



รายงานวิจัยฉบับสมบูรณ์

โครงการ ผลของฟิสิกส์ที่ระดับพลังงานสูงต่อเอกภพที่สังเกตได้

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สนับสนุนโดยสำนักงานกองทุนสนับสนุนการวิจัย

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กิตติกรรมประกาศ

ผู้ทำวิจัยขอขอบคุณ อาจารย์ อภิสิทธิ์ อึ้งกิจจานุกิจ อาจารย์ท่านอื่นๆและนิสิตในกลุ่ม
วิจัยทฤษฎีของฟิสิกส์พลังงานสูงและจักรวาลวิทยาภาควิชาฟิสิกส์คณะวิทยาศาสตร์จุฬาลงกรณ์
มหาวิทยาลัย ตลอดจนภาควิชาฟิสิกส์ คณะวิทยาศาสตร์ และ จุฬาลงกรณ์มหาวิทยาลัย ที่ให้
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ต่อ อาจารย์ บุรินทร์ กำจัดภัย สำหรับความช่วยเหลือในการเป็นนักวิจัยที่ปรึกษาให้กับ
โครงการวิจัยนี้และสำหรับคำแนะนำที่เป็นประโยชน์ในการทำวิจัย นอกจากนี้ผู้ทำวิจัยขอขอบคุณ
อาจารย์ที่ภาควิชาฟิสิกส์มหาวิทยาลัยเกษตรศาสตร์ที่ให้การช่วยเหลือทางด้าน computer
ประสิทธิภาพสูงในการทำวิจัยนี้ โดยภาควิชาฟิสิกส์คณะวิทยาศาสตร์มหาวิทยาลัยเกษตรศาสตร์
เป็นสถานที่ทำงานใหม่ของผู้ทำวิจัยและได้ทำงานวิจัยหลายส่วนที่นี้ ท้ายที่สุดผู้ทำวิจัยขอแสดง
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Abstract

We study some consequences of high energy physics on the observable universe. We are interested in two independent topics. First we consider the coincidence problem in the interacting holographic dark energy model. To alleviate the coincidence problem, we suppose that holographic dark energy has an interaction with dark matter. We analyze the cosmic evolution for this interacting dark energy model and compare the region of parameters of the interaction terms for which the cosmic evolution has an attractor within the parameter region that satisfies the observational constraints. We have shown that for holographic dark energy model, which is inspired by holographic principle in high energy theory, a general form of the interaction between holographic dark energy and dark matter can lead to alleviation of the coincidence problem for restricted range of parameters. This implies that the introduction of interaction between dark energy and dark matter in holographic dark energy model is not so effective in alleviating coincidence problem in as the usual dark energy models.

In the second topic, we consider the effects of spacetime noncommutativity on statistical properties of Cosmic Microwave Background (CMB). We investigate the primordial power spectrum of the density perturbations based on the assumption that spacetime is noncommutative in the early stage of inflation. For k-inflation model, we show that the deviation from rotational invariance of the primordial power spectrum due to spacetime noncommutativity depends on the noncommutative length scale and sound speed of inflaton. We use five-year WMAP CMB maps to constrain the contributions from spacetime noncommutativity in this power spectrum and find that the upper bound for the noncommutative length scale should be less than 10^{-27} cm at 99.7 % confidence level.

Keywords: cosmology, interacting dark energy, noncommutative inflation, observational constraints on cosmological models

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ชื่อโครงการ ผลของฟลักซ์ที่ระดับพลังงานสูงต่อเอกภพที่สังเกตได้

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ระยะเวลาโครงการ 2 ปี

บทคัดย่อ

ผู้ทำวิจัยศึกษาผลเกี่ยวเนื่องจากฟลักซ์พลังงานสูงต่อเอกภพที่สังเกตได้ โดยมีความสนใจในสองหัวข้อที่แตกต่างกัน ในหัวข้อแรกศึกษาปัญหา coincidence ในแบบจำลอง holographic ของพลังงานมืด เพื่อแก้ปัญหา coincidence จึงสมมุติว่าพลังงานมืดมีอันตรกิริยากับสสารมืด เมื่อวิเคราะห์การวิวัฒนาการของเอกภพในกรณีนี้แล้วเปรียบเทียบช่วงของ parameter ของอันตรกิริยาที่ทำให้การวิวัฒนาการของเอกภพมีจุดจุด กล่าวคือเกิดการแก้ปัญหา coincidence กับช่วงของ parameter ที่สอดคล้องกับข้อมูลจากการสังเกตการณ์ พบว่าในกรณีของแบบจำลองพลังงานมืดนี้ อันตรกิริยาระหว่างพลังงานมืดและสสารมืดที่มีรูปแบบทั่วไปทำให้เกิดการแก้ปัญหา coincidence ได้สำหรับช่วงแคบๆของ parameter เท่านั้น นี่แสดงว่าในกรณีของแบบจำลองพลังงานมืดนี้การสมมุติว่าพลังงานมืดและสสารมืดมีอันตรกิริยากันไม่ช่วยแก้ปัญหา coincidence ได้อย่างดีเหมือนแบบจำลองพลังงานมืดทั่วไป

ในหัวข้อที่สองผู้ทำวิจัยศึกษาผลของการไม่ commute กันของกาลอวกาศต่อสมบัติเชิงสถิติของ Cosmic Microwave Background (CMB) โดยใช้สมมุติฐานว่ามี การไม่ commute กันของกาลอวกาศในช่วงแรกของ inflation ในเอกภพยุคเริ่มต้นแล้วคำนวณหา power spectrum ของความผันผวนในการกระจายตัวของสสารและพลังงานในเอกภพยุคนี้ สำหรับกรณีของแบบจำลอง k-inflation ผลจากการคำนวณแสดงให้เห็นว่าการเบี่ยงเบนจากการมีสมมาตรในการหมุนของ power spectrum อันเป็นผลมาจากการไม่ commute กันของกาลอวกาศขึ้นกับ noncommutative length scale และ sound speed ของinflaton นอกจากนี้เมื่อใช้แผนที่ CMB จากผลห้าปีของ WMAP มาจำกัดปริมาณการไม่ commute กันของกาลอวกาศใน power spectrum พบว่า noncommutative length scale ควรมีค่าไม่เกิน 10^{-27} ซม. ที่ระดับความเชื่อมั่น 99.7 %

คำหลัก จักรวาลวิทยา อันตรกิริยาระหว่างพลังงานมืดและสสารมืด การไม่ commute กันของกาลอวกาศในแบบจำลอง inflation การใช้ข้อมูลจากการสังเกตการณ์เพื่อจำกัดรูปแบบของแบบจำลองเอกภพ

1. บทนำ

จากการสังเกตการณ์ทางฟิสิกส์ดาราศาสตร์ที่มีความแม่นยำมากขึ้นในปัจจุบันทำให้เรามีข้อมูลและรู้ถึงสมบัติของเอกภพมากขึ้น สมบัติของเอกภพที่กล่าวถึงนี้เป็นสมบัติของเอกภพที่สังเกตได้ (observable universe) [1] เนื่องจากการเคลื่อนที่ของสสารพลังงานตลอดจนข้อมูลในอวกาศไม่สามารถมีความเร็วเกินกว่าความเร็วของแสงในสุญญากาศดังนั้นส่วนของเอกภพที่แสงใช้เวลานานกว่าอายุของเอกภพในการเดินทางจากส่วนนั้นมาถึงเราจึงถูกตัดขาดจากเราและเอกภพในส่วนที่เราสังเกตได้ก็เป็นส่วนของเอกภพที่แสงจากส่วนนั้นสามารถเดินทางมาถึงเราได้ ในยุคเริ่มต้นของเอกภพความหนาแน่นของสสารและพลังงานจะมีค่าสูงมากปรากฏการณ์ทางฟิสิกส์ที่เกิดในเอกภพยุคนี้จึงเป็นไปตามทฤษฎีของฟิสิกส์พลังงานสูงซึ่งเป็นที่เชื่อกันโดยทั่วไปว่าปรากฏการณ์เหล่านี้อาจทิ้งร่องรอยหรือผลพวงบางอย่างในเอกภพที่สังเกตได้ [2] ดังนั้นการศึกษาเอกภพจึงอาจช่วยให้เข้าใจหรือช่วยทดสอบทฤษฎีของฟิสิกส์พลังงานสูงในบางประเด็นได้ซึ่งในโครงการวิจัยนี้จะศึกษาผลของปรากฏการณ์ทางฟิสิกส์พลังงานสูงบางอย่างที่มีต่อเอกภพที่สังเกตได้

ประเด็นหนึ่งที่ศึกษาในโครงการนี้คือศึกษาว่าปัญหา coincidence ในแบบจำลอง holographic ของพลังงานมืดซึ่งเป็นแบบจำลองที่สร้างจากสมมติฐานในฟิสิกส์พลังงานสูง [3] สามารถถูกแก้ไขได้ดีเพียงใดด้วยการสมมุติว่าพลังงานมืดมีอันตรกิริยากับสสารมืด โดยทั่วไปปัญหาสำคัญหนึ่งของแบบจำลองพลังงานมืดซึ่งเป็นแบบจำลองที่ใช้อธิบายการขยายตัวด้วยความเร่งในยุคปัจจุบันของเอกภพคือปัญหา coincidence [4] ซึ่งเป็นปัญหาที่ว่าเพราะสาเหตุใดความหนาแน่นพลังงานของสสารมืดและพลังงานมืดจึงมีค่าอยู่ในระดับเดียวกัน ณ เวลาปัจจุบัน ทั้งที่ความหนาแน่นพลังงานของสองสิ่งนี้ลดลงด้วยอัตราที่แตกต่างกันเมื่อเอกภพวิวัฒน์ในเวลา นั่นคือความหนาแน่นพลังงานของสสารมืดมีค่ามากกว่าความหนาแน่นพลังงานของพลังงานมืดมากในอดีตและเป็นไปในทางกลับกันในอนาคต แนวทางหนึ่งที่ช่วยแก้ปัญหา coincidence ได้ดีในระดับหนึ่งสำหรับแบบจำลองพลังงานมืดทั่วไปคือการสมมุติว่าสสารมืดและพลังงานมืดมีอันตรกิริยากัน [5] แต่จากการศึกษาแบบจำลอง holographic ของพลังงานมืดในโครงการนี้พบว่าการสมมุติให้สสารมืดและพลังงานมืดมีอันตรกิริยากันช่วยแก้ปัญหา coincidence ได้ไม่ดีนัก

อีกประเด็นหนึ่งที่ศึกษาในโครงการคือศึกษาว่าการไม่ commute กันของกาลอวกาศในเอกภพยุคเริ่มต้นมีผลมากน้อยเพียงใดต่อสมบัติเชิงสถิติของ CMB และค่าที่สอดคล้องกับข้อมูลจากการวัด CMB ของ parameter ที่เกี่ยวข้องกับการไม่ commute กันของกาลอวกาศควรเป็นเช่นใด ในช่วงเริ่มต้นของยุค inflation ซึ่งเป็นส่วนหนึ่งของเอกภพยุคเริ่มต้นความหนาแน่นพลังงานของเอกภพอาจมีค่าเข้าใกล้สเกล Planck [6] อันจะทำให้มีผลจากฟิสิกส์ที่ระดับสเกล Planck ปรากฏในสมบัติของความผันผวนในการกระจายตัวของสสารและพลังงานในเอกภพ ปรากฏการณ์ทางฟิสิกส์ที่ระดับสเกล Planck ที่ศึกษาในโครงการนี้คือปรากฏการณ์ของการไม่ commute กันของกาลอวกาศอันเป็นผลมาจากธรรมชาติ quantum ของความโน้มถ่วง [7 - 8] จากการศึกษาที่เคยมีมาแสดงให้เห็นว่าการไม่ commute กันของกาลอวกาศสามารถทำให้ power

spectrum ของความผันผวนในการกระจายตัวของสสารและพลังงานเสียดสมมาตรในการหมุน [9] ซึ่งจะทำให้สถิติของ CMB ไม่ isotropic [10] จากการศึกษาในโครงการพบว่า การไม่ isotropic ของสถิติของ CMB อันเป็นผลมาจากการไม่ commute กันของกาลอวกาศสามารถตรวจพบได้ จากการวิเคราะห์แผนที่ของ CMB แต่ข้อมูลของ CMB ในปัจจุบันยังมีความละเอียดไม่พอที่จะใช้ทดสอบว่ามีการไม่ commute ของกาลอวกาศในเอกภพยุคเริ่มต้นหรือไม่ อย่างไรก็ตามข้อมูลของ CMB ก็สามารถใช้จำกัดค่า parameter ที่เกี่ยวข้องกับการไม่ commute กันของกาลอวกาศได้ดี

2. วิธีดำเนินการ

2.1 วิธีดำเนินการสำหรับการศึกษาปัญหา coincidence ในแบบจำลอง holographic ของพลังงานมืด

2.1.1 คำนวณหาสมการ autonomous ที่บรรยายการวิวัฒนาการของการแผ่รังสี สสารมืด baryon และ พลังงานมืดแบบ holographic ในเอกภพ โดยสมมติว่าพลังงานมืดและสสารมืดมีอันตรกิริยากัน

2.1.2 หาจุด fixed ของสมการข้างต้นและวิเคราะห์ความเสถียรของจุดเหล่านี้ ซึ่งช่วงของ parameter ของอันตรกิริยาที่ทำให้เกิดจุด fixed ที่เสถียรจะเป็นช่วงของ parameter ที่อาจทำให้เกิดการแก้ปัญหา coincidence ได้

2.1.3 หาช่วงของ parameter ของอันตรกิริยาที่สอดคล้องกับข้อมูลจากการสังเกตการณ์

2.1.4 เปรียบเทียบช่วงของ parameter จากข้อ 2.1.2 และ 2.1.3 เพื่อพิจารณาว่าอันตรกิริยาระหว่างสสารมืดและพลังงานมืดช่วยแก้ปัญหา coincidence ได้ดีเพียงใด

2.2 วิธีดำเนินการสำหรับการศึกษาผลของการไม่ commute ของกาลอวกาศต่อสมบัติเชิงสถิติของ CMB

