$$\xi_s \frac{d}{dx} F_s(0) = \gamma[PF_s(0) - e^{-i\varphi} RF_s(d_s)], \tag{27}$$

$$\xi_s \frac{d}{dx} F_s(d_s) = \gamma[e^{i\varphi} RF_s(0) - PF_s(d_s)], \tag{28}$$

with

$$P = \widetilde{k}_f \xi_f [\coth(\widetilde{k}_f d_f) + \gamma_b \widetilde{k}_f \xi_f]/M, \tag{29}$$

$$R = \widetilde{k}_f \xi_f / (M \sinh(\widetilde{k}_f d_f)), \tag{30}$$

$$M = 1 + 2\gamma_b \widetilde{k}_f \xi_f \coth(\widetilde{k}_f d_f) + \gamma_b^2 (\widetilde{k}_f \xi_f)^2, \tag{31}$$

are the material functions of the F layers. The parameters γ and γ_b represent a leakage of paired electrons from S to F, and a jump at the interfaces due to the transparency of materials, respectively [17]. The phase shift φ has been introduced to account for the pi-phase state.

In order to determine the upper critical fields and the superconducting critical temperatures, we have to solve the above equations for $F_s(x,\omega)$ and $\Delta(x)$ self-consistently subject to the boundary conditions. Instead of $F_s(x,\omega)$, let us consider the diffusive kernel $Q_\omega(x,x')$ for the S layer, and its differential equations. The exact multimode solution can be found by employing the method of eigenfunction expansion, that is

$$Q_\omega(x,x') = \sum_{m=-\infty}^{\infty} Q_\omega(q_m,x') \cos(q_m x), \tag{32}$$

with $q_m = m\pi/d_s$. The nontrivial solution of the transformed $\Delta(x)$ equation obeys the secular equation

$$\det \left| \delta_{mm'} - \lambda \frac{T}{T_{cs}} \sum_\omega L_{mm'}(\omega) \right| = 0, \tag{33}$$

where we have denoted

$$L_{mm'}(\omega) = \pi T_{cs} \int_0^{d_s} dx' Q_\omega(q_m,x') \cos(q_{m'} x'). \tag{34}$$

Having obtained $Q_\omega(q_m,x')$, by the Fourier transformation of the differential equation of $Q_\omega(x,x')$, $L_{mm'}(\omega)$ is easily evaluated and it final form depends on the field orientations and the boundary conditions of the layered structures. The straightforward calculations of the matrix element $L_{mm'}(\omega)$ in the parallel field orientation yield

$$\begin{aligned} L^{bi}_{mm'\|}(\omega) = {} & \frac{\delta_{mm'}}{\xi_s^2(k_{s\|}^2 + q_m^2)} - \frac{\Omega_\|(\omega)}{\xi_s^4(k_{s\|}^2 + q_m^2)(k_{s\|}^2 + q_{m'}^2)} \\ & + \frac{2\Omega_\|(\omega) h_{c2\|}^2}{\xi_s^2(k_{s\|}^2 + q_m^2)} \\ & \times \sum_{l=-\infty}^{\infty} \left(\frac{x_0}{d_s} v_l^+ + \left(1 - \frac{x_0}{d_s}\right)(-1)^l v_l^- \right) L^{bi}_{lm'\|} \\ & - \frac{2h_{c2\|}^2}{\xi_s^2(k_{s\|}^2 + q_m^2)} \\ & \times \sum_{l \neq m} \left(\frac{x_0}{d_s} + \left(1 - \frac{x_0}{d_s}\right)(-1)^{m+l} \right) \frac{L^{bi}_{lm'\|}}{\xi_s^2(q_m - q_l)^2}, \end{aligned} \tag{35}$$

for the bilayer, where

$$\Omega_\|(\omega) = \frac{W(\omega)}{(d_s/\xi_s)\left(1 + \frac{W(\omega)}{k_{s\|}\xi_s} \coth(k_{s\|} d_s)\right)}, \tag{36}$$

$$v_l^\pm = \sum_{m \neq l, m=-\infty}^{\infty} \frac{(\pm 1)^m}{\xi_s^4 (q_m - q_l)^2 (k_{s\|}^2 + q_m^2)}, \tag{37}$$

$$k_{s\|}^2 = k_s^2 + \left(\frac{h_{c2\|} d_s}{\xi_s^2} \right)^2 \left(\frac{1}{3} - \frac{x_0}{d_s} + \left(\frac{x_0}{d_s} \right)^2 \right), \tag{38}$$

with $h_{c2\|} = 2\pi H \xi_s^2/\phi_0$ is the dimensionless parallel orbital magnetic field, and

$$\begin{aligned} L^{sl}_{mm'\|} = {} & \frac{\delta_{mm'}}{\xi_s^2(k_{s\|}^2 + q_m^2)} \\ & - \frac{\gamma X_{mm'\|}}{(d_s/\xi_s) Y_\| \xi_s^4 (k_{s\|}^2 + q_m^2)(k_{s\|}^2 + q_{m'}^2)} \\ & + \frac{2\gamma h_{c2\|}^2}{(d_s/\xi_s) Y_\| \xi_s^2 (k_{s\|}^2 + q_m^2)} \\ & \times \sum_{l=-\infty}^{\infty} \left(\frac{x_0}{d_s} Z_{ml}^+ + \left(1 - \frac{x_0}{d_s}\right)(-1)^l Z_{ml}^- \right) L^{sl}_{lm'\|} \\ & - \frac{2h_{c2\|}^2}{\xi_s^2(k_{s\|}^2 + q_m^2)} \\ & \times \sum_{l \neq m} \left(\frac{x_0}{d_s} + \left(1 - \frac{x_0}{d_s}\right)(-1)^{m+l} \right) \frac{L^{sl}_{lm'\|}}{\xi_s^2(q_m - q_l)^2}, \end{aligned} \tag{39}$$

for the superlattice, where

$$\begin{aligned} X_{mm'\|} = {} & [1 + (-1)^{m+m'}]P - [(-1)^{m'} e^{-i\varphi} + (-1)^m e^{i\varphi}]R \\ & + \frac{\gamma[1 + (-1)^{m+m'}]}{k_{s\|}\xi_s \sinh(k_{s\|} d_s)} (\cosh(k_{s\|} d_s) - (-1)^m)(P^2 - R^2), \end{aligned} \tag{40}$$

$$\begin{aligned} Y_\| = {} & 1 + \frac{2\gamma}{k_{s\|}\xi_s \sinh(k_{s\|} d_s)} (P \cosh(k_{s\|} d_s) - \cos\varphi R) \\ & + \left(\frac{\gamma}{k_{s\|}\xi_s} \right)^2 (P^2 - R^2), \end{aligned} \tag{41}$$

$$\begin{aligned} Z_{ml}^\pm = {} & (v_l^\pm + (-1)^m v_l^\mp)P - ((-1)^m e^{i\varphi} v_l^\pm + e^{-i\varphi} v_l^\mp)R \\ & + \frac{\gamma[v_l^\pm + (-1)^m v_l^\mp]}{k_\| \xi_s \sinh(k_{s\|} d_s)} (\cosh(k_{s\|} d_s) - (-1)^m)(P^2 - R^2). \end{aligned} \tag{42}$$

The analogous formulae for the perpendicular field orientation are easily obtained by the following procedure: (i) replace $k_\|$ by $k_\perp$, (ii) change the subscript $\|$ in $\Omega_\|$, $X_{mm'\|}$, and $Y_\|$, to $\perp$, and (iii) take $L_{lm'}(\omega)$ to be zero.

An analytical approach using the secular Eq. (33) in the single mode approximation leads to the Abrikosov–Gorkov like-formula [18] and to the determination of the phase diagram (H,T) under the assumptions that (i) the S layer is thin $d_s/\xi_s \ll 1$, and (ii) the propagating momentum in F can be approximated to be frequency independent, so

$$\widetilde{k}_f \approx \frac{1}{\xi_f}\sqrt{\frac{1/\tau_m + 1/\tau_{so} + i\sqrt{I^2 - (1/\tau_{so})^2}}{\pi T_{cs}}}.$$

Then the (0,0) element of (33) yields the simple formula

$$\ln t = \psi\left(\frac{1}{2}\right) - \mathrm{Re}\,\psi\left(\frac{1}{2} + \frac{\varrho(t)}{2t}\right), \qquad (43)$$

where $t = T/T_{cs}$ is the reduced temperature, $\psi(x)$ is the digamma function, and the complex pair-breaking parameters

$$\varrho(t) = \varrho(t_c) + h_{c2\perp}(t), \qquad (44)$$

$$\varrho(t) = \varrho(t_c) + h_{c2\|}^2(t)\left(\frac{d_s}{\xi_s}\right)^2\left(\frac{1}{3} - \frac{x_0}{d_s} + \left(\frac{x_0}{d_s}\right)^2\right), \qquad (45)$$

for the perpendicular and the parallel magnetic fields, respectively. Eqs. (44) and (45) contain both contributions from the strong ferromagnetic exchange field $\varrho(t_c)$ and the orbital field effects depending on the orientations of applied magnetic fields either perpendicular or parallel to the layered planes. The orbital field completely vanishes at a transition point $t = t_c$ so (43) implies the equation for the variation of the superconducting transition temperature as a function of material parameters,

$$\varrho^{bi}(t_c) = \frac{\gamma(\xi/d_s)}{\gamma_b + \coth(\widetilde{k}_f d_f)/\widetilde{k}_f\xi_f}, \qquad (46)$$

$$\varrho^{sl}(t_c, \varphi) = \frac{2\gamma}{(d_s/\xi_s)}(P - \cos\varphi R) + \gamma^2(P^2 - R^2). \qquad (47)$$

Near T_c, we find that the universal ratio $h_{c2\|}^2/h_{c2\perp}$ does not depend on temperature, material parameters of the F film, pair-breaking scattering rate, number of layers, and phase shift angle, i.e.

$$\frac{h_{c2\|}^2}{h_{c2\perp}} = \frac{1}{(d_s/\xi_s)^2\left(\frac{1}{3} - \frac{x_0}{d_s} + \left(\frac{x_0}{d_s}\right)^2\right)}. \qquad (48)$$

By taking the nucleation center at the mid S film, $x_0/d_s = 1/2$, which corresponds to the decoupled regime, we reobtain the result of Radović et al. [19].

3. Numerical results

We present the numerical calculations of the upper critical fields and critical temperatures of F/S bilayers (Figs. 1 and 2) and F/S multilayers (Figs. 3–6). For the bilayered structure the $(h_{c2\|}, t)$ phase diagrams are drawn to investigate the dimensional crossover behavior by varying the parameters associated with the spin–orbit scattering $1/I\tau_{so}$, and the spin–flip scattering $1/I\tau_m$, in Figs. 1 and 2, respectively. We consider the case of weak ferromagnet ($I/\pi T_{cs} = 10$), weak proximity effect ($\gamma = 0.2$), and low boundary resistivity ($\gamma_b = 0.1$). The position of superconducting nucleus $x_0/d_s = 0.55$ corresponds to the highest field $h_{c2\|}(t)$,

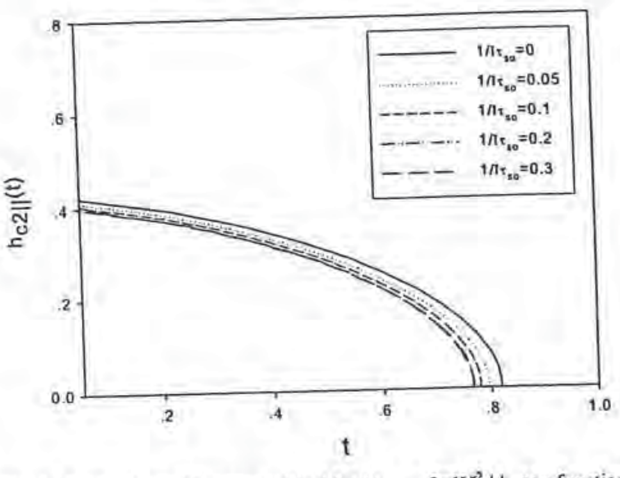

Fig. 1. Reduced parallel upper critical field $h_{c2\|} = 2\pi H\xi_s^2/\phi_0$ as a function of reduced temperature $t = T/T_{cs}$ for several values of $1/I\tau_{so}$. $I/\pi T_{cs} = 10$, $d_s/\xi_s = 4$, $d_f/\xi_f = 2$, $\gamma = 0.2$, $\gamma_b = 0.1$, $x_0/d_s = 0.55$, and $1/I\tau_m = 0$.

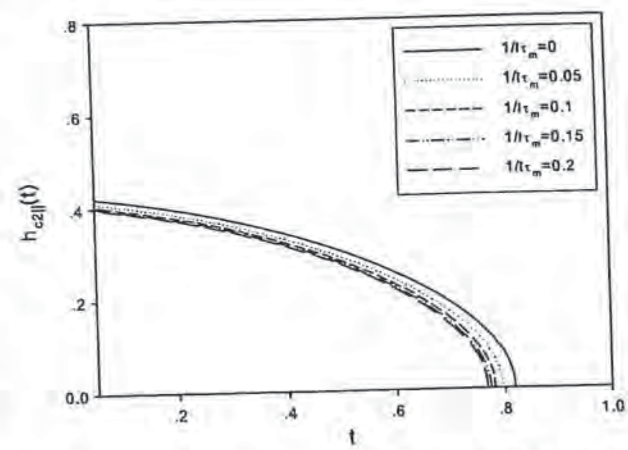

Fig. 2. Reduced parallel upper critical field $h_{c2\|}$ versus reduced temperature t with varying $1/I\tau_m$. $1/I\tau_{so} = 0$, and other parameters are the same as in Fig. 1.

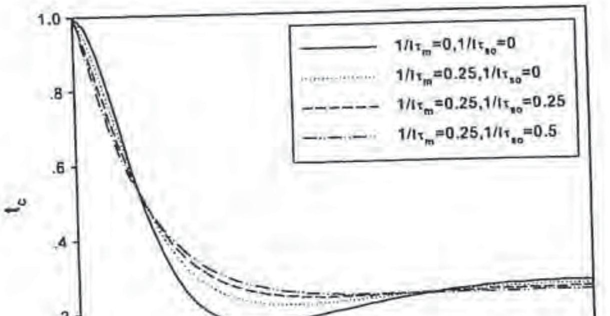

Fig. 3. The reduced superconducting transition temperature $t_c = T_c/T_{cs}$ as a function of the reduced ferromagnetic layer thickness d_f/ξ_f with varying $1/I\tau_{so}$, and $1/I\tau_m$. $I/\pi T_{cs} = 30$, $d_s/\xi_s = 3$, $\gamma = 0.2$, $\gamma_b = 0.3$, and $\varphi = 0$ (zero-phase).

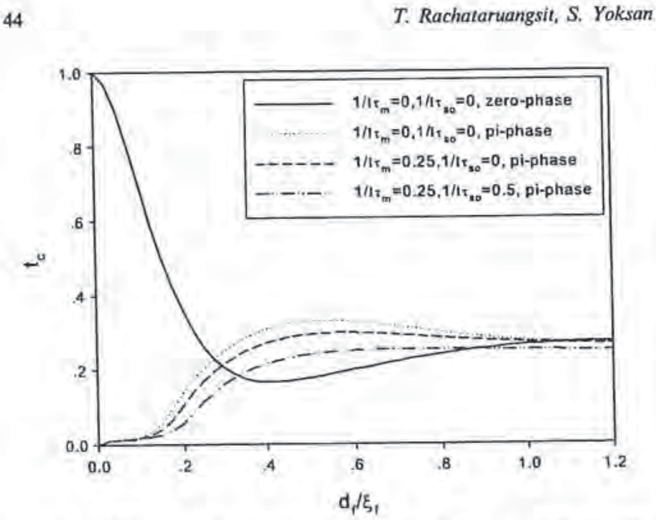

Fig. 4. $t_c(d_f/\xi_f)$ curves between zero-phase ($\varphi = 0$) and pi-phase ($\varphi = \pi$) for several values of $1/l\tau_{so}$, and $1/l\tau_m$. The values of other parameters are the same as in Fig. 3.

