\mathcal{H}^{6-}$ gets a contribution from the two-form fields.

In order to consistently remove A^{6-} , we truncate the two-form fields to only B^{18} and B^{78} . With the symmetry of $\text{AdS}_2 \times \Sigma_2$ background and a particular choice of tensor gauge transformations

$$B^{MN} \rightarrow B^{MN} + d\Xi^{MN}, \quad (70)$$

we will take the ansatz for the two-forms to be

$$B^{78} = B(r)F(\theta)d\theta \wedge d\phi,$$

$$B^{18} = \tilde{B}(r)F(\theta)d\theta \wedge d\phi. \quad (71)$$

With the explicit form of the embedding tensor, we can compute the covariant field strengths

$$\begin{aligned} \mathcal{H}^{3+} &= \mathcal{A}_t^{3'} dr \wedge dt + (p^3 + 4B)F(\theta)d\theta \wedge d\phi, \\ \mathcal{H}^{6+} &= \mathcal{A}_t^{6'} dr \wedge dt + (p^6 - 4\tilde{B})F(\theta)d\theta \wedge d\phi, \\ \mathcal{H}^{9+} &= \mathcal{A}_t^{9'} dr \wedge dt + p^9 F(\theta)d\theta \wedge d\phi, \\ \mathcal{H}^{3-} &= \tilde{\mathcal{A}}_t^{3'} dr \wedge dt + (e_3 - 2\sqrt{2}\tilde{B})F(\theta)d\theta \wedge d\phi, \\ \mathcal{H}^{6-} &= -6\sqrt{2}kBF(\theta)d\theta \wedge d\phi, \\ \mathcal{H}^{9-} &= \tilde{\mathcal{A}}_t^{9'} dr \wedge dt + (e_9 - 2\sqrt{2}B)F(\theta)d\theta \wedge d\phi. \end{aligned} \quad (72)$$

Note the non-vanishing covariant field strength $\mathcal{H}^{6-}$, as mentioned above, due to the contribution from the two-form fields despite $A^{6-} = 0$.

Equations arising from (15) and (16) are explicitly given in the appendix. They can be solved by imposing the following conditions:

$$\begin{aligned} Z'_3 &= 0, \quad \Phi' = 2Z_1Z'_3 - 2Z_3Z'_1, \\ B'F(\theta)dr \wedge d\theta \wedge d\phi &= \sqrt{2}e^{-4(2U+V_1)} \\ &\times (3k * A^{9+} + *A^{6+} - \sqrt{2} * A^{3-}), \\ \tilde{B}'F(\theta)dr \wedge d\theta \wedge d\phi &= 4Z_1e^{-4(2U+V_1)} \\ &\times (3k * A^{9+} + *A^{6+} - \sqrt{2} * A^{3-}). \end{aligned} \quad (73)$$

The first condition implies that Z_3 is constant. As mentioned above, this allows one to set $Z_3 = 0$. The second condition then requires that Φ is constant. We can also set $\Phi = 0$. Together with $\beta = 0$, we are left with only six scalars ($U, V_1, V_2, \chi, Z_1, Z_2$) or equivalently ($\tilde{U}_1, \tilde{U}_2, \tilde{V}, \chi, Z_1, Z_2$).

We move to the last two conditions in (73). First of all, the $dt \wedge dr \wedge d\theta$ component gives

$$3kp^9 + p^6 - \sqrt{2}e_3 = 0 \quad (74)$$

while the $dr \wedge d\theta \wedge d\phi$ component leads to first-order differential equations for B and $\tilde{B}$

$$B' = \sqrt{2}e^{-4(2U+V_1)+2g-f}(3k\mathcal{A}_t^9 + \mathcal{A}_t^6 - \sqrt{2}\tilde{\mathcal{A}}_t^3), \quad (75)$$

$$\tilde{B}' = -4Z_1e^{-4(2U+V_1)+2g-f}(3k\mathcal{A}_t^9 + \mathcal{A}_t^6 - \sqrt{2}\tilde{\mathcal{A}}_t^3). \quad (76)$$

After solving all of the Yang–Mills equations and Bianchi identities, we now consider the duality equation for electric and magnetic vector fields. These equations whose explicit form is given in the appendix lead to the relations between $(\mathcal{A}_t^{M'}, \tilde{\mathcal{A}}_t^{M'})$ and scalars. We can accordingly express the former in terms of the latter. These relations are given by

$$\begin{aligned} \mathcal{A}_t^{3'} &= e^{f-2g-2(U+V_1)-3V_2}[e^{4U+2V_2} \\ &[e_3 + \sqrt{2}e_9Z_2 - 4BZ_2 \\ &+ \chi(p^3 + 4B + \sqrt{2}Z_2)] \\ &+ Z_2^2[2(e_3 + p^3\chi) + \sqrt{2}Z_2(e_9 + p^9\chi)] \\ &- 4Z_2B(3k - 2\chi Z_2 + Z_2^2)] \end{aligned}$$

$$-2\sqrt{2}\tilde{B}(e^{4U+2V_2} + 2Z_2\chi + 2Z_2^2) + \sqrt{2}p^6Z_2\chi], \quad (77)$$

$$\begin{aligned} \mathcal{A}_t^{6'} &= e^{f-2g-2(2U+V_1)-3V_2} \\ &[(2\sqrt{2}B - e_9 - p^9\chi)e^{8U+4V_2} - p^6Z_2^2\chi \\ &- e^{4U+2V_2}Z_2[\sqrt{2}e_3 - 4\tilde{B} + 2e_9Z_2 + \chi(\sqrt{2}p^3 + 2p^9Z_2)] \\ &+ 4\tilde{B}Z_2^2(\chi + Z_2) - Z_2^3[\sqrt{2}(e_3 + p^3\chi) + Z_2(e_9 + p^9\chi)] \\ &+ 4\sqrt{2}BZ_2e^{4U+2V_2}(Z_2 - \chi) + 2\sqrt{2}BZ_2^2(3k - 2\chi Z_2 + Z_2^2)], \end{aligned} \quad (78)$$

$$\begin{aligned} \mathcal{A}_t^{9'} &= -e^{f-2g-2(2U+V_1)-3V_2} \\ &[Z_2(\sqrt{2}e_3 - 4\tilde{B} + e_9Z_2) - 2\sqrt{2}B(3k - 2\chi Z_2 + Z_2^2) \\ &+ \chi(p^6 - 4\tilde{B} + \sqrt{2}Z_2 + p^9Z_2^2)], \end{aligned} \quad (79)$$

$$\begin{aligned} \tilde{\mathcal{A}}_t^{3'} &= \frac{e^{f-2g-2V_1-V_2}}{Z_2}[-e^{4V_1+2V_2}[\sqrt{2}e^{4U+2V_2}p^9 \\ &+ Z_2(p^3 + 4B + \sqrt{2}p^9Z_2)] \\ &+ \chi Z_2[e_3 - 2\sqrt{2}\tilde{B} + \sqrt{2}e_9Z_2 - 4BZ_2 \\ &+ \chi(p^3 + 4B + \sqrt{2}p^9Z_2)] \\ &+ \chi e^{4U+2V_2}[\sqrt{2}(e_9 + p^9\chi) - 4B]], \end{aligned} \quad (80)$$

$$\begin{aligned} \tilde{\mathcal{A}}_t^{9'} &= \frac{e^{f-2g-2V_1-V_2}}{Z_2^2}[e^{4(U+V_1+V_2)}p^9 - e^{4U+2V_1}\chi(e_9 - 2\sqrt{2}B + p^9\chi) \\ &- \chi Z_2[\sqrt{2}e_3 - 4\tilde{B} + 4\sqrt{2}B(\chi - Z_2) + 2e_9Z_2 \\ &+ \chi(\sqrt{2}p^3 + 2p^9Z_2)] \\ &+ e^{4V_1+2V_2}Z_2(\sqrt{2}p^3 + 4\sqrt{2}B + 2p^9Z_2)]. \end{aligned} \quad (81)$$

It turns out that only $\mathcal{A}_t^9$, $\mathcal{A}_t^6$ and $\tilde{\mathcal{A}}_t^3$ appear in other equations, while the remaining ones only appear through their derivatives. Therefore, these fields can be integrated out.

3.2 BPS equations for $SO(2)$ invariant scalars

We now use the ansatz for all the fields given in the previous section to set up the BPS equations for finding supersymmetric solutions. We will use Majorana representation for the gamma matrices in which all γ_μ are real, and

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 \quad (82)$$

is purely imaginary. We then have, for example,

$$\begin{aligned} \epsilon^i &= \frac{1}{2}(1 + \gamma_5)\epsilon_M^i, \\ \epsilon_i &= \frac{1}{2}(1 - \gamma_5)\epsilon_M^i \end{aligned} \quad (83)$$

with ϵ_M^i being four-component Majorana spinors. It follows that $\epsilon_i = (\epsilon^i)^*$.

We first consider the gravitino transformations. As in other holographic solutions involving twisted compactifications of the dual SCFTs, the strategy is to use the gauge connection to cancel the spin connection on Σ_2 . Equations from $\delta\psi_\theta^i = 0$ and $\delta\psi_\phi^i = 0$ then reduce to the same equation. The gauge connection enters the covariant derivative of ϵ^i through the composite connection Q_j^i . With the $SO(2)$ singlet scalars,

we find that Q_j^i takes the form of

$$Q_j^i = \frac{1}{2} \hat{A} \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (84)$$

where $\hat{A}$ is given by

$$\hat{A} = \sqrt{2} e^{-2(2U+V_1)} (3kA^{9+} + A^{6+} - \sqrt{2}A^{3-} - 4e^{4U+2V_1}A^{9+}). \quad (85)$$

From the form of Q_i^j , we can see that supersymmetry corresponding to $\epsilon^{3,4}$ is broken for spherical and hyperbolic Σ_2 since we cannot cancel the spin connections along $\epsilon^{3,4}$. The $N = 4$ supersymmetry is then broken to $N = 2$.

After using the condition (74) in the $Q_{\hat{\phi}_i}^j$ components, the twist is achieved by imposing the projection

$$\gamma^{\hat{\theta}\hat{r}} \epsilon^{\hat{i}} = \epsilon^{\hat{i}} \epsilon^{\hat{j}} \quad (86)$$

provided that we impose the following twist condition:

$$2\sqrt{2}\kappa p^9 = 1. \quad (87)$$

Indices $\hat{i}, \hat{j} = 1, 2$ denote the Killing spinors corresponding to the unbroken supersymmetry. From Eq. (86), the chirality condition on $\epsilon^{\hat{i}}$ implies that

$$\gamma^{\hat{\theta}\hat{r}} \epsilon^{\hat{i}} = -i \epsilon^{\hat{i}} \epsilon^{\hat{j}}. \quad (88)$$

With these projections, we can write the $\delta\psi_{\hat{\theta}}^i = 0$ equation, which is the same as $\delta\psi_{\hat{\phi}}^i$ equation, as

$$g' \gamma_{\hat{r}} \epsilon^{\hat{i}} - \frac{2}{3} A_1^{\hat{i}\hat{j}} \epsilon_{\hat{j}} + \frac{i}{2} (\mathcal{V}_{\alpha})^* \mathcal{V}_M^{\hat{i}\hat{j}} \times (i \mathcal{H}_{\hat{\theta}\hat{r}}^{M\alpha} - \mathcal{H}_{\hat{\theta}\hat{\phi}}^{M\alpha}) \epsilon_{\hat{j}}^{\hat{k}} \epsilon_{\hat{k}} = 0 \quad (89)$$

where we have multiplied the resulting equation by $\gamma^{\hat{\theta}}$. We further impose the projector

$$\gamma_{\hat{r}} \epsilon^{\hat{i}} = e^{i\Lambda} \delta^{\hat{i}\hat{j}} \epsilon_{\hat{j}} \quad (90)$$

in which $e^{i\Lambda}$ is an r -dependent phase. By Eq. (88), this projector implies

$$\gamma_{\hat{\theta}} \epsilon^{\hat{i}} = i e^{i\Lambda} \epsilon^{\hat{i}\hat{j}} \epsilon_{\hat{j}}. \quad (91)$$

It should be noted that there are only two independent projectors given in (86) and (90). Therefore, the entire flows preserve $\frac{1}{4}$ supersymmetry. On the other hand, the $\text{AdS}_2 \times \Sigma_2$ vacua is $\frac{1}{2}$ supersymmetric since the $\gamma_{\hat{r}}$ projection is not needed for constant scalars.

As a next step, we introduce the “superpotential” $\mathcal{W}$ and “central charge” $\mathcal{Z}$ defined, respectively, by the eigenvalues of

$$\frac{2}{3} A_1^{\hat{i}\hat{j}} = \mathcal{W}_i \delta^{\hat{i}\hat{j}} \quad (92)$$

and

$$-\frac{i}{2} (\mathcal{V}_{\alpha})^* \mathcal{V}_M^{\hat{i}\hat{j}} (i \mathcal{H}_{\hat{\theta}\hat{r}}^{M\alpha} - \mathcal{H}_{\hat{\theta}\hat{\phi}}^{M\alpha}) \epsilon_{\hat{j}}^{\hat{k}} = \mathcal{Z}_i \delta^{\hat{i}\hat{k}}. \quad (93)$$

It should be emphasized that no summation is implied in the above two equations.

With all these, we obtain the BPS equation from $\delta\psi_{\hat{\theta}}^i = 0$ by the equation

$$e^{i\Lambda} g' - \mathcal{W}_i - \mathcal{Z}_i = 0 \quad (94)$$

which gives

$$g' = |\mathcal{W}_i + \mathcal{Z}_i| \quad \text{and} \quad e^{i\Lambda} = \frac{\mathcal{W}_i + \mathcal{Z}_i}{|\mathcal{W}_i + \mathcal{Z}_i|}. \quad (95)$$

Using all of these results, we find that the equation $\delta\psi_{\hat{\theta}}^i = 0$ gives

$$e^{i\Lambda} (f' + i \hat{A}_t e^{-f}) - \mathcal{W}_i + \mathcal{Z}_i = 0. \quad (96)$$

Taking the real and imaginary parts leads to the following BPS equations:

$$f' = \text{Re}[e^{-i\Lambda} (\mathcal{W}_i - \mathcal{Z}_i)] \quad (97)$$

and

$$\hat{A}_t = e^f \text{Im}[e^{-i\Lambda} (\mathcal{W}_i - \mathcal{Z}_i)]. \quad (98)$$

We now come to the equation $\delta\psi_{\hat{r}}^i = 0$, which gives the r -dependence of the Killing spinors. When combined with the equation $\delta\psi_{\hat{\theta}}^i = 0$, this equation reads

$$2\epsilon^{\hat{i}'} - f' - i \hat{A}_t e^{-f} \epsilon^{\hat{i}} = 0, \quad (99)$$

which can be solved by

$$\epsilon^{\hat{i}} = e^{\frac{f}{2} + \frac{i}{2} \int \hat{A}_t e^{-f} dr} \tilde{\epsilon}^{\hat{i}}. \quad (100)$$

$\tilde{\epsilon}^{\hat{i}}$ are constant spinors satisfying the projections

$$\gamma_{\hat{r}} \tilde{\epsilon}^{\hat{i}} = \delta^{\hat{i}\hat{j}} \tilde{\epsilon}_{\hat{j}}, \quad \gamma_{\hat{\theta}\hat{\phi}} \tilde{\epsilon}^{\hat{i}} = \epsilon^{\hat{i}}_{\hat{j}} \tilde{\epsilon}^{\hat{j}}. \quad (101)$$

Using the $\gamma_{\hat{r}}$ projector, we obtain the following BPS equations from $\delta\chi^i$ and $\delta\lambda_a^i$:

$$-e^{i\Lambda} \epsilon^{\alpha\beta} \mathcal{V}_{\alpha} \mathcal{V}'_{\beta} \delta_{\hat{i}\hat{j}} - \frac{4i}{3} A_2^{\hat{j}\hat{i}} + i \mathcal{V}_{\alpha} \mathcal{V}_M^{\hat{k}\hat{j}} \epsilon^{\hat{k}\hat{i}} (i \mathcal{H}_{\hat{\theta}\hat{r}}^{M\alpha} + \mathcal{H}_{\hat{\theta}\hat{\phi}}^{M\alpha}) = 0, \quad (102)$$

$$\mathcal{V}_a^M \mathcal{V}_M^{ij'} e^{-i\Lambda} + \frac{1}{4} \mathcal{V}_{\alpha} \mathcal{V}_{Ma} (\mathcal{H}_{\hat{\theta}\hat{r}}^{M\alpha} + i \mathcal{H}_{\hat{\theta}\hat{\phi}}^{M\alpha}) \delta_i^j \delta_{\hat{j}}^{\hat{i}'} \epsilon^{\hat{i}\hat{j}} + A_{2aj}^i = 0. \quad (103)$$

Note that there are four equations from $\delta\lambda_a^i$ for each value of $a = 1, 2, 3$, but $\delta\lambda_a^{i=3,4}$ we do not get any contribution from the gauge fields. However, the scalars appearing in these

equations cannot be consistently set to zero since $A_{2aj}{}^i$ is not diagonal in ij indices.

It should be pointed out that the $N = 3$ supersymmetric AdS₄ vacuum corresponds to the Killing spinors $\epsilon^{2,3,4}$ while ϵ^1 is the Killing spinor of the $N = 1$ AdS₄ critical point. In the next section, we will look for possible AdS₂ $\times$ Σ_2 solutions to the above BPS equations. As mentioned before, in the twist given above, the supersymmetry corresponding to $\epsilon^{3,4}$ is broken. Therefore, the resulting AdS₂ $\times$ Σ_2 solutions will preserve only two supercharges or half of the $N = 1$ supersymmetry corresponding to either ϵ^1 or ϵ^2 . We will analyze these two cases separately.

4 Supersymmetric AdS₂ $\times$ Σ_2 solutions

In this section, we look for the AdS₂ $\times$ Σ_2 fixed points of the above BPS flow equations with constant scalars. These solutions should correspond to IR fixed points of the RG flows from twisted compactifications of the dual $N = 3$ and $N = 1$ SCFTs in three dimensions. They also describe near horizon geometries of BPS black holes arising from M2-branes wrapped on Σ_2 . Before giving the solutions, we first discuss the conditions for obtaining the AdS₂ fixed points.

At the AdS₂ $\times$ Σ_2 geometries, the scalars are constant, and we can choose the gauge in which $A_t^{M\alpha} \sim 0$. Furthermore, the warped factor $g(r)$ is required to be constant, $g'(r) = 0$. Let r_h be the position of the horizon, we can summarize the conditions for AdS₂ $\times$ Σ_2 solutions and their properties as follows:

$$\begin{aligned} f(r_h) &= \frac{r_h}{L_{\text{AdS}_2}}, \quad e^{g(r_h)} = L_{\Sigma_2}, \\ \text{Im}[e^{-i\Lambda}(\mathcal{W}_i - \mathcal{Z}_i)] &= 0, \\ |\mathcal{W}_i + \mathcal{Z}_i| &= 0, \quad \frac{4}{3}A_2^{\hat{i}\hat{j}} = \mathcal{V}_\alpha \mathcal{V}_M^{\hat{i}\hat{k}} \epsilon^{\hat{k}\hat{j}} (i\mathcal{H}_{\hat{0}\hat{r}}^{M\alpha} + \mathcal{H}_{\hat{\theta}\hat{\phi}}^{M\alpha}), \\ \frac{i}{4}\mathcal{V}_\alpha \mathcal{V}_{Ma} (-i\hat{H}_{\hat{0}\hat{r}}^{M\alpha} + \mathcal{H}_{\hat{\theta}\hat{\phi}}^{M\alpha}) \epsilon^{\hat{i}\hat{j}} &= -A_{2a\hat{j}}^{\hat{i}} \\ A_{2a\hat{j}}^{\hat{i}} &= 0, \quad \hat{j} = 3, 4, \end{aligned} \quad (104)$$

where L_{AdS_2} and L_{Σ_2} are, respectively, the radii of AdS₂ and Σ_2 . These conditions can be viewed as attractor equations for the scalars at the black hole horizon.

4.1 Solutions in the $N = 3$ case

We begin with the $N = 3$ case. The AdS₂ $\times$ Σ_2 solutions will describe the fixed points of the RG flows from $N = 3$ SCFTs dual to the N^{010} compactification of 11-dimensional supergravity to supersymmetric CFT₁'s dual to the AdS₂ $\times$ Σ_2 geometries. These flows are examples of the twisted compactifications of the $N = 3$ SCFT on Σ_2 .

In this case, the superpotential and central charge are given in terms of the redefined scalars ($\tilde{U}_1, \tilde{U}_2, \tilde{V}$) by

$$\begin{aligned} \mathcal{W}_2 &= \frac{1}{2}e^{-\frac{1}{2}(4\tilde{U}_1+2\tilde{U}_2+\tilde{V})}[e^{2\tilde{U}_2} + 4e^{\tilde{U}_1+\tilde{U}_2} - 2e^{\tilde{U}_2+\tilde{V}} + 4e^{\tilde{U}_1+\tilde{V}} \\ &\quad - 3k + 2iZ_2e^{\tilde{U}_2} + 4iZ_2e^{\tilde{U}_1} - 4iZ_1(e^{\tilde{U}_2} + e^{\tilde{V}} + iZ_2) \\ &\quad - 2iZ_2e^{\tilde{V}} - Z_2^2 + 2\chi(2ie^{\tilde{U}_1} - ie^{\tilde{U}_2} + 2Z_1 + Z_2)], \quad (105) \\ \mathcal{Z}_2 &= \frac{1}{4}e^{-\frac{1}{2}(4\tilde{U}_1+2\tilde{U}_2+\tilde{V})}[2e_3e^{\tilde{U}_2} - \sqrt{2}ie_9e^{2\tilde{U}_2} + 2ie_3\chi + 2p^3\chi e^{\tilde{U}_2} \\ &\quad - \sqrt{2}ip^9\chi(e^{2\tilde{U}_2} + 3k) - 4\sqrt{2}\tilde{B}[e^{\tilde{U}_2} + e^{\tilde{V}} + i(\chi + Z_2)] \\ &\quad + 2ie_3Z_2 + 2\sqrt{2}e_9Z_2e^{\tilde{U}_2} + 2ip^3\chi Z_2 + 2\sqrt{2}p^9\chi Z_2e^{\tilde{U}_2} \\ &\quad + \sqrt{2}i(e_9 + p^9\chi)Z_2^2 + 4iB(e^{2\tilde{U}_2} - 2e^{\tilde{U}_2+\tilde{V}} - 3k) \\ &\quad + 4B[2\chi(e^{\tilde{U}_2} + iZ_2) + Z_2(e^{\tilde{V}} - 2e^{\tilde{U}_2} - iZ_2)] \\ &\quad + e^{\tilde{V}}(2e_3 - 3\sqrt{2}p^9 - \sqrt{2}p^9e^{2\tilde{U}_2} + 2p^3Z_2 + \sqrt{2}p^9Z_2^2) \\ &\quad - 2ie^{\tilde{U}_2+\tilde{V}}(p^3 + \sqrt{2}p^9Z_2)] \quad (106) \end{aligned}$$

in which the subscript 2 on $\mathcal{W}_2$ and $\mathcal{Z}_2$ refers to the superpotential and central charge associated to the Killing spinor ϵ^2 .

The BPS equations are given by

$$\begin{aligned} f' &= \text{Re}[e^{-i\Lambda}(\mathcal{W}_2 - \mathcal{Z}_2)], \\ e^{i\Lambda} &= \frac{\mathcal{W}_2 + \mathcal{Z}_2}{|\mathcal{W}_2 + \mathcal{Z}_2|}, \quad (107) \end{aligned}$$

$$g' = |\mathcal{W}_2 + \mathcal{Z}_2|, \quad (108)$$

$$\begin{aligned} e^{i\Lambda}\tilde{V}' - ie^{-\tilde{V}+i\Lambda}\chi' &= \frac{1}{2}\left[e^{-\frac{\tilde{V}}{2}-\tilde{U}_2-2\tilde{U}_1}[2e^{\tilde{U}_2} + 8e^{2\tilde{U}_1} - 6k + Z_2(8Z_1 - 2Z_2)] \right. \\ &\quad \left. - e^{-2g-2\tilde{U}_1+\frac{\tilde{V}}{2}}[4e^{2g} + 2e^{2\tilde{U}_1}(p^3 + 4B + \sqrt{2}p^9Z_2)] \right. \\ &\quad \left. + 4\chi(2Z_1 + Z_2)e^{-\frac{\tilde{V}}{2}-\tilde{U}_2-2\tilde{U}_1} + \sqrt{2}e_9e^{\tilde{U}_2-2g-\frac{\tilde{V}}{2}}\right] \\ &\quad + \frac{1}{2}e^{-2g-\tilde{U}_2-\frac{\tilde{V}}{2}}[\sqrt{2}Z_2(4\tilde{B} - e_9Z_2) - 2e_3(\chi + Z_2) + 4\sqrt{2}\chi\tilde{B} \\ &\quad - 4B(e^{2\tilde{U}_2} - 3k + 2\chi Z_2 - Z_2^2) + \sqrt{2}p^9\chi(e^{\tilde{U}_2} + 3k) \\ &\quad - Z_2\chi(2p^3 + \sqrt{2}p^9Z_2)] \\ &\quad - \frac{i}{2}e^{-\tilde{U}_2-\frac{\tilde{V}}{2}}[4e^{\tilde{U}_2-2\tilde{U}_1}(Z_2 - 2Z_1 - \chi) - 4e^{\tilde{V}-2\tilde{U}_1}(2Z_1 + Z_2) \\ &\quad - 2e^{\tilde{U}_2-2g}[Z_2(\sqrt{2}e_9 - 4B - 2\sqrt{2}\tilde{B}) + \chi(p^3 + 4B + \sqrt{2}p^9Z_2)] \\ &\quad + e^{\tilde{V}-2g}[2e_3 - 4\sqrt{2}\tilde{B} - \sqrt{2}p^9(3k + e^{2\tilde{U}_2}) - 4\sqrt{2}\tilde{B} \\ &\quad + Z_2(2p^3 + 8B + \sqrt{2}p^9Z_2)] - 2e^{\tilde{U}_2-2g}e_3], \quad (109) \\ e^{-i\Lambda}\tilde{U}_2' + ie^{-\tilde{U}_2-i\Lambda}Z_2' &= \frac{1}{2}e^{-2g-\tilde{U}_2-2\tilde{U}_1-\frac{\tilde{V}}{2}}[2e^{2(g+\tilde{U}_2)} + \sqrt{2}ie_9e^{2(\tilde{U}_1+\tilde{U}_2)} + 6ke^{2g} \\ &\quad - 2ie_3\chi e^{2\tilde{U}_1} + \sqrt{2}ip^9\chi e^{2(\tilde{U}_1+\tilde{U}_2)} + 3\sqrt{2}ikp^9\chi e^{2\tilde{U}_1} \\ &\quad + 8iZ_2e^{2g+\tilde{U}_1} - 2ie_3Z_2e^{2\tilde{U}_1} - 4\chi Z_2e^{2g} - 2ip^3\chi Z_2e^{2\tilde{U}_1} \\ &\quad - 8Z_1Z_2e^{2g} + 2Z_2^2e^{2g} - \sqrt{2}ie_9Z_2^2e^{2\tilde{U}_1} - 8\chi Z_1e^{2g} \\ &\quad - 4iBe^{2\tilde{U}_1}[e^{2\tilde{U}_2} - 3k + Z_2(2\chi - Z_2 - 2ie^{\tilde{V}})] + 8i\chi e^{2g+\tilde{U}_1} \\ &\quad + 4\sqrt{2}\tilde{B}e^{2\tilde{U}_1}(e^{\tilde{V}} + i\chi + iZ_2) - \sqrt{2}ip^9\chi Z_2^2e^{2\tilde{U}_1} \\ &\quad - 4ie^{2g+\tilde{V}}(2Z_1 + Z_2) - Z_2e^{2\tilde{U}_1+\tilde{V}}(2p^3 + \sqrt{2}p^9Z_2) \\ &\quad - e^{\tilde{U}_1+\tilde{V}}[8e^{2g} + e^{\tilde{U}_1}[2e_3 - \sqrt{2}p^9(e^{2\tilde{U}_2} + 3k)]]], \quad (110) \end{aligned}$$

$$\begin{aligned}
& e^{-i\Lambda} \tilde{U}'_1 - i e^{-\tilde{U}_1 - i\Lambda} Z'_1 \\
& = e^{-\tilde{U}_2 - 2\tilde{U}_1 - \frac{\tilde{V}}{2}} [2e^{\tilde{U}_2 + \tilde{V}} - e^{2\tilde{U}_2} - 2e^{\tilde{U}_1} (e^{\tilde{U}_2} + e^{\tilde{V}}) + 3k \\
& \quad - 4i Z_1 (e^{\tilde{U}_2} + e^{\tilde{V}} - i Z_2) + 2i \chi (e^{\tilde{U}_1} - e^{\tilde{U}_2} + 2i Z_1 + i Z_2) \\
& \quad + 2i Z_2 (e^{\tilde{U}_2} + e^{\tilde{U}_1} - e^{\tilde{V}}) + Z_2^2]
\end{aligned} \quad (111)$$

where we have used the relation (74) to express p^6 in terms of p^9 and e_3 .

To obtain the complete flow solutions, we have to solve these equations together with the two-form equations (75), (76) and the equations for the gauge fields (77)–(81) as well as the algebraic constraint given by Eq. (98). These equations are very complicated even with the numerical technique not to mention the analytic solutions. In what follows, we will present only the $\text{AdS}_2 \times \Sigma_2$ solutions and will not give the numerical flow solutions which may be obtained by suitable boundary conditions. In principle, the horizon is characterized by the values of the scalars as functions of the electric and magnetic charges. However, due to the complexity of the BPS equations, it is more convenient to solve the horizon conditions for the charges in terms of the scalar fields although inverting the solutions to express the scalars in terms of the charges is desirable.

In the present case, although it is straightforward to solve the above equations for $(B, \tilde{B}, \chi, Z_1, p^9, p^3, e_3, e_9)$ in terms of $(\tilde{U}_1, \tilde{U}_2, \tilde{V}, Z_2)$, the resulting expressions turn out to be cumbersome and not very illuminating. Accordingly, we refrain from giving the general result here but instead present some solutions with specific values of the parameters. These are obtained from truncating the full result and represent some examples of $\text{AdS}_2 \times \Sigma_2$ geometries within the solution space.

Examples of $\text{AdS}_2 \times \Sigma_2$ solutions are as follows:

- We begin with a simple solution with vanishing pseudoscalars. In the M-theory point of view, only scalars coming from the 11-dimensional metric are turned on. The solution is given by

$$\begin{aligned}
& k = \frac{1}{5}, \quad \chi = Z_1 = Z_2 = 0, \quad e_9 = 0, \\
& \tilde{V} = \frac{1}{2} \ln \left[\frac{27}{5} \right], \quad \tilde{U}_1 = \frac{1}{2} \ln \left[\frac{27}{80} \right], \\
& \tilde{U}_2 = -\frac{1}{2} \ln \left[\frac{5}{3} \right], \quad \tilde{B} = \frac{1}{20} (5\sqrt{2}e_3 - 27p^9), \\
& g = \frac{1}{2} \ln \left[-\frac{81}{80} \sqrt{\frac{3}{10}} \kappa p^9 \right], \\
& B = -\frac{p^3}{4}, \quad L_{\text{AdS}_2} = \frac{3^{\frac{9}{4}}}{32(5)^{\frac{3}{4}}}.
\end{aligned} \quad (112)$$

It is clearly seen that only the hyperbolic horizon ($\kappa = -1$) is possible otherwise $g(r_h)$ will become complex. Therefore, we find that this is an $\text{AdS}_2 \times H^2$ solution.

- We next consider a solution with scalars and pseudoscalars turned on. In the 11-dimensional context, the solution involves scalar fields from both the metric and the four-form field. This solution is characterized by

$$\begin{aligned}
& k = 1, \quad Z_1 = Z_2 = \tilde{U} = 0, \\
& \tilde{U} = \tilde{V} = \ln \left[\frac{12}{7} \right], \\
& p^3 = \frac{41e_9 + 220p^9}{41\sqrt{2}}, \quad B = -\frac{41e_9 + 136p^9}{164\sqrt{2}}, \\
& \tilde{B} = \frac{e_3}{2\sqrt{2}} - \frac{111}{41} p^9, \\
& \chi = -\frac{1}{7}, \quad g = \frac{1}{2} \ln \left[-2^{\frac{5}{2}} \kappa p^9 \sqrt{\frac{21}{41}} \right], \\
& L_{\text{AdS}_2} = \frac{\sqrt{21}}{19}.
\end{aligned} \quad (113)$$

This solution is also $\text{AdS}_2 \times H^2$.

- As a final example, we consider a solution with more scalars turned on and hence more general than the previous two solutions. This solution is given by

$$\begin{aligned}
& Z_1 = 0, \quad Z_2 = -\frac{2\sqrt{k}}{7}, \quad \chi = -\frac{\sqrt{k}}{7}, \\
& \tilde{U}_1 = \tilde{U}_2 = \frac{1}{2} \ln k, \\
& p^3 = \frac{128,447k - 104,895}{4,116\sqrt{2k}} p^9, \\
& e_9 = \frac{32,723k - 13,923}{4,116\sqrt{2k}} p^9, \\
& \tilde{B} = \frac{e_3}{2\sqrt{2}} + \frac{567 - 667k}{98} p^9, \\
& g = \frac{1}{2} \ln \left[\frac{21(1-k)\sqrt{k} \kappa p^9}{2\sqrt{2}} \right], \\
& \tilde{V} = \ln(2\sqrt{k}), \quad B = -25p^9 \left[\frac{3,809k - 2,961}{16,464\sqrt{2k}} \right], \\
& L_{\text{AdS}_2} = \frac{k^{\frac{3}{4}}}{3\sqrt{2}}.
\end{aligned} \quad (114)$$

In this case, the flux parameter k is not fixed, and there are two types of solutions, $\text{AdS}_2 \times S^2$ and $\text{AdS}_2 \times H^2$, depending on the value of k . For $k > 1$, we have an $\text{AdS}_2 \times H^2$ solution with $\kappa = -1$ while the solution with $k < 1$ is $\text{AdS}_2 \times S^2$ for which $\kappa = 1$.

4.2 Solutions in the $N = 1$ case

We now repeat a similar analysis for the $N = 1$ case in which the $N = 1$ AdS₄ vacuum arises from the squashed N^{010} manifold. This critical point exists only for $k < 0$, and the AdS₂ $\times$ Σ_2 solutions would be IR fixed points of the twisted compactifications of the dual $N = 1$ SCFT. The superpotential and central charge are given by

$$\begin{aligned} \mathcal{W}_1 = & \frac{1}{2} e^{-\tilde{U}_2 - 2\tilde{U}_1 - \frac{\tilde{V}}{2}} [e^{2\tilde{U}_2} - 4e^{\tilde{U}_1 + \tilde{U}_2} - 2e^{\tilde{V}} (e^{\tilde{U}_2} + 2e^{\tilde{U}_1}) \\ & + 4Z_1(Z_2 - ie^{\tilde{U}_2} - ie^{\tilde{V}}) \\ & - 3k + iZ_2(2e^{\tilde{U}_2} - 4e^{\tilde{U}_1} - 2e^{\tilde{V}} + iZ_2) \\ & + 2\chi(2Z_1 + Z_2 - ie^{\tilde{U}_2} - 2ie^{\tilde{U}_1})], \end{aligned} \quad (115)$$

$$\begin{aligned} \mathcal{Z}_1 = & \frac{1}{4} e^{-2g - \tilde{U}_2 - \frac{\tilde{V}}{2}} \\ & \times [2e_3(e^{\tilde{U}_2} + i\chi) - \sqrt{2}ie_9e^{2\tilde{U}_2} + 2p^3\chi e^{\tilde{U}_2} - 3\sqrt{2}ikp^9\chi \\ & - \sqrt{2}ip^9\chi e^{2\tilde{U}_2} - 4\sqrt{2}\tilde{B}(e^{\tilde{U}_2} + e^{\tilde{V} + i\chi + iZ_2}) + 2ie_3Z_2 \\ & + 2\sqrt{2}e_9Z_2e^{\tilde{U}_2} + 2ip^3\chi Z_2 + 2\sqrt{2}p^9\chi Z_2 + \sqrt{2}ie_9Z_2^2 \\ & + \sqrt{2}ip^9\chi Z_2^2 + 4B[2\chi(e^{\tilde{U}_2} + iZ_2) + i(e^{2\tilde{U}_2} - 2e^{\tilde{U}_2 + \tilde{V}} - 3k)] \\ & + 4BZ_2(2e^{\tilde{V}} - 2e^{\tilde{U}_2} - iZ_2) - 2ie^{\tilde{U}_2 + \tilde{V}}(p^3 + \sqrt{2}p^9Z_2) \\ & + e^{\tilde{V}}(2e_3 - 6\sqrt{2}p^9 - \sqrt{2}p^9e^{2\tilde{U}_2} + 2p^3Z_2 + \sqrt{2}p^9Z_2^2)]. \end{aligned} \quad (116)$$

The procedure is essentially the same, so we will just present the result of AdS₂ $\times$ Σ_2 solutions and leave the explicit form of the corresponding BPS equations to the appendix. In this case, it turns out to be more difficult to find the solutions in particular we have not found any solutions without the pseudoscalars turned on. With some effort, we obtain the following solutions:

- We begin with a simple solution in which all scalars have the same value as the $N = 1$ supersymmetric AdS₄ vacuum

$$\begin{aligned} k = & -\frac{18}{11}, \quad Z_1 = Z_2 = \chi = 0, \\ \tilde{U}_1 = \tilde{U}_2 = & \ln 5 - \frac{1}{2} \ln \left[\frac{55}{6} \right], \quad \tilde{V} = -\frac{1}{2} \ln \left[\frac{55}{6} \right], \\ B = & -\frac{p^3}{4}, \quad \tilde{B} = \frac{e_3}{2\sqrt{2}}, \quad e_9 = -\frac{14p^3}{5\sqrt{2}}, \\ g = & \frac{1}{2} \ln \left[-\frac{10}{11} \sqrt{\frac{15}{11}} \kappa p^9 \right], \\ L_{\text{AdS}_2} = & \frac{5^{\frac{5}{4}}}{2^{\frac{5}{4}} \left(3^{\frac{1}{4}} \right) \left(11^{\frac{3}{4}} \right)}. \end{aligned} \quad (117)$$

The solution is of the AdS₂ $\times$ H^2 form.

- We now give a more complicated solution

$$\begin{aligned} k = & -\frac{18}{11}, \quad Z_1 = \chi = 0, \\ \tilde{U}_1 = \tilde{V} = & \ln \left[7\sqrt{-\frac{3k}{319}} \right], \\ p^3 = & \sqrt{\frac{3}{638}} \left(\frac{p^9}{3, 190\sqrt{-k}} \right) \\ & (567, 365k - 1, 002, 298), \\ B = & \sqrt{\frac{3}{638}} \left(\frac{p^9}{89, 320\sqrt{-k}} \right) \\ & (13, 987, 355k - 27, 368, 286), \\ \tilde{B} = & \frac{e_3}{2\sqrt{2}} + \frac{3p^9}{8, 932} (63, 162 - 32, 267k), \\ Z_2 = & -5\sqrt{-\frac{3k}{319}}, \\ g = & \ln \left[7 \left(\frac{3}{638} \right)^{\frac{1}{4}} \sqrt{(k-2)\sqrt{-k}\kappa p^9} \right], \\ \tilde{U}_2 = & \frac{1}{2} \ln \left[-\frac{588k}{319} \right], \\ L_{\text{AdS}_2} = & \frac{21(3^{\frac{1}{4}})}{11} \sqrt{\frac{7}{21}} \left(\frac{2}{29} \right)^{\frac{3}{4}}. \end{aligned} \quad (118)$$

This solution also gives AdS₂ $\times$ H^2 geometry. To show that this leads to real solutions, we explicitly give one example of the possible solutions

$$\begin{aligned} Z_1 = \chi = & 0, \quad e_9 = 54.35, \quad p^3 = -11.56, \\ \tilde{U}_1 = \tilde{V} = & -0.14, \\ \tilde{U}_2 = & 0.55, \quad Z_2 = -0.62, \quad B = 10.66, \\ \tilde{B} = & -13.77 + 0.35e_3, \\ g = & 1.06. \end{aligned} \quad (119)$$

4.3 Uplift formulas

We end this section by giving the uplift formulas for embedding the previously found AdS₂ $\times$ Σ_2 solutions in 11 dimensions. We first identify the vector and tensor fields in the $N = 4$ gauged supergravity and those obtained from the dimensional reduction of 11-dimensional supergravity on a tri-sasakian manifold

$$\begin{aligned} A_1^3 = & \sqrt{2}A^{9+}, \quad a_1^3 = -\sqrt{2}A^{6+}, \quad c_1^3 = A^{3+}, \\ \tilde{a}_1^3 = & -A^{3-}, \\ \tilde{c}_1^3 = & \sqrt{2}A^{9-}, \quad a_2^{12} = \sqrt{2}B^{18}, \quad c_2^3 = B^{78}. \end{aligned} \quad (120)$$

With this identification and the ansatz for the scalars and vector fields, the 11-dimensional metric and the four-form

field are given by

$$\begin{aligned}
 ds_{11}^2 = & e^{-\frac{1}{3}(4\tilde{U}_1+2\tilde{U}_2+\tilde{V})}[-e^{2f}dt^2 + dr^2 \\
 & + e^{2g}(d\theta^2 + F(\theta)^2 d\phi^2)] \\
 & + e^{\frac{1}{3}(2\tilde{U}_1+\tilde{U}_2-\tilde{V})}ds^2(B_{\text{QK}}) + e^{\frac{2}{3}(\tilde{U}_1-\tilde{U}_2+\tilde{V})} \\
 & \times [(\eta^1)^2 + (\eta^2)^2] + e^{\frac{2}{3}(\tilde{V}-2\tilde{U}_1-2\tilde{U}_2)} \\
 & \times (\eta^3 + \sqrt{2}\mathcal{A}_t^9 dt - \sqrt{2}p^9 F'(\theta)d\phi)^2
 \end{aligned} \quad (121)$$

and

$$\begin{aligned}
 G_4 = & -[6ke^{-(4\tilde{U}_1+2\tilde{U}_2+\tilde{V})+f+2g} - \sqrt{2}B\mathcal{A}_t^9 - \sqrt{2}B'\mathcal{A}_t^9] \\
 & \times F(\theta)dt \wedge dr \wedge d\theta \wedge d\phi \\
 & + B'F(\theta)dr \wedge d\theta \wedge d\phi \wedge \eta^3 + dZ_1 \wedge (\eta^1 \wedge J^1 + \eta^2 \\
 & \wedge J^2)[\sqrt{2}(\tilde{\mathcal{A}}_t^9 + \chi\mathcal{A}_t^9)dr \wedge dt + \sqrt{2}(e_9 + \chi p^9 \\
 & - \sqrt{2}B)F(\theta)d\theta \wedge d\phi] \wedge \eta^1 \wedge \eta^2 \\
 & \times [(\mathcal{A}_t^3 + \sqrt{2}Z_2\mathcal{A}_t^9)dr \\
 & \wedge dt + (p^3 + \sqrt{2}p^9 Z_2 + 2B)F(\theta)d\theta \wedge d\phi] \wedge J^3 \\
 & + 2(\chi + 2Z_1)\eta^1 \wedge \eta^2 \wedge J^3 + (dZ_2 \wedge J^3 + d\chi \\
 & \wedge \eta^2 \wedge \eta^2) \wedge (\eta^3 - \sqrt{2}p^9 F(\theta)d\phi) \\
 & + 2[(\mathcal{A}_t^3 + \sqrt{2}\tilde{\mathcal{A}}_t^9)dt \\
 & - (\sqrt{2}e_9 + p^3)F(\theta)d\phi + 4(2Z_1 + Z_2)\text{vol}(B_{\text{QK}}) \\
 & + (\chi + Z_2)(\eta^3 + \sqrt{2}\mathcal{A}_t^9 dt - \sqrt{2}p^9 F(\theta)d\phi)] \\
 & \wedge (\eta^1 \wedge J^2 - \eta^2 \wedge J^1).
 \end{aligned} \quad (122)$$

5 Conclusions

In this paper, we have found a number of $\text{AdS}_2 \times \Sigma_2$ solutions in $N = 4$ gauged supergravity with $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ gauge group. The solutions can be uplifted to M-theory since the $N = 4$ gauged supergravity is a consistent truncation of 11-dimensional supergravity on a tri-sasakian manifold. These $\text{AdS}_2 \times \Sigma_2$ geometries are expected to arise from the near horizon limit of certain dyonic BPS black holes which can be identified as holographic RG flows from twisted compactifications of the dual $N = 1, 3$ SCFTs in the UV to superconformal quantum mechanics corresponding to the AdS_2 geometry in the IR. We have found that most of the solutions have hyperbolic horizons, but some of them have spherical horizons depending on the values of the four-form flux parameter. These solutions provide examples of AdS_2 geometries from M-theory compactified on a tri-sasakian manifold such as N^{010} and are hopefully useful in the holographic study of the $N = 1, 3$ Chern–Simons–Matter theories in three dimensions. They should also be useful in the study of black hole entropy along the line of recent results in [37–39]. In this aspect, the near horizon solutions given here are enough although we have not constructed the full black hole solutions, numerically. It would be interesting to compute the

topologically twisted index in the dual $N = 1, 3$ SCFTs and compare with the black hole entropy computed from the area of the horizon $A \sim L_{\Sigma_2}^2$.

The solutions found here might constitute only a small number of all possible solutions due to the complexity of the resulting BPS equations. It could be interesting to look for more solutions or even to identify all possible black hole solutions to this $N = 4$ gauged supergravity similar to the analysis in $N = 2$ gauged supergravity. For the case of N^{010} manifold, there exists an invariant two-form in addition to the universal forms on a generic tri-sasakian manifold. This leads to an additional vector multiplet, called a Betti multiplet, in $N = 4$ gauged supergravity. This vector multiplet corresponds to a baryonic symmetry in the dual SCFTs. Finding a reduction that includes the Betti multiplet and $SU(3)$ non-singlet fields would be very useful in order to find more interesting black hole and other holographic solutions. We leave all these issues for future work.

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Appendix A: Useful formulas

In this appendix, we collect some convention on 't Hooft matrices and details on Yang–Mills equations and complicated BPS equations in the $N = 1$ case.

A.1: 't Hooft matrices

In converting $SO(6)$ vector indices m, n to chiral spinor indices i, j , we use the following 't Hooft matrices:

$$\begin{aligned}
 G_1^{ij} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \\
 G_2^{ij} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},
 \end{aligned}$$

$$\begin{aligned}
G_3^{ij} &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \\
G_4^{ij} &= \begin{bmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix}, \\
G_5^{ij} &= \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix}, \\
G_6^{ij} &= \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}.
\end{aligned} \quad (123)$$

A.2: Field equations of gauge fields

In this section, we present the full equations of motion for the gauge fields $A^{M\alpha}$. Equation (15) gives

$$\begin{aligned}
- * \mathcal{D} \mathcal{H}^{3-} &= e^{-4(2U+V_1)} [4Z_1(\Phi' + 2Z_3 Z_1') \\
&\quad - 4e^{4U+2V_1} Z_3' - 8Z_1^2 Z_3'] dr \\
&\quad - 8Z_1 e^{-4(2U+V_1)} (2A^{3-} - \sqrt{2}A^{6+} - 3\sqrt{2}kA^{9+}),
\end{aligned} \quad (124)$$

$$\begin{aligned}
* \mathcal{D} \mathcal{H}^{6-} &= 3\sqrt{2}k e^{-4(2U+V_1)} (\Phi' + 2Z_3 Z_1' - 2Z_1 Z_3') dr \\
&\quad + 12k e^{-4(2U+V_1)} (3kA^{9+} + A^{6+} - \sqrt{2}A^{3-}),
\end{aligned} \quad (125)$$

$$\begin{aligned}
* \mathcal{D} \mathcal{H}^{9-} &= \sqrt{2}k e^{-4(2U+V_1)} (\Phi' + 2Z_3 Z_1' - 2Z_1 Z_3') dr \\
&\quad + 4e^{-4(2U+V_1)} (3kA^{9+} + A^{6+} - \sqrt{2}A^{3-})
\end{aligned} \quad (126)$$

while Eq. (16) leads to

$$\begin{aligned}
- * \mathcal{D} \mathcal{H}^{3+} &= 2e^{-4(2U+V_1)} (\Phi' + 2Z_3 Z_1' - 2Z_1 Z_3') dr \\
&\quad + 4e^{-4(2U+V_1)} (3kA^{9+} + A^{6+} - \sqrt{2}A^{3-}),
\end{aligned} \quad (127)$$

$$\begin{aligned}
* \mathcal{D} \mathcal{H}^{6+} &= 4\sqrt{2}k e^{-4(2U+V_1)} \\
&\quad [e^{4U+2V_1} Z_3' + 2Z_1^2 Z_3' - Z_1(\Phi' + 2Z_3 Z_1')] dr \\
&\quad - 16Z_1 e^{-4(2U+V_1)} (3kA^{9+} + A^{6+} - \sqrt{2}A^{3-}),
\end{aligned} \quad (128)$$

$$* \mathcal{D} \mathcal{H}^{9+} = 0 \quad (129)$$

For equations obtained from (17), it is more convenient to express them in the following combinations:

$$\begin{aligned}
\mathcal{H}^{9-} &= e^{-4U+2V_1-V_2} (Z_2^2 * \mathcal{H}^{9+} + * \mathcal{H}^{6+} \\
&\quad + \sqrt{2}Z_2 * \mathcal{H}^{3+}) - \chi \mathcal{H}^{9+},
\end{aligned} \quad (130)$$

$$\begin{aligned}
Z_2^2 \mathcal{H}^{9-} + \mathcal{H}^{6-} + \sqrt{2}Z_2 \mathcal{H}^{3-} &= e^{4U+2V_1+3V_2} * \mathcal{H}^{9+} \\
&\quad - \chi (Z_2^2 \mathcal{H}^{9+} + \mathcal{H}^{6+} + \sqrt{2}Z_2 \mathcal{H}^{3+}),
\end{aligned} \quad (131)$$

$$\begin{aligned}
\sqrt{2}Z_2 \mathcal{H}^{9-} + \mathcal{H}^{3-} &= -\chi (\sqrt{2}Z_2 \mathcal{H}^{9+} + \mathcal{H}^{3+}) \\
&\quad - e^{2V_1+V_2} (\sqrt{2}Z_2 * \mathcal{H}^{9+} + * \mathcal{H}^{3+}).
\end{aligned} \quad (132)$$

A.3: BPS equations for the $N = 1$ case

In this section, we collect all the relevant BPS equations in the $N = 1$ case. These are given by

$$\begin{aligned}
e^{-i\Lambda} \tilde{U}'_1 + i e^{-\tilde{U}_1 - i\Lambda} Z'_1 \\
&= e^{-\tilde{U}_2 - 2\tilde{U}_1 - \frac{\tilde{V}}{2}} [2e^{\tilde{U}_1 + \tilde{U}_2} - e^{2\tilde{U}_2} + 2e^{\tilde{V}} (e^{\tilde{U}_2} + e^{\tilde{U}_1}) + 3k \\
&\quad - 4iZ_1(e^{\tilde{U}_2} + e^{\tilde{V}} - iZ_2) - 2i\chi(e^{\tilde{U}_2} + e^{\tilde{U}_1} - 2iZ_1 - iZ_2) \\
&\quad + Z_2[Z_2 - 2i(e^{\tilde{V}} + e^{\tilde{U}_1} - e^{\tilde{U}_2})]],
\end{aligned} \quad (133)$$

$$\begin{aligned}
e^{-i\Lambda} \tilde{U}'_2 + i e^{-\tilde{U}_2 - i\Lambda} Z'_2 \\
&= \frac{1}{2} e^{-2g - \tilde{U}_2 - 2\tilde{U}_1 - \frac{\tilde{V}}{2}} [2e^{2(g+\tilde{U}_2)} + \sqrt{2}ie_9 e^{2(\tilde{U}_1 + \tilde{U}_2)} + 6ke^{2g} \\
&\quad - 2ie_3 \chi e^{2\tilde{U}_1} + \sqrt{2}ip^9 \chi e^{2(\tilde{U}_1 + \tilde{U}_2)} + 3\sqrt{2}kp^9 \chi e^{2\tilde{U}_1} \\
&\quad - 8iZ_2 e^{2g + \tilde{U}_1} - 2ie_3 Z_2 e^{2\tilde{U}_1} - 4\chi Z_2 e^{2g} - 2ip^3 \chi Z_2 e^{2\tilde{U}_1} \\
&\quad - 8Z_1 Z_2 e^{2g} + 2Z_2^2 e^{2g} - \sqrt{2}ie_9 Z_2^2 e^{2\tilde{U}_1} - \sqrt{2}ip^9 \chi Z_2^2 e^{2\tilde{U}_1} \\
&\quad - 4iBe^{2\tilde{U}_1} [e^{2\tilde{U}_2} - 3k + Z_2(2\chi - Z_2 - 2ie^{\tilde{V}})] - 8i\chi e^{2g + \tilde{U}_1} \\
&\quad - 4ie^{2g + \tilde{V}} (2Z_1 + Z_2) + 4\sqrt{2}\tilde{B}e^{2\tilde{U}_1} [e^{\tilde{V}} + i(\chi + Z_2)] \\
&\quad + e^{\tilde{U}_1 + \tilde{V}} [8e^{2g} + e^{\tilde{U}_1} (\sqrt{2}p^9 (e^{2\tilde{U}_2} + 3k) - 2e_3)] \\
&\quad - 8\chi Z_1 e^{2g} - Z_2 e^{2\tilde{U}_1 + \tilde{V}} (2p^3 + \sqrt{2}p^9 Z_2)],
\end{aligned} \quad (134)$$

$$\begin{aligned}
e^{i\Lambda} \tilde{V}' - i e^{-\tilde{V} + i\Lambda} \chi' \\
&= \frac{1}{2} e^{-2g - \tilde{U}_2 - 2\tilde{U}_1 - \frac{\tilde{V}}{2}} [2e^{2(g+\tilde{U}_2)} - 8e^{2g + \tilde{U}_2 + \tilde{U}_1} + 2ie_3 e^{\tilde{U}_2 + 2\tilde{U}_1} \\
&\quad + \sqrt{2}e_9 e^{2(\tilde{U}_1 + \tilde{U}_2)} - 4i\chi e^{2g + \tilde{U}_2} - 8i\chi e^{2g + \tilde{U}_1} - 2e_3 \chi e^{2\tilde{U}_1} \\
&\quad + 2ip^3 \chi e^{\tilde{U}_2 + 2\tilde{U}_1} + \sqrt{2}p^9 \chi e^{2(\tilde{U}_1 + \tilde{U}_2)} + 3\sqrt{2}kp^9 \chi e^{2\tilde{U}_1} \\
&\quad + 4e^{2g} (2\chi Z_1 + iZ_2 e^{\tilde{U}_2}) - 2Z_2 e^{\tilde{U}_1} (4ie^{2g} + e_3 e^{\tilde{U}_1}) - 2Z_2^2 e^{2g} \\
&\quad + 2\sqrt{2}ie_9 Z_2 e^{\tilde{U}_2 + 2\tilde{U}_1} + 4\chi Z_2 e^{2g} - 2p^3 \chi Z_2 e^{2\tilde{U}_1} - 6ke^{2g} \\
&\quad + 2\sqrt{2}ip^9 \chi Z_2 e^{\tilde{U}_2 + 2\tilde{U}_1} + 8Z_1 Z_2 e^{2g} - \sqrt{2}e_9 Z_2^2 e^{2\tilde{U}_1} \\
&\quad + 4\sqrt{2}\tilde{B}e^{2\tilde{U}_1} [Z_2 + \chi - i(e^{\tilde{U}_2} - e^{\tilde{V}})] - 4Be^{2\tilde{U}_1} (e^{2\tilde{U}_2 + \tilde{V}} - 3k) \\
&\quad - 4Be^{2\tilde{U}_1} [2iZ_2 (e^{\tilde{U}_2} + e^{\tilde{V}}) - Z_2^2 + 2\chi (Z_2 - ie^{\tilde{U}_2})] \\
&\quad + ie^{2\tilde{U}_1 + \tilde{V}} [6\sqrt{2}p^9 - 2e_3 + \sqrt{2}p^9 - 2p^3 Z_2 - \sqrt{2}p^9 Z_2^2 \\
&\quad + 2ie^{\tilde{U}_2} (p^3 + \sqrt{2}p^9 Z_2)] + 4e^{2g + \tilde{V}} [e^{\tilde{U}_2} + i(2Z_1 + Z_2)] \\
&\quad + 8e^{2g + \tilde{U}_1 + \tilde{V}} - 8iZ_1 e^{2g + \tilde{U}_2} - \sqrt{2}p^9 \chi Z_2^2 e^{2\tilde{U}_1}]
\end{aligned} \quad (135)$$

where

$$e^{i\Lambda} = \frac{\mathcal{W}_1 + \mathcal{Z}_1}{|\mathcal{W}_1 + \mathcal{Z}_1|}. \quad (136)$$

These equations need to be solved together with the following equations:

$$\begin{aligned}
f' &= \text{Re}[e^{-i\Lambda} (\mathcal{W}_1 - \mathcal{Z}_1)], \\
g' &= |\mathcal{W}_1 + \mathcal{Z}_1|, \quad \hat{A}_t = e^f \text{Im}[e^{-i\Lambda} (\mathcal{W}_1 - \mathcal{Z}_1)]
\end{aligned} \quad (137)$$

and the two-form equations (75) and (76).

