รายงานวิจัยฉบับสมบูรณ์

โครงการ: ทฤษฎีอินทีเกรตตามวิถีของพลาสมารอนในหนึ่งมิติ สองมิติ และสามมิติ

โดย ผู้ช่วยศาสตราจารย์ ดร.อุดมศิลป์ ปิ่นสุข

สิงหาคม 2546

สัญญาเลขที่ TRG4580110

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ผู้ช่วยศาสตราจารย์ ดร.อุดมศิลป์ ปิ่นสุข
ภาควิชาฟิสิกส์ คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

สนับสนุนโดยสำนักงานกองทุนสนับสนุนการวิจัย

กิตติกรรมประกาศ

ผู้วิจัยขอแสดงความขอบคุณนักวิจัยที่ปรึกษา ศาสตราจารย์ ดร.วิรุฬห์ สายคณิต ที่ได้กรุณาแนะนำ ชี้นำ ให้ความกระจ่าง และคอยช่วยเหลือ ดูแล ตลอดเวลาที่ผู้วิจัยดำเนินโครงการนี้จนสำเร็จลุล่วงไปได้ด้วยดี

ขอขอบคุณภาควิชาฟิสิกส์ คณะวิทยาศาสตร์ และจุฬาลงกรณ์มหาวิทยาลัยที่ได้เอื้อเฟื้อสถานที่ทำวิจัยตลอดโครงการ

ขอบคุณสำนักงานกองทุนสนับสนุนการวิจัยสำหรับทุนส่งเสริมนักวิจัยรุ่นใหม่ ที่ผู้วิจัยได้รับการอุดหนุนตลอดระยะเวลาการทำโครงการ

ขอบคุณนางสาวเอกอนงค์ สุขเกษม ที่ได้คอยสนับสนุนและเป็นกำลังใจอย่างสม่ำเสมอ และยังช่วยผลิตงานเอกสารที่สำคัญหลายๆ ชิ้น

ท้ายที่สุด ขอกราบขอบพระคุณมารดาของผู้วิจัย ที่ได้ประทานร่างกายที่แข็งแรง สติปัญญาที่ดี และจิตใจที่เข้มแข็ง จนสามารถฝ่าฟันอุปสรรคนานับประการ และทำให้โครงการรวมทั้งเหตุการณ์อื่นๆ ในชีวิตผ่านพ้นไปได้ด้วยดีเสมอมา

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ชื่อนักวิจัย: ผู้ช่วยศาสตราจารย์ ดร.อุดมศิลป์ ปิ่นสุข
ภาควิชาฟิสิกส์ คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย
E-mail Address: may@astro.phys.sc.chula.ac.th
ระยะเวลาโครงการ: 1 ปี

บทคัดย่อ

เราใช้ทฤษฎีอินทีเกรดตามวิถีของฟายน์แมน เพื่อศึกษาอันตรกิริยาระหว่างอิเล็กตรอนและพลาสมอน หรือในสถานะการณ์พิเศษจะเรียกเฉพาะว่า พลาสมารอน เราคำนวณตัวแผ่กระจาย พลังงานสถานะพื้น และมวลยังผล ทฤษฎีของเราเป็นบททั่วไป นั่นคือสมบัติของระบบอิเล็กตรอนพลาสมอนที่คำนวณได้นี้จะขึ้นกับความหนาแน่นของก๊าซอิเล็กตรอนเพียงอย่างเดียว เพื่อทดสอบความถูกต้องของทฤษฎี เราพิจารณาก๊าซอิเล็กตรอนที่ความหนาแน่นต่างๆ ผู้วิจัยพบว่า พลังงานสถานะพื้นจากทฤษฎีของเราให้ผลใกล้เคียงกับผลที่มีผู้คำนวณแล้วแต่ใช้วิธีที่ซับซ้อนกว่า เนื่องจากทฤษฎีของเราเป็นบททั่วไป จึงสามารถคำนวณผลในระบบอื่นๆ ได้ อย่างเช่น สารกึ่งตัวนำ ฉนวน และของเหลว เราขยายทฤษฎีเพื่ออธิบายการเคลื่อนที่ของประจุไฟฟ้าใดๆ ในวัสดุควบแน่นได้ เราใช้โพสิตรอนเป็นกรณีศึกษาและพบว่า มวลยังผลจากทฤษฎีของเราให้ผลใกล้เคียงกับทฤษฎีที่ซับซ้อนกว่า

คำหลัก: อินทีเกรดตามวิถี พลาสมารอน ก๊าซอิเล็กตรอน

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Investigator: Assistant Professor Dr.Udomsilp Pinsook

Department of Physics, Faculty of Science, Chulalongkorn University

E-mail Address: may@astro.phys.sc.chula.ac.th

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Abstract

We use Feynman path integration for studying the interaction between electron and plasmon, in some special cases called the plasmaron. We calculate the propagator, the ground state energy and the effective mass. Our theory is stated in general, that is the calculated properties of the electron-plasmon system are depended only upon the density of the electron gas. For verifying our theory, we consider the electron gas in various densities. We find that the ground state energy calculated by our theory is in a good agreement with that of a much more sophisticated method previously calculated. Due to its generality, it can be used to calculate the properties of other systems, such as semiconductors, insulators and liquids. We expand our theory to explain the motion of a general charge particle in condensed matter. We choose the positron as a case study and find that its effective mass claculated from our theory is close to that from a more complicated theory.

Keywords: path integration, plasmaron, electron gas

วัตถุประสงค์ของโครงการ

1. เพื่อคำนวณตัวแผ่กระจายของระบบอิเล็กตรอนพลาสมอนด้วยวิธีอินทีเกรตตามวิถีของฟายน์แมน
2. เพื่อคำนวณ พลังงานสถานะพื้น และมวลยังผลของระบบอิเล็กตรอนพลาสมอน
3. เพื่อนำทฤษฎีที่ได้มาสร้างแบบจำลองในการอธิบายปรากฏการณ์ทางฟิสิกส์ อย่างเช่น อันตรกิริยาระหว่างกลุ่มอิเล็กตรอน มวลยังผลของโพสิตรอนในของแข็ง
4. เพื่อเผยแผ่งานวิจัยสู่วงการวิทยาศาสตร์ในระดับนานาชาติ

1 Introduction

The interaction among electrons is extremely important because the world is made of the Pauli exclusion principle of electrons. Without electrons, life cannot exist. There were many theoretical approaches attempted to explain the electron-electron interaction [1]. This work intend to study only a part of the electron-electron interaction, i.e. the electron-plasmon interaction.