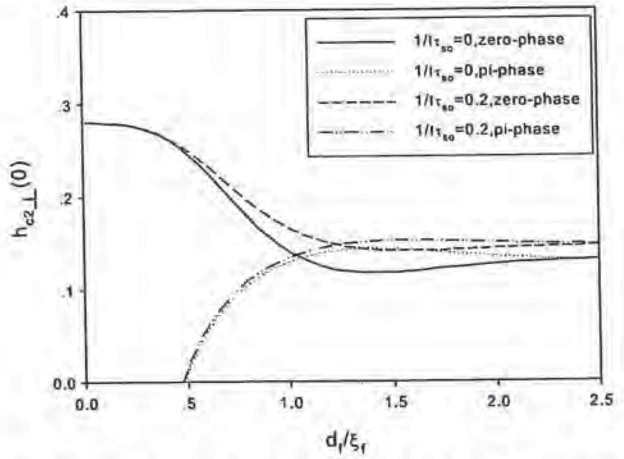

Fig. 5. Reduced perpendicular upper critical field at zero temperature $h_{c2\perp}(0)$ as a function of reduced ferromagnetic layer thickness d_f/ξ_f between zero-phase and pi-phase for two values of $1/l\tau_{so} = 0$, and 0.2. The other parameters are $I/\pi T_{cs} = 10$, $d_s/\xi_s = 3$, $\gamma = 0.1$, and $\gamma_b = 0$.

when the film thicknesses are $d_s/\xi_s = 4$ for S and $d_f/\xi_f = 2$ for F. Since for the thicker S layer the parallel upper critical field $h_{c2\|}(t)$ displays a 2D feature independent of the exchange field strength and the F layer thickness, we find that a 2D behavior is still retained for any values of the scattering rates either spin–orbit or spin–flip. Both pair-breaking scatterers provide identical results, and in general, $h_{c2\|}(t)$ decreases as $1/\tau_{so}$ or $1/\tau_m$ increases and becomes saturated when $1/\tau_{so}$, $1/\tau_m \approx 0.2I$. This means we cannot distinguish which pair-breaking mechanism is responsible for the reduction of the parallel critical field $h_{c2\|}(t)$. Though the spin–orbit term is coupled to the exchange field through the coupling parameter α_ω which suppresses the oscillations of the pair amplitude in F as defined by the imaginary part of $\tilde{k}_f$, whereas its real part which also contains the spin–flip term implies an extra decay of the pair amplitude. As already mentioned above, a 2D behavior is not altered in

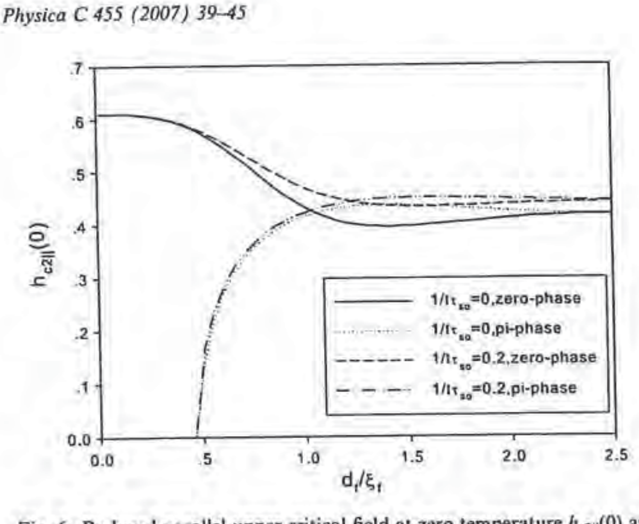

Fig. 6. Reduced parallel upper critical field at zero temperature $h_{c2\|}(0)$ as a function of reduced ferromagnetic layer thickness d_f/ξ_f between zero-phase and pi-phase for two values of $1/l\tau_{so} = 0$, and 0.2. The other parameters are the same as in Fig. 5.

any way when the exchange field changes, therefore the only important factor which causes the effect will come from the real part of $\tilde{k}_f$ which in turn makes the Cooper pair amplitude decay spatially and becomes conduction electrons in the F region and consequently this results in the reduction of $h_{c2\|}(t)$.

For multilayer systems, we examine the influence of the spin-dependent scattering on both the critical temperature and the zero temperature upper critical fields as a function of the ferromagnetic layer thickness. In Fig. 3, we show the variations of $t_c(d_f)$ in a zero-phase ($\varphi = 0$) for several values of the pair-breaking parameters $1/l\tau_{so}$, and $1/l\tau_m$. The influence of the scattering processes emerge when the exchange field is sufficiently strong ($I/\pi T_{cs} = 30$). Therefore the S layer thickness is not supposed to be thin ($d_s/\xi_s = 3$). In the absence of scattering processes with a perfect transparency ($\gamma_b = 0$), $t_c(d_f)$ exhibits the nonmonotonic decay within a single F layer. Upon the introduction of the spin–orbit and the spin–flip scattering, t_c decreases at first near the interface. An enhancement of t_c occurs later inside F with less pronounced nonmonotonicity. As is well known [9], the spin–orbit interaction produces a smaller exchange field and gives a higher critical temperature resulting from an increase of the oscillation period of the pair amplitude. However, the spin–flip scattering plays the same role as the spin–orbit does. Furthermore, Fig. 4 demonstrates the competition between a zero-phase and a pi-phase. Unfortunately, the highest critical temperature in the pi-phase state is most probable in the absence of the pair-breaking processes. Our obtained result is therefore in contrast to the previous study by Oh et al. [10] who argued that the pi-phase shift solution yields higher critical temperatures as the spin–orbit scattering increases. We would like to point out here that our solutions for t_c and $h_{c2\perp}$ are the correct ones because our formulae are found to agree perfectly with the case of an ordinary ferromagnet i.e., without the spin-dependent scattering process [15,16].

[9–14]. Many experimental reports showed good agreement with the theory [15–18], but some works provided the opposite results [19,20]. Another interesting effect is the Radovic pi-phase state in SC/FM multilayers where the theory [21] predicted that the phase shift of the pair amplitudes between the adjacent SC layers can take either the zero-phase or the pi-phase [22–25]. The observations by Jiang et al. [26,27], Obi et al. [28], and Shelukhin et al. [29] verified this prediction.

All above-mentioned effects involve only the case of a homogeneous exchange interaction in the FM layer where it is assumed that the uniformity magnetic moments exist throughout the sample and orient in the specific direction. This type of magnetization causes the pair function in SC to penetrate into FM for a short distance of the order of the exchange length $\xi_I = \sqrt{D_f/I}$, where D_f is the diffusion constant and I is the exchange field strength, and the pair amplitude oscillates spatially as a result of the broken time-reversal symmetry of the spin singlet pair. However, the situation changes drastically if the magnetization is not homogeneous and its direction does not point along the spin quantization axis. In this case a new type of the triplet condensate arises [5]. Apart from the triplet component with zero spin magnetic moment projection $S_z = 0$ which has the short coherence length ξ_I like the singlet condensate, there are also other triplet components with nonzero spin projections $S_z = \pm 1$ which are independent of the exchange field and can penetrate the FM over a long-range scale of the order of $\xi_f = \sqrt{D_f/2\pi T_{c0}}$, where T_{c0} is the isolated superconducting critical temperature, without being destroyed by the impurity scattering. This type of the triplet condensate can survive in the diffusive metals when the following conditions are met: (i) the SC layer is a conventional s-wave spin singlet superconductor, (ii) the phonon mediated electron–electron pairing interaction is in the s-wave channel, (iii) the majority and minority spin bands are treated to be identical, (iv) the induced triplet pairing is independent of the momentum direction in order to be consistent with the diffusion process but odd in the Matsubara frequency, (v) the short-range triplet component is generated when the exchange field is uniform in FM, and (vi) the long-range triplet components require the inhomogeneous magnetization to induce them. The idea of the odd frequency triplet condensate was first suggested a long time ago by Berezinskii [30] and later has been applied to SC/FM systems [31,32].

In the case of a homogeneous magnetization, the influence of the non-collinear magnetization orientation in FM/SC/FM trilayers on the T_c has been studied theoretically in Refs. [33–35]. However, in real situations the magnetic structure consists of a domain with alternate regions of fixed magnetization and each domain is separated by a wall. Inside the wall the region is characterized by the presence of the inhomogeneous magnetization, and the long-range triplet components can arise inevitably. The consideration of a realistic domain wall structure is mathematically very complicated. Therefore to reduce the complexity of the problem it is quite suitable to study a local inhomogeneous exchange field in a SC/FM bilayer system.

A simple model of a rotating magnetization in the FM layer has been investigated recently by many authors [31,32,36–41]. Usually, there are two types of the spiral magnetic orders, rotating in the plane and perpendicular to the SC/FM interface. A Bloch-type spiral structure is typically modeled by the magnetization vector lying in the plane of the FM layer and rotating with increasing length in the perpendicular direction. This model was considered in Refs. [31,32,36]. Another type is a Néel-like spiral structure, where the magnetization rotates in the plane and is constant in the direction normal to the film. This structure was analyzed theoretically in Refs. [37–41]. Physically, this latter model is more relevant for the bilayer system and might be the basic mechanism responsible for the enhancement of the superconductivity in the presence of domain walls during the switching of the magnetization [42,43].

All the previous theoretical studies were done within the framework of the quasiclassical Green function and the system is in the dirty limit [44–47]. Our particular interest is paid to the works of Champel and Eschrig [39,40] who developed a theory of triplet proximity effect in the Matsubara representation and investigated the effect of the Néel type spiral magnetic order on the T_c of SC/FM hybrid structures. Surprisingly, the authors considered only the homogeneous case and concluded that the long-range triplet components are absent, although the in-plane spiral magnetic structure is inhomogeneous. It is worth mentioning that the inhomogeneity of the magnetic moments unavoidably induces the long-range triplet components and causes the modulation of the pair amplitude in the transverse direction in a form of the FFLO (Fulde–Ferrel–Larkin–Ovchinnikov) phase [48,49].

To rectify this situation we find it necessary to reinvestigate the SC/FM bilayer with emphasis on the possible formation of the long-range triplet pair. This paper is organized as follows. In Section 2, we present a formulation of triplet proximity effect by extending the Takahashi–Tachiki theory [50] within the de Gennes correlation function approach [51,52], so the spin orbit interaction can be included in a phenomenological way [53]. The obtained Usadel transport-like equations agree perfectly with Refs. [39,40] in the absence of the spin orbit scattering, where Ref. [34] does not. In Section 3, we study the model of the inhomogeneous Néel type spiral exchange field in the FM/SC bilayer and calculate exactly the secular equation to determine T_c as a function of a rotating spiral wave vector and an in-plane momentum which represents the modulation of the pair amplitude in the transverse direction. In Section 4, we perform the numerical calculations of the T_c variations to show the behavior of the induced long-range triplet components. Finally, conclusions are presented in Section 5.

While $\hat{U}_{so}$ preserves the time-reversal invariance

$$\hat{K}^{\dagger}\hat{U}_{so}\hat{K} = +\hat{U}_{so}. \tag{14}$$

To resolve this difficulty, one uses the trick due to Auvil et al. [53], that is a combination of $\hat{K}$ and its transpose $\hat{K}^{tr}$ equals zero at the time $t = 0$. Hence we obtain the equation of motion of $\hat{K}^{\dagger}$

$$\frac{\partial}{\partial t}\hat{K}^{\dagger}(t) = \mathrm{i}(\hat{U}_{ex}\hat{K}^{\dagger}(t) + \hat{K}^{\dagger}(t)\hat{K}(0)\hat{U}_{ex}\hat{K}^{\dagger}(0)) - \frac{1}{\tau'}(\hat{K}^{\dagger}(t) + \hat{K}^{\dagger tr}(t)), \tag{15}$$

where τ' is the phenomenological spin orbit scattering time.

Combining (11) and (15) to obtain the equation of motion for the matrix correlation function $\hat{g}_{\xi=0}(\mathbf{r}, \mathbf{s}, t)$, one arrives at the following equation

$$\begin{aligned}\frac{\partial}{\partial t}\hat{g}_{\xi=0}(\mathbf{r}, \mathbf{s}, t) &= D(\mathbf{r})\nabla^2\hat{g}_{\xi=0}(\mathbf{r}, \mathbf{s}, t) - \mathrm{i}(\mathbf{I}(\mathbf{r}) \cdot \hat{\sigma}\hat{g}_{\xi=0}(\mathbf{r}, \mathbf{s}, t) \\ &\quad + \hat{g}_{\xi=0}(\mathbf{r}, \mathbf{s}, t)\mathbf{I}(\mathbf{r}) \cdot \hat{\sigma}) - \frac{1}{\tau'}(\hat{g}_{\xi=0}(\mathbf{r}, \mathbf{s}, t) \\ &\quad - \sigma_y\hat{g}^{tr}_{\xi=0}(\mathbf{r}, \mathbf{s}, t)\sigma_y).\end{aligned} \tag{16}$$

Utilizing (8), (10) and (16) to yield the differential equation for the matrix kernel $\hat{Q}_{\omega}(\mathbf{r},\mathbf{s})$ in the diffusive regime, one has

$$\begin{aligned}&\left(|\omega| - \frac{1}{2}D(\mathbf{r})\nabla^2\right)\hat{Q}_{\omega}(\mathbf{r}, \mathbf{s}) + \frac{\mathrm{i}}{2}(\mathbf{I}(\mathbf{r}) \cdot \hat{\sigma}\hat{Q}_{\omega}(\mathbf{r}, \mathbf{s}) \\ &\quad + \hat{Q}_{\omega}(\mathbf{r}, \mathbf{s})\mathbf{I}(\mathbf{r}) \cdot \hat{\sigma}) + \frac{1}{2\tau'}(\hat{Q}_{\omega}(\mathbf{r}, \mathbf{s}) - \sigma_y\hat{Q}^{tr}_{\omega}(\mathbf{r}, \mathbf{s})\sigma_y) \\ &= \sigma_0\delta(\mathbf{r} - \mathbf{s}).\end{aligned} \tag{17}$$

In the limiting case, where the exchange field points in a particular direction, we choose the z-axis as the spin quantization axis so that

$$\mathbf{I}(\mathbf{r}) \cdot \hat{\sigma} = I(\mathbf{r})\sigma_z, \tag{18}$$

and introduce the scalar kernels $Q_{\omega}(\mathbf{r}, \mathbf{s})$ and $R_{\omega}(\mathbf{r},\mathbf{s})$ through the relations

$$Q_{\omega}(\mathbf{r}, \mathbf{s}) = \frac{1}{2}\mathrm{Tr}\hat{Q}_{\omega}(\mathbf{r}, \mathbf{s}), \tag{19}$$

and

$$R_{\omega}(\mathbf{r}, \mathbf{s}) = \frac{1}{2}\mathrm{Tr}\sigma_z\hat{Q}_{\omega}(\mathbf{r}, \mathbf{s}). \tag{20}$$

Applying (18)–(20) to (17), we obtain the coupled differential equations

$$\left(|\omega| - \frac{1}{2}D(\mathbf{r})\nabla^2\right)Q_{\omega}(\mathbf{r}, \mathbf{s}) + \mathrm{i}I(\mathbf{r})R_{\omega}(\mathbf{r}, \mathbf{s}) = \delta(\mathbf{r} - \mathbf{s}), \tag{21}$$

$$\left(|\omega| - \frac{1}{2}D(\mathbf{r})\nabla^2 + \frac{1}{\tau'}\right)R_{\omega}(\mathbf{r}, \mathbf{s}) + \mathrm{i}I(\mathbf{r})Q_{\omega}(\mathbf{r}, \mathbf{s}) = 0, \tag{22}$$

which were first derived by Takahashi and Tachiki [50] in the absence of the spin orbit interaction, and some time later have been generalized by Auvil et al. [53] to take into account the spin orbit scattering. We will show in this section that the auxiliary kernel $R_{\omega}(\mathbf{r}, \mathbf{s})$ is indeed the short-range triplet pair function.