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Supersymmetric RG flows and Janus from type II orbifold compactification

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Abstract We study holographic RG flow solutions within four-dimensional $N = 4$ gauged supergravity obtained from type IIA and IIB string theories compactified on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with gauge, geometric and non-geometric fluxes. In type IIB non-geometric compactifications, the resulting gauged supergravity has $ISO(3) \times ISO(3)$ gauge group and admits an $N = 4$ AdS₄ vacuum dual to an $N = 4$ superconformal field theory (SCFT) in three dimensions. We study various supersymmetric RG flows from this $N = 4$ SCFT to $N = 4$ and $N = 1$ non-conformal field theories in the IR. The flows preserving $N = 4$ supersymmetry are driven by relevant operators of dimensions $\Delta = 1, 2$ or alternatively by one of these relevant operators, dual to the dilaton, and irrelevant operators of dimensions $\Delta = 4$ while the $N = 1$ flows in addition involve marginal deformations. Most of the flows can be obtained analytically. We also give examples of supersymmetric Janus solutions preserving $N = 4$ and $N = 1$ supersymmetries. These solutions should describe two-dimensional conformal defects within the dual $N = 4$ SCFT. Geometric compactifications of type IIA theory give rise to $N = 4$ gauged supergravity with $ISO(3) \ltimes U(1)^6$ gauge group. In this case, the resulting gauged supergravity admits an $N = 1$ AdS₄ vacuum. We also numerically study possible $N = 1$ RG flows to non-conformal field theories in this case.

1 Introduction

Along the line of research within the context of the AdS/CFT correspondence, the study of holographic RG flows is of particular interest since the original proposal in [1]. There is much work exploring this type of holographic solutions in various space-time dimensions with different numbers of

supersymmetry. In this paper, we will particularly consider holographic RG flows within three-dimensional superconformal field theories (SCFTs) using gauged supergravity in four dimensions. This might give some insight to the dynamics of strongly coupled SCFTs in three dimensions and related brane configurations in string/M-theory.

Most of the previously studied holographic RG flows have been found within the maximal $N = 8$ gauged supergravities [2–9]. Many of these solutions describe various deformations of the $N = 8$ SCFTs arising from M2-brane world-volume proposed in [10, 11]. Similar study in the case of lower number of supersymmetry is, however, less known. For example, a number of RG flow solutions have appeared only recently in $N = 3$ and $N = 4$ gauged supergravities [12–14]. In this work, we will give more solutions of this type from the half-maximal $N = 4$ gauged supergravity.

$N = 4$ supergravity allows for coupling to an arbitrary number of vector multiplets. With n vector multiplets, there are $6n + 2$ scalars, 2 from gravity and $6n$ from vector multiplets, parametrized by $SL(2, \mathbb{R})/SO(2) \times SO(6, n)/SO(6) \times SO(n)$ coset. The $N = 4$ gauged supergravity has been constructed for a long time in [15, 16]. Gaugings constructed in [15] are called electric gaugings since only electric $n + 6$ vector fields appearing in the ungauged Lagrangian gauge a subgroup of $SO(6, n)$. These vector fields transform as a fundamental representation of $SO(6, n)$. The scalar potential of the resulting gauged supergravity constructed in this way does not possess any AdS₄ critical points [17, 18]. This is not the case for the construction in [16] in which non-trivial $SL(2, \mathbb{R})$ phases have been included.

The most general gauging in which both electric vector fields and their magnetic dual can participate has been constructed in [19] using the embedding tensor formalism. A general gauge group is a subgroup of the full duality group $SL(2, \mathbb{R}) \times SO(6, n)$ with the vector fields and their magnetic dual transforming as a doublet of $SL(2, \mathbb{R})$. In this

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work, we will consider $N = 4$ gauged supergravity obtained from compactifications of type II string theories with various fluxes given in [20]; for other work along this line, see for example [21–23].

In [20], the scalar potential arising from flux compactifications of type IIA and IIB theories on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ within a truncation to $SO(3)$ singlet scalars has been considered, and some AdS_4 critical points together with their properties have been given. For type IIB non-geometric compactification, the vacuum structure is very rich even with only a few number of fluxes turned on. Among these vacua, there exists an $N = 4$ AdS_4 critical point dual to an $N = 4$ SCFT with $SO(3) \times SO(3)$ global symmetry. In addition, the full classification of vacua from type IIA geometric compactification has also been given. In this case, there exist a number of stable non-supersymmetric AdS_4 critical points as well as an $N = 1$ AdS_4 vacuum; see [24] for an $N = 1$ supersymmetric AdS_4 vacuum in massive type IIA theory.

We are particularly interested in $N = 4$ and $N = 1$ AdS_4 critical points from these two compactifications. They correspond to $N = 4$ and $N = 1$ SCFTs in three dimensions with global symmetries $SO(3) \times SO(3)$ and $SO(3)$, respectively. We will look for possible supersymmetric deformations within these two SCFTs in the form of RG flows to non-conformal phases preserving some amount of supersymmetry. These deformations are described by supersymmetric domain walls in the $N = 4$ gauged supergravity. In the case of $N = 1$ SCFT arising from massive type IIA theory, non-supersymmetric RG flows to conformal fixed points in the IR have been recently found in [25].

For type IIB compactification, we will also consider supersymmetric Janus solutions describing $(1+1)$ -dimensional conformal interfaces in the $N = 4$ SCFT. This type of solutions breaks conformal symmetry in three dimensions but preserves a smaller conformal symmetry on the lower-dimensional interface. Similar to the RG flow solutions, there are only a few examples of these solutions within the context of four-dimensional gauged supergravities [14, 26–28], see also [29–31] for examples of higher-dimensional solutions. They also play an important role in the holographic study of interface and boundary CFTs [32, 33]. We will give more examples of these solutions in $N = 4$ gauged supergravity obtained from non-geometric flux compactification.

The paper is organized as follows. In Sect. 2, we review relevant formulas and introduce some notations for $N = 4$ gauged supergravity in the embedding tensor formalism. In Sects. 3 and 4, we give a detailed analysis of supersymmetric RG flow and Janus solutions obtained from non-geometric type IIB compactification. Similar study of RG flows from geometric type IIA compactification will be given in Sect. 5. We finally give some conclusions and comments on the results in Sect. 6. We have also included an Appendix con-

taining more details on the conventions and the explicit form of complicated equations.

2 $N = 4$ gauged supergravity coupled to six vector multiplets

We first review relevant information and necessary formulas of four-dimensional $N = 4$ gauged supergravity which is the framework we use to find supersymmetric solutions. We mainly follow the most general gauging of $N = 4$ supergravity in the embedding tensor formalism given in [19] in which more details on the construction can be found. $N = 4$ supersymmetry allows for coupling the supergravity multiplet to an arbitrary number of vector multiplets. We will begin with a general formulation of $N = 4$ gauged supergravity with n vector multiplets and later specify to the case of six vector multiplets.

In half-maximal $N = 4$ supergravity, the supergravity multiplet consists of the graviton $e_{\mu}^{\hat{\mu}}$, four gravitini ψ_{μ}^i , six vectors A_{μ}^m , four spin- $\frac{1}{2}$ fields χ^i and one complex scalar τ consisting of the dilaton ϕ and the axion χ . The complex scalar can be parametrized by $SL(2, \mathbb{R})/SO(2)$ coset. The supergravity multiplet can couple to an arbitrary number n of vector multiplets, and each vector multiplet contains a vector field A_{μ} , four gaugini λ^i and six scalars ϕ^m . Similar to the dilaton and the axion in the gravity multiplet, the $6n$ scalar fields in these vector multiplets can be parametrized by $SO(6, n)/SO(6) \times SO(n)$ coset.

We will use the following convention on various indices appearing throughout the paper. Space-time and tangent space indices are denoted, respectively, by $\mu, \nu, \dots = 0, 1, 2, 3$ and $\hat{\mu}, \hat{\nu}, \dots = 0, 1, 2, 3$. The $SO(6) \sim SU(4)$ R-symmetry indices will be described by $m, n = 1, \dots, 6$ for the $SO(6)$ vector representation and $i, j = 1, 2, 3, 4$ for the $SO(6)$ spinor or $SU(4)$ fundamental representation. The n vector multiplets will be labeled by indices $a, b = 1, \dots, n$. All fields in the vector multiplets then carry an additional index in the form of $(A_{\mu}^a, \lambda^{ia}, \phi^{ma})$. Fermionic fields and the supersymmetry parameters transform in the fundamental representation of $SU(4)_R \sim SO(6)_R$ R-symmetry and are subject to the chirality projections

$$\gamma_5 \psi_{\mu}^i = \psi_{\mu}^i, \quad \gamma_5 \chi^i = -\chi^i, \quad \gamma_5 \lambda^i = \lambda^i. \quad (1)$$

On the other hand, for the fields transforming in the anti-fundamental representation of $SU(4)_R$, we have

$$\gamma_5 \psi_{\mu i} = -\psi_{\mu i}, \quad \gamma_5 \chi_i = \chi_i, \quad \gamma_5 \lambda_i = -\lambda_i. \quad (2)$$

Gaugings of the matter-coupled $N = 4$ supergravity can be efficiently described by using the embedding tensor. This tensor encodes all the information as regards the

embedding of any gauge group G_0 in the global or duality symmetry group $G = SL(2, \mathbb{R}) \times SO(6, n)$ in a G covariant way. According to the analysis in [19], a general gauging can be described by two components of the embedding tensor $\xi^{\alpha M}$ and $f_{\alpha MNP}$ with $\alpha = (+, -)$ and $M, N = (m, a) = 1, \dots, n+6$ denoting fundamental representations of $SL(2, \mathbb{R})$ and $SO(6, n)$, respectively. The electric vector fields $A^{+M} = (A_\mu^m, A_\mu^a)$, appearing in the ungauged Lagrangian, and their magnetic dual A^{-M} form a doublet under $SL(2, \mathbb{R})$ denoted by $A^{\alpha M}$. A particular electric-magnetic frame in which the $SO(2) \times SO(6, n)$ symmetry, with $SO(2) \subset SL(2, \mathbb{R})$, is manifest in the action can always be chosen. In this frame, A^{+M} and A^{-M} have charges $+1$ and -1 under this $SO(2)$.

In general, a subgroup of both $SL(2, \mathbb{R})$ and $SO(6, n)$ can be gauged, and the magnetic vector fields can also participate in the gauging. Furthermore, it has been shown in [17], see also [18], that purely electric gaugings do not admit AdS_4 vacua unless an $SL(2, \mathbb{R})$ phase is included [16]. The latter is, however, incorporated in the magnetic component f_{-MNP} [19]. Accordingly, we will consider only gaugings involving both electric and magnetic vector fields in order to obtain AdS_4 vacua. We will see that gauged supergravities obtained from type II compactifications are precisely of this form.

The gauge covariant derivative can be written as

$$D_\mu = \nabla_\mu - g A_\mu^{\alpha M} \Theta_{\alpha M}^{NP} t_{NP} + g A_\mu^{M(\alpha} \epsilon^{\beta)\gamma} \xi_{\gamma M} t_{\alpha\beta} \quad (3)$$

where ∇_μ is the usual space-time covariant derivative including the spin connection. t_{MN} and $t_{\alpha\beta}$ are $SO(6, n)$ and $SL(2, \mathbb{R})$ generators which can be chosen as

$$(t_{MN})_P^Q = 2\delta_{[M}^Q \eta_{N]P}, \quad (t_{\alpha\beta})_\gamma^\delta = 2\delta_{(\alpha}^\delta \epsilon_{\beta)\gamma} \quad (4)$$

with $\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}$ and $\epsilon^{+-} = 1$. $\eta_{MN} = \text{diag}(-1, -1, -1, -1, -1, -1, 1, \dots, 1)$ is the $SO(6, n)$ invariant tensor, and g is the gauge coupling constant that can be absorbed in the embedding tensor Θ .

The embedding tensor component $\Theta_{\alpha MNP}$ can be written in terms of $\xi^{\alpha M}$ and $f_{\alpha MNP}$ components as

$$\Theta_{\alpha MNP} = f_{\alpha MNP} - \xi_{\alpha[N} \eta_{P]M}. \quad (5)$$

To define a consistent gauging, the embedding tensor has to satisfy a quadratic constraint. This ensures that the gauge generators

$$X_{\alpha M} = \Theta_{\alpha MNP} t^{NP} - \xi_M^\beta t_{\alpha\beta} \quad (6)$$

form a closed algebra.

In this work, we will consider solutions with only the metric and scalars non-vanishing. In addition, we will consider gaugings with only $f_{\alpha MNP}$ non-vanishing. Therefore, we

will set all vector fields and $\xi_{\alpha M}$ to zero from now on. In particular, this simplifies the full quadratic constraint to

$$f_{\alpha R[MN} f_{\beta PQ]}^R = 0, \quad \epsilon^{\alpha\beta} f_{\alpha MNR} f_{\beta PQ}^R = 0. \quad (7)$$

For electric gaugings, these relations reduce to the usual Jacobi identity for $f_{MNP} = f_{+MNP}$ as shown in [15, 16].

The scalar coset manifold $SL(2, \mathbb{R})/SO(2) \times SO(6, n)/SO(6) \times SO(n)$ can be described by the coset representative $(\mathcal{V}_\alpha, \mathcal{V}_M^A)$ with $A = (m, a)$. The first factor can be parametrized by

$$\mathcal{V}_\alpha = \frac{1}{\sqrt{\text{Im}\tau}} \begin{pmatrix} \tau \\ 1 \end{pmatrix} \quad (8)$$

or equivalently by a symmetric 2×2 matrix

$$M_{\alpha\beta} = \text{Re}(\mathcal{V}_\alpha \mathcal{V}_\beta^*) = \frac{1}{\text{Im}\tau} \begin{pmatrix} |\tau|^2 & \text{Re}\tau \\ \text{Re}\tau & 1 \end{pmatrix}. \quad (9)$$

Note also that $\text{Im}(\mathcal{V}_\alpha \mathcal{V}_\beta^*) = \epsilon_{\alpha\beta}$. The complex scalar τ can also be written in terms of the dilaton ϕ and the axion χ as

$$\tau = \chi + i e^\phi. \quad (10)$$

For the $SO(6, n)/SO(6) \times SO(n)$ factor, we introduce another coset representative $\mathcal{V}_M^A$ transforming by left and right multiplications under $SO(6, n)$ and $SO(6) \times SO(n)$, respectively. From the splitting of the index $A = (m, a)$, we can write the coset representative as $\mathcal{V}_M^A = (\mathcal{V}_M^m, \mathcal{V}_M^a)$. Being an element of $SO(6, n)$, the matrix $\mathcal{V}_M^A$ satisfies the relation

$$\eta_{MN} = -\mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \quad (11)$$

As in the $SL(2, \mathbb{R})/SO(2)$ factor, we can parametrize the $SO(6, n)/SO(6) \times SO(n)$ coset in terms of a symmetric matrix

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \quad (12)$$

We are now in a position to give the bosonic Lagrangian with the vector fields and auxiliary two-form fields vanishing

$$e^{-1} \mathcal{L} = \frac{1}{2} R + \frac{1}{16} \partial_\mu M_{MN} \partial^\mu M^{MN} - \frac{1}{4(\text{Im}\tau)^2} \partial_\mu \tau \partial^\mu \tau^* - V \quad (13)$$

where e is the vielbein determinant. The scalar potential can be written in terms of scalar coset representatives and the embedding tensor as

$$V = \frac{g^2}{16} \left[f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta} \right]$$

$$\times \left[\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left(\frac{2}{3} \eta^{MQ} - M^{MQ} \right) \eta^{NR} \eta^{PS} \right] - \frac{4}{9} f_{\alpha MNP} f_{\beta QRS} \epsilon^{\alpha\beta} M^{MNPQRS} \quad (14)$$

where M^{MN} is the inverse of M_{MN} , and M^{MNPQRS} is defined by

$$M_{MNPQRS} = \epsilon_{mnpqrs} \mathcal{V}_M^m \mathcal{V}_N^n \mathcal{V}_P^p \mathcal{V}_Q^q \mathcal{V}_R^r \mathcal{V}_S^s \quad (15)$$

with indices raised by η^{MN} .

Before giving an explicit parametrization of the scalar coset, we give fermionic supersymmetry transformations of $N = 4$ gauged supergravity which play an important role in subsequent analyses. These are given by

$$\delta \psi_\mu^i = 2D_\mu \epsilon^i - \frac{2}{3} g A_1^{ij} \gamma_\mu \epsilon_j, \quad (16)$$

$$\delta \chi^i = i \epsilon^{\alpha\beta} \mathcal{V}_\alpha D_\mu \mathcal{V}_\beta \gamma^\mu \epsilon^i - \frac{4}{3} i g A_2^{ij} \epsilon_j, \quad (17)$$

$$\delta \lambda_a^i = 2i \mathcal{V}_a^M D_\mu \mathcal{V}_M^{ij} \gamma^\mu \epsilon_j + 2i g A_{2aj}^i \epsilon^j. \quad (18)$$

The fermion shift matrices, appearing in fermionic mass-like terms in the gauged Lagrangian, are defined by

$$\begin{aligned} A_1^{ij} &= \epsilon^{\alpha\beta} (\mathcal{V}_\alpha)^* \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_2^{ij} &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_{2ai}^j &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_a^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jk} f_{\beta MN}^P \end{aligned} \quad (19)$$

where $\mathcal{V}_M^{ij}$ is defined in terms of the 't Hooft symbols G_m^{ij} and $\mathcal{V}_M^m$ as

$$\mathcal{V}_M^{ij} = \frac{1}{2} \mathcal{V}_M^m G_m^{ij} \quad (20)$$

and similarly for its inverse

$$\mathcal{V}_{ij}^M = -\frac{1}{2} \mathcal{V}_m^M (G_m^{ij})^*. \quad (21)$$

G_m^{ij} convert an index m in vector representation of $SO(6)$ to an anti-symmetric pair of indices $[ij]$ in the $SU(4)$ fundamental representation. They satisfy the relations

$$G_{mij} = -(G_m^{ij})^* = -\frac{1}{2} \epsilon_{ijkl} G_m^{kl}. \quad (22)$$

The explicit form of these matrices can be found in the appendix.

We finally note the expression for the scalar potential written in terms of A_1 and A_2 tensors as

$$V = -\frac{1}{3} A_1^{ij} A_{1ij} + \frac{1}{9} A_2^{ij} A_{2ij} + \frac{1}{2} A_{2ai}^j A_{2a}^i{}_j. \quad (23)$$

It follows that unbroken supersymmetry corresponds to an eigenvalue of A_1^{ij} , α , satisfying $V_0 = -\frac{\alpha^2}{3}$ where V_0 is the value of the scalar potential at the vacuum, the cosmological constant.

We now consider the case of $n = 6$ vector multiplets. Possible gauge groups are accordingly subgroups of $SO(6, 6)$ for $\xi_{\alpha M} = 0$. Following [20], we restrict ourselves to solutions preserving at least $SO(3)$ subgroup of the full gauge group. The residual $SO(3)$ symmetry is embedded in $SO(6, 6)$ as a diagonal subgroup of $SO(3) \times SO(3) \times SO(3) \times SO(3)$ with the four factors of $SO(3)$ being subgroups of $SO(6) \times SO(6) \subset SO(6, 6)$. The 36 scalars within $SO(6, 6)/SO(6) \times SO(6)$ coset transform as $(\mathbf{6}, \mathbf{6})$ under $SO(6) \times SO(6)$ compact subgroup. The above embedding of $SO(3) \times SO(3)$ in $SO(6)$ is given by

$$\mathbf{6} \rightarrow (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}). \quad (24)$$

This implies that the 36 scalars transform as

$$(\mathbf{6}, \mathbf{6}) \rightarrow 4 \times (\mathbf{1} + \mathbf{3} + \mathbf{5}) \quad (25)$$

under the unbroken $SO(3) \sim [SO(3) \times SO(3) \times SO(3) \times SO(3)]_{\text{diag}}$. We see that there are four $SO(3)$ singlets. We will denote these scalars by $(\varphi_1, \varphi_2, \chi_1, \chi_2)$ as in [20]. In addition, we will also use the explicit parametrization given in [20]. This gives the coset representative

$$\mathcal{V}_M^A = \begin{pmatrix} e^T & 0 \\ B e^T & e^{-1} \end{pmatrix} \otimes \mathbb{I}_3 \quad (26)$$

where the two 2×2 matrices e and B are defined by

$$e = e^{\frac{1}{2}(\varphi_1 + \varphi_2)} \begin{pmatrix} 1 & \chi_2 \\ 0 & e^{-\varphi_2} \end{pmatrix}, \quad B = \begin{pmatrix} 0 & \chi_1 \\ -\chi_1 & 0 \end{pmatrix}. \quad (27)$$

Explicitly, the $SO(6, 6)/SO(6) \times SO(6)$ coset representative consisting of all $SO(3)$ singlet scalars is given by

$$\begin{aligned} \mathcal{V}_M^A &= \begin{pmatrix} e^{\frac{1}{2}(\varphi_1 + \varphi_2)} & 0 & 0 & 0 \\ e^{\frac{1}{2}(\varphi_1 + \varphi_2)} \chi_2 & e^{\frac{1}{2}(\varphi_1 - \varphi_2)} & 0 & 0 \\ e^{\frac{1}{2}(\varphi_1 + \varphi_2)} \chi_1 \chi_2 & e^{\frac{1}{2}(\varphi_1 - \varphi_2)} \chi_1 & e^{-\frac{1}{2}(\varphi_1 + \varphi_2)} & -e^{\frac{1}{2}(\varphi_2 - \varphi_1)} \chi_2 \\ -e^{\frac{1}{2}(\varphi_1 + \varphi_2)} \chi_1 & 0 & 0 & e^{\frac{1}{2}(\varphi_2 - \varphi_1)} \end{pmatrix} \\ &\otimes \mathbb{I}_3. \end{aligned} \quad (28)$$

It should also be noted that there are two scalars which are singlet under $SO(3) \times SO(3) \subset [SO(3) \times SO(3)]_{\text{diag}} \times [SO(3) \times SO(3)]_{\text{diag}}$ as can be seen by taking the tensor product of the representation $\mathbf{6}$ in (24) giving rise to two singlets $(\mathbf{1}, \mathbf{1})$ of $SO(3) \times SO(3)$. These two singlets correspond to φ_1 and φ_2 .

Similarly, the $SL(2, \mathbb{R})/SO(2)$ coset representative will be parametrized by

$$\mathcal{V}_\alpha = e^{\varphi_g/2} \begin{pmatrix} \chi_g - i e^{-\varphi_g} \\ 1 \end{pmatrix}. \quad (29)$$

With all these and the definition $\phi^i = (\varphi_g, \varphi_1, \varphi_2, \chi_g, \chi_1, \chi_2)$, the scalar kinetic terms can be found to be

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= -\frac{1}{2} K_{ij} \partial_\mu \phi^i \partial^\mu \phi^j \\ &= -\frac{1}{4} (\partial_\mu \varphi_g \partial^\mu \varphi_g \\ &\quad + 3 \partial_\mu \varphi_1 \partial^\mu \varphi_1 + 3 \partial_\mu \varphi_2 \partial^\mu \varphi_2 + e^{2\varphi_g} \partial_\mu \chi_g \partial^\mu \chi_g \\ &\quad + 3 e^{2\varphi_1} \partial_\mu \chi_1 \partial^\mu \chi_1 + 3 e^{2\varphi_2} \partial_\mu \chi_2 \partial^\mu \chi_2) \end{aligned} \quad (30)$$

where we have defined the scalar metric K_{ij} , which will play a role in writing the BPS equations.

The four $SO(3)$ singlet scalars in $SO(6, 6)/SO(6) \times SO(6)$ correspond to non-compact generators of $SO(2, 2) \subset SO(6, 6)$ that commute with the $SO(3)$ symmetry. It is convenient to split indices $M = (AI)$ for $A = 1, 2, 3, 4$ and $I = 1, 2, 3$. This implies that the $SO(6, 6)$ fundamental representation decomposes as $(4, 3)$ under $SO(2, 2) \times SO(3)$. In terms of (AI) indices, the embedding tensor can be written as

$$f_{\alpha MNP} = f_{\alpha AIBJCK} = \Lambda_{\alpha ABC} \epsilon_{IJK} \quad (31)$$

with $\Lambda_{\alpha ABC} = \Lambda_{\alpha(ABC)}$. In particular, the quadratic constraints read

$$\epsilon^{\alpha\beta} \Lambda_{\alpha AB}{}^C \Lambda_{\beta DEC} = 0, \quad \Lambda_{(\alpha A[B}{}^C \Lambda_{\beta)D]EC} = 0. \quad (32)$$

The $SO(6, 6)$ fundamental indices M, N can also be decomposed into $(m, \bar{m})$, $m, \bar{m} = 1, 2, \dots, 6$. In connection with the internal manifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, the index m is used to label the T^6 coordinates and split into (a, i) such that $a = 1, 3, 5$ and $i = 2, 4, 6$. Similar decomposition is also in use for $\bar{m} = (\bar{a}, \bar{i})$. All together, indices A, B can be written as $A = (1, 2, 3, 4) = (a, i, \bar{a}, \bar{i})$. Indices $I, J = 1, 2, 3$ label the three T^2 's inside $T^6 \sim T^2 \times T^2 \times T^2$.

The $SO(6, 6)$ invariant tensor η_{MN} and its inverse are chosen to be

$$\eta_{MN} = \eta^{MN} = \begin{pmatrix} 0 & \mathbb{I}_6 \\ \mathbb{I}_6 & 0 \end{pmatrix}. \quad (33)$$

This leads to some extra projections on the negative and positive eigenvalues of η_{MN} . For example, in order to compute M_{MNPQRS} in the scalar potential defined by equation (15), we need to project the second index of $\mathcal{V}_M^A$ by using the projection matrix

$$R = \frac{1}{\sqrt{2}} \begin{pmatrix} -\mathbb{I}_6 & \mathbb{I}_6 \\ \mathbb{I}_6 & \mathbb{I}_6 \end{pmatrix}. \quad (34)$$

Finally, we will also set the gauge coupling $g = \frac{1}{2}$ as in [20].

3 RG flows from type IIB non-geometric compactification

We begin with a non-geometric compactification of type IIB theory on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$. This involves the fluxes of NS and RR three-form fields (H_3, F_3) and non-geometric (P, Q) fluxes. This compactification admits a locally geometric description although it is non-geometric in nature.

From the result of [20], the effective $N = 4$ gauged supergravity theory is not unique. In this paper, we will only consider the gauged supergravity admitting the maximally supersymmetric $N = 4$ AdS₄ vacuum. In this case, all the gauge and non-geometric fluxes lead to the following components of the embedding tensor:

$$\begin{aligned} f_{-i\bar{j}\bar{k}} &= \Lambda_{-444} = -\lambda, & f_{+\bar{a}\bar{b}\bar{c}} &= \Lambda_{+333} = \lambda, \\ f_{-i\bar{j}k} &= \Lambda_{-244} = -\lambda, & f_{+a\bar{b}\bar{c}} &= \Lambda_{+133} = \lambda \end{aligned} \quad (35)$$

for a constant λ . The first and second lines correspond to (H_3, F_3) and (P, Q) fluxes, respectively. As shown in [20], the gauge group arising from this embedding tensor is $ISO(3) \times ISO(3) \sim [SO(3) \ltimes T^3] \times [SO(3) \ltimes T^3]$. This gauge group is embedded in $SO(6, 6)$ via the $SO(3, 3) \times SO(3, 3)$ subgroup.

Using this embedding tensor and the explicit form of the scalar coset representative given in the previous section, we find the scalar potential

$$\begin{aligned} V &= \frac{1}{32} e^{\varphi_1 - 3\varphi_2 - \varphi_g} \lambda^2 [e^{2\varphi_1} - 3e^{2\varphi_2} + 6e^{\varphi_1 + 2\varphi_2 + \varphi_g} \\ &\quad - 18e^{3\varphi_2 + \varphi_g} - 3e^{4\varphi_2 + 2\varphi_g} \\ &\quad - 2e^{2\varphi_1 + 3\varphi_2 + \varphi_g} (1 + 3\chi_1^2) + 3e^{2(\varphi_1 + \varphi_2)} (\chi_1 - \chi_2)^2 \\ &\quad - 12e^{5\varphi_2 + \varphi_g} \chi_2^2 \\ &\quad + 3e^{6\varphi_2} \chi_2^4 + e^{2\varphi_1 + 6\varphi_2} \chi_2^4 (\chi_2 - 3\chi_1)^2 \\ &\quad + 3e^{2\varphi_1 + 4\varphi_2} \chi_2^2 (\chi_2 - 2\chi_1)^2 - 3e^{2(\varphi_2 + \varphi_g)} \chi_g^2 \\ &\quad + 6e^{\varphi_1 + 4\varphi_2 + \varphi_g} (1 + \chi_2^2) + e^{2(\varphi_1 + \varphi_g)} \chi_g^2 \\ &\quad + 3e^{2(\varphi_1 + \varphi_2 + \varphi_g)} (\chi_1 - \chi_2)^2 \chi_g^2 \\ &\quad + 3e^{6\varphi_2 + 2\varphi_g} \chi_2^2 (-1 + \chi_2 \chi_g)^2 \\ &\quad + 3e^{2(\varphi_1 + 2\varphi_2 + \varphi_g)} (\chi_1 - 2\chi_1 \chi_2 \chi_g + \chi_2^2 \chi_g)^2 \\ &\quad + e^{2(\varphi_1 + 3\varphi_2 + \varphi_g)} [1 + \chi_2^3 \chi_g - 3\chi_1 \chi_2 (-1 + \chi_2 \chi_g)]^2]. \end{aligned} \quad (36)$$

Table 1 Scalar masses at the $N = 4$ supersymmetric AdS₄ critical point with $SO(3) \times SO(3)$ symmetry and the corresponding dimensions of the dual operators

Scalar fields	$m^2 L^2$	Δ
φ_g, χ_g	-2	1, 2
φ_1, φ_2	4	4
χ_1, χ_2	0	3

This potential admits a trivial critical point at which all scalars vanish. The cosmological constant is given by

$$V_0 = -\frac{3}{8}\lambda^2. \quad (37)$$

At this critical point, we find the scalar masses as in Table 1. In the table, we also give the corresponding dimensions of the dual operators. The AdS₄ radius is given by $L = \frac{2\sqrt{2}}{\lambda}$. Note that we have used different convention for scalar masses from that used in [20]. The masses given in Table 1 are obtained by multiplying the masses given in [20] by 3.

This AdS₄ critical point preserves $N = 4$ supersymmetry as can be checked from the A_1^{ij} tensor. It should also be emphasized here that this critical point has $SO(3) \times SO(3)$ symmetry which is the maximal compact subgroup of $ISO(3) \times ISO(3)$ gauge group.

To set up the BPS equations for finding supersymmetric RG flow solutions, we first give the metric ansatz

$$ds^2 = e^{2A(r)} dx_{1,2}^2 + dr^2 \quad (38)$$

where $dx_{1,2}^2$ is the flat Minkowski metric in three dimensions.

We will use the Majorana representation for gamma matrices with all γ^μ real and γ_5 purely imaginary. This choice implies that ϵ_i is a complex conjugate of ϵ^i . All scalar fields will be functions of only the radial coordinate r . To solve the BPS conditions coming from setting $\delta\chi^i = 0$ and $\delta\lambda_a^i = 0$, we need the following projection:

$$\gamma_r \epsilon^i = e^{i\Lambda} \epsilon^i. \quad (39)$$

From the $\delta\psi_{\mu i} = 0$ conditions for $\mu = 0, 1, 2$, we find

$$A' = \pm W, \quad e^{i\Lambda} = \pm \frac{\mathcal{W}}{W} \quad (40)$$

where $W = |\mathcal{W}|$, and $'$ denotes the r -derivative. These equations are obtained by solving real and imaginary parts of $\delta\psi_{\mu i} = 0$ separately; see for more details [12, 27]. The superpotential $\mathcal{W}$ is defined by

$$\mathcal{W} = \frac{1}{3}\alpha \quad (41)$$

where α is the eigenvalue of A_1^{ij} corresponding to the unbroken supersymmetry. We will choose a definite sign for the A'

equation and $e^{i\Lambda}$ such that the $N = 4$ critical point identified with the $N = 4$ SCFT in the UV corresponds to $r \rightarrow \infty$.

For all scalars non-vanishing, the $N = 4$ supersymmetry is broken to $N = 1$ corresponding to the Killing spinor ϵ^1 . The superpotential for this unbroken $N = 1$ supersymmetry is given by

$$\begin{aligned} \mathcal{W} = & \frac{1}{4\sqrt{2}} e^{\frac{1}{2}(\varphi_1 - 3\varphi_2 - \varphi_g)} [e^{\varphi_2} [e^{\varphi_2 + \varphi_g} (-e^{\varphi_1 + \varphi_2} \lambda \\ & - 3\lambda(i + e^{\varphi_1} \chi_1)(i + e^{\varphi_2} \chi_2)) \\ & - e^{\varphi_1} \lambda(i + e^{\varphi_2} \chi_2)^3 (i + e^{\varphi_g} \chi_g) \\ & + 3\lambda(i + e^{\varphi_1} \chi_1)(i + e^{\varphi_2} \chi_2)^2 (i + e^{\varphi_g} \chi_g)]] \end{aligned} \quad (42)$$

from which we find

$$\begin{aligned} W = & \frac{1}{8\sqrt{2}} \lambda e^{\frac{1}{2}(\varphi_1 - 3\varphi_2 - \varphi_g)} [((-3e^{\varphi_2} (-e^{\varphi_1} + 2e^{\varphi_2} \\ & + e^{2\varphi_2 + \varphi_g}) \chi_2 - e^{\varphi_1 + 3\varphi_2} \chi_2^3 \\ & + e^{\varphi_g} (e^{\varphi_1} - 3e^{\varphi_2}) \chi_g + 3e^{2\varphi_2 + \varphi_g} \\ & \times (-e^{\varphi_1} + e^{\varphi_2}) \chi_2^2 \chi_g + 3e^{\varphi_1 + \varphi_2} \chi_1 (-1 \\ & - e^{\varphi_2 + \varphi_g} + e^{2\varphi_2} \chi_2^2 + 2e^{2\varphi_2 + \varphi_g} \chi_2 \chi_g))^2 \\ & + [e^{\varphi_1} (-1 + 3e^{2\varphi_2} \chi_2^2) \\ & - e^{\varphi_1 + \varphi_2 + \varphi_g} \chi_2 (-3 + e^{2\varphi_2} \chi_2^2) \chi_g \\ & + e^{\varphi_2} (3 + 3e^{\varphi_2 + \varphi_g} - e^{\varphi_1 + 2\varphi_2 + \varphi_g} \\ & - 3e^{2\varphi_2} \chi_2^2 - 6e^{\varphi_2 + \varphi_g} \chi_2 \chi_g \\ & + 3e^{\varphi_1} \chi_1 (-e^{\varphi_2} (2 + e^{\varphi_2 + \varphi_g}) \chi_2 \\ & - e^{\varphi_g} \chi_g + e^{2\varphi_2 + \varphi_g} \chi_2^2 \chi_g))]^2]^{\frac{1}{2}}. \end{aligned} \quad (43)$$

The scalar potential can be written in terms of W as

$$V = -2K^{ij} \frac{\partial W}{\partial \phi^i} \frac{\partial W}{\partial \phi^j} - 3W^2, \quad (44)$$

and, as usual, the BPS equations from $\delta\chi^i = 0$ and $\delta\lambda_a^i = 0$ can be written as

$$\phi^{i'} = 2K^{ij} \frac{\partial W}{\partial \phi^j}. \quad (45)$$

K^{ij} is the inverse of the scalar kinetic metric defined in (30). The explicit form of these equations is rather complicated and will not be given here. However, they can be found in the appendix.

It is also straightforward to check that these BPS equations solve the second order field equations. Furthermore, there exist a number of interesting subtruncations keeping some subsets of these $SO(3)$ singlets. We will firstly discuss these truncations and consider the full $SO(3)$ singlet sector at the end of this section.

3.1 RG flows with $N = 4$ supersymmetry

We begin with RG flow solutions preserving $N = 4$ supersymmetry to $N = 4$ non-conformal field theories in the IR. The analysis of BPS conditions $\delta\psi_{\mu i} = 0$, $\delta\chi^i = 0$ and $\delta\lambda_a^i = 0$ shows that there are two possibilities in order to preserve $N = 4$ supersymmetry. The first one is to truncate out $\varphi_{1,2}$ and $\chi_{1,2}$. The second possibility is to keep only the three dilatons φ_g and $\varphi_{1,2}$ by setting $\chi_g = \chi_{1,2} = 0$.

3.1.1 $N = 4$ RG flows by relevant deformations

From Table 1, we see that (φ_g, χ_g) correspond to relevant deformations by operators of dimensions 1 or 2. The BPS equations admit a consistent truncation to these two scalars. By setting $\varphi_{1,2} = \chi_{1,2} = 0$, we obtain a set of simple BPS equations

$$\varphi'_g = -\frac{\lambda e^{-\frac{\varphi_g}{2}}}{2\sqrt{2}} \frac{(e^{2\varphi_g} + e^{2\varphi_g} \chi_g^2 - 1)}{\sqrt{(1 + e^{\varphi_g})^2 + e^{2\varphi_g} \chi_g^2}}, \quad (46)$$

$$\chi'_g = -\frac{\lambda e^{-\frac{\varphi_g}{2}}}{\sqrt{2}} \frac{\chi_g}{\sqrt{(1 + e^{\varphi_g})^2 + e^{2\varphi_g} \chi_g^2}}, \quad (47)$$

$$A' = \frac{\lambda e^{-\frac{\varphi_g}{2}}}{4\sqrt{2}} \sqrt{(1 + e^{\varphi_g})^2 + e^{2\varphi_g} \chi_g^2}. \quad (48)$$

Since (φ_g, χ_g) are scalars in $SL(2, \mathbb{R})/SO(2)$, they are $SO(6, 6)$ singlets and hence $SO(3) \times SO(3)$ invariant. All solutions to these equations then preserve the full $SO(3) \times SO(3)$ symmetry. Moreover, equations $\delta\lambda_a^i = 0$ are identically satisfied, and it can be checked that $N = 4$ supersymmetry is unbroken since equations $\delta\chi^i = 0$ and $\delta\psi_{\mu i} = 0$ hold for all ϵ^i satisfying the $\gamma_{\hat{r}}$ projector (39). We should clarify here the convention on the number of supersymmetry. In four dimensions, the $\gamma_{\hat{r}}$ projector reduce the number of supercharges from 16 to 8. The latter corresponds to $N = 4$ supersymmetry in three dimensions. On the other hand, the AdS_4 vacuum preserves all 16 supercharges corresponding to $N = 4$ superconformal symmetry in three dimensions containing $8 + 8 = 16$ supercharges.

We begin with an even simpler solution with $\chi_g = 0$ which, from the above equations, is clearly a consistent truncation. In this case, we end up with the BPS equations

$$\varphi'_g = -\frac{\lambda}{2\sqrt{2}} e^{-\frac{\varphi_g}{2}} (e^{\varphi_g} - 1), \quad (49)$$

$$A' = \frac{\lambda}{4\sqrt{2}} e^{-\frac{\varphi_g}{2}} (1 + e^{\varphi_g}). \quad (50)$$

The solution to these equations is easily found to be

$$\varphi_g = \ln[e^{\frac{r\lambda}{2\sqrt{2}}+C} - 1] - \ln[e^{\frac{r\lambda}{2\sqrt{2}}+C} + 1], \quad (51)$$

$$A = \ln[e^{\frac{r\lambda}{2\sqrt{2}}+C} - 1] - \frac{r\lambda}{2\sqrt{2}} \quad (52)$$

with C being an integration constant. The additive integration constant for A has been neglected since it can be absorbed by scaling $dx_{1,2}^2$ coordinates. In addition, the constant C can also be removed by shifting the r coordinate.

At large r , we find, as expected for dual operators of dimensions $\Delta = 1, 2$,

$$\varphi_g \sim e^{-\frac{\lambda r}{2\sqrt{2}}} \sim e^{-\frac{r}{L}}. \quad (53)$$

The solution is singular as $r \rightarrow -\frac{2\sqrt{2}C}{\lambda}$ since $\varphi_g \rightarrow -\infty$. Near this singularity, we find

$$\varphi_g \sim A \sim \ln \left[r + \frac{2\sqrt{2}C}{\lambda} \right]. \quad (54)$$

The scalar potential is bounded above with $V \rightarrow -\infty$. Therefore, the singularity of this solution is physical by the criterion given in [34]. The solution then describes an RG flow from the dual $N = 4$ SCFT to an $N = 4$ non-conformal field theory with unbroken $SO(3) \times SO(3)$ symmetry. The metric in the IR is given by

$$ds^2 = (\lambda r + 2\sqrt{2}C)^2 dx_{1,2}^2 + dr^2 \quad (55)$$

where we have absorbed some constants to $dx_{1,2}^2$ coordinates.

We then consider possible flows solution with $\chi_g \neq 0$. By introducing a new variable ρ via

$$\frac{d\rho}{dr} = \frac{\chi_g}{\sqrt{1 - C\chi_g} + \sqrt{1 - \chi_g(2C + \chi_g)}} \quad (56)$$

we find the following solution to Eqs. (46)–(48):

$$\varphi_g = -\frac{1}{2} \ln[1 - 2C\chi_g - \chi_g^2], \quad (57)$$

$$A = -\ln \chi_g + \frac{1}{4} \ln[1 - 2C\chi_g - \chi_g^2] + \frac{1}{2} \ln \left[1 - C\chi_g + \sqrt{1 - 2C\chi_g - \chi_g^2} \right], \quad (58)$$

$$\begin{aligned} & \rho \lambda [1 - \chi_g(2C + \chi_g)]^{\frac{3}{4}} \\ &= 4(2)^{\frac{1}{4}} (C + \chi_g - \sqrt{1 + C^2}) \\ & \times \left[\frac{1 + C^2 + \sqrt{1 + C^2}(C + \chi_g)}{1 + C^2} \right]^{\frac{3}{4}} \\ & \times {}_2F_1 \left(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{\chi_g + \sqrt{1 + C^2} - C}{2\sqrt{1 + C^2}} \right) \end{aligned} \quad (59)$$

where ${}_2F_1$ is the hypergeometric function.

The solution interpolates between the $N = 4$ AdS_4 vacuum as $r \rightarrow \infty$ and a singular geometry at a finite value of r .

There are two possibilities for the IR singularities. The first one is given by

$$\chi_g \sim \chi_0, \quad \varphi_g \sim -2 \ln \left[\frac{\sqrt{2} r \lambda (1 + \chi_0^2) - 4 \chi_0 C}{8 \chi_0} \right],$$

$$A \sim \frac{\chi_0}{\sqrt{1 + \chi_0}} \ln[\sqrt{2} r \lambda (1 + \chi_0^2) - 4 \chi_0 C] \quad (60)$$

where χ_0 is a constant. In this case, we have $\varphi_g \rightarrow \infty$ and $\chi_g \rightarrow \text{constant}$ as $\sqrt{2} r \lambda (1 + \chi_0^2) \rightarrow 4 \chi_0 C$. It should be noted here that the constant C in these equations is not the same as in the full solution given in (57)–(59).

Another possibility is given by

$$\varphi_g \sim 2 \ln(\sqrt{2} r \lambda + 4C), \quad \chi_g \sim \frac{\tilde{C}}{4C + \sqrt{2} r \lambda},$$

$$A \sim \ln(\sqrt{2} r \lambda + 4C). \quad (61)$$

In this case, as $\sqrt{2} r \lambda \rightarrow -4C$, we have $\varphi_g \rightarrow -\infty$ and $\chi_g \rightarrow \pm\infty$ depending on the sign of the constant $\tilde{C}$. Both of these singularities lead to $V \rightarrow -\infty$, so they are physical.

3.1.2 $N = 4$ RG flows by relevant and irrelevant deformations

We now consider RG flows with $N = 4$ supersymmetry with $\chi_g = \chi_{1,2} = 0$. Recall that φ_1 and φ_2 are $SO(3) \times SO(3)$ singlets, we still have solutions with $SO(3) \times SO(3)$ unbroken along the flows. It should also be noted that the truncation $\chi_{1,2} = 0$ is consistent only for $\chi_g = 0$. This implies that $N = 4$ supersymmetry does not allow turning on the operators dual to χ_g and $\varphi_{1,2}$ simultaneously. It would be interesting to see the implication of this in the dual $N = 4$ SCFT.

In this case, the BPS equations reduce to

$$\varphi'_g = \frac{\lambda}{4\sqrt{2}} e^{\frac{1}{2}(\varphi_1 - 3\varphi_2 - \varphi_g)} \times (3e^{\varphi_2} - e^{\varphi_1} - 3e^{2\varphi_2 + \varphi_g} + e^{\varphi_1 + 3\varphi_2 + \varphi_g}), \quad (62)$$

$$\varphi'_1 = \frac{\lambda}{4\sqrt{2}} (e^{\varphi_1} - e^{\varphi_2} - e^{2\varphi_2 + \varphi_g} + e^{\varphi_1 + 3\varphi_2 + \varphi_g}), \quad (63)$$

$$\varphi'_2 = \frac{\lambda}{4\sqrt{2}} (e^{\varphi_2} - e^{\varphi_1} - e^{2\varphi_2 + \varphi_g} + e^{\varphi_1 + 3\varphi_2 + \varphi_g}), \quad (64)$$

$$A' = -\frac{\lambda}{8\sqrt{2}} e^{\frac{1}{2}(\varphi_1 - 3\varphi_2 - \varphi_g)} \times (e^{\varphi_1} - 3e^{\varphi_2} - 3e^{2\varphi_2 + \varphi_g} + e^{\varphi_1 + 3\varphi_2 + \varphi_g}). \quad (65)$$

These equations can be analytically solved by introducing new variables

$$\tilde{\varphi}_1 = \varphi_1 - \varphi_2, \quad \tilde{\varphi}_2 = \varphi_1 + \varphi_2 \quad (66)$$

in terms of which the BPS equations become

$$\tilde{\varphi}'_1 = \frac{\lambda}{2\sqrt{2}} e^{\frac{1}{2}(\tilde{\varphi}_1 + \varphi_g)} (e^{\tilde{\varphi}_1} - 1), \quad (67)$$

$$\tilde{\varphi}'_2 = \frac{\lambda}{2\sqrt{2}} e^{\frac{1}{2}(\tilde{\varphi}_2 - \varphi_g)} (e^{\tilde{\varphi}_2} - 1), \quad (68)$$

$$\varphi'_g = \frac{\lambda}{4\sqrt{2}} e^{-\frac{\varphi_g}{2}} (3e^{\frac{\tilde{\varphi}_2}{2}} - e^{\frac{3}{2}\tilde{\varphi}_2} - 3e^{\frac{1}{2}\tilde{\varphi}_1 + \varphi_g} + e^{\frac{3}{2}\tilde{\varphi}_1 + \varphi_g}), \quad (69)$$

$$A' = -\frac{\lambda}{8\sqrt{2}} e^{-\frac{\varphi_g}{2}} (e^{\frac{3}{2}\tilde{\varphi}_1 + \varphi_g} + e^{\frac{3}{2}\tilde{\varphi}_2} - 3e^{\frac{\tilde{\varphi}_2}{2}} - 3e^{\frac{1}{2}\tilde{\varphi}_1 + \varphi_g}). \quad (70)$$

By combining all of these equations, we find that

$$\frac{dA}{d\tilde{\varphi}_1} - \frac{1}{2} \frac{d\varphi_g}{d\tilde{\varphi}_1} = \frac{3 - e^{\tilde{\varphi}_1}}{2(e^{\tilde{\varphi}_1} - 1)}, \quad (71)$$

$$\frac{dA}{d\tilde{\varphi}_2} + \frac{1}{2} \frac{d\varphi_g}{d\tilde{\varphi}_2} = \frac{3 - e^{\tilde{\varphi}_2}}{2(e^{\tilde{\varphi}_2} - 1)} \quad (72)$$

which can be solved by the following solution:

$$\varphi_g = \frac{3}{2}(\tilde{\varphi}_1 - \tilde{\varphi}_2) - \ln(1 - e^{\tilde{\varphi}_1}) + \ln(1 - e^{\tilde{\varphi}_2}), \quad (73)$$

$$A = \frac{\varphi_g}{2} - \frac{3}{2}\tilde{\varphi}_1 + \ln(1 - e^{\tilde{\varphi}_1}). \quad (74)$$

In this solution, we have fixed the integration constant for φ_g to zero since at the AdS_4 critical point $\varphi_g = \tilde{\varphi}_1 = \tilde{\varphi}_2 = 0$. The integration constant for A is irrelevant.