The plasmons are the quantized harmonic oscillations. They exist naturally in real metals and any electron gas. Because of the many-body nature of the problem, any charge particle interacting with plasmons cannot be solved accurately by an ordinary quantum theory. In this work, we propose that a charge particle interacting with plasmons can be modeled by Fröhlich-type Lagrangian. This Lagrangian was employed to study the system of an electron interacting with plasmons [2, 3]. It has been shown that this kind of Lagrangian can be solved analytically by using Feynman variational path integral. Also, a similar approach has been used for examining the polaron problem. It gives the best analytical results in the polaron case [1]. From the path integral method, we obtain the corresponding propagator and, then the ground state energy and the effective mass.

Our theory can be generalized to study the interaction between a fermion particle and plasmons. The formulation for a hole-plasmon or a positron-plasmon system should be exactly the same. A slow positron in solids is an important example. This is because the interaction between a positron and a solid can be decomposed into, for example, positron-plasmon [4, 5] and positron-phonon [6] interactions. It was pointed out that the positron-plasmon interaction is dominant in semiconductors [7]. Positrons are quite technologically important. Many theoretical works have been devoted for the calculations of the positron's ground state energy [8], band structures [7, 9, 10], effective mass [4, 5, 11, 12], diffusion coefficient and mobility [13] in condensed matter. These calculations are useful for the interpretation of positron annihilation measurements. These measurements are of great interests as they reveal some information of defects in solids [14, 15], and they are non-destructive and hence favorable.

To verify our theory, we examine two special cases. The first case is the Coulomb-hole self-energy of the electron and the ground state energy of the plasmaron. The results are compared with a more sophisticated work proposed by Overhauser [16]. The second case is the motion of a positron in solids. We compare our results with some existing experimental data. The effective mass is important as it can be related to the mobility and the diffusion coefficient of the positron [13]. In addition, the results from theoretical approaches [4, 5, 11, 12] and experimental determination [17, 18] are not in a good agreement. Some possible discrepancies and some extension using path integral formulations are discussed.

This report is organized as follows: the models of the charge-plasmon interaction are in section 2, the general Feynman path integral theory for the charge-plasmon system is in section 3, and calculation results and discussions are in section 4. Conclusions are laid in section 5.

2 Models of the charge-plasmon interaction

The interaction between a charge particle and plasmon field can be modeled by a Fröhlich-type Lagrangian [2, 19] as

$$L = \frac{1}{2}m_c\dot{r}^2 + \sum_{\mathbf{q}}\frac{1}{2}m_e\left(\dot{Q}_{\mathbf{q}}^2 - \omega_q^2 Q_{\mathbf{q}}^2\right) - \sum_{\mathbf{q}}\sqrt{\frac{2m_e\omega_q}{V}}g_q\mathbf{Q}_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{1}$$

where m_c is the charge mass, $\mathbf{r}$ is the position vector of the charge particle, m_e is the bare electron mass as electrons are the bases of the plasmons, $\mathbf{Q}_{\mathbf{q}}$ are the plasmon generalized coordinates and $\dot{\mathbf{Q}}_{\mathbf{q}}$ are their conjugate momenta, $\mathbf{q}$ denote the plasmon wavevectors, ω_q is the plasmon spectrum, V is the volume of the system. The system is assumed isotropic so that the dispersion of the plasmons is directionally independent. The first term in Eq. (1) describes the kinetic energy of the charge particle. The second term is the harmonic representation of the plasmon modes. The third term is the interaction between the charge particle and the plasmons. The strength of the interaction is enclosed in a coupling function g_q. The remaining task is to select a suitable ω_q and g_q.

In the present work, we intend to use a simplified picture for the many-body interaction. Thus, the plasmon spectrum ω_q and the coupling function g_q are taken from the so-called plasmon-pole approximation. In this approximation, the electron-hole excitations are absorbed into the effective plasmon excitations. The f sum rules are imposed as a particle conservation rule. There are several plasmon-pole models for metals [16, 19], semiconductors [20, 21] and liquids [22]. For the purpose of illustration, we choose a simple model introduced by Lundqvist [19]. He proposed that an electron gas can be described by a simplified dielectric function,

$$\varepsilon\left(q,\omega\right) = \frac{\omega_q^2 - \omega^2}{\omega_q^2 - \omega_p^2 - \omega^2}. \tag{2}$$

This dielectric function is not a full extension of the celebrated random phase approximation (RPA) but rather an approximation, and it was chosen to satisfy the f sum rules [19]. The plasmon spectrum is

$$\omega_q^2 = \omega_p^2 + \frac{4}{3}k_F^2q^2 + q^4. \tag{3}$$

The coupling function can be written as

$$g_q^2 = \frac{8\pi}{q^2}\left(\frac{\partial\varepsilon(q,\omega)}{\partial\omega}\right)^{-1}_{\omega=\omega_p} = \frac{4\pi\omega_p^2}{q^2\omega_q}. \tag{4}$$

This model is simple and yet allow us to find some useful analytic solutions. It also gives reasonable ground state energy and correlation energy of the electron gas in the metallic densities [19].

Another interesting model was suggested by Overhauser [16]. He noticed that RPA does not account for the exchange and correlation effects of the electron gas. Thus he included some correction terms into the dielectric function, which can be expressed as

$$\varepsilon(q) = 1 + \frac{P(x)}{1 - G(x)P(x)}, \tag{5}$$

where

$$G(x) = \frac{1.1x^2}{\sqrt{1 + 10x^2 + 1.5x^4}}, \tag{6}$$

$$P(x) = \frac{1}{\pi k_F x^2}\left(\frac{1}{2} + \frac{1 - x^2}{4x}\ln\left|\frac{1+x}{1-x}\right|\right), \tag{7}$$

and $x = \frac{q}{2k_F}$. The form of Eq. (5) is similar to incorporating the local field corrections and the vertex corrections into the dielectric function. This model is phenomenological because the exact corrections are not known. Nevertheless, Overhauser showed that his dielectric function reproduced correct correlation energy in the metallic density range [16]. The plasmon spectrum is then

$$\omega_q^2 = \frac{\omega_p^2 \varepsilon(q)}{\varepsilon(q) - 1}. \tag{8}$$

The coupling function is slightly modified as

$$g_q' = g_q(1 - G(x)), \tag{9}$$

where g_q is defined in Eq. (4).

Throughout this work, we adopt the Bohr radius as the unit of length and Rydbergs (Ry) as the unit of energy. In these units, the long wavelength plasmon frequency $\omega_p = \frac{3.4641}{r_s^{3/2}}$ and the magnitude of Fermi wave vector $k_F = \frac{1.9192}{r_s}$, where r_s is a parameter defined from the electron gas density $n = (\frac{4}{3}\pi r_s^3)^{-1}$. The charge particle states will be labeled by $\mathbf{k}$ and the plasmon states will be labeled by $\mathbf{q}$.

It is worthy of note that the Lagrangian for a positive or negative charge particle is the same as in Eq. (1). Only appropriate dielectric response must be selected accordingly. To a crude approximation, the dielectric response for an electron in the electron gas should be the same as that for a positron in the electron gas. Some refinement to the interaction model has been suggested, i.e. for a positron in an electron gas, the coupling function must be multiplied by some enhancement factor and the plasmon dispersion relation must be modified accordingly [16, 23, 24]. This refinement will be taken into account in our subsequent works.