2.2.1 คำนวณหา power spectrum ของความผันผวนในการกระจายตัวของสสารและพลังงานโดยใช้แบบจำลอง k-inflation และสมมุติฐานที่ว่ากาลอวกาศในยุคเริ่มต้นของเอกภพไม่ commute กัน

2.2.2 คำนวณหา covariance matrix ของ สัมประสิทธิ์ของการกระจาย harmonic ของความผันผวนในอุณหภูมิของ CMB จาก power spectrum ข้างต้น

2.2.3 ตรวจสอบว่าการไม่ commute กันของกาลอวกาศที่ปรากฏใน covariance matrix ทำให้เกิดผลที่วัดได้หรือไม่ในแผนที่ CMB และใช้แผนที่ CMB จากผลห้าปีของ WMAP [11] เพื่อจำกัดค่าของ parameter ที่เกี่ยวข้องกับการไม่ commute กันของกาลอวกาศ

3. ผลที่ได้

3.1 ผลจากการศึกษาปัญหา coincidence ในแบบจำลอง holographic ของพลังงานมืด

3.1.1 รูปแบบทั่วไปหนึ่งของอันตรกิริยาระหว่างสสารมืดและพลังงานมืดบรรยายได้ด้วยสมการ

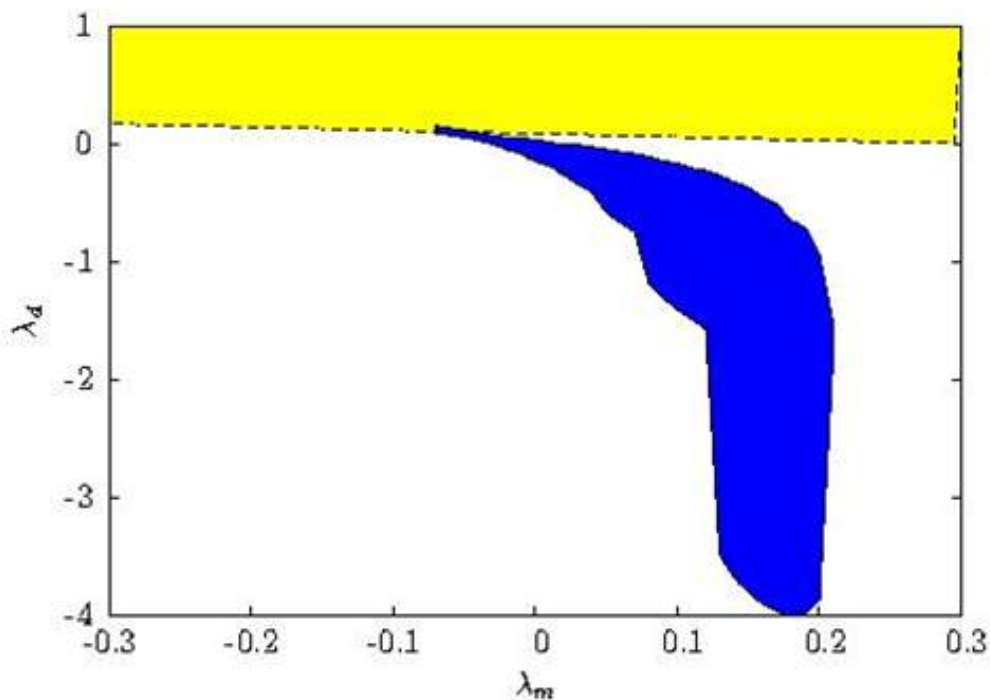
$Q = 3H(\lambda_d \rho_d + \lambda_c \rho_c)$ โดย ρ_d, ρ_c คือความหนาแน่นพลังงานของพลังงานมืดและสสารมืด H คือ Hubble parameter และ λ_d, λ_c คือ parameter ของอันตรกิริยา ช่วงของ parameter ที่ทำให้เกิดจุด fixed ที่เสถียรคือ

$$\lambda_d \Omega_d + \lambda_c \Omega_c > \Omega_c / 3 \quad (1)$$

โดย Ω_c, Ω_d คืออัตราส่วนของความหนาแน่นพลังงานของสสารมืดและพลังงานมืดต่อพลังงานวิกฤติที่เวลาปัจจุบัน

3.1.2 ช่วงของ parameter ที่สอดคล้องกับข้อมูลจากการสังเกตการณ์คือพื้นที่สีฟ้าในรูปที่ 1

3.1.3 ช่วงของ parameter ที่ทำให้เกิดการแก้ปัญหา coincidence คือช่วงของ parameter ที่อยู่ในบริเวณที่พื้นที่สีฟ้าทับกับพื้นที่สีเหลือง โดยพื้นที่สีเหลืองคือช่วงของ parameter ที่สอดคล้องกับสมการ 1



รูป 1 รูปแสดงช่วงของ parameter ของอันตรกิริยาที่ทำให้เกิดการแก้ปัญหา coincidence โดยพื้นที่สีฟ้าแสดงช่วงของ parameter ที่สอดคล้องกับข้อมูลจากการสังเกตการณ์และพื้นที่สีเหลืองแสดงช่วงของ parameter ที่สอดคล้องกับสมการ 1 ในรูปนี้ใช้ค่า $\Omega_c = 0.27$ และ $\Omega_d = 0.73$ ซึ่งสอดคล้องกับข้อมูลจากการสังเกตการณ์

จากรูปจะเห็นได้ว่าในกรณีที่พิจารณาปัญหา coincidence สามารถถูกแก้ได้สำหรับช่วงแคบ ๆ ของ parameter เท่านั้นซึ่งแสดงให้เห็นว่าปัญหา coincidence ถูกแก้ได้ไม่ดีนัก

3.2 ผลจากการศึกษาผลของการไม่ commute ของกาลอวกาศต่อสมบัติเชิงสถิติของ CMB

3.2.1 โดยการใช้สมมติฐานว่ามีการไม่ commute กันของกาลอวกาศในยุคเริ่มต้นของ inflation จะได้ว่า power spectrum ของความผันผวนในการกระจายตัวของสสารและพลังงานสำหรับแบบจำลอง k-inflation สามารถเขียนในรูป

$$P(\vec{k}) = P(k) \left(1 - \nu \frac{H^4 L_s^4}{c_s^2} \sin^2 o \right) \quad (2)$$

โดย $P(k)$ คือ power spectrum ของความผันผวนในกรณีที่กาลอวกาศ commute กัน L_s คือ noncommutative length scale c_s คือ sound speed ของ inflaton ν คือ parameter ที่เกี่ยวข้องกับอัตราเร็วในการขยายตัวของเอกภพ $\vec{k}$ คือเวกเตอร์ของเลขคลื่นของความผันผวนใน Fourier space และ o แสดงมุมระหว่างเวกเตอร์ $\vec{k}$ กับ basis ที่สามของมัน k_3 พจน์ที่สองในวงเล็บทางด้านขวามือของสมการข้างต้นเกิดจากการไม่ commute กันของกาลอวกาศซึ่งพจน์นี้ทำให้ power spectrum ทางด้านซ้ายมือขึ้นกับทิศทางของเวกเตอร์ $\vec{k}$ นั่นคือการไม่ commute กันของกาลอวกาศทำให้ power spectrum เสียสมมาตรในการหมุน

3.2.2 เพื่อความสะดวกในการศึกษาเชิงทฤษฎีเราเขียนความผันผวนในอุณหภูมิของ CMB ในรูปของการกระจาย harmonic

$$\Delta_T = T(\hat{n}) - T_0 = \sum_{l,m} a_{lm} Y_{lm}(\hat{n}) \quad (3)$$

โดย $T(\hat{n})$ คืออุณหภูมิที่วัดในทิศทาง $\hat{n}$ T_0 คืออุณหภูมิเฉลี่ยและ $Y_{lm}(\hat{n})$ คือ spherical harmonic functions ความผันผวนในอุณหภูมิ เป็นตัวแปรสุ่มซึ่งสมบัติเชิงสถิติบรรยายได้ด้วย correlation function ใน harmonic space two-point correlation function ของความผันผวนในอุณหภูมิขึ้นกับ covariance matrix ของ a_{lm} โดยการใช้ power spectrum ในสมการ 2 covariance matrix ของ lm เขียนได้ดังสมการ

$$\langle a_{lm} a_{l'm'}^* \rangle = \delta_{ll'} \delta_{mm'} C_l + \text{off diagonal elements} \quad (4)$$

โดย C_l คือ angular power spectrum ของ CMB ในกรณีที่ power spectrum ในสมการ 2 มีสมมาตรในการหมุนกล่าวคือพจน์ที่มาจากการไม่ commute กันของกาลอวกาศเป็นศูนย์ covariance matrix จะมีเฉพาะ diagonal elements และขึ้นกับ C_l เท่านั้น การที่ covariance matrix ของ a_{lm} มี off-diagonal elements แสดงให้เห็นว่า two-point correlation function ของ ΔT ขึ้นกับทิศทางนั่นคือสถิติของ CMB ไม่ isotropic

3.2.3 จากการวิเคราะห์แผนที่ CMB จากผลห่าปีของ WMAP แสดงให้เห็นว่าการไม่ isotropic ในสถิติของ CMB ต้องมีค่าจำกัดด้วยเหตุนี้ noncommutative length scale ควรมีค่าไม่เกิน 10^{-27} cm ที่ระดับความเชื่อมั่น 99.7%

4. สรุป

โดยทั่วไปปัญหา coincidence ในแบบจำลองพลังงานมืดสามารถถูกแก้ได้ด้วยวิธีการสมมุติว่ามีอันตรกิริยากันระหว่างสสารมืดและพลังงานมืดแต่จากการศึกษาแบบจำลอง holographic ของพลังงานมืดพบว่าอันตรกิริยาระหว่างสสารมืดและพลังงานมืดช่วยแก้ปัญหา coincidence ในแบบจำลองพลังงานมืดนี้ได้ไม่ตึงเครียด อย่างไรก็ตามผู้ทำวิจัยเชื่อว่าถ้าปรับปรุงรูปแบบของอันตรกิริยาอย่างเหมาะสมปัญหา coincidence อาจถูกแก้ได้ดีขึ้น

การไม่ commute กันของกาลอวกาศซึ่งเป็นผลมาจากธรรมชาติ quantum ของความโน้มถ่วงในเอกภพยุคเริ่มต้นสามารถทำให้เกิดผลที่วัดได้ใน CMB และโดยการวิเคราะห์แผนที่ CMB จากผลห่าปีของ WMAP พบว่า noncommutative length scale ควรมีค่าไม่เกิน 10^{-27} ซม. ที่ระดับความเชื่อมั่น 99.7% สำหรับแบบจำลอง k-inflation ซึ่งเป็นแบบจำลองทั่วไปของ inflation

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2. รายงานการวิจัยที่ส่งไปตีพิมพ์ที่ JCAP

The coincidence problem and interacting holographic dark energy

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Abstract. We study the dynamical behaviour of the interacting holographic dark energy model whose interaction term is $Q = 3H(\lambda_d \rho_d + \lambda_c \rho_c)$, where ρ_d and ρ_c are the energy densities of dark energy and cold dark matter respectively. To satisfy the observational constraints from type Ia supernovae, the cosmic microwave background shift parameter and baryon acoustic oscillation measurements, if $\lambda_c = \lambda_d$ or $\lambda_d, \lambda_c > 0$, the cosmic evolution will only reach the attractor in the future and the ratio ρ_c/ρ_d cannot be slowly varying at present. Since the cosmic attractor can be reached in the future even when the present values of the cosmological parameters do not satisfy the observational constraints, the coincidence problem is not really alleviated in this case. However, if $\lambda_c \neq \lambda_d$ and they are allowed to be negative, the ratio ρ_c/ρ_d can be slowly varying at present and the cosmic attractor can be reached near the present epoch. Hence, the alleviation of the coincidence problem is attainable in this case. The alleviation of the coincidence problem in this case is still attainable when confronting this model with Sloan Digital Sky Survey data.

Keywords: dark energy theory, classical tests of cosmology

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1. Introduction

Observations suggest that the expansion of the universe is accelerating [1, 2]. The acceleration of the universe may be explained by supposing that the present universe is dominated by a mysterious form of energy whose pressure is negative, known as dark energy. One of the weaknesses of the dark energy model is the coincidence problem, which is the problem why the dark energy density and matter density are of the same order of magnitude in the present epoch although they evolve differently during the expansion of the universe. A possible way to alleviate the coincidence problem is to suppose that there is an interaction between matter and dark energy. The cosmic coincidence can then be alleviated by appropriate choice of the form of the interaction between matter and dark energy leading to a nearly constant ratio $r = \rho_c/\rho_d$ during the present epoch [3]–[5] or giving rise to an attractor of the cosmic evolution at late time [6, 7]. Since the existence of the cosmic attractor implies constant r but the attractor does not always occur at the present epoch, we first find a range of dark energy parameters for which the attractor occurs and then check the evolution of r during the present epoch.

On the basis of holographic ideas [8, 9], one can determine the dark energy density in terms of the horizon radius of the universe. This type of dark energy is holographic dark energy [10]–[14]. Choosing the Hubble radius as the cosmological horizon, the present amount of dark energy density agrees with observations. Nevertheless, dark energy evolves like matter at present, so it cannot lead to an accelerated expansion. However, if the particle horizon is chosen to be the cosmological horizon, the equation of state parameter of dark energy can become negative but not negative enough to drive an accelerating universe. The situation is better when one uses the event horizon as the cosmological horizon. In this case, dark energy can drive the present accelerated expansion, and the coincidence problem can be resolved by assuming an appropriate number of e-foldings of inflation. Roughly speaking, the coincidence problem can be resolved because the size of the cosmological horizon during the present epoch depends on the amount of e-folds of inflation, and the amount of holographic dark energy depends on the horizon size. Nevertheless, the second law of thermodynamics will be violated if $w_d < -1$ [9, 15]. Hence, w_d should not cross the boundary $w_d = -1$. The boundary $w_d = -1$ can be

crossed if dark energy interacts with matter. Since now the horizon size has a dependence on the interaction terms, the alleviation of cosmic coincidence should also depend on the interaction term.

In this work we suppose that the holographic dark energy interacts only with cold dark matter (CDM) and treat baryons as non-interacting matter components. Our objective is to compare the region of dark energy parameters for which the cosmic evolution has an attractor within the parameter region that satisfies the observational constraints from combined analysis of SNIa data [16], the CMB shift parameter [17] and BAO measurements [18]. The results of the comparison can tell us about the range of parameters that alleviate the cosmic coincidence.