Combining Eqs. (67) and (68), we obtain after substituting for φ_g

$$\frac{d\tilde{\varphi}_1}{d\tilde{\varphi}_2} = e^{2(\tilde{\varphi}_1 - \tilde{\varphi}_2)} \quad (75)$$

whose solution is given by

$$\tilde{\varphi}_1 = -\frac{1}{2} \ln(e^{-2\tilde{\varphi}_2} - C_1). \quad (76)$$

Near the AdS_4 critical point, we have $\tilde{\varphi}_1 \sim \tilde{\varphi}_2 \sim 0$, which requires that $C_1 = 0$. This choice leads to $\tilde{\varphi}_2 = \tilde{\varphi}_1$, which implies $\varphi_2 = 0$ and $\varphi_g = 0$. We see that the flow does not involve φ_g and is driven purely by an irrelevant operator of dimension 4 dual to φ_1 . In this case, the $N = 4$ SCFT dual to the AdS_4 vacuum is expected to appear in the IR. Note also that equation (69) is consistent for $\varphi_g = 0$ if and only if $\tilde{\varphi}_2 = \tilde{\varphi}_1$ as being the case here.

Finally, we can solve Eq. (67) for $\tilde{\varphi}_1(r)$

$$\frac{\lambda r}{2\sqrt{2}} = 2e^{-\frac{\tilde{\varphi}_1}{2}} + \ln(1 - e^{-\frac{\tilde{\varphi}_1}{2}}) - \ln(1 + e^{-\frac{\tilde{\varphi}_1}{2}}) + C. \quad (77)$$

The solution is singular as $r \rightarrow \frac{2\sqrt{2}C}{\lambda}$. Near this singularity, the solution becomes

$$\begin{aligned}\tilde{\varphi}_1 &\sim \tilde{\varphi}_2 \sim -\frac{2}{3} \ln \frac{3}{2} \left[C - \frac{\lambda r}{2\sqrt{2}} \right], \\ A &\sim -\frac{1}{2} \tilde{\varphi}_1 \sim \frac{1}{3} \ln \frac{3}{2} \left[C - \frac{\lambda r}{2\sqrt{2}} \right].\end{aligned}\quad (78)$$

This singularity leads to $V \rightarrow \infty$, so the solution is unphysical.

We end the discussion of this truncation by giving some comments on possible subtruncations. From Eqs. (67) and (68), we easily see that setting $\tilde{\varphi}_1 = 0$ or $\tilde{\varphi}_2 = 0$ is a consistent truncation. This is equivalent to setting $\varphi_2 = \pm\varphi_1$. In this case, the solution is found to be

$$\begin{aligned}\varphi_g &= \pm \ln \left[\frac{e^{-\varphi_1} - C_1 e^{3\varphi_1}}{2 - 2e^{2\varphi_1}} \right], \\ A &= -\frac{7}{2} \varphi_1 + \frac{1}{2} \ln(1 - e^{2\varphi_1}) + \frac{1}{2} \ln(1 - C_1 e^{4\varphi_1}), \\ \frac{\lambda \rho}{4\sqrt{2}} &= e^{-\varphi_1} + \frac{1}{2} \ln(1 - e^{-\varphi_1}) - \frac{1}{2} \ln(1 + e^{-\varphi_1}) + C\end{aligned}\quad (79)$$

where the new radial coordinate ρ is defined by $d\rho = e^{-\frac{\varphi_g}{2}} dr$.

We see that in this case φ_g is non-trivial along the flow. In order to make the solution approach the AdS₄ critical point with $\varphi_g \sim \varphi_1 \sim 0$, we need to choose $C_1 = 1$. This gives

$$\varphi_g = \pm \ln \cosh \varphi_1. \quad (80)$$

The solution is singular for $\rho \rightarrow \frac{4\sqrt{2}C}{3\lambda}$ with $\varphi_1 \rightarrow \infty$. In this limit, we find

$$\begin{aligned}\varphi_g &\sim \pm \varphi_1 \quad \varphi_1 \sim -\ln \left[C - \frac{3\lambda \rho}{4\sqrt{2}} \right], \\ A &\sim \frac{1}{2} \ln \left[C - \frac{3\lambda \rho}{4\sqrt{2}} \right].\end{aligned}\quad (81)$$

Both of these singularities lead to $V \rightarrow \infty$, so they are also unphysical.

3.2 RG flows with $N = 1$ supersymmetry

We now consider a class of RG flow solutions preserving $N = 1$ supersymmetry and breaking the $SO(3) \times SO(3)$ to its diagonal subgroup. This is achieved by turning on the marginal deformations corresponding to χ_1 and χ_2 to the solutions. As in the $N = 4$ case, there is a consistent subtruncation to only irrelevant and marginal deformations with only φ_1 and χ_1 non-vanishing. We will consider this case first and then look for the most general solutions with all six $SO(3)$ singlet scalars non-vanishing. It should be noted that

the truncation with only φ_2 and χ_2 non-vanishing is not consistent. This is also an interesting feature to look for in the dual field theory.

3.2.1 $N = 1$ RG flows by marginal and irrelevant deformations

By setting $\varphi_g = \chi_g = \varphi_2 = \chi_2 = 0$ in the BPS equations, we obtain

$$\varphi'_1 = -\frac{\lambda}{2\sqrt{2}} e^{\frac{\varphi_1}{2}} \frac{(3 - 4e^{\varphi_1} + e^{2\varphi_1} + 9\chi_1^2 e^{2\varphi_1})}{\sqrt{(e^{\varphi_1} - 3)^2 + 9\chi_1^2 e^{2\varphi_1}}}, \quad (82)$$

$$\chi'_1 = -\frac{3\lambda}{2\sqrt{2}} \frac{\chi_1 e^{\frac{\varphi_1}{2}}}{\sqrt{(e^{\varphi_1} - 3)^2 + 9\chi_1^2 e^{2\varphi_1}}}, \quad (83)$$

$$A' = \frac{\lambda}{4\sqrt{2}} e^{\frac{\varphi_1}{2}} \sqrt{(e^{\varphi_1} - 3)^2 + 9\chi_1^2 e^{2\varphi_1}}. \quad (84)$$

We are not able to analytically solve these equations in full generality, so we will look for numerical solutions in this case.

Note that further truncation to only φ_1 gives rise to the following BPS equations:

$$\varphi'_1 = \frac{\lambda}{2\sqrt{2}} e^{\frac{\varphi_1}{2}} (e^{\varphi_1} - 1) \quad \text{and} \quad A' = -\frac{\lambda}{4\sqrt{2}} e^{\frac{\varphi_1}{2}} (e^{\varphi_1} - 3), \quad (85)$$

with the solution

$$\begin{aligned}A &= -\frac{3}{2} \varphi_1 + \ln(1 - e^{\varphi_1}), \\ \frac{\lambda \rho}{2\sqrt{2}} &= 2e^{-\frac{\varphi_1}{2}} + \ln \left(1 - e^{-\frac{\varphi_1}{2}} \right) - \ln \left(1 + e^{-\frac{\varphi_1}{2}} \right).\end{aligned}\quad (86)$$

This is nothing but the solution of the previous section for $\tilde{\varphi}_2 = \tilde{\varphi}_1$. Therefore, we will not further discuss this solution.

For non-vanishing χ_1 , we need to find the solutions numerically. An example of these solutions is given in Fig. 1.

The asymptotic behavior of this solution can be determined from the BPS equations at large φ_1 as follows:

$$\begin{aligned}\chi_1 &\sim \chi_0, \quad \varphi_1 \sim -\frac{2}{3} \ln \left(r\lambda\sqrt{2 + 18\chi_0^2} - 4C_1 \right), \\ A &\sim \frac{1}{3} \ln \left(r\lambda\sqrt{2 + 18\chi_0^2} - 4C_1 \right)\end{aligned}\quad (87)$$

where χ_0 is a constant. This singularity leads to $V \rightarrow \infty$, implying that it is unphysical. We have in addition checked this by a numerical analysis which consistently shows a diverging scalar potential near the singularity.

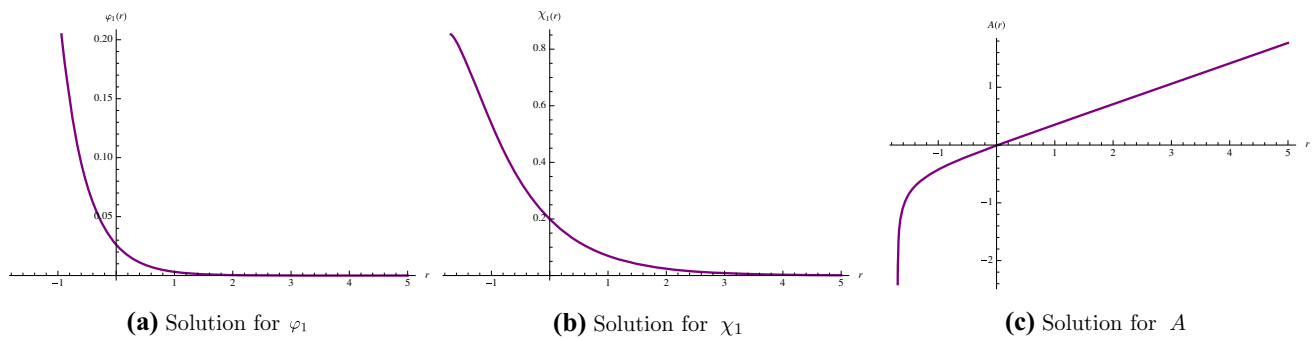


Fig. 1 An $N = 1$ RG flow with marginal and irrelevant deformations from type IIB compactification

3.2.2 $N = 1$ RG flows by relevant, marginal and irrelevant deformations

After considering various consistent truncations, we end this section by considering $N = 1$ RG flow solutions with all six $SO(3)$ singlet scalars turned on. The resulting RG flows will be driven by all types of possible deformations namely marginal, irrelevant and relevant. In this case, we need to use a numerical analysis due to the complexity of the full set of BPS equations given in the Appendix. Similar to the analysis of [9], there could be many possible IR singularities due to the competition between various deformations both by operators and vacuum expectation values (vev) present in the UV $N = 4$ SCFT. Some examples of these solutions are given in Fig. 2. In the figure, we have given solutions for three different values of the flux parameter λ for comparison.

From Fig. 2, we see a singularity in the IR end of the flow while near $r \rightarrow \infty$ the flow approaches the UV $N = 4$ AdS₄. The numerical analysis shows that the singularity is of a bad type according to the criterion of [34] since it leads to $V \rightarrow \infty$.

4 Supersymmetric Janus solutions

In this section, we look at another type of solutions with an AdS₃-sliced domain wall ansatz, obtained by replacing the flat metric $dx_{1,2}^2$ by an AdS₃ metric of radius ℓ ,

$$ds^2 = e^{2A(r)} \left(e^{\frac{2\xi}{\ell}} dx_{1,1}^2 + d\xi^2 \right) + dr^2. \quad (88)$$

The solution, called Janus solution, describes a conformal interface of co-dimension one within the SCFT dual to the AdS₄ critical point. This solution breaks the three-dimensional conformal symmetry $SO(2, 3)$ to that on the $(1 + 1)$ -dimensional interface $SO(2, 2)$.

In this case, the resulting BPS equations will get modified compared to the RG flow case. First of all, the analysis of $\delta\psi_\mu^i = 0$ equations requires an additional $\gamma_{\hat{\xi}}$ projection

$$\gamma_{\hat{\xi}} \epsilon_i = i\kappa e^{i\Lambda} \epsilon^i \quad (89)$$

while the $\gamma_{\hat{r}}$ projector in $\delta\chi^i = 0$ and $\delta\lambda_a^i = 0$ equations is still given by Eq. (39) but with the phase $e^{i\Lambda}$ modified to

$$e^{i\Lambda} = \frac{\mathcal{W}}{A' + \frac{i\kappa}{\ell} e^{-A}}. \quad (90)$$

Furthermore, the integrability conditions for $\delta\psi_{0,1}^i = 0$ equations lead to

$$A'^2 + \frac{1}{\ell^2} e^{-2A} = W^2. \quad (91)$$

As expected, these two equations reduce to $A' = \pm W$ and $e^{i\Lambda} = \frac{\mathcal{W}}{A'} = \pm \frac{\mathcal{W}}{W}$ in the limit $\ell \rightarrow \infty$.

The constant κ , with $\kappa^2 = 1$, imposes the chirality condition on the Killing spinors corresponding to the unbroken supersymmetry on the $(1 + 1)$ -dimensional interface. The detailed analysis of these equations can be found for example in [27]. Unlike the RG flow case, the Killing spinors depend on both r and ξ coordinates; see for more details [26].

We have seen that the analysis of RG flow solutions with all six $SO(3)$ singlet scalars turned on involves a very complicated set of BPS equations. Since the BPS equations for supersymmetric Janus solutions are usually more complicated than those of the RG flows, we will not perform the full analysis with all $SO(3)$ singlet scalars but truncate the BPS equations to two consistent truncations, with (φ_g, χ_g) and (φ_1, χ_1) non-vanishing. As in other cases studied in [14, 26, 27], truncations to only dilatons or scalars without the axions or pseudoscalars are not consistent with the Janus BPS equations, or equivalently Janus solutions require non-trivial pseudoscalars.

4.1 $N = 4$ Janus solution

We first consider the Janus solution with only the dilaton and axion in the gravity multiplet non-vanishing. In this case, the BPS conditions $\delta\lambda_a^i = 0$ are automatically satisfied by

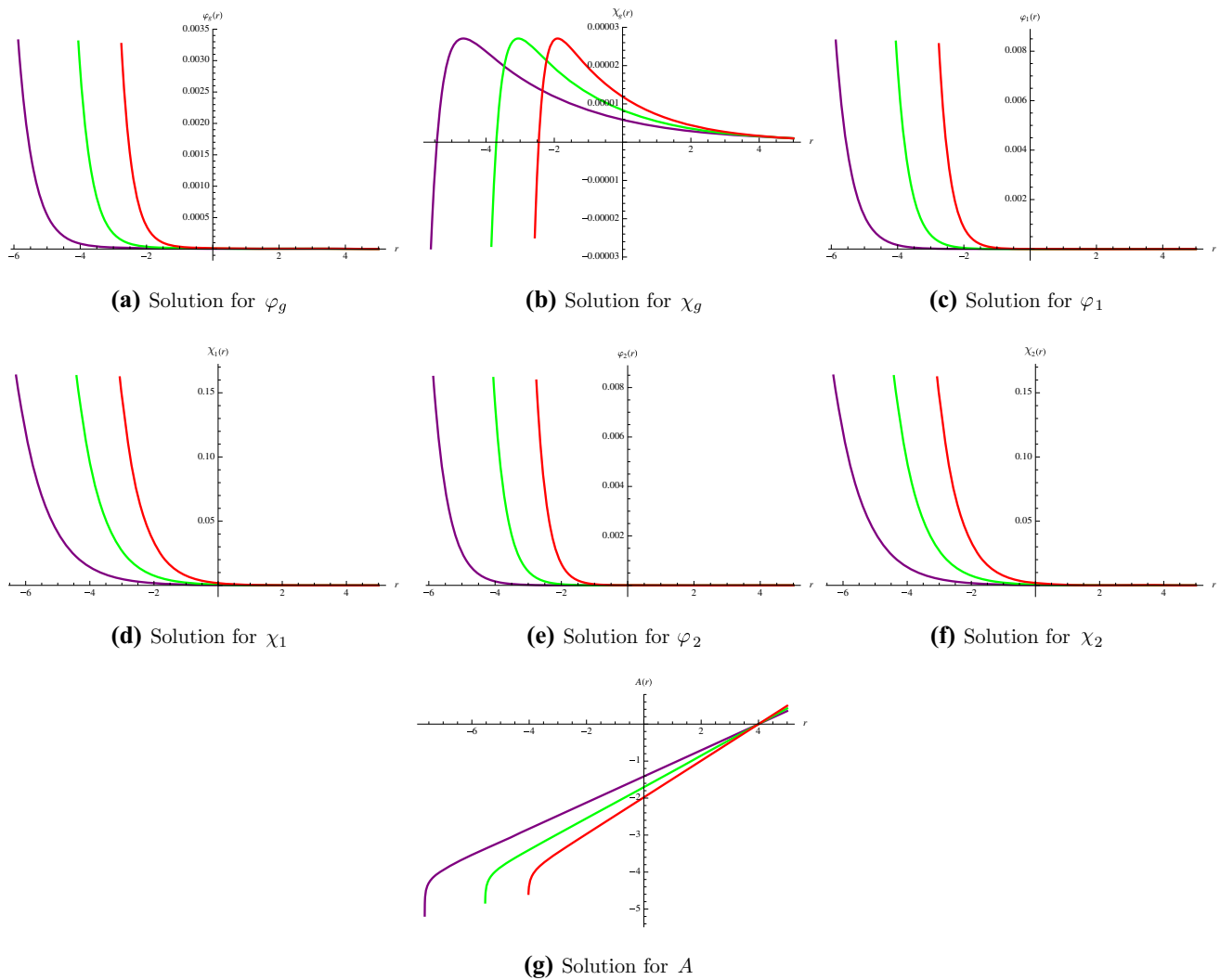


Fig. 2 $N = 1$ RG flow solutions from type IIB compactification with all $SO(3)$ singlet scalars and $\lambda = 1$ (purple), $\lambda = 1.2$ (green) and $\lambda = 1.4$ (red)

setting $\varphi_{1,2} = \chi_{1,2} = 0$. By using the phase (90) in $\delta\chi^i = 0$ equations and separating real and imaginary parts, we obtain the following BPS equations:

$$\begin{aligned} \varphi'_g &= -4 \frac{A'}{W} \frac{\partial W}{\partial \varphi_g} - 4\kappa e^{-\varphi_g} \frac{e^{-A}}{\ell W} \frac{\partial W}{\partial \chi_g} \\ &= \frac{-2\ell A' (e^{2\varphi_g} - 1 + 2\chi_g^2 e^{2\varphi_g}) - 4\kappa e^{\varphi_g - A} \chi_g}{\ell [(1 + e^{\varphi_g})^2 + \chi_g^2 e^{2\varphi_g}]}, \end{aligned} \quad (92)$$

$$\begin{aligned} \chi'_g &= -4 \frac{A'}{W} e^{-2\varphi_g} \frac{\partial W}{\partial \chi_g} + 4\kappa e^{-\varphi_g} \frac{e^{-A}}{\ell W} \frac{\partial W}{\partial \varphi_g} \\ &= \frac{2\kappa e^{-A - \varphi_g} (e^{2\varphi_g} - 1 + \chi_g^2 e^{2\varphi_g}) - 4\ell \chi_g A'}{\ell [(1 + e^{\varphi_g})^2 + \chi_g^2 e^{2\varphi_g}]}, \end{aligned} \quad (93)$$

$$0 = A'^2 + \frac{e^{-2A}}{\ell^2} - \frac{\lambda^2}{32} e^{-\varphi_g} [(1 + e^{\varphi_g})^2 + \chi_g^2 e^{2\varphi_g}] \quad (94)$$

where we have also included the gravitini equations from (91). In terms of the superpotential

$$W = \frac{\lambda}{4\sqrt{2}} e^{-\frac{\varphi_g}{2}} \sqrt{(1 + e^{\varphi_g})^2 + \chi_g^2 e^{2\varphi_g}}, \quad (95)$$

these equations take a similar form as in the other four-dimensional Janus solutions in [14, 26, 27]. These equations solve all the BPS conditions for any ϵ^i , $i = 1, 2, 3, 4$. Therefore, any solutions to these equations will preserve $N = 4$ supersymmetry. We solve these equations numerically with an example of the solutions given in Fig. 3.

From Fig. 3, we see that the solution interpolates between $N = 4$ AdS₄ vacua at both $r \rightarrow \pm\infty$. This solution is then interpreted as a (1+1)-dimensional conformal interface within the $N = 4$ SCFT. The interface preserves $N = (4, 0)$ supersymmetry on the interface due to the sign choice $\kappa = 1$,

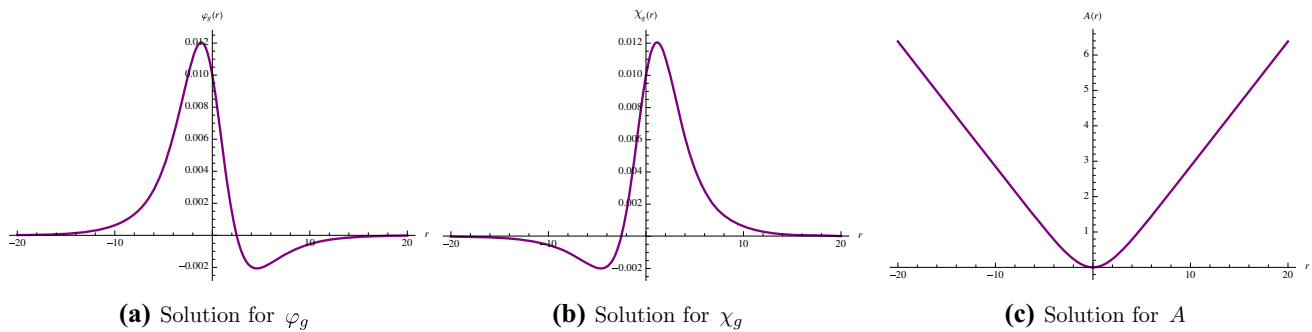


Fig. 3 $N = 4$ Janus solution from type IIB compactification within a truncation to φ_g and χ_g with $\kappa = \lambda = 1$ and $\ell = 2\sqrt{2}$

and $SO(3) \times SO(3)$ symmetry remains unbroken throughout the solution.

4.2 $N = 1$ Janus solution

The truncation keeping only φ_1 and χ_1 is still consistent in the case of Janus BPS equations. In contrast to the previous truncation, any solutions to these equations will break $N = 4$ supersymmetry to $N = 1$ and preserve only $SO(3)$ diagonal subgroup of the full $SO(3) \times SO(3)$ symmetry of the $N = 4$ AdS₄ vacuum.

The real superpotential for this truncation is given by

$$W = \frac{\lambda}{4\sqrt{2}} e^{\frac{\varphi_1}{2}} \sqrt{(e^{\varphi_1} - 3)^2 + 9\chi_1^2 e^{2\varphi_1}} \quad (96)$$

in terms of which the BPS equations can be written as

$$\begin{aligned} \varphi_1' &= -\frac{4}{3} \frac{A'}{W} \frac{\partial W}{\partial \varphi_1} - \frac{4}{3} \kappa e^{-\varphi_1} \frac{e^{-A}}{\ell W} \frac{\partial W}{\partial \chi_1} \\ &= \frac{2\ell A' (4e^{2\varphi_1} - 3 - 9\chi_1^2 e^{2\varphi_1} - e^{2\varphi_1}) - 12\kappa e^{\varphi_1 - A} \chi_1}{\ell [(e^{\varphi_1} - 3)^2 + 9\chi_1^2 e^{2\varphi_1}]}, \end{aligned} \quad (97)$$

$$\begin{aligned} \chi_1' &= -\frac{4}{3} \frac{A'}{W} e^{-2\varphi_1} \frac{\partial W}{\partial \chi_1} + \frac{4}{3} \kappa e^{-\varphi_1} \frac{e^{-A}}{\ell W} \frac{\partial W}{\partial \varphi_1} \\ &= \frac{2\kappa e^{-A - \varphi_1} (3 - 4e^{\varphi_1} + e^{2\varphi_1} + 9\chi_1^2 e^{2\varphi_1}) - 12\ell \chi_1 A'}{\ell [(e^{\varphi_1} - 3)^2 + 9\chi_1^2 e^{2\varphi_1}]}, \end{aligned} \quad (98)$$

$$0 = A'^2 + \frac{e^{-2A}}{\ell^2} - \frac{\lambda^2}{32} e^{\varphi_1} [(e^{\varphi_1} - 3)^2 + 9\chi_1^2 e^{2\varphi_1}]. \quad (99)$$

Unlike the previous case, an intensive numerical search has not found any solutions interpolating between AdS₄ vacua in the limits $r \rightarrow \pm\infty$. All of the solutions found here are singular Janus in the sense that they connect singular domain walls at two finite values of the radial coordinate. We give an example of these solutions in Fig. 4.

This solution could be interpreted as a conformal interface between two $N = 1$ non-conformal phases of the dual $N = 4$ SCFT. However, the singularities are of the bad type.

An uplift to type IIB theory would be needed in order to decide whether the solution is physically acceptable in the ten-dimensional context.

5 RG flows from type IIA geometric compactification

We now carry out a similar analysis for a geometric compactification of type IIA theory. The procedure is essentially the same, so we will omit unnecessary details. In this case, the compactification only involves gauge (H_3, F_0, F_2, F_4, F_6) and geometric (ω) fluxes. However, the fluxes are more complicated and lead to many components of the embedding tensor compared to the type IIB case

$$H_{ijk} \sim f_{-\bar{a}\bar{b}\bar{c}} = \Lambda_{-333} = \frac{\sqrt{6}}{3} \lambda,$$

$$F_{aibjck} \sim f_{+\bar{a}\bar{b}\bar{c}} = \Lambda_{+333} = -\frac{3\sqrt{10}}{2} \lambda,$$

$$F_{aibj} \sim f_{+\bar{a}\bar{b}\bar{k}} = \Lambda_{+334} = \frac{\sqrt{6}}{2} \lambda,$$

$$F_{ai} \sim f_{+\bar{a}\bar{j}\bar{k}} = \Lambda_{+344} = \frac{\sqrt{10}}{6} \lambda,$$

$$F_0 \sim f_{+\bar{i}\bar{j}\bar{k}} = \Lambda_{+444} = \frac{5\sqrt{6}}{6} \lambda,$$

$$H_{abk} \sim f_{+\bar{a}\bar{b}\bar{k}} = \Lambda_{+233} = \frac{\sqrt{6}}{3} \lambda,$$

$$\omega_{ij}{}^c \sim f_{-\bar{a}\bar{b}\bar{k}} = \Lambda_{-334} = \frac{\sqrt{10}}{3} \lambda,$$

$$\omega_{ka}{}^j = \omega_{bk}{}^i = \omega_{bc}{}^a \sim f_{+\bar{a}\bar{j}\bar{k}} = f_{+\bar{i}\bar{b}\bar{k}}$$

$$= f_{+\bar{a}\bar{b}\bar{c}} = \Lambda_{+234} = \Lambda_{+133} = \sqrt{10} \lambda. \quad (100)$$

In the above equations, we have also given the form field corresponding to each flux component.

The resulting gauged $N = 4$ supergravity has a non-semisimple group $ISO(3) \ltimes U(1)^6$ and admits the minimal $N = 1$ AdS₄ vacuum at which the gauge group is broken down to $SO(3)$ compact subgroup. The corresponding superpotential for the unbroken $N = 1$ supersymmetry is given by

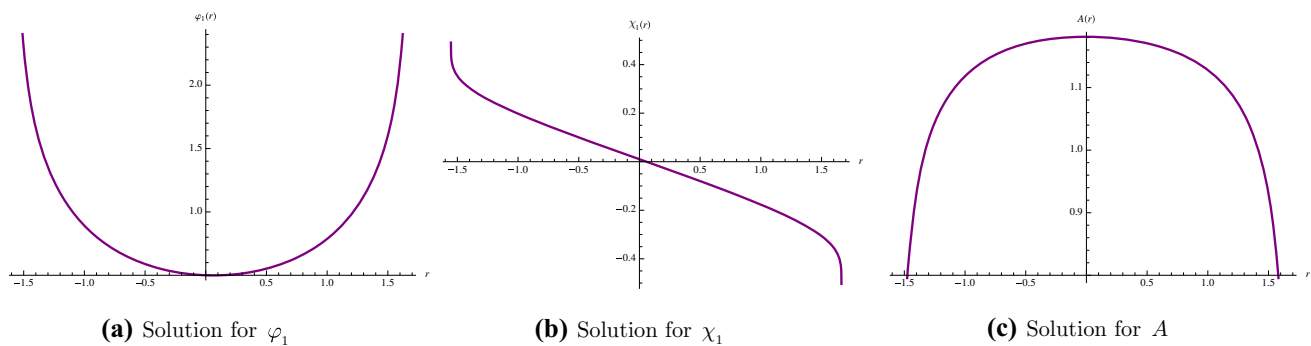


Fig. 4 $N = 1$ Janus solution from type IIB compactification within a truncation to φ_1 and χ_1 with $\kappa = \lambda = \ell = 1$

$$\begin{aligned} \mathcal{W} = & \frac{\lambda}{24} e^{\frac{1}{2}(\varphi_1 - 3\varphi_2 + \varphi_g)} [2e^{\varphi_1 + 2\varphi_2 - \varphi_g} \\ & \times [3\sqrt{5}i + e^{2\varphi_2}(\sqrt{3} + 3\sqrt{5}\chi_2)](i + \chi_g e^{\varphi_g}) \\ & - 5\sqrt{3}e^{\varphi_1}(i + e^{\varphi_2}\chi_2)^3 - 3\sqrt{5}e^{\varphi_1 + \varphi_2}(i + e^{\varphi_2}\chi_2)^2 \\ & - 9\sqrt{3}ie^{\varphi_1 + 2\varphi_2} \\ & + 18\sqrt{5}e^{2\varphi_2}(i + e^{\varphi_1}\chi_1)(i + e^{\varphi_2}\chi_2) \\ & + 6\sqrt{3}ie^{3\varphi_2} + 9\sqrt{5}e^{\varphi_1 + 3\varphi_2} \\ & + 6\sqrt{3}e^{\varphi_1 + 3\varphi_2}\chi_1 - 9\sqrt{3}\chi_2e^{\varphi_1 + 3\varphi_2}]. \end{aligned} \quad (101)$$

The scalar potential can be written in terms of $W = |\mathcal{W}|$ as

$$V = -\frac{1}{2}K^{ij}\frac{\partial W}{\partial\phi^i}\frac{\partial W}{\partial\phi^j} - \frac{3}{4}W^2. \quad (102)$$

Its explicit form is given in the appendix.

When all scalars vanish, there is an $N = 1$ AdS₄ vacuum with the cosmological constant

$$V_0 = -\lambda^2. \quad (103)$$

The six scalars have squared masses as follows:

$$m^2L^2 : 0, -2, 4 \pm \sqrt{6}, \frac{1}{3}(47 \pm \sqrt{159}). \quad (104)$$

All of these values are in agreement with [20] after changing to our convention including a factor of 3.

As in the type IIB case, the BPS equations obtained from supersymmetry variations can be written as

$$A' = W, \quad \varphi^{i'} = K^{ij}\frac{\partial W}{\partial\phi^j}. \quad (105)$$

However, the resulting equations are much more complicated than those from type IIB compactification. We will then not give them in this paper. Furthermore, we have not found any consistent subtruncation within this set of equations. In the following, we will only give examples of holographic RG flows from the $N = 1$ SCFT dual to the above AdS₄ critical point to non-conformal $N = 1$ field theories in the IR. These

numerical solutions are shown in Fig. 5 with three different values of the flux parameter λ as in the IIB case.

As in the IIB case, we have numerically analyzed the scalar potential near the singularity and found that it leads to $V \rightarrow \infty$, which implies the singularity is unphysical.

6 Conclusions and discussions

We have found many supersymmetric RG flows and examples of Janus solutions in $N = 4$ gauged supergravities obtained from flux compactifications of type II string theories. These solutions describe supersymmetric deformations and conformal interfaces within the dual $N = 4$ and $N = 1$ SCFTs in three dimensions. Many of the flow solutions have been obtained analytically which should be useful for further investigation.

In type IIB non-geometric compactification, the gauged supergravity has $ISO(3) \times ISO(3)$ gauge group and admits an $N = 4$ AdS₄ vacuum dual to an $N = 4$ SCFT with global symmetry $SO(3) \times SO(3)$. We have found two classes of supersymmetric RG flows. The first one preserves $N = 4$ supersymmetry, and the global $SO(3) \times SO(3)$ symmetry is unbroken. This type of solutions can be obtained by turning on only the dilaton and axion in the gravity multiplet dual to relevant operators of dimensions $\Delta = 1, 2$. In this case, the flows are accordingly driven by relevant operators. Another possibility for preserving $N = 4$ supersymmetry is to truncate out all axions or pseudoscalars. The resulting RG flows are driven by relevant and irrelevant operators of dimensions $\Delta = 1, 2$ and $\Delta = 4$, respectively. When the axions in the vector multiplets, corresponding to marginal deformations, are turned on, the flows break $N = 4$ supersymmetry to $N = 1$ and break $SO(3) \times SO(3)$ symmetry to their $SO(3)$ diagonal subgroup. We have given numerically the flows driven by marginal and irrelevant operators and the most general deformations in the presence of all types of deformations, relevant, marginal and irrelevant. It has been pointed out in [20] that the vacuum structure of type IIB com-

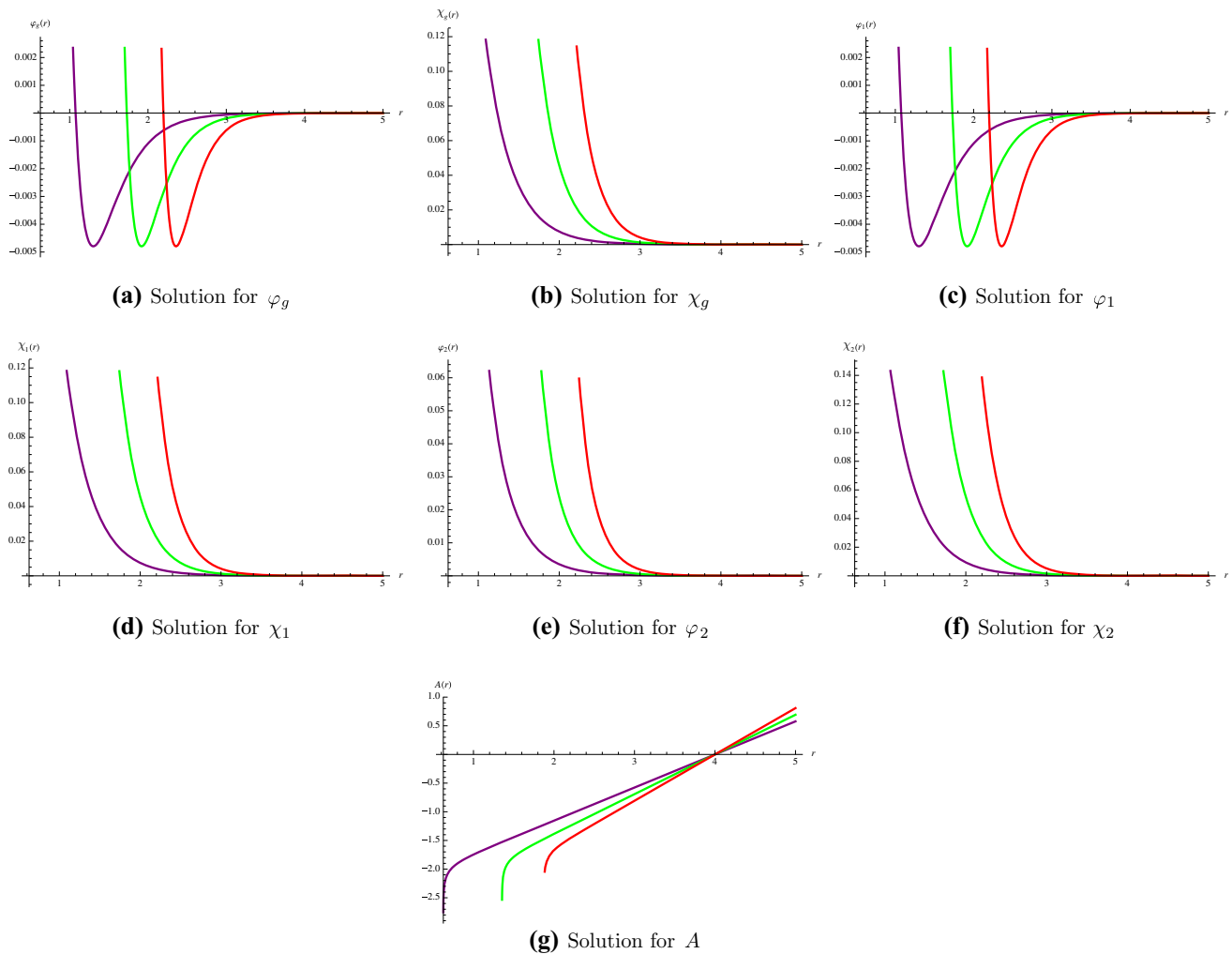


Fig. 5 $N = 1$ RG flow solutions from type IIA compactification with $\lambda = 1$ (purple), $\lambda = 1.2$ (green) and $\lambda = 1.4$ (red)

pactification is very rich. The solutions found in this paper show that the number of supersymmetric deformations of these vacua is also enormous.

Within this type IIB compactification, we have also given Janus solutions preserving $N = 4$ and $N = 1$ supersymmetry. These correspond to $(1 + 1)$ -dimensional conformal interfaces preserving $SO(3) \times SO(3)$ and $SO(3)$ symmetry. For the $N = 4$ solution, we have given a numerical solution interpolating between AdS_4 vacua on the two sides of the interface, called regular Janus. This solution gives a holographic dual of a conformal interface in $N = 4$ SCFT. For the $N = 1$ case, we have not found this type of solutions but the singular Janus, interpolating between $N = 1$ non-conformal phases of the dual $N = 4$ SCFT. The situation is very similar to the $N = 1$ Janus solutions studied in [14]. It would be interesting to have a definite conclusion about the existence of regular Janus solutions in these two $N = 4$ gauged supergravities.

In this non-geometric compactification, it is useful to give some comments about the holographic interpretation of the results. Due to its non-geometric nature, the stringy origin of the $N = 4$ gauged supergravity is presently not well understood. This makes the meaning of the resulting solutions in terms of RG flows in the dual SCFT unclear. However, working in four-dimensional gauged supergravity has an obvious advantage in the sense that the whole formulation of $N = 4$ gauged supergravity is virtually unchanged for all gaugings from both geometric and non-geometric compactifications. Therefore, the approach used here can be carried out for all other gaugings regardless of their higher-dimensional origins. On the other hand, the full interpretation of the results in higher-dimensional contexts calls for further study. Hopefully, the results presented here could be useful along this line of investigations.

We have also carried out the same analysis in a geometric compactification of type IIA theory resulting in $N = 4$ gauged supergravity with $ISO(3) \ltimes U(1)^6$. The gauged

supergravity admits an $N = 1$ AdS₄ vacuum dual to an $N = 1$ SCFT in three dimensions. Due to the lack of further consistent subtruncation, we have numerically found examples of holographic RG flows to $N = 1$ non-conformal field theories. Similar to the solutions in the type IIB case, the flows are driven by relevant, marginal and irrelevant operators in a more complicated manner. It should be pointed out that the massless scalars dual to marginal deformations considered in this paper are not the Goldstone bosons corresponding to the symmetry breaking $ISO(3) \times ISO(3) \rightarrow SO(3) \times SO(3)$ and $ISO(3) \times U(1)^6 \rightarrow SO(3)$. The Goldstone bosons transform non-trivially under the residual symmetry groups $SO(3) \times SO(3)$ and $SO(3)$ while the massless scalars considered in the solutions are singlets. Therefore, they are truly marginal deformations in the $N = 1$ and $N = 4$ SCFTs. Note also that, in the type IIB case, these marginal deformations break $N = 4$ supersymmetry in consistent with the fact that all maximally supersymmetric AdS₄ vacua of $N = 4$ gauged supergravity have no moduli preserving $N = 4$ supersymmetry [35].

There are many possibilities for further investigations. First of all, it would be interesting to identify the $N = 1$ and $N = 4$ SCFTs dual to the $N = 1$ and $N = 4$ AdS₄ vacua. This should allow us to identify the dual operators driving the RG flows obtained holographically in this paper. It could be interesting to look for more general Janus solutions in type IIB compactification with more scalars turned on and also look for similar solutions in type IIA compactification. Another direction would be to uplift the solutions found here to ten dimensions. This could be used to identify the g_{00} component of the ten-dimensional metric and checked whether the unphysical singularities by the criterion of [34] are physical by the criterion of [36]. Finally, it would be of particular interest to further explore type IIB compactification with more general fluxes than those considered in [20]. This could enlarge the solution space of both AdS₄ vacua and their deformations including possible flow solutions between two AdS₄ vacua. We leave these issues for future work.

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Appendix A: Useful formulas

In this appendix, we collect all of the conventions about 't Hooft symbols, the scalar potential coming from type IIA

geometric compactification and complicated BPS equations arising from type IIB non-geometric compactification with all $SO(3)$ singlet scalars non-vanishing.

A.1: 't Hooft symbols

To convert an $SO(6)$ vector index m to a pair of anti-symmetric $SU(4)$ indices $[ij]$, we use the following 't Hooft symbols:

$$\begin{aligned} G_1^{ij} &= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, & G_2^{ij} &= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \\ G_3^{ij} &= \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, & G_4^{ij} &= \begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix}, \\ G_5^{ij} &= \begin{bmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix}, & G_6^{ij} &= \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (106)$$

These matrices satisfy the relation

$$G_{mij} = -\frac{1}{2}\epsilon_{ijkl}G_m^{kl} = -(G_m^{ij})^*. \quad (107)$$

A.2: BPS equations for type IIB compactification

In this section, we give the full BPS equations for the non-geometric compactification of type IIB theory. These equations are given by

$$\begin{aligned} \varphi'_g &= -\frac{1}{32W}e^{\varphi_1-3\varphi_2-\varphi_g}\lambda^2[-e^{2\varphi_1}-9e^{2\varphi_2} \\ &\quad + 6e^{\varphi_1+\varphi_2} + e^{2(\varphi_1+3\varphi_2+\varphi_g)} \\ &\quad + 9e^{4\varphi_2+2\varphi_g} - 6e^{\varphi_1+5\varphi_2+2\varphi_g} \\ &\quad + 6e^{3\varphi_2+2\varphi_g}(2e^{\varphi_1}-3e^{\varphi_2}-e^{2\varphi_1+\varphi_2} \\ &\quad + 2e^{\varphi_1+2\varphi_2})\chi_2\chi_g + 2e^{5\varphi_2+2\varphi_g} \\ &\quad \times (6e^{\varphi_1}-9e^{\varphi_2}+e^{2\varphi_1+\varphi_2})\chi_2^3\chi_g + e^{2(\varphi_1+\varphi_g)}\chi_g^2 \\ &\quad + 9e^{2(\varphi_2+\varphi_g)}\chi_g^2 - 6e^{\varphi_1+\varphi_2+2\varphi_g}\chi_g^2 \\ &\quad + 3e^{4\varphi_2}(e^{2\varphi_1}+3e^{2\varphi_2}-2e^{\varphi_1+\varphi_2})\chi_2^4(-1 \\ &\quad + e^{2\varphi_g}\chi_g^2) + e^{2\varphi_1+6\varphi_2}\chi_2^6(-1+e^{2\varphi_g}\chi_g^2) \\ &\quad + 3e^{2\varphi_2}\chi_2^2(-e^{2\varphi_1}-6e^{2\varphi_2}+4e^{\varphi_1+\varphi_2} \\ &\quad + 3e^{4\varphi_2+2\varphi_g}+e^{2\varphi_g}(e^{2\varphi_1}+6e^{2\varphi_2} \\ &\quad - 4e^{\varphi_1+\varphi_2})\chi_g^2) + 9e^{2(\varphi_1+\varphi_2)}\chi_1^2(1 \\ &\quad + e^{2\varphi_2}\chi_2^2)(-1+e^{2(\varphi_2+\varphi_g)}-2e^{2(\varphi_2+\varphi_g)}\chi_2\chi_g \end{aligned}$$

$$\begin{aligned}
& + e^{2\varphi_g} \chi_g^2 + e^{2\varphi_2} \chi_2^2 (-1 + e^{2\varphi_g} \chi_g^2)) \\
& - 6e^{2(\varphi_1+\varphi_2)} \chi_1 (-e^{2\varphi_g} (-1 + e^{2\varphi_2}) \chi_g \\
& + e^{4\varphi_2+2\varphi_g} \chi_2^2 \chi_g - e^{4\varphi_2+2\varphi_g} \chi_2^4 \chi_g \\
& - \chi_2 (1 + e^{4\varphi_2+2\varphi_g} - e^{2\varphi_g} \chi_g^2) \\
& + 2e^{2\varphi_2} \chi_2^3 (-1 + e^{2\varphi_g} \chi_g^2) + e^{4\varphi_2} \chi_2^5 (-1 + e^{2\varphi_g} \chi_g^2) \big], \\
\end{aligned} \tag{108}$$

$$\begin{aligned}
\chi'_g = & -\frac{1}{16W} e^{\varphi_1-3\varphi_2-\varphi_g} \lambda^2 [3e^{3\varphi_2} (2e^{\varphi_1} - 3e^{\varphi_2} \\
& - e^{2\varphi_1+\varphi_2} + 2e^{\varphi_1+2\varphi_2}) \chi_2 \\
& + e^{5\varphi_2} (6e^{\varphi_1} - 9e^{\varphi_2} + e^{2\varphi_1+\varphi_2}) \chi_2^3 \\
& + (e^{\varphi_1} - 3e^{\varphi_2})^2 \chi_g + 3e^{2\varphi_2} (e^{2\varphi_1} + 6e^{2\varphi_2} \\
& - 4e^{\varphi_1+\varphi_2}) \chi_2^2 \chi_g + 3e^{4\varphi_2} (e^{2\varphi_1} + 3e^{2\varphi_2} \\
& - 2e^{\varphi_1+\varphi_2}) \chi_2^4 \chi_g + e^{2\varphi_1+6\varphi_2} \chi_2^6 \chi_g \\
& + 9e^{2(\varphi_1+\varphi_2)} \chi_1^2 (1 + e^{2\varphi_2} \chi_2^2) (-e^{2\varphi_2} \chi_2 + \chi_g \\
& + e^{2\varphi_2} \chi_2^2 \chi_g) - 3e^{2(\varphi_1+\varphi_2)} \chi_1 (1 \\
& - e^{2\varphi_2} + e^{4\varphi_2} \chi_2^2 - e^{4\varphi_2} \chi_2^4 + 2\chi_2 \chi_g \\
& + 4e^{2\varphi_2} \chi_2^3 \chi_g + 2e^{4\varphi_2} \chi_2^5 \chi_g) \big], \\
\end{aligned} \tag{109}$$

$$\begin{aligned}
\varphi'_1 = & -\frac{1}{32W} e^{\varphi_1-3\varphi_2-\varphi_g} \lambda^2 [e^{2\varphi_1} + 3e^{2\varphi_2} \\
& - 4e^{\varphi_1+\varphi_2} - 4e^{\varphi_1+2\varphi_2+\varphi_g} + 6e^{3\varphi_2+\varphi_g} \\
& + e^{2(\varphi_1+3\varphi_2+\varphi_g)} + 2e^{2\varphi_1+3\varphi_2+\varphi_g} \\
& - 4e^{\varphi_1+4\varphi_2+\varphi_g} + 3e^{4\varphi_2+2\varphi_g} - 4e^{\varphi_1+5\varphi_2+2\varphi_g} \\
& + 2e^{3\varphi_2+2\varphi_g} (4e^{\varphi_1} - 3e^{\varphi_2} - 3e^{2\varphi_1+\varphi_2} \\
& + 4e^{\varphi_1+2\varphi_2}) \chi_2 \chi_g + 2e^{5\varphi_2+2\varphi_g} (4e^{\varphi_1} \\
& - 3e^{\varphi_2} + e^{2\varphi_1+\varphi_2}) \chi_2^3 \chi_g + e^{2(\varphi_1+\varphi_g)} \chi_g^2 \\
& + 3e^{2(\varphi_2+\varphi_g)} \chi_g^2 - 4e^{\varphi_1+\varphi_2+2\varphi_g} \chi_g^2 \\
& + e^{2\varphi_1+6\varphi_2} \chi_2^6 (1 + e^{2\varphi_g} \chi_g^2) + e^{2\varphi_2} \chi_2^2 (3e^{2\varphi_1} \\
& + 6e^{2\varphi_2} - 8e^{\varphi_1+\varphi_2} + 6e^{3\varphi_2+\varphi_g} \\
& - 6e^{2\varphi_1+3\varphi_2+\varphi_g} + 4e^{\varphi_1+4\varphi_2+\varphi_g} \\
& + 3e^{4\varphi_2+2\varphi_g} + e^{2\varphi_g} (3e^{2\varphi_1} + 6e^{2\varphi_2} \\
& - 8e^{\varphi_1+\varphi_2}) \chi_g^2) + e^{4\varphi_2} \chi_2^4 (3e^{2\varphi_1} + 3e^{2\varphi_2} \\
& - 4e^{\varphi_1+\varphi_2} + 4e^{\varphi_1+2\varphi_2+\varphi_g} \\
& + e^{2\varphi_g} (3e^{2\varphi_1} + 3e^{2\varphi_2} - 4e^{\varphi_1+\varphi_2}) \chi_g^2) \\
& + 9e^{2(\varphi_1+\varphi_2)} \chi_1^2 (1 + e^{2\varphi_2} \chi_2^2) ((1 + e^{\varphi_2+\varphi_g})^2 \\
& - 2e^{2(\varphi_2+\varphi_g)} \chi_2 \chi_g + e^{2\varphi_g} \chi_g^2 + e^{2\varphi_2} \chi_2^2 (1 + e^{2\varphi_g} \chi_g^2)) \\
& - 6e^{2(\varphi_1+\varphi_2)} \chi_1 (-e^{2\varphi_g} (-1 \\
& + e^{2\varphi_2}) \chi_g + e^{4\varphi_2+2\varphi_g} \chi_2^2 \chi_g \\
& - e^{4\varphi_2+2\varphi_g} \chi_2^4 \chi_g + e^{4\varphi_2} \chi_2^5 (1 + e^{2\varphi_g} \chi_g^2) \\
& + 2e^{2\varphi_2} \chi_2^3 (1 + e^{\varphi_2+\varphi_g}
\end{aligned}$$

$$\begin{aligned}
& + e^{2\varphi_g} \chi_g^2) + \chi_2 (1 + 2e^{\varphi_2+\varphi_g} - 2e^{3\varphi_2+\varphi_g} \\
& - e^{4\varphi_2+2\varphi_g} + e^{2\varphi_g} \chi_g^2) \big], \\
\end{aligned} \tag{110}$$

$$\begin{aligned}
\varphi'_2 = & -\frac{1}{32W} e^{\varphi_1-3\varphi_2-\varphi_g} \lambda^2 [-e^{2\varphi_1} - 3e^{2\varphi_2} \\
& + 4e^{\varphi_1+\varphi_2} + 2e^{\varphi_1+2\varphi_2+\varphi_g} \\
& + e^{2(\varphi_1+3\varphi_2+\varphi_g)} - 2e^{\varphi_1+4\varphi_2+\varphi_g} \\
& + 3e^{4\varphi_2+2\varphi_g} - 4e^{\varphi_1+5\varphi_2+2\varphi_g} + 2e^{4\varphi_2+2\varphi_g} (-3 \\
& - e^{2\varphi_1} + 4e^{\varphi_1+\varphi_2}) \chi_2 \chi_g \\
& + 2e^{5\varphi_2+2\varphi_g} (4e^{\varphi_1} - 9e^{\varphi_2} + e^{2\varphi_1+\varphi_2}) \chi_2^3 \chi_g \\
& - e^{2(\varphi_1+\varphi_g)} \chi_g^2 - 3e^{2(\varphi_2+\varphi_g)} \chi_g^2 \\
& + 4e^{\varphi_1+\varphi_2+2\varphi_g} \chi_g^2 + e^{2\varphi_1+6\varphi_2} \chi_2^6 (1 + e^{2\varphi_g} \chi_g^2) \\
& + e^{2\varphi_2} \chi_2^2 (-e^{2\varphi_1} + 6e^{2\varphi_2} + 12e^{3\varphi_2+\varphi_g} \\
& - 4e^{2\varphi_1+3\varphi_2+\varphi_g} + 6e^{\varphi_1+4\varphi_2+\varphi_g} \\
& + 9e^{4\varphi_2+2\varphi_g} - e^{2\varphi_g} (e^{2\varphi_1} - 6e^{2\varphi_2}) \chi_g^2) \\
& + e^{4\varphi_2} \chi_2^4 (e^{2\varphi_1} + 9e^{2\varphi_2} - 4e^{\varphi_1+\varphi_2} \\
& + 6e^{\varphi_1+2\varphi_2+\varphi_g} + e^{2\varphi_g} (e^{2\varphi_1} + 9e^{2\varphi_2} \\
& - 4e^{\varphi_1+\varphi_2}) \chi_g^2) - 2e^{2(\varphi_1+\varphi_2)} \chi_1 (-e^{2\varphi_g} (1 \\
& + e^{2\varphi_2}) \chi_g + 3e^{4\varphi_2+2\varphi_g} \chi_2^2 \chi_g - 3e^{4\varphi_2+2\varphi_g} \chi_2^4 \chi_g \\
& + 3e^{4\varphi_2} \chi_2^5 (1 + e^{2\varphi_g} \chi_g^2) \\
& + 2e^{2\varphi_2} \chi_2^3 (1 + 2e^{\varphi_2+\varphi_g} + e^{2\varphi_g} \chi_g^2) - \chi_2 (1 + 4e^{3\varphi_2+\varphi_g} \\
& + 3e^{4\varphi_2+2\varphi_g} + e^{2\varphi_g} \chi_g^2)) \\
& + 3e^{2(\varphi_1+\varphi_2)} \chi_1^2 (-1 + e^{2(\varphi_2+\varphi_g)} - 2e^{2(\varphi_2+\varphi_g)} \chi_2 \chi_g \\
& - 6e^{4\varphi_2+2\varphi_g} \chi_2^3 \chi_g - e^{2\varphi_g} \chi_g^2 \\
& + 3e^{4\varphi_2} \chi_2^4 (1 + e^{2\varphi_g} \chi_g^2) + e^{2\varphi_2} \chi_2^2 (2 + 4e^{\varphi_2+\varphi_g} \\
& + 3e^{2(\varphi_2+\varphi_g)} + 2e^{2\varphi_g} \chi_g^2) \big], \\
\end{aligned} \tag{111}$$

$$\begin{aligned}
\chi'_1 = & \frac{1}{16W} e^{\varphi_1-\varphi_2-\varphi_g} \lambda^2 [-e^{2\varphi_g} (-1 + e^{2\varphi_2}) \chi_g \\
& + e^{4\varphi_2+2\varphi_g} \chi_2^2 \chi_g - e^{4\varphi_2+2\varphi_g} \chi_2^4 \chi_g \\
& + e^{4\varphi_2} \chi_2^5 (1 + e^{2\varphi_g} \chi_g^2) + 2e^{2\varphi_2} \chi_2^3 \\
& \times (1 + e^{\varphi_2+\varphi_g} + e^{2\varphi_g} \chi_g^2) + \chi_2 (1 + 2e^{\varphi_2+\varphi_g} \\
& - 2e^{3\varphi_2+\varphi_g} - e^{4\varphi_2+2\varphi_g} + e^{2\varphi_g} \chi_g^2) \\
& - 3\chi_1 (1 + e^{2\varphi_2} \chi_2^2) ((1 + e^{\varphi_2+\varphi_g})^2 \\
& - 2e^{2(\varphi_2+\varphi_g)} \chi_2 \chi_g + e^{2\varphi_g} \chi_g^2 \\
& + e^{2\varphi_2} \chi_2^2 (1 + e^{2\varphi_g} \chi_g^2) \big], \\
\end{aligned} \tag{112}$$

$$\begin{aligned}
\chi'_2 = & -\frac{1}{16W} e^{\varphi_1-3\varphi_2-\varphi_g} \lambda^2 [e^{\varphi_2+2\varphi_g} (2e^{\varphi_1} - 3e^{\varphi_2} \\
& - e^{2\varphi_1+\varphi_2} + 2e^{\varphi_1+2\varphi_2}) \chi_g \\
& + e^{3\varphi_2+2\varphi_g} (6e^{\varphi_1} - 9e^{\varphi_2}
\end{aligned}$$

$$\begin{aligned}
& + e^{2\varphi_1+\varphi_2})\chi_2^2\chi_g + e^{2\varphi_1+4\varphi_2}\chi_2^5(1 + e^{2\varphi_g}\chi_g^2) \\
& + \chi_2(e^{2\varphi_1} + 6e^{2\varphi_2} - 4e^{\varphi_1+\varphi_2} \\
& + 6e^{3\varphi_2+\varphi_g} - 2e^{2\varphi_1+3\varphi_2+\varphi_g} + 2e^{\varphi_1+4\varphi_2+\varphi_g} \\
& + 3e^{4\varphi_2+2\varphi_g} + e^{2\varphi_g}(e^{2\varphi_1} + 6e^{2\varphi_2} \\
& - 4e^{\varphi_1+\varphi_2})\chi_g^2) + 2e^{2\varphi_2}\chi_2^3(e^{2\varphi_1} + 3e^{2\varphi_2} \\
& - 2e^{\varphi_1+\varphi_2} + 2e^{\varphi_1+2\varphi_2+\varphi_g} + e^{2\varphi_g}(e^{2\varphi_1} \\
& + 3e^{2\varphi_2} - 2e^{\varphi_1+\varphi_2})\chi_g^2) - e^{2\varphi_1}\chi_1(1 \\
& + 2e^{\varphi_2+\varphi_g} - 2e^{3\varphi_2+\varphi_g} - e^{4\varphi_2+2\varphi_g} \\
& + 2e^{4\varphi_2+2\varphi_g}\chi_2\chi_g - 4e^{4\varphi_2+2\varphi_g}\chi_2^3\chi_g \\
& + e^{2\varphi_g}\chi_g^2 + 5e^{4\varphi_2}\chi_2^4(1 + e^{2\varphi_g}\chi_g^2) \\
& + 6e^{2\varphi_2}\chi_2^2(1 + e^{\varphi_2+\varphi_g} + e^{2\varphi_g}\chi_g^2)) \\
& + 3e^{2(\varphi_1+\varphi_2)}\chi_1^2(-e^{2\varphi_g}\chi_g - 3e^{2(\varphi_2+\varphi_g)}\chi_2^2\chi_g \\
& + 2e^{2\varphi_2}\chi_2^3(1 + e^{2\varphi_g}\chi_g^2) \\
& + \chi_2(2 + 2e^{\varphi_2+\varphi_g} + e^{2(\varphi_2+\varphi_g)} + 2e^{2\varphi_g}\chi_g^2))) \quad (113)
\end{aligned}$$

where W is given in (43).