3 Feynman path integral theory of an electron-plasmon system

By using path integration technique [25, 26], Sa-yakanit *et.al.* [2] showed that the corresponding action of Eq. (1), after integrating out the plasmon coordinates, can be

written as

$$S = \frac{1}{2}m_c \int_0^t d\tau \dot{r}^2(\tau) + \frac{1}{16\pi^3} \int dq g_q^2 \int_0^t \int_0^t d\tau d\sigma \frac{\cos[\omega_q(t/2-u)]}{\sin[\omega_q t/2]} e^{i\mathbf{q}\cdot(\mathbf{r}(\tau)-\mathbf{r}(\sigma))}, \tag{10}$$

where $u = |\tau - \sigma|$. This action describes a system of a particle moving in a non-local potential field. This potential field reflects the disturbance caused by the presence of the particle and affecting the particle's motion at later time.

However, the action in Eq. (10) is too complicated to be solved. By using variational principle, we have freedom to choose a simpler action and use variational technique to obtain some useful information. In a previous work [2], a polaron-like action was chosen, which is

$$S_0 = \frac{1}{2}m_c \int_0^t d\tau \dot{r}^2(\tau) - \frac{\kappa\Omega}{8} \int_0^t \int_0^t d\tau d\sigma \frac{\cos[\Omega t/2-u]}{\sin[\Omega t/2]} |\mathbf{r}(\tau) - \mathbf{r}(\sigma)|^2. \tag{11}$$

This action is quadratic and composes of two variational parameters, i.e. κ and Ω which are related to Feynman parameters C and w by $\kappa = \frac{4C}{w}$ and $\Omega = w$. The first term in Eq. (11) is the kinetic energy of the particle. The second term is a non-local harmonic potential in which κ is an effective coupling strength between the particle and an external harmonic force with an effective harmonic frequency of Ω. This trial action is a good approximation to the action in Eq. (10) when $|\mathbf{r}(\tau) - \mathbf{r}(\sigma)|$ is fairly small or there is only a short interaction time available, as discussed by Feynman [25, 26]. From this action, Feynman stated that it leads to the construction of a approximate propagator [25, 26]. Furthermore, the propagator already contains all the information of the system, such as the ground state energy and the effective mass. Thus knowing the propagator is equivalent to solving the problem.

Next, we discuss briefly how to obtain the approximate propagator. From the action in Eq. (10), the corresponding propagator is

$$G(\mathbf{r}_2, \mathbf{r}_1; t) = \int_{\mathbf{r}_1}^{\mathbf{r}_2} D[\mathbf{r}(t)]\, e^{iS}. \tag{12}$$

This propagator is complicated and we cannot find its solutions directly. Thus we impose the variational principle and choose a polaron-like action as in Eq. (11). Its corresponding propagator is

$$G_0(\mathbf{r}_2, \mathbf{r}_1; t) = \int_{\mathbf{r}_1}^{\mathbf{r}_2} D[\mathbf{r}(t)]\, e^{iS_0}. \tag{13}$$

The chosen trial action is in a quadratic form so that it can be solved analytically. From the cumulant expansion, we find that the true propagator can be approximated in term of the trial action as

$$G(\mathbf{r}_2, \mathbf{r}_1; t) \approx G_0(\mathbf{r}_2, \mathbf{r}_1; t) e^{i\langle S - S_0\rangle_{S_0}}, \tag{14}$$

where $\langle ... \rangle_{S_0}$ denotes an average with respect to S_0. Eq. (14) can be solved in term of r_1, r_2 and the variational parameters. The mathematical detail has been given many times in the literature [2, 27, 28, 29, 30]. Here we simply quote the solutions which are

$$G_0(\mathrm{r}_2, \mathrm{r}_1; t) = \left(\frac{m_c}{2\pi i t}\right)^{3/2} \left(\frac{\nu \sin[\Omega t/2]}{\Omega \sin[\nu t/2]}\right)^3 \times \exp\left\{-\frac{i m_c}{2t}\left(\frac{(1-\rho^2)\nu t}{2}\cot\left[\frac{\nu t}{2}\right] + \rho^2\right)|\mathrm{r}_2 - \mathrm{r}_1|^2\right\}. \tag{15}$$

The average of S and S_0 can be decomposed as

$$< S - S_0 >_{S_0} = < S >_{S_0} - < S_0 >_{S_0}, \tag{16}$$

and

$$< S >_{S_0} = \frac{1}{16\pi^3} \int dq g_q^2 \int_0^t \int_0^t d\tau d\sigma \frac{\cos[\omega_q(t/2-u)]}{\sin[\omega_q t/2]} e^{-\frac{1}{2}iq^2 A + iq\cdot(\mathrm{r}_2 - \mathrm{r}_1)B}, \tag{17}$$

and

$$< S_0 >_{S_0} = \frac{3i(1-\rho^2)}{2}\left(\frac{\nu t}{2}\cot[\frac{\nu t}{2}] - 1\right) - \frac{\kappa\Omega}{8}\int_0^t \int_0^t d\tau d\sigma \frac{\cos[\Omega t/2 - u]}{\sin[\Omega t/2]}(\mathrm{r}_2 - \mathrm{r}_1)^2 B^2, \tag{18}$$

where

$$A = \frac{(1-\rho^2)}{m_c}\left\{\frac{2\sin[\nu u/2]\sin[\nu(t-u)/2]}{\nu \sin[\nu t/2]} + \frac{\Omega^2}{\nu^2 - \Omega^2}\frac{u(t-u)}{t}\right\}, \tag{19}$$

and

$$B = (1-\rho^2)\left\{\frac{\sin[\nu u/2]\cos[\nu(t-(\tau+\sigma))/2]}{\sin[\nu t/2]} + \frac{\Omega^2}{\nu^2 - \Omega^2}\frac{u}{t}\right\}. \tag{20}$$

The diagonal part of $G(\mathrm{r}_2, \mathrm{r}_1; t)$ can be explicitly written as

$$G(\mathrm{r}, \mathrm{r}; t) = \left(\frac{m_c}{2\pi i t}\right)^{3/2} \left(\frac{\nu \sin[\Omega t/2]}{\Omega \sin[\nu t/2]}\right)^3 \exp\left\{\frac{3(1-\rho^2)}{2}\left(\frac{\nu t}{2}\cot[\frac{\nu t}{2}] - 1\right)\right\} \times \exp\left\{\frac{i}{16\pi^3}\int dq g_q^2 \int_0^t \int_0^t d\tau d\sigma \frac{\cos[\omega_q(t/2-u)]}{\sin[\omega_q t/2]} e^{-\frac{1}{2}iq^2 A}\right\}. \tag{21}$$

