บทที่ 6
สรุปผลการวิจัย

จากการใช้แบบจำลองต่างๆ 3 แบบคือ แบบจำลองแบบหลุมศักย์ 2 หลุม แบบจำลองแบบไม่สมมาตร และแบบจำลองแบบสองแถบ ศึกษาสมบัติของแมกนีเซียมไดโบไรด์ โดยใช้แบบจำลองแบบหลุมศักย์ 2 หลุม ศึกษาอัตราส่วนของช่องว่างพลังงาน ต่ออุณหภูมิวิกฤติของแมกนีเซียมไดโบไรด์ ใช้แบบจำลองแบบไม่สมมาตรศึกษาความจุความร้อน และใช้แบบจำลองแบบสองแถบพลังงานศึกษาอุณหภูมิวิกฤติและสัมประสิทธิ์ของไอโซโทป พบว่าสามารถอธิบายสมบัติของแมกนีเซียมไดโบไรด์ได้ดี

แต่จากผลการทดลองและคำนวณเชิงตัวเลขและทางทฤษฎีล่าสุดพบว่าแมกนีเซียมไดโบไรด์มีสมบัติของตัวนำยวดยิ่งแบบสองแถบพลังงานที่มีความไม่สมมาตร และในการวิจัยครั้งนี้ผู้วิจัยใช้แบบจำลองแบบสองแถบพลังงานที่คำนึงผลของอันตรกิริยาทั้งแบบ phonon และ non-phonon interaction รวมทั้งผลของ interband interaction เข้าไปด้วย ซึ่งให้ผลการคำนวณที่ดี โดยมีค่าตัวแปรต่างๆ คือ ค่าคงตัวของการคู่ควบ อุณหภูมิวิกฤติ สัมประสิทธิ์ของไอโซโทป สอดคล้องกับผลการทดลอง อย่างไรก็ตามผู้วิจัยไม่ได้คำนึงถึงผลของความไม่สมมาตรเข้าไปด้วย จึงไม่ทราบว่าความไม่สมมาตรมีผลต่อสมบัติของแมกนีเซียมไดโบไรด์มากหรือน้อยอย่าง จึงไม่สามารถอธิบายสมบัติอื่นได้ครบถ้วน ซึ่งจะต้องทำการวิจัยต่อไป

เอกสารอ้างอิง

phys. stat. sol. (b), 1–5 (2003) / **DOI** 10.1002/pssb.200301884

Effect of in-plane anisotropy on specific heat jump of MgB_2

P. Udomsamuthirun[*], **S. Rakpanich,** and **S. Yoksan**

Department of Physics, Faculty of Science, Srinakhariwirot University,
Sukumvit 23 Road, Bangkok 10110, Thailand

Received by P. Deák 17 March 2003, revised 23 July 2003, accepted 5 August 2003
Published online 10 October 2003

PACS 65.40.Ba, 74.20.Fg, 74.70.Ad

An analytical result for the normalized jump of the heat capacity $\Delta C/C_N$ is derived for in-plane anisotropic s-wave superconductors within the framework of the weak-coupling BCS approach. Our results show that within the anisotropy gap model the value of $\Delta C/C_N$ must be less than the BCS value and the effect of in-plane asymmetry will reduce or increase the value of $\Delta C/C_N$ of in-plane symmetry depending on its parameters.

1 Introduction

The specific heat of MgB_2 [1] has shown the difference of amplitude and sharpness between the BCS curve and the data and has been interpreted that MgB_2 is a two-gap superconductor. One gap is found to be larger (Δ_σ) and the other smaller (Δ_π) than $1.76\,k_B T_c$ [2]. Δ_σ approximately follows the BCS-like curve with non-standard gap ratio $2\Delta_\sigma/k_B T_c \approx 4.18$ but Δ_π shows a marked reduction with respect to BCS-like behavior with $2\Delta_\pi/k_B T_c \approx 1.59$ [3]. A larger gap Δ_σ is associated with a two- dimensional σ band due to p_x and p_y electrons of B atoms and the smaller Δ_π is associated with a three-dimensional π band due to p_z electrons of B atoms. These are weakly hybridized with electron orbitals of Mg atoms. The σ band is strongly coupled to in-plane B-atom vibration and the inter-band impurity scattering is strongly reduced due to the different symmetry of σ and π bands [4]; therefore the diffusion from one band to the other is inhibited. This preserves the T_c suppression and the large anisotropy of the energy gaps. Within these observations, MgB_2 has shown the influence of both two bands and gap anisotropy. The concept of multiband superconductivity was first introduced by Suhl et al. [5] and Moskalenko [6] in the case of large disparity of the electron−phonon interaction for the different Fermi-surface sheets. The isotope effect of MgB_2 established that the interaction responsible for the formation of pairing is mediated by phonons [7, 8] and nuclear magnetic resonance showed that the symmetry of Cooper pairs is s-wave [9].

In Seneor et al.'s paper [10], they found that the order parameter of MgB_2 must be of the anisotropic s-wave with uniaxial symmetry or of the anisotropic s-wave with in-plane anisotropy. Mishonov et al. [11, 12] considered the temperature dependence of the specific heat of a clean anisotropic gap superconductor, which is applicable to MgB_2. In Hass and Maki [13] and Posazhennikova et al. [14], models for the anisotropy gap with uniaxial symmetry in MgB_2 were proposed. The central issue in their research is to propose an analytical model for analyzing the thermodynamic behavior by assuming a spherical Fermi surface; the anisotropy gap functions are introduced as $\Delta(k) = \Delta(1+az^2)/(1+a)$ in [13] and $\Delta(k) = \Delta/\sqrt{1+az^2}$ in [14], where $z = \cos\theta$, θ is the polar angle, and a is an anisotropy parameter that

[*] Corresponding author: e-mail: udomsamut55@yahoo.com

can be determined experimentally from the ratio of the gap in the *z*-axis and the gap in the *ab*-plane. Using these two models, Mishonov et al. [15, 16] derived analytical results for the normalized jump of the heat capacity $\Delta C/C_N$. They find that the anisotropy parameter a decreases the BCS $\Delta C/C_N$ result.

In order to describe the three-dimensional superconducting nature in MgB_2, we consider the gap function of MgB_2 depending on its axis parameters in three dimensions and follow the weak-coupling BCS approach to derive an analytical expression for $\Delta C/C_N$ for an anisotropic s-wave with in-plane anisotropy.

2 Anisotropy gap model

Because MgB_2 is an anisotropic s-wave superconductor, we can use the BCS gap equation as

$$\Delta_k = \frac{1}{2}\sum_{k'} \frac{V_{kk'}\Delta_{k'}}{\sqrt{\varepsilon_{k'}^2+\Delta_{k'}^2}} \tanh\left(\frac{\sqrt{\varepsilon_{k'}^2+\Delta_{k'}^2}}{2T}\right), \quad (1)$$

where every symbol has its usual meaning.

