รายงานวิจัยฉบับสมบูรณ์

โครงการ การศึกษาเกี่ยวกับการโฟลว์แบบโฮโลกราฟิกและหัวข้อที่เกี่ยวข้อง

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ดร. ปริญญา การดำริห์ จุฬาลงกรณ์มหาวิทยาลัย

สนับสนุนโดยสำนักงานกองทุนสนับสนุนการวิจัย

และจุฬาลงกรณ์มหาวิทยาลัย

กิตติกรรมประกาศ

ตลอดระยะเวลา 2 ปีของการทำงานวิจัยในโครงการนี้ หัวหน้าโครงการวิจัยผู้รับทุนขอแสดงความขอบคุณสำนักงานกองทุนสนับสนุนการวิจัย (สกว.) และจุฬาลงกรณ์มหาวิทยาลัยที่ให้การสนับสนุนการวิจัยของหัวหน้าโครงการมาด้วยดีตลอดระยะเวลาในการทำวิจัย ขอขอบคุณผู้ช่วยศาตราจารย์ ดร. อรรถกฤต ฉัตรภูติ สำหรับการเป็นนักวิจัยที่ปรึกษาและผู้ร่วมงาน ขอบคุณนาย บุญพิทักษ์ งามวัฒนากุลและนายเขม อุปถัมภากุล ที่เป็นผู้ร่วมงานและให้การช่วยเหลือทางด้านคอมพิวเตอร์ ขอบคุณคณะกรรมการบริหารภาควิชาฟิสิกส์และคณะวิทยาศาสตร์สำหรับบรรยากาศการทำงานที่เอื้อต่อการจัดสรรค์เวลาในการทำงานวิจัยรวมทั้งเจ้าหน้าที่ของสกว. ทุกฝ่ายที่ช่วยให้การดำเนินงานต่างๆ ในโครงการวิจัยเป็นไปด้วยความเรียบร้อย และในท้ายที่สุดนี้ โครงการวิจัยนี้คงไม่สามารถประสบความสำเร็จและเกิดผลลัพธ์ดังปรากฏอยู่ในรายงานฉบับนี้ได้หากไม่ได้รับการสนับสนุน กำลังใจ และความช่วยเหลือจากครอบครัวของหัวหน้าโครงการทั้งภรรยาที่คอยทวงถามว่าปีนี้เขียนบทความวิจัยไปกี่ฉบับแล้ว และบิดามารดาที่คอยรับฟังและให้การสนับสนุนตลอดเวลาที่งานวิจัยประสบความล้มเหลวหรือผิดพลาด รวมทั้งผู้มีส่วนเกี่ยวข้องอีกจำนวนมากที่ไม่สามารถระบุชื่อได้หมดในที่นี้ หัวหน้าโครงการวิจัยผู้รับทุนขอขอบ คุณไว้ ณ ที่นี้โดยทั่วกัน

Abstract

Project Code: TRG5680010

Project Title: Studies of Holographic RG Flows and Related Topics

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Project Period: 2 years

The project concerns with the studies of holographic RG flows and related topics within the framework of gauged supergravities in various space-time dimensions. The theories considered here are three-dimensional gauged supergravities with N=2,5,6,8,10 supersymmetries and the matter-coupled half-maximal gauged supergravities in six and seven dimensions. The corresponding scalar potential for each theory is explicitly computed, and the analysis of possible anti-de Sitter (AdS) vacua together with holographic RG flow solutions is carried out. The results from the research project are the discovery of new gauged supergravity theories in three and seven dimensions. In particular, a new embedding of N=2 SO(4) gauged supergravity in seven dimensions in eleven-dimensional supergravity is obtained. A large class of N=2 three-dimensional gauged supergravities from wrapped D3-branes in type IIB string theory is discovered. Among these results, novel supersymmetric AdS_7 backgrounds and supersymmetric RG flows are identified within the half-maximal gauged supergravity with topological mass term for the three-form field in the gravity multiplet. A class of supersymmetric RG flows, describing supersymmetric deformations of N=(1,0) superconformal field theories (SCFTs) in six dimensions and N=2 SCFTs in five dimensions, to non-conformal gauge theories and lower dimensional SCFTs is given. The result also provides new AdS_4 and AdS_5 solutions dual to certain SCFTs in three and four dimensions within the context of gauged supergravities. All of the outcomes of this project will be useful in the research involving embedding lower dimensional gauged supergravities in higher dimensions and holographic studies of strongly coupled gauge theories in various dimensions.

Keywords: Gauged supergravity, AdS/CFT correspondence, Gauge/gravity correspondence, Holographic RG flow

บทคัดย่อ

รหัสโครงการ: TRG5680010

ชื่อโครงการ: การศึกษาเกี่ยวกับการโฟลว์แบบโฮโลกราฟิกและหัวข้อที่เกี่ยวข้อง

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ระยะเวลาโครงการ: 2 ปี

โครงการวิจัยนี้เป็นการศึกษาการโฟลว์แบบโฮโลกราฟิกและหัวข้อที่เกี่ยวข้องภายในขอบเขตของทฤษฎีเกจซูเปอร์กราวิตี้ในกาลอวกาศหลากหลายมิติ ทฤษฎีที่พิจาณาในโครงการนี้ประกอบด้วยทฤษฎีเกจซูเปอร์กราวิตี้ในสามมิติที่มีจำนวนซูเปอร์ซิมเมทรี N=2,5,6,8,10 และทฤษฎีเกจซูเปอร์กราวิตี้ในหกและเจ็ดมิติที่มีซูเปอร์ซิมเมทรีเป็นครึ่งหนึ่งของค่าสูงสุดและมีการคู่ควบกับสนามสสาร ค่าพลังงานศักย์ของสนามสเกลาร์ภายในทฤษฎีทั้งหมดนี้ได้รับการคำนวณอย่างชัดแจ้งควบคู่ไปกับการวิเคราะห์หาคำตอบสุญญากาศแบบแอนติ-เดอ ซิตเตอร์ (AdS) และคำตอบแบบ RG โฟลว์ที่เป็นไปได้ ผลการวิจัยที่ได้จากโครงการวิจัยนี้ประกอบด้วยการค้นพบทฤษฎีเกจซูเปอร์กราวิตี้ในสามและเจ็ดมิติจำนวนมาก โดยเฉพาะการค้นพบทฤษฎีเกจซูเปอร์กราวิตี้ N=2 ที่มีเกจกรุ๊ป SO(4) จากทฤษฎีซูเปอร์กราวิตี้ในสิบเอ็ดมิติ การค้นพบทฤษฎีเกจซูเปอร์กราวิตี้ในสามมิติจำนวนมากจากการพันรอบของ D3-เบรนในทฤษฎีสตริงแบบ IIB นอกจากนี้ยังมีการค้นพบคำตอบพื้นหลังแบบ AdS_7 และคำตอบแบบ RG โฟลว์ในทฤษฎีเกจซูเปอร์กราวิตี้เจ็ดมิติที่มีซูเปอร์ซิมเมทรีครึ่งหนึ่งของค่าสูงสุดและมีพจน์มวลสำหรับสนาม 3-ฟอร์มในมัลติเพล็ทความโน้มถ่วง ทั้งยังค้นพบคำตอบแบบ RG โฟลว์จำนวนมากที่อธิบายการบิดเบือนทฤษฎีสนามแบบคอนฟอร์มอล (SCFT) ที่มีซูเปอร์ซิมเมทรี N=(1,0) ในหกมิติและ N=2 ในห้ามิติไปยังทฤษฎีสนามแบบเกจที่ไม่มีสมมาตรคอนฟอร์มอลและทฤษฎีสนามแบบคอนฟอร์มอลที่มีมิติต่ำกว่า ผลลัพธ์ที่ได้ถือเป็นการค้นพบคำตอบแบบ AdS_4 และ AdS_5 ชนิดใหม่ที่สัมพันธ์กับทฤษฎีสนามแบบคอนฟอร์มอลในสามและสี่มิติภายในบริบทของทฤษฎีเกจซูเปอร์กราวิตี้ ผลการวิจัยทั้งหมดที่เกิดขึ้นมีประโยชน์ในการศึกษาวิจัยเกี่ยวกับการค้นหาทฤษฎีเกจซูเปอร์กราวิตี้จากกาลอวกาศที่มีมิติสูงกว่าและการศึกษาทฤษฎีเกจในหลากหลายมิติที่มีการคู่ควบรุนแรงในเชิงโฮโลกราฟิก

คำหลัก: Gauged supergravity, AdS/CFT correspondence, Gauge/gravity correspondence, Holographic RG flow

บทนำ

ผลลัพธ์สำคัญประการหนึ่งในทฤษฎีซูเปอร์สตริงคือความสอดคล้องแบบ AdS/CFT ซึ่งเป็นดูออลิตี้ระหว่างทฤษฎีสตริงบนอวกาศแอนติ-เดอ ซิตเตอร์ (AdS) กับทฤษฎีสนามแบบคอนฟอร์มอล (CFT) บนขอบเขตของอวกาศ AdS ความสัมพันธ์เชิงโฮโลกราฟิกดังกล่าวมีด้านหนึ่งของความสัมพันธ์เป็นทฤษฎีในกาลอวกาศ D มิติและมีอีกด้านหนึ่งเป็นทฤษฎีใน D-1 มิติ โดยได้รับการเสนอขึ้นครั้งแรกใน [1] และได้รับการปรับปรุงจนมีรูปแบบที่รัดกุมยิ่งขึ้นใน [2]

หลังจากค้นพบหลักความสอดคล้องนี้ไม่นาน ได้มีงานวิจัยอีกเป็นจำนวนมากที่ขยายขอบเขตของหลักการดังกล่าวให้ครอบคลุมกว้างขวางยิ่งขึ้น แนวทางในการขยายขอบเขตแบบหนึ่งคือความสอดคล้องแบบเกจ/ความโน้มถ่วงหรือที่บางครั้งเรียกว่าความสอดคล้องแบบ non-AdS/ non-CFT ในกรณีนี้ คำตอบพื้นหลังของความโน้มถ่วงไม่จำเป็นต้องเป็นอวกาศ AdS ที่มีไอโซเมทรีสอดคล้องกับสมมาตรคอนฟอร์มอลที่ขอบเขต ทฤษฎีสนามแบบเกจที่เป็นคู่กับพื้นหลังชนิดนี้จึงไม่มีสมมาตรคอนฟอร์มอล เป็นผลให้ทฤษฎีสนามที่ได้มีลักษณะใกล้เคียงกับทฤษฎีเกจ ของแบบจำลองมาตรฐานที่ใช้ในทฤษฎีของฟิสิกส์อนุภาคมากยิ่งขึ้น ประโยชน์ที่สำคัญประการหนึ่งของ ความสอดคล้องทั้งสองแบบนี้คือสามารถใช้ศึกษาทฤษฎีสนามที่มีการคู่ควบรุนแรงได้ เนื่องจากความสอดคล้องนี้เป็นดูออลิตี้แบบเข้ม/อ่อน ในความหมายที่ว่าทฤษฎีด้านหนึ่งมีการคู่ควบสูงส่วนอีกด้านหนึ่งมีการคู่ควบอย่างอ่อนๆ จึงใช้ความสอดคล้องนี้ศึกษาทฤษฎีเกจที่มีการคู่ควบรุนแรงได้โดยการศึกษาทฤษฎีความโน้มถ่วงที่มีการคู่ควบอย่างอ่อน ซึ่งบ่อยครั้งจะเป็นทฤษฎีแบบคลาสสิกที่มีซูเปอร์ซิมเมทรีภายใต้การประมาณที่สมเหตุสมผล

การศึกษาเกี่ยวกับความสอดคล้องแบบเกจ/ความโน้มถ่วงที่สำคัญมากประการหนึ่งคือการศึกษา RG โฟลว์แบบโฮโลกราฟิก ในกรณีนี้ ทฤษฎีสนามที่เป็นคู่กับความโน้มถ่วงจะไม่เป็นทฤษฎีแบบคอนฟอร์มอล แต่มีบางลิมิตเป็นทฤษฎีแบบคอนฟอร์มอล จุดที่ทฤษฎีสนามกลายเป็นทฤษฎีแบบคอนฟอร์มอลเรียกว่าจุดคงที่แบบคอนฟอร์มอล (conformal fixed point) หรืออาจเรียกสั้นๆ ว่าจุดคงที่ (fixed point)

ด้วยกระบวนการนี้ สามารถศึกษาแง่มุมต่างๆ ของทฤษฎีสนามได้โดยพิจารณาทฤษฎีดังกล่าวที่จุดคอนฟอร์มอลหนึ่งในระดับ UV หรือที่ระดับพลังงานสูง ณ จุดนี้ คำตอบในทฤษฎีความโน้มถ่วงจะเป็นอวกาศ AdS จากนั้นพิจารณาการรบกวนทฤษฎีสนามแบบคอนฟอร์มอลนี้โดยเปอเรเตอร์หรือค่าคาดหวังในสุญญากาศ (vacuum expectation value) ที่ทำให้ทฤษฎีสนามในระดับ UV เกิด RG โฟลว์ไปยังทฤษฎีสนามแบบคอนฟอร์มอลอีกทฤษฎีหนึ่งในระดับ IR ซึ่งอาจเป็นจุดคงที่อีกจุดหนึ่งของทฤษฎีสนามตั้งต้นที่กำลังพิจารณาอยู่ก็ได้ นอกจากนี้ทฤษฎีในระดับ IR อาจเป็นทฤษฎีที่ไม่มีสมมาตรคอนฟอร์มอลได้เช่นเดียวกัน ในกรณีนี้ คำตอบในทฤษฎีความโน้มถ่วงที่ได้จะเป็นคำตอบโดเมนวอลล์ (domain wall) [3-5] การโฟลว์ในลักษณะนี้เรียกว่าการโฟลว์แบบไม่คอนฟอร์มอล (non-conformal flow)

ตัวอย่างแรกเริ่มที่แสดงถึงหลักความสอดคล้องแบบ AdS/CFT คือความสอดคล้องกันระหว่างทฤษฎีสตริงแบบ IIB บนอวกาศ AdS_5xS^5 กับทฤษฎีสนามคอนฟอร์มอล N=4 ซูเปอร์หยาง-มิลล์ (SYM) ในสี่มิติ ผลการศึกษาในลำดับต่อมาแสดงให้เห็นการจับคู่แบบหนึ่งต่อหนึ่ง

ระหว่างสนามภายในทฤษฎียังผล 10 มิติของทฤษฎีสตริงแบบ IIB คือซูเปอร์กราวิตี้แบบ IIB ที่มีมิติม้วนตัวอยู่ในรูปของทรงกลมห้ามิติ (S^5) และโอเปอเรเตอร์ในทฤษฎี N=4 SYM ผลการศึกษาแสดงให้เห็นอย่างชัดเจนว่าทั้งสองทฤษฎีสอดรับกันเป็นอย่างดี นับตั้งแต่นั้นเป็นต้นมา ได้มีการศึกษาเกี่ยวกับการรบกวนทฤษฎี N=4 SYM ที่รักษาซูเปอร์ซิมเมทรีในรูปของ RG โฟลว์เกิดขึ้นเป็นจำนวนมาก ตัวอย่างเช่นใน [6-8] นอกจากนี้ ยังมีการศึกษาเกี่ยวกับการบิดเบือนที่ทำลายซูเปอร์ซิมเมทรีอีกด้วย [9] ผลลัพธ์ที่ได้ส่งผลให้เกิดความรู้ความเข้าใจเกี่ยวกับสมบัติของทฤษฎี N=4 SYM ในแง่มุมต่างๆ ได้อย่างที่ไม่เคยมีมาก่อน โดยผลลัพธ์เกือบทั้งหมดเกิดขึ้นจากการศึกษาทฤษฎีเกจซูเปอร์กราวิตี้ในห้ามิติที่มีเกจกรุ๊ป SO(6) ซึ่งคาดว่าเป็นทฤษฎียังผลของทฤษฎีซูเปอร์กราวิตี้แบบ IIB ในสิบมิติที่ทำการลดมิติลงบน S^5 ซึ่งแสดงให้เห็นว่าทฤษฎีเกจซูเปอร์กราวิตี้เป็นเครื่องมือสำคัญสำหรับการศึกษาหลักโฮโลกราฟิก AdS/CFT

ทฤษฎีซูเปอร์กราวิตี้ (supergravity) คือทฤษฎีที่อธิบายความโน้มถ่วงในรูปแบบที่มีสมมาตรซูเปอร์ซิมเมทรี (supersymmetry) ซึ่งเป็นสมมาตรระหว่างอนุภาคโบซอน (ที่มีสปินเป็นจำนวนเต็ม) และอนุภาคเฟอร์มิออน (ที่มีสปินเป็นครึ่งหนึ่งของจำนวนเต็ม) ทฤษฎีซูเปอร์กราวิตี้มีสุญญากาศที่รักษาซูเปอร์ซิมเมทรีเป็นกาลอวกาศมินคอฟสกี หากต้องการสุญญากาศแบบ AdS จำเป็นต้องใช้ทฤษฎีซูเปอร์กราวิตี้ในรูปแบบที่ถูกเกจ (gauged) ในความหมายที่ว่าสมมาตรที่เหมือนกันทั่วทั้งหมด (global) จะกลายเป็นสมมาตรแบบเฉพาะที่ (local) เป็นผลให้เกิดการบิดเบือนของทฤษฎีซูเปอร์กราวิตี้ที่เรียกว่า ทฤษฎีเกจซูเปอร์กราวิตี้ (gauged supergravity) การบิดเบือนดังกล่าวรักษาซูเปอร์ซิมเมทรี จึงได้ทฤษฎีสุดท้ายที่มีซูเปอร์ซิมเม ทรี นอกจากนี้ ยังมีการบิดเบือนอีกแบบหนึ่งคือ แมสสีฟซูเปอร์กราวิตี้ (massive supergravity) ซึ่งเกิดจากการเพิ่มพจน์มวลให้กับสนามของดิฟเฟอเรนเชียลฟอร์ม ในหลายๆ กรณี ทั้งทฤษฎีเกจซูเปอร์กราวิตี้และทฤษฎีแมสสีฟซูเปอร์กราวิตี้ต่างก็ให้สุญญากาศแบบ AdS ที่มีซูเปอร์ซิมเมทรีได้เช่นกัน

ในที่นี้ จะมุ่งเน้นไปยังการศึกษาทฤษฎีสนามแบบคอนฟอร์มอลในสอง ห้าและหกมิติโดยหลักการทั่วไปของความสอดคล้องแบบเกจ/ความโน้มถ่วง การศึกษาดังกล่าวต้องใช้ทฤษฎีเกจซูเปอร์กราวิตี้ในสาม หกและเจ็ดมิติ ตามลำดับ เครื่องมือสำคัญที่ใช้ในการศึกษาทฤษฎีสนามสองมิติคือทฤษฎีเกจซูเปอร์กราวิตี้สามมิติ งานวิจัยที่ศึกษาตามแนวทางนี้คือ [10-15] เนื่องจากโครงสร้างของทฤษฎีสนามแบบคอนฟอร์มอลสองมิติเป็นที่รู้กันดี การศึกษาความสอดคล้อง AdS_3/CFT_2 จึงมีความสำคัญในแง่ที่อาจช่วยให้เกิดความเข้าใจอย่างลึกซึ้งเกี่ยวกับทฤษฎีความโน้มถ่วงควอนตัมได้ อีกทั้งทฤษฎีความโน้มถ่วงในสามมิติก็เป็นทฤษฎีที่ไม่ซับซ้อนดังเช่นในมิติที่สูงกว่า นักฟิสิกส์ส่วนใหญ่จึงคาดว่าการศึกษาความสอดคล้อง AdS_3/CFT_2 จะเป็นจุดเริ่มต้นในการทำความเข้าใจหลักการพื้นฐานของความสอดคล้องแบบ AdS/CFT ได้ นอกจากนี้ ความสอดคล้อง AdS_3/CFT_2 ยังมีความสำคัญในการศึกษาเอนโทรปีของหลุมดำอีกด้วย [16]

การศึกษาเกี่ยวกับการโฟลว์แบบโฮโลกราฟิกในมิติอื่นๆ เช่น ในทฤษฎีสนามหกมิติเกิดขึ้นใน [17] งานวิจัยนี้ศึกษาทฤษฎีเกจซูเปอร์กราวิตี้ N=2 ในเจ็ดมิติที่ไม่มีการคู่ควบกับ

สนามสสาร [18] การศึกษาการโฟลว์ในทฤษฎีสนามห้ามิติโดยใช้ทฤษฎีเกจซูเปอร์กราวิตี้หกมิติ N=(1,1) ที่ไม่มีการคู่ควบกับสนามสสาร [19] ทั้งการโฟลว์ระหว่างทฤษฎีสนามคอนฟอร์มอลห้ามิติและการโฟลว์ไปยังทฤษฎีสนามคอนฟอร์มอลในมิติที่ต่ำกว่ารวมทั้งการโฟลว์ไปยังทฤษฎีสนามแบบไม่คอนฟอร์มอลเกิดขึ้นใน [20,21] RG โฟลว์ระหว่างทฤษฎีสนามคอนฟอร์มอลแบบมีซูเปอร์ซิมเมทรีและไม่มีซูเปอร์ซิมเมทรีได้รับการศึกษาใน [22] โดยใช้ทฤษฎีเกจซูเปอร์กราวิตี้ N=(1,1) ที่มีการคู่ควบกับสนามเวกเตอร์ 3 มัลติเพล็ท [23]

การโฟลว์ที่น่าสนใจอีกแบบหนึ่งคือการโฟลว์ข้ามมิติ (across dimension) กล่าวคือทฤษฎีสนามคอนฟอร์มอลถูกรบกวนให้เกิดการโฟลว์ลงไปยังทฤษฎีคอนฟอร์มอลในมิติที่ต่ำกว่า คำตอบในทฤษฎีซูเปอร์กราวิตี้สำหรับกรณีนี้จะเป็นโดเมนวอลล์ที่มีลิมิตหนึ่งเป็น AdS_D และอีกลิมิตหนึ่งเป็น AdS_d ที่ $d<D$ การตีความในทฤษฎีสนามคือ ทฤษฎีคอนฟอร์มอลในมิติที่สูงกว่าเกิดการม้วนมิติแบบบิด (twisted compactification) เป็นผลให้ทฤษฎียังผลที่พลังงานต่ำไม่รับรู้ถึงมิติที่ม้วนตัวอยู่ จึงได้ทฤษฎีในระดับ IR เป็นทฤษฎีสนามที่มีมิติลดลง การศึกษาดังกล่าวมีประโยชน์สำหรับศึกษาทฤษฎีสนามในมิติสูงๆ ที่มักจะมีความซับซ้อนสูงกว่าทฤษฎีในมิติที่ต่ำกว่าเช่นทฤษฎีสนามคอนฟอร์มอล N=(2,0) ในหกมิติเป็นต้น

ในโครงการวิจัยนี้ จะพิจารณาการโฟลว์ของทฤษฎีสนามแบบคอนฟอร์มอลในกาลอวกาศสอง ห้าและหกมิติทุกแบบที่ได้กล่าวถึงไปข้างต้น โดยมีทฤษฎีเกจซูเปอร์กราวิตี้ที่เกี่ยวข้องดังต่อไปนี้ ทฤษฎีเกจซูเปอร์กราวิตี้ในสามมิติที่พิจารณาในงานวิจัยนี้คือทฤษฎีที่มีซูเปอร์ซิมเมทรี N=5,6,8,10 โดยทฤษฎีทั้งหมดนี้สร้างขึ้นใน [24] ส่วนทฤษฎีเกจซูเปอร์กราวิตี้ในหกและเจ็ดมิติที่มีซูเปอร์ซิมเมทรีครึ่งหนึ่งของค่าสูงสุดและมีการคู่ควบกับสนามสสารจะใช้ทฤษฎีที่สร้างขึ้นใน [23] และ [25] ตามลำดับ

ระเบียบวิธีวิจัย

ในการศึกษาเกี่ยวกับการโฟลว์แบบโฮโลกราฟิกรวมทั้งหัวข้ออื่นๆ ที่เกี่ยวข้อง จำเป็นต้องกำหนดทฤษฎีเกจซูเปอร์กราวิตี้ที่จะใช้เป็นจุดตั้งต้นเสียก่อน การโฟลว์แบบโฮโลกราฟิกที่มีซูเปอร์ซิมเมทรีในทฤษฎีสนาม d มิติอธิบายได้ด้วยคำตอบภายในทฤษฎีเกจซูเปอร์กราวิตี้ d+1 มิติที่เป็นโดเมนวอลล์ชนิด BPS หลังจากระบุทฤษฎีเกจซูเปอร์กราวิตี้ที่จะใช้แล้ว ขั้นตอนต่อไปคือการคำนวณหาค่าพลังงานศักย์ของสนามสเกลาร์รวมทั้งจุดวิกฤตที่เป็นไปได้ จุดวิกฤตที่สนใจจะเป็นแบบไม่ชัด (non trivial) กล่าวคือ มีค่าสนามสเกลาร์ไม่เป็นศูนย์ จุดวิกฤตแบบแอนติ-เดอซิตเตอร์ (AdS) จะสอดคล้องกับจุดคงที่ของทฤษฎีสนาม ซึ่งเป็นจุดที่ทฤษฎีสนามมีสมมาตรแบบคอนฟอร์มอล จากนั้นค้นหาคำตอบแบบ BPS ที่เชื่อมโยงระหว่างจุดวิกฤตที่ได้ คำตอบดังกล่าวจะได้รับการตีความเป็น RG โฟลว์จากจุดคงที่ในระดับ UV ไปยังจุดคงที่ในระดับ IR หรือกล่าวได้อีกอย่างหนึ่งว่า เป็นการโฟลว์จากทฤษฎีสนามแบบคอนฟอร์มอลหนึ่งในระดับ UV ไปยังอีกทฤษฎีหนึ่งในระดับ IR

ในกระบวนการคำนวณ จะใช้โปรแกรมคอมพิวเตอร์สำเร็จรูป Mathematica เนื่องจากการคำนวณเกี่ยวข้องกับพีชคณิตของเมทริกซ์ขนาดใหญ่จำนวนมาก อีกทั้งสมการ BPS ที่

เกี่ยวข้องก็มีเป็นจำนวนมากและเป็นสมการเชิงอนุพันธ์ที่ต้องสอดคล้องกันทั้งหมด ในบางกรณีอาจมีสมการ BPS ที่ต้องแก้ถึงหนึ่งร้อยสมการ จึงแทบเป็นไปไม่ได้เลยที่จะแก้สมการดังกล่าวโดยไม่อาศัยโปรแกรมคอมพิวเตอร์

จากที่กล่าวมาทั้งหมด อาจสรุปขั้นตอนการดำเนินงานวิจัยได้ดังนี้

1. กำหนดทฤษฎีเกจซูเปอร์กราวิตี้ที่เหมาะสม
2. ศึกษาโครงสร้างของทฤษฎีเกจซูเปอร์กราวิตี้ดังกล่าวพร้อมทั้งคำนวณหาพลังงานศักย์ของสนามสเกลาร์
3. ค้นหาจุดวิกฤตของพลังงานศักย์ที่ได้
4. ศึกษาคุณลักษณะของจุดวิกฤตที่ได้เช่น สมมาตรที่ไม่สูญหายและจำนวนซูเปอร์ซิมเมทรีเป็นต้น
5. สร้างสมการ BPS จากการแปรผันของซูเปอร์ซิมเมทรีในสนามเฟอร์มิออน
6. แก้สมการ BPS ที่ได้เพื่อหาคำตอบแบบ RG โฟลว์ซึ่งอาจอยู่ในรูปแบบเชิงวิเคราะห์หรือเชิงตัวเลข
7. ศึกษาสมบัติของคำตอบ RG โฟลว์ที่ได้เช่น พฤติกรรมของคำตอบที่ได้ในลิมิตที่เข้าใกล้จุดวิกฤติทั้งในระดับ UV และ IR เพื่อวิเคราะห์หามิติของโอเปอเรเตอร์ที่เป็นตัวผลักดันให้เกิดการโฟลว์หรือคำนวณหาฟังก์ชันสหสัมพันธ์ (correlation function) ของโอเปอเรเตอร์ที่สนใจ เป็นต้น