Our system is isotropic and thus translational invariant. Therefore $G(\mathrm{r}, \mathrm{r}; t) \equiv G(0, 0; t)$. By using the identity

$$\int_0^t \int_0^t d\tau d\sigma f(u) = 2\int_0^t dz(t-z)f(z), \tag{22}$$

we get

$$G(0, 0; t) = \left(\frac{m_c}{2\pi i t}\right)^{3/2} \left(\frac{\nu \sin[\Omega t/2]}{\Omega \sin[\nu t/2]}\right)^3 \exp\left\{\frac{3(1-\rho^2)}{2}\left(\frac{\nu t}{2}\cot[\frac{\nu t}{2}] - 1\right)\right\} \times \exp\left\{\frac{i}{8\pi^3}\int dq g_q^2 \int_0^t dz(t-z)\frac{\cos[\omega_q(t/2-z)]}{\sin[\omega_q t/2]} e^{-\frac{1}{2}iq^2 C}\right\}, \tag{23}$$

where

$$C = \frac{(1-\rho^2)}{m_c}\left\{\frac{2\sin[\nu z/2]\sin[\nu(t-z)/2]}{\nu\sin[\nu t/2]} + \frac{\Omega^2}{\nu^2-\Omega^2}\frac{z(t-z)}{t}\right\}. \tag{24}$$

To avoid dealing with some oscillating behavior in the solutions, we transform $t \rightarrow -iT$. This transformation is valid as long as spin is not explicitly taken into account, as discussed by Feynman [25].

For the system of lower dimensions, i.e. one or two dimension, the propagator can be modified as

$$\begin{aligned} G(0,0;t) = &\left(\tfrac{m_c}{2\pi i t}\right)^{d/2}\left(\tfrac{\nu\sin[\Omega t/2]}{\Omega\sin[\nu t/2]}\right)^{d}\exp\left\{\tfrac{d(1-\rho^2)}{2}\left(\tfrac{\nu t}{2}\cot[\tfrac{\nu t}{2}]-1\right)\right\} \\ &\times\exp\left\{\tfrac{i}{8\pi^3}\int dq g_q^2\int_0^t dz(t-z)\tfrac{\cos[\omega_q(t/2-z)]}{\sin[\omega_q t/2]}e^{-\frac{1}{2}iq^2C}\right\}, \end{aligned} \tag{25}$$

where d is the dimension of the system. The plasmon state q, the dielectric function $\varepsilon(q)$ and the plasmon dispersion relation ω_q must be modified accordingly. Some models for these quantities have been proposed by many authors [20, 31]. They also suggested that the effects of plasmon become more significant in lower dimensions. We will extensively study these lower dimension systems in our subsequent works.

We concentrate now on the three dimensional system. In order to find some ground state properties, we let $T \rightarrow \infty$, the ground state energy E_0 and the effective mass m_c^*/m_c can be extracted from

$$G(0,0;T\rightarrow\infty) \simeq \varphi_0^*(0)\varphi_0(0)e^{-E_0T} = \left(\frac{m_c^*}{2\pi T}\right)^{3/2}e^{-E_0T}. \tag{26}$$

This propagator describes the charge particle and the plasmons which are initially in their ground states, then they are virtually excited and interact with each other, and eventually, after some time T later, they go back to their ground states. This picture is analogous to the self-energy scheme in quantum many-body theory [1].

From the approximate propagator in Eqs. (23) and (26), the ground state energy is written as [2, 3]

$$E_0 = \frac{3}{4}\nu(1-\rho)^2 - \frac{1}{2\pi^2\nu}\int dq q^2 g_q^2 e^{-\beta}\int_0^\infty dx\exp(\beta e^{-x}-\mu x), \tag{27}$$

where $\nu^2 = \Omega^2 + \frac{\kappa}{m_c}$, $\rho = \frac{\Omega}{\nu}$, $\beta = \frac{q^2(1-\rho^2)}{2m_c\nu}$ and $\mu = \frac{\omega_q}{\nu} + \frac{q^2\rho^2}{2m_c\nu}$. The variational parameters κ and Ω are now absorbed into another set of adjustable parameters ν and ρ. The system is assumed spherical symmetry. Thus we can replace $\int d$q by $\int 4\pi^2 q^2 dq$. According to the propagator $G(0,0;T)$, E_0 can be interpreted as the energy of the particle which is moving stochastically in response to the changes in the potential field created by the motion of the particle itself and then goes back to its initial position after a duration T. For $T \rightarrow \infty$, the particle has an average zero momentum. Hence, E_0 can be compared with the energy of a particle in a compatible physical system at $k = 0$

or at Γ point where the particle has slowest dynamics and its wavefunction has highest symmetry. It is readily to see that this method is quite suitable for the problem of a thermalized positron in solids because the positron is slowing moving and it occupies the Γ state [7, 10].

The limits of $\int dq$ in Eq. (27) are worth discussing. Overhauser [16] show that if the charge particle is an electron and we perform $\int_0^{k_F} dq$, E_0 is a major contribution to the correlation energy of the electron gas at $k = 0$. The energy state of this system is quite complicated and sometimes referred to as the plasmaron. The concept of the plasmaron is working well in poorly conductive semiconductors and insulators. If we perform $\int_0^{\infty} dq$, E_0 is the Coulomb-hole self-energy of the electron gas at $k = 0$ [3, 16].

For a positron, we have no prior reasons to exclude short wavelength plasmons. Thus we simply perform $\int_0^{\infty} dq$. Then E_0 should be the lowest possible ground state energy for the positron-plasmon interaction in our model. The reason is as follows. Firstly, it is readily seen that the first term, the second term and its integrand in Eq. (27) are always positive. Owing to the minus sign in front of the second term, for a given set of variational parameters, the lowest E_0 is where the second term is largest. Performing $\int_0^{\infty} dq$ with appropriate variational parameters will give the largest possible value of the second term and hence the lowest possible E_0.

Next, the effective mass can also be extracted from Eqs. (23) and (26). Sa-yakanit [29] showed that if we expand the propagator Eq. (26) in a series of $\frac{1}{T}$ where $\frac{1}{T} \to 0$, we get the low energy limit effective mass which is [3]

$$\frac{m_c^*}{m_c} = \frac{1}{\rho^2} \exp\left[\rho^2 - 1 + \frac{\rho^2}{3\pi^2\nu^3} \int dq q^4 g_q^2 e^{-\beta} \int_0^{\infty} dx x^2 \exp(\beta e^{-x} - \mu x)\right]. \quad (28)$$

Note that we do not assume here that $m_c = m_e$. Therefore, we can implicitly take the effects of a periodic potential or a phonon interaction into the account. This is a major improvement in our theory.