2. The autonomous equations

In this section, we derive the first-order differential equations that describe the evolutions of radiation, baryon, CDM and dark energy densities in the universe. By analysing these equations, one can estimate the asymptotic evolution of the universe. To proceed, we start from the Friedmann equation

$$H^2 + \frac{K}{a^2} = \frac{1}{3\bar{m}_p} (\rho_r + \rho_b + \rho_c + \rho_d), \quad (1)$$

where H is the Hubble parameter and the subscripts r, b, c and d correspond to the radiation, baryons, CDM and dark energy respectively. The parameter K denotes the curvature of the universe, where $K = -1, 0, +1$ for the close, flat and open universe respectively. The above equation can be written in terms of the density parameters $\Omega_K = K/(a^2 H^2)$ and $\Omega_\alpha = \rho_\alpha/(3\bar{m}_p^2 H^2)$ as

$$1 + \Omega_K = \sum_{\alpha=r,b,c,d} \Omega_\alpha = \Omega_r + \Omega_b + \Omega_c + \Omega_d. \quad (2)$$

The index α runs over the four species, namely radiation, baryons, CDM and dark energy. We now derive the autonomous equations for the dynamical variables Ω_K and Ω_α . Differentiating $\Omega_K = K/(a^2 H^2)$ with respect to $\ln a$, we get

$$\Omega'_K = -\frac{2K}{H} \left(\frac{\dot{a}}{a^3 H^2} + \frac{\dot{H}}{a^2 H^3} \right) = -2\Omega_K \left(1 + \frac{\dot{H}}{H^2} \right), \quad (3)$$

where the prime and dot denote the derivatives with respect to $\ln a$ and time respectively. From the definition of the density parameter, one can show that

$$\Omega'_\alpha = \Omega_\alpha \left(\frac{\dot{\rho}_\alpha}{H \rho_\alpha} - 2 \frac{\dot{H}}{H^2} \right). \quad (4)$$

To study the evolution of the universe at late time, we will search for the fixed points of the above autonomous equations and check the stability of these fixed points. The fixed points of equations (3) and (4) are the points $(\Omega_{Kc}, \Omega_{\alpha c})$ at which

$$\Omega'_K = \Omega'_\alpha = 0. \quad (5)$$

It follows from equation (3) that $\Omega'_K = 0$ at $\Omega_K = 0$ or $1 + \dot{H}/H^2 = 0$. Hence, possible fixed points at $\Omega_K \neq 0$ correspond to the non-accelerating universe, i.e. $1 + \dot{H}/H^2 \propto \ddot{a} = 0$.

Since the expansion of the universe is accelerating today, we consider only the fixed points at $\Omega_K = 0$, and therefore neglect Ω_K in our consideration for simplicity.

Now we come to the case of interacting holographic dark energy and use equation (4) to obtain the autonomous equation for this case. In the holographic dark energy scenario, the energy density of dark energy is related to the cosmological horizon L by

$$\rho_d = 3c^2 \bar{m}_p^2 L^{-2}, \quad (6)$$

where c is a positive constant. Differentiating the above equation with respect to time, we obtain

$$\dot{\rho}_d = -2\rho_d \frac{\dot{L}}{L}. \quad (7)$$

We take the cosmological horizon to be the event horizon, which is defined as $R_e(t) = a(t) \int_t^\infty d\tilde{t}/a(\tilde{t})$. Hence,

$$\frac{\dot{L}}{L} = \frac{\dot{R}_e}{R_e} = H - \frac{1}{R_e}. \quad (8)$$

We therefore get

$$\dot{\rho}_d = -2H\rho_d + 2\frac{\rho_d^{3/2}}{\sqrt{3c}\bar{m}_p}. \quad (9)$$

When dark energy has an interaction with CDM, the continuity equations yield

$$\dot{\rho}_c = -3H\rho_c + Q, \quad (10)$$

$$\dot{\rho}_d = -3H(1 + w_d)\rho_d - Q, \quad (11)$$

where $Q = 3H(\lambda_d\rho_d + \lambda_c\rho_c)$. Usually, one supposes that $Q > 0$ because the second law of thermodynamics might be violated if energy transfers from matter to dark energy ($Q < 0$). However, for generality, we will not restrict Q to be positive in our consideration. Comparing equation (9) with equation (11), we get

$$w_d = -\frac{1}{3} - 2\frac{\sqrt{\Omega_d}}{3c} - \frac{\lambda_d\Omega_d + \lambda_c\Omega_c}{\Omega_d}. \quad (12)$$

We assume that radiation and baryons have no interaction with dark energy, so that they obey the continuity equations

$$\dot{\rho}_r = -4H\rho_r \quad \text{and} \quad \dot{\rho}_b = -3H\rho_b. \quad (13)$$

From equation (1), one can show that

$$2\frac{\dot{H}}{H^2} = \frac{(H^2)^\cdot}{H^3} = -3 + \Omega_d + 2\frac{\Omega_d^{3/2}}{c} + 3(\lambda_d\Omega_d + \lambda_c\Omega_c) - \Omega_r. \quad (14)$$

In the above equation, we set $\Omega_K = 0$. Using equations (4), (9), (13) and (14), we obtain

$$\Omega'_d = \Omega_d \left(1 + 2\frac{\Omega_d^{1/2}}{c} - \Omega_d - 2\frac{\Omega_d^{3/2}}{c} - 3(\lambda_d\Omega_d + \lambda_c\Omega_c) + \Omega_r \right), \quad (15)$$

$$\Omega'_c = \Omega_c \left(\frac{3}{\Omega_c}(\lambda_d\Omega_d + \lambda_c\Omega_c) - \Omega_d - 2\frac{\Omega_d^{3/2}}{c} - 3(\lambda_d\Omega_d + \lambda_c\Omega_c) + \Omega_r \right), \quad (16)$$

$$\Omega'_r = \Omega_r \left(-1 - \Omega_d - 2\frac{\Omega_d^{3/2}}{c} - 3(\lambda_d\Omega_d + \lambda_c\Omega_c) + \Omega_r \right), \quad (17)$$

$$\Omega'_b = \Omega_b \left(-\Omega_d - 2\frac{\Omega_d^{3/2}}{c} - 3(\lambda_d\Omega_d + \lambda_c\Omega_c) + \Omega_r \right). \quad (18)$$

The fixed points of the above equations are $(\Omega_d, \Omega_c, \Omega_r, \Omega_b) = (0, 0, \Omega_{rc}, 0)$, $(0, 0, 0, \Omega_{bc})$ and $(\Omega_{dc}, \Omega_{cc}, 0, 0)$. Since we are interested in the late time evolution of the universe, we will consider only the fixed point $(\Omega_{dc}, \Omega_{cc}, 0, 0)$. This fixed point can occur at late time, i.e., about the present or in the future, because Ω_r/Ω_c usually decreases with time and Ω_b/Ω_c can decrease with time if $Q > 0$. Using equations (15) and (16), the relation between Ω_{dc} and Ω_{cc} can be written as

$$1 + 2\frac{\Omega_{dc}^{1/2}}{c} = \frac{3}{\Omega_{cc}}(\lambda_d\Omega_{dc} + \lambda_c\Omega_{cc}). \quad (19)$$

From equation (2), we have $\Omega_{dc} + \Omega_{cc} = 1$ and hence

$$1 - 3\lambda_c + 2\frac{\Omega_{dc}^{1/2}}{c} - (1 + 3\lambda_d - 3\lambda_c)\Omega_{dc} - 2\frac{\Omega_{dc}^{3/2}}{c} = 0. \quad (20)$$

The solution of equation (20) gives Ω_{dc} in terms of λ_d , λ_c and c . Instead of finding the solution of this equation, we will use equation (19) to compute the cosmological parameters of interest in the following section. The basic idea is that there are various values of c , λ_d and λ_c that satisfy equation (19) for given Ω_{dc} and Ω_{cc} . Changes in the values of c , λ_d and λ_c lead to a different cosmological evolutions, i.e. a different w_d for a given Ω_{dc} and Ω_{cc} .

The stability of this fixed point can be investigated by linearizing equations (15)–(18) around the fixed point and studying how the fluctuations around the fixed point evolve in time. If the amplitude of the fluctuations decreases in time, the fixed point is a stable fixed point or attractor. Linearizing equations (15)–(18) around $(\Omega_{dc}, \Omega_{cc}, 0, 0)$, we get

$$\delta\Omega'_d = \left(\frac{\Omega_{dc}^{1/2}}{c} - \Omega_{dc} - 3\frac{\Omega_{dc}^{3/2}}{c} - 3\lambda_d\Omega_{dc} \right) \delta\Omega_d - 3\lambda_c\Omega_{dc}\delta\Omega_c + \Omega_{dc}\delta\Omega_r, \quad (21)$$

$$\delta\Omega'_c = \left(-1 - 2\frac{\Omega_{dc}^{1/2}}{c} + 3\lambda_c\Omega_{dc} \right) \delta\Omega_c + \left(3\lambda_d - \Omega_{cc} - 3\Omega_{cc}\frac{\Omega_{dc}^{1/2}}{c} - 3\lambda_d\Omega_{cc} \right) \delta\Omega_d + \Omega_{cc}\delta\Omega_r, \quad (22)$$

$$\delta\Omega'_r = -2 \left(1 + \frac{\Omega_{dc}^{1/2}}{c} \right) \delta\Omega_r, \quad (23)$$

$$\delta\Omega'_b = - \left(1 + 2 \frac{\Omega_{dc}^{1/2}}{c} \right) \delta\Omega_b, \quad (24)$$

where $\delta\Omega_\alpha = \Omega_\alpha - \Omega_{ac}$ denote fluctuations around the fixed point. The above linear equations can be written as

$$\delta\Omega'_\alpha = M_{\alpha\beta} \delta\Omega_\beta, \quad (25)$$

where α and β run over the four species. The eigenvalues of the matrix M govern how the amplitude of the fluctuations around the fixed point changes with time. The fixed point is a stable, saddle or unstable point if all the eigenvalues are negative, some of the eigenvalues are positive or all the eigenvalues are positive, respectively (since we are dealing with real eigenvalues). The eigenvalues of the matrix M are

$$\begin{aligned} \lambda_1 &= \frac{\Omega_{dc}^{1/2}}{c} - (1 + 3\lambda_d - 3\lambda_c)\Omega_{dc} - 3\frac{\Omega_{dc}^{3/2}}{c}, \\ \lambda_2 = \lambda_4 &= -1 - 2\frac{\Omega_{dc}^{1/2}}{c}, \quad \lambda_3 = -2 \left(1 + \frac{\Omega_{dc}^{1/2}}{c} \right). \end{aligned} \quad (26)$$

Hence, the fixed point $(\Omega_{dc}, \Omega_{cc}, 0, 0)$ is a stable point when $\lambda_1 < 0$ and a saddle point when $\lambda_1 > 0$. Using equation (19), λ_1 can be written as

$$\lambda_1 = \frac{3(\lambda_d\Omega_{dc} + \lambda_c\Omega_{cc})}{\Omega_{cc}} \left(\frac{1}{2} - \Omega_{dc} \right) - \frac{1}{2} - \frac{\Omega_{dc}^{3/2}}{c} - 3(\lambda_d - \lambda_c)\Omega_{dc}. \quad (27)$$

It is not easy to determine the sign of λ_1 in general. Thus, we will determine it in particular cases in the next section.

3. The attractor of cosmic evolution

We now consider the fixed point and stability of the cosmic evolution around the present epoch. For simplicity, we first consider the case where $\lambda_d = \lambda_c = b^2$. In this case, equation (19) becomes

$$1 + 2\frac{\Omega_{dc}^{1/2}}{c} = \frac{3b^2}{\Omega_{cc}}. \quad (28)$$

Since $c > 0$, equation (28) implies that $b^2 \geq \Omega_{cc}/3$. This is the lower limit of b^2 for the existence of the fixed point. From equations (12) and (28), it is easy to show that

$$w_d = -\frac{b^2}{\Omega_{cc}\Omega_{dc}}. \quad (29)$$

The equation of state parameter of the universe is defined as

$$w = \frac{p_{\text{total}}}{\rho_{\text{total}}} = \sum_{\alpha=r,b,m,d} w_\alpha \Omega_\alpha, \quad (30)$$

where $w_\alpha = p_\alpha/\rho_\alpha$. Since $w_b = w_c = 0$ and Ω_r can be neglected at late time, we have $w = w_d\Omega_d$ and therefore

$$w = -\frac{b^2}{\Omega_{cc}}. \quad (31)$$

From the lower bound of b^2 , we get $w \leq 1/3$. This means that the fixed point corresponds to cosmic acceleration. Moreover, one can see that the universe will be in the phantom phase, i.e., $w < -1$, if $b^2 > \Omega_{cc}$ or equivalently $\Omega_{dc}^{1/2} > c$. Since the observations seem to indicate that $w_d < -1$ today, we find the value of b^2 that makes $w_d < -1$ at the fixed point. It follows from equation (29) that $w_d < -1$ if $b^2 > \Omega_{cc}\Omega_{dc}$. We now check the stability of the fixed point. In this case, equation (27) becomes

$$\lambda_1 = \frac{3b^2}{\Omega_{cc}} \left(\frac{1}{2} - \Omega_{dc} \right) - \frac{1}{2} - \frac{\Omega_{dc}^{3/2}}{c}. \quad (32)$$

It can be seen that it is not easy to find a point at which λ_1 changes sign. However, λ_1 is negative for any b^2 or c if $\Omega_{dc} > 1/2$, i.e. $\Omega_{dc} > \Omega_{cc}$. This means that the fixed point at which $\Omega_{dc} > \Omega_{cc}$ is a stable fixed point or attractor, since the fixed point will occur when Q is positive and $\Omega_r = \Omega_b = 0$. At the present epoch Ω_r is small and can be neglected, and with positive Q the ratio Ω_b/Ω_c decreases with time, so the fixed point $(\Omega_{dc}, \Omega_{cc}, 0, 0)$ will be reached in the future. From equations (15)–(18), one can see that if b^2 is large, Ω_b can decrease quickly with time compared with Ω_c . As a result, the fixed point can be reached quickly near the present epoch. However, the CDM dominated epoch will disappear and the baryon fraction will be larger than the CDM fraction in the last scattering epoch due to the rapid decrease of ρ_b/ρ_c with time. This is excluded by the observed peak height ratio of the CMB power spectrum. Thus, on the basis of observations, the fixed point cannot be reached near the present for this case. We will see in the next section that small b^2 or equivalently small Ω_{cc} is required by observations. Substituting $\Omega_{cc} = 1 - \Omega_{dc}$ into equation (28), we get

$$1 - 3b^2 + 2\frac{\Omega_{dc}^{1/2}}{c} - \Omega_{dc} - 2\frac{\Omega_{dc}^{3/2}}{c} = 0. \quad (33)$$

The above third-degree polynomial for $\Omega_{dc}^{1/2}$ will have one positive real root if $1 > 3b^2$. This root is the previously considered fixed point. In contrast, if $1 \leq 3b^2$ the above equation will have one additional real root. This root gives another fixed point at smaller Ω_{dc} . It is not hard to show that this fixed point is not stable and we will not consider it here.

