รายงานวิจัยฉบับสมบูรณ์

โครงการการแพร่ระบาดทางการเงินและตัวเลือกกลุ่มการลงทุนที่ดีที่สุด

โดย ไทยศิริ เวทไว

มิถุนายน 2561

สัญญาเลขที่ RSA5980065

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สนับสนุนโดยสำนักงานกองทุนสนับสนุนการวิจัยและ
จุฬาลงกรณ์มหาวิทยาลัย

บทคัดย่อ

รหัสโครงการ : RSA5980065

ชื่อโครงการ : การแพร่ระบาดทางการเงินและตัวเลือกกลุ่มการลงทุนที่ดีที่สุด

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งานวิจัยนี้ศึกษาการแพร่ระบาดทางการเงินของตลาดทุนระหว่างประเทศและระหว่างธนาคาร ในส่วนแรกนักวิจัยเสนอแบบจำลองของการแพร่ระบาดทางการเงินของตลาดทุนระหว่างประเทศ ในแบบจำลองนี้ปัจจัยพื้นฐานของแต่ละประเทศมีความสัมพันธ์กันผ่านช่องทางทางการค้าและการเงิน ผลการศึกษาโดยใช้ข้อมูลของประเทศไทยกับประเทศญี่ปุ่น ฮ่องกง และสหรัฐอเมริกาพบว่าช่องทางทางการเงินเป็นช่องทางของการแพร่ระบาดของตลาดทุนระหว่างประเทศ และปัจจัยพื้นฐานที่อ่อนแอของประเทศจะทำให้ได้รับผลกระทบที่สูงขึ้น ในส่วนที่สองนักวิจัยเสนอแบบจำลองของระบบธนาคาร โดยธนาคารมีความสัมพันธ์กันผ่านการกู้ยืมระหว่างธนาคารและการถือสินทรัพย์สภาพคล่องต่ำประเภทเดียวกัน ธนาคารอาจจะได้รับความสูญเสียที่เฉพาะสำหรับธนาคารเองหรือเป็นความสูญเสียเชิงระบบ เมื่อมีความสูญเสียเกิดขึ้น ธนาคารจะทำการปรับสัดส่วนการถือครองสินทรัพย์ตามระดับความเสี่ยงและผลตอบแทนที่ทางธนาคารต้องการ ผลการศึกษาพบว่าธนาคารที่กลัวความเสี่ยงสูงจะช่วยลดผลกระทบด้านราคาเมื่อได้รับผลกระทบจากความสูญเสีย แต่จะเป็นผู้ซื้อสินทรัพย์ที่ไม่ดีหากไม่ได้รับผลกระทบ ในทางกลับกันธนาคารที่มีความกลัวความเสี่ยงต่ำจะช่วยเพิ่มความสูญเสียด้านราคาได้มากเมื่อได้รับผลกระทบ แต่จะเป็นผู้ซื้อที่ดีหากไม่ได้รับผลกระทบจึงช่วยลดผลกระทบด้านราคาได้ดี

คำหลัก : การแพร่ระบาดทางการเงิน ช่องทางการส่งผ่านความสูญเสีย ตลาดทุนระหว่างประเทศ ระบบสถาบันธนาคาร การเทขายสินทรัพย์

Abstract

Project Code : RSA5980065

Project Title : Financial Contagion and Optimal Portfolio Choice

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Project Period : 2 years

This research project studies financial contagion between international stock markets and between banks. In the first part, we propose a contagion model of International stock markets in which countries are connected through their fundamental linkages such as trade and financial linkages. Using the data between Thailand and three other couries, namely, Japan, Hong Kong and the US, we find that the financial linkage is a channel of contagion, and weak domestic fundamental amplifies the effect of the trasmitted shocks. In the second part, we propose a model of banking system in which banks are connected through interbank liabilities and common illiquid asset holdings. Banks are subject to bank-specific and market-wide shocks. After arrivals of shocks, banks adjust their portfolios based on their risk-adjusted return preferences. We find that banks with high level of risk-aversion help absorb the shocks, reducing the fire sale effect, but they are poor buyers when they are unaffected by the shocks. On the other hand, banks with low level of risk-aversion amplify the effect of fire sale when they are hit by shocks, but act as good buyers, reducing the fire sale effect, when they are safe from shocks.

Keywords : Financial contagion, channel of contagion, international stock markets, banking systems, fire sale

Executive Summary

Since the 1990s, there has been many evidence of financial contagion, for the 1994 Mexican peso crisis, the 1997 Asian financial crisis, the 1998 Russian financial crisis, and the 1998-2002 Argentine Great Depression, are characterised as financial contagion. In general terms, financial contagion can be defined as the spread of shocks in a financial market across borders, either across different asset markets or in asset markets across different countries. The 1997 Asian financial crisis, for example, was originally started in Thailand with a collapse of the Thai currency. The crisis was then transmitted to other financial markets in Asia, especially in the currency and the stock markets, and also to the financial markets in the United States, Europe and Russia. The first part of this research project focuses on the cross-country contagion in the stock markets.

The international stock contagion has drastic impacts in various ways. In the investment perspective, without financial contagion, a globally diversified investment strategy would reduce portfolio risk and increase expected returns. But in the presence of financial contagion, a downside shock originated in one country leads to a significant drop in asset prices in other countries. As a result, the stock market contagion, if ignored, would lead to an underestimation of risk and eliminates the benefits of international portfolio diversification. So, global investors should take possible contagion into account, and modify their multinational investment strategy. Further, the existence of financial contagion puts a threat to country's economic and financial stability. Since enhancing the country's stabilities are the main challenges for policymakers, a better understanding of stock market contagion is needed in order to impose effective polices and regulations. A better understanding of financial contagion allows the policy makers to detect and prepare for shocks that would spill over, and reduce the contagion effect in the country. Therefore, understanding financial contagion is critically important and urgent.

The first part of this research project aims to develop a framework of fundamental-based contagion in the stock market. By defining the term *fundamental* as the health of the economy, we assume that return characteristics of a country is are determined by the country's domestic fundamental. In the international context, with countries' fundamentals being interconnected through fundamental linkages: trades and financial linkages, a domestic shock in one country could spread globally. During a crisis, banks and corporations suffer from withdrawals of foreign funds, investment spending significantly drops, trade financing dries up, and exports subsequently collapse. We study the role of the trade and financial linkages in *the transmission of shock*. Moreover, we introduce another effect in the contagion called *the shock amplification* by the domestic fundamental. The domestic fundamental plays a crucial role in the vulnerability of the country to an external shock. Holding everything else constant, countries with weak fundamental are likely to experience a more severe contagion effect than the ones whose the fundamental is sound. In other words, the weak fundamental amplifies the effects of transferred shocks.

With the proposed framework, we conduct an empirical study to study the international stock market contagion between Thailand and Japan, Hong Kong and the US, and aim to answer the following three questions: (1) Is there an amplification of shock by the domestic fundamental in Thailand?, (2) Are trade and financial linkages transmissions channels of shock?, and (3) Shocks from which country would transmit to Thailand the most? To reach to the answers of these questions, the proposed model is estimated using Markov Chain Monte Carlo (MCMC). We find that financial linkage is the only channel that transmits shocks. In contrast, trade linkage is somewhat an indirect measure of the Thai fundamental. We also find significant evidence of the shock amplification effect in Thailand, and that Thailand was sensitive to changes in the fundamental of the US the most. The effect of the changes in the US fundamental on the Thai fundamental and stock market returns became more pronounced during the 2008 global financial crisis.

Although financial contagion has been a famous topic of study in the past decade, our work contributes to the literatures in two ways. First, we propose that the effect of the two transmission channels of financial contagion, trade and financial linkages, is amplified by the domestic fundamental, while recent studies mainly focus on the role of trade and financial linkages and role of fundamental in explaining financial contagion separately. This enables us to investigate the interaction between each component. Second, unlike past studies that measure the contagion directly through relationship between countries' stock markets, we propose explicitly model how contagion occurs through countries' fundamentals, and how shocks in fundamentals affect stock market returns.

In the second part of our research project, we focus on financial contagion and the fire sale effect inside a banking system. Asset fire sales can be a major cause of a financial crisis. When banks are hit by unexpected shocks, either directly or indirectly through contagion channels, banks can become insolvent and need to liquidate all of their assets, the major of which are illiquid loans. Banks that survive but experience losses need to sell some of their assets to pay for the losses. Because in general banks' liquid assets are low-risk assets such as cash equivalents and banks' illiquid assets are high-risk assets such as loans, risk-averse banks do not sell only liquid assets, but also part of their illiquid assets to re-optimize the risk-adjusted return of their portfolios after the shocks. If there are few potential buyers who are willing to pay for the assets, the fire sale prices can be much worse. The system that tends to keep some potential buyers untouched from a result of a shock can be a solution to the fire sale problem. This *self-rescue* feature avoids seeking for help from the outsiders such as the government or the central bank, and thus reduces the burden on the taxpayers.

In this part of the project we present an equilibrium model of financial contagion in banking networks that allows survival banks to act as potential buyers. Banks in our model are risk averse, and hence they optimize their portfolios of cash and various types of illiquid loans based on a risk-adjusted return basis. A bank with a low level of risk aversion, or an *aggressive bank*, holds a larger portion of illiquid risky loans per unit of the bank's equity. So when the equity value of the bank decreases, it reduces its loan holding more aggressively than banks that are more risk averse, or *conservative banks*. When a shock hits an aggressive bank, it can originate fire sales of illiquid loans. On the other hand, if most aggressive bank are not affected by the shock, they act as potential buyers who are willing to take a large amount of loans given a small reduction in the price, and thus help save the network from asset fire sales.

Banks in our model are also linked through interbank liabilities. If an aggressive bank, holding a large amount of loans, is hit by a large shock and becomes insolvent, the loss will be propagated to its interbank creditors who will need to sell off some of their loans, if not all. If those creditors are relatively more aggressive compared to the other banks in the network, then the sales of illiquid loans can be huge, while the non-creditor banks, who are relatively more conservative, require a deep discount in the prices in order to generate enough demand to meet the huge supply of loans. In this case, both sellers and buyers amplify the fire sale effects. On the other hand, if the creditors are the relatively more conservative banks, the loan sales will be less and the non-creditor banks, which are relatively more aggressive, would take all of the loans given just a small discount. Thus, the self-rescue ability of the network relies partly on how conservative and aggressive banks are linked through the interbank liabilities.

When the loan markets and banks are divided into multiple sectors, the role of banks as potential buyers can be significant during a crisis of a sector. In our model, different banks have different expertise in managing different types of loans. Managing the loan type in their expertise requires a low cost, while managing loans outside their expertise is costly. This high cost creates a barrier for banks to hold loans outside their expertise and divides the loan markets and banks into sectors. In this situation, our model shows that when a small shock hits a bank in one sector, the other banks in the sector will act as the potential buyers providing the self-rescue mechanism of the sector. However, when the sector is hit by a large shock damaging many banks in the sector, it is difficult to avoid a sharp drop in the loan price as banks do not play their role as helpful potential buyers. This can happen, for example, if the default risk of the loans in the sector jumps up markedly, affecting the equity values of all banks in the sector. If the loan price falls enough to outweight the high cost, then, and only then, banks outside the failing sector will step in and act as the potential buyers to rescue the failing sector when it is most needed.

Separating loan markets and banks into sectors helps create the *secondary potential buyers*, and thus enhances the self-rescue mechanism of the system. When banks are separated into loan sectors, a shock to one type of loans does not cause losses to banks outside the sector through the asset price channel. Thus, it keeps those outside banks safe and ready to step in to save the sector once the time comes. Each sector now acts as secondary potential buyers for the other sectors. However, interbank liability linkages may exist between banks that belong to different sectors. This channel of contagion weaken the role as the secondary potential buyers of the outside banks as losses from the failing sector can be transmitted to the banks outside the sector. Another factor that can weaken the secondary potential buyer role of the outside banks is the default correlation. Default correlations between loans from different sectors create negative hedging demands due to the substitution effect. That is, when outside banks step in to buy loans from the failing sector, they reduce the holdings of the loans in their sector as they are substitute goods, causing the price to drop. The negative hedging demand is large when the correlation is high and the fire sale loans are attractive.

This result provides an interesting policy suggestion. The regulatory body can divide loan markets into non-overlapping sectors, and require banks to choose an area of expertise (i.e. the loan sector) in which the banks are allowed to run their businesses as usual. Banks running the business outside their declared area of expertise are required to pay a large amount of fee. This is to reduce the incentive of banks to create undesired contagion channel across the sectors. As a result, banks from one sector act as the secondary potential buyers for the others. The regulator should also set a limit for interbank liabilities between banks from different areas of expertise. Once a crisis is about to happen in one sector in which the fire sale can bring down the sector, the regulator may search for financially healthy sectors. If there is not any healthy sector, then it might be better to keep the fee high to avoid a cross-sector contagion and limit the losses. If there are financially healthy sectors, the regulator may choose to lower the fee for the financially healthy sectors to allow the healthy banks to step in and save the failing sector. The regulator can choose the fee level that does not cause a serious effect on the healthy sectors. Once this happens; however, the healthy sectors are contaminated by the failing sector, and the self-rescue system will not function for the next crisis. So the regulator should use this as a temporary solution and try to bring the system back to normal soon before the next crisis.

PART I

Roles of Domestic Fundamentals and Cross-Country Linkages on International Stock Market Contagion

I Objectives

1. To develop a fundamental-based model of financial contagion between international stock markets in which shocks are transmitted through cross-country linkages, and the effect of contagion depends on the strength of the domestic fundamentals.

2. To develop a Markov Chain Monte Carlo estimation algorithm for the contagion model that uses returns data and cross-country linkages data that can jointly estimate the global market with a large number of countries.

3. To conduct an empirical study between Thai market and its major peer countries to determine the roles of trade and financial linkages in transmitting shocks, and the role of domestic fundamentals in amplifying the contagion effect.

II Methodology

A Model

We develop a financial contagion model with n countries. Let $R_{i,t}$ and $F_{i,t}$ denote the stock market return and the fundamental of country i at time t, respectively, where the higher the value of F implies the stronger the fundamental.

Assume that the stock market return of country i at time $t+1$, $R_{i,t+1}$, follows the following dynamic:

$$R_{i,t+1} = \alpha_i^R + \beta_i^R F_{i,t} + e^{-\frac{1}{2}F_{i,t}} \epsilon_{i,t+1}^R \tag{1}$$

where α_i^R, β_i^R and γ_i^R are constants. $\epsilon_{i,t+1}^R$ is the domestic shock in the stock market return of country i at time $t+1$, and $\epsilon_t^R = \left[\epsilon_{1,t}^R, \epsilon_{2,t}^R, \ldots, \epsilon_{n,t}^R\right]$ is jointly normally distributed with mean 0 and covariance matrix $\Sigma^R = \left[\sigma_{ij}^R\right]$. Conditional on the current fundamental $F_{i,t}$, the expected stock market return is $\alpha_i^R + \beta_i^R F_{i,t}$ and the variance is $e^{-F_{i,t}}\sigma_{ii}^R$. That is, both mean and variance of return $R_{i,t+1}$ depend on the current fundamental $F_{i,t}$.

The change in the fundamental of country i at time $t+1$, $\Delta F_{i,t+1}$, follows the following dynamic:

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}}\left[\alpha_i^F - \beta_i^F F_{i,t} + \sum_{j\neq i}^{n}\left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t}\right)\cdot F_{j,t}\right] + \epsilon_{i,t+1}^F \tag{2}$$

where $\kappa_i, \alpha_i^F, \beta_i^F, \beta_{ij}^{(0)}, \beta_{ij}^{(1)}$, and $\beta_{ij}^{(2)}$ are constants. $\epsilon_t^F = \left[\epsilon_{1,t}^F, \ldots, \epsilon_{n,t}^F\right]$ is jointly normally distributed with mean 0 and covariance matrix Σ^F. At the equilibrium, we would have

$$\alpha_i^F - \beta_i^F F_{i,t} + \sum_{j\neq i}^{n}(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t})\cdot F_{j,t} = 0 \tag{3}$$

and hence the expected change in the fundamental $\Delta F_{i,t}$ is 0. When the quantity in (3) is non-zero, $F_{i,t+1}$ tend to move in the direction to adjust back to the equilibrium. Additionally, $\Delta F_{i,t+1}$ can be driven by fundamental shock $\epsilon_{i,t+1}^F$ causing the fundamental to deviate from the equilibrium.

To see what drives the change in fundamental of country i, let's first assume that $\kappa_i^F = \beta_{ij}^{(1)} = \beta_{ij}^{(2)} = 0$, then (2) reduces to

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \sum_{j\neq i}^{n}\beta_{ij}^{(0)} F_{j,t} + \epsilon_{i,t+1}^F. \tag{4}$$

This gives an error correction model of order 0, where the change in fundamental of country i, $\Delta F_{i,t+1}$, depends on its own and other countries' fundamental. The first part $\alpha_i^F - \beta_i^F F_{i,t}$ indicates the mean-reversion mechanism of $F_{i,t+1}$, where α_i^F/β_i^F and β_i^F measure the long-run mean and the speed of the reversion, respectively. The second part $\sum_{j\neq i}^{n}\beta_{ij}^{(0)} F_{j,t}$ measures the movement of $F_{i,t+1}$ that comes from the fundamentals of other countries $F_{j,t}, j = 1, \ldots, n$, and $j \neq i$. The constant $\beta_{ij}^{(0)}$ measures the degree of interdependence of country i to country j. When $\beta_{ij}^{(0)}$ is large, the equilibrium value of the fundamental of country i depends heavily on the fundamental of country j.

Then, if $\beta_{ij}^{(1)}$ and $\beta_{ij}^{(2)}$ are not zero, (2) becomes

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \sum_{j=\neq i}^{n} \left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t}\right) F_{j,t} + \epsilon_{i,t+1}^F. \tag{5}$$

Here, the transmission of shocks in the fundamental from country j to country i is explained by the trade and financial linkages of country i to country j, $TL_{ij,t}$ and $FL_{ij,t}$. Parameters $\beta_{ij}^{(1)}$ and $\beta_{ij}^{(2)}$ indicate the significance of trade linkage and financial linkage in transmitting the shocks in the fundamental from country j to country i, and parameter $\beta_{ij}^{(0)}$ is for other transmission channels of shocks beyond trade and financial linkages.

Now, when $\kappa_i \neq 0$, shocks in the fundamental that is transmitted to country i is scaled or amplified by $e^{-\kappa_i F_{i,t}}$, and the significance of this amplification effect is measured by the parameter κ_i. A positive value of κ_i implies that the domestic fundamental plays an important role in amplifying the shock when the fundamental is weak, and reducing the shock effect when the fundamental is strong.

Note that the domestic shocks in country i's fundamental and stock market return, $\epsilon_{i,t}^F$ and $\epsilon_{i,t}^R$ are not necessarily independent. Let Σ^{RF} denote the covariance matrix of $\left[\epsilon_t^R, \epsilon_t^F\right]$, and assume that $cov(\epsilon_{i,t}^F, \epsilon_{j,t}^R) = 0$, for $i \neq j$.

B Estimation

To estimate the proposed model, we employ the Metropolis-within-Gibbs technique, which is a hybrid between two Markov chain Monte Carlo (MCMC) techniques: Metropolis-Hastings and Gibbs sampling. This hybrid MCMC method increases efficiency, and, more importantly, provides more practical solutions of complex models. The detailed estimation procedure and the posterior distribution of the MCMC can be found in Appendix A of the attached manuscript.

In general, Markov chain Monte Carlo (MCMC) estimation uses *Monte Carlo* method to generate a *Markov chain*, and with enough simulations, the distribution of the chain would converge to the posterior distribution. According to the Bayes' formula, the posterior distribution of the parameters, $\varphi = \{\alpha_i^R, \beta_i^R, \kappa_i, \alpha_i^F, \beta_i^F, \beta_{ij}^{(0)}, \beta_{ij}^{(1)}, \beta_{ij}^{(2)}, \Sigma^{RF}\}$, where $i, j = 1, \ldots, n$ and $j \neq i$, and the latent fundamental, $F = \{F_{i,t}\}$, $t = 0, \ldots, T-1$ given the equity market return, $R = \{R_{i,t}\}$, and the trade and financial linkages, $L = \{TL_{ij,t}, FL_{ij,t}\}$, where $i, j = 1, \ldots, n$, $j \neq i$, and $t = 0, \ldots, T$, can be written as

$$p(\,\varphi, F \,|\, R, L\,) \propto p(R, F \,|\, \varphi\, L)\, p(\,\varphi \,|\, L\,),$$

where $p(R, F \,|\, \varphi\, L\,)$ is the likelihood function, and $p(\,\varphi \,|\, L\,)$ is the prior distribution.

Basing on Gibbs sampling, each parameter in this paper is iteratively sampled one at a time, conditioned on all other parameters.

Parameters: $p(\,\varphi_k \,|\, \varphi_{\sim k}, F, R, L\,)$, for $k = 1, \ldots, K$

Fundamental: $p(\,F_t \,|\, \varphi, F_{\sim t}, R, L\,)$, for $t = 1, \ldots, T-1$

where $\varphi_{\sim k}$ denotes the set of all parameters except the parameter φ_k, and K is the number of parameters, and $F_{\sim t}$ denotes the set of all fundamental except F_t. The samples are drawn based on the usual conjugate prior of Normal and Inverse-Wishart distributions for all parameters except κ_i and F_t, for which the Metropolis-Hasting is employed. For each iteration of the Metropolis-Hasting, a proposal value is generated and accepted with the acceptance probability. If the value is accepted, the parameter will take that value in that iteration. But if the value is rejected, then the value of that parameter remains unchanged.

According to the Markov chain theory, for a large enough number of iterations N, the distributions would converge to the posterior distributions. However, the samples simulated at early iterations may not be good representatives of the actual posterior distribution, so they are usually dropped out. Then for large enough m, where $m < N$, the expected value of each parameter, φ_k, and the expected value of F at each time t, F_t, are approximated as

$$E[\,\varphi_k \,|\, R, L\,] \approx \frac{1}{N-m}\left(\sum_{z=m+1}^{N} \varphi_k^{(z)}\right), \qquad \text{and} \qquad E[\,F_t \,|\, R, L\,] \approx \frac{1}{N-m}\left(\sum_{z=m+1}^{N} F_t^{(z)}\right),$$

where $\varphi_k^{(z)}$ and $F_t^{(z)}$ are the samples of φ_k and F_t at the iteration z, respectively.

III Results and Discussion

We now provide an empirical study of the international stock market contagion between Thailand and its major peers, including Japan, the US, and Hong Kong. However, because of the data limitation, we consider the models with two countries: Japan - Thailand, Hong Kong - Thailand, and the US - Thailand.

A Data

To estimate the model parameters, data on the stock market returns and data on the trade and financial linkages are required. We provide the details of each variable below. All the data used in this research are sampled on a monthly basis. The sample period is from January 2006 to November 2017, which gives a total of 143 observations for each series.

B Stock market returns

This paper uses major stock index returns, which include Nikkei 225 (Japan), S&P 500 (the US), HSI (Hong Kong), and SET (Thailand), to proxy returns in the stock market of each country. We use log-returns computed from the adjusted closed price sourced from Bloomberg. The summary statistics of monthly stock market returns over the sample period is shown in Panel A of Table I. The Thai stock index, SET, has the highest average monthly return of 0.91%, followed by HSI, S&P 500, and the Japanese stock market, Nikkei 225, has the lowest average return of 0.39%. The HSI monthly return is also the most volatile with the standard deviation of 6.20%, followed by SET, Nikkei 225, and S&P 500 with the lowest standard deviation of 4.13%. All of the stock index returns have a negative skewness and a positive kurtosis.

C Trade linkages

Thailand's trade linkage to Japan, Hong Kong, and the US are proxied by the 12-month average of total trade values between Thailand and the three countries relative to the size of the Thai economy, measured by GDP. The reason for using the 12-month average value is to capture the yearly trade cycle. The monthly USD bilateral imports and exports values are sourced from the Bank of Thailand Statistics, and the USD value of the monthly GDP is sourced from the World Bank Statistics. Finally, to make the result interpretation easier, the 12-month average trade value per GDP is standardised by its mean and standard deviation over the sample period.

The summary statistics of the raw monthly trade linkage per GDP over the sample

Table I
Data Summary Statistics

	Mean (%)	S.D. (%)	Skewness	Kurtosis	Max (%)	Min (%)
			A: Stock Market Returns			
Japan (Nikkei 225)	0.3861	5.7792	-1.0067	2.9198	12.201	-27.216
Hong Kong (HSI)	0.7524	6.2019	-0.7516	2.3932	16.743	-25.388
The US (S&P 500)	0.7022	4.1310	-1.0640	3.0714	10.363	-18.383
Thailand (SET)	0.9068	5.9202	-1.8930	9.7195	14.473	-35.770
			B: Trade Linkage			
Japan -Thailand	2.0213	0.3381	0.4360	-0.1503	2.9151	1.3036
Hong Kong - Thailand	0.4648	0.0994	0.7871	0.2494	0.7865	0.2996
The US - Thailand	1.2491	0.1355	-0.0730	0.3477	1.6517	0.8923
Thailand - Japan	0.0874	0.0128	-0.5007	-0.1180	0.1098	0.0507
Thailand - Hong Kong	0.4659	0.1015	0.7557	0.0438	0.7647	0.2914
Thailand - The US	0.0094	0.0024	0.7725	-0.2147	0.0155	0.0055
			C: Financial Linkage			
Japan -Thailand	7.4548	4.0365	0.1153	-1.2772	13.293	0.2125
Hong Kong - Thailand	7.0559	1.3986	1.3126	3.5297	9.0751	0.8553
The US - Thailand	2.2744	1.8430	1.4179	1.8430	8.8111	0.1958
Thailand - Japan	0.3521	0.2411	0.5397	-1.2337	0.7737	0.0073
Thailand - Hong Kong	7.0516	1.3172	1.6578	4.9683	8.8459	0.8713
Thailand - The US	0.0167	0.0147	1.6377	2.2359	0.0684	0.0022

period is shown in Panel B of Table 1. Between Japan, Hong Kong, and the US, Thailand has the highest average total trade linkage with Japan of 2.02%, followed by with the US of 1.25%, and the trade linkage of Thailand to Hong Kong has the lowest average of 0.46%. Similarly, the standard deviation of the Thai trade linkage to Japan is the highest, then the US, and lastly Hong Kong. On the other hand, comparing the trade linkages of Japan, Hong Kong, and the US to Thailand, Hong Kong's trade linkage is the highest on average and most volatile, and the US's trade linkage is lowest.

D Financial linkages

Two main measures are used to proxy the financial links of Thailand to Japan, Hong Kong, and the US, including USD value of the Foreign Direct Investment positions (inward and outward) and USD value of the Equities and Debt Securities Investment Portfolio positions (assets and liabilities). Both of the two data sets are sourced from Bank of Thailand Statistics. Similar to the way TL_t is computed, Thailand's financial linkage to Japan, Hong Kong, and the US are proxied by the standardised value of the 12-month average of the sum of the two measures relative to the Thai GDP.

The summary statistics of the raw monthly financial linkage per GDP over the sample period is shown in Panel C of Table 1. Between Japan, Hong Kong, and the US, Thailand has the highest average total financial linkage with Japan of 7.45%, followed by with Hong Kong of 7.06%, and the financial linkage of Thailand to the US has the lowest average of 2.27%. However, comparing the financial linkages of Japan, Hong Kong, and the US to Thailand, Hong Kong's financial linkage is the highest on average, of 7.05%, and most volatile, and the US's financial linkage is lowest, of below 0.02%.

E Sub-Models

Although our model can handle many countries at once, but given the data limitation, we consider the model with two countries. The dynamics of the fundamental of country i at time $t+1$ is reduced to:

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \left[\alpha_i^F - \beta_i^F F_{i,t} + \left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t} \right) \cdot F_{j,t} \right] + \epsilon_{i,t+1}^F, \qquad (6)$$

where $\epsilon_{t+1}^F = [\epsilon_{1,t+1}^F, \epsilon_{2,t+1}^F]'$ is jointly normally distributed and i.i.d. across time.

This paper aims to study the financial contagion through the fundamental, which consists of a total of three effects: the effects of trade and financial linkages on the shock transmission and the shock amplification effect. The following sub-models of equation (6) are fitted to study each effect separately and to study the effects when tested with the other effects.

Model 1: Effect of Trade Linkage on Shock Transmission

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} \right) \cdot F_{j,t} + \epsilon_{i,t+1}^F, \qquad (7)$$

Model 2: Effect of Financial Linkage on Shock Transmission

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \left(\beta_{ij}^{(0)} + \beta_{ij}^{(2)} FL_{ij,t} \right) \cdot F_{j,t} + \epsilon_{i,t+1}^F, \qquad (8)$$

Model 3: Effect of Shock Amplification

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \left[\alpha_i^F - \beta_i^F F_{i,t} + \beta_{ij}^{(0)} F_{j,t} \right] + \epsilon_{i,t+1}^F, \qquad (9)$$

Model 4: Effects of Trade and Financial Linkages on Shock Transmission

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t}\right) \cdot F_{j,t} + \epsilon_{i,t+1}^F, \tag{10}$$

Model 5: Effects of Trade Linkage on Shock Transmission and Shock Amplification

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \left[\alpha_i^F - \beta_i^F F_{i,t} + \left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t}\right) \cdot F_{j,t}\right] + \epsilon_{i,t+1}^F, \tag{11}$$

Model 6: Effects of Financial Linkage on Shock Transmission and Shock Amplification

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \left[\alpha_i^F - \beta_i^F F_{i,t} + \left(\beta_{ij}^{(0)} + \beta_{ij}^{(2)} FL_{ij,t}\right) \cdot F_{j,t}\right] + \epsilon_{i,t+1}^F, \tag{12}$$

Finally, the last model, **Model 7** is equation (6), which includes all of the three effects.

The estimated parameters of the equations for Thailand in the Japan - Thailand, Hong Kong - Thailand, and the US - Thailand models are shown in Tables 2 - 4. The detailed parameter estimates of the models of each country pair are given in Appendix B of the attached manuscript.

F Role of fundamentals

Consider α_i^R and β_i^R where α_i^R is the constant component of the expected returns, and β_i^R measures the effect of the domestic fundamental on the expected returns of the stock market. Although the estimated values of α_i^R are mostly insignificant, the estimated values of β_i^R are positive in all models across all three countries, implying that the domestic fundamental has a positive impact on returns, as stronger fundamentals (higher $F_{i,t}$) lead to higher expected return of the stock market.

Figure 1 shows the plot of the estimated fundamental of Thailand against the SET returns from January 2006 to December 2017. The Thai fundamental and the SET returns movements, as shown in Figure 1, are consistent with our hypothesis. For instance, between July 2007 and September 2008, which was the time the Thai fundamental was low, was also the time that the returns in the SET declined to its lowest and highly volatile. Then, from September 2013 to December 2017, while the Thai fundamental was improving, the SET returns became more stable.

Figure 1. The Thai Fundamental VS SET Returns: Jan/06 - Dec/17

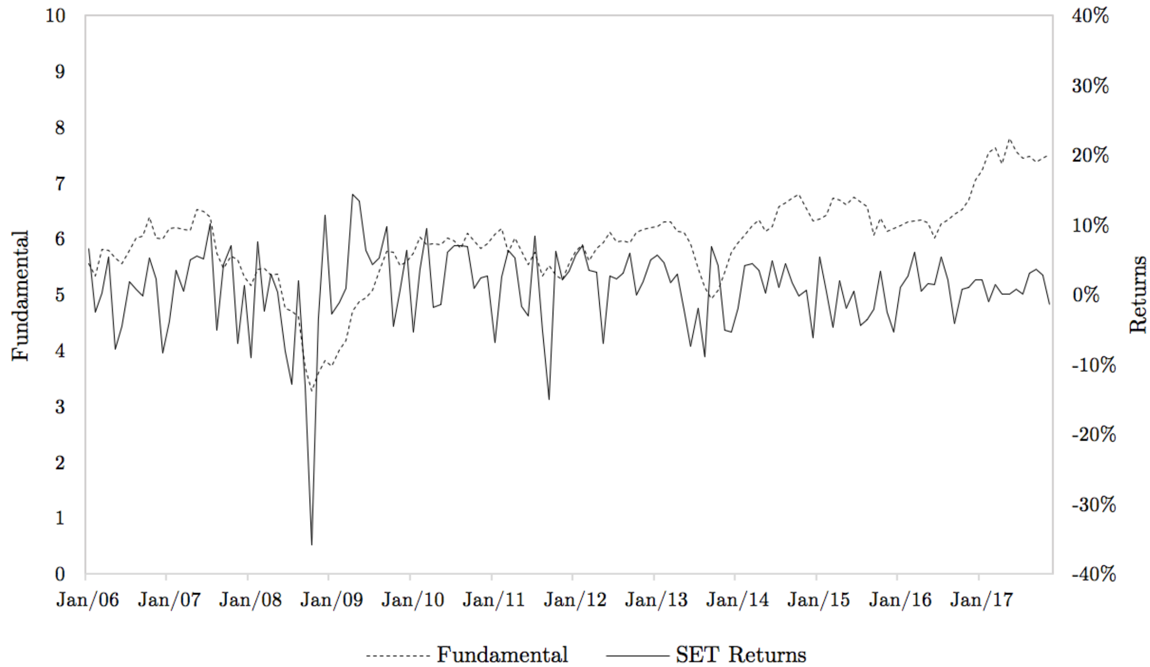


Table II

Key Posterior Estimates of Japan - Thailand models

This table presents the posterior means of the model parameters of equations for Thailand in Japan - Thailand models and the significance levels, where the values marked with *, ** and *** are statistically significantly different from zero at the 10%, 5% and 1% significance levels, respectively.

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
α^R_{TH}	0.0070	0.0059	0.0038	0.0051	0.0050	0.0018	0.0081
	**	**					
β^R_{TH}	0.1349	0.1333	0.0696	0.0922	0.1032	0.1429	0.1076
	**	**	**	**	**	***	**
α^F_{TH}	1.3611	-1.0820	0.0269	0.6345	0.6341	0.9248	0.1579
	**		***				
β^F_{TH}	0.2014	0.2232	0.1059	0.1904	2.1205	0.9990	1.4107
	***	***	***	***	***	***	***
κ_{TH}	-	-	0.8566	-	0.8875	0.8883	0.9461
			***		***	***	***
$\beta^{(0)}_{TH,JP}$	0.0342	0.4219	0.3930	0.0881	2.6450	1.9622	3.6193
	***	***	***	***	***	***	***
$\beta^{(1)}_{TH,JP}$	-0.0064	-	-	-0.0094	-1.0261	-	-1.5383
	***			***			
$\beta^{(2)}_{TH,JP}$	-	0.0269	-	0.0153	-	3.4193	1.8140
		***		***		**	**

Table III
Key Posterior Estimates of Hong Kong - Thailand models

This table presents the posterior means of the model parameters of equations for Thailand in the Hong Kong - Thailand model and the significance levels, where the values marked with *, ** and *** are statistically significantly different from zero at the 10%, 5% and 1% significance levels, respectively.

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
α^R_{TH}	0.0071	0.0108	0.0035	0.0079	0.0052	0.0012	0.0072
	**	**	**	**			**
β^R_{TH}	0.1324	0.1105	0.1214	0.1008	0.1322	0.1556	0.1027
	***	***	**	**	**	***	**
α^F_{TH}	0.5058	-0.0284	-0.2146	0.1920	-0.1632	0.1084	0.4839
β^F_{TH}	0.2953	0.2442	0.5613	0.0653	1.7821	0.4556	1.0722
	***	***	***	***	***	****	***
κ_{TH}	-	-	0.8977	-	0.8865	0.9711	0.8790
			***		***	***	***
$\beta^{(0)}_{TH,HK}$	0.2270	0.2542	1.2833	0.0371	2.5558	1.4242	1.5973
	***	***	***	***	***	***	***
$\beta^{(1)}_{TH,HK}$	-0.0042	-	-	-0.1381	-0.4025	-	-0.0753
	**			**			
$\beta^{(2)}_{TH,HK}$	-	0.0035	-	0.1394	-	0.5575	0.0086
		**		***		**	**

G Amplification Effects

First we consider the mean reversion property of the fundamentals: α_i^F and β_i^F explain how the domestic fundamental at time t, $F_{i,t}$, affects its own movement at time $t+1$, $\Delta F_{i,t+1}$. According to the model, ignoring all the other terms, the fundamentals would converge back to the mean level α_i^F/β_i^F at the spreed of β_i^F. The estimated values of β_i^F are significantly positive in all models, which conclude that the fundamental processes of all three countries follow a mean-reversion assumption.

The domestic fundamental $F_{i,t}$ also affects its movement $\Delta F_{i,t+1}$ through the amplification effect, and such effect is measured by the parameter κ_i. The estimated values of κ_i of all the models are significantly positive, which reveals the amplification effect. In other words, weak fundamental (low $F_{i,t}$) would make the value $e^{-\kappa F_{i,t}}$ high, and, consequently, amplifies the contagion effect.

Table IV
Key Posterior Estimates of the US - Thailand models

This table presents the posterior means of the model parameters of equations for Thailand in the US - Thailand models and the significance levels, where the values marked with *, ** and *** are statistically significantly different from zero at the 10%, 5% and 1% significance levels, respectively.