ผลการวิจัย

งานวิจัยเกี่ยวกับทฤษฎีเกจซูเปอร์กราวิตี้ในกาลอวกาศสามมิติมีผลงานวิจัยตีพิมพ์ทั้งสิ้น 4 ฉบับ [26-29] โดยมีรายละเอียดของผลลัพธ์ที่ได้ดังต่อไปนี้

ผลการวิจัยพบทฤษฎีเกจซูเปอร์กราวิตี้ N=2 ในสามมิติเป็นจำนวนมากจากการพันรอบของ D3-เบรนจากทฤษฎีซูเปอร์กราวิตี้แบบ IIB ในสิบมิติ ผลลัพธ์ที่ได้มีประโยชน์ต่อการศึกษาโฮโลกราฟิกของกระบวนการ c-extremization ที่ใช้ในการกำหนดค่าที่แน่ชัดของประจุศูนย์กลาง (central charge) ในทฤษฎีสนามคอนฟอร์มอล N=(2,0) ในสองมิติ [26]

สำหรับเกจซูเปอร์กราวิตี้ที่มีซูเปอร์ซิมเมทรี N=5,6,10 [27,28] มีการค้นพบทฤษฎีเกจซูเปอร์กราวิตี้แบบใหม่ๆ ที่มีเกจกรุ๊ปหลากหลายรูปแบบรวมทั้งทฤษฎีที่อาจหาต้นกำเนิดในมิติที่สูงกว่าได้ โดยทฤษฎีดังกล่าวมีเกจกรุ๊ปในรูปแบบ non-semisimple ผลการวิจัยที่ได้ช่วยเติมเต็มช่องว่างในแวดวงวิจัยและเพิ่มตัวอย่างทฤษฎีเกจซูเปอร์กราวิตี้สามมิติที่ได้จากการลดมิติของทฤษฎีในมิติที่สูงกว่าให้มากยิ่งขึ้น เนื่องจากการลดมิติดังกล่าวยังเป็นปริศนาและมีตัวอย่างที่เป็นไปได้น้อยมากในปัจจุบัน สำหรับทฤษฎีเกจซูเปอร์กราวิตี้ N=8 ซึ่งมีสุญญากาศสอดคล้องกับทฤษฎีสนามแบบคอนฟอร์มอลในสองมิติที่มีซูเปอร์ซิมเมทรี N=(4,4) ขนาดใหญ่ [29] ผลการวิจัยค้นพบจุดวิกฤตเสถียรที่ไม่มีซูเปอร์ซิมเมทรีจำนวนหนึ่ง ผลลัพธ์ที่ได้แสดงถึงคอนฟอร์มอลเฟสแบบต่างๆ ของทฤษฎีคอนฟอร์มอล N=(4,4) ซึ่งมีความสำคัญทั้งในแง่ของการเป็นทฤษฎีสนามยังผลสำหรับระบบ D1-D5-เบรนแบบคู่และในฟิสิกส์ของหลุมดำ

สำหรับการศึกษาทฤษฎีเกจซูเปอร์กราวิตี้หกมิติแบบ F(4) มีผลงานวิจัยตีพิมพ์หนึ่งฉบับ [30] ในงานวิจัยนี้ ได้ทำการศึกษาการบิดเบือนที่รักษาซูเปอร์ซิมเมทรีแล้วเกิดการโฟลว์จากทฤษฎีสนามคอนฟอร์มอล N=2 ไปยังทฤษฎีสนามแบบไม่คอนฟอร์มอล N=2 SYM ในห้ามิติ ผลการวิจัยที่ได้นับเป็นผลลัพธ์แรกที่แสดงถึงการรบกวนแบบรักษาซูเปอร์ซิมเมทรีจากทฤษฎีเกจซูเปอร์กราวิตี้แบบ F(4) ที่มีการคู่ควบกับสนามสสารตั้งแต่มีการศึกษาการบิดเบือนแบบไม่รักษาซูเปอร์ซิมเมทรีใน [22]

งานวิจัยเกี่ยวกับทฤษฎีเกจซูเปอร์กราวิตี้ N=2 ในเจ็ดมิติที่มีการคู่ควบกับสนามสสารมีผลงานวิจัยตีพิมพ์ทั้งสิ้น 4 ฉบับ [31-34] ผลการวิจัยเริ่มต้นจากการค้นพบคำตอบแบบ AdS_7 ที่มีซูเปอร์ซิมเมทรีแบบใหม่ในรอบหลายปีที่ผ่านมาจากทฤษฎีเกจซูเปอร์กราวิตี้ N=2 ที่มีพจน์มวลทางโทโพโลจีและมีเกจกรุ๊ป SO(4) ผลการศึกษาแสดงให้เห็นจุดคงที่แบบใหม่ในทฤษฎีสนามแบบคอนฟอร์มอล N=(1,0) รวมทั้งการบิดเบือนทฤษฎี N=(1,0) ในระดับ UV ที่ได้รับการค้นพบก่อนหน้านี้ไปยังจุดคงที่ที่ค้นพบใหม่จาก RG โฟลว์ ที่รักษาซูเปอร์ซิมเมทรีทั้งหมดไว้ [31]

การค้นพบดังกล่าวเป็นผลให้เกิดงานวิจัยค้นหาวิธีการลดมิติของทฤษฎีสตริงหรือทฤษฎีเอ็มเพื่อสร้างคำตอบ AdS_7 ชนิดใหม่ที่ได้ การวิจัยในลำดับต่อมาส่งผลให้ค้นพบวิธีการลดมิติแบบใหม่ของทฤษฎีซูเปอร์กราวิตี้สิบเอ็ดมิติลงมายังเจ็ดมิติที่ให้ทฤษฎีเกจซูเปอร์กราวิตี้ N=2 และมีเกจกรุ๊ป SO(4) ซึ่งเป็นการลดมิติรูปแบบใหม่ที่ได้จากการตัดทอน (truncate) บางส่วนของการลดมิติบนทรงกลมสี่มิติ (S^4) ซึ่งให้ทฤษฎีเกจซูเปอร์กราวิตี้ N=4 ที่มีเกจกรุ๊ป SO(5) [32]

การศึกษาเกี่ยวกับ RG โฟลว์ได้รับการขยายผลออกไปยังเกจกรุ๊ปแบบ non-compact ใน [33] โดยผลการวิจัยแสดงให้เห็นสุญญากาศแบบ AdS_7 ชนิดอีกจำนวนหนึ่งรวมทั้งจุด คงที่ที่มีรูปแบบเป็น AdS_5 และสามารถตีความได้เป็นทฤษฎีสนามคอนฟอร์มอลสี่มิติ งานวิจัยนี้จึงมีประโยชน์ต่อการศึกษาทฤษฎีสนามในสี่มิติด้วย ทั้งยังเป็นการจัดจำแนกสุญญากาศของเกจกรุ๊ปแบบ non-compact ที่เป็นไปได้ทั้งหมดของทฤษฎีเกจซูเปอร์กราวิตี้ N=2 อีกด้วย

ในช่วงปลายของโครงการวิจัย งานวิจัยได้มุ่งเน้นไปยังการศึกษาแบบรวบยอดโดยนำผลงานวิจัยที่ได้มาก่อนหน้านี้สังเคราะห์เป็นงานวิจัยใหม่ [34] โดยงานวิจัยดังกล่าวเริ่มต้นจากการศึกษาการม้วนมิติแบบบิดของทฤษฎีสนามคอนฟอร์มอล N=(1,0) ภายในขอบเขตของทฤษฎีเกจซูเปอร์กราวิตี้ที่มีเกจกรุ๊ป SO(4) จุดประสงค์ของงานวิจัยนี้คือค้นหาคำตอบ AdS_5 และ AdS_4 ซึ่งอธิบายทฤษฎีสนามคอนฟอร์มอลในสี่และสามมิติ ตามลำดับ นอกจากนี้ยังทำการศึกษาการโฟลว์จากทฤษฎีสนามคอนฟอร์มอล N=(1,0) ในหกมิติที่ค้นพบใน [31] ลงมายังสี่มิติและสามมิติพร้อมทั้งยกระดับคำตอบที่ได้ขึ้นไปยังสิบเอ็ดมิติโดยใช้ผลลัพธ์ของ [32] อีกด้วย

บทสรุปและบทวิจารณ์

งานวิจัยทั้งหมดที่เกิดขึ้นในโครงการวิจัยนี้ครอบคลุมการศึกษาทฤษฎีสนามคอนฟอร์มอลเชิงโฮโลกราฟิกในกาลอวกาศมิติต่างๆ ในหลากหลายแง่มุม ทั้งการค้นพบจุดวิกฤตใหม่ การโฟลว์ระหว่างทฤษฎีสนามแบบคอนฟอร์มอลในมิติเดียวกันทั้งที่รักษาและทำลายซูเปอร์ซิมเมทรี การโฟลว์ไปยังทฤษฎีสนามที่ไม่มีสมมาตรคอนฟอร์มอลและการโฟลว์ข้ามมิติไปยังทฤษฎีสนามคอนฟอร์มอลในมิติที่ต่ำกว่า ผลการวิจัยที่ได้เติมเต็มช่องว่างในแวดวงวิจัยที่เกี่ยวข้องในแง่ของการสร้างคำตอบในทฤษฎีความโน้มถ่วงเพื่อใช้ศึกษาทฤษฎีสนามที่มีการคู่ควบรุนแรงต่อไปในอนาคต ทั้งยังเป็นการบุกเบิกงานวิจัยเกี่ยวกับโฮโลกราฟิกของทฤษฎีสนามคอนฟอร์มอลในหกและห้ามิติซึ่งกำลังได้รับความสนใจอย่างมากในแวดวงวิจัยปัจจุบันและยังมีอีกหลายๆ ประเด็นที่ยังไม่ชัดเจน

อย่างไรก็ตามงานวิจัยที่ได้จากโครงการนี้เป็นเพียงจุดเริ่มต้นเท่านั้น ยังมีเรื่องที่ต้องค้นคว้าวิจัยอีกมากเช่น ผลลัพธ์จากการลดมิติที่ได้ใน [32] ยังไม่สามารถใช้ยกระดับคำตอบ AdS_7 ที่ค้นพบใหม่ใน [31] ได้ เนื่องจากคำตอบดังกล่าวต้องการให้ค่าคงที่การคู่ควบของกรุ๊ป SU(2) ทั้งสองกรุ๊ปใน SO(4) มีค่าไม่เท่ากัน แต่ผลลัพธ์ของ [32] เป็นการลดมิติเฉพาะในกรณีที่ค่าคงที่ทั้งสองเท่ากันเท่านั้น นอกจากนี้ ในปัจจุบันยังไม่มีวิธีการลดมิติที่เป็นต้นกำเนิดของทฤษฎีเกจซูเปอร์กราวิตี้แบบ F(4) ในกรณีที่มีการคู่ควบกับสนามสสาร ผลลัพธ์ที่ได้ใน [22] จึงยังไม่สามารถตีความในบริบทของทฤษฎีสตริงหรือทฤษฎีเอ็มได้ ประการสุดท้าย ทฤษฎีสนามคอนฟอร์มอลที่เป็นคู่กับคำตอบของทฤษฎีเกจซูเปอร์กราวิตี้ที่ค้นพบในโครงการวิจัยนี้เกือบทั้งหมดยังไม่เป็นที่รู้จักกันในแวดวงวิจัย จึงมีงานที่ต้องศึกษาวิจัยอีกมากเพื่อระบุทฤษฎีสนามคอนฟอร์มอลที่แน่ชัดต่อไป ทั้งหมดนี้จึงเป็นแนวทางการวิจัยในอนาคตที่เป็นไปได้

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Output จากโครงการวิจัยที่ได้รับทุนจากสกว.

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   1.6 Auttakit Chatrabhuti, Parinya Karndumri and Boonpithak Ngamwatthanakul, *New N=5,6, 3D gauged supergravities and holography*, JHEP 01 (2014) **159**
   1.7 Parinya Karndumri, *Deformations of large N=(4,4) 2D SCFT from 3D gauged supergravity*, JHEP 05 (2014) **087**
   1.8 Parinya Karndumri,*1/2-BPS domain wall from N = 10 three dimensional gauged supergravity*, JHEP 11 (2013) **023**
   1.9 Parinya Karndumri and Eoin O Colgain, *3D supergravity from wrapped D3-branes*, JHEP 10 (2013) **094**

2. การนำผลงานวิจัยไปใช้ประโยชน์
   ในเชิงวิชาการ มีการสร้างนักวิจัยใหม่คือนาย บุญพิทักษ์ งามวัฒนากุล ซึ่งขณะนี้เป็นนิสิตปริญญาเอกและอยู่ระหว่างดำเนินการสอบวิทยานิพนธ์ปริญญาเอก ณ ภาควิชาฟิสิกส์ คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย
3. อื่นๆ (เช่น หนังสือ การจดสิทธิบัตร)
   หนังสือตำราวิชาการเรื่อง “ทฤษฎีสัมพัทธภาพทั่วไป” โดยขณะนี้อยู่ระหว่างดำเนินการตีพิมพ์

ภาคผนวก

ตลอดระยะเวลา 2 ปีที่ดำเนินโครงการวิจัยนี้ มีผลการวิจัยเกิดขึ้นหลายเรื่องด้วยกัน โดยมีบทความวิจัยที่ตีพิมพ์เผยแพร่ในวารสารทางวิชาการระดับนานาชาติทั้งสิ้น 9 ฉบับ และได้แนบบทความวิจัยทั้งหมดนี้ไว้ในรายงานฉบับนี้แล้ว บทความวิจัยทั้งหมดได้รับการตีพิมพ์ในวารสาร Journal of High Energy Physics และ Physical Review D ซึ่งเป็นวารสารทางวิชาการที่มีคุณภาพสูงในฐานข้อมูล ISI และ Scopus โดยมีค่า impact factor ในปี 2013 เท่ากับ 6.220 และ 4.864 ตามลำดับ

ผลงานวิจัยที่เกิดขึ้นในโครงการวิจัยนี้ส่งผลให้หัวหน้าโครงการวิจัยผู้รับทุนได้รับรางวัลนักวิทยาศาสร์รุ่นใหม่ประจำปี 2557 จากมูลนิธิส่งเสริมวิทยาศาสตร์และเทคโนโลยีในพระบรมราชูปถัมภ์ อีกทั้งหัวหน้าโครงการยังได้รับเชิญให้เข้าร่วมบรรยายในการประชุมวิชาการระดับนานาชาติดังต่อไปนี้

1. Invited speaker ในการประชุมวิชาการวิทยาศาสตร์และเทคโนโลยีแห่งประเทศไทยครั้งที่ 40 (STT 40) ภายใต้หัวข้อ “N=2 seven-dimensional gauged supergravity from M-theory”
2. Invited speaker ในการประชุมวิชาการ Siam Physics Congress (SPC) 2015 ครั้งที่ 10 ภายใต้หัวข้อ “N=2 seven-dimensional SO(4) gauged supergravity from eleven dimensions”

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3D supergravity from wrapped D3-branes

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Abstract: AdS_3 solutions dual to $\mathcal{N} = (0,2)$ SCFTs arise when D3-branes wrap Kähler two-cycles in manifolds with SU(4) holonomy. Here we review known AdS_3 solutions and identify the corresponding three-dimensional gauged supergravities, solutions of which uplift to type IIB supergravity. In particular, we discuss gauged supergravities dual to twisted compactifications on Riemann surfaces of both $\mathcal{N} = 4$ SYM and $\mathcal{N} = 1$ SCFTs with Sasaki-Einstein duals. We check in each case that c-extremization gives the exact central charge and R symmetry. For completeness, we also study AdS_3 solutions from intersecting D3-branes, generalise recent KK reductions of Detournay & Guica and identify the underlying gauged supergravities. Finally, we discuss examples of null-warped AdS_3 solutions to three-dimensional gauged supergravity, all of which embed in string theory.

Keywords: Supergravity Models, Supersymmetric Effective Theories, Gauge-gravity correspondence, AdS-CFT Correspondence

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Contents

1 Introduction

Gauged supergravity is a very useful tool in many areas of string theory such as flux compactifications and the AdS/CFT correspondence (see [1] for a review). Due to these applications, gauged supergravities in various dimensions as well as their Kaluza-Klein (KK) dimensional reductions have been extensively explored. It is well known that lower-dimensional gauged supergravities can be obtained from dimensional reductions of higher-dimensional theories. Up to now, many examples have appeared and amongst them, [2, 3] and [5–8] are recognizable primary examples. In this paper, we are interested in gauged supergravities in three dimensions in order to incorporate both the principle of c-extremization and null-warped AdS_3 solutions.

The complete classification of Chern-Simons gauged supergravities in three dimensions has been given in [9]. Most theories constructed in this formulation have no known higher-dimensional origin. The three-dimensional gauged supergravities obtainable from dimensional reductions form a small part, with non-semisimple gauge groups, in this classification [10]. Unlike in higher-dimensional analogues, only a few examples of three-dimensional gauged supergravities, which play an important role in $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence, have been obtained by dimensional reductions [11–13]. In this paper, we will extend this list with more examples of gauged supergravities in three dimensions arising from wrapped D3-branes in type IIB supergravity.

Recently, c-extremization for $\mathcal{N} = (0, 2)$ two-dimensional SCFT's has been proposed and various examples of gravity duals in five- and seven-dimensional gauged supergravities exhibited [14, 15]. Recall that c-extremization is a procedure that allows one to single out the correct $\mathrm{U}(1)_R$ symmetry of the CFT from the mixing with other U(1) symmetries. Soon after, c-extremization was formulated purely in the context of the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence by explicitly showing that, in the presence of a gauged $\mathrm{SO}(2)_R \sim \mathrm{U}(1)_R$ R symmetry, the so-called T tensor of the three-dimensional gauged supergravity can be extremized leading to the exact central charge and R symmetry [16]. This realization is similar to how a-maximization of four-dimensional SCFT's [17] can be encoded in five-dimensional gauged supergravity [18] in the context of the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence.[1] Interestingly, in three dimensions, not only is the central charge reproduced, but the moment maps comprising the T tensor give information about the exact R symmetry. In this work we will provide more details of the results quoted in [16] and also exhibit another (related) example by considering twists of generic SCFT's with Sasaki-Einstein duals.

In three dimensions, where a vector is dual to a scalar, the matter coupled supergravity theory can be formulated purely in terms of scalar fields resulting in a non-linear sigma model coupled to supergravity. $\mathcal{N} = 2$ supersymmetry in three dimensions requires the scalar target manifold to be Kähler. Gaugings of the theory are implemented by the embedding tensor specifying the way in which the gauge group is embedded in the global symmetry group. In general, the moment map of the embedding tensor, given by scalar matrices $\mathcal{V}$, determines the T tensor which plays an important role in computing the scalar potential and supersymmetry transformations. As a general result, $\mathcal{N} = 2$ supersymmetry allows any proper subgroup of the symmetry to be gauged. Furthermore, there is a possibility of other deformations through a holomorphic superpotential W. The scalar potential generally gets contributions from both the T tensor and the superpotential. However, any gauging of the R symmetry requires vanishing W.

The particular higher-dimensional theories we choose to reduce can all be motivated from the perspective of ten dimensions. From either an analysis of the Killing spinor equations [20], or by following wrapped D-brane intuition [21], it is known that supersymmetric AdS_3 solutions supported by the five-form RR flux of type IIB supergravity, or in other words, those corresponding to wrapped D3-branes, have seven-dimensional internal manifolds Y_7 and bear some resemblance to Sasaki-Einstein metrics. More precisely, Y_7

[1] A concrete realization is presented in [19].

can be expressed locally in terms of a natural U(1) fibration (the R symmetry) over a six-dimensional Kähler base that is subject to a single differential condition

$$\Box R = \frac{1}{2}R^2 - R_{ij}R^{ij}, \tag{1.1}$$

where R and R_{ij} are, respectively, the Ricci scalar and Ricci tensor of the metric of the Kähler manifold. Through the supersymmetry conditions [20, 21], the Ricci scalar R is related to an overall warp factor for the ten-dimensional space-time.

Of course the above equation can be simplified considerably by assuming that the Kähler manifold is also Einstein, but in general, solutions with non-trivial warp factors can be difficult to find. A search for IIB solutions tailored to this context can be found in [22], where a solution originally found in [23] was recovered. The challenges here are reminiscent of generalisations of direct-product AdS_4 and AdS_5 solutions to warped products. To support this observation, we recall that, for an Ansatz covering the most general supersymmetric warped AdS_5 solutions of type IIB supergravity [24], the only warped geometry[2] noted by the authors beyond the special case of Sasaki-Einstein was the Pilch-Warner solution [26]. On a more recent note, warped AdS_4 solutions of eleven-dimensional supergravity generalising Sasaki-Einstein have been found [27, 28]. In the face of these difficulties, it is a pleasant surprise to witness the ease at which supersymmetric solutions with warp factors can be constructed in five-dimensional supergravity through twisted compactifications on a constant curvature Riemann surface $\Sigma_{\mathfrak{g}}$ of genus $\mathfrak{g}$ and how the principle of c-extremization accounts for the central charge and exact R symmetry of the dual $\mathcal{N} = (0, 2)$ SCFT [14, 15].

c-extremization aside, we can further motivate the study of three-dimensional gauged supergravities through the continued interest in "null warped" AdS_3 space-times. Over the last few years, we have witnessed a hive of activity surrounding warped AdS_3 space-times and their field theory duals [29], primarily in Topologically Massive Gravity (TMG) [30, 31]. Indeed, the mere existence of these solutions and the fact that they are deformations of AdS_3 with $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)$ isometry, raises very natural questions about the putative dual CFT. Since relatively little is known about these theories, the common approach is to extract information holographically from warped AdS_3 solutions. To date, in three dimensions, warped AdS_3 solutions have cropped up in a host of diverse settings, including of course, solutions [29, 32, 33] to TMG, solutions [34] to New Massive Gravity [35], Higher-Spin Gravity [36], topologically gauged CFTs [37] and three-dimensional gravity with a Chern-Simons (CS) Maxwell term [38], where the latter is embeddable in string theory. As we shall see, within the last class of three-dimensional theories, one also finds gauged supergravities.

Indeed, "null warped" AdS_3 are central to efforts to generalise AdS/CFT to a non-relativistic setting, where holography may be applicable to condensed matter theory via a class of Schrödinger space-times. Taking the catalyst from [39, 40], through fledgling embeddings in string theory [41–44], various attempts have been made to provide a working description of non-relativistic holography. On one hand, one may wish to start with a

[2] A class of solutions can be generated via TsT transformations [25] starting from $AdS_5 \times S^5$, but as the transformation only acts on the internal S^5, the final solution is not warped.

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recognisable theory with Schrödinger symmetry, such as a non-relativistic limit [45, 46] of ABJM [47], but holographic studies [48–50] fail to capture the required high degree of supersymmetry. On the other hand, if one starts from gravity solutions with Schrödinger symmetry, one may be more pragmatic and obtain an effective description of the dual non-relativistic CFT, valid at large N and strong coupling [51].[3] Similar points of view were also advocated in [55–57]. Whether the dual theory is a genuine CFT as proposed in [29], or some warped CFT, is an open question drawing considerable attention.[4]

The structure of the rest of this paper is as follows. In section 2, we present an overview of our knowledge of supersymmetric AdS_3 geometries arising from wrapped D3-branes. In section 2.2, we focus on geometries with a U(1) R symmetry dual to $\mathcal{N} = (0, 2)$ SCFT's and present known examples preserving at least four supersymmetries, all of which will correspond to the vacua of the gauged supergravities we discuss later. In section 3, we provide more details of the KK reduction reported in [16]. In section 4.1, we present the three-dimensional gauged supergravity corresponding to a twisted compactification of an $\mathcal{N} = 1$ SCFT with a generic Sasaki-Einstein dual. In section 4.2, we generalise the KK reductions discussed in [38] and identify the corresponding gauged supergravities. In section 5 we present some simple constructions of null-warped AdS_3, or alternatively Schrödinger geometries with dynamical exponent $z = 2$, before discussing some open avenues for future study in section 6.

2 AdS_3 from wrapped D3-branes

2.1 Review of wrapped D3-branes

In this section we review supersymmetric AdS_3 geometries arising from D3-branes wrapping calibrated two-cycles in manifolds with SU(2), SU(3) and SU(4) holonomy. To this end, we follow the general ten-dimensional classification presented in [21] and later indicate where particular explicit solutions fit into the bigger picture. The approach of [21] builds on earlier work concerning wrapped M5-branes [59, 60] and M2-branes [61].

We recall that the general "wrapped-brane" strategy [59] involves first assuming that AdS_3 geometries start off as warped products of the form

$$\mathrm{d}s^2_{10} = L^{-1}\mathrm{d}s^2\left(\mathbb{R}^{1,1}\right) + \mathrm{d}s^2\left(\mathcal{M}_8\right), \tag{2.1}$$

where both the warp factor L and the metric on $\mathcal{M}_8$ are independent of the Minkowski factor. Here the Minkowski space-time should be regarded as the unwrapped part of the D3-brane, and as expected, the D3-branes source a self-dual RR five-form flux $F_5 = \Theta + *_{10}\Theta$ invariant under the symmetries of the Minkowski factor.

For the particular geometries of interest to us, the metric and the flux for the geometry may be expressed as [21]

$$\begin{aligned} \mathrm{d}s^2_{10} &= L^{-1}\mathrm{d}s^2\left(\mathbb{R}^{1,1}\right) + \mathrm{d}s^2\left(\mathcal{M}_{2d}\right) + L\mathrm{d}s^2\left(\mathbb{R}^{8-2d}\right), \\ \Theta &= \mathrm{vol}\left(\mathbb{R}^{1,1}\right) \wedge \mathrm{d}\left(L^{-1}J_{2d}\right), \end{aligned} \tag{2.2}$$

[3]Separately it has been argued [52, 53] that generic non-relativistic quantum field theories have a holographic description in terms of Hořava gravity [54].

[4]See [58] for a recent discussion.

wrapped brane	manifold	supersymmetry	R symmetry
Kähler 2-cycle	CY_2	$\mathcal{N}=(4,4)$	$\mathrm{SO}(4)\times\mathrm{U}(1)$
Kähler 2-cycle	CY_3	$\mathcal{N}=(2,2)$	$\mathrm{U}(1)\times\mathrm{U}(1)$
Kähler 2-cycle	CY_4	$\mathcal{N}=(0,2)$	$\mathrm{U}(1)$

Table 1. Wrapped D3-brane geometries and their supersymmetry.

where $d=2,3,4$. In each case we require the existence of globally defined $\mathrm{SU}(d)$ structures, specified by everywhere non-zero forms J_{2d},Ω_{2d} on $\mathcal{M}_{2d}$. The accompanying torsion conditions follow from the $\mathrm{SU}(4)\ltimes\mathbb{R}^8$ case of [62], with the conditions for smaller structure groups being determined through decompositions of the form

$$\begin{aligned} J_{2d+2} &= J_{2d}\pm e^{2d+1}\wedge e^{2d+2}, \\ \Omega_{2d+2} &= \Omega_{2d}\wedge\left(e^{2d+1}\pm ie^{2d+2}\right). \end{aligned} \tag{2.3}$$

As explained in detail in [21], the supersymmetry conditions for AdS_3 space-times may then be derived by introducing an AdS_3 radial coordinate r, writing the (unit radius) AdS_3 metric in the form

$$\mathrm{d}s^2\left(AdS_3\right)=e^{-2r}\mathrm{d}s^2\left(\mathbb{R}^{1,1}\right)+\mathrm{d}r^2, \tag{2.4}$$

redefining the warp factor, $L=e^{2r}\lambda$, and performing a frame rotation of the form

$$\lambda^{-\frac{1}{2}}\mathrm{d}r=\sin\theta\,\hat{u}+\cos\theta\,\hat{v}, \tag{2.5}$$

where θ parametrises the frame-rotation, which is further assumed to be independent of the AdS_3 radial coordinate, and $\hat{u},\hat{v}$ are respectively unit one-forms on $\mathcal{M}_{2d}$ and the overall transverse space.[5] Omitting various technicalities associated to this frame-rotation one arrives at a simple but effective derivation of the supersymmetry conditions for various AdS_3 space-times of type IIB supergravity. A summary of the outcome may be encapsulated in table 1 which we reproduce from [21].