We assume the pairing interaction as

$$V_{kk'} = V_0 f(k)\, f(k'). \quad (2)$$

Here $f(k)$ is the anisotropy function and V_0 is the coupling constant.

Based on the pairing interaction, an anisotropy gap is given by

$$\Delta_k = \Delta(T)\, f(k), \quad (3)$$

where $\Delta(T)$ is temperature dependent and $f(z) = (1+az^2)/(1+a)$ as in [13] or $f(z) = 1/\sqrt{1+az^2}$ as in [14]. These two models consider only the effect of anisotropy in the *c*-axis.

Inserting Eqs. (2) and (3) into Eq. (1), we get the gap equation averaged over all directions of Δ_k as

$$\frac{1}{N_0V_0} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi \int_0^{\omega_D} \mathrm{d}\varepsilon \frac{f^2(\theta,\phi)}{\sqrt{\varepsilon^2+\Delta^2(T)\, f^2(\theta,\phi)}} \tanh\left(\frac{\sqrt{\varepsilon^2+\Delta^2(T)\, f^2(\theta,\phi)}}{2T}\right). \quad (4)$$

Here we use a constant density of states N_0, θ is the polar angle, ϕ is the azimuthal angle, and the factor $1/4\pi$ is the normalization factor.

In the vicinity of T_c where $(T_c - T)/T_c \ll 1$ the gap is small compared with temperature. Let $\ln(T/T_c) \approx (T - T_c)/T_c$; we find that

$$\Delta^2(T) = \left(\frac{T_c - T}{T_c}\right)\frac{8\pi^2T_c^2\langle f^2(\theta,\phi)\rangle}{7\zeta(3)\,\langle f^4(\theta,\phi)\rangle}, \quad (5)$$

where $\langle f^2(\theta,\phi)\rangle = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} \mathrm{d}\theta\, \mathrm{d}\phi\, \sin\theta f^2(\theta,\phi)$ and $\zeta(x)$ is the Riemann zeta function.

The thermodynamic potential density in the normal state and the superconducting state are Ω_n and Ω_s respectively. Their relation can be written as [17]

$$\Omega_s - \Omega_n = -\frac{7}{16}\frac{\zeta(3)\, N_0}{\pi^2T_c^2}\langle f^4(\theta,\phi)\rangle\, \Delta^4(T). \quad (6)$$

Inserting Eq. (5) into Eq. (6), we get

$$\Omega_{\mathrm{s}} - \Omega_{\mathrm{n}} = -\frac{8N_0\pi^2T_{\mathrm{c}}^2}{14\zeta(3)}\left(1-\frac{T}{T_{\mathrm{c}}}\right)^2 \frac{\langle f^2(\theta,\phi)\rangle^2}{\langle f^4(\theta,\phi)\rangle}\,. \tag{7}$$

By the relation $\Omega_{\mathrm{s}} - \Omega_{\mathrm{n}} = -(1/8\pi)\,H_{\mathrm{c}}^2$ with H_{c} the critical magnetic field, we get

$$H_{\mathrm{c}}(T) = \sqrt{\frac{32\pi N_0}{7\zeta(3)}}\pi T_{\mathrm{c}}\left(1-\frac{T}{T_{\mathrm{c}}}\right)\frac{\langle f^2(\theta,\phi)\rangle}{\sqrt{\langle f^4(\theta,\phi)\rangle}}\,. \tag{8}$$

And, by the relation $\Delta C = \frac{T_{\mathrm{c}}}{4\pi}\left(\frac{\mathrm{d}H_{\mathrm{c}}}{\mathrm{d}T}\right)^2\Big|_{T=T_{\mathrm{c}}}$, the normalized jump of the heat capacity at $T = T_{\mathrm{c}}$ is found to be

$$\frac{\Delta C}{C_{\mathrm{N}}} = 1.43\frac{\langle f^2(\theta,\phi)\rangle^2}{\langle f^4(\theta,\phi)\rangle}\,. \tag{9}$$

Equation (9) is Pokrovskii's result for the reduced heat-capacity jump [12, 16, 18].

3 The specific heat jump of MgB_2

To show the effect of the type of in-plane anisotropy on $\Delta C/C_{\mathrm{N}}$, we make some comparisons between different models applied to MgB_2. We first consider the case of the symmetry gap $f(\theta,\phi) = 1$. Equation (9) then gives $\Delta C/C_{\mathrm{N}} = 1.43$, which is the BCS result.

In the case of

$$\Delta(\theta,\phi) = \Delta_0(1 + a(\sin\theta\cos\theta)^2 + b(\sin\theta\sin\theta)^2)\,, \tag{10}$$

this gives

$$f(\theta,\phi) = 1 + a(\sin\theta\cos\theta)^2 + b(\sin\theta\sin\theta)^2\,. \tag{11}$$

Here Δ_0, a, and b are the gap parameters in the c-axis, a-axis, and b-axis, respectively. Within this model, we get $\Delta_z = \Delta_0$, $\Delta_x = \Delta_0(1+a)$, and $\Delta_y = \Delta_0(1+b)$. For $a = b$, we get $f(\theta) = 1 + a\ \sin^2\theta$, which agrees with Hass and Maki's model [13] in the case of an anisotropic s-wave with uniaxial symmetry.

Next we let $b = a + \delta$ and insert Eq. (11) into Eq. (9); the normalized jump of the heat capacity is

$$\frac{\Delta C}{C_{\mathrm{N}}} = 1.43\frac{\left(1+\frac{4}{3}a+\frac{8}{15}a^2+\frac{2}{3}\delta+\frac{8}{15}a\delta+\frac{1}{5}\delta^2\right)^2}{\left[\begin{array}{l}1+\frac{8}{3}a+\frac{16}{5}a^2+\frac{64}{35}a^3+\frac{128}{315}a^4+\frac{4}{3}\delta+\frac{16}{5}a\delta+\frac{96}{35}a^2\delta\\ +\frac{256}{315}a^3\delta+\frac{6}{5}\delta^2+\frac{72}{35}a\delta^2+\frac{32}{35}a^2\delta^2+\frac{4}{7}\delta^3+\frac{32}{63}a\delta^3+\frac{1}{9}\delta^4\end{array}\right]}\,. \tag{12}$$

Our Eq. (12) can be reduced to Eq. (1) of [14] by putting $\delta = 0$ and $a = -a'/(1+a)$; a' is the gap parameter of [15]. The numerical calculation of Eq. (12) is shown in Fig. 1, where $\Delta C/C_{\mathrm{N}}$ shows the maximum value when $a = 0$ for all δ. When $-1 < a < 0$, $\Delta C/C_{\mathrm{N}}$ decreases with increasing δ/a and, as $a > 0$, $\Delta C/C_{\mathrm{N}}$ also decreases with increasing $|\delta/a|$.