Let us now consider the case where $\lambda_d \neq \lambda_c$. In this case, equation (19) becomes

$$1 + 2\frac{\Omega_{dc}^{1/2}}{c} = \frac{3}{\Omega_{cc}} (\lambda_d\Omega_{dc} + \lambda_c\Omega_{cc}). \quad (34)$$

Hence, the fixed point occurs when $\lambda_d\Omega_{dc} + \lambda_c\Omega_{cc} > \Omega_{cc}/3$. Using equations (12) and (34), we get

$$w_d = -\frac{\lambda_d\Omega_{dc} + \lambda_c\Omega_{cc}}{\Omega_{cc}\Omega_{dc}}, \quad (35)$$

and therefore

$$w = -\frac{\lambda_d \Omega_{dc} + \lambda_c \Omega_{cc}}{\Omega_{cc}}. \quad (36)$$

Like for the case when $\lambda_d = \lambda_c$, one can show that the fixed point occurs when $w_d < -1/(3\Omega_{dc})$ or $w < -1/3$. Since the terms $\lambda_d \Omega_{dc}$ and $\lambda_c \Omega_{cc}$ from the interaction dominate at different times, the parameters λ_d and λ_c can be chosen such that the baryon fraction is smaller than the CDM fraction during the last scattering epoch although Ω_{cc} need not be very small. Thus, in this case, the attractor can be reached faster than for the case of $\lambda_c = \lambda_d$. From equation (27), we see that the fixed point is the stable fixed point when $\Omega_{dc} > \Omega_{cc}$ and λ_c is not much larger than λ_d .

4. The observational constraints

We now check the range of parameters for which the cosmic evolution has an attractor for compatibility with observations. We constrain the parameters of the interacting holographic dark energy using the latest observational data from SNIa [16] combined with the CMB shift parameter derived from three-year WMAP [17] and the baryon acoustic oscillations (BAO) from SDSS LRG [18].

The SNIa observations measure the apparent magnitude m of a supernova and its redshift z . The apparent magnitude m is related to the distance modulus μ and luminosity distance d_L of the supernova by

$$\mu(z) = m(z) - M = 5 \log_{10}(d_L(z)/\text{Mpc}) + 25, \quad (37)$$

where M is the absolute magnitude of the supernova. For flat spacetime, the luminosity distance is given by

$$d_L(z) = H_0^{-1}(1+z) \int_0^z \frac{d\tilde{z}}{E(\tilde{z})}. \quad (38)$$

Here, H_0 is the present value of the Hubble parameter and $E(z) = H(z)/H_0$. To constrain the holographic dark energy model, we perform a χ^2 fit for the parameters $\lambda_d, \lambda_c, c, \Omega_{c0}, \Omega_{b0}$, where the subscript 0 denotes the present value. For Ω_{r0} , we compute its value from the CMB and neutrino temperatures. Using equation (2), one can compute Ω_{d0} from Ω_{c0}, Ω_{b0} and Ω_{r0} . We have to include radiation in our consideration because it should not be neglected when we compute the CMB shift parameter. For the SNIa data, the parameter H_0 is the nuisance parameter which needs to be marginalized out. Since the fit of holographic dark energy to SNIa data is sensitive to H_0 [19], we have to add constraints from other observations to improve the fit. For this reason, we include the constraints from the CMB shift parameter and BAO measurements in our analysis.

The CMB shift parameter is a quantity derived from CMB data that has been shown to be model independent, so it can be used to constrain cosmological models. The CMB shift parameter is defined as [20]

$$R = \Omega_{m0}^{1/2} \int_0^{z_{\text{CMB}}} \frac{d\tilde{z}}{E(\tilde{z})}, \quad (39)$$

where $\Omega_{m0} = \Omega_{c0} + \Omega_{b0}$ and $z_{\text{CMB}} = 1089$ is the redshift at recombination. The estimated value of R from three-year WMAP data is 1.70 ± 0.03 . From the SDSS data, the

measurement of the BAO peak in the distribution of SDSS LRG can be used to derive the model independent parameter A which is defined as [18]

$$A = \Omega_{m0}^{1/2} E(z_{\text{BAO}})^{-1/3} \left[\frac{1}{z_{\text{BAO}}} \int_0^{z_{\text{BAO}}} \frac{d\tilde{z}}{E(\tilde{z})} \right]^{2/3}, \quad (40)$$

where $z_{\text{BAO}} = 0.35$. The estimated A is $A = 0.469(n_s/0.98)^{-0.35} \pm 0.017$. According to the three-year WMAP data, the scalar spectral index is chosen to be $n_s = 0.95$.

We first consider the case where $\lambda_d = \lambda_c = b^2$. For simplicity, we start by neglecting baryons in our consideration. Hence the attractor becomes $(\Omega_d, \Omega_c, \Omega_r) = (\Omega_{dc}, \Omega_{cc}, 0)$, where Ω_{dc} and Ω_{cc} satisfy equation (28). The region of b^2 and Ω_{cc} for which the fixed point exists and the 99.7% confidence regions from the combined constraints are shown in figure 1. From this figure, one sees that a small b^2 is required by observations, so that the attractor that satisfies the observational constraints occurs at low Ω_{cc} . This implies that the attractor cannot occur at present, where $\Omega_c \approx 0.3$. In the future, Ω_c can become small, so the fixed point can exist. Nevertheless, if the attractor is reached in the far future, the cosmic coincidence may not be alleviated because the present values of Ω_c and w_d may not satisfy the observational constraints. In the following consideration, we will study how the ratio r evolves during the present epoch to check the possibility of alleviating the cosmic coincidence. Since the physical fixed point cannot occur if $b^2 < 0$, we perform another fit by supposing that $b^2 \geq 0$. The result is shown in figure 1. It can be seen that the above conclusions for the case of arbitrary b^2 are also valid for the case of $b^2 \geq 0$.

The situation changes a bit when we include baryons in our consideration. From the previous section, we know that the cosmic evolution reaches the attractor at $(\Omega_d, \Omega_c, \Omega_b, \Omega_r) = (\Omega_{dc}, \Omega_{cc}, 0, 0)$. Since the present value of Ω_b does not vanish, this attractor cannot occur at present. However, the attractor can occur in the future because the ratio Ω_b/Ω_c decreases with time due to the positive Q . It can be seen that the ratio Ω_b/Ω_c decreases faster when b^2 increases. According to the observational constraints, a small b^2 is also required in this case, so that the attractor is slowly reached in the future. In order to perform a fit for this case, we use a prior $\Omega_{b0} = 0.047 \pm 0.006$ from the one-year WMAP data [21] because Ω_{b0} cannot be constrained very well by using only SNIa data, the CMB shift parameter and BAO measurements. It can be seen from figure 1 that the shape of the confidence contours does not change much when we include non-interacting baryons in our consideration. Obviously, the contours move to the left when the amount of Ω_{b0} increases. This is because the amount of Ω_{c0} decreases.

We now consider the case of arbitrary λ_d and λ_c . We also perform a χ^2 fit for the case with baryons and without baryons. For the case where baryons are neglected, it follows from figure 2 that the attractor can occur at present ($\Omega_{cc} \approx 0.3$) for a narrow range of λ_d and λ_c . Nevertheless, a small Ω_{cc} is required by observations if we restrict λ_d and λ_c to being positive. This implies that the attractor must occur in the future. Similarly to the case for $\lambda_d = \lambda_c = b^2$, the attractor cannot be reached at present if baryons are included in the consideration. Nevertheless, for suitable values of λ_c and λ_d which satisfy the observational constraints, the attractor in this case can be reached faster than for the case of $\lambda_c = \lambda_d$.

In order to check whether the cosmic coincidence can be alleviated for this form of interaction, we study the evolution of r during the present epoch. Since the evolutions

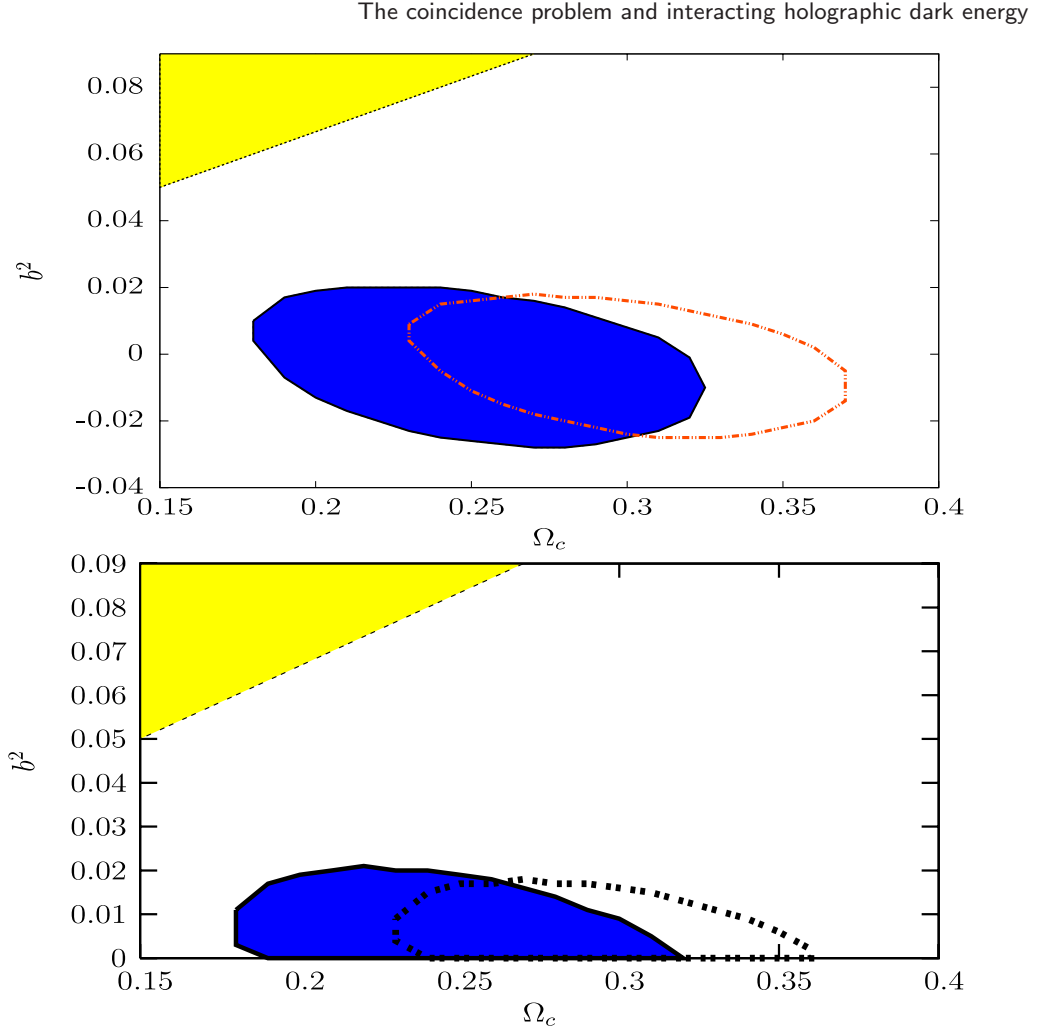


Figure 1. A region of parameters b^2 and Ω_c in which the cosmic evolution has a late time attractor (the yellow regions above the dashed lines), and the 99.7% confidence levels from the combined analysis of SNIa data, the CMB shift parameter and BAO measurements. For the yellow region, Ω_c refers to Ω_{cc} , but Ω_c refers to Ω_{c0} for the confidence levels. The upper panel shows the case of arbitrary b^2 , while the lower panel shows the case where $b^2 \geq 0$. The blue regions represent the confidence regions for the case that includes baryons and prior $\Omega_{b0} = 0.047 \pm 0.006$, while the thick dotted lines represent the confidence levels for the case where baryons are neglected.

of r for the cases with baryons and without baryons have nearly the same features, we consider only the latter case. We first consider the case where $b^2 = \lambda_c = \lambda_d$. Setting b^2 , c and Ω_{c0} equal to their best fit values, i.e. $b^2 = -0.004$, $c = 0.84$ and $\Omega_{c0} = 0.3$, the evolutions of r and $r' = \dot{r}/(rH)$ are plotted in figure 3. From figure 3, we see that $|r'| > 1$ during the present epoch because Ω_{c0} is quite different from Ω_{cc} . Due to a negative b^2 , r and Ω_c become negative and consequently reach the attractor at late time. Recall that $\Omega_c > 0$ at the attractor if $b^2 \geq \Omega_{cc}/3$. To solve the coincidence problem, the ratio r should vary slowly during the present epoch such that $|r'| \lesssim 1$ today [4]. The present value of $|r'|$ will decrease if the value of Ω_{cc} gets closer to the value of Ω_{c0} , i.e. the cosmic evolution