A.3: Scalar potential from type IIA compactification

The scalar potential obtained from a geometric compactification of type IIA theory is given by

$$\begin{aligned}
V = & \frac{1}{192}e^{\varphi_1-3\varphi_2-\varphi_g}\lambda^2[20e^{2\varphi_1+4\varphi_2} \\
& + 25e^{2(\varphi_1+\varphi_g)} - 240e^{\varphi_1+4\varphi_2+\varphi_g} - 180e^{4\varphi_2+2\varphi_g} \\
& + 5e^{2(\varphi_1+\varphi_2+\varphi_g)}(1 + 2\sqrt{15}\chi_2 + 15\chi_2^2) \\
& + 12e^{6\varphi_2+2\varphi_g}(1 + 2\sqrt{15}\chi_2 + 15\chi_2^2) \\
& + e^{2\varphi_1+6\varphi_2}(4 + 8\sqrt{15}\chi_2 + 60\chi_2^2) \\
& + e^{2(\varphi_1+2\varphi_2+\varphi_g)}[180\chi_1^2 - 12\chi_1(3\sqrt{15} \\
& + 5\chi_2(2 + \sqrt{15}\chi_2) - 10\chi_g) \\
& + 3(9 + 4\sqrt{15}\chi_2 - 4\sqrt{15}\chi_g) + 5[4\sqrt{15}\chi_2^3 \\
& + 15\chi_2^4 - 8\chi_2\chi_g + 4\chi_g^2 + \chi_2^2(22 - 4\sqrt{15}\chi_g)]] \\
& + e^{2(\varphi_1+3\varphi_2+\varphi_g)}[135 \\
& - 54\sqrt{15}\chi_2 + 10\sqrt{15}\chi_2^5 \\
& + 25\chi_2^6 + 12\sqrt{15}\chi_g - 4\chi_2^3(3\sqrt{15} + 60\chi_1 + 20\chi_g) \\
& + 8\chi_2(3\chi_1 + \chi_g)(18 + 3\sqrt{15}\chi_1 + \sqrt{15}\chi_g) \\
& - 5\chi_2^4(-21 + 12\sqrt{15}\chi_1 \\
& + 4\sqrt{15}\chi_g) + 4[9\chi_1(\sqrt{15} + \chi_1) \\
& + 6\chi_1\chi_g + \chi_g^2] + \chi_2^2[-9 - 40\sqrt{15}\chi_g \\
& + 60[9\chi_1^2 - 2\chi_1(\sqrt{15} - 3\chi_g) + \chi_g^2]]]. \quad (114)
\end{aligned}$$

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Holographic RG flows and AdS_5 black strings from 5D half-maximal gauged supergravity

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Abstract We study five-dimensional $N = 4$ gauged supergravity coupled to five vector multiplets with compact and non-compact gauge groups $U(1) \times SU(2) \times SU(2)$ and $U(1) \times SO(3, 1)$. For $U(1) \times SU(2) \times SU(2)$ gauge group, we identify $N = 4$ AdS_5 vacua with $U(1) \times SU(2) \times SU(2)$ and $U(1) \times SU(2)_{\text{diag}}$ symmetries and analytically construct the corresponding holographic RG flow interpolating between these critical points. The flow describes a deformation of the dual $N = 2$ SCFT driven by vacuum expectation values of dimension-two operators. In addition, we study $AdS_3 \times \Sigma_2$ geometries, for Σ_2 being a two-sphere S^2 or a two-dimensional hyperbolic space H^2 , dual to twisted compactifications of $N = 2$ SCFTs with flavor symmetry $SU(2)$. We find a number of $AdS_3 \times H^2$ solutions preserving eight supercharges for different twists from $U(1) \times U(1) \times U(1)$ and $U(1) \times U(1)_{\text{diag}}$ gauge fields. We numerically construct various RG flow solutions interpolating between $N = 4$ AdS_5 critical points and these $AdS_3 \times H^2$ geometries in the IR. The solutions can also be interpreted as supersymmetric black strings in asymptotically AdS_5 space. These types of holographic solutions are also studied in non-compact $U(1) \times SO(3, 1)$ gauge group. In this case, only one $N = 4$ AdS_5 vacuum exists, and we give an RG flow solution from this AdS_5 to a singular geometry in the IR corresponding to an $N = 2$ non-conformal field theory. An $AdS_3 \times H^2$ solution together with an RG flow between this vacuum and the $N = 4$ AdS_5 are also given.

1 Introduction

AdS_5/CFT_4 correspondence has attracted much attention since the first proposal of the AdS/CFT correspondence

in [1]. Various aspects of the very well-understood duality between type IIB theory on $AdS_5 \times S^5$ and $N = 4$ Super Yang-Mills (SYM) theory in four dimensions are captured by $N = 8$ $SO(6)$ gauged supergravity in five dimensions which is a consistent truncation of type IIB supergravity on S^5 [2]. One aspect of the AdS/CFT correspondence that has been extensively studied is holographic RG flows. There are many previous works considering these solutions both in $N = 8$ five-dimensional gauged supergravity and directly in type IIB string theory, see for example [3–7].

Results along this direction with less supersymmetry have also appeared in [8–11]. In this case, gauged supergravities in five dimensions with $N < 8$ supersymmetry provide a very useful framework. In this paper, we consider holographic RG flows in half-maximal $N = 4$ gauged supergravity coupled to vector multiplets. This gauged supergravity has global symmetry $SO(1, 1) \times SO(5, n)$, n being the number of vector multiplets. Gaugings of a subgroup $G_0 \subset SO(1, 1) \times SO(5, n)$ have been constructed in an $SO(1, 1) \times SO(5, n)$ covariant manner using the embedding tensor formalism in [12], see also [13]. The resulting solutions should describe RG flows arising from perturbing $N = 2$ superconformal field theories (SCFTs) by turning on some operators or their expectation values. Holographic solutions describing these $N = 2$ SCFTs and their deformations are less known compared to the $N = 4$ SYM. The results of this paper will give more examples of supersymmetric RG flow solutions and should hopefully shed some light on strongly coupled dynamics of $N = 2$ SCFTs.

We will consider $N = 4$ gauged supergravity coupled to five vector multiplets. This $N = 4$ gauged supergravity has a possibility of embedding in ten dimensions since the ungauged supergravity can be obtained via a T^5 reduction of $N = 1$ supergravity in ten dimensions similar to $N = 4$ supergravity in four dimensions coupled to six vector multiplets that descends from $N = 1$ ten-dimensional supergravity

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compactified on a T^6 . However, it should be emphasized that the gaugings considered here have no known higher dimensional origin to date. We mainly focus on domain wall solutions interpolating between $N = 4$ AdS_5 vacua or between an AdS_5 vacuum and a singular domain wall corresponding to a non-conformal field theory. These types of solutions have been extensively studied in half-maximal gauged supergravities in various space-time dimensions, see [10, 11, 14–21] for an incomplete list. The solutions involve only the metric and scalar fields.

We will also study solutions with some vector fields non-vanishing. These solutions interpolate between the above mentioned supersymmetric AdS_5 vacua and $AdS_3 \times \Sigma_2$ geometries in the IR in which Σ_2 is a two-sphere (S^2) or a two-dimensional hyperbolic space (H^2). Holographically, the resulting solutions describe twisted compactifications of the dual $N = 2$ SCFTs to two-dimensional SCFTs as first studied in [22]. A number of these flows across dimensions have been found within $N = 8$ gauged supergravity and its truncations in [23–27], see also a universal result in [28] and solutions obtained directly from type IIB theory in [29]. To the best of our knowledge, solutions of this type have not appeared before in the framework of $N = 4$ gauged supergravity coupled to vector multiplets, see however [30] for similar solutions in pure $N = 4$ gauged supergravity. Our results should give a generalization of the universal RG flows across dimensions in [28] by turning on the twists from flavor symmetries.

In addition, $AdS_3 \times \Sigma_2$ geometries can arise as near horizon limits of black strings. Therefore, flow solutions interpolating between AdS_5 and $AdS_3 \times \Sigma_2$ should describe black strings in asymptotically AdS_5 space. Similar solutions in $N = 2$ gauged supergravity have been considered in [31–35]. We will give solutions of this type in $N = 4$ gauged supergravity. The solutions presented here will provide further examples of supersymmetric AdS_5 black strings and might be useful for both holographic studies of twisted $N = 2$ SCFTs on Σ_2 and certain dynamical aspects of black strings.

The paper is organized as follows. In Sect. 2, we review $N = 4$ gauged supergravity in five dimensions coupled to vector multiplets using the embedding tensor formalism. In Sect. 3, a compact $U(1) \times SU(2) \times SU(2)$ gauge group is considered. Supersymmetric AdS_5 vacua and RG flows interpolating between them are given. A number of $AdS_3 \times H^2$ solutions will also be given along with numerical RG flows interpolating between the previously identified AdS_5 vacua and these $AdS_3 \times H^2$ geometries. In Sect. 4, we repeat the analysis for a non-compact $U(1) \times SO(3, 1)$ gauge group. An RG flow from $N = 2$ SCFT dual to a supersymmetric AdS_5 vacuum to a singular geometry dual to a non-conformal field theory is considered. A supersymmetric $AdS_3 \times H^2$ geometry and an RG flow between this vacuum and the AdS_5

critical point will also be given. We end the paper with some conclusions and comments in Sect. 5.

2 Five dimensional $N = 4$ gauged supergravity coupled to vector multiplets

In this section, we review the structure of five dimensional $N = 4$ gauged supergravity coupled to vector multiplets. We mainly focus on relevant formulae to find supersymmetric solutions. More details on the construction of $N = 4$ gauged supergravity can be found in [12] and [13].

In five dimensions, $N = 4$ gravity multiplet consists of the graviton $e_{\mu}^{\hat{\mu}}$, four gravitini $\psi_{\mu i}$, six vectors A^0 and A_{μ}^m , four spin- $\frac{1}{2}$ fields χ_i and one real scalar Σ , the dilaton. Space-time and tangent space indices are denoted respectively by $\mu, \nu, \dots = 0, 1, 2, 3, 4$ and $\hat{\mu}, \hat{\nu}, \dots = 0, 1, 2, 3, 4$. The $SO(5) \sim USp(4)$ R-symmetry indices are described by $m, n = 1, \dots, 5$ for the $SO(5)$ vector representation and $i, j = 1, 2, 3, 4$ for the $SO(5)$ spinor or $USp(4)$ fundamental representation.

$N = 4$ supersymmetry allows the gravity multiplet to couple to an arbitrary number n of vector multiplets. Each vector multiplet contains a vector field A_{μ} , four gaugini λ_i and five scalars ϕ^m . The n vector multiplets will be labeled by indices $a, b = 1, \dots, n$. Components fields in the n vector multiplets are accordingly denoted by $(A_{\mu}^a, \lambda_i^a, \phi^{ma})$. The $5n$ scalar fields parametrized the $SO(5, n)/SO(5) \times SO(n)$ coset. Combining the gravity and vector multiplets, we have $6 + n$ vector fields denoted by $A_{\mu}^{\mathcal{M}} = (A_{\mu}^0, A_{\mu}^m, A_{\mu}^a)$ and $5n + 1$ scalars. All fermionic fields are symplectic Majorana spinors subject to the condition

$$\xi_i = \Omega_{ij} C (\bar{\xi}^j)^T \quad (1)$$

with C and Ω_{ij} being the charge conjugation matrix and $USp(4)$ symplectic form, respectively.

To describe the $SO(5, n)/SO(5) \times SO(n)$ coset, we introduce a coset representative $\mathcal{V}_M^A$ transforming under the global $G = SO(5, n)$ and the local $H = SO(5) \times SO(n)$ by left and right multiplications, respectively. We use indices $M, N, \dots = 1, 2, \dots, 5 + n$. The local H indices $A, B, \dots$ can be split into $A = (m, a)$. We can then write the coset representative as

$$\mathcal{V}_M^A = (\mathcal{V}_M^m, \mathcal{V}_M^a). \quad (2)$$

The matrix $\mathcal{V}_M^A$, an element of $SO(5, n)$, satisfies the relation

$$\eta_{MN} = \mathcal{V}_M^A \mathcal{V}_N^B \eta_{AB} = -\mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \quad (3)$$

$\eta_{MN} = \text{diag}(-1, -1, -1, -1, -1, 1, \dots, 1)$ is the $SO(5, n)$ invariant tensor. In addition, the $SO(5, n)/SO(5) \times SO(n)$ coset can also be described in term of a symmetric matrix

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a \quad (4)$$

which is manifestly invariant under the $SO(5) \times SO(n)$ local symmetry.

The full global symmetry of $N = 4$ supergravity coupled to n vector multiplets is $SO(1, 1) \times SO(5, n)$. The $SO(1, 1) \sim \mathbb{R}^+$ factor is identified with the coset space described by the dilaton Σ . Gaugings can be efficiently described, in an $SO(1, 1) \times SO(5, n)$ covariant manner, by using the embedding tensor formalism. $N = 4$ supersymmetry allows three components of the embedding tensor ξ^M , $\xi^{MN} = \xi^{[MN]}$ and $f_{MNP} = f_{[MNP]}$. The existence of supersymmetric AdS_5 vacua requires $\xi^M = 0$, see [36] for more detail. Since, in this paper, we are only interested in supersymmetric AdS_5 vacua and solutions interpolating between these vacua or solutions asymptotically approaching AdS_5 , we will restrict ourselves to the gaugings with $\xi^M = 0$.

With $\xi^M = 0$, the gauge group is entirely embedded in $SO(5, n)$. The gauge generators in the fundamental representation of $SO(5, n)$ can be written in terms of the $SO(5, n)$ generators $(t_{MN})_P{}^Q = \delta_{[M}^Q \eta_{N]P}$ as

$$\begin{aligned} (X_M)_N{}^P &= -f_M^{QR} (t_{QR})_N{}^P = f_{MN}{}^P \quad \text{and} \\ (X_0)_N{}^P &= -\xi^{QR} (t_{QR})_N{}^P = \xi_N{}^P. \end{aligned} \quad (5)$$

As a result, the covariant derivative reads

$$D_\mu = \nabla_\mu + A_\mu^M X_M + A_\mu^0 X_0 \quad (6)$$

where ∇_μ is the usual space-time covariant derivative including the spin connection. It should be noted that the definition of ξ^{MN} and f_{MNP} includes the gauge coupling constants. Furthermore, $SO(5, n)$ indices $M, N, \dots$ are lowered and raised by η_{MN} and its inverse η^{MN} .

In order to define a consistent gauge group, generators $X_M = (X_0, X_M)$ must form a closed subalgebra of $SO(5, n)$. This requires ξ^{MN} and f_{MNP} to satisfy the quadratic constraints

$$f_{R[MN} f_{PQ]}{}^R = 0 \quad \text{and} \quad \xi_M{}^Q f_{QNP} = 0. \quad (7)$$

The first condition is the usual Jacobi identity. From the result of [36], gauge groups that admit $N = 4$ supersymmetric AdS_5 vacua are generally of the form $U(1) \times H_0 \times H$. The $U(1)$ is gauged by A_μ^0 while $H \subset SO(n + 3 - \dim H_0)$ is a compact group gauged by vector fields in the vector multiplets. H_0 is a non-compact group gauged by three of the graviphotons and $\dim H_0 - 3$ vectors from the vector multiplets. In addition, H_0 must contain an $SU(2)$ subgroup. For simple groups, H_0 can be $SU(2) \sim SO(3)$, $SO(3, 1)$ and $SL(3, \mathbb{R})$.

The bosonic Lagrangian of a general gauged $N = 4$ supergravity coupled to n vector multiplets can be written as

$$e^{-1} \mathcal{L} = \frac{1}{2} R - \frac{1}{4} \Sigma^2 M_{MN} \mathcal{H}_{\mu\nu}^M \mathcal{H}^{N\mu\nu} - \frac{1}{4} \Sigma^{-4} \mathcal{H}_{\mu\nu}^0 \mathcal{H}^{0\mu\nu}$$

$$\begin{aligned} & - \frac{3}{2} \Sigma^{-2} D_\mu \Sigma D^\mu \Sigma + \frac{1}{16} D_\mu M_{MN} D^\mu M^{MN} \\ & - V + e^{-1} \mathcal{L}_{\text{top}} \end{aligned} \quad (8)$$

where e is the vielbein determinant. $\mathcal{L}_{\text{top}}$ is the topological term which we will not give the explicit form here due to its complexity. The covariant gauge field strength tensors read

$$\mathcal{H}_{\mu\nu}^M = 2\partial_{[\mu} A_{\nu]}^M + X_{\mathcal{N}\mathcal{P}}{}^M A_\mu^{\mathcal{N}} A_\nu^{\mathcal{P}} + Z^{\mathcal{MN}} B_{\mathcal{N}\mu\nu} \quad (9)$$

where

$$\begin{aligned} X_{MN}{}^P &= -f_{MN}{}^P, \quad X_{M0}{}^0 = 0, \quad X_{0M}{}^N = -\xi_M{}^N, \\ Z^{MN} &= \frac{1}{2} \xi^{MN}, \quad Z^{0M} = -Z^{M0} = 0. \end{aligned} \quad (10)$$

The two-form fields do not have kinetic terms and satisfy the first-order field equation

$$Z^{\mathcal{MN}} \left[\frac{1}{6\sqrt{2}} \epsilon_{\mu\nu\rho\lambda\sigma} \mathcal{H}_{\mathcal{N}}^{(3)\rho\lambda\sigma} - \mathcal{M}_{\mathcal{N}\mathcal{P}} \mathcal{H}_{\mu\nu}^{\mathcal{P}} \right] = 0 \quad (11)$$

with $\mathcal{H}^{(3)}$ defined by

$$\begin{aligned} Z^{\mathcal{MN}} \mathcal{H}_{\mathcal{N}\mu\nu\rho}^{(3)} &= Z^{\mathcal{MN}} \left[3D_{[\mu} B_{\nu\rho]\mathcal{N}} + 6d_{\mathcal{N}\mathcal{P}\mathcal{Q}} A_{[\mu}^{\mathcal{P}} \right. \\ & \quad \times \left. \left(\partial_\nu A_{\rho]}^{\mathcal{Q}} + \frac{1}{3} X_{\mathcal{RS}}{}^{\mathcal{Q}} A_\nu^{\mathcal{R}} A_{\rho]}^{\mathcal{S}} \right) \right] \end{aligned} \quad (12)$$

and $d_{0MN} = d_{MN0} = d_{M0N} = \eta_{MN}$. These two form fields arise from vector fields that transform non-trivially under the $U(1)$ part of the gauge group.

The scalar potential is given by

$$\begin{aligned} V &= -\frac{1}{4} \left[f_{MNP} f_{QRS} \Sigma^{-2} \left(\frac{1}{12} M^{MQ} M^{NR} M^{PS} \right. \right. \\ & \quad - \frac{1}{4} M^{MQ} \eta^{NR} \eta^{PS} + \frac{1}{6} \eta^{MQ} \eta^{NR} \eta^{PS} \Big) \\ & \quad + \frac{1}{4} \xi_{MN} \xi_{PQ} \Sigma^4 \left(M^{MP} M^{NQ} - \eta^{MP} \eta^{NQ} \right) \\ & \quad \left. + \frac{\sqrt{2}}{3} f_{MNP} \xi_{QR} \Sigma M^{MNPQRS} \right] \end{aligned} \quad (13)$$

where M^{MN} is the inverse of M_{MN} , and M^{MNPQRS} is obtained from

$$M_{MNPQR} = \epsilon_{mnpqr} \mathcal{V}_M^m \mathcal{V}_N^n \mathcal{V}_P^p \mathcal{V}_Q^q \mathcal{V}_R^r \quad (14)$$

by raising the indices with η^{MN} .

Fermionic supersymmetry transformations are given by

$$\begin{aligned} \delta\psi_{\mu i} &= D_\mu \epsilon_i + \frac{i}{\sqrt{6}} \Omega_{ij} A_1^{jk} \gamma_\mu \epsilon_k \\ & - \frac{i}{6} \left(\Omega_{ij} \Sigma \mathcal{V}_M^{jk} \mathcal{H}_{\nu\rho}^M - \frac{\sqrt{2}}{4} \delta_i^k \Sigma^{-2} \mathcal{H}_{\nu\rho}^0 \right) (\gamma_\mu^{\nu\rho} \\ & - 4\delta_\mu^\nu \gamma^\rho) \epsilon_k, \end{aligned} \quad (15)$$

$$\begin{aligned} \delta\chi_i = & -\frac{\sqrt{3}}{2}i\Sigma^{-1}D_\mu\Sigma\gamma^\mu\epsilon_i + \sqrt{2}A_2^{kj}\epsilon_k \\ & -\frac{1}{2\sqrt{3}}\left(\Sigma\Omega_{ij}\mathcal{V}_M^{jk}\mathcal{H}_{\mu\nu}^M \right. \\ & \left. +\frac{1}{\sqrt{2}}\Sigma^{-2}\delta_i^k\mathcal{H}_{\mu\nu}^0\right)\gamma^{\mu\nu}\epsilon_k, \end{aligned} \quad (16)$$

$$\begin{aligned} \delta\lambda_i^a = & i\Omega^{jk}(\mathcal{V}_M^aD_\mu\mathcal{V}_{ij}^M)\gamma^\mu\epsilon_k + \sqrt{2}\Omega_{ij}A_2^{akj}\epsilon_k \\ & -\frac{1}{4}\Sigma\mathcal{V}_M^a\mathcal{H}_{\mu\nu}^M\gamma^{\mu\nu}\epsilon_i. \end{aligned} \quad (17)$$

In the above equations, the fermion shift matrices are defined by

$$\begin{aligned} A_1^{ij} = & -\frac{1}{\sqrt{6}}\left(\sqrt{2}\Sigma^2\Omega_{kl}\mathcal{V}_M^{ik}\mathcal{V}_N^{jl}\xi^{MN} \right. \\ & \left. +\frac{4}{3}\Sigma^{-1}\mathcal{V}^{ik}_M\mathcal{V}^{jl}_N\mathcal{V}^P_{kl}f^{MN}_P\right), \\ A_2^{ij} = & \frac{1}{\sqrt{6}}\left(\sqrt{2}\Sigma^2\Omega_{kl}\mathcal{V}_M^{ik}\mathcal{V}_N^{jl}\xi^{MN} \right. \\ & \left. -\frac{2}{3}\Sigma^{-1}\mathcal{V}^{ik}_M\mathcal{V}^{jl}_N\mathcal{V}^P_{kl}f^{MN}_P\right), \\ A_2^{aij} = & -\frac{1}{2}\left(\Sigma^2\mathcal{V}_M^a\mathcal{V}_N^{ij}\xi^{MN} \right. \\ & \left. -\sqrt{2}\Sigma^{-1}\Omega_{kl}\mathcal{V}_M^a\mathcal{V}_N^{ik}\mathcal{V}_P^{jl}f^{MNP}\right). \end{aligned} \quad (18)$$

$\mathcal{V}_M^{ij}$ is defined in term of $\mathcal{V}_M^m$ as

$$\mathcal{V}_M^{ij} = \frac{1}{2}\mathcal{V}_M^m\Gamma_m^{ij} \quad (19)$$

where $\Gamma_m^{ij} = \Omega^{ik}\Gamma_{mk}^j$ and Γ_{mi}^j are $SO(5)$ gamma matrices. Similarly, the inverse $\mathcal{V}_{ij}^M$ can be written as

$$\mathcal{V}_{ij}^M = \frac{1}{2}\mathcal{V}_m^M(\Gamma_m^{ij})^* = \frac{1}{2}\mathcal{V}_m^M\Gamma_m^{kl}\Omega_{ki}\Omega_{lj}. \quad (20)$$

The covariant derivative on ϵ_i is given by

$$D_\mu\epsilon_i = \partial_\mu\epsilon_i + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon_i + Q_{\mu i}^j\epsilon_j \quad (21)$$

where the composite connection is defined by

$$\begin{aligned} Q_{\mu i}^j = & \mathcal{V}_{ik}^M\partial_\mu\mathcal{V}_M^{kj} - A_\mu^0\xi^{MN}\mathcal{V}_{Mik}\mathcal{V}_N^{kj} \\ & -A_\mu^M\mathcal{V}_{ik}^N\mathcal{V}^{kjP}f_{MNP}. \end{aligned} \quad (22)$$

Before considering various supersymmetric solutions, we note here the relation between the scalar potential and the fermion shift matrices A_1 and A_2

$$V = -A_1^{ij}A_{1ij} + A_2^{ij}A_{2ij} + A_2^{aij}A_{2ij}^a. \quad (23)$$

Raising and lowering of indices $i, j, \dots$ by Ω^{ij} and Ω_{ij} are also related to complex conjugation for example $A_{1ij} = \Omega_{ik}\Omega_{jl}A_1^{kl} = (A_1^{ij})^*$.

3 Supersymmetric RG flows in $U(1) \times SU(2) \times SU(2)$ gauge group

We begin with a compact gauge group $U(1) \times SU(2) \times SU(2)$. In order to gauge this group, we need to couple the gravity multiplet to at least three vector multiplets. Components of the embedding tensor for this gauge group are given by

$$\xi^{MN} = g_1\left(\delta_2^M\delta_1^N - \delta_1^M\delta_2^N\right), \quad (24)$$

$$f_{\tilde{m}+2, \tilde{n}+2, \tilde{p}+2} = -g_2\epsilon_{\tilde{m}\tilde{n}\tilde{p}}, \quad \tilde{m}, \tilde{n}, \tilde{p} = 1, 2, 3, \quad (25)$$

$$f_{abc} = g_3\epsilon_{abc}, \quad a, b, c = 1, 2, 3 \quad (26)$$

where g_1, g_2 and g_3 are the corresponding coupling constants for each factor in $U(1) \times SU(2) \times SU(2)$.

To parametrize the scalar coset $SO(5, n)/SO(5) \times SO(n)$, we introduce a basis for $GL(5+n, \mathbb{R})$ matrices

$$(e_{MN})_{PQ} = \delta_{MP}\delta_{NQ} \quad (27)$$

in terms of which $SO(5, n)$ non-compact generators are given by

$$Y_{ma} = e_{m, a+5} + e_{a+5, m}, \quad m = 1, 2, \dots, 5, \quad a = 1, 2, \dots, n. \quad (28)$$

We will mainly consider the case of $n = 5$ vector multiplets, but the results can be straightforwardly extended to the case of $n > 5$.

3.1 RG flows between $N = 4$ supersymmetric AdS_5 critical points

We will consider scalar fields that are singlets of $U(1) \times SU(2)_{\text{diag}} \subset U(1) \times SU(2) \times SU(2)$. Under $SO(5) \times SO(5) \subset SO(5, 5)$, the scalars transform as $(\mathbf{5}, \mathbf{5})$. With the above embedding of the gauge group in $SO(5, 5)$, the scalars transform under $U(1) \times SU(2) \times SU(2)$ gauge group as

$$\begin{aligned} & 2 \times (\mathbf{1}, \mathbf{1})_{+2} + 2 \times (\mathbf{1}, \mathbf{1})_{-2} + (\mathbf{1}, \mathbf{3})_{+2} + (\mathbf{1}, \mathbf{3})_{-2} \\ & + 2 \times (\mathbf{3}, \mathbf{1})_0 + (\mathbf{3}, \mathbf{3})_0 \end{aligned} \quad (29)$$

and transform under $U(1) \times SU(2)_{\text{diag}}$ as

$$\mathbf{1}_0 + 2 \times \mathbf{1}_{+2} + 2 \times \mathbf{1}_{-2} + 3 \times \mathbf{3}_0 + \mathbf{3}_{+2} + \mathbf{3}_{-2} + \mathbf{5}_0 \quad (30)$$

where the subscript denotes the $U(1)$ charges. According to this decomposition, there is one singlet corresponding to the following $SO(5, 5)$ non-compact generator

$$Y_s = Y_{31} + Y_{42} + Y_{53}. \quad (31)$$

Using the coset representative parametrized by

$$\mathcal{V} = e^{\phi Y_s}, \quad (32)$$

we find the scalar potential for ϕ and Σ as follow

$$V = \frac{1}{32\Sigma^2} \left[32\sqrt{2}g_1g_2\Sigma^3 \cosh^3 \phi - 9(g_2^2 + g_3^2) \cosh(2\phi) - 8(g_2^2 - g_3^2 - 4\sqrt{2}g_1g_3\Sigma^3 \sinh^3 \phi - g_2g_3 \sinh^3 \phi) + (g_2^2 + g_3^2) \cosh(6\phi) \right]. \quad (33)$$

This potential admits two $N = 4$ supersymmetric AdS_5 critical points. The first one is given by

$$\phi = 0 \quad \text{and} \quad \Sigma = \left(-\frac{g_2}{\sqrt{2}g_1} \right)^{\frac{1}{3}}. \quad (34)$$

This critical point is invariant under the full gauge symmetry $U(1) \times SU(2) \times SU(2)$ since Σ is a singlet of the whole $SO(5, 5)$ global symmetry. Furthermore, we can rescale Σ , or equivalently set $g_2 = -\sqrt{2}g_1$ to bring this critical point located at $\Sigma = 1$. The cosmological constant, the value of the scalar potential at the critical point, is

$$V_0 = -3g_1^2. \quad (35)$$

Another supersymmetric critical point is given by

$$\phi = \frac{1}{2} \ln \left[\frac{g_3 - g_2}{g_3 + g_2} \right] \quad \text{and} \quad \Sigma = \left(\frac{g_2g_3}{g_1\sqrt{2(g_3^2 - g_2^2)}} \right)^{\frac{1}{3}}. \quad (36)$$

This critical point also preserves the full $N = 4$ supersymmetry but has only $U(1) \times SU(2)_{\text{diag}}$ symmetry due to the non-vanishing scalar ϕ . The cosmological constant for this critical point is

$$V_0 = -3 \left(\frac{g_1g_2^2g_3^2}{2(g_3^2 - g_2^2)} \right)^{\frac{2}{3}}. \quad (37)$$

This second $N = 4$ AdS_5 critical point has been shown to exist in [11] when an additional $SU(2)$ dual to a flavor symmetry of the dual $N = 2$ SCFT is present.

We now analyze the BPS equations arising from setting supersymmetry transformations of fermions to zero. We first define $\mathcal{V}_M^{ij}$ with the following explicit choice of $SO(5)$ gamma matrices Γ_{mi}^j

$$\begin{aligned} \Gamma_1 &= -\sigma_2 \otimes \sigma_2, & \Gamma_2 &= i\mathbb{I}_2 \otimes \sigma_1, & \Gamma_3 &= \mathbb{I}_2 \otimes \sigma_3, \\ \Gamma_4 &= \sigma_1 \otimes \sigma_2, & \Gamma_5 &= \sigma_3 \otimes \sigma_2 \end{aligned} \quad (38)$$

where σ_i , $i = 1, 2, 3$ are the usual Pauli matrices.

Since we are interested only in solutions with only the metric and scalars non-vanishing, we will set all the vector and two-form fields to zero. The metric ansatz is given by the standard domain wall

$$ds^2 = e^{2A(r)} dx_{1,3}^2 + dr^2 \quad (39)$$

with $dx_{1,3}^2$ being the metric on four dimensional Minkowski space. In addition, the scalars Σ and ϕ as well as the Killing spinors ϵ_i are functions of only the radial coordinate r .

We begin with the $\delta\psi_{\mu i} = 0$ conditions for $\mu = 0, 1, 2, 3$ which lead to

$$A' \gamma_r \epsilon_i + i \sqrt{\frac{2}{3}} \Omega_{ij} A_1^{jk} \epsilon_k = 0 \quad (40)$$

where $'$ denotes the r -derivative. Multiply this equation by $A' \gamma_r$ and iterate, we find

$$A'^2 \epsilon_i + M_i^k M_k^j \epsilon_j = 0 \quad (41)$$

for $M_i^j = \sqrt{\frac{2}{3}} \Omega_{ik} A_1^{kj}$. The above equation has non-vanishing solutions for ϵ_i if $M_i^k M_k^j \propto \delta_i^j$. We will write

$$M_i^k M_k^j = -|W|^2 \delta_i^j \quad (42)$$

where W will be identified with the superpotential. When substitute this result in Eq. (41), we find

$$A' = \pm |W|. \quad (43)$$

On the other hand, Eq. (40) leads to the projection condition on ϵ_i

$$\gamma_r \epsilon_i = \pm i I_i^j \epsilon_j \quad (44)$$

where I_i^j is defined via

$$M_i^j = |W| I_i^j. \quad (45)$$

The condition $\delta\psi_{\hat{r}i} = 0$ gives the usual r -dependent Killing spinors of the form $\epsilon_i = e^{\frac{A}{2}} \epsilon_{0i}$ for constant spinors ϵ_{0i} satisfying (44). Using the projector (44) in conditions $\delta\chi_i = 0$ and $\delta\lambda_i^a = 0$, we can derive the first order flow equations for Σ and ϕ .

For the coset representative in (32), we find the superpotential

$$W = \frac{1}{6} \Sigma^{-1} \left(\sqrt{2}g_1 \Sigma^3 - 2g_2 \cosh^3 \phi - 2g_3 \sinh^3 \phi \right). \quad (46)$$

The matrix I_i^j in the γ_r projection is given by

$$i I_i^j = (\sigma_2 \otimes \sigma_3)_i^j. \quad (47)$$

The scalar kinetic term reads

$$\mathcal{L}_{\text{kin}} = -\frac{3}{2} \Sigma^{-2} \Sigma'^2 - \frac{3}{2} \phi'^2. \quad (48)$$

The scalar potential (33) can be written in term of the superpotential as

$$V = \frac{3}{2} \left(\frac{\partial W}{\partial \phi} \right)^2 + \frac{3}{2} \Sigma^2 \left(\frac{\partial W}{\partial \Sigma} \right)^2 - 6W^2. \quad (49)$$

By choosing the sign choice such that the $U(1) \times SU(2) \times SU(2)$ is identified with $r \rightarrow \infty$, the BPS conditions from $\delta\chi_i$ and $\delta\lambda_i^a$ reduce to the following equations

$$\phi' = -\frac{\partial W}{\partial \phi} = \Sigma^{-1} \cosh \phi \sinh \phi (g_2 \cosh \phi + g_3 \sinh \phi), \quad (50)$$

$$\Sigma' = -\Sigma^2 \frac{\partial W}{\partial \Sigma} = -\frac{1}{3} \left(\sqrt{2} g_1 \Sigma^3 + g_2 \cosh^3 \phi + g_3 \sinh^3 \phi \right). \quad (51)$$

It can be readily verified that the critical points given in (34) and (36) are also critical point of W and solve Eqs. (50) and (51). These critical points are then $N = 4$ supersymmetric. Together with the A' equation

$$A' = -\frac{1}{6} \Sigma^{-1} (\sqrt{2} g_1 \Sigma^3 - 2g_2 \cosh^3 \phi - 2g_3 \sinh^3 \phi), \quad (52)$$

we have the full set of BPS equations to be solved for RG flows interpolating between the two supersymmetric AdS_5 critical points. It can be verified that these BPS equations imply the second-order field equations.

By treating ϕ as an independent variable, we can solve for $\Sigma(\phi)$ and $A(\phi)$ as follow

$$\Sigma = -\frac{e^{\frac{\phi}{3}} (g_3 - g_3 e^{2\phi} - g_2 - g_2 e^{2\phi})^{\frac{1}{3}}}{\left[(e^{4\phi} - 1)C - 2\sqrt{2}g_1 \right]^{\frac{1}{3}}}, \quad (53)$$

$$A = -\frac{1}{3}\phi + \frac{1}{2} \ln(1 - e^{4\phi}) - \frac{1}{2} \ln(g_3 - g_3 e^{2\phi} - g_2 - g_2 e^{2\phi}) - \frac{1}{6} \ln \left[g_2(1 - e^{2\phi}) - g_3(1 + e^{2\phi}) \right]. \quad (54)$$

We have neglected an irrelevant additive integration constant in A . The constant C will be chosen in such a way that Σ approaches the second AdS_5 vacuum. This requires $C = -\frac{g_1(g_3+g_2)^2}{\sqrt{2}g_2g_3}$ leading to the final form of the solution for Σ

$$\Sigma = \left[\frac{\sqrt{2}g_2g_3e^{\phi}}{g_1(g_2 - g_2e^{2\phi} - g_3 - g_3e^{2\phi})} \right]^{\frac{1}{3}}. \quad (55)$$

Finally, the solution for $\phi(r)$ is given by

$$g_2g_3\rho = g_3 \ln \left[\frac{1 - e^{\phi}}{1 + e^{\phi}} \right] - 2g_2 \tanh^{-1} e^{\phi} + 2\sqrt{g_3^2 - g_2^2} \tanh^{-1} \left[e^{\phi} \sqrt{\frac{g_3 + g_2}{g_3 - g_2}} \right] \quad (56)$$

Table 1 Scalar masses at the $N = 4$ supersymmetric AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry and the corresponding dimensions of the dual operators

Scalar field representations	$m^2 L^2$	Δ
$(\mathbf{1}, \mathbf{1})_0$	-4	2
$(\mathbf{1}, \mathbf{1})_{\pm 2}$	-3×4	3
$(\mathbf{1}, \mathbf{3})_{\pm 2}$	-3×6	3
$(\mathbf{3}, \mathbf{1})_0$	-4×6	2
$(\mathbf{3}, \mathbf{3})_0$	-4×9	2

Table 2 Scalar masses at the $N = 4$ supersymmetric AdS_5 critical point with $U(1) \times SU(2)_{\text{diag}}$ symmetry and the corresponding dimensions of the dual operators

Scalar field representations	$m^2 L^2$	Δ
$\mathbf{1}_0$	-4	2
$\mathbf{1}_0$	12	6
$\mathbf{1}_{\pm 2}$	-3×4	3
$\mathbf{3}_{\pm 2}$	5×6	5
$\mathbf{3}_0$	-4×6	2
$\mathbf{3}_0$	0×3	4
$\mathbf{5}_0$	0×5	4

where the new radial coordinate ρ is defined by $\frac{d\rho}{dr} = \Sigma^{-1}$. This solution is the same as that obtained in [11] and has a very similar structure to solutions obtained from half-maximal gauged supergravities in seven and six dimensions [14, 16].

Near the UV $N = 4$ critical point, we find

$$\phi \sim \Sigma \sim e^{-\sqrt{2}g_1 r} \sim e^{-\frac{2r}{L_{UV}}} \quad (57)$$

where the AdS_5 radius is given by $L_{UV} = \sqrt{-\frac{6}{V_0}} = \frac{\sqrt{2}}{g_1}$. This behavior implies that the RG flow dual to this solution is driven by vacuum expectation values of operators with dimension $\Delta = 2$. Near the IR point, we find

$$\phi \sim e^{\frac{2r}{L_{IR}}}, \quad \Sigma \sim e^{-\frac{2r}{L_{IR}}} \quad (58)$$

where

$$L_{IR} = \left[\frac{2^{\frac{5}{2}} (g_3^2 - g_2^2)}{g_1 (g_2 g_3)^2} \right]^{\frac{1}{3}}. \quad (59)$$

The operator dual to ϕ becomes irrelevant in the IR with dimension $\Delta = 6$ while the operator dual to Σ has dimension $\Delta = 2$ as in the UV. For completeness, we give masses for all scalars at both critical points in Tables 1 and 2. In these tables, the singlets $(\mathbf{1}, \mathbf{1})_0$ and one of the $\mathbf{1}_0$ with $m^2 L^2 = -4$ in Table 2 corresponds to Σ . The massless scalars $\mathbf{3}_0$ in Table 2 are Goldstone bosons corresponding to the symmetry breaking $SU(2) \times SU(2) \rightarrow SU(2)_{\text{diag}}$. The massless scalars $\mathbf{5}_0$ are dual to marginal operators in the dual $N = 2$ SCFT. It should also be noted that most of the results in this section have already been found in [11]. However, the full scalar

mass spectra are new results that have not been studied in [11].

3.2 Supersymmetric RG flows from $N = 2$ SCFTs to two dimensional SCFTs

We now consider another type of solutions namely solutions interpolating between supersymmetric AdS_5 vacua identified previously and $AdS_3 \times \Sigma_2$ geometries. In the present consideration, Σ_2 is a two-sphere (S^2) or a two-dimensional hyperbolic space (H^2).

We begin with the metric ansatz for the $\Sigma_2 = S^2$ case

$$ds^2 = e^{2f(r)} dx_{1,1}^2 + dr^2 + e^{2g(r)} (d\theta^2 + \sin^2 \theta d\phi^2) \quad (60)$$

where $dx_{1,1}^2$ is the flat metric in two dimensions. It is useful to note the components of the spin connection

$$\begin{aligned} \omega^{\hat{\mu}\hat{r}} &= f' e^{\hat{\mu}}, & \omega^{\hat{\theta}\hat{r}} &= g' e^{\hat{\theta}}, \\ \omega^{\hat{\phi}\hat{r}} &= g' e^{\hat{\phi}}, & \omega^{\hat{\phi}\hat{\theta}} &= e^{-g} \cot \theta e^{\hat{\phi}} \end{aligned} \quad (61)$$

with the obvious choice of vielbein

$$e^{\hat{\mu}} = e^f dx^\mu, \quad e^{\hat{r}} = dr, \quad e^{\hat{\theta}} = e^g d\theta, \quad e^{\hat{\phi}} = e^g \sin \theta d\phi \quad (62)$$

for $\hat{\mu} = 0, 1$.

To preserve some amount of supersymmetry, we impose a twist condition by cancelling the spin connection on S^2 with some gauge connections. We will consider abelian twists from $U(1) \times U(1) \times U(1) \subset U(1) \times SU(2) \times SU(2)$ and its $U(1) \times U(1)_{\text{diag}}$ subgroup. The corresponding gauge fields are denoted by (A^0, A^5, A^8) . Note that turning on A^0 and A^5 correspond to a twist along the R-symmetry $U(1) \times SU(2)$ of the dual $N = 2$ SCFTs while a non-vanishing A^8 is related to turning on the gauge field of $SU(2)$ flavor symmetry. The latter cannot be used as a twist since the Killing spinor is neutral under this symmetry.

An effect of the twisting procedure is to cancel $\omega^{\hat{\theta}\hat{\phi}}$ on S^2 . The BPS conditions $\delta\psi_{i\hat{\theta}} = 0$ and $\delta\psi_{i\hat{\phi}} = 0$ then lead to the same BPS equation. In order to achieve this, we consider the ansatz for the gauge fields

$$A^{\mathcal{M}=0,5,8} = a_{\mathcal{M}} \cos \theta d\phi. \quad (63)$$

We consider two type of solutions with unbroken gauge symmetry $U(1) \times U(1) \times U(1)$ and $U(1) \times U(1)_{\text{diag}}$. We begin with a simpler case of $U(1) \times U(1) \times U(1)$ invariant sector consisting of four singlet scalars Σ and φ_i , $i = 1, 2, 3$. The latter correspond to the $SO(5, 5)$ non-compact generators Y_{53} , Y_{54} and Y_{55} . The coset representative is then given by

$$\mathcal{V} = e^{\varphi_1 Y_{53}} e^{\varphi_2 Y_{54}} e^{\varphi_3 Y_{55}}. \quad (64)$$

A straightforward computation gives relevant components of the covariant derivative on the Killing spinors ϵ_i

$$\begin{aligned} D_{\hat{\phi}} \epsilon_i &= \cdots + \frac{1}{2} e^{-g} \cot \theta \left[\gamma_{\hat{\theta}\hat{\phi}} \epsilon_i - i a_0 g_1 (\sigma_2 \otimes \sigma_3)_i{}^j \epsilon_j \right. \\ &\quad \left. + i a_5 g_2 (\sigma_1 \otimes \sigma_1)_i{}^j \epsilon_j \right]. \end{aligned} \quad (65)$$

In order to cancel the spin connection, we need to impose the conditions

$$i \gamma_{\hat{\theta}\hat{\phi}} \epsilon_i = a_0 g_1 (\sigma_2 \otimes \sigma_3)_i{}^j \epsilon_j - a_5 g_2 (\sigma_1 \otimes \sigma_1)_i{}^j \epsilon_j. \quad (66)$$

Consistency with $(i \gamma_{\hat{\theta}\hat{\phi}})^2 = \mathbb{I}_4$ requires the conditions

$$(g_1 a_0)^2 + (g_2 a_5)^2 = 1 \quad \text{and} \quad g_1 g_2 a_0 a_5 = 0. \quad (67)$$

The second condition implies, for non-vanishing g_1 and g_2 , either $a_0 = 0$ or $a_5 = 0$ for which the first condition gives $g_2 a_5 = \pm 1$ or $g_1 a_0 = \pm 1$, respectively. These two possibilities correspond respectively to the α -twist and β -twist studied in [37], see also a discussion in [28].

For $a_0 = 0$ and $g_2 a_5 = \pm 1$, the condition (66) becomes a projector

$$i \gamma_{\hat{\theta}\hat{\phi}} \epsilon_i = \mp (\sigma_1 \otimes \sigma_1)_i{}^j \epsilon_j. \quad (68)$$

For $a_5 = 0$ and $g_1 a_0 = \pm 1$, we find

$$i \gamma_{\hat{\theta}\hat{\phi}} \epsilon_i = \pm (\sigma_2 \otimes \sigma_3)_i{}^j \epsilon_j. \quad (69)$$

It should be noted that we can make a definite sign choice for the twist condition and the $\gamma_{\hat{\theta}\hat{\phi}}$ projector. The other possibility can be obtained by changing the sign of a_0 or a_5 together with a sign change in the $\gamma_{\hat{\theta}\hat{\phi}}$ projector. In the remaining part of this paper, we will choose the twist conditions and $\gamma_{\hat{\theta}\hat{\phi}}$ projector with the upper sign.

For the $U(1) \times U(1)_{\text{diag}}$ sector with the $U(1)_{\text{diag}}$ being a diagonal subgroup of $U(1) \times U(1) \subset SU(2) \times SU(2)$, we have five singlets from the vector multiplet scalars corresponding to the following non-compact generators of $SO(5, 5)$

$$\begin{aligned} \hat{Y}_1 &= Y_{31} + Y_{42}, & \hat{Y}_2 &= Y_{53}, & \hat{Y}_3 &= Y_{32} - Y_{41}, \\ \hat{Y}_4 &= Y_{54}, & \hat{Y}_5 &= Y_{55} \end{aligned} \quad (70)$$

giving rise to the coset representative

$$\mathcal{V} = e^{\phi_1 \hat{Y}_1} e^{\phi_2 \hat{Y}_2} e^{\phi_3 \hat{Y}_3} e^{\phi_4 \hat{Y}_4} e^{\phi_5 \hat{Y}_5}. \quad (71)$$

The result of the analysis is the same as in the previous case but with an additional condition imposed on the flux parameters a_5 and a_8

$$g_2 a_5 = g_3 a_8 \quad (72)$$

implementing the $U(1)_{\text{diag}}$ gauge symmetry. It turns out that, in both cases, all two-form fields can be consistently set to zero provided that A^1 and A^2 vanish.

For the H^2 case, we simply change $\sin \theta$ to $\sinh \theta$ in the metric (60) and take the gauge fields to be $A^{\mathcal{M}} = a_{\mathcal{M}} \cosh \theta d\phi$. The twist procedure works as in the S^2 case. However, due to the opposite sign in the covariant field strengths $\mathcal{H}^{\mathcal{M}} = dA^{\mathcal{M}}$, the resulting BPS equations for the two cases are related to each other by a sign change in the twist parameters $a_{\mathcal{M}}$.

3.2.1 Flow solutions with $U(1) \times U(1) \times U(1)$ symmetry

With the coset representative (64), the scalar potential and the superpotential are given respectively by

$$V = -\frac{1}{2}\Sigma^{-2} \left(g_2^2 - 2\sqrt{2}g_1g_2\Sigma^3 \cosh \varphi_1 \cosh \varphi_2 \cosh \varphi_3 \right) \quad (73)$$

and

$$W = \frac{1}{6}\Sigma^{-1} \left(\sqrt{2}g_1\Sigma^3 - 2g_2 \cosh \varphi_1 \cosh \varphi_2 \cosh \varphi_3 \right) \quad (74)$$

The scalar kinetic term reads

$$\mathcal{L}_{\text{kin}} = -\frac{3}{2}\Sigma^{-2}\Sigma'^2 - \frac{1}{2}\cosh^2 \varphi_2 \cosh^2 \varphi_3 \varphi_1'^2 - \frac{1}{2}\cosh^2 \varphi_3 \varphi_2'^2 - \frac{1}{2}\varphi_3'^2. \quad (75)$$

The scalar potential can also be written in term of the superpotential as

$$V = \frac{3}{2}\Sigma^2 \left(\frac{\partial W}{\partial \Sigma} \right)^2 + \frac{9}{2}\cosh^{-2} \varphi_2 \cosh^{-2} \varphi_3 \left(\frac{\partial W}{\partial \varphi_1} \right)^2 + \frac{9}{2}\cosh^{-2} \varphi_3 \left(\frac{\partial W}{\partial \varphi_2} \right)^2 + \frac{9}{2}\left(\frac{\partial W}{\partial \varphi_3} \right)^2 - 6W^2. \quad (76)$$

It can be easily checked that setting $\varphi_2 = \varphi_3 = 0$ is a consistent truncation. Moreover, the result with non-vanishing φ_2 and φ_3 is not significantly different from that with $\varphi_2 = \varphi_3 = 0$. Therefore, we will further simplify the computation by using this truncation and set $\varphi_1 = \varphi$.