Returning to the central Eq. (17) and define the matrix pair amplitude

$$\hat{F}(\mathbf{r}, \omega) = \int d^3s\hat{Q}_{\omega}(\mathbf{r}, \mathbf{s})\mathrm{i}\sigma_y\Delta(\mathbf{s}), \tag{23}$$

we immediately obtain the equation for the triplet proximity effect, including the spin orbit scattering potential.

$$\begin{aligned}&\left(|\omega| - \frac{1}{2}D(\mathbf{r})\nabla^2\right)\hat{F}(\mathbf{r}, \omega) + \frac{\mathrm{i}}{2}(\mathbf{I}(\mathbf{r}) \cdot \hat{\sigma}\hat{F}(\mathbf{r}, \omega) \\ &\quad + \hat{F}(\mathbf{r}, \omega)\sigma_y\mathbf{I}(\mathbf{r}) \cdot \hat{\sigma}\sigma_y) + \frac{1}{2\tau'}(\hat{F}(\mathbf{r}, \omega) + \hat{F}^{tr}(\mathbf{r}, \omega)) \\ &= \mathrm{i}\sigma_y\Delta(\mathbf{r}).\end{aligned} \tag{24}$$

Going further, by expanding $\hat{F}(\mathbf{r}, \omega)$ along the superfluid pairing states [55]

$$\hat{F}(\mathbf{r}, \omega) = [F_s(\mathbf{r}, \omega) + \mathbf{F}_t(\mathbf{r}, \omega) \cdot \hat{\sigma}]\mathrm{i}\sigma_y, \tag{25}$$

where $F_s(\mathbf{r}, \omega)$ and $\mathbf{F}_t(\mathbf{r}, \omega)$ are the scalar spin singlet and the vector spin triplet pair amplitudes, respectively. In the (S^2, S_z) spin representation of the composite electrons, $S = 0$ case corresponds to the spin singlet state with the zero spin projection on the quantization axis $S_z = 0$, while $S = 1$ case defines the spin triplet state with the spin projections $S_z = 0, \pm 1$. Thus F_s and F_{tz} are the probability amplitudes of finding a paired electron on the spin quantization axis which are the most probable, when the exchange field is fixed in space. $\mp F_{tx} + \mathrm{i}F_{ty}$ corresponds to the spin projections ± 1, and may be calculated if the exchange field is inhomogeneous.

Inserting (25) into (24), we have the scalar and the vector equations

$$\left(|\omega| - \frac{1}{2}D(\mathbf{r})\nabla^2\right)F_s(\mathbf{r}, \omega) + \mathrm{i}\mathbf{I}(\mathbf{r}) \cdot \mathbf{F}_t(\mathbf{r}, \omega) = \Delta(\mathbf{r}), \tag{26}$$

$$\left(|\omega| - \frac{1}{2}D(\mathbf{r})\nabla^2 + \frac{1}{\tau'}\right)\mathbf{F}_t(\mathbf{r}, \omega) + \mathrm{i}\mathbf{I}(\mathbf{r})F_s(\mathbf{r}, \omega) = 0. \tag{27}$$

The spin orbit interaction itself does not have any effect on the spin singlet pair because $\hat{U}_{so}$ preserves the time-reversal symmetry. It acquires the spin exchange interaction to break a Cooper pair and generates the spin triplet pair. As a result, the spin orbit interaction attributes to spin singlet pair amplitude via the spin triplet components by coupling with the exchange field. This mechanism was explained by Demler et al. [56]. Note that the effect of the spin orbit interaction on the triplet superconductivity has already been obtained by Bergeret et al. [5], using the quasiclassical Green functions method in the Keldysh representation.

In the absence of the spin orbit interaction, (26) and (27) can be reduced to the equations as derived by Champel and Eschrig [39,40] in which the quasiclassical approach in the Matsubara representation has been used. However, apart from the normalization condition of the energy integrated 4×4 matrix Green function, this approach also requires an additional condition of the particle-hole symmetry in equilibrium to produce the proper form of the Usadel transport-like equations. Another triplet pairing basis was presented by Fominov et al. in Ref. [34], but their formulation cannot be applied in a general case, because when transformed to the formal basis, one gets the term $-\hat{F}\sigma_z\mathbf{I}\cdot\hat{\sigma}\sigma_z$ instead of $\hat{F}\sigma_y\mathbf{I}\cdot\hat{\sigma}\sigma_y$ in (24), which is evidently incorrect.

Let us consider the special case when the exchange field points along the z-axis. Within this regard, only the singlet pair amplitude $F_s(\mathbf{r},\omega)$ and the short-range triplet component $F_{tz}(\mathbf{r},\omega)$ are coupled together. By comparing F_s and F_{tz} with (21) and (22), we obtain the relations $F_s(\mathbf{r},\omega)\propto Q_\omega(\mathbf{r},\mathbf{s})$ and $F_{tz}(r,\omega)\propto R_\omega(\mathbf{r},\mathbf{s})$. It is worth noting that F_s and F_{tz} are incorporated into the Usadel equations and have been widely used to study the non-monotonic T_c behavior of SC/FM proximity systems [21,57–59].

We now discuss the possibility of the triplet pair formation and its implications. The supplemented condition to be determined is the self-consistent s-wave gap Eq. (4) which reads as

$$\Delta(\mathbf{r})\mathrm{i}\sigma_y = \pi TN(\mathbf{r})V(\mathbf{r})\sum_\omega \hat{F}(\mathbf{r},\omega). \tag{28}$$

Here the matrix pair amplitude $\hat{F}(\mathbf{r},\omega)$ contains the anti-symmetric $\mathrm{i}\sigma_y$, and symmetric $\hat{\sigma}\mathrm{i}\sigma_y$ matrices as defined in (25). The spatial functions $\Delta(\mathbf{r})$ and $F_s(\mathbf{r})$ must have an even symmetry, whereas $\mathbf{F}_t(\mathbf{r})$ must posses an odd symmetry, in order to satisfy the Pauli principle. In this respect, it seems that the linear combination of the singlet part and the triplet components is forbidden since $\hat{F}(-\mathbf{r})\neq\hat{F}(\mathbf{r})$. Nevertheless, by imposing the even spatial parity of $\hat{F}(\mathbf{r},\omega)$ and taking the even frequency symmetry of $F_s(\mathbf{r},\omega)$ and the odd frequency symmetry of $\mathbf{F}_t(\mathbf{r},\omega)$ into account, a new type of the spin triplet pair amplitude arises, and is called as Berezinskii's odd frequency triplet condensate [30]. Within this consideration the mixed pairing states are possible and (28) becomes

$$\Delta(\mathbf{r}) = 2\pi TN(\mathbf{r})V(\mathbf{r})\sum_{\omega>0} F_s(\mathbf{r},\omega). \tag{29}$$

The spatial variation of the order parameter still has a conventional form and tells us about the nature of the pairing symmetry that why there is only the spin singlet state responsible for the superconducting phase. However, the odd triplet condensate may exist through the hybrid FM/SC proximity system.

Therefore in this paper (26), (27) and (29) are proved to constitute the set of triplet proximity effect. The pairing functions in each layer are connected by the boundary conditions at the FM/SC interface [60,61]

$$\begin{aligned}&\xi_s\nabla\hat{F}^s = \xi_f\gamma\nabla\hat{F}^f,\\ &\hat{F}^s = \hat{F}^f - \xi_f\gamma_b\hat{n}_f\cdot\nabla\hat{F}^f,\end{aligned} \tag{30}$$

and, at the outer surfaces

$$\nabla\hat{F}^{s(f)} = 0. \tag{31}$$

The superscript $s(f)$ on the matrix pair amplitude $\hat{F}$ belongs to the SC(FM) layer, $\xi_{s(f)}=\sqrt{D_{s(f)}/2\pi T_{c0}}$ is the corresponding coherence length. $\gamma=\rho_s\xi_s/\rho_f\xi_f$ and γ_b are the resistivity mismatch between the two layers in contact, and the boundary resistance which characterize, respectively, the pair leakage and the pair jumping from SC into FM in the direction outward normal to the interface $\hat{n}_f$.

3. Inhomogeneous Néel type exchange field

In this section we calculate the critical temperature T_c of a FM/SC bilayer in the exact multimode method. We take the x-axis perpendicular to the layered planes and represents the direction of the propagation of induced pair amplitudes. The layered planes occupy the regions $-d_f<x<0$ for FM, and $0<x<d_s$ for SC. It is assumed that the superconducting order parameter does not exist in the FM layer, only the pair amplitude is permitted to induce the pairing via the proximity effect. The model of the inhomogeneous magnetization is of the Néel type, that is it rotates in an in-plane of the FM layer with a spiral wave vector Q,

$$\mathbf{I}(y) = I(0,\sin Qy,\cos Qy). \tag{32}$$

Within this configuration the homogeneous fixed exchange field corresponds to the $Q=0$ limit. The spin quantization axis points in the z-direction and, of course, there always exist the singlet and the short-range triplet pair amplitudes. When $Q\neq 0$, we may expect the occurrence of the induced long-range triplet components in the FM layer. The dependence of T_c on Q indicates the manifestation of the induced triplet superconductivity and is our main purpose in carrying out this configuration.

Employing the transformation as introduced in Refs. [39,40]

$$F_{t\pm}(x,y,\omega) = \frac{1}{2}(F_{tz}(x,y,\omega)\pm \mathrm{i}F_{ty}(x,y,\omega))\mathrm{e}^{\mp\mathrm{i}Qy}, \tag{33}$$

one obtains the set of Eqs. (26) and (27) which read as

$$\left[\omega-\frac{1}{2}D(\partial_x^2+\partial_y^2)\right]F_s+\mathrm{i}I(F_{t+}+F_{t-})=\Delta(x,y), \tag{34}$$

$$\left[\omega+\frac{2}{\tau_{so}}-\frac{1}{2}D(\partial_x^2+\partial_y^2\pm 2\mathrm{i}Q\partial_y-Q^2)\right]F_{t\pm}+\frac{\mathrm{i}}{2}IF_s=0. \tag{35}$$

Here F_{tx} has been disregarded because it does not coupled with F_s. The phenomenological spin orbit scattering parameter $1/\tau'$ is set to be $2/\tau_{so}$, where

$$\frac{1}{\tau_{so}}=\frac{2\pi}{3}n_{imp}N(0)\int\frac{\mathrm{d}\Omega}{4\pi}|u_{so}|^2\sin^2\theta,$$

n_{imp} is the impurity concentration [62].

Treating the problem in one dimension and assuming the separable form of the pair amplitude

$$F_j(x,y,\omega)=\mathrm{e}^{\mathrm{i}py}F_j(x,\omega),\quad j=s,t\pm \tag{36}$$

as well as for the pair potential $\Delta(x,y)$, one finds that the wave vector p represents the modulation of the pair amplitudes in an in-plane direction and is directly attributed to an inhomogeneous superconducting state or the FFLO [48,49] phase. Therefore we shall take p as a free parameter and varies it in the range $0 < p < Q$.

By virtue of (36) the effective one dimensional equations are

$$\left[\omega - \frac{1}{2}D(\partial_x^2 - p^2)\right]F_s(x) + iI(F_{t+}(x) + F_{t-}(x)) = \Delta(x), \quad (37)$$

$$\left[\omega + \frac{2}{\tau_{so}} - \frac{1}{2}D(\partial_x^2 - (p \pm Q)^2)\right]F_{t\pm}(x) + \frac{i}{2}IF_s(x) = 0, \quad (38)$$

$$\Delta(x) = 2\pi T N(x)V(x)\sum_{\omega>0} F_s(x,\omega). \quad (39)$$

The boundary conditions still have the canonical form as follows

$$\xi_s\partial_x F_s^s(x=0) = W_{fp}(Q,\omega)F_s^s(x=0), \quad (40)$$

$$\partial_x F_s^s(x=d_s) = 0, \quad (41)$$

and

$$W_{fp}(Q,\omega) = \frac{\gamma\xi_f\partial_x F_s^f(x=0)}{F_s^f(x=0) + \gamma_b\xi_f\partial_x F_s^f(x=0)}. \quad (42)$$

The interface boundary function $W_{fp}(Q, \omega)$ contains information of the singlet pair in FM. Nevertheless, as can be seen from (37) and (38), the singlet and triplet components are unavoidably coupled together. Thus the other sets of the boundary conditions are

$$\partial_x F_{t\pm}^s(d_s) = 0, \quad (43)$$

$$\partial_x F_{s,t\pm}^f(-d_f) = 0, \quad (44)$$

$$\gamma\xi_f\partial_x F_{t\pm}^f(0) - \frac{\xi_s\partial_x F_{t\pm}^s(0)}{F_{t\pm}^s(0)}(F_{t\pm}^f(0) + \gamma_b\xi_f\partial_x F_{t\pm}^f(0)) = 0. \quad (45)$$

In SC ($0 < x < d_s$) we have

$$(\partial_x^2 - k_{sp}^2)F_s^s(x,\omega) = -\frac{1}{\pi T_{c0}\xi_s^2}\Delta(x), \quad (46)$$

$$\Delta(x) = 2\pi T\lambda_s\sum_{\omega>0} F_s^s(x,\omega), \quad (47)$$

for the singlet part, where $\lambda_s = N_s V_s$ is the BCS coupling constant, and

$$(\partial_x^2 - k_{tp\pm}^2)F_{t\pm}^s(x,\omega) = 0, \quad (48)$$

for the triplet components. The corresponding spectra are

$$k_{sp}^2 = \frac{\omega}{\pi T_{c0}\xi_s^2} + p^2, \quad (49)$$

$$k_{tp\pm}^2 = \frac{\omega}{\pi T_{c0}\xi_s^2} + (p \pm Q)^2. \quad (50)$$

Using the outer boundary condition (43) yields the solution of (48)

$$F_{t\pm}^s(x,\omega) = F_{t\pm}^s(0)\cosh[k_{tp\pm}(x - d_s)]. \quad (51)$$

For the singlet part, (46) and (47) have to be solved self-consistently subject to the boundary conditions (40) and (41).

Let us consider the kernel $Q_\omega(x, x')$ instead of $F_s^s(x,\omega)$, whose the differential equation has the delta function source

$$(\partial_x^2 - k_{sp}^2)Q_\omega(x,x') = -\frac{1}{\pi T_{c0}\xi_s^2}\delta(x - x'), \quad (52)$$

and the same the boundary conditions as $F_s^s(x,\omega)$, i.e., (40) and (41). Due to the even spatial parity of the singlet function, $Q_\omega(x,x')$ and $\Delta(x)$ are expanded in Fourier cosine series

$$Q_\omega(x,x') = \sum_{m=-\infty}^{\infty} Q_\omega(q_m, x')\cos(q_m x), \quad (53)$$

$$\Delta(x) = \sum_{m=-\infty}^{\infty} \Delta(q_m)\cos(q_m x), \quad (54)$$

with $q_m = m\pi/d_s$. Transformation of (52) to obtain $Q_\omega(q_m, x')$ through

$$Q_\omega(q_m, x') = \frac{1}{d_s}\int_0^{d_s} dx Q_\omega(x,x')\cos(q_m x), \quad (55)$$

one finds that the self-consistency Eq. (47) in Fourier space is given by

$$\Delta(q_m) = 2\pi T\lambda_s\sum_{m'=-\infty}^{\infty}\sum_{\omega>0}\int_0^{d_s} dx' Q_\omega(q_m, x')\cos(q_{m'}x'). \quad (56)$$

The non-trivial solution of (56) obeys the secular equation

$$\det\|\delta_{mm'} - 2\lambda_s\frac{T}{T_{c0}}\sum_{\omega>0} L_{mm'}(\omega)\| = 0, \quad (57)$$

where

$$L_{mm'}(\omega) = \frac{\delta_{mm'}}{\xi_s^2(k_{sp}^2 + q_m^2)} - \frac{\Omega(\omega)}{\xi_s^4(k_{sp}^2 + q_m^2)(k_{sp}^2 + q_{m'}^2)}, \quad (58)$$

$$\Omega(\omega) = \frac{W_{fp}(Q,\omega)}{(d_s/\xi_s)(1 + W_{fp}(Q,\omega)/A_{sp})}, \quad (59)$$

with

$$A_{sp} = k_{sp}\xi_s\tanh(k_{sp}d_s), \quad (60)$$

as for the triplet case

$$A_{tp\pm} = k_{tp\pm}\xi_s\tanh(k_{tp\pm}d_s). \quad (61)$$

The remaining task is to find the explicit form of the interface boundary function $W_{fp}(Q,\omega)$.