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
α^R_{TH}	0.0079	0.0083	0.0077	0.0076	0.0072	0.0039	0.0031
	**	**	**	**	**	***	**
β^R_{TH}	0.0841	0.0776	0.0600	0.0980	0.0888	0.2254	0.1204
	**	**	**	**	**	***	**
α^F_{TH}	-0.2110	1.0937	-0.1511	1.0324	0.3289	0.8760	0.9053
	**	**	**	**	**	***	**
β^F_{TH}	0.1127	0.3240	0.9773	0.3957	1.0093	0.3040	1.3246
	**	**	**	**	**	***	**
κ_{TH}	-	-	0.8979	-	0.8867	0.9716	0.8791
			***		***	***	***
$\beta^{(0)}_{TH,US}$	0.0705	0.1354	1.9371	0.2018	1.5235	3.2153	2.8203
	**	**	**	**	**	***	**
$\beta^{(1)}_{TH,US}$	-0.0766	-	-	-0.0862	-0.6941	-	-0.9561
	**	**	**	**	**	***	**
$\beta^{(2)}_{TH,US}$	-	0.0169	-	0.0224	-	2.1642	0.4711
	**	**	**	**	**	***	**

H Roles of financial and trade linkages

Another factor that affects the movements of the fundamental, $\Delta F_{i,t+1}$, is the fundamental of the other country j, $F_{j,t}$, and the degree to which the $F_{j,t}$ affects $\Delta F_{i,t+1}$, in this paper, depends on the level of trade and financial linkages, $TL_{ij,t}$ and $FL_{ij,t}$. The parameters $\beta^{(1)}_{ij}$ and $\beta^{(2)}_{ij}$ measure the role of trade linkage and the role of financial linkage in transmitting the shock from country j to country i, respectively. Finally, the parameter $\beta^{(0)}_{i}$ measures effect of non-fundamental linkages. If $\beta^{(1)}_{ij} > 0$ and $\beta^{(2)}_{ij} > 0$, it would implies that trade and financial linkages are the transmission channels of shocks. However, $\beta^{(2)}_{ij}$ is the only parameter whose estimated values are positive in all models, while estimated values of $\beta^{(1)}_{ij}$ are negative. Therefore, the financial linkage is a transmission channel of shocks, and the trade linkage is not. The results on the trade linkage effect support earlier findings of Blanchard, Das and Faruqee (2010), Rose and Spiegel (2011), and Berkmen *et al.* (2012) that conclude that trade linkage did not have a significant role in shock transmission in recent financial crises.

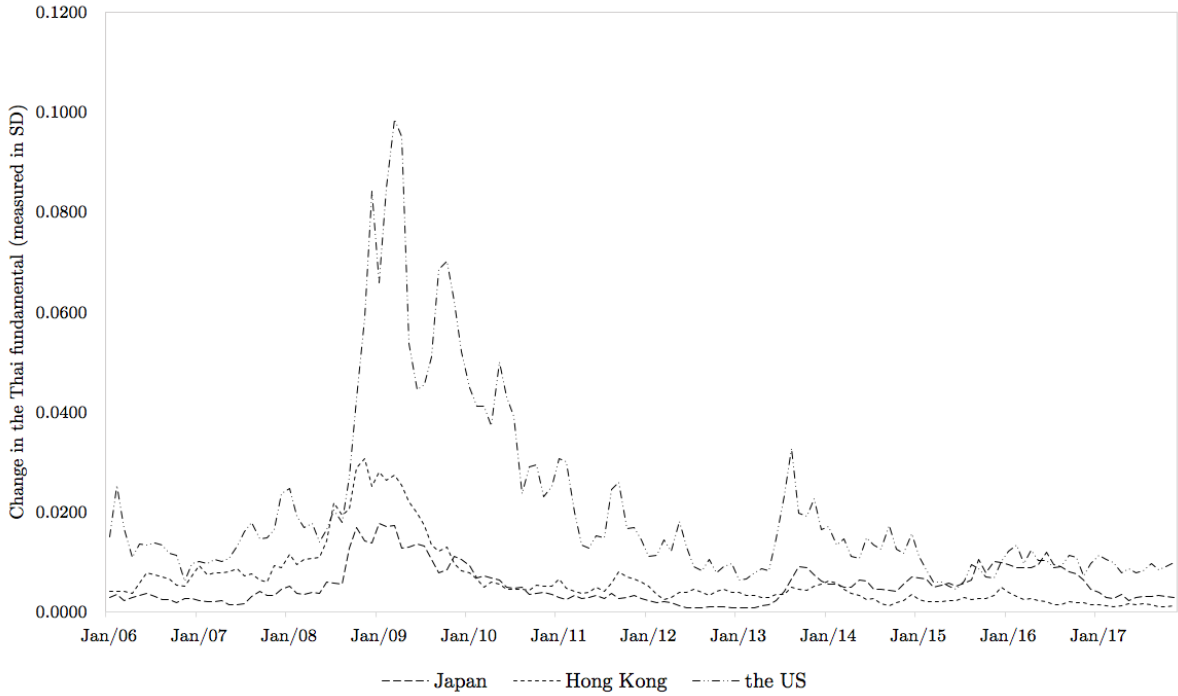
To further investigate the role of the trade linkage, observe that the estimated values of $\beta^{(1)}_{TH,JP}$, $\beta^{(1)}_{TH,HK}$, and $\beta^{(1)}_{TH,US}$ are only significant in Model 1, the model that examines only the trade linkage effect, and Model 4, the model that includes both trade and financial linkages. The estimated values are insignificant in the models with the amplification effect (Model 5 and Model 7). Moreover, the estimated values of $\beta^{(1)}_{TH,JP}$, $\beta^{(1)}_{TH,HK}$, and $\beta^{(1)}_{TH,US}$ that are significant are all negative. Therefore, basing on the traditional definition, trade linkage is not transmission channel of shocks to Thailand. The fact that the estimated values of the $\beta^{(1)}_{TH,JP}$, $\beta^{(1)}_{TH,HK}$, and $\beta^{(1)}_{TH,US}$ are significantly negative in the models without the amplification effect and are insignificant in the models with the amplification effect means that $\beta^{(1)}_{TH,j}$, where $j = JP, HK$, and, US and κ_{TH} somewhat explain similar things. To explain the root of the result, we hypothesize that the trade linkage is an indirect measure of the fundamental. By nature of Thailand's trade activities, Thailand imports raw materials from the countries it exports its final products to. So, if an importing country's fundamental is good, there would be more demand for the Thai exports, as a results, the values of both the Thai exports and imports would be high, which makes the Thai fundamental better as well. Meanwhile, a good fundamental implies low contagion. The opposite is true for the case of bad fundamentals. Thus, when the effect of trade linkage on the contagion is tested by itself, the results reveal a negative relationship between the trade linkage and the contagion. However, such relationship becomes insignificant when the effect of trade linkage is tested along with the amplification effect, and this is because the amplification effect dominates the trade linkage's effect. Therefore, this hypothesis supports the findings.

Unlike other studies that find no effect on shock transmission through trade linkage (Blanchard *et al.*, 2010, Rose and Spiegel, 2011, and Berkmen *et al.*, 2012), by including the amplification effect of the domestic fundamental in the model, this paper can explain the reason for such findings and also the role of the trade linkage in the financial contagion mechanism.

I Sensitivity of Thai fundamental and return distribution

There are two ways the Japanese, the Hong Kong, and the US fundamentals affect Thailand. First is the impact on the Thai fundamental, and second is the impact on Thailand's stock market, SET, returns. Each impact is discussed below.

Figure 2. Changes in the Thai Fundamental: Jan/06 - Dec/17



First of all, the levels to which an increase in the fundamentals of Japan, Hong Kong, and the US affects Thai fundamental's movements, or

$$e^{-\kappa_{TH} F_{TH,t}} \cdot \left(\beta^{(0)}_{TH,j} + \beta^{(1)}_{TH,j} TL_{TH,j,t} + \beta^{(2)}_{TH,j} FL_{TH,j,t}\right) \cdot \sigma^F_j / \sigma^F_{TH},$$

where $j = JP, HK$, and US from January 2006 to December 2017 is plotted in Figure 2. This measures changes in the Thai fundamental, in terms of standard deviation, when the fundamental of the other country j increases by one standard deviation. Throughout the entire sample period, January 2006 to December 2017, the Thai fundamental was affected by changes in the US fundamental the most. However, from the plot, the US fundamental's effect on the Thai fundamental rose greatly during the 2008 global financial crisis, and gradually declined back to the level close to the effect level of the Japanese and the Hong Kong fundamentals.

Figure 3. Changes in the SET average return: Jan/06 - Dec/17

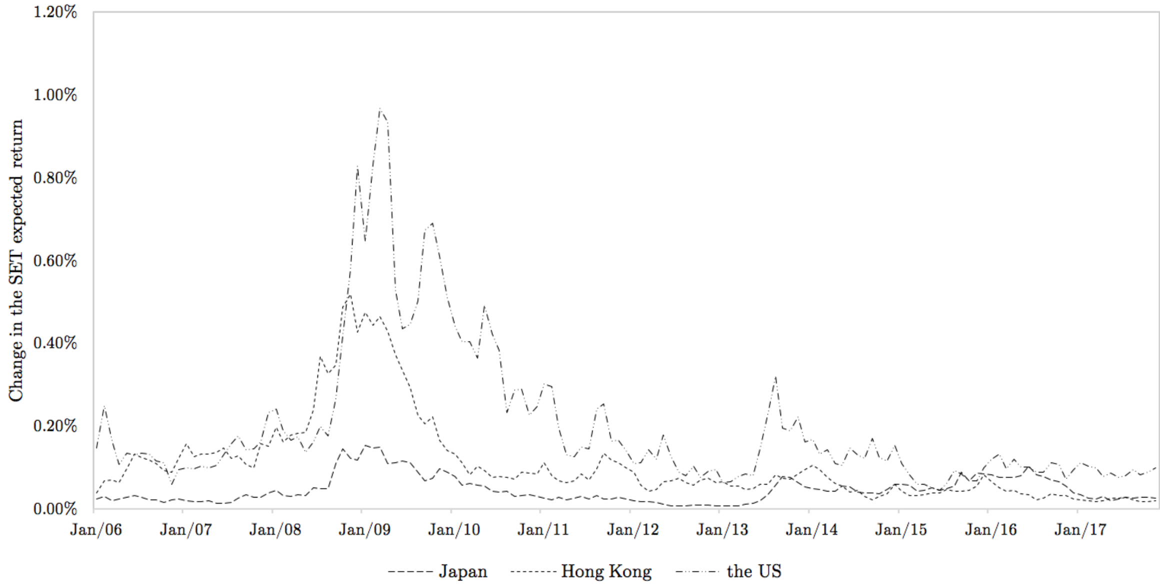
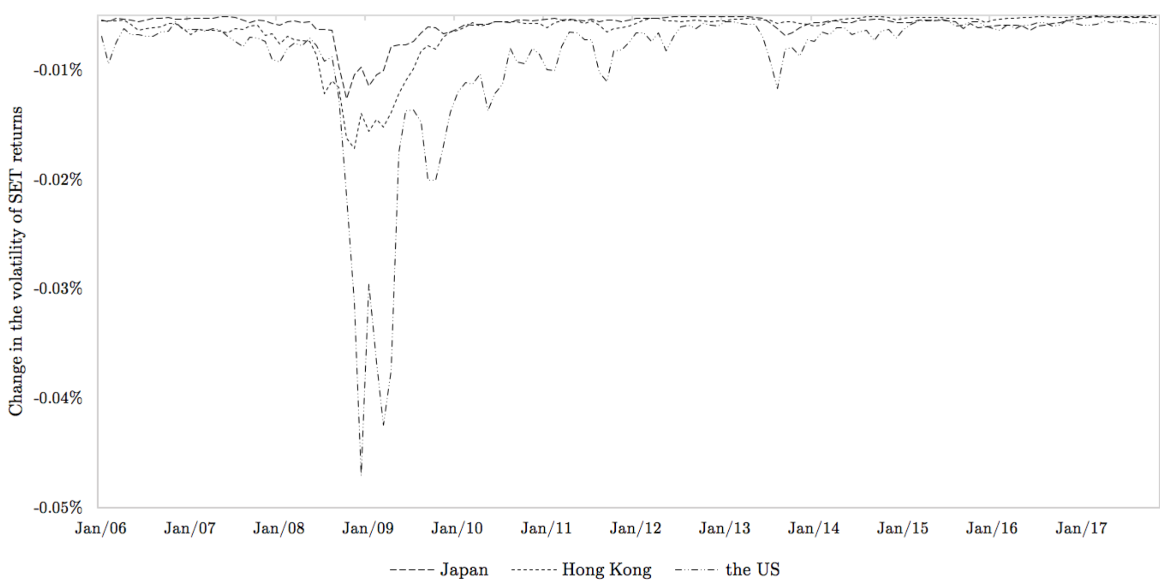


Figure 4. Change in SET returns' volatility: Jan/06 - Dec/17



Secondly, the levels to which an increase in the fundamentals of Japan, Hong Kong, and the US affects the SET average return, which is

$$\beta_{TH}^{R} \cdot e^{-\kappa_{TH} F_{TH,t}} \cdot \left(\beta_{TH,j}^{(0)} + \beta_{TH,j}^{(1)} TL_{TH,j,t} + \beta_{TH,j}^{(2)} FL_{TH,j,t}\right) \cdot \sigma_j^F,$$

and the return volatility, which is

$$-\frac{1}{2} \cdot \sigma_{TH}^{R} \cdot e^{-\frac{1}{2} F_{TH,t}} \cdot e^{-\kappa_{TH} F_{TH,t}} \cdot \left(\beta_{TH,j}^{(0)} + \beta_{TH,j}^{(1)} TL_{TH,j,t} + \beta_{TH,j}^{(2)} FL_{TH,j,t}\right) \cdot \sigma_j^F$$

are shown in Figure 3 and Figure 4, respectively. These measure the changes in the expected return and volatility of the SET per one standard deviation increase in the fundamental of country j. From the plots, from August 2006 to November 2008, the Hong Kong and the US fundamentals had the greatest impact on the SET return's distribution. However, after the global financial crisis until January 2011, the movements of the SET return's average and volatility were influenced by the US fundamental the most, and the effect of the US fundamental converged down to the level close to the effect level of the Japanese and the Hong Kong fundamentals.

IV Conclusions and Suggestions

This paper develops a framework and a model of the fundamental-based contagion to study about the international stock market contagion. First, this paper assumes that the stock market returns move according to the domestic fundamental, where the average stock market return is higher and the returns are less volatile in countries with good fundamentals than countries with poor fundamental. Then, through the fundamental, this paper examines (1) the transmission mechanism of shock through trade and financial linkages whether larger linkages leads to more co-movement between countries' fundamentals, and (2) the amplification effect of the domestic fundamentals on the contagion, particularly, whether countries with weak fundamentals suffer from external shocks more greatly than the ones with strong fundamentals.

This paper conducts an empirical study on the international stock market contagion between Japan - Thailand, Hong Kong - Thailand and the US - Thailand. Using MCMC estimation technique, the results reveals that the amplification effect of the domestic fundamental is significant in Thailand. Further, it is found that shocks are transmitted to Thailand through only the financial linkage. In sum, all else being equal, when countries are subject to the contagion risk given the financial linkage, it is more likely that countries with better fundamental would be subject to lower contagion risk. Similarly, with low level of linkages, countries may seem safe from contagion risk, but the weak domestic economy of the countries may trigger huge losses to them. One of the advantages of including the effects of shock transmission through trade and financial linkages and the shock amplification effect in the model is that, while the results of this paper show no evidence of shock transmission through trade linkage, which is consistent with the findings of recent researches, this paper can give an explanation for the results as trade linkage is an indirect measure of the fundamental.

Finally, this paper finds that the effect levels of the fundamentals of Japan, Hong Kong, and the US on Thailand's fundamental and stock market return distribution vary over time. The country whose fundamental Thailand's fundamental and stock market returns' distribution was affected by the most was the US. But the effect of the changes in the US fundamental on the Thai fundamental and stock market returns became more pronounced during the 2008 global financial crisis, and gradually declined to the level close to the effect level of the Japanese and the Hong Kong fundamentals.

The findings of this research have several implications. First, the possibility of financial contagion could eliminate the benefits of international portfolio diversification. Knowing which countries shock would transmit to Thailand the most and least can be useful for their risk management and diversification strategies. Second, preventing financial contagion is one of the key objectives of policy makers. Knowing that the health of the economy plays an important role the degree to which the country affects from external shocks, and those shocks are transmitted through financial linkage and not trade linkage would help the policy makers in various ways.

PART II

Potential Buyers and Fire Sales in Financial Networks

I Objectives

1. To develop an equilibrium-based contagion model of banking system in which banks are interconnected through interbank liabilities and holding of common illiquid assets which function as channels of contagion.

2. To study the fire sale effect inside the banking system that interacts with the network topology of interbank liabilities.

3. To study the roles of banks inside the banking system in amplifying and absorbing shocks based on their risk aversion levels, cash holding, asset holding, operating cost, and their liability networks.

4. To derive a policy implication that helps maintain the system stability of the banking network, and avoid using the tax payers' money to rescue the failing system.

II Methodology

A Setup

Consider a three-period ($t = 0, 1, 2$) financial system with N *banks*. At time 0, each bank i holds a liquid asset or *cash* with value c_i and a portfolio of illiquid assets or *loans*. There are K types of loans which are for non-bank borrowers such as auto loans and credit card loans. These loans mature at time 2 with random payoffs, and do not pay intermediate payments. Each bank i is endowed with $\theta_{i,k}$ units of type-k loans at time 0. Banks do not create new loans after time 0. In our model, cash represents a liquid asset portion of a bank that normally provides low return with minimal risk (zero risk and zero return in our setting). On the other hand, illiquid loans represent a majority portion of the bank's asset that are typically riskier and have higher returns.

In addition to cash and loans made for non-bank borrowers, banks are endowed with shorter-term *interbank loans* between each other. These interbank loans mature at time 1. Let $l_{i,j}$ denote the claim of bank j on the asset of bank i at the maturity of the interbank loan. The interbank claims at time 1 can be summarized by matrix $L = [l_{i,j}]$ where $l_{i,i} = 0$. We assume that no new interbank loans are created after time 0.

At time 0, each bank i is also financed by deposits of d_i. We assume that the interbank liabilities and deposits are of equal seniority, and that interest rate is normalized to zero. So cash and deposits earn no interest. To summarize, the asset side of the balance sheet of each bank consists of cash, illiquid loans, and interbank loans, while the debt side of the bank consists of deposits and interbank liabilities. The equity value of the bank is equal to the asset value minus the debt value. We assume that each bank has positive equity at time 0; that is, all banks are solvent.

At time 1, the system is subject to unexpected shocks, which can be in various forms. A shock can be a *bank shock* that comes in as a surprise expense of one particular bank, reducing the bank's net worth. Such shocks could be due to frauds, litigation costs, or settlements of lawsuits.[1] A shock can also be categorized as an *asset shock* such as an increase in the default probability of one type of loans. An adverse shock to the default probability causes the loan price to drop. We model that mechanism below. After shocks are realized, banks settle their interbank liabilities by using cash or repayments obtained from their interbank claims, or by selling their illiquid loans, or a combination of them.

Suppose that after the shocks the market price of type-k illiquid loan is $p_k, k = 1, \ldots, K$. Let $x_{i,j}$ denote the amount that bank i repays its interbank liability to bank j for $j \neq i$. If the value of the total asset of bank i is less than the value of its total debt, then the bank is insolvent and must liquidate all of its assets and distribute the proceeds to all of its creditors proportional to the face values. Otherwise, bank i repays the interbank liability in full. Let $L_i = d_i + \sum_{u \neq i}^{N} l_{i,u}$ denote the total debt of bank i. Then the amount that bank i

[1]For example, the fraud in the Barings Bank caused it to collapse in 1995. The 2016 annual report of the Royal Bank of Scotland reports the loss of over 5.8 billion pounds for litigation and conduct costs.

repays to bank j is equal to

$$x_{i,j} = \frac{l_{i,j}}{L_i} \min \left\{ L_i, c_i + \sum_{k=1}^{K} p_k \theta_{i,k} + \sum_{u \neq i}^{N} x_{u,i} - v_i \right\} \tag{13}$$

We assume that each bank carries over its cash and deposits from time 0 to time 1. Given all the shocks and the market prices of illiquid loans $p = [p_1, \ldots, p_K]'$, the collection of $[x_{i,j}]$ for $i \neq j$ that satisfies (13) for each $i = 1, \ldots, N$ simultaneously is said to be an *equilibrium repayment* at the price vector p.

Let

$$e_i = \max \left\{ 0, c_i + \sum_{k=1}^{K} p_k \theta_{i,k} + \sum_{u \neq i}^{N} x_{u,i} - v_i - d_i - \sum_{u \neq i}^{N} l_{i,u} \right\} \tag{14}$$

denote the equity value of bank i. Banks that are solvent ($e_i > 0$) can now adjust their asset portfolio by buying or selling illiquid loans. We assume that each solvent bank chooses a portfolio of liquid and illiquid assets to maximize its risk-adjusted return on equity based on a mean-variance utility. Let $\hat{\theta}_{i,k}$ denote the number of units of tyep-k loan held by bank i after the adjustment, and $\tilde{R}_{i,k}(\hat{\theta}_{i,k})$ denote the values of the bank i's portfolio of the type-k illiquid loans realized at time 2. The change in the equity value from time 1 to time 2 comes from the change in the value of illiquid loans, and the cost of managing the loans. So the return on equity of bank i is

$$ROE_i = \frac{\sum_{k=1}^{K} (\tilde{R}_{i,k}(\hat{\theta}_{i,k}) - \hat{\theta}_{i,k} p_k - \hat{\theta}_{i,k} f_{i,k})}{e_i} \tag{15}$$

where $f_{i,k}$ is the cost of managing one unit of type-k loan of bank i. The cost matrix $F = [f_{i,k}]$ is used to define *expertise* of banks for different types of loans. Now the optimization problem of bank i is given by

$$[\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,K}] = \arg\max \left\{ E[ROE_i] - \frac{\gamma_i}{2} Var(ROE_i) \right\} \tag{16}$$

subject to the budget constraint

$$\sum_{k=1}^{K} p_k \hat{\theta}_{i,k} \leq e_i + d_i \tag{17}$$

where γ_i denotes the risk-aversion parameter of bank i. Note that banks are not allowed to hold short positions on loans and hence we also need the no-short-position constraint

$$\hat{\theta}_{i,k} \geq 0, \quad k = 1, \ldots, K. \tag{18}$$

We provide the details of $\tilde{R}_{i,k}(\theta_{i,k})$ below.

We assume that illiquid loans are traded inside the financial system with N banks. So the loan prices are determined endogenously based on the market clearing condition. That is, the price vector $p = [p_1, \ldots, p_K]'$ is said to be an *equilibrium price* if the demand and supply of each loan type are equal:

$$\sum_{i=1}^{N} \theta_{i,k} = \sum_{i=1}^{N} \hat{\theta}_{i,k}, \quad k = 1, \ldots, K. \tag{19}$$

B Default correlations and loan payoff distribution

There are K types of illiquid loans. At time 2, a loan of type k repays the creditor the full amount of \$1 with probability $1 - \lambda_k$ or defaults and pays nothing to the creditor with probability λ_k for $\lambda_k \in (0, 1)$. We assume that loan defaults are correlated and we model the default correlation with a Gaussian copula model. Specifically, let $M_k = \sum_{i=1}^{N} \theta_k$ denote the total number of type-k loans that are available in the system, and $\tilde{r}_{m,k}$ denote the payoff of loan m in type k, $m = 1, \ldots, M_k$. The Gaussian copula framework models default correlation through common factors. Let $\tilde{Z}_0, \tilde{Z}_1, \ldots, \tilde{Z}_K$ be independent standard normal random variables such that $\tilde{Z}_0$ represents the *market* factor, and $\tilde{Z}_k$ represents type-k factor for $k = 1, \ldots, K$. For each loan m of type k, let

$$\tilde{Y}_{m,k} = \alpha \tilde{Z}_0 + \beta_k \tilde{Z}_k + \sqrt{1 - \alpha^2 - \beta_k^2}\,\tilde{\epsilon}_{m,k} \tag{20}$$

where $\tilde{\epsilon}_{m,k}$'s are i.i.d. standard normal random variables for $m = 1, \ldots, M_k, k = 1, \ldots, K$ and are independent of $\tilde{Z}_0, \ldots, \tilde{Z}_K$. The parameters α and β_k are such that $\alpha \geq 0, \beta_k > 0$ and $\alpha^2 + \beta_k^2 < 1$. Observe that each $\tilde{Y}_{m,k}$ is also a standard normal random variable, and they are correlated. The correlations between $\tilde{Y}_{m,k}$'s are used to determine the default correlations between loan payoffs $\tilde{r}_{m,k}$'s based on the following relationship:

$$\tilde{r}_{m,k} = 0 \quad \text{if and only if} \quad \tilde{Y}_{m,k} \leq \Phi^{-1}(\lambda_k)$$

where $\Phi(y)$ is the cumulative distribution function of standard normal distribution at y. So loan m in type k defaults if and only if $\tilde{Y}_{m,k} \leq \Phi^{-1}(\lambda_k)$. Note that from the standard normal distribution of $\tilde{Y}_{m,k}$, the default probability is $P(\tilde{Y}_{m,k} \leq \Phi^{-1}(\lambda_k)) = \lambda_k$, as it must.

The correlations between loans depend on parameters $\alpha, \beta_1, \ldots, \beta_K$. For loans m and m' in the same type k, the correlation between $\tilde{Y}_{m,k}$ and $\tilde{Y}_{m',k}$ is $\alpha^2 + \beta_k^2$, while for loans m and m' of different types k and k', correlation between $\tilde{Y}_{m,k}$ and $\tilde{Y}_{m',k'}$ is α^2. In other words, loans of different types are less correlated than loans of the same types, and the difference is determined by β_k. With different values in $\beta_1, \ldots, \beta_K$, we can have loan types that have higher default correlation, such as corporate loans within a particular sector, and loan types that have lower default correlation, such as student loans. We have the following results:

Proposition 1 *Let $\tilde{R}_{i,k}(\theta) = \sum_{m=1}^{\theta} \tilde{r}_{m,k}$ and $\tilde{R}_{i,k'}(\theta') = \sum_{m=1}^{\theta'} \tilde{r}_{m,k'}$ denote the payoffs at time 2 of the portfolio of θ units of type-k loans, and portfolio of θ' units of type-k' loans, respectively. We have*

$$E\left[\tilde{R}_{i,k}(\theta)\right] = \theta(1-\lambda_k) \tag{21}$$

$$Var\left(\tilde{R}_{i,k}(\theta)\right) = \theta^2[\psi_k - (1-\lambda_k)^2] + \theta[1-\lambda_k-\psi_k] \tag{22}$$

$$Cov\left(\tilde{R}_{i,k}(\theta), \tilde{R}_{i,k'}(\theta')\right) = \theta\theta'[\Psi_{k,k'} - (1-\lambda_k)(1-\lambda_{k'})], \quad k \neq k' \tag{23}$$

where

$$\psi_k = \Phi_2(\Phi^{-1}(1-\lambda_k), \Phi^{-1}(1-\lambda_k); \alpha^2+\beta_k^2)$$
$$\Psi_{k,k'} = \Phi_2(\Phi^{-1}(1-\lambda_k), \Phi^{-1}(1-\lambda_{k'}); \alpha^2)$$

and $\Phi_2(y_1, y_2; \rho)$ is the bivariate cumulative distribution function at (y_1, y_2) of standard normal random variables with correlation ρ.[2]

The expected value in (21) is simply the number of loans multiplied by the probability that a type-k loan does not default. The variance in (22) has two terms. The first term is quadratic in the number of loans, while the second term is linear. To understand this, note first that the variance of the payoff of *one unit* of a type-k loan is $\lambda_k(1-\lambda_k)$. This variance

[2] Note that we have assumed that θ is an integer, but we will rely on the same formulas (21) - (23) even when θ is real. The error from rounding the number should not change the conclusions of the consequent analyses.

is broken into $\psi_k - (1-\lambda_k)^2$ and $1-\lambda_k-\psi_k$, the sum of which is $\lambda_k(1-\lambda_k)$. When the term $\psi_k - (1-\lambda_k)^2$ at the loan level is aggregated to the portfolio level, it gives a quadratic function of θ, while the term $1-\lambda_k-\psi_k$ gives rise to a liner function at the portfolio level. As the value of ψ_k increases from $(1-\lambda_k)^2$ when loan defaults are independent to $1-\lambda_k$ when they are perfectly correlated, we can view $\psi_k-(1-\lambda_k)^2$ as a *variance-based* measure of how loans are close to perfect correlation, and view $1-\lambda_k-\psi_k$ as a variance-based measure of how loans are close to independence. So the portfolio variance given by (22) suggests that the higher the correlation, the stronger the quadratic term, and the weaker the linear term. Finally, if loans of different types default independently ($\alpha = 0$), then $\Psi_{k,k'} = (1-\lambda_k)(1-\lambda_{k'})$ and thus the covariance between the values of loan portfolios in (23) is zero. It is positive if the defaults of loans of different types are positively correlated.

III Results and Discussion: Banks' optimal portfolios

When an adverse bank shock, such as an unexpected litigation cost, hits a bank, it reduces the bank's net worth. The bank may use cash or cash equivalents to pay for the cost, reducing the liquid asset portion in the bank's balance sheet. Because liquid assets such as cash and cash equivalents are considered as risk-free or low-risk assets, this results in the bank's asset portfolio that overweights the risky loans, increasing the risk of the bank's portfolio relative to the smaller equity value. Likewise, when an adverse asset shock hits the bank, the value of the risky loans and hence the equity value reduce, making the risk profile of the bank's portfolio deviate from the optimal level. Banks are risk-averse but profit-seeking institutions. So the changes in the proportion of risk-free/risky assets relative to its equity require banks to re-adjust their asset holdings to achieve a better risk-return trade-off.

In this section we consider the banks' optimization problems and their optimal portfolios of liquid and illiquid assets. We start with the simplest case with one type of loans. Then we study the interaction between types of loans from the case of two types of loans.

A One loan type

Assume that there is only one type of illiquid loans ($K = 1$). From (15) - (16) and (21) - (22), the objective of bank i is to maximize the following risk-adjusted return on equity

$$V_i(\theta) = \frac{\theta(1-\lambda) - \theta p - \theta f_i}{e_i} - \frac{\gamma_i}{2}\left(\frac{\theta^2[\psi - (1-\lambda)^2] + \theta[1-\lambda-\psi]}{e_i^2}\right) \tag{24}$$

subject to the budget constraint $\theta p \leq e_i + d_i$ and no-short-position constraint $\theta \geq 0$ where we have dropped the subscript k for simplicity.

To understand the optimal number of loans held by the bank, let's suppose for the moment that the constraints are not binding, and let $\bar{\theta}_i$ denote the optimal solution for the unconstrained problem derived from the first order condition:

$$\begin{aligned}\bar{\theta}_i &= \frac{e_i}{\gamma_i}\left[\frac{(1-\lambda) - p - f_i}{\psi - (1-\lambda)^2}\right] - \frac{1}{2}\left[\frac{1-\lambda-\psi}{\psi - (1-\lambda)^2}\right] \\ &= \frac{e_i}{\gamma_i}\left[\frac{(1-\lambda) - p - f_i}{\lambda(1-\lambda)}\right]\left(\frac{\lambda(1-\lambda)}{\psi - (1-\lambda)^2}\right) - \frac{1}{2}\left[\frac{1-\lambda-\psi}{\psi - (1-\lambda)^2}\right]. \end{aligned} \tag{25}$$

Observe that $\bar{\theta}_i$ has two components. The first component has the mean-variance spirit. It suggests that the bank should hold more loans if it has large equity value (large e_i), low risk aversion (small γ_i), or if each loan has high expected profit after cost (high $(1-\lambda) - p - f_i$), and low variance (low $\lambda(1-\lambda)$). This mean-variance term is scaled up by a factor of $\lambda(1-\lambda)/[\psi - (1-\lambda)^2] \geq 1$. Given a fixed default probability λ, this factor increases as the default correlation decreases (smaller $\psi-(1-\lambda)^2$). This is intuitive as a lower correlation provides better risk-return trade-off and hence increases the demand for loans. That is, banks should hold more loans if each loan provides a good risk-adjusted return (mean-variance term), and the portfolio has low risk as loan defaults are less correlated (scaling factor term).

This first component of the optimal loan holding comes from two effects. The first effect is the *wealth effect* through the equity value e_i. The more equity the bank has, the larger the loan demand. The second effect is the *price effect* in the expected profit. When the price increases, the expected return decreases, reducing the loan demand. What is interesting is that banks also hold loans. So as the price increases, the equity values of banks increase and create more demands for loans due to the wealth effect. On the other hand, the price effect reduces the demands. The combination of these two effects from all banks in the system determines the final price at an equilibrium, as discussed in Section IV.

The second component of the optimal holding is a downward adjustment. Its value depends on how correlated loan defaults are. The lower the correlation, the higher the size of the adjustment. When the equity values of banks are large (e.g. millions or billions of dollars), the effect of the second component is minimal. However, when banks are close to insolvency, the second component can be relatively significant.

Now let us bring back the budget and no-short-position constraints. Because the objective function (16) is quadratic in θ, it is easy to obtain the optimal solution with the constraints from the solution of the unconstrained problem. The following proposition gives the result.

Proposition 2 *The optimal loan holding for bank i when there is one type of loans is*

$$\hat{\theta}_i = \begin{cases} 0 & \textit{if } \bar{\theta}_i \leq 0 \\ \bar{\theta}_i & \textit{if } 0 < \bar{\theta}_i < (e_i + d_i)/p \\ (e_i + d_i)/p & \textit{if } \bar{\theta}_i \geq (e_i + d_i)/p \end{cases} . \tag{26}$$

So when the bank is close to insolvency (the second downward adjustment component dominates), or when it has higher cost of managing loans ($f_i > 1 - \lambda - p$), it is optimal for the bank not to hold any loans. In contrast, when loans are very attractive, the number of loans held is capped by the budget constraint. This implies that the bank does not hold cash. However, such a situation rarely occurs because the equity decreases when the price decreases as the bank holds loans. With a large leverage ratio in the banking industry ($d_i \gg e_i$), the constraint is unlikely to be binding.

Assume that there are two types of loans ($K = 2$). The bank i's problem is to maximize the following objective function

$$\begin{aligned} V_i(\theta_1, \theta_2) = & \sum_{k=1}^{2} \left(\frac{\theta_k(1-\lambda_k) - \theta_k p_k - \theta_k f_{i,k}}{e_i} \right) \\ & - \frac{\gamma_i}{2} \sum_{k=1}^{K} \left(\frac{\theta_k^2[\psi_k - (1-\lambda_k)^2] + \theta_k[1-\lambda_k-\psi_k]}{e_i^2} \right) \\ & - \gamma_i \left(\frac{\theta_1\theta_2[\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)]}{e_i^2} \right) \end{aligned}$$

subject to the budget constraint $\theta_1 p_1 + \theta_2 p_2 \leq e_i + d_i$ and no-short-position constraint $\theta_k \geq 0, k = 1, 2$. Again, we first consider the case without constraints. The first order condition yields the following optimal solution for the unconstrained problem:

$$\begin{bmatrix} \theta_{i,1}^* \\ \theta_{i,2}^* \end{bmatrix} = \frac{1}{1 - \eta_{1,2}[\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)]} \begin{bmatrix} \bar{\theta}_{i,1} - \eta_{1,2}(\psi_2 - (1-\lambda_2)^2)\bar{\theta}_{i,2} \\ \bar{\theta}_{i,2} - \eta_{1,2}(\psi_1 - (1-\lambda_1)^2)\bar{\theta}_{i,1} \end{bmatrix} \tag{27}$$

where

$$\eta_{1,2} = \frac{\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)}{(\psi_1 - (1-\lambda_1)^2)(\psi_2 - (1-\lambda_2)^2)}$$

and $\bar{\theta}_{i,k}$ is $\bar{\theta}_i$ given by (25) in the one-type case with subscript k for the type-k loans. The term $\eta_{1,2}$ contains the correlation between defaults of type-1 loans and type-2 loans. Its value is zero when type-1 loans and type-2 loans are independent ($\alpha = 0$) and goes up as the correlation goes up (but keeping $\alpha^2 + \beta_k^2$ constant). To see the interaction between the two types of loans on the loan demand, let us focus on the demand of type-1 loans, or $\theta_{i,1}^*$. There are two components in $\theta_{i,1}^*$. The first one is the optimal holding from the one-type case $\bar{\theta}_{i,1}$ and the second component represents the hedging demand between the two types of loans. Suppose that $\bar{\theta}_{i,1}$, $\bar{\theta}_{i,2}$ and $\theta_{i,1}^*$ are positive. If type-1 and type-2 loans are independent, the second term disappears as $\eta_{1,2} = 0$, and in this case $\theta_{i,k}^* = \bar{\theta}_{i,k}$. When the correlation is positive, the second component creates a negative hedging demand on type-1 loans. The hedging demand depends on how attractive the type-2 loans are. The more attractive they are (large $\bar{\theta}_{i,2}$), the more negative the hedging demand, given everything else constant. That is, there is a *substitution effect* between the two loan types if the correlation is positive.

This substitution effect plays an important role on the cross-asset contagion channel. Suppose that a fire sale of type-1 loans from one bank reduces the price of the loans, making them more attractive (price effect), which in turn reduces the demand of type-2 loans of other banks due to the larger (more negative) hedging demand (substitution effect). This may trigger a fire sale on type-2 loans, and hence a contagion across the two loan types. The outcome can be much worse as there are interbank liability and common holding of type-1 loans channels that can transmit losses to other banks, following the fire sales of type-1 and type-2 loans. Lower equity values further reduces the loan demands (wealth effect) and reinforce the asset fire sales. We discuss the effect of contagion through different channels and their interaction in the subsequent sections.