As can be seen from the above table, in each case the cycle being wrapped is the same, but as the dimensionality of the Calabi-Yau n-fold (CY_n) increases, the preserved supersymmetry decreases. For D3-branes wrapping Kähler two-cycles in CY_2 manifolds, one can generically have $\mathrm{SO}(4)\times\mathrm{U}(1)$ R symmetry provided the radial direction (2.5) involves a rotation. Upon analytic continuation, one recovers the half-BPS LLM solutions [63] with isometry $\mathbb{R}\times\mathrm{SO}(4)\times\mathrm{SO}(4)\times\mathrm{U}(1)$, however there appear to be no known AdS_3 space-times in this class. On the contrary, when $\theta=0$, i.e. when the radial direction is purely transverse, one recovers the well known $AdS_3\times S^3\times CY_2$ solution[6] with R symmetry $\mathrm{SO}(4)$. In either case the supersymmetry is $\mathcal{N}=(4,4)$.

[5]For $\mathrm{SU}(4)$ structure manifolds there is no transverse space so there $\theta=\pi/2$.

[6]Specialising to $CY_2=T^4$ and performing T-dualities we arrive at the usual form of the D1-D5 near-horizon sourced by three-form RR flux. We also remark that the geometry sourced by five-form flux and three-form flux are also related via fermionic T-duality [64] as explained in [65].

For D3-branes wrapping Kähler two-cycles in CY_3, supersymmetry is reduced to $\mathcal{N} = (2,2)$, while the associated R symmetry group is $\mathrm{U}(1) \times \mathrm{U}(1)$. Examples of these space-times can be found in the literature [66, 67]. Finally, for D3-branes wrapping Kähler two-cycles in CY_4 the dual SCFTs preserve $\mathcal{N} = (0,2)$ supersymmetry and the $\mathrm{U}(1)$ Killing direction is dual to the R symmetry. A rich set of examples of these geometries exist in the literature [14, 15, 22, 23, 67, 68]. In the notation of [21], the metric and flux may be expressed as

$$\begin{aligned} \mathrm{d}s^2_{10} &= \lambda^{-1}\mathrm{d}s^2\left(AdS_3\right) + \lambda \mathrm{d}s^2\left(\mathcal{M}_6\right) + \lambda^{-1}\left(\mathrm{d}\psi + B\right)^2, & (2.6)\\ \Theta &= \mathrm{vol}\left(AdS_3\right) \wedge \left[\mathrm{d}\left(\lambda^{-2}(\mathrm{d}\psi + B)\right) - 2\lambda^{-1}J\right], & (2.7) \end{aligned}$$

where ∂_ψ is the Killing vector dual to the R symmetry. The SU(3) structure manifold $\mathcal{M}_6$ is subject to the the conditions [21]:

$$\begin{aligned} \mathrm{d}J &= 0, & (2.8)\\ J^2 \wedge \mathrm{d}B &= \frac{2}{3}\lambda^2 J^3, & (2.9)\\ \mathrm{d}\Omega &= 2i\left(\mathrm{d}\psi + B\right) \wedge \Omega\,. & (2.10) \end{aligned}$$

The first condition implies that $\mathcal{M}_6$ is a Kähler manifold, while the last condition simply identifies the Ricci form $\mathcal{R} = 2\mathrm{d}B$.

2.2 D3-branes with $\mathcal{N} = (0,2)$ SCFTs duals

Now that we have covered AdS_3 space-times arising from D3-branes wrapping Kähler two-cycles in Calabi-Yau manifolds in a general manner, here we focus on the particular case where the manifold is CY_4. Since this case preserves the least amount of supersymmetry, it includes geometries dual to two-dimensional SCFTs with $\mathcal{N} = (2,2)$ and $\mathcal{N} = (4,4)$ supersymmetry as special cases.

While the characterisation of wrapped D3-branes [21] presented in the previous section offers a welcome sense of overview, henceforth we switch to the notation of [22], which is itself based on the work of [20]. The generic AdS_3 solutions corresponding to wrapped D3-branes are then of the form [22],

$$\begin{aligned} \mathrm{d}s^2 &= L^2\left[e^{2A}\mathrm{d}s^2\left(AdS_3\right) + \frac{1}{4}e^{2A}\left(\mathrm{d}z + P\right)^2 + e^{-2A}\mathrm{d}s^2\left(\mathcal{M}_6\right)\right],\\ F_5 &= L^4\,\mathrm{vol}(AdS_3) \wedge \left[\frac{1}{2}J - \frac{1}{8}\mathrm{d}\left(e^{4A}\left(\mathrm{d}z + P\right)\right)\right]\\ &\quad + \frac{1}{16}L^4\left[J \wedge \mathcal{R} \wedge \left(\mathrm{d}z + P\right) + \frac{1}{2} *_6 \mathrm{d}R\right], \qquad (2.11) \end{aligned}$$

where L is an overall scale factor, $*_6$ refers to Hodge duality with respect to the metric of the Kähler space, $\mathrm{d}P = \mathcal{R}$, with $\mathcal{R}$ being the Ricci form on $\mathcal{M}_6$.[7] The warp factor is related to

[7]The Ricci form is defined by $\mathcal{R}_{ij} = \frac{1}{2}R_{ijkl}J^{kl}$, where R_{ijkl} is the Riemann tensor. Recall also that the Ricci scalar R and the Ricci tensor R_{ij} may be expressed in terms of the Ricci form as $R = J^{ij}\mathcal{R}_{ij}$ and $R_{ij} = -J_i{}^k\mathcal{R}_{kj}$.

the Ricci scalar through $8e^{-4A} = R$, a relation that can be inferred from (2.9). The closure of F_5 leads to the differential condition on the curvature (1.1). Finally, to make direct comparison with the previous incarnation of this solution (2.6), one can simply redefine

$$\lambda = e^{-2A}, \quad z = 2\psi, \quad P = 2B, \quad \Omega = e^{iz}\tilde{\Omega}, \tag{2.12}$$

where we have added a tilde to differentiate between complex forms. The five-form fluxes (2.7) and (2.11) are related up to a factor of -4 and follow from the choice of normalisation adopted in [20]. This point should be borne in mind when making comparisons.

Examples. To get better acquainted with the form of the general soution, we can consider some supersymmetric solutions that will correspond later to the vacua of our gauged supergravities. We begin with the well-known $AdS_3 \times S^3 \times T^4$ solution corresponding to the near-horizon geometry of two intersecting D3-branes. Via T-duality it is related to the D1-D5 near-horizon where the geometry is supported by a RR three-form.

To rewrite the solution in terms of the general description (2.11), we take

$$\begin{aligned} A &= 0, \\ \mathrm{d}z + P &= (\mathrm{d}\phi_3 - \cos\phi_1 \mathrm{d}\phi_2)\,, \\ \mathrm{d}s^2(\mathcal{M}_6) &= \mathrm{d}s^2(T^4) + \frac{1}{4}\left(\mathrm{d}\phi_1^2 + \sin^2\phi_1 \mathrm{d}\phi_2^2\right), \end{aligned} \tag{2.13}$$

where ϕ_i parametrise the coordinates on the S^3 normalised to unit radius, the same radius as the AdS_3 factor. Despite this solution fitting into the general ten-dimensional framework, it preserves sixteen supercharges and is dual to a SCFT with $\mathcal{N} = (4,4)$ supersymmetry.

Before illustrating the most general solution of [14, 15] in its ten-dimensional guise, we can satisfy the required supersymmetry condition

$$a_1 + a_2 + a_3 = -\kappa, \tag{2.14}$$

where κ is the curvature of the Riemann surface $\Sigma_{\mathfrak{g}}$, more simply through setting all the a_i equal, $a_i = \frac{1}{3}$, and taking the Riemann surface to be a unit radius Hyperbolic space, $\kappa = -1$. This solution originally featured in [67]. With these simplifications the solution reads

$$\begin{aligned} \mathrm{d}s^2 &= \frac{4}{9}\mathrm{d}s^2(AdS_3) + \frac{1}{3}\mathrm{d}s^2(H^2) + \sum_{i=1}^{3} \mathrm{d}\mu_i^2 + \mu_i^2\left(\mathrm{d}\varphi_i + \hat{A}\right)^2, \\ F_5 &= (1 + *)\left[-\frac{32}{81}\,\mathrm{vol}(AdS_3)\wedge\mathrm{vol}(H^2) - \frac{4}{27}\,\mathrm{vol}(AdS_3)\wedge\sum_{i=1}^{3}\mathrm{d}\left(\mu_i^2\right)\wedge\left(\mathrm{d}\varphi_i + \hat{A}\right)\right], \end{aligned} \tag{2.15}$$

where the μ_i are constrained so that $\sum_{i=1}^{3}\mu_i^2 = 1$. Note now that all A_i are equal, $A_i = \hat{A}$, and $\mathrm{d}\hat{A} = -\frac{1}{3}\,\mathrm{vol}(H^2)$. It is easy to determine the one-form $K = \frac{1}{2}e^{2A}(\mathrm{d}z + P)$ corresponding to the R symmetry direction

$$K = \frac{2}{3}\left[\sum_{i=1}^{3}\mu_i^2\left(\mathrm{d}\varphi_i + \hat{A}\right)\right], \tag{2.16}$$

and check that it has the correct norm $K^2 = e^{2A} = \frac{4}{9}$ [20]. Taking into account the factor of -4 in the definitions of the flux, and also setting $L = 1$, we then learn from comparing (2.11) with (2.15) that

$$-\frac{32}{81}\,\mathrm{vol}(H^2) - \frac{4}{27}\sum_{i=1}^{3}\mathrm{d}(\mu_i^2)\wedge(\mathrm{d}\varphi_i + \hat{A}) = -2J + \mathrm{d}(e^{2A}K). \tag{2.17}$$

One can then determine J

$$J = \frac{4}{27}\,\mathrm{vol}\left(H^2\right) + \frac{2}{9}\sum_{i=1}^{3}\mathrm{d}\left(\mu_i^2\right)\wedge\left(\mathrm{d}\varphi_i + \hat{A}\right), \tag{2.18}$$

which comes with the correct factor of $\mathrm{vol}(H^2)$,

$$\begin{aligned}\mathrm{d}s^2\left(\mathcal{M}_6\right) = \frac{4}{27}\mathrm{d}s^2(H^2) + \frac{4}{9}\Big[&\mathrm{d}\mu_1^2 + \mathrm{d}\mu_2^2 + \mathrm{d}\mu_3^2 + \mu_1^2\mu_2^2\left(\mathrm{d}\varphi_1 - \mathrm{d}\varphi_2\right)^2 \\ &+ \mu_1^2\mu_3^2\left(\mathrm{d}\varphi_1 - \mathrm{d}\varphi_3\right)^2 + \mu_2^2\mu_3^2\left(\mathrm{d}\varphi_2 - \mathrm{d}\varphi_3\right)^2\Big],\end{aligned} \tag{2.19}$$

so that $\mathrm{vol}(\mathcal{M}_6) = \frac{1}{3!}J^3$. Observe also that J is independent of K since $\mu_i \mathrm{d}\mu_i = 0$ follows from the fact that the μ_i are constrained. In addition, the final difference in angular coordinates $\varphi_2 - \varphi_3$ can be written as a linear combination of the other two, so we only have four directions separate from those along the H^2. As a further consistency check, we have confirmed that the Ricci scalar for $\mathcal{M}_6$ is $R = 8e^{-4A}$, in line with our expectations.

We can now repeat for general a_i subject to the single constraint (2.14). This also comprises the only example we discuss where the warp factor A is not a constant. In the notation of [14, 15], the ten-dimensional solution is

$$\mathrm{d}s^2 = \Delta^{\frac{1}{2}}\left[e^{2f}\mathrm{d}s^2\left(AdS_3\right) + e^{2g}\mathrm{d}s^2\left(\Sigma_{\mathfrak{g}}\right)\right] + \Delta^{-\frac{1}{2}}\sum_{i=1}^{3}X_i^{-1}\left(\mathrm{d}\mu_i^2 + \mu_i^2\left(\mathrm{d}\varphi_i + A^i\right)^2\right), \tag{2.20}$$

$$F_5 = (1+*)\,\mathrm{vol}\left(AdS_3\right)\wedge\sum_{i=1}^{3}e^{3f+2g}\left[2X^i\left(X_i^2\mu_i^2 - \Delta\right)\mathrm{vol}\left(\Sigma_{\mathfrak{g}}\right) - \frac{a_i}{2\,e^{4g}X_i^2}\mathrm{d}\left(\mu_i^2\right)\wedge\left(\mathrm{d}\varphi + A_i\right)\right],$$

where

$$\Delta = \sum_{i=1}^{3}X_i\mu_i^2, \qquad X_1X_2X_3 = 1, \tag{2.21}$$

and as before the μ_i are constrained. The constrained scalars X_i can be expressed in terms of two scalars φ_i in the following way

$$X_1 = e^{-\frac{1}{2}\left(\frac{2}{\sqrt{6}}\varphi_1 + \sqrt{2}\varphi_2\right)}, \qquad X_2 = e^{-\frac{1}{2}\left(\frac{2}{\sqrt{6}}\varphi_1 - \sqrt{2}\varphi_2\right)}, \qquad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}. \tag{2.22}$$

To give the full form of the solution one also needs to specify the values of the various warp factors e^f, e^g and scalars X_i [14]:[8]

$$\begin{aligned} e^f &= \frac{2}{X_1+X_2+X_3}, & e^{2g} &= \frac{a_1X_2+a_2X_1}{2}, \\ X_1X_3^{-1} &= \frac{a_1\left(a_2+a_3-a_1\right)}{a_3\left(a_1+a_2-a_3\right)}, & X_2X_3^{-1} &= \frac{a_2\left(a_1+a_3-a_2\right)}{a_3\left(a_1+a_2-a_3\right)}. \end{aligned} \tag{2.23}$$

From the higher-dimensional perspective afforded to us here, the canonical R symmetry corresponds with the Killing vector [15]

$$\partial_\psi = 2\sum_{i=1}^{3}\frac{X_i}{X_1+X_2+X_3}\partial_{\varphi_i}. \tag{2.24}$$

Again, one is in a position to determine the dual one-form

$$K = e^f\Delta^{-\frac{1}{2}}\sum_{i=1}^{3}\mu_i^2\left(\mathrm{d}\varphi_i + A_i\right), \tag{2.25}$$

and confirm that it squares correctly $K^2 = e^{2A} = \Delta^{\frac{1}{2}}e^{2f}$. Proceeding in the same fashion as above, one can then determine J

$$J = \sum_{i=1}^{3}\frac{1}{4}\left[-\frac{\Theta}{a_i\left(2a_i+\kappa\right)}e^{3f}\mathrm{d}\left(\mu_i^2\right)\wedge\left(\mathrm{d}\varphi_i+A_i\right) + 2a_i\left(2a_i+\kappa\right)\frac{\Theta}{\Pi}\mu_i^2e^{3f}\,\mathrm{vol}\left(\Sigma_\mathfrak{g}\right)\right], \tag{2.26}$$

where we have adopted the notation of [15], namely

$$\begin{aligned} \Theta &= a_1^2+a_2^2+a_3^2 - 2\left(a_1a_2+a_1a_3+a_2a_3\right), \\ \Pi &= \left(-a_1+a_2+a_3\right)\left(a_1-a_2+a_3\right)\left(a_1+a_2-a_3\right). \end{aligned} \tag{2.27}$$

The accompanying expression for the manifold $\mathcal{M}_6$ is

$$\begin{aligned} \mathrm{d}s^2\left(\mathcal{M}_6\right) = \Delta e^{2g+2f}\mathrm{d}s^2\left(\Sigma_\mathfrak{g}\right) + e^{2f}\Big[&X_1^{-1}\mathrm{d}\mu_1^2 + X_2^{-1}\mathrm{d}\mu_2^2 + X_3^{-1}\mathrm{d}\mu_3^2 \\ &+\frac{X_3}{\Delta}\mu_1^2\mu_2^2\left(X_2\mathrm{D}\varphi_1 - X_1\mathrm{D}\varphi_2\right)^2 + \frac{X_2}{\Delta}\mu_1^2\mu_3^2\left(X_3\mathrm{D}\varphi_1 - X_1\mathrm{D}\varphi_3\right)^2 \\ &+\frac{X_1}{\Delta}\mu_2^2\mu_3^2\left(X_3\mathrm{D}\varphi_2 - X_2\mathrm{D}\varphi_3\right)^2\Big], \end{aligned} \tag{2.28}$$

where we have further defined $\mathrm{D}\varphi_i = \mathrm{d}\varphi_i + A_i$. One can check it is consistent with the expression for J and furthermore that one recovers the previous expressions upon simplification, i.e. setting $a_i = \frac{1}{3}$, $\kappa = -1$.

These solutions will all be utilised later when we come to discuss three-dimensional gauged supergravities with vacua corresponding to the above supersymmetric solutions. In the next section, we begin by discussing an example of a generic reduction, in other words one where the warp factor is not a constant, by providing further details of the reduction and resulting three-dimensional $\mathcal{N}=2$ supergravity initially reported in [16].

[8] The solutions with $\mathfrak{g}=1$ were studied in [69], while for $\mathfrak{g}=0$, $\mathfrak{g}>1$, modulo issues related to the range of the parameters, the solutions can be mapped to (4.6) of [70] through interchanging the scalars $\phi_1 \leftrightarrow -\phi_2$ and redefining the parameters accordingly $a_i = -\epsilon m_i/(m_1+m_2+m_3)$, where $\epsilon = 1$ for $\Sigma_\mathfrak{g} = S^2$ and $\epsilon = -1$ for $\Sigma_\mathfrak{g} = H^2$.

3 An example of a generic reduction

In this section we illustrate an example of a generic reduction, where we use the word "generic" to draw a line between dimensional reductions with non-trivial warp factors from the ten-dimensional perspective, and those that are direct products. Recall that, in addition to the famous KK reductions based on spheres [2, 3, 5–8], which give rise to maximal gauged supergravities in lower dimensions, generic KK reductions based on gaugings of R symmetry groups, notably gaugings of U(1) R symmetry [71, 72] and SU(2) R symmetry [73, 74] exist despite the internal space not being a sphere. This observation leads to the natural conjecture [72] that gaugings of R symmetry groups are intimately connected to the existence of consistent KK dimensional reductions. Here should be no exception, so we expect that one can gauge the existing U(1) R symmetry present in (2.11) and reduce to three dimensions.

However, in contrast to similar reductions to four and five dimensions, for instance [71, 72], here in addition to retaining the gauge field from the R symmetry gauging, we also require an additional scalar so that the three-dimensional gauged supergravity fits into the structure of $\mathcal{N} = 2$ gauged supergravity as laid out in [9]. More concretely, we require an even number of scalars to constitute a Kähler scalar manifold. While the reduction we discuss presently assumes additional structure for the $\mathcal{M}_6$, i.e. the existence of a Riemann surface, it would be interesting to identify truly generic reductions without having to specify the internal six-dimensional Kähler manifold.

Here we will present further details of the dimensional reduction from five-dimensional $\mathrm{U}(1)^3$ gauged supergravity to three-dimensional $\mathcal{N} = 2$ gauged supergravity reported in [16]. While not being the most general reduction, from the ten-dimensional vantage point it provides a neat example of a reduction where the warp factor, and the associated Ricci scalar of the internal $\mathcal{M}_6$, is not a constant. We also do not need to address the full embedding of the three-dimensional theory in ten dimensions, since we can work with the $\mathrm{U}(1)^3$ gauged supergravity in five dimensions.

The bosonic sector of the action for five-dimensional $\mathrm{U}(1)^3$ gauged supergravity can be found in [75]. It arises as a consistent reduction from type IIB on S^5, so it is directly connected to ten dimensions[9] via the equations of motion, and corresponds to the special case where only the $\mathrm{SO}(2)^3$ Cartan subgroup of SO(6) is gauged. The action reads

$$\begin{aligned}\mathcal{L}_5 = R * \mathbf{1} - \frac{1}{2}\sum_{i=1}^{2} \mathrm{d}\varphi_i \wedge *\mathrm{d}\varphi_i - \frac{1}{2}\sum_{i=1}^{3} X_i^{-2} F^i \wedge *F^i \\ + 4g^2 \sum_{i=1}^{3} X_i^{-1}\,\mathrm{vol}_5 + F^1 \wedge F^2 \wedge A^3,\end{aligned} \tag{3.1}$$

[9]The bosonic sector also appears as a reduction from $D = 11$ supergravity [76] where it is based on the existence of near-horizon black holes [77]. Interestingly, one can start from $D = 11$ and reduce to $D = 4$ $\mathrm{U}(1)^4$ gauged supergravity, which, for consistency, requires $F^i \wedge F^j = 0$. Taking a near-horizon limit prescribed in [77] one finds the bosonic sector of $D = 5$ $\mathrm{U}(1)^3$ gauged supergravity, without such a condition.

where g is the gauge coupling and the constrained scalars X_i we have defined earlier (2.22). From varying the potential with respect to the scalars it is easy to see that there is only a single supersymmetric AdS_5 vacuum at $X_i = 1$.

As commented in [75], or by inspection from the equations of motion in appendix C, one can consistently truncate the theory by setting first $\varphi_2 = 0$ implying that $X_1 = X_2 = X_3^{-1/2}$. This truncation is consistent provided $F^1 = F^2$. Furthermore, one can take an additional step and set $\varphi_1 = 0$ leading to minimal gauged supergravity in five dimensions.

Dimensional reduction. As it turns out, this dimensional reduction can be performed consistently at the level of the action. Simply put, this means that we can adopt the space-time metric Ansatz

$$\mathrm{d}s_5^2 = e^{-4C}\mathrm{d}s_3^2 + e^{2C}\mathrm{d}s^2(\Sigma_{\mathfrak{g}}) \tag{3.2}$$

where $\Sigma_{\mathfrak{g}}$ is a constant curvature Riemann surface of genus $\mathfrak{g}$ and we have used C to denote the scalar warp factor in five dimensions. In addition, we have orchestrated the warp factors so that we arrive directly in Einstein frame in three dimensions.

The metric on the Riemann surface may be expressed as

$$\mathrm{d}s^2\left(\Sigma_{\mathfrak{g}}\right) = e^{2h}\left(\mathrm{d}x^2 + \mathrm{d}y^2\right), \tag{3.3}$$

where the function h depends on the curvature κ of the Riemann surface. It is respectively, $h = -\log\left((1+x^2+y^2)/2\right)$ $(\kappa = 1)$, $h = \log(2\pi)/2$ $(\kappa = 0)$ and $h = -\log(y)$ $(\kappa = -1)$, depending on whether the genus is $\mathfrak{g} = 0, \mathfrak{g} = 1$, or $\mathfrak{g} > 1$. In addition, one takes the following Ansatz for the field strengths,

$$F^i = G^i - a_i\,\mathrm{vol}\left(\Sigma_{\mathfrak{g}}\right), \tag{3.4}$$

where closure of F^i ensures that a_i are constants and G^i is closed, $G^i = \mathrm{d}B^i$.

In doing the reduction at the level of the action the following expression for the five-dimensional Ricci scalar is useful

$$R*\mathbf{1} = R*_3\mathbf{1} - 6\mathrm{d}C\wedge *_3\mathrm{d}C + 2\kappa e^{-6C}*_3\mathbf{1}. \tag{3.5}$$

The resulting three-dimensional action in Einstein frame is

$$\begin{aligned}\mathcal{L}^{(3)} = {} & R*_3\mathbf{1} - 6\mathrm{d}C\wedge *_3\mathrm{d}C - \frac{1}{2}\sum_{i=1}^{2}\mathrm{d}\varphi_i\wedge *_3\mathrm{d}\varphi_i - \frac{1}{2}e^{4C}\sum_{i=1}^{3}X_i^{-2}G^i\wedge *_3 G^i \\ & + \left(\sum_i^3\left[4g^2e^{-4C}X_i^{-1} - \frac{1}{2}e^{-8C}a_i^2X_i^{-2}\right] + 2\kappa e^{-6C}\right)*\mathbf{1} + \mathcal{L}^{(3)}_{\text{top}},\end{aligned} \tag{3.6}$$

where the topological term takes the form

$$\mathcal{L}^{(3)}_{\text{top}} = a_1B^2\wedge G^3 + a_2B^3\wedge G^1 + a_3B^1\wedge G^2. \tag{3.7}$$

We remark that the reduction and the resulting potential appeared previously in [78]. In appendix C, we have confirmed that it is indeed consistent.

Dualising the action. Now that we have the action, we would like to rewrite it in the form of a three-dimensional non-linear sigma model coupled to supergravity so that we can make contact with three-dimensional gauged supergravities in the literature [9]. We take our first steps in that direction by dualising the gauge fields, or more appropriately, their field strengths, and replacing them with scalars:

$$\begin{aligned} G^1 &= X_1^2 e^{-4C} * \mathrm{D}Y_1, \qquad & \mathrm{D}Y_1 &= \mathrm{d}Y_1 + a_3 B^2 + a_2 B^3, \\ G^2 &= X_2^2 e^{-4C} * \mathrm{D}Y_2, \qquad & \mathrm{D}Y_2 &= \mathrm{d}Y_2 + a_1 B^3 + a_3 B^1, \\ G^3 &= X_3^2 e^{-4C} * \mathrm{D}Y_3, \qquad & \mathrm{D}Y_3 &= \mathrm{d}Y_3 + a_1 B^2 + a_2 B^1. \end{aligned} \tag{3.8}$$

Through these redefinitions, we can recast the action (3.6) in the following form

$$\begin{aligned} \mathcal{L}^{(3)} = R * \mathbf{1} - 6\mathrm{d}C \wedge *\mathrm{d}C - \frac{1}{2}\sum_{i=1}^{2} \mathrm{d}\varphi_i \wedge *\mathrm{d}\varphi_i - \frac{1}{2} e^{-4C} \sum_{i=1}^{3} X_i^2 \mathrm{D}Y_i \wedge *\mathrm{D}Y_i \\ + \mathcal{L}^{(3)}_{\mathrm{pot}} + a_1 B^2 \wedge G^3 + a_2 B^3 \wedge G^1 + a_3 B^1 \wedge G^2, \end{aligned} \tag{3.9}$$

where we have omitted the explicit form of the potential as it will play no immediate role. We have also dropped all subscripts for Hodge duals on the understanding that we are now confining our interest to three dimensions. Note that the Chern-Simons terms are untouched and when we vary with respect to B^i we recover the duality conditions (3.8), so it should be clear that the equations of motion are the same and we have just rewritten the action.

At this point, before blindly stumbling on, we will attempt to motivate the expected gauged supergravity. Firstly, we know from the Killing spinor analysis in [15] that the AdS_3 solutions generically preserve four supersymmetries, meaning we are dealing with $\mathcal{N} = 2$ supersymmetry in three dimensions. Indeed, for $\mathcal{N} = 2$, we have precisely an SO(2) R symmetry group under which the gravitini transform and in this case the target space is a Kähler manifold with the scalars pairing into complex conjugates. Naturally, a prerequisite for a Kähler manifold is that we have an even number of scalars, and we observe that after dualising, this is indeed the case. So, we will now push ahead and identify some features of the $\mathcal{N} = 2$ gauged supergravity.

To identify the scalar manifold it is good to diagonalise the scalars by redefining them in the following way

$$\begin{aligned} W_1 &= 2C + \frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2, \\ W_2 &= 2C + \frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2, \\ W_3 &= 2C - \frac{2}{\sqrt{6}}\varphi_1 . \end{aligned} \tag{3.10}$$

In terms of the original X_i these new scalars are simply $e^{W_i} = e^{2C} X_i^{-1}$.