In FM ($-d_f < x < 0$) we have

$$\left[-\partial_x^2 + \frac{\omega}{\pi T_{c0}\xi_f^2} + p^2\right]F_s^f(x) + \frac{2i}{\xi_I^2}(F_{t+}^f(x) + F_{t-}^f(x)) = 0, \quad (62)$$

$$\left[-\partial_x^2 + \frac{1}{\pi T_{c0}\xi_f^2}(\omega + \frac{2}{\tau_{so}}) + (p \pm Q)^2\right]F_{t\pm}^f(x) + \frac{i}{\xi_I^2}F_s^f(x) = 0, \quad (63)$$

where $\xi_I = \sqrt{D_f/I}$ is the exchange length which is originated from the broken time-reversal symmetry. Naturally,

$$W_{f,p=0}(Q,\omega) = \frac{(\eta_+ + \eta_-)B_{f+}B_{f-} + (\eta_+ B_{f+} + \eta_- B_{f-})A_t}{\eta_+ B_{f-} + \eta_- B_{f+} + (\eta_+ + \eta_-)A_t}, \quad (86)$$

where $\eta_\pm$,$B_{f\pm}$, and A_t are, respectively, the short-hand notations of $\eta_{p=0,\pm}$, $B_{f,p=0,\pm}$, and $A_{t,p=0,\pm}$. It is clear that the long-range mode is absent in (86) and this means superconductivity is localized in the spin singlet state [41]. At this point we are aware of the theoretical results presented in Refs. [34,35] that they might be incorrect. In those papers the authors studied the T_c of FM/SC/FM trilayer as a function of the relative orientations of the homogeneous exchange fields and reached the conclusions that the long-range triplet arises from intermediate orientations.

4. Numerical results

We perform the numerical calculations of the critical temperatures T_c of a FM/SC bilayer as functions of a rotating wave vector Q (Figs. 1 and 2) and a FM layer of thickness d_f (Figs. 3–5) using the secular Eq. (57) and the associated functions as given by (58), (59) and (80). Throughout the investigations we use the dimensionless quantities such as the reduced critical temperature $t_C = T_c/T_{c0}$, the dimensionless rotating wave vector $Q\xi_s$, the normalized FM layer thickness d_f/ξ_f, etc. Parameter values in all phase diagrams are the following $I/\pi T_{c0} = 10$, $d_s/\xi_s = 2, \gamma = 0.2$, by taking $\xi_s = \xi_f$. Also, we let $d_f/\xi_f = 0.5$, and $\gamma_b = 0$ in Figs. 1 and 2, where we plot t_C versus $Q\xi_s$ to investigate the variations of the induced long-range triplet behavior as a function of in-plane momentum parameter p/Q, with and without the associated spin orbit scattering rate, e.g., $1/\tau_{so\ I} = 0$ in Fig. 1, and $1/\tau_{so\ I} = 0.2$ in Fig. 2. Furthermore, the t_C versus d_f/ξ_f curves are drawn in Figs. 3–5 to examine the occurrence of induced superconductivity in the FM layer. Here we take $\gamma_b = 0$, $Q\xi_s = 0.5$, and vary the ratio p/Q as shown

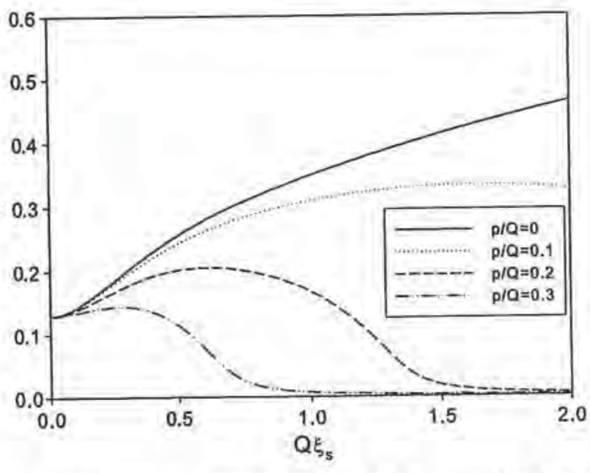

Fig. 1. Reduced critical temperature $t_C = T_c/T_{c0}$ as a function of dimensionless rotating wave vector $Q\xi_s$ for several values of p/Q. $I/\pi T_{c0} = 10$, $d_s/\xi_s = 2$, $d_f/\xi_f = 0.5$, $\gamma = 0.2$, $\gamma_b = 0$, $\xi_s = \xi_f$, and $1/I\tau_{so} = 0$.

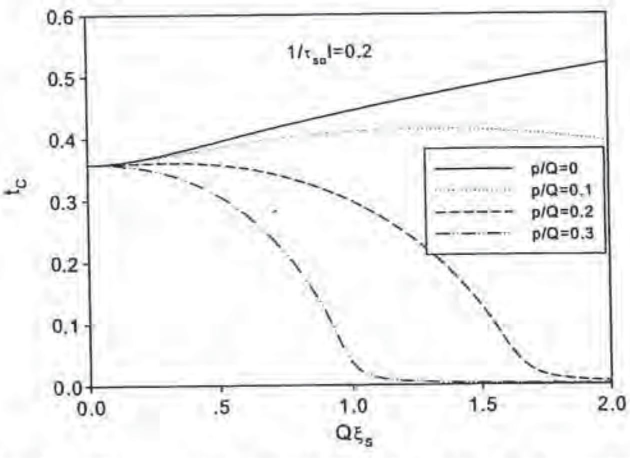

Fig. 2. $t_C(Q\xi_s)$ curves with varying p/Q, $1/I\tau_{so} = 0.2$, and other parameters are the same as in Fig. 1.

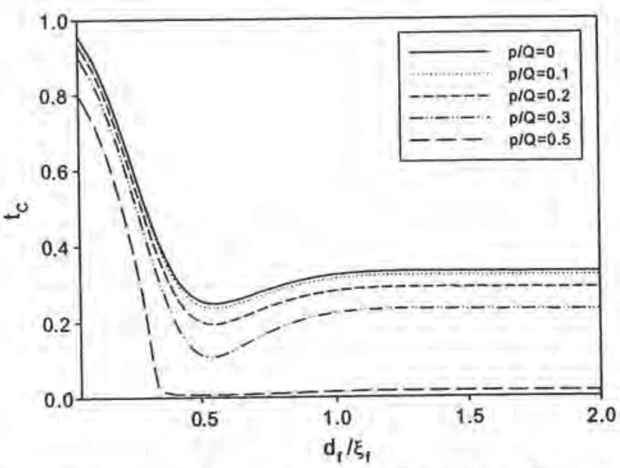

Fig. 3. Reduced critical temperature t_C as a function of the reduced ferromagnetic layer thickness d_f/ξ_f with varying p/Q, $I/\pi T_{c0} = 10$, $d_s/\xi_s = 2$, $\gamma = 0.2$, $\gamma_b = 0$, $Q\xi_s = 0.5$, $\xi_s = \xi_f$, and $1/I\tau_{so} = 0$.

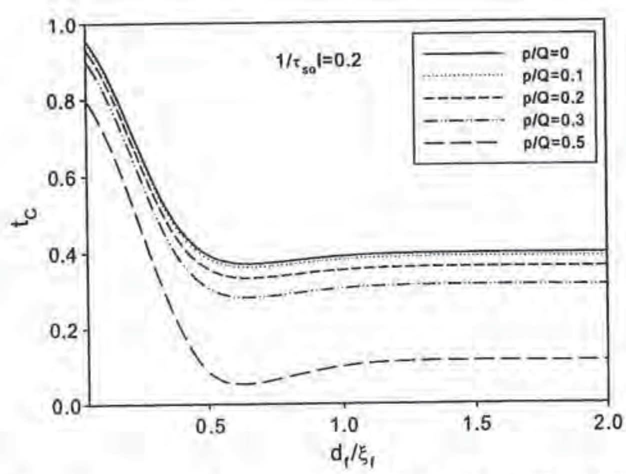

Fig. 4. $t_c(d_f/\xi_f)$ curves for several values of p/Q, $1/I\tau_{so} = 0.2$, and the values of other parameters are the same as in Fig. 3.

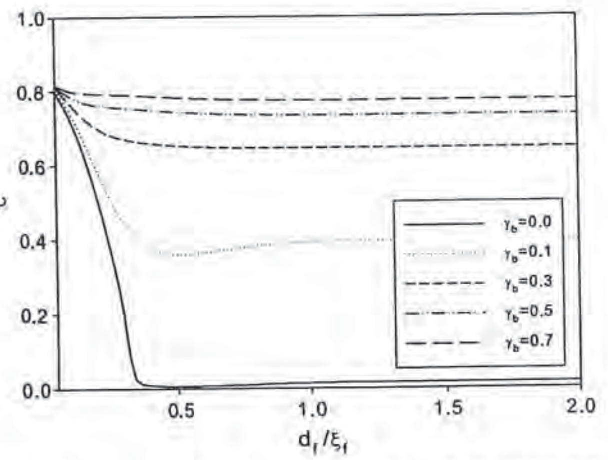

Fig. 5. $t_c(d_f/\xi_f)$ curves with varying γ_b, $p/Q = 0.5$, and other parameters are the same as in Fig. 3.

in Figs. 3 and 4, with $1/\tau_{so\ I} = 0$ and 0.2, respectively. In Fig. 5, several t_C curves for each value of γ_b are presented to show the non-monotonicity with fixed parameters $Q\xi_s = 0.5$, $p/Q = 0.5$, and $1/\tau_{so\ I} = 0$. Within these parameter values, we find no superconductivity at all for a thin S layer ($d_s/\xi_s \leqslant 1$). For $d_s/\xi_s \approx 1.5$, the switching phenomenon between the normal state and the superconducting state occurs at a certain value of $Q = Q_{cr}$, i.e., there is an onset of superconductivity when $Q > Q_{cr}$ (this feature has already been demonstrated in Refs. [39,40] and will not be reproduced here).

Before discussing the obtained results in detail, we recall that the case with the finite Q value ($Q \neq 0$) represents the inhomogeneity of the exchange field and the parameter p describes the inhomogeneous superconducting state as the pair amplitudes modulate spatially perpendicular to the proximity direction (or parallel to the interface plane) and it is connected inevitably to the emergence of the induced long-range triplet correlation, when $p \neq 0$. Typical inhomogeneous exchange fields build a domain structure called the cryptoferromagnetic state as was first proposed by Anderson and Suhl [65]. Their hypothesis suggested a possible mechanism for the existence of superconductivity with a ferromagnetic state in magnetic domains. A possibility of the cryptoferromagnetic state was investigated theoretically in Refs. [37,66] and, in particular, experimentally by Mühge et al. [67] in an attempt to explain the observed magnetically dead layer from the magnetization measurements. It is well known that a T_c suppression is caused by the destructive influence of the exchange field. However, the magnetic moments may be weakened in some regions inside the FM layer so the effective exchange field is minimum but remains finite and superconductivity may coexist with the cryptoferromagnetic state in the system. Another aspect of the cryptoferromagnetic state has been developed in the context of domain wall superconductivity where the inhomogeneous rotating exchange field is confined inside the walls which separate the domains. However, the consideration of the full domain wall structure is far beyond the scope of this work, since we are interested only in the possibility for the occurrence of the induced long-range triplet component in hybrid structures.

In Fig. 1 the variations of t_C vs. $Q\xi_s$ show many significant features depending on p/Q. For the $p = 0$ curve, t_C rises significantly as $Q\xi_s$ increases and it is the highest critical temperature, here the long-range triplet is absent from the nucleation of superconductivity as already stated in (86). Other curves ($p \neq 0$) describe the inhomogeneous superconducting state, it is evident that the onset of the long-range triplet emerges when the exchange field in the FM layer starts to rotate. We find that t_C is very sensitive to the degree of inhomogeneity p/Q which results in its rapid reduction with more pronounced non-monotonicity as p/Q increases. It indicates that the onset of the long-range triplet is less energetically favorable than the short-range triplet. However, the non-vanishing t_C for a finite interval of $Q\xi_s$ of some curves may explain a possible origin of the magnetically dead layer which corresponds to the cryptoferromagnetic state. Though the long-range triplet provides t_C of lower value than the short-range one, yet it is more relevant to a real situation since naturally, the exchange interaction plays the role of the pair breaker which tends to suppress superconductivity.

Fig. 2 illustrates the influence of the spin orbit interaction on t_C. We can see that it makes the behavior of t_C less oscillatory since the exchange field strength is reduced considerably which results in weakening the influence of the pair breaker so that the pair amplitudes can survive a longer distance to the order of a decay length $\sqrt{\tau_{so} D_f}$. Because the spectrum of the long-range triplet component k_{fp0}^2 depends on $1/\tau_{so}$, this leads to the suppression of superconductivity in the $p \neq 0$ case. An enhancement of t_C reveals clearly the important role of the spin orbit scattering. At small values of $Q\xi_s$, as the exchange field starts to modulate the degree of inhomogeneity p/Q is very small there exists only the induced short-range triplet, so all curves merge. Upon the onset of the long-range triplet, the t_C evolution separate from each other as $Q\xi_s$ increases. We can see that the scale $Q\xi_s \approx 0.5$ is sufficient to distinguish the long-range from the short-range triplet.

The non-monotonic behavior of t_C as a function of d_f/ξ_f with varying p/Q are shown in Figs. 3 and 4 to display identical features as the boundary resistance does. Here the interface is assumed to be perfectly transparent ($\gamma_b = 0$) and the choice $Q\xi_s = 0.5$ is selected to ensure the contribution of the long-range triplet. In Fig. 3 the non-monotonicity is found to be restricted within a finite range of p/Q, i.e., $0 < p/Q < 0.3$, the monotonic t_C decay is obtained when $p/Q = 0.5$. Various characterizations of t_C can be observed for a FM layer thickness. Fig. 4 shows the influence of the spin orbit scattering, it produces significant effect inside the FM layer rather than at the interface. As expected, the presence of the spin orbit interaction is antagonistic to the non-monotonic behavior. We also note that the non-monotonic character can be restored from the monotonic decay in the $p/Q = 0.5$ case.

Finally, Fig. 5 demonstrates the interesting and feasible case of the long-range triplet contribution when one allows the pair jumping at the interface. Though our model of the exchange field has an inhomogeneous spiral magnetic order, yet the t_C behavior yields the same result as that obtained from the homogeneous fixed exchange field.

5. Conclusion

We have presented a formulation of the triplet proximity effect within the de Gennes correlation function approach, and applied it to study the behavior of the diffusive FM/SC bilayered structure. The spin orbit interaction is also incorporated into the Usadel transport-like equation, and it found that this interaction tends to break the singlet pair indirectly through the exchange interaction by mixing with it the triplet pairs. The long-range triplet pair correlation which is defined as the non-zero spin projection on the spin quantization axis is found to be generated in the presence of the inhomogeneous exchange field. We use the model of the Néel type spiral magnetic order that rotates in the plane and has no contribution in the perpendicular direction to the interface. The linearized self-consistent order parameter equation has been solved exactly in the multimode method, and the secular equation is derived to determine the dependence of the critical temperature T_c on the spiral wave vector Q and the ferromagnetic layer thickness d_f. The inhomogeneous superconducting state is characterized by the in-plane modulating wave vector p and directly measures the degree of inhomogeneity of the system. We have shown that the induced long-range triplet arises when the pair amplitudes are modulated spatially in the transverse direction.