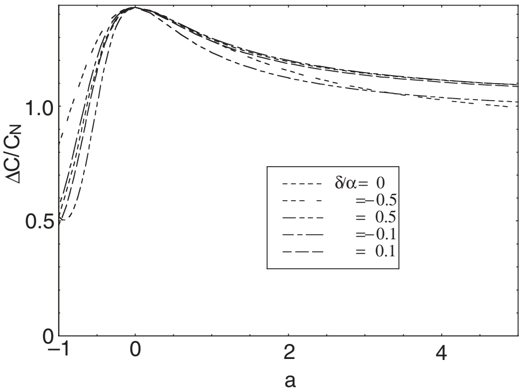


Fig. 1 Plot of $\Delta C/C_{\mathrm{N}}$ versus gap parameter a in the a-axis at various values of δ/a in the case $\Delta(\theta,\phi) = \Delta_0(1+a(\sin\theta\cos\theta)^2 + b(\sin\theta\sin\theta)^2)$, where $b = a + \delta$. At $\delta/a = 0$, we get Mishonov et al. [15] result.

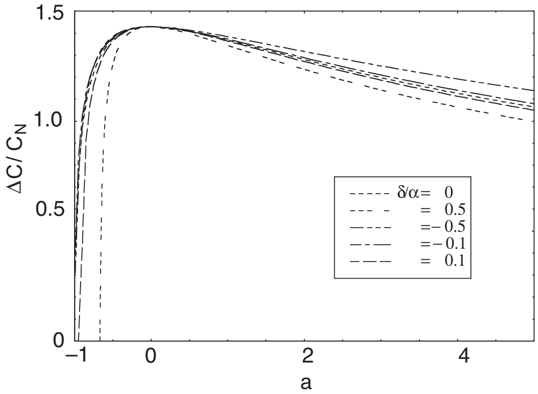


Fig. 2 Plot of $\Delta C/C_{\mathrm{N}}$ versus gap parameter a in the a-axis at various values of δ/a in the case $\Delta(\theta,\phi) = \Delta_0/\sqrt{1+a(\sin\theta\cos\phi)^2 + b(\sin\theta\sin\phi)^2}$, where $b = a + \delta$. At $\delta/a = 0$, we get Mishonov et al. [16] result.

For another case of interest, we consider the anisotropy gap in the form

$$\Delta(\theta,\phi) = \frac{\Delta_0}{\sqrt{1+a(\sin\theta\cos\phi)^2 + b(\sin\theta\sin\phi)^2}}, \tag{13}$$

which gives

$$f(\theta,\phi) = \frac{1}{\sqrt{1+a(\sin\theta\cos\phi)^2 + b(\sin\theta\sin\phi)^2}}. \tag{14}$$

Here Δ_0, a, and b are the gap parameters along the c-axis, a-axis, and b-axis, respectively. Within this model, we get $\Delta_z = \Delta_0, \Delta_x = \Delta_0/(1+a)$, and $\Delta_y = \Delta_0/(1+b)$. For $a = b$, we get $f(\theta) = 1/\sqrt{1+a\sin^2\theta}$, which agrees with Posazhennikova et al.'s model [14] in the case of an anisotropic s-wave with uniaxial symmetry.

As before, we let $b = a + \delta$ and insert Eq. (14) into Eq. (9); the normalized jump of the heat capacity is

$$\frac{\Delta C}{C_{\mathrm{N}}} = 1.43\times 2\,\frac{\left[\dfrac{\tanh^{-1}\sqrt{\dfrac{a}{1+a}}}{\sqrt{a(1+a)}} + \dfrac{\delta}{2}\left(-\dfrac{\tanh^{-1}\sqrt{\dfrac{a}{1+a}}}{\sqrt{a}(1+a)^{3/2}} + \sum_{k=0}^{\infty}\dfrac{ka^{k-1}}{(2k+1)\,(1+a)^{k+2}}\right)\right]^2}{\left[\dfrac{1}{\sqrt{(1+a)\,(1+a+\delta)}} + \dfrac{\tanh^{-1}\sqrt{\dfrac{a}{1+a}}}{\sqrt{a}(1+a)^{3/2}} + \dfrac{\delta}{2}\left(-\dfrac{2\tanh^{-1}\sqrt{\dfrac{a}{1+a}}}{\sqrt{a}(1+a)^{5/2}} + \sum_{k=0}^{\infty}\dfrac{ka^{k-1}}{(2k+1)\,(1+a)^{k+3}}\right)\right]}. \tag{15}$$

Equation (15) can be reduced to Eq. (1) of [16] by putting $\delta = 0$ and $a = -a'/(1+a')$; a' is the gap parameter of [16]. Numerical calculations of Eq. (15) are shown in Fig. 2. The maximum value of $\Delta C/C_{\mathrm{N}}$ occurs at $a = 0$. When $-1 < a < 0$ and $\delta/a > 0$, $\Delta C/C_{\mathrm{N}}$ decreases while δ/a increases but there is no change in $\Delta C/C_{\mathrm{N}}$ in the case $\delta/a < 0$ and $a = 0$. Figure 2 shows that $\Delta C/C_{\mathrm{N}}$ decreases as δ/a increases.

4 Discussion and conclusions

In both cases under consideration, we find an anisotropy effect on $\Delta C/C_N$, which shows a maximum value at $a = 0$ and, when $\delta/a > 0$, $\Delta C/C_N$ will decrease as δ/a increases. This result indicates that the effect of anisotropy in the *ab*-plane decreases $\Delta C/C_N$.

Based on the experimental data on MgB_2 of Seneor et al. [10], they found a maximum gap $\Delta_z = 8$ meV for the *c*-axis film and a minimum gap $\Delta_{xy} = 5$ meV; this gives $\Delta_{xy}/\Delta_z = 0.625$. Our model predicts that if there is a gap symmetry in the *ab*-plane, i.e. $a = b$, the value of $\Delta C/C_N$ should be 1.28 and, if there is an asymmetry of the gap in the *ab*-plane, $a \neq b$, the value of $\Delta C/C_N$ should be slightly more or less than this depending on its a and b parameters. Because of $\Delta C/C_N$ depending on the parameter, we think that the effect of in-plane asymmetry may be the cause of non-sharpness in the specific heat curve at T_c.

In conclusion, we have derived analytical results for the normalized jump of the heat capacity of $\Delta C/C_N$ of MgB_2 with an anisotropy gap by including the effect of in-plane gap anisotropy within the framework of the weak-coupling BCS approach. We find that, within the three-dimensional anisotropy, the value of $\Delta C/C_N$ is less than the BCS value, and the effect of the in-plane asymmetry will reduce or increase the value of $\Delta C/C_N$ of the in-plane symmetry depending on its parameters.