To finish this section, we consider the cases when one or both of the no-short-position constraints are binding. Similar to the case of one type of loan, the budget constraint is unlikely to be binding at any equilibrium given sufficiently large leverage ratios. With the Lagrange multiplier technique, we obtain the following optimal holdings of the portfolio of loans:

Proposition 3 *Suppose the budget constraint is not binding for the case of two loan types. Then the optimal loan holdings for bank i is*

$$(\hat{\theta}_{i,1}, \hat{\theta}_{i,2}) = \begin{cases} (\theta^*_{i,1}, \theta^*_{i,2}) & \text{if } \theta^*_{i,1} > 0 \text{ and } \theta^*_{i,2} > 0 \\ (\bar{\theta}_{i,1},\ 0\ ) & \text{if } \theta^*_{i,1} > 0 \text{ and } \theta^*_{i,2} \leq 0 \\ (\ 0\ , \bar{\theta}_{i,2}) & \text{if } \theta^*_{i,1} \leq 0 \text{ and } \theta^*_{i,2} > 0 \\ (\ 0\ ,\ 0\ ) & \text{if } \theta^*_{i,1} \leq 0 \text{ and } \theta^*_{i,2} \leq 0 \end{cases}. \tag{28}$$

We now show that the negative demand always reduces the optimal holdings. Consider the case when $\theta^*_{i,1}$ is positive. From (27), it must be that

$$\bar{\theta}_{i,1} > \eta_{1,2}(\psi_2 - (1 - \lambda_2)^2)\bar{\theta}_{i,2}.$$

Hence, for $\eta_{1,2} > 0$, we have

$$\begin{aligned}\theta^*_{i,2} &= \frac{\bar{\theta}_{i,2} - \eta_{1,2}(\psi_1 - (1-\lambda_1)^2)\bar{\theta}_{i,1}}{1 - \eta_{1,2}[\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)]} \\ &< \frac{\bar{\theta}_{i,2} - \eta_{1,2}(\psi_1 - (1-\lambda_1)^2)\left(\eta_{1,2}(\psi_2 - (1-\lambda_2)^2)\bar{\theta}_{i,2}\right)}{1 - \eta_{1,2}[\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)]} \\ &= \bar{\theta}_{i,2}.\end{aligned}$$

Similarly, if $\theta^*_{i,2} > 0$, we have $\bar{\theta}_{i,1} > \theta^*_{i,1}$. Thus, when $\theta^*_{i,1}$ and $\theta^*_{i,2}$ are both positive, the optimal holding of the type-k loans is $\hat{\theta}_{i,k} = \theta^*_{i,k} \geq \bar{\theta}_{i,k}$ where the inequality holds if, and only if, the loans of different types have zero default correlation, or $\eta_{1,2} = 0$. Thus, the hedging demand always reduces the optimal loan holdings.

IV Results and Discussion: Equilibrium Analysis

Prior literature on fire sales in financial networks typically assumes an inverse demand function characterizing the price change as a function of the aggregate sales. This implicitly suggests that fire sale assets are sold to buyers outside the financial system. This contradicts to the fact that a large portion of the bank assets are loans which are costly to manage by non-bankers. So it is wiser for banks to sell their assets to other banks that are more efficient buyers who are willing to pay higher prices for the assets. This situation is reasonable particularly when a shock hits one bank, and the remaining banks have sufficient funds to buy the troubled bank's assets. When there is an adverse asset shock to one type of loans, demands for the loans from banks holding that type of loans will decrease due to the wealth effect, but other banks that do not hold that loan type can be potential buyers, willing to pay for the loans, and hence help reduce the effect of the fire sale. We study the effect of potential buyers in the banking networks by allowing the prices of illiquid loans to be determined endogenously. We first provide the formal definition of our equilibrium.

Definition 1 *Given the banks' balance sheets* ($c = [c_i], \theta = [\theta_{i,k}], d = [d_i], L = [l_{i,j}]$), *banks' characteristics* ($\gamma = [\gamma_i], F = [f_{i,k}]$), *and the illiquid loans default distribution* ($\lambda = [\lambda_k], \alpha, \beta = [\beta_k]$), *an equilibrium triplet of repayments, holdings, and prices* ($X = [x_{i,j}], \hat{\theta} = [\hat{\theta}_{i,k}], p = [p_k]$) *at time* 1 *is such that*

1. *Repayment equilibrium: Equations (13)-(14) hold for all banks* $i = 1, \ldots, N$,

2. *Bank optimization: Each bank i maximizes mean-variance utility (16) subject to constraints (17)-(18), $i = 1, \dots, N$.*

3. *Market clearing: Equation (19) holds.*

To study the effect of shocks and roles of banks on transmitting and absorbing shocks, we assume that without shocks to the system, there is an equilibrium in which all the interbanks liabilities are fully repaid at time 1. That is, without shocks, all banks are solvent ($x_{i,j} = l_{i,j}$). In other words, there is a price vector $p = [p_k]$ and a loan holding matrix $\hat{\theta} = [\hat{\theta}_{i,k}]$ such that the triplet $(L, \hat{\theta}, p)$ is an equilibrium triplet for the network at time 1.

A One loan type

This section considers a network with one type of loans ($K = 1$), and hence we drop subscript k that refers to the loan type. We consider three scenarios: before shocks, after a bank shock, and after an asset shock.

A.1 Before shocks

Consider the case of no shocks with an equilibrium triplet $(L, \hat{\theta}, p)$ for which all banks are solvent. Let

$$\bar{c}_i = c_i + \sum_{j \neq i}^{N} l_{j,i} - d_i - \sum_{j \neq i}^{N} l_{i,j}$$

denote the excess cash position over the deposit after the repayment settlement of bank i. This value is typically negative as cash c_i is much smaller than deposit d_i, and interbank liabilities $l_{i,j}$'s are relatively small. The equity value of bank i is the sum of the value of the loan portfolio and its excess cash:

$$e_i = \theta_i p + \bar{c}_i > 0. \tag{29}$$

We assume further that none of the banks is close to be insolvent ($e_i \gg 0$), and that all banks are equally good at managing loans and thus have the same managing cost

$f_i \equiv f < 1 - \lambda - p$.[3] So it follows from (25) that $\hat{\theta}_i = \bar{\theta}_i > 0, i = 1, \ldots, N$. Now consider the total demand of loans in the system. From (25) and (29), we have the total loan demand is

$$\begin{aligned}
\Theta_D &= \sum_{i=1}^{N} \left\{ \frac{e_i}{\gamma_i} \left[\frac{(1-\lambda) - p - f}{\psi - (1-\lambda)^2} \right] - \frac{1}{2} \left[\frac{1 - \lambda - \psi}{\psi - (1-\lambda)^2} \right] \right\} \\
&= \sum_{i=1}^{N} \left\{ \left(\frac{\theta_i p + \bar{c}_i}{\gamma_i} \right) \left[\frac{(1-\lambda) - p - f}{\psi - (1-\lambda)^2} \right] - \frac{1}{2} \left[\frac{1 - \lambda - \psi}{\psi - (1-\lambda)^2} \right] \right\} \\
&= -\left(\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i} \right) \frac{p^2}{\psi - (1-\lambda)^2} + \left(\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1 - \lambda - f) - \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} \right) \frac{p}{\psi - (1-\lambda)^2} \\
&\quad + \left(\sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} \right) \left[\frac{1 - \lambda - f}{\psi - (1-\lambda)^2} \right] - \frac{N}{2} \left[\frac{1 - \lambda - \psi}{\psi - (1-\lambda)^2} \right]. \qquad (30)
\end{aligned}$$

As we can see, the total demand in (30) is a concave quadratic function of p. This is due to the combination of the wealth effect and the price effect. It is easy to see that

$$\bar{p} = \frac{1}{2}(1 - \lambda - f - \zeta)$$

is the price at which the price effect and the wealth effect of the total demand are equal ($\partial\Theta_D/\partial p = 0$) where

$$\zeta = \frac{\sum_{i=1}^{N} \bar{c}_i/\gamma_i}{\sum_{i=1}^{N} \theta_i/\gamma_i}. \qquad (31)$$

When the price is lower than $\bar{p}$, an increase in the price increases the loan demand as the wealth effect dominates the price effect. When the price is higher than $\bar{p}$, an increase in the price lowers the loan demand as the price effect dominates the wealth effect. This demand behavior applies to each individual bank's loan demand in (25) as well. Precisely, the price at which both price and wealth effects equal for the demand of bank i is

$$\bar{p}_i = \frac{1}{2}\left(1 - \lambda - f - \frac{\bar{c}_i}{\theta_i}\right).$$

Let

$$\Theta_S = \sum_{i=1}^{N} \theta_i$$

[3] We use the differences in the managing costs in the case of multiple loan types to consider the effect of banks' expertise on the stability of the network.

denote the total number of units of loans available in the system. This represents the total supply of loans. The market clearing condition (19) dictates that the demand and supply are equal at each equilibrium: $\Theta_D = \Theta_S$. Since the total supply is fixed and the total demand is a concave quadratic function of p, we have the following result:

Theorem 1 *Suppose none of the banks is insolvent and the repayment matrix is $X = L$ in the case of one loan type. Then an equilibrium price exists if, and only if,*

$$(1-\lambda-f+\zeta)^2 \geq 4\left[\frac{(\psi-(1-\lambda)^2)\Theta_S + \frac{N}{2}(1-\lambda-\psi)}{\sum_{i=1}^N \theta_i/\gamma_i}\right], \tag{32}$$

and in that case the equilibrium prices are given by

$$p = \frac{1}{2}(1-\lambda-f-\zeta) \pm \frac{1}{2}\left\{(1-\lambda-f+\zeta)^2 - 4\left[\frac{(\psi-(1-\lambda)^2)\Theta_S + \frac{N}{2}(1-\lambda-\psi)}{\sum_{i=1}^N \theta_i/\gamma_i}\right]\right\}^{1/2}. \tag{33}$$

The equilibrium price is unique if, and only if, the inequality in (32) is binding and the equilibrium price is $p = (1-\lambda-f-\zeta)/2$.

To further investigate the equilibrium prices given by (33), we apply the first-order Taylor approximation for the function of the form $f(x) = (a^2 - x)^{1/2}$ around the point $x = 0$ to the last term in (33): $f(x) \approx |a| - x/2|a|$ where a is a constant. From the inequality in (29) and (31), it is clear that $\zeta > -p$. From the assumption that $\bar{\theta}_i > 0$, (25) implies that $1-\lambda-f-p > 0$. So we have $1-\lambda-f+\zeta > 0$. Thus, the Taylor approximation gives the following two equilibrium prices:

$$p^h \approx 1-\lambda-f-\frac{(\psi-(1-\lambda)^2)\Theta_S + \frac{N}{2}(1-\lambda-\psi)}{\displaystyle\sum_{i=1}^N \frac{\theta_i}{\gamma_i}(1-\lambda-f) + \sum_{i=1}^N \frac{\bar{c}_i}{\gamma_i}} \tag{34}$$

$$p^l \approx -\frac{\displaystyle\sum_{i=1}^N \frac{\bar{c}_i}{\gamma_i}}{\displaystyle\sum_{i=1}^N \frac{\theta_i}{\gamma_i}} + \frac{(\psi-(1-\lambda)^2)\Theta_S + \frac{N}{2}(1-\lambda-\psi)}{\displaystyle\sum_{i=1}^N \frac{\theta_i}{\gamma_i}(1-\lambda-f) + \sum_{i=1}^N \frac{\bar{c}_i}{\gamma_i}}. \tag{35}$$

Note that $p^h > \bar{p} > p^l > 0$.

As it will become clear later that an asset fire sale may occur at the equilibrium price p^h, but not at p^l, we will focus on p^h. From (34), the equilibrium price p^h is equal to the expected payoff after cost, or $1-\lambda-f$, minus the *premium*. The premium is the expected profit required by the banks for holding the risky loans. This premium depends on the *riskiness* of the loan

$$U = (\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1-\lambda-\psi). \tag{36}$$

This riskiness U is the combination of the variance component measuring the closeness to the perfect correlation $\phi - (1-\lambda)^2$ and the variance component measuring the closeness to the independence $1-\lambda-\psi$. When the number of loans available in the system is large, the first term is important. On the other hand, the second becomes large when the number of banks in the system is large. The risk premium is high when the loan has high level of riskiness.

The premium also depends on the *risk-aversion-adjusted wealth* of the banking system. To see this recall that equity of bank i is

$$e_i = \theta_i p + \bar{c}_i.$$

As p^h is an approximated price, we replace p by its expected payoff after cost, which is $1-\lambda-f$. This gives

$$e_i \approx \theta_i(1-\lambda-f) + \bar{c}_i.$$

Each equity value is scaled by the bank's risk aversion parameter γ_i as the one unit of equity of a more risk-averse bank (high γ) is worth less than that of a less risk-averse bank (low γ) in terms of the loan demand (see (25)). Then we sum over all banks to get the risk-aversion-adjusted wealth

$$\sum_{i=1}^{N} \frac{e_i}{\gamma_i} \approx \sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1-\lambda-f) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i}.$$

When the risk-aversion-adjusted wealth of the system is high, banks have more cash to pay for the loans, pushing the price up, and thus a lower premium. In the subsequent sections, we can explain changes in the equilibrium price p^h based on the changes in the expected payoff and/or the premium (risk and wealth).

A.2 After a bank shock

Let us first focus on the equilibrium price p^h, and study the change in the equilibrium price p^h in response to a bank shock. As mentioned earlier, $p^h > \bar{p}$ so the price effect of the aggregate demand is stronger than the wealth effect. If $\bar{c}_i/\theta_i$ is close to ζ, this is true for the individual bank's demand too. For the discussion below, we assume that the price effect is stronger than the wealth effect for each individual bank's demand at the equilibrium price p^h.

Suppose there is an adverse bank shock of size v_j hitting bank j. Let $\bar{\theta}_j(v_j)$ denote the value of $\bar{\theta}_j$ after an adverse shock of size v_j on bank j. This notation is used similarly for the other variables. If v_j is sufficiently small so that bank j still has positive holding in the loans, or $\bar{\theta}_j(v_j) > 0$, then the value of the excess cash of bank j after the shock is $\bar{c}_j(v_j) = \bar{c}_j - v_j$. Thus the new equilibrium price is

$$p^h(v_j) \approx 1 - \lambda - f - \frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1-\lambda-\psi)}{\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1-\lambda-f) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}. \tag{37}$$

So after a small bank shock v_j, the equilibrium price becomes lower due to a lower risk-aversion-adjusted wealth. What is interesting is that the effect of a shock of the same size on the equilibrium price depends on the risk-aversion parameter of the bank being hit. A small shock hitting a *conservative* bank (high γ) yields a smaller impact on the equilibrium price than a shock of the same size hitting an *aggressive* bank (low γ). This is due to the lower sensitivity of the loan demand to a one-unit decrease in the equity value, which can be seen in (25). Since everything remains the same for all other banks $j' \neq j$ except all banks see the new equilibrium price, each bank adjusts its loan holding based purely on the price change. As the price effect dominates the wealth effect at p^h, all of the other banks $j' \neq j$ act as the potential buyers and increase their loan holdings in response to the lower price. Thus, bank j has to hold fewer loans at the new equilibrium. Note that without the shock, the price effect of bank j is stronger than the wealth effect. But since the external shock v_j reduces the equity value in addition to the effect from the lower equilibrium price, it results in a loan sell-off for bank j.

Let us consider a larger shock. Suppose that v_j is large enough to make $e_j(v_j) < 0$, but not enough to make the other banks insolvent. So bank j sells all of its loans at a fire sale price. In addition, it spreads the loss to its *neighbor* banks (the banks that hold interbank claims on the assets of bank j) through the interbank liability linkages. The loss to bank $j' \neq j$ due to the direct interbank liability with bank j is

$$l_{j,j'} \min\left\{\frac{v_j - (\theta_j p^h(v_j) + \bar{c}_j)}{L_j}, 1\right\}.$$

The new equilibrium price is

$$\begin{aligned} p^h(v_j) \approx\ & 1 - \lambda - f \\ & - \frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1 - \lambda - \psi)}{\sum_{i \neq j}^{N} \frac{\theta_i}{\gamma_i}(1 - \lambda - f) + \sum_{i \neq j}^{N} \frac{\bar{c}_i}{\gamma_i} - \sum_{i \neq j}^{N} \frac{l_{j,i}}{\gamma_i} \min\left\{\frac{v_j - (\theta_j p^h(v_j) + \bar{c}_j)}{L_j}, 1\right\}}. \end{aligned} \tag{38}$$

The contagion through the interbank liability channel reduces the equity values of neighbor banks that hold claims on the asset of bank j. The reduction in the equity values lowers their demands for loans, and causes the price to drop further. The impact on the price depends on the neighbor bank j''s ratio $l_{j,j'}/\gamma_{j'}$. The impact is large if the neighbor bank has a large claim on the asset of bank j and it is an aggressive bank (small γ). This suggests that interbank liabilities between aggressive banks amplify the fire sale effect in the network.

Now consider p^l. When there is a small bank shock of size v_j on bank j, the new equilibrium price is

$$p^l(v_j) \approx -\frac{\sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}{\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}} + \frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1 - \lambda - \psi)}{\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1 - \lambda - f) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}$$

which is higher than the equilibrium price before the shock, or p^l, where, similar to the case of p^h, we have assumed that all of the banks are solvent after this small shock. Assuming that at p^l the wealth effect dominates the price effect for all banks, we have the increase in the price results in higher demands for loans for all other banks $j' \neq j$. As we can see this equilibrium does not correspond to a fire sale as the selling price increases after a shock arrives. In addition, banks should agree to choose the equilibrium with the higher price to maximize their net worth. So we will focus on p^h from now on.

A.3 After an asset shock

This section considers the effect of an asset shock on the equilibrium price. Suppose the default probability of each loan increases from λ to λ' and none of the banks are insolvent after the shock. It can be shown that the terms $\psi-(1-\lambda)^2$ and $1-\lambda-\psi$ are increasing in λ for $\lambda \in (0, 0.5)$ but are decreasing in λ for $\lambda \in (0.5, 1)$.[4] Since the typical values of λ are less than 0.5, p^h decreases as the default probability increases (see (34)). When the price effect is stronger than the wealth effect as we assume here, each bank should tend to increase its holdings in the loans following the shock. However, as the expected value declines and the risk rises, the worsen loan characteristic reduces the loan demands and this brings the price to the new equilibrium. When the price drops, so do banks' equity values. Once the default probability is large enough, it may trigger a default of a bank, and the losses are transmitted through the interbank liability linkages, further reducing the equity of other banks. This reinforces the fire sale in the network.

Let $p^*(i)$ denote the price at which bank i's equity reaches zero due to the increase in the default probability. We have

$$p^*(i) = p^h - \frac{e_i}{\theta_i}$$

where p^h and e_i are the equilibrium price and the bank's equity before the shock, respectively. Let us call $p^*(i)$ the *critical price* of bank i. When the default probability rises, the bank that has the highest critical price can be insolvent first. As we can see, that *critical bank* is the bank that initially holds the largest number of loans per one unit of its equity value. If all banks initially hold the loans at the optimal holding level as suggested by (25), the most aggressive bank with the lowest risk aversion parameter tends to be insolvent first when the default probability increases. If the critical bank has large liabilities with other aggressive banks in the network, the contagion effect is much larger once it becomes insolvent.

B *Two loan types with equal costs*

We now consider the case with two types of loans ($K = 2$). Here we assume that all banks have the same level of expertise in managing loans, and hence the same managing costs. That is, we assume that $f_{i,k} \equiv f_k < 1 - \lambda_k$, for $k = 1, 2$ and $i = 1, \ldots, N$. Again we

[4] This can be seen from the fact that $\frac{\partial\psi}{\partial\lambda} = -2\Phi\left(\Phi^{-1}(1-\lambda)\sqrt{\frac{1-\rho}{1+\rho}}\right)$ where $\rho = \alpha^2 + \beta^2$.

consider the equilibrium prices before shocks, after a bank shock, and after an asset shock. The main difference between the one-type and two-type cases we consider here is that when there are two types of loans, the demands of the loans of different types interact through the hedging demand and the wealth effect, causing the cross-asset contagion.

B.1 Before shocks

Let $(L, \hat{\theta}, p)$ denote the equilibrium triplet at time 1 before shocks, and assume that at the equilibrium all banks hold both types of loans, or $\hat{\theta}_{i,k} = \theta^*_{i,k} > 0$ for all $i = 1, \ldots, N$ and $k = 1, 2$. From (27), this implies that $\bar{\theta}_{i,k} > 0$ for all i and k. Using the optimal holding condition (27) and the clearing condition (19), it can be shown that the equilibrium price vector $p = [p_1, p_2]'$ satisfies the following system of equations:

$$U_1 = (1 - \lambda_1 - f_1 - p_1) \left[\left(\sum_{i=1}^{N} \frac{\theta_{i,1}}{\gamma_i} \right) p_1 + \left(\sum_{i=1}^{N} \frac{\theta_{i,2}}{\gamma_i} \right) p_2 + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} \right] \tag{39}$$

$$U_2 = (1 - \lambda_2 - f_2 - p_2) \left[\left(\sum_{i=1}^{N} \frac{\theta_{i,1}}{\gamma_i} \right) p_1 + \left(\sum_{i=1}^{N} \frac{\theta_{i,2}}{\gamma_i} \right) p_2 + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} \right] \tag{40}$$

where

$$U_1 = (\psi_1 - (1-\lambda_1)^2)\Theta_{S,1} + \frac{N}{2}(1 - \lambda_1 - \psi_1) + (\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2))\Theta_{S,2}$$

$$U_2 = (\psi_2 - (1-\lambda_2)^2)\Theta_{S,2} + \frac{N}{2}(1 - \lambda_2 - \psi_2) + (\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2))\Theta_{S,1}$$

and $\Theta_{S,k} = \sum_{i=1}^{N} \theta_{i,k}$ denotes the total number of type-k loans available in the system. The quantity U_k captures the risk of type-k loans similar to U given by (36) for the one-type case. The difference is that U_k contains an additional term due to the risk from the hedging demand, which is zero if the default correlation between the two types of loans is zero.

Observe from (39) - (40) that the loan prices at the equilibrium have linear relationship:

$$\frac{1 - \lambda_1 - f_1 - p_1}{U_1} = \frac{1 - \lambda_2 - f_2 - p_2}{U_2}. \tag{41}$$

This relationship suggests that the expected profits per unit risk of the two loan types are equal at each equilibrium. Solving (39) - (40), we obtain the following results:

Theorem 2 *Suppose none of the banks is insolvent and the repayment matrix is $X = L$ in the case of two loan types with $f_{i,k} = f_k$ for all $i = 1, \ldots, N, k = 1, 2$. Then an equilibrium price vector exists if, and only if,*

$$(1 - \lambda_k - f_k + \zeta_k)^2 \geq \frac{4U_k^2}{\left(\sum_{i=1}^N \theta_{i,k}/\gamma_i\right) U_k + \left(\sum_{i=1}^N \theta_{i,k'}/\gamma_i\right) U_{k'}}, \tag{42}$$

for $k, k' = 1, 2$ and $k \neq k'$, and in that case the equilibrium price vectors $p = [p_1, p_2]'$ are given by

$$p_1 = \frac{1}{2}(1 - \lambda_1 - f_1 - \zeta_1) \pm \frac{1}{2}\left\{(1 - \lambda_1 - f_1 + \zeta_1)^2 - \frac{4U_1^2}{\left(\sum_{i=1}^N \theta_{i,1}/\gamma_i\right) U_1 + \left(\sum_{i=1}^N \theta_{i,2}/\gamma_i\right) U_2}\right\}^{1/2} \tag{43}$$

where

$$\zeta_1 = \frac{\displaystyle\sum_{i=1}^N \frac{\theta_{i,2}}{\gamma_i}\left[U_1(1 - \lambda_2 - f_2) - U_2(1 - \lambda_1 - f_1)\right] + \sum_{i=1}^N \frac{\bar{c}_i}{\gamma_i}}{\displaystyle\sum_{i=1}^N \frac{\theta_{i,1}}{\gamma_i} U_1 + \sum_{i=1}^N \frac{\theta_{i,2}}{\gamma_i} U_2} \tag{44}$$

and the corresponding p_2 can be determined from the linear relationship (41).

As mentioned, we focus on the price vector at which the fire sales may occur after a shock. Using the first-order Taylor approximation as in the one-type case, the interested equilibrium price vector $p^h = [p_1^h, p_2^h]'$ is given by

$$p_k^h \approx 1 - \lambda_k - f_k - \frac{U_k}{\displaystyle\sum_{i=1}^N \frac{\theta_{i,1}}{\gamma_i}(1 - \lambda_1 - f_1) + \sum_{i=1}^N \frac{\theta_{i,2}}{\gamma_i}(1 - \lambda_2 - f_2) + \sum_{i=1}^N \frac{\bar{c}_i}{\gamma_i}} \tag{45}$$

for $k = 1, 2$. This is similar to p^h of the one-type case given by (34). However, the denominator of the premium term now contains the expected payoff after cost of both types of loans. In addition, the numerator in the premium term, or U_k, has an extra hedging demand component which links the default probability of one type of loans to the price of the other type of loans. We discuss the implications below.

B.2 After a bank shock

Consider a small adverse bank shock on bank j of size v_j. Assume that this shock does not cause any insolvency in the banking system. The equilibrium price changes from $p^h = [p_1^h, p_2^h]'$ as given by (45) to

$$p_k^h(v_j) \approx 1 - \lambda_k - f_k - \frac{U_k}{\sum_{i=1}^{N} \frac{\theta_{i,1}}{\gamma_i}(1-\lambda_1-f_1) + \sum_{i=1}^{N} \frac{\theta_{i,2}}{\gamma_i}(1-\lambda_2-f_2) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}.$$

As we can see, the effect is similar to the one-type case; that is, the prices of both types of loans reduce due to the lower wealth in the system, and the effect is large if the bank being hit is an aggressive bank. However, comparing the reduction in the prices, we can see that the price of the loan type that has higher level of riskiness (larger U_k) reduces more. We can also see this from taking the difference due to the shock on both sides of (41) to get

$$\frac{p_1^h - p_1^h(v_j)}{U_1} = \frac{p_2^h - p_2^h(v_j)}{U_2} \quad \Rightarrow \quad p_1^h(v_j) - p_1^h = \frac{U1}{U_2}(p_2^h(v_j) - p_2^h).$$

So the price of the loan with a higher level of riskiness is more sensitive to a bank shock, and the effect is large if the shocks hit an aggressive bank.

Now if the shock is large enough to make bank j become insolvent, but all the other banks are not, the result is similar to the one-type case as given in (38). That is, the intetbank liabilities of the insolvent bank transmit losses to its neighbor banks, and the impact to the loan prices is large if the liabilities are large and the neighbor banks are aggressive banks.

B.3 After an asset shock

In this section we focus on how a shock in the default probability of one type of loans causes the change in the price of the other type of loans. Let us assume for the moment that defaults of different types of loans are independent ($\alpha = 0$). Now suppose the default probability of the type-1 loan increases from λ_1 to λ_1'. As mentioned earlier in the one-type case, $\psi_1 - (1-\lambda_1)^2$ and $1 - \lambda_1 - \psi_1$ are increasing in λ_1 when $\lambda_1 < 0.5$. Thus, U_1 increases as the default probability λ_1 increases. We assume further that the increase in the default risk does not cause any banks to become insolvent. Based on (45) for $k = 1$, it is clear that, the equilibrium price of type-1 loan decreases. This is due to the lower expected payoff, higher level of riskiness, and lower wealth in the system. The higher default risk of the type-1 loans also reduces the price of the type-2 loans as can be seen in (45) for $k = 2$. This cross-asset contagion comes from the wealth effect in the denominator of the type-2 loan's premium term.

Note that the cross-asset contagion always occurs as the term $\sum_{i=1}^{N} \theta_{i,1}/\gamma_i$ in the equilibrium price p_2^h is always positive. To see how this happens, we note that when the defaults of the two types of loans are uncorrelated ($\alpha = 0$), $\eta_{1,2} = 0$ and thus the loan demand is $\hat{\theta}_{i,k} = \bar{\theta}_{i,k}$, which is assumed to be positive for all $i = 1, \ldots, N$ and $k = 1, 2$. Now consider $\bar{\theta}_{i,k}$ as given by (25) with the subscript $k = 1, 2$. Let us assume that $e_i \gg 0$ so that the negative adjustment term in (25) is insignificant. As λ_1 increases, the type-1 loan characteristic becomes worsen as the risk $(\psi_1 - (1 - \lambda_1)^2)$ rises and the expected payoff $(1 - \lambda_1)$ declines. This makes $\bar{\theta}_{i,1}$ lower. So the demand for the type-1 loans decreases, and consequently the price of the type-1 loans has to drop to make the expected profit go up to bring the demand back to the balance. But once the price of the type-1 loans decreases, the equity value of each bank holding the type-1 loans decreases, and this reduces demand $\bar{\theta}_{i,2}$ for the type-2 loans due to the wealth effect. As a result, the price of the type-2 loans has to drop to make the expected profit higher and bring the demand back to the balance.

Now consider a little more extreme case in which the banks are divided into two non-overlapping groups, one holding only the type-1 loans and the other holding only the type-2 loans at time 0. The cross-asset contagion still occurs in this case as long as each bank has a demand for both types of the loans at time 1 before the shock. The magnitude of the effect of the cross-asset contagion from type-1 loans to type-2 loans depends, however, on the initial banks' holdings of type-1 loans at time 0, which is $\sum_{i=1}^{N} \theta_{i,1}/\gamma_i$. If initially the majority of the type-1 loans are held by aggressive banks (low γ), the impact of the cross-asset contagion from type-1 to type-2 is large, while the impact is smaller if most of the type-1 loans are held by conservative banks. The latter is unlikely if banks try to hold optimal number of loans at time 0 as aggressive banks tend to hold more loans. In Section C, we discuss the cases where banks may hold only one type of loans at an equilibrium before a shock due to different expertise. In that case, the results can be different.

When the default correlation between the two types is not zero ($\alpha > 0$), the hedging demand term $\Psi_{1,2} - (1 - \lambda_1)(1 - \lambda_2)$ in U_2 can transmit the effect of the increase in the default probability of type-1 loans to the price of type-2 loans. However, the relationship between $\Psi_{1,2}-(1-\lambda_1)(1-\lambda_2)$ and λ_1 is not monotone for typical values of λ_1, λ_2 and α. So it is possible that the hedging demand term can strengthen or weaken the contagion effect.

C Two loan types with bank expertise

In this section we assume that each bank has its own expertise in managing one particular type of loans. Let $\mathbb{N}_k$ denote the set of banks that have an expertise in managing type-k loans, $k=1,2$. We assume that each bank belongs to either $\mathbb{N}_1$ or $\mathbb{N}_2$, but not both. We call banks that are in $\mathbb{N}_k$ as type-k *expert banks* and those that are not in $\mathbb{N}_k$ as type-k *non-expert* banks. So the banking industry is divided into two sectors defined by $\mathbb{N}_1$ and $\mathbb{N}_2$. The cost associated with managing the loans of type k for type-k expert banks is zero, while the cost for every type-k non-expert bank is $f_k > 0$, which is the same for all non-expert banks. Let N_k denotes the number of banks in $\mathbb{N}_k$. We assume that there is at least one bank for each sector, or $N_k > 0$ for both $k = 1, 2$.

We assume further than the cost of managing loans of type k is so large that it is not optimal for type-k non-expert banks to hold type-k loans in their portfolios at an equilibrium before a shock. We also assume that the initial holdings of type-k loans for type-k non-expert banks are zero due to the high managing cost, or $\theta_{j,k} = 0$ for all $j \notin \mathbb{N}_k$. We are interested in how the loan sectors play a role on contagion risk in the banking network. Again, we consider the equilibrium prices before shocks, after a bank shock, and after an asset shock.

C.1 Before shocks

Because we assume that banks in sector k do not hold loans of the other types at an equilibrium before shocks, it must be that $\theta^*_{j,k} \le 0$ for $j \notin \mathbb{N}_k$. From (28), we have $\hat{\theta}_{i,k} = \bar{\theta}_{i,k}$ and $\hat{\theta}_{i,k'} = 0$ for $i \in \mathbb{N}_k$ and $k' \neq k$. Thus, the clearing condition (19) gives

$$\Theta_{S,k} = \left[\left(\sum_{i\in\mathbb{N}_k} \frac{\theta_{i,k}}{\gamma_i}\right) p_k + \sum_{i\in\mathbb{N}_k} \frac{\bar{c}_i}{\gamma_i}\right] \left(\frac{1-\lambda_k - p_k}{\psi_k^2 - (1-\lambda_k)^2}\right) - \frac{N_k}{2}\left(\frac{1-\lambda_k-\psi_k}{\psi_k^2-(1-\lambda_k)^2}\right)$$

for $k = 1, 2$. Note that we have used the fact that $\theta_{i,k} = 0$ for $i \notin \mathbb{N}_k$. As we can see, the equilibrium price for each type of loans can be determined independently as the equations for $k = 1, 2$ are decoupled. Thus, as long as banks do not have demands for loans outside their expertise, the equilibrium price of each loan type is determined based on the banks in the sector. This reduces the problem into two independent one-asset equilibrium problems. Hence, we have the equilibrium price of type-k loans is

$$p_k^h \approx 1 - \lambda_k - \frac{(\psi_k - (1-\lambda_k)^2)\Theta_{S,k} + \frac{N_k}{2}(1-\lambda_k-\psi_k)}{\displaystyle\sum_{i\in\mathbb{N}_k} \frac{\theta_{i,k}}{\gamma_i}(1-\lambda_k) + \sum_{i\in\mathbb{N}_k} \frac{\bar{c}_i}{\gamma_i}}.$$

As we can see, the price of type-1 loans does not depend on the default probability of type-2 loans, nor the default correlation. It does not depend on the information about the banks in sector 2 either. This holds true as long as $\theta_{i,2}^* \leq 0$ for all $i \in \mathbb{N}_2$. So the contagion across banks and loan types is different from the one considered in Section B.

C.2 After a bank shock

Suppose there is an adverse small bank shock of size v_j on bank j in sector 1. We assume that after the shock none of the banks are insolvent, and that the price is still high for the banks in sector 2 to buy type-1 loans. In this case, the new equilibrium price for type-1 loans reduces to

$$p_1^h(v_j) \approx 1 - \lambda_1 - \frac{(\psi_1 - (1-\lambda_1)^2)\Theta_{S,1} + \frac{N_1}{2}(1-\lambda_1-\psi_1)}{\displaystyle\sum_{i\in\mathbb{N}_1} \frac{\theta_{i,1}}{\gamma_i}(1-\lambda_1) + \sum_{i\in\mathbb{N}_1} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}$$

while the price of type-2 loans remains the same. That is, there is no contagion from sector 1 to sector 2. The price of type-1 loans reduces because the shock reduces the equity value of bank j, causing the bank to sell off some of the loans, pushing the price down to make it more attractive for the other type-1 expert banks to increase their demands. This is the same as the one-type case.

Now let's assume that the shock v_j is large enough to make bank j insolvent, but not any other banks in the system. Assume further that at the new equilibrium prices, it is not optimal for banks in one sector to buy loans in the other sector. Under these conditions, the new equilibrium price of the type-1 loans is similar to (38) in the one-type case in which the impact on the price depends on the sizes of the interbank liabilities and the risk aversion parameters of the neighbor banks. That is, the new equilibrium is

$$p_1^h(v_j) \approx 1 - \lambda_1 \\ - \frac{(\psi_1 - (1-\lambda_1)^2)\Theta_{S,1} + \frac{N_1}{2}(1-\lambda_1-\psi_1)}{\sum_{i\in\mathbb{N}_1, i\neq j} \frac{\theta_{i,1}}{\gamma_i}(1-\lambda_1) + \sum_{i\in\mathbb{N}_1, i\neq j} \frac{\bar{c}_i}{\gamma_i} - \sum_{i\in\mathbb{N}_1, i\neq j} \frac{l_{j,i}}{\gamma_i} \min\left\{\frac{v_j - (p_1^h(v_j)\theta_{j,i} + \bar{c}_j)}{L_j}, 1\right\}}.$$

As we can see, only the parameters describing the banking network inside sector 1 are involved.

Let us look at the price of type-2 loans. Only if there is a bank in sector 2 that is an interbank creditor of bank j, the loss of bank j can be transmitted to sector 2 through the interbank channel. This transmitted loss reduces the wealth in the sector, causing the equilibrium price for the type-2 loans to drop. The new equilibrium type-2 price is

$$p_2^h(v_j) \approx 1 - \lambda_2 - \frac{(\psi_2 - (1-\lambda_2)^2)\Theta_{S,2} + \frac{N_2}{2}(1-\lambda_2-\psi_2)}{\sum_{i\in\mathbb{N}_2} \frac{\theta_{i,2}}{\gamma_i}(1-\lambda_2) + \sum_{i\in\mathbb{N}_2} \frac{\bar{c}_i}{\gamma_i} - \sum_{i\in\mathbb{N}_2} \frac{l_{j,i}}{\gamma_i} \min\left\{\frac{v_j - (p_1^h(v_j)\theta_{j,i} + \bar{c}_j)}{L_j}, 1\right\}}.$$

In addition to the liability sizes $l_{j,i}$ and the risk aversion parameters γ_i of the interbank creditors i in sector 2, the reduction in the type-2 loan price depends also on the price of the type-1 loans after the shock or $p_1^h(v_j)$. So the more sensitive $p_1^h(v_j)$ to the shock, the higher the impact the shock has on $p_2^h(v_j)$. Because the new equilibrium price $p_1^h(v_j)$ depends on the information of all the banks in sector 1, the contagion effect from sector 1 to sector 2 depends on the information of all banks in sector 1 and how they are related. For example, if the insolvent bank j has two interbank creditors, which are bank i in sector 1 and bank i' in sector 2. Given the shock v_j, the change in the price of the type-2 loans depends not only on the information about bank j and the liability link between bank j and bank i' in sector 2, but also the liability link between bank j and bank i in sector 1 as well as the risk aversion parameter of bank i.

When this type of shock gets larger, and more type-1 expert banks become insolvent, the losses from sector 1 can be transmitted to sector 2 via the interbank liabilities between the insolvent banks in sector 1 and the banks in sector 2. So even if there is no direct interbank liability from bank j to any bank in sector 2, the loss originated from the shock on bank j may eventually affect the price of the type-2 loans if there is a liability path starting from bank j to a bank in sector 2.