Numerical results demonstrate the possibility of the cryptoferromagnetic state in favor of superconductivity and may be used to explain a possible origin of the magnetically dead layer. The graphs clearly show the evolution of T_c with respect to Q that it is enhanced greatly in the presence of the spin orbit interaction with less oscillatory characters. The single ferromagnetic layer thickness is a necessary condition for observing various characterizations of T_c behavior. The effect of the boundary resistance γ_b on T_c provides the same feature as in the homogeneous fixed exchange field case. We conclude that the induced long-range triplet pair condensate is feasible and may be observed through a FM/SC bilayer system.

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References

[1] L.N. Bulaevskii, A.I. Buzdin, M.L. Kulic, S.V. Panjukov, Adv. Phys. 34 (1985) 175.
[2] Yu.A. Izyumov, Yu.N. Proshin, M.G. Khusainov, Phys. Usp. 45 (2002) 109.
[3] A.A. Golubov, M.Yu. Kupriyanov, E. Il'ichev, Rev. Mod. Phys. 76 (2004) 411.
[4] A.I. Buzdin, Rev. Mod. Phys. 77 (2005) 935.
[5] F.S. Bergeret, A.F. Volkov, K.B. Efetov, Rev. Mod. Phys. 77 (2005) 1321.
[6] I.F. Lyuksyutov, V.L. Pokrovsky, Adv. Phys. 54 (2005) 67.
[7] A.I. Buzdin, A.V. Vedyayev, N.V. Ryzhanova, Europhys. Lett. 48 (1999) 686.
[8] L.R. Tagirov, Phys. Rev. Lett. 83 (1999) 2058.
[9] I. Baladiè, A.I. Buzdin, N. Ryzhanova, A.V. Vedyayev, Phys. Rev. B 63 (2001) 054518.
[10] I. Baladiè, A.I. Buzdin, Phys. Rev. B 67 (2003) 014523.
[11] C.-Y. You, Ya.B. Bazaliy, J.Y. Gu, S.-J. Oh, L.M. Litvak, S.D. Bader, Phys. Rev. B 70 (2004) 014505.
[12] B. Krunavakarn, W. Sritrakool, S. Yoksan, Physica C 406 (2004) 46.
[13] K. Halterman, O.T. Valls, Phys. Rev. B 72 (2005) 060514 (R).
[14] M. Faurè, A.I. Buzdin, D. Gusakova, Physica C 454 (2007) 61.
[15] J.Y. Gu, C.-Y. You, J.S. Jiang, J. Pearson, Ya.B. Bazaliy, S.D. Bader, Phys. Rev. Lett. 89 (2002) 267001.
[16] A. Potenza, C.H. Marrows, Phys. Rev. B 71 (2005) 180503 (R).
[17] I.C. Moraru, W.P. Pratt Jr., N.O. Birge, Phys. Rev. Lett. 96 (2006) 037004.
[18] I.C. Moraru, W.P. Pratt Jr., N.O. Birge, Phys. Rev. B 74 (2006) 220507(R).
[19] A.Yu. Rusanov, S. Habraken, J. Aarts, Phys. Rev. B 73 (2006) 060505(R).
[20] R. Steiner, P. Ziemann, Phys. Rev. B 74 (2006) 094504.
[21] Z. Radoviè, M. Ledvij, L. Dobrosavljeviè-Grujiè, A.I. Buzdin, J.R. Clem, Phys. Rev. B 44 (1991) 759.
[22] K. Kuboya, K. Takanaka, Phys. Rev. B 57 (1998) 6022.
[23] B. Krunavakarn, S. Yoksan, Physica C 440 (2006) 25.
[24] Y.N. Proshin, A. Zimin, N.G. Fazleev, M.G. Khusainov, Phys. Rev. B 73 (2006) 184514.
[25] P.H. Barsic, O.T. Valls, K. Halterman, Phys. Rev. B 75 (2007) 104502.
[26] J.S. Jiang, D. Davidoviè, D.H. Reich, C.L. Chein, Phys. Rev. Lett. 74 (1995) 314.
[27] J.S. Jiang, D. Davidoviè, D.H. Reich, C.L. Chein, Phys. Rev. B 54 (1996) 6119.
[28] Y. Obi, M. Ikebe, H. Fujishiro, Phys. Rev. Lett. 94 (2005) 057008.
[29] V. Shelukhin, A. Tsukernik, M. Karpovski, Y. Blum, K.B. Efetov, A.F. Volkov, T. Champel, M. Eschrig, T. Löfwander, G. Schön, A. Palevski, Phys. Rev. B 73 (2006) 174506.
[30] V.L. Berezinskii, JETP Lett. 20 (1974) 287.
[31] F.S. Bergeret, A.F. Volkov, K.B. Efetov, Phys. Rev. Lett. 86 (2001) 4096.
[32] A. Kadigrobov, R. Shekhter, M. Jonson, Europhys. Lett. 54 (2001) 394.
[33] F.S. Bergeret, A.F. Volkov, K.B. Efetov, Phys. Rev. B 68 (2003) 064513.
[34] Y.V. Fominov, A.A. Golubov, M.Yu. Kupriyanov, JETP Lett. 77 (2003) 510.
[35] T. Löfwander, T. Champel, M. Eschrig, Phys. Rev. B 75 (2007) 014512.
[36] A.F. Volkov, A. Anischanka, K.B. Efetov, Phys. Rev. B 73 (2006) 104412.
[37] F.S. Bergeret, K.B. Efetov, A.I. Larkin, Phys. Rev. B 62 (2000) 11872.
[38] A.F. Volkov, Y.V. Fominov, K.B. Efetov, Phys. Rev. B 72 (2005) 184504.
[39] T. Champel, M. Eschrig, Phys. Rev. B 71 (2005) 220506(R).
[40] T. Champel, M. Eschrig, Phys. Rev. B 72 (2005) 054523.

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Influence of pair breaking effects on the long-range odd triplet superconductivity in a ferromagnet/superconductor bilayer

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Abstract

The spin-dependent potential together with magnetic impurity and the spin-orbit scatterings were incorporated into the de Gennes-Takahashi-Tachiki theory of a diffusive superconductor-and-ferromagnetic metal to derive a formulation of the odd triplet superconductivity proximity effect. It is found that when the spin exchange interaction is inhomogeneous, i.e., the Neel spiral magnetic order, a new type of triplet condensate is generated, due to the broken time-reversal invariance. The triplet amplitude still contains the s-wave state, similar to the conventional singlet pairing, but the frequency symmetry must be odd to obey the Pauli's exclusion principle. As a result, the self-consistent order parameter contains only the singlet pair amplitude. The superconducting critical temperature of the bilayer is obtained in the single mode approximation and takes the form of the Abrikosov-Gorkov formula. The necessary condition for the occurrence of the induced long-range triplet component in the ferromagnet layer is characterized by the modulation of the pair amplitudes in the transverse direction. The possibility of the cryptoferromagnetic state, corresponding to the finite value of the spiral wave vector, is demonstrated in favor of the superconductivity. In addition, the influence of the magnetic impurity and the spin-orbit scattering is to decrease the decay length and to increase the oscillation period of the pair amplitudes which in turn enhances the critical temperature but in a less pronounced non-monotonicity manner.

Keywords: *odd triplet superconductivity, ferromagnetism, proximity effect, transition temperature, impurity scattering, spin-orbit scattering*

1. Introduction

The great progress in nano-fabricated technology in the past decade has been paid to the study of hybrid heterostructures of different materials. In the sandwiched structures composed of a superconductor (SC) and a normal metal (N), the Cooper pairs can physically penetrate the N side in a monotonic decaying manner to a certain length and so the induced superconductivity can therefore occur in N. This phenomenon is called the proximity effect. If the N metal is substituted by a ferromagnetic metal (FM), the Cooper pairs can be affected by the influence of the magnetic ordering. This leads to an unusual behavior of the superconducting condensate in the FM region.

The most striking phenomena of the FM/SC system manifests itself in the Buzdin-Tagirov spin switch effect (Buzdin, 1999; Tagirov, 1999) and the Radovic pi-phase state (Radovic,1991) where the first one exists in a FM/SC/FM trilayer in which theories predicted that an antiparallel magnetization in the FM layers could provide the higher critical temperature than a parallel configuration (Baladie, 2001; Baladie, 2003; You, 2004; Krunavakarn, 2004; Halterman, 2005; Faure, 2007). While the latter phenomenon, occurs in SC/FM multilayers, demonstates that the phase shift of the pair amplitudes between the adjacent SC layers can take either the zero-phase or the pi-phase (Kuboya, 1998; Krunavakarn, 2006; Proshin, 2006; Barsic, 2007; Jiang, 1995; Jiang, 1996; Obi, 2005; Shelukhin, 2006).

The model of a homogeneous exchange interaction in FM has been widely used and predicts the pair amplitude function in the SC to penetrate into the FM for a short distance. However, if the magnetization is inhomogeneous, the situation will change drastically. A new type of the triplet condensate arises which results in a long range triplet component of Cooper pairs.

The objective of this study is to investigate the influence of the pair breaking effects and the inhomogeneous magnetization on the long-range triplet superconductivity, which has never been clarified until now. The motivation is to investigate the physics of the FM/SC bilayer with emphasis on the possible formation of the long-range triplet pair. In Section 2, we present a formulation of the generalized triplet proximity effect by extending the Takahashi-Tachiki (1986) theory within the de Gennes's correlation function approach (de Gennes, 1966), so the spin-orbit scattering and the magnetic impurity scattering can be included in a phenomenological way (Auvil, 1989). The obtained Usadel transport-like equations are found to agree perfectly with those of Champel and Eschrig (2005) in the absence of the pair breaking scatterers. In Section 3, we study the model of the inhomogeneous Neel type spiral exchange field in the FM/SC bilayer and solve the critical temperature (T_C) equation as a function of a rotating spiral wave vector and an in-plane momentum which represents the modulation of the pair amplitude in the transverse direction. Finally, conclusions are presented in Section 4.

2. General formalism

We use the generalized de Gennes-Takahashi-Tachiki theory to derive the FM/SC odd triplet proximity effect. According to the de Gennes' correlation function formalism, the motion of the normal electrons at temperature T is controlled by the diffusion process. Suppose the system is characterized by the position-dependent material parameters such as the density of states $N(r)$, the diffusion coefficient $D(r)$, and the phonon mediated electron-electron pairing interaction $V(r)$ in the s-wave channel. Near the second order phase transition the superconducting order parameter $\Delta(r)$ in the s-wave state takes the linearized form

$$\Delta(\vec{r}) = \pi T N(\vec{r}) V(\vec{r}) \sum_{\omega} \int d^3 s Q_\omega(\vec{r},\vec{s}) \Delta(\vec{s}) \qquad (1)$$

where $\omega = (2n+1)\pi T$ with $n = 0, \pm 1, \pm 2, \ldots,$ and T is the temperature.

The scalar kernel $Q_\omega(\vec{r},\vec{s})$ is related to the matrix kernel $\hat{Q}_\omega(\vec{r},\vec{s})$ by the relation

$$Q_\omega(\vec{r},\vec{s}) = \frac{1}{2} Tr \hat{Q}_\omega(\vec{r},\vec{s}) \qquad (2)$$

where Tr means the diagonal summation.

The equation of motion for the matrix kernel in the diffusive regime is governed by the equation

$$\left(|\omega| + \frac{1}{\tau_\omega} - \frac{1}{2} D\Pi^2\right)\hat{Q}_\omega + \frac{i}{2}(\vec{I}\cdot\hat{\sigma}\hat{Q}_\omega + \hat{Q}_\omega \vec{I}\cdot\hat{\sigma}) +$$

$$\frac{1}{\tau_\omega}\left(\hat{Q}_\omega - \sigma_2 \hat{Q}_\omega^{\ tr} \sigma_2\right) = \delta(\vec{r}-\vec{s}) \qquad (3)$$

In the above equation, the gauge invariant operator $\vec{\Pi} = \nabla - (2ie/c)\vec{A}$ describes the electromagnetic coupling through the vector potential $\vec{A}$, the spin exchange interaction $\vec{I}\cdot\hat{\sigma}$ represents the electron spins $\hat{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ coupling to the vector exchange field $\vec{I}(\vec{r})$. The scattering rates due to the magnetic impurity τ_m and the spin-orbit term τ_{so}, are calculated by averaging the Matsubara Green's functions over the impurity sites in the Born approximation, and δ is the delta function.

The Usadel transport-like equation can be obtained from (3) by defining the matrix pair amplitude

$$\hat{F}(\vec{r},\omega) = \int d^3 s \hat{Q}_\omega(\vec{r},\vec{s}) i\sigma_2 \Delta(\vec{s}) \qquad (4)$$

However, it is useful to expand the matrix pair amplitude along the superfluid pairing states

$$\hat{F}(\vec{r},\omega) = \left[F_s + \vec{F}_t\cdot\hat{\sigma}\right] i\sigma_2 \qquad (5)$$

where F_s and $\vec{F}_t = (F_{tx}, F_{ty}, F_{tz})$ are respectively the scalar spin singlet and the vector spin triplet. Inserting (4) into (3) and separating along the pairing basis, using (5), we then have the scalar and the vector equations

$$\left(|\omega| + \frac{1}{\tau_m} - \frac{D}{2}\Pi^2\right) F_s + i\vec{I}\cdot\vec{F}_t = \Delta(\vec{r}) \qquad (6)$$

$$\left(|\omega| + \frac{1}{\tau_m} - \frac{D}{2}\Pi^2 + \frac{2}{\tau_{so}}\right)\vec{F}_t + i\vec{I} F_s = 0 \qquad (7)$$

The obtained set of equations (6) and (7) are the Usadel transport-like equations (Usadel, 1970) which include the perturbation effects on the superconducting phase, namely, the orbital diamagnetism, the vector exchange field, the magnetic impurity and the spin-orbit scattering.

To discuss the pairing symmetry, we write down (1) in the form

$$\Delta(\vec{r})i\sigma_2 = \pi TN(\vec{r})V(\vec{r})\sum\hat{F}(\vec{r},\omega) \quad (8)$$

Therefore $\hat{F}$ contains the antisymmetric $i\sigma_2$ and symmetric $\hat{\sigma}i\sigma_2$ matrices. According to the Pauli's exclusion principle, the spatial functions Δ and F_s will have an even symmetry whereas $\vec{F}_t$ possesses an odd symmetry. So $\hat{F}(\vec{r})$ does not satisfy the condition $\hat{F}(-\vec{r}) = \hat{F}(\vec{r})$. Nevertheless by imposing the even spatial parity of $\hat{F}(\vec{r},\omega)$ and taking the even symmetry of $F_s(\vec{r},\omega)$ and the odd frequency symmetry of $F_t(\vec{r},\omega)$ into account, a new type of the spin triplet pair amplitude arises, and is the so called Berezinskii's odd frequency triplet condensate (Berezinskii, 1974). Thus (8) becomes

$$\Delta(\vec{r}) = \pi TN(\vec{r})V(\vec{r})\sum F_s(\vec{r},\omega) \quad (9)$$

which still possesses a conventional form and explain why there is only the spin singlet pairing responsible for the superconducting phase.