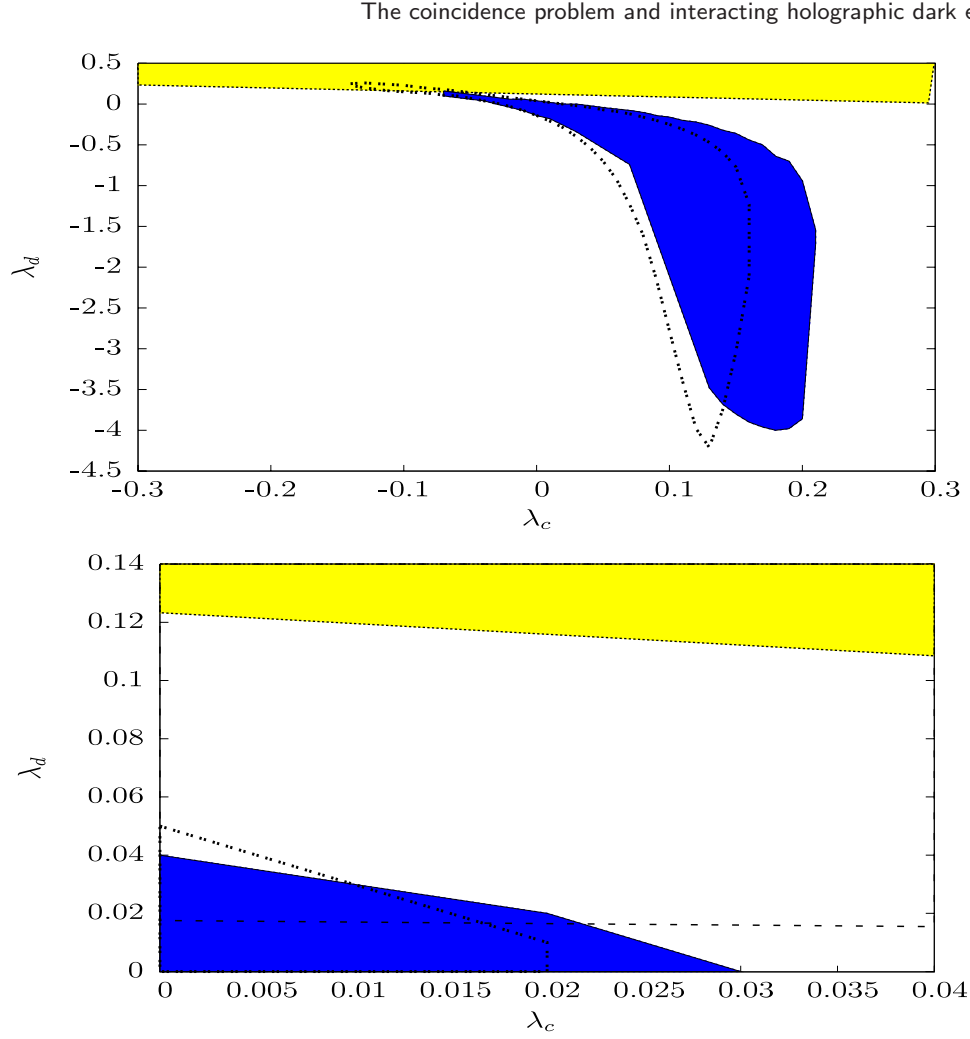


Figure 2. A region of parameters λ_d and λ_c for which the cosmic evolution has a late time attractor (the yellow regions above the dashed lines), and the 99.7% confidence levels from the combined analysis of SNIa data, the CMB shift parameter and BAO measurements. The upper panel shows the case for arbitrary λ_d and λ_c , while the lower panel shows the case where $\lambda_d, \lambda_c \geq 0$. The blue regions represent the confidence regions for the case that includes baryons and prior $\Omega_{b0} = 0.047 \pm 0.006$, while the thick dotted lines represent the confidence levels for the case where baryons are neglected. The dashed lines are plotted by setting $\Omega_{cc} = 0.27$, but the thick dashed line in the lower panel is plotted by setting $\Omega_{cc} = 0.05$. We note that the region above the thick dashed line also represents the region for which the cosmic evolution has an attractor.

reaches the attractor near the present. According to the yellow region in figure 1, the value of Ω_{cc} will increase and get closer to Ω_{c0} if b^2 increases. However, $|r'|$ during the present epoch will be smaller than 1 only when b^2 is larger than the observational bound. The evolution of r for the case where Ω_{cc} is close to Ω_{c0} is shown in figure 3. In this case, the values of c and Ω_{c0} are the best fit values, while the value of b^2 is larger than the observational bound but is inside the yellow region in figure 1. From figure 3 we see that $|r'| < 1$ during the present epoch in this case but the amount of Ω_c during the early

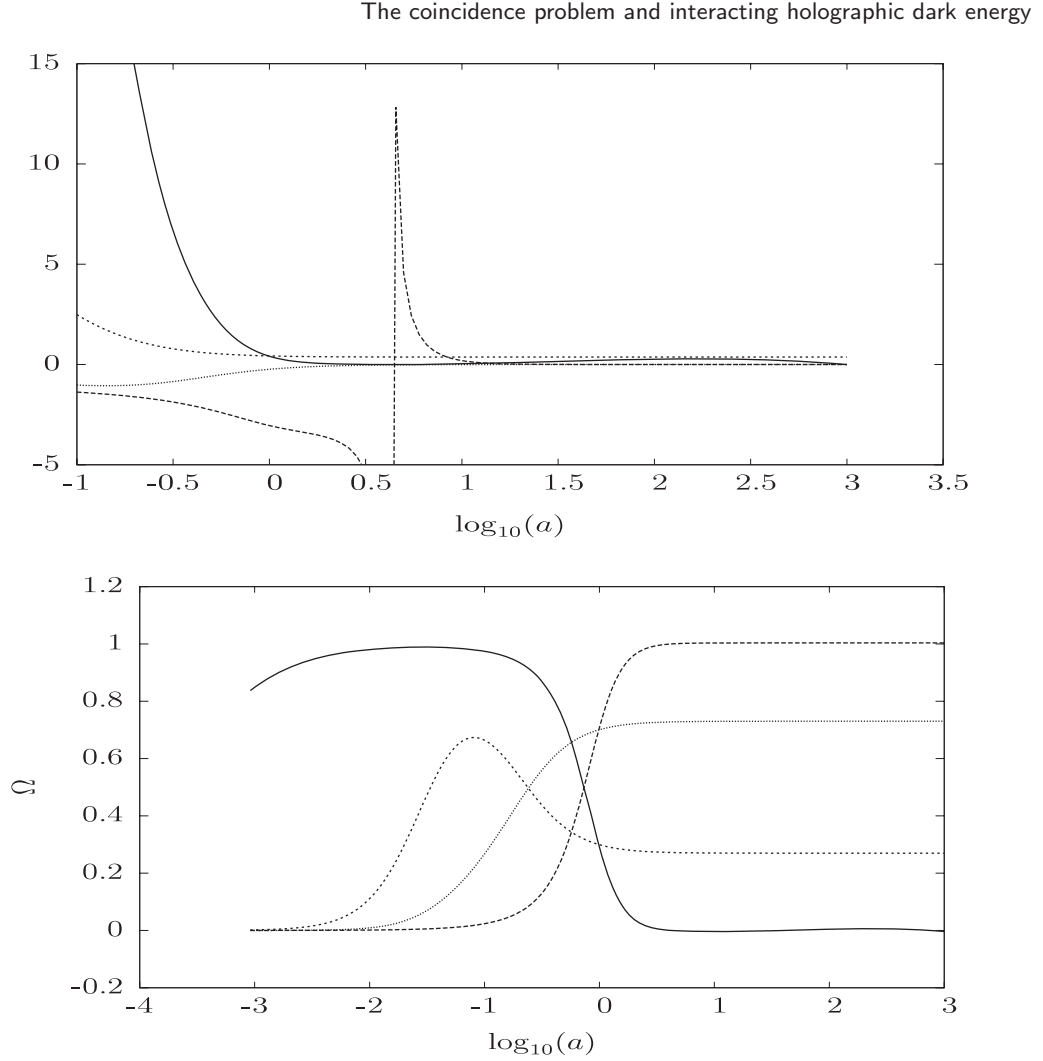


Figure 3. The upper panel shows the evolutions of r (solid and dashed lines) and $\dot{r}/(rH)$ (long dashed and dotted lines), while the lower panel shows the evolutions of Ω_c (solid and dashed lines) and Ω_d (long dashed and dotted lines). The solid and long dashed lines correspond to the case where the values of b^2 , c and Ω_{c0} are equal to their best fit values, while the dashed and dotted lines correspond to the case where b^2 is chosen such that Ω_{cc} is close to Ω_{c0} , i.e. $\Omega_{cc} = 0.27$.

time is too small. For this reason, this case does not satisfy the observational constraints. Hence, in the case of $\lambda_c = \lambda_d$, the coincidence problem is not really alleviated for this form of interaction terms.

Next, we consider the case where $\lambda_c \neq \lambda_d$. We first set c , λ_c , λ_d and Ω_{c0} equal to their best fit values, i.e. $c = 0.37$, $\lambda_c = 0.08$, $\lambda_d = -0.45$ and $\Omega_{c0} = 0.27$. This choice of the parameter value is outside the yellow region in figure 2. Since the interaction term Q is dominated by $3H\lambda_d\Omega_d$ during the present epoch, Q becomes negative at present. Hence, r and also Ω_c become negative and consequently reach the attractor at late time. Of course, the attractor in this case does not correspond to the physical attractor. The evolutions of r and Ω_c are shown in figure 4. Similarly to the case for $\lambda_c = \lambda_d$, the present value of $|r'|$ for this choice of parameter values is larger than 1 because Ω_{cc} is quite different from Ω_{c0} .

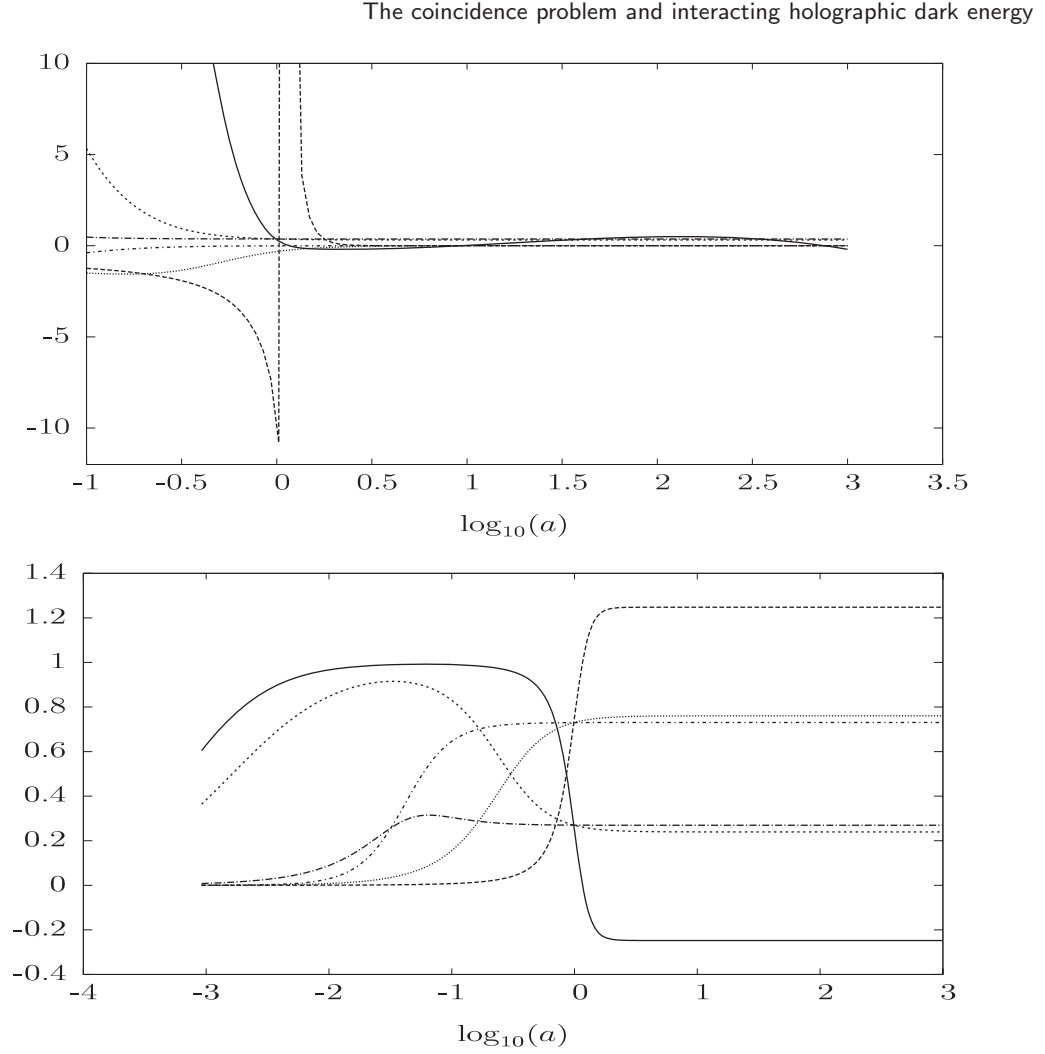


Figure 4. The upper panel shows the evolutions of r (solid, dashed and long dashed-dotted lines) and $\dot{r}/(rH)$ (long dashed, dotted and dashed-dotted lines), while the lower panel shows the evolutions of Ω_c (solid, dashed and long dashed-dotted lines) and Ω_d (long dashed, dotted and dashed-dotted lines). The solid and long dashed lines correspond to the case where the values of λ_c , λ_d , c and Ω_{c0} are equal to their best fit values, the dashed and dotted lines correspond to the case where c , λ_c and λ_d are chosen such that Ω_{cc} is close to Ω_{c0} , i.e. $\Omega_{cc} = 0.24$, and the long dashed-dotted and dashed-dotted lines correspond to the case where c , λ_c and λ_d are chosen such that $\Omega_{cc} = \Omega_{c0}$.

We now keep Ω_{c0} fixed and choose the new values of parameters c , λ_c and λ_d such that the observational constraints are satisfied and the cosmic attractor occurs at $\Omega_{cc} = 0.24$ near the present epoch. This choice of the parameter values is inside the intersection between the yellow region and the confidence region in figure 2. From figure 4, we see that the present value of $|r'|$ is smaller than 1 and the attractor is reached near the present epoch for this choice of the parameter values. On the basis of the soft coincidence idea, the coincidence problem can be alleviated in this case. The coincidence problem is better alleviated if $\Omega_{cc} = \Omega_{c0}$ or equivalently if the cosmic attractor is reached at the present.