We first consider the case with $a_0 = 0$. By using the $\gamma_{\hat{r}}$ projection (44) with the matrix I_i^j given in (47), we find the following BPS equations

$$\varphi' = \frac{1}{2}\Sigma^{-1}e^{-\varphi-2g} [g_2e^{2g}(e^{2\varphi} - 1) - \kappa\Sigma^2(a_5 - a_8) + \kappa(a_5 + a_8)e^{2\varphi}], \quad (77)$$

$$\Sigma' = -\frac{1}{3}\left[\sqrt{2}g_1\Sigma^3 + g_2 \cosh \varphi - \kappa e^{-2g}\Sigma^2(a_5 \cosh \varphi + a_8 \sinh \varphi) \right], \quad (78)$$

$$g' = \frac{1}{6}\Sigma^{-1}\left[\sqrt{2}g_1\Sigma^3 - 2g_2 \cosh \varphi - \kappa 4e^{-2g}\Sigma^2(a_5 \cosh \varphi + a_8 \sinh \varphi) \right], \quad (79)$$

$$f' = \frac{1}{6}\Sigma^{-1}\left[\sqrt{2}g_1\Sigma^3 + 2\kappa g_2 \cosh \varphi + 2\kappa e^{-2g}\Sigma^2(a_5 \cosh \varphi + a_8 \sinh \varphi) \right]. \quad (80)$$

The sign choices $\kappa = +1$ and $\kappa = -1$ correspond to $\Sigma_2 = S^2$ and $\Sigma_2 = H^2$, respectively. We will use this convention throughout the paper.

The $AdS_3 \times \Sigma_2$ vacua are characterized by the conditions $g' = \varphi' = \Sigma' = 0$ and $f' = \frac{1}{L_{AdS_3}}$. It turns out that the above equations admit any AdS_3 solutions only for $a_8 = 0$ and $\kappa = -1$. In this case, we find that any constant value of φ leads to an $AdS_3 \times H^2$ solution of the form

$$\varphi = \varphi_0, \quad \Sigma = -\left(\frac{\sqrt{2}g_2 \cosh \varphi_0}{g_1} \right)^{\frac{1}{3}}, \quad g = \frac{1}{6} \ln \left[\frac{2a_5 \cosh^2 \varphi_0}{g_1^2 g_2} \right], \quad L_{AdS_3} = \left(\frac{\sqrt{2}}{g_1 g_2^2 \cosh^2 \varphi_0} \right)^{\frac{1}{3}} \quad (81)$$

where φ_0 is a constant. This solution preserves eight supercharges or $N = 4$ in three dimensions due to the $\gamma_{\hat{\theta}\hat{\phi}}$ projector. On the other hand, the entire flow solution will preserve only four supercharges due to an additional $\gamma_{\hat{r}}$ projector.

For $\varphi_0 = 0$, the solution has $U(1) \times U(1) \times SU(2)$ symmetry due to the vanishing A^8 while the solution with $\varphi_0 \neq 0$ has smaller symmetry $U(1) \times U(1) \times U(1)$. The resulting $AdS_3 \times H^2$ geometry should be dual to a two dimensional $N = (2, 2)$ SCFT with $SU(2)$ or $U(1)$ flavor symmetry depending on the value of φ_0 . An asymptotic analysis near the $AdS_3 \times H^2$ critical point shows that φ is dual to a marginal operator in the two-dimensional SCFT. The central charge of the dual SCFT can also be computed by [38]

$$c = \frac{3L_{AdS_3}}{2G_3} = \frac{3L_{AdS_3} \text{vol}(H^2)}{2G_5} e^{2g_0} = \frac{3\text{vol}(H^2)a_5^{\frac{1}{3}}}{\sqrt{2}g_1g_2G_5} \quad (82)$$

which is independent of φ_0 . g_0 is the value of $g(r)$ at the AdS_3 critical point. For H_2 being a genus $g > 1$ Riemann surface, we have $\text{vol}(H^2) = 4\pi(g - 1)$.

Examples of numerical flow solutions interpolating between $N = 4$ supersymmetric AdS_5 and $N = 4$ supersymmetric $AdS_3 \times H^2$ with different values of φ_0 are given in Fig. 1. The solution with $\varphi_0 = 0$ is effectively the same as that studied in [28] which is in turn obtained from the solutions in [30] by turning off the $U(1)$ gauge field. In this case, the matter multiplets can be decoupled. Solutions with

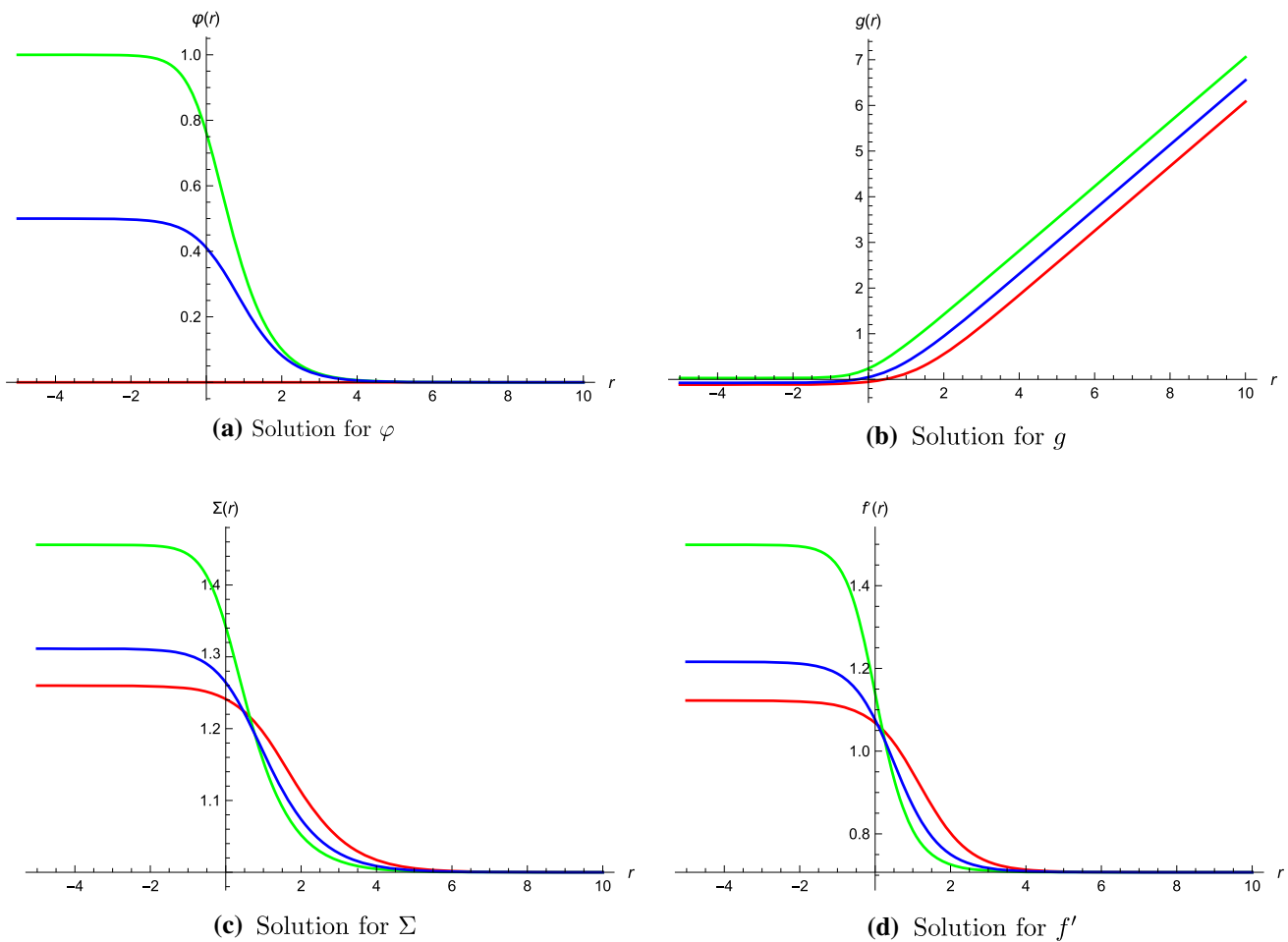


Fig. 1 RG flows from $N = 4$ AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to $N = 4$ $AdS_3 \times H^2$ geometries in the IR with $g_1 = 1$ and $\varphi_0 = 0$ (red), $\varphi_0 = 0.5$ (blue) and $\varphi_0 = 1$ (green)

$\varphi_0 \neq 0$ are only possible in the matter-coupled gauged supergravity and have not previously appeared.

For the case of $a_5 = 0$, we also find that the BPS conditions require $a_8 = 0$. The resulting BPS equations read

$$\varphi' = g_2 \Sigma^{-1} \sinh \varphi, \quad (83)$$

$$\Sigma' = -\frac{\sqrt{2}}{3} \Sigma^{-1} \left(\kappa a_0 e^{-2g} + g_1 \Sigma^4 \right) - \frac{1}{3} g_2 \cosh \varphi, \quad (84)$$

$$g' = \frac{1}{6} \Sigma^{-2} \left(\sqrt{2} g_1 \Sigma^4 - 2 g_2 \Sigma \cosh \varphi - 2 \kappa \sqrt{2} a_0 e^{-2g} \right), \quad (85)$$

$$f' = \frac{1}{6} \Sigma^{-2} \left(\sqrt{2} g_1 \Sigma^4 - 2 g_2 \Sigma \cosh \varphi + \sqrt{2} \kappa a_0 e^{-2g} \right). \quad (86)$$

Note also that the BPS equation for φ does not involve a_0 since φ is neutral under A^0 . In this case, AdS_3 vacua do not exist. For $\varphi' = g' = 0$, we find a singular behavior of Σ at a finite value of r

$$\Sigma \sim \frac{1}{\sqrt{\sqrt{2} g_1 r - C}} \quad (87)$$

for some constant C . This has also been pointed out in [28].

3.2.2 Flow solutions with $U(1) \times U(1)_{diag}$ symmetry

In this case, there are five singlet scalars from $SO(5, 5)/SO(5) \times SO(5)$ coset with the coset representative given by (71). Together with Σ , there are in total six singlet scalars, and the computation is much more complicated than the previous case. We will again make a truncation by setting $\phi_4 = \phi_5 = 0$ in the following analysis. The scalar potential with this truncation is given by

$$V = \frac{1}{16} \Sigma^{-2} \left[8 \sqrt{2} g_1 (g_2 \cosh \phi_2 - g_3 \sinh \phi_2) + 4(g_3^2 - g_2^2) + g_2 g_3 \sinh(2\phi_2) (2 \cosh^2(2\phi_1) \cosh(4\phi_3) + \cosh(4\phi_1) - 3) + 8 \sqrt{2} g_1 \cosh(2\phi_1) \cosh(2\phi_3) \Sigma^3 (g_2 \cosh \phi_2 + g_3 \sinh \phi_2) \right]$$

$$+(g_2^2 + g_3^2) \cosh(2\phi_2)(\cosh^2(2\phi_1) \cosh^2(2\phi_3) - 1) - 4(g_2^2 + g_3^2) \cosh(2\phi_1) \cosh(2\phi_3)]. \quad (88)$$

This can be written in term of the superpotential as

$$V = \frac{3}{2} \Sigma^2 \left(\frac{\partial W}{\partial \Sigma} \right)^2 + \frac{9}{4} \cosh^{-2}(2\phi_3) \left(\frac{\partial W}{\partial \phi_1} \right)^2 + \frac{9}{2} \left(\frac{\partial W}{\partial \phi_2} \right)^2 + \frac{9}{4} \left(\frac{\partial W}{\partial \phi_3} \right)^2 - 6W^2 \quad (89)$$

where the superpotential in this case is given by

$$W = \frac{1}{6} \Sigma^{-1} \left[\sqrt{2} g_1 \Sigma^3 - g_2 \cosh \phi_2 + g_3 \sinh \phi_2 - \cosh(2\phi_1) \cosh(2\phi_3)(g_2 \cosh \phi_2 + g_3 \sinh \phi_2) \right]. \quad (90)$$

It can be verified that this superpotential admits two critical points given in Eqs. (34) and (36). When $\phi_1 = \phi_3 = 0$, this is the $U(1) \times U(1) \times U(1)$ invariant sector. For $\phi_3 = 0$ and $\phi_1 = \phi_2$, we reobtain the $U(1) \times SU(2)_{\text{diag}}$ invariant scalars which admit the second $N = 4$ AdS_5 critical point with $U(1) \times SU(2)_{\text{diag}}$ symmetry.

We firstly consider the twist from A^0 gauge field. For $a_5 = 0$, the $U(1)_{\text{diag}}$ symmetry also demands $a_8 = 0$. The BPS equations for ϕ_1, ϕ_2 and ϕ_3 will not depend on the twist parameter a_0 since they are not charged under A^0 . Therefore, the only possibility to have AdS_3 vacua is to set these scalars to their values at the two AdS_5 critical points. Setting all $\phi_i = 0$ for $i = 1, 2, 3$ dose not lead to any AdS_3 solutions as in the previous case. The other choice namely $\phi_3 = 0$ and $\phi_1 = \phi_2 = \frac{1}{2} \ln \left[\frac{g_3 - g_2}{g_3 + g_2} \right]$ does not give rise to any AdS_3 vacua either. Therefore, we will not give the explicit form of the BPS equations in this case.

We now consider the twist from A^5 and A^8 gauge fields. In this case, we do find some AdS_3 solutions. The BPS equations read

$$\phi_1' = \frac{1}{2} \Sigma^{-1} \text{sech}(2\phi_3) \sinh(2\phi_1)(g_2 \cosh \phi_2 + g_3 \sinh \phi_2), \quad (91)$$

$$\phi_2' = \frac{1}{2} \Sigma^{-1} \left[\cosh(2\phi_1) \cosh(2\phi_3)(g_3 \cosh \phi_2 + g_2 \sinh \phi_2) - g_3 \sinh \phi_2 - 2\kappa \Sigma^2 e^{-2g} (a_8 \cosh \phi_2 + a_5 \sinh \phi_2) + g_2 \sinh \phi_2 \right], \quad (92)$$

$$\phi_3' = \frac{1}{2} \Sigma^{-1} \cosh(2\phi_1) \sinh(2\phi_3)(g_2 \cosh \phi_2 + g_3 \sinh \phi_2), \quad (93)$$

$$\Sigma' = \frac{1}{6} [g_3 \sinh \phi_2 - \cosh(2\phi_2) \cosh(2\phi_3)(g_2 \cosh \phi_2 + g_3 \sinh \phi_2) - 2\kappa e^{-2g} \Sigma^2 (a_5 \cosh \phi_2 + a_8 \sinh \phi_2) - 2\sqrt{2} \Sigma^3 - g_2 \cosh \phi_2], \quad (94)$$

$$g' = \frac{1}{6} \Sigma^{-1} \left[g_3 \sinh \phi_2 - \cosh(2\phi_1) \cosh(2\phi_3)(g_2 \cosh \phi_2 + g_3 \sinh \phi_2) - g_2 \cosh \phi_2 + 4\kappa e^{-2g} \Sigma^2 (a_5 \cosh \phi_2 + a_8 \sinh \phi_2) + \sqrt{2} g_1 \Sigma^3 \right], \quad (95)$$

$$f' = \frac{1}{6} \Sigma^{-1} \left[g_3 \sinh \phi_2 - \cosh(2\phi_1) \cosh(2\phi_3)(g_2 \cosh \phi_2 + g_3 \sinh \phi_2) - g_2 \cosh \phi_2 - 2\kappa e^{-2g} \Sigma^2 (a_5 \cosh \phi_2 + a_8 \sinh \phi_2) + \sqrt{2} g_1 \Sigma^3 \right] \quad (96)$$

for which there is a relation $g_2 a_5 = g_3 a_8$ to be imposed.

We find that these equations admit $AdS_3 \times \Sigma_2$ solutions only for $\kappa = -1$. The $AdS_3 \times H^2$ solutions are given as follow.

- I. The simplest solution is obtained by setting $\phi_i = 0$, $i = 1, 2, 3$ and

$$\Sigma = - \left(\frac{\sqrt{2} g_2}{g_1} \right)^{\frac{1}{3}}, \quad g = \frac{1}{6} \ln \left[\frac{2a_5}{g_1^2 g_2} \right], \quad L_{AdS_3} = \left(\frac{\sqrt{2}}{g_1 g_2^2} \right)^{\frac{1}{3}}. \quad (97)$$

- II. One of the $AdS_3 \times H^2$ solutions with vector multiplet scalars non-vanishing is given by

$$\phi_1 = \phi_2 = \frac{1}{2} \ln \left[\frac{g_3 - g_2}{g_3 + g_2} \right], \quad \phi_3 = 0, \quad \Sigma^3 = - \frac{\sqrt{2} g_2 g_3}{g_1 \sqrt{g_3^2 - g_2^2}}, \quad g = \frac{1}{6} \ln \left[\frac{a_5^3 (g_3^2 - g_2^2)^2}{g_1^2 g_2 g_3^4} \right], \quad L_{AdS_3} = \left(\frac{\sqrt{2} (g_3^2 - g_2^2)}{g_1 g_2^2 g_3^2} \right)^{\frac{1}{3}}. \quad (98)$$

- III. There is another $AdS_3 \times H^2$ solution located at

$$\phi_1 = 0, \quad \phi_2 = \phi_3 = \frac{1}{2} \ln \left[\frac{g_3 - g_2}{g_3 + g_2} \right], \quad \Sigma^3 = - \frac{\sqrt{2} g_2 g_3}{g_1 \sqrt{g_3^2 - g_2^2}}, \quad g = \frac{1}{6} \ln \left[\frac{2a_5^3 (g_3^2 - g_2^2)^2}{g_1^2 g_2 g_3^4} \right], \quad L_{AdS_3} = \left(\frac{\sqrt{2} (g_3^2 - g_2^2)}{g_1 g_2^2 g_3^2} \right)^{\frac{1}{3}}. \quad (99)$$

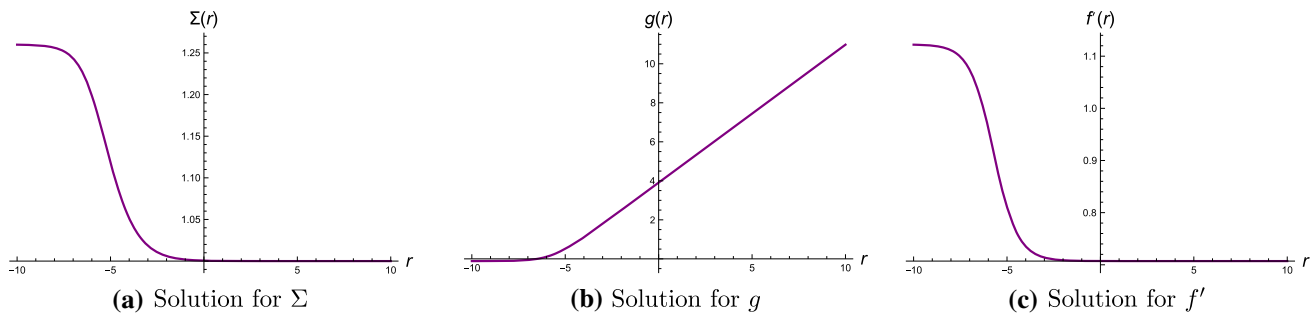


Fig. 2 An RG flow between $N = 4$ AdS_5 vacuum with $U(1) \times SU(2) \times SU(2)$ symmetry and $N = 4$ $AdS_3 \times H^2$ vacuum with $U(1) \times U(1)_{\text{diag}}$ symmetry in (97) for $g_1 = 1$

All of these solutions preserve eight supercharges corresponding to $N = 4$ supersymmetry in three dimensions or equivalently $N = (2, 2)$ in the dual two dimensional SCFTs. It should also be noted that critical points II and III appear to be related by a permutation of ϕ_i . However, the solution with $\phi_2 = 0$ does not exist since this also requires $\phi_1 = \phi_3 = 0$ and $a_8 = a_5 = 0$.

The next step is to find RG flow solutions interpolating between $N = 4$ supersymmetric AdS_5 critical points and the above $AdS_3 \times H^2$ geometries. We first consider a simple case of the flow to $AdS_3 \times H^2$ critical point I with $\phi_1 = \phi_2 = \phi_3 = 0$. The BPS equations simplify considerably to

$$\Sigma' = -\frac{1}{3} \left(\sqrt{2} g_1 \Sigma^3 + g_2 - a_5 e^{-2g} \Sigma^2 \right), \quad (100)$$

$$g' = \frac{1}{6} \Sigma^{-1} \left(\sqrt{2} g_1 \Sigma^3 - 2g_2 - 4a_5 e^{-2g} \Sigma^2 \right), \quad (101)$$

$$f' = \frac{1}{6} \Sigma^{-1} \left(\sqrt{2} g_1 \Sigma^3 - 2g_2 + 2a_5 e^{-2g} \Sigma^2 \right). \quad (102)$$

We can partially solve these equations analytically and find a relation between solutions of g and Σ of the form

$$2a_5 g + 4a_5 \ln \Sigma = e^{2g} \Sigma^{-2} (g_2 + \sqrt{2} g_1 \Sigma^3). \quad (103)$$

However, the complete solutions can only be found numerically. In this case, the solutions reduce to the universal flows across dimensions considered in [28]. An example of these solutions is given in Fig. 2.

$AdS_3 \times H^2$ critical point II is more interesting in the sense that it can be connected to both of the $N = 4$ AdS_5 vacua. In order to obtain RG flow solutions, we set $\phi_3 = 0$ which is a consistent truncation. An example of flows from AdS_5 with $U(1) \times SU(2) \times SU(2)$ symmetry to $AdS_3 \times H^2$ critical point II is given in Fig. 3. With suitable boundary conditions, we can find a solution that flows from AdS_5 with $U(1) \times SU(2) \times SU(2)$ symmetry and approaches AdS_5 with $U(1) \times SU(2)_{\text{diag}}$ symmetry before reaching the $AdS_3 \times H^2$ critical point II. A solution of this type is shown in Fig. 4.

Similarly, we can set $\phi_1 = 0$ and find a numerical solution interpolating between AdS_5 vacuum with $U(1) \times SU(2) \times SU(2)$ symmetry and $AdS_3 \times H^2$ critical point III. The result

is shown in Fig. 5. We have also verified that all of these solutions satisfy the corresponding field equations.

4 Supersymmetric RG flows in $U(1) \times SO(3, 1)$ gauge group

In this section, we consider a non-compact gauge group $U(1) \times SO(3, 1)$ with the embedding tensor

$$\xi^{MN} = g_1 (\delta_2^M \delta_1^N - \delta_1^M \delta_2^N), \quad (104)$$

$$f_{345} = f_{378} = -f_{468} = -f_{567} = -g_2. \quad (105)$$

At the vacuum, the $U(1) \times SO(3, 1)$ gauge group will be broken down to its maximal compact subgroup $U(1) \times SO(3) \subset U(1) \times SO(3, 1)$. Under this unbroken symmetry, there is one scalar singlet from $SO(5, 5)/SO(5) \times SO(5)$ corresponding to the non-compact generator

$$Y = Y_{31} + Y_{42} - Y_{53}. \quad (106)$$

With the usual parametrization of the coset representative of the form

$$L = e^{\phi Y}, \quad (107)$$

the scalar potential is given by

$$V = \frac{1}{16} \Sigma^{-2} e^{-6\phi} g_2 \left[\left(1 + 3e^{4\phi} - 16e^{6\phi} + 3e^{8\phi} + e^{12\phi} \right) g_2 - 4\sqrt{2} e^{3\phi} \left(1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi} \right) g_1 \Sigma^3 \right]. \quad (108)$$

This potential admits only one $N = 4$ supersymmetric AdS_5 vacuum due to the absence of flavor symmetry in the dual $N = 2$ SCFT in agreement with the result of [11]. This critical point is located at

$$\phi = 0 \quad \text{and} \quad \Sigma = - \left(\frac{g_2}{\sqrt{2} g_1} \right)^{1/3}. \quad (109)$$

As in the $U(1) \times SU(2) \times SU(2)$ gauge group, we can rescale Σ such that $\Sigma = 1$ at the AdS_5 vacuum. Equivalently, we can choose the value of g_2 to be $g_2 = -\sqrt{2} g_1$. With this choice, the cosmological constant and AdS_5 radius are given by

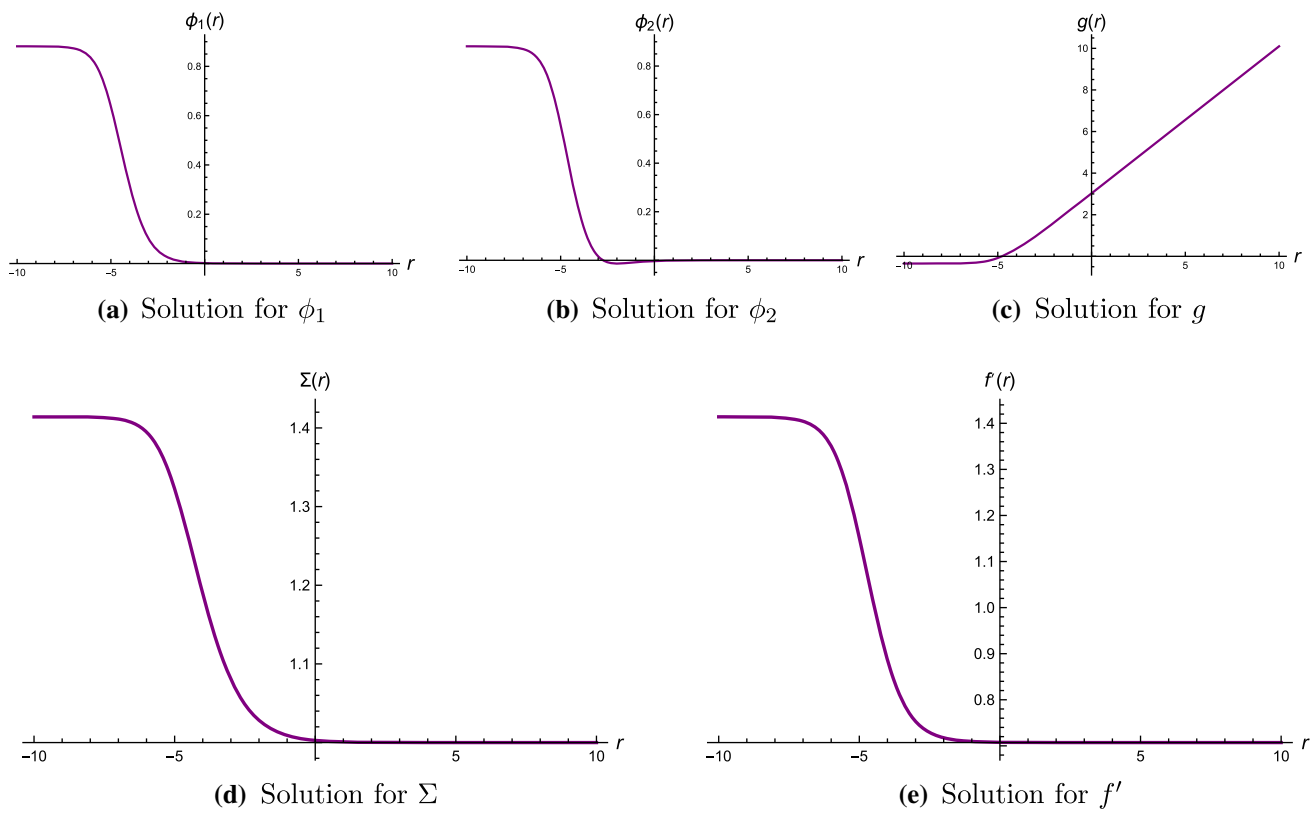


Fig. 3 An RG flow from AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to $AdS_3 \times H^2$ critical point II for $g_1 = 1$ and $g_3 = 2g_1$

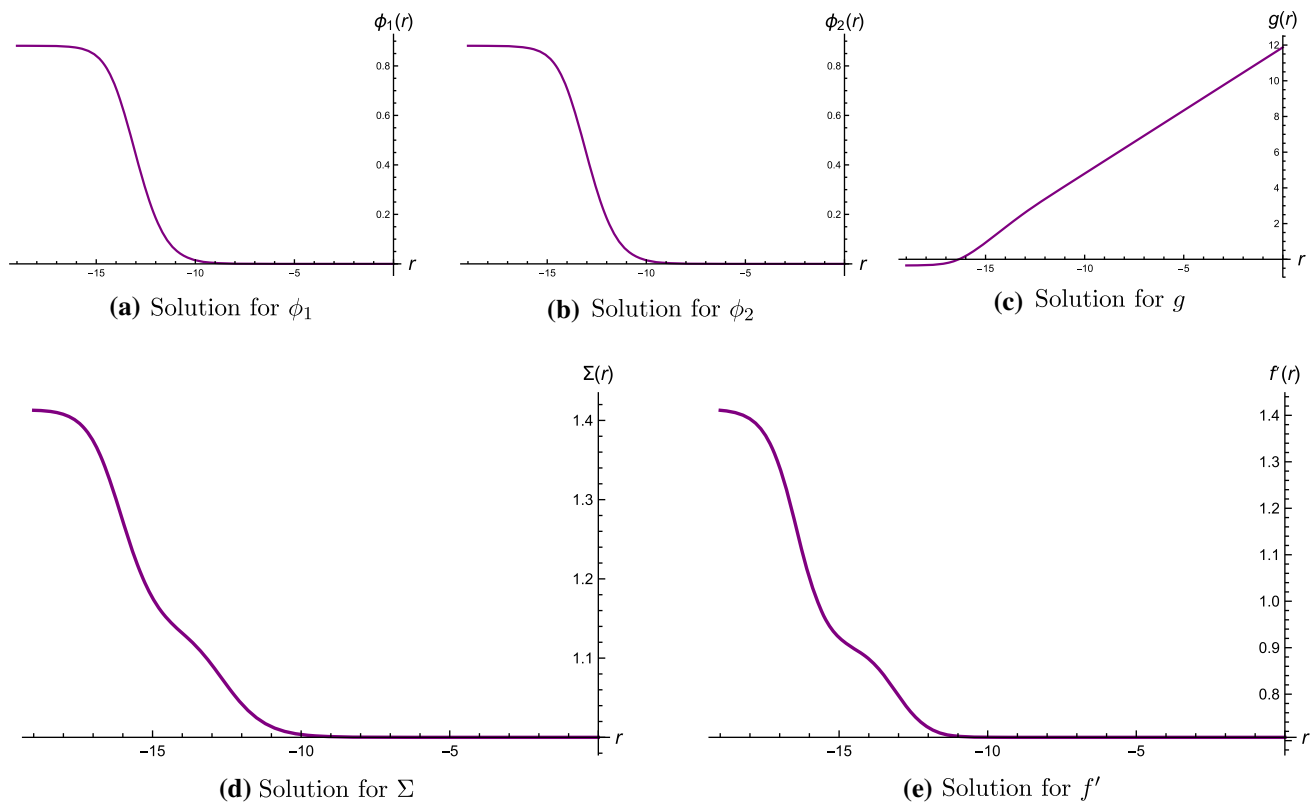


Fig. 4 An RG flow from AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to AdS_5 critical point with $U(1) \times SU(2)_{\text{diag}}$ symmetry and finally to $AdS_3 \times H^2$ critical point II for $g_1 = 1$ and $g_3 = 2g_1$

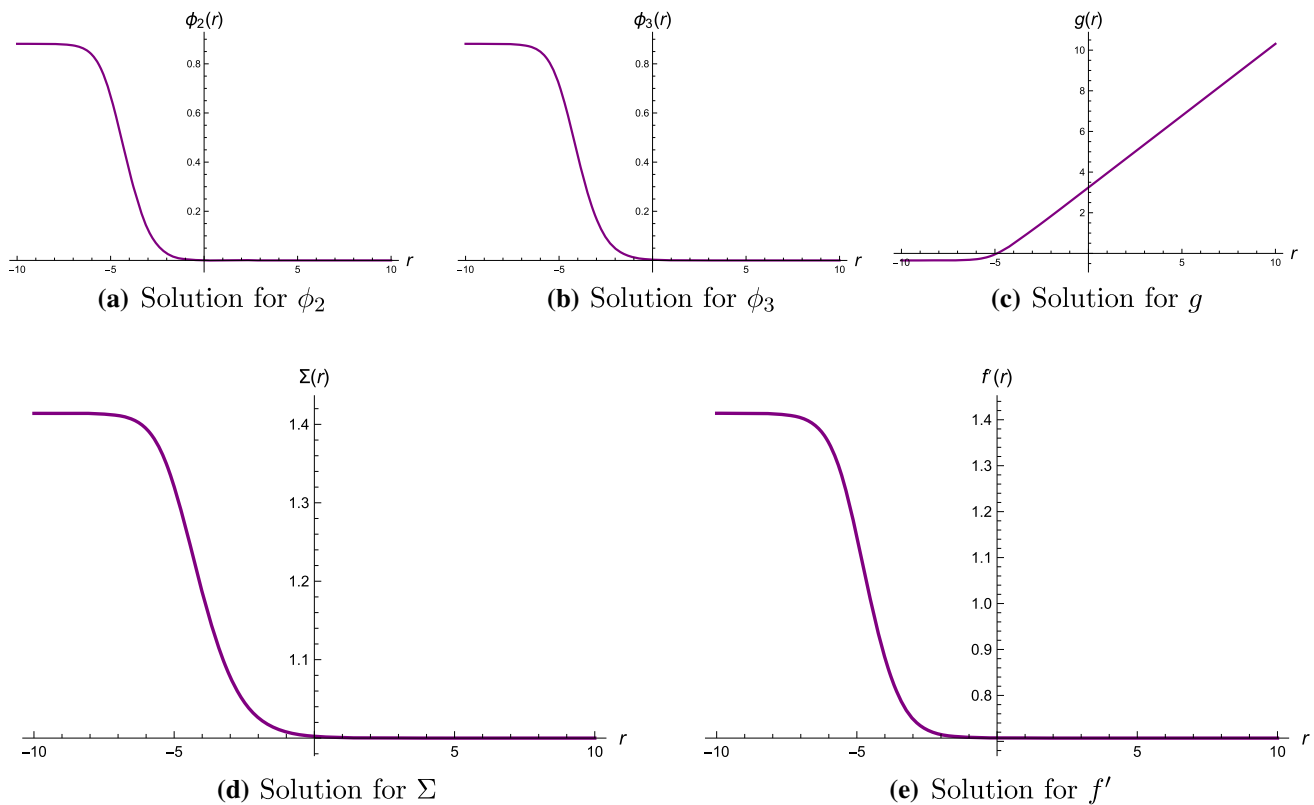


Fig. 5 An RG flow from AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to $AdS_3 \times H^2$ critical point III for $g_1 = 1$ and $g_3 = 2g_1$

Table 3 Scalar masses at the $N = 4$ supersymmetric AdS_5 critical point with $U(1) \times SO(3)$ symmetry and the corresponding dimensions of the dual operators for the non-compact $U(1) \times SO(3, 1)$ gauge group

Scalar field representations	$m^2 L^2$	Δ
$\mathbf{1}_0$	-4	2
$\mathbf{1}_0$	12	6
$\mathbf{1}_{\pm 2}$	-3×4	3
$\mathbf{3}_{\pm 2}$	5×6	5
$\mathbf{3}_0$	-4×6	2
$\mathbf{3}_0$	0×3	4
$\mathbf{5}_0$	0×5	4

$$V_0 = -3g_1^2 \quad \text{and} \quad L^2 = -\frac{6}{V_0} = \frac{2}{g_1^2}. \quad (110)$$

Scalar masses at this vacuum are given in Table 3. The spectrum is the same as that of the $N = 4$ AdS_5 with $U(1) \times SU(2)_{\text{diag}}$ symmetry in the compact gauge group. Massless scalars in $\mathbf{3}_0$ representation are Goldstone bosons of the symmetry breaking $SO(3, 1) \rightarrow SO(3)$.

4.1 BPS equations and holographic RG flow solutions

Since there is only one supersymmetric AdS_5 critical point, supersymmetric RG flows between AdS_5 critical points do not exist. We will look for solution describing a domain wall

with one limit being the AdS_5 critical point identified above and another limit being a singular geometry dual to an $N = 2$ non-conformal field theory.

With the same procedure as before, the superpotential in this case reads

$$W = \frac{e^{-3\phi} (1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi}) g_2 + 2\sqrt{2}g_1 \Sigma^3}{12\Sigma}. \quad (111)$$

It can be easily verified that W has only one critical point. The potential can be written in term of the superpotential as

$$V = \frac{3}{2} \left[\Sigma^2 \left(\frac{\partial W}{\partial \Sigma} \right)^2 + \left(\frac{\partial W}{\partial \phi} \right)^2 \right] - 6W^2. \quad (112)$$

The BPS equations for this gauge group are given by

$$\begin{aligned} \phi' &= - \left(\frac{\partial W}{\partial \phi} \right) = \frac{e^{-3\phi} (e^{2\phi} - e^{4\phi} + e^{6\phi} - 1) g_2}{4\Sigma}, \\ \Sigma' &= -\Sigma^2 \left(\frac{\partial W}{\partial \Sigma} \right) = \frac{1}{12} e^{-3\phi} \\ &\quad \times \left[4\sqrt{2}e^{3\phi} g_1 \Sigma^3 - (1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi}) g_2 \right], \\ A' &= W = \frac{e^{-3\phi} (1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi}) g_2 + 2\sqrt{2}g_1 \Sigma^3}{12\Sigma}. \end{aligned} \quad (113)$$

By combining these equations, the flow equations for the warp factor A and the dilaton Σ can be written as

$$\Sigma'(\phi) = \frac{\Sigma \left[4\sqrt{2}e^{3\phi}g_1\Sigma^3 - g_2(1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi}) \right]}{3g_2(e^{2\phi} - e^{4\phi} + e^{6\phi} - 1)}, \quad (114)$$

$$A'(\phi) = -\frac{(1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi})g_2 + 2\sqrt{2}e^{3\phi}g_1\Sigma^3}{3(e^{2\phi} - e^{4\phi} + e^{6\phi} - 1)g_2}. \quad (115)$$

The solution for Σ can be readily obtained

$$\Sigma = -\left[g_2 \frac{e^\phi(e^{2\phi} - 1)}{g_2 C_1(1 + e^{4\phi}) - \sqrt{2}g_1} \right]^{1/3}. \quad (116)$$

To make the flow approach the AdS_5 critical point, we choose the constant C_1 to be $C_1 = \frac{g_1}{\sqrt{2}g_2}$. This leads to

$$\Sigma = -\left[\frac{\sqrt{2}g_2 e^\phi}{g_1(1 + e^{2\phi})} \right]^{1/3} \quad (117)$$

which clearly gives $\Sigma = -\left(\frac{g_2}{\sqrt{2}g_1}\right)^{1/3}$ for $\phi = 0$.

With the solution for $\Sigma(\phi)$, the solution for A can be straightforwardly obtained. The result is

$$A = \frac{1}{6} \left[2\phi + 3 \ln(1 - e^{2\phi}) + \ln(1 + e^{2\phi}) - 3 \ln(1 + e^{4\phi}) \right]. \quad (118)$$

Finally, by redefining the radial coordinate r to ρ via $\frac{d\rho}{dr} = \Sigma^{-1}$, we find the solution for $\phi(\rho)$

$$2g_2\rho = 2 \ln(1 - e^\phi) - 2 \ln(1 + e^\phi) + \sqrt{2} \left[\ln(\sqrt{2}e^\phi + e^{2\phi} + 1) - \ln(\sqrt{2}e^\phi - e^{2\phi} - 1) \right] \quad (119)$$

where an additive integration constant has been discarded.

As $r \rightarrow \infty$, we find

$$\Sigma \sim e^{-\frac{2r}{L}} \quad \text{and} \quad \phi \sim e^{\frac{2r}{L}}. \quad (120)$$

The operator dual to ϕ is irrelevant as indicated by the value of $m^2 L^2 = 12$ in Table 3. From the solution (119), $\phi \rightarrow \pm\infty$ at a finite value of ρ . Explicitly, we find that, as $\phi \rightarrow \pm\infty$,

$$\phi \sim \frac{1}{3} \ln \left[C + \frac{3g_2\rho}{4} \right] \quad \text{and} \quad \phi \sim -\ln \left[C - \frac{g_1\rho}{2} \right] \quad (121)$$

for some constant C . In both cases, $\Sigma \rightarrow 0$ and $V \rightarrow \infty$. As a result, these singularities are unphysical by the criterion of [39].

4.2 RG flows to $AdS_3 \times \Sigma_2$ geometries

We now restrict ourselves to scalars which are invariant under $SO(2) \subset SO(3) \subset SO(3, 1)$ whose generator is $(T_5)_M^N = f_{5M}^N$. There are in total five singlets from $SO(5, 5)/SO(5) \times SO(5)$. However, as in the case of $U(1) \times SU(2) \times SU(2)$ gauge group, we can truncate this set to just three singlets corresponding to the following non-compact generators

$$\tilde{Y}_1 = Y_{31} + Y_{42}, \quad \tilde{Y}_2 = Y_{32} - Y_{41}, \quad \tilde{Y}_3 = Y_{53}. \quad (122)$$

The coset representative is given by

$$L = e^{\phi_1 \tilde{Y}_1} e^{\phi_2 \tilde{Y}_2} e^{\phi_3 \tilde{Y}_3}, \quad (123)$$

and the potential reads

$$V = \frac{1}{16} \Sigma^{-2} \left[g_2^2 [\cosh(4\phi_1 - 2\phi_3) + \cosh(4\phi_1 + 2\phi_3) + \cosh(2\phi_3) (6 + 4 \cosh(4\phi_2) \sinh^2(2\phi_1))] - 16g_2^2 + 16\Sigma^3 \sqrt{2}g_1g_2 [\cosh \phi_3 + \cosh(2\phi_2) \sinh(2\phi_1) \sinh(\phi_3)] \right] \quad (124)$$

which admits only a single supersymmetric critical point at which all vector multiplet scalars vanish.

The metric ansatz is still given by (60). We will consider the twists obtained from turning on $U(1) \times U(1) \subset U(1) \times SO(3, 1)$ gauge fields along Σ_2 . These gauge fields will be denoted by A^0 and A^5 . As in the previous section, the twists from A^0 and A^5 cannot be turned on simultaneously. Furthermore, the A^0 twist does not lead to $AdS_3 \times \Sigma_2$ solutions. We will therefore consider only the twist from A^5 . It turns out that the two-form fields can also be consistently set to zero provided that we set the gauge fields $A^1 = A^2 = 0$.

With the same ansatz as in (63), together with the projectors (44) and (68), we find the following BPS equations after using the twist condition $g_2 a_5 = 1$

$$f' = -\frac{1}{24\Sigma} e^{-2\phi_1 - 2\phi_2 - \phi_3 - 2g} \left[e^{2g} (1 - e^{4\phi_1} + e^{4\phi_2} + 4e^{2(\phi_1 + \phi_2)} - e^{4(\phi_1 + \phi_2)} - e^{2\phi_3} + 4e^{2(\phi_1 + \phi_2 + \phi_3)} + e^{4\phi_1 + 2\phi_3} - e^{4\phi_2 + 2\phi_3} + e^{4\phi_1 + 4\phi_2 + 2\phi_3}) g_2 - 4\kappa a_5 e^{2(\phi_1 + \phi_2)} (1 + e^{2\phi_3}) \Sigma^2 - 4\sqrt{2} e^{2\phi_1 + 2\phi_2 + \phi_3 + 2g} g_1 \Sigma^3 \right], \quad (125)$$

$$g' = -\frac{1}{24\Sigma} e^{-2\phi_1 - 2\phi_2 - \phi_3 - 2g} \left[e^{2g} (1 - e^{4\phi_1} + e^{4\phi_2} + 4e^{2(\phi_1 + \phi_2)} - e^{4(\phi_1 + \phi_2)} - e^{2\phi_3} + 4e^{2(\phi_1 + \phi_2 + \phi_3)} + e^{4\phi_1 + 2\phi_3} - e^{4\phi_2 + 2\phi_3} + e^{4\phi_1 + 4\phi_2 + 2\phi_3}) g_2 + 8\kappa a_5 e^{2(\phi_1 + \phi_2)} (1 + e^{2\phi_3}) \Sigma^2 - 4\sqrt{2} e^{2\phi_1 + 2\phi_2 + \phi_3 + 2g} g_1 \Sigma^3 \right], \quad (126)$$

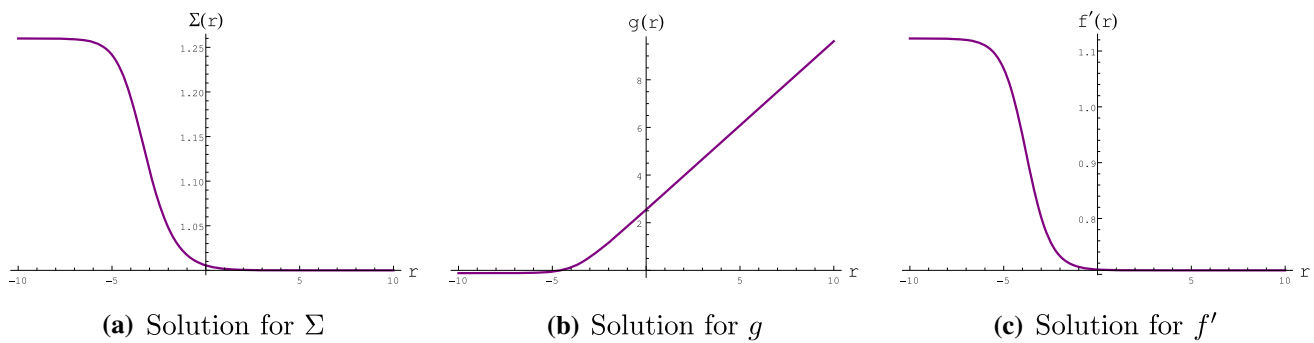


Fig. 6 An RG flow from $N = 4$ AdS_5 critical point with $U(1) \times SO(3)$ symmetry to $AdS_3 \times H^2$ geometry in the IR from $U(1) \times SO(3, 1)$ gauge group and $g_1 = 1$

$$\begin{aligned} \Sigma' = & -\frac{1}{24}e^{-2\phi_1-2\phi_2-\phi_3-2g} \left[e^{2g} \left(1 - e^{4\phi_1} + e^{4\phi_2} \right. \right. \\ & + 4e^{2(\phi_1+\phi_2)} - e^{4(\phi_1+\phi_2)} - e^{2\phi_3} \\ & + 4e^{2(\phi_1+\phi_2+\phi_3)} + e^{4\phi_1+2\phi_3} - e^{4\phi_2+2\phi_3} \\ & \left. \left. + e^{4\phi_1+4\phi_2+2\phi_3} \right) g_2 - 4\kappa a_5 e^{2(\phi_1+\phi_2)} (1 + e^{2\phi_3}) \Sigma^2 \right. \\ & \left. + 8\sqrt{2}e^{2\phi_1+2\phi_2+\phi_3+2g} g_1 \Sigma^3 \right], \end{aligned} \quad (127)$$

$$\phi_1' = \frac{e^{-2\phi_1+2\phi_2-\phi_3} (1 + e^{4\phi_1}) (e^{2\phi_3} - 1) g_2}{2(1 + e^{4\phi_2}) \Sigma}, \quad (128)$$

$$\phi_2' = \frac{e^{-2\phi_1-2\phi_2-\phi_3} (e^{4\phi_1} - 1) (e^{4\phi_2} - 1) (e^{2\phi_3} - 1) g_2}{8\Sigma}, \quad (129)$$

$$\begin{aligned} \phi_3' = & \frac{1}{8\Sigma}e^{-2\phi_1-2\phi_2-\phi_3-2g} \left[e^{2g} \left(e^{4\phi_1} - e^{4\phi_2} \right. \right. \\ & - 4e^{2(\phi_1+\phi_2)} + e^{4(\phi_1+\phi_2)} - e^{2\phi_3} - 1 \\ & + 4e^{2(\phi_1+\phi_2+\phi_3)} + e^{4\phi_1+2\phi_3} - e^{4\phi_2+2\phi_3} \\ & \left. \left. + e^{4\phi_1+4\phi_2+2\phi_3} \right) g_2 + 4\kappa a_5 e^{2(\phi_1+\phi_2)} (e^{2\phi_3} - 1) \Sigma^2 \right]. \end{aligned} \quad (130)$$

Unlike the compact gauge group considered in the previous section, the above equations admit only one $AdS_3 \times H^2$ solution given by

$$\begin{aligned} \phi_1 = \phi_2 = \phi_3 = 0, \quad \Sigma = & -\left(\frac{\sqrt{2}g_2}{g_1} \right)^{1/3}, \\ g = & -\frac{1}{2} \ln \left[\frac{1}{a_5} \left(\frac{g_1^2 g_2}{2} \right)^{1/3} \right], \quad L_{AdS_3} = \left(\frac{\sqrt{2}}{g_1 g_2^2} \right)^{1/3}. \end{aligned} \quad (131)$$

To find an RG flow solution interpolating between this $AdS_3 \times H^2$ and the AdS_5 critical point (109), we can consistently set all ϕ_i , $i = 1, 2, 3$, to zero and $\kappa = -1$. The

remaining BPS equations read

$$\begin{aligned} f' = & -\frac{2g_2 + 2a_5 e^{-2g} \Sigma^2 - \sqrt{2}g_1 \Sigma^3}{6\Sigma}, \\ g' = & -\frac{2g_2 - 4a_5 e^{-2g} \Sigma^2 - \sqrt{2}g_1 \Sigma^3}{6\Sigma}, \\ \Sigma' = & -\frac{1}{3} \left(g_2 + a_5 e^{-2g} \Sigma^2 + \sqrt{2}g_1 \Sigma^3 \right). \end{aligned} \quad (132)$$

A numerical solution to these equations is shown in Fig. 6. Similar to an analogous solution in the compact gauge group, this solution is the same as the universal RG flow considered in [28] since it does not involve scalars from vector multiplets.

5 Conclusions and discussions

We have studied gauged $N = 4$ supergravity in five dimensions coupled to five vector multiplets with compact and non-compact gauge groups $U(1) \times SU(2) \times SU(2)$ and $U(1) \times SO(3, 1)$. For $U(1) \times SU(2) \times SU(2)$ gauge group, we have recovered two supersymmetric $N = 4$ AdS_5 vacua with $U(1) \times SU(2) \times SU(2)$ and $U(1) \times SU(2)_{\text{diag}}$ symmetries together with the RG flow interpolating between them found in [11]. However, we have also given the full mass spectra for scalar fields at both critical points which have not been studied in [11]. These should be useful in the holographic context since it provides information about dimensions of operators dual to the supergravity scalars. For $U(1) \times SO(3, 1)$ gauge group, there is only one $N = 4$ supersymmetric AdS_5 critical point with vanishing vector multiplet scalars. We have given an RG flow solution from an $N = 2$ SCFT dual to this vacuum to a non-conformal field theory dual to a singular geometry. However, this singularity is unphysical within the framework of $N = 4$ gauged supergravity. It would be interesting to embed this solution in ten or eleven dimensions and further investigate whether this singularity is resolved or has any physical interpretation in the context of string/M-theory.

We have also considered $AdS_3 \times \Sigma_2$ solutions by turning on gauge fields along Σ_2 . We have found that in order to preserve eight supercharges, the twists from the $U(1)$ factor in the gauge group and the Cartan $U(1) \subset SU(2)$, denoted by the parameters a_0 and a_5 , cannot be performed simultaneously. It should also be noted that for less supersymmetric solutions, both a_0 and a_5 can be non-vanishing such as $\frac{1}{4}$ -BPS solutions found in [30] for pure $N = 4$ gauged supergravity with $U(1) \times SU(2)$ gauge group. It would also be interesting to look for more general solutions of this type.

For $U(1) \times SU(2) \times SU(2)$ gauge group, we have identified a number of $AdS_3 \times H^2$ solutions preserving eight supercharges. We have given numerical RG flow solutions from the two AdS_5 vacua to these $AdS_3 \times H^2$ geometries. For $U(1) \times SO(3, 1)$ gauge group, there is one $AdS_3 \times H^2$ solution when all scalars from vector multiplets vanish. The solution preserves eight supercharges similar to the solutions in the compact gauge group. A numerical RG flow between this solution and the $N = 4$ AdS_5 vacuum has also been given. All of these solutions describe twisted compactifications of $N = 2$ SCFTs on H^2 and should be of interest in holographic studies of $N = 2$ SCFTs in four dimensions and in the context of supersymmetric black strings. It is noteworthy that the space of AdS_5 and AdS_3 solutions in the compact gauge group is much richer than that of the non-compact gauge group. This is in line with similar studies of half-maximal gauged supergravities in other dimensions.