Acknowledgement We acknowledge financial support by the Thai Research Fund.

References

[1] F. Bouquet, Y. Wang, R. A. Fisher, et al., Europhys. Lett. **56**, 856 (2001).
F. Bouquet, R. A. Fisher, N. E. Phillips, et al., Phys. Rev. Lett. **87**, 047001 (2001).
Y. Wang, T. Plackowski, and A. Junod, Physica C **355**, 179 (2001).
F. Bouquet, Y. Wang, I. Sheikin, et al., Physica C **385**, 192 (2003).
[2] R. Combescot, Europhys. Lett. **43**, 701 (1998).
[3] R. S. Gonnelli, et al., cond-mat/0209472v2.
[4] I. I. Mazin, et al., Phys. Rev. Lett. **89**, 107002 (2002).
[5] H. Suhl, B. T. Matthias, and L. R. Walker, Phys. Rev. Lett. **3**, 552 (1959).
[6] V. A. Moskalenko, M. E. Palistrant, and V. M. Vakalyuk, Sov. Phys. Usp. **34**, 717 (1991).
[7] S. L. Budko, G. Lapertot, C. Petrovic, et al., Phys. Rev. Lett. **86**, 1877 (2001).
[8] D. H. Hinks, H. Claus, and J. D. Jorgensen, Nature **411**, 457 (2001).
[9] H. Kotegawa, K. Ishida, Y. Kitaoka, et al., Phys. Rev. Lett. **87**, 127001 (2001).
[10] P. Seneor, et al., Phys. Rev. B **65**, 012505 (2002).
[11] T. M. Mishonov, et al., cond-mat/0212491v3.
[12] T. M. Mishonov and E. Penev, Int. J. Mod. Phys. B **16**, 3573 (2002).
[13] S. Hass and K. Maki, Phys. Rev. B **65**, 020502(R) (2001).
[14] A. I. Posazhennikova, T. Dahm, and K. Maki, cond-mat/0204272.
[15] T. M. Mishonov, E. S. Penev, and J. O. Indekeu, cond-mat/0204545v1.
[16] T. M. Mishonov, E. S. Penev, and J. O. Indekeu, cond-mat/0205104v3.
[17] A. L. Fetter and J. D. Walecka, Quantum Theory of Many-particle Systems (McGraw-Hill, New York, 1971).
[18] V. L. Pokrovskii and M. S. Ryvkin, Zh. Exp. Teor. Fiz. **43**, 92 (1962).
V. L. Pokrovskii, Zh. Exp. Teor. Fiz. **40**, 641 (1961).

Influence of Interband Interaction on Isotope Effect Exponent of MgB_2 Superconductors

P.Udomsamuthirun[1],C.Kumvongsa [2] ,A.Burakorn[1], P.Changkanarth[1]

[1] Department of Physics, Faculty of Science, Srinakharinwirot University,Bangkok 10110,Thailand.E-mail: udomsamut55@yahoo.com

[2] Department of Basic Science, School of Science, The University of the Thai Chamber of Commerce, Dindaeng , Bangkok 10400, Thailand.

Keywords: isotope effect exponent, superconductors, interband interaction
PACS : 74.20.Fg, 74.70.Ad.

Abstract

The exact formula of T_c's equation and the isotope effect exponent of two-band s-wave superconductors in weak-coupling limit are derived by considering the influence of interband interaction . The paring interaction in each band consisted of 2 parts : the electron-phonon interaction and non-electron-phonon interaction are included in our model. The isotope effect exponent of MgB_2, $\alpha = 0.3$ with $T_c \approx 40$ K ,can be found in the weak coupling regime and interband interaction of electron-phonon interaction show more effect on isotope effect exponent than of non-phonon interaction .

1.Introduction

The isotope effect exponent, α , is one of the most interesting properties of superconductors. In the conventional BCS theory $\alpha = 0.5$ for all element.
In high-T_c superconductors , experimenter found that α is smaller than 0.5[1-3]. This unusual small value leads to suggestion that the pairing interaction might be predominantly of electronic origin with a possible small phononnic contribution[4]. To explain the unusual isotope effect in high-T_c superconductors, many models have been proposed such as the van Hove singularity[5-7], anharmonic phonon[8,9], pairing-breaking effect[10], and pseudogap[11,12].

The discovery of [13] of superconductivity in MgB_2 with a high critical temperature , $T_c \approx 39$ K, has attracted a lot of considerable attention. Various experiments [14-21] suggest the existence of mutiband in MgB_2 superconductors.The gap values $\Delta(k)$ cluster into two groups at low temperature, a small value of ≈ 2.5 meV and a large value of of $\approx$ 7 meV. The calculation of the electron structure [22-26] support this conclusion. The Fermi surface consists of four sheets : two three-dimensional sheets from the π bonding and antibonding bands $(2p_z)$, and two nearly cylindrical sheets from the two-dimensional σ band $(2p_{x,y})$ [24,27]. There is a large difference in the electron-phonon coupling on different Fermi surface sheet and this fact leads to multiband description of superconductivity. The average electron-phonon coupling strength is found to be small values[14-16].Ummarino et al.[28] proposed that MgB_2 is the weak coupling two band phononic system where the Coulomb pseudopotential and the interchannel paring mechanism are key terms to interpret the superconductivity state. Garland[29] has shown that Coulomb potential in the d-orbitals of transition metal reduce the isotope exponent whereas sp-metals generally shown a nearly full isotope effect. So for sp-metal as MgB_2 , the Coulomb effect could not be account to explain the reduced of isotope exponent.

Budko et al.[30] and Hinks et al.[31] measured the boron isotope exponent and estimated as $\alpha_B = 0.26 \pm 0.03$ and nearly zero magnesium isotope effect. The boron isotope exponent is closed to that obtained for the YNi_2B_2C and $LuNi_2B_2C$ borocarbideds [32,33] where theoretical work[34] suggested that the phonons responsible for the superconductivity are high-frequency boron optical modes. This observation is consistent with a phonon-mediated BCS superconducting mechanism that boron phonon modes are playing an important role.

The theory of thermodynamic and transport properties of MgB_2 was made in the framework of the two band BCS model [35-43]. Zhitomirsky and Dao[44] derive the Ginzburg-Landau functional for two gap superconductors from the microscopic BCS model and then investigate the magnetic properties . The concept of multiband superconductors was first introduced by Suhl[45] and Moskalenke[46] in case of large disparity of the electron-phonon interaction for different Fermi-surface sheets.

The purpose of this paper is to derive the exact formula of T_c 's equation and the isotope effect exponent of two-band superconductors in weak-coupling limit by considering the influence of interband interaction . The paring interaction in each band consisted of 2 parts : a attractive electron-phonon interaction and a attractive non-electron-phonon interaction are included in our model.