Unfortunately, the parameter values that make $\Omega_{cc} = \Omega_{c0}$ do not satisfy the observational constraints. The evolutions of r and Ω_c for this case are shown in figure 4. From the figure, it is clear that this case is ruled out by observations. We note that the present value of $|r'|$ will increase if we include baryons in the above consideration.

Finally, we estimate whether $|r'|$ can be smaller than 1 at present if we constrain the parameters of dark energy using the SDSS matter power spectrum. Here, we will not perform a complete fit for this data set because the nature of the perturbations in holographic dark energy is not yet completely understood. To write down the evolution equations for the density perturbation and compute the matter power spectrum for this interacting dark energy model, we use the assumption below. From the results in [22], we suppose that the perturbation in holographic dark energy can be neglected when $kR_e/a \gg 1$, where k is the wavenumber of the perturbation modes. Using equations (1) and (6) and supposing that radiation can be neglected during matter domination, one can show that $H^{-1}/R_e = \sqrt{\Omega_d}/c$, i.e. the Hubble radius is smaller than the event horizon if $\sqrt{\Omega_d} < c$. Since we use the data from SDSS which measures the matter power spectrum on scales smaller than the Hubble radius, we neglect the perturbation in holographic dark energy in the calculation of the matter power spectrum. We write the perturbed interaction term using the formulae in [23, 24], so that the evolution equations for the perturbation in CDM are

$$\begin{aligned} \frac{d\Delta_c}{d\eta} &= -kV_c + 3\mathcal{H}\Psi \left(\lambda_c + \lambda_d \frac{\rho_d}{\rho_c} \right) - 3 \frac{d\Phi}{d\eta} \left(\lambda_c + \lambda_d \frac{\rho_d}{\rho_c} \right) - 3\mathcal{H}\lambda_d \Delta_c \frac{\rho_d}{\rho_c}, \\ \frac{dV_c}{d\eta} &= -\mathcal{H}V_c + k\Psi, \end{aligned} \quad (41)$$

where Δ_c and V_c are the gauge-invariant density contrast and velocity perturbation of CDM, Ψ and Φ are the metric perturbations, $\mathcal{H} = a^{-1}(da/d\eta)$ and η is the conformal time. We solve the above equations, compute the matter power spectrum and compare the matter power spectrum obtained with SDSS data using CMBEASY [25]. By checking the value of χ^2 , we have found that the best fit parameters for this data set are different from the best fit parameters from the observational constraints on the homogeneous universe. Instead of searching for the best fit parameters for this data set, we roughly check the viability of the parameters by comparing the matter power spectrum of the models considered with the matter power spectrum for the Λ CDM model whose parameters are taken from the best fit value for three-year WMAP and SDSS data [26]. In figure 5, we plot the fractional difference in matter power spectrum between the interacting holographic dark energy and Λ CDM models. From this figure, we see that for suitable ranges of dark energy parameters, $|r'|$ can be smaller than 1 at present and the difference in matter power spectrum can be smaller than the error for the matter power spectrum of SDSS data. This implies that the alleviation of the coincidence problem by this interacting dark energy model is not excluded by SDSS data.

5. Conclusions

For the interacting holographic dark energy model, we study the fixed points and their stability, and compare a range of model parameters for which an attractor exists with the 99.7% confidence levels from the combined analysis of SNIa data, the CMB shift

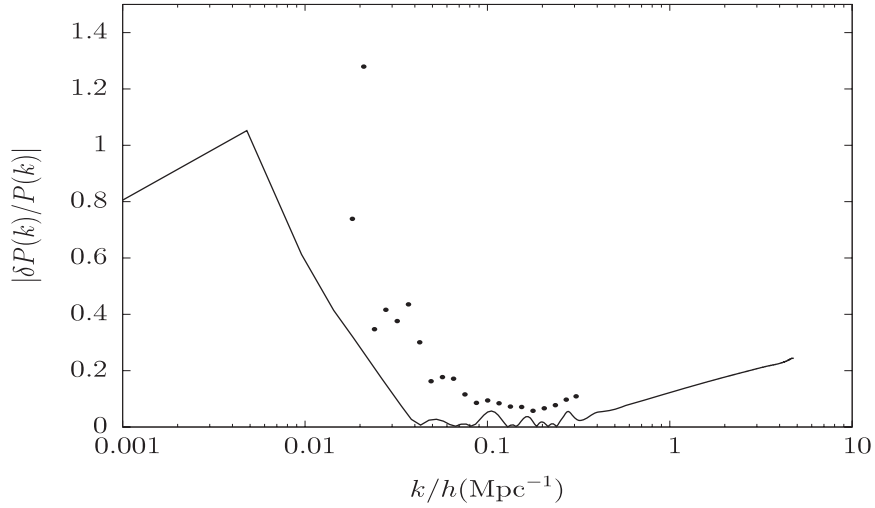


Figure 5. The fractional difference in matter power spectrum $|\delta P(k)/P(k)|$ between the interacting holographic dark energy and Λ CDM models. For this plot, the parameters of dark energy are chosen such that $|r'| \approx 0.9$ at present. The fractional error for the matter power spectrum $|\delta P(k)/P(k)| = \text{error}/\text{spectrum}$ of SDSS data is represented by dots.

parameter and BAO measurements. Neglecting baryons, the observational constraints require that the value of Ω_c at the attractor point must be small if $\lambda_d = \lambda_c$ or $\lambda_d, \lambda_c > 0$. This implies that the cosmic evolution will reach the attractor point in the future when Ω_c becomes small. In this case, r cannot be slowly varying during the present epoch and the cosmic attractor cannot be reached near the present. Hence, the coincidence problem is not really alleviated for this case. However, if λ_d and λ_c are allowed to be negative, the cosmic evolution can reach the attractor near the present epoch for a narrow range of λ_d and λ_c . Therefore, it is possible to alleviate the coincidence problem in this case. Including baryons in our consideration, the attractor of the cosmic evolution cannot occur at present due to the non-vanishing baryon fraction. According to observations, the fixed point in this case is possible only when Q is small and positive. Hence, the fixed point will be slowly reached in the future. These results indicate that for the interacting holographic dark energy model with the interaction terms considered here, the cosmic coincidence problem cannot be alleviated very well. We also briefly considered the constraint from the SDSS matter power spectrum on the dark energy parameters. We have found that the parameters ranges that lead to the alleviation of the cosmic coincidence are allowed by SDSS data.

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CMB constraints on spacetime noncommutativity in inflationary scenario

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Abstract.

We investigate the primordial power spectrum of the density perturbations based on the assumption that spacetime is noncommutative in the early stage of inflation, and constrain the contribution from spacetime noncommutativity on the CMB anisotropies. Due to the spacetime noncommutativity, the primordial power spectrum can lose rotational invariance. Using the power law k-inflation model, we show that the deviation from rotational invariance of the primordial power spectrum depends on the ratio L_s^4/c_S^2 , where L_s is the noncommutative length scale and c_S is the sound speed of inflaton. We compute the covariance matrix for the harmonic coefficients of the CMB anisotropies from this direction-dependent primordial power spectrum, and constrain the contributions from the spacetime noncommutativity on this covariance matrix using five-year WMAP CMB maps. We find that the upper bound for the ratio $L_s c_S^{1/(\beta-1)}/\beta^{1/2}$ is 1.4×10^{-28} cm at 99.7% confidence level. Taking the values of c_S and β to be known precisely, and with β being the inverse of the slow-roll parameter, the upper bound for L_s is estimated to be less than 10^{-27} cm at 99.7% confidence level.

Keywords: inflation.

1. Introduction

The inflationary cosmology [1, 2, 3] is the scenario of the very early universe. It provides a successful mechanism for generating nearly scale invariant primordial density perturbations, that give rise to galaxy formation and temperature anisotropies in the CMB which are in agreement with observation [4]. If the period of inflation is sufficiently longer than that required for solving the horizon and flatness problems, such that the wavelengths of perturbations which are observed today emerged from the Planck regime in the early stages of inflation, the physics on trans-Planckian scales should leave an imprint on the primordial density perturbations [5, 6]. Here, we consider the imprint of trans-Planckian physics based on noncommutative spacetime.

Near the Planck scale, the properties of spacetime are expected to be modified due to the quantum nature of gravity [7]. It has been shown that a consequence of string theory which is a promising candidate of quantum gravity, is that the spacetime is noncommutative [8]

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}(x), \quad (1)$$

where $\Theta^{\mu\nu}$ is an antisymmetric tensor.

The influences of spacetime noncommutativity on the feature of power spectrum of primordial fluctuations have been studied by many authors [9] - [16]. For the case where $\Theta^{ij} = 0$ but $\Theta^{0i} \neq 0$ [9, 10, 11], the contribution from spacetime noncommutativity can lead to the running of the spectral index of the primordial power spectrum. For the case where $\Theta^{ij} \neq 0$ but $\Theta^{0i} = 0$ [12, 13], the primordial power spectrum can become direction-dependent, and consequently the statistics of CMB fluctuations becomes anisotropic. We are interested in this spacetime noncommutativity induced statistical anisotropy. @@ Usually, the statistics of the CMB temperature fluctuations is supposed to be isotropic. Hence, if the non-Gaussianity of the CMB fluctuations is assumed to be negligible, the statistical properties of the CMB fluctuations will be completely described by the angular power spectrum [17]. However, recently there are many attempts to check whether the statistics of the CMB fluctuations is perfectly isotropic by searching for the statistical anisotropy contributions in the CMB sky maps [18, 19, 20]. In the case where the statistics of the CMB fluctuations is anisotropic, the angular power spectrum does not contain all the information about the statistical properties of the CMB fluctuations even when the Gaussianity of the CMB fluctuations is assumed. Some of the estimators for quantifying the statistical anisotropy contributions in the CMB fluctuations have been proposed in [21, 17, 22]. According to [18, 19], the statistics of the observed CMB fluctuations does not deviate from isotropy significantly.

In this work, we constrain the contributions from spacetime noncommutativity on CMB temperature fluctuations using five-year WMAP CMB maps. In the next section, we compute the primordial power spectrum of the power law k-inflation by taking the spacetime noncommutativity of the form $\Theta^{ij} \neq 0$ and $\Theta^{0i} = 0$. In section 3, we compute the covariance matrix for the harmonic coefficients of the CMB temperature fluctuations

$\langle a_{lm}^* a_{l'm'} \rangle$, and constrain the contributions from spacetime noncommutativity using CMB maps. Finally, we conclude in section 4.

2. The contributions from spacetime noncommutativity

We now investigate the contributions from spacetime noncommutativity on the primordial power spectrum of the k-inflaton whose action is given by [23]

$$S = \int d^4x \sqrt{-g} \left[\frac{\bar{m}_p^2 R}{2} + P(X, \phi) \right], \quad (2)$$

where $\bar{m}_p = (8\pi G)^{-1/2}$ is the reduced Planck mass, ϕ is the inflaton field and $X = (1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. The effect of spacetime noncommutativity can be taken into account in the action (2) by replacing the ordinary products in the action with the star products. In curved spacetime, the star product can be expanded as [12]

$$f \star g \equiv \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{i}{2} \right)^k \Theta^{\mu_1\nu_1} \dots \Theta^{\mu_k\nu_k} (D_{\mu_1} \dots D_{\mu_k} f) (D_{\nu_1} \dots D_{\nu_k} g), \quad (3)$$

where D_μ is the covariant derivative. In our consideration, we suppose that the nonzero components of $\Theta^{\mu\nu}$ are $\Theta^{12} = -\Theta^{21} = 1/(\Lambda^2 a^2)$, where a is the cosmic scale factor and $\Lambda^{-1} = L_s$ is the noncommutative length scale [12].

To study the evolution of density perturbations during inflation, one splits the metric and the inflaton field in the action into the background and the perturbed parts, e.g., for the inflaton field we write $\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$. The action for the perturbed field can be obtained by expanding the action (2) around the background. In our consideration the effect of spacetime noncommutativity can be incorporated by just replacing the ordinary product between perturbed variables in the action with the star product. Since we consider the case where $\Theta^{0i} = 0$, it follows from eq. (3) that the star product of the background quantity with any field variable is just the ordinary product between functions.