There are a number of future works extending our results presented here. It is interesting to consider flow solutions with non-vanishing two-form fields similar to the recently found solutions in seven and six dimensions in [40–42]. These solutions will also give a description of conformal defects in the dual $N = 2$ SCFTs. Furthermore, finding Janus solution within this $N = 4$ gauged supergravity is also of particular interest. This can be done by an analysis similar to that initiated in [43] and [44]. Up to now, this type of solutions has only appeared in $N = 8$ and $N = 2$ gauged supergravities, see for example [45, 46].

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Supersymmetric AdS_5 black holes and strings from 5D $N = 4$ gauged supergravity

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Abstract We study supersymmetric $AdS_3 \times \Sigma_2$ and $AdS_2 \times \Sigma_3$ solutions, with $\Sigma_2 = S^2, H^2$ and $\Sigma_3 = S^3, H^3$, in five-dimensional $N = 4$ gauged supergravity coupled to five vector multiplets. The gauge groups considered here are $U(1) \times SU(2) \times SU(2)$, $U(1) \times SO(3, 1)$ and $U(1) \times SL(3, \mathbb{R})$. For $U(1) \times SU(2) \times SU(2)$ gauge group admitting two supersymmetric $N = 4$ AdS_5 vacua, we identify a new class of $AdS_3 \times \Sigma_2$ and $AdS_2 \times H^3$ solutions preserving four supercharges. Holographic RG flows describing twisted compactifications of $N = 2$ four-dimensional SCFTs dual to the AdS_5 vacua to the SCFTs in two and one dimensions dual to these geometries are numerically given. The solutions can also be interpreted as supersymmetric black strings and black holes in asymptotically AdS_5 spaces with near horizon geometries given by $AdS_3 \times \Sigma_2$ and $AdS_2 \times H^3$, respectively. These solutions broaden previously known black brane solutions including half-supersymmetric AdS_5 black strings recently found in $N = 4$ gauged supergravity. Similar solutions are also studied in non-compact gauge groups $U(1) \times SO(3, 1)$ and $U(1) \times SL(3, \mathbb{R})$.

1 Introduction

Black branes of different spatial dimensions play an important role in the development of string/M-theory. They lead to many insightful results such as the construction of gauge theories in various dimensions and the celebrated AdS/CFT correspondence [1]. According to the latter, black branes in asymptotically AdS spaces are of particular interest since they are dual to RG flows across dimensions from superconformal field theories (SCFTs) dual to the asymptotically AdS spaces to lower-dimensional fixed points dual to the

near horizon geometries [2]. Recently, a new approach for computing microscopic entropy of AdS_4 black holes has been introduced based on twisted partition functions of three-dimensional SCFTs [3–11]. This has also been applied to AdS black holes in other dimensions [12–18].

In this paper, we are interested in supersymmetric black holes and black strings in asymptotically AdS_5 spaces from five-dimensional $N = 4$ gauged supergravity coupled to vector multiplets constructed in [19, 20] using the embedding tensor formalism [21–23]. These solutions have near horizon geometries of the forms $AdS_2 \times \Sigma_3$ and $AdS_3 \times \Sigma_2$, respectively. We will consider Σ_3 in the form of a three-sphere (S^3) and a three-dimensional hyperbolic space (H^3). Similarly, Σ_2 will be given by a two-sphere (S^2) and a two-dimensional hyperbolic space (H^2), or a Riemann surface of genus $g > 1$. Similar solutions have previously been found in minimal and maximal gauged supergravities, see for example [24–32]. This type of solutions has also appeared in pure $N = 4$ gauged supergravity in [33], and recently, half-supersymmetric black strings with hyperbolic horizons have been found in matter-coupled $N = 4$ gauged supergravity with compact $U(1) \times SU(2) \times SU(2)$ and non-compact $U(1) \times SO(3, 1)$ gauge groups [34].

We will look for more general solutions of AdS_5 black strings with both hyperbolic and spherical horizons and preserving $\frac{1}{4}$ of the $N = 4$ supersymmetry in five dimensions. The solutions interpolate between $N = 4$ supersymmetric AdS_5 vacua of the gauged supergravity and near horizon geometries of the form $AdS_3 \times \Sigma_2$. In addition, we will look for supersymmetric black holes interpolating between AdS_5 vacua and near horizon geometries $AdS_2 \times \Sigma_3$. According to the AdS/CFT correspondence, these solutions describe RG flows across dimensions from the dual $N = 2$ SCFTs to two- and one-dimensional SCFTs in the IR. The IR SCFTs are obtained via twisted compactifications of $N = 2$ SCFTs in four dimensions. Many solutions of this type have been

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found in various space-time dimensions, see [35–47] for an incomplete list.

We mainly consider $N = 4$ gauged supergravity coupled to five vector multiplets with gauge groups entirely embedded in the global symmetry $SO(5, 5)$. We will also restrict ourselves to gauge groups that lead to supersymmetric AdS_5 vacua. These gauge groups have been shown in [48] to take the form of $U(1) \times H_0 \times H$ with the $U(1)$ gauged by the graviphoton that is a singlet under $USp(4) \sim SO(5)$ R-symmetry. The $H \subset SO(n + 3 - \dim H_0)$ is a compact group gauged by vector fields in the vector multiplets, and H_0 is a non-compact group gauged by three of the graviphotons and $\dim H_0 - 3$ vectors from the vector multiplets. The remaining two graviphotons in the fundamental representation of $SO(5)$ are dualized to massive two-form fields. In addition, H_0 must contain an $SU(2)$ subgroup. For the case of five vector multiplets, possible gauge groups that admit supersymmetric AdS_5 vacua and can be embedded in $SO(5, 5)$ are $U(1) \times SU(2) \times SU(2)$, $U(1) \times SO(3, 1)$ and $U(1) \times SL(3, \mathbb{R})$. We will look for AdS_5 black string and black hole solutions in all of these gauge groups.

The paper is organized as follow. In Sect. 2, we review $N = 4$ gauged supergravity in five dimensions coupled to vector multiplets using the embedding tensor formalism. In Sect. 3, we find supersymmetric $AdS_3 \times \Sigma_2$ solutions preserving four supercharges and give numerical RG flow solutions interpolating between these geometries and supersymmetric AdS_5 vacua. An $AdS_2 \times H^3$ solution together with an RG flow interpolating between AdS_5 vacua and this geometry will also be given. In Sects. 4 and 5, we repeat the same analysis for non-compact $U(1) \times SO(3, 1)$ and $U(1) \times SL(3, \mathbb{R})$ gauge groups. Since the $U(1) \times SL(3, \mathbb{R})$ gauge group has not been studied in [34], we will discuss its construction and supersymmetric AdS_5 vacuum in detail. The full scalar mass spectrum at this critical point will also be given. This should be useful in the holographic context since it contains information on dimensions of operators dual to supergravity scalars. We end the paper with some conclusions and comments in Sect. 6.

2 Five dimensional $N = 4$ gauged supergravity coupled to vector multiplets

In this section, we briefly review the structure of five dimensional $N = 4$ gauged supergravity coupled to vector multiplets with the emphasis on formulae relevant for finding supersymmetric solutions. The detailed construction of $N = 4$ gauged supergravity can be found in [19, 20].

The $N = 4$ gravity multiplet consists of the graviton $e_{\mu}^{\hat{\mu}}$, four gravitini $\psi_{\mu i}$, six vectors A^0 and A_{μ}^m , four spin- $\frac{1}{2}$ fields χ_i and one real scalar Σ , the dilaton. Space-

time and tangent space indices are denoted respectively by $\mu, \nu, \dots = 0, 1, 2, 3, 4$ and $\hat{\mu}, \hat{\nu}, \dots = 0, 1, 2, 3, 4$. The $SO(5) \sim USp(4)$ R-symmetry indices are described by $m, n = 1, \dots, 5$ for the $SO(5)$ vector representation and $i, j = 1, 2, 3, 4$ for the $SO(5)$ spinor or $USp(4)$ fundamental representation. The gravity multiplet can couple to an arbitrary number n of vector multiplets. Each vector multiplet contains a vector field A_{μ} , four gaugini λ_i and five scalars ϕ^m . The n vector multiplets will be labeled by indices $a, b = 1, \dots, n$, and the components fields within these vector multiplets will be denoted by $(A_{\mu}^a, \lambda_i^a, \phi^{ma})$. From both gravity and vector multiplets, there are in total $6 + n$ vector fields which will be denoted by $A_{\mu}^{\mathcal{M}} = (A_{\mu}^0, A_{\mu}^m, A_{\mu}^a)$. All fermionic fields are described by symplectic Majorana spinors subject to the following condition

$$\xi_i = \Omega_{ij} C (\bar{\xi}^j)^T \quad (1)$$

with C and Ω_{ij} being respectively the charge conjugation matrix and $USp(4)$ symplectic form.

The $5n$ scalar fields from the vector multiplets parametrize the $SO(5, n)/SO(5) \times SO(n)$ coset. To describe this coset manifold, we introduce a coset representative $\mathcal{V}_M^A$ transforming under the global $SO(5, n)$ and the local $SO(5) \times SO(n)$ by left and right multiplications, respectively. We use indices $M, N, \dots = 1, 2, \dots, 5 + n$ for global $SO(5, n)$ indices. The local $SO(5) \times SO(n)$ indices $A, B, \dots$ will be split into $A = (m, a)$. We can accordingly write the coset representative as

$$\mathcal{V}_M^A = (\mathcal{V}_M^m, \mathcal{V}_M^a). \quad (2)$$

The matrix $\mathcal{V}_M^A$ is an element of $SO(5, n)$ and satisfies the relation

$$\eta_{MN} = \mathcal{V}_M^A \mathcal{V}_N^B \eta_{AB} = -\mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a \quad (3)$$

with $\eta_{MN} = \text{diag}(-1, -1, -1, -1, -1, 1, \dots, 1)$ being the $SO(5, n)$ invariant tensor. Equivalently, the $SO(5, n)/SO(5) \times SO(n)$ coset can also be described in term of a symmetric matrix

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a \quad (4)$$

which is manifestly invariant under the $SO(5) \times SO(n)$ local symmetry.

Gaugings promote a given subgroup G_0 of the full global symmetry $SO(1, 1) \times SO(5, n)$ of $N = 4$ supergravity coupled to n vector multiplets to be a local symmetry. These gaugings are efficiently described by using the embedding tensor formalism. $N = 4$ supersymmetry allows three components of the embedding tensor ξ^M , $\xi^{MN} = \xi^{[MN]}$ and $f_{MNP} = f_{[MNP]}$ [19]. The first component ξ^M describes the embedding of the gauge group in the $SO(1, 1) \sim \mathbb{R}^+$ factor identified with the coset space parametrized by the dilaton Σ . From the result of [48], the existence of $N = 4$ supersymmetric AdS_5 vacua requires $\xi^M = 0$. In this paper, we are

only interested in solutions that are asymptotically AdS_5 , so we will restrict ourselves to the gaugings with $\xi^M = 0$.

For $\xi^M = 0$, the gauge group is entirely embedded in $SO(5, n)$ with the gauge generators given by

$$(X_M)_N{}^P = -f_M{}^{QR}(t_{QR})_N{}^P = f_{MN}{}^P \quad \text{and} \\ (X_0)_N{}^P = -\xi^{QR}(t_{QR})_N{}^P = \xi_N{}^P. \quad (5)$$

The matrices $(t_{MN})_P{}^Q = \delta_{[M}^Q \eta_{N]P}$ are $SO(5, n)$ generators in the fundamental representation. The full covariant derivative reads

$$D_\mu = \nabla_\mu + A_\mu^M X_M + A_\mu^0 X_0 \quad (6)$$

where ∇_μ is the usual space-time covariant derivative. We use the convention that the definition of ξ^{MN} and f_{MNP} includes the gauge coupling constants. Note also that $SO(5, n)$ indices $M, N, \dots$ are lowered and raised by η_{MN} and its inverse η^{MN} , respectively.

Generators $X_M = (X_0, X_M)$ of a consistent gauge group must form a closed subalgebra of $SO(5, n)$. This requires ξ^{MN} and f_{MNP} to satisfy the quadratic constraints, see [19],

$$f_{R[MN} f_{PQ]}{}^R = 0 \quad \text{and} \quad \xi_M{}^Q f_{QNP} = 0. \quad (7)$$

Gauge groups that admit $N = 4$ supersymmetric AdS_5 vacua generally take the form of $U(1) \times H_0 \times H$, see [48] for more detail. The $U(1)$ is gauged by A_μ^0 while $H \subset SO(n+3 - \dim H_0)$ is a compact group gauged by vector fields in the vector multiplets. H_0 is a non-compact group gauged by three of the graviphotons and $\dim H_0 - 3$ vectors from the vector multiplets. H_0 must also contain an $SU(2)$ subgroup. For simple groups, H_0 can be $SU(2) \sim SO(3)$, $SO(3, 1)$ and $SL(3, \mathbb{R})$.

In the embedding tensor formalism, there are two-form fields $B_{\mu\nu M}$ that are introduced off-shell. These two-form fields do not have kinetic terms and couple to vector fields via a topological term. They satisfy a first-order field equation given by, see [19] for more detail,

$$\xi^{\mathcal{MN}} \left[\frac{1}{6\sqrt{2}} \epsilon_{\mu\nu\rho\lambda\sigma} \mathcal{H}_{\mathcal{N}}^{(3)\rho\lambda\sigma} - \mathcal{M}_{\mathcal{NP}} \mathcal{H}_{\mu\nu}^{\mathcal{P}} \right] = 0 \quad (8)$$

in which $\mathcal{M}_{00} = \Sigma^{-4}$, $\mathcal{M}_{0M} = 0$ and $\mathcal{M}_{MN} = \Sigma^2 M_{MN}$. The field strength $\mathcal{H}_{\mathcal{M}}^{(3)}$ is defined by

$$\xi^{\mathcal{MN}} \mathcal{H}_{\mu\nu\rho\mathcal{N}}^{(3)} = \xi^{\mathcal{MN}} \left[3D_{[\mu} B_{\nu\rho]\mathcal{N}} + 6d_{\mathcal{NPQ}} A_{[\mu}^{\mathcal{P}} \right. \\ \left. \times \left(\partial_\nu A_{\rho]}^{\mathcal{Q}} + \frac{1}{3} X_{RS}{}^{\mathcal{Q}} A_\nu^R A_{\rho]}^S \right) \right] \quad (9)$$

with $d_{0MN} = d_{MN0} = d_{M0N} = \eta_{MN}$ and

$$X_{MN}{}^P = -f_{MN}{}^P, \quad X_{M0}{}^0 = 0, \quad X_{0M}{}^N = -\xi_M{}^N. \quad (10)$$

In all of the solutions considered here, the Chern–Simons term in Eq. (9) vanish due to a particular form of the ansatz for the gauge fields. In addition, the term $\mathcal{M}_{\mathcal{NP}} \mathcal{H}_{\mu\nu}^{\mathcal{P}}$ in Eq.

(8) also vanish provided that the gauge fields A^1 and A^2 are set to zero. With all these, the two-form fields can be consistently truncated out. We will accordingly set all the two-form fields to zero from now on.

The bosonic Lagrangian of a general gauged $N = 4$ supergravity coupled to n vector multiplets can accordingly be written as

$$e^{-1} \mathcal{L} = \frac{1}{2} R - \frac{1}{4} \Sigma^2 M_{MN} \mathcal{H}_{\mu\nu}^M \mathcal{H}^{N\mu\nu} - \frac{1}{4} \Sigma^{-4} \mathcal{H}_{\mu\nu}^0 \mathcal{H}^{0\mu\nu} \\ - \frac{3}{2} \Sigma^{-2} D_\mu \Sigma D^\mu \Sigma + \frac{1}{16} D_\mu M_{MN} D^\mu M^{MN} \\ - V + e^{-1} \mathcal{L}_{\text{top}} \quad (11)$$

where e is the vielbein determinant. $\mathcal{L}_{\text{top}}$ is the topological term whose explicit form will not be given here since, given our ansatz for the gauge fields, it will not play any role in the present discussion.

With vanishing two-form fields, the covariant gauge field strength tensors read

$$\mathcal{H}_{\mu\nu}^{\mathcal{M}} = 2\partial_{[\mu} A_{\nu]}^{\mathcal{M}} + X_{\mathcal{NP}}{}^{\mathcal{M}} A_\mu^{\mathcal{N}} A_\nu^{\mathcal{P}}. \quad (12)$$

The scalar potential is given by

$$V = -\frac{1}{4} \left[f_{MNP} f_{QRS} \Sigma^{-2} \left(\frac{1}{12} M^{MQ} M^{NR} M^{PS} - \frac{1}{4} M^{MQ} \eta^{NR} \eta^{PS} \right. \right. \\ \left. \left. + \frac{1}{6} \eta^{MQ} \eta^{NR} \eta^{PS} \right) + \frac{1}{4} \xi_{MNP} \xi_{QRS} \Sigma^4 (M^{MP} M^{NQ} - \eta^{MP} \eta^{NQ}) \right. \\ \left. + \frac{\sqrt{2}}{3} f_{MNP} \xi_{QRS} \Sigma M^{MNPQRS} \right] \quad (13)$$

where M^{MN} is the inverse of M_{MN} , and M^{MNPQRS} is obtained from

$$M_{MNPQR} = \epsilon_{mnpqr} \mathcal{V}_M{}^m \mathcal{V}_N{}^n \mathcal{V}_P{}^p \mathcal{V}_Q{}^q \mathcal{V}_R{}^r \quad (14)$$

by raising the indices with η^{MN} .

Supersymmetry transformations of fermionic fields $(\psi_{\mu i}, \chi_i, \lambda_i^a)$ are given by

$$\delta\psi_{\mu i} = D_\mu \epsilon_i + \frac{i}{\sqrt{6}} \Omega_{ij} A_1^{jk} \gamma_\mu \epsilon_k \\ - \frac{i}{6} \left(\Omega_{ij} \Sigma \mathcal{V}_M{}^{jk} \mathcal{H}_{\nu\rho}^M - \frac{\sqrt{2}}{4} \delta_i^k \Sigma^{-2} \mathcal{H}_{\nu\rho}^0 \right) (\gamma_\mu{}^{\nu\rho} - 4\delta_\mu^\nu \gamma^\rho) \epsilon_k, \quad (15)$$

$$\delta\chi_i = -\frac{\sqrt{3}}{2} i \Sigma^{-1} D_\mu \Sigma \gamma^\mu \epsilon_i + \sqrt{2} A_2^{kj} \epsilon_k \\ - \frac{1}{2\sqrt{3}} \left(\Sigma \Omega_{ij} \mathcal{V}_M{}^{jk} \mathcal{H}_{\mu\nu}^M + \frac{1}{\sqrt{2}} \Sigma^{-2} \delta_i^k \mathcal{H}_{\mu\nu}^0 \right) \gamma^{\mu\nu} \epsilon_k, \quad (16)$$

$$\delta\lambda_i^a = i \Omega^{jk} (\mathcal{V}_M{}^a D_\mu \mathcal{V}_{ij}{}^M) \gamma^\mu \epsilon_k + \sqrt{2} \Omega_{ij} A_2^{ak} \epsilon_k \\ - \frac{1}{4} \Sigma \mathcal{V}_M{}^a \mathcal{H}_{\mu\nu}^M \gamma^{\mu\nu} \epsilon_i \quad (17)$$

in which the fermion shift matrices are defined by

$$\begin{aligned} A_1^{ij} &= -\frac{1}{\sqrt{6}} \left(\sqrt{2} \Sigma^2 \Omega_{kl} \mathcal{V}_M^{ik} \mathcal{V}_N^{jl} \xi^{MN} \right. \\ &\quad \left. + \frac{4}{3} \Sigma^{-1} \mathcal{V}^{ik}_M \mathcal{V}^{jl}_N \mathcal{V}^P_{kl} f^{MN}_P \right), \\ A_2^{ij} &= \frac{1}{\sqrt{6}} \left(\sqrt{2} \Sigma^2 \Omega_{kl} \mathcal{V}_M^{ik} \mathcal{V}_N^{jl} \xi^{MN} \right. \\ &\quad \left. - \frac{2}{3} \Sigma^{-1} \mathcal{V}^{ik}_M \mathcal{V}^{jl}_N \mathcal{V}^P_{kl} f^{MN}_P \right), \\ A_2^{aij} &= -\frac{1}{2} \left(\Sigma^2 \mathcal{V}_M^a \mathcal{V}_N^{ij} \xi^{MN} \right. \\ &\quad \left. - \sqrt{2} \Sigma^{-1} \Omega_{kl} \mathcal{V}_M^a \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f^{MNP} \right). \end{aligned} \quad (18)$$

In these equations, $\mathcal{V}_M^{ij}$ is defined in term of $\mathcal{V}_M^m$ as

$$\mathcal{V}_M^{ij} = \frac{1}{2} \mathcal{V}_M^m \Gamma_m^{ij} \quad (19)$$

where $\Gamma_m^{ij} = \Omega^{ik} \Gamma_{mk}^j$ and Γ_{mi}^j are $SO(5)$ gamma matrices. Similarly, the inverse element $\mathcal{V}_{ij}^M$ can be written as

$$\mathcal{V}_{ij}^M = \frac{1}{2} \mathcal{V}_m^M (\Gamma_m^{ij})^* = \frac{1}{2} \mathcal{V}_m^M \Gamma_m^{kl} \Omega_{ki} \Omega_{lj}. \quad (20)$$

In the subsequent analysis, we use the following explicit choice of $SO(5)$ gamma matrices Γ_{mi}^j given by

$$\begin{aligned} \Gamma_1 &= -\sigma_2 \otimes \sigma_2, \quad \Gamma_2 = i\mathbb{I}_2 \otimes \sigma_1, \quad \Gamma_3 = \mathbb{I}_2 \otimes \sigma_3, \\ \Gamma_4 &= \sigma_1 \otimes \sigma_2, \quad \Gamma_5 = \sigma_3 \otimes \sigma_2 \end{aligned} \quad (21)$$

where $\sigma_i, i = 1, 2, 3$ are the usual Pauli matrices.

The covariant derivative on ϵ_i reads

$$D_\mu \epsilon_i = \partial_\mu \epsilon_i + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon_i + Q_{\mu i}^j \epsilon_j \quad (22)$$

where the composite connection is defined by

$$\begin{aligned} Q_{\mu i}^j &= \mathcal{V}_{ik}^M \partial_\mu \mathcal{V}_M^{kj} - A_\mu^0 \xi^{MN} \mathcal{V}_{Mik} \mathcal{V}_N^{kj} \\ &\quad - A_\mu^M \mathcal{V}_{ik}^N \mathcal{V}^{kjP} f_{MNP}. \end{aligned} \quad (23)$$

In this work, we mainly focus on the case of $n = 5$ vector multiplets. To parametrize the scalar coset $SO(5, 5)/SO(5) \times SO(5)$, it is useful to introduce a basis for $GL(10, \mathbb{R})$ matrices

$$(e_{MN})_{PQ} = \delta_{MP} \delta_{NQ} \quad (24)$$

in terms of which $SO(5, 5)$ non-compact generators are given by

$$\begin{aligned} Y_{ma} &= e_{m,a+5} + e_{a+5,m}, \quad m = 1, 2, \dots, 5, \\ a &= 1, 2, \dots, 5. \end{aligned} \quad (25)$$

3 $U(1) \times SU(2) \times SU(2)$ gauge group

For a compact $U(1) \times SU(2) \times SU(2)$ gauge group, components of the embedding tensor are given by

$$\xi^{MN} = g_1 (\delta_2^M \delta_1^N - \delta_1^M \delta_2^N), \quad (26)$$

$$f_{\tilde{m}+2, \tilde{n}+2, \tilde{p}+2} = -g_2 \epsilon_{\tilde{m}\tilde{n}\tilde{p}}, \quad \tilde{m}, \tilde{n}, \tilde{p} = 1, 2, 3, \quad (27)$$

$$f_{abc} = g_3 \epsilon_{abc}, \quad a, b, c = 1, 2, 3 \quad (28)$$

where g_1, g_2 and g_3 are the coupling constants for each factor in $U(1) \times SU(2) \times SU(2)$.

The scalar potential obtained from truncating the scalars from vector multiplets to $U(1) \times SU(2)_{\text{diag}} \subset U(1) \times SU(2) \times SU(2)$ singlets has been studied in [34]. There is one $U(1) \times SU(2)_{\text{diag}}$ singlet from the $SO(5, 5)/SO(5) \times SO(5)$ coset corresponding to the following $SO(5, 5)$ non-compact generator

$$Y_s = Y_{31} + Y_{42} + Y_{53}. \quad (29)$$

With the coset representative given by

$$\mathcal{V} = e^{\phi Y_s}, \quad (30)$$

the scalar potential can be computed to be

$$\begin{aligned} V &= \frac{1}{32 \Sigma^2} \left[32 \sqrt{2} g_1 g_2 \Sigma^3 \cosh^3 \phi - 9(g_2^2 + g_3^2) \cosh(2\phi) \right. \\ &\quad \left. - 8(g_2^2 - g_3^2 - 4\sqrt{2} g_1 g_3 \Sigma^3 \sinh^3 \phi - g_2 g_3 \sinh^3 \phi) \right. \\ &\quad \left. + (g_2^2 + g_3^2) \cosh(6\phi) \right]. \end{aligned} \quad (31)$$

The potential admits two $N = 4$ supersymmetric AdS_5 critical points given by

$$\text{i} : \phi = 0, \quad \Sigma = 1, \quad V_0 = -3g_1^2 \quad (32)$$

$$\text{ii} : \phi = \frac{1}{2} \ln \left[\frac{g_3 - g_2}{g_3 + g_2} \right], \quad \Sigma = \left(\frac{g_2 g_3}{g_1 \sqrt{2(g_3^2 - g_2^2)}} \right)^{\frac{1}{3}},$$

$$V_0 = -3 \left(\frac{g_1 g_2^2 g_3^2}{2(g_3^2 - g_2^2)} \right)^{\frac{2}{3}}. \quad (33)$$

In critical point i, we have set $g_2 = -\sqrt{2}g_1$ to make this critical point occur at $\Sigma = 1$. However, we will keep g_2 explicit in most expressions for brevity. Critical point i is invariant under the full gauge symmetry $U(1) \times SU(2) \times SU(2)$ while critical point ii preserves only $U(1) \times SU(2)_{\text{diag}}$ symmetry due to the non-vanishing scalar ϕ . V_0 denotes the cosmological constant, the value of the scalar potential at a critical point.

3.1 Supersymmetric black strings

We now consider vacuum solutions of the form $AdS_3 \times \Sigma_2$ with Σ_2 being S^2 or H^2 . A number of $AdS_3 \times H^2$ solutions that preserve eight supercharges together with RG flows

interpolating between them and supersymmetric AdS_5 critical points have already been given in [34]. In this section, we look for more general solutions that preserve only four supercharges.

We begin with the metric ansatz for the $\Sigma_2 = S^2$ case

$$ds^2 = e^{2f(r)} dx_{1,1}^2 + dr^2 + e^{2g(r)} (d\theta^2 + \sin^2 \theta d\phi^2) \quad (34)$$

where $dx_{1,1}^2$ is the flat metric in two dimensions. For $\Sigma_2 = H^2$, the metric is given by

$$ds^2 = e^{2f(r)} dx_{1,1}^2 + dr^2 + e^{2g(r)} (d\theta^2 + \sinh^2 \theta d\phi^2). \quad (35)$$

As $r \rightarrow \infty$, the metric becomes locally AdS_5 with $f(r) \sim g(r) \sim \frac{r}{L_{AdS_5}}$ while the near horizon geometry is characterized by the conditions $f(r) \sim \frac{r}{L_{AdS_3}}$ and constant $g(r)$, or equivalently $g'(r) = 0$.

To preserve some amount of supersymmetry, we perform a twist by cancelling the spin connection along the Σ_2 by some suitable choice of gauge fields. We will first consider abelian twists from the $U(1) \times U(1) \times U(1)$ subgroup of the $U(1) \times SU(2) \times SU(2)$ gauge symmetry. The gauge fields corresponding to this subgroup will be denoted by (A^0, A^5, A^8) . The ansatz for these gauge fields will be chosen as

$$A^{\mathcal{M}=0,5,8} = a_{\mathcal{M}} \cos \theta d\phi. \quad (36)$$

for the S^2 case and

$$A^{\mathcal{M}=0,5,8} = a_{\mathcal{M}} \cosh \theta d\phi. \quad (37)$$

for the H^2 case.

3.1.1 Solutions with $U(1) \times U(1) \times U(1)$ symmetry

There are three singlets from the $SO(5, 5)/SO(5) \times SO(5)$ coset corresponding to the $SO(5, 5)$ non-compact generators Y_{53} , Y_{54} and Y_{55} . However, these can be consistently truncated to only a single scalar with the coset representative given by

$$\mathcal{V} = e^{\varphi Y_{53}}. \quad (38)$$

We now begin with the analysis for $\Sigma_2 = S^2$. With the relevant component of the spin connection $\omega^{\hat{\phi}\hat{\theta}} = e^{-g} \cot \theta e^{\hat{\phi}}$, we find the covariant derivative of ϵ_i along the $\hat{\phi}$ direction

$$D_{\hat{\phi}} \epsilon_i = \dots + \frac{1}{2} e^{-g} \cot \theta \left[\gamma_{\hat{\phi}\hat{\theta}} \epsilon_i - i a_0 g_1 (\sigma_2 \otimes \sigma_3)_i{}^j \epsilon_j + i a_5 g_2 (\sigma_1 \otimes \sigma_1)_i{}^j \epsilon_j \right] \quad (39)$$

where $\dots$ refers to the term involving g' that is not relevant to the present discussion. Note also that a_8 does not appear in the above equation since A^8 is not part of the R-symmetry

under which the gravitini and supersymmetry parameters are charged.

For half-supersymmetric solutions considered in [34], it has been shown that the twists from A^0 and A^5 can not be performed simultaneously, and there exist only $AdS_3 \times H^2$ solutions. However, if we allow for an extra projector such that only $\frac{1}{4}$ of the original supersymmetry is unbroken, it is possible to keep both the twists from A^0 and A^5 non-vanishing. To achieve this, we note that

$$i\sigma_2 \otimes \sigma_3 = i(\sigma_1 \otimes \sigma_1)(\sigma_3 \otimes \sigma_2). \quad (40)$$

We then impose the following projector to make the two terms with a_0 and a_5 in (39) proportional

$$(\sigma_3 \otimes \sigma_2)_i{}^j \epsilon_j = -\epsilon_i. \quad (41)$$

To cancel the spin connection, we then impose another projector

$$i\gamma_{\hat{\theta}\hat{\phi}} \epsilon_i = -(\sigma_1 \otimes \sigma_1)_i{}^j \epsilon_j. \quad (42)$$

and the twist condition

$$a_0 g_1 + a_5 g_2 = 1. \quad (43)$$

It should be noted that the condition (43) reduces to that of [34] for either $a_0 = 0$ or $a_5 = 0$. However, the solutions in this case preserve only four supercharges, or $N = 2$ supersymmetry in three dimensions, due to the additional projector (41).

To setup the BPS equations, we also need the γ_r projection due to the radial dependence of scalars. Following [34], this projector is given by

$$\gamma_r \epsilon_i = I_i{}^j \epsilon_j \quad (44)$$

with $I_i{}^j$ defined by

$$I_i{}^j = (\sigma_2 \otimes \sigma_3)_i{}^j. \quad (45)$$

The covariant field strength tensors for the gauge fields in (36) can be straightforwardly computed, and the result is

$$\mathcal{H}^{\mathcal{M}} = -a_{\mathcal{M}} \sin \theta d\theta \wedge d\phi. \quad (46)$$

For $\Sigma_2 = H^2$, the cancellation of the spin connection $\omega^{\hat{\phi}\hat{\theta}} = e^{-g} \coth \theta e^{\hat{\phi}}$ is again achieved by the gauge field ansatz (37) using the conditions (41), (42) and (43). On the other hand, the covariant field strengths are now given by

$$\mathcal{H}^{\mathcal{M}} = a_{\mathcal{M}} \sinh \theta d\theta \wedge d\phi. \quad (47)$$

which have opposite signs to those of the S^2 case. This results in a sign change of the parameter (a_0, a_5, a_8) in the corresponding BPS equations.

With all these, we obtain the following BPS equations

$$\begin{aligned} \varphi' &= \frac{1}{2} \Sigma^{-1} e^{-\varphi-2g} \left[g_2 e^{2g} (e^{2\varphi} - 1) \right. \\ &\quad \left. - \kappa \Sigma^2 (a_5 - a_8 - e^{2\varphi} (a_5 + a_8)) \right], \end{aligned} \quad (48)$$

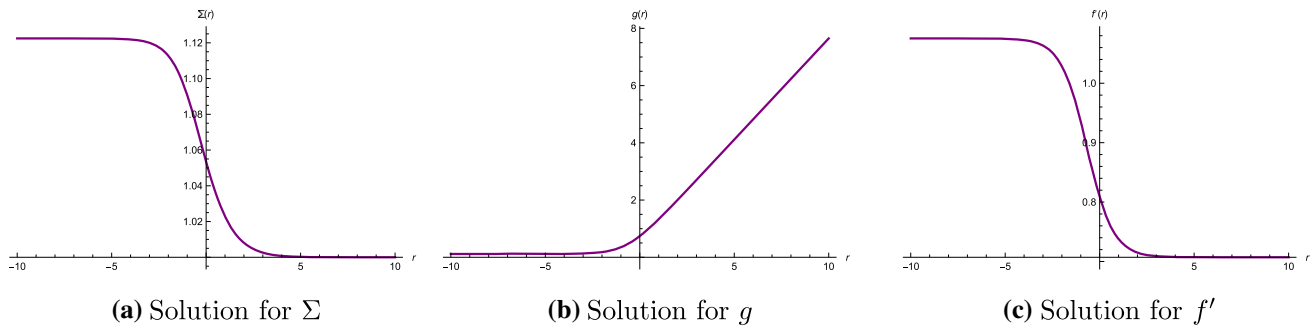


Fig. 1 An RG flow from $N = 4$ AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to $N = 2$ $AdS_3 \times S^2$ geometry in the IR with $U(1) \times U(1) \times SU(2)$ symmetry and $g_1 = 1$, $a_5 = 1$ and $a_8 = 0$

$$\Sigma' = -\frac{1}{3}(\sqrt{2}g_1\Sigma^3 + g_2 \cosh \varphi) + \frac{1}{3}\Sigma^{-1}e^{-2g}[-\sqrt{2}\kappa a_0 + \kappa \Sigma^3(a_5 \cosh \varphi + a_8 \sinh \varphi)], \quad (49)$$

$$g' = \frac{1}{6}\Sigma^{-2} \left[\sqrt{2}g_1\Sigma^4 - 2\sqrt{2}\kappa a_0 e^{-2g} - 2g_2 \cosh \varphi \Sigma - 4\kappa \Sigma^3 e^{-2g}(a_5 \cosh \varphi + a_8 \sinh \varphi) \right], \quad (50)$$

$$f' = \frac{1}{6}\Sigma^{-2} \left[\sqrt{2}g_1\Sigma^4 + \sqrt{2}\kappa a_0 e^{-2g} - 2g_2 \cosh \varphi \Sigma + 2\kappa \Sigma^3 e^{-2g}(a_5 \cosh \varphi + a_8 \sinh \varphi) \right]. \quad (51)$$

In these equations, $\kappa = 1$ and $\kappa = -1$ refer to $\Sigma_2 = S^2$ and $\Sigma_2 = H^2$, respectively. It can also be readily verified that these equations also imply the second order field equations.

We now look for AdS_3 solutions from the above BPS equations. These solutions are characterized by the conditions $g' = \varphi' = \Sigma' = 0$ and $f' = \frac{1}{L_{AdS_3}}$. We find the following AdS_3 solutions.

- For $\varphi = 0$, AdS_3 solutions only exist for $a_8 = 0$ and are given by

$$\Sigma = \frac{2^{\frac{1}{6}}\kappa}{(a_5 g_1)^{\frac{1}{3}}}, \quad g = \frac{1}{6} \ln \left(\frac{2a_5^4}{g_1^2} \right),$$

$$L_{AdS_3} = \frac{2^{\frac{7}{6}}a_5^{\frac{2}{3}}}{g_1^{\frac{1}{3}}(1 - \kappa a_5 g_2)}. \quad (52)$$

This should be identified with similar solutions of pure $N = 4$ gauged supergravity found in [33]. Since a_8 and φ vanish in this case, the AdS_3 solution has a larger symmetry $U(1) \times U(1) \times SU(2)$. Note also that unlike half-supersymmetric solutions that exist only for $\Sigma_2 = H^2$, both $\Sigma_2 = S^2$ and $\Sigma_2 = H^2$ are possible by appropriately chosen values of a_0 , a_5 and g_1 , recall that $g_2 = -\sqrt{2}g_1$.

- For $\varphi \neq 0$, we find a class of solutions

$$\varphi = \frac{1}{2} \ln \left[\frac{(a_5 - a_8)(a_0 g_1 - a_8 g_2)}{(a_5 + a_8)(a_0 g_1 + a_8 g_2)} \right],$$

$$\Sigma = \left(\frac{\sqrt{2}\kappa a_0}{\sqrt{(a_5^2 - a_8^2)(a_0^2 g_1^2 - a_8^2 g_2^2)}} \right)^{\frac{1}{3}},$$

$$g = \frac{1}{6} \ln \left[\frac{2a_0^2(a_5^2 - a_8^2)}{a_0^2 g_1^2 - a_8^2 g_2^2} \right],$$

$$L_{AdS_3} = \frac{2^{\frac{7}{6}}a_0^{\frac{1}{3}}(a_5^2 - a_8^2)^{\frac{1}{3}}(a_0^2 g_1^2 - a_8^2 g_2^2)^{\frac{1}{3}}}{a_0 g_1(1 - \kappa a_5 g_2) - \kappa g_2^2 a_8^2}. \quad (53)$$

Note that when $a_8 = 0$, we recover the AdS_3 solutions in (52). As in the previous solution, it can also be verified that these AdS_3 solutions exist for both $\Sigma_2 = S^2$ and $\Sigma_2 = H^2$.

Examples of numerical solutions interpolating between $N = 4$ AdS_5 vacuum with $U(1) \times SU(2) \times SU(2)$ symmetry to these $AdS_3 \times \Sigma_2$ are shown in Figs. 1 and 2. At large r , the solutions are asymptotically $N = 4$ supersymmetric AdS_5 critical point i given in (32). It should also be noted that the flow solutions preserve only two supercharges due to the γ_r projector imposed along the flow.

3.1.2 Solutions with $U(1) \times U(1)_{diag}$ symmetry

We now move to a set of scalars with smaller unbroken symmetry $U(1) \times U(1)_{diag}$ with $U(1)_{diag}$ being a diagonal subgroup of $U(1) \times U(1) \subset SU(2) \times SU(2)$. As pointed out in [34], there are five singlets from the vector multiplet scalars but these can be truncated to three scalars corresponding to the following non-compact generators of $SO(5, 5)$

$$\hat{Y}_1 = Y_{31} + Y_{42}, \quad \hat{Y}_2 = Y_{53}, \quad \hat{Y}_3 = Y_{32} - Y_{41}. \quad (54)$$

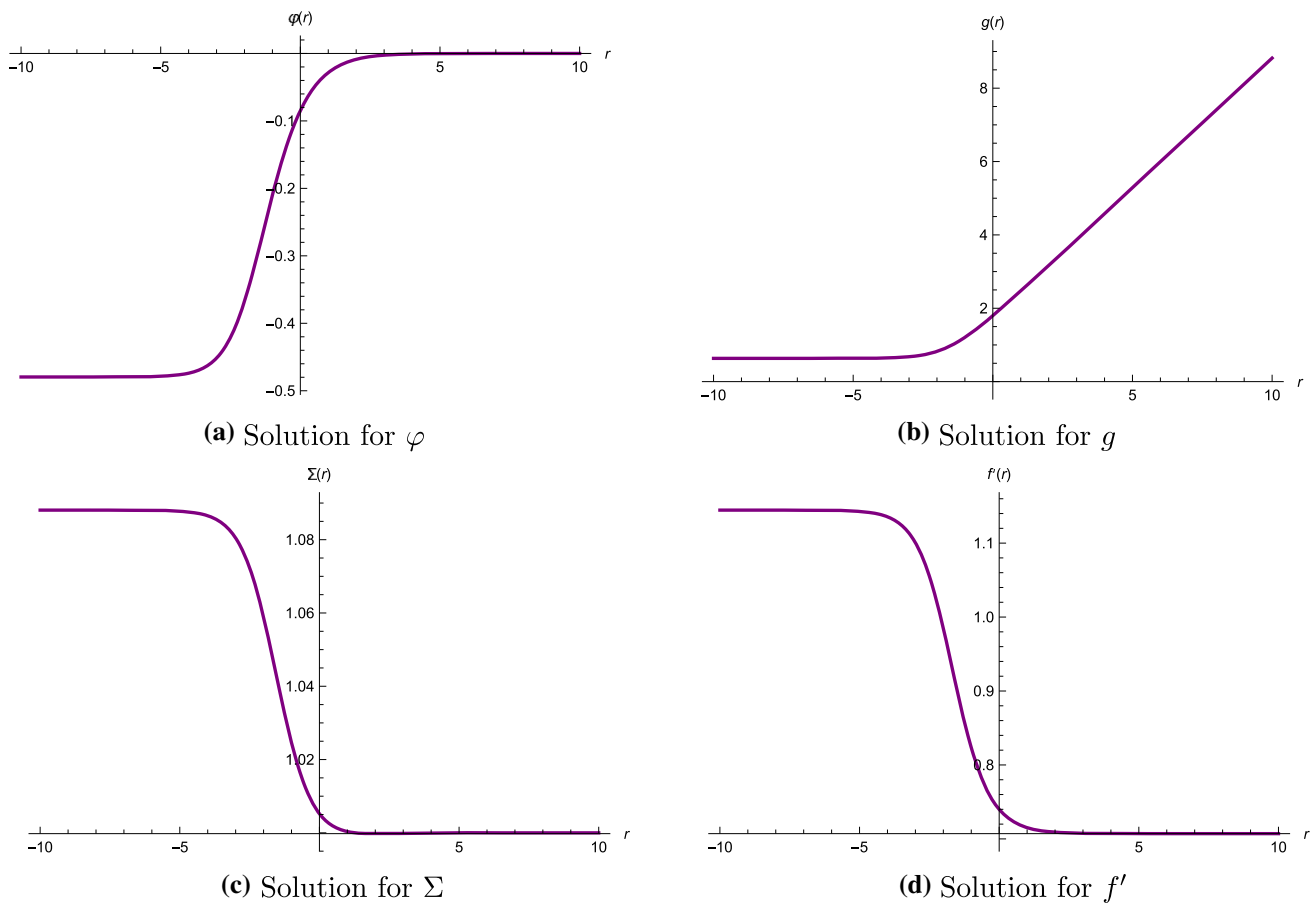


Fig. 2 An RG flow from $N = 4$ AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to $N = 2$ $AdS_3 \times S^2$ geometry in the IR with $U(1) \times U(1) \times U(1)$ symmetry and $g_1 = 1$, $a_5 = 2$ and $a_8 = -1$

The coset representative is then given by

$$\mathcal{V} = e^{\phi_1 \hat{Y}_1} e^{\phi_2 \hat{Y}_2} e^{\phi_3 \hat{Y}_3}. \quad (55)$$

To implement the $U(1)_{\text{diag}}$ gauge symmetry, we impose an additional condition on the parameters a_5 and a_8 as follow

$$g_2 a_5 = g_3 a_8. \quad (56)$$

We can repeat the previous analysis for the $U(1) \times U(1) \times U(1)$ twists, and the result is the same as in the previous case with the twist condition (43) and projectors (41), (42) and (44).

With the same procedure as in the previous case, we obtain the following BPS equations

$$\phi'_1 = \frac{1}{2} \Sigma^{-1} \text{sech}(2\phi_3) \sinh(2\phi_1) (g_2 \cosh \phi_2 + g_3 \sinh \phi_2), \quad (57)$$

$$\begin{aligned} \phi'_2 = & \frac{1}{2} \Sigma^{-1} \cosh(2\phi_1) \cosh(2\phi_3) (g_2 \sinh \phi_2 + g_3 \cosh \phi_2) \\ & + \frac{1}{2} \Sigma^{-1} (g_2 \sinh \phi_2 - g_3 \cosh \phi_2) \\ & + \frac{a_5 \kappa}{g_3} e^{-2g} \Sigma (g_2 \cosh \phi_2 + g_3 \sinh \phi_2), \end{aligned} \quad (58)$$

$$\begin{aligned} \phi'_3 = & \frac{1}{2} \Sigma^{-1} \cosh(2\phi_1) \sinh(2\phi_3) \\ & \times (g_2 \cosh \phi_2 + g_3 \sinh \phi_2), \end{aligned} \quad (59)$$

$$\begin{aligned} \Sigma' = & -\frac{1}{6g_3} \Sigma^{-1} e^{-2g} \left[-2\kappa a_5 \Sigma^3 (g_3 \cosh \phi_2 \right. \\ & + g_2 \sinh \phi_2) + 2\sqrt{2}\kappa g_3 a_0 \\ & + e^{2g} g_3 \Sigma \left[\cosh(2\phi_1) \cosh(2\phi_3) \right. \\ & \times (g_2 \cosh \phi_2 + g_3 \sinh \phi_2) \\ & \left. \left. g_2 \cosh \phi_2 - g_3 \sinh \phi_2 + 2\sqrt{2}g_1 \Sigma^3 \right] \right], \end{aligned} \quad (60)$$

$$\begin{aligned} g' = & \frac{1}{6g_3} \Sigma^{-2} \left[g_3 \Sigma (g_3 \sinh \phi_2 \right. \\ & - g_2 \cosh \phi_2) - 2\sqrt{2}\kappa a_0 g_3 e^{-2g} \\ & - \Sigma \cosh(2\phi_1) \cosh(2\phi_3) (g_2 \cosh \phi_2 \\ & + g_3 \sinh \phi_2) + \sqrt{2}g_1 g_3 \Sigma^4 \\ & \left. - 4\kappa a_5 e^{-2g} \Sigma^3 (g_3 \cosh \phi_2 + g_2 \sinh \phi_2) \right], \end{aligned} \quad (61)$$

$$\begin{aligned} f' = & \frac{1}{6g_3} \Sigma^{-2} \left[g_3 \Sigma (g_3 \sinh \phi_2 \right. \\ & - g_2 \cosh \phi_2) + \sqrt{2}\kappa a_0 g_3 e^{-2g} \end{aligned}$$

$$\begin{aligned}
& -\Sigma \cosh(2\phi_1) \cosh(2\phi_3)(g_2 \cosh \phi_2 \\
& + g_3 \sinh \phi_2) + \sqrt{2}g_1g_3\Sigma^4 \\
& + 2\kappa a_5 e^{-2g}\Sigma^3(g_3 \cosh \phi_2 + g_2 \sinh \phi_2) \Big]. \quad (62)
\end{aligned}$$

From these equations, we find the following $AdS_3 \times \Sigma_2$ solutions.

- For $\phi_1 = \phi_3 = 0$, there is a family of AdS_3 solutions given by

$$\begin{aligned}
\text{I : } \phi_2 &= \frac{1}{2} \ln \left[\frac{(g_2 - g_3)(g_2^2 a_5 - a_0 g_1 g_3)}{(g_2 + g_3)(g_2^2 a_5 + a_0 g_1 g_3)} \right], \\
g &= \frac{1}{6} \ln \left[\frac{2a_0^2 a_5^4 (g_3^2 - g_2^2)^2}{g_3^2 (a_0^2 g_1^2 g_3^2 - a_5^2 g_2^4)} \right], \\
\Sigma &= - \left[\frac{\sqrt{2} a_0 g_3^2}{a_5 \sqrt{(g_3^2 - g_2^2)(a_0^2 g_1^2 g_3^2 - a_5^2 g_2^4)}} \right]^{\frac{1}{3}}. \quad (63)
\end{aligned}$$

We refrain from giving the explicit form of L_{AdS_3} at this vacuum due to its complexity.

- For $\phi_3 = 0$, we find

$$\begin{aligned}
\text{II : } \phi_2 = \phi_1 &= \frac{1}{2} \ln \left[\frac{g_3 - g_2}{g_3 + g_2} \right], \\
\Sigma &= \left[\frac{\sqrt{2} \kappa g_3}{g_1 a_5 \sqrt{g_3^2 - g_2^2}} \right]^{\frac{1}{3}}, \\
g &= \frac{1}{6} \ln \left[\frac{2a_5^4 (g_3^2 - g_2^2)^2}{g_1^2 g_3^4} \right], \\
L_{AdS_3} &= \left[\frac{8\sqrt{2} a_5^2 (g_3^2 - g_2^2)}{g_1 g_3^2 (1 - \kappa a_5 g_2)^3} \right]^{\frac{1}{3}}. \quad (64)
\end{aligned}$$

- Finally, for $\phi_1 = 0$, we find

$$\begin{aligned}
\text{III : } \phi_2 = \phi_3 &= \frac{1}{2} \ln \left[\frac{g_3 - g_2}{g_3 + g_2} \right], \\
\Sigma &= \left[\frac{\sqrt{2} \kappa g_3}{g_1 a_5 \sqrt{g_3^2 - g_2^2}} \right]^{\frac{1}{3}}, \\
g &= \frac{1}{6} \ln \left[\frac{2a_5^4 (g_3^2 - g_2^2)^2}{g_1^2 g_3^4} \right], \\
L_{AdS_3} &= \left[\frac{8\sqrt{2} a_5^2 (g_3^2 - g_2^2)}{g_1 g_3^2 (1 - \kappa a_5 g_2)^3} \right]^{\frac{1}{3}}. \quad (65)
\end{aligned}$$

Unlike the previous case, at large r , we find that solutions to these BPS equations can be asymptotic to any of the two $N = 4$ supersymmetric AdS_5 vacua i and ii given in (32) and (33). Therefore, we can have RG flows from the two AdS_5 vacua to any of these $AdS_3 \times \Sigma_2$ solutions. Some examples of these solutions for $\Sigma_2 = S^2$ are given in Figs. 3, 4, 5 and 6.

3.2 Supersymmetric black holes

We now move to another type of solutions, supersymmetric AdS_5 black holes. We will consider near horizon geometries of the form $AdS_2 \times \Sigma_3$ for $\Sigma_3 = S^3$ and $\Sigma_3 = H^3$. The twist procedure is still essential to preserve supersymmetry. For the S^3 case, we take the metric to be

$$\begin{aligned}
ds^2 &= -e^{2f(r)} dt^2 + dr^2 + e^{2g(r)} \\
&\times \left[d\psi^2 + \sin^2 \psi (d\theta^2 + \sin^2 \theta d\phi^2) \right]. \quad (66)
\end{aligned}$$

With the following choice of vielbein

$$\begin{aligned}
e^{\hat{t}} &= e^f dt, \quad e^{\hat{r}} = dr, \quad e^{\hat{\psi}} = e^g d\psi, \\
e^{\hat{\theta}} &= e^g \sin \psi d\theta, \quad e^{\hat{\phi}} = e^g \sin \psi \sin \theta d\phi, \quad (67)
\end{aligned}$$

we obtain non-vanishing components of the spin connection

$$\begin{aligned}
\omega^{\hat{t}}_{\hat{r}} &= f' e^{\hat{t}}, \quad \omega^{\hat{\psi}}_{\hat{r}} = g' e^{\hat{\psi}}, \quad \omega^{\hat{\theta}}_{\hat{r}} = g' e^{\hat{\theta}}, \quad \omega^{\hat{\phi}}_{\hat{r}} = g' e^{\hat{\phi}}, \\
\omega^{\hat{\phi}}_{\hat{\theta}} &= e^{-g} \frac{\cot \theta}{\sin \psi} e^{\hat{\phi}}, \\
\omega^{\hat{\phi}}_{\hat{\psi}} &= e^{-g} \cot \psi e^{\hat{\phi}}, \quad \omega^{\hat{\theta}}_{\hat{\psi}} = e^{-g} \cot \psi e^{\hat{\theta}}. \quad (68)
\end{aligned}$$

We then turn on gauge fields corresponding to the $U(1) \times SU(2)_{\text{diag}} \subset U(1) \times SU(2) \times SU(2)$ symmetry and consider scalar fields that are singlet under $U(1) \times SU(2)_{\text{diag}}$. Using the coset representative (30), we find components of the composite connection that involve the gauge fields

$$\begin{aligned}
Q_i{}^j &= -\frac{i}{2} g_1 A^0 (\sigma_2 \otimes \sigma_3)_i{}^j + \frac{i}{2} g_2 \left[A^3 (\sigma_2 \otimes \mathbb{I}_2)_i{}^j \right. \\
&\quad \left. - A^4 (\sigma_3 \otimes \sigma_1)_i{}^j + A^5 (\sigma_1 \otimes \sigma_1)_i{}^j \right]. \quad (69)
\end{aligned}$$

The components of the spin connection on S^3 that need to be cancelled are $\omega^{\hat{\phi}}_{\hat{\theta}}$, $\omega^{\hat{\phi}}_{\hat{\psi}}$ and $\omega^{\hat{\theta}}_{\hat{\psi}}$. To impose the twist, we set $A^0 = 0$ and take the $SU(2)_{\text{diag}}$ gauge fields to be

$$\begin{aligned}
A^3 &= a_3 \cos \psi d\theta, \quad A^4 = a_4 \cos \theta d\phi, \\
A^5 &= a_5 \cos \psi \sin \theta d\phi \quad (70)
\end{aligned}$$

together with $A^{3+m} = \frac{g_2}{g_3} A^m$ for $m = 3, 4, 5$.