Now let us consider another possible outcome from the shock v_j. Assume that after the shock, none of the banks is insolvent and the resulting equilibrium price of the type-1 loans is low enough to make it attractive to bank i' in sector 2 to hold some positive number of type-1 loans, but it is not attractive enough for the other banks in sector 2. Suppose for the moment that the defaults of type-1 and type-2 loans are uncorrelated, and thus $\eta_{1.2} = 0$. From (27) and (28) we have the optimal loan holdings of bank i' are $(\hat{\theta}_{i',1}, \hat{\theta}_{i',2}) = (\bar{\theta}_{i',1}, \bar{\theta}_{i',2})$. That is, bank i' does not change the holding in type-2 loans, but increases the holding of type-1 loans from zero to $\bar{\theta}_{i',1}$. As a consequence, the market for type-2 loans is not affected by the shock, resulting in the same equilibrium price for the type-2 loans. On the other hand, there is a loss in the equity of bank j in sector 1 and an additional demand for type-1 loans from bank i' originally from sector 2. Thus the new equilibrium price of the type-1 loans satisfies

$$\begin{aligned}
\Theta_{S,1} = & \left[\left(\sum_{i\in\mathbb{N}_1} \frac{\theta_{i,1}}{\gamma_i}\right) p_1 + \sum_{i\in\mathbb{N}_1} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}\right] \left(\frac{1-\lambda_1-p_1}{\psi_1^2-(1-\lambda_1)^2}\right) - \frac{N_1}{2}\left(\frac{1-\lambda_1-\psi_1}{\psi_1^2-(1-\lambda_1)^2}\right) \\
& + \left[\left(\frac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \frac{\bar{c}_{i'}}{\gamma_{i'}}\right] \left(\frac{1-\lambda_1-f_1-p_1}{\psi_1^2-(1-\lambda_1)^2}\right) - \frac{1}{2}\left(\frac{1-\lambda_1-\psi_1}{\psi_1^2-(1-\lambda_1)^2}\right) \\
= & \left[\left(\sum_{i\in\mathbb{N}_1} \frac{\theta_{i,1}}{\gamma_i}\right) p_1 + \sum_{i\in\mathbb{N}_1} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j} + \left(\frac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \frac{\bar{c}_{i'}}{\gamma_{i'}}\right] \left(\frac{1-\lambda_1-p_1}{\psi_1^2-(1-\lambda_1)^2}\right) \\
& - \frac{N_1+1}{2}\left(\frac{1-\lambda_1-\psi_1}{\psi_1^2-(1-\lambda_1)^2}\right) - \left[\left(\frac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \frac{\bar{c}_{i'}}{\gamma_{i'}}\right] \left(\frac{f_1}{\psi_1^2-(1-\lambda_1)^2}\right)
\end{aligned}$$

Using the first-order Taylor approximation as in the one-type case, we have the new equilibrium price for type-1 loans is

$$p_1^h(v_j) \approx 1-\lambda_1 - \frac{(\psi_1^2-(1-\lambda_1)^2)\Theta_{S,1} + \frac{N_1+1}{2}(1-\lambda_1-\psi_1) + \left[\left(\frac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \frac{\bar{c}_{i'}}{\gamma_{i'}}\right] f_1}{\left(\sum_{i\in\mathbb{N}_1} \frac{\theta_{i,1}}{\gamma_i}\right)(1-\lambda_1) + \sum_{i\in\mathbb{N}_1} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j} + \left[\left(\frac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \frac{\bar{c}_{i'}}{\gamma_{i'}}\right]}. \tag{46}$$

It is easy to show that the additional demand from bank i' for the type-1 loans helps reduce the effect of the shock on the type-1 loan price given that it is optimal for the type-2 expert bank i' to enter into sector 1. Moreover, as loan defaults are uncorrelated, this does not hurt the price of the type-2 loans. To further understand this situation, let us identify the bank in sector 2 that actually is bank i'. To do this, consider a type-2 expert bank i. We can rewrite (25) for bank i after the shock as follows:

$$\bar{\theta}_{i,1}(v_j) = \left(\frac{\theta_{i,2}p_2 + \bar{c}_i}{\gamma_i}\right)\left[\frac{(1-\lambda_1) - p_1^h(v_j) - f_1}{\psi_1 - (1-\lambda_1)^2}\right] - \frac{1}{2}\left[\frac{1-\lambda_1-\psi_1}{\psi_1 - (1-\lambda_1)^2}\right]. \tag{47}$$

As $p_1^h(v_j)$ decreases, the value of $\bar{\theta}_{i,1}(v_j)$ increases as there is no wealth effect for bank i in sector 2. So the bank that has the largest $\bar{\theta}_{i,1}(v_j)$ for $i \in \mathbb{N}_2$ is the bank i'. It is easy to see from (47) that it is the bank with the largest equity to risk aversion parameter ratio (e/γ) among the type-2 expert banks as all the banks in sector 2 has the same managing cost of f_1.

Now if loan defaults are correlated, or $\eta_{1,2} > 0$, the demand for type-1 loans from bank i' will lead to a decline in the demand for type-2 loans from bank i' due to the negative hedging demand. As the price effect is stronger than the wealth effect at the equilibrium we are interested in, the price of the type-2 loans must drop to bring the type-2 loan demand up to meet the total supply. Hence this creates a cross-asset contagion purely through the hedging demand. We do not require any interbank liabilities, nor do we require a bank to hold both types of loans at time 0 to act as a channel to transmit the effect from one type of loans to the other type of loans through the reduction in the equity value of the bank.

Now consider another alternative outcome. Suppose that the shock v_j causes a bank in sector 1 to become insolvent, and the insolvent bank has interbank liabilities with some type-2 expert banks. The resulting equilibrium prices depends on these liabilities. If banks that hold claims on the insolvent bank j are the ones with low e/γ ratios, then it is possible that the type-2 expert banks with the largest e/γ ratio will find the drop in the type-1 loan price attractive enough to buy them into the bank's balance sheet, reducing the effect on the new equilibrium price of the type-1 loans, but at the same time causing the contagion to the price of the type-2 loans due to the negative hedging demand. On the contrary, if banks that hold claims on the insolvent bank j are the ones with highest e/γ ratios, the reduction in the equity values of those type-2 expert banks could reduce the possibility for them to be the potential buyers of type-1 loans. This results in a worse outcome for the price of the type-1 loans as no new buyers from sector 2. This suggests that interbank liabilities of this type weaken the role as the potential buyers of the banks with largest e/γ.

C.3 After an asset shock

Suppose the default probability of the type-1 loan increases from λ_1 to λ_1'. As long as the new equilibrium price of the type-1 loans does not fall enough to attract type-2 expert banks to buy type-1 loans, and there are no losses transmitted through the liability linkages to banks in sector 2, this does not affect the equilibrium price of the type-2 loans. But once one of those scenarios occurs, the price of the type-2 loans reduces due to either the negative hedging demand, provided that $\eta_{1,2} > 0$, or the reduction in the equity values of some type-2 expert banks similar to the case of a bank shock discussed above.

The above discussion leads to an interesting policy implication. Suppose that the costs of managing loans are low, but the regulator would like to separate banks and loan markets into non-overlapping sectors to limit the effect of contagion. As a consequence, the regulator may allow banks to choose their areas of expertise or sectors where they can run their businesses as usual. However, banks need to pay a huge regulatory fee to do the business outside their selected areas of expertise. When a sector is hit by a small shock, the banks in the sector can function as potential buyers to self-rescue the sector from fire sales. When the shock is large, and there are not many banks in the sector that can function as the potential buyers, the regulator can initiate the self-rescue mission by allowing the secondary potential buyers from the other healthy sectors to step in and buy the assets, reducing the effect of fire sales in the failing sector. The regulator can choose the right value of $f_{i,k}$ to allow enough funds from other sectors to flow into the failing sector, providing support to the loan price in the failing sector. At the same time the regulator needs to avoid the unintended contagion effects due to the negative hedging demands.

Once the cross-sector rescue mission has been accomplished, the healthy sector is now contaminated by the fire sale loans, and cannot function as secondary potential buyers for the next crisis. So the regulator should use this as a temporary solution to reduce the effect of fire sales, and start to bring everything back to normal and be ready for the next crisis.

V Conclusions and Suggestions

When an adverse shock hits a bank, causing it to become insolvent, the bank needs to sell all of its assets, the majority of which are illiquid loans. This can cause the loan prices to drop, reducing the mark-to-market values of other banks holding the same types of loans. The loss of the insolvent bank can also be transmitted to other banks through the interbank liability linkages, reducing the net worth of its neighbor banks. The banks affected by these two channels of contagion will re-adjust their portfolios in response to lower equity values, and start to sell more illiquid loans into the markets. If banks are highly connected either through the liability linkages or the common loan holdings, then most of the banks in the system will suffer from the losses and cannot function as the potential buyers, reducing the self-rescue ability of the system.

We study a financial system in which banks in the system may create self-rescue ability. We find that aggressive banks can become good potential buyers if they are not affected by a shock as they are willing to buy a large amount of loans given a small discount. At the same time, they can become fire sale initiators even if they are hit by a relatively small shock. Interbank liabilities between these aggressive banks can also amplify the contagion effect and the effect of fire sales as they adjust the portfolios markedly following losses in their equity values. So it is better to avoid having interbank liabilities between those aggressive banks. We also find that prices of loans that have higher risk are more sensitive to a shock in the system. So having aggressive banks holding these high risk loans would accelerate the contagion effect, once it occurs. Unfortunately, aggressive banks tend to hold a large amount of loans, including the high risk loans, so we need some regulatory policies to help reduce the potential damages caused by these aggressive banks.

Contagion across loan types can occur from many channels. A shock to one particular bank may trigger a fire sale of one loan type, lowering the loan price. Banks that hold the same type of loans will lose their equity values and start to sell loans of other types in their portfolios to re-adjust their portfolios' risk-adjusted returns. This creates the cross-asset contagion. Alternatively, a drop of the price of one loan type makes it more attractive to healthy banks to buy the loans. As loan defaults are positively correlated, the substitute effect creates a negative hedging demand, requiring the banks to reduce the holdings of certain types of loans in their portfolios upon buying another type of loans.

Finally, we study the system in which banks and loan markets are separated into sectors based on their areas of expertise defined by the cost of managing loans. We find that small shocks in one sector do not cause contagion to the others as long as the interbank liabilities between the sectors are not available and the cost for entering an area outside of the banks' expertise is sufficiently high. In this case, the banks in the sector experiencing a small shock need to act as potential buyers for their own sectors. Once the shock is large, causing the price to drop enough, then banks from other sectors may function as potential buyers for the failing sector. Based on this observation, we propose a policy that separates banks and loans into sectors, limiting the contagion effects between groups of banks and types of loans. At the same time, we create secondary potential buyers that are ready to step in and save the failing sector when it is most needed. This type of policy can be achieved by imposing regulatory fees that keep them separated during good times, and allow them to rescue their peers during bad times, creating a self-rescue system.

Outputs

1. Two manuscripts to be submitted to international peer-reviewed journals attached in the appendix.
2. Computer codes to estimate parameters of fundamental-based models.
3. Presentation at PIER Field Workshop, Bank of Thailand.
4. Presentation at Stock Exchange of Thailand.

Appendix: Two Manuscripts

Roles of Domestic Fundamentals and Cross-Country Linkages on International Stock Market Contagion*

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Abstract

We develop a novel fundamental-based contagion model of international stock markets. In this model, the characteristics of each stock market return is determined by the country's *hidden* domestic fundamental. These fundamentals are connected through cross-country linkages such as trade and financial linkages. The mechanism of the contagion consists of two effects: the transmission of shocks and the shock amplification effects. The stronger the cross-country linkages, the larger the shocks are transmitted, and the weaker the domestic fundamentals, the larger the shocks are amplified. An empirical study on the international stock market contagion between Japan - Thailand, Hong Kong - Thailand and the US - Thailand reveals that financial linkage is the only transmission channel of shock to Thailand and that there is a significant evidence of the effect of shock amplification by the weak Thai fundamental. Trade linkages, on the other hand, are just an indirect measure of the Thai fundamental.

Key words– fundamental-based contagion, stock market contagion, trade and financial linkages, shock amplification.

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1 Introduction

Since the 1990s, there has been many evidence of financial contagion, for the 1994 Mexican peso crisis, the 1997 Asian financial crisis, the 1998 Russian financial crisis, and the 1998-2002 Argentine Great Depression, are characterised as financial contagion. In general terms, financial contagion can be defined as the spread of shocks in a financial market across borders, either across different asset markets or in asset markets across different countries. The 1997 Asian financial crisis, for example, was originally started in Thailand with a collapse of the Thai currency. The crisis was then transmitted to other financial markets in Asia, especially in the currency and the stock markets, and also to the financial markets in the United States, Europe and Russia. This paper focuses on the cross-country contagion in the stock markets.

The international stock contagion has drastic impacts in various ways. In the investment perspective, without financial contagion, a globally diversified investment strategy would reduce portfolio risk and increase expected returns. But in the presence of financial contagion, a downside shock originated in one country leads to a significant drop in asset prices in other countries. As a result, the stock market contagion, if ignored, would lead to an underestimation of risk and eliminates the benefits of international portfolio diversification. So, global investors should take possible contagion into account, and modify their multinational investment strategy. Further, the existence of financial contagion puts a threat to country's economic and financial stability. Since enhancing the country's stabilities are the main challenges for policymakers, a better understanding of stock market contagion is needed in order to impose effective polices and regulations. A better understanding of financial contagion allows the policy makers to detect and prepare for shocks that would spill over, and reduce the contagion effect in the country. Therefore, understanding financial contagion is critically important and urgent.

This paper aims to develop a framework of fundamental-based contagion in the stock market. By defining the term *fundamental* as the health of the economy, this paper assumes that return characteristics of a country is are determined by the country's domestic fundamental. In the international context, with countries' fundamentals being interconnected through fundamental linkages: trades and financial linkages, a domestic shock in one country could spread globally. During a crisis, banks and corporations suffer from withdrawals of foreign funds, investment spending significantly drops, trade financing dries up, and exports subsequently collapse. This paper studies the role of the trade and financial linkages in *the transmission of shock*. Moreover, this paper introduces another effect in the contagion called *the shock amplification* by the domestic fundamental. The domestic fundamental plays a crucial role in the vulnerability of the country to an external shock. Holding everything else constant, countries with weak fundamental are likely to experience a more severe contagion effect than the ones whose the fundamental is sound. In other words, the weak fundamental amplifies the effects of transferred shocks.

With the proposed framework, we conduct an empirical study to study the international stock market contagion between Thailand and Japan, Hong Kong and the US, and aim to answer the following three questions: (1) Is there an amplification of shock by the domestic fundamental in Thailand?, (2) Are trade and financial linkages transmissions channels of shock?, and (3) Shocks from which country would transmit to Thailand the most? To reach to the answers of these questions, the proposed model is estimated using Markov Chain Monte Carlo (MCMC). We find that financial linkage is the only channel that transmits shocks. In contrast, trade linkage is somewhat an indirect measure of the Thai fundamental. We also find significant evidence of the shock amplification effect in Thailand, and that Thailand was sensitive to changes in the fundamental of the US the most. The effect of the changes in the US fundamental on the Thai fundamental and stock market returns became more pronounced during the 2008 global financial crisis.

Although financial contagion has been a famous topic of study in the past decade, this paper contributes to the literatures in two ways. First, this paper proposes that the effect of the two transmission channels of financial contagion, trade and financial linkages, is amplified by the domestic fundamental, while recent studies mainly focus on the role of trade and financial linkages and role of fundamental in explaining financial contagion separately. This enables us to investigate the interaction between each component. Second, unlike past studies that measure the contagion directly through relationship between countries' stock markets, we propose explicitly model how contagion occurs through countries' fundamentals, and how shocks in fundamentals affect stock market returns.

The remainder of the paper is organized as follows. Section 2 reviews previous literatures. Section 3 explains our conceptual framework and develops our model. Section 4 provides details of our estimation method. Section 5 provides empirical results. Lastly, Section 6 concludes this paper.

2 Literature Review

2.1 Definitions of Financial Contagion

Although there is a growing literature on financial contagion, there is no consensus on what the term means. This is partly because historical crises exhibit little to no similarity, and each shock is observed to be transmitted across countries differently. Consequently, researches employ different approaches to measure and test for the contagion effect.

The World Bank Group (2009) proposes three definitions of financial contagion: a broad, a restrictive, and a very restrictive definition. Firstly, according to the broad definition, financial contagion is the spillover effect or the transmission of shocks across countries. Under this definition, the financial contagion can occur in both good times and bad times. This definition is referred to as fundamental-based contagion or interdependence in the literature as trade and financial linkages are considered as the channels of financial contagion. Under this definition, Corsetti *et al.* (2005) develop a model of financial contagion, through which country-specific shocks spread to other countries in the region or around the world via trade using microeconomic foundation. This definition is also employed in the works of Diebold and Yilmaz (2009) and Diebold and Yilmaz (2012) in studying the interdependence of asset returns and volatility spillovers.

Secondly, according to the restrictive definition, financial contagion is the excess co-movement or the correlation after controlling for common shocks and trade and financial linkages. Investor behaviour usually accounts for the excess co-movement under this definition. Eichengreen *et al.* (1995,1996) find evidence of financial contagion in this context in past currency crises.

Thirdly, according to the very restrictive definition, financial contagion occurs during a crisis when there is a shift in correlation. This definition is sometimes called shift-contagion (Forbes and Rigobon, 2001) or pure contagion (Masson, 1999). Numerous literatures adopt this definition and test for financial contagion by comparing changes in relationship between markets before and after crises. Under this definition, studies of King and Wadhwani (1990) and Lee and Kim (1993) are ones of the early studies to measure financial contagion via the shift in the correlation coeffcients pre- and post-crisis. Other studies that also use this definition of financial contagion are Forbes and Rigobon (2002) and Gravelle *et al.* (2006).

Other than that, there are other ways the contagion is defined. Goldstein (1998) divides causes of financial contagion into fundamental and panics. Fundamental-based contagion or spillover is a transmission mechanism of shocks caused by fundamental linkages. Whereas, panic-based contagion is a result of self-fulfilling expectation from investors' actions when they are panic.

The term financial contagion in this paper is defined basing on the Broad definition of financial contagion of the World Bank Group (2009) and the fundamental-based contagion of Goldstein (2013), where shocks spread across countries because of fundamental linkages and fundamental that have adverse impacts on country's fundamental and its stock market returns.

2.2 Channels of Financial Contagion

2.2.1 Trade linkages

In terms of trade linkages, countries that rely heavily on exports are more likely to be affected by shocks in their major trading partner countries due to two reasons, a decrease in demand and a loss in competitiveness. During a financial crisis, demand for imports of the trading partners drops significantly, which has a direct impact on the state of economy of the export-dependent countries. Also, a financial crisis in one economy causes a large depreciation of the currency; as a consequence, it affects other countries' relative export competitiveness. Many literatures, for example Eichengreen *et al.* (1996), Glick and Rose (1999), and Forbes (2002) show that trade linkages are empirically a main determinant of financial contagion.

Although trade linkages were one of the most significant channels of crisis transmission in 1971-1997, recent literatures (Blanchard *et al.*, 2010, Rose and Spiegel, 2011, Berkmen *et al.*, 2012) do not find empirical evidence of the transmission of the 2008 Global Financial Crisis through trade linkages. Cheewatrakoolpong and Manprasert (2015) state that direct trade alone cannot capture the total trade linkages, and find that the total trade linkages remains an important transmission channel when account for indirect trade.

2.2.2 Financial linkages

Similarly, for financial linkages, countries with a large amount of foreign borrowings, lending, and investments are exposed to the contagion risk, as a crisis causes a large decline in inflow, or even an outflow of credits and capital. According to Claessens *et al.* (2000), when investors incur capital loss from a decline in stock prices during a crisis, they may need to rebalance their portfolio or improve their liquidity position by withdrawing their investments in other countries. In the same manner, because of a decline in loan quality in one country, banks may need to reduce their exposures to risks in other countries, especially in countries with high risk. Caramazza *et al.* (2004), Kaminsky and Reinhart (2000), and Van Rijckeghem and Weder (2003) find empirical evidence that commercial banks contribute to financial contagion. Accordingly, Kaminsky *et al.* (2003) report that when stock prices fall in one country, mutual funds would also reduce their stock holding in other countries, which partially cause a shock to spread.

2.2.3 Investor behaviour

Investor behaviour plays a major role in financial contagion. Investors in the financial markets worldwide are neither perfectly rational nor perfectly irrational, and such behaviour is another cause for a crisis to transmit from one country to another.

A wake-up call hypothesis proposed by Goldstein (1998) states that a crisis originated in one country serves as a wake-up call for investors to reassess other countries' state of economy, and reevaluate their investments in the other countries. The investors may make their judgments according to some known economic indicators, and rationally sell off their assets in the countries weak fundamental, or the ones that with strong fundamental linkages with the crisis-originated countries. However, obtaining information can be costly, so some investors remain uninformed, and choose to observe and follow actions of the rest of the market, but some investors sell off assets just to rebalance their portfolio or because they panic, and not because of the information. The most commonly used framework to test for financial contagion through this channel is a global games framework. Bernard and Ouarda (2013) find a strong evidence of herding behaviour contributed to the global financial crisis in 2007.

In summary, there are three channels of financial contagion. The first two channels, trade and financial linkages, can sometime be called the "fundamental linkages". These linkages are account for the interdependence across countries. The other factor that affects financial contagion is investor behaviour, and although this factor is proven to be another key element affecting financial contagion, it is difficult to measure the behaviour of investors quantitatively. Thus, this paper only considers that the transmission of shocks through trade and financial linkages, and omits the investor behaviour factor, which is consistent with the way financial contagion is defined here.

2.3 The Importance of Domestic Fundamentals

A country-specific fundamental is a factor of financial contagion that is often ignored by literatures. Domestic fundamentals reflect the vulnerability of countries to external financial shocks and the severity of the crisis transmission. Countries with weak fundamentals have a higher risk of shock spillovers.

Kaminsky and Reinhart (1999) find the relationship between the soundness of fundamentals and crises; a crisis tends to arise when the fundamentals are weak. Sachs *et al.* (1996) test whether the fundamentals explain which countries suffer more from international crises. They find that excess credit creation and real exchange rate misalignments are the key sources of the problem. Similarly, Berg and Pattillo (1999) find that the growth rate of domestic credit, a real exchange rate value, and the reserves to money supply ratio are the most important indicators of economic vulnerability to shocks. According to a study of Berg (1999), the countries that were affected by financial crisis in the 1990s shared similar weak fundamentals, particularly, real effective exchange rate appreciation, high current account deficit, and large short-term debts have been identified as important factors associated with a regional crisis.

There are various indicators that measure domestic fundamentals: GDP growth, unemployment rate, current account to GDP ratio, public debt to GDP ratio, and bank credit growth. A study of Claessens *et al.* (2010) finds that house price appreciation, bank credit growth, and the magnitude of current account significantly explain the crisis spillovers. Likewise, Rose and Spiegel (2011) conclude that with a high current account deficit, country would suffer from external shocks more severely. In sum, countries with sound fundamentals are more likely to be less vulnerable to external financial shocks, whereas, a small external shock may have a great adverse impact in the country when it is having a weak economy.

However, recent empirical literatures reveal contradicting conclusions on the role of the domestic fundamentals in explaining financial contagion. Eichengreen and Gupta (2015) find no evidence that stronger fundamentals would protect countries from financial contagion, but they do find that emerging economies that experience great exchange rate appreciation and widening current accounts are usually affected by financial contagion more severely. However, the study states that size of financial market of the country is the main factor that influences the severity of the financial contagion, as it is found that it is easier for investors to rebalance their portfolios in relatively large and liquid financial markets. Similarly, Aizenman *et al.* (2016) do not find any significant evidence that the soundness of domestic fundamentals provides any protection to countries from the contagion effects in the short-term.

This research differs from other literatures in two ways. First, this research assumes that the process of the fundamental is a latent process that determines the stock market return characteristics. Second, this research proposes the role of the fundamental in amplifying the severity of the contagion effect.

2.4 Empirical Approaches

Since the late 1990s, financial contagion has received much attention by economists and researchers. A large amount of theories, and many econometric techniques have been developed to study the contagion effect in the financial market. Various approaches are adopted to test for financial contagion to serve different purposes of studies.

By defining financial contagion as an increase in cross-country linkages after a crisis, the correlation/co-movement analysis compares the correlation between two countries between the periods of crisis and the stable periods. If there is a significant increase in the cross-country correlation during a crisis, then there is an evidence of financial contagion. This analysis mainly focuses on examining the existence of financial contagion; hence no channel of financial contagion is identified.

King and Wadhwani (1990) are the first to apply the correlation method to measure financial contagion, and they find that the correlations between returns in the stock market of the United States, the United Kingdom, and Japan increase after the 1987 stock market crash. Many literatures extend the correlation coefficient analysis to study financial contagion between various financial markets, countries, and for different crises (Lee and Kim, 1993 and Calvo and Reinhart, 1996).

However, Forbes and Rigobon (2002) argue that results from previous literature are subject to heteroskedasticity bias. This is because the cross-country correlation coefficients are conditional on market volatility (volatility normally increases during a crisis). The results after adjusting for the bias imply that there is no increase unconditional correlation coefficient during the 1997 Asian crisis, 1994 Mexican peso crisis, and 1987 stock market crash. Therefore, basing on the correlation-based definition of financial contagion, they conclude that there is only interdependence from high market comovements in all period, and no contagion.

Another method that is used in the literature is GARCH (Generalised Autoregressive Regressive Conditional Heteroskedasticity). GARCH can be employed to estimate the variance-covariance matrix of the transmission mechanism across countries. GARCH models take into account the fact that volatility of asset returns varies over time, and tends to cluster during a crisis. Several types of multivariate GARCH models are employed to analyse comovements, volatility spillovers, and financial contagion.

Dynamic conditional correlations (DCC) model introduced by Engle (2002) is developed from Bollerslev (1990) Constant Conditional Correlation (CCC) model, in which the DDC is based on the same decomposition of the covariance matrix as the CCC model, but it assumes time-varying GARCH-type structure of the correlations. The DDC model is widely used in researches to test for financial contagion. Using DCC-GARCH model, Wang *et al.* (2006) provide an evidence of contagion effect of the 1997 Asian financial crisis on China, Hong Kong, Taiwan and Thailand, from stock market shocks, and an increase in stock return volatility after the crisis. The same approach is used in Chiang *et al.* (2007) to nine Asian stock markets, confirming the evidence of contagion. But the DCC framework is proven to be inadequate for higher dimensional models.

Many refinements of the DCC model have recently been proposed. Among others, Cappiello *et al.* (2006), introduce an additional term in the DCC equation to account for asymmetric effects, called Asymmetric Generalised Dynamic Conditional Correlation (AG-DCC) model. The model is used to examine the asymmetries in conditional variances and correlation dynamic, and capture the heterogeneity in the data. Kenourgios *et al.* (2010) apply the AG-DCC model, and find that the correlations of stock markets in Brazil, Russia, India and China with stock markets in the U.S. and UK (developed countries) increase during crises. The AG-DCC results reveal that there is a contagion effect during number of crises, and that the developing countries are more likely to suffer from financial contagion. However, using the same model on stock market returns of France, Germany, UK, Ireland, Italy, Spain and Greece, Tamakoshi *et al.* (2012) find a fall in the dynamic correlation, and thus conclude that there is no evidence of financial contagion during the European sovereign debt crisis.

Although the multivariate GARCH framework provides a key tool to model financial contagion, the GARCH-type model flexibility is insufficient to capture complex dynamics. Stochastic volatility (SV) model offers increased flexibility over GARCH-type specifications since they assume separate innovation processes for the conditional mean and the conditional variance of the observables. The main characteristic of these models is that the volatility is modelled as an unobserved latent variable.

Hwang *et al.* (2007) point out that there are permanent effects of shocks such as permanent changes in the economy that the standard SV models can not capture, and thus, introduce Markov-Switching SV (MSSV) model that uses a Markov chain to control the possibility of regime changes. However, this specification is not commonly used in recent literatures of financial contagion because the specification of a fixed number of regimes and the assumption that the Markov chain is recurrent, so is assumed a positive probability of returning to a previous regime. Another limitation in the application of this model is that it is formulated only in a univariate specification, and thus it is not possible to identify common patterns in several markets using this model.

Recently, there has been a growing literature adopting the jump-diffusion process into the stochastic volatility model to explain the financial contagion as the effects on the stock prices are observed to have some jumps (large negative drops) during episodes of shocks. Many forms of model with jump have been introduced. Aït-Sahalia *et al.* (2015) introduce a model of financial contagion using mutually exciting jump processes, where a jump in a market of one country increases the jump intensity in that market (self-excitation) and also across the country (cross-excitation). This model incorporates Heston's stochastic volatility with conditional Hawke-process jump intensity. Then, Polson and Scott (2011) propose a model of financial contagion that accounts for explosive, mutually exciting shock to market volatility, explosive stochastic volatility (ESV) model, assuming that the random shocks to volatility are heavy-tailed and correlated cross-sectionally, both with each other and with returns. In addition, Laurini and Mauad (2015) introduce a multivariate stochastic volatility model with a common jump factor to capture the common jumps in times of financial contagion. However, the results of these jump-diffusion models do not provide any analysis about the factors of financial contagion.

Alternatively, financial contagion can be viewed as a significant increase in the probability of shocks in a country, conditional on information about the occurrence of crisis elsewhere. Ones of the first studies that employ the conditional probability are Eichengreen *et al.* (1996) and Sachs *et al.* (1996), where they examine whether the likelihood of a crisis is higher conditional on the information that crises occur in other countries. This approach has some clear advantages: first, it permits statistical tests of the existence of contagion, and second, these tests can also try to investigate the channels through which contagion may occur (Dornbusch *et al.*, 2000). However, these tests do not allow testing whether there have been structural breaks in the transmission mechanisms of crises, and therefore, one cannot straightforwardly distinguish crisis-contingent and non-crisis-contingent propagation channels. Using the conditional probability approach, De Gregorio and Valdes (2001) compare the financial contagion of the the 1982 debt, the 1994 Mexican and the 1997 Asian crises, and find that the 1994 Mexican crisis is the least contagious crisis among the three. But the similar comparison conducted by Caramazza *et al.* (2004) yield a different conclusion, as they find that the 1994 Mexican, the contagion of the 1997 Asian and the Russian crises does not differ much. The findings of other studies that use the conditional probability approach yield mixed results.

In contrast to the literature, this paper develops a new framework to model interdependence of fundamentals based on their equilibrium relationship, and proposes the role of domestic fundamentals in contagion in amplifying the transmitted shocks when the economy is bad, and these fundamentals serve as a protecting shield from the shocks when the economy is good.

3 Conceptual Framework and Contagion Model

The conceptual framework is developed to demonstrate the design of the model of financial contagion proposed in this paper. The objective is to clarify the concept of financial contagion by discussing the roles of trade and financial linkages, and domestic fundamental in explaining financial contagion.

Figure 1. Conceptual Framework

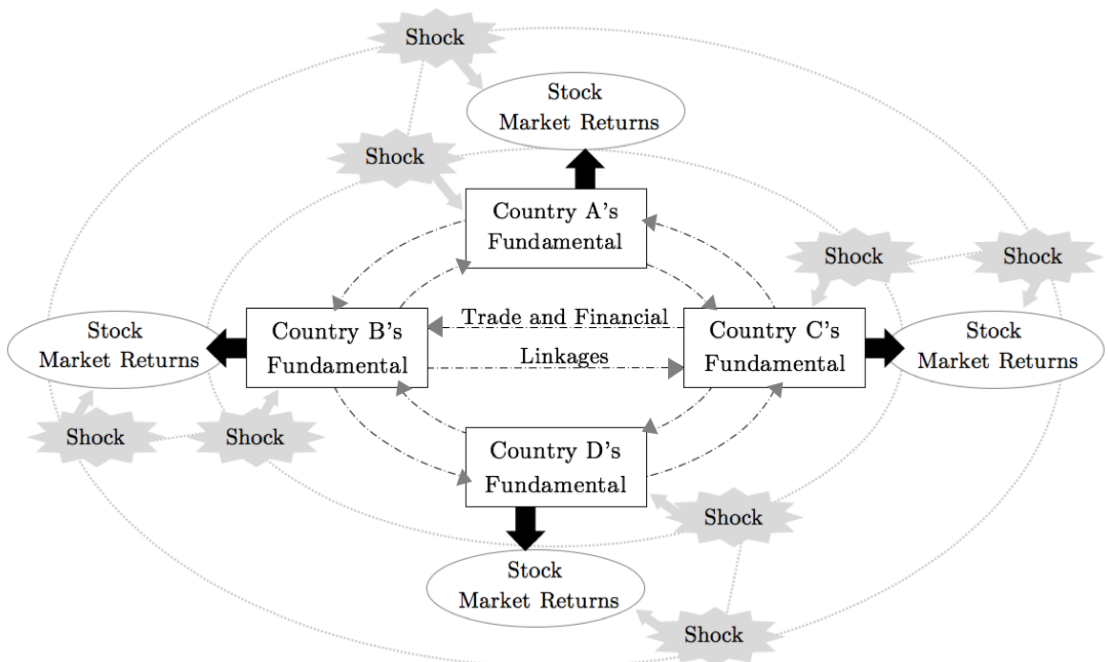


Figure 1 illustrates the network of countries and the mechanism of financial contagion under the framework of this paper. First of all, this paper defines the term *fundamental* as the health of the economy, which is assumed to be a factor driving the country's stock market return distribution, where the stock market in countries with good fundamental would have a high average return and low volatility. In this conceptual framework, countries are connected via trade and financial linkages, causing the fundamental of countries co-move, even in the equilibrium. An occurrence of adverse shocks in fundamental in one country causes its fundamental to deviate from the equilibrium, which then has negative impacts on the stock market returns. If the shock spills over to another country through the trade and financial linkages adversely affecting that country's fundamental and hence its stock market returns, the spread of the shocks is referred to as "financial contagion". However, not all shocks would be transmitted to other countries nor would those countries always be affected at the same level of severity.

When there is a shock in country A's fundamental, there are two main factors that determine contagion of the shock to country B, C and D: the size of the trade and financial linkages per GDP of each country to country A and the strength of the fundamental of each country. Note that the size of the linkages per GDP of country A to B and that of country B to A are not the same as it depends on the size of each country's economy.

3.1 Transmission of Shocks through Fundamental Linkages

Firstly, the shock in country A would be transmitted to other countries through the trade and financial linkages. These fundamental linkages represent (relative) connectedness of one country to another, which determine the tendency and size of the shock that would spill over. If country B has a large linkage with country A, it is more likely to be sensitive to shocks in country A, compared to countries with smaller linkages. On the other hand, despite not having any linkages to country A, country D may also be affected by the shock since the shock could spread to country B and/or C, and eventually reaches country D. In sum, this paper introduces the first mechanism of financial contagion, which is the transmission of shock, as countries are linked through the trade and financial linkages.

3.1.1 The effects of trade on the transmission of shock

Suppose country A imports goods from country B. If there is a shock in country A, the demand for imports of country A will decline. Thus, country B receives lower income from exports, and this causes adverse impacts on the country B's economy. At the same time, if country A exports good to country C, a shock in country A causes the country's currency to depreciate, and that will benefit country C, as prices of its imported goods are now lower. If the goods that country C imports from country A are raw materials for production, it would mean that its cost of production will reduce, thus, higher profit. Or if the goods that country C imports are for consumption, the lower price of imported goods will allow people in country C to consume more. Therefore, the trade link can also have a positive effect on importing country. Either of these two effects could dominate, so it depends on whether the country is an importing or an exporting country, and its trade behaviour.

3.1.2 The effects of investments on the transmission of shock

According to Milesi-Ferretti, *et al.* (2010), the financial linkage between countries can be measured in various dimensions, including portfolio investments on equities and debt securities and Foreign Direct Investment (FDI).

The effects of portfolio investment level on the transmission of shock are contributed in two ways. First, countries with a large position of assets in foreign countries would hurt when there is an adverse shock in the countries they invest in as their portfolio values and wealth reduce. Second, for the countries whose assets are held by foreigners, they would also hurt when there is an adverse shock in other countries from capital outflows.

The effects of FDI on the transmission of shock, however, is not as obvious. There are two ways in which shocks are transmitted through FDI. First, countries whose companies that expand their operations or own companies in foreign countries would also be affected by the shocks in those countries. For example, because Japanese automotive companies based their car manufacturing in Thailand, they were also adversely affected when there was a massive flood in Thailand in 2011. On the other hand, shocks could be transmitted from the foreign to the host countries when a crisis in the foreign country causes foreign companies to close down factories in the host country and withdraw their investments. Lin and Ye (2017) state that the role FDI in transmitting the shock is even more significant for developing countries because, in practice, inward FDIs are more embraced while there are stricter restrictions on portfolio investments.

3.2 Amplification of Shocks by Domestic Fundamentals

Once the shock is transmitted to another country through the two linkages, the effect of the shock depends on the soundness of the fundamental of that country at the time. Given that the shock spreads from country A to country B and C, and the two countries have the same size of trade and financial linkages to country A, but under the conceptual framework of this paper, the two countries may not experience an equal severity of the shock. If country B has poor fundamental at the time, the shock from country A would lead to a more severe shock to country B's fundamental and a greater decline in stock return in country B. Whereas good fundamental of country C can be served as a protection, thus reducing impacts of the shock in fundamental and returns in the stock market in country C. Thus, the second mechanism of financial contagion introduced in this paper is the amplification of shock influenced by each country's domestic fundamental.

In addition, this paper allows market shocks of each country to be correlated. This correlation could be due to the common exposure of the countries' fundamental to, for example, changes in oil price. Similarly, shocks in the stock market of each country can be correlated based on the factors beyond the scope of this paper. This could be the sentiment on the global stock market, and the correlation of this type may represent the herding behaviour of investors. This paper also allows shocks in the fundamental and the stock market to be correlated within the same country. This models the rapid adjustment in the stock price in response to unexpected shocks in the fundamental of the country. All these correlations are represented by the dashed lines in Figure 1.

All in all, this conceptual framework describes the mechanism of financial contagion, as well as unfolds how the level of the trade and financial linkages between two countries, and the soundness of the domestic fundamental explain financial contagion between two countries.

Based on the conceptual framework, this paper proposes a multivariate model of financial contagion that contains both the transmission and the amplification mechanisms as we describe next.

3.3 Contagion Model

Consider a model with n countries. Let $R_{i,t}$ and $F_{i,t}$ denote the stock market return and the fundamental of country i at time t, respectively, where the higher the value of F implies the stronger the fundamental.