Instead of directly expanding the action (2) around the background, we start from the second order perturbed action for k-inflaton [24, 25],

$$\delta S^{(2)} = \frac{1}{2} \int d^4x a z^2 \left[\dot{\zeta}^2 + c_s^2 \partial_i \zeta \partial^i \zeta \right], \quad (4)$$

where the dot denotes derivative with respect to time, ζ is the curvature perturbation in the comoving gauge, $z = a(P + \rho)^{1/2}/(c_s H)$, $H = \dot{a}/a$ is the Hubble parameter and c_s is the sound speed, defined by $c_s^2 = P_{,X}/\rho_{,X}$. Here, the subscript $,X$ denotes a derivative with respect to X and ρ is the energy density of the inflaton. Using the relation $(H/\dot{\phi}_0)\delta\phi = \alpha\delta\phi = \zeta$ between the scalar field perturbation in the uniform curvature gauge and the curvature perturbation in the comoving gauge, the action for the perturbed inflaton field in the uniform curvature gauge can be written as

$$S = \frac{1}{2} \int d^4x a z^2 \alpha^2 \left[\dot{\delta\phi}^2 + c_s^2 \partial_i \delta\phi \partial^i \delta\phi - \left[\left(\frac{\dot{a}}{a} + 2\frac{\dot{z}}{z} \right) \frac{\dot{\alpha}}{\alpha} + \frac{\ddot{\alpha}}{\alpha} \right] \delta\phi^2 \right]. \quad (5)$$

Replacing the products between the perturbed field variables with the star products, the above action becomes

$$S = S_0 + \delta S_K + \delta S_V + \mathcal{O}(\Theta^4), \quad (6)$$

where δS_K and δS_V represent the lowest nonzero corrections ($\mathcal{O}(\Theta^2)$) to the kinetic and potential terms in S respectively. We follow the procedure in [12] to compute the star products in the action (5). The correction to the potential and kinetic terms are then given by

$$\delta S_V = -\frac{m^2}{16} \int d^4x \frac{az^2\alpha^2}{\Lambda^4} H^2 (\partial_1 \delta\phi \partial^1 \delta\phi + \partial_2 \delta\phi \partial^2 \delta\phi), \quad (7)$$

where $m^2 = (H + 2\dot{z}/z)(\dot{\alpha}/\alpha) + \ddot{\alpha}/\alpha$,

$$\delta S_K = \frac{1}{16} \int d^4x \frac{az^2\alpha^2}{\Lambda^4} H^2 \left[\partial_p \partial_0 \delta\phi \partial^p \partial^0 \delta\phi + c_S^2 \partial_p \partial_i \delta\phi \partial^p \partial^i \delta\phi - 2H \partial^p \delta\phi \partial_0 \partial_p \delta\phi + H^2 \partial_p \delta\phi \partial^p \delta\phi \right], \quad (8)$$

where $p = 1, 2$. Expressing the action (6) in terms of the new variable $v = z\zeta = z\alpha\delta\phi$ and expanding v in Fourier space as

$$v(\mathbf{x}, t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} (\mathbf{a}_{\mathbf{k}} v_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.), \quad (9)$$

where $\mathbf{a}_{\mathbf{k}}, \mathbf{a}_{\mathbf{k}}^\dagger$ satisfy the canonical commutation relations, the action (6), evaluated in the vacuum after normal ordering, becomes

$$S = \int d\eta d^3\mathbf{k} \left[|v'_{\mathbf{k}}|^2 + \left(\frac{z''}{z} - c_S^2 k^2 \right) |v_{\mathbf{k}}|^2 - \frac{H^2 k_\perp^2}{8\Lambda^4 a^2} (|v'_{\mathbf{k}}|^2 - c_S^2 k^2 |v_{\mathbf{k}}|^2 + \left(\left(\frac{a''}{a} - 2(Ha)^2 \right) \left(5 + \frac{\dot{\rho}_{,X}}{H\rho_{,X}} \right) + \frac{z''}{z} \right) |v_{\mathbf{k}}|^2 \right], \quad (10)$$

where the prime denotes derivative with respect to the conformal time $\eta = \int da/a$ and $k_\perp^2 = k_1^2 + k_2^2$. In the above action, we have used $\rho + P = 2Xc_S^2\rho_{,X}$. The expression for $\dot{\rho}_{,X}/(H\rho_{,X})$ in the above action can be simplified, once the model of k-inflation is specified. Hence, in the following consideration we will perform the calculation using a power law k-inflation model. For power law inflation, the scale factor evolves as

$$a \propto t^\beta, \quad (11)$$

where β is a constant parameter, so that $H = \beta/t$. From the Friedman equation, $H^2 = \rho/(3\bar{m}_p^2)$, one sees that the energy density of power law inflaton evolves as

$$\rho = 3\bar{m}_p^2 \beta^2 \frac{1}{t^2}. \quad (12)$$

It follows from the continuity equation, $\dot{\rho} = -3H(\rho + P)$ and eq. (12), that

$$\Gamma\rho = (\rho + P) = 2XP_{,X}, \quad (13)$$

where Γ is constant. Since X and c_S are constant for the power law k-inflation, one can show that

$$\frac{\dot{\rho}_{,X}}{H\rho_{,X}} = \frac{\dot{P}_{,X}}{HP_{,X}} = \frac{\dot{\rho}}{H\rho} = -\frac{2}{\beta}. \quad (14)$$

Since $(\rho + P)^{1/2}$ and H evolve much slower than a during inflation, we suppose that $z''/z \approx a''/a$. Using eq. (11), it can be shown that $a''/a = \beta(2\beta - 1)(1 - \beta)^{-2}/\eta^2$ and $Ha = \beta/(1 - \beta)\eta$. The action (10) can be written in terms of the variable

$$y_{\mathbf{k}}(\eta) = (1 - \tilde{\epsilon}^2(\eta)k_{\perp}^2)^{1/2}v_{\mathbf{k}}(\eta), \quad (15)$$

where $\tilde{\epsilon}^2 = H^2/(8\Lambda^4 a^2)$, up to second order in ϵ as

$$S = \int d\eta d^3\mathbf{k} \left[|y'_{\mathbf{k}}|^2 - \left(c_S^2 k_{\mathbf{k}}^2 - \frac{\beta(2\beta - 1)}{(1 - \beta)^2 \eta^2} + \epsilon^2 k_{\perp}^2 \left(1 - \frac{1}{\beta} + \frac{5}{\beta^2} \right) + \mathcal{O}(\epsilon^4) \right) |y_{\mathbf{k}}|^2 \right]. \quad (16)$$

Here, we have set $\epsilon^2 = \tilde{\epsilon}^2 \beta^2 / ((1 - \beta)^2 \eta^2) = H^4 / (8\Lambda^4)$, and have expanded $\epsilon^2 k_{\perp}^2 / (1 - \tilde{\epsilon}^2 k_{\perp}^2) \approx \epsilon^2 k_{\perp}^2 + \mathcal{O}(\epsilon^4)$. Let us now make some approximation. For the power law k-inflation model, the relation between the spectral index of the primordial density perturbation and β is $n_s - 1 = 2/(1 - \beta)$. Hence, using the best fit value of n_s from the five-year WMAP results, we get $\beta = 2/(1 - 0.96) + 1 \approx 51$ and $\epsilon^2 \propto H^4 \propto 1/\eta^{4/(1-\beta)} \propto \eta^{0.08}$. Since ϵ^2 evolves much slower than the third term on the RHS of the action (16), we take ϵ^2 to be approximately constant. Moreover, for this value of β , we have $1 - 1/\beta + 5/\beta^2 \approx 1.02$, so that we can set this term equal to one without significantly changing the final result. Using the above approximation, the evolution equation for $y_{\mathbf{k}}$ can be written in the form of the Bessel equation as

$$y''_{\mathbf{k}} + \left[c_S^2 k^2 \gamma^2 - \left(\nu^2 - \frac{1}{4} \right) \frac{1}{\eta^2} \right] y_{\mathbf{k}} = 0, \quad (17)$$

where $\nu = (3/2) + 1/(\beta - 1)$, $\gamma^2 = 1 + \epsilon^2 \sin^2(\theta)/c_S^2$ and $\sin^2(\theta) = k^2/k_{\perp}^2$ denotes the angle between the vectors $\mathbf{k}$ and $\hat{k}_3$. Following the standard procedure, the solution of eq. (17) is given by [26]

$$y_{\mathbf{k}} = \frac{\sqrt{\pi}}{2} e^{i(\nu + \frac{1}{2})\frac{\pi}{2}} \sqrt{-\eta} H_{\nu}^{(1)}(-c_S k \gamma \eta), \quad (18)$$

where $H_{\nu}^{(1)}$ is the Hankel function of the first kind. The argument of the Hankel function can be written as

$$-c_S k \gamma \eta \approx -c_S k \eta \left(1 + \frac{1}{2} \frac{\epsilon^2 \sin^2(\theta)}{c_S^2} \right) = \xi + \delta\xi, \quad (19)$$

where $\delta\xi$ denotes a small contribution from $\epsilon^2 \sin^2(\theta)/2c_S^2$. We expand the Hankel function around ξ as

$$\begin{aligned} H_{\nu}^{(1)}(-c_S k \gamma \eta) &\simeq H_{\nu}^{(1)}(\xi) + \frac{dH_{\nu}^{(1)}(\xi')}{d\xi'} \Big|_{\xi} \delta\xi \\ &= H_{\nu}^{(1)}(\xi) + \left(\nu H_{\nu}^{(1)}(\xi) - H_{\nu+1}^{(1)}(\xi) \xi \right) \frac{\delta\xi}{\xi}. \end{aligned} \quad (20)$$

Since we are interested in the long wavelength perturbation modes, we express the hankel function in the asymptotic form [26],

$$H_{\nu}^{(1)}(\xi \ll 1) \sim \sqrt{2/\pi} e^{-i\frac{\pi}{2}} 2^{\nu-\frac{3}{2}} \frac{\Gamma(\nu)}{\Gamma(3/2)} \xi^{-\nu}, \quad (21)$$

so that the curvature perturbation is given by

$$\zeta_{\mathbf{k}} = \frac{v_{\mathbf{k}}}{z} \simeq e^{i(\nu-\frac{1}{2})\frac{\pi}{2}} 2^{(\nu-\frac{3}{2})} \frac{\Gamma(\nu)}{\Gamma(3/2)} \times \frac{(-c_S k \eta)^{\frac{1}{2}-\nu}}{z(\eta) \sqrt{2c_S k}} \left(1 - \nu \frac{\epsilon^2 \sin^2(\theta)}{2c_S^2} \right), \quad (22)$$

and finally we obtain the primordial power spectrum

$$\begin{aligned} \mathcal{P}_{\mathbf{k}}^\zeta &= \left[2^{\frac{2}{\beta-1}} \left| \frac{\Gamma(\nu)}{\Gamma(3/2)} \right|^2 \left(\frac{\beta-1}{\beta} \right)^{\frac{2\beta}{\beta-1}} \right] \frac{GH_1^2}{\pi} c_S^{\frac{1+\beta}{1-\beta}} \times \\ &\quad \beta \left(\frac{k}{k_1} \right)^{\frac{2}{1-\beta}} \left(1 - \nu \frac{\epsilon^2 \sin^2(\theta)}{c_S^2} \right), \\ &= A_s(\beta, c_S) \left(\frac{k}{k_1} \right)^{\frac{2}{1-\beta}} \left(1 - \nu \frac{\epsilon^2 \sin^2(\theta)}{c_S^2} \right), \end{aligned} \quad (23)$$

where the subscript 1 denotes the evaluation at the time when the wavenumber k_1 crosses the horizon. It can be seen that the obtained power spectrum is direction-dependent due to the spacetime noncommutativity. The magnitude of the noncommutative contribution depends on the angle between the vectors $\mathbf{k}$ and k_3 , and also depends on the sound speed for the k-inflation. Neglecting the contributions from spacetime noncommutativity, the above power spectrum gives rise to the usual power spectrum as in [24].

3. The CMB constraints

In this section, we will constrain the contributions from spacetime noncommutativity on CMB anisotropies using the CMB data. It is well known that if the primordial power spectrum is direction-dependent, the statistics of CMB will become anisotropic, i.e., the two-point function of the temperature fluctuations is no longer rotationally invariant. Here, we consider only the two-point function because we assume the non-Gaussianity of the CMB fluctuations to be negligible. In addition to spacetime noncommutativity, the direction-dependent primordial power spectrum can also be a consequence of many phenomena in the early universe, for example see [27] - [28]. To compare theoretical prediction with the observation, it is convenient to expand the temperature anisotropy into spherical harmonics

$$\Delta T(\hat{n}) = T(\hat{n}) - T_0 = \sum_{l,m} a_{lm} Y_{lm}(\hat{n}), \quad (24)$$

where T_0 is the mean temperature of the CMB. If we write the direction-dependent primordial power spectrum as [27, 22]

$$\mathcal{P}(\mathbf{k}) = A(k) \left[1 + \sum_{LM} g_{LM} Y_{LM}(\hat{k}) \right], \quad (25)$$

the covariance matrix of a_{lm} will be

$$\langle a_{l_1 m_1} a_{l_2 m_2}^* \rangle = \delta_{l_1 l_2} \delta_{m_1 m_2} C_{l_1} + \sum_{LM} \Xi_{l_2 m_2 LM}^{l_1 m_1} D_{l_1 l_2}^{LM}. \quad (26)$$

Here, C_{l_1} is the CMB angular power spectrum, given by

$$C_{l_1} = (4\pi)^2 \int_0^\infty dk k^2 A(k) [T_{l_1}(k)]^2, \quad (27)$$

where T_{l_1} is the CMB transfer function which is taken to be rotationally invariant. It is known that if the two-point function of the temperature anisotropy is not rotationally invariant, the covariance matrix of a_{lm} will not be diagonal. The off diagonal elements of the covariance matrix appear in the second term of eq. (26) This term is given by

$$D_{l_1 l_2}^{LM} = (4\pi)^2 (-i)^{l_1 - l_2} \int_0^\infty dk k^2 A(k) g_{LM} T_{l_1}(k) T_{l_2}(k), \quad (28)$$

and

$$\Xi_{l_1 m_1 l_2 m_2}^{l_3 m_3} = \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)}{4\pi(2l_3 + 1)}} \mathfrak{C}_{l_1 0 l_2 0}^{l_3 0} \mathfrak{C}_{l_1 m_1 l_2 m_2}^{l_3 m_3}, \quad (29)$$

where $\mathfrak{C}_{l_1 m_1 l_2 m_2}^{l_3 m_3}$ are the Clebsch-Gordan coefficients. Comparing the power spectrum in eq. (25) with the one in eq. (23), we find that the non zero components of g_{LM} are

$$g_{00} = -\frac{4\sqrt{\pi}}{3} \frac{\nu \epsilon^2}{c_s^2} \quad \text{and} \quad g_{20} = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\nu \epsilon^2}{c_s^2}. \quad (30)$$

We see that the contributions from spacetime noncommutativity also influence the diagonal elements of the covariance matrix, i.e. they modify the amplitude of C_l . Substituting g_{LM} from eq. (30) into eq. (26), we obtain the covariance matrix

$$\begin{aligned} \langle a_{l_1 m_1} a_{l_2 m_2}^* \rangle \simeq C_{l_1} \Big\{ & \delta_{l_1 l_2} \delta_{m_1 m_2} \left[1 + \frac{\nu \epsilon^2}{c_s^2 (2l_1 + 1)} \times \right. \\ & \left. \left(\frac{(l_1 + 1)^2 - m_1^2}{2l_1 + 3} + \frac{l_1^2 - m_1^2}{2l_1 - 1} - 1 \right) \right] \\ & - \left(\delta_{l_1 - 2l_2} \delta_{m_1 m_2} \frac{\nu \epsilon^2 C_{l_1 l_2}}{c_s^2 (2l_1 - 1)} \times \right. \\ & \left. \left. \sqrt{\frac{((l_1 - 1)^2 - m_1^2)(l_1^2 - m_1^2)}{(2l_1 - 3)(2l_1 + 1)} + l_1 \leftrightarrow l_2} \right) \right\}, \end{aligned} \quad (31)$$

where $C_{l_1 l_2} = (4\pi)^2 \int_0^\infty dk k^2 A(k) T_{l_1}(k) T_{l_2}(k) / C_{l_1}$.