2. Model and calculation

The properties of MgB_2 suggest the two-band s-wave superconductors(σ-band and π-band). And in each band ,it may have two energy gaps . To recover this fact, we make the assumption that the paring interaction consists of 2 parts : a attractive electron-phonon interaction and a attractive non-electron-phonon interaction in σ-band and π-band , and the $\sigma-\pi$ scattering of interband pairs. The Hamiltonian of the corresponding system is taken in the form

$$H = H_\pi + H_p + H_{p\pi} \qquad (1)$$

where H_π, H_p and $H_{p\pi}$ are the Hamiltonian of π band , σ band and interband respectively that

$$H_\pi = \sum_{k\sigma} \varepsilon_{k\sigma}\pi^+_{k\sigma}\pi_{k\sigma} - \sum_{kk'} V_{\pi\pi kk'}\pi^+_{k\uparrow}\pi^+_{-k\downarrow}\pi_{-k'\downarrow}\pi_{k'\uparrow} \qquad (2.1)$$

$$H_p = \sum_{k\sigma} \varepsilon_{k\sigma}p^+_{k\sigma}p_{k\sigma} - \sum_{kk'} V_{ppkk'}p^+_{k\uparrow}p^+_{-k\downarrow}p_{-k'\downarrow}p_{k'\uparrow} \qquad (2.2)$$

$$H_{p\pi} = -\sum_{kk'} V_{p\pi kk'}(p^+_{k\uparrow}p^+_{-k\downarrow}\pi_{-k'\downarrow}\pi_{k'\uparrow} + \pi^+_{k\uparrow}\pi^+_{-k\downarrow}p_{-k'\downarrow}p_{k'\uparrow}) \qquad (2.3)$$

Here we use the standard meaning of parameters and $V_{\pi\pi kk'}$, $V_{ppkk'}$, $V_{p\pi kk'}$ are the attractive interaction potential in σ band and π band ,and interband respectively .

By performing a BCS mean field analysis of Eq.(1) and applying standard techniques, we obtain the gap equation as

$$\Delta_{pk} = -\sum_{k'} V_{ppkk'}\frac{\Delta_{pk'}}{2\sqrt{\varepsilon^2_{pk'}+\Delta^2_{pk'}}}\tanh\left(\frac{\sqrt{\varepsilon^2_{pk'}+\Delta^2_{pk'}}}{2T}\right) - \sum_{k'} V_{p\pi kk'}\frac{\Delta_{\pi k'}}{2\sqrt{\varepsilon^2_{\pi k'}+\Delta^2_{\pi k'}}}\tanh\left(\frac{\sqrt{\varepsilon^2_{\pi k'}+\Delta^2_{\pi k'}}}{2T}\right) \qquad (3.1)$$

$$\Delta_{\pi k} = -\sum_{k'} V_{\pi\pi kk'}\frac{\Delta_{\pi k'}}{2\sqrt{\varepsilon^2_{\pi k'}+\Delta^2_{\pi k'}}}\tanh\left(\frac{\sqrt{\varepsilon^2_{\pi k'}+\Delta^2_{\pi k'}}}{2T}\right) - \sum_{k'} V_{\pi pkk'}\frac{\Delta_{pk'}}{2\sqrt{\varepsilon^2_{pk'}+\Delta^2_{pk'}}}\tanh\left(\frac{\sqrt{\varepsilon^2_{pk'}+\Delta^2_{pk'}}}{2T}\right) \qquad (3.2)$$

In each band, the paring interaction consists of 2 parts[47,48] : an attractive electron-phonon interaction V_{ph} and an attraction non-electron-phonon interaction U_c . ω_D and ω_c is the characteristic energy cutoff of the Debye phonon and non-phonon respectively.The interaction potential $V_{kk'}$ may be written as

$$V_{ikk'} = -V^i_{ph} - U^i_C \qquad \text{for} \quad 0 < |\varepsilon| < \omega_D$$
$$= -U^i_c \qquad \text{for} \quad \omega_D < |\varepsilon| < \omega_c \quad \text{and } i = p, \pi, p\pi$$

For such as the interaction the superconducting order parameter can be written as

$$\Delta_{jk} = \Delta_{j1} \qquad \text{for} \quad 0 < |\varepsilon| < \omega_D$$
$$= \Delta_{j2} \qquad \text{for} \quad \omega_D < |\varepsilon| < \omega_c \text{ and } j = p, \pi$$

3. T_C 's Equation

In this section, the exact formula of T_C 's equation of two- band s-wave superconductors is derived . At $T = T_C$ and constant density of state $N(\varepsilon_k) = N(0)$, Eq.(3) become

$$\begin{pmatrix}\Delta_{\pi 1}\\ \Delta_{p1}\\ \Delta_{\pi 2}\\ \Delta_{p2}\end{pmatrix} = \begin{pmatrix}(\lambda_\pi+\mu_\pi)I_1 & (\lambda_{\pi p}+\mu_{\pi p})I_1 & \mu_\pi I_2 & \mu_{\pi p}I_2\\ (\lambda_{\pi p}+\mu_{\pi p})I_1 & (\lambda_p+\mu_p)I_1 & \mu_{\pi p}I_2 & \mu_p I_2\\ \mu_\pi I_1 & \mu_{\pi p}I_1 & \mu_\pi I_2 & \mu_{\pi p}I_2\\ \mu_{\pi p}I_1 & \mu_p I_1 & \mu_{\pi p}I_2 & \mu_p I_2\end{pmatrix}\begin{pmatrix}\Delta_{\pi 1}\\ \Delta_{p1}\\ \Delta_{\pi 2}\\ \Delta_{p2}\end{pmatrix} \tag{6}$$

Here
$$I_1 = \int_0^{\omega_D} d\varepsilon \frac{\tanh(\varepsilon/2T_C)}{\varepsilon} \tag{7.1}$$

$$I_2 = \int_{\omega_D}^{\omega_C} d\varepsilon \frac{\tanh(\varepsilon/2T_C)}{\varepsilon} \tag{7.2}$$

and

$$\lambda_\pi = N_\pi(0)V_{ph}^{\pi}, \lambda_p = N_p(0)V_{ph}^{p}, \lambda_{\pi p} = N_\pi(0)V_{ph}^{\pi p} = N_p(0)V_{ph}^{p\pi}$$

and
$$\mu_\pi = N_\pi(0)U_C^{\pi}, \mu_p = N_p(0)U_C^{p}, \mu_{\pi p} = N_\pi(0)U_C^{\pi p} = N_p(0)U_C^{p\pi}$$

are the coupling constants.