3.3.1 Stock Market Returns

Assume that the stock market return of country i at time $t+1$, $R_{i,t+1}$, follows the following dynamic:

$$R_{i,t+1} = \alpha_i^R + \beta_i^R F_{i,t} + e^{-\frac{1}{2}F_{i,t}} \epsilon_{i,t+1}^R \tag{1}$$

where α_i^R, β_i^R and γ_i^R are constants. $\epsilon_{i,t+1}^R$ is the domestic shock in the stock market return of country i at time $t+1$, and $\epsilon_t^R = \left[\epsilon_{1,t}^R, \epsilon_{2,t}^R, \ldots, \epsilon_{n,t}^R\right]$ is jointly normally distributed with mean 0 and covariance matrix $\Sigma^R = \left[\sigma_{ij}^R\right]$. Conditional on the current fundamental $F_{i,t}$, the expected stock market return is $\alpha_i^R + \beta_i^R F_{i,t}$ and the variance is $e^{-F_{i,t}}\sigma_{ii}^R$. That is, both mean and variance of return $R_{i,t+1}$ depend on the current fundamental $F_{i,t}$.

3.3.2 Fundamental

The change in the fundamental of country i at time $t+1$, $\Delta F_{i,t+1}$, follows the following dynamic:

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \left[\alpha_i^F - \beta_i^F F_{i,t} + \sum_{j\neq i}^{n} \left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t}\right) \cdot F_{j,t} \right] + \epsilon_{i,t+1}^F \tag{2}$$

where $\kappa_i, \alpha_i^F, \beta_i^F, \beta_{ij}^{(0)}, \beta_{ij}^{(1)}$, and $\beta_{ij}^{(2)}$ are constants. $\epsilon_t^F = \left[\epsilon_{1,t}^F, \ldots, \epsilon_{n,t}^F\right]$ is jointly normally distributed with mean 0 and covariance matrix Σ^F. At the equilibrium, we would have

$$\alpha_i^F - \beta_i^F F_{i,t} + \sum_{j\neq i}^{n} (\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t}) \cdot F_{j,t} = 0 \tag{3}$$

and hence the expected change in the fundamental $\Delta F_{i,t}$ is 0. When the quantity in (3) is non-zero, $F_{i,t+1}$ tend to move in the direction to adjust back to the equilibrium. Additionally, $\Delta F_{i,t+1}$ can be driven by fundamental shock $\epsilon_{i,t+1}^F$ causing the fundamental to deviate from the equilibrium.

To see what drives the change in fundamental of country i, let's first assume that $\kappa_i^F = \beta_{ij}^{(1)} = \beta_{ij}^{(2)} = 0$, then (2) reduces to

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \sum_{j\neq i}^{n} \beta_{ij}^{(0)} F_{j,t} + \epsilon_{i,t+1}^F. \tag{4}$$

This gives an error correction model of order 0, where the change in fundamental of country i, $\Delta F_{i,t+1}$, depends on its own and other countries' fundamental. The first part $\alpha_i^F - \beta_i^F F_{i,t}$ indicates the mean-reversion mechanism of $F_{i,t+1}$, where α_i^F/β_i^F and β_i^F measure the long-run mean and the speed of the reversion, respectively. The second part $\sum_{j\neq i}^{n} \beta_{ij}^{(0)} F_{j,t}$ measures the movement of $F_{i,t+1}$ that comes from the fundamentals of other countries $F_{j,t}, j = 1, \ldots, n$, and $j \neq i$. The constant $\beta_{ij}^{(0)}$ measures the degree of interdependence of country i to country j. When $\beta_{ij}^{(0)}$ is large, the equilibrium value of the fundamental of country i depends heavily on the fundamental of country j.

Then, if $\beta_{ij}^{(1)}$ and $\beta_{ij}^{(2)}$ are not zero, (2) becomes

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \sum_{j=\neq i}^{n} \left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t} \right) F_{j,t} + \epsilon_{i,t+1}^F. \tag{5}$$

Here, the transmission of shocks in the fundamental from country j to country i is explained by the trade and financial linkages of country i to country j, $TL_{ij,t}$ and $FL_{ij,t}$. Parameters $\beta_{ij}^{(1)}$ and $\beta_{ij}^{(2)}$ indicate the significance of trade linkage and financial linkage in transmitting the shocks in the fundamental from country j to country i, and parameter $\beta_{ij}^{(0)}$ is for other transmission channels of shocks beyond trade and financial linkages.

Now, when $\kappa_i \neq 0$, shocks in the fundamental that is transmitted to country i is scaled or amplified by $e^{-\kappa_i F_{i,t}}$, and the significance of this amplification effect is measured by the parameter κ_i. A positive value of κ_i implies that the domestic fundamental plays an important role in amplifying the shock when the fundamental is weak, and reducing the shock effect when the fundamental is strong.

Note that the domestic shocks in country i's fundamental and stock market return, $\epsilon_{i,t}^F$ and $\epsilon_{i,t}^R$ are not necessarily independent. Let Σ^{RF} denote the covariance matrix of $\left[\epsilon_t^R, \epsilon_t^F\right]$, and assume that $cov(\epsilon_{i,t}^F, \epsilon_{j,t}^R) = 0$, for $i \neq j$.

4 Estimation Method

To estimate the proposed model, we employ the Metropolis-within-Gibbs technique, which is a hybrid between two Markov chain Monte Carlo (MCMC) techniques: Metropolis-Hastings and Gibbs sampling. This hybrid MCMC method increases efficiency, and, more importantly, provides more practical solutions of complex models. The detailed estimation procedure and the posterior distribution of the MCMC can be found in Appendix A.

In general, Markov chain Monte Carlo (MCMC) estimation uses *Monte Carlo* method to generate a *Markov chain*, and with enough simulations, the distribution of the chain would converge to the posterior distribution. According to the Bayes' formula, the posterior distribution of the parameters, $\varphi = \{\alpha_i^R, \beta_i^R, \kappa_i, \alpha_i^F, \beta_i^F, \beta_{ij}^{(0)}, \beta_{ij}^{(1)}, \beta_{ij}^{(2)}, \Sigma^{RF}\}$, where $i, j = 1, \ldots, n$ and $j \neq i$, and the latent fundamental, $F = \{F_{i,t}\}$, $t = 0, \ldots, T-1$ given the equity market return, $R = \{R_{i,t}\}$, and the trade and financial linkages, $L = \{TL_{ij,t}, FL_{ij,t}\}$, where $i, j = 1, \ldots, n$, $j \neq i$, and $t = 0, \ldots, T$, can be written as

$$p(\,\varphi, F \,|\, R, L\,) \propto p(R, F \,|\, \varphi\, L)\, p(\,\varphi \,|\, L\,),$$

where $p(R, F \,|\, \varphi\, L\,)$ is the likelihood function, and $p(\,\varphi \,|\, L\,)$ is the prior distribution.

Basing on Gibbs sampling, each parameter in this paper is iteratively sampled one at a time, conditioned on all other parameters.

Parameters: $p(\,\varphi_k \,|\, \varphi_{\sim k}, F, R, L\,)$, for $k = 1, \ldots, K$

Fundamental: $p(\,F_t \,|\, \varphi, F_{\sim t}, R, L\,)$, for $t = 1, \ldots, T-1$

where $\varphi_{\sim k}$ denotes the set of all parameters except the parameter φ_k, and K is the number of parameters, and $F_{\sim t}$ denotes the set of all fundamental except F_t. The samples are drawn based on the usual conjugate prior of Normal and Inverse-Wishart distributions for all parameters except κ_i and F_t, for which the Metropolis-Hasting is employed. For each iteration of the Metropolis-Hasting, a proposal value is generated and accepted with the acceptance probability. If the value is accepted, the parameter will take that value in that iteration. But if the value is rejected, then the value of that parameter remains unchanged.

According to the Markov chain theory, for a large enough number of iterations N, the distributions would converge to the posterior distributions. However, the samples simulated at early iterations may not be good representatives of the actual posterior distribution, so they are usually dropped out. Then for large enough m, where $m < N$, the expected value of each parameter, φ_k, and the expected value of F at each time t, F_t, are approximated as

$$E[\,\varphi_k \,|\, R, L\,] \approx \frac{1}{N-m}\left(\sum_{z=m+1}^{N} \varphi_k^{(z)}\right), \qquad \text{and} \qquad E[\,F_t \,|\, R, L\,] \approx \frac{1}{N-m}\left(\sum_{z=m+1}^{N} F_t^{(z)}\right),$$

where $\varphi_k^{(z)}$ and $F_t^{(z)}$ are the samples of φ_k and F_t at the iteration z, respectively.

5 Empirical Study

We now provide an empirical study of the international stock market contagion between Thailand and its major peers, including Japan, the US, and Hong Kong. However, because of the data limitation, we consider the models with two countries: Japan - Thailand, Hong Kong - Thailand, and the US - Thailand.

5.1 Data

To estimate the model parameters, data on the stock market returns and data on the trade and financial linkages are required. We provide the details of each variable below. All the data used in this research are sampled on a monthly basis. The sample period is from January 2006 to November 2017, which gives a total of 143 observations for each series.

5.1.1 Stock market returns

This paper uses major stock index returns, which include Nikkei 225 (Japan), S&P 500 (the US), HSI (Hong Kong), and SET (Thailand), to proxy returns in the stock market of each country. We use log-returns computed from the adjusted closed price sourced from Bloomberg. The summary statistics of monthly stock market returns over the sample period is shown in Panel A of Table 1. The Thai stock index, SET, has the highest average monthly return of 0.91%, followed by HSI, S&P 500, and the Japanese stock market, Nikkei 225, has the lowest average return of 0.39%. The HSI monthly return is also the most volatile with the standard deviation of 6.20%, followed by SET, Nikkei 225, and S&P 500 with the lowest standard deviation of 4.13%. All of the stock index returns have a negative skewness and a positive kurtosis.

5.1.2 Trade linkages

Thailand's trade linkage to Japan, Hong Kong, and the US are proxied by the 12-month average of total trade values between Thailand and the three countries relative to the size of the Thai economy, measured by GDP. The reason for using the 12-month average value is to capture the yearly trade cycle. The monthly USD bilateral imports and exports values are sourced from the Bank of Thailand Statistics, and the USD value of the monthly GDP is sourced from the World Bank Statistics. Finally, to make the result interpretation easier, the 12-month average trade value per GDP is standardised by its mean and standard deviation over the sample period.

The summary statistics of the raw monthly trade linkage per GDP over the sample period is shown in Panel B of Table 1. Between Japan, Hong Kong, and the US, Thailand has the highest average total trade linkage with Japan of 2.02%, followed by with the US of 1.25%, and the trade linkage of Thailand to Hong Kong has the lowest average of 0.46%. Similarly, the standard deviation of the Thai trade linkage to Japan is the highest, then the US, and lastly Hong Kong. On the other hand, comparing the trade linkages of Japan, Hong

Table 1
Data Summary Statistics

	Mean (%)	S.D. (%)	Skewness	Kurtosis	Max (%)	Min (%)
			A: Stock Market Returns			
Japan (Nikkei 225)	0.3861	5.7792	-1.0067	2.9198	12.201	-27.216
Hong Kong (HSI)	0.7524	6.2019	-0.7516	2.3932	16.743	-25.388
The US (S&P 500)	0.7022	4.1310	-1.0640	3.0714	10.363	-18.383
Thailand (SET)	0.9068	5.9202	-1.8930	9.7195	14.473	-35.770
			B: Trade Linkage			
Japan -Thailand	2.0213	0.3381	0.4360	-0.1503	2.9151	1.3036
Hong Kong - Thailand	0.4648	0.0994	0.7871	0.2494	0.7865	0.2996
The US - Thailand	1.2491	0.1355	-0.0730	0.3477	1.6517	0.8923
Thailand - Japan	0.0874	0.0128	-0.5007	-0.1180	0.1098	0.0507
Thailand - Hong Kong	0.4659	0.1015	0.7557	0.0438	0.7647	0.2914
Thailand - The US	0.0094	0.0024	0.7725	-0.2147	0.0155	0.0055
			C: Financial Linkage			
Japan -Thailand	7.4548	4.0365	0.1153	-1.2772	13.293	0.2125
Hong Kong - Thailand	7.0559	1.3986	1.3126	3.5297	9.0751	0.8553
The US - Thailand	2.2744	1.8430	1.4179	1.8430	8.8111	0.1958
Thailand - Japan	0.3521	0.2411	0.5397	-1.2337	0.7737	0.0073
Thailand - Hong Kong	7.0516	1.3172	1.6578	4.9683	8.8459	0.8713
Thailand - The US	0.0167	0.0147	1.6377	2.2359	0.0684	0.0022

Kong, and the US to Thailand, Hong Kong's trade linkage is the highest on average and most volatile, and the US's trade linkage is lowest.

5.1.3 Financial linkages

Two main measures are used to proxy the financial links of Thailand to Japan, Hong Kong, and the US, including USD value of the Foreign Direct Investment positions (inward and outward) and USD value of the Equities and Debt Securities Investment Portfolio positions (assets and liabilities). Both of the two data sets are sourced from Bank of Thailand Statistics. Similar to the way TL_t is computed, Thailand's financial linkage to Japan, Hong Kong, and the US are proxied by the standardised value of the 12-month average of the sum of the two measures relative to the Thai GDP.

The summary statistics of the raw monthly financial linkage per GDP over the sample period is shown in Panel C of Table 1. Between Japan, Hong Kong, and the US, Thailand has the highest average total financial linkage with Japan of 7.45%, followed by with Hong Kong of 7.06%, and the financial linkage of Thailand to the US has the lowest average of 2.27%. However, comparing the financial linkages of Japan, Hong Kong, and the

US to Thailand, Hong Kong's financial linkage is the highest on average, of 7.05%, and most volatile, and the US's financial linkage is lowest, of below 0.02%.

5.2 Sub-Models

Although our model can handle many countries at once, but given the data limitation, we consider the model with two countries. The dynamics of the fundamental of country i at time $t+1$ is reduced to:

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \Big[\alpha_i^F - \beta_i^F F_{i,t} + \big(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t} \big) \cdot F_{j,t} \Big] + \epsilon_{i,t+1}^F, \tag{6}$$

where $\epsilon_{t+1}^F = [\epsilon_{1,t+1}^F, \epsilon_{2,t+1}^F]'$ is jointly normally distributed and i.i.d. across time.

This paper aims to study the financial contagion through the fundamental, which consists of a total of three effects: the effects of trade and financial linkages on the shock transmission and the shock amplification effect. The following sub-models of equation (6) are fitted to study each effect separately and to study the effects when tested with the other effects.

Model 1: Effect of Trade Linkage on Shock Transmission

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \big(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} \big) \cdot F_{j,t} + \epsilon_{i,t+1}^F, \tag{7}$$

Model 2: Effect of Financial Linkage on Shock Transmission

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \big(\beta_{ij}^{(0)} + \beta_{ij}^{(2)} FL_{ij,t} \big) \cdot F_{j,t} + \epsilon_{i,t+1}^F, \tag{8}$$

Model 3: Effect of Shock Amplification

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \Big[\alpha_i^F - \beta_i^F F_{i,t} + \beta_{ij}^{(0)} F_{j,t} \Big] + \epsilon_{i,t+1}^F, \tag{9}$$

Model 4: Effects of Trade and Financial Linkages on Shock Transmission

$$\Delta F_{i,t+1} = \alpha_i^F - \beta_i^F F_{i,t} + \big(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t} \big) \cdot F_{j,t} + \epsilon_{i,t+1}^F, \tag{10}$$

Model 5: Effects of Trade Linkage on Shock Transmission and Shock Amplification

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \Big[\alpha_i^F - \beta_i^F F_{i,t} + \big(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} \big) \cdot F_{j,t} \Big] + \epsilon_{i,t+1}^F, \tag{11}$$

Model 6: Effects of Financial Linkage on Shock Transmission and Shock Amplification

$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \left[\alpha_i^F - \beta_i^F F_{i,t} + \left(\beta_{ij}^{(0)} + \beta_{ij}^{(2)} FL_{ij,t} \right) \cdot F_{j,t} \right] + \epsilon_{i,t+1}^F, \tag{12}$$

Finally, the last model, **Model 7** is equation (6), which includes all of the three effects.

5.3 Results and Discussion

The estimated parameters of the equations for Thailand in the Japan - Thailand, Hong Kong - Thailand, and the US - Thailand models are shown in Tables 2 - 4. The detailed parameter estimates of the models of each country pair are given in Appendix B.

5.3.1 Role of fundamentals

Consider α_i^R and β_i^R where α_i^R is the constant component of the expected returns, and β_i^R measures the effect of the domestic fundamental on the expected returns of the stock market. Although the estimated values of α_i^R are mostly insignificant, the estimated values of β_i^R are positive in all models across all three countries, implying that the domestic fundamental has a positive impact on returns, as stronger fundamentals (higher $F_{i,t}$) lead to higher expected return of the stock market.

Figure 2 shows the plot of the estimated fundamental of Thailand against the SET returns from January 2006 to December 2017. The Thai fundamental and the SET returns movements, as shown in Figure 2, are consistent with our hypothesis. For instance, between July 2007 and September 2008, which was the time the Thai fundamental was low, was also the time that the returns in the SET declined to its lowest and highly volatile. Then, from September 2013 to December 2017, while the Thai fundamental was improving, the SET returns became more stable.

Figure 2. The Thai Fundamental VS SET Returns: Jan/06 - Dec/17

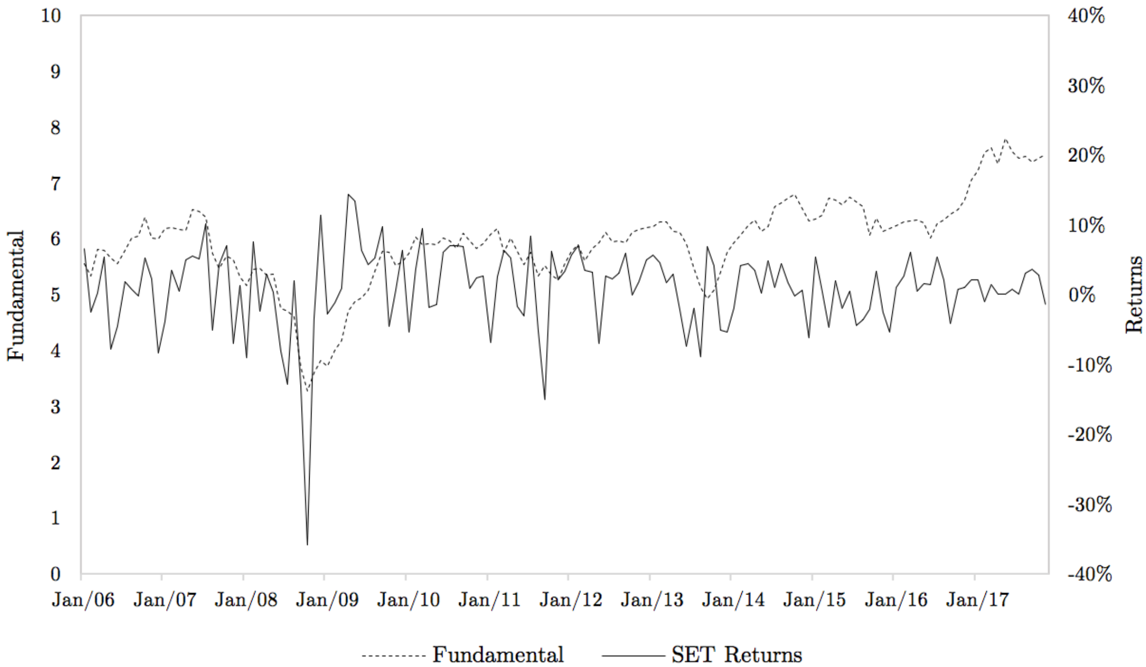


Table 2
Key Posterior Estimates of Japan - Thailand models

This table presents the posterior means of the model parameters of equations for Thailand in Japan - Thailand models and the significance levels, where the values marked with *, ** and *** are statistically significantly different from zero at the 10%, 5% and 1% significance levels, respectively.

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
α_{TH}^R	0.0070	0.0059	0.0038	0.0051	0.0050	0.0018	0.0081
	**	**					
β_{TH}^R	0.1349	0.1333	0.0696	0.0922	0.1032	0.1429	0.1076
	**	**	**	**	**	***	**
α_{TH}^F	1.3611	-1.0820	0.0269	0.6345	0.6341	0.9248	0.1579
	**		***				
β_{TH}^F	0.2014	0.2232	0.1059	0.1904	2.1205	0.9990	1.4107
	***	***	***	***	***	***	***
κ_{TH}	-	-	0.8566	-	0.8875	0.8883	0.9461
			***		***	***	***
$\beta_{TH,JP}^{(0)}$	0.0342	0.4219	0.3930	0.0881	2.6450	1.9622	3.6193
	***	***	***	***	***	***	***
$\beta_{TH,JP}^{(1)}$	-0.0064	-	-	-0.0094	-1.0261	-	-1.5383
	***			***			
$\beta_{TH,JP}^{(2)}$	-	0.0269	-	0.0153	-	3.4193	1.8140
		***		***		**	**

Table 3
Key Posterior Estimates of Hong Kong - Thailand models

This table presents the posterior means of the model parameters of equations for Thailand in the Hong Kong - Thailand model and the significance levels, where the values marked with *, ** and *** are statistically significantly different from zero at the 10%, 5% and 1% significance levels, respectively.

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
α^R_{TH}	0.0071	0.0108	0.0035	0.0079	0.0052	0.0012	0.0072
	**	**	**	**			**
β^R_{TH}	0.1324	0.1105	0.1214	0.1008	0.1322	0.1556	0.1027
	***	***	**	**	**	***	**
α^F_{TH}	0.5058	-0.0284	-0.2146	0.1920	-0.1632	0.1084	0.4839
β^F_{TH}	0.2953	0.2442	0.5613	0.0653	1.7821	0.4556	1.0722
	***	***	***	***	***	****	***
κ_{TH}	-	-	0.8977	-	0.8865	0.9711	0.8790
			***		***	***	***
$\beta^{(0)}_{TH,HK}$	0.2270	0.2542	1.2833	0.0371	2.5558	1.4242	1.5973
	***	***	***	***	***	***	***
$\beta^{(1)}_{TH,HK}$	-0.0042	-	-	-0.1381	-0.4025	-	-0.0753
	**			**			
$\beta^{(2)}_{TH,HK}$	-	0.0035	-	0.1394	-	0.5575	0.0086
		**		***		**	**

5.3.2 Amplification Effects

First we consider the mean reversion property of the fundamentals: α_i^F and β_i^F explain how the domestic fundamental at time t, $F_{i,t}$, affects its own movement at time $t+1$, $\Delta F_{i,t+1}$. According to the model, ignoring all the other terms, the fundamentals would converge back to the mean level α_i^F/β_i^F at the spreed of β_i^F. The estimated values of β_i^F are significantly positive in all models, which conclude that the fundamental processes of all three countries follow a mean-reversion assumption.

The domestic fundamental $F_{i,t}$ also affects its movement $\Delta F_{i,t+1}$ through the amplification effect, and such effect is measured by the parameter κ_i. The estimated values of κ_i of all the models are significantly positive, which reveals the amplification effect. In other words, weak fundamental (low $F_{i,t}$) would make the value $e^{-\kappa F_{i,t}}$ high, and, consequently, amplifies the contagion effect.

Table 4
Key Posterior Estimates of the US - Thailand models

This table presents the posterior means of the model parameters of equations for Thailand in the US - Thailand models and the significance levels, where the values marked with *, ** and *** are statistically significantly different from zero at the 10%, 5% and 1% significance levels, respectively.

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
α^R_{TH}	0.0079	0.0083	0.0077	0.0076	0.0072	0.0039	0.0031
	**	**	**	**	**	***	**
β^R_{TH}	0.0841	0.0776	0.0600	0.0980	0.0888	0.2254	0.1204
	**	**	**	**	**	***	**
α^F_{TH}	-0.2110	1.0937	-0.1511	1.0324	0.3289	0.8760	0.9053
	**	**	**	**	**	***	**
β^F_{TH}	0.1127	0.3240	0.9773	0.3957	1.0093	0.3040	1.3246
	**	**	**	**	**	***	**
κ_{TH}	-	-	0.8979	-	0.8867	0.9716	0.8791
			***		***	***	***
$\beta^{(0)}_{TH,US}$	0.0705	0.1354	1.9371	0.2018	1.5235	3.2153	2.8203
	**	**	**	**	**	***	**
$\beta^{(1)}_{TH,US}$	-0.0766	-	-	-0.0862	-0.6941	-	-0.9561
	**	**	**	**	**	***	**
$\beta^{(2)}_{TH,US}$	-	0.0169	-	0.0224	-	2.1642	0.4711
	**	**	**	**	**	***	**

5.3.3 Roles of financial and trade linkages

Another factor that affects the movements of the fundamental, $\Delta F_{i,t+1}$, is the fundamental of the other country j, $F_{j,t}$, and the degree to which the $F_{j,t}$ affects $\Delta F_{i,t+1}$, in this paper, depends on the level of trade and financial linkages, $TL_{ij,t}$ and $FL_{ij,t}$. The parameters $\beta^{(1)}_{ij}$ and $\beta^{(2)}_{ij}$ measure the role of trade linkage and the role of financial linkage in transmitting the shock from country j to country i, respectively. Finally, the parameter $\beta^{(0)}_i$ measures effect of non-fundamental linkages. If $\beta^{(1)}_{ij} > 0$ and $\beta^{(2)}_{ij} > 0$, it would implies that trade and financial linkages are the transmission channels of shocks. However, $\beta^{(2)}_{ij}$ is the only parameter whose estimated values are positive in all models, while estimated values of $\beta^{(1)}_{ij}$ are negative. Therefore, the financial linkage is a transmission channel of shocks, and the trade linkage is not. The results on the trade linkage effect support earlier findings of Blanchard, Das and Faruqee (2010), Rose and Spiegel (2011), and Berkmen *et al.* (2012) that conclude that trade linkage did not have a significant role in shock transmission in recent financial crises.

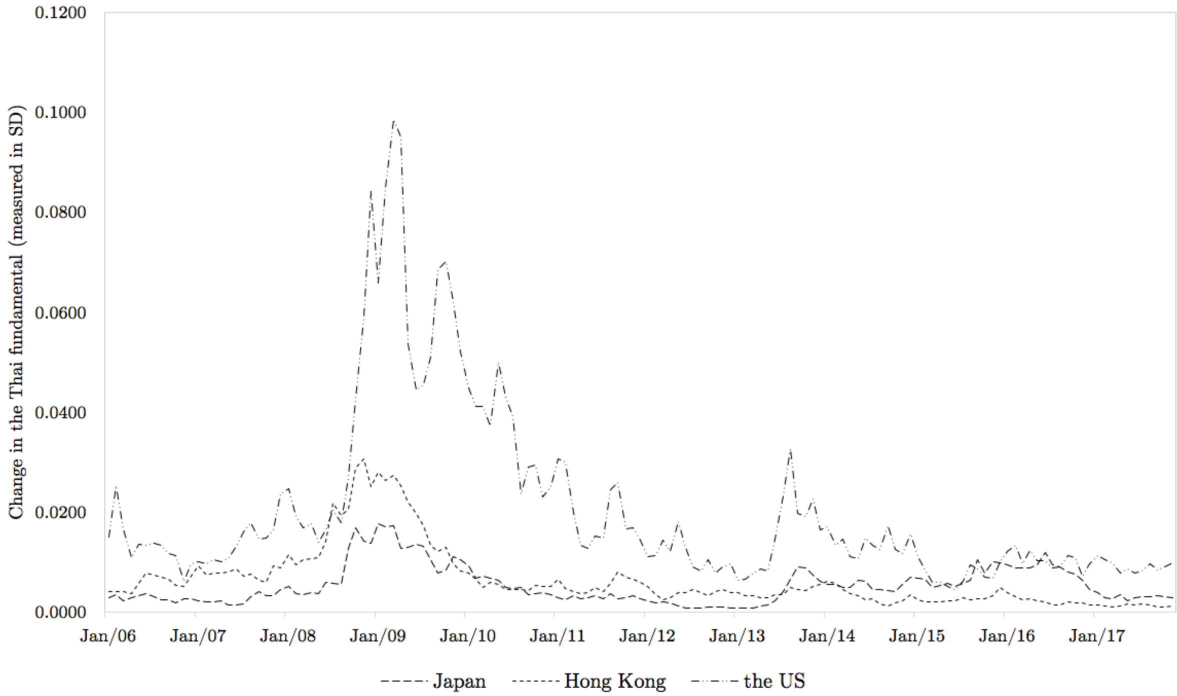
To further investigate the role of the trade linkage, observe that the estimated values of $\beta^{(1)}_{TH,JP}$, $\beta^{(1)}_{TH,HK}$, and $\beta^{(1)}_{TH,US}$ are only significant in Model 1, the model that examines only the trade linkage effect, and Model 4, the model that includes both trade and financial linkages. The estimated values are insignificant in the models with the amplification effect (Model 5 and Model 7). Moreover, the estimated values of $\beta^{(1)}_{TH,JP}$, $\beta^{(1)}_{TH,HK}$, and $\beta^{(1)}_{TH,US}$ that are significant are all negative. Therefore, basing on the traditional definition, trade linkage is not transmission channel of shocks to Thailand. The fact that the estimated values of the $\beta^{(1)}_{TH,JP}$, $\beta^{(1)}_{TH,HK}$, and $\beta^{(1)}_{TH,US}$ are significantly negative in the models without the amplification effect and are insignificant in the models with the amplification effect means that $\beta^{(1)}_{TH,j}$, where $j = JP, HK$, and, US and κ_{TH} somewhat explain similar things. To explain the root of the result, we hypothesize that the trade linkage is an indirect measure of the fundamental. By nature of Thailand's trade activities, Thailand imports raw materials from the countries it exports its final products to. So, if an importing country's fundamental is good, there would be more demand for the Thai exports, as a results, the values of both the Thai exports and imports would be high, which makes the Thai fundamental better as well. Meanwhile, a good fundamental implies low contagion. The opposite is true for the case of bad fundamentals. Thus, when the effect of trade linkage on the contagion is tested by itself, the results reveal a negative relationship between the trade linkage and the contagion. However, such relationship becomes insignificant when the effect of trade linkage is tested along with the amplification effect, and this is because the amplification effect dominates the trade linkage's effect. Therefore, this hypothesis supports the findings.

Unlike other studies that find no effect on shock transmission through trade linkage (Blanchard *et al.*, 2010, Rose and Spiegel, 2011, and Berkmen *et al.*, 2012), by including the amplification effect of the domestic fundamental in the model, this paper can explain the reason for such findings and also the role of the trade linkage in the financial contagion mechanism.

5.3.4 Sensitivity of Thai fundamental and return distribution

There are two ways the Japanese, the Hong Kong, and the US fundamentals affect Thailand. First is the impact on the Thai fundamental, and second is the impact on Thailand's stock market, SET, returns. Each impact is discussed below.

Figure 3. Changes in the Thai Fundamental: Jan/06 - Dec/17



First of all, the levels to which an increase in the fundamentals of Japan, Hong Kong, and the US affects Thai fundamental's movements, $e^{-\kappa_{TH} F_{TH,t}} \cdot \left(\beta^{(0)}_{TH,j} + \beta^{(1)}_{TH,j} TL_{TH,j,t} + \beta^{(2)}_{TH,j} FL_{TH,j,t}\right) \cdot \sigma^F_j / \sigma^F_{TH}$, where $j = JP, HK$, and US from January 2006 to December 2017 is plotted in Figure 3. This measures changes in the Thai fundamental, in terms of standard deviation, when the fundamental of the other country j increases by one standard deviation. Throughout the entire sample period, January 2006 to December 2017, the Thai fundamental was affected by changes in the US fundamental the most. However, from the plot, the US fundamental's effect on the Thai fundamental rose greatly during the 2008 global financial crisis, and gradually declined back to the level close to the effect level of the Japanese and the Hong Kong fundamentals.

Figure 4. Changes in the SET average return: Jan/06 - Dec/17

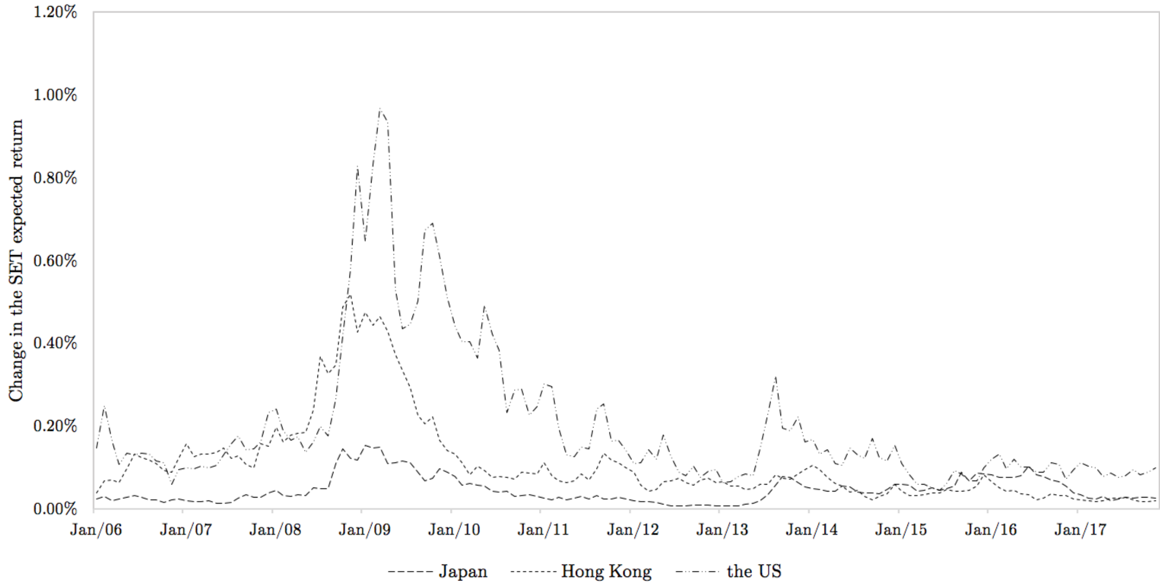
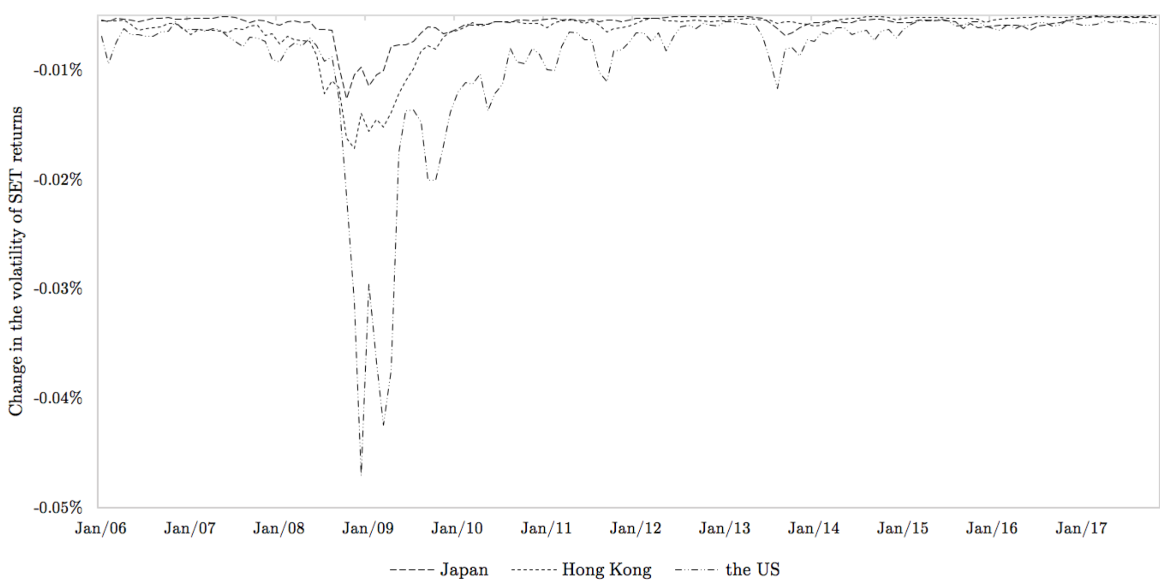


Figure 5. Change in SET returns' volatility: Jan/06 - Dec/17



Secondly, the levels to which an increase in the fundamentals of Japan, Hong Kong, and the US affects the SET average return, which is

$$\beta_{TH}^{R} \cdot e^{-\kappa_{TH} F_{TH,t}} \cdot \left(\beta_{TH,j}^{(0)} + \beta_{TH,j}^{(1)} TL_{TH,j,t} + \beta_{TH,j}^{(2)} FL_{TH,j,t}\right) \cdot \sigma_{j}^{F},$$

and the return volatility, which is

$$-\frac{1}{2} \cdot \sigma_{TH}^{R} \cdot e^{-\frac{1}{2} F_{TH,t}} \cdot e^{-\kappa_{TH} F_{TH,t}} \cdot \left(\beta_{TH,j}^{(0)} + \beta_{TH,j}^{(1)} TL_{TH,j,t} + \beta_{TH,j}^{(2)} FL_{TH,j,t}\right) \cdot \sigma_{j}^{F}$$

are shown in Figure 4 and Figure 5, respectively. These measure the changes in the expected return and volatility of the SET per one standard deviation increase in the fundamental of country j. From the plots, from August 2006 to November 2008, the Hong Kong and the US fundamentals had the greatest impact on the SET return's distribution. However, after the global financial crisis until January 2011, the movements of the SET return's average and volatility were influenced by the US fundamental the most, and the effect of the US fundamental converged down to the level close to the effect level of the Japanese and the Hong Kong fundamentals.

6 Conclusions

This paper develops a framework and a model of the fundamental-based contagion to study about the international stock market contagion. First, this paper assumes that the stock market returns move according to the domestic fundamental, where the average stock market return is higher and the returns are less volatile in countries with good fundamentals than countries with poor fundamental. Then, through the fundamental, this paper examines (1) the transmission mechanism of shock through trade and financial linkages whether larger linkages leads to more co-movement between countries' fundamentals, and (2) the amplification effect of the domestic fundamentals on the contagion, particularly, whether countries with weak fundamentals suffer from external shocks more greatly than the ones with strong fundamentals.