In applications to the layered structures, the materials parameters in each layer are treated separately which means that the pairing functions are connected by the boundary conditions at the interfaces

$$\xi_s \nabla\hat{F}^s = \gamma\xi_f \nabla\hat{F}^f, \hat{F}^s = \hat{F}^f - \gamma_b\xi_f \hat{n}\cdot\nabla\hat{F}^f \quad (10)$$

and at the outer surfaces

$$\nabla\hat{F}^{s,f} = 0 \quad (11)$$

The superscript s(f) refers to the SC(FM) layer,

the diffusion length $\xi_{s(f)} = \sqrt{D_{s(f)}/2\pi T_{C0}}$

where T_{C0} is the isolated superconducting critical temperature. The parameter γ is the resistivity mismatch between the two layers in contact. The boundary resistance γ_b characterizes the pair jumping from SC to FM in the direction outward normal to the interface.

3. Critical temperature of FM/SC bilayers

Assuming that the dirty-limit conditions are held and there is no external magnetic field applying to the system, so the FM/SC proximity problems are well described by the generalized Usadel equations (6) and (7) without the vector potential, and (9).

To calculate the critical temperature of a FM/SC bilayer we model two infinite slabs located in the y-z plane so the x-axis represents the direction of the pair amplitude propagation from SC $(0 < x < d_s)$ to FM $(-d_f < x < 0)$. The inhomogeneous exchange field inside FM is the spiral magnetic order that rotates in an in-plane with a spiral wave vector Q,

$$\vec{I}(y) = I(0, \sin Qy, \cos Qy) \quad (12)$$

Within this configuration, the Q=0 limit corresponds to the homogeneous fixed exchange field which has the spin magnetization vector pointing constantly along the z-direction. Therefore the crypto-ferromagnetic model corresponds to a finite Q value.

Using the standard method it can be shown that an analytical expression of the reduced critical temperature $t_c = T_c/T_{co}$ takes the form like the Abrikosov-Gorkov (1961) formula which reads

$$\ln t = \Psi\left(\tfrac{1}{2}\right) - \Psi\left(\tfrac{1}{2} + \tfrac{1}{2t}\left[\left(p\xi_s\right)^2 + \tfrac{W_f(Q)}{d_s/\xi_s}\right]\right) \quad (13)$$

Where $\Psi(x)$ is the digamma function, $t = T/T_c$

And $W_f(Q)$ is the boundary function that contains all information involved in the system,

$$W_f(Q) = \frac{\left(\eta_{p+} + \eta_{p-}\right)B_{fp+}B_{fp-}}{\eta_{p+}B_{fp-} + \eta_{p-}B_{fp+} + \zeta^2 N_{fp}} \quad (14)$$

with

$$N_{fp} = \frac{\eta_{p+}B_{fp+} + \eta_{p-}B_{fp-}}{\eta_{p+}\eta_{p-}} - \frac{\left(\eta_{p+} + \eta_{p-}\right)B_{fp+}B_{fp-}}{\eta_{p+}\eta_{p-}B_{fp0}}$$

and

$$B_{fpj} = \frac{\gamma}{\gamma_b + \coth\left(k_{fpj}d_f\right)/k_{fpj}\xi_f}$$

The spectrum $k_{fp}^2 = (k_{fp0}^2, k_{fp\pm}^2)$ consists of the long-range mode k_{fp0}^2 and the short-range ones $k_{fp\pm}^2$, namely

$$k_{fp0}^2 = \frac{1}{\pi T_{c0}\xi_f^2}\left(\frac{1}{\tau_m}+\frac{2}{\tau_{so}}\right)+p^2+Q^2$$

$$k_{fp\pm}^2 = \frac{1}{\pi T_{c0}\xi_f^2}\frac{1}{\tau_m}+p^2 \pm \frac{2i}{\xi_I^2}\eta_{p\mp}$$

For the long-range mode, the spectrum $k_{fp\pm}^2$ does not depends on the exchange length, $\xi_I = \sqrt{D_f/I}$ provides that the pair amplitudes having a long penetration length in a decay manner of the order of the characteristic length $1/k_{fp0}$. The pair breaking scattering rates τ_m and τ_{so} as well as the in-plane momentum p and the spiral wave vector Q give an identical feature, in reducing the decay length $1/k_{fp0}$. For the short-range mode, the spectrum $k_{fp\pm}$ is a complex quantity,

$$\eta_{p\pm} = \sqrt{1-\zeta^2-\eta_{so}^2}$$

with $\eta_{so} = \eta + 1/I\tau_{so}, \eta = (Q\xi_I)^2/4$,
therefore the corresponding characteristic length $1/k_{fp\pm}$ contains the penetration length and the oscillation length given by the real part and the imaginary part, respectively.

At this stage we summarize the obtained results as follows; (i) the inhomogeneous exchange field and the in-plane momentum are the important ingredients for the appearance of the induced long-range triplet, (ii) the magnetic impurity scattering flips the spins of paired electrons, in both the singlet and the triplet states, so the penetration lengths are reduced, and (iii) the spin-orbit scattering displays two distinct features simultaneously, it gives an extra decay of the long-range mode, while for the short-range modes the exchange field strength is dissipated by coupling with the spin-orbit interaction as a result the oscillation period of the pair amplitude is increased.

4. Conclusion

We have presented a formulation of the odd triplet superconductivity proximity effect within the de Gennes correlation function approach, and applied it to study the behavior of the diffusive FM/SC bilayered structure. The spin-orbit interaction and the magnetic impurity scattering are also incorporated into the Usadel transport-like equation. We found that the spin-orbit interaction tends to break the singlet pair indirectly through the exchange interaction by mixing it with the triplet pair, while the magnetic impurity flips all spin states. The long-range triplet pair correlation which is defined as the non-zero spin projection on the spin quantization axis is found to be generated in the presence of the inhomogeneous exchange field. We use the model of the Neel type spiral magnetic order that rotates in the plane and has no contribution in the perpendicular direction to the interface. The linearized self-consistent order parameter equation has been solved approximately in the single mode, and the critical temperature equation is derived to determine the dependence of the critical temperature T_C on the spiral wave vector Q and the ferromagnetic layer thickness d_f. The inhomogeneous superconducting state is characterized by the in-plane modulating wave vector p. We have shown that the induced long-range triplet arises when the pair amplitudes are modulated spatially in the transverse direction.

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6. References

Abrikosov, A.A., & Gorkov, L.P. (1961). Contribution to the theory of superconducting alloys with paramagnetic impurities. *Sov. Phys. JETP*, 12(6), 1243-1253.

Auvil, P.R., Ketterson, J.B., & Song, S.N. (1989). Generalized de Gennes-Takahashi-Tachiki proximity effect theory. *J. Low. Temp. Phys*, 74(1/2), 103-117.

Baladie, I., Buzdin, A.I., Ryzhanova, N., & Vedyayev, A.V. (2001). Interplay of superconductivity and magnetism in superconductor/ferromagnet structures. *Phys. Rev. B*, 63, 054518.

Baladie, I., & Buzdin, A.I. (2003). Thermodynamic properties of ferromagnet/superconductor/ferromagnet nanostructures. *Phys. Rev. B*, 67, 014523.

Berezinskii, V.L. (1974). New model of the anisotropic phase of superfluid He3. *JETP Lett.*, 20(9), 287-289.

Buzdin, A.I., Vedyayev, A.V., & Ryzhanova, N.V. (1999). Spin orientation dependent superconductivity in S/F/S structures. *Europhys. Lett.*, 48(6), 686-691.

Barsic, P.H., Valls, O.T., & Halterman, K. (2007). Thermodynamic and phase diagrams of layerd superconductor/ferromagnet nanostructures. *Phys. Rev. B*, 75, 104502.

Champel, T., & Eschrig, M. (2005). Effect of an inhomogeneous exchange field on the proximity effect in disordered superconductor-ferromagnet hybrid structures. *Phys. Rev. B*, 72, 054523.

de Gennes, P.G. (1966). *Superconductivity of Metals and Alloys*, New York, USA: Benjamin.

Faure, M., Buzdin, A.I., & Gusakova, D. (2007). On the theory of ferromagnet/ superconductor heterostructures. *Physica C*, 454, 61-69.

Halterman, K., & Valls, O.T. (2005). Nanoscale ferromagnet-superconductor-ferromagnet switches controlled by magnetization orientation. *Phys. Rev. B*, 72, 060514(R).

Jiang, J.S., Davidovic, D., Reich, D.H., & Chein, C.L. (1995). Oscillatory superconducting transition temperature in Nb/Gd multilayers. *Phys. Rev. Lett.*, 74(2), 314-317.

Jiang, J.S., Davidovic, D., Reich, D.H., & Chein, C.L. (1996). Superconducting transition in Nb/Gd/Nb trilayers. *Phys. Rev. B*, 54(9), 6119-6121.

Krunavakarn, B., Sritrakool, W., & Yoksan, S. (2004). Nonmonotonic critical temperature in ferromagnet/ superconductor/ferromagnet trilayers. *Physica C*, 406, 46-52.

Krunavakarn, B., & Yoksan, S. (2006). Upper critical fields of ferromagnet/ superconductor layered structures. *Physica C*, 440, 25-34.

Kuboya, K., & Takanaka, K. (1998). Upper critical field and transition temperature of superconducting/ferromagnetic superlattices. *Phys. Rev. B*, 57(10), 6022-6028.

Obi, Y., Ikebe, M., & Fujishiro, H. (2005). Evidence for zero-and pi-phase order parameters of superconducting Nb/Co tri- and pentalayers from the oscillatory behavior of the transition temperature. *Phys. Rev. Lett.*, 94, 057008.

Proshin, Y.N., Zimin, A., Fazleev, N.G., & Khusainov, M.G. (2006). Hierarchy of critical temperatures in four-layered ferromagnet/superconductor nanostructures and control devices. *Phys. Rev. B*, 73, 184514.

Radovic, Z., Ledvij, M., Dobrosavljevic-Grujic, L., Shelukhin, V., Tsukernik, A., Karpovski, M., Blum, Y., Efetov, K.B., Volkov, A.F., Champel, T., Eschrig, M., Lofwander, T., Schon, G., & Palevski, A. (2006). Observation of periodic pi-phase shifts in ferromagnet-superconductor multilayers. *Phys. Rev. B*, 73, 174506.

Tagirov, L.R. (1999). Low-field superconducting spin switch based on a superconductor/ ferromagnet multilayers. *Phys. Rev. Lett*, 83(10), 2058-2061.

Takahashi, S., & Tachiki, M. (1986). Theory of the upper critical field of superconducting superlattices. *Phys. Rev. B*, 33(7), 4620-4631.

Usadel, K.D. (1970). Generalized diffusion equation for superconducting alloys. *Phys. Rev. Lett.*, 25(8), 507-509.

You, C.-Y., Bazaliy, Ya.B., Gu, J.Y., Oh, S.-J., Litvak, L.M., & Bader, S.D. (2004). Magnetization-orientation dependence of the superconducting transition temperature calculated for F/S/F trilayer structures. *Phys. Rev. B*, 70, 014505.

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Vortex contributed the critical temperature oscillations in hybrid superconductor-ferromagnet coaxial cylinders

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Abstract

The s-wave triplet Usadel equations which include the odd frequency triplet condensate are employed to study the oscillatory behavior of the superconducting critical temperature T_c in multiply connected hybrid systems e.g., a ferromagnetic core surrounded by a superconducting shell. This geometry allows us to treat the angular momentum number L of the Cooper pair as the vorticity parameter. The spiral exchange field in ferromagnet rotates in the z-plane which is characterized by a spiral wave vector Q, and its direction varies within an in-plane of the ferromagnetic core. The induced superconductivity in the ferromagnet core is controlled by the electric contact at the boundary $r = d_f$, for which we are interested in the switching of the superconducting states among the various vorticity numbers. The peculiar superconducting states with $L \neq 0$ resemblance to the pi-phase state in the case of bilayer systems. We demonstrate the switching phenomena by solving the Usadel equations together with the self-consistent order parameter subject to the Kripriyanov-Lukichev boundary conditions at the contact surface $r = d_f$, to obtain the secular equation in the multimode method for determine the oscillatory behavior of T_c in the ferromagnet core. The Abrikosov-Gorkov like-formula is also obtained in the single mode approximation.

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Keywords: Vortex; Triplet superconductivity; Ferromagnetism; Proximity effect; Transition temperature;

1 Introduction

There are the similarity between the vortex states and the proximity effect in hybrid superconductor (SC) and ferromagnet (FM) structures [1, 2]. The first one is associated with the orbital effect and causes the oscillation of the critical temperature T_c in an applied magnetic field H which is known as the Little-Park effect in multiply connected superconducting sample [3]. The latter one is responsible for the interplay between the ferromagnetic moment and the superconducting order parameter. The important feature of the proximity effect manifests itself in the oscillating or reentrant behavior of T_c as a function of a ferromagnetic layer thickness in FM/SC structures [4].

The characters of both effects differ in nature, namely, for the vortex state the interaction of Cooper pairs with the orbital magnetic field resulting the switching between vortex states characterized by different winding numbers, meanwhile, for the proximity effect the interaction of Cooper pairs with the exchange field causes the penetration of Cooper pairs inside FM layer. However, though there are different, the common situation is therefore the switching phenomenon of the Cooper pair wavefunction states. So the interplay between the orbital and exchange effects is of particular interest to determine whether the exchange interaction can stimulate the superconducting states with a nonzero vorticity in the absence of an applied magnetic field.

The attention is paid to the behavior of the superconducting critical temperature with different vorticities in multiply connected domains in which we choose a sample to be a ferromagnetic core covered by a superconducting shell. Within this selected geometry the angular momentum L of the Cooper pair wavefunction can represent the vorticity parameter. Thus the switching states between different vorticities will display the nonmonotonic T_c behavior.

Our purpose is to study the possibility of the occurrence of vortex oscillations in hybrid SC/FM coaxial cylinders by investigating the behavior of the superconducting critical temperatures with different vorticities. This work may be considered as an extensive studies of Samokhvalov et. al. [5, 6] where we taking into account the model of the spiral magnetization inside the FM core and using the Usadel equations that contain both singlet and triplet pair amplitudes [7]. We determine T_c of the proximity system by solving the secular equation in an exact manner. This paper is organized as follows. In Section 2, the formulation of the generalized Usadel equations with the boundary conditions are presented. In Section 3, we give the detail calculations of T_c exactly in the multimode method. Finally, conclusions are presented in Section 4.

2 Model and Formulation

In the dirty-limit condition the elastic electron-scattering time τ, the critical temperature T_c, and the exchange length h satisfy $T_c\tau \ll 1$ and $h\tau \ll 1$ so an appropriate method to calculate T_c is based on the Usadel equations [8]. It is well known that the Usadel approach is suitable to apply to the proximity effect system when the layers are spatially separated, the interplay between superconductivity and ferromagnetic ordering is feasible. In each layer the exchange field exists only in the ferromagnet whereas the superconducting order parameter is localized in the superconductor. However, below T_c the pair potential can leakage into the ferromagnet through the contact surface. Hence the superconducting properties are modified by the proximity effect.

Near the second order phase transition, the generalized Usadel equations (in the absence of the external magnetic field) take the linearized form [7, 9]

$$\begin{aligned}(\omega - \frac{D}{2}\nabla^2)F_s(\boldsymbol{r},\omega) + i\boldsymbol{I}(\boldsymbol{r})\cdot\boldsymbol{F}_t(\boldsymbol{r},\omega) &= \Delta(\boldsymbol{r}), &(1)\\ (\omega - \frac{D}{2}\nabla^2)\boldsymbol{F}_t(\boldsymbol{r},\omega) + i\boldsymbol{I}(\boldsymbol{r})F_s(\boldsymbol{r},\omega) &= 0, &(2)\end{aligned}$$

where F_s and $\boldsymbol{F}_t = (F_x, F_y, F_z)$ are, respectively, the scalar spin singlet and the vector spin triplet pair amplitudes. The Matsubara frequencies $\omega = (2n+1)\pi T$, with $n = 0, \pm 1, \pm 2, \ldots$, and the parameter D is the diffusion constant which labeled either D_s or D_f.

The order parameter Δ and the pair amplitude F_s in the spin singlet s-wave state are related self-consistently

$$\Delta(\boldsymbol{r}) = 2\pi T \lambda \sum_{\omega>0} F_s(\boldsymbol{r}, \omega), \tag{3}$$

where λ is the dimensionless BCS coupling constant which exists only in the SC layer.