Solving the secular equation , the appropriate solution is,

$$I_1 = \frac{A}{B+\sqrt{C^2 - D}} \tag{8}$$

that

$$A = 2(-1+I_2\mu_p)(-1+I_2\mu_\pi) - 2I_2^2\mu_{\pi p}^2$$

$$B = \mu_p + \mu_\pi + 2I_2(-\mu_p\mu_\pi + \mu_{\pi p}^2) + (\lambda_p+\lambda_\pi)((-1+I_2\mu_p)(-1+I_2\mu_\pi) - I_2^2\mu_{\pi p}^2))$$

$$C = \lambda_p + \lambda_\pi + \mu_p - I_2\mu_p(\lambda_p+\lambda_\pi) + \mu_\pi(1 + I_2^2\mu_p(\lambda_p+\lambda_\pi) - I_2(\lambda_p+\lambda_\pi+2\mu_p)) - I_2\mu_{\pi p}^2(-2 + I_2(\lambda_p+\lambda_\pi))$$

$$D = 4((-1+I_2\mu_p)(-1+I_2\mu_\pi) - I_2^2\mu_{\pi p}^2)[\lambda_\pi\mu_p - 2\lambda_{\pi p}\mu_{\pi p} - (-1+I_2\lambda_\pi)(\mu_p\mu_\pi - \mu_{\pi p}^2) + \lambda_p[(-1+I_2\mu_p)(-\mu_\pi + \lambda_\pi(-1+I_2\mu_\pi)) - I_2\mu_{\pi p}^2(-1+I_2\lambda_\pi)] + \lambda_{\pi p}^2(-1+I_2(\mu_p+\mu_\pi) + I_2^2(-\mu_p\mu_\pi + \mu_{\pi p}^2))]$$

Eq.(8) is the T_c's equation of two-band s-wave superconductors.

4. The isotope effect exponent

In harmonic approximation , $\omega_D \alpha M^{-1/2}$, and ω_c does not depend on mass. The isotope effect exponent can be derived from the equation

$$\alpha = -\frac{d\ln T_c}{d\ln M} = \frac{1}{2}\frac{\omega_D}{T_c}\frac{dT_c}{d\omega_D} \tag{9}$$

where M is the mass of the atom constituting the specimen under consideration.

Using Eq.(6) and Eq.(9), we can the isotope effect exponent as below

$$\alpha = \frac{(1/2)}{\dfrac{\tanh(\omega_c/2T_c)}{\tanh((\omega_D/2T_c))}\dfrac{(\mu_\pi D' + \mu_p E' + \mu_{\pi p}^2 F')}{(\lambda_\pi A' + I_1\lambda_{\pi p}B' + \lambda_p C')} - 1} \tag{10}$$

Here

$$A' = -[-1 + \mu_p(I_1 + I_2)][-1 + \mu_\pi(I_1 - I_2)] + (I_1^2 - I_2^2)\mu_{\pi p}^2$$

$$B' = \lambda_{\pi p}[2(-1 + I_2\mu_p)(-1 + I_2\mu_p) + I_1(\mu_p + \mu_\pi - 2I_2\mu_p\mu_\pi)] + 4\mu_{\pi p} + 2(I_1 - I_2)I_2\lambda_{\pi p}\mu_{\pi p}^2$$

$$C' = (-1 + I_2\mu_p)(-1 + I_2\mu_\pi) + I_2^2\mu_{\pi p}^2 + I_1^2[-\mu_\pi + \mu_p(-1 + 2I_2\mu_\pi) - 2I_2\mu_{\pi p}^2]$$
$$+ I_1[\mu_p - \mu_\pi + 2\lambda_\pi(-(-1 + I_2\mu_p)(-1 + I_2\mu_\pi) + I_2^2\mu_{\pi p}^2)]$$

$$D' = I_1^2\lambda_{\pi p}^2 - (-1 + I_1\lambda_p)(-1 + I_1\lambda_\pi)$$

$$E' = -1 + I_1(\lambda_\pi + \lambda_p(1 - I_1\lambda_\pi) + I_1\lambda_{\pi p}^2) + \mu_\pi F$$

$$F' = 2I_2 + I_1(2 - 2I_2(\lambda_p + \lambda_\pi) + I_1(-\lambda_\pi + \lambda_p(-1 + 2I_2\lambda_\pi) - 2I_2\lambda_{\pi p}^2)$$

Eq.(8) and Eq.(10) can be easily reduced to the T_c's equation and isotope exponent of BCS theory .

In Figure.(1), we plot a three dimensional graph of the isotope exponent Eq.(10) versus the interband coupling constant $\lambda_{\pi p}$ and $\mu_{\pi p}$. Depending on the measured Debye frequency ω_D =64.3 meV [30,49] and $T_c \approx 40$ K , the parameters are $T_c = 40$ K, $\omega_D = 745$ K , $\omega_c = 1.5\omega_D$,$\lambda_\pi = \lambda_p = 0.05$, $\mu_\pi = \mu_p = 0.05$. The isotope effect exponent is tend to 0.5 at large values of phonon and low value of non-phonon interband coupling constant . We calculate Eq.(8) and Eq.(10) numerically to find isotope effect exponent of MgB_2, $\alpha = 0.3$ with $T_c \approx 40$ K that many ranges of coupling constant agree with these conditions, example as $\mu_\pi = \mu_p = 0.05$, $\lambda_{\pi p} = 0.05$,$\mu_{\pi p} = 0.142$,$0.034 < \lambda_p < 0.114$, and $0.01 < \lambda_\pi < 0.1$. In Figure.(2), we show the effect of interband coupling constant on isotope effect exponent . The interband interaction of electron-phonon interaction show more effect on isotope exponent than of non-phonon interaction and both of them increase the isotope effect exponent in the same way .

5.Conclusions

The exact formula of T_c's equation and the isotope effect exponent of two-band s-wave superconductors in weak-coupling limit are derived by considering the influence of interband interaction . The paring interaction in each band consisted of 2 parts : a attractive electron-phonon interaction and a attractive non-electron-phonon interaction are included .We find isotope effect exponent of MgB_2, $\alpha = 0.3$ with $T_c \approx 40$ K in many ranges of coupling constant. These strength values of the coupling parameters indicate that the MgB_2 superconductor is in the weak coupling regime. The interband interaction of electron-phonon interaction show more effect on isotope exponent than of non-phonon interaction.

Acknowledgement The author would like to thank Thailand Research Fund for financial support and the university of the Thai Chamber of Commerce for partial financial support and the useful discussions from Prof.Dr.Suthat Yoksan.