This paper conducts an empirical study on the international stock market contagion between Japan - Thailand, Hong Kong - Thailand and the US - Thailand. Using MCMC estimation technique, the results reveals that the amplification effect of the domestic fundamental is significant in Thailand. Further, it is found that shocks are transmitted to Thailand through only the financial linkage. In sum, all else being equal, when countries are subject to the contagion risk given the financial linkage, it is more likely that countries with better fundamental would be subject to lower contagion risk. Similarly, with low level of linkages, countries may seem safe from contagion risk, but the weak domestic economy of the countries may trigger huge losses to them. One of the advantages of including the effects of shock transmission through trade and financial linkages and the shock amplification effect in the model is that, while the results of this paper show no evidence of shock transmission through trade linkage, which is consistent with the findings of recent researches, this paper can give an explanation for the results as trade linkage is an indirect measure of the fundamental.

Finally, this paper finds that the effect levels of the fundamentals of Japan, Hong Kong, and the US on Thailand's fundamental and stock market return distribution vary over time. The country whose fundamental Thailand's fundamental and stock market returns' distribution was affected by the most was the US. But the effect of the changes in the US fundamental on the Thai fundamental and stock market returns became more pronounced during the 2008 global financial crisis, and gradually declined to the level close to the effect level of the Japanese and the Hong Kong fundamentals.

The findings of this research have several implications. First, the possibility of financial contagion could eliminate the benefits of international portfolio diversification. Knowing which countries shock would transmit to Thailand the most and least can be useful for their risk management and diversification strategies. Second, preventing financial contagion is one of the key objectives of policy makers. Knowing that the health of the economy plays an important role the degree to which the country affects from external shocks, and those shocks are transmitted through financial linkage and not trade linkage would help the policy makers in various ways.

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Appendix A: Details of MCMC Algorithm

A1. Dynamics

Recall the return and fundamental equations (1) and (2):

$$R_{i,t+1} = \alpha_i^R + \beta_i^R F_{i,t} + e^{-\frac{1}{2}F_{i,t}} \epsilon_{i,t+1}^R$$
$$\Delta F_{i,t+1} = e^{-\kappa_i F_{i,t}} \left[\alpha_i^F - \beta_i^F F_{i,t} + \sum_{j \neq i}^{n} \left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t} \right) \cdot F_{j,t} \right] + \epsilon_{i,t+1}^F$$

for $i = 1, \dots, n$ and $t = 0, \dots, T-1$. Now let

$$R_t = \begin{bmatrix} R_{1,t} \\ \vdots \\ R_{n,t} \end{bmatrix}, \quad F_t = \begin{bmatrix} F_{1,t} \\ \vdots \\ F_{n,t} \end{bmatrix}, \quad D_t = \begin{bmatrix} R_t \\ F_t \end{bmatrix}, \quad \epsilon_t^R = \begin{bmatrix} \epsilon_{1,t}^R \\ \vdots \\ \epsilon_{n,t}^R \end{bmatrix}, \quad \epsilon_t^F = \begin{bmatrix} \epsilon_{1,t}^F \\ \vdots \\ \epsilon_{n,t}^F \end{bmatrix}$$

and let

$$\alpha^R = \begin{bmatrix} \alpha_1^R \\ \vdots \\ \alpha_n^R \end{bmatrix}, \quad \beta^R = \begin{bmatrix} \beta_1^R \\ \vdots \\ \beta_n^R \end{bmatrix}, \quad \alpha^F = \begin{bmatrix} \alpha_1^F \\ \vdots \\ \alpha_n^F \end{bmatrix},$$

$$\beta_v^{F0} = vec\left(\begin{bmatrix} -\beta_1^F & \beta_{12}^{(0)} & \cdots & \beta_{1n}^{(0)} \\ \beta_{21}^{(0)} & -\beta_2^F & \cdots & \beta_{2n}^{(0)} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n1}^{(0)} & \beta_{n2}^{(0)} & \cdots & \beta_n^F \end{bmatrix}' \right),$$

$$\beta_v^{12} = vec\left(\begin{bmatrix} \beta_{12}^{(1)} & \beta_{13}^{(1)} & \cdots & \beta_{1n}^{(1)} & \beta_{12}^{(2)} & \beta_{13}^{(2)} & \cdots & \beta_{1n}^{(2)} \\ \beta_{21}^{(1)} & \beta_{23}^{(1)} & \cdots & \beta_{2n}^{(1)} & \beta_{21}^{(2)} & \beta_{23}^{(2)} & \cdots & \beta_{2n}^{(2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \beta_{n1}^{(1)} & \beta_{n2}^{(1)} & \cdots & \beta_{nn-1}^{(1)} & \beta_{n1}^{(2)} & \beta_{n2}^{(2)} & \cdots & \beta_{nn-1}^{(2)} \end{bmatrix}' \right).$$

We can rewrite the return and fundamental equations for all countries in the following matrix form:

$$D_{t+1} = A_t + B_t\eta + \epsilon^D_{t+1}, \quad t = 0, \ldots, T-1$$

where

$$A_t = \begin{bmatrix} 0 \\ F_t \end{bmatrix}, \quad B_t = \begin{bmatrix} B_{1,t} & B_{2,t} & B_{3,t} & B_{4,t} & B_{5,t} \end{bmatrix}, \quad \eta = \begin{bmatrix} \alpha^R \\ \beta^R \\ \alpha^F \\ \beta_v^{F0} \\ \beta_v^{12} \end{bmatrix}, \quad \epsilon^D_{t+1} = (\Sigma^D_t)^{1/2} \begin{bmatrix} \epsilon^R_{t+1} \\ \epsilon^F_{t+1} \end{bmatrix}.$$

To provide the detail of the notations above, we first define

$$\begin{aligned}
diag(TL_{i\cdot,t}) &= diag(TL_{i1,t}, \ldots, TL_{in,j}) \\
diag(FL_{i\cdot,t}) &= diag(FL_{i1,t}, \ldots, FL_{in,j}) \\
diag(F_t) &= diag(F_{1,t}, \ldots, F_{n,t}) \\
diag(e^{-\kappa F_t}) &= diag(e^{-\kappa_1 F_{1,t}}, \ldots, e^{-\kappa_n F_{n,t}}) \\
diag(e^{-F_t/2}) &= diag(e^{-F_{1,t}/2}, \ldots, e^{-F_{n,t}/2}) \\
F_{\sim i,t} &= [F_{1,t}, \ldots, F_{i-1,t}, F_{i+1,t}, \ldots, F_{n,t}]' \\
\mathbb{L}_{i,t} &= [diag(TL_{i\cdot,t}) \; diag(FL_{i\cdot,t})]
\end{aligned}$$

and

$$\mathbb{F}_t = \begin{bmatrix} F_t' & 0 & \cdots & 0 \\ 0 & F_t' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F_t' \end{bmatrix}.$$

We have

$$B_{1,t} = \begin{bmatrix} I_{n\times n} \\ 0_{n\times n} \end{bmatrix}, \quad B_{2,t} = \begin{bmatrix} diag(F_t) \\ 0_{n\times n} \end{bmatrix}, \quad B_{3,t} = \begin{bmatrix} 0_{n\times n} \\ diag(e^{-\kappa F_t}) \end{bmatrix}, \quad B_{4,t} = \begin{bmatrix} 0_{n\times n^2} \\ diag(e^{-\kappa F_t}) \times \mathbb{F}_t \end{bmatrix}$$

$$B_{5,t} = diag(e^{-\kappa F_t}) \begin{bmatrix} F'_{\sim 1,t}\mathbb{L}_{1,t} & 0 & \cdots & 0 \\ 0 & F'_{\sim 2,t}\mathbb{L}_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F'_{\sim n,t}\mathbb{L}_{n,t} \end{bmatrix}$$

and

$$\Sigma_t^D = \begin{bmatrix} diag(e^{-F_t/2}) & 0_{n\times n} \\ 0_{n\times n} & I_{n\times n} \end{bmatrix} \Sigma^{RF} \begin{bmatrix} diag(e^{-F_t/2}) & 0_{n\times n} \\ 0_{n\times n} & I_{n\times n} \end{bmatrix}.$$

We re-parameterize Σ^{RF} as follows:

$$\Sigma^{RF} = \begin{bmatrix} C\Sigma^F C' + \Omega & C\Sigma^F \\ \Sigma^F C' & \Sigma^F \end{bmatrix}$$

where C and Ω are $n \times n$ matrices, and Ω is positive definite.

A2. Likelihood Function

Given the data $L = \{TL_{i,t}, FL_{i,t}, i = 1, \ldots, n,\ t = 0, \ldots, T-1\}$ and parameter set φ, we obtain the likelihood function of $D = (R, F) = (R_{i,t}, F_{i,s}, i = 1, \ldots, n,\ t = 1, \ldots, T,\ s = 0, \ldots, T\}$ as follows:

$$P(D|\varphi, L) \propto P(F_0|\varphi, L) \prod_{t=1}^{T} P(D_t|F_{t-1}, L_{t-1}, \varphi).$$

Assuming the diffuse prior for F_0:

$$P(F_0|\varphi, L) \propto 1,$$

we have

$$P(D|\varphi, L) \propto e^{-\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n} F_{i,t-1}} |\Sigma^F|^{-T/2} |\Omega|^{-T/2} e^{-\frac{1}{2}\sum_{t=1}^{T}(D_t - A_{t-1} - B_{t-1}\eta)'(\Sigma_{t-1}^{D})^{-1/2}(D_t - A_{t-1} - B_{t-1}\eta)}.$$

A3. Priors

We assume the following prior distributions:

$$
\begin{aligned}
\eta &\sim N(\mu_\eta, \Sigma_\eta) \\
\kappa &\sim N(\mu_\kappa, \Sigma_\kappa) \\
C_v|\Omega &\sim N(\mu_{C_v}, \Sigma_{C_v} \otimes \Omega) \\
\Sigma^F &\sim IW(\nu_{\Sigma^F}, \Gamma_{\Sigma^F}) \\
\Omega &\sim IW(\nu_\Omega, \Gamma_\Omega)
\end{aligned}
$$

where

$$\kappa = [\kappa_1, \ldots, \kappa_n]', \quad C_v = vec(C).$$

A4. Posteriors and Gibb's

Let

$$\hat{\epsilon}_t^R = [\hat{\epsilon}_{1,t}^R, \ldots, \hat{\epsilon}_{n,t}^R]', \quad \hat{\epsilon}_t^F = [\hat{\epsilon}_{1,t}^F, \ldots, \hat{\epsilon}_{n,t}^F]'$$

where

$$
\begin{aligned}
&\hat{\epsilon}_{i,t}^R = e^{\frac{1}{2}F_{i,t}}(R_{i,t+1} - \alpha_i^R - \beta_i^R F_{i,t}), \\
&\hat{\epsilon}_{i,t}^F = F_{i,t+1} - F_{i,t} - e^{-\kappa_i F_{i,t}} \left[\alpha_i^F - \beta_i^F F_{i,t} + \sum_{j \neq i}^{n} \left(\beta_{ij}^{(0)} + \beta_{ij}^{(1)} TL_{ij,t} + \beta_{ij}^{(2)} FL_{ij,t} \right) \cdot F_{j,t} \right].
\end{aligned}
$$

We perform the Gibb's sampling based on the following posterior distributions:

Posterior of η

$$\eta|\varphi_{\sim\eta}, R, F, L \sim N(\hat{\mu}_\eta, \hat{\Sigma}_\eta)$$

where

$$
\begin{aligned}
\hat{\Sigma}_\eta &= \left(\Sigma_\eta^{-1} + \sum_{t=1}^{T} B_{t-1}' (\Sigma_{t-1}^D)^{-1} B_{t-1} \right)^{-1} \\
\hat{\mu}_\eta &= \hat{\Sigma}_\eta \left[\Sigma_\eta^{-1} \mu_\eta + \sum_{t=1}^{T} B_{t-1}' (\Sigma_{t-1}^D)^{-1} (D_t - A_{t-1}) \right],
\end{aligned}
$$

Posterior of C_v

$$C_v|\varphi_{\sim C_v}, R, F, L \sim N(\hat{\mu}_{C_v}, \hat{\Sigma}_{C_v})$$

where

$$\hat{\Sigma}_{C_v} = (\Sigma_{C_v}^{-1} + \sum_{t=1}^{T} \hat{\epsilon}_t^F \hat{\epsilon}_t^{F\prime})^{-1} \otimes \Omega$$
$$\hat{\mu}_{C_v} = \hat{\Sigma}_{C_v} vec\left(\Omega^{-1}\left[\mu_C \Sigma_{C_v}^{-1} + \sigma_{t=1}^{T} \hat{\epsilon}_t^R \hat{\epsilon}_t^{F\prime}\right]\right).$$

Posterior of Σ^F

$$\Sigma^F|\varphi_{\sim\Sigma^F}, R, F, L \sim IW(\hat{\nu}_{\Sigma^F}, \hat{\Gamma}_{\Sigma^F})$$

where

$$\hat{\nu}_{\Sigma^F} = \nu_{\Sigma^F} + T$$
$$\hat{\Gamma}_{\Sigma^F} = \Gamma_{\Sigma^F} + \sum_{t=1}^{T} \hat{\epsilon}_t^F \hat{\epsilon}_t^{F\prime}.$$

Posterior of Ω

$$\Omega|\varphi_{\sim\Omega}, R, F, L \sim IW(\hat{\nu}_{\Omega}, \hat{\Gamma}_{\Omega})$$

where

$$\hat{\nu}_{\Omega} = \nu_{\Omega} + T + n$$
$$\hat{\Gamma}_{\Omega} = \Gamma_{\Omega} + (C - \mu_C)\Sigma_{C_v}^{-1}(C - \mu_C)' + \sum_{t=1}^{T}(\hat{\epsilon}_t^R - C\hat{\epsilon}_t^F)(\hat{\epsilon}_t^R - C\hat{\epsilon}_t^F)'$$

with

$$\mu_{C_v} = vec(\mu_C).$$

A A5. Metropolis-Hasting

We perform the Metropolis-Hasting steps for κ and F_t as follows:

MH for κ

Given the current value of κ, we use the independent proposal distribution to draw κ^*

$$\kappa^* \sim U(-1, 1)$$

and accept it with probability

$$\rho(\kappa^*, \kappa) = \min\left\{1, \frac{h(\kappa^*)}{h(\kappa)}\right\}$$

where

$$\begin{aligned} h(x) = \exp\Bigg\{ & -\frac{1}{2}(x-\mu_\kappa)'\Sigma_\kappa^{-1}(x-\mu_\kappa)' \\ & -\frac{1}{2}\sum_{t=1}^{T}(D_t - A_{t-1} - B_{t-1}(x)\eta)'(\Sigma_{t-1}^D)^{-1}((D_t - A_{t-1} - B_{t-1}(x)\eta)\Bigg\} \end{aligned}$$

and $B_{t-1}(x)$ is B_{t-1} with κ being replaced by x.

MH for F_0

Given the current value of F_0, we use the random walk proposal distribution to draw F_0^*

$$F_0^* = F_0 + u, \quad u \sim N(0, \Sigma_u)$$

and accept it with probability

$$\rho_0(F_0^*, F_0) = \min\left\{1, \frac{h_0(F_0^*)}{h_0(F_0)}\right\}$$

where

$$h_0(x) = \exp\left\{\frac{1}{2}\sum_{i=1}^{n}x_i - \frac{1}{2}(D_1 - A_0(x) - B_0(x)\eta)'(\Sigma_0^D(x))^{-1}(D_1 - A_0(x) - B_0(x)\eta)\right\}$$

and $A_0(x), B_0(x)$ and $\Sigma_0^D(x)$ are A_0, B_0 and Σ_0^D with F_0 being replaced by x.

MH for F_T

Given the current value of F_T, we use the random walk proposal distribution to draw F_T^*

$$F_T^* = F_T + u, \quad u \sim N(0, \Sigma_u)$$

and accept it with probability

$$\rho_T(F_T^*, F_T) = \min\left\{1, \frac{h_T(F_T^*)}{h_T(F_T)}\right\}$$

where

$$h_T(x) = \exp\left\{-\frac{1}{2}(D_T(x) - A_{T-1} - B_{T-1}\eta)'(\Sigma_{T-1}^D)^{-1}(D_T(x) - A_{T-1} - B_{T-1}\eta)\right\}$$

and $D_T(x)$ is D_T with F_T being replaced by x.

MH for $F_t, t = 1, \ldots, T-1$

Given the current value of F_t, we use the random walk proposal distribution to draw F_t^*

$$F_t^* = F_t + u, \quad u \sim N(0, \Sigma_u)$$

and accept it with probability

$$\rho_t(F_t^*, F_t) = \min\left\{1, \frac{h_t(F_t^*)}{h_t(F_t)}\right\}$$

where

$$\begin{aligned} h_t(x) = \exp\Bigg\{ & \frac{1}{2}\sum_{i=1}^{n} x_i - \frac{1}{2}(D_{t+1} - A_t(x) - B_t(x)\eta)'(\Sigma_t^D(x))^{-1}(D_{t+1} - A_t(x) - B_t(x)\eta) \\ & -\frac{1}{2}(D_t(x) - A_{t-1} - B_{t-1}\eta)'(\Sigma_{t-1}^D)^{-1}(D_t(x) - A_{t-1} - B_{t-1}\eta)\Bigg\} \end{aligned}$$

and $A_t(x), B_t(x), \Sigma_t^D(x)$ and $D_t(x)$ are A_t, B_t, Σ_t^D and D_t with F_t being replaced by x.

Appendix B: Posterior Means

This section provides the posterior means of the model parameters and their standard errors (in parentheses).

Table 5
All Posterior Estimates of Japan - Thailand Models

Parameter	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6		Model 7	
	JP	TH	JP	TH	JP	TH	JP	TH	JP	TH	JP	TH	JP	TH
α_i^R	0.0032	0.0070	0.0038	0.0059	0.0033	0.0038	0.0044	0.0051	0.0042	0.0050	-0.0024	0.0018	0.0023	0.0081
	(0.0042)	(0.0043)	(0.0034)	(0.0027)	(0.0043)	(0.0032)	(0.00049)	(0.0055)	(0.0044)	(0.0053)	(0.0035)	(0.0020)	(0.0029)	(0.0078)
β_i^R	0.1853	0.1349	0.1014	0.1333	0.0746	0.0696	0.0953	0.0922	0.0873	0.1032	0.1371	0.1429	0.1337	0.1076
	(0.1031)	(0.0680)	(0.0523)	(0.0672)	(0.0325)	(0.0294)	(0.0431)	(0.0416)	(0.0387)	(0.0519)	(0.0801)	(0.0874)	(0.0677)	(0.0545)
$\mathrm{var}(\epsilon_i^R)$	0.7388	0.8590	0.7435	0.8921	0.6991	0.7384	0.6748	0.6993	0.7212	0.8430	0.7111	0.8993	0.6989	0.7918
	(0.2153)	(0.3281)	(0.2406)	(0.3313)	(0.1886)	(0.2649)	(0.1863)	(0.1989)	(0.2101)	(0.2834)	(0.0208)	(0.2911)	(0.1920)	(0.2246)
$\mathrm{cov}(\epsilon_i^R, \epsilon_j^R)$	0.3194		0.3530		0.2899		0.3145		0.3492		0.3261		0.3523	
	(0.1342)		(0.1151)		(0.1471)		(0.1339)		(0.1520)		(0.1366)		(0.1439)	
α_i^F	6.0734	1.3611	4.9092	-1.0820	2.9781	0.0269	4.2261	0.6345	7.6213	0.6341	9.6752	0.9248	2.1583	0.1579
	(1.2104)	(0.6341)	(1.3935)	(0.8862)	(0.9224)	(0.0103)	(1.2945)	(0.6211)	(2.7194)	(0.6883)	(3.7701)	(0.9088)	(0.7641)	(0.1218)
β_i^F	1.4028	0.2014	1.0184	0.2232	0.6441	0.1059	1.0070	0.1904	1.6871	2.1205	3.8861	0.9990	0.5385	1.4107
	(0.2832)	(0.0731)	(0.2855)	(0.0780)	(0.2119)	(0.0398)	(0.2024)	(0.0653)	(0.6414)	(0.7957)	(1.2061)	(0.3615)	(0.1932)	(0.2756)
κ_i	-	-	-	-	0.0938	0.8566	-	-	0.1402	0.8875	0.1429	0.8883	0.1403	0.9461
					(0.0252)	(0.3374)			(0.0513)	(0.3509)	(0.0521)	(0.3411)	(0.0498)	(0.3527)
$\beta_{ij}^{(0)}$	0.3276	0.0342	0.1573	0.4219	0.1235	0.3930	0.2724	0.0881	0.3487	2.6450	1.1328	1.9622	0.1574	3.6193
	(0.0718)	(0.0093)	(0.0670)	(0.1423)	(0.0643)	(0.1152)	(0.0931)	(0.0318)	(0.1144)	(0.9866)	(0.6419)	(0.6622)	(0.0038)	(0.9936)
$\beta_{ij}^{(1)}$	-0.0136	-0.0064	-	-	-	-	-0.0158	-0.0094	-0.0264	-1.0261	-	-	-0.0010	-1.5383
	(0.0031)	(0.0011)					(0.0050)	(0.0023)	(0.0458)	(1.1093)			(0.0045)	(1.5564)
$\beta_{ij}^{(2)}$	-	-	-0.0052	0.0269	-	-	-0.0011	0.0153	-	-	-0.0700	3.4193	0.0074	1.8140
			(0.0060)	(0.0089)			(0.0018)	(0.0021)			(0.1194)	(1.0084)	(0.0013)	(0.8228)
$\mathrm{var}(\epsilon_i^F)$	0.0731	0.0483	0.0528	0.0489	0.0976	0.0792	0.1419	0.0806	0.0668	0.0616	0.0528	0.0486	0.0519	0.0284
	(0.0225)	(0.0089)	(0.0208)	(0.0108)	(0.0853)	(0.0429)	(0.0921)	(0.0758)	(0.0322)	(0.0301)	(0.0343)	(0.0288)	(0.0267)	(0.0220)
$\mathrm{cov}(\epsilon_i^F, \epsilon_j^F)$	0.0349		0.0103		0.0365		0.0294		0.0134		0.0327		0.0284	
	(0.0191)		(0.0082)		(0.0244)		(0.0152)		(0.0097)		(0.0201)		(0.0146)	
$\mathrm{cov}(\epsilon_i^R, \epsilon_i^F)$	0.1790	0.2465	0.0693	0.2260	0.1786	0.2246	0.3245	0.2091	0.1662	0.2132	0.1015	0.1456	0.0871	0.1410
	(0.0623)	(0.0938)	(0.0597)	(0.0871)	(0.0622)	(0.0906)	(0.1514)	(0.1103)	(0.0588)	(0.1491)	(0.0872)	(0.0843)	(0.0549)	(0.0821)

Table 6
All Posterior Estimates of Hong Kong - Thailand Models

Parameter	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6		Model 7	
	HK	TH	HK	TH	HK	TH	HK	TH	HK	TH	HK	TH	HK	TH
α_i^R	0.0112	0.0071	0.0135	0.0108	0.0042	0.0035	0.0113	0.0079	0.0074	0.0052	-0.0010	0.0012	0.0034	0.0072
	(0.0044)	(0.0012)	(0.0148)	(0.0113)	(0.0050)	(0.0032)	(0.0097)	(0.0072)	(0.0074)	(0.0055)	(0.0009)	(0.0024)	(0.0028)	(0.0067)
β_i^R	0.1138	0.1324	0.0848	0.1105	0.1146	0.1214	0.0511	0.1008	0.0980	0.1322	0.1263	0.1556	0.1190	0.1027
	(0.0791)	(0.0582)	(0.0334)	(0.0420)	(0.0466)	(0.0391)	(0.0188)	(0.0550)	(0.0219)	(0.0585)	(0.0497)	(0.0631)	(0.0410)	(0.0376)
$\mathrm{var}(\epsilon_i^R)$	0.8815	0.8837	0.8511	0.9740	0.8154	0.4222	0.6748	0.6993	0.7212	0.8430	0.7111	0.8993	0.6989	0.7918
	(0.3459)	(0.3501)	(0.2209)	(0.3812)	(0.3991)	(0.2005)	(0.1954)	(0.2107)	(0.2682)	(0.3091)	(0.2380)	(0.2907)	(0.3195)	(0.3896)
$\mathrm{cov}(\epsilon_i^R, \epsilon_j^R)$	0.4495		0.4617		0.7623		0.4501		0.3922		0.4910		0.7493	
	(0.1948)		(0.2113)		(0.3852)		(0.1082)		(0.1920)		(0.2106)		(0.3259)	
α_i^F	0.6321	0.5058	0.9833	-0.0284	1.1759	-0.2146	0.0772	0.1920	0.5974	-0.1632	0.3022	0.1084	-0.0180	0.4839
	(0.0244)	(0.5271)	(0.8994)	(0.0312)	(0.9592)	(0.3320)	(0.0818)	(0.1922)	(0.5814)	(0.2984)	(0.2887)	(0.9918)	(0.017)	(0.5202)
β_i^F	0.2471	0.2953	0.3970	0.2442	1.5341	0.5613	0.3917	0.0653	0.4271	1.7821	1.4650	0.4556	4.0531	1.0722
	(0.1118)	(0.1309)	(0.1828)	(0.1031)	(0.6240)	(0.0032)	(0.1088)	(0.0305)	(0.1924)	(0.5453)	(0.4028)	(0.1210)	(1.7315)	(0.455)
κ_i	-	-	-	-	0.8650	0.8977	-	-	0.9803	0.8865	0.9072	0.9711	0.6743	0.8790
					(0.0726)	(0.0705)			(0.0329)	(0.0262)	(0.0228)	(0.0091)	(0.0214)	(0.0138)
$\beta_{ij}^{(0)}$	0.1304	0.2270	0.2218	0.2542	2.0834	1.2833	0.3670	0.0371	2.0113	2.5558	1.9773	1.4242	3.8581	1.5973
	(0.0551)	(0.1080)	(0.1003)	(0.1209)	(0.9188)	(0.5402)	(0.1755)	(0.0099)	(0.8838)	(1.0301)	(0.9530)	(0.6033)	(1.7582)	(0.8243)
$\beta_{ij}^{(1)}$	0.0008	-0.0042	-	-	-	-	-0.0616	-0.1381	-0.1952	-0.4025	-	-	0.2901	-0.0753
	(0.0009)	(0.0011)					(0.0583)	(0.6212)	(0.1990)	(0.3809)			(0.3174)	(0.0782)
$\beta_{ij}^{(2)}$	-	-	0.0101	0.0035	-	-	-0.0427	0.1394	-	-	0.3224	0.5575	0.0822	0.0086
			(0.0089)	(0.0012)			(0.0130)	(0.0572)			(0.1162)	(0.1884)	(0.0319)	(0.0035)
$\mathrm{var}(\epsilon_i^F)$	0.0515	0.0660	0.0866	0.0452	0.0716	0.0703	0.1095	0.1286	0.0840	0.0336	0.0548	0.1036	0.0503	0.0624
	(0.0219)	(0.0248)	(0.0385)	(0.0114)	(0.0182)	(0.0155)	(0.0461)	(0.0549)	(0.0350)	(0.0097)	(0.0183)	(0.0612)	(0.0258)	(0.0279)
$\mathrm{cov}(\epsilon_i^F, \epsilon_j^F)$	0.0241		0.0311		0.0247		0.0303		0.0341		0.0307		0.0295	
	(0.0183)		(0.0125)		(0.0104)		(0.0180)		(0.0200)		(0.0148)		(0.0159)	
$\mathrm{cov}(\epsilon_i^R, \epsilon_i^F)$	0.2774	0.2089	0.1309	0.2647	0.1634	0.2474	0.0983	0.1108	0.1072	0.2930	0.1259	0.1364	0.0929	0.1127
	(0.1813)	(0.1309)	(0.0445)	(0.1087)	(0.0782)	(0.1040)	(0.0391)	(0.0402)	(0.0311)	(0.0891)	(0.0562)	(0.0598)	(0.0224)	(0.0350)

Table 7
All Posterior Estimates of the US - Thailand Models

Parameter	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6		Model 7	
	US	TH	US	TH	US	TH	US	TH	US	TH	US	TH	US	TH
α_i^R	0.0080	0.0079	0.1012	0.0083	0.0104	0.0077	0.0098	0.0076	0.0086	0.0072	0.0063	0.0039	0.0077	0.0031
	(0.0044)	(0.0041)	(0.0713)	(0.0042)	(0.0073)	(0.0032)	(0.0059)	(0.0050)	(0.0044)	(0.0040)	(0.0047)	(0.0042)	(0.0039)	(0.0033)
β_i^R	0.0679	0.0841	0.0512	0.0776	0.0111	0.0600	0.0215	0.0980	0.0099	0.0888	0.3110	0.2254	0.0663	0.1204
	(0.0203)	(0.0277)	(0.0184)	(0.0199)	(0.0041)	(0.0328)	(0.0141)	(0.0257)	(0.0028)	(0.0431)	(0.0990)	(0.0922)	(0.0015)	(0.0449)
$\text{var}(\epsilon_i^R)$	0.7388	0.8590	0.9564	0.7847	0.9020	0.7811	0.7809	0.6990	0.7712	0.9205	0.8597	0.8041	0.8138	0.7821
	(0.2910)	(0.3116)	(0.3419)	(0.3982)	(0.3909)	(0.1196)	(0.1308)	(0.1852)	(0.2983)	(0.9192)	(0.4703)	(0.4439)	(0.4122)	(0.3984)
$\text{cov}(\epsilon_i^R, \epsilon_j^R)$	0.4491		0.4124		0.3297		0.4295		0.4810		0.4422		0.3986	
	(0.2078)		(0.1653)		(0.2712)		(0.2583)		(0.2291)		(0.3122)		(0.2105)	
α_i^F	0.8003	-0.2110	0.1008	1.0937	1.1463	-0.1511	0.6893	1.0324	0.2201	0.3289	0.0681	0.8760	0.2327	0.9053
	(0.8924)	(0.2109)	(0.2866)	(0.7660)	(1.4091)	(0.3223)	(0.2180)	(0.4408)	(0.0190)	(0.3692)	(0.0613)	(0.8299)	(0.3712)	(0.9061)
β_i^F	0.2951	0.1127	0.1429	0.3240	3.3111	0.9773	0.4106	0.3957	0.6744	1.0093	0.1431	0.3040	0.9043	1.3246
	(0.0938)	(0.0415)	(0.0611)	(0.0830)	(1.0488)	(0.3229)	(0.1606)	(0.1721)	(0.2110)	(0.5824)	(0.0893)	(0.2001)	(0.3819)	(0.5422)
κ_i	-	-	-	-	0.8651	0.8979	-	-	0.9804	0.8867	0.9072	0.9716	0.6748	0.8791
					(0.0229)	(0.0300)			(0.0296)	(0.0213)	(0.0234)	(0.0213)	(0.0271)	(0.0473)
$\beta_{ij}^{(0)}$	0.1934	0.0705	0.1362	0.1354	3.3040	1.9371	0.3433	0.2018	1.7090	1.5235	3.8259	3.2153	2.5141	2.8203
	(0.0753)	(0.0119)	(0.0540)	(0.0533)	(0.9952)	(0.8737)	(0.0821)	(0.0696)	(0.7388)	(0.6640)	(1.0356)	(1.0112)	(0.9981)	(0.9993)
$\beta_{ij}^{(1)}$	0.0066	-0.0766	-	-	-	-	0.0034	-0.0862	-3.3054	-0.6941	-	-	-2.0658	-0.9561
	(0.0181)	(0.0139)					(0.0928)	(0.0331)	(3.3050)	(0.7301)			(2.2933)	(0.9894)
$\beta_{ij}^{(2)}$	-	-	0.0002	0.0169	-	-	0.0031	0.0224	-	-	1.5199	2.1642	1.5686	0.4711
			(0.0002)	(0.0066)			(0.0031)	(0.0125)			(1.6106)	(1.7110)	(1.6890)	(0.2881)
$\text{var}(\epsilon_i^F)$	0.0770	0.1279	0.0565	0.0584	0.0672	0.0831	0.0592	0.0812	0.1103	0.1228	0.0680	0.0744	0.0891	0.0925
	(0.0392)	(0.0504)	(0.0410)	(0.0472)	(0.0418)	(0.0446)	(0.0192)	(0.0505)	(0.0492)	(0.0568)	(0.0294)	(0.0409)	(0.0301)	(0.0318)
$\text{cov}(\epsilon_i^F, \epsilon_j^F)$	0.0534		0.0348		0.0365		0.0294		0.0134		0.0327		0.0284	
	(0.0191)		(0.0077)		(0.0244)		(0.0152)		(0.0097)		(0.0201)		(0.0146)	
$\text{cov}(\epsilon_i^R, \epsilon_i^F)$	0.1560	0.3572	0.3848	0.4021	0.3413	0.2139	0.4512	0.5911	0.3010	0.3219	0.1835	0.2004	0.3416	0.3294
	(0.0881)	(0.1209)	(0.2204)	(0.1941)	(0.1239)	(0.0544)	(0.1749)	(0.2840)	(0.1793)	(0.1980)	(0.1021)	(0.1629)	(0.1984)	(0.1402)

Potential Buyers and Fire Sales in Financial Networks*

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Abstract

We model contagion in financial networks using an equilibrium approach. Banks in the networks are risk averse and optimize loan holdings. Contagion can occur through the interbank liabilities, and asset prices. Banks in the system are allowed to act as potential buyers buying illiquid loans from troubled banks. We find that banks with low risk aversion, the *aggressive banks*, can be helpful as good potential buyers and harmful as risk amplifiers. The system is subject to greater risk when aggressive banks hold claims between each other, and when they hold high risk loans as their prices are sensitive to shocks. When banks and loan markets are separated into non-overlapping sectors based on their areas of expertise defined by the cost of managing different types of loans, a shock in one sector is not transmitted to another sector if there are not liability linkages and the cost is sufficiently high. We use this observation to suggest a policy that separates banks during good times to limit unexpected contagion, and allows them to act as *secondary* potential buyers to save their peers during bad times, creating a self-rescue system that puts no burden on the taxpayers.

Key words– potential buyers, fire sales, contagion, illiquidity, banking network.

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1 Introduction

Asset fire sales can be a major cause of a financial crisis. When banks are hit by unexpected shocks, either directly or indirectly through contagion channels, banks can become insolvent and need to liquidate all of their assets, the major of which are illiquid loans. Banks that survive but experience losses need to sell some of their assets to pay for the losses. Because in general banks' liquid assets are low-risk assets such as cash equivalents and banks' illiquid assets are high-risk assets such as loans, risk-averse banks do not sell only liquid assets, but also part of their illiquid assets to re-optimize the risk-adjusted return of their portfolios after the shocks.[1] How much banks sell off their illiquid loans depends on the banks' attitude toward risk.[2] If there are few potential buyers who are willing to pay for the assets, the fire sale prices can be much worse (Shleifer and Vishny, 2011). The system that tends to keep some potential buyers untouched from a result of a shock can be a solution to the fire sale problem. This *self-rescue* feature avoids seeking for help from the outsiders such as the government or the central bank, and thus reduces the burden on the taxpayers.

In this paper we present an equilibrium model of financial contagion in banking networks that allows survival banks to act as potential buyers. Banks in our model are risk averse, and hence they optimize their portfolios of cash and various types of illiquid loans based on a risk-adjusted return basis. A bank with a low level of risk aversion, or an *aggressive bank*, holds a larger portion of illiquid risky loans per unit of the bank's equity. So when the equity value of the bank decreases, it reduces its loan holding more aggressively than banks that are more risk averse, or *conservative banks*. When a shock hits an aggressive bank, it can originate fire sales of illiquid loans. On the other hand, if most aggressive bank are not affected by the shock, they act as potential buyers who are willing to take a large amount of loans given a small reduction in the price, and thus help save the network from asset fire sales.

Banks in our model are also linked through interbank liabilities. If an aggressive bank, holding a large amount of loans, is hit by a large shock and becomes insolvent, the loss will be propagated to its interbank creditors who will need to sell off some of their loans, if not all. If those creditors are relatively more aggressive compared to the other banks in the network, then the sales of illiquid loans can be huge, while the non-creditor banks, who are relatively more conservative, require a deep discount in the prices in order to generate enough demand to meet the huge supply of loans. In this case, both sellers and buyers amplify the fire sale

[1] See Pyle (1971) for an example of banks using portfolio management to determine optimal allocation for the banks' assets.

[2] Ratti (1980) and Angelini (2000) provide evidence that banks are risk averse.

effects. On the other hand, if the creditors are the relatively more conservative banks, the loan sales will be less and the non-creditor banks, which are relatively more aggressive, would take all of the loans given just a small discount. Thus, the self-rescue ability of the network relies partly on how conservative and aggressive banks are linked through the interbank liabilities.

When the loan markets and banks are divided into multiple sectors, the role of banks as potential buyers can be significant during a crisis of a sector. In our model, different banks have different expertise in managing different types of loans. Managing the loan type in their expertise requires a low cost, while managing loans outside their expertise is costly. This high cost creates a barrier for banks to hold loans outside their expertise and divides the loan markets and banks into sectors. In this situation, our model shows that when a small shock hits a bank in one sector, the other banks in the sector will act as the potential buyers providing the self-rescue mechanism of the sector. However, when the sector is hit by a large shock damaging many banks in the sector, it is difficult to avoid a sharp drop in the loan price as banks do not play their role as helpful potential buyers. This can happen, for example, if the default risk of the loans in the sector jumps up markedly, affecting the equity values of all banks in the sector. If the loan price falls enough to outweight the high cost, then, and only then, banks outside the failing sector will step in and act as the potential buyers to rescue the failing sector when it is most needed.