4 Calculation results and discussions

4.1 The electron-plasmon system

The main purpose of this study is to evaluate the ground state energy and the effective mass. According to the variational principle, in order to obtain the ground state properties of this system, we insert ω_q from Eq. (3) or (8) and g_q from Eq. (4) or (9) into Eq. (27), and solve for ν and ρ which yield the lowest E_0. Then we insert these ν and ρ into Eq. (28) to get the corresponding m_c^*/m_c. It is worth mentioning here that the integrals in these equations cannot be evaluated analytically. Thus we resort to a standard numerical integration and a numerical minimization [32]. The numerical minimization is adapted from the simulated annealing method which has a potential to find the global minimum in multidimensional problems. In this subsection, we exam-

r_s	ν	ρ	E_0 (Ry)	m_c^*/m_c
0.00001	finite	1	-489.77	1
0.0001	finite	1	-152.57	1
0.001	finite	1	-46.49	1
0.01	finite	1	-13.49	1
0.1	236.46	0.999	-3.56	1.002
1	7.03	0.989	-8.21×10^{-1}	1.024
2	2.46	0.979	-5.14×10^{-1}	1.046
5	6.09×10^{-1}	0.952	-2.72×10^{-1}	1.108
10	2.22×10^{-1}	0.916	-1.67×10^{-1}	1.203
100	9.59×10^{-3}	0.600	-3.23×10^{-2}	2.897
1000	7.24×10^{-4}	0.245	-6.37×10^{-3}	18.592
10000	6.06×10^{-5}	0.101	-1.22×10^{-3}	119.877
100000	4.96×10^{-6}	0.044	-2.26×10^{-4}	727.316

Table 1: Shows ν, ρ, E_0 and m_c^*/m_c as a function of r_s. The plasmon spectrum and the corresponding coupling function are taken from Lundqvist's dielectric model. We set $m_c = m_e$.

ine the electron case. Thus it is appropriate to use $m_c = m_e$. Some selected results are shown in table 1 and 2.

According to table 1, we can see firstly, that ν is large at small r_s and becomes finite but irrelevant at very small r_s. It tends to zero quickly as r_s increases. Secondly, $\rho \approx 1$ at small r_s and it is getting gradually smaller as r_s increases. Thirdly, E_0 is very large at small r_s and very small at large r_s. Lastly, $m_c^*/m_c \approx 1$ at small r_s and it is sharply increasing at very large r_s.

According to table 2, we can see that E_0 from the two dielectric models behaves similarly, although E_0 from Overhauser's dielectric model is a little higher. This is because the local field and the vertex corrections slightly alter the dynamics of the electron gas and also affect the coupling strength of the electron-plasmon interaction.

As mentioned earlier, the Lagrangian and hence the Hamiltonain of the physical system we have discussed here can be related to the Coulomb-hole self-energy. Therefore, the results of the Coulomb-hole self-energy at $k = 0$ from Overhauser's work [16] using his own dielectric model are shown in table 2. Even though, Overhauser's theory and ours originate from different approaches and have very different mathematical structures, it can be readily seen that with the same dielectric model, E_0 from our theory is in a very good agreement with Overhauser's Coulomb-hole self-energy at $k = 0$. The ground state of the plasmaron at $k = 0$ [19] are also given in the table for comparison.

Since both dielectric models have been shown to give the similar results. Thus we

r_s	Lundqvist	Overhauser	Coulomb-hole ($k=0$) [16]	Plasmaron ($k=0$) [19]
0.1	-3.56	-3.55	-3.55	5.20
1	-8.21×10^{-1}	-7.96×10^{-1}	-7.95×10^{-1}	1.09
2	-5.14×10^{-1}	-4.93×10^{-1}	-4.87×10^{-1}	5.94×10^{-1}
5	-2.72×10^{-1}	-2.48×10^{-1}	-2.47×10^{-1}	2.47×10^{-1}
10	-1.67×10^{-1}	-1.46×10^{-1}	-1.45×10^{-1}	1.23×10^{-1}
100	-3.23×10^{-2}	-2.29×10^{-2}	-2.22×10^{-2}	1.19×10^{-2}

Table 2: Shows E_0 from the different dielectric models. For comparison, the Coulomb-hole self-energy from Eq. (48) in Ref. [16] and the ground state energy of the plasmaron from Eq. (26) in Ref. [19] are also given.

have chosen Lundqvist's model for further investigation because it is simpler and easier to handle. By adopting Lundqvist's dielectric function, E_0 can be written explicitly as

$$E_0 = \frac{3}{4}\nu(1-\rho)^2 - \frac{2\omega_p^2}{\pi\nu}\int_0^\infty dq \frac{e^{-\beta}}{\omega_q}\int_0^\infty dx \exp\left(\beta e^{-x} - \mu x\right). \tag{29}$$

In the next two steps, we will study this E_0 in some asymptotic limits, such as in a very high electron density, i.e. r_s is vanishingly small, and in a very low density, i.e. r_s is very large. In these limits, we find that Eq. (29) can be evaluated analytically.

High density limit

In this limit, $r_s \to 0$, we find from the numerical results that ν is large and finite and $\rho = 1$ so that the first term in Eq. (29) vanishes, $\beta = 0$ and

$$\int_0^\infty dx \exp(\beta e^{-x} - \mu x) = \frac{1}{\mu}. \tag{30}$$

Moreover, ω_p and k_F become very large so that q^4 has less significant contribution to ω_q and

$$\mu\nu \approx \omega_q \approx \sqrt{\omega_p^2 + \frac{4}{3}k_F^2 q^2}. \tag{31}$$

Consequently,

$$E_0 \approx -\frac{2\omega_p^2}{\pi\nu}\int_0^\infty dq \frac{1}{\omega_q \mu} \approx -\frac{2\omega_p^2}{\pi}\int_0^\infty dq \frac{1}{\omega_p^2 + \frac{4}{3}k_F^2 q^2}. \tag{32}$$

It is straightforward to perform the definite integral and writing E_0 as a function of r_s. Then we get

$$E_0 \approx -\frac{1.56}{\sqrt{r_s}}, \tag{33}$$

r_s	numerical	high density limit	low density limit	error (%)
0.00001	-489.77	-493.32	-	0.72
0.0001	-152.57	-156.00	-	2.25
0.001	-46.49	- 49.33	-	6.11
1000	-6.37×10^{-3}	-	-7.42×10^{-3}	16.53
10000	-1.22×10^{-3}	-	-1.32×10^{-3}	8.20
100000	-2.26×10^{-4}	-	-2.35×10^{-4}	3.86

Table 3: Shows E_0 in the high density limit, Eq. (33) and in the low density limit, Eq. (37), compared with the numerical results of Eq. (29). The relative error is also given.

which yields less than 1% error compared with the numerical value at $r_s = 10^{-5}$, see table 3. We expect that the error would reduce as r_s is getting closer to 0. Most of all, we see that E_0 is not a constant as $r_s \to 0$ but rather diverges as $\frac{1}{\sqrt{r_s}}$.