6.Refference

1. B.Batlogg et al., Phys.Rev.Lett. **58**, 2333(1987).
2. D.E.Moris et al., Phys.Rev. B **37**, 5936(1988).
3. S.Hoen et al., Phys.Rev.B **39**,2269(1989).
4. F.Marsiglio, R.Akis, and J.P.Carbotte, Solid State Commun. **64**,905(1987).
5. J.Labbe and J.Bok, Europhys.Lett. **3**,1225(1987)
6. C.C.Tsuei,D.M.Newns,C.C.Chi, and P.C.Pattnaik, Phys.Rev.Lett. **65**,2724(1990).
7. R.J.Radtke and M.R.Norman, Phys.Rev. B **50**,9554(1994).
8. H.B.Schuttler and C.H.Pao, Phys.Rev.Lett. **75**,4504(1995).
9. L.Pietronero and S.Strassler, Europhys.Lett. **18**, 627(1992).
10. J.P.Carbotte, M.Greeson, and A.Perez-gonzalez, Phys.Rev.Lett. **66**,1789(1991).
11. T.Dahm, Phys.Rev.B **61**, 6381(2000).
12. P.Udomsamuthirun, Phys.Stat.Sol.(b) **226**,315(2001).

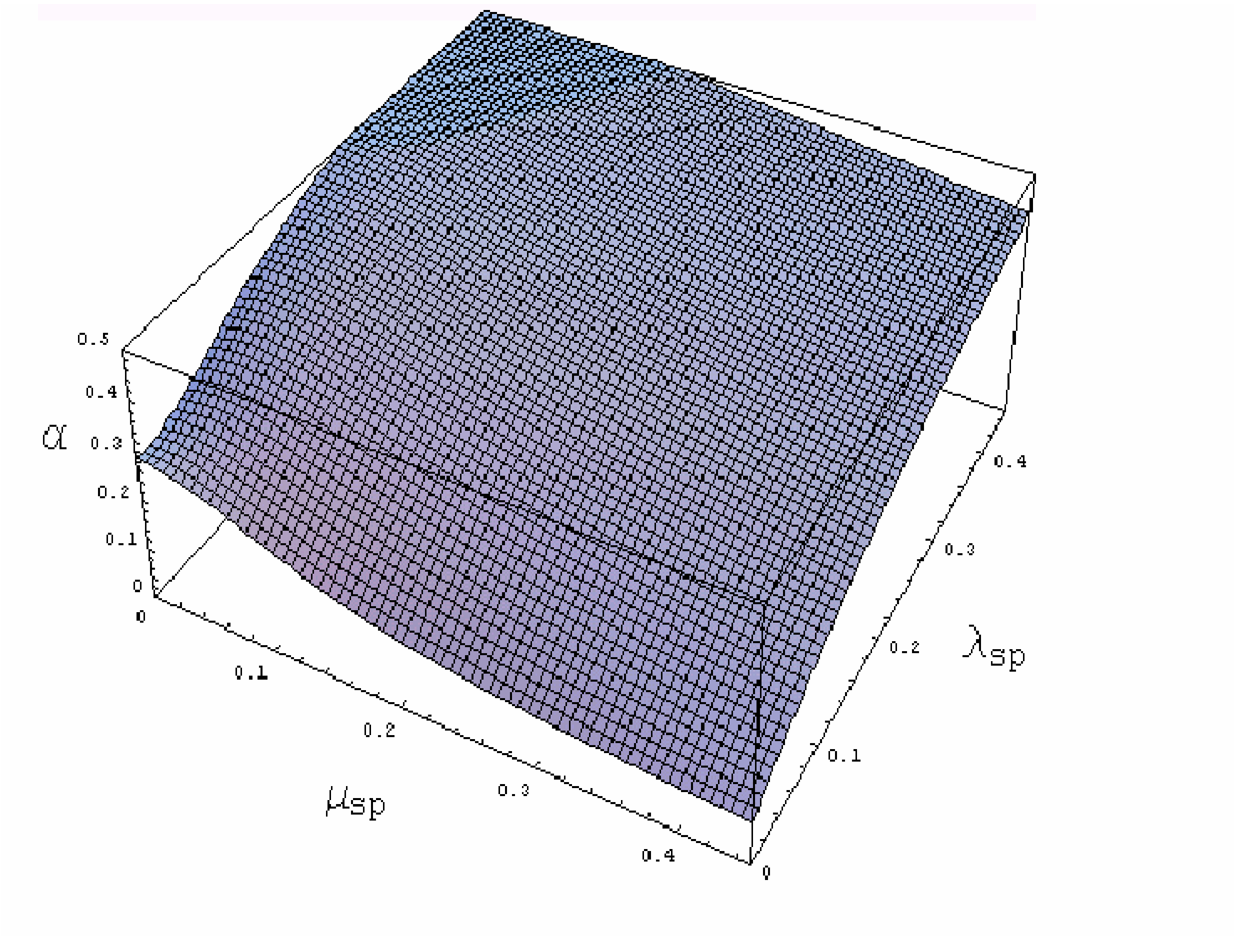
13 J.Nagamatsu,N.Nakagawa,T.Muranaka,Y.Zenitani and J.Akimutsu, Nature(London) **410**,63(2001).
14 F.Bouquet, Y.Wang,R.A.Fisher et al., Europhys,Lett. **56**,856(2001).
15.F.Bouquet,R.A.Fisher,N.E.Phillips et al., Phys.Rev.Lett. **87**,047001(2001).
16.Y.Wang,T.Plackowski, and A.Junod, Physica C **355**,179(2001).
17.F.Bouquet,Y.Wang,I.Sheikin et al., Physica C **385**,192(2003).
18.P.Szabo et al.,Phys.Rev.Lett. **87**,137005(2001).
19.X.K.Chen et al.,Phys.Rev.Lett. **87**,157002(2001).
20.S.Tsuda et al., Phys.Rev.Lett. **87**,177006(2001).
21.F.Giubileo et al.,Phys.Rev.Lett. **87**,177008(2001).
22.J.M.An and W.E.Pickett,Phys.Rev.Lett. **86**,4366(2001).
23.J.Kortus et al.,Phys.Rev.Lett. **86**,4656(2001).
24.A.Y.Liu,I.I.Mazin, and J.Kortus, Phys.Rev.Lett. **87**,087005(2001).
25.N.I.Medvedeva et al. Phys.Rev. B **64**,020502(2001).
26 I.I.Mazin and V.P.Antropo, Physica C **385**,49(2003).
27.Y.Kong et al.,Phys.Rev. B **64**, 020501(2001).
28 .G.A.Ummario,R.S.Gonnelli,S.Massidda, and A.Bianconi,cond-mat/0310284v1
29 J.W.Garland,Phys.Rev.Lett. **11**,114(1963).
30S.L.Budko,G.Lapertot,C.Petrovic et al., Phys.Rev.Lett. **86**,1877(2001).
31 D.G.Hinks,H.Claus and J.D.Jorgensen, Nature **411**,457(2001).
32 D.D.Lawrie and J.P.Franck, Physica(Amsterdam) **245C**, 159(1995).
33 K.O.Cheon,I.R.Fisher, and P.C.Canfield, Physica(Amsterdam) **312C**, 35(1999).
34 L.F.Mattheiss,T.Siegrist, and R.J.Cava, Solid State Commun. **91**, 587(1994).
35 A.A.Golubov et al.,J.Phys.:Condens.Matter **14**,1353(2002).
36 A.Brinkman et al.,Phys.Rev.B **65**,180517(2002).
37 I.I.Mazin et al.,Phys.Rev.Lett. **89**,1072002(2002).
38 A.Nakai, M.Ichioka, and K.Machida, J.Phys.Soc.Jpn. **71**,23(2002).
39 P.Miranovic, K.Machida, and V.G.Kogan, J.Phys.Soc.Jpn. **72**,221(2003).
40 T.Dahm and N.Scopohl, Phys.Rev.Lett. **91**,017001(2003).
41 A.Gurevich, Phys.Rev. B **67**,184515(2003).
42 A.A.Golubov and A.E.Koshelev, cond-mat/0303237
43 A.E.Koshelev and A.A.Golubov, Phys.Rev.Lett. **90**,177002(2003).