The vector exchange field $\boldsymbol{I}$ is chosen to be the spiral type which rotates in an in-plane of the FM layer with a spiral wave vector Q,

$$\boldsymbol{I} = h(\cos Qz, \sin Qz, 0). \tag{4}$$

By means of the proximity the pair amplitude functions are connected by the boundary conditions [10, 11]

$$\xi_s \nabla F^s = \gamma \xi_f \nabla F^f, \tag{5}$$

$$F^s = F^f - \gamma_b \xi_f \hat{n} \cdot \nabla F^f, \tag{6}$$

at the interface and

$$\nabla F^{s,f} = 0, \tag{7}$$

at the vacuum surface. The superscripts s and f on F are labeled to distinguish SC from FM layers. $\xi_{s,f} = \sqrt{D_{s,f}/2\pi T_{c0}}$ is the corresponding diffusion length, where T_{c0} is the isolated superconducting critical temperature. The parameter γ is the resistivity mismatch between the two layers in contact which usually $\gamma < 1$, as a result of pair leakage from SC to FM. γ_b is the boundary resistance which represents the pair jumping from SC to FM in the direction outward normal to the interface $\hat{n}$.

Geometry of the FM/SC system consists of a ferromagnetic cylinder $r < d_f$ covered by a superconducting shell $d_f < r < d_f + d_s$. This coaxial cylinder has a height c. In such geometry the angular momentum number L serves as the vorticity parameter then we introduce [5]

$$\Delta(\boldsymbol{r}) = e^{iL\theta}\Delta(r,z), \quad F(\boldsymbol{r},\omega) = e^{iL\theta}F(r,z,\omega) \tag{8}$$

where $L = 0$ corresponds to the non-vertex state but $L \geq 1$ represents the contribution of the vortex energy.

When the spiral exchange field $\boldsymbol{I}(\boldsymbol{r})$ is substituted into the Usadel equations, the triplet component F_z can be disregarded because it is not coupled to F_s. Then by introduce the Champel-Eschrig transformation

$$F_x \pm iF_y = 2e^{\pm iQz}F_\pm, \tag{9}$$

we arrive at the effective Usadel equations

$$\left[\omega - \frac{D}{2}\nabla^2\right] F_s(\boldsymbol{r},\omega) + ih\left[F_+(\boldsymbol{r},\omega) + F_-(\boldsymbol{r},\omega)\right] = \Delta(\boldsymbol{r}), \tag{10}$$

$$\left[\omega - \frac{D}{2}(\nabla^2 \pm 2iQ\partial_z - Q^2)\right] F_\pm(\boldsymbol{r},\omega) + \frac{ih}{2} F_s(\boldsymbol{r},\omega) = 0. \tag{11}$$

Within the transformation (9), the boundary conditions still possess the canonical form as follows

$$\xi_s \partial_r F_s^s = W F_s^s, \tag{12}$$

at the FM/SC interface $r = d_f$, where

$$W = \left.\frac{\gamma \xi_f \partial_r F_s^f}{F_s^f + \gamma_b \xi_f \partial_r F_s^f}\right|_{r=d_f}, \tag{13}$$

is the boundary function. Other boundary conditions are

$$\partial_r F_s^s = 0, \quad \partial_r F_\pm^s = 0, \tag{14}$$

at the vacuum/SC surface $r = d_f + d_s$, and

$$\gamma \xi_f \partial_r F_\pm^f = \frac{\xi_s \partial_r F_\pm^s}{F_\pm^s}\left[F_\pm^f + \gamma_b \xi_f \partial_r F_\pm^f\right], \tag{15}$$

at $r = d_f$. Therefore (12)-(15) are the boundary conditions at the lateral sides ($r = d_f, d_f + d_s$), there are also have the boundary conditions at the top $z = c$, and bottom $z = 0$

$$\left.\partial_z F_{s,\pm}^{s,f}\right|_{z=0,c} = 0. \tag{16}$$

Having introduced the vorticity parameter L, next let us observe that the eigenfunctions $\cos(q_m z)$, where $q_m = m\pi/c$ with $m = 0, \pm 1, \ldots$, satisfy the Neumann type boundary conditions at $z = 0$ and $z = c$, then the z-dependent of $\Delta(\boldsymbol{r})$ and $F(\boldsymbol{r}, \omega)$ can be expanded as

$$\Delta(\boldsymbol{r}) = e^{iL\theta} \sum_{m=-\infty}^{\infty} \cos(q_m z)\Delta(r, q_m), \tag{17}$$

$$F(\boldsymbol{r}, \omega) = e^{iL\theta} \sum_{m=-\infty}^{\infty} \cos(q_m z) F(r, q_m, \omega). \tag{18}$$

The effective Usadel equations (10) and (11) subject to the boundary condition (16) can be further simplified by using the expansion (17) and (18)

$$\left[\omega - \frac{D}{2}\left(\frac{1}{r}\partial_r(r\partial_r) - \frac{L^2}{r^2} - q_m^2\right)\right] F_s + ih\left[F_+ + F_-\right] = \Delta, \tag{19}$$

$$\left[\omega - \frac{D}{2}\left(\frac{1}{r}\partial_r(r\partial_r) - \frac{L^2}{r^2} - q_m^2 - Q^2\right)\right] F_\pm + \frac{ih}{2} F_s = 0. \tag{20}$$

Therefore $F_{s,\pm}$ and Δ depend only on the radial coordinate and can be solved when information of layers are specified explicitly.

In FM ($0 < r < d_f$) we have $D = D_f$ and $\Delta = 0$. To obtain the pair amplitude solutions, relating $F_\pm^f$ to F_s^f through $F_\pm^f(r, q_m, \omega) = \Gamma_\pm F_s^f(r, q_m, \omega)$. It is found that $\Gamma_+ = \Gamma_- = \pm\eta_\mp/2$, where

$$\eta_\pm = \sqrt{1 - \eta^2} \pm i\eta, \quad \eta = \frac{1}{4}(Q\xi_h)^2,$$

with $\xi_h = \sqrt{D_f/h}$ is the exchange length. Hence the pair amplitudes are orm of the boundary function W.

sing (15) to eliminate the constant

$$\frac{A_s(\eta_+ B_{f+} + \eta_- B_{f-})}{- A_s(\eta_+ + \eta_-)}, \quad (28)$$

$$\frac{- K_L'(k_s d_f) I_L'(k_s[d_f + d_s])}{- K_L(k_s d_f) I_L'(k_s[d_f + d_s])}, \quad (29)$$

$$\frac{I_L'(k_{f\pm} d_f)}{{}_b k_{f\pm} \xi_f I_L'(k_{f\pm} d_f)}. \quad (30)$$

e Calculations

ty effect on the critical temperature

to solve the effective radial Usadel

radial gap equation

$$\sum_{\omega>0} F_s^s(r, q_m, \omega) \quad (31)$$

ndary conditions (12) and (14). Since

l due to the absence of the magnetic

oing over to the case of a thin disk.

given in terms of the modified Bessel

$$F_s^f(r, q_m, \omega) = C_+ I_L($$
$$F_\pm^f(r, q_m, \omega) = \eta_- C_+ I$$

where the wave vector $k_{f\pm}$ satisfies

$$k_{f\pm}^2 = q_m^2 +$$

In SC ($d_f < r < d_f + d_s$) the

$D_s, h = Q = 0$, and the wave vector

$$k_s^2 = q_m^2$$

The effective radial Usadel equation

$$\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) - \frac{L^2}{r^2} -\right.$$
$$\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) - \frac{L^2}{r^2} -\right.$$

The solution of (26) subject to the

$$F_\pm^s(r, q_m, \omega) = I_L(k_s r) K_L'(k_s[d_f$$

where $K_L(x)$ is the modified Bessel

notation $f' = df/dx$ has been used.

Then we can consider the homogeneous solutions along z-axis by putting the value $m = 0$ in $q_m = m\pi/c$ which gives the maximum T_c value than the other m's.

Introduce the radial eigenfunctions

$$R_n(r) = \cos(\alpha_n \ln \frac{r}{d_f}) \tag{32}$$

defined in the range $d_f < r < d_f + d_s$, with $\alpha_n = n\pi/\ln(1 + d_s/d_f), n = 0, \pm 1, \pm 2, \ldots$. Using $R_n(r)$ as a basis to expand $F_s^s(r, \omega)$ and $\Delta(r)$,

$$F_s^s(r,\omega) = \sum_{n=-\infty}^{\infty} F_n(\omega) R_n(r), \tag{33}$$

$$\Delta(r) = \sum_{n=-\infty}^{\infty} \Delta_n R_n(r). \tag{34}$$

The radial gap equation (31) can be transformed as

$$\Delta_n = 2\pi T \lambda \sum_{\omega > 0} F_n(\omega). \tag{35}$$

Meanwhile, the Fourier transform of $F_s^s(r, \omega)$ reads

$$F_n(\omega) = \frac{1}{\ln(1 + d_s/d_f)} \int_{d_f}^{d_f + d_s} F_s^s(r,\omega) R_n(r) \frac{dr}{r}, \tag{36}$$

where we have used the orthogonality relation

$$\int_{d_f}^{d_f + d_s} R_n(r) R_m(r) \frac{dr}{r} = \ln(1 + \frac{d_s}{d_f}) \delta_{nm}.$$

In order to obtain $F_s^s(r,\omega)$ let us consider the diffusive kernel $G(r,r')$ whose its differential equation has the same form as (25) except that $\Delta(r)$ is replaced by $\delta(r-r')/r$. By virtue of $G(r,r')$ we can express $F_s^s(r,\omega)$ as an integral equation

$$F_s^s(r,\omega) = \int_{d_f}^{d_f+d_s} G(r,r')\Delta(r')r'dr'. \tag{37}$$

Employing an eigenfunction expansion method [12] to solve for $G(r,r')$ and then $F_s^s(r,\omega)$. By utilizing (35),(36) and (37) together, we obtain the secular equation

$$\det|\delta_{nm} - 2\lambda\frac{T}{T_{c0}}(\frac{d_f}{\xi_s})^2\sum_{\omega>0}\Lambda_{nm}(\omega)| = 0, \tag{38}$$

where

$$\Lambda_{nm}(\omega) = \frac{\delta_{nm}}{\mu_n^2} - \frac{\Omega_L}{(d_s/d_f)\mu_n^2\mu_m^2}, \tag{39}$$

$$\mu_n = \sqrt{(\frac{n\pi}{d_s/d_f})^2 + (k_s d_f)^2 + L^2}, \tag{40}$$

$$\Omega_L = \frac{(d_f W/\xi_s)}{1+(d_f W S/\xi_s)}, \tag{41}$$

and

$$S = \frac{1}{\sqrt{(k_s d_f)^2 + L^2}}\coth(\frac{d_s}{d_f}\sqrt{(k_s d_f)^2 + L^2}). \tag{42}$$

The critical temperature T_c is determined from the secular equation (38) for which the highest T_c corresponds to the physical value.

We will show that the secular equation (38) leads to the Abrikosov-Gorkov like-formula [13] in the single mode approximation. By treating only the zero mode $(n,m)=(0,0)$ in (38), we obtain

$$\ln\left(\frac{T_c}{T_{c0}}\right)=\Psi\left(\frac{1}{2}\right)-\Psi\left(\frac{1}{2}+\frac{T_{c0}}{2T_c}\rho_L\right), \tag{43}$$

where Ψ is the digamma function and the pair-breaking parameter

$$\rho_L=\frac{L^2}{(d_f/\xi_s)^2}+\frac{W}{(d_s/\xi_s)} \tag{44}$$

also containing the contribution from the unusual $L\neq 0$ ground state. Here (43) has been derived from the following assumptions: (i) the SC shell width, d_s, is very thin comparable to the FM core radius, d_f, so we can use the approximation $\coth x\approx 1/x$ in the the summation S, and (ii) the wave vector $k_{f\pm}$ is assumed to be ω independent or equivalent to the limit of strong ferromagnet $h/\pi T_{c0}\geq 1$, so $k_{f\pm}^2\approx 2(\eta\pm i\sqrt{1-\eta^2})/\xi_h^2$. Consequently, the boundary function W that appeared in (44) does not depend on ω and has the expression approximately as

$$W\approx\frac{\sqrt{1-\eta^2}|B_{f+}|^2}{\sqrt{1-\eta^2}\mathrm{Re}B_{f+}+\eta\mathrm{Im}B_{f+}}. \tag{45}$$

For simplicity, if we neglect the effect of boundary resistivity ($\gamma_b\to 0$) the term B_{f+} above reads

$$B_{f+}=\gamma\left[\frac{L}{(d_f/\xi_f)}+k_{f+}\xi_f\frac{I_{L+1}(k_{f+}d_f)}{I_L(k_{f+}d_f)}\right]. \tag{46}$$

It is obvious that $W(Q \neq 0) < W(Q = 0)$ then we can speculate the T_c behavior should be less oscillatory but tends towards a monotonic decay. Thus the spiral type ferromagnet decreases the decay length and increases the oscillation period, which is the typical pair-weakening effect.

4 Conclusion

We have applied the generalized Usadel transport-like equations that include the odd triplet component, to study the proximity effect in multiply connected regions, namely, the hybrid FM/SC coaxial cylinders. A model of the spiral magnetic order which rotates inside the ferromagnetic core was considered to investigate the switching phenomena between the superconducting states with different vorticities. We have solved the effective radial Usadel equations together with the self-consistent gap equation exactly in the multimode method to obtain the secular equation that determine the dependence of the critical temperature T_c on material parameters. Unfortunately, within the Champel-Eschrig transformation that is the merely exact soluble method, the long-range triplet mode is absent. This means that only the short-range modes can survive in FM. However, the spiral type ferromagnet plays the role of the pair-weakening effect i.e., it decreases the decay length and increases the oscillation period of the pair amplitudes in FM. The zeroth mode of the secular equation, with some simplified arguments, yields the Abrikosov-Gorkov like-formula and clearly shows the switching phenomena which can be seen from the vorticity L through the depairing parameter of the reduced critical temperature equation. Numerical results exhibit the vortex states in favor of superconductivity in some ranges of the FM core radius. Thus the induced superconductivity between vortex and non-vortex states can be realized in the proximity system.

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References

[1] A.I. Buzdin, *Rev. Mod. Phys.* **77** (2005) 935.

[2] I.F. Lyuksyutov, V.L. Pokrovsky, *Adv. Phys.* **54** (2005) 67.

[3] W. Little, R. D. Park, *Phys. Rev. Lett.* **9** (1962) 9.

[4] Z. Radović, M. Ledvij, L. Dobrosavljević-Grujić, A. I. Buzdin, J. R. Clem, *Phys. Rev. B* **44** (1991) 759.

[5] A. V. Samokhvalov, A. S. Melnikov, A. I. Buzdin, *Phys. Rev. B* **76** (2007) 184519.

[6] A. V. Samokhvalov, A. S. Melnikov, J-P. Ader, A. I. Buzdin, *Phys. Rev. B* **79** (2009) 174502.

[7] T. Champel, M. Eschrig, *Phys. Rev. B* **71** (2005) 220506(R).

[8] K. D. Usadel, *Phys. Rev. Lett.* **25** (1970) 507.

[9] T. Rachataruangsit, S. Yoksan, *Physica C* **467** (2007) 156.

[10] M. Yu. Kupriyanov, V. F. Lukichev, *Sov. Phys. JETP* **67** (1988) 1163.

[11] J. Aarts, J. M. E. Geers, E. Brück, A. A. Golubov, R. Coehoorn, *Phys. Rev. B* **56** (1997) 2779.

[12] B. Krunavakarn, S. Yoksan, *Physica C* **440** (2006) 25.

[13] A. A. Abrikosov, L. P. Gorkov, *Sov. Phys. JETP* **12** (1961) 1243.