Low density limit

In this limit, $r_s \to \infty$, $\nu \to 0$ and $\rho \to 0$ so that $\beta = \frac{q^2}{\nu}$, $\mu = \frac{\omega_q}{\nu}$ and the first term in Eq. (29) can be neglected. In a closer examination, we find that

$$\exp(\beta e^{-x} - \mu x) \leq \exp(\beta - \mu x) \tag{34}$$

for all x. Thus

$$E_0 \leq -\frac{2\omega_p^2}{\pi\nu}\int_0^\infty dq \frac{1}{\omega_q \mu} = -\frac{2\omega_p^2}{\pi}\int_0^\infty dq \frac{1}{\omega_q^2}. \tag{35}$$

We also notice that the term which contains q^2 has a smaller contribution to ω_q than the q^4 term. Thus, we have

$$E_0 \approx -\frac{2\omega_p^2}{\pi}\int_0^\infty dq \frac{1}{\omega_p^2 + q^4}, \tag{36}$$

which can be readily integrated out and written in term of r_s as

$$E_0 \approx -\frac{1.32}{r_s^{3/4}}, \tag{37}$$

which yields less than 4% error compared with the numerical value at $r_s = 10^5$, see table 3.

We have some comments on the behavior of m_c^*/m_c in the present work as it would reflect a phase transition of the dressed electron from one physical regime to another

between very different scales of r_s. We can see from table 1 that in the high density limit (small r_s) the electron is mobile, i.e. m_c^*/m_c is close to unity, whereas in the low density limit (large r_s) it becomes self-trapped, i.e. m_c^*/m_c is exceedingly large. In addition, we have shown that E_0 behaves differently between these two extreme limits. One possible realization of our findings is the fact that in a degenerate electron gas, the total energy can be decomposed into the kinetic, the screened exchange and the Coulomb-hole parts. Furthermore, at very high density, the dynamics of electrons are dominated by the kinetic energy while at very low density, the electrons are localized. This is just coinciding with the behavior of m_c^*/m_c in our results. We believe that these findings might be an indication of Wigner crystallization.

4.2 The positron-plasmon system

In this subsection, we show that the interaction model and Feynman variational path integration can also be applied for the calculation of the effective mass of a slow positron in metals. The metallic densities of the electron gas are a density in which r_s is between 2.0-6.0. According to the variational principle, in order to obtain the ground state properties of a slow positron in metals, we insert ω_q from Eq. (3) and g_q from Eq. (4) into Eq. (27), and solve for ν and ρ which yield the lowest E_0. Then we insert these ν and ρ into Eq. (28) to get the corresponding m_c^*/m_c. The expressions (27) and (28) cannot be evaluated analytically. Again, we resort to a standard numerical integration and a simulated annealing minimization [32].

r_s	ν	ρ	E_0 (Ry)	m_c^*/m_c	m_c^*/m_e
1	7.05	0.988	-0.791	1.020	0.816
2	2.55	0.978	-0.494	1.039	0.831
3	1.37	0.968	-0.373	1.057	0.846
4	8.90×10^{-1}	0.959	-0.305	1.074	0.859
5	6.33×10^{-1}	0.950	-0.261	1.091	0.873
6	6.15×10^{-1}	0.931	-0.232	1.110	0.888
10	2.27×10^{-1}	0.911	-0.160	1.173	0.938

Table 4: Shows ν, ρ, E_0, m_c^*/m_c and m_c^*/m_e as a function of r_s. The plasmon spectrum and the coupling function are taken from Lundqvist's model. We set $m_c/m_e = 0.8$.

Apart from ω_q and g_q, another input parameter in this theory is the ratio m_c/m_e. It has been shown that m_c/m_e differs from unity because it depends on band structures [7, 9, 10] or phonon effects [6]. For illustration, we explore three different values of m_c/m_e, i.e. 0.8, 1.0 and 1.2. The results are shown in Table 4-6. It can be readily seen that the ground state energy decreases as r_s increases and it increases a little as m_c/m_e increases. The effective mass, m_c^*/m_c increases as r_s increases and as m_c/m_e

increases. Moreover, at fixed r_s, ν and ρ are different for a different m_c/m_e. Evidently, this shows the internal effects of m_c/m_e to the ground state properties.

r_s	ν	ρ	E_0 (Ry)	m_c^*/m_c	m_c^*/m_e
1	7.03	0.989	-0.821	1.024	1.024
2	2.46	0.979	-0.514	1.046	1.046
3	1.31	0.969	-0.388	1.067	1.067
4	8.65×10^{-1}	0.961	-0.318	1.088	1.088
5	6.09×10^{-1}	0.952	-0.272	1.108	1.108
6	4.74×10^{-1}	0.945	-0.239	1.127	1.127
10	2.22×10^{-1}	0.916	-0.167	1.203	1.203

Table 5: Same as table 1 but with $m_c/m_e = 1.0$.

A similar approach to our theory was introduced by Hamann [12]. By using the many-body techniques and Feynman diagrams, he evaluated m_c^*/m_c of a positron in metals. Some selected values of m_c^*/m_c from Hamann's work are shown in Table 7. We find that those m_c^*/m_c in his calculations are 3-8% higher than ours. This is because he used a slightly different dielectric function. However, the trend is quite the same.

r_s	ν	ρ	E_0 (Ry)	m_c^*/m_c	m_c^*/m_e
1	7.13	0.990	-0.843	1.027	1.232
2	2.38	0.980	-0.528	1.052	1.262
3	1.29	0.971	-0.400	1.076	1.291
4	8.43×10^{-1}	0.963	-0.328	1.099	1.319
5	6.02×10^{-1}	0.955	-0.280	1.122	1.346
6	4.64×10^{-1}	0.948	-0.247	1.144	1.373
10	2.16×10^{-1}	0.920	-0.172	1.229	1.475

Table 6: Same as table 1 but with $m_c/m_e = 1.2$.

There were a number of experiments which aimed to determine m_c^*/m_c of a thermalized positron in metals [17, 18]. Some selected values of m_c^*/m_c from the experiments are also shown in Table 7. We find that those m_c^*/m_c from the experiments are quite large, i.e. almost twice larger than those from our calculations. It should be pointed out that m_c^*/m_c from the experiments should include all the interactions in metals. The large m_c^*/m_c could result from other effects, such as positron-phonon or positron-defect interaction. An experiment showed that the positron-polaron also has a large effective mass [33]. Other experiments suggested that positrons are easily

trapped in the vicinity of defects [14, 15]. A path integral theory showed that interacting with defects can modify the effective mass [27, 28]. Moreover, the effective mass determination experiments were carried out over a range of high temperature. They cannot be done at very low temperature because a positron annihilates with an electron before thermalization. At the present, our theory does not include temperature effects. However, the extension of our theory to include the temperature effects should be straightforward. This was done in the case of polaron by modifying the approximate propagator to account for the effects of some thermally excited states [34, 35]. It was suggested that the plasmon spectrum can be modified to include the effect of temperature as well [31]. All these thermal modifications will be considered in our future works.

r_s	m_c^*/m_c (Ours)	m_c^*/m_c (Hamann [12])	r_s	m_c^*/m_c (Experiments [17, 18])
2	1.05	1.08	3.25	1.8
3	1.07	1.12	3.93	1.8
4	1.09	1.15	4.86	2.1
5	1.11	1.19	5.20	2.3
6	1.13	1.22	5.63	2.5

Table 7: Some selected values of m_c^*/m_c from Hamann's work and from the experiments compared with our results where $m_c/m_e = 1.0$.