44 M.E. Zhitomirsky and V.H.Dao, Phys.Rev.B **69**,054508(2004).
45 H.Suhl,B.T.Matthias, and L.R.Walker, Phys.Rev.Lett. **3**,552(1959).
46 V.A.Moskalenko, M.E.Palistrant, and V.M.Vakalyuk, Sov.Phys.Usp. **34**,717(1991).
47 S.Yoksan Solid State Commun. **78**, 233(1991).
48 L.L.Daemen and A.W.Overhauser, Phys.Rev. B**41**, 7182(1990)
49 Ch.Walti et al.,Phys.Rev. B **65**,172515(2001).

Figure Caption

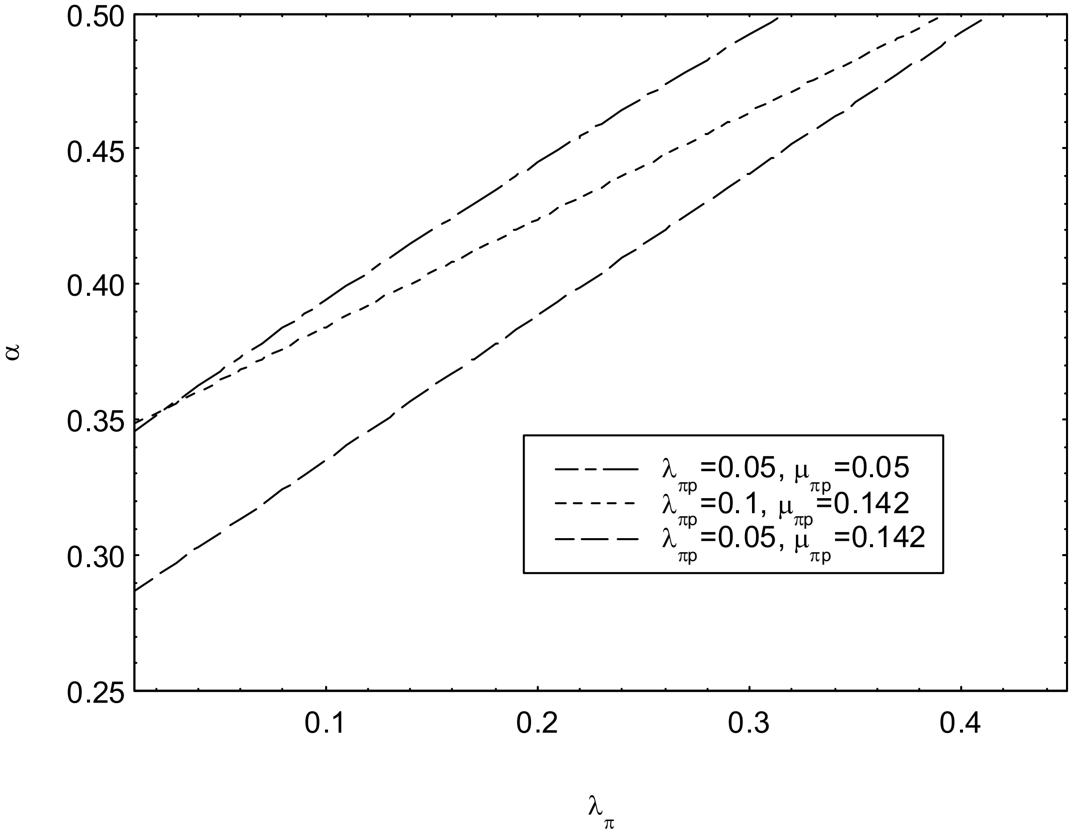
Figure(1). Plot graph of the isotope exponent Eq.(10) versus the interband coupling constant $\lambda_{\pi p}$ and $\mu_{\pi p}$, the parameters are $T_c = 40$ K, $\omega_D = 745$ K, $\omega_c = 1.5\omega_D$, $\lambda_\pi = \lambda_p = 0.05$, $\mu_\pi = \mu_p = 0.05$.

Figure.(2). We show the effect of interband coupling constant on isotope effect exponent. The parameters are $T_c = 40$ K, $\omega_D = 745$ K, $\omega_c = 1.5\omega_D$, $\lambda_p = 0.1$, $\mu_\pi = \mu_p = 0.05$.



Figure(1).

Udomsamuthirun et al.



Figure(2).

Udomsamuthirun et al.

Output ของโครงการ

1.ผลงานตีพิมพ์ในระดับต่างประเทศ 2 เรื่อง ดังนี้

-Udomsamuthirun P.,Rakpanich S.,and Yoksan S.,(2003),"Effect of in-plane anisotropy on specific heat of MgB_2",Phys.Stat.Sol.(b) **240**:591-595.

-เรื่อง" Influence of Interband Interaction on Isotope Effect Exponent of MgB_2 Superconductors" submit to Physica Status Solidi(b) และนำเสนอทาง web-site ใน http://xxx.lanl.gov/abs/cond-mat/0406496 .

2.ผลงานตีพิมพ์ในระดับในประเทศ 2 เรื่อง

-พงษ์แก้ว อุดมสมุทรหิรัญ และ อดุลย์ บุราครม," อัตราส่วนของช่องว่างพลังงานกับอุณหภูมิวิกฤติของตัวนำยวดยิ่ง MgB_2",การประชุมทางวิชาการของมหาวิทยาลัยเกษตรศาสตร์ ครั้งที่ 41(2545) 261-267 (ตามเอกสารแนบหมายหมายเลข)

-สมบัติบางประการของ MgB_2จะนำเสนอ วทท.(กำลังจัดเตรียม)

3.มีนักศึกษาระดับปริญญาโทของภาควิชาฟิสิกส์ คณะวิทยาศาสตร์ มหาวิทยาลัยศรีนครินทรวิโรฒ ที่ทำงานวิจัยในด้านนี้สำเร็จการศึกษา 3 คน