With these redefinitions, the Kähler manifold now assumes the simple form

$$\mathcal{L}^{(3)}_{\mathrm{scalar}} = -\frac{1}{2}\sum_{i=1}^{3} \left[\mathrm{d}W_i \wedge *\mathrm{d}W_i + e^{-2W_i}\mathrm{D}Y_i \wedge *\mathrm{D}Y_i\right] \tag{3.11}$$

and we are in a position to identify it as $[\mathrm{SU}(1,1)/\mathrm{U}(1)]^3$. The Kähler structure of the scalar target space can be made fully explicit through the introduction of the Kähler potential of the form

$$\mathcal{K} = -\sum_{i=1}^{3} \log\left(\Re z_i\right) , \tag{3.12}$$

where we have introduced complex coordinates $z_i = e^{W_i} + iY_i$. This means that the metric for the manifold is $g_{i\bar{i}} = \partial_i \partial_{\bar{i}} \mathcal{K} = \frac{1}{4} e^{-2W_i}$, where $\partial_i = \partial_{z_i}, \partial_{\bar{i}} = \partial_{\bar{z}_i}$.

Having identified the scalar manifold and the Kähler potential, we turn our attention to the scalar potential. In the language of three-dimensional $\mathcal{N}=2$ gauged supergravity [9], the scalar potential is comprised of two components, a T tensor and a superpotential W:

$$\mathcal{L}^{(3)}_{\text{pot}} = 8T^2 - 8g^{i\bar{i}} \partial_i T \partial_{\bar{i}} T + 8e^{\mathcal{K}} |W|^2 - 2g^{i\bar{i}} e^{\mathcal{K}} D_i W D_{\bar{i}} \bar{W}, \tag{3.13}$$

where the Kähler covariant derivative is $D_i W \equiv \partial_i W + \partial_i \mathcal{K} W$ and W is holomorphic, so $\partial_i \bar{W} = \partial_{\bar{i}} W = 0$. While W plays a natural role when eleven-dimensional supergravity is reduced on $S^2 \times CY_3$ to three dimensions [11], whenever the R symmetry is gauged, consistency demands that $W = 0$. Thus, to make contact with the literature, we face the simpler task of identifying the correct T tensor and making sure that the potential is recovered.

After rewriting the scalars, the potential takes the more symmetric form

$$\begin{aligned}\mathcal{L}^{(3)}_{\text{pot}} &= 4g^2 \left[e^{-W_1 - W_3} + e^{-W_2 - W_3} + e^{-W_1 - W_2} \right] + 2\kappa e^{-W_1 - W_2 - W_3} \\ &\quad - \frac{1}{2} \left[a_1^2 \, e^{-2(W_2 + W_3)} + a_2^2 \, e^{-2(W_1 + W_3)} + a_3^2 \, e^{-2(W_1 + W_2)} \right] .\end{aligned} \tag{3.14}$$

Note that in performing the reduction we have not been picky about supersymmetry and *a priori*, neglecting the gauge coupling g, which can be set to one, the constants κ and a_i are unrelated. However, setting $g = 1$ for simplicity, one can find the appropriate T tensor

$$T = -\frac{1}{4} \left[a_1 e^{-W_2 - W_3} + a_2 e^{-W_1 - W_3} + a_3 e^{-W_1 - W_2} \right] + \frac{1}{2} \left[e^{-W_1} + e^{-W_2} + e^{-W_3} \right], \tag{3.15}$$

and check that it reproduces the potential on the nose provided (2.14) is satisfied. This is precisely the condition identified in [14, 15] for supersymmetry to be preserved. Though it happens that the existence of what is commonly referred to as a "superpotential", in this case T, could conceivably be related to some fake supersymmetry structure for the theory, the fact that we recover the supersymmetry condition is reassuring. In fact, in appendix C.1 we reduce some of the Killing spinor equations and show that they also lead to the same T tensor. Thus, once the potential (and also T) is extremised, the Killing spinor equations are satisfied.

Central charge and exact R symmetry. At this stage it should be obvious that we have a potential with a supersymmetric critical point provided condition (2.14) holds. Furthermore, once we extremise T, we in turn extremise the potential and arrive at the supersymmetric AdS_3 vacuum. As discussed in [16], the extremization of the T tensor offers a natural supergravity counterpart for c-extremization [14, 15]. Recall that c-extremization has been proposed for SCFTs with $\mathcal{N} = (0,2)$ supersymmetry as a means to identify the exact central charge and R symmetry where ambiguities exist due to the U(1) R symmetry mixing with other global U(1) symmetries that may be present.

Like the trial c-function proposed in [14, 15], T is also quadratic and comes from squaring the moment maps $\mathcal{V}^i$

$$T = 2\mathcal{V}^i \Theta_{ij} \mathcal{V}^j, \tag{3.16}$$

contracted with the embedding tensor Θ_{ij} [9], where the index i ranges over the various U(1) symmetries, which for the immediate example, $i = 1, 2, 3$. In addition, since the embedding tensor also appears in the Chern-Simons terms in the action, it also related to the 't Hooft anomaly coefficients which appear in the trial c-function for c-extremization [14, 15]. Indeed, for the class of wrapped D3-brane geometries discussed in [14, 15] this can all be made precise through the relations [16]

$$c_R = 3\eta_\Sigma d_G T^{-1}, \quad R = 2\,\mathcal{V}^i T^{-1} Q_i\,, \tag{3.17}$$

where c_R is the exact central charge, R is the exact R symmetry, η_Σ is related to the volume of the Riemann surface, $\eta_\Sigma = 2\pi\,\mathrm{vol}(\Sigma_{\mathfrak{g}})$, d_G is the dimension of the gauge group and Q_i denotes the charges corresponding to the U(1) currents.

All that remains to do is simply to identify the minimum of the potential by extremising T. The critical point of T corresponds to the following values for the scalars:

$$\begin{aligned} W_1 &= \ln\left[\frac{a_2 a_3}{a_2 + a_3 - a_1}\right], \qquad W_2 = \ln\left[\frac{a_1 a_3}{a_1 + a_3 - a_2}\right] \\ W_3 &= \ln\left[\frac{a_1 a_2}{a_1 + a_2 - a_3}\right]. \end{aligned} \tag{3.18}$$

Once written in terms of C, φ_1 and φ_2 or in terms of C and X_i, this precisely gives the AdS_3 critical point of [14]. Then, slotting the critical value of T into the (3.17), we arrive at the exact central charge and R symmetry,

$$c_R = -12\eta_\Sigma N^2 \frac{a_1 a_2 a_3}{\Theta}, \tag{3.19}$$

$$R = \frac{2a_i\,(2a_i + \kappa)}{\Theta}\,, \tag{3.20}$$

where we have made use of (2.27) to display the result. In deriving (3.19) we have used the fact that the dimension of the gauge group at large N is $d_G = N^2$, while for (3.20) it is good to use the fact that the moment map is $\mathcal{V}_i = \frac{1}{4} e^{-W_i}$. The central charge and R symmetry agree with those quoted in [14, 15] and reproduce the coefficients of the Killing vector corresponding to the R symmetry (2.24).

4 Less generic reductions

Experience suggests that it is much easier to construct KK reduction Ansätze for direct product solutions than those that are warped products. This should come as no surprise since warped products are often more involved and consequently it may not be easy to identify a symmetry principle to guide the construction of a fitting Ansatz. For dimensional reductions from ten or eleven dimensions to five-dimensional gauged supergravities admitting AdS_5 vacua, the restrictions are quite clear. Starting with coset reductions [5–8, 79, 80], through generic Sasaki-Einstein reductions [81–84] to the more general cases, the richness of the reduced theory gradually decreases until one is left with minimal gauged supergravity [71, 72]. For warped AdS_5 solutions, only reductions to minimal gauged supergravity are known, with a notable exception being KK reductions [85] based on $Y^{p,q}$ spaces [86, 87], which when uplifted to eleven dimensions, the vacua correspond to warped solutions.

In this section we will discuss KK reductions to three dimensions confined to the special case where the Kähler manifold is a product of Kähler-Einstein spaces. As a direct consequence, (1.1) simplifies to

$$R^2 = 2R_{ij}R^{ij}. \tag{4.1}$$

A nice treatment of this special case can be found in [22] which we follow. We take the internal Kähler manifold to be a product of a set of two-dimensional Kähler-Einstein metrics

$$\mathrm{d}s^2\left(\mathcal{M}_6\right) = \sum_{i=1}^{3} \mathrm{d}s^2\left(KE_2^{(i)}\right). \tag{4.2}$$

Since $\mathcal{M}_6$ now has constant curvature, it is easy to satisfy (4.1). The Ricci form for $\mathcal{M}_6$ takes the form

$$\mathcal{R} = \sum_{i=1}^{3} l_i J_i\,, \tag{4.3}$$

where J_i are the Kähler forms of the constituent metrics and the constants l_i are zero, positive or negative depending on whether the metric is locally that on T^2, S^2 or H^2. We also have the one-form connection $P = \sum_i P_i$ with $\mathrm{d}P = \sum_i l_i J_i$. Slotting (4.3) into (4.1) we find a single constraint on the l_i

$$l_1 l_2 + l_1 l_3 + l_2 l_3 = 0\,, \tag{4.4}$$

and discover that the overall warp factor is determined,

$$e^{-4A} = \frac{1}{8}R = \frac{1}{4}\sum_i l_i\,. \tag{4.5}$$

Finally, the expression for the five-form flux (2.11) simplifies and assumes the following form

$$F_5 = (1+*)\,L^4\,\mathrm{vol}\left(AdS_3\right)\wedge\frac{1}{2\sum_i l_i}\left[J_1\left(l_2+l_3\right)+J_2\left(l_1+l_3\right)+J_3\left(l_1+l_2\right)\right]. \tag{4.6}$$

We now can make some comments. Demanding that the ten-dimensional space-time has the correct signature, we require $R > 0$ from (4.5). In the light of (4.4), this means that the potential solutions are constrained to be either $S^2\times T^4$ or $S^2\times S^2\times H^2$. The first option here corresponds to the famous intersecting D3-branes solution, while the second case was considered in [67]. We note that when the $KE_2^{(i)}$ space is H^2, it is a well-known fact that one can quotient the space without breaking supersymmetry leading to a compact Riemann surface with genus $\mathfrak{g} > 1$. The Ricci tensor for these solutions can be found in appendix D.

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4.1 Twists of SCFTs with Sasaki-Einstein duals

In this section we will discuss KK reductions on the first class of products of Kähler-Einstein spaces by confining our attention to spaces with curvature, $l_i \neq 0$. For simplicity, we will take $l_1 = l_2$, and the requirement that the scalar curvature of the internal $\mathcal{M}_6$ be positive (4.5) subject to (4.4) means that there is only one case, namely $\mathcal{M}_6 = H^2 \times KE_4$, where KE_4 is a positively curved Kähler-Einstein manifold.[10] For concreteness, we take $(l_1, l_2, l_3) = (2, 2, -1)$ so that the H^2 is canonically normalised.

Our next task is to construct a ten-dimensional Ansatz. While we could begin from scratch, we can incorporate some results from the literature as, in the end, a natural question concerns how they may be related. So we opt to kill two birds with one stone by simply reducing the IIB reduction on a generic Sasaki-Einstein five-manifold SE_5 [81–84] further to three dimensions on a constant curvature Riemann surface (H^2). We will follow the notation of [82] and subsequent comments are in the context of that work.

To achieve our goal, we make two simplifications. Firstly, we truncate out the complex two-form L_2, since as our internal space is now six-dimensional, a complex $(2,0)$-form, Ω_2, is less natural. We can easily replace it with a field coupling to the complex $(3,0)$-form Ω_3 via the five-form flux, but this will simply give us an additional complex scalar. More importantly, one can ask what is the fate of the complex scalars ξ and χ under dimensional reduction. Recall that they feature prominently in embeddings of holographic superconductors [88] (see also [89, 90]). However, since ξ, χ couple to the graviphoton A_1, it is not possible to twist A_1 in the usual way to produce a supersymmetric AdS_3 vacuum without truncating out ξ and χ. As such, we will have nothing to say about models for holographic superconductivity here. Moreover, as the same fields support the non-supersymmetric Romans' vacuum in five dimensions, we do not expect to find an analogue in three dimensions that follows from the reduction procedure.

The five-dimensional action in Einstein frame can be found in (3.10) of [82]. With the above simplifications taken onboard, for completeness, we reproduce the kinetic term

$$\begin{aligned}\mathcal{L}^{(5)}_{\text{kin}} = {} & R\ \text{vol}_5 - \frac{28}{3}\text{d}U \wedge *\text{d}U - \frac{8}{3}\text{d}U \wedge *\text{d}V - \frac{4}{3}\text{d}V \wedge *\text{d}V - \tfrac{1}{2}e^{2\phi}\text{d}a \wedge *\text{d}a \\ & - \tfrac{1}{2}\text{d}\phi \wedge *\text{d}\phi - 2e^{-8U}K_1 \wedge *K_1 - e^{-4U-\phi}H_1 \wedge *H_1 - e^{-4U+\phi}G_1 \wedge *G_1 \\ & - \tfrac{1}{2}e^{\frac{8}{3}(U+V)}F_2 \wedge *F_2 - e^{-\frac{4}{3}(U+V)}K_2 \wedge *K_2 - \tfrac{1}{2}e^{\frac{4}{3}(2U-V)-\phi}H_2 \wedge *H_2 \\ & - \tfrac{1}{2}e^{\frac{4}{3}(2U-V)+\phi}G_2 \wedge *G_2 - \tfrac{1}{2}e^{\frac{4}{3}(4U+V)-\phi}H_3 \wedge *H_3 - \tfrac{1}{2}e^{\frac{4}{3}(4U+V)+\phi}G_3 \wedge *G_3\,,\end{aligned} \tag{4.7}$$

the scalar potential

$$\mathcal{L}^{(5)}_{\text{pot}} = \left[24e^{-\frac{2}{3}(7U+V)} - 4e^{\frac{4}{3}(-5U+V)} - 8e^{-\frac{8}{3}(4U+V)}\right]\text{vol}_5\,, \tag{4.8}$$

[10] Suitable choices for KE_4 include $S^2 \times S^2$, $\mathbb{C}P^2$ and del Pezzo dP_k, $k = 3, \ldots, 8$.

and the topological terms are given by the expression

$$\begin{aligned}\mathcal{L}^{(5)}_{\text{top}} &= -A_1 \wedge K_2 \wedge K_2 - (\mathrm{d}k - 2E_1 - 2A_1) \wedge [\mathrm{d}B_2 \wedge (\mathrm{d}c - 2C_1) + (\mathrm{d}b - 2B_1) \wedge \mathrm{d}C_2] \\ &+ A_1 \wedge (\mathrm{d}k - 2E_1) \wedge [(\mathrm{d}b - 2B_1) \wedge \mathrm{d}C_1 - \mathrm{d}B_1 \wedge (\mathrm{d}c - 2C_1)] \\ &+ 2A_1 \wedge dE_1 \wedge (\mathrm{d}b - 2B_1) \wedge (\mathrm{d}c - 2C_1) + A_1 \wedge (\mathrm{d}b - 2B_1) \wedge (\mathrm{d}c - 2C_1) \wedge F_2 \\ &- 4C_2 \wedge \mathrm{d}B_2 \,. \end{aligned} \tag{4.9}$$

In turn, the above fields can be written in terms of various potentials and scalars in five dimensions

$$\begin{aligned} G_1 &= \mathrm{d}c - 2C_1 - a\mathrm{d}b + 2aB_1, \\ H_1 &= \mathrm{d}b - 2B_1, \\ K_1 &= \mathrm{d}k - 2E_1 - 2A_1, \\ F_2 &= \mathrm{d}A_1, \\ G_2 &= \mathrm{d}C_1 - a\mathrm{d}B_1, \\ H_2 &= \mathrm{d}B_1, \\ K_2 &= \mathrm{d}E_1 + \frac{1}{2}(\mathrm{d}b - 2B_1) \wedge (\mathrm{d}c - 2C_1) \,, \end{aligned} \tag{4.10}$$

thus ensuring the that ten-dimensional Bianchi identities (appendix A) for the fluxes hold. In total we have 7 scalars U, V, k, b, c including the axion a and dilaton ϕ, 4 one-form potentials A_1, B_1, C_1, E_1 and 2 two-form potentials B_2, C_2.

Dimensional reduction. Having introduced the five-dimensional theory, we are in a position to push ahead with the same reduction as section 3 to three dimensions on a constant curvature Riemann surface $\Sigma_{\mathfrak{g}}$. We consider the usual metric Ansatz[11]

$$\mathrm{d}s_5^2 = e^{-4C}\mathrm{d}s_3^2 + e^{2C}\mathrm{d}s^2(\Sigma_{\mathfrak{g}}) \,, \tag{4.11}$$

where warp factors have been chosen so that we end up in Einstein frame, and for the moment, we will assume that we have a constant curvature Riemann surface and not specify its curvature κ. Supersymmetry will later dictate that $\kappa < 0$. As for the rest of the fields, the five-dimensional scalars reduce to three-dimensional scalars. The fact that the field strengths H_1, G_1 appear in the Einstein equation mean that we cannot twist with respect to B_1 and C_1 since such a twisting is inconsistent with the assumption that the Riemann surface is constantly curved. This leaves A_1 and E_1, or their field strengths, which we twist in the following way

$$\begin{aligned} K_2 &= -\epsilon \,\mathrm{vol}(\Sigma_{\mathfrak{g}}) + \tilde{K}_2 \,, \\ F_2 &= \epsilon \,\mathrm{vol}(\Sigma_{\mathfrak{g}}) + \tilde{F}_2 \,, \end{aligned} \tag{4.12}$$

where tildes denote three-dimensional field strengths. ϵ is dictated to be a constant through $F_2 = \mathrm{d}A_1$ and no twisting along K_1 imposes the requirement that we twist K_2 in the

[11] Here C without subscript will denote the scalar warp factor and C_1 is a one-form.

opposite way. This latter point is also in line with our expectation that one can further truncate the theory to minimal gauged supergravity through $K_1 = 0, K_2 = -F_2$ in five dimensions [82].

Since we are not twisting B_1, C_1, the field strengths G_1, H_1, G_2, H_2 reduce directly to three dimensions. On the contrary, we can consider a decomposition for the three-form field strengths G_3, H_3 on the condition that we respect the symmetries of $\Sigma_{\mathfrak{g}}$. So we can decompose

$$C_2 = e\,\mathrm{vol}\,(\Sigma_{\mathfrak{g}}) + \tilde{C}_2, \quad B_2 = f\,\mathrm{vol}\,(\Sigma_{\mathfrak{g}}) + \tilde{B}_2\,, \tag{4.13}$$

leading to two new scalars e, f in the process. The corresponding field strengths can then be written as

$$\begin{aligned} G_3 &= M_1 \wedge \mathrm{vol}\,(\Sigma_{\mathfrak{g}}) + g\,\mathrm{vol}_3, \qquad & M_1 &= \mathrm{d}e - a\mathrm{d}f + \frac{1}{2}\epsilon\,(\mathrm{d}c - 2C_1 - a\mathrm{d}b + 2aB_1)\,, \\ H_3 &= N_1 \wedge \mathrm{vol}\,(\Sigma_{\mathfrak{g}}) + h\,\mathrm{vol}_3, \qquad & N_1 &= \mathrm{d}f + \frac{1}{2}\epsilon\,(\mathrm{d}b - 2B_1)\,. \end{aligned} \tag{4.14}$$

One can check that this choice is consistent with the closure of the Bianchi identities.

The scalars g, h are, up to an integration constants λ_1, λ_2, set by the equations of motion

$$\begin{aligned} g &= -4e^{-\frac{4}{3}(4U+V)-\phi-8C}\,(\lambda_1 + f) \\ h &= 4e^{-\frac{4}{3}(4U+V)+\phi-8C}\,(\lambda_2 + e - a\,(\lambda_1 + f))\,. \end{aligned} \tag{4.15}$$

We will normalise these so that $\lambda_i = 1$.

We now reduce directly at the level of the action and take care to check in appendix E that one gets the same result from reducing the equations of motion, thus guaranteeing the consistency of the reduction. Dropping tildes, as only the three-dimensional fields remain, the resulting kinetic terms are

$$\begin{aligned} \mathcal{L}^{(3)}_{\mathrm{kin}} = R\,\mathrm{vol}_3 &- 6\mathrm{d}C\wedge*\mathrm{d}C - \frac{28}{3}\mathrm{d}U\wedge*\mathrm{d}U - \frac{8}{3}\mathrm{d}U\wedge*\mathrm{d}V - \frac{4}{3}\mathrm{d}V\wedge*\mathrm{d}V - \frac{1}{2}e^{2\phi}\mathrm{d}a\wedge*\mathrm{d}a \\ &- \frac{1}{2}\mathrm{d}\phi\wedge*\mathrm{d}\phi - 2e^{-8U}K_1\wedge*K_1 - e^{-4U+\phi}G_1\wedge*G_1 - e^{-4U-\phi}H_1\wedge*H_1 \\ &- \frac{1}{2}e^{\frac{4}{3}(4U+V)+\phi-4C}M_1\wedge*M_1 - \frac{1}{2}e^{\frac{4}{3}(4U+V)-\phi-4C}N_1\wedge*N_1 - e^{-\frac{4}{3}(U+V)+4C}K_2\wedge*K_2 \\ &- \frac{1}{2}e^{\frac{8}{3}(U+V)+4C}F_2\wedge*F_2 - \frac{1}{2}e^{\frac{4}{3}(2U-V)+\phi+4C}G_2\wedge*G_2 - \frac{1}{2}e^{\frac{4}{3}(2U-V)-\phi+4C}H_2\wedge*H_2\,, \end{aligned} \tag{4.16}$$

while those of the scalar potential take the form

$$\begin{aligned} \mathcal{L}^{(3)}_{\mathrm{pot}} = e^{-4C}\Big[& 2\kappa e^{-2C} + 24e^{-\frac{2}{3}(7U+V)} - 4e^{\frac{4}{3}(-5U+V)} - 8e^{-\frac{8}{3}(4U+V)} \\ & - \frac{1}{2}\epsilon^2 e^{-4C}\left(e^{\frac{8}{3}(U+V)} + 2e^{-\frac{4}{3}(U+V)}\right) - 8e^{-\frac{4}{3}(4U+V)-\phi-4C}\,(1+f)^2 \\ & - 8e^{-\frac{4}{3}(4U+V)+\phi-4C}\,(1+e-a\,(1+f))^2\Big]\,\mathrm{vol}_3\,. \end{aligned} \tag{4.17}$$

The topological term is then given by the expression

$$\mathcal{L}^{(3)}_{\rm top} = 2\epsilon A_1 \wedge K_2 - 4\,(1+e)\,A_1 \wedge \mathrm{d}B_1 + 4\,(1+f)\,A_1 \wedge \mathrm{d}C_1$$
$$- \epsilon E_1 \wedge K_2 + 2E_1 \wedge \left[\mathrm{d}f \wedge (\mathrm{d}c - 2C_1) - \mathrm{d}e \wedge (\mathrm{d}b - 2B_1) + \frac{3}{4}\epsilon\,(\mathrm{d}b - 2B_1) \wedge (\mathrm{d}c - 2C_1)\right]$$
$$+ 2k\left[\left(\mathrm{d}f + \frac{1}{2}\epsilon(\mathrm{d}b - 2B_1)\right) \wedge \mathrm{d}C_1 - \left(\mathrm{d}e + \frac{1}{2}\epsilon\,(\mathrm{d}c - 2C_1)\right) \wedge \mathrm{d}B_1\right]. \qquad (4.18)$$

Now is an opportune time to identify the supersymmetric AdS_3 vacuum. This can be done by comparing directly with (6.9) of [22] (see also [23]). For concreteness we can take $KE_4 = S^2 \times S^2$ to exhibit the explicit solution, but one can consider other choices. The form of the space-time metric before rescaling is

$$\mathrm{d}s^2 = L^2 \left[\frac{2}{\sqrt{3}}\mathrm{d}s^2\,(AdS_3) + \frac{\sqrt{3}}{2}\left(\frac{\mathrm{d}x^2 + \mathrm{d}y^2}{y^2}\right) + \frac{\sqrt{3}}{2}\sum_{i=1}^{2}\frac{1}{2}\left(\mathrm{d}\theta_i^2 + \sin^2\theta_i \mathrm{d}\phi_i^2\right)\right.$$
$$\left. \frac{1}{2\sqrt{3}}\left(\mathrm{d}z - \frac{\mathrm{d}x}{y} - \sum_i \cos\theta_i \mathrm{d}\phi_i\right)^2\right], \qquad (4.19)$$

where AdS_3 is normalised to unit radius and all normalisations for the H^2, parametrised by (x, y), and two S^2's, parametrised by (θ_i, ϕ_i) are now explicit. We have also reintroduced an overall scale factor L. We omit the five-form flux as it will not provide any new information and it is enough to compare the ten-dimensional metrics.

To make meaningful comparison with the KK reduction Ansatz of [82], we need to compare with the following space-time Ansatz

$$\mathrm{d}s^2 = e^{-\frac{2}{3}(4U+V)}\left[e^{-4C}\mathrm{d}s_3^2 + e^{2C}\mathrm{d}s^2\,(\Sigma_\mathfrak{g})\right] + e^{2U}\mathrm{d}s^2\,(KE_4) + e^{2V}\,(\eta + A_1)^2\,, \qquad (4.20)$$

where $\mathrm{d}\eta = 2J$ and the Kähler-Einstein metric g_{ij} with positive curvature is normalised so that $R_{ij} = 6g_{ij}$. To make the connection, we first rescale the KE_4 factor in (4.19) by a factor of three, take $L^2 = 2/\left(3\sqrt{3}\right)$ and make the following identifications

$$(\eta + A_1) = \frac{1}{3}\left(\mathrm{d}z - \cos\theta_1 \mathrm{d}\phi_1 - \cos\theta_2 \mathrm{d}\phi_2 - \frac{\mathrm{d}x}{y}\right). \qquad (4.21)$$

The supersymmetric AdS_3 vacuum can then be identified

$$U = V = 0, \qquad C = -\frac{1}{2}\log 3, \qquad e = f = -1\,, \qquad (4.22)$$

where $\kappa = -1$, since the H^2 was normalised to unit radius, and $\epsilon = -\frac{1}{3}$ follows from (4.21). One can indeed check that this choice leads to a critical point of the potential and that the AdS_3 radius of the three-dimensional space-time is $\ell = \frac{2}{9}$.