5 Conclusions

In this work, we have shown that Feynman path integral can be applied to a system of an electron or a positron interacting with plasmons in a Fermi Gas. By choosing an appropriate dielectric model, the ground state energy and the effective mass can be calculated. Some physical systems can be studied by choosing an appropriate integral limits for the plasmon modes. For an electron, we choose to study the Coulomb-hole self-energy. Our results are in good agreement with the Coulomb-hole self energy at $k = 0$ from Overhauser's work [16].The theory predicts a phase transition, *i.e.* the dressed electron is mobile in the high density limit but it exhibits a self-trapping behavior in the low density limit. This should be an indication, or at least a precursor, of Wigner crystallization. For a positron, we study its effective mass as it interacts with plasmons in solids. We find that our results are in agreement with a similar approach but not with experiments. The electron-plasmon interaction is important because it supports some experimental findings in metals [36] and in magneto-optical properties of semiconductors [37]. Our theory is proposed in general and can be readily extend to lower dimensionality. Some studies show that the electron-plasmon interaction is more significant in 2D systems [20, 31, 38], such as in a quantum well [39], than in

3D system. Furthermore, it has been suggested that the electron-plasmon interaction might assist Cooper pairing in superconductors [40]. We believe that this report will open a possibility for investigating these frontier physical systems in the future.

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Output จากโครงการวิจัยที่ได้รับทุนจากสกว.

1. ผลงานตีพิมพ์ในวารสารวิชาการนานาชาติ

- Udomsilp Pinsook and Virulh Sa-yakanit, "Application of Feynman path integration to the electron-plasmon interaction", Physica status solidi (b), 2003, volume 237, page 82-89. (ดูภาคผนวก)
- Udomsilp Pinsook, Virulh Sa-yakanit and Thiti Bovornratanaraks, "Effect of charge-plasmon interaction to the effective mass of a charge particle in solids", International journal of modern physics (b), submitted. (ดูภาคผนวก)

2. การนำผลงานวิจัยไปใช้ประโยชน์

- เชิงสาธารณะ สร้างกระแสความสนใจในระดับนานาชาติเกี่ยวกับการเคลื่อนที่ของอิเล็กตรอนหรือประจุไฟฟ้าใดๆ ในวัสดุควบแน่น อันตรกิริยาระหว่างอิเล็กตรอนและพลาสมอนอาจเป็นกลไกอันหนึ่งของตัวนำยิ่งยวดอุณหภูมิสูง ได้มีการเสนอผลงานทั้งในรูปแบบของการประชุมวิชาการระดับนานาชาติ และบทความวิจัยในวารสารวิชาการระดับนานาชาติ พบว่าได้รับความสนใจอย่างมาก
- เชิงวิชาการ มีการพัฒนาการเรียนการสอนระดับปริญญาบัณฑิตและมหาบัณฑิตในวิชาควอนตัมหลายวัตถุและวิชาฟิสิกส์ของวัสดุควบแน่น สามารถใช้ฝึกนิสิตในการทำงานวิจัยเชิงทฤษฎีและผลิตบุคลากรระดับมหาบัณฑิตได้ด้วย

3. การเสนอผลงานในที่ประชุมวิชาการระดับนานาชาติ
(มีตัวอย่างเนื้อหาที่ใช้เสนอในที่ประชุมในภาคผนวก)

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Application of Feynman path integration to the electron–plasmon interaction

U. Pinsook* and V. Sa-yakanit

Department of Physics, Faculty of Science, Chulalongkorn University, Bangkok 10330, Thailand

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Dedicated to Professor Jozef T. Devreese on the occasion of his 65th birthday

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We study a system of an electron interacting with plasmons using Feynman path integral method. The Lagrangian and then the action which describe the motion of the electron interacting with a harmonic field of the plasmons are derived. The interaction strength is embedded in a specific dielectric function and a corresponding plasmon spectrum. We have studied two models of the dielectric media proposed by Lundqvist and Overhauser. By using numerical integration and minimization, we evaluate the ground state energy and the effective mass of the system as a function of r_s. We found that both models give similar ground state energy. There is a possibility that the theory would predict a phase transition in this system. The analytic expansion of the ground state energy in some extreme limits are investigated. The applications of theory to some excitations in a Fermi gas are discussed.

1 Introduction The interaction among electrons is extremely important because the world is made of the Pauli exclusion principle of electrons. Without electrons, life cannot exist. There were many theoretical approaches attempted to explain the electron–electron interaction [1]. This work presents an alternative method for studying a part of the electron–electron interaction, i.e. the electron–plasmon interaction.

The plasmons are the quantized harmonic oscillations. They exist naturally in real metals and any electron gas. The interaction between an electron and plasmons can be described by Fröhlich Hamiltonian [2]. This problem is analogous to the polaron problem [3]. However, the polaron problem involves only two constant parameters; a phonon frequency and a coupling parameter. In the electron–plasmon or the hole–plasmon interaction, the problem becomes more complicated because the coupling strength is depended on the momentum of the plasmons [3, 4]. In this study, we choose Feynman variational path integral method as a calculation tool. It is a non-perturbative method. Hence it has a potential to give more accurate calculations. Feynman had shown that this method can accurately describe the properties of the polaron for the whole range of its coupling constants [5, 6]. Sa-yakanit had also shown that the same method was successfully applied to study a number of frontier physics problems such as disordered systems, heavily doped semiconductors, anharmonic crystals and etc. (see for example [7]). We will show that Feynman method is still an efficient tool for such a complicated problem like the electron–plasmon system.

* Corresponding author: e-mail: may@deathstar.phys.sc.chula.ac.th

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Note that our theory can as well be generalized to study the interaction between a fermion particle and plasmons. However, for simplicity, we will concentrate on an electron–plasmon system as a case study. The formulation for a hole–plasmon system should be exactly the same.

This paper is organized as follows: the models of the electron–plasmon interaction are in Section 2, the general Feynman path integral theory for the electron–plasmon system is in Section 3, and calculation results and discussions are in Section 4.