We next constrain the contributions from spacetime noncommutativity in this covariance matrix using the five-year WMAP foreground-reduced maps [30]. Since the spacetime noncommutativity also influences the diagonal elements of the covariance matrix and $\epsilon \propto H^2$, we use eq. (23) to write the contribution from spacetime noncommutativity as

$$\frac{3\epsilon^2}{2c_s^2} \simeq A_s^2 \Sigma, \quad (32)$$

where A_s is the amplitude of the primordial power spectrum and

$$\Sigma \simeq \frac{3}{16} \left(\frac{\pi}{G\beta} \right)^2 L_s^4 c_s^{\frac{4}{\beta-1}}. \quad (33)$$

In the above two equations, we have used $\beta \gg 1$. We adopt the procedures in [19] to compute the posterior probability of the parameters Σ and A_s , given the observed temperature anisotropies $\mathbf{a}$,

$$P(\bar{\sigma}|\mathbf{a}) \propto L(\mathbf{a}|\bar{\sigma})P(\bar{\sigma}), \quad (34)$$

where $\bar{\sigma} = \Sigma, A_s$ is the set of parameters, $L(\mathbf{a}|\bar{\sigma})$ is the likelihood and $P(\bar{\sigma})$ is the prior. Since the galactic contamination cannot be completely removed from some regions of the sky, one does not have the full-sky CMB maps with well-defined error properties. To reduce the galactic contamination, one masks the contaminated regions as

$$c_i = M_i \Delta T_i, \quad (35)$$

where i is the pixel index, c_i is the masked CMB map, ΔT_i is the full-sky map and M_i is a mask which is zero at the contaminated points and is one elsewhere. The above relation can be written in harmonic space as

$$c_{lm} = M_{lm,l'm'} b_{l'm'}, \quad (36)$$

where the matrix $M_{lm,l'm'}$ is given by

$$M_{lm,l'm'} = \sum_{LM} M_{LM} \Xi_{LMl'm'}^{lm}. \quad (37)$$

Here, c_{lm} , M_{lm} and b_{lm} are the spherical harmonic coefficients of c_i , M_i and ΔT_i respectively. Moreover, due to the instrument noise, the finite beams resolution and the discreteness of the temperature maps, the contributions to the unmasked CMB map come from the sum of the instrument noise with the convolution between the signal of the CMB anisotropies and the window function, such that

$$\Delta \mathcal{T} = W \mathbf{a} + \mathcal{N}, \quad (38)$$

where W is the window function and $\mathcal{N}$ is the instrument noise. We suppose that the contributions from the beam and the pixel asymmetries are negligible. Using eqs. (36) and (38), the covariance matrix of the masked temperature multipoles can be written as

$$C_{lm,l'm'} = \sum_{l_1 m_1 l_2 m_2} M_{lm,l_1 m_1} \left[W_{l_1} \langle a_{l_1 m_1} a_{l_2 m_2}^* \rangle W_{l_2} + N_{l_1 m_1, l_2 m_2} \right] M_{l_2 m_2, l' m'}^*, \quad (39)$$

where $N_{l_1 m_1, l_2 m_2}$ is the pixel noise covariance matrix, given by

$$N_{l_1 m_1, l_2 m_2} = \Delta a \sum_{LM} \Xi_{LMl_2 m_2}^{l_1 m_1} N_{LM}. \quad (40)$$

Here, Δa is the area of each pixel in the temperature map, and N_{LM} is defined as

$$N_{LM} = \sum_i \Delta a \frac{\sigma_0^2}{n_i^{\text{obs}}} Y_{LM}(\hat{r}_i), \quad (41)$$

where σ_0 is the rms noise of a single observation and n_i^{obs} is the number of observations of pixel i .

In order to compute the likelihood, the inversion of the covariance matrix is required. Since the inversion of the large matrix is time consuming, we avoid to inverse the large covariance matrix by writing the likelihood function in terms of the reduced bipolar coefficients instead of c_{lm} . The reduced bipolar coefficients are defined as [18]

$$A_{LM} = \sum_{l_1 m_1 l_2 m_2} (-1)^{m_2} d_{l_1 m_1} d_{l_2 m_2}^* \mathfrak{C}_{l_1 m_1 l_2 -m_2}^{LM}, \quad (42)$$

where d_{lm} are the harmonic coefficients of the temperature anisotropies. For the full-sky and noiseless case, the mean of the reduced bipolar coefficients for the noncommutative k-inflation can be computed using eq. (31), and the non zero components are

$$\langle A_{00} \rangle = \sum_l (-1)^l C_l \sqrt{2l+1} \left(1 - \frac{2}{3} A_s^2 \Sigma \right), \quad (43)$$

$$\begin{aligned} \langle A_{20} \rangle = & -2A_s^2 \Sigma \sum_l (-1)^l C_l \sqrt{\frac{l(l+1)(2l+1)}{45(2l-1)(2l+3)}} \\ & - 2A_s^2 \Sigma \sum_{l \geq 4} (-1)^l C_l \sqrt{\frac{2l(l-1)}{15(2l-1)}}. \end{aligned} \quad (44)$$

In the more realistic case, the mask and the instrument noise must be taken into account, so that the reduced bipolar coefficients are computed from c_{lm} in eq. (36) and their mean is

$$\langle A_{LM} \rangle = \sum_{l_1 l_2 m_1 m_2} (-1)^{m_2} C_{l_1 m_1, l_2 m_2} \mathfrak{C}_{l_1 m_1 l_2 -m_2}^{LM}. \quad (45)$$

It can be seen from the above equation that due to the effect of the mask, $\langle A_{00} \rangle$ and $\langle A_{20} \rangle$ will not be the only non zero components of $\langle A_{LM} \rangle$.

We define

$$\delta A_{LM} = A_{LM} - \langle A_{LM} \rangle, \quad (46)$$

and write the covariance matrix for δA_{LM} as

$$\begin{aligned} \langle \delta A_{LM} \delta A_{L'M'}^* \rangle &= \mathcal{C}_{LM, L'M'} \\ &= \sum_{l_1 m_1 l_2 m_2} \sum_{l_3 m_3 l_4 m_4} \mathfrak{C}_{l_1 m_1 l_2 m_2}^{LM} \mathfrak{C}_{l_3 m_3 l_4 m_4}^{L'M'} \\ &\quad \times \left(C_{l_1 m_1, l_3 m_3} C_{l_2 m_2, l_4 m_4} \right. \\ &\quad \left. + C_{l_1 m_1, l_4 m_4} C_{l_2 m_2, l_3 m_3} \right). \end{aligned} \quad (47)$$

Therefore, the posterior probability function takes the form

$$\begin{aligned} P(\bar{\sigma} | A_{LM}) &\propto \frac{\exp(-\frac{1}{2} \mathcal{Z}^2)}{\det^{1/2} \mathcal{C}} P(A_s) \\ \mathcal{Z}^2 &= \sum_{L M L' M'} \delta A_{LM} (\mathcal{C}^{-1})_{LM, L'M'} \delta A_{L'M'}^*. \end{aligned} \quad (48)$$

Here, we have used a flat prior on Σ and a Gaussian prior on A_s . For the Gaussian prior, the mean and variance of A_s are taken from the five-year WMAP results [33]. Since the contributions from spacetime noncommutativity mainly appear in the $\langle A_{00} \rangle$ and $\langle A_{20} \rangle$, we restrict the multipole index L of A_{LM} to be less than 4. We note that the parallel computing of covariance matrices $C_{l_1 m_1, l_2 m_2}$ and $C_{LM, L'M'}$ can be easily implemented in the Healpix package [31]. The data that are used to compute δA_{LM} are the five-year WMAP foreground-reduced V2 and W1 differential assembly temperature maps [30]. These maps are masked using the band-limited masks in [19]. According to [19], we limit the multipole index of c_{lm} and M_{LM} to be $l \leq 62$ and $L \leq 92$ respectively. The covariance matrix $\mathcal{C}$ can be computed from the covariance matrix in eq. (26). Since the spacetime noncommutativity does not affect the cosmic evolution after the inflationary epoch, we can use CMBEASY [32] to compute the CMB transfer function and c_l by supposing that the cosmic evolution after inflation obeys the Λ CDM model whose parameters are taken from the best fit value of the five-year WMAP results [33]. However, recall that the amplitude of the primordial power spectrum is treated as a free parameter. We compute the posterior probability function for $\bar{\sigma}$ and marginalize it over A_s to obtain the marginalized posterior probability function for Σ . The marginalized posterior probability functions for Σ obtained from V2 and W1 maps have a peak at negative Σ , i.e. negative c_S^2 . However we restrict ourselves to the case where $c_S^2 > 0$, so that the confidence intervals for Σ can be obtained from the areas under the curves of The marginalized posterior probability functions in figure 1.

Since the parameters β and c_S cannot be constrained using the above analysis, the upper bound for the noncommutative length scale L_s can be computed from the upper bound for Σ if the values of β and c_S are specified. For simplicity, we suppose that the values of β and c_S are known precisely, so that the upper bound for L_s can be written as $L_s < 1.4 \times 10^4 \beta^{1/2} c_S^{1/(1-\beta)} / m_p \text{Gev}^{-1} \sim 1.4 \times 10^{-28} \beta^{1/2} c_S^{1/(1-\beta)} \text{cm}$ at 99.7% confidence level. Here, m_p is the Planck mass. It can be seen that if inflaton evolves more slowly, the upper bound for L_s will increase and the dependence of this upper bound on c_S becomes weaker. We note that β is the inverse of the slow-roll parameter. As shown in [24], the ratio of the tensor to scalar perturbations amplitudes r depends on c_S . We roughly estimate the values of β and c_S using the values of n_s and r from the five-year WMAP results, and obtain $L_s < 10^{-27} \text{cm}$ at 99.7% confidence level.

	68%CL	95%CL	99.9%CL
W1	0.024	0.048	0.070
V2	0.029	0.055	0.78

Table 1. The upper bound for the parameter $\Sigma \times 10^{18}$ from W1 and V2 maps.

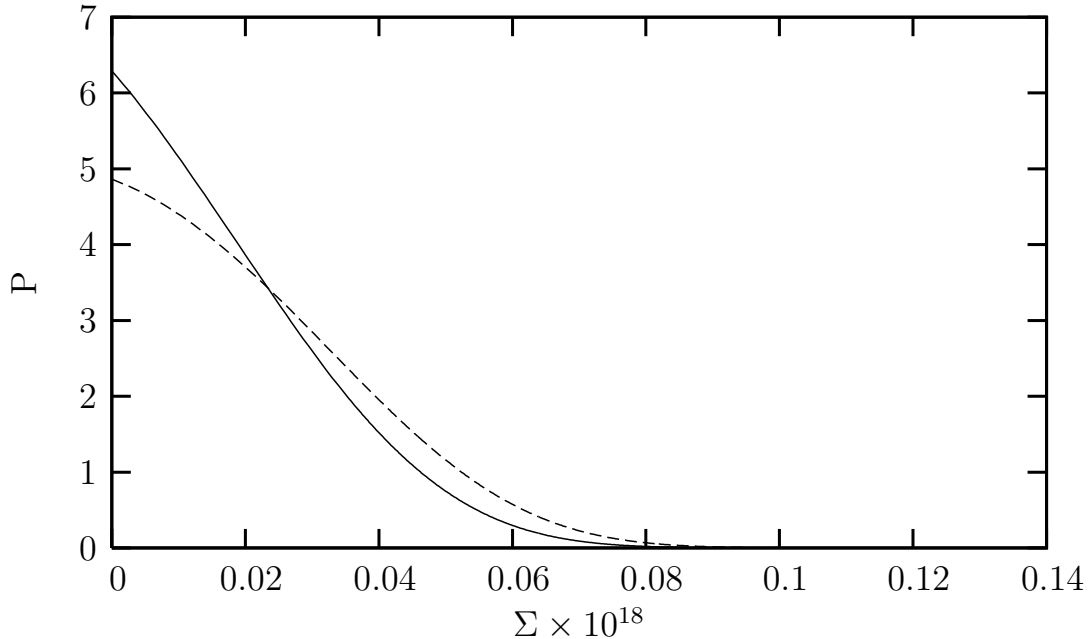


Figure 1. The marginalized posterior probability functions for Σ . The solid line represents the probability function from W1 map, while the long dashed line represents the probability function from V2 map. Here, Σ is restricted to be positive.

4. Conclusions

In this work, we study the effect of spacetime noncommutativity on the rotational invariance of the primordial power spectrum, and constrain the contributions from this effect using CMB data. For the power law k-inflation model, the deviation from rotational invariance of the primordial power spectrum due to the spacetime noncommutativity effect depends on the factor $\nu H_1^4 L_s^4 / (8c_S^2)$. This result will give rise to the result in [12] if $c_S = 1$, and will be invalid if the Hubble parameter does not evolve sufficiently slow during inflation.

Since the primordial power spectrum is direction-dependent in our consideration, the covariance matrix for the harmonic coefficients of the CMB temperature anisotropies has off diagonal elements. In our case, these off diagonal elements arise from the spacetime noncommutativity contributions. As is well known, this implies that statistics of the CMB anisotropies become anisotropic. The spacetime noncommutativity also contributes to the diagonal elements of the covariant matrix suggesting that the noncommutative contribution also modifies the amplitude of the CMB angular power spectrum.

Both contributions from spacetime noncommutativity are simultaneously constrained using five-year WMAP foreground-reduced V2 and W1 maps. The upper bound for the ratio $L_s c_S^{1/(\beta-1)} / \beta^{1/2}$ is approximately $1.4 \times 10^{-28} \text{ cm}$ at 99.7% confidence level. If we suppose that the values of β and c_S are known precisely, the upper bound for the noncommutative length scale can be written as $L_s < 1.4 \times 10^{-28} \beta^{1/2} c_S^{1/(1-\beta)} \text{ cm}$ at 99.7%

confidence level. This shows that if inflaton evolves more slowly, the upper bound for L_s will increase and the dependence of this upper bound on c_S becomes weaker. Estimating the values of β and c_S using the best fit cosmological parameters from the five-year WMAP results, we obtain $L_s < 10^{-27}$ cm at 99.7% confidence level.

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