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กรรมการพิจารณาตำแหน่งวิชาการของมหาวิทยาลัยเทคโนโลยีสุรนารี

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3.	งานเขียน	1	ศาสตร์พิศวง	สำนักพิมพ์ สารคดี	2546	
4.	งานเขียน	1	ปริศนานานาสัตว์	สำนักพิมพ์ ปาเจรา	2547	
5.	งานเขียน	1	มหัศจรรย์แห่งพืช	สำนักพิมพ์ ปาเจรา	2547	
6.	งานเขียน	1	ศาสตร์ปริศนา	สำนักพิมพ์ สารคดี	2547	
7.	งานเขียน	1	อารยธรรมลี้ลับ	สำนักพิมพ์ สารคดี	2547	
8.	งานเขียน	1	โภชนา-โรคา น่าฉงน	สำนักพิมพ์ สารคดี	2547	
9.	งานเขียน	1	ไอน์สไตน์ 1 ศตวรรษแห่งปีมหัศจรรย์	สำนักพิมพ์ สารคดี	2548	ร่วมกับนักเขียนอื่นอีก 2 ท่าน
10.	งานเขียน	1	สารพัดสัตว์โลก	สำนักพิมพ์ สารคดี	2548	
11.	งานเขียน	1	นักวิทยาศาสตร์ผู้ยิ่งใหญ่	สำนักพิมพ์ ปาเจรา	2548	

12.	งานเขียน	1	อัจฉริยะนักวิทย์	สำนักพิมพ์ สารคดี	2548	
13.	งานเขียน	1	100 ผู้พิชิตรางวัลโนเบล	สำนักพิมพ์ ปาเจรา	2549	ร่วมกับนักเขียนอื่นอีก 4 ท่าน
14.	งานเขียน	1	ไขปริศนาวิทยาการ	สำนักพิมพ์ สารคดี	2549	
15.	งานเขียน	1	ถอดรหัสอัจฉริยะดาวินชี	สำนักพิมพ์ สารคดี	2550	ร่วมกับนักเขียนอื่นอีก 2 ท่าน
16.	งานเขียน	1	อัจฉริยะนักวิทย์เล่ม 2	สำนักพิมพ์ สารคดี	2550	
17.	งานเขียน	1	ศิลปินอัจฉริยะ	สำนักพิมพ์ สารคดี	2550	
18.	งานเขียน	1	สุดยอดนักฟิสิกส์โลก	สำนักพิมพ์ สารคดี	2554	
19.	งานเขียน	1	สุดยอดนักเคมีโลก	สำนักพิมพ์ สารคดี	2554	
20.	งานเขียน	1	สุดยอดนักผจญภัย	สำนักพิมพ์ สารคดี	2555	

ลำดับที่	ประเภทของผลงาน	จำนวนผลงาน	ชื่อผลงาน	สำนักพิมพ์หรือแหล่งสนับสนุน	ปีที่พิมพ์เผยแพร่	
1.	งานแปล	1	ดาวฤกษ์และดาวเคราะห์	สำนักพิมพ์ ปาเจรา	2548	
2.	งานแปล	1	วิถีความเป็นไปแห่งเอกภพ	สำนักพิมพ์ ปาเจรา	2553	

ลำดับที่	ประเภทของผลงาน	จำนวนผลงาน	ชื่อผลงาน	สำนักพิมพ์หรือแหล่งสนับสนุน	ปีที่พิมพ์เผยแพร่	
1.	งานบรรณาธิการ	1	สำรวจโลกวิทยาศาสตร์: การเคลื่อนที่ แรง และพลังงาน	บริษัท เพียร์สัน เอ็ดดูเคชั่น อินโดไชน่า จำกัด	2546	
2.	งานบรรณาธิการ	1	สำรวจโลกวิทยาศาสตร์: แสง และเสียง	บริษัท เพียร์สัน เอ็ดดูเคชั่น อินโดไชน่า จำกัด	2546	
3.	งานบรรณาธิการ	1	100 ความเชื่อ 100 ความจริง	สำนักพิมพ์ ปาเจรา	2550	

ลำดับที่	ประเภทของผลงาน	จำนวนผลงาน	ชื่อผลงาน	สำนักพิมพ์หรือแหล่งสนับสนุน	ปีที่พิมพ์เผยแพร่	
1.	หนังสือตำรา	1	กลศาสตร์คลาสสิก	มหาวิทยาลัยศรีนครินทรวิโรฒ	2536	
2.	หนังสือตำรา	1	กลศาสตร์ตัวอย่าง	มหาวิทยาลัยศรีนครินทรวิโรฒ	2538	
3.	หนังสือตำรา	1	กลศาสตร์นิวตัน: เฉลยโจทย์	มหาวิทยาลัยศรีนครินทรวิโรฒ	2545	

ลำดับที่	ประเภทของผลงาน	จำนวนผลงาน	ชื่อผลงาน	สำนักพิมพ์หรือแหล่งสนับสนุน	ปีที่พิมพ์เผยแพร่	
1.	งานวิจัย	1	Calculations of Host Nuclear Relaxation in Dilute Kondo alloys for $T<<T_k$	Solid State Communications 27: 287-289 ทุนรัฐบาลไทย	1978	

2.	งานวิจัย	1	Spatial Variations of Order Parameter near Anderson Impurity in Superconductors	Journal de Physique 39: C6-874-875 ทุนรัฐบาลไทย	1978	
3.	งานวิจัย	1	Spatial Variations of Order Parameter near Anderson Impurity with high Kondo Temperature	Physical Review B 18: 4768-4779 ทุนรัฐบาลไทย	1978	
4.	งานวิจัย	1	Anomalous Specific Heat and Nuclear Spin Relaxation in A1-Transition Metal Alloys	Physics Letters 70 A: 122-124 ทุนรัฐบาลไทย	1979	
5.	งานวิจัย	1	Spatial Variations of the Order Parameter around a Kondo Impurity for $T<T_c$	Physics Letters 78 A: 416-418 ทุนส่วนตัว	1980	
6.	งานวิจัย	1	Transition Temperature of Two-Band Superconductors with Anderson Impurities	Journal of Low Temperature Physics 24: 217-226 ทุนส่วนตัว	1981	
7.	งานวิจัย	1	Specific Heat of Two-Band Superconductors with Anderson Impurities	Journal of Low Temperature Physics 44: 247-257 ทุนส่วนตัว	1981	
8.	งานวิจัย	1	Transition Temperature of Proximity-Effect Sandwich Systems Containing Kondo Impurities	Journal of Low Temperature Physics 49: 33-42 ทุนส่วนตัว	1982	ฉบับที่ดีที่สุด คือ เรื่องที่ 8 มีประโยชน์ต่อ นานาชาติ เหตุผล: มีความสำคัญทำให้ได้รับเลือก

						เป็นนักวิจัยและทุนวิจัยของ Natural Science and Engineering Research Council of Canada ปี 2534 – 2536 และได้รับทุนวิจัยของ Abdus Salam International Center for Theoretical Physics ที่ Italy ปี 2534 - 2546
9.	งานวิจัย	1	Critical Temperature of Two-Band Superconductor Containing Kondo Impurities	Physics Letters 91 A: 469-471 ทุน ICTP, Italy	1982	
10.	งานวิจัย	1	Specific Heat Discontinuity of a Proximity-Effect Sandwich System Containing Kondo Impurities	Journal of Low Temperature Physics 51: 569-579 ทุน ICTP, Italy	1983	
11.	งานวิจัย	1	Kondo Impurities in Strong Coupling Superconductors	Journal of Low Temperature Physics 56: 13-23 ทุน NSERC, Canada	1984	
12.	งานวิจัย	1	Shiba-Rusinov Theory of Magnetic Impurities in Superconductors Beyond s-Wave Scattering: Eliashberg Formalism	Physical Review B 30: 2659-2665 ทุน NSERC, Canada	1984	

13.	งานวิจัย	1	Shiba-Rusinov Theory of Magnetic Impurities in Anisotropic Superconductors: Eliasberg Formalism	Physical Review B 30: 2917-2919 ทุน NSERC, Canada	1984	
14.	งานวิจัย	1	Transition Temperature of Anisotropic Superconductors with Kondo Impurities	Solid State Communications 51: 175-177 ทุน NSERC, Canada	1984	
15.	งานวิจัย	1	Transition Temperature of Anisotropic Superconductor with Kondo Impurities	Journal of Low Temperature Physic 60: 405-414 ทุน ICTP, Italy	1985	
16.	งานวิจัย	1	Superconducting Composite: Calculation of the Localized Single-Particle Excitations	Solid State Communications 56: 763-765 ทุน NSERC, Canada	1985	

17.	งานวิจัย	1	Influence of Kondo Effect on the Specific Heat Jump of Anisotropic Superconductors	Solid State Communications 57: 113-117ทุน NSERC, Canada	1986	
18.	งานวิจัย	1	Localized Single-Particle Excitations and the Density of State for Superconducting Composites	Journal of Low Temperature Physics 63: 447-458 ทุน NSERC, Canada	1986	
19.	งานวิจัย	1	Excitation Spectrum and the Density of States for a superconducting Bilayer Composite	Journal of Low Temperature Physics 66:115-125 ทุน NSERC, Canada	1987	
20.	งานวิจัย	1	Free Energy Formula for a Superconducting Alloy with Localized State in the Energy Gap	Physical Review B 37: 9710-9711 ทุน NSERC, Canada	1988	
21.	งานวิจัย	1	Isotope Effect in High T_c Superconductors	Solid State Communications 78: 233-237 ทุน STDB, ประเทศไทย	1991	

22.	งานวิจัย	1	Role of Bounding Layers in High T_c Superconductors	Journal of Low Temperature Physics 83: 117-124 ทุน STDB, ประเทศไทย	1991	
23.	งานวิจัย	1	Cooper Pairing in Oxide Superconductors	Solid State Communications 79: 417-420 ทุน STDB, ประเทศไทย	1991	
24.	งานวิจัย	1	Effect of Gap Anisotropy and Plasmons on the Critical Temperature of High-T_c Superconductors	Solid State Communications 83: 431-434 ทุน STDB, ประเทศไทย	1992	
25.	งานวิจัย	1	Transition Temperature of Proximity-Effect Antiferromagnetic Superconductor Sanwiches	Solid State Communications 89: 453-458 ทุน ICTP, Italy	1994	
26.	งานวิจัย	1	Energy Scale of the Pairing Model with Energy-Dependent Density of States	Journal of Superconductivity 9: 167-170 ทุน สวทช, ประเทศไทย	1996	

27.	งานวิจัย	1	The ratio $2\Delta_0/kT_c$ in a Van Hove Superconductor	Journal of Superconductivity 9: 485-486 ทุน สวทช, ประเทศไทย	1996	
28.	งานวิจัย	1	The ratio $2\Delta_0/kT_c$ in BCS Superconductivity	Journal of Superconductivity 9: 603-604 ทุน สวทช, ประเทศไทย	1996	
29.	งานวิจัย	1	Gap-to-T_c Ratio as a Function of the Fermi Level Shift	Journal of Superconductivity 10: 1-2 ทุน สวทช, ประเทศไทย	1997	
30.	งานวิจัย	1	Effect of Orthorhombic Distortion and Second-Nearest Neighbor Hopping on Gap-to-T_c Ratio	Journal of Superconductivity 10: 189-191 ทุน สวทช, ประเทศไทย	1997	
31.	งานวิจัย	1	Critical Temperature of a Non-Fermi Superconductor with an Energy-Dependant Density of States	Physical Review B 56: 8298-8302 ทุน สวทช, ประเทศไทย	1997	

32.	งานวิจัย	1	Pair-breaking Effect in Van Hove Superconductor	Journal of Superconductivity 10: 503-506 ทุน สวทช, ประเทศไทย	1997	
33.	งานวิจัย	1	The Gap-to-T_c Ratio of a Van Hove Superconductor	Journal of Superconductivity 11: 271-273 ทุน สวทช, ประเทศไทย	1998	
34.	งานวิจัย	1	Reduced Gap Ratio of High-T_c Cuprates within d-Wave Two-Dimensional Van Hove Scenario	Journal of Superconductivity 11: 429-432 ทุน สวทช, ประเทศไทย	1998	
35.	งานวิจัย	1	Specific Heat Jump at T_c of High-T_c Superconductors: Effect of Van Hove Singularity	Proceedings of the First Regional Conference on Magnetic and Superconducting Materials, Tehran Vol. A 289-294 ทุน สกว, ประเทศไทย	1999	

36.	งานวิจัย	1	Dependence of the Gap-Ratio on the Fermi Level Shift in a Van Hove Superconductor	International Journal of Modern Physic B 14: 2127-2133 ทุน สกว, ประเทศไทย	2000	
37.	งานวิจัย	1	Dependence of the Gap-Ratio on the Fermi Level Shift in a Van Hove Superconductor	Journal of Superconductivity 13: 33-36 ทุน สกว, ประเทศไทย	2000	
38.	งานวิจัย	1	Specific Heat Jump at T_c of High Superconductors: Effect of Van Hove Singularity	Journal of Superconductivity 13: 41-46 ทุน สกว, ประเทศไทย	2000	
39.	งานวิจัย	1	Thermodynamic Properties of a BCS Superconductor	Physica C 338: 305-315 ทุน สกว, ประเทศไทย	2000	
40.	งานวิจัย	1	Free Energy Formula for a BCS Superconductor near zero Temperature	Physica B 321: 353-355 ทุน สกว, ประเทศไทย	2002	
41.	งานวิจัย	1	Thermodynamic Properties of a BCS Superconductor in the Zero Temperature	International Journal of Modern Physics B 16: 3615-3621 ทุน สกว, ประเทศไทย	2002	

42.	งานวิจัย	1	Effect of in-Plane Anisotropy on Specific Heat Jump of MgB_2	Phys Stat Sol (b) 240: 591-595 ทุน สกว, ประเทศไทย	2003	
43.	งานวิจัย	1	Thermodynamic Properties of a d-Wave Superconductor near Zero Temperature	Journal of Superconductivity 17: 297- ทุน สกว, ประเทศไทย	2004	
44.	งานวิจัย	1	Critical Temperature of Ferromagnetic/ Superconductor/ Ferromagnet Trilayers	Physics Letters A 322: 396-401 ทุน สกว, ประเทศไทย	2004	
45.	งานวิจัย	1	Nonmonotoxic Critical Temperature in Ferromagnet/ Superconductor/ Ferromagnet Trilayers	Physica C 406: 46-52 ทุน สกว, ประเทศไทย	2004	
46.	งานวิจัย	1	Effect of Density of States on Isotope Effect Exponent of Two-Band Superconductors	Physica C 425: 149-154 ทุน สกว, ประเทศไทย	2005	
47.	งานวิจัย	1	H_{C2} of Anisotropy Two-Band Superconductors by Ginzburg Landau Approach	Physica C 434: 62-66 ทุน สกว, ประเทศไทย	2006	

48.	งานวิจัย	1	Upper Critical Fields of Ferromagnet/ Superconductor/ Ferromagnet layered Structures	Physica C 440: 25-34 ทุน สกว, ประเทศไทย	2006	
49.	งานวิจัย	1	Pair Breaking Effects on Upper Critical Fields in Ferromagnet/ Superconductor Layered Structures	Physica C 455: 39-45 ทุน สกว, ประเทศไทย	2007	
50.	งานวิจัย	1	Spin Orbit Scattering Effect on Long-Range Odd Frequency Triplet Pairing in Ferromagnet/ Superconductor Bilayers	Physica C 467: 156-166 ทุน สกว, ประเทศไทย	2007	
51.	งานวิจัย	1	Influence of pair breaking effects on the long-range odd triplet superconductivity in a ferromagnet/superconductor bilayer	Rangsit Journal of Arts and Sciences July-December 121-125	2011	
52.	งานวิจัย	1	Vortex Contributed the critical temperature oscillations in hybrid superconductor-ferromagnet coaxial cylinders	Submitted for publication in Physica C	2013	