Further truncation & supergravity. In this subsection we consider the above action with the three-form fluxes truncated out by setting $b = c = B_1 = C_1 = B_2 = C_2 = 0$, $e = f = -1$. Even from the ten-dimensional perspective, it is known that it is always consistent to perform this truncation to just the metric, fields in the five-form flux and the axion and dilaton.[12]

We now recast the simpler action in the more familiar language of three-dimensional gauged supergravity. In part this will involve dualising the one-form potentials. To do so we redefine the following fields

$$\begin{aligned} K_2 &= e^{\frac{4}{3}(U+V)-4C} * \mathrm{D}Y_2 \,, \qquad && \mathrm{D}Y_2 = \mathrm{d}Y_2 + \tilde{B}_2 \\ F_2 &= e^{-\frac{8}{3}(U+V)-4C} * \mathrm{D}Y_3 \,, \qquad && \mathrm{D}Y_3 = \mathrm{d}Y_3 + \tilde{B}_3 \,, \end{aligned} \tag{4.23}$$

while, at the same time, adding the following additional CS terms

$$\delta\mathcal{L}^{(3)}_{\text{top}} = 2\tilde{B}_2 \wedge K_2 + \tilde{B}_3 \wedge F_2 \,. \tag{4.24}$$

The covariant derivatives are chosen so that the equations of motion are still satisfied once $\tilde{B}_i$ are integrated out. We can then redefine K_1

$$K_1 = \frac{1}{2}\mathrm{D}Y_1, \qquad \mathrm{D}Y_1 = (\mathrm{d}Y_1 - 4E_1 - 4A_1) \,, \tag{4.25}$$

and finally introduce the following scalars

$$W_1 = -4U, \qquad W_2 = \frac{2}{3}(U+V) - 2C, \qquad W_3 = -\frac{4}{3}(U+V) - 2C \,. \tag{4.26}$$

The scalar manifold is now $[\mathrm{SU}(1,1)/\mathrm{U}(1)]^4$, which should be familiar from previous analysis, and the kinetic term for the action becomes

$$\begin{aligned} \mathcal{L}_{\text{kin}} = &-\frac{1}{2}\mathrm{d}W_1 \wedge *\mathrm{d}W_1 - \frac{1}{2}e^{2W_1}\mathrm{D}Y_1 \wedge *\mathrm{D}Y_1 - \mathrm{d}W_2 \wedge *\mathrm{d}W_2 - e^{2W_2}\mathrm{D}Y_2 \wedge *\mathrm{D}Y_2 \\ &- \frac{1}{2}\mathrm{d}W_3 \wedge *\mathrm{d}W_3 - \frac{1}{2}e^{2W_3}\mathrm{D}Y_3 \wedge *\mathrm{D}Y_3 - \frac{1}{2}\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2}e^{2\phi}\mathrm{d}a \wedge *\mathrm{d}a \,. \end{aligned} \tag{4.27}$$

We can thus introduce the complex coordinates

$$z_i = e^{-W_i} + iY_i \,, \qquad i = 1,2,3, \qquad z_4 = e^{-\phi} + ia \,, \tag{4.28}$$

allowing us explicitly to write the Kähler potential $\mathcal{K}$ as

$$\mathcal{K} = -\log(\Re z_1) - 2\log(\Re z_2) - \log(\Re z_3) - \log(\Re z_4) \,. \tag{4.29}$$

While we could have made this point earlier, it is now clear that the axion a and the dilaton ϕ decouple completely and can be truncated out. They also do not feature in the scalar potential.

In terms of the other scalars the potential takes the form

$$\begin{aligned} \mathcal{L}_{\text{pot}} = \Big[&2\kappa e^{2W_2+W_3} + 24 e^{W_1+W_2+W_3} - 4e^{2(W_1+W_2)} - 8e^{2(W_1+W_3)} \\ &- \frac{1}{2}\epsilon^2 \left(e^{4W_2} + 2e^{2(W_2+W_3)} \right) \Big] \mathrm{vol}_3 \,. \end{aligned} \tag{4.30}$$

[12] In performing this truncation we remove the six scalars coming from the RR and NS three-form fluxes. In general, it is possible to see that one always has an $\mathrm{SU}(1,1)/\mathrm{U}(1)$ factor, but it is not clear if the remaining twelve scalars constitute a Kähler manifold. It is also possible that the vacuum spontaneously breaks $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 2$, for example [82] in five dimensions. We leave this point to future work.

We can then work out the corresponding T tensor in terms of ϵ and κ,

$$T = -\frac{\epsilon}{4}e^{2W_2} - \frac{\epsilon}{2}e^{W_2+W_3} - e^{W_1+W_2} + e^{W_1+W_3} - \frac{\kappa}{2\epsilon}e^{W_3}\,. \tag{4.31}$$

We note that κ and ϵ are not independent and we require $\kappa = 3\epsilon$ so that the T tensor reproduces the potential. Once they are identified in this way, and taking into account the fact that $\kappa < 0, \epsilon < 0$, one finds a vacuum at

$$W_1 = 0, \qquad W_2 = W_3 = -\log(-\epsilon) \Rightarrow U = V = 0, \qquad C = \frac{1}{2}\log(-\epsilon)\,. \tag{4.32}$$

Setting $\epsilon = -\frac{1}{3}$, we arrive at the result quoted previously.

Central charge and R symmetry. In fact we have already discussed the central charge for this case as it corresponds to a particular example in section 3, namely $a_i = \frac{1}{3}, \kappa = -1$, thus ensuring that (2.14) is satisfied. However, to avoid the onerous task of rescaling metrics and comparing solutions, we can simply recalculate the central charge using the standard holographic prescription [91, 92]

$$c_R = \frac{3\ell}{2G^{(3)}}\,, \tag{4.33}$$

where ℓ is the AdS_3 radius and $G^{(3)}$ the three-dimensional Newton's constant. Using the conventions of [14, 15] where $G^{(3)} = 1/(4\eta_\Sigma N^2)$, one can check that the result agrees with (3.19) when $a_i = \frac{1}{3}$.

It is also of interest here to ask about the R symmetry? The ten-dimensional origin of our reduction makes it clear that there is only a single U(1) R symmetry, so there is no ambiguity. However, without this insight, we can ask what the three-dimensional theory can tell us about the R symmetry. Once we truncate out K_1, we have essentially two U(1) symmetries and the moment maps $\mathcal{V}^i$ associated to these can be worked out by comparing the T tensor (3.16) with the CS term in the action. We find that the components of the embedding tensor are $\Theta_{23} = 2\epsilon, \Theta_{22} = -2\epsilon$ and, for agreement, the moment maps are

$$\mathcal{V}^2 = \frac{1}{4}e^{W_2}, \qquad \mathcal{V}^3 = -\frac{1}{4}e^{W_3}\,, \tag{4.34}$$

where $i = 2, 3$ label the U(1)'s associated to the gauge fields E_1 and A_1 respectively. We can then extract the R symmetry

$$R = -\frac{2}{3}\mathrm{U}(1)_2 + \frac{2}{3}\mathrm{U}(1)_3\,, \tag{4.35}$$

where we have again used indices to distinguish the U(1)'s. We can now compare to our earlier result (3.20) by inserting $a_i = \frac{1}{3}$ and one arrives at the same numbers, up to a relative sign. This relative sign can be traced to the relative sign in (4.12) and by simply changing the sign of A_1 in ten dimensions, one can find perfect agreement.

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4.2 Intersecting D3-branes

In this section we discuss dimensional reductions to three dimensions for intersecting D3-branes. Some of the work presented here will not be new and will recover the recent work of [38]. Although we could approach this task directly from a ten-dimensional Ansatz, it is handier to make use of an intermediate reduction to six dimensions on a Calabi-Yau two-fold [93], details of which can be found in appendix F.

As such, we adopt the same strategy as [38], but an important distinction is that we will not impose truncations directly in six dimensions and then reduce. Instead, we will reduce directly so that we can unify the reductions presented in [38]. In addition, we will make statements about the underlying gauged supergravity, an aspect that was overlooked in [38]. Note that it is expected that the three-dimensional gauged supergravity be a theory with $\mathcal{N} = 4$ supersymmetry, so that the scalar manifold is a product of quaternionic manifolds [9], but this falls outside of our scope here and we hope to address this question in future work. Finally, we remark that these reductions are related to those of [11] via T-duality and uplift, a point that is fleshed out in appendix B.

So the task now is to perform the reduction on S^3, written as a Hopf-fibration, from the six-dimensional theory presented in [93] to extract a three-dimensional gauged supergravity. Strictly speaking we are then doing a reduction on the D1-D5 near-horizon or its S-dual F1-NS5, so further T-dualities along $CY_2 = T^2 \times T^2$ will be required to recover the intersecting D3-brane vacuum discussed previously. We will come to this point in due course.

Dimensional reduction. Starting from the six-dimensional theory in appendix F, we adopt the natural space-time Ansatz

$$\mathrm{d}s_6^2 = e^{-4U-2V}\mathrm{d}s_3^2 + \frac{1}{4}e^{2U}\mathrm{d}s^2\left(S^2\right) + \frac{1}{4}e^{2V}\left(\mathrm{d}z + P + A_1\right), \tag{4.36}$$

where U, V are warp factors and A_1 is a one-form with legs on the three-dimensional space-time. Our Ansatz fits into the overarching description for supersymmetric AdS_3 solutions from wrapped D3-branes presented earlier with choice $(l_1, l_2, l_3) = (0, 0, 4)$. In contrast to [38], this means that $P = -\cos\theta\mathrm{d}\phi$ so that $\mathrm{d}P = \mathrm{vol}(S^2) = 4J_3$. In addition, $A = 0$ follows from (4.5).

For the three-form fluxes, we consider the following Ansatz

$$\begin{aligned} F_3 &= G_0\frac{1}{2}\left(\mathrm{d}z + P + A_1\right)\wedge J_3 + G_1\wedge J_3 + G_2\wedge\frac{1}{2}\left(\mathrm{d}z + P + A_1\right) + ge^{-6U-3V}\,\mathrm{vol}_3 \quad (4.37)\\ H_3 &= \sin\alpha\left(\mathrm{d}z + P + A_1\right)\wedge J_3 + H_1\wedge J_3 + H_2\wedge\frac{1}{2}\left(\mathrm{d}z + P + A_1\right) + he^{-6U-3V}\,\mathrm{vol}_3\,, \end{aligned}$$

where the Bianchi identities (see appendix A) determine the following:

$$\begin{aligned} G_0 &= 2\left(\cos\alpha - \sin\alpha\chi_1\right),\\ G_1 &= \mathrm{d}c - \chi_1\mathrm{d}b - 2\left(C_1 - \chi_1 B_1\right) - \left(\cos\alpha - \sin\alpha\chi_1\right)A_1,\\ G_2 &= \mathrm{d}C_1 - \chi_1\mathrm{d}B_1,\\ H_1 &= \mathrm{d}b - 2B_1 - \sin\alpha A_1,\\ H_2 &= \mathrm{d}B_1\,. \end{aligned} \tag{4.38}$$

Here χ_1 is the scalar axion of type IIB supergravity and we have introduced the constant α, scalars b, c and one-form potentials B_1, C_1. The remaining scalars, $\phi_i, \chi_i = 1, 2$, of the six-dimensional theory simply descend to become three-dimensional scalars.

We now plug our Ansatz into the equations of motion of the six-dimensional theory (F.3)–(F.9), the details of which can be found in appendix F. In the process one determines the form for g, h:

$$g = 2e^{-\phi_1+\phi_2-V-2U}\left(\cos\alpha + \sin\alpha\chi_2\right), \tag{4.39}$$

$$h = 2e^{\phi_1+\phi_2-V-2U}\left[\sin\alpha - \cos\alpha\chi_2 + \left(\cos\alpha + \sin\alpha\chi_2\right)\chi_1\right], \tag{4.40}$$

where we have normalised the integration constants for later convenience.

One finds that the equations of motion all come from varying the following three-dimensional action:

$$\mathcal{L}^{(3)} = \mathcal{L}^{(3)}_{\rm kin} + \mathcal{L}^{(3)}_{\rm pot} + \mathcal{L}^{(3)}_{\rm top}\,, \tag{4.41}$$

where the kinetic term is

$$\begin{aligned}\mathcal{L}^{(3)}_{\rm kin} = {}& R\,{\rm vol}_3 - \frac{1}{2}{\rm d}\phi_1\wedge *{\rm d}\phi_1 - \frac{1}{2}e^{2\phi_1}{\rm d}\chi_1\wedge *{\rm d}\chi_1 - \frac{1}{2}{\rm d}\phi_2\wedge *{\rm d}\phi_2\\ &- \frac{1}{2}e^{2\phi_2}{\rm d}\chi_2\wedge *{\rm d}\chi_2 - 6{\rm d}U\wedge *{\rm d}U - 4{\rm d}U\wedge *{\rm d}V - 2{\rm d}V\wedge *{\rm d}V\\ &- \frac{1}{2}e^{-\phi_1-\phi_2-4U}H_1\wedge *H_1 - \frac{1}{2}e^{\phi_1-\phi_2-4U}G_1\wedge *G_1 - \frac{1}{2}e^{-\phi_1-\phi_2+4U}H_2\wedge *H_2\\ &- \frac{1}{2}e^{\phi_1-\phi_2+4U}G_2\wedge *G_2 - \frac{1}{8}e^{4U+4V}F_2\wedge *F_2\,,\end{aligned} \tag{4.42}$$

and the scalar potential takes the form

$$\begin{aligned}\mathcal{L}^{(3)}_{\rm pot} = \Big[&8e^{-6U-2V} - 2e^{-8U} - 2e^{\phi_1+\phi_2-8U-4V}\left[\sin\alpha - \cos\alpha\chi_2 + \left(\cos\alpha+\sin\alpha\chi_2\right)\chi_1\right]^2\\ &- 2e^{-\phi_1+\phi_2-8U-4V}\left(\cos\alpha+\sin\alpha\chi_2\right)^2 - 2e^{-\phi_1-\phi_2-8U-4V}\sin^2\alpha\\ &- 2e^{\phi_1-\phi_2-8U-4V}\left(\cos\alpha - \sin\alpha\chi_1\right)^2\Big]\,{\rm vol}_3\,.\end{aligned} \tag{4.43}$$

Finally, the topological term takes the simple form

$$\mathcal{L}^{(3)}_{\rm top} = \chi_2\left(H_1\wedge G_2 - G_1\wedge H_2\right) - \left(\cos\alpha C_1 + \sin\alpha B_1\right)\wedge F_2\,. \tag{4.44}$$

When $U = V = \phi_i = \chi_i = c = b = A_1 = B_1 = C_1 = 0$, the above scalar potential has a critical point corresponding to either the D1-D5 near-horizon, its S-dual, or a one parameter interpolating vacuum. We have chosen the integration constants so that an $SL(2,\mathbb{R})$ transformation, parametrised by the constant α,

$$\begin{pmatrix} C_2\\ B_2\end{pmatrix} \to \begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix} C_2\\ B_2\end{pmatrix} \tag{4.45}$$

takes one from the vacuum supported by a RR three-form flux ($\alpha = 0$) to the vacuum supported by a NS three-form flux ($\alpha = \frac{\pi}{2}$). In each case the AdS_3 radius is unity. It is known more generally that the effect of an $SL(2,\mathbb{R})$ transformation is simply to rotate the Killing spinors [94],[13] so supersymmetry is unaffected.

[13]In this immediate context, see [65].

Ten-dimensional picture. As we have reached our three-dimensional theory through the result of two steps, a reduction on a Calabi-Yau two-fold [93] and a further reduction generalising the recent work of [38], here we wish to pause to consider the higher-dimensional picture. We would also like to recast the KK reduction Ansatz in terms of the generic form of wrapped D3-branes. Specialising to $CY_2 = T^2 \times T^2$, we can perform two T-dualities along the second T^2 leading to the following NS sector with the metric in string frame:

$$\begin{aligned}
\mathrm{d}s^2_{10} &= e^{\frac{1}{2}(\phi_1+\phi_2)}\left[e^{-4U-2V}\mathrm{d}s^2_3 + \frac{1}{4}e^{2U}\mathrm{d}s^2\left(S^2\right) + \frac{1}{4}e^{2V}\left(\mathrm{d}z+P+A_1\right)^2\right] \qquad (4.46)\\
&\quad + e^{\frac{1}{2}(\phi_1-\phi_2)}\mathrm{d}s^2\left(T^2_1\right) + e^{-\frac{1}{2}(\phi_1-\phi_2)}\mathrm{d}s^2\left(T^2_2\right),\\
H_3 &= \left[2\sin\alpha J_3 + H_2\right]\wedge\frac{1}{2}\left(\mathrm{d}z+P+A_1\right) + H_1\wedge J_3 + he^{-6U-3V}\,\mathrm{vol}_3, \qquad (4.47)\\
\tilde{\phi} &= \frac{1}{2}\left(\phi_1+\phi_2\right), \qquad (4.48)
\end{aligned}$$

where $\tilde{\phi}$ is the new ten-dimensional dilaton. Note that the three-form flux H_3 is not affected by the T-duality. The accompanying RR fluxes then take the form

$$\begin{aligned}
F_5 &= \left[G_0 J_2\wedge J_3 + ge^{\phi_1-\phi_2+2U+V}J_1\wedge J_3\right]\wedge\frac{1}{2}\left(\mathrm{d}z+P+A_1\right) + e^{\phi_1-\phi_2+4U}*G_2\wedge J_1\wedge J_3\\
&\quad + G_1\wedge J_2\wedge J_3 + \left[G_2\wedge J_2 - e^{\phi_1-\phi_2-4U}*G_1\wedge J_1\right]\wedge\frac{1}{2}\left(\mathrm{d}z+P+A_1\right)\\
&\quad + G_0 e^{\phi_1-\phi_2-8U-4V}\,\mathrm{vol}_3\wedge J_1 + ge^{-6U-3V}\,\mathrm{vol}_3\wedge J_2\,,\\
F_3 &= \mathrm{d}\chi_1\wedge J_2 - \mathrm{d}\chi_2\wedge J_1\,, \qquad (4.49)
\end{aligned}$$

where $J_1 = \mathrm{vol}(T^2_1)$, $J_2 = \mathrm{vol}(T^2_2)$ and, as before, $J_3 = \frac{1}{4}\,\mathrm{vol}(S^2)$ and there is no axion, $F_1 = 0$.

Further truncations. Even if we dualise the gauge fields in the action (4.42), since we have an odd number of scalars and $\mathcal{N}=2$ supergravity in three dimensions has a Kähler scalar manifold, one will need to truncate out some fields to find a gauged supergravity description. In this subsection we consider some further truncations and make contact with the work of [38] in the process.

Setting $\alpha = \chi_i = c = A_1 = C_1 = 0$, $\phi_1 = \phi_2 = \phi$, $U = -V$, and finally employing the following identification

$$B_1 = \hat{A} \qquad (4.50)$$

one can check that our action can be brought to the form of (4.7) of [38]:

$$\begin{aligned}
\mathcal{L}^{(3)} &= R\,\mathrm{vol}_3 + \left(4e^{-4U} - 2e^{-8U}\right)\mathrm{vol}_3 - \mathrm{d}\phi\wedge*\mathrm{d}\phi - 4\mathrm{d}U\wedge*\mathrm{d}U\\
&\quad - \frac{1}{2}e^{-2\phi-4U}H_1\wedge*H_1 - \frac{1}{2}e^{-2\phi+4U}H_2\wedge*H_2\,. \qquad (4.51)
\end{aligned}$$

Note we have set $\ell = 1$ for simplicity, but this can be reinstated if one rescales the radius of the Hopf-fibre S^3 correctly. We have also retained the scalar field b, which one is required to set to zero to make direct connection with [38].

The reduction of [38], where the six-dimensional space-time is supported solely by RR flux, involves setting $\phi_1 = -\phi_2 = \phi$, $\chi_i = b = \alpha = B_1 = 0$. Making the further identifications

$$C_1 = -\hat{A}\,, \qquad A_1 = 2A\,, \tag{4.52}$$

one arrives at

$$\begin{aligned}\mathcal{L}^{(3)} =&\, R\,\mathrm{vol}_3 - \mathrm{d}\phi\wedge *\mathrm{d}\phi - 6\mathrm{d}U\wedge *\mathrm{d}U - 4\mathrm{d}U\wedge *\mathrm{d}V - 2\mathrm{d}V\wedge *\mathrm{d}V \\ &- \frac{1}{2}e^{2\phi-4U}\left(\mathrm{d}c+2\left(\hat{A}-A\right)\right)\wedge *\left(\mathrm{d}c+2\left(\hat{A}-A\right)\right) - \frac{1}{2}e^{2\phi+4U}\hat{F}\wedge *\hat{F} - \frac{1}{2}e^{4U+4V}F\wedge *F \\ &+ \left[8e^{-6U-2V} - 2e^{-8U} - 2e^{-2\phi-8U-4V} - 2e^{2\phi-8U-4V}\right]\mathrm{vol}_3 + 2\hat{A}\wedge F\,. \end{aligned}\tag{4.53}$$

Once one sets $c = 0$ one can again confirm this is the same action as (4.17) of [38]. A further truncation of action ($\phi = 0 = A, U = -V$) permits warped black string solutions, the holographic interpretation of which was considered in [95].[14]

An obvious truncation not discussed in [38] is the truncation to just the NS sector. In some sense this may be regarded as the S-dual of the truncation we have just discussed. We can do this by setting $\alpha = \frac{\pi}{2}$, $\chi_i = c = C_1 = 0$ and $\phi_1 = \phi_2 = \tilde{\phi}$. The resulting action is

$$\begin{aligned}\mathcal{L}^{(3)} =&\, R\,\mathrm{vol}_3 - \mathrm{d}\tilde{\phi}\wedge *\mathrm{d}\tilde{\phi} - 6\mathrm{d}U\wedge *\mathrm{d}U - 4\mathrm{d}U\wedge *\mathrm{d}V - 2\mathrm{d}V\wedge *\mathrm{d}V \\ &- \frac{1}{2}e^{-2\tilde{\phi}-4U}H_1\wedge *H_1 - \frac{1}{2}e^{-2\tilde{\phi}+4U}H_2\wedge *H_2 - \frac{1}{8}e^{4U+4V}F_2\wedge *F_2 \\ &+ \left[8e^{-6U-2V} - 2e^{-8U} - 2e^{-2\phi-8U-4V} - 2e^{2\phi-8U-4V}\right]\mathrm{vol}_3 - B_1\wedge F_2\,. \end{aligned}\tag{4.54}$$

Up to a rewriting, $b = c, A_1 = 2A, B_1 = -\hat{A}, \tilde{\phi} = -\phi$, this action is identical to (4.53).

Rewriting the supergravity. Here we identify the underlying gauged supergravities. As a warm-up we consider the action (4.51), but make a conversion from the three-dimensional Yang-Mills (YM) Lagrangian to a Chern-Simons Lagrangian following general prescriptions given in [9] (see also [10, 96]). This procedure replaces every YM gauge field with two gauge fields and a new scalar field. This allows us to trade the following Yang-Mills term in the action

$$\mathcal{L}^{(3)}_{\mathrm{YM}} = -\frac{1}{2}e^{-2\phi+4U}H_2\wedge *H_2 \tag{4.55}$$

with the terms

$$\mathcal{L}^{(3)}_{\mathrm{CS}} = -\frac{1}{2}e^{2\phi-4U}\mathrm{D}\tilde{\phi}\wedge *\mathrm{D}\tilde{\phi} + H_2\wedge\tilde{B}_1\,, \tag{4.56}$$

where $\mathrm{D}\tilde{\phi} = \mathrm{d}\tilde{\phi} - \tilde{B}_1$ and we now have two gauge fields $B_1, \tilde{B}_1$ and an additional scalar $\tilde{\phi}$. Varying with respect to $\tilde{B}_1$ we get

$$H_2 + e^{2\phi-4U} * \mathrm{D}\tilde{\phi} = 0\,, \tag{4.57}$$

which, on choosing the gauge $\tilde{\phi} = 0$, we can integrate out $\tilde{B}_1$ to recover the original Lagrangian. The equation of motion following from varying B_1 now reads

$$\mathrm{d}\tilde{B}_1 + 2e^{-2\phi-4U} * H_1 = 0\,, \tag{4.58}$$

[14] It is easier to start with the action in [95] and use the EOM for $\hat{A}$ to find the form for the action above.

which can be shown to be equivalent to that of the original Lagrangian once one imposes (4.57). The equation of motion for $\tilde{\phi}$ is trivially satisfied through (4.57).

With these changes, the scalar kinetic term of the full Lagrangian (4.51) is given by

$$\mathcal{L}^{(3)}_{\text{kin}} = -\mathrm{d}\phi \wedge *\mathrm{d}\phi - 4\mathrm{d}U \wedge *\mathrm{d}U - \frac{1}{2}e^{-2\phi-4U} H_1 \wedge *H_1 - \frac{1}{2}e^{2\phi-4U}\mathrm{D}\tilde{\phi} \wedge *\mathrm{D}\tilde{\phi} \tag{4.59}$$

where as before $H_1 = \mathrm{d}b - 2B_1$. We redefine all of the scalars through

$$Y_1 = \tilde{\phi}, \qquad Y_2 = b, \qquad W_1 = \phi - 2U, \qquad W_2 = -\phi - 2U, \tag{4.60}$$

so that the scalar kinetic term becomes

$$\mathcal{L}^{(3)}_{\text{kin}} = -\frac{1}{2}\sum_{i=1}^{2}\left[\mathrm{d}W_i \wedge *\mathrm{d}W_i + e^{2W_i}\mathrm{D}Y_i \wedge *\mathrm{D}Y_i\right] . \tag{4.61}$$

The corresponding scalar manifold is clearly $[\mathrm{SU}(1,1)/\mathrm{U}(1)]^2$ and the Kähler potential is $\mathcal{K} = -\sum_i \log(\Re z_i)$, where $z_i = e^{-W_i} + iY_i$. In terms of W_i, the scalar potential becomes

$$\mathcal{L}^{(3)}_{\text{pot}} = \left[4e^{W_1+W_2} - 2e^{2(W_1+W_2)}\right] \mathrm{vol}_3 \ . \tag{4.62}$$

The corresponding T tensor is found to be

$$T = \frac{1}{2}\left(e^{W_1} + e^{W_2} - e^{W_1+W_2}\right) \tag{4.63}$$

with only one critical point at $W_1 = W_2 = 0$. Here it is not immediately obvious that this is the only option. Recall that for $\mathcal{N} = 2$ gauged supergravity, when the R symmetry is gauged, no holomorphic superpotential can appear [9]. Now when the R symmetry is not gauged, as is the case here, one can consider replacing the T tensor with the free energy $F = -T \pm e^{\mathcal{K}/2}W$. However, since $e^{\mathcal{K}} = e^{W_1+W_2}$, we can see that a problem arises with W being holomorphic, so this does not appear to be an option.

We now move onto the second action that results from truncating out all the NS three-form flux fields. Referring to (4.46), (4.49), this means that we set $\alpha = b = B_1 = \chi_i = 0$. With this simplification, one further observes that it is consistent to set $\phi_1 = -\phi_2 = \phi$. This is simply (4.53) with the scalar c reinstated and A_1 and C_1 rewritten accordingly, $A_1 = 2A$, $C_1 = -\hat{A}$.

We can now diagonalise the scalars by redefining them

$$W_1 = -\phi - 2U, \qquad W_2 = \phi - 2U, \qquad W_3 = -2U - 2V\,, \tag{4.64}$$

leading to canonically normalised kinetic terms:

$$\mathcal{L}^{(3)}_{\text{kin}} = -\frac{1}{2}\sum_{i=1}^{3}\left[\mathrm{d}W_i \wedge *\mathrm{d}W_i + e^{2W_i}\mathrm{D}Y_i \wedge *\mathrm{D}Y_i\right] . \tag{4.65}$$

In the process we have redefined $Y_2 = c$ so that $\mathrm{D}Y_2 = \mathrm{d}Y_2 + 2(\hat{A} - A)$ and in addition dualised the one-form potentials, $A, \hat{A}$ so that

$$\begin{aligned} \hat{F} &= e^{-2\phi-4U} * \mathrm{D}Y_1, \qquad && \mathrm{D}Y_1 = \mathrm{d}Y_1 + B_1, \\ F &= e^{-4U-4V} * \mathrm{D}Y_3, \qquad && \mathrm{D}Y_3 = \mathrm{d}Y_3 + B_3. \end{aligned} \tag{4.66}$$

As should be customary at this stage, we have to add a corresponding CS term so the new topological term is

$$\mathcal{L}^{(3)}_{\text{top}} = 2\hat{A} \wedge F + B_1 \wedge \hat{F} + B_3 \wedge F\,. \tag{4.67}$$

Introducing complex coordinates in the usual fashion, $z_i = e^{-W_i} + iY_i$, $i = 1, 2, 3$, the Kähler potential for the scalar manifold is $\mathcal{K} = -\sum_i \log(\Re z_i)$.

In terms of our new scalars W_i, the potential takes a simple form and is symmetric in all the scalars W_i:

$$\mathcal{L}^{(3)}_{\text{pot}} = 2\left[4e^{W_1+W_2+W_3} - e^{2(W_1+W_3)} - e^{2(W_1+W_2)} - e^{2(W_2+W_3)}\right] \text{vol}_3\,. \tag{4.68}$$

A suitable choice for the corresponding T tensor is

$$T = -e^{W_2} + \frac{1}{2}\left(e^{W_1+W_2} - e^{W_1+W_3} + e^{W_2+W_3}\right), \tag{4.69}$$

though symmetry dictates that there are other choices and we can send $W_1 \to W_2 \to W_3 \to W_1$ to uncover the other options. Regardless of how we choose T, the critical point is located at $W_i = 0$. Since the R symmetry is gauged, we do not expect a holomorphic superpotential.