Separating loan markets and banks into sectors helps create the *secondary potential buyers*, and thus enhances the self-rescue mechanism of the system. When banks are separated into loan sectors, a shock to one type of loans does not cause losses to banks outside the sector through the asset price channel. Thus, it keeps those outside banks safe and ready to step in to save the sector once the time comes. Each sector now acts as secondary potential buyers for the other sectors. However, interbank liability linkages may exist between banks that belong to different sectors. This channel of contagion weaken the role as the secondary potential buyers of the outside banks as losses from the failing sector can be transmitted to the banks outside the sector. Another factor that can weaken the secondary potential buyer role of the outside banks is the default correlation. Default correlations between loans from different sectors create negative hedging demands due to the substitution effect. That is, when outside banks step in to buy loans from the failing sector, they reduce the holdings of the loans in their sector as they are substitute goods, causing the price to drop. The negative hedging demand is large when the correlation is high and the fire sale loans are attractive.

This result provides an interesting policy suggestion. The regulatory body can divide loan markets into non-overlapping sectors, and require banks to choose an area of expertise (i.e. the loan sector) in which the banks are allowed to run their businesses as usual.[3] Banks running the business outside their declared area of expertise are required to pay a large amount of fee. This is to reduce the incentive of banks to create undesired contagion channel across the sectors. As a result, banks from one sector act as the secondary potential buyers for the others. The regulator should also set a limit for interbank liabilities between banks from different areas of expertise. Once a crisis is about to happen in one sector in which the fire sale can bring down the sector, the regulator may search for financially healthy sectors. If there is not any healthy sector, then it might be better to keep the fee high to avoid a cross-sector contagion and limit the losses. If there are financially healthy sectors, the regulator may choose to lower the fee for the financially healthy sectors to allow the healthy banks to step in and save the failing sector. The regulator can choose the fee level that does not cause a serious effect on the healthy sectors. Once this happens; however, the healthy sectors are contaminated by the failing sector, and the self-rescue system will not function for the next crisis. So the regulator should use this as a temporary solution and try to bring the system back to normal soon before the next crisis.

Our work is related to the literature of potential buyers during fire sales. Shleifer and Vishny (1992) develop an equilibrium model of two firms to show that the price of an asset in liquidation can fall below the value in best use because the depression causes one firm to liquidate its asset, while the other firm in the same industry, who values the asset and is the potential buyer, also has trouble raising fund during the depression period. The asset thus has to be sold to the outsider who does not know how to manage, and is willing to pay at a lower price. See also Shleifer and Vishny (2011) for the role of the firms inside the industry as the potential buyers and when they do not function well during financial crises.

Acharya and Yorulmazer (2008) consider a system of banks and outside investors in which the assets of the failed banks are auctioned to surviving banks and the outside investors. As the number of failed banks increases, the number of surviving banks decreases and the surviving banks do not have enough funds to buy all the assets, allowing the outside investors, who are inefficient users of the assets, to purchase the assets. They show that a bail out policy gives incentives for banks to herd ex-ante and in turn increases the risk of the bank failures. On the contrary, providing capital to surviving banks to acquire the failed banks' assets when the number of failed banks is large leads banks to differentiate their loan exposures. Acharya et al. (2011) extend the model and show that providing the support to surviving banks conditional on their liquid asset holdings gives incentives for banks to hold more liquid assets. In contrast to the earlier works, we propose a resolution of financial meltdown that

[3] An area of expertise may consist of many types of loans. However, each area of expertise cannot overlap.

relies on the capital inside the system by keeping them separated when the economy is sound, but allowing them to act as the secondary support when the primary potential buyers fail to rescue their sectors.

Modeling interbank liabilities linkages using network models has recently been of great interests in the contagion literature. Eisenberg and Noe (2001) provide a model of debt clearing among defaulted and non-defaulted firms, and show the existence and the condition for the uniqueness of the repayment vector. Acemoglu et al. (2015) analyze the network stability conditional on a shock, and show that when the shock is small, a more densely connected network provides better stability due to risk-sharing benefits, while when the shock is large, the dense connection creates fragility of the network by propagating shocks throughout the network. This is consistent with the experiment of Gai and Kapadia (2010) who report that financial networks are robust-but-fragile. Demange (2016) develop a threat index that accounts for firm's characteristic and the links to the other firms in the network to identify optimal intervention policies.

The literature on the financial networks of cross-liability has been extended to include the contagion channel due to asset prices. Cifuentes et al. (2005) propose a network model of interconnected financial institutions that are subject to regulatory solvency constraints. In their model, the institutions may have to sell off their illiquid assets after being hit by a shock to satisfy the constraints. As the assets are illiquid, the asset sales cause the prices to drop and further reduce the mark-to-mark values of the institutions' capital, triggering another round of asset sales. They model the price changes in response to the asset sales using an inverse demand function, and hence implicitly assume that assets are sold to institutions outside the network. Cecchetti et al. (2016) and Feinstein (2017) extend the work of Cifuentes et al. (2005) to include multiple illiquid assets and provide a proof of the existence of an equilibrium. Following the framework of Cifuentes et al. (2005), Chen et al. (2016) study the interaction between the contagion channels through the liability linkages and the asset prices. They conclude that illiquidity channel has a great potential to cause systemic-wide contagion. Greenwood et al. (2015) study the effect of fire sales in banking networks but do not consider interbank liability linkages. They assume that banks sell assets to adjust the leverage ratios back to their target levels after being hit by negative shocks. Their study shows that the systemic risk of a banking system is large if the volatile and illiquid assets are held by the most levered banks, and they suggest that illiquid assets with low volatility should be isolated from the risky ones so that they are not contaminated by those assets.

All of these works share the same assumptions embedded in the model of Cifuentes et al. (2005). That is, they assume that financial institutes sell assets purely due to the liquidity constraints or the target leverage, and the assets are sold to outsiders using an assumed inverse demand function. This differs from our model in which the financial institutes make decision on asset holdings based on a risk-adjusted returns utility, reflecting the profit-seeking and risk-averse nature of financial firms.[4] In addition, the assets in our model are sold to the institutions inside the network, and the asset prices are determined endogenously. This allows us to study a more complete picture of fire sales which are originated from the unbalance between the low demand and high supply. For a recent review of other related financial contagion literature, we refer the reader to Glasserman and Young (2016).

The rest of this paper is organized as follows: Section 2 outlines the banking network model with two channels of contagion: interbank liabilites and asset prices. Section 3 derives the optimal asset holdings of banks. This characterizes the demands of banks which are crucial to understand the roles of banks as sellers and buyers of illiquid assets. Section 4 provides analysis of equilibrium prices before and after shocks hit the system. We conclude in Section 5.

2 Financial Network Model

2.1 Setup

Consider a three-period ($t = 0, 1, 2$) financial system with N *banks*. At time 0, each bank i holds a liquid asset or *cash* with value c_i and a portfolio of illiquid assets or *loans*. There are K types of loans which are for non-bank borrowers such as auto loans and credit card loans. These loans mature at time 2 with random payoffs, and do not pay intermediate payments. Each bank i is endowed with $\theta_{i,k}$ units of type-k loans at time 0. Banks do not create new loans after time 0. In our model, cash represents a liquid asset portion of a bank that normally provides low return with minimal risk (zero risk and zero return in our setting). On the other hand, illiquid loans represent a majority portion of the bank's asset that are typically riskier and have higher returns.

In addition to cash and loans made for non-bank borrowers, banks are endowed with shorter-term *interbank loans* between each other. These interbank loans mature at time 1. Let $l_{i,j}$ denote the claim of bank j on the asset of bank i at the maturity of the interbank loan. The interbank claims at time 1 can be summarized by matrix $L = [l_{i,j}]$ where $l_{i,i} = 0$. We assume that no new interbank loans are created after time 0.

[4] Aldasoro et al. (2016) consider banking networks that incorporate three channels of contagion: liquidity hoarding, interbank liabilities and fire sales. Similar to ours, they assume that banks are risk averse and maximize the expected utility of the banks' profits. However, they do not consider buyers as in our model.

At time 0, each bank i is also financed by deposits of d_i. We assume that the interbank liabilities and deposits are of equal seniority, and that interest rate is normalized to zero. So cash and deposits earn no interest. To summarize, the asset side of the balance sheet of each bank consists of cash, illiquid loans, and interbank loans, while the debt side of the bank consists of deposits and interbank liabilities. The equity value of the bank is equal to the asset value minus the debt value. We assume that each bank has positive equity at time 0; that is, all banks are solvent.

At time 1, the system is subject to unexpected shocks, which can be in various forms. A shock can be a *bank shock* that comes in as a surprise expense of one particular bank, reducing the bank's net worth. Such shocks could be due to frauds, litigation costs, or settlements of lawsuits.[5] A shock can also be categorized as an *asset shock* such as an increase in the default probability of one type of loans. An adverse shock to the default probability causes the loan price to drop. We model that mechanism below. After shocks are realized, banks settle their interbank liabilities by using cash or repayments obtained from their interbank claims, or by selling their illiquid loans, or a combination of them.

Suppose that after the shocks the market price of type-k illiquid loan is $p_k, k = 1, \ldots, K$. Let $x_{i,j}$ denote the amount that bank i repays its interbank liability to bank j for $j \neq i$. If the value of the total asset of bank i is less than the value of its total debt, then the bank is insolvent and must liquidate all of its assets and distribute the proceeds to all of its creditors proportional to the face values. Otherwise, bank i repays the interbank liability in full. Let $L_i = d_i + \sum_{u \neq i}^{N} l_{i,u}$ denote the total debt of bank i. Then the amount that bank i repays to bank j is equal to

$$x_{i,j} = \frac{l_{i,j}}{L_i} \min \left\{ L_i, c_i + \sum_{k=1}^{K} p_k \theta_{i,k} + \sum_{u \neq i}^{N} x_{u,i} - v_i \right\} \tag{1}$$

We assume that each bank carries over its cash and deposits from time 0 to time 1. Given all the shocks and the market prices of illiquid loans $p = [p_1, \ldots, p_K]'$, the collection of $[x_{i,j}]$ for $i \neq j$ that satisfies (1) for each $i = 1, \ldots, N$ simultaneously is said to be an *equilibrium repayment* at the price vector p.

Let

$$e_i = \max \left\{ 0, c_i + \sum_{k=1}^{K} p_k \theta_{i,k} + \sum_{u \neq i}^{N} x_{u,i} - v_i - d_i - \sum_{u \neq i}^{N} l_{i,u} \right\} \tag{2}$$

[5] For example, the fraud in the Barings Bank caused it to collapse in 1995. The 2016 annual report of the Royal Bank of Scotland reports the loss of over 5.8 billion pounds for litigation and conduct costs.

denote the equity value of bank i. Banks that are solvent ($e_i > 0$) can now adjust their asset portfolio by buying or selling illiquid loans. We assume that each solvent bank chooses a portfolio of liquid and illiquid assets to maximize its risk-adjusted return on equity based on a mean-variance utility. Let $\hat{\theta}_{i,k}$ denote the number of units of tyep-k loan held by bank i after the adjustment, and $\tilde{R}_{i,k}(\hat{\theta}_{i,k})$ denote the values of the bank i's portfolio of the type-k illiquid loans realized at time 2. The change in the equity value from time 1 to time 2 comes from the change in the value of illiquid loans, and the cost of managing the loans. So the return on equity of bank i is

$$ROE_i = \frac{\sum_{k=1}^{K}(\tilde{R}_{i,k}(\hat{\theta}_{i,k}) - \hat{\theta}_{i,k}p_k - \hat{\theta}_{i,k}f_{i,k})}{e_i} \tag{3}$$

where $f_{i,k}$ is the cost of managing one unit of type-k loan of bank i. The cost matrix $F = [f_{i,k}]$ is used to define *expertise* of banks for different types of loans. Now the optimization problem of bank i is given by

$$[\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,K}] = \arg\max \left\{ E[ROE_i] - \frac{\gamma_i}{2} Var(ROE_i) \right\} \tag{4}$$

subject to the budget constraint

$$\sum_{k=1}^{K} p_k \hat{\theta}_{i,k} \leq e_i + d_i \tag{5}$$

where γ_i denotes the risk-aversion parameter of bank i. Note that banks are not allowed to hold short positions on loans and hence we also need the no-short-position constraint

$$\hat{\theta}_{i,k} \geq 0, \quad k = 1, \ldots, K. \tag{6}$$

We provide the details of $\tilde{R}_{i,k}(\theta_{i,k})$ below.

We assume that illiquid loans are traded inside the financial system with N banks. So the loan prices are determined endogenously based on the market clearing condition. That is, the price vector $p = [p_1, \ldots, p_K]'$ is said to be an *equilibrium price* if the demand and supply of each loan type are equal:

$$\sum_{i=1}^{N} \theta_{i,k} = \sum_{i=1}^{N} \hat{\theta}_{i,k}, \quad k = 1, \ldots, K. \tag{7}$$

2.2 Default correlations and loan payoff distribution

There are K types of illiquid loans. At time 2, a loan of type k repays the creditor the full amount of \$1 with probability $1-\lambda_k$ or defaults and pays nothing to the creditor with probability λ_k for $\lambda_k \in (0,1)$. We assume that loan defaults are correlated and we model the default correlation with a Gaussian copula model. Specifically, let $M_k = \sum_{i=1}^{N} \theta_k$ denote the total number of type-k loans that are available in the system, and $\tilde{r}_{m,k}$ denote the payoff of loan m in type k, $m = 1, \ldots, M_k$. The Gaussian copula framework models default correlation through common factors. Let $\tilde{Z}_0, \tilde{Z}_1, \ldots, \tilde{Z}_K$ be independent standard normal random variables such that $\tilde{Z}_0$ represents the *market* factor, and $\tilde{Z}_k$ represents type-k factor for $k = 1, \ldots, K$. For each loan m of type k, let

$$\tilde{Y}_{m,k} = \alpha \tilde{Z}_0 + \beta_k \tilde{Z}_k + \sqrt{1-\alpha^2-\beta_k^2}\,\tilde{\epsilon}_{m,k} \tag{8}$$

where $\tilde{\epsilon}_{m,k}$'s are i.i.d. standard normal random variables for $m = 1, \ldots, M_k, k = 1, \ldots, K$ and are independent of $\tilde{Z}_0, \ldots, \tilde{Z}_K$. The parameters α and β_k are such that $\alpha \geq 0, \beta_k > 0$ and $\alpha^2 + \beta_k^2 < 1$. Observe that each $\tilde{Y}_{m,k}$ is also a standard normal random variable, and they are correlated. The correlations between $\tilde{Y}_{m,k}$'s are used to determine the default correlations between loan payoffs $\tilde{r}_{m,k}$'s based on the following relationship:

$$\tilde{r}_{m,k} = 0 \quad \text{if and only if} \quad \tilde{Y}_{m,k} \leq \Phi^{-1}(\lambda_k)$$

where $\Phi(y)$ is the cumulative distribution function of standard normal distribution at y. So loan m in type k defaults if and only if $\tilde{Y}_{m,k} \leq \Phi^{-1}(\lambda_k)$. Note that from the standard normal distribution of $\tilde{Y}_{m,k}$, the default probability is $P(\tilde{Y}_{m,k} \leq \Phi^{-1}(\lambda_k)) = \lambda_k$, as it must.

The correlations between loans depend on parameters $\alpha, \beta_1, \ldots, \beta_K$. For loans m and m' in the same type k, the correlation between $\tilde{Y}_{m,k}$ and $\tilde{Y}_{m',k}$ is $\alpha^2 + \beta_k^2$, while for loans m and m' of different types k and k', correlation between $\tilde{Y}_{m,k}$ and $\tilde{Y}_{m',k'}$ is α^2. In other words, loans of different types are less correlated than loans of the same types, and the difference is determined by β_k. With different values in $\beta_1, \ldots, \beta_K$, we can have loan types that have higher default correlation, such as corporate loans within a particular sector, and loan types that have lower default correlation, such as student loans. We have the following results:

Proposition 1 *Let $\tilde{R}_{i,k}(\theta) = \sum_{m=1}^{\theta} \tilde{r}_{m,k}$ and $\tilde{R}_{i,k'}(\theta') = \sum_{m=1}^{\theta'} \tilde{r}_{m,k'}$ denote the payoffs at time 2 of the portfolio of θ units of type-k loans, and portfolio of θ' units of type-k' loans,* *respectively. We have*

$$E\left[\tilde{R}_{i,k}(\theta)\right] = \theta(1-\lambda_k) \tag{9}$$

$$Var\left(\tilde{R}_{i,k}(\theta)\right) = \theta^2[\psi_k - (1-\lambda_k)^2] + \theta[1-\lambda_k-\psi_k] \tag{10}$$

$$Cov\left(\tilde{R}_{i,k}(\theta), \tilde{R}_{i,k'}(\theta')\right) = \theta\theta'[\Psi_{k,k'} - (1-\lambda_k)(1-\lambda_{k'})], \quad k \neq k' \tag{11}$$

where

$$\psi_k = \Phi_2(\Phi^{-1}(1-\lambda_k), \Phi^{-1}(1-\lambda_k); \alpha^2+\beta_k^2)$$
$$\Psi_{k,k'} = \Phi_2(\Phi^{-1}(1-\lambda_k), \Phi^{-1}(1-\lambda_{k'}); \alpha^2)$$

and $\Phi_2(y_1, y_2; \rho)$ is the bivariate cumulative distribution function at (y_1, y_2) of standard normal random variables with correlation ρ.[6]

The expected value in (9) is simply the number of loans multiplied by the probability that a type-k loan does not default. The variance in (10) has two terms. The first term is quadratic in the number of loans, while the second term is linear. To understand this, note first that the variance of the payoff of *one unit* of a type-k loan is $\lambda_k(1-\lambda_k)$. This variance is broken into $\psi_k - (1-\lambda_k)^2$ and $1-\lambda_k-\psi_k$, the sum of which is $\lambda_k(1-\lambda_k)$. When the term $\psi_k - (1-\lambda_k)^2$ at the loan level is aggregated to the portfolio level, it gives a quadratic function of θ, while the term $1-\lambda_k-\psi_k$ gives rise to a liner function at the portfolio level. As the value of ψ_k increases from $(1-\lambda_k)^2$ when loan defaults are independent to $1-\lambda_k$ when they are perfectly correlated, we can view $\psi_k-(1-\lambda_k)^2$ as a *variance-based* measure of how loans are close to perfect correlation, and view $1-\lambda_k-\psi_k$ as a variance-based measure of how loans are close to independence. So the portfolio variance given by (10) suggests that the higher the correlation, the stronger the quadratic term, and the weaker the linear term. Finally, if loans of different types default independently ($\alpha = 0$), then $\Psi_{k,k'} = (1-\lambda_k)(1-\lambda_{k'})$ and thus the covariance between the values of loan portfolios in (11) is zero. It is positive if the defaults of loans of different types are positively correlated.

[6] Note that we have assumed that θ is an integer, but we will rely on the same formulas (9) - (11) even when θ is real. The error from rounding the number should not change the conclusions of the consequent analyses.

3 Banks' optimal portfolios

When an adverse bank shock, such as an unexpected litigation cost, hits a bank, it reduces the bank's net worth. The bank may use cash or cash equivalents to pay for the cost, reducing the liquid asset portion in the bank's balance sheet. Because liquid assets such as cash and cash equivalents are considered as risk-free or low-risk assets, this results in the bank's asset portfolio that overweights the risky loans, increasing the risk of the bank's portfolio relative to the smaller equity value. Likewise, when an adverse asset shock hits the bank, the value of the risky loans and hence the equity value reduce, making the risk profile of the bank's portfolio deviate from the optimal level. Banks are risk-averse but profit-seeking institutions. So the changes in the proportion of risk-free/risky assets relative to its equity require banks to re-adjust their asset holdings to achieve a better risk-return trade-off.

In this section we consider the banks' optimization problems and their optimal portfolios of liquid and illiquid assets. We start with the simplest case with one type of loans. Then we study the interaction between types of loans from the case of two types of loans.

3.1 One loan type

Assume that there is only one type of illiquid loans ($K = 1$). From (3) - (4) and (9) - (10), the objective of bank i is to maximize the following risk-adjusted return on equity

$$V_i(\theta) = \frac{\theta(1-\lambda) - \theta p - \theta f_i}{e_i} - \frac{\gamma_i}{2}\left(\frac{\theta^2[\psi - (1-\lambda)^2] + \theta[1-\lambda-\psi]}{e_i^2}\right) \tag{12}$$

subject to the budget constraint $\theta p \leq e_i + d_i$ and no-short-position constraint $\theta \geq 0$ where we have dropped the subscript k for simplicity.

To understand the optimal number of loans held by the bank, let's suppose for the moment that the constraints are not binding, and let $\bar{\theta}_i$ denote the optimal solution for the unconstrained problem derived from the first order condition:

$$\begin{aligned} \bar{\theta}_i &= \frac{e_i}{\gamma_i}\left[\frac{(1-\lambda) - p - f_i}{\psi - (1-\lambda)^2}\right] - \frac{1}{2}\left[\frac{1-\lambda-\psi}{\psi-(1-\lambda)^2}\right] \\ &= \frac{e_i}{\gamma_i}\left[\frac{(1-\lambda) - p - f_i}{\lambda(1-\lambda)}\right]\left(\frac{\lambda(1-\lambda)}{\psi - (1-\lambda)^2}\right) - \frac{1}{2}\left[\frac{1-\lambda-\psi}{\psi-(1-\lambda)^2}\right]. \end{aligned} \tag{13}$$

Observe that $\bar{\theta}_i$ has two components. The first component has the mean-variance spirit. It suggests that the bank should hold more loans if it has large equity value (large e_i), low risk aversion (small γ_i), or if each loan has high expected profit after cost (high $(1-\lambda) - p - f_i$), and low variance (low $\lambda(1-\lambda)$). This mean-variance term is scaled up by a factor of

$\lambda(1-\lambda)/[\psi-(1-\lambda)^2] \geq 1$. Given a fixed default probability λ, this factor increases as the default correlation decreases (smaller $\psi-(1-\lambda)^2$). This is intuitive as a lower correlation provides better risk-return trade-off and hence increases the demand for loans. That is, banks should hold more loans if each loan provides a good risk-adjusted return (mean-variance term), and the portfolio has low risk as loan defaults are less correlated (scaling factor term).

This first component of the optimal loan holding comes from two effects. The first effect is the *wealth effect* through the equity value e_i. The more equity the bank has, the larger the loan demand. The second effect is the *price effect* in the expected profit. When the price increases, the expected return decreases, reducing the loan demand. What is interesting is that banks also hold loans. So as the price increases, the equity values of banks increase and create more demands for loans due to the wealth effect. On the other hand, the price effect reduces the demands. The combination of these two effects from all banks in the system determines the final price at an equilibrium, as discussed in Section 4.

The second component of the optimal holding is a downward adjustment. Its value depends on how correlated loan defaults are. The lower the correlation, the higher the size of the adjustment. When the equity values of banks are large (e.g. millions or billions of dollars), the effect of the second component is minimal. However, when banks are close to insolvency, the second component can be relatively significant.

Now let us bring back the budget and no-short-position constraints. Because the objective function (4) is quadratic in θ, it is easy to obtain the optimal solution with the constraints from the solution of the unconstrained problem. The following proposition gives the result.

Proposition 2 *The optimal loan holding for bank i when there is one type of loans is*

$$\hat{\theta}_i = \begin{cases} 0 & \text{if } \bar{\theta}_i \leq 0 \\ \bar{\theta}_i & \text{if } 0 < \bar{\theta}_i < (e_i + d_i)/p \\ (e_i + d_i)/p & \text{if } \bar{\theta}_i \geq (e_i + d_i)/p \end{cases} . \tag{14}$$

So when the bank is close to insolvency (the second downward adjustment component dominates), or when it has higher cost of managing loans ($f_i > 1-\lambda-p$), it is optimal for the bank not to hold any loans. In contrast, when loans are very attractive, the number of loans held is capped by the budget constraint. This implies that the bank does not hold cash. However, such a situation rarely occurs because the equity decreases when the price decreases as the bank holds loans. With a large leverage ratio in the banking industry ($d_i \gg e_i$), the constraint is unlikely to be binding.

3.2 Two loan types

Assume that there are two types of loans ($K = 2$). The bank i's problem is to maximize the following objective function

$$\begin{aligned} V_i(\theta_1, \theta_2) = &\sum_{k=1}^{2} \left(\frac{\theta_k(1-\lambda_k) - \theta_k p_k - \theta_k f_{i,k}}{e_i} \right) \\ &- \frac{\gamma_i}{2} \sum_{k=1}^{K} \left(\frac{\theta_k^2[\psi_k - (1-\lambda_k)^2] + \theta_k[1-\lambda_k-\psi_k]}{e_i^2} \right) \\ &- \gamma_i \left(\frac{\theta_1\theta_2[\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)]}{e_i^2} \right) \end{aligned}$$

subject to the budget constraint $\theta_1 p_1 + \theta_2 p_2 \leq e_i + d_i$ and no-short-position constraint $\theta_k \geq 0, k = 1, 2$. Again, we first consider the case without constraints. The first order condition yields the following optimal solution for the unconstrained problem:

$$\begin{bmatrix} \theta_{i,1}^* \\ \theta_{i,2}^* \end{bmatrix} = \frac{1}{1 - \eta_{1,2}[\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)]} \begin{bmatrix} \bar{\theta}_{i,1} - \eta_{1,2}(\psi_2 - (1-\lambda_2)^2)\bar{\theta}_{i,2} \\ \bar{\theta}_{i,2} - \eta_{1,2}(\psi_1 - (1-\lambda_1)^2)\bar{\theta}_{i,1} \end{bmatrix} \tag{15}$$

where

$$\eta_{1,2} = \frac{\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)}{(\psi_1 - (1-\lambda_1)^2)(\psi_2 - (1-\lambda_2)^2)}$$

and $\bar{\theta}_{i,k}$ is $\bar{\theta}_i$ given by (13) in the one-type case with subscript k for the type-k loans. The term $\eta_{1,2}$ contains the correlation between defaults of type-1 loans and type-2 loans. Its value is zero when type-1 loans and type-2 loans are independent ($\alpha = 0$) and goes up as the correlation goes up (but keeping $\alpha^2 + \beta_k^2$ constant). To see the interaction between the two types of loans on the loan demand, let us focus on the demand of type-1 loans, or $\theta_{i,1}^*$. There are two components in $\theta_{i,1}^*$. The first one is the optimal holding from the one-type case $\bar{\theta}_{i,1}$ and the second component represents the hedging demand between the two types of loans. Suppose that $\bar{\theta}_{i,1}$, $\bar{\theta}_{i,2}$ and $\theta_{i,1}^*$ are positive. If type-1 and type-2 loans are independent, the second term disappears as $\eta_{1,2} = 0$, and in this case $\theta_{i,k}^* = \bar{\theta}_{i,k}$. When the correlation is positive, the second component creates a negative hedging demand on type-1 loans. The hedging demand depends on how attractive the type-2 loans are. The more attractive they are (large $\bar{\theta}_{i,2}$), the more negative the hedging demand, given everything else constant. That is, there is a *substitution effect* between the two loan types if the correlation is positive.

This substitution effect plays an important role on the cross-asset contagion channel. Suppose that a fire sale of type-1 loans from one bank reduces the price of the loans, making them more attractive (price effect), which in turn reduces the demand of type-2 loans of other banks due to the larger (more negative) hedging demand (substitution effect). This may trigger a fire sale on type-2 loans, and hence a contagion across the two loan types. The outcome can be much worse as there are interbank liability and common holding of type-1 loans channels that can transmit losses to other banks, following the fire sales of type-1 and type-2 loans. Lower equity values further reduces the loan demands (wealth effect) and reinforce the asset fire sales. We discuss the effect of contagion through different channels and their interaction in the subsequent sections.

To finish this section, we consider the cases when one or both of the no-short-position constraints are binding. Similar to the case of one type of loan, the budget constraint is unlikely to be binding at any equilibrium given sufficiently large leverage ratios. With the Lagrange multiplier technique, we obtain the following optimal holdings of the portfolio of loans:

Proposition 3 *Suppose the budget constraint is not binding for the case of two loan types. Then the optimal loan holdings for bank i is*

$$(\hat{\theta}_{i,1}, \hat{\theta}_{i,2}) = \begin{cases} (\theta^*_{i,1}, \theta^*_{i,2}) & \text{if } \theta^*_{i,1} > 0 \text{ and } \theta^*_{i,2} > 0 \\ (\bar{\theta}_{i,1},\ 0\ ) & \text{if } \theta^*_{i,1} > 0 \text{ and } \theta^*_{i,2} \leq 0 \\ (\ 0\ , \bar{\theta}_{i,2}) & \text{if } \theta^*_{i,1} \leq 0 \text{ and } \theta^*_{i,2} > 0 \\ (\ 0\ ,\ 0\ ) & \text{if } \theta^*_{i,1} \leq 0 \text{ and } \theta^*_{i,2} \leq 0 \end{cases}. \tag{16}$$

We now show that the negative demand always reduces the optimal holdings. Consider the case when $\theta^*_{i,1}$ is positive. From (15), it must be that

$$\bar{\theta}_{i,1} > \eta_{1,2}(\psi_2 - (1-\lambda_2)^2)\bar{\theta}_{i,2}.$$

Hence, for $\eta_{1,2} > 0$, we have

$$\begin{aligned} \theta^*_{i,2} &= \frac{\bar{\theta}_{i,2} - \eta_{1,2}(\psi_1 - (1-\lambda_1)^2)\bar{\theta}_{i,1}}{1 - \eta_{1,2}[\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)]} \\ &< \frac{\bar{\theta}_{i,2} - \eta_{1,2}(\psi_1 - (1-\lambda_1)^2)\left(\eta_{1,2}(\psi_2 - (1-\lambda_2)^2)\bar{\theta}_{i,2}\right)}{1 - \eta_{1,2}[\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2)]} \\ &= \bar{\theta}_{i,2}. \end{aligned}$$

Similarly, if $\theta^*_{i,2} > 0$, we have $\bar{\theta}_{i,1} > \theta^*_{i,1}$. Thus, when $\theta^*_{i,1}$ and $\theta^*_{i,2}$ are both positive, the optimal holding of the type-k loans is $\hat{\theta}_{i,k} = \theta^*_{i,k} \geq \bar{\theta}_{i,k}$ where the inequality holds if, and only if, the loans of different types have zero default correlation, or $\eta_{1,2} = 0$. Thus, the hedging demand always reduces the optimal loan holdings.

4 Equilibrium Analysis

Prior literature on fire sales in financial networks typically assumes an inverse demand function characterizing the price change as a function of the aggregate sales. This implicitly suggests that fire sale assets are sold to buyers outside the financial system. This contradicts to the fact that a large portion of the bank assets are loans which are costly to manage by non-bankers. So it is wiser for banks to sell their assets to other banks that are more efficient buyers who are willing to pay higher prices for the assets. This situation is reasonable particularly when a shock hits one bank, and the remaining banks have sufficient funds to buy the troubled bank's assets. When there is an adverse asset shock to one type of loans, demands for the loans from banks holding that type of loans will decrease due to the wealth effect, but other banks that do not hold that loan type can be potential buyers, willing to pay for the loans, and hence help reduce the effect of the fire sale. We study the effect of potential buyers in the banking networks by allowing the prices of illiquid loans to be determined endogenously. We first provide the formal definition of our equilibrium.

Definition 1 *Given the banks' balance sheets* ($c = [c_i], \theta = [\theta_{i,k}], d = [d_i], L = [l_{i,j}]$)*, banks' characteristics* ($\gamma = [\gamma_i], F = [f_{i,k}]$)*, and the illiquid loans default distribution* ($\lambda = [\lambda_k], \alpha, \beta = [\beta_k]$)*, an equilibrium triplet of repayments, holdings, and prices* ($X = [x_{i,j}], \hat{\theta} = [\hat{\theta}_{i,k}], p = [p_k]$) *at time* 1 *is such that*

1. *Repayment equilibrium: Equations (1)-(2) hold for all banks* $i = 1, \dots, N$*,*
2. *Bank optimization: Each bank* i *maximizes mean-variance utility (4) subject to constraints (5)-(6),* $i = 1, \dots, N$*.*
3. *Market clearing: Equation (7) holds.*

To study the effect of shocks and roles of banks on transmitting and absorbing shocks, we assume that without shocks to the system, there is an equilibrium in which all the interbanks liabilities are fully repaid at time 1. That is, without shocks, all banks are solvent ($x_{i,j} = l_{i,j}$). In other words, there is a price vector $p = [p_k]$ and a loan holding matrix $\hat{\theta} = [\hat{\theta}_{i,k}]$ such that the triplet $(L, \hat{\theta}, p)$ is an equilibrium triplet for the network at time 1.

4.1 One loan type

This section considers a network with one type of loans ($K = 1$), and hence we drop subscript k that refers to the loan type. We consider three scenarios: before shocks, after a bank shock, and after an asset shock.

4.1.1 Before shocks

Consider the case of no shocks with an equilibrium triplet $(L, \hat{\theta}, p)$ for which all banks are solvent. Let

$$\bar{c}_i = c_i + \sum_{j\neq i}^{N} l_{j,i} - d_i - \sum_{j\neq i}^{N} l_{i,j}$$

denote the excess cash position over the deposit after the repayment settlement of bank i. This value is typically negative as cash c_i is much smaller than deposit d_i, and interbank liabilities $l_{i,j}$'s are relatively small. The equity value of bank i is the sum of the value of the loan portfolio and its excess cash:

$$e_i = \theta_i p + \bar{c}_i > 0. \tag{17}$$

We assume further that none of the banks is close to be insolvent ($e_i \gg 0$), and that all banks are equally good at managing loans and thus have the same managing cost $f_i \equiv f < 1 - \lambda - p$.[7] So it follows from (13) that $\hat{\theta}_i = \bar{\theta}_i > 0, i = 1, \ldots, N$. Now consider the total demand of loans in the system. From (13) and (17), we have the total loan demand is

$$\begin{aligned}
\Theta_D &= \sum_{i=1}^{N} \left\{ \frac{e_i}{\gamma_i} \left[\frac{(1-\lambda) - p - f}{\psi - (1-\lambda)^2} \right] - \frac{1}{2} \left[\frac{1-\lambda-\psi}{\psi-(1-\lambda)^2} \right] \right\} \\
&= \sum_{i=1}^{N} \left\{ \left(\frac{\theta_i p + \bar{c}_i}{\gamma_i} \right) \left[\frac{(1-\lambda) - p - f}{\psi - (1-\lambda)^2} \right] - \frac{1}{2} \left[\frac{1-\lambda-\psi}{\psi-(1-\lambda)^2} \right] \right\} \\
&= -\left(\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i} \right) \frac{p^2}{\psi - (1-\lambda)^2} + \left(\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1-\lambda-f) - \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} \right) \frac{p}{\psi-(1-\lambda)^2} \\
&\quad + \left(\sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} \right) \left[\frac{1-\lambda-f}{\psi-(1-\lambda)^2} \right] - \frac{N}{2} \left[\frac{1-\lambda-\psi}{\psi-(1-\lambda)^2} \right].
\end{aligned} \tag{18}$$

As we can see, the total demand in (18) is a concave quadratic function of p. This is due to

[7] We use the differences in the managing costs in the case of multiple loan types to consider the effect of banks' expertise on the stability of the network.

the combination of the wealth effect and the price effect. It is easy to see that

$$\bar{p} = \frac{1}{2}(1 - \lambda - f - \zeta)$$

is the price at which the price effect and the wealth effect of the total demand are equal ($\partial\Theta_D/\partial p = 0$) where

$$\zeta = \frac{\sum_{i=1}^{N} \bar{c}_i/\gamma_i}{\sum_{i=1}^{N} \theta_i/\gamma_i}. \tag{19}$$

When the price is lower than $\bar{p}$, an increase in the price increases the loan demand as the wealth effect dominates the price effect. When the price is higher than $\bar{p}$, an increase in the price lowers the loan demand as the price effect dominates the wealth effect. This demand behavior applies to each individual bank's loan demand in (13) as well. Precisely, the price at which both price and wealth effects equal for the demand of bank i is

$$\bar{p}_i = \frac{1}{2}\left(1 - \lambda - f - \frac{\bar{c}_i}{\theta_i}\right).$$

Let

$$\Theta_S = \sum_{i=1}^{N} \theta_i$$

denote the total number of units of loans available in the system. This represents the total supply of loans. The market clearing condition (7) dictates that the demand and supply are equal at each equilibrium: $\Theta_D = \Theta_S$. Since the total supply is fixed and the total demand is a concave quadratic function of p, we have the following result:

Theorem 1 *Suppose none of the banks is insolvent and the repayment matrix is $X = L$ in the case of one loan type. Then an equilibrium price exists if, and only if,*

$$(1 - \lambda - f + \zeta)^2 \geq 4\left[\frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1 - \lambda - \psi)}{\sum_{i=1}^{N} \theta_i/\gamma_i}\right], \tag{20}$$

and in that case the equilibrium prices are given by

$$p = \frac{1}{2}(1 - \lambda - f - \zeta) \pm \frac{1}{2}\left\{(1 - \lambda - f + \zeta)^2 - 4\left[\frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1 - \lambda - \psi)}{\sum_{i=1}^{N} \theta_i/\gamma_i}\right]\right\}^{1/2}. \tag{21}$$

The equilibrium price is unique if, and only if, the inequality in (20) is binding and the equilibrium price is $p = (1 - \lambda - f - \zeta)/2$.

To further investigate the equilibrium prices given by (21), we apply the first-order Taylor approximation for the function of the form $f(x) = (a^2 - x)^{1/2}$ around the point $x = 0$ to the last term in (21): $f(x) \approx |a| - x/2|a|$ where a is a constant. From the inequality in (17) and (19), it is clear that $\zeta > -p$. From the assumption that $\bar{\theta}_i > 0$, (13) implies that $1 - \lambda - f - p > 0$. So we have $1 - \lambda - f + \zeta > 0$. Thus, the Taylor approximation gives the following two equilibrium prices:

$$p^h \approx 1 - \lambda - f - \frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1 - \lambda - \psi)}{\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1 - \lambda - f) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i}} \tag{22}$$

$$p^l \approx -\frac{\sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i}}{\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}} + \frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1 - \lambda - \psi)}{\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1 - \lambda - f) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i}}. \tag{23}$$

Note that $p^h > \bar{p} > p^l > 0$.