2 Models of an electron–plasmon interaction Firstly, we consider an electron moving in a dielectric medium such as a degenerate electron gas or a jellium. The electron and the dielectric medium will interact with each other. The spectrum of excitation modes will determine the strength of the interaction. However, there are two elementary excitations in the electron gas; plasmons and electron–hole pairs. Calculations involving the electron–hole pairs are quite tedious [4] and we shall try to avoid. Lundqvist [3] suggested that the whole excitation spectrum can be replaced by a modified plasmon spectrum. This modified plasmon spectrum obeys the f sum rules [1, 3, 4]. Those sum rules limit the contributions of plasmons to the dielectric function and hence serve as a convenient interpolation scheme. This approximation is sometimes called the plasmon pole approximation. The main aim of this Section is to discuss some selected dielectric models and their corresponding plasmon spectra.

The first model was introduced by Lundqvist [3]. He considered the electron gas in the random phase approximation (RPA) and proposed that the dielectric medium in this case can be described by a simplified dielectric function,

$$\varepsilon(q,\omega) = \frac{\omega_p^2(q) - \omega^2}{\omega_p^2(q) - \omega_p^2 - \omega^2} . \tag{1}$$

This dielectric function is not a full cover of RPA but rather an approximation, and it was chosen to satisfy the sum rules [3]. The plasmon spectrum is

$$\omega_p^2(q) = \omega_p^2 + \tfrac{4}{3}k_F^2 q^2 + q^4 . \tag{2}$$

The coupling function can be written as

$$g^2(q) = \frac{8\pi}{q^2}\left(\frac{1}{\partial\varepsilon(q,\omega)/\partial\omega}\right)_{\omega=\omega_p} = \frac{4\pi\omega_p^2}{q^2\omega_p(q)} . \tag{3}$$

Note that in order to compare our results with some existing works, we need to follow some natural units widely used in the physics of the atomic scale, i.e. $\hbar = 1$, $e^2 = 2$, $\frac{\hbar^2}{2m} = 1$. In these units, the long wavelength plasmon frequency $\omega_p = \frac{3.4641}{r_s^{3/2}}$ and the magnitude of Fermi wave vector $k_F = \frac{1.9192}{r_s}$, where r_s is the density parameter and the electron density n is $1/\frac{4}{3}\pi r_s^3$. Throughout this paper, the electron states will be labeled by k and the plasmon states will be labeled by q.

The second model was suggested by Overhauser [4]. He noticed that RPA does not account for the exchange and correlation effects of the electron gas. Thus he included some correction terms into the dielectric function, which can be expressed as

$$\varepsilon(q) = 1 + \frac{P(x)}{1 - G(x)P(x)} , \tag{4}$$

where

$$G(x) = \frac{1.1x^2}{\sqrt{1 + 10x^2 + 1.5x^4}} , \tag{5}$$

$$P(x) = \frac{1}{\pi k_F x^2}\left(\frac{1}{2} + \frac{1 - x^2}{4x}\ln\left|\frac{1 + x}{1 - x}\right|\right) , \tag{6}$$

and $x = \frac{q}{2k_F}$. The form of Eq. (4) is similar to incorporating the local field corrections and the vertex corrections into the dielectric function. This model is phenomenological because the exact corrections are not known. Nevertheless, Overhauser showed that his dielectric function reproduced correct correlation energy in the metallic density range [4]. The plasmon spectrum is then

$$\omega_p^2(q) = \frac{\omega_p^2 \varepsilon(q)}{\varepsilon(q) - 1} . \tag{7}$$

The coupling function is slightly modified as

$$g'(q) = g(q)\,(1 - G(x))\,, \tag{8}$$

where $g(q)$ is defined in Eq. (3).

It is worth noting that these dielectric functions are in fact valid for any test charged particle. Hence, they can be used for describing the interaction of an electron with plasmons or a hole with plasmons.

In the next section, we will discuss Feynman path integral method for the electron–plasmon system. The information needed in Feynman method is the plasmon spectra and the coupling functions defined in Eqs. (2), (3), (7), and (8).

3 Feynman path integral theory of an electron–plasmon system In this Section, we will give a general description of the path integral theory of the electron–plasmon system. Our theory can be stated without referring to a specific interaction model. Firstly, we construct the Lagrangian of the electron–plasmon system [2] which is of Fröhlich-type;

$$L = \frac{1}{2} m \dot{r}^2 + \sum_q \frac{M}{2} (\dot{Q}_q^2 - \omega_p^2(q)\, Q_q^2) - \sum_q \sqrt{\frac{2M\omega_p(q)}{V}}\, g(q)\, Q_q e^{iq\cdot r}\,, \tag{9}$$

where m is the electron mass, r is the position vector of the electron, $\frac{1}{2}m\dot{r}^2$ is the electron kinetic energy, M is the plasmon mass which we do not know but it will disappear during the path integration process, Q_q are the plasmon generalized coordinates and $\dot{Q}_q$ are their conjugate momenta, q denote plasmon wavevectors, V is the volume of the system. The system is assumed isotropic so that the dispersion of the plasmons is directionally independent. This Lagrangian represents a coupled system of the electron and the plasmons. The first term in the Lagrangian describes the kinetic energy of the electron. The second term is the harmonic representation of the plasmon modes. The third term is the interaction between the electron and the plasmons. The strength of the interaction is enclosed in $g(q)$. We can choose $g(q)$ and its corresponding $\omega_p(q)$ from the previous section. We will see later that both models give similar results.

Following Feynman's method, Sa-yakanit et al. [2] showed that the corresponding action is

$$S = \frac{m}{2}\int_0^t d\tau\, \dot{r}^2(\tau) + \frac{1}{4\pi^2}\int_0^\infty dq\, q^2 g^2(q) \int_0^t\int_0^t d\tau\, d\sigma \frac{\cos\left[\omega_p(q)\,(t/2 - |\tau - \sigma|)\right]}{\sin\left[\omega_p(q) t/2\right]} e^{iq\cdot(r(\tau) - r(\sigma))}. \tag{10}$$

The system is assumed spherical symmetry. Thus we have replaced $\int dq$ by $\int 4\pi^2 q^2 dq$. Choices of the integration limits are depended on a physical system we choose to study. A trivial choice is $\int_0^\infty dq$ and the action in Eq. (10) can be used for describing the Coulomb-hole self-energy [4]. In order to study the plasmaron, a hole dressed by plasmons in a Fermi gas, a constraint on a hole in a state k and plasmons in states q must be imposed, i.e. $|k + q| < k_F$ [3, 4]. This is because a hole cannot be excited to a state above the Fermi surface. For simplicity, we will use the trivial choice for general discussions from now on.

From the path integral's point of view, Eq. (10) shows that the system of the electron interacting with the plasmons can be reduced to a system of a particle in a potential field. This potential field is described by the second term in Eq. (10), which is non-local in time. The classical picture is that the