5 Null-warped AdS_3 solutions

Recently, it has been noted that null-warped AdS_3 solutions, or equivalently geometries exhibiting Schrödinger symmetry with $z = 2$, can be found in three-dimensional theories that arise as consistent reductions based on the D1-D5 (or its S-dual) near-horizon geometries of type IIB supergravity [38]. In section 4.2, we identified the relevant theories in the gauged supergravity literature and here we will discuss some of the solutions. Prior to [38], it was noted that non-relativistic geometries with dynamical exponent $z = 4$ could be found in an $\mathcal{N} = 2$ gauged supergravity that is the consistent KK reduction of eleven-dimensional supergravity on $S^2 \times CY_3$ [11].[15] We will now address a natural question by scanning the other gauged supergravities we have identified for non-relativistic solutions with dynamical exponent z.

Before doing so, we recall some facts about Schrödinger solutions in three dimensions. Starting from an AdS_3 vacuum, solutions with dynamical exponent z arise as solutions to Chern-Simons theories where the relevant equation is

$$\mathrm{d} *_3 F + \frac{\kappa}{\ell} F = 0\,, \tag{5.1}$$

with $F = \mathrm{d}A$ and ℓ denotes the AdS_3 radius. Taking the derivative of (5.1), we see that κ must be a constant. Adopting the usual form of the space-time Ansatz

$$\mathrm{d}s^2 = \ell^2\left(-\lambda^2 r^z \mathrm{d}u^2 + 2r\mathrm{d}u\mathrm{d}v + \frac{\mathrm{d}r^2}{4r^2}\right), \tag{5.2}$$

the Einstein equation, through the components of the Ricci tensor:[16]

$$R_{+-} = -\frac{2}{\ell^2}, \qquad R_{++} = \frac{\lambda^2}{\ell^2} 2z\,(z-1)\,r^{z-1}, \qquad R_{--} = 0\,, \tag{5.3}$$

[15] These were mistakenly labelled null-warped AdS_3, but this label should be reserved solely for the $z = 2$ case in the literature.

[16] We have used the dreibein $e^+ = \ell\, r^{\frac{1}{2}}\mathrm{d}u, e^- = \ell\, r^{\frac{1}{2}}\left(\mathrm{d}v - \frac{1}{2}\lambda^2 r^{z-1}\mathrm{d}u\right), e^r = \ell\, \frac{\mathrm{d}r}{2r}$.

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determines the constant κ in terms of the dynamical exponent, $\kappa = z$. Observe here that λ is an arbitrary constant that can either be set to unity through rescaling the metric, or when set to zero, one recovers the unwarped AdS_3 vacuum.

Now the task of searching for new solutions becomes a very accessible goal; one simply has to identify ℓ and compare the equations of motion of the theory with (5.1) to extract κ and thus z. For the gauged supergravity discussed in section 3, namely the theory given by the action (3.6), the AdS_3 radius is

$$\ell = \frac{1}{2T} = -\frac{2a_1 a_2 a_3}{\Theta}\,, \tag{5.4}$$

which in general depends on the parameters a_i. For simplicity, we confine our search to the case where $G_i = G$, i.e. they are all equal. After changing frame to Einstein frame, consistency of the three equations (C.5) then places constraints on a_i:

$$\{a_1 = a_2 = a_3\}\,, \quad \left\{a_1 = a_2 = \frac{2}{7}a_3\right\}, \quad \left\{a_1 = a_3 = \frac{2}{7}a_2\right\}, \quad \left\{a_2 = a_3 = \frac{2}{7}a_1\right\}. \tag{5.5}$$

Combining these with the condition for a supersymmetric vacuum (2.14), one reaches the conclusion that good AdS_3 solutions exist only for $\Sigma_{\mathfrak{g}} = H^2$.[17] The two independent choices we find are

$$(a_1, a_2, a_3) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right), \qquad (a_1, a_2, a_3) = \left(\frac{7}{11}, \frac{2}{11}, \frac{2}{11}\right), \tag{5.6}$$

where one is free to consider various cyclic permutations of the latter. The first choice leads to the non-integral value $z = \frac{4}{3}$ with $\ell = \frac{2}{9}$. The second choice does produce an integer, namely $z = 18$ with $\ell = \frac{8}{11}$. Thus, within the limited scope of our search, we do not find any null-warped AdS_3 $(z = 2)$ solutions here.

Moving on, we can turn to the gauged supergravity corresponding to twisted compactifications of $\mathcal{N} = 1$ SCFTs, namely (4.16). A particular case of this we have already covered above. Referring the reader to equations (E.1) and (E.6), if one truncates consistently to just K_1, K_2 and F_2, and regardless of how one further truncates to an equation bearing resemblance to (5.1), one finds the dynamical exponent $z = \frac{4}{3}$. This should not come as a surprise as once one truncates to these fields, the theory should correspond to five-dimensional $\mathrm{U}(1)^3$ theory where one identifies two of the gauge fields and truncates out a scalar.

However, for the action (4.16), we do have other options. As we are considering a null space-time, it is consistent to truncate to just the scalar c and one-form C_1 with the various other scalars taking their vacuum values. Obviously, this is not a consistent truncation in general, but since we assume $G_2 \wedge * G_2 = G_1 \wedge * G_1 = M_1 \wedge * M_1 = 0$ in this case, we do not have to worry about the consistency of equations such as (E.5), (E.7) and (E.8). Note that M_1 is not independent and is related to G_1, $M_1 = \frac{\epsilon}{2} G_1$. This in turn means that, in addition to the Einstein equation, we only have two flux equations

$$\mathrm{d} * G_1 = 0, \quad \mathrm{d} * G_2 - \frac{9}{\ell} * G_1 = 0\,, \tag{5.7}$$

[17] One can compare the values of a_i against figure 1 of [15].

where we have used $\ell = \frac{2}{9}$ and $e^{2C} = \frac{1}{3}$. If we further truncate to set $*G_1 = -\frac{2}{9}G_2$, then we can find null-warped AdS_3 solutions with $z = 2$. This allows us to determine c which can be set consistently to zero. In the notation of section 4.1, the solution may be expressed as

$$\begin{aligned} \mathrm{d}s^2 &= \ell^2 \left(-r^z \mathrm{d}u^2 + 2r\mathrm{d}u\mathrm{d}v + \frac{\mathrm{d}r^2}{4r^2} \right), \\ C_1 &= \frac{2}{3}\ell\, r\, \mathrm{d}u\,, \end{aligned} \tag{5.8}$$

where we have rescaled C_1 so that $\lambda = 1$.

We can also consider deformations for AdS_3 supported by the scalar b and one-form B_1. This involves consistently truncating the action (4.16) to N_1, H_1 and H_2 and since this may be regarded as the S-dual of the truncation presented immediately above, we recover the same solution.

For some sense of completeness, we also touch upon the existence of solutions for the theory arising from a dimensional reduction on $S^2 \times T^4$ from ten dimensions presented in section 4.2. Schrödinger solutions based on the D1-D5 near-horizon, or its S-dual F1-NS5, have already been the focus of considerable attention in the literature. Not only have solutions been constructed directly in ten dimensions [55], but examples in the three-dimensional setting have also been identified in [38]. Though not mentioned in [38], an S-duality transformation is all that is required to generate an example supported purely by the NS sector provided one starts with the RR supported two-parameter family of [38]. Rather than take this path, we will work directly with our reduced theory and employ an appropriate Ansatz. We will also make use of a further truncation.

Starting from the action in section 4.2, we take $\alpha = \frac{\pi}{2}$ and truncate out various fields $U = V = \phi_i = \chi_i = a = c = C_1 = 0$. This corresponds to setting the scalars to their AdS_3 vacuum ($\ell = 1$) values and the choice of α is appropriate for a vacuum supported solely by NS flux. Further truncating out A_1 leads to the condition $*H_1 = H_2$, leading to the equations of motion:

$$\begin{aligned} \mathrm{d} * H_2 &= -2H_2\,, \\ R_{\mu\nu} &= -2g_{\mu\nu} + H_{2\mu\rho}H_{2\nu}{}^{\rho}\,, \end{aligned} \tag{5.9}$$

where we have used the fact that B_1 is null. Note that the CS equation is now in the accustomed form (5.1), so we can be confident we have a null-warped solution. It is then a straightforward exercise to provide the explicit form of the solution that satisfies these equations of motion:

$$\begin{aligned} \mathrm{d}s^2 &= -r^z \mathrm{d}u^2 + 2r\mathrm{d}u\mathrm{d}v + \frac{\mathrm{d}r^2}{4r^2}, \\ B_1 &= r\,\mathrm{d}u\,. \end{aligned} \tag{5.10}$$

It would be interesting to see if any solutions can be generated through applying TsT [25] transformations, such as those considered in [97].

6 Outlook

Our primary motivation for this work stems from [16] where five-dimensional $\mathrm{U}(1)^3$ gauged supergravity was dimensionally reduced on a Riemann surface and the lower-dimensional theory re-expressed in terms of the language of three-dimensional gauged supergravity [9]. As explained in section 3, the T tensor presents a natural supergravity counterpart to the quadratic trial function for the central charge presented in [14, 15] and it is a striking feature that the T tensor, through the embedding tensor, knows about the exact R symmetry. Without recourse to the higher-dimensional solution, this provides a natural way to identify the exact central charge and R symmetry directly in three dimensions.

Since any solution to this particular three-dimensional gauged supergravity uplifts to the $\mathrm{U}(1)^3$ theory in five dimensions, which is itself a reduction of type IIB supergravity [75], we have also taken the opportunity to step back and address consistent KK reductions to three dimensions for wrapped D3-brane geometries. As reviewed in section 2, the origin of supersymmetric AdS_3 geometries in type IIB can be traced to D3-branes wrapping Kähler two-cycles in Calabi-Yau manifolds, with CFTs of interest to c-extremization, namely those with $\mathcal{N} = (0, 2)$ supersymmetry, resulting when a two-cycle in a Calabi-Yau four-fold is wrapped. All AdS_3 solutions of this form fall into the general classification of supersymmetric geometries presented in [20] and at the heart of each supersymmetric geometry is a six-dimensional Kähler manifold $\mathcal{M}_6$, satisfying the differential condition (1.1).

Not only does this condition appear in the flux equations of motion, but the Einstein equation is satisfied through imposing this condition. This makes the task of finding a fully generic KK reduction, in contrast to the case studied in section 3, where one assumes the presence of a Riemann surface, an inviting problem. It is expected that one can gauge the $\mathrm{U}(1)$ R symmetry and reduce to three dimensions in line with the conjecture of [72] that gaugings of R symmetry groups always lead to consistent reductions to lower-dimensional gauged supergravities. What is not clear at this moment is whether a truly "generic" reduction - one working at the level of the supersymmetry conditions - on $\mathcal{M}_6$ exists, thus mimicking general reductions to five dimensions discovered in [71, 72], or whether one needs to specify more structure for the $\mathcal{M}_6$. An added subtlety here is that since the reduced theory is expected to fit into $\mathcal{N} = 2$ gauged supergravity, it is not enough simply to retain a gauge field coming from an R symmetry gauging and an extra degree of freedom is required.

Naturally enough, what we have discussed here just pertains to D3-branes and AdS_3 vacua also arise in eleven-dimensional supergravity arising from wrapped M5-branes. It is then fitting to consider KK reductions from eleven dimensions to three-dimensional gauged supergravity. While supersymmetric AdS_3 solutions can be found by considering twists of seven-dimensional supergravity [15, 98, 99], more general solutions are expected to fit into the general classification of supersymmetric solutions presented in [59, 60]. A particular case discussed in [15], namely seven-dimensional supergravity reduced on $H^2 \times H^2$, we have already considered[18] and we will report on M5-brane analogues in future work [100].

In addition to the c-extremization angle, another thread to our story concerns the search for null-warped AdS_3 or Schrödinger ($z = 2$) solutions. While it is likely that we

[18] It corresponds to $\mathcal{N} = 2$ supergravity with Kähler manifold $[\mathrm{SU}(1, 1)/\mathrm{U}(1)]^4$.

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have recovered some known solutions, and found solutions with more general z, we believe that the solutions based on $H^2 \times KE_4$ internal geometries are new. What remains is to check whether they preserve supersymmetry, and indeed the identification of the Killing spinor equations for the reduced theories in sections 4.1 and 4.2 needs to be considered if one is to discuss supersymmetric solutions. The reduction in section 3 aside, we have simply focused on the bosonic sector and the equations of motion. It may also be interesting to study families of Schrödinger solutions interpolating between the D1-D5 vacuum and F1-NS5 vacuum directly in three dimensions. This would presumably overlap with the higher-dimensional examples presented in [55]. It is expected that some supersymmetry is preserved.

Combining the principle of c-extremization [14, 15], which can be understood in terms of three-dimensional supergravity [16], and the fact that null-warped AdS_3 solutions clearly exist, it is worth considering if c-extremization can be extended to warped AdS_3. The most immediate setting to address this question is the theory of section 3, however, as we have seen, the simplest solutions appear to preclude solutions with $z = 2$. A more thorough search for null-warped solutions is warranted. If they do not exist, one can imagine starting from a more involved theory in five dimensions that includes the $\mathrm{U}(1)^3$ gauged supergravity. Evidently, the more involved reductions based on $H^2 \times KE_4$ and $S^2 \times T^4$ allow solutions, so it can be expected that this question can be addressed in future work.

It would equally be interesting to look for a holographic analogue of c-extremization in two dimensions.[19] Starting from eleven dimensions, one can reduce to four dimensions [75] retaining the Cartan subgroup $\mathrm{U}(1)^4$ of the R symmetry group. Relevant solutions are already known [70, 78], and the two-dimensional theory one gets from twisted compactifications on Riemann surfaces are likely to be in the literature, for example [101], and may be related to BFSS matrix quantum mechanics [102]. At a quick glance, it looks like we have some of the jigsaw pieces in place.

One of the potentially interesting avenues for future study is to explore the connection between supersymmetric black holes in five dimensions and null-warped AdS_3 space-times. For non-relativistic geometries with $z = 4$, it was noted in [11] that these geometries naturally appear when one considers a general class of five-dimensional supersymmetric black holes and strings and then reduces on an S^2. The corresponding picture for the known null-warped solutions can also be worked out. It would be interesting to extend recent studies of the classical motion of strings in warped AdS_3 backgrounds [103] to higher-dimensional black holes.

Finally, we are aware of string theory embeddings of holographic superconductors in four and five dimensions [88–90], where an important element in the construction is the presence of charged scalars that couple to the complex form of the internal Kähler-Einstein manifold. To date, there is no example of an embedding of the bottom-up model considered in [104], though strong similarities between the supersymmetric geometries here and Sasaki-Einstein manifolds suggest that this may be a good place to look. So far we have been unable to find a consistent reduction based on $\mathcal{M}_6 = S^2 \times T^4$ or $\mathcal{M}_6 = H^2 \times KE_4$, but one could hope to address the problem perturbatively. Such an approach was adopted in [105].

[19] We are grateful to N. Halmagyi for suggesting this possibility.

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A Type IIB supergravity conventions

Our conventions for type IIB supergravity follow those of [82], which for completeness, we reproduce here. Restricting ourselves to the bosonic sector of type IIB supergravity, the field content consists of RR n-forms $F_{(n)}$, $n = 1, 3, 5$, the NS form $H_{(3)}$, the dilaton Φ and the metric. The forms satisfy the Bianchi identities

$$\mathrm{d}F_{(5)} + F_{(3)} \wedge H_{(3)} = 0\,, \tag{A.1}$$
$$\mathrm{d}F_{(3)} + F_{(1)} \wedge H_{(3)} = 0\,, \tag{A.2}$$
$$\mathrm{d}F_{(1)} = 0\,, \tag{A.3}$$
$$\mathrm{d}H_{(3)} = 0\,, \tag{A.4}$$

which can be satisfied through the introduction of potentials $C_{(n-1)}$, $B_{(2)}$. In terms of these potentials, the forms are $F_{(5)} = \mathrm{d}C_{(4)} - C_{(2)} \wedge H_{(3)}, F_{(3)} = \mathrm{d}C_{(2)} - C_{(0)} H_{(3)}, F_{(1)} = \mathrm{d}C_{(0)}$, $H_{(3)} = \mathrm{d}B_{(2)}$. In addition to the self-duality condition on the five-form, $*F_{(5)} = F_{(5)}$, the equations of motion take the form:

$$\mathrm{d}\left(e^{\Phi} * F_{(3)}\right) - F_{(5)} \wedge H_{(3)} = 0\,, \tag{A.5}$$
$$\mathrm{d}\left(e^{2\Phi} * F_{(1)}\right) + e^{\Phi} H_{(3)} \wedge *F_{(3)} = 0\,, \tag{A.6}$$
$$\mathrm{d}\left(e^{-\Phi} * H_{(3)}\right) - e^{\Phi} F_{(1)} \wedge *F_{(3)} - F_{(3)} \wedge F_{(5)} = 0\,, \tag{A.7}$$
$$\mathrm{d} * \mathrm{d}\Phi - e^{2\Phi} F_{(1)} \wedge *F_{(1)} + \frac{1}{2} e^{-\Phi} H_{(3)} \wedge *H_{(3)} - \frac{1}{2} e^{\Phi} F_{(3)} \wedge *F_{(3)} = 0\,, \tag{A.8}$$
$$\begin{aligned} R_{MN} = {} & \frac{1}{2}\partial_M C_{(0)} \partial_N C_{(0)} + \frac{1}{2}\partial_M \Phi \partial_N \Phi + \frac{1}{96} F_{MPQRS} F_N^{\ PQRS} \\ & \frac{1}{4} e^{-\Phi} \left(H_M^{\ PQ} H_{NPQ} - \frac{1}{12} g_{MN} H^{PQR} H_{PQR} \right), \\ & \frac{1}{4} e^{\Phi} \left(F_M^{\ PQ} F_{NPQ} - \frac{1}{12} g_{MN} F^{PQR} F_{PQR} \right). \end{aligned} \tag{A.9}$$

B Connection between [11] and [38]

In this section we will discuss the connection between two dimensional reductions from higher-dimensional supergravities to three-dimensional theories that have appeared in the literature. Both theories admit supersymmetric Schrödinger solutions, however, for those based on the D1-D5 near-horizon [38] the dynamical exponent $z = 2$ appears, while the dynamical exponent quoted in [11] is $z = 4$.

Recall that these theories support AdS_3 vacua whose higher-dimensional manifestations are $AdS_3 \times S^3 \times CY_2$ geometries of type IIB supergravity and $AdS_3 \times S^2 \times CY_3$ geometries of eleven-dimensional supergravity, respectively. Specialising to the case where the Calabi-Yau three-fold is a direct product involving a torus T^2, $CY_3 = CY_2 \times T^2$, it is a well-known fact that the geometries are related via dimensional reduction and T-duality. This raises a question about the difference in the quoted dynamical exponents. Here we address that issue and show that a sub-truncation of [38] and [11] is common and that amongst the $z = 2$ solutions presented in [38], one can also find a $z = 4$ solution.

We start by considering the KK reduction Ansatz from eleven-dimensions. The solution appearing in [11] has a higher-dimensional manifestation of the form

$$\begin{aligned} \mathrm{d}s^2_{11} &= e^{-4W}\mathrm{d}s^2_3 + e^{2W}\mathrm{d}s^2\left(S^2\right) + \mathrm{d}s^2\left(CY_2\right) + \mathrm{d}x_5^2 + \mathrm{d}x_6^2\,, \\ G_4 &= \left(\alpha\,\mathrm{vol}(S^2) + H_2\right) \wedge \left(J_{CY_2} + \mathrm{d}x_5 \wedge \mathrm{d}x_6\right)\,, \end{aligned} \tag{B.1}$$

where we have consistently truncated out the fields f, V, B_1 leaving just a scalar W and one-form potential B_2, where $H_2 = \mathrm{d}B_2$. Here (x_5, x_6) label coordinates on the T^2 and α is a constant. Plugging this Ansatz into the equations of motion of eleven-dimensional supergravity one finds [11]

$$\mathrm{d}\left(e^{4W} *_3 H_2\right) = -2\alpha H_2\,, \tag{B.2}$$

$$\mathrm{d} *_3 \mathrm{d}W = \frac{1}{2}e^{W} H_2 \wedge *_3 H_2 + \left(e^{-6W} - \alpha^2 e^{-8W}\right)\mathrm{vol}_3\,, \tag{B.3}$$

and the Einstein equation which we omit.

Dimensional reduction on x_6 and T-duality on x_5 leads to the following IIB KK reduction Ansatz

$$\mathrm{d}s^2_{10} = e^{-4W}\mathrm{d}s^2_3 + e^{2W}\mathrm{d}s^2\left(S^2\right) + \mathrm{d}s^2\left(CY_2\right) + \left(\mathrm{d}x_5 - \alpha\cos\theta\mathrm{d}\phi + B_2\right)^2\,, \tag{B.4}$$

$$F_5 = (1 + *_{10})\left[\alpha\,\mathrm{vol}\left(S^2\right) \wedge J_{CY_2} + J_{CY_2} \wedge H_2\right] \wedge \left(\mathrm{d}x_5 - \alpha\cos\theta\mathrm{d}\phi + B_2\right)\,,$$

where (θ, ϕ) parametrise the two-sphere S^2 and all other fields, including the dilaton are zero.

At this point it is easier to compare with the ten-dimensional uplift [93] of the six-dimensional Ansatz considered in [38] to get our bearings. After rescaling the metric to make the transition to string frame, the ten-dimensional space-time may be written as

$$\begin{aligned} \mathrm{d}s^2_{10} &= e^{\frac{\phi_1}{2}+\frac{\phi_2}{2}}\mathrm{d}s^2_6 + e^{\frac{\phi_1}{2}-\frac{\phi_2}{2}}\mathrm{d}s^2\left(CY_2\right)\,, \\ \mathrm{d}s^2_6 &= e^{-4U-2V}\mathrm{d}s^2_3 + \frac{1}{4}e^{2U}\mathrm{d}s^2\left(S^2\right) + \frac{1}{4}e^{2V}\left(\mathrm{d}\psi + \cos\theta\mathrm{d}\phi + 2A\right)^2\,, \end{aligned} \tag{B.5}$$

where we have set the length-scale ℓ corresponding to the AdS_3 radius to unity for simplicity. To compare the metrics we note that we require the following identifications:

$$\phi = \phi_1 = \phi_2 = -2V, \qquad e^W = \frac{1}{2}e^U, \qquad 2x_5 = \psi, \qquad \alpha = -\frac{1}{2}, \quad B_2 = A. \tag{B.6}$$

While this places us in the class of consistent reductions in section 4.2 of [38], the added condition that the dilaton ϕ is zero tells us that the scalars ϕ, V appearing in equations (B.25) and (B.29) of [38] are zero. These equations together then tell us that the two gauge fields appearing in [38] should be identified $A = \pm\hat{A}$. For $CY_2 = T^4$, the RR-sector is then simply related via T-duality.

The choice $A = \hat{A}$ immediately leads to the condition $F^2 = 0$ through (B.25), however there is another option. We can choose $A = -\hat{A}$ with the further relation

$$A = \frac{1}{4}e^{4U} *_3 F\,. \tag{B.7}$$

With this relation one can then satisfy oneself that (B.27) and the U equation from (B.29) of [38] can be identified with (B.2) and (B.3) above, meaning that this particular sub-truncation of both reductions is the same.

Indeed, since the higher-dimensional AdS_3 solutions can be related via dimensional reduction and T-duality, it is expected that the KK reductions are also related at some level.

C Details of reduction of $D = 5$ U(1)^3 gauged supergravity

Here we begin by recording the five-dimensional equations of motion one gets from varying the action (3.1). The equations of motion for the gauge fields A_i, $i = 1, 2, 3$, are

$$\begin{aligned} \mathrm{d}\left(X_1^{-2} * F^1\right) &= F^2 \wedge F^3, \\ \mathrm{d}\left(X_2^{-2} * F^2\right) &= F^1 \wedge F^3, \\ \mathrm{d}\left(X_3^{-2} * F^3\right) &= F^1 \wedge F^2, \end{aligned} \tag{C.1}$$

and those of the scalars are given by

$$\begin{aligned} \mathrm{d} * \mathrm{d}\varphi_1 &= \frac{1}{\sqrt{6}}\left(X_1^{-2}F^1 \wedge *F^1 + X_2^{-2}F^2 \wedge *F^2 - 2X_3^{-2}F^3 \wedge *F^3\right) \\ &\quad - g^2\frac{4}{\sqrt{6}}\left(X_1^{-1} + X_2^{-1} - 2X_3^{-1}\right)\mathrm{vol}_5\,, \end{aligned} \tag{C.2}$$

$$\mathrm{d} * \mathrm{d}\varphi_2 = \frac{1}{\sqrt{2}}\left(X_1^{-2}F^1 \wedge *F^1 - X_2^{-2}F^2 \wedge *F^2\right) - g^2 2\sqrt{2}\left(X_1^{-1} - X_2^{-1}\right)\mathrm{vol}_5\,.$$

Finally, the Einstein equation reads

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{2}\sum_{i=1}^{2}\partial_\mu\varphi_i\partial_\nu\varphi_i + \frac{1}{2}\sum_{i=1}^{3}X_i^{-2}\left(F^i_{\mu\rho}F^{i\ \rho}_{\nu} - \frac{1}{6}g_{\mu\nu}F^i_{\rho\sigma}F^{i\,\rho\sigma}\right) \\ &\quad - g_{\mu\nu}\frac{4}{3}g^2\sum_{i=1}^{3}X_i^{-1}\,. \end{aligned} \tag{C.3}$$

The reduction at the level of the equations of motion is most simply performed be first reducing on the internal space, in this case a Riemann surface $\Sigma_{\mathfrak{g}}$, and then rescaling the external space-time to go to Einstein frame. Thus, here we consider the initial Ansatz for the five-dimensional space-time

$$\mathrm{d}s_5^2 = \mathrm{d}s_3^2 + e^{2C}\mathrm{d}s^2\left(\Sigma_{\mathfrak{g}}\right), \tag{C.4}$$

where C is a scalar warp factor depending on the coordinates of the three-dimensional space-time.

To reduce the gauge field strengths we consider the Ansatz (3.4). The equations of motion for the gauge fields now reduce as

$$\begin{aligned}
\mathrm{d}\left(X_1^{-2}e^{2C} *_3 G^1\right) &= -\left(a_3 G^2 + a_2 G^3\right), \\
\mathrm{d}\left(X_2^{-2}e^{2C} *_3 G^2\right) &= -\left(a_3 G^1 + a_1 G^3\right), \\
\mathrm{d}\left(X_3^{-2}e^{2C} *_3 G^3\right) &= -\left(a_1 G^2 + a_2 G^1\right).
\end{aligned} \tag{C.5}$$

From the scalar equations of motion, we find

$$\begin{aligned}
\mathrm{d}\left(e^{2C} *_3 \mathrm{d}\varphi_1\right) = {}& \frac{1}{\sqrt{6}}e^{2C}\Big[X_1^{-2}\left(G^1\wedge *_3 G^1 + a_1^2 e^{-4C}\,\mathrm{vol}_3\right) + X_2^{-2}\Big(G^2\wedge *_3 G^2 \\
& + a_2^2 e^{-4C}\,\mathrm{vol}_3\Big) - 2X_3^{-2}\left(G^3\wedge *_3 G^3 + a_3^2 e^{-4C}\,\mathrm{vol}_3\right)\Big] \\
& - g^2\frac{4}{\sqrt{6}}e^{2C}\left(X_1^{-1} + X_2^{-1} - 2X_3^{-1}\right)\mathrm{vol}_3, \\
\mathrm{d}\left(e^{2C} *_3 \mathrm{d}\varphi_2\right) = {}& \frac{1}{\sqrt{2}}e^{2C}\Big[X_1^{-2}\left(G^1\wedge *_3 G^1 + a_1^2 e^{-4C}\,\mathrm{vol}_3\right) - X_2^{-2}\Big(G^2\wedge *_3 G^2 \\
& + a_2^2 e^{-4C}\,\mathrm{vol}_3\Big)\Big] - 2\sqrt{2}g^2 e^{2C}\left(X_1^{-1} - X_2^{-1}\right)\mathrm{vol}_3 .
\end{aligned} \tag{C.6}$$

The Einstein equation along the Riemann surface presents us with another scalar equation of motion, this time for C:

$$-\nabla_\mu\nabla^\mu C - 2\partial_\mu A\partial^\mu C + e^{-2C}\kappa = \frac{1}{2}\sum_{i=1}^{3}X_i^{-2}\left(\frac{2}{3}a_i^2 e^{-4C} - \frac{1}{6}G^i_{\rho\sigma}G^{i\,\rho\sigma}\right) - \frac{4}{3}g^2\sum_{i=1}^{3}X_i^{-1}, \tag{C.7}$$

where κ is the curvature of the Riemann surface.