As it will become clear later that an asset fire sale may occur at the equilibrium price p^h, but not at p^l, we will focus on p^h. From (22), the equilibrium price p^h is equal to the expected payoff after cost, or $1 - \lambda - f$, minus the *premium*. The premium is the expected profit required by the banks for holding the risky loans. This premium depends on the *riskiness* of the loan

$$U = (\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1 - \lambda - \psi). \tag{24}$$

This riskiness U is the combination of the variance component measuring the closeness to the perfect correlation $\phi - (1 - \lambda)^2$ and the variance component measuring the closeness to the independence $1 - \lambda - \psi$. When the number of loans available in the system is large, the first term is important. On the other hand, the second becomes large when the number of banks in the system is large. The risk premium is high when the loan has high level of riskiness.

The premium also depends on the *risk-aversion-adjusted wealth* of the banking system.

To see this recall that equity of bank i is

$$e_i = \theta_i p + \bar{c}_i.$$

As p^h is an approximated price, we replace p by its expected payoff after cost, which is $1 - \lambda - f$. This gives

$$e_i \approx \theta_i(1 - \lambda - f) + \bar{c}_i.$$

Each equity value is scaled by the bank's risk aversion parameter γ_i as the one unit of equity of a more risk-averse bank (high γ) is worth less than that of a less risk-averse bank (low γ) in terms of the loan demand (see (13)). Then we sum over all banks to get the risk-aversion-adjusted wealth

$$\sum_{i=1}^{N} \frac{e_i}{\gamma_i} \approx \sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1 - \lambda - f) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i}.$$

When the risk-aversion-adjusted wealth of the system is high, banks have more cash to pay for the loans, pushing the price up, and thus a lower premium. In the subsequent sections, we can explain changes in the equilibrium price p^h based on the changes in the expected payoff and/or the premium (risk and wealth).

4.1.2 After a bank shock

Let us first focus on the equilibrium price p^h, and study the change in the equilibrium price p^h in response to a bank shock. As mentioned earlier, $p^h > \bar{p}$ so the price effect of the aggregate demand is stronger than the wealth effect. If $\bar{c}_i/\theta_i$ is close to ζ, this is true for the individual bank's demand too. For the discussion below, we assume that the price effect is stronger than the wealth effect for each individual bank's demand at the equilibrium price p^h.

Suppose there is an adverse bank shock of size v_j hitting bank j. Let $\bar{\theta}_j(v_j)$ denote the value of $\bar{\theta}_j$ after an adverse shock of size v_j on bank j. This notation is used similarly for the other variables. If v_j is sufficiently small so that bank j still has positive holding in the loans, or $\bar{\theta}_j(v_j) > 0$, then the value of the excess cash of bank j after the shock is $\bar{c}_j(v_j) = \bar{c}_j - v_j$.

Thus the new equilibrium price is

$$p^h(v_j) \approx 1 - \lambda - f - \frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1-\lambda-\psi)}{\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1-\lambda-f) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}. \tag{25}$$

So after a small bank shock v_j, the equilibrium price becomes lower due to a lower risk-aversion-adjusted wealth. What is interesting is that the effect of a shock of the same size on the equilibrium price depends on the risk-aversion parameter of the bank being hit. A small shock hitting a *conservative* bank (high γ) yields a smaller impact on the equilibrium price than a shock of the same size hitting an *aggressive* bank (low γ). This is due to the lower sensitivity of the loan demand to a one-unit decrease in the equity value, which can be seen in (13). Since everything remains the same for all other banks $j' \neq j$ except all banks see the new equilibrium price, each bank adjusts its loan holding based purely on the price change. As the price effect dominates the wealth effect at p^h, all of the other banks $j' \neq j$ act as the potential buyers and increase their loan holdings in response to the lower price. Thus, bank j has to hold fewer loans at the new equilibrium. Note that without the shock, the price effect of bank j is stronger than the wealth effect. But since the external shock v_j reduces the equity value in addition to the effect from the lower equilibrium price, it results in a loan sell-off for bank j.

Let us consider a larger shock. Suppose that v_j is large enough to make $e_j(v_j) < 0$, but not enough to make the other banks insolvent. So bank j sells all of its loans at a fire sale price. In addition, it spreads the loss to its *neighbor* banks (the banks that hold interbank claims on the assets of bank j) through the interbank liability linkages. The loss to bank $j' \neq j$ due to the direct interbank liability with bank j is

$$l_{j,j'} \min\left\{\frac{v_j - (\theta_j p^h(v_j) + \bar{c}_j)}{L_j}, 1\right\}.$$

The new equilibrium price is

$$\begin{aligned} p^h(v_j) &\approx 1 - \lambda - f \\ &\quad - \frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1-\lambda-\psi)}{\sum_{i\neq j}^{N} \frac{\theta_i}{\gamma_i}(1-\lambda-f) + \sum_{i\neq j}^{N} \frac{\bar{c}_i}{\gamma_i} - \sum_{i\neq j}^{N} \frac{l_{j,i}}{\gamma_i} \min\left\{\frac{v_j - (\theta_j p^h(v_j) + \bar{c}_j)}{L_j}, 1\right\}}. \end{aligned} \tag{26}$$

The contagion through the interbank liability channel reduces the equity values of neighbor banks that hold claims on the asset of bank j. The reduction in the equity values lowers their demands for loans, and causes the price to drop further. The impact on the price depends on the neighbor bank j''s ratio $l_{j,j'}/\gamma_{j'}$. The impact is large if the neighbor bank has a large claim on the asset of bank j and it is an aggressive bank (small γ). This suggests that interbank liabilities between aggressive banks amplify the fire sale effect in the network.

Now consider p^l. When there is a small bank shock of size v_j on bank j, the new equilibrium price is

$$p^l(v_j) \approx -\frac{\sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}{\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}} + \frac{(\psi - (1-\lambda)^2)\Theta_S + \frac{N}{2}(1-\lambda-\psi)}{\sum_{i=1}^{N} \frac{\theta_i}{\gamma_i}(1-\lambda-f) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}$$

which is higher than the equilibrium price before the shock, or p^l, where, similar to the case of p^h, we have assumed that all of the banks are solvent after this small shock. Assuming that at p^l the wealth effect dominates the price effect for all banks, we have the increase in the price results in higher demands for loans for all other banks $j' \neq j$. As we can see this equilibrium does not correspond to a fire sale as the selling price increases after a shock arrives. In addition, banks should agree to choose the equilibrium with the higher price to maximize their net worth. So we will focus on p^h from now on.

4.1.3 After an asset shock

This section considers the effect of an asset shock on the equilibrium price. Suppose the default probability of each loan increases from λ to λ' and none of the banks are insolvent after the shock. It can be shown that the terms $\psi - (1-\lambda)^2$ and $1 - \lambda - \psi$ are increasing in λ for $\lambda \in (0, 0.5)$ but are decreasing in λ for $\lambda \in (0.5, 1)$.[8] Since the typical values of λ are less than 0.5, p^h decreases as the default probability increases (see (22)). When the price effect is stronger than the wealth effect as we assume here, each bank should tend to increase its holdings in the loans following the shock. However, as the expected value declines and the risk rises, the worsen loan characteristic reduces the loan demands and this brings the price to the new equilibrium. When the price drops, so do banks' equity values. Once the default probability is large enough, it may trigger a default of a bank, and the losses are transmitted through the interbank liability linkages, further reducing the equity of other banks. This reinforces the fire sale in the network.

Let $p^*(i)$ denote the price at which bank i's equity reaches zero due to the increase in

[8]This can be seen from the fact that $\frac{\partial \psi}{\partial \lambda} = -2\Phi\left(\Phi^{-1}(1-\lambda)\sqrt{\frac{1-\rho}{1+\rho}}\right)$ where $\rho = \alpha^2 + \beta^2$.

the default probability. We have

$$p^*(i) = p^h - \frac{e_i}{\theta_i}$$

where p^h and e_i are the equilibrium price and the bank's equity before the shock, respectively. Let us call $p^*(i)$ the *critical price* of bank i. When the default probability rises, the bank that has the highest critical price can be insolvent first. As we can see, that *critical bank* is the bank that initially holds the largest number of loans per one unit of its equity value. If all banks initially hold the loans at the optimal holding level as suggested by (13), the most aggressive bank with the lowest risk aversion parameter tends to be insolvent first when the default probability increases. If the critical bank has large liabilities with other aggressive banks in the network, the contagion effect is much larger once it becomes insolvent.

4.2 Two loan types with equal costs

We now consider the case with two types of loans ($K = 2$). Here we assume that all banks have the same level of expertise in managing loans, and hence the same managing costs. That is, we assume that $f_{i,k} \equiv f_k < 1 - \lambda_k$, for $k = 1, 2$ and $i = 1, \ldots, N$. Again we consider the equilibrium prices before shocks, after a bank shock, and after an asset shock. The main difference between the one-type and two-type cases we consider here is that when there are two types of loans, the demands of the loans of different types interact through the hedging demand and the wealth effect, causing the cross-asset contagion.

4.2.1 Before shocks

Let $(L, \hat{\theta}, p)$ denote the equilibrium triplet at time 1 before shocks, and assume that at the equilibrium all banks hold both types of loans, or $\hat{\theta}_{i,k} = \theta^*_{i,k} > 0$ for all $i = 1, \ldots, N$ and $k = 1, 2$. From (15), this implies that $\bar{\theta}_{i,k} > 0$ for all i and k. Using the optimal holding condition (15) and the clearing condition (7), it can be shown that the equilibrium price vector $p = [p_1, p_2]'$ satisfies the following system of equations:

$$U_1 = (1 - \lambda_1 - f_1 - p_1) \left[\left(\sum_{i=1}^{N} \frac{\theta_{i,1}}{\gamma_i} \right) p_1 + \left(\sum_{i=1}^{N} \frac{\theta_{i,2}}{\gamma_i} \right) p_2 + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} \right] \tag{27}$$

$$U_2 = (1 - \lambda_2 - f_2 - p_2) \left[\left(\sum_{i=1}^{N} \frac{\theta_{i,1}}{\gamma_i} \right) p_1 + \left(\sum_{i=1}^{N} \frac{\theta_{i,2}}{\gamma_i} \right) p_2 + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} \right] \tag{28}$$

where

$$U_1 = (\psi_1 - (1-\lambda_1)^2)\Theta_{S,1} + \frac{N}{2}(1-\lambda_1-\psi_1) + (\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2))\Theta_{S,2}$$

$$U_2 = (\psi_2 - (1-\lambda_2)^2)\Theta_{S,2} + \frac{N}{2}(1-\lambda_2-\psi_2) + (\Psi_{1,2} - (1-\lambda_1)(1-\lambda_2))\Theta_{S,1}$$

and $\Theta_{S,k} = \sum_{i=1}^N \theta_{i,k}$ denotes the total number of type-k loans available in the system. The quantity U_k captures the risk of type-k loans similar to U given by (24) for the one-type case. The difference is that U_k contains an additional term due to the risk from the hedging demand, which is zero if the default correlation between the two types of loans is zero.

Observe from (27) - (28) that the loan prices at the equilibrium have linear relationship:

$$\frac{1-\lambda_1-f_1-p_1}{U_1} = \frac{1-\lambda_2-f_2-p_2}{U_2}. \tag{29}$$

This relationship suggests that the expected profits per unit risk of the two loan types are equal at each equilibrium. Solving (27) - (28), we obtain the following results:

Theorem 2 *Suppose none of the banks is insolvent and the repayment matrix is $X = L$ in the case of two loan types with $f_{i,k} = f_k$ for all $i = 1, \ldots, N, k = 1, 2$. Then an equilibrium price vector exists if, and only if,*

$$(1-\lambda_k - f_k + \zeta_k)^2 \geq \frac{4U_k^2}{\left(\sum_{i=1}^N \theta_{i,k}/\gamma_i\right) U_k + \left(\sum_{i=1}^N \theta_{i,k'}/\gamma_i\right) U_{k'}}, \tag{30}$$

for $k, k' = 1, 2$ and $k \neq k'$, and in that case the equilibrium price vectors $p = [p_1, p_2]'$ are given by

$$p_1 = \frac{1}{2}(1-\lambda_1-f_1-\zeta_1) \pm \frac{1}{2}\left\{(1-\lambda_1-f_1+\zeta_1)^2 - \frac{4U_1^2}{\left(\sum_{i=1}^N \theta_{i,1}/\gamma_i\right) U_1 + \left(\sum_{i=1}^N \theta_{i,2}/\gamma_i\right) U_2}\right\}^{1/2} \tag{31}$$

where

$$\zeta_1 = \frac{\sum_{i=1}^N \frac{\theta_{i,2}}{\gamma_i}\left[U_1(1-\lambda_2-f_2) - U_2(1-\lambda_1-f_1)\right] + \sum_{i=1}^N \frac{\bar{c}_i}{\gamma_i}}{\sum_{i=1}^N \frac{\theta_{i,1}}{\gamma_i} U_1 + \sum_{i=1}^N \frac{\theta_{i,2}}{\gamma_i} U_2} \tag{32}$$

and the corresponding p_2 can be determined from the linear relationship (29).

As mentioned, we focus on the price vector at which the fire sales may occur after a shock. Using the first-order Taylor approximation as in the one-type case, the interested equilibrium price vector $p^h = [p_1^h, p_2^h]'$ is given by

$$p_k^h \approx 1 - \lambda_k - f_k - \frac{U_k}{\sum_{i=1}^{N} \frac{\theta_{i,1}}{\gamma_i}(1-\lambda_1-f_1) + \sum_{i=1}^{N} \frac{\theta_{i,2}}{\gamma_i}(1-\lambda_2-f_2) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i}} \tag{33}$$

for $k = 1, 2$. This is similar to p^h of the one-type case given by (22). However, the denominator of the premium term now contains the expected payoff after cost of both types of loans. In addition, the numerator in the premium term, or U_k, has an extra hedging demand component which links the default probability of one type of loans to the price of the other type of loans. We discuss the implications below.

4.2.2 After a bank shock

Consider a small adverse bank shock on bank j of size v_j. Assume that this shock does not cause any insolvency in the banking system. The equilibrium price changes from $p^h = [p_1^h, p_2^h]'$ as given by (33) to

$$p_k^h(v_j) \approx 1 - \lambda_k - f_k - \frac{U_k}{\sum_{i=1}^{N} \frac{\theta_{i,1}}{\gamma_i}(1-\lambda_1-f_1) + \sum_{i=1}^{N} \frac{\theta_{i,2}}{\gamma_i}(1-\lambda_2-f_2) + \sum_{i=1}^{N} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}.$$

As we can see, the effect is similar to the one-type case; that is, the prices of both types of loans reduce due to the lower wealth in the system, and the effect is large if the bank being hit is an aggressive bank. However, comparing the reduction in the prices, we can see that the price of the loan type that has higher level of riskiness (larger U_k) reduces more. We can also see this from taking the difference due to the shock on both sides of (29) to get

$$\frac{p_1^h - p_1^h(v_j)}{U_1} = \frac{p_2^h - p_2^h(v_j)}{U_2} \quad \Rightarrow \quad p_1^h(v_j) - p_1^h = \frac{U1}{U_2}(p_2^h(v_j) - p_2^h).$$

So the price of the loan with a higher level of riskiness is more sensitive to a bank shock, and the effect is large if the shocks hit an aggressive bank.

Now if the shock is large enough to make bank j become insolvent, but all the other banks are not, the result is similar to the one-type case as given in (26). That is, the intetbank liabilities of the insolvent bank transmit losses to its neighbor banks, and the impact to the loan prices is large if the liabilities are large and the neighbor banks are aggressive banks.

4.2.3 After an asset shock

In this section we focus on how a shock in the default probability of one type of loans causes the change in the price of the other type of loans. Let us assume for the moment that defaults of different types of loans are independent ($\alpha = 0$). Now suppose the default probability of the type-1 loan increases from λ_1 to λ_1'. As mentioned earlier in the one-type case, $\psi_1 - (1-\lambda_1)^2$ and $1 - \lambda_1 - \psi_1$ are increasing in λ_1 when $\lambda_1 < 0.5$. Thus, U_1 increases as the default probability λ_1 increases. We assume further that the increase in the default risk does not cause any banks to become insolvent. Based on (33) for $k = 1$, it is clear that, the equilibrium price of type-1 loan decreases. This is due to the lower expected payoff, higher level of riskiness, and lower wealth in the system. The higher default risk of the type-1 loans also reduces the price of the type-2 loans as can be seen in (33) for $k = 2$. This cross-asset contagion comes from the wealth effect in the denominator of the type-2 loan's premium term.

Note that the cross-asset contagion always occurs as the term $\sum_{i=1}^{N} \theta_{i,1}/\gamma_i$ in the equilibrium price p_2^h is always positive. To see how this happens, we note that when the defaults of the two types of loans are uncorrelated ($\alpha = 0$), $\eta_{1,2} = 0$ and thus the loan demand is $\hat{\theta}_{i,k} = \bar{\theta}_{i,k}$, which is assumed to be positive for all $i = 1, \ldots, N$ and $k = 1, 2$. Now consider $\bar{\theta}_{i,k}$ as given by (13) with the subscript $k = 1, 2$. Let us assume that $e_i \gg 0$ so that the negative adjustment term in (13) is insignificant. As λ_1 increases, the type-1 loan characteristic becomes worsen as the risk ($\psi_1 - (1-\lambda_1)^2$) rises and the expected payoff ($1 - \lambda_1$) declines. This makes $\bar{\theta}_{i,1}$ lower. So the demand for the type-1 loans decreases, and consequently the price of the type-1 loans has to drop to make the expected profit go up to bring the demand back to the balance. But once the price of the type-1 loans decreases, the equity value of each bank holding the type-1 loans decreases, and this reduces demand $\bar{\theta}_{i,2}$ for the type-2 loans due to the wealth effect. As a result, the price of the type-2 loans has to drop to make the expected profit higher and bring the demand back to the balance.

Now consider a little more extreme case in which the banks are divided into two non-overlapping groups, one holding only the type-1 loans and the other holding only the type-2 loans at time 0. The cross-asset contagion still occurs in this case as long as each bank has a demand for both types of the loans at time 1 before the shock. The magnitude of the effect of the cross-asset contagion from type-1 loans to type-2 loans depends, however, on the initial banks' holdings of type-1 loans at time 0, which is $\sum_{i=1}^{N} \theta_{i,1}/\gamma_i$. If initially the majority of the type-1 loans are held by aggressive banks (low γ), the impact of the cross-asset contagion from type-1 to type-2 is large, while the impact is smaller if most of the type-1 loans are held by conservative banks. The latter is unlikely if banks try to hold optimal number of loans at time 0 as aggressive banks tend to hold more loans. In Section 4.3, we discuss the cases where banks may hold only one type of loans at an equilibrium before a shock due to different expertise. In that case, the results can be different.

When the default correlation between the two types is not zero ($\alpha > 0$), the hedging demand term $\Psi_{1,2} - (1 - \lambda_1)(1 - \lambda_2)$ in U_2 can transmit the effect of the increase in the default probability of type-1 loans to the price of type-2 loans. However, the relationship between $\Psi_{1,2} - (1 - \lambda_1)(1 - \lambda_2)$ and λ_1 is not monotone for typical values of λ_1, λ_2 and α. So it is possible that the hedging demand term can strengthen or weaken the contagion effect.

4.3 Two loan types with bank expertise

In this section we assume that each bank has its own expertise in managing one particular type of loans. Let $\mathbb{N}_k$ denote the set of banks that have an expertise in managing type-k loans, $k = 1, 2$. We assume that each bank belongs to either $\mathbb{N}_1$ or $\mathbb{N}_2$, but not both. We call banks that are in $\mathbb{N}_k$ as type-k *expert banks* and those that are not in $\mathbb{N}_k$ as type-k *non-expert* banks. So the banking industry is divided into two sectors defined by $\mathbb{N}_1$ and $\mathbb{N}_2$. The cost associated with managing the loans of type k for type-k expert banks is zero, while the cost for every type-k non-expert bank is $f_k > 0$, which is the same for all non-expert banks. Let N_k denotes the number of banks in $\mathbb{N}_k$. We assume that there is at least one bank for each sector, or $N_k > 0$ for both $k = 1, 2$.

We assume further than the cost of managing loans of type k is so large that it is not optimal for type-k non-expert banks to hold type-k loans in their portfolios at an equilibrium before a shock. We also assume that the initial holdings of type-k loans for type-k non-expert banks are zero due to the high managing cost, or $\theta_{j,k} = 0$ for all $j \notin \mathbb{N}_k$. We are interested in how the loan sectors play a role on contagion risk in the banking network. Again, we consider the equilibrium prices before shocks, after a bank shock, and after an asset shock.

4.3.1 Before shocks

Because we assume that banks in sector k do not hold loans of the other types at an equilibrium before shocks, it must be that $\theta^*_{j,k} \leq 0$ for $j \notin \mathbb{N}_k$. From (16), we have $\hat{\theta}_{i,k} = \bar{\theta}_{i,k}$ and

$\hat{\theta}_{i,k'} = 0$ for $i \in \mathbb{N}_k$ and $k' \neq k$. Thus, the clearing condition (7) gives

$$\Theta_{S,k} = \left[\left(\sum_{i \in \mathbb{N}_k} \frac{\theta_{i,k}}{\gamma_i} \right) p_k + \sum_{i \in \mathbb{N}_k} \frac{\bar{c}_i}{\gamma_i} \right] \left(\frac{1 - \lambda_k - p_k}{\psi_k^2 - (1-\lambda_k)^2} \right) - \frac{N_k}{2} \left(\frac{1 - \lambda_k - \psi_k}{\psi_k^2 - (1-\lambda_k)^2} \right)$$

for $k = 1, 2$. Note that we have used the fact that $\theta_{i,k} = 0$ for $i \notin \mathbb{N}_k$. As we can see, the equilibrium price for each type of loans can be determined independently as the equations for $k = 1, 2$ are decoupled. Thus, as long as banks do not have demands for loans outside their expertise, the equilibrium price of each loan type is determined based on the banks in the sector. This reduces the problem into two independent one-asset equilibrium problems. Hence, we have the equilibrium price of type-k loans is

$$p_k^h \approx 1 - \lambda_k - \frac{(\psi_k - (1-\lambda_k)^2)\Theta_{S,k} + \frac{N_k}{2}(1 - \lambda_k - \psi_k)}{\displaystyle\sum_{i \in \mathbb{N}_k} \frac{\theta_{i,k}}{\gamma_i}(1-\lambda_k) + \sum_{i \in \mathbb{N}_k} \frac{\bar{c}_i}{\gamma_i}}.$$

As we can see, the price of type-1 loans does not depend on the default probability of type-2 loans, nor the default correlation. It does not depend on the information about the banks in sector 2 either. This holds true as long as $\theta_{i,2}^* \leq 0$ for all $i \in \mathbb{N}_2$. So the contagion across banks and loan types is different from the one considered in Section 4.2.

4.3.2 After a bank shock

Suppose there is an adverse small bank shock of size v_j on bank j in sector 1. We assume that after the shock none of the banks are insolvent, and that the price is still high for the banks in sector 2 to buy type-1 loans. In this case, the new equilibrium price for type-1 loans reduces to

$$p_1^h(v_j) \approx 1 - \lambda_1 - \frac{(\psi_1 - (1-\lambda_1)^2)\Theta_{S,1} + \frac{N_1}{2}(1 - \lambda_1 - \psi_1)}{\displaystyle\sum_{i \in \mathbb{N}_1} \frac{\theta_{i,1}}{\gamma_i}(1-\lambda_1) + \sum_{i \in \mathbb{N}_1} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}}$$

while the price of type-2 loans remains the same. That is, there is no contagion from sector 1 to sector 2. The price of type-1 loans reduces because the shock reduces the equity value of bank j, causing the bank to sell off some of the loans, pushing the price down to make it more attractive for the other type-1 expert banks to increase their demands. This is the same as the one-type case.

Now let's assume that the shock v_j is large enough to make bank j insolvent, but not any other banks in the system. Assume further that at the new equilibrium prices, it is not optimal for banks in one sector to buy loans in the other sector. Under these conditions, the new equilibrium price of the type-1 loans is similar to (26) in the one-type case in which the impact on the price depends on the sizes of the interbank liabilities and the risk aversion parameters of the neighbor banks. That is, the new equilibrium is

$$p_1^h(v_j) \approx 1 - \lambda_1 - \frac{(\psi_1 - (1-\lambda_1)^2)\Theta_{S,1} + \frac{N_1}{2}(1 - \lambda_1 - \psi_1)}{\sum_{i \in \mathbb{N}_1, i \neq j} \frac{\theta_{i,1}}{\gamma_i}(1-\lambda_1) + \sum_{i \in \mathbb{N}_1, i \neq j} \frac{\bar{c}_i}{\gamma_i} - \sum_{i \in \mathbb{N}_1, i \neq j} \frac{l_{j,i}}{\gamma_i} \min\left\{ \frac{v_j - (p_1^h(v_j)\theta_{j,i} + \bar{c}_j)}{L_j}, 1 \right\}}.$$

As we can see, only the parameters describing the banking network inside sector 1 are involved.

Let us look at the price of type-2 loans. Only if there is a bank in sector 2 that is an interbank creditor of bank j, the loss of bank j can be transmitted to sector 2 through the interbank channel. This transmitted loss reduces the wealth in the sector, causing the equilibrium price for the type-2 loans to drop. The new equilibrium type-2 price is

$$p_2^h(v_j) \approx 1 - \lambda_2 - \frac{(\psi_2 - (1-\lambda_2)^2)\Theta_{S,2} + \frac{N_2}{2}(1 - \lambda_2 - \psi_2)}{\sum_{i \in \mathbb{N}_2} \frac{\theta_{i,2}}{\gamma_i}(1-\lambda_2) + \sum_{i \in \mathbb{N}_2} \frac{\bar{c}_i}{\gamma_i} - \sum_{i \in \mathbb{N}_2} \frac{l_{j,i}}{\gamma_i} \min\left\{ \frac{v_j - (p_1^h(v_j)\theta_{j,i} + \bar{c}_j)}{L_j}, 1 \right\}}.$$

In addition to the liability sizes $l_{j,i}$ and the risk aversion parameters γ_i of the interbank creditors i in sector 2, the reduction in the type-2 loan price depends also on the price of the type-1 loans after the shock or $p_1^h(v_j)$. So the more sensitive $p_1^h(v_j)$ to the shock, the higher the impact the shock has on $p_2^h(v_j)$. Because the new equilibrium price $p_1^h(v_j)$ depends on the information of all the banks in sector 1, the contagion effect from sector 1 to sector 2 depends on the information of all banks in sector 1 and how they are related. For example, if the insolvent bank j has two interbank creditors, which are bank i in sector 1 and bank i' in sector 2. Given the shock v_j, the change in the price of the type-2 loans depends not only on the information about bank j and the liability link between bank j and bank i' in sector 2, but also the liability link between bank j and bank i in sector 1 as well as the risk aversion parameter of bank i.

When this type of shock gets larger, and more type-1 expert banks become insolvent, the losses from sector 1 can be transmitted to sector 2 via the interbank liabilities between the insolvent banks in sector 1 and the banks in sector 2. So even if there is no direct interbank liability from bank j to any bank in sector 2, the loss originated from the shock on bank j may eventually affect the price of the type-2 loans if there is a liability path starting from bank j to a bank in sector 2.

Now let us consider another possible outcome from the shock v_j. Assume that after the shock, none of the banks is insolvent and the resulting equilibrium price of the type-1 loans is low enough to make it attractive to bank i' in sector 2 to hold some positive number of type-1 loans, but it is not attractive enough for the other banks in sector 2. Suppose for the moment that the defaults of type-1 and type-2 loans are uncorrelated, and thus $\eta_{1.2} = 0$. From (15) and (16) we have the optimal loan holdings of bank i' are $(\hat{\theta}_{i',1}, \hat{\theta}_{i',2}) = (\bar{\theta}_{i',1}, \bar{\theta}_{i',2})$. That is, bank i' does not change the holding in type-2 loans, but increases the holding of type-1 loans from zero to $\bar{\theta}_{i',1}$. As a consequence, the market for type-2 loans is not affected by the shock, resulting in the same equilibrium price for the type-2 loans. On the other hand, there is a loss in the equity of bank j in sector 1 and an additional demand for type-1 loans from bank i' originally from sector 2. Thus the new equilibrium price of the type-1 loans satisfies

$$\begin{aligned}
\Theta_{S,1} = & \left[\left(\sum_{i\in\mathbb{N}_1} \frac{\theta_{i,1}}{\gamma_i}\right) p_1 + \sum_{i\in\mathbb{N}_1} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j}\right]\left(\frac{1-\lambda_1-p_1}{\psi_1^2-(1-\lambda_1)^2}\right) - \frac{N_1}{2}\left(\frac{1-\lambda_1-\psi_1}{\psi_1^2-(1-\lambda_1)^2}\right) \\
& + \left[\left(\frac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \frac{\bar{c}_{i'}}{\gamma_{i'}}\right]\left(\frac{1-\lambda_1-f_1-p_1}{\psi_1^2-(1-\lambda_1)^2}\right) - \frac{1}{2}\left(\frac{1-\lambda_1-\psi_1}{\psi_1^2-(1-\lambda_1)^2}\right) \\
= & \left[\left(\sum_{i\in\mathbb{N}_1} \frac{\theta_{i,1}}{\gamma_i}\right) p_1 + \sum_{i\in\mathbb{N}_1} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j} + \left(\frac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \frac{\bar{c}_{i'}}{\gamma_{i'}}\right]\left(\frac{1-\lambda_1-p_1}{\psi_1^2-(1-\lambda_1)^2}\right) \\
& - \frac{N_1+1}{2}\left(\frac{1-\lambda_1-\psi_1}{\psi_1^2-(1-\lambda_1)^2}\right) - \left[\left(\frac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \frac{\bar{c}_{i'}}{\gamma_{i'}}\right]\left(\frac{f_1}{\psi_1^2-(1-\lambda_1)^2}\right)
\end{aligned}$$

Using the first-order Taylor approximation as in the one-type case, we have the new equilibrium price for type-1 loans is

$$p_1^h(v_j) \approx 1-\lambda_1 - \frac{(\psi_1^2-(1-\lambda_1)^2)\Theta_{S,1} + \dfrac{N_1+1}{2}(1-\lambda_1-\psi_1) + \left[\left(\dfrac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \dfrac{\bar{c}_{i'}}{\gamma_{i'}}\right] f_1}{\left(\displaystyle\sum_{i\in\mathbb{N}_1} \frac{\theta_{i,1}}{\gamma_i}\right)(1-\lambda_1) + \displaystyle\sum_{i\in\mathbb{N}_1} \frac{\bar{c}_i}{\gamma_i} - \frac{v_j}{\gamma_j} + \left[\left(\dfrac{\theta_{i',2}}{\gamma_{i'}}\right) p_2 + \dfrac{\bar{c}_{i'}}{\gamma_{i'}}\right]}. \tag{34}$$

It is easy to show that the additional demand from bank i' for the type-1 loans helps reduce the effect of the shock on the type-1 loan price given that it is optimal for the type-2 expert bank i' to enter into sector 1. Moreover, as loan defaults are uncorrelated, this does not hurt the price of the type-2 loans. To further understand this situation, let us identify the bank in sector 2 that actually is bank i'. To do this, consider a type-2 expert bank i. We can rewrite (13) for bank i after the shock as follows:

$$\bar{\theta}_{i,1}(v_j) = \left(\frac{\theta_{i,2}p_2 + \bar{c}_i}{\gamma_i}\right)\left[\frac{(1-\lambda_1) - p_1^h(v_j) - f_1}{\psi_1 - (1-\lambda_1)^2}\right] - \frac{1}{2}\left[\frac{1-\lambda_1-\psi_1}{\psi_1 - (1-\lambda_1)^2}\right]. \tag{35}$$

As $p_1^h(v_j)$ decreases, the value of $\bar{\theta}_{i,1}(v_j)$ increases as there is no wealth effect for bank i in sector 2. So the bank that has the largest $\bar{\theta}_{i,1}(v_j)$ for $i \in \mathbb{N}_2$ is the bank i'. It is easy to see from (35) that it is the bank with the largest equity to risk aversion parameter ratio (e/γ) among the type-2 expert banks as all the banks in sector 2 has the same managing cost of f_1.

Now if loan defaults are correlated, or $\eta_{1,2} > 0$, the demand for type-1 loans from bank i' will lead to a decline in the demand for type-2 loans from bank i' due to the negative hedging demand. As the price effect is stronger than the wealth effect at the equilibrium we are interested in, the price of the type-2 loans must drop to bring the type-2 loan demand up to meet the total supply. Hence this creates a cross-asset contagion purely through the hedging demand. We do not require any interbank liabilities, nor do we require a bank to hold both types of loans at time 0 to act as a channel to transmit the effect from one type of loans to the other type of loans through the reduction in the equity value of the bank.

Now consider another alternative outcome. Suppose that the shock v_j causes a bank in sector 1 to become insolvent, and the insolvent bank has interbank liabilities with some type-2 expert banks. The resulting equilibrium prices depends on these liabilities. If banks that hold claims on the insolvent bank j are the ones with low e/γ ratios, then it is possible that the type-2 expert banks with the largest e/γ ratio will find the drop in the type-1 loan price attractive enough to buy them into the bank's balance sheet, reducing the effect on the new equilibrium price of the type-1 loans, but at the same time causing the contagion to the price of the type-2 loans due to the negative hedging demand. On the contrary, if banks that hold claims on the insolvent bank j are the ones with highest e/γ ratios, the reduction in the equity values of those type-2 expert banks could reduce the possibility for them to be the potential buyers of type-1 loans. This results in a worse outcome for the price of the type-1 loans as no new buyers from sector 2. This suggests that interbank liabilities of this type weaken the role as the potential buyers of the banks with largest e/γ.

4.3.3 After an asset shock

Suppose the default probability of the type-1 loan increases from λ_1 to λ_1'. As long as the new equilibrium price of the type-1 loans does not fall enough to attract type-2 expert banks to buy type-1 loans, and there are no losses transmitted through the liability linkages to banks in sector 2, this does not affect the equilibrium price of the type-2 loans. But once one of those scenarios occurs, the price of the type-2 loans reduces due to either the negative hedging demand, provided that $\eta_{1,2} > 0$, or the reduction in the equity values of some type-2 expert banks similar to the case of a bank shock discussed above.

The above discussion leads to an interesting policy implication. Suppose that the costs of managing loans are low, but the regulator would like to separate banks and loan markets into non-overlapping sectors to limit the effect of contagion. As a consequence, the regulator may allow banks to choose their areas of expertise or sectors where they can run their businesses as usual. However, banks need to pay a huge regulatory fee to do the business outside their selected areas of expertise. When a sector is hit by a small shock, the banks in the sector can function as potential buyers to self-rescue the sector from fire sales. When the shock is large, and there are not many banks in the sector that can function as the potential buyers, the regulator can initiate the self-rescue mission by allowing the secondary potential buyers from the other healthy sectors to step in and buy the assets, reducing the effect of fire sales in the failing sector. The regulator can choose the right value of $f_{i,k}$ to allow enough funds from other sectors to flow into the failing sector, providing support to the loan price in the failing sector. At the same time the regulator needs to avoid the unintended contagion effects due to the negative hedging demands.

Once the cross-sector rescue mission has been accomplished, the healthy sector is now contaminated by the fire sale loans, and cannot function as secondary potential buyers for the next crisis. So the regulator should use this as a temporary solution to reduce the effect of fire sales, and start to bring everything back to normal and be ready for the next crisis.

5 Conclusion

When an adverse shock hits a bank, causing it to become insolvent, the bank needs to sell all of its assets, the majority of which are illiquid loans. This can cause the loan prices to drop, reducing the mark-to-market values of other banks holding the same types of loans. The loss of the insolvent bank can also be transmitted to other banks through the interbank liability linkages, reducing the net worth of its neighbor banks. The banks affected by these two channels of contagion will re-adjust their portfolios in response to lower equity values, and start to sell more illiquid loans into the markets. If banks are highly connected either through the liability linkages or the common loan holdings, then most of the banks in the system will suffer from the losses and cannot function as the potential buyers, reducing the self-rescue ability of the system.

We study a financial system in which banks in the system may create self-rescue ability. We find that aggressive banks can become good potential buyers if they are not affected by a shock as they are willing to buy a large amount of loans given a small discount. At the same time, they can become fire sale initiators even if they are hit by a relatively small shock. Interbank liabilities between these aggressive banks can also amplify the contagion effect and the effect of fire sales as they adjust the portfolios markedly following losses in their equity values. So it is better to avoid having interbank liabilities between those aggressive banks. We also find that prices of loans that have higher risk are more sensitive to a shock in the system. So having aggressive banks holding these high risk loans would accelerate the contagion effect, once it occurs. Unfortunately, aggressive banks tend to hold a large amount of loans, including the high risk loans, so we need some regulatory policies to help reduce the potential damages caused by these aggressive banks.

Contagion across loan types can occur from many channels. A shock to one particular bank may trigger a fire sale of one loan type, lowering the loan price. Banks that hold the same type of loans will lose their equity values and start to sell loans of other types in their portfolios to re-adjust their portfolios' risk-adjusted returns. This creates the cross-asset contagion. Alternatively, a drop of the price of one loan type makes it more attractive to healthy banks to buy the loans. As loan defaults are positively correlated, the substitute effect creates a negative hedging demand, requiring the banks to reduce the holdings of certain types of loans in their portfolios upon buying another type of loans.

Finally, we study the system in which banks and loan markets are separated into sectors based on their areas of expertise defined by the cost of managing loans. We find that small shocks in one sector do not cause contagion to the others as long as the interbank liabilities between the sectors are not available and the cost for entering an area outside of the banks' expertise is sufficiently high. In this case, the banks in the sector experiencing a small shock need to act as potential buyers for their own sectors. Once the shock is large, causing the price to drop enough, then banks from other sectors may function as potential buyers for the failing sector. Based on this observation, we propose a policy that separates banks and loans into sectors, limiting the contagion effects between groups of banks and types of loans. At the same time, we create secondary potential buyers that are ready to step in and save the failing sector when it is most needed. This type of policy can be achieved by imposing regulatory fees that keep them separated during good times, and allow them to rescue their peers during bad times, creating a self-rescue system.

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