By considering the covariant derivative of ϵ_i along θ and ϕ directions, we find that the twist is achieved by imposing the following conditions

$$g_2 a_3 = g_2 a_4 = g_2 a_5 = 1 \quad (71)$$

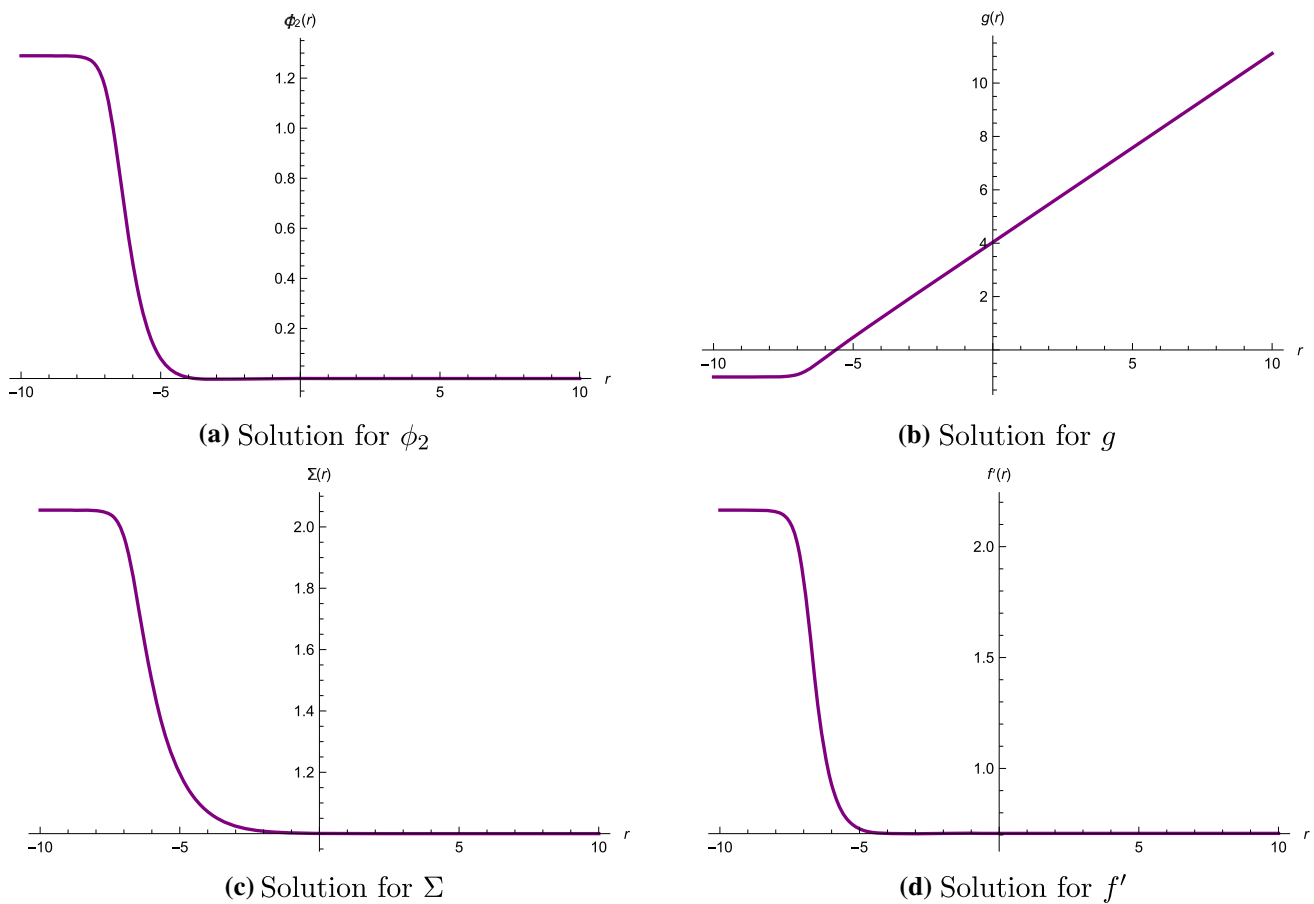


Fig. 3 An RG flow from AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to $AdS_3 \times S^2$ critical point I for $g_1 = 1$, $g_3 = 2g_1$ and $a_5 = \frac{1}{4}$

and projectors

$$\begin{aligned} i\gamma_{\hat{\theta}\hat{\psi}}\epsilon_i &= (\sigma_2 \otimes \mathbb{I}_2)_i^j \epsilon_j, & i\gamma_{\hat{\theta}\hat{\phi}}\epsilon_i &= (\sigma_3 \otimes \sigma_1)_i^j \epsilon_j, \\ i\gamma_{\hat{\phi}\hat{\psi}}\epsilon_i &= (\sigma_1 \otimes \sigma_1)_i^j \epsilon_j. \end{aligned} \quad (72)$$

Note that the last projector is not independent of the first two. Therefore, the AdS_2 solutions preserve four supercharges of the original supersymmetry. Condition (71) also implies $a_3 = a_4 = a_5$. We will then set $a_3 = a_4 = a_5 = a$ from now on. Using the definition (12), we find the gauge covariant field strengths

$$\begin{aligned} \mathcal{H}^3 &= -ae^{-2g}e^{\hat{\psi}} \wedge e^{\hat{\theta}}, & \mathcal{H}^4 &= -ae^{-2g}e^{\hat{\theta}} \wedge e^{\hat{\phi}}, \\ \mathcal{H}^5 &= -ae^{-2g}e^{\hat{\psi}} \wedge e^{\hat{\phi}} \end{aligned} \quad (73)$$

and $\mathcal{H}^{3+m} = \frac{g_2}{g_3} \mathcal{H}^m$ for $m = 3, 4, 5$.

For $\Sigma_3 = H^3$, we use the metric ansatz

$$ds^2 = -e^{2f}dt^2 + dr^2 + \frac{e^{2g}}{y^2}(dx^2 + dy^2 + dz^2) \quad (74)$$

with non-vanishing components of the spin connection

$$\begin{aligned} \omega^{\hat{x}}_{\hat{r}} &= g'e^{\hat{x}}, & \omega^{\hat{y}}_{\hat{r}} &= g'e^{\hat{y}}, & \omega^{\hat{z}}_{\hat{r}} &= g'e^{\hat{z}}, \\ \omega^{\hat{x}}_{\hat{y}} &= -e^{-g}e^{\hat{x}}, & \omega^{\hat{z}}_{\hat{y}} &= -e^{-g}e^{\hat{z}}, & \omega^{\hat{t}}_{\hat{r}} &= f'e^{\hat{t}} \end{aligned} \quad (75)$$

where various components of the vielbein are given by

$$\begin{aligned} e^{\hat{t}} &= e^f dt, & e^{\hat{r}} &= dr, & e^{\hat{x}} &= \frac{e^g}{y} dx, \\ e^{\hat{y}} &= \frac{e^g}{y} dy, & e^{\hat{z}} &= \frac{e^g}{y} dz. \end{aligned} \quad (76)$$

Since there are only two components, $\omega^{\hat{x}}_{\hat{y}}$ and $\omega^{\hat{z}}_{\hat{y}}$, of the spin connection to be cancelled in the twisting process, we turn on the following $SU(2)$ gauge fields

$$A^3 = \frac{a}{y} dx, \quad A^4 = 0, \quad A^5 = \frac{\tilde{a}}{y} dz \quad (77)$$

and $A^{m+3} = \frac{g_2}{g_3} A^m$, for $m = 3, 4, 5$.

Repeating the same analysis as in the S^3 case, we find the twist conditions

$$g_2 a = g_2 \tilde{a} = 1 \quad (78)$$

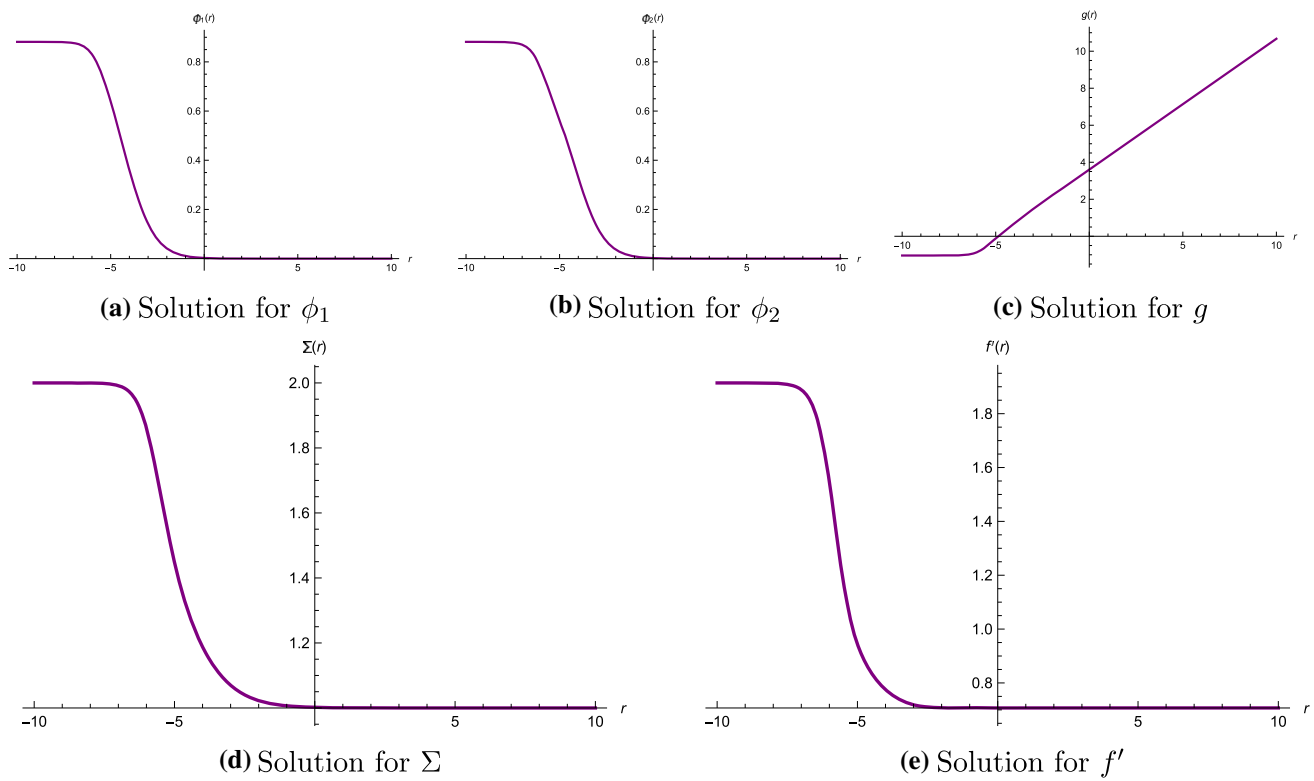


Fig. 4 An RG flow from AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to $AdS_3 \times S^2$ critical point II for $g_1 = 1$, $g_3 = 2g_1$ and $a_5 = \frac{1}{4}$

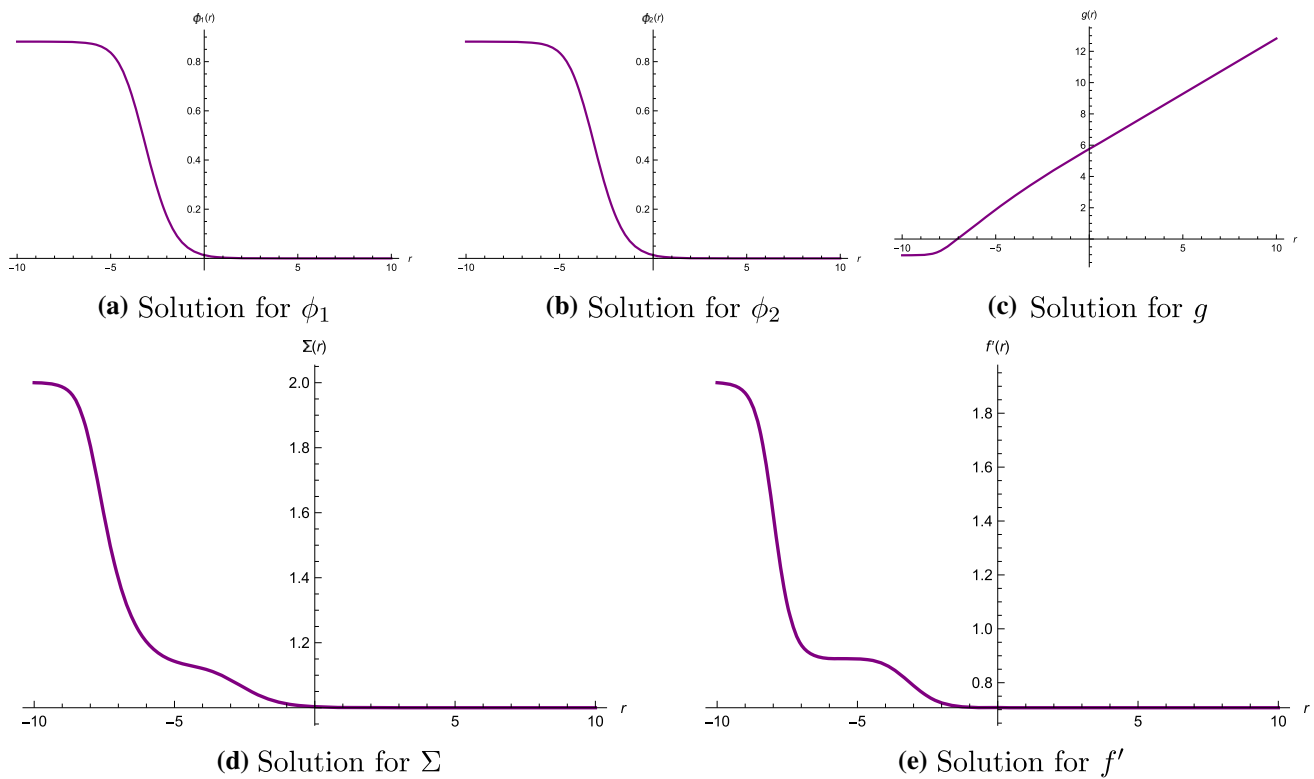


Fig. 5 An RG flow from AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to AdS_5 critical point with $U(1) \times SU(2)_{\text{diag}}$ symmetry and finally to $AdS_3 \times S^2$ critical point II for $g_1 = 1$, $g_3 = 2g_1$ and $a_5 = \frac{1}{4}$

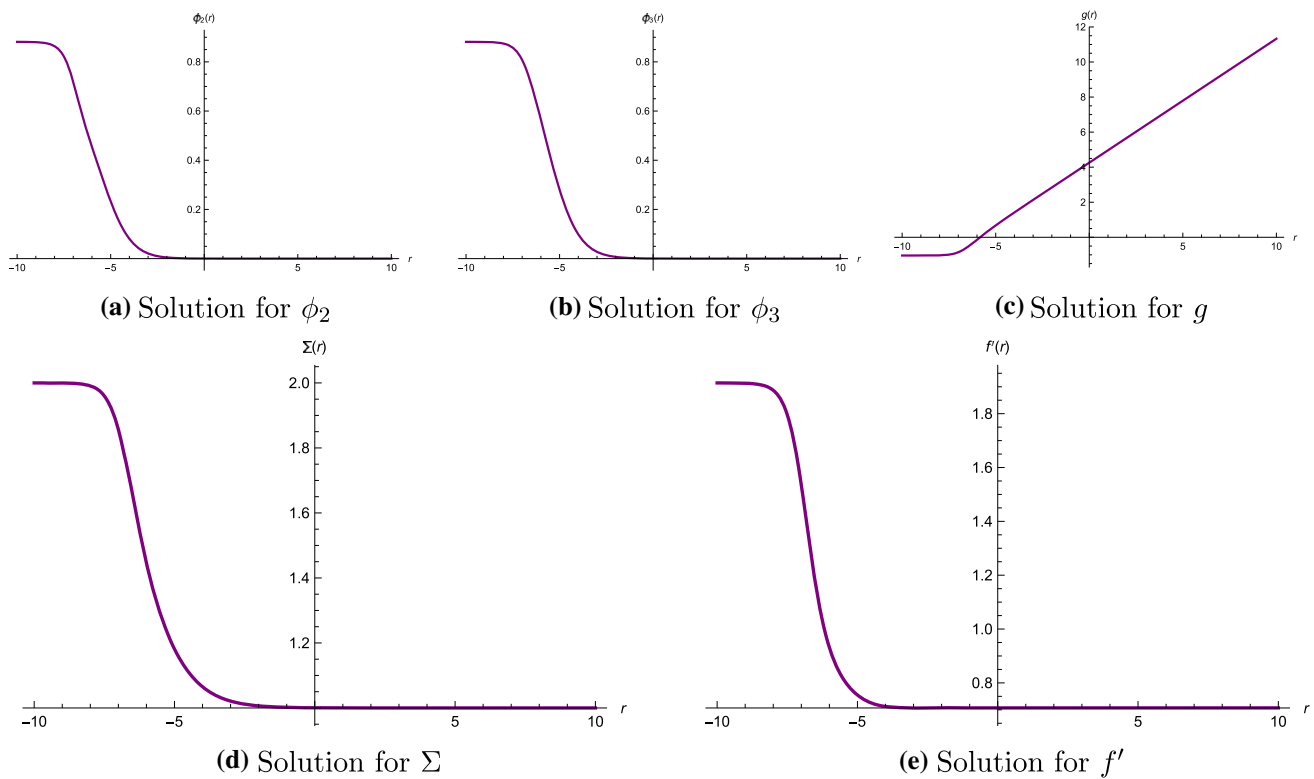


Fig. 6 An RG flow from AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to $AdS_3 \times S^2$ critical point III for $g_1 = 1$, $g_3 = 2g_1$ and $a_5 = \frac{1}{4}$

and projectors

$$\begin{aligned} i\gamma_{\hat{y}\hat{x}}\epsilon_i &= (\sigma_2 \otimes \mathbb{I}_2)_i^j \epsilon_j, & i\gamma_{\hat{y}\hat{z}}\epsilon_i &= (\sigma_1 \otimes \sigma_1)_i^j \epsilon_j, \\ i\gamma_{\hat{x}\hat{z}}\epsilon_i &= (\sigma_3 \otimes \sigma_1)_i^j \epsilon_j. \end{aligned} \quad (79)$$

The last projector is not needed for the twist with $A^4 = 0$. In addition, it follows from the first two projectors as in the S^3 case. The twist condition (78) again implies that $\tilde{a} = a$, and the covariant field strengths in this case are given by

$$\begin{aligned} \mathcal{H}^3 &= ae^{-2g} e^{\hat{x}} \wedge e^{\hat{y}}, & \mathcal{H}^4 &= ae^{-2g} e^{\hat{z}} \wedge e^{\hat{x}}, \\ \mathcal{H}^5 &= ae^{-2g} e^{\hat{z}} \wedge e^{\hat{y}} \end{aligned} \quad (80)$$

and $\mathcal{H}^{m+3} = \frac{g_2}{g_3} \mathcal{H}^m$, for $m = 3, 4, 5$. Note that although $A^4 = 0$, we have non-vanishing $\mathcal{H}^4$ due to the non-abelian nature of $SU(2)$ field strengths.

With all these ingredients, the following BPS equations are straightforwardly obtained

$$\begin{aligned} \phi' &= \frac{1}{8g_3} \Sigma^{-1} e^{-3\phi-2g} [g_2 - g_3 + e^{2\phi} (g_2 + g_3)] \\ &\quad \times [g_3 e^{2g} (e^{4\phi} - 1) + 4\kappa a e^{2\phi} \Sigma^2], \end{aligned} \quad (81)$$

$$\begin{aligned} \Sigma' &= -\frac{1}{3} [g_2 \cosh^3 \phi + g_3 \sinh^3 \phi + \sqrt{2} g_1 \Sigma^3] \\ &\quad + \frac{\kappa}{g_3} a e^{-2g} \Sigma^2 (g_3 \cosh \phi + g_2 \sinh \phi), \end{aligned} \quad (82)$$

$$\begin{aligned} g' &= -\frac{1}{3} \Sigma^{-1} (g_2 \cosh^3 \phi + g_3 \sinh^3 \phi) + \frac{1}{3} g_1 \Sigma^2 \\ &\quad - \frac{\kappa}{g_3} a e^{-2g} \Sigma (g_3 \cosh \phi + g_2 \sinh \phi), \end{aligned} \quad (83)$$

$$\begin{aligned} f' &= -\frac{1}{3} \Sigma^{-1} (g_2 \cosh^3 \phi + g_3 \sinh^3 \phi) + \frac{1}{3} g_1 \Sigma^2 \\ &\quad + \frac{\kappa}{g_3} a e^{-2g} \Sigma (g_3 \cosh \phi + g_2 \sinh \phi). \end{aligned} \quad (84)$$

As in the AdS_3 solutions, $\kappa = 1$ and $\kappa = -1$ corresponds to $\Sigma_3 = S^3$ and $\Sigma_3 = H^3$, respectively.

It turns out that only $\kappa = -1$ leads to an AdS_2 solution given by

$$\begin{aligned} \phi &= \frac{1}{2} \ln \left[\frac{g_3 - g_2}{g_3 + g_2} \right], & \Sigma &= - \left[\frac{2\sqrt{2} g_2 g_3}{g_1 \sqrt{g_3^2 - g_2^2}} \right]^{\frac{1}{3}}, \\ g &= \frac{1}{2} \ln \left[\frac{2a(g_3^2 - g_2^2)^{\frac{2}{3}}}{g_1^{\frac{2}{3}} g_2^{\frac{1}{3}} g_3^{\frac{4}{3}}} \right], & L_{AdS_2} &= \frac{(g_3^2 - g_2^2)^{\frac{1}{3}}}{\sqrt{2} g_1^{\frac{1}{3}} g_2^{\frac{2}{3}} g_3^{\frac{2}{3}}}. \end{aligned} \quad (85)$$

This solution preserves $N = 4$ supersymmetry in two dimensions and $U(1) \times SU(2)_{\text{diag}}$ symmetry. As $r \rightarrow \infty$, $f \sim g \sim r$, solutions to the above BPS equations are locally asymptotic to either of the $N = 4$ AdS_5 vacua in (32) and (33). RG flow solutions interpolating between these AdS_5

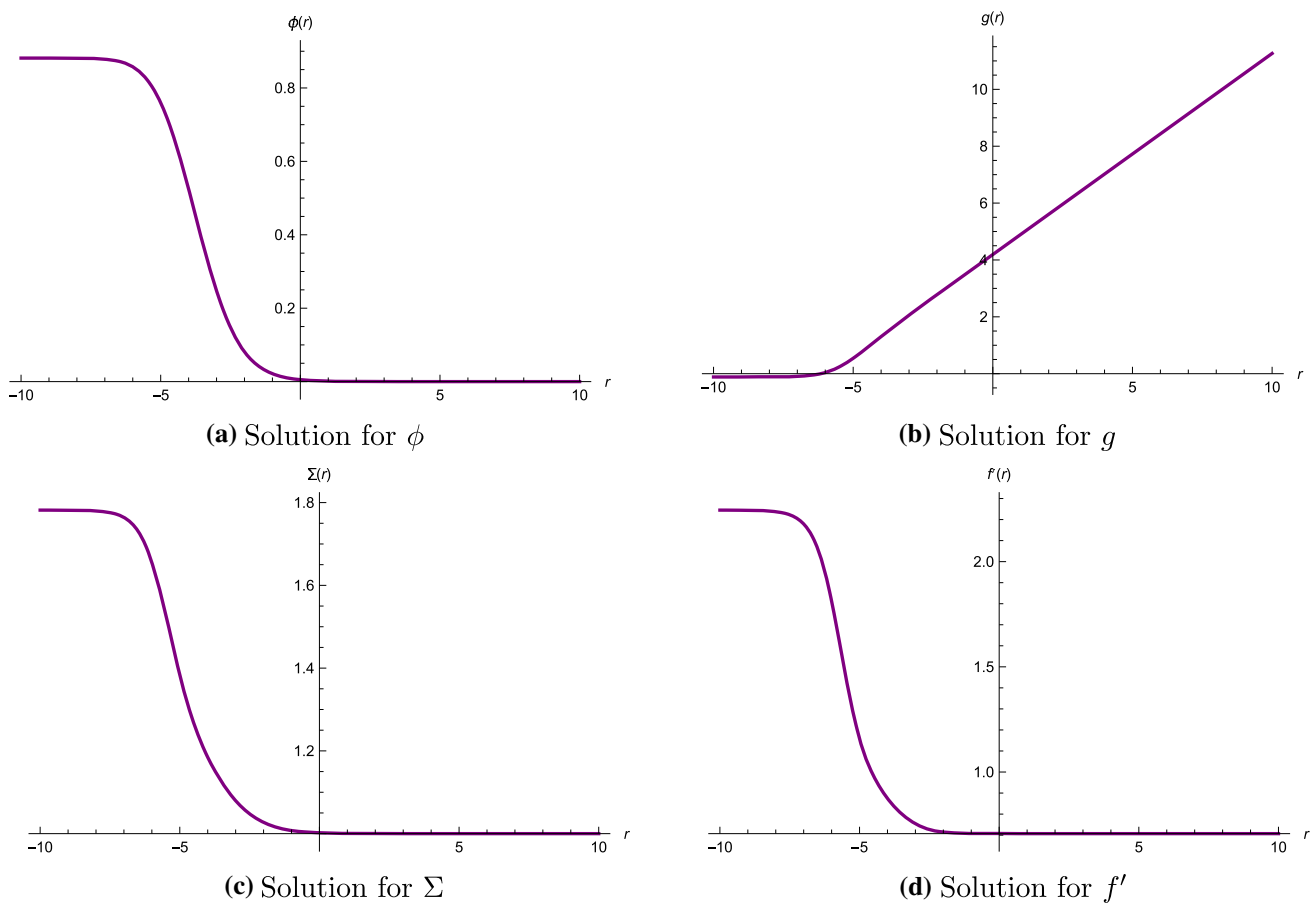


Fig. 7 An RG flow from AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to $AdS_2 \times H^3$ critical point for $g_1 = 1$ and $g_3 = 2g_1$

vacua and the $AdS_2 \times H^3$ solution in (85) are shown in Figs. 7 and 8. In particular, the flow in Fig. 8 connects three critical points similar to the solution given in the previous section.

We end this section by a comment on the possibility of turning on the twist from A^0 along with those from the $SU(2)_{\text{diag}}$ gauge fields. As in the previous section, if we impose an additional projector

$$(\mathbb{I}_2 \otimes \sigma_3)_i^j \epsilon_j = -\epsilon_i, \quad (86)$$

the projection matrix of the A^0 term in the composite connection (69) will be proportional to that of A^3 . We will consider the S^3 case for concreteness and take the ansatz for A^0 to be

$$A^0 = a_0 \cos \psi d\theta \quad (87)$$

and proceed as in the $A^0 = 0$ case. This results in the projectors given in (72) and the twist conditions

$$g_2 a_4 = g_2 a_5 = 1 \quad \text{and} \quad g_1 a_0 + g_2 a_3 = 1. \quad (88)$$

We can see that at this stage the parameter a_3 needs not be equal to a_4 and a_5 . However, consistency of the BPS equations from $\delta \lambda_i^a$ conditions require $a_3 = a_4 = a_5$ and hence $a_0 = 0$ by the conditions in (88). This is because A^0 does

not appear in $\delta \lambda_i^a$ variation. The resulting BPS equations then reduce to those of the previous case with $A^0 = 0$. So, we conclude that the A^0 twist cannot be turned on along with the $SU(2)_{\text{diag}}$ twists.

4 $U(1) \times SO(3, 1)$ gauge group

For non-compact $U(1) \times SO(3, 1)$ gauge group, components of the embedding tensor are given by

$$\xi^{MN} = g_1 (\delta_2^M \delta_1^N - \delta_1^M \delta_2^N), \quad (89)$$

$$f_{345} = f_{378} = -f_{468} = -f_{567} = -g_2. \quad (90)$$

This gauge group has already been studied in [34]. The scalar potential admits one supersymmetric $N = 4$ AdS_5 vacuum in which all scalars from vector multiplets vanish and $\Sigma = 1$ after choosing $g_2 = -\sqrt{2}g_1$. At the vacuum, the gauge group is broken down to its maximal compact subgroup $U(1) \times SO(3)$. A holographic RG flow from this critical point to a non-conformal field theory in the IR and a flow to $AdS_3 \times H^2$ vacuum preserving eight supercharges have also been studied in [34]. In this case, $AdS_3 \times S^2$ solutions do not exist.

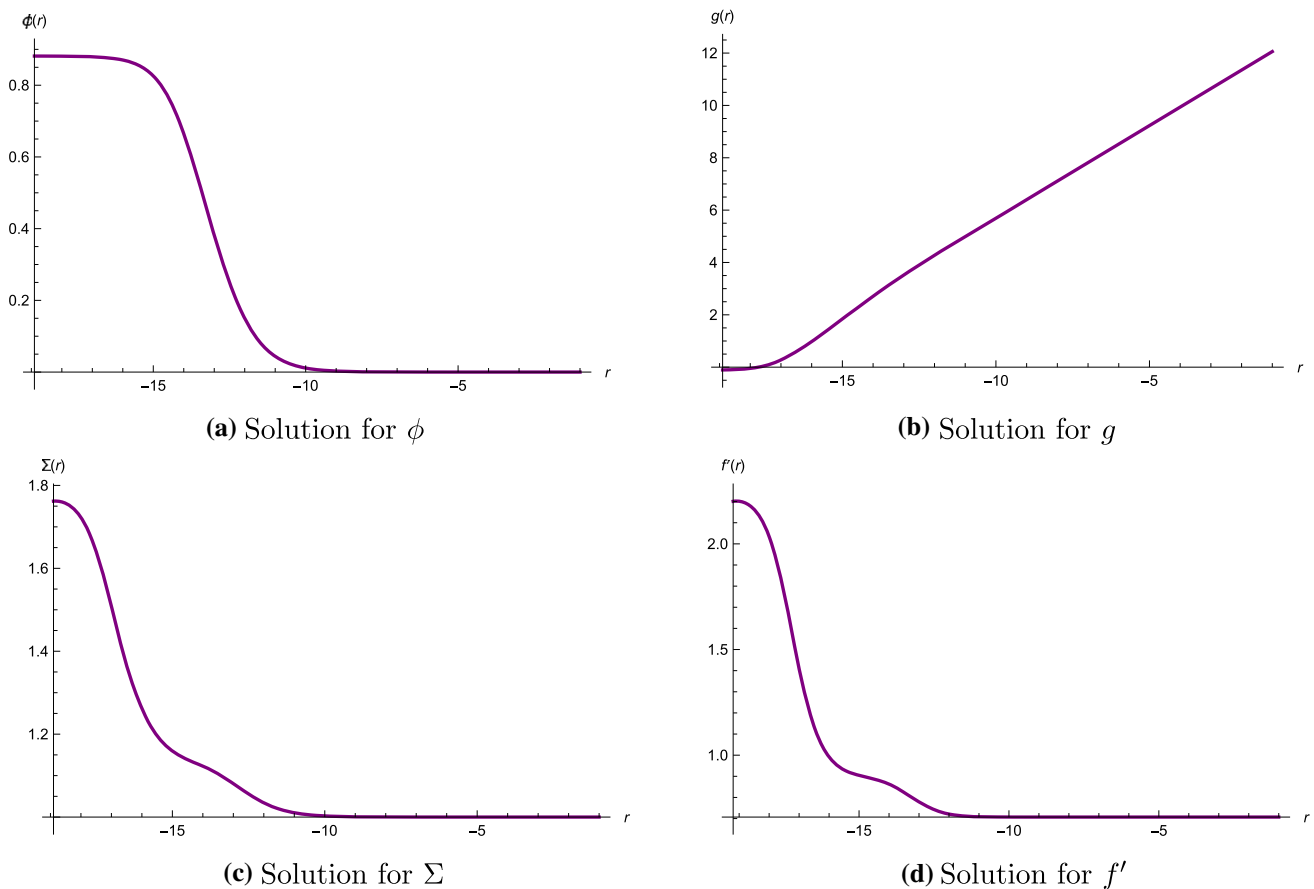


Fig. 8 An RG flow from AdS_5 critical point with $U(1) \times SU(2) \times SU(2)$ symmetry to AdS_5 critical point with $U(1) \times SU(2)_{\text{diag}}$ symmetry and finally to $AdS_2 \times H^3$ critical point for $g_1 = 1$ and $g_3 = 2g_1$

In this section, we will study $AdS_3 \times \Sigma_2$ and $AdS_2 \times \Sigma_3$ solutions preserving four supercharges. The analysis is closely parallel to that performed in the previous section, so we will give less detail in order to avoid repetition.

4.1 Supersymmetric black strings

We will use the same metric ansatz as in Eqs. (34) and (35) and consider the twist from $U(1) \times U(1)$ gauge fields. The second $U(1)$ is a subgroup of the $SO(3) \subset SO(3, 1)$. There are in total five scalars that are singlet under this $U(1) \times U(1)$, but as in the compact $U(1) \times SU(2) \times SU(2)$ gauge group, these can be truncated to three singlets corresponding to the following $SO(5, 5)$ non-compact generators

$$\tilde{Y}_1 = Y_{31} + Y_{42}, \quad \tilde{Y}_2 = Y_{32} - Y_{41}, \quad \tilde{Y}_3 = Y_{53}. \quad (91)$$

With the embedding tensor (90), the compact $SO(3)$ symmetry is generated by X_3 , X_4 and X_5 generators.

Using the coset representative of the form

$$L = e^{\phi_1 \tilde{Y}_1} e^{\phi_2 \tilde{Y}_2} e^{\phi_3 \tilde{Y}_3}, \quad (92)$$

we can repeat all the analysis of the previous section by using the ansatz for the gauge fields

$$A^0 = a_0 \cos \theta d\phi \quad \text{and} \quad A^5 = a_5 \cos \theta d\phi, \quad (93)$$

for $\Sigma_2 = S^2$ and

$$A^0 = a_0 \cosh \theta d\phi \quad \text{and} \quad A^5 = a_5 \cosh \theta d\phi, \quad (94)$$

for $\Sigma_2 = H^2$. The result is similar to the compact case with the projectors (41) and (42) and the twist condition (43).

Using the γ_r projection (44), the BPS equations in this case read

$$\begin{aligned} f' = & -\frac{1}{24\Sigma^2} e^{-2\phi_1 - \phi_2 - 2(\phi_3 + g)} \left[e^{2g} \left[1 - e^{4\phi_1} \right. \right. \\ & - e^{2\phi_2} + e^{4\phi_1 + 2\phi_2} + e^{4\phi_3} + 4e^{2(\phi_1 + \phi_3)} \\ & - e^{4(\phi_1 + \phi_3)} + 4e^{2(\phi_1 + \phi_2 + \phi_3)} \\ & \left. \left. - e^{2\phi_2 + 4\phi_3} + e^{4\phi_1 + 2\phi_2 + 4\phi_3} \right] g_2 \Sigma \right. \\ & - 4\sqrt{2}\kappa a_0 e^{2\phi_1 + \phi_2 + 2\phi_3} - 4\kappa a_5 e^{2(\phi_1 + \phi_3)} (1 + e^{2\phi_2}) \Sigma^3 \\ & \left. - 4\sqrt{2}e^{2\phi_1 + \phi_2 + 2(\phi_3 + g)} g_1 \Sigma^4 \right], \quad (95) \end{aligned}$$

$$g' = \frac{1}{24\Sigma^2} e^{-2\phi_1 - \phi_2 - 2(\phi_3 + g)} \left[-e^{2g} \left[1 - e^{4\phi_1} - e^{2\phi_2} + e^{4\phi_1 + 2\phi_2} + e^{4\phi_3} + 4e^{2(\phi_1 + \phi_3)} - e^{4(\phi_1 + \phi_3)} + 4e^{2(\phi_1 + \phi_2 + \phi_3)} - e^{2\phi_2 + 4\phi_3} + e^{4\phi_1 + 2\phi_2 + 4\phi_3} \right] g_2 \Sigma - 8\kappa\sqrt{2}a_0 e^{2\phi_1 + \phi_2 + 2\phi_3} - 8\kappa a_5 e^{2(\phi_1 + \phi_3)} (1 + e^{2\phi_2}) \Sigma^3 + 4\sqrt{2}e^{2\phi_1 + \phi_2 + 2(\phi_3 + g)} g_1 \Sigma^4 \right], \quad (96)$$

$$\Sigma' = \frac{1}{24\Sigma} e^{-2\phi_1 - \phi_2 - 2(\phi_3 + g)} \left[-e^{2g} (1 - e^{4\phi_1} - e^{2\phi_2} + e^{4\phi_1 + 2\phi_2} + e^{4\phi_3} + 4e^{2(\phi_1 + \phi_3)} - e^{4(\phi_1 + \phi_3)} + 4e^{2(\phi_1 + \phi_2 + \phi_3)} - e^{2\phi_2 + 4\phi_3} + e^{4\phi_1 + 2\phi_2 + 4\phi_3}) g_2 \Sigma - 8\kappa\sqrt{2}a_0 e^{2\phi_1 + \phi_2 + 2\phi_3} + 4\kappa a_5 e^{2(\phi_1 + \phi_3)} (1 + e^{2\phi_2}) \Sigma^3 - 8\sqrt{2}e^{2\phi_1 + \phi_2 + 2(\phi_3 + g)} g_1 \Sigma^4 \right], \quad (97)$$

$$\phi'_1 = \frac{e^{-2\phi_1 - \phi_2 + 2\phi_3} (1 + e^{4\phi_1}) (e^{2\phi_2} - 1) g_2}{2(1 + e^{4\phi_3}) \Sigma}, \quad (98)$$

$$\phi'_2 = \frac{1}{8\Sigma} e^{-2\phi_1 - \phi_2 - 2(\phi_3 + g)} \left[e^{2g} (e^{4\phi_1} - e^{2\phi_2} + e^{4\phi_1 + 2\phi_2} - e^{4\phi_3} - 4e^{2(\phi_1 + \phi_3)} + e^{4(\phi_1 + \phi_3)} - 1 + 4e^{2(\phi_1 + \phi_2 + \phi_3)} - e^{2\phi_2 + 4\phi_3} + e^{4\phi_1 + 2\phi_2 + 4\phi_3}) g_2 + 4\kappa a_5 e^{2(\phi_1 + \phi_3)} (e^{2\phi_2} - 1) \Sigma^2 \right], \quad (99)$$

$$\phi'_3 = \frac{e^{-2\phi_1 - \phi_2 - 2\phi_3} (e^{4\phi_1} - 1) (e^{2\phi_2} - 1) (e^{4\phi_3} - 1) g_2}{8\Sigma}. \quad (100)$$

This set of equations admits an AdS_3 solution given by

$$\phi_2 = \phi_3 = 0, \quad \Sigma = \left(\frac{\sqrt{2}\kappa}{a_5 g_1} \right)^{\frac{1}{3}}, \quad g = \frac{1}{3} \ln \left(\frac{\sqrt{2}a_5^2}{g_1} \right), \quad L_{AdS_3} = \left(\frac{\sqrt{2}a_5^2}{g_1} \right)^{\frac{1}{3}} \frac{2}{(1 - \kappa a_5 g_2)}. \quad (101)$$

As in the compact case, Σ_2 can be either S^2 or H^2 , depending on the values of a_5, a_0, g_1 and g_2 such that the twist condition (43) is satisfied. This is in contrast to the half-supersymmetric solution found in [34] for which only $\Sigma_2 = H^2$ is possible.

To find a domain wall interpolating between the AdS_5 vacuum to this $AdS_3 \times \Sigma_2$ solution, we further truncate the

BPS equations by setting $\phi_i = 0$ for $i = 1, 2, 3$. The resulting equations are given by

$$f' = \frac{1}{6\Sigma^2} e^{-2g} \left(\sqrt{2}\kappa a_0 - 2e^{2g} g_2 \Sigma + 2\kappa a_5 \Sigma^3 - \sqrt{2}e^{2g} g_1 \Sigma^4 \right), \quad (102)$$

$$g' = -\frac{1}{6\Sigma^2} e^{-2g} \left(2\sqrt{2}\kappa a_0 + 2e^{2g} g_2 \Sigma + 4\kappa a_5 \Sigma^3 + \sqrt{2}e^{2g} g_1 \Sigma^4 \right), \quad (103)$$

$$\Sigma' = -\frac{1}{3\Sigma} e^{-2g} \left(\sqrt{2}\kappa a_0 + e^{2g} g_2 \Sigma - \kappa a_5 \Sigma^3 + \sqrt{2}e^{2g} g_1 \Sigma^4 \right). \quad (104)$$

An example of numerical solutions is shown in Fig. 9.

4.2 Supersymmetric black holes

We now consider $AdS_2 \times \Sigma_3$ solutions within this non-compact gauge group. We will look for solutions with $U(1) \times SO(3) \subset U(1) \times SO(3, 1)$ symmetry. There is one $U(1) \times SO(3)$ singlet from the $SO(5, 5)/SO(5) \times SO(5)$ coset corresponding to the non-compact generator

$$Y = Y_{31} + Y_{42} - Y_{53}. \quad (105)$$

The coset representative can be written as

$$L = e^{\phi Y}. \quad (106)$$

Using the metric ansatz (66) and (74) together with the gauge fields (70) and (77), we find that the twist can be implemented by using the projectors given in (72). Furthermore, the twist condition also implies that $a_3 = a_4 = a_5 = a$ with $g_2 a = 1$, and the twist from A^0 cannot be turned on. The $AdS_2 \times \Sigma_3$ solutions preserve four supercharges.

Using the projector (44), we can derive the following BPS equations

$$f' = \frac{1}{12\Sigma} \left[e^{-3\phi} (1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi}) g_2 + 6\kappa a e^{-\phi - 2g} (1 + e^{2\phi}) \Sigma^2 + 2\sqrt{2}g_1 \Sigma^3 \right], \quad (107)$$

$$g' = \frac{1}{12\Sigma} \left[e^{-3\phi} (1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi}) g_2 - 6\kappa a e^{-\phi - 2g} (1 + e^{2\phi}) \Sigma^2 + 2\sqrt{2}g_1 \Sigma^3 \right], \quad (108)$$

$$\Sigma' = \frac{1}{12} e^{-3\phi - 2g} \left[e^{2g} (1 - 3e^{2\phi} - 3e^{4\phi} + e^{6\phi}) g_2 + 6\kappa a e^{2\phi} (1 + e^{2\phi}) \Sigma^2 - 4\sqrt{2}e^{3\phi + 2g} g_1 \Sigma^3 \right], \quad (109)$$

$$\phi' = -\frac{1}{4\Sigma} e^{-3\phi - 2g} \times \left(e^{2\phi} - 1 \right) \left(e^{2g} (1 + e^{4\phi}) g_2 - 2\kappa a e^{2\phi} \Sigma^2 \right). \quad (110)$$

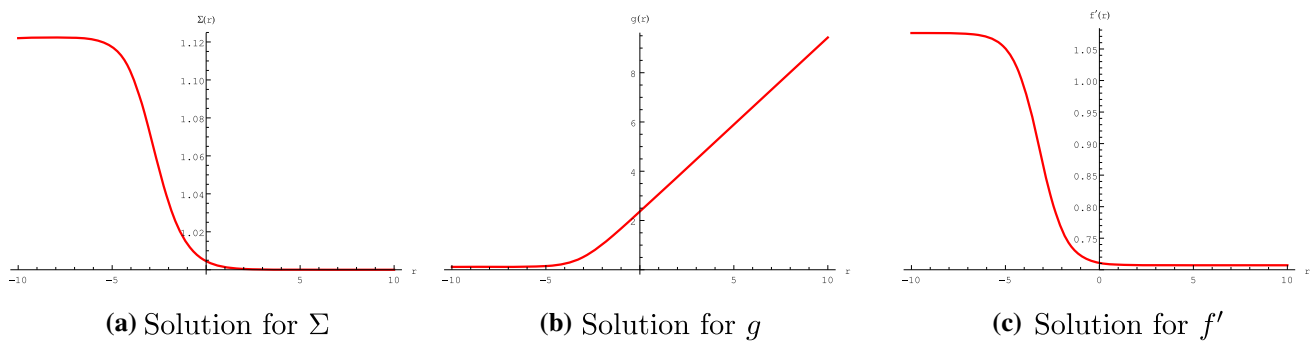


Fig. 9 An RG flow solution from supersymmetric AdS_5 with $U(1) \times SO(3)$ symmetry to $AdS_3 \times S^2$ geometry in the IR for $U(1) \times SO(3, 1)$ gauge group and $g_1 = 1, a_5 = 1$

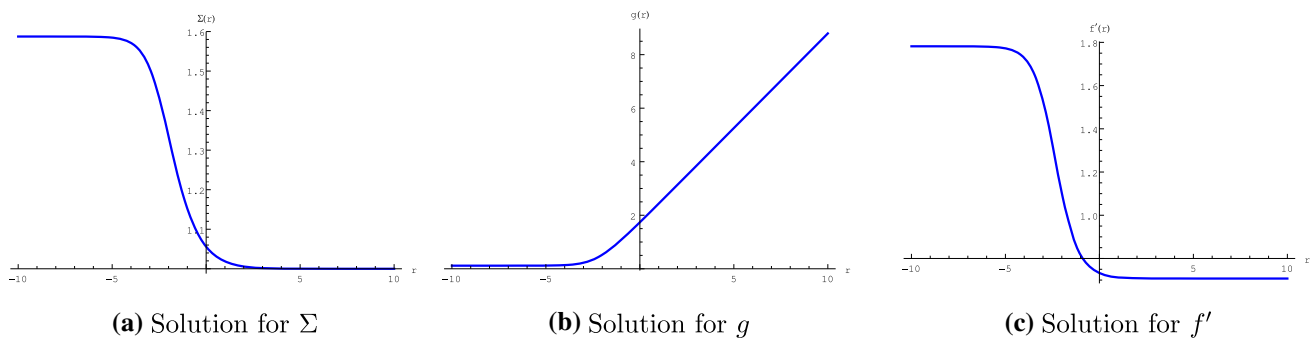


Fig. 10 An RG flow solution from AdS_5 with $U(1) \times SO(3)$ symmetry to $AdS_2 \times H^3$ geometry in the IR for $U(1) \times SO(3, 1)$ gauge group and $g_1 = 1$

These equations admit one $AdS_2 \times H^3$ solution given by

$$\begin{aligned} \phi &= 0, \quad \Sigma = -\sqrt{2} \left(\frac{g_2}{g_1} \right)^{\frac{1}{3}} \\ g &= -\frac{1}{2} \ln \left[\frac{(g_1^2 g_2)^{\frac{1}{3}}}{2a} \right], \quad L_{AdS_2} = \frac{1}{\sqrt{2} (g_1 g_2^2)^{\frac{1}{3}}} \end{aligned} \quad (111)$$

while $AdS_2 \times S^3$ solutions do not exist.

By setting $\phi = 0$, we find a numerical solution to the above BPS equations as shown in Fig. 10.

5 $U(1) \times SL(3, \mathbb{R})$ gauge group

In this section, we consider non-compact $U(1) \times SL(3, \mathbb{R})$ gauge group. This has not been studied in [34], so we will give more detail about the construction of this gauged supergravity and possible supersymmetric AdS_5 vacua.

Components of the embedding tensor for this gauge group are given by

$$\xi^{MN} = g_1 (\delta_2^M \delta_1^N - \delta_1^M \delta_2^N), \quad (112)$$

$$\begin{aligned} f_{345} &= f_{389} = f_{468} = f_{497} = f_{569} = f_{578} = -g_2, \\ f_{367} &= 2g_2, \quad f_{4,9,10} = f_{5,8,10} = \sqrt{3}g_2. \end{aligned} \quad (113)$$

f_{MN}^P can be extracted from $SL(3, \mathbb{R})$ generators $(\frac{i\lambda_2}{2}, \frac{i\lambda_5}{2}, \frac{i\lambda_7}{2}, \frac{\lambda_1}{2}, \frac{\lambda_3}{2}, \frac{\lambda_4}{2}, \frac{\lambda_6}{2}, \frac{\lambda_8}{2})$ with $\lambda_i, i = 1, 2, \dots, 8$ being the usual Gell-Mann matrices. The compact $SO(3) \subset SL(3, \mathbb{R})$ symmetry is generated by X_3, X_4 and X_5 .

5.1 Supersymmetric AdS_5 vacuum

The $SL(3, \mathbb{R})$ factor is embedded in $SO(3, 5) \subset SO(5, 5)$ such that its adjoint representation is identified with the fundamental representation of $SO(3, 5)$. The $SO(3) \subset SL(3, \mathbb{R})$ is embedded in $SL(3, \mathbb{R})$ such that $\mathbf{3} \rightarrow \mathbf{\bar{3}}$. Decomposing the adjoint representation of $SO(3, 5)$ to $SL(3, \mathbb{R})$ and $SO(3)$, we find that the 25 scalars transform under $SO(3) \subset SL(3, \mathbb{R})$ as

$$2(\mathbf{1} \times \mathbf{5}) + \mathbf{3} \times \mathbf{5} = \mathbf{3} + \mathbf{3} \times \mathbf{5} + \mathbf{7}. \quad (114)$$

Unlike the $U(1) \times SO(3, 1)$ gauge group, there is no singlet under the compact $SO(3)$ symmetry. Taking into account the embedding of the $U(1)$ factor in the gauge group as described in (112), we find the transformation of the scalars under $U(1) \times SO(3)$

$$\mathbf{3}_0 + \mathbf{5}_0 + \mathbf{7}_0 + \mathbf{5}_2 + \mathbf{5}_{-2} \quad (115)$$

with the subscript denoting the $U(1)$ charges.

Table 1 Scalar masses at the $N = 4$ supersymmetric AdS_5 critical point with $U(1) \times SO(3)$ symmetry and the corresponding dimensions of dual operators for the non-compact $U(1) \times SL(3, \mathbb{R})$ gauge group. The scalars are organized into representations of $U(1) \times SO(3)$ with the singlet corresponding to the dilaton Σ

Scalar field representations	$m^2 L^2$	Δ
$\mathbf{1}_0$	-4	2
$\mathbf{3}_0$	32	8
$\mathbf{5}_0$	0	4
$\mathbf{7}_0$	12	6
$\mathbf{5}_{-2}$	21	7
$\mathbf{5}_2$	21	7

It can be readily verified by studying the corresponding scalar potential or recalling the result of [48] that this $U(1) \times SL(3, \mathbb{R})$ gauge group admits a supersymmetric $N = 4$ AdS_5 vacuum at which all scalars from vector multiplets vanish with

$$\Sigma = 1 \quad \text{and} \quad V_0 = -3g_1^2. \quad (116)$$

We have, as in other gauge groups, set $g_2 = -\sqrt{2}g_1$ to bring this vacuum to the value of $\Sigma = 1$. All scalar masses at this vacuum are given in Table 1. Massless scalars in $\mathbf{5}_0$ representation are Goldstone bosons corresponding to the symmetry breaking $SL(3, \mathbb{R}) \rightarrow SO(3)$.

5.2 Supersymmetric black strings

We now consider $U(1) \times U(1) \subset U(1) \times SO(3) \subset U(1) \times SL(3, \mathbb{R})$ invariant scalars. We will choose the $U(1) \subset SO(3)$ generator to be X_5 . From the vector multiplets, there are three singlet scalars corresponding to the following non-compact generators

$$\bar{Y}_1 = Y_{31} - Y_{44}, \quad \bar{Y}_2 = Y_{41} + Y_{34}, \quad \bar{Y}_3 = \sqrt{3}Y_{52} - Y_{55}. \quad (117)$$

The coset representative can be written as

$$L = e^{\phi_1 \bar{Y}_1} e^{\phi_2 \bar{Y}_2} e^{\phi_3 \bar{Y}_3} \quad (118)$$

which gives rise to the scalar potential

$$\begin{aligned} V = & \frac{1}{16\Sigma^2} e^{-4(\phi_2 + \phi_3)} g_2 \left[(3 + 6e^{4\phi_2} \right. \\ & + 3e^{8\phi_2} + 3e^{8\phi_3} - 32e^{4(\phi_2 + \phi_3)} + 3e^{8(\phi_2 + \phi_3)} \\ & \left. + 6e^{4\phi_2 + 8\phi_3}) g_2 - 4\sqrt{2}e^{2(\phi_2 + \phi_3)} \right. \\ & \times (\sqrt{3} - 2e^{2\phi_2} - \sqrt{3}e^{4\phi_2} - \sqrt{3}e^{4\phi_3} \\ & \left. + \sqrt{3}e^{4(\phi_2 + \phi_3)} - 2e^{2\phi_2 + 4\phi_3}) g_1 \Sigma^3 \right]. \end{aligned} \quad (119)$$

Notice that V doesn't depend on ϕ_1 , consistent with the fact that ϕ_1 is part of the Goldstone bosons in $\mathbf{5}_0$ representation.