Finally, the Einstein equation in three dimensions may be written as

$$\begin{aligned}
R_{\mu\nu} = {}& 2\left(\nabla_\nu\nabla_\mu C + \partial_\mu C\partial_\nu C\right) + \sum_{i=1}^{2}\partial_\mu\varphi_i\partial_\nu\varphi_i + \frac{1}{2}\sum_{i=1}^{3}X_i^{-2}\left(G^i_{\mu\rho}G^{i\;\rho}_{\nu} - \frac{1}{6}g_{\mu\nu}G^i_{\rho\sigma}G^{i\,\rho\sigma}\right) \\
& - \frac{1}{6}g_{\mu\nu}\sum_{i=1}^{3}\left(a_i^2 e^{-4C}X_i^{-2} + 8g^2 X_i^{-1}\right).
\end{aligned} \tag{C.8}$$

The above equations can be shown to result from varying the action

$$\begin{aligned}\mathcal{L}^{(3)} = e^{2C}\Bigg[& R *_3 \mathbf{1} + 2\mathrm{d}C \wedge *_3\mathrm{d}C - \frac{1}{2}\sum_{i=1}^{2}\mathrm{d}\varphi_i \wedge *_3\mathrm{d}\varphi_i - \frac{1}{2}\sum_{i=1}^{3} X_i^{-2} G^i \wedge *_3 G^i\Bigg] \\ & + \left(\sum_{i=1}^{3}\left[4g^2 e^{2C} X_i^{-1} - \frac{1}{2}e^{-2C} a_i^2 X_i^{-2}\right] + 2\kappa\right) *_3 \mathbf{1} + \mathcal{L}^{(3)}_{\text{top}}\,,\end{aligned} \tag{C.9}$$

where the topological term is

$$\mathcal{L}^{(3)}_{\text{top}} = a_1 B^2 \wedge G^3 + a_2 B^3 \wedge G^1 + a_3 B^1 \wedge G^2\,. \tag{C.10}$$

Here B^i is the one-form potential for G^i, $G^i = \mathrm{d}B^i$.

Now, to go to Einstein frame we just need to do a conformal transformation, $g_{\mu\nu} = e^{-4C}\hat{g}_{\mu\nu}$. This leads to the Einstein frame action (3.6) quoted in the text.

In checking the Einstein equation we have made use of the following Ricci tensor components

$$\begin{aligned} R_{\mu\nu} &= \bar{R}_{\mu\nu} - 2\left(\nabla_\nu\nabla_\mu C + \partial_\nu C \partial_\mu C\right), \\ R_{mn} &= \left[\kappa e^{-2C} - \nabla_\mu\nabla^\mu C - 2\partial_\mu C\partial^\mu C\right]\delta_{mn}\,, \end{aligned} \tag{C.11}$$

where μ,ν label space-time directions and m,n correspond to directions on the Riemann surface.

C.1 Killing spinor equations

We would like to confirm that the T tensor (3.16) can be extracted directly from the Killing spinor equations via reduction. In a related context, a similar calculation appeared in [19] and in that context assisted the identification of a five-dimensional prepotential. Our motivation here is the same.

We adopt the conventions for the Killing spinor equations in $D = 5$ from (F.1) of [15] (see also [66]), and in some sense, up to some additional fields, the calculation here is almost identical to appendix F of [15]. We work with the natural vielbein

$$e^\mu = e^{-2C}\bar{e}^\mu, \quad e^a = e^C\bar{e}^a\,, \tag{C.12}$$

where $\mu = 0,1,2$ label three-dimensional space-time directions and $a = 3,4$ denote directions along the Riemann surface. Our Ansatz for the flux follows from (3.4).

For the Killing spinor we make the choice

$$\epsilon = e^{\beta C}\xi \otimes \eta\,, \tag{C.13}$$

where β is a constant we will fix later. We use the following decomposition of the five-dimensional gamma matrices

$$\gamma^\mu = \rho^\mu \otimes \sigma^3, \qquad \gamma^3 = 1 \otimes \sigma^1, \qquad \gamma^4 = 1 \otimes \sigma^2. \tag{C.14}$$

As in [15], where one has $\gamma_{34}\epsilon = i\epsilon$, following decomposition, we have $\sigma^3\eta = \eta$.

Inserting the Ansatz into the Killing spinor equations we arrive at

$$2\delta\psi_3 = \left[\gamma_3{}^{\mu} e^{2C}\partial_\mu C + \sum_{i=1}^{3}\left(X_i\gamma_3 + \frac{i}{3}e^{-2C}a_iX_i^{-1}\gamma_4 + \frac{i}{12}e^{4C}\gamma_3{}^{\mu\nu}X_i^{-1}G^i_{\mu\nu}\right)\right]e^{\beta C}\xi\otimes\eta\,, \tag{C.15}$$

$$\begin{aligned}\sqrt{6}\delta\chi_{(1)} &= \left[\frac{1}{8}\sum_{i=1}^{2}X_i^{-1}\left(e^{4C}G^i_{\mu\nu}\gamma^{\mu\nu} - 2ia_ie^{-2C}\right) - \frac{1}{4}X_3^{-1}\left(e^{4C}G^3_{\mu\nu}\gamma^{\mu\nu} - 2ia_3e^{-2C}\right)\right.\\ &\qquad\left. + \frac{i}{2}\left(-X_1 - X_2 + 2X_3\right) - i\frac{\sqrt{6}}{4}e^{2C}\partial_\mu\varphi_1\gamma^\mu\right]e^{\beta C}\xi\otimes\eta\,, \end{aligned}\tag{C.16}$$

$$\begin{aligned}\sqrt{2}\delta\chi_{(2)} &= \left[\frac{1}{8}X_1^{-1}\left(e^{4C}G^1_{\mu\nu}\gamma^{\mu\nu} - 2ia_1e^{-2C}\right) - \frac{1}{8}X_2^{-1}\left(e^{4C}G^2_{\mu\nu}\gamma^{\mu\nu} - 2ia_2e^{-2C}\right)\right.\\ &\qquad\left. + \frac{i}{2}\left(-X_1 + X_2\right) - i\frac{\sqrt{2}}{4}e^{2C}\partial_\mu\varphi_2\gamma^\mu\right]e^{\beta C}\xi\otimes\eta\,. \end{aligned}\tag{C.17}$$

Note, in contrast to [15] where scalars with raised and lowered indices are employed, here our X_i are simply those in (2.22). As a consistency check, (C.15), (C.16), (C.17) agree with (3.20) of [15] when $G^i = 0$ and $\phi_i = \phi_i(r), C = g(r)$.

Taking various linear combinations we can write

$$\begin{aligned}4\gamma^3\delta\psi_3 + \frac{2}{3}\sqrt{6}i\delta\chi_{(1)} + 2\sqrt{2}i\delta\chi_{(2)} &= \delta_\epsilon\lambda^1\otimes\eta\,,\\ 4\gamma^3\delta\psi_3 + \frac{2}{3}\sqrt{6}i\delta\chi_{(1)} - 2\sqrt{2}i\delta\chi_{(2)} &= \delta_\epsilon\lambda^2\otimes\eta\,,\\ 4\gamma^3\delta\psi_3 - \frac{4}{3}\sqrt{6}i\delta\chi_{(1)} &= \delta_\epsilon\lambda^3\otimes\eta\end{aligned}\tag{C.18}$$

leading to the variations (constant $\beta = -2$)

$$\begin{aligned}\delta_\epsilon\lambda^1 &= \left[\rho^\mu\partial_\mu W_1 + \frac{i}{2}X_1^{-1}e^{2C}G^1_{\mu\nu}\rho^{\mu\nu} + e^{-4C}\left(2e^{2C}X_1 - a_2X_2^{-1} - a_3X_3^{-1}\right)\right]\xi,\\ \delta_\epsilon\lambda^2 &= \left[\rho^\mu\partial_\mu W_2 + \frac{i}{2}X_2^{-1}e^{2C}G^2_{\mu\nu}\rho^{\mu\nu} + e^{-4C}\left(2e^{2C}X_2 - a_1X_1^{-1} - a_3X_3^{-1}\right)\right]\xi,\\ \delta_\epsilon\lambda^3 &= \left[\rho^\mu\partial_\mu W_3 + \frac{i}{2}X_1^{-3}e^{2C}G^3_{\mu\nu}\rho^{\mu\nu} + e^{-4C}\left(2e^{2C}X_3 - a_1X_1^{-1} - a_2X_2^{-1}\right)\right]\xi\,.\end{aligned}\tag{C.19}$$

Dualising G^i as instructed in the text, the above equations can be condensed into a single equation

$$\delta_\epsilon\lambda^a = 2E_i{}^a\left(\rho^\mu D_\mu z^i - 2\partial^i T\right), \tag{C.20}$$

which is the expected form for the Killing spinor equation for the spinor fields [9, 11] and we see that the T tensor (3.16) features. $E_i{}^a$, $a = 1,2,3$, is the complex dreibein defined through $g_{i\bar{i}} = E_i{}^aE_{\bar{i}a}$, where $E_{\bar{i}a} = (E_i{}^a)^*$.

D Curvature for Kähler-Einstein space-times

Working in Einstein frame, we adopt the following Ansatz for the space-time

$$ds^2_{10} = e^{2A}ds^2\left(\mathcal{M}_3\right) + e^{2A}\frac{1}{4}e^{2W}\left(dz + P + A_1\right)^2 + e^{-2A}\sum_{a=1}^{3}e^{2V_a}ds^2\left(KE_2^{(a)}\right), \tag{D.1}$$

where A is a *constant* overall factor, we have dropped the overall scale L appearing in (2.11) and W, V_a, $a = 1,2,3$ denote scalar warp factors. A_1 is a one-form living on the three-dimensional space-time $\mathcal{M}_3$.

We adopt the natural orthonormal frame

$$e^\mu = e^A \, \bar{e}^\mu, \qquad e^z = e^{A+W} \frac{1}{2} \left(\mathrm{d}z + P + A_1 \right), \qquad e^i = e^{-A+V_a} \, \bar{e}^i, \tag{D.2}$$

where $\mu = 0,1,2$ label AdS_3 directions and $i = 3,\ldots,8$ correspond to directions along the internal Kähler-Einstein spaces.

With constant A, the spin-connection for the metric may be written as

$$\begin{aligned}
\omega^\mu{}_\nu &= \bar{\omega}^\mu{}_\nu - \frac{1}{4} e^{-A+W} (F_2)^\mu{}_\nu e^z, \\
\omega^i{}_j &= \bar{\omega}^i{}_j - \frac{1}{4} e^{3A+W-2V_a} l_a (J_a)^i{}_j e^z, \\
\omega^\mu{}_z &= -e^{-A} \partial^\mu W e^z - \frac{1}{4} e^{-A+W} (F_2)^\mu{}_\rho e^\rho, \\
\omega^i{}_\mu &= e^{-A} \partial_\mu V_a e^i, \\
\omega^i{}_z &= -\frac{1}{4} e^{3A+W-2V_a} l_a (J_a)^i{}_j e^j.
\end{aligned} \tag{D.3}$$

Using the above spin-connection one can calculate the Ricci-form

$$\begin{aligned}
R_{\mu\nu} &= e^{-2A} \left[\bar{R}_{\mu\nu} - (\nabla_\nu \nabla_\mu W + \partial_\mu W \partial_\nu W) - \sum_{a=1}^{3} 2 \left(\nabla_\nu \nabla_\mu V_a + \partial_\mu V_a \partial_\nu V_a \right) - \frac{1}{8} e^{2W} F_{2\,\mu\rho} F_{2\,\nu}{}^\rho \right], \\
R_{zz} &= \frac{1}{8} e^{6A+2W} \sum_{a=1}^{3} e^{-4V_a} l_a^2 - e^{-2A} \left(\nabla_\mu \nabla^\mu W + \partial_\mu W \partial^\mu W \right) - 2 \partial^\mu W \sum_{a=1}^{3} e^{-2A} \partial_\mu V_a \\
&\quad + \frac{1}{16} e^{-2A+2W} F_{2\,\rho\sigma} F_2^{\rho\sigma}, \\
R_{11} = R_{22} &= e^{-2A} \left[-\nabla_\mu \nabla^\mu V_1 - \partial_\mu W \partial^\mu V_1 - 2 \partial_\mu V_1 \sum_{i=a}^{3} \partial^\mu V_a \right] + l_1 e^{2A-2V_1} - \frac{1}{8} l_1^2 e^{4A+2W-4V_1}, \\
R_{33} = R_{44} &= e^{-2A} \left[-\nabla_\mu \nabla^\mu V_2 - \partial_\mu W \partial^\mu V_2 - 2 \partial_\mu V_2 \sum_{i=a}^{3} \partial^\mu V_a \right] + l_2 e^{2A-2V_2} - \frac{1}{8} l_2^2 e^{4A+2W-4V_2}, \\
R_{55} = R_{66} &= e^{-2A} \left[-\nabla_\mu \nabla^\mu V_3 - \partial_\mu W \partial^\mu V_3 - 2 \partial_\mu V_3 \sum_{a=1}^{3} \partial^\mu V_a \right] + l_3 e^{2A-2V_3} - \frac{1}{8} l_3^2 e^{4A+2W-4V_3}, \\
R_{\mu z} &= -\frac{1}{4} e^{-2W-2(V_1+V_2+V_3)} \nabla_\rho \left(e^{3W+2(V_1+V_2+V_3)} F_2{}^\rho{}_\mu \right),
\end{aligned} \tag{D.4}$$

where all other terms are zero.

E Details of reduction on $H^2 \times KE_4$

In this section we record equations of motion of the dimensionally reduced three-dimensional theory. This will be useful for testing the consistency of the reduction. We begin with the Bianchi identities. The Bianchi identities for the three-form fluxes $F_{(3)}$ and $H_{(3)}$ are trivially satisfied using the expressions in the text. The Bianchi for $F_{(5)}$ is partially satisfied, with the remaining equations being:

$$\mathrm{d}\left(e^{-\frac{4}{3}(U+V)+4C}*K_2\right)-4e^{-8U}*K_1+\epsilon\left(K_2-F_2\right)-N_1\wedge G_1-H_1\wedge M_1=0,$$
$$\mathrm{d}\left(e^{-8U}*K_1\right)+\frac{1}{2}N_1\wedge G_2-\frac{1}{2}H_2\wedge M_1=0\,. \qquad \text{(E.1)}$$

The equations of motion for $F_{(3)}$ and $H_{(3)}$ give respectively the equations

$$\mathrm{d}\left(e^{\frac{4}{3}(4U+V)+\phi-4C}*M_1\right)-4h\,\mathrm{vol}_3+2H_2\wedge K_1-2H_1\wedge K_2=0\,,$$
$$\mathrm{d}\left(e^{\frac{4}{3}(2U-V)+\phi+4C}*G_2\right)-4e^{-4U+\phi}*G_1-\epsilon e^{\frac{4}{3}(4U+V)+\phi-4C}*M_1$$
$$+ge^{\frac{4}{3}(4U+V)+\phi+8C}F_2+2N_1\wedge K_1+2e^{-\frac{4}{3}(U+V)+4C}H_1\wedge *K_2=0\,,$$
$$\mathrm{d}\left(e^{-4U+\phi}*G_1\right)-N_1\wedge K_2+\epsilon h\,\mathrm{vol}_3+e^{-\frac{4}{3}(U+V)+4C}H_2\wedge *K_2+2e^{-8U}H_1\wedge *K_1=0\,, \qquad \text{(E.2)}$$

and

$$\mathrm{d}\left(e^{\frac{4}{3}(4U+V)-\phi-4C}*N_1\right)+4g\,\mathrm{vol}_3-2G_2\wedge K_1+2G_1\wedge K_2-e^{\frac{4}{3}(4U+V)+\phi-4C}\mathrm{d}a\wedge *M_1=0,$$
$$\mathrm{d}\left(e^{\frac{4}{3}(2U-V)-\phi+4C}*H_2\right)-4e^{-4U-\phi}*H_1-\epsilon e^{\frac{4}{3}(4U+V)-\phi-4C}*N_1+he^{\frac{4}{3}(4U+V)-\phi+8C}F_2$$
$$-2M_1\wedge K_1-2e^{-\frac{4}{3}(U+V)+4C}G_1\wedge *K_2-e^{\frac{4}{3}(2U-V)+\phi+4C}\mathrm{d}a\wedge *G_2=0,$$
$$\mathrm{d}\left(e^{-4U-\phi}*H_1\right)+M_1\wedge K_2-\epsilon g\,\mathrm{vol}_3-e^{-\frac{4}{3}(U+V)+4C}G_2\wedge *K_2-2e^{-8U}G_1\wedge *K_1$$
$$-e^{-4U+\phi}\mathrm{d}a\wedge *G_1=0\,. \qquad \text{(E.3)}$$

The axion and dilaton equation are respectively

$$\mathrm{d}\left(e^{2\phi}*\mathrm{d}a\right)+e^{\frac{4}{3}(4U+V)+\phi-4C}N_1\wedge *M_1-e^{\frac{4}{3}(4U+V)+\phi+8C}gh\,\mathrm{vol}_3+e^{\frac{4}{3}(2U-V)+\phi+4C}H_2\wedge *G_2$$
$$2e^{-4U+\phi}H_1\wedge *G_1=0\,, \qquad \text{(E.4)}$$

and

$$\mathrm{d}*\mathrm{d}\phi-e^{2\phi}\mathrm{d}a\wedge *\mathrm{d}a+\frac{1}{2}e^{\frac{4}{3}(4U+V)-4C}\left[e^{-\phi}N_1\wedge *N_1-e^{\phi}M_1\wedge *M_1\right]$$
$$-\frac{1}{2}e^{\frac{4}{3}(4U+V)+8C}\left[e^{-\phi}h^2-e^{\phi}g^2\right]\mathrm{vol}_3+\frac{1}{2}e^{\frac{4}{3}(2U-V)+4C}\left[e^{-\phi}H_2\wedge *H_2-e^{\phi}G_2\wedge *G_2\right]$$
$$+e^{-4U}\left[e^{-\phi}H_1\wedge *H_1-e^{\phi}G_1\wedge *G_1\right]=0\,. \qquad \text{(E.5)}$$

The equations of motion for A_1, U and V are

$$\mathrm{d}\left(e^{\frac{8}{3}(U+V)+4C}*F_2\right)-2\epsilon K_2-8e^{-8U}*K_1+e^{\frac{4}{3}(4U+V)+8C}\left[e^{-\phi}hH_2+e^{\phi}gG_2\right]=0, \qquad \text{(E.6)}$$

$$\mathrm{d}*\mathrm{d}U+e^{-8U}K_1\wedge *K_1-\frac{1}{8}e^{\frac{4}{3}(4U+V)-4C}\left[e^{-\phi}N_1\wedge *N_1+e^{\phi}M_1\wedge *M_1\right] \qquad \text{(E.7)}$$
$$+\frac{1}{8}e^{\frac{4}{3}(4U+V)+8C}\left[e^{-\phi}h^2+e^{\phi}g^2\right]\mathrm{vol}_3-\frac{1}{8}e^{\frac{4}{3}(2U-V)+4C}\left[e^{-\phi}H_2\wedge *H_2+e^{\phi}G_2\wedge *G_2\right]$$
$$+\frac{1}{4}e^{-4U}\left[e^{-\phi}H_1\wedge *H_1+e^{\phi}G_1\wedge *G_1\right]+e^{-4C}\left(-6e^{-\frac{2}{3}(7U+V)}+2e^{\frac{4}{3}(-5U+V)}+4e^{-\frac{8}{3}(4U+V)}\right)=0,$$

$$
\begin{aligned}
&\mathrm{d}*\mathrm{d}V-\frac{1}{8}e^{\frac{4}{3}(4U+V)-4C}\left[e^{-\phi}N_1\wedge *N_1+e^{\phi}M_1\wedge *M_1\right]+\frac{1}{8}e^{\frac{4}{3}(4U+V)+8C}\left[e^{-\phi}h^2+e^{\phi}g^2\right]\mathrm{vol}_3\\
&\quad-\frac{1}{2}e^{\frac{8}{3}(U+V)+4C}F_2\wedge *F_2+\frac{1}{2}e^{-\frac{4}{3}(U+V)+4C}K_2\wedge *K_2-e^{-8U}K_1\wedge *K_1\\
&\quad-\frac{1}{2}\epsilon^2e^{\frac{8}{3}(U+V)-8C}\,\mathrm{vol}_3+\frac{1}{2}\epsilon^2e^{-\frac{4}{3}(U+V)-8C}\,\mathrm{vol}_3+\frac{3}{8}e^{\frac{4}{3}(2U-V)+4C}\left[e^{-\phi}H_2\wedge *H_2+e^{\phi}G_2\wedge *G_2\right]\\
&\quad-\frac{1}{4}e^{-4U}\left[e^{-\phi}H_1\wedge *H_1+e^{\phi}G_1\wedge *G_1\right]+e^{-4C}\left(-4e^{\frac{4}{3}(-5U+V)}+4e^{-\frac{8}{3}(4U+V)}\right)\mathrm{vol}_3=0\,. \qquad \text{(E.8)}
\end{aligned}
$$

F Details of reduction on $S^2\times T^4$

IIB reduced on CY_2. Here we briefly review the KK reduction Ansatz of type IIB on a Calabi-Yau two-fold that featured in [93]. The KK Ansatz in Einstein frame is

$$
\begin{aligned}
\mathrm{d}s_{10}^2&=e^{\frac{1}{2}\phi_2}\mathrm{d}s_6^2+e^{-\frac{1}{2}\phi_2}\mathrm{d}s^2\left(CY_2\right),\\
F_{(5)}&=\mathrm{vol}\left(CY_2\right)\wedge \mathrm{d}\chi_2+e^{2\phi_2}*_6\mathrm{d}\chi_2\,, \qquad \text{(F.1)}
\end{aligned}
$$

and all other fields of type IIB supergravity simply reduce to six dimensions. This Ansatz thus leads to extra scalars in addition to the axion χ_1 and dilaton ϕ_1 of type IIB supergravity, one corresponding to a breathing mode ϕ_2, and another axion χ_2 coming from the self-dual five-form flux. The six-dimensional action is

$$
\begin{aligned}
e^{-1}\mathcal{L}=R-\sum_{i=1}^{2}\frac{1}{2}\left(\partial\phi_i\right)^2-\sum_{i=1}^{2}\frac{1}{2}e^{2\phi_i}\left(\partial\chi_i\right)^2-\frac{1}{12}e^{-\phi_1-\phi_2}H_3^2\\
-\frac{1}{12}e^{\phi_1-\phi_2}F_3^2-\chi_2\mathrm{d}B_2\wedge \mathrm{d}C_2, \qquad \text{(F.2)}
\end{aligned}
$$

where $H_3=\mathrm{d}B_2$ and $F_3=\mathrm{d}C_2-\chi_1\mathrm{d}B_2$. Some sign changes relative to [93] follow from the difference in conventions. The equations of motion are:

$$\mathrm{d}\left(e^{\phi_1-\phi_2}*_6F_3\right)-\mathrm{d}\chi_2\wedge \mathrm{d}B_2=0, \qquad \text{(F.3)}$$

$$\mathrm{d}\left(e^{-\phi_1-\phi_2}*_6H_3\right)-e^{\phi_1-\phi_2}\mathrm{d}\chi_1\wedge *_6F_3+\mathrm{d}\chi_2\wedge F_3=0, \qquad \text{(F.4)}$$

$$\mathrm{d}\left(e^{2\phi_1}*_6d\chi_1\right)+e^{\phi_1-\phi_2}\mathrm{d}B_2\wedge *_6F_3=0, \qquad \text{(F.5)}$$

$$\mathrm{d}\left(e^{2\phi_2}*_6d\chi_2\right)-\mathrm{d}B_2\wedge \mathrm{d}C_2=0, \qquad \text{(F.6)}$$

$$\mathrm{d}*_6d\phi_1-e^{2\phi_1}\mathrm{d}\chi_1\wedge *_6\mathrm{d}\chi_1+\frac{1}{2}e^{-\phi_1-\phi_2}H_3\wedge *_6H_3-\frac{1}{2}e^{\phi_1-\phi_2}F_3\wedge *F_3=0, \qquad \text{(F.7)}$$

$$\mathrm{d}*_6d\phi_2-e^{2\phi_2}\mathrm{d}\chi_2\wedge *_6\mathrm{d}\chi_2+\frac{1}{2}e^{-\phi_1-\phi_2}H_3\wedge *_6H_3+\frac{1}{2}e^{\phi_1-\phi_2}F_3\wedge *F_3=0, \qquad \text{(F.8)}$$

$$
\begin{aligned}
R_{\mu\nu}=&\frac{1}{2}\sum_{i=1}^{2}\left(\partial_\mu\phi_i\partial_\nu\phi_i+e^{2\phi_i}\partial_\mu\chi_i\partial_\nu\chi_i\right)\\
&+\frac{1}{4}e^{-\phi_1-\phi_2}\left(H_{3\mu\rho_1\rho_2}H_{3\nu}{}^{\rho_1\rho_2}-\frac{1}{6}g_{\mu\nu}H_{3\rho_1\rho_2\rho_3}H_3{}^{\rho_1\rho_2\rho_3}\right)\\
&+\frac{1}{4}e^{\phi_1-\phi_2}\left(F_{3\mu\rho_1\rho_2}F_{3\nu}{}^{\rho_1\rho_2}-\frac{1}{6}g_{\mu\nu}F_{3\rho_1\rho_2\rho_3}F_3{}^{\rho_1\rho_2\rho_3}\right). \qquad \text{(F.9)}
\end{aligned}
$$

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Reduction to three dimensions. To reduce the above equations of motion to three dimensions we substitute in our six-dimensional space-time Ansatz

$$\mathrm{d}s_6^2 = \mathrm{d}s_3^2 + \frac{1}{4}e^{2U}\mathrm{d}s^2\left(S^2\right) + \frac{1}{4}e^{2V}\left(\mathrm{d}z + P + A_1\right), \tag{F.10}$$

and expressions for the three-form field strengths (4.37). From (F.3) and (F.4) we get

$$\mathrm{d}\left(e^{\phi_1-\phi_2+2U+V}g - 2\chi_2\sin\alpha\right) = 0, \tag{F.11}$$

$$\mathrm{d}\left(e^{\phi_1-\phi_2+V-2U} * G_1\right) + \mathrm{d}\chi_2\wedge H_2 = 0, \tag{F.12}$$

$$\mathrm{d}\left(e^{\phi_1-\phi_2-V+2U} * G_2\right) - 2e^{\phi_1-\phi_2+V-2U} * G_1 + \frac{1}{2}ge^{\phi_1-\phi_2+2U+V}F_2 - \mathrm{d}\chi_2\wedge H_1 = 0\,, \tag{F.13}$$

and

$$\mathrm{d}\left(e^{-\phi_1-\phi_2+V+2U}h\right) - e^{\phi_1-\phi_2+V+2U}g\mathrm{d}\chi_1 + G_0\mathrm{d}\chi_2 = 0, \tag{F.14}$$

$$\mathrm{d}\left(e^{-\phi_1-\phi_2+V-2U} * H_1\right) - e^{-\phi_1-\phi_2+V-2U}\mathrm{d}\chi_1\wedge *G_1 - \mathrm{d}\chi_2\wedge G_2 = 0, \tag{F.15}$$

$$\begin{aligned}\mathrm{d}\left(e^{-\phi_1-\phi_2-V+2U} * H_2\right) &- 2e^{-\phi_1-\phi_2+V-2U} * H_1 + \frac{1}{2}he^{-\phi_1-\phi_2+V+2U}F_2 \\ &- e^{\phi_1-\phi_2-V+2U}\mathrm{d}\chi_1\wedge *G_2 + \mathrm{d}\chi_2\wedge G_1 = 0\,.\end{aligned} \tag{F.16}$$

We can now solve (F.11) and (F.14) to determine g and h

$$g = 2e^{-\phi_1+\phi_2-V-2U}\left(\cos\alpha + \sin\alpha\chi_2\right), \tag{F.17}$$

$$h = 2e^{\phi_1+\phi_2-V-2U}\left[\sin\alpha - \cos\alpha\chi_2 + (\cos\alpha + \sin\alpha\chi_2)\chi_1\right]. \tag{F.18}$$

In the process we have chosen the integration constants for convenience.