It can be verified that this potential admits only one supersymmetric AdS_5 critical point at $\phi_1 = \phi_2 = \phi_3 = 0$ and $\Sigma = 1$ for $g_2 = -\sqrt{2}g_1$.

We first consider $AdS_3 \times \Sigma_2$ solutions preserving eight supercharges. We will omit some detail since the same analysis has been carried out in [34]. By turning on gauge fields A^0 and A^5 along Σ_2 and performing the twist in Eq. (39) by imposing only one projector

$$i\gamma_{\hat{\theta}\hat{\phi}}\epsilon_i = a_0 g_1 (\sigma_2 \otimes \sigma_3)_i^j \epsilon_j - a_5 g_2 (\sigma_1 \otimes \sigma_1)_i^j \epsilon_j, \quad (120)$$

we find that consistency of this projection condition, namely $(i\gamma_{\hat{\theta}\hat{\phi}})^2 = \mathbb{I}_4$, implies $a_0 a_5 = 0$, see [34] for more detail. Therefore, for half-supersymmetric solutions, the twists from A^0 and A^5 cannot be turned on simultaneously. Furthermore, as shown in [34], see also a similar discussion in [39], the twist with $a_5 = 0$ does not lead to an AdS_3 fixed point. We will accordingly consider only the case of $a_0 = 0$ and $a_5 \neq 0$ which leads to the twist condition $a_5 g_2 = 1$ and the projector

$$i\gamma_{\hat{\theta}\hat{\phi}}\epsilon_i = -(\sigma_1 \otimes \sigma_1)_i^j \epsilon_j. \quad (121)$$

The resulting BPS equations read

$$\begin{aligned} f' = & \frac{1}{12\Sigma} e^{-2(\phi_2 + \phi_3 + g)} \left[e^{2g} (\sqrt{3} - 2e^{2\phi_2} \right. \\ & - \sqrt{3}e^{4\phi_2} - \sqrt{3}e^{4\phi_3} + \sqrt{3}e^{4(\phi_2 + \phi_3)} \\ & \left. - 2e^{2\phi_2 + 4\phi_3}) g_2 + 2\kappa a_5 e^{2\phi_2} (1 + e^{4\phi_3}) \Sigma^2 \right. \\ & \left. + 2\sqrt{2}e^{2(\phi_2 + \phi_3 + g)} g_1 \Sigma^3 \right], \end{aligned} \quad (122)$$

$$\begin{aligned} g' = & \frac{1}{12\Sigma} e^{-2(\phi_2 + \phi_3 + g)} \left[e^{2g} (\sqrt{3} - 2e^{2\phi_2} \right. \\ & - \sqrt{3}e^{4\phi_2} - \sqrt{3}e^{4\phi_3} + \sqrt{3}e^{4(\phi_2 + \phi_3)} \\ & \left. - 2e^{2\phi_2 + 4\phi_3}) g_2 - 4\kappa a_5 e^{2\phi_2} (1 + e^{4\phi_3}) \Sigma^2 \right. \\ & \left. + 2\sqrt{2}e^{2(\phi_2 + \phi_3 + g)} g_1 \Sigma^3 \right], \end{aligned} \quad (123)$$

$$\begin{aligned} \Sigma' = & \frac{1}{12} e^{-2(\phi_2 + \phi_3 + g)} \left[e^{2g} (\sqrt{3} - 2e^{2\phi_2} \right. \\ & - \sqrt{3}e^{4\phi_2} - \sqrt{3}e^{4\phi_3} + \sqrt{3}e^{4(\phi_2 + \phi_3)} \\ & \left. - 2e^{2\phi_2 + 4\phi_3}) g_2 + 2\kappa a_5 e^{2\phi_2} (1 + e^{4\phi_3}) \Sigma^2 \right. \\ & \left. - 4\sqrt{2}e^{2(\phi_2 + \phi_3 + g)} g_1 \Sigma^3 \right], \end{aligned} \quad (124)$$

$$\phi'_1 = 0 \quad (125)$$

$$\phi'_2 = -\frac{\sqrt{3}e^{-2(\phi_2 + \phi_3)} (1 + e^{4\phi_2}) (e^{4\phi_3} - 1) g_2}{4\Sigma}, \quad (126)$$

$$\begin{aligned} \phi'_3 = & -\frac{1}{8\Sigma} e^{-2(\phi_2 + \phi_3 + g)} \left[e^{2g} (2e^{2\phi_2} - \sqrt{3} \right. \\ & + \sqrt{3}e^{4\phi_2} - \sqrt{3}e^{4\phi_3} + \sqrt{3}e^{4(\phi_2 + \phi_3)} \\ & \left. - 2e^{2\phi_2 + 4\phi_3}) g_2 - 2\kappa a_5 e^{2\phi_2} (e^{4\phi_3} - 1) \Sigma^2 \right]. \end{aligned} \quad (127)$$

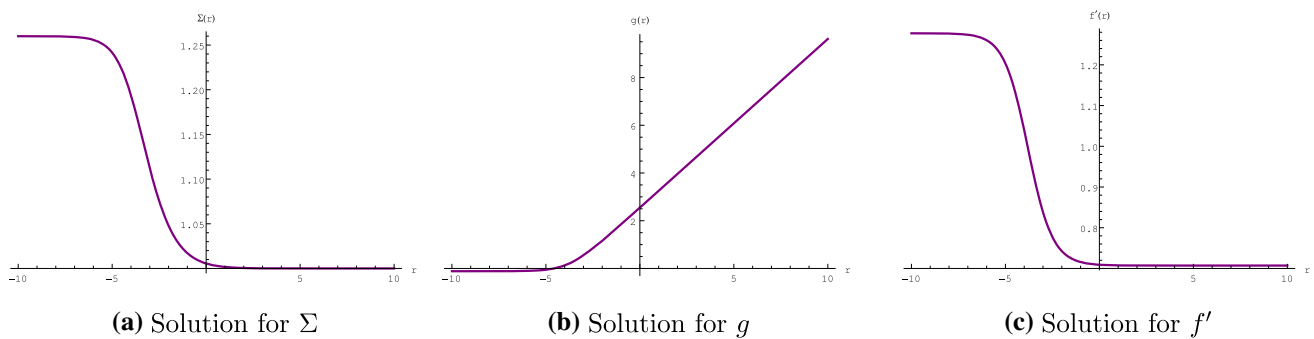


Fig. 11 An RG flow solution from AdS_5 with $U(1) \times SO(3)$ symmetry to $N = 4$ $AdS_3 \times H^2$ geometry in the IR for $U(1) \times SL(3, \mathbb{R})$ gauge group and $g_1 = 1$

The Killing spinors ϵ_i are subject to the projection conditions (44) and

$$i\gamma_{\hat{\theta}\hat{\phi}}\epsilon_i = -(\sigma_1 \otimes \sigma_1)_i^j \epsilon_j. \quad (128)$$

As in the $U(1) \times SO(3, 1)$ gauge group studied in [34], there is only one supersymmetric $AdS_3 \times H^2$ critical point given by

$$\begin{aligned} \phi_1 = \phi_2 = \phi_3 = 0, \quad \Sigma = -\left(\frac{\sqrt{2}g_2}{g_1}\right)^{\frac{1}{3}}, \\ g = -\frac{1}{2} \ln \left[\frac{1}{a_5} \left(\frac{g_1^2 g_2}{2} \right)^{\frac{1}{3}} \right] \quad L_{AdS_3} = \left(\frac{\sqrt{2}}{g_1 g_2^2} \right)^{\frac{1}{3}}. \end{aligned} \quad (129)$$

This solution is dual to a two-dimensional $N = (2, 2)$ SCFT. By setting $\phi_1 = \phi_2 = \phi_3 = 0$, we find a domain wall interpolating between this critical point and the supersymmetric AdS_5 as shown in Fig. 11.

We now move to $AdS_3 \times \Sigma_2$ solutions preserving four supercharges. The analysis follows the same line as in the previous two gauge groups, so we will be very brief in this section. By the same analysis as in the previous two gauge groups, we obtain the following BPS equations

$$\begin{aligned} f' &= \frac{1}{12\Sigma^2} e^{-2(\phi_2+\phi_3+g)} \left[2\sqrt{2}\kappa a_0 e^{2(\phi_2+\phi_3)} \right. \\ &\quad \left. - e^{2g} \left(\sqrt{3} - 2e^{2\phi_2} - \sqrt{3}e^{4\phi_2} - \sqrt{3}e^{4\phi_3} \right. \right. \\ &\quad \left. \left. + \sqrt{3}e^{4(\phi_2+\phi_3)} - 2e^{2\phi_2+4\phi_3} \right) g_2 \Sigma \right. \\ &\quad \left. + 2\kappa a_5 e^{2\phi_2} \left(1 + e^{4\phi_3} \right) \Sigma^3 \right. \\ &\quad \left. + 2\sqrt{2}e^{2(\phi_2+\phi_3+g)} g_1 \Sigma^4 \right], \\ g' &= -\frac{1}{12\Sigma^2} e^{-2(\phi_2+\phi_3+g)} \left[4\sqrt{2}\kappa a_0 e^{2(\phi_2+\phi_3)} \right. \\ &\quad \left. + e^{2g} \left(\sqrt{3} - 2e^{2\phi_2} - \sqrt{3}e^{4\phi_2} - \sqrt{3}e^{4\phi_3} \right. \right. \end{aligned} \quad (130)$$

$$\begin{aligned} &\left. + \sqrt{3}e^{4(\phi_2+\phi_3)} - 2e^{2\phi_2+4\phi_3} \right) g_2 \Sigma \\ &\left. + 4\kappa a_5 e^{2\phi_2} \left(1 + e^{4\phi_3} \right) \Sigma^3 \right. \\ &\left. - 2\sqrt{2}e^{2(\phi_2+\phi_3+g)} g_1 \Sigma^4 \right], \end{aligned} \quad (131)$$

$$\begin{aligned} \Sigma' &= \frac{1}{12\Sigma} e^{-2(\phi_2+\phi_3+g)} \left[-4\sqrt{2}\kappa a_0 e^{2(\phi_2+\phi_3)} \right. \\ &\quad \left. + e^{2g} \left(\sqrt{3} - 2e^{2\phi_2} - \sqrt{3}e^{4\phi_2} - \sqrt{3}e^{4\phi_3} \right. \right. \\ &\quad \left. \left. + \sqrt{3}e^{4(\phi_2+\phi_3)} - 2e^{2\phi_2+4\phi_3} \right) g_2 \Sigma \right. \\ &\quad \left. + 2\kappa a_5 e^{2\phi_2} \left(1 + e^{4\phi_3} \right) \Sigma^3 \right. \\ &\quad \left. - 4\sqrt{2}e^{2(\phi_2+\phi_3+g)} g_1 \Sigma^4 \right], \end{aligned} \quad (132)$$

$$\phi'_1 = 0, \quad (133)$$

$$\phi'_2 = -\frac{\sqrt{3}e^{-2(\phi_2+\phi_3)} (1 + e^{4\phi_2}) (e^{4\phi_3} - 1) g_2}{4\Sigma}, \quad (134)$$

$$\begin{aligned} \phi'_3 &= -\frac{1}{8\Sigma} e^{-2(\phi_2+\phi_3+g)} \left[e^{2g} \left(2e^{2\phi_2} - \sqrt{3} \right. \right. \\ &\quad \left. \left. + \sqrt{3}e^{4\phi_2} - \sqrt{3}e^{4\phi_3} + \sqrt{3}e^{4(\phi_2+\phi_3)} \right. \right. \\ &\quad \left. \left. - 2e^{2\phi_2+4\phi_3} \right) g_2 - 2\kappa a_5 e^{2\phi_2} (e^{4\phi_3} - 1) \Sigma^2 \right]. \end{aligned} \quad (135)$$

These equations admit one supersymmetric $AdS_3 \times \Sigma_2$ solution given by

$$\begin{aligned} \phi_2 = \phi_3 = 0, \quad \Sigma &= \left(\frac{\sqrt{2}\kappa}{a_5 g_1} \right)^{\frac{1}{3}}, \\ g &= \frac{1}{3} \ln \left(\frac{\sqrt{2}a_5^2}{g_1} \right), \quad L_{AdS_3} = \left(\frac{\sqrt{2}a_5^2}{g_1} \right)^{\frac{1}{3}} \frac{2}{(1 - \kappa a_5 g_2)}, \end{aligned} \quad (136)$$

and a domain wall interpolating between this critical point and the supersymmetric AdS_5 is shown in Fig. 12. It should

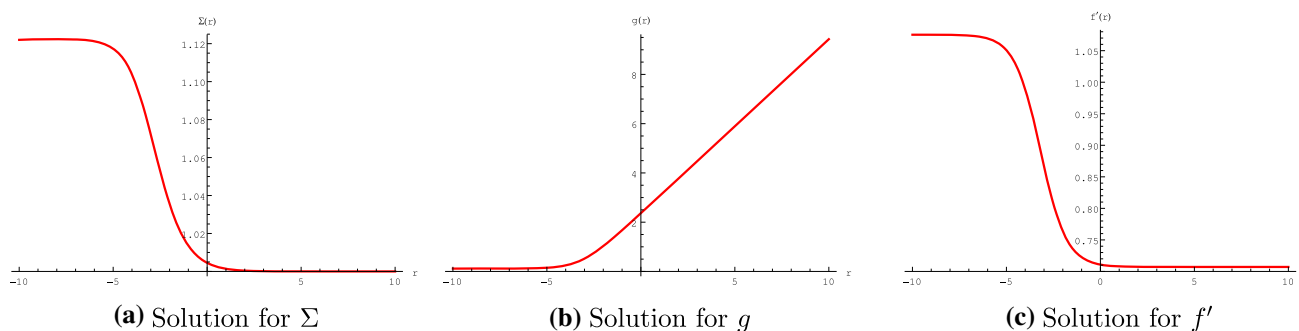


Fig. 12 An RG flow solution from AdS_5 with $U(1) \times SO(3)$ symmetry to $N = 2$ $AdS_3 \times S^2$ geometry in the IR for $U(1) \times SL(3, \mathbb{R})$ gauge group and $g_1 = 1$, $a_5 = 1$

also be noted that this $AdS_3 \times \Sigma_2$ solution is the same as in $U(1) \times SO(3, 1)$ gauge group.

5.3 Supersymmetric black holes

We end this section with an analysis of $AdS_2 \times \Sigma_3$ solutions and domain walls connecting these solutions to the supersymmetric AdS_5 . In order to preserve supersymmetry, $SO(3) \subset SL(3, \mathbb{R})$ gauge fields must be turned on. However, in the present case, there is no $SO(3)$ singlet scalar from the vector multiplets. After using the twist condition $g_2 a = 1$ and projectors in (72) and (79) together with the ansatz for the gauge fields in (70) and (77), we obtain the BPS equations

$$f' = -\frac{1}{6\Sigma} \left(2g_2 - 6\kappa a e^{-2g} \Sigma^2 - \sqrt{2} g_1 \Sigma^3 \right), \quad (137)$$

$$g' = -\frac{1}{6\Sigma} \left(2g_2 + 6\kappa a e^{-2g} \Sigma^2 - \sqrt{2} g_1 \Sigma^3 \right), \quad (138)$$

$$\Sigma' = -\frac{1}{3} \left(g_2 - 3\kappa a e^{-2g} \Sigma^2 + \sqrt{2} g_1 \Sigma^3 \right). \quad (139)$$

These equations turn out to be the same as in the $SO(3, 1)$ case after setting all the scalars from vector multiplets to zero. A single $AdS_2 \times H^3$ critical point is again given by (111).

6 Conclusions and discussions

We have found a new class of supersymmetric black strings and black holes in asymptotically AdS_5 space within $N = 4$ gauged supergravity in five dimensions coupled to five vector multiplets with gauge groups $U(1) \times SU(2) \times SU(2)$, $U(1) \times SO(3, 1)$ and $U(1) \times SL(3, \mathbb{R})$. These generalize the previously known black string solutions preserving eight supercharges by including more general twists along Σ_2 . Furthermore, unlike the half-supersymmetric solutions which only exhibit hyperbolic horizons, the $\frac{1}{4}$ -supersymmetric

black strings can have both S^2 and H^2 horizons. On the other hand, the AdS_5 black holes only feature H^3 horizons.

For $U(1) \times SU(2) \times SU(2)$ gauge group, we have identified a number of $AdS_3 \times \Sigma_2$ solutions preserving four supercharges. The solutions have $U(1) \times U(1) \times U(1)$ and $U(1) \times U(1)_{\text{diag}}$ symmetries and correspond to $N = (0, 2)$ SCFTs in two dimensions. We have given many examples of numerical RG flow solutions from the two supersymmetric AdS_5 vacua to these $AdS_3 \times \Sigma_2$ geometries. We have also found a supersymmetric $AdS_2 \times H^3$ solution describing the near horizon geometry of a supersymmetric black hole in AdS_5 . For $U(1) \times SO(3, 1)$ and $U(1) \times SL(3, \mathbb{R})$ gauge groups, all $AdS_3 \times \Sigma_2$ and $AdS_2 \times H^3$ solutions exist only for vanishing scalar fields from vector multiplets and have the same form for both gauge groups.

It would be interesting to compute twisted partition functions and twisted indices in the dual $N = 2$ SCFTs compactified on Σ_2 and Σ_3 . These should provide a microscopic description for the entropy of the aforementioned black strings and black holes in AdS_5 space. On the other hand, it is also interesting to find supersymmetric rotating AdS_5 black holes similar to the solutions found in minimal and maximal gauged supergravities [49, 50] or black holes with horizons in the form of a squashed three-sphere [51–53]. Furthermore, embedding these solutions in string/M-theory is of particular interest and should give a full holographic interpretation for the RG flows across dimensions identified here.

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Supersymmetric AdS_6 vacua in six-dimensional $N = (1, 1)$ gauged supergravity

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ABSTRACT: We study fully supersymmetric AdS_6 vacua of half-maximal $N = (1, 1)$ gauged supergravity in six space-time dimensions coupled to n vector multiplets. We show that the existence of AdS_6 backgrounds requires that the gauge group is of the form $G' \times G'' \subset \text{SO}(4, n)$ where $G' \subset \text{SO}(3, m)$ and $G'' \subset \text{SO}(1, n - m)$. In the AdS_6 vacua this gauge group is broken to its maximal compact subgroup $\text{SO}(3) \times H' \times H''$ where $H' \subset \text{SO}(m)$ and $H'' \subset \text{SO}(n - m)$. Furthermore, the $\text{SO}(3)$ factor is the R-symmetry gauged by three of the four graviphotons. We further show that the AdS_6 vacua have no moduli that preserve all supercharges. This is precisely in agreement with the absence of supersymmetric marginal deformations in holographically dual five-dimensional superconformal field theories.

KEYWORDS: Supergravity Models, AdS-CFT Correspondence, Gauge-gravity correspondence

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Contents

1	Introduction	1
2	$N = (1, 1)$ gauged supergravity in six dimensions	2
3	Maximally supersymmetric AdS_6 vacua	5
4	Moduli space of supersymmetric AdS_6 vacua	8
5	Conclusions	9

1 Introduction

Supersymmetric anti-de Sitter (AdS) vacua and their moduli spaces of gauged supergravities are of particular interest in the AdS/CFT correspondence [1]. The AdS vacua correspond to conformal fixed points of the holographically dual field theories while the moduli spaces describe the conformal manifolds near these fixed points [2, 3]. The latter encode useful information about the exactly marginal deformations of the corresponding superconformal field theories (SCFTs).

AdS backgrounds of gauged supergravities and their moduli spaces have been studied in various space-time dimensions with different numbers of supercharges. In this paper we exclusively focus on the half-maximal gauged $N = (1, 1)$ supergravity in six space-time dimensions ($d = 6$) and their maximally supersymmetric AdS_6 backgrounds.¹ This supergravity is also known as $F(4)$ supergravity and was first constructed in [6]. It is non-chiral and can be coupled to an arbitrary number n of vector multiplets. Each vector multiplet contains four scalars and together with the dilaton in the gravity multiplet, they parametrize the $(4n + 1)$ -dimensional coset manifold $\mathbb{R}^+ \times \text{SO}(4, n)/\text{SO}(4) \times \text{SO}(n)$. The corresponding gauged supergravity was constructed in [7, 8] by extending the pure $F(4)$ supergravity using the geometric group manifold approach. [7, 8] also showed that for a gauge group $\text{SU}(2)_R \times G$ and $G \subset \text{SO}(n)$ a maximally supersymmetric AdS_6 vacuum exists where the full $\text{SU}(2)_R \times G$ symmetry is realized at the origin of the scalar manifold. This vacuum could be identified with the near horizon geometry of the D4-D8 brane system [9]. For the case of $n = 3$ vector multiplets and $G = \text{SO}(3)$, another non-trivial AdS_6 vacuum breaking the $\text{SU}(2)_R \times \text{SO}(3)$ symmetry to $\text{SO}(3)_{\text{diag}}$ and preserving the full $N = (1, 1)$ supersymmetry has been identified in [10].

In this paper we do not specify the gauge group upfront but instead follow the strategy of [11–15] in that we first determine the general conditions on the parameters of the gauged

¹In fact, the $N = (1, 1)$ supergravity is the only gauged supergravity in $d = 6$ that admits maximally supersymmetric AdS_6 backgrounds [4]. This in turn is consistent with the known classification of the AdS superalgebras given in [5].

supergravity such that AdS_6 backgrounds which preserve all supercharges can exist. In half-maximal supergravities it is then possible to also give all possible gauge groups that can have such vacua. Concretely we find that the gauge group has to be of the form $G' \times G'' \subset \text{SO}(4, n)$ where $G' \subset \text{SO}(3, m)$ and $G'' \subset \text{SO}(1, n - m)$. In the AdS_6 vacua this gauge group is broken to its maximal compact subgroup $\text{SO}(3) \times H' \times H''$ where $H' \subset \text{SO}(m)$ and $H'' \subset \text{SO}(n - m)$. The $\text{SO}(3) \sim \text{SU}(2)$ factor precisely is the R-symmetry and it is gauged by three of the four graviphotons. Finally, we derive the necessary conditions for the existence of a supersymmetric moduli space near these vacua. For the case at hand we find that no moduli space is possible which is again consistent with the fact that the holographically dual SCFTs have no supersymmetric exactly marginal deformations.

In the AdS/CFT correspondence, AdS_6 vacua are also relevant for studying strongly coupled five-dimensional SCFTs arising from the dynamics of D4-D8 branes [9, 16]. The interpretation in terms of AdS_6 geometry in [17] inspired various studies considering gravity duals of these SCFTs including a recent generalization to quiver gauge theories in [18]. Finding AdS_6 solutions in type II and eleven dimensional supergravities also deserves detailed investigations.² In this paper, however, we stay in $d = 6$ throughout the analysis leaving the higher dimensional origins of these vacua for future work.

The paper is organized as follow. In section 2, we set the stage for our analysis and recall the relevant features of $N = (1, 1)$ gauged supergravity. The conditions for the existence of maximally supersymmetric AdS_6 vacua are then derived in section 3, and the analysis of the moduli space is carried out in section 4. We finally end the paper by giving some conclusions and comments on the results in section 5.

2 $N = (1, 1)$ gauged supergravity in six dimensions

In this section, we briefly review $N = (1, 1)$ gauged supergravity coupled to n vector multiplets in order to set up the notation for the later analysis. More details on this gauged supergravity can be found in [7, 8]. We will follow most of the conventions in these two references.

The possible supermultiplets are the gravitational multiplet and n vector multiplets given respectively by

$$(e_\mu^\alpha, \psi_\mu^A, A_\mu^\alpha, B_{\mu\nu}, \chi^A, \sigma) \quad \text{and} \quad (A_\mu, \lambda_A, \phi^\alpha)^I. \quad (2.1)$$

The bosonic fields of the supergravity multiplet are given by the graviton e_μ^a , the dilaton σ , four graviphotons A_μ^α , and a two-form field $B_{\mu\nu}$ while each vector multiplet contains a vector, A_μ , and four scalars, ϕ^α . Two sets of indices $\alpha, \beta, \dots = 0, 1, 2, 3$ and $I, J, \dots = 1, \dots, n$ label the $n + 4$ vector fields. Space-time and tangent space indices are denoted respectively by $\mu, \nu = 0, \dots, 5$ and $a, b = 0, \dots, 5$. We will also follow the mostly minus space-time signature $(+ - - - -)$ of [7, 8].

The fermionic fields consist of two gravitini ψ_μ^A , two spin- $\frac{1}{2}$ fields χ^A and $2n$ gauginos λ_A^I . All of these fields and the supersymmetry parameter ϵ^A are eight-component

²See [19–22] for recent results along this direction and references therein.

pseudo-Majorana spinors and transform in the fundamental representation of the $SU(2)_R \sim USp(2)_R$ R-symmetry denoted by indices $A, B = 1, 2$.

The dilaton and the $4n$ scalars $\phi^{\alpha I}$ of the vector multiplets span the coset manifold

$$\mathbb{R}^+ \times SO(4, n)/SO(4) \times SO(n). \quad (2.2)$$

The second factor can in turn be parametrized by the coset representative L^Λ_Σ with $\Lambda, \Sigma, \dots = 1, 2, \dots, n+4$. It is convenient to split the indices transforming under the compact group $SO(4) \times SO(n)$ as $\Lambda = (\alpha, I)$ and further under the $SO(3)_R \times SO(n)$ as $\Lambda = (0, r, I)$ with $r, s, \dots = 1, 2, 3$. The $SO(3)_R$ is identified with the diagonal subgroup of $SO(3) \times SO(3) \sim SO(4)$. The coset representative can be accordingly decomposed as

$$L^\Lambda_\Sigma = (L^\Lambda_\alpha, L^\Lambda_I) = (L^\Lambda_0, L^\Lambda_r, L^\Lambda_I). \quad (2.3)$$

Furthermore, all of the $n+4$ vector fields will be collectively denoted by $A^\Lambda_\mu = (A^0_\mu, A^r_\mu, A^I_\mu)$. Being $SO(4, n)$ matrices, the L^Λ_Σ satisfy the relation

$$\eta_{\Lambda\Sigma} = L^0_\Lambda L^0_\Sigma + L^i_\Lambda L^i_\Sigma - L^I_\Lambda L^I_\Sigma \quad (2.4)$$

with $\eta_{\Lambda\Sigma} = (1, 1, 1, 1, -1, -1, \dots, -1)$.

We now turn to the gauged version of this supergravity. The most complete gauged $N = (1, 1)$ supergravity up to now is given in [7, 8]. As in seven dimensions, the full $SO(4, n)$ covariant formulation in terms of the embedding tensors has not been worked out yet although the corresponding components of the embedding tensor have been identified in [23] using the Kac-Moody approach. In this paper, we will restrict ourselves to the gauged supergravity constructed in [7, 8].

Gauging is implemented by making a particular subgroup G of $SO(4, n)$ local such that the adjoint representation of G can be embedded in the fundamental representation, $\mathbf{n} + \mathbf{4}$, of $SO(4, n)$, and $\eta_{\Lambda\Sigma}$ contains the Cartan-Killing form of the gauge group. Consistency with supersymmetry requires that the structure constants are totally anti-symmetric, i.e. $f_{\Lambda\Sigma\Pi} = f_{\Lambda\Sigma}{}^\Gamma \eta_{\Gamma\Pi} = f_{[\Lambda\Sigma\Pi]}$. In the embedding tensor formalism, this condition is called the linear constraint.

The $f_{\Lambda\Sigma}{}^\Gamma$ appear as structure constants in the gauge algebra

$$[T_\Lambda, T_\Sigma] = f_{\Lambda\Sigma}{}^\Gamma T_\Gamma \quad (2.5)$$

in which T_Λ are gauge generators. These structure constants must satisfy the Jacobi identity

$$f_{[\Sigma\Gamma}{}^\Delta f_{\Lambda]\Delta}{}^\Pi = 0 \quad (2.6)$$

which in the embedding tensor formalism is the so-called quadratic constraint. In general, this constraint comes from the requirement that the gauge generators, obtained from appropriate projections of the global symmetry generators by the embedding tensor, form a closed Lie algebra of the corresponding gauge group.

The bosonic Lagrangian with only the metric and scalars non-vanishing reads

$$e^{-1} \mathcal{L} = -\frac{1}{4} R + \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{4} (P^{I0}_\mu P^\mu_{I0} + P^{Ir}_\mu P^\mu_{Ir}) - V, \quad (2.7)$$

where the scalar kinetic term is written in terms of the Maurer-Cartan one-forms

$$P^I_0 = (L^{-1})^I_{\Lambda} (dL^{\Lambda}_0 + f^{\Lambda}_{\Gamma\Pi} A^{\Gamma} L^{\Pi}_0), \quad P^I_r = (L^{-1})^I_{\Lambda} (dL^{\Lambda}_r + f^{\Lambda}_{\Gamma\Pi} A^{\Gamma} L^{\Pi}_r). \quad (2.8)$$

The scalar potential V is given by

$$\begin{aligned} V = & -5 \left[\frac{1}{144} (Ae^{\sigma} + 6me^{-3\sigma} L_{00})^2 + \frac{1}{16} (B_i e^{\sigma} - 2me^{-3\sigma} L_{0i})^2 \right] \\ & + \frac{1}{144} (Ae^{\sigma} - 18me^{-3\sigma} L_{00})^2 + \frac{1}{16} (B_i e^{\sigma} + 6me^{-3\sigma} L_{0i})^2 \\ & + \frac{1}{4} (C^I_t C_{It} + 4D^I_t D_{It}) e^{2\sigma} - m^2 e^{-6\sigma} L_{0I} L^{0I} \end{aligned} \quad (2.9)$$

where m is the mass of the two-form in the gravitational multiplet and we abbreviated

$$\begin{aligned} A &= \epsilon^{rst} K_{rst}, & B^i &= \epsilon^{ijk} K_{jk0}, \\ C_I{}^t &= \epsilon^{trs} K_{rIs}, & D_{It} &= K_{0It}, \end{aligned} \quad (2.10)$$

with the “dressed” structure constants given by

$$\begin{aligned} K_{rst} &= f_{\Lambda\Sigma\Pi} L^{\Lambda}_r (L^{-1})^{\Sigma}_s L^{\Pi}_t, & K_{rs0} &= f_{\Lambda\Sigma\Pi} L^{\Lambda}_r (L^{-1})^{\Sigma}_s L^{\Pi}_0, \\ K_{rIt} &= f_{\Lambda\Sigma\Pi} L^{\Lambda}_r (L^{-1})^{\Sigma}_I L^{\Pi}_t, & K_{0It} &= f_{\Lambda\Sigma\Pi} L^{\Lambda}_0 (L^{-1})^{\Sigma}_I L^{\Pi}_t. \end{aligned} \quad (2.11)$$

The supersymmetry transformations of the fermions which will play an important role in the following analysis are given by

$$\begin{aligned} \delta\psi_{A\mu} &= D_{\mu}\epsilon_A + S_{AB}\gamma_{\mu}\epsilon^B, \\ \delta\chi_A &= \frac{i}{2}\gamma^{\mu}\partial_{\mu}\sigma\epsilon_A + N_{AB}\epsilon^B, \\ \delta\lambda^I_A &= -iP^I_{ri}\sigma^{rAB}\partial_{\mu}\phi^i\gamma^{\mu}\epsilon_B + iP^I_{0i}\epsilon^{AB}\partial_{\mu}\phi^i\gamma^7\gamma^{\mu}\epsilon_B + M^I_{AB}\epsilon^B, \end{aligned} \quad (2.12)$$

where the fermion-shift matrices are defined as

$$\begin{aligned} S_{AB} &= \frac{i}{24} [Ae^{\sigma} + 6me^{-3\sigma}(L^{-1})_{00}] \epsilon_{AB} - \frac{i}{8} [B_t e^{\sigma} - 2me^{-3\sigma}(L^{-1})_{t0}] \gamma^7 \sigma^t_{AB}, \\ N_{AB} &= \frac{1}{24} [Ae^{\sigma} - 18me^{-3\sigma}(L^{-1})_{00}] \epsilon_{AB} + \frac{1}{8} [B_t e^{\sigma} + 6me^{-3\sigma}(L^{-1})_{t0}] \gamma^7 \sigma^t_{AB}, \\ M^I_{AB} &= (-C^I_t + 2i\gamma^7 D^I_t) e^{\sigma} \sigma^t_{AB} - 2me^{-3\sigma} (L^{-1})^I_0 \gamma^7 \epsilon_{AB}. \end{aligned} \quad (2.13)$$

In the present convention, the anti-symmetric matrix $\epsilon_{AB} = -\epsilon_{BA}$ is taken to be $\epsilon_{12} = \epsilon^{12} = 1$. The σ^t_{AB} matrices are related to the usual Pauli matrices σ^{tA}_B by the relation³

$$\sigma^t_{AB} = \sigma^{tC}_B \epsilon_{CA}. \quad (2.14)$$

Finally, the chirality matrix γ_7 is defined by

$$\gamma_7 = i\gamma_0\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5 \quad (2.15)$$

with $\gamma_7^2 = -\mathbb{I}$ and $\gamma_7^T = -\gamma_7$.

³Note that $\sigma^t_{AB} = \sigma^t_{(AB)}$.

3 Maximally supersymmetric AdS₆ vacua

We now determine the maximally supersymmetric AdS₆ vacua preserving all sixteen supercharges. In order to do so, we impose that the following conditions vanish for all supercharges in the background

$$\langle \delta \psi_{\mu A} \rangle = 0, \quad \langle \delta \chi_A \rangle = 0, \quad \langle \delta \lambda_A^I \rangle = 0. \quad (3.1)$$

Due to the symmetries of $\sigma_{AB}^t = \sigma_{(AB)}^t$ and $\epsilon_{AB} = \epsilon_{[AB]}$, the linear independence of γ_7 and $\mathbb{I}$ and by using (2.12) and (2.13) we infer that the second and third equations imply

$$\langle A e^\sigma - 18 m e^{-3\sigma} (L^{-1})_{00} \rangle = 0, \quad (3.2)$$

$$\langle e^{-3\sigma} (L^{-1})_0^I \rangle = 0, \quad (3.3)$$

$$\langle B_t e^\sigma + 6 m e^{-3\sigma} (L^{-1})_{t0} \rangle = 0, \quad (3.4)$$

$$\langle C_t^I \rangle = \langle D_t^I \rangle = 0. \quad (3.5)$$

From (2.10) we learn that the first condition in (3.5) is equivalent to

$$\epsilon^{trs} K_{rIs} = 0. \quad (3.6)$$

The second condition in (3.5) yields $K_{0It} = 0$ so that together we have

$$K_{rIs} = K_{0It} = 0. \quad (3.7)$$

Using (2.10) we can rewrite condition (3.2) as

$$\epsilon^{rst} K_{rst} = 18 m e^{-4\langle \sigma \rangle} \langle (L^{-1})_{00} \rangle \quad (3.8)$$

which is solved by

$$K_{rst} = g \epsilon_{rst} \quad (3.9)$$

for an arbitrary SU(2)_R gauge coupling g . We can accordingly determine the background value of the dilaton by inserting (3.9) into (3.8)

$$e^{-4\langle \sigma \rangle} \langle (L^{-1})_{00} \rangle = \frac{g}{3m}. \quad (3.10)$$

The remaining conditions (3.3) and (3.4) give

$$\langle (L^{-1})_0^I \rangle = 0, \quad \langle B_t \rangle = -6 m e^{-4\langle \sigma \rangle} \langle (L^{-1})_{t0} \rangle. \quad (3.11)$$

Using the component-(0I) and -(0i) of the relation (2.4) and the identity $L^{-1} = \eta L^T \eta$, we find that $L_{0I} = 0$ implies $L_{0i} = 0$ and thus

$$\langle B_t \rangle = 0. \quad (3.12)$$

Using the definition of B_t given in (2.10) we thus arrive at

$$K_{jk0} = 0. \quad (3.13)$$

By taking the (00)-component of the relation (2.4), we find that $L_{0I} = L_{0i} = 0$ also implies $L_{00} = 1$. Inserting the results obtained so far into (2.9) we conclude that the background value of the scalar potential (related to the cosmological constant) in an AdS_6 vacuum is given by

$$\langle V \rangle = -20m^2 \left(\frac{g}{3m} \right)^{\frac{3}{2}}. \quad (3.14)$$

We see that AdS vacua do not exist for $m = 0$ as already pointed out in [7, 8].⁴ This is very similar to AdS backgrounds of half-maximal supergravities in seven dimensions [13, 24, 25]. Note also that by shifting the value of $\langle \sigma \rangle$ we can choose $g = 3m$ as in [7, 8].

In order to continue let us recall that we are left with the unconstrained structure constants

$$K_{rst}, \quad K_{rIJ}, \quad K_{0IJ}, \quad K_{IJK}, \quad (3.15)$$

whose choice specify the particular supergravity at hand. We can now use the quadratic constraint to determine the corresponding gauge groups. These are the gauge groups which can occur in the supergravities that admit maximally supersymmetric AdS_6 vacua. For the case at hand the quadratic constraint reduces to the usual Jacobi identity given in (2.6).

As a warm up let us first consider the simple situation where $K_{rIJ} = K_{0IJ} = K_{IJK} = 0$ and we only have K_{rst} non-zero. In this case, equation (2.6) reduces to the Jacobi identity of an $\text{SO}(3)$ algebra with the structure constants $K_{rst} = g\epsilon_{rst}$. We then simply recover the pure $F(4)$ gauged supergravity with an $\text{SU}(2) \sim \text{SO}(3)$ gauge group constructed in [6].

For $K_{rIJ} = K_{0IJ} = 0$ but $K_{IJK} \neq 0$, the condition (2.6) gives rise to two separate Jacobi identities for K_{rst} and K_{IJK} which correspond to two commuting compact groups. The gauge group is accordingly $\text{SO}(3) \times H$ with $H \subset \text{SO}(n)$ and compact. This gauge group and the resulting AdS_6 vacuum together with the dual five-dimensional SCFT have already been studied in [7, 8].

As a next step let us also take $K_{rIJ} \neq 0$ but still have $K_{0IJ} = 0$. In this case the $\text{SO}(3)$ -singlet graviphoton A^0 decouples from all other gauge bosons. This is very similar to the seven-dimensional case studied in [13] where the gauge groups are embedded in $\text{SO}(3, n) \subset \text{SO}(4, n)$. If one additionally assumes that the gauge group is semi-simple one can in fact list all possibilities. The Cartan-Killing form of these gauge groups must be embeddable in the $\text{SO}(3, n)$ invariant tensor $\eta = (\delta_{rs}, -\delta_{IJ})$ which imposes a strong constraint. Furthermore, the existence of supersymmetric AdS_6 vacua requires that the gauge groups must contain $\text{SO}(3)$ as a subgroup. The only possible semisimple gauge groups are then given by

$$\tilde{G} \times H \quad (3.16)$$

where $\tilde{G} = \text{SO}(3), \text{SO}(3, 1)$ or $\text{SL}(3, \mathbb{R})$ and $H \subset \text{SO}(n + 3 - \dim \tilde{G})$ is a compact group.

We finally consider the most general case with all structure constants in (3.15) non-zero. Follow a similar analysis in [14] we introduce the gauge generators embedded in $\text{SO}(4, n)$ as

$$(T_\Lambda)_\Gamma^\Pi = f_\Lambda^{\Sigma\Delta} (t_{\Sigma\Delta})_\Gamma^\Pi = f_{\Lambda\Gamma}^\Pi \quad (3.17)$$

⁴Recall that m is the mass parameter of the two-form in the gravitational multiplet.

where $(t_{\Sigma\Delta})_{\Gamma}^{\Pi} = \delta_{[\Sigma}^{\Pi} \eta_{\Delta]\Gamma}$ are $\text{SO}(4, n)$ generators in the vector representation. Splitting the indices $\Lambda = (0, i, I)$ decomposes the gauge generators as

$$(T_0)_{\Gamma}^{\Pi} = f_{0\Gamma}^{\Pi}, \quad (T_i)_{\Gamma}^{\Pi} = f_{i\Gamma}^{\Pi}, \quad (T_I)_{\Gamma}^{\Pi} = f_{I\Gamma}^{\Pi}, \quad (3.18)$$

which couple to the vector fields A^0 , A^i and A^I , respectively.

It is more convenient to write down the various independent components of the Jacobi identity. They read

$$K_{[ij}{}^l K_{k]l}{}^m = 0, \quad (3.19)$$

$$K_{iJ}{}^I K_{IK}{}^J + K_{Ki}{}^I K_{IJ}{}^J + K_{JK}{}^r K_{ri}{}^J = 0, \quad (3.20)$$

$$K_{iJ}{}^I K_{IK}{}^L + K_{Ki}{}^I K_{IJ}{}^L + K_{JK}{}^I K_{Ii}{}^L = 0, \quad (3.21)$$

$$K_{0I}{}^J K_{Jj}{}^K + K_{Ij}{}^J K_{J0}{}^K = 0, \quad (3.22)$$

$$K_{IJ}{}^K K_{K0}{}^L + K_{0I}{}^K K_{KJ}{}^L + K_{J0}{}^K K_{KI}{}^L = 0, \quad (3.23)$$

$$K_{[IJ}{}^0 K_{K]0}{}^M + K_{[IJ}{}^r K_{K]r}{}^M + K_{[IJ}{}^L K_{K]L}{}^M = 0. \quad (3.24)$$

The first two relations (3.19), (3.20) imply that the $\text{SO}(3)$ generators T_i have non-vanishing elements in both $\text{SO}(3)$ and $\text{SO}(n)$ blocks. We therefore split the indices $I, J, K, \dots$ into two sets $\hat{I}, \hat{J}, \hat{K} = 1, \dots, m$ and $\tilde{I}, \tilde{J}, \tilde{K} = 1, \dots, n - m$ such that only the $\hat{I}, \hat{J}, \hat{K}$ indices mix with r, s, t indices. Or, in other word, we have $K_{r\hat{I}\hat{J}} \neq 0$ and $K_{r\tilde{I}\tilde{J}} = 0$. With this convention the $\text{SO}(3)$ generators take the form

$$T_i = \begin{pmatrix} 0 & & & \\ & K_{ij}{}^k & & \\ & & K_{i\hat{J}}{}^{\hat{K}} & \\ & & & 0_{n-m} \end{pmatrix}, \quad (3.25)$$

where 0_n indicates an $n \times n$ zero matrix.

The relation (3.22) corresponds to $[T_i, T_0] = 0$ and thus T_0 and T_i cannot have common I, J, K indices or equivalently $K_{0\hat{I}\hat{J}} = K_{0\tilde{I}\tilde{J}} = 0$. This determines the T_0 generator to be

$$T_0 = \begin{pmatrix} 0 & & & \\ & 0_3 & & \\ & & 0_m & \\ & & & K_{0\tilde{J}}{}^{\tilde{K}} \end{pmatrix}. \quad (3.26)$$

Equation (3.21) and the $(\hat{I}, \hat{J}, \hat{K}, \hat{M})$ components of relation (3.24) imply that the $T_{\hat{I}}$ generators are given by

$$T_{\hat{I}} = \begin{pmatrix} 0 & & & \\ & 0_3 & K_{\hat{I}r}{}^{\hat{K}} & \\ & K_{\hat{I}\hat{J}}{}^r & K_{\hat{I}\hat{J}}{}^{\hat{K}} & \\ & & & 0_{n-m} \end{pmatrix}. \quad (3.27)$$

Therefore, the $(T_i, T_{\hat{I}})$ generators together form a non-compact group $G' \subset \text{SO}(3, m)$, $m \leq n$.

Finally, the relation (3.23) and the $(\tilde{I}, \tilde{J}, \tilde{K}, \tilde{M})$ components of relation (3.24) determine the structure of $T_{\tilde{I}}$ to be

$$T_{\tilde{I}} = \begin{pmatrix} 0 & & K_{\tilde{I}0}^{\tilde{K}} \\ & 0_3 & \\ & & 0_m \\ K_{\tilde{I}\tilde{J}}^0 & & K_{\tilde{I}\tilde{J}}^{\tilde{K}} \end{pmatrix}. \quad (3.28)$$

These generators together with T_0 form another non-compact group $G'' \subset \text{SO}(1, n-m)$. We then conclude that the general gauge group admitting maximally supersymmetric AdS_6 vacua take the form

$$G' \times G'' \quad (3.29)$$

where $G' \subset \text{SO}(3, m)$ and $G'' \subset \text{SO}(1, n-m)$. In an AdS_6 background, the gauge group is broken to its maximal compact subgroup $\text{SO}(3) \times H' \times H''$ in which $H' \subset \text{SO}(m)$ and $H'' \subset \text{SO}(n-m)$.

To confirm this, we inspect the massive vector fields arising from the above symmetry breaking. Defining $A^{\hat{I}} = (L^{-1})^{\hat{I}}_{\Lambda} A^{\Lambda}$ and $A^{\tilde{I}} = (L^{-1})^{\tilde{I}}_{\Lambda} A^{\Lambda}$, we find that various components of the Maurer-Cartan one-form P^I_{α} are given by

$$\begin{aligned} P^{\hat{I}}_0 &= (L^{-1})^{\hat{I}}_{\Lambda} dL^{\Lambda}_0, & P^{\hat{I}}_r &= (L^{-1})^{\hat{I}}_{\Lambda} dL^{\Lambda}_r + K^{\hat{I}}_{j_r} A^j, \\ P^{\tilde{I}}_0 &= (L^{-1})^{\tilde{I}}_{\Lambda} dL^{\Lambda}_0 + K^{\tilde{I}}_{j_0} A^j, & P^{\tilde{I}}_r &= (L^{-1})^{\tilde{I}}_{\Lambda} dL^{\Lambda}_r. \end{aligned} \quad (3.30)$$

By computing the scalar kinetic terms, we can indeed see that there is precisely one massive vector field corresponding to each non-compact generators $K_{\hat{I}\hat{J}}^r$ and $K_{\tilde{I}\tilde{J}}^0$. These massive vectors correspond to the broken non-compact generators of the full gauge group.

4 Moduli space of supersymmetric AdS_6 vacua

In this section, we determine the flat directions of the scalar potential V which preserve all 16 supercharges. These are the moduli of the AdS_6 backgrounds corresponding to supersymmetric marginal deformations of the five-dimensional superconformal field theories dual to the AdS_6 vacua identified in the previous section.

Similar to the analysis of [12–14], a necessary condition for the existence of these moduli can be determined by considering possible deformations of the supersymmetry conditions (3.1) near the AdS_6 vacua. By varying the conditions in (3.5), we find

$$\delta(e^{4\sigma} A) = 4\langle A \rangle \delta\sigma + e^{4\langle\sigma\rangle} \delta A = 0, \quad (4.1)$$

$$\delta C^I_t = \delta D^I_t = \delta B_t = 0. \quad (4.2)$$

We now introduce a parametrization of the variation of the coset representative with respect to the $4n$ scalars $\phi^{\alpha I}$

$$\delta L^{\Lambda}_{\alpha} = \langle L^{\Lambda}_I \rangle \delta\phi^{\alpha I}, \quad \delta L^{\Lambda}_I = \langle L^{\Lambda}_{\alpha} \rangle \delta\phi^{\alpha I} \quad (4.3)$$

and their inverse

$$\delta(L^{-1})^\Lambda_\alpha = -\langle(L^{-1})^\Lambda_I\rangle\delta\phi^{\alpha I}, \quad \delta(L^{-1})^\Lambda_I = -\langle(L^{-1})^\Lambda_\alpha\rangle\delta\phi^{\alpha I}. \quad (4.4)$$

Using these relations and the decomposition of indices $\alpha = (0, r)$, we find

$$\delta L^\Lambda_0 = \langle L^\Lambda_I \rangle \delta\phi^{0I}, \quad \delta L^\Lambda_i = \langle L^\Lambda_I \rangle \delta\phi^{iI}, \quad \delta L^\Lambda_I = \langle L^\Lambda_0 \rangle \delta\phi^{0I} + \langle L^\Lambda_r \rangle \delta\phi^{rI} \quad (4.5)$$

and

$$\begin{aligned} \delta(L^{-1})^\Lambda_0 &= -\langle(L^{-1})^\Lambda_I\rangle\delta\phi^{0I}, & \delta(L^{-1})^\Lambda_i &= -\langle(L^{-1})^\Lambda_I\rangle\delta\phi^{iI}, \\ \delta(L^{-1})^\Lambda_I &= -\langle(L^{-1})^\Lambda_0\rangle\delta\phi^{0I} - \langle(L^{-1})^\Lambda_r\rangle\delta\phi^{rI}. \end{aligned} \quad (4.6)$$

With the help of these relations, we can rewrite the conditions (4.1) and (4.2) as

$$0 = \delta(e^{4\sigma} A) = 4e^{4\langle\sigma\rangle} \langle A \rangle \delta\sigma + 3e^{4\langle\sigma\rangle} \langle C_{Ir} \rangle \delta\phi^{rI}, \quad (4.7)$$

$$0 = \delta B_t = \langle C_{It} \rangle \delta\phi^{0I} + 2\epsilon_{rtk} \langle D_{Ik} \rangle \delta\phi^{rI}, \quad (4.8)$$

$$0 = \delta C^I_t = 2\epsilon^{trs} K_{[rIJ} \delta\phi_{s]J} - \epsilon^{trs} K_{r0s} \delta\phi^{0I} - \epsilon^{trs} K_{ris} \delta\phi^{iI}, \quad (4.9)$$

$$0 = \delta D^I_t = K_{0It} \delta\phi^{0I} + K_{0IJ} \delta\phi^{tJ} - K_{0rt} \delta\phi^{rI} \quad (4.10)$$

where

$$K_{0IJ} = f_{\Lambda\Sigma\Pi} L^\Lambda_0 (L^{-1})^\Sigma_I L^\Pi_J, \quad K_{rIJ} = f_{\Lambda\Sigma\Pi} L^\Lambda_r (L^{-1})^\Sigma_I L^\Pi_J. \quad (4.11)$$

Using the AdS₆ conditions (3.5), (3.7) and (3.13) obtained in the previous section, we find

$$\delta\sigma = 0, \quad K_{0\tilde{I}\tilde{J}} \delta\phi_{t\tilde{J}} = 0, \quad K_{rst} \delta\phi_{t\tilde{I}} = 0, \quad 2\epsilon^{rst} K_{[r\tilde{I}\tilde{J}} \delta\phi_{s]\tilde{J}} + K_{rst} \delta\phi_{t\tilde{I}} = 0. \quad (4.12)$$

From these conditions, we immediately obtain $\delta\phi_{t\tilde{I}} = 0$ for $K_{rst} \neq 0$.

The last equation in (4.12) is similar to the one considered in [12, 13], and it has been shown in [12] that this equation has general solutions of the form

$$\delta\phi_{s\tilde{I}} = K_{s\tilde{I}J} \lambda^{\tilde{J}}. \quad (4.13)$$

The remaining scalars that are not fixed by the above conditions are $\delta\phi_{0\tilde{I}}$. We can readily recognize that $\delta\phi_{s\tilde{I}}$ and $\delta\phi_{0\tilde{I}}$ correspond to Goldstone bosons of the symmetry breaking $G' \times G'' \rightarrow \text{SO}(3) \times H' \times H''$, with $H' \subset \text{SO}(m)$ and $H'' \subset \text{SO}(n-m)$ in the AdS₆ vacuum. These massless scalars are eaten by the massive gauge fields mentioned in the previous section. Thus, all of the flat directions correspond to Goldstone bosons and no moduli exist. This in turn is consistent with the fact that there are no marginal deformations preserving all supersymmetry in the dual five-dimensional SCFTs.

5 Conclusions

In this paper, we have analyzed the general conditions for the existence of maximally supersymmetric AdS₆ vacua in the $N = (1, 1)$ half-maximal gauged supergravities in six dimensions. We have found that three of the graviphotons have to gauge an $\text{SU}(2)_R$ R-symmetry while the forth one can be used to gauge a commuting non-compact group.

The fact that the $SU(2)_R$ R-symmetry must be gauged is similar to the results in $d = 4, 6, 7$. This is in general a necessary condition for the existence of AdS vacua as shown in [4]. It is also consistent with the important role played by the corresponding R-symmetry in the dual field theories [16]. Furthermore, all vacua we have identified have no flat directions which preserve all supercharges corresponding to the absence of supersymmetric exactly marginal deformations in the dual five-dimensional SCFTs.

We end the paper by briefly giving some comments on the $\mathbb{R}^+ \times SO(4, n)$ covariant formulation in term of the embedding tensor. As shown in [23], there are two components of the embedding tensor given by ξ^Λ and $f_{\Lambda\Sigma\Gamma}$ as well as a massive deformation of the two-form field. The ξ^Λ is involved in gauging of the $\mathbb{R}^+$ factor. Due to many similarities between the six-dimensional $N = (1, 1)$ gauged supergravity considered here and the $N = 2$ gauged supergravity in seven dimensions, we expect that the $\mathbb{R}^+$ gauging and the massive deformation could not be turned on simultaneously. Therefore, the existence of maximally supersymmetric AdS_6 vacua would require $\xi^\Lambda = 0$ as shown in [13] for the seven-dimensional case. It would be of particular interest to explore this issue in particular to construct the complete gauging of $N = (1, 1)$ supergravity in the embedding tensor formulation.

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