From (F.5) and (F.6) we get the following two equations:

$$\begin{aligned}\mathrm{d}\left(e^{2\phi_1+2U+V} * \mathrm{d}\chi_1\right) &+ \left[2\sin\alpha G_0 e^{\phi_1-\phi_2-2U-V} - ghe^{\phi_1-\phi_2+2U+V}\right]\mathrm{vol}_3 \\ &+ e^{\phi_1-\phi_2+V-2U}H_1\wedge *G_1 + e^{\phi_1-\phi_2-V+2U}H_2\wedge *G_2 = 0\,,\end{aligned} \tag{F.19}$$

$$\mathrm{d}\left(e^{2\phi_1+2U+V} * \mathrm{d}\chi_2\right) + \left[hG_0 - 2\sin\alpha g\right]\mathrm{vol}_3 + H_1\wedge G_2 - G_1\wedge H_2 = 0\,. \tag{F.20}$$

The final two scalar equations give

$$\begin{aligned}&\mathrm{d}\left(e^{2U+V} * \mathrm{d}\phi_1\right) - e^{2\phi_1+2U+V}\mathrm{d}\chi_1\wedge *\mathrm{d}\chi_1 + \frac{1}{2}e^{-\phi_2-2U-V}\left[4e^{-\phi_1}\sin^2\alpha - e^{\phi_1}G_0^2\right]\mathrm{vol}_3 \\ &+ \frac{1}{2}e^{-\phi_2-2U+V}\left[e^{-\phi_1}H_1\wedge *H_1 - e^{\phi_1}G_1\wedge *G_1\right] + \frac{1}{2}e^{-\phi_2+2U-V}\left[e^{-\phi_1}H_2\wedge *H_2 - e^{\phi_1}G_2\wedge *G_2\right] \\ &- \frac{1}{2}e^{-\phi_2+2U+V}\left[e^{-\phi_1}h^2 - e^{\phi_1}g^2\right]\mathrm{vol}_3 = 0,\end{aligned} \tag{F.21}$$

$$\begin{aligned}&\mathrm{d}\left(e^{2U+V} * \mathrm{d}\phi_2\right) - e^{2\phi_2+2U+V}\mathrm{d}\chi_2\wedge *\mathrm{d}\chi_2 + \frac{1}{2}e^{-\phi_2-2U-V}\left[4e^{-\phi_1}\sin^2\alpha + e^{\phi_1}G_0^2\right]\mathrm{vol}_3 \\ &+ \frac{1}{2}e^{-\phi_2-2U+V}\left[e^{-\phi_1}H_1\wedge *H_1 + e^{\phi_1}G_1\wedge *G_1\right] + \frac{1}{2}e^{-\phi_2+2U-V}\left[e^{-\phi_1}H_2\wedge *H_2 + e^{\phi_1}G_2\wedge *G_2\right] \\ &- \frac{1}{2}e^{-\phi_2+2U+V}\left[e^{-\phi_1}h^2 + e^{\phi_1}g^2\right]\mathrm{vol}_3 = 0\,.\end{aligned} \tag{F.22}$$

We now only have to work out the Einstein equation. Taking into account a change in how we define scalars, namely $W \to V, V_1 \to U$, we can use the Ricci tensor appearing in (D.4). We simply have to take note of the fact that the S^2 is normalised so that $l_1 = 4$, in which case $A = 0$.

From the Einstein equation, we get the following equations:

$$
\begin{aligned}
&2e^{2V-4U}\,\mathrm{vol}_3 - \mathrm{d}*\mathrm{d}V - \mathrm{d}V\wedge*\mathrm{d}V - 2\mathrm{d}V\wedge*\mathrm{d}U + \frac{1}{8}e^{2V}F_2\wedge*F_2 \qquad \text{(F.23)}\\
&= \left[\frac{1}{4}e^{-\phi_2-2V-4U}\left(4e^{-\phi_1}\sin^2\alpha + e^{\phi_1}G_0^2\right) + \frac{1}{4}e^{-\phi_2}\left(e^{-\phi_1}h^2 + e^{\phi_1}g^2\right)\right]\mathrm{vol}_3\\
&\quad + \frac{1}{4}e^{-\phi_2-2V}\left[e^{-\phi_1}H_2\wedge*H_2 + e^{\phi_1}G_2\wedge*G_2\right] - \frac{1}{4}e^{-\phi_2-4U}\left[e^{-\phi_1}H_1\wedge*H_1 + e^{\phi_1}G_1\wedge*G_1\right]\\
&\left(4e^{-2U} - 2e^{2V-4U}\right)\mathrm{vol}_3 - \mathrm{d}*\mathrm{d}U - \mathrm{d}U\wedge*\mathrm{d}V - 2\mathrm{d}U\wedge*\mathrm{d}U \qquad \text{(F.24)}\\
&= \left[\frac{1}{4}e^{-\phi_2-2V-4U}\left(4e^{-\phi_1}\sin^2\alpha + e^{\phi_1}G_0^2\right) + \frac{1}{4}e^{-\phi_2}\left(e^{-\phi_1}h^2 + e^{\phi_1}g^2\right)\right]\mathrm{vol}_3\\
&\quad - \frac{1}{4}e^{-\phi_2-2V}\left[e^{-\phi_1}H_2\wedge*H_2 + e^{\phi_1}G_2\wedge*G_2\right] + \frac{1}{4}e^{-\phi_2-4U}\left[e^{-\phi_1}H_1\wedge*H_1 + e^{\phi_1}G_1\wedge*G_1\right]\\
&\frac{1}{2}e^{-2U-2V}\mathrm{d}\left(e^{3V+2U}*F_2\right) = 2\sin\alpha e^{-\phi_1-\phi_2-4U-V}*H_1 + G_0e^{\phi_1-\phi_2-4U-V}*G_1\\
&\qquad\qquad - e^{-\phi_1-\phi_2-V}hH_2 - e^{\phi_1-\phi_2-V}gG_2\,. \qquad \text{(F.25)}
\end{aligned}
$$

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$\frac{1}{2}$-BPS Domain wall from $N = 10$ three dimensional gauged supergravity

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Abstract: We explicitly construct $N = 10$ Chern-Simons gaged supergravity in three dimensions with non-semisimple gauge group $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$. The gauge group is embedded in $E_{6(-14)}$ which is the isometry group of the 32-dimensional scalar manifold $E_{6(-14)}/\mathrm{SO}(10) \times \mathrm{U}(1)$. The resulting theory is on-shell equivalent to SO(5) Yang-Mills gauged supergravity coming from dimensional reduction on S^1 of SO(5) $N = 5$ gauged supergravity in four dimensions. We discuss the spectrum of the corresponding reduction. The $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ gauged supergravity, describing the reduced theory, admits a $\frac{1}{2}$-BPS domain wall vacuum solution whose explicit form is also given. This provides an example of a domain wall in non-maximal gauged supergravity.

Keywords: Supergravity Models, Gauge-gravity correspondence, AdS-CFT Correspondence

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Contents

1 Introduction

Chern-Simons gauged supergravity in three dimensions has a very rich structure due to the duality between scalars and vectors in three dimensions. There are many possible gauge groups since there is no restriction on the number of vector fields that act as gauge fields [1, 2], or equivalently, no restriction on the dimension of the gauge group provided that it can be embedded in the global symmetry group and consistent with supersymmetry. Any number of vector fields can be introduced via Chern-Simons terms which do not give rise to extra degrees of freedom. The theory is also useful in the study of AdS$_3$/CFT$_2$ correspondence, see for example [3] for a nice review.

To understand AdS$_3$/CFT$_2$ correspondence in the context of string/M theory, the embedding of three dimensional gauged supergravity in ten or eleven dimensions is required. The usual procedure to obtain lower dimensional supergravities from higher dimensional theories is the Kaluza-Klein (KK) dimensional reduction. The general U-duality covariant formulation of three dimensional gauged supergravities is in the form of Chern-Simons theory in which the gauge fields enter the Lagrangian through the Chern-Simons terms [4]. On the other hand, dimensional reductions result in Yang-Mills type gauged supergravity in which gauge kinetic terms are in the form of conventional Yang-Mills terms. The known class of Chern-Simons gauge groups that gives equivalent Yang-Mills type theory is of non-semisimple type [5]. Any Yang-Mills type Lagrangian can be rewritten in the Chern-Simons form by introducing two gauge fields and a compensating scalar for each Yang-Mills gauge field. This makes non-semisimple gauge groups more interesting in finding effective theories of string/M theory in three dimensions.

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Some embeddings of three dimensional gauged supergravities into higher dimensions have appeared so far. These examples include $N = 2, 4, 8, 16$ gauged supergravities from reductions on spheres and Calabi-Yau manifold in [6–12] and recently various $N = 2$ theories from wrapped D3-branes of [13]. In this paper, we will give another example of this embedding namely $N = 10$ gauged supergravity with $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ gauge group. Due to the above mentioned equivalent between Chern-Simons and Yang-Mills type gauged supergravities, this should potentially describe $N = 5$ gauged supergravity in four dimensions with gauged group SO(5) reduced on S^1. The latter has been constructed in [14]. It has been shown in [15] that the theory admits two AdS_4 critical points, an $N = 5$ supersymmetric point with SO(5) gauge symmetry and a non-supersymmetric point with SO(3) residual gauge symmetry. The theory has also been studied in the context of holographic superconductor in [16]. The non-supersymmetric critical point is perturbatively stable with all mass-squares above the BF-bound.

Unlike the four dimensional analogue which has maximally supersymmetric AdS_4 ground state, we will find that the reduced theory in three dimensions admits only a $\frac{1}{2}$-BPS domain wall as a vacuum solution. This is in contrast to compact and non-compact gaugings of the same theory studied in [17] that admits maximally supersymmetric AdS_3 critical points. The loss of supersymmetry after S^1 reduction has been pointed out in the context of non-semisimple gaugings in three dimensions in [10]. A general result on S^1 reduction of AdS spaces has been given in [18]. There are many known $\frac{1}{2}$-BPS domain walls in higher dimensional gauged supergravities, see for example [19–24] as well as in lower dimensions, see [25] and [26] for three- and two-dimensional solutions. These domain walls are important in the context of the DW/QFT correspondence [27–29] which is a generalization to non-conformal field theories of the original AdS/CFT correspondence [30]. They are also useful in the study of domain wall/cosmology [31–33].

The paper is organized as follow. In section 2, we review the general structure of N extended gauged supergravities in three dimensions including all relevant formulae and notations. The $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ gauged supergravity and the associated domain wall solution are discussed in section 3. We then discuss possible higher dimensional origin of the resulting theory from S^1 dimensional reduction of $N = 5$ SO(5) gauged supergravity in four dimensions. We finally give some conclusions and comments in section 5. All details and explicit calculations are given in appendix A. In appendix B, we will explore possible non-semisimple gauge groups of $N = 9$ gauged supergravity in three dimensions.

2 $N = 10$ gauged supergravity in three dimensions with non-semisimple gauge groups

Before going to the detail of the construction, we briefly review the general structure of three dimensional gauged supergravities and apply it to the construction of $N = 10$ gauged supergravity with non-semisimple gauge group $\mathrm{SO}(10) \ltimes \mathbf{T}^{10}$. We will keep the number of supersymmetry to be N for conveniences and later set $N = 10$. In general, the matter coupled supergravity in three dimensions is in the form of a non-linear sigma model coupled to supergravity. For $N > 4$, supersymmetry demands that the scalar target manifold must be a symmetric space of the form G/H in which G and H are the global symmetry group and its maximal compact subgroup, respectively [34]. In particular, for $N > 8$, supersymmetry determines the scalar manifold uniquely. In the present case of $N = 10$, the scalar manifold is given by the coset space $E_{6(-14)}/\mathrm{SO}(10)\times\mathrm{U}(1)$ which is a 32-dimensional Kahler manifold.

Coupling of the sigma model to N-extended supergravity requires the presence of $N-1$ almost complex structures f^P, $P = 2,\dots,N$ on the scalar manifold. The tensors $f^{IJ} = f^{[IJ]}$, $I,J = 1,\dots,N$, constructed by the relation

$$f^{1P} = -f^{P1} = f^P, \qquad f^{PQ} = f^{[P}f^{Q]}\,. \tag{2.1}$$

generate the $\mathrm{SO}(N)$ R-symmetry in a spinor representation under which scalar fields transform. On symmetric scalar manifolds of the form G/H, the maximal compact subgroup $H = \mathrm{SO}(N)\times H'$ contains the R-symmetry $\mathrm{SO}(N)$ and another compact subgroup H' commuting with $\mathrm{SO}(N)$. In $N = 10$ theory, the group H' is simply $\mathrm{U}(1)$. The G-generators $t^{\mathcal{M}}$, $\mathcal{M} = 1,\dots,\mathrm{dim}G$, can be split into (T^{IJ}, X^α) generating, respectively, $\mathrm{SO}(N)\times H'$ and non-compact generators Y^A corresponding to $\dim G - \dim H$ scalars. The global symmetry group G is characterized by the following algebra

$$\begin{aligned}
\left[T^{IJ},T^{KL}\right] &= -4\delta^{[I[K}T^{L]J]}, & \left[T^{IJ},Y^A\right] &= -\frac{1}{2}f^{IJ,AB}Y_B,\\
\left[X^\alpha,X^\beta\right] &= f^{\alpha\beta}_{\ \ \gamma}X^\gamma, & \left[X^\alpha,Y^A\right] &= h^\alpha{}_B{}^AY^B,\\
\left[Y^A,Y^B\right] &= \frac{1}{4}f^{AB}_{IJ}T^{IJ}+\frac{1}{8}C_{\alpha\beta}h^{\beta AB}X^\alpha\,.
\end{aligned} \tag{2.2}$$

The tensors f^{IJ} are related to $\mathrm{SO}(N)$ gamma matrices, $\Gamma^I_{A\dot{A}}$ in which A and $\dot{A}$ label spinor and conjugate spinor representations, respectively, by

$$f^{IJ} = -\frac{1}{2}\Gamma^{IJ} = -\frac{1}{4}\left(\Gamma^I\Gamma^J-\Gamma^J\Gamma^I\right). \tag{2.3}$$

$C_{\alpha\beta}$ and $f^{\alpha\beta}_{\ \ \gamma}$ are H' invariant tensor and H' structure constants, respectively. The H' group is generated in the $\mathrm{SO}(N)$ spinor representation by matrices $h^\alpha{}_B{}^A$. The coset manifold whose coordinates are given by $d = \dim(G/H)$ scalar fields ϕ^i, $i = 1,\dots,d$ can be described by a coset representative L. The usual formulae for a coset space are

$$L^{-1}t^{\mathcal{M}}L = \frac{1}{2}\mathcal{V}^{\mathcal{M}}{}_{IJ}T^{IJ}+\mathcal{V}^{\mathcal{M}}{}_\alpha X^\alpha+\mathcal{V}^{\mathcal{M}}{}_AY^A, \tag{2.4}$$

$$L^{-1}\partial_iL = \frac{1}{2}Q_i^{IJ}T^{IJ}+Q_i^\alpha X^\alpha+e_i^AY^A \tag{2.5}$$

which will be useful later on. e_i^A is the vielbein on the scalar manifold while Q_i^{IJ} and Q_i^α are $\mathrm{SO}(N)\times H'$ composite connections. Scalar matrices $\mathcal{V}$ will be used to define the moment maps below.

Gaugings of supergravities in various space-time dimensions are efficiently described in a G-covariant way by the so-called embedding tensor formalism [1]. In essence, the embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$ is a symmetric gauge invariant tensor that acts as a projector from the global symmetry group G to a particular gauge group. Gauge covariant derivatives describing the minimal coupling of the gauge fields $A^{\mathcal{M}}_\mu$ to other fields also involve the embedding tensor. For example, the covariant derivative on scalar fields is given by

$$\mathcal{D}_\mu\phi^i = \partial_\mu\phi^i + g\Theta_{\mathcal{M}\mathcal{N}}A^{\mathcal{M}}_\mu X^{\mathcal{N}i} \tag{2.6}$$

where $X^{\mathcal{N}i}$ are Killing vectors generating isometries on the scalar manifold and g is the gauge coupling constant.

In order to define a viable gauging, the embedding tensor has to satisfy the so-called quadratic constraint

$$\Theta_{\mathcal{P}\mathcal{L}}f^{\mathcal{K}\mathcal{L}}{}_{(\mathcal{M}}\Theta_{\mathcal{N})\mathcal{K}} = 0, \tag{2.7}$$

which is the requirement that the gauge generators $\Theta_{\mathcal{M}\mathcal{N}}t^{\mathcal{N}}$ form a closed algebra, or equivalently the gauge group is a proper subgroup of G. Furthermore, for supersymmetry to be preserved in the gauging process, the embedding tensor needs to satisfy the projection constraint

$$\mathbb{P}_{R_0}\Theta_{\mathcal{M}\mathcal{N}} = 0\,. \tag{2.8}$$

This condition comes from supersymmetry, but it should be noted that the constraint in this form is obtained by regarding the scalar manifold to be a symmetric space.

It is useful to introduce the T-tensor given by the moment map of the embedding tensor by scalar matrices $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$, obtained from (2.4),

$$T_{\mathcal{A}\mathcal{B}} = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\Theta_{\mathcal{M}\mathcal{N}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}}\,. \tag{2.9}$$

The T-tensor transforms under the maximal compact subgroup H and consists of various components such as $T^{IJ,KL}$, $T^{IJ,A}$ and $T^{A,B}$. Since fermions transform under H, the fermion couplings will be written in term of the T-tensor or linear combinations of its components as we will see below. For any supersymmetric gauging, supersymmetry requires only that the T-tensor satisfies the projection

$$\mathbb{P}_{\boxplus}T^{IJ,KL} = 0 \tag{2.10}$$

where $\boxplus$ is the Riemann tensor-like representation of SO(N). In the case of symmetric scalar manifolds which are of interest in this paper, this constraint can be lifted to the constraint on the embedding tensor given in (2.8) in which the G-representation R_0, branched under SO(N), contains $\boxplus$ representation of SO(N). Any subgroup of G whose embedding tensor satisfies the above constraints is called admissible gauge group.

In general, gaugings need some modifications to the original ungauged Lagrangian by fermionic mass-like terms and a scalar potential, at order g and g^2, respectively. Also, the supersymmetry transformation rules need to be modified at order g. In what follow, we will need the scalar potential and fermionic supersymmetry transformations. They are written in terms of the A^{IJ}_1 and A^{IJ}_{2i} tensors which are in turn constructed from various components of the T-tensor

$$A^{IJ}_1 = -\frac{4}{N-2}T^{IM,JM} + \frac{2}{(N-1)(N-2)}\delta^{IJ}T^{MN,MN}, \tag{2.11}$$

$$A^{IJ}_{2j} = \frac{2}{N}T^{IJ}{}_j + \frac{4}{N(N-2)}f^{M(Im}{}_j T^{J)M}{}_m + \frac{2}{N(N-1)(N-2)}\delta^{IJ}f^{KL}{}_j{}^m T^{KL}{}_m\,. \tag{2.12}$$

The scalar potential is simply given by

$$V=-\frac{4}{N}g^2\left(A_1^{IJ}A_1^{IJ}-\frac{1}{2}Ng^{ij}A_{2i}^{IJ}A_{2j}^{IJ}\right). \tag{2.13}$$

The metric g_{ij} on the target manifold is related to the vielbein by $g_{ij}=e_i^Ae_j^A$. We also note here that the quadratic constraint (2.7) can be written in terms of A_1^{IJ} and A_{2i}^{IJ} as

$$2A_1^{IK}A_1^{KJ}-NA_2^{IKi}A_{2i}^{JK}=\frac{1}{N}\delta^{IJ}\left(2A_1^{KL}A_1^{KL}-NA_2^{KLi}A_{2i}^{KL}\right). \tag{2.14}$$

The fermionic field content of the N extended supergravity in three dimensions consists of N gravitini ψ_μ^I and d spin-$\frac{1}{2}$ fields χ^{iI}. The latter is written in an overcomplete basis and subject to the projection constraint

$$\chi^{iI}=\frac{1}{N}\left(\delta^{IJ}\delta_j^i-f^{IJi}{}_j\right)\chi^{jJ} \tag{2.15}$$

giving rise to d independent χ^{iI} fields. The fermions χ^{iI} can be redefined such that they transform in a conjugate spinor representation of SO(N) via [4]

$$\chi^{\dot{A}}=\frac{1}{N}e_i^A\Gamma^I_{A\dot{A}}\chi^{iI}. \tag{2.16}$$

The corresponding supersymmetry transformations are as follow:

$$\delta\psi_\mu^I=\mathcal{D}_\mu\epsilon^I+gA_1^{IJ}\gamma_\mu\epsilon^J, \tag{2.17}$$

$$\delta\chi^{iI}=\frac{1}{2}(\delta^{IJ}\mathbf{1}-f^{IJ})^i{}_j\not{\mathcal{D}}\phi^j\epsilon^J-gNA_2^{JIi}\epsilon^J \tag{2.18}$$

where only relevant terms are given and

$$\mathcal{D}_\mu\epsilon^I=\partial_\mu\epsilon^I+\frac{1}{4}\omega_\mu^{ab}\gamma_{ab}+\partial_\mu\phi Q_i^{IJ}\epsilon^I+g\Theta_{\mathcal{M}\mathcal{N}}A_\mu^{\mathcal{M}}\mathcal{V}^{\mathcal{N}IJ}\epsilon^J. \tag{2.19}$$

Gauge groups of interest to us are non-semisimple groups of the form $G_0\ltimes\mathbf{T}^{\dim G}$. The translational symmetry $\mathbf{T}^{\dim G}$ consists of $\dim G$ commuting generators which transform as an adjoint representation under G_0. This type of gauge groups gives rise to the on-shell equivalent Yang-Mills gauged supergravity coming from dimensional reductions of some higher dimensional theory. The $G_0\ltimes\mathbf{T}^{\dim G}$ gauge group whose generators are respectively J^m and T^m, $m=1,\ldots,\dim G$ is characterized by the following algebra

$$[J^m,J^n]=f^{mn}{}_kJ^k,\qquad [J^m,T^n]=f^{mn}{}_kT^k,\qquad [T^m,T^n]=0 \tag{2.20}$$

where $f^{mn}{}_k$ are G_0 structure constants. We will denote the G_0 and $\mathbf{T}^{\dim G}$ parts of the gauge group by a and b, respectively. As shown in [5], the corresponding embedding tensor consists of two parts, one with the coupling between a and b types $\Theta_{\rm ab}$ and the other with the coupling between b and b types $\Theta_{\rm bb}$. The full embedding tensor can be written as

$$\Theta=g_1\Theta_{\rm ab}+g_2\Theta_{\rm bb} \tag{2.21}$$

with g_1 and g_2 being the coupling constants. Supersymmetry constraint (2.8) may impose some relation on g_1 and g_2 such that eventually there is only one coupling. Both Θ_{ab} and Θ_{bb} are given by the Cartan-Killing form of G_0, $\eta^{G_0}_{mn}$, which is non-degenerate since G_0 is semisimple. The above information is sufficient for our discussion in this paper. The interested readers are invited to consult [4] and [5] for more a detailed discussion about three dimensional gauged supergravity with non-semisimple gauge groups.

3 $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ gauged supergravity and $\frac{1}{2}$-BPS domain wall solution

In this section, we explicitly construct $N = 10$ gauged supergravity with $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ gauge group. We begin with the scalar manifold $E_{6(-14)}/\mathrm{SO}(10) \times \mathrm{U}(1)$ and use E_6 generators given in [35] and [36]. The non-compact form $E_{6(-14)}$ is constructed by using the "Weyl unitarity trick". We follow the same construction and notation as in [17] to which we refer the readers for more details.

The 78 generators of E_6 constructed in [36] are labeled by c_i, $i = 1, \dots, 78$. The $\mathrm{SO}(10)$ R-symmetry is generated by c_i, $i = 1, \dots, 21, 30, \dots 36, 45, \dots, 52, 71, \dots, 78$ and $\tilde{c}_{53}$. We need to relabel these generators to the form of T^{IJ} in our $\mathrm{SO}(N)$ covariant formalism. This has already been done in [17], but we will repeat it in appendix A for convenience. The group $H' = \mathrm{U}(1)$ is generated by $\tilde{c}_{70}$ whose definition and that of $\tilde{c}_{53}$ can be found in appendix A.

The non-compact generators can be identified as

$$Y^A = \begin{cases} ic_{A+21} & \text{for } A = 1, \dots, 8 \\ ic_{A+28} & \text{for } A = 9, \dots, 16 \\ ic_{A+37} & \text{for } A = 17, \dots, 32 \end{cases} . \tag{3.1}$$

We can then use (2.2) to extract the tensors f^{IJ} whose components are computed by

$$f^{IJ}_{AB} = -\frac{1}{3}\mathrm{Tr}\left(\left[T^{IJ}, Y^A\right] Y^B\right). \tag{3.2}$$

Notice that the generators have normalizations $\mathrm{Tr}(T^{IJ}T^{IJ}) = -6$ and $\mathrm{Tr}(Y^AY^A) = 6$, no sum on IJ and A.

We now construct generators of the gauge group $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$. This group is embedded in $\mathrm{USp}(4,4) \subset E_{6(-14)}$. The maximal compact subgroup $\mathrm{USp}(4) \times \mathrm{USp}(4) \subset \mathrm{USp}(4,4)$ is identified as the $\mathrm{SO}(5) \times \mathrm{SO}(5)$ subgroup of the R-symmetry $\mathrm{SO}(10)$. Recall that the 32 scalars transform as $\mathbf{16}^+ + \mathbf{16}^-$ under $\mathrm{SO}(10) \times \mathrm{U}(1)$. Under $\mathrm{SO}(5) \times \mathrm{SO}(5)$, the scalars transform as

$$\mathbf{16}^+ + \mathbf{16}^- = (\mathbf{4}, \mathbf{4})^+ + (\mathbf{4}, \mathbf{4})^- . \tag{3.3}$$

We then identify $\mathrm{SO}(5)$ part of the gauge group as the diagonal subgroup $\mathrm{SO}(5)_{\mathrm{diag}} \subset \mathrm{SO}(5) \times \mathrm{SO}(5)$ under which scalars transform as

$$\begin{aligned} \mathbf{16}^+ + \mathbf{16}^- &= (\mathbf{4} \times \mathbf{4})^+ + (\mathbf{4} \times \mathbf{4})^- \\ &= (\mathbf{1} + \mathbf{10} + \mathbf{5})^+ + (\mathbf{1} + \mathbf{10} + \mathbf{5})^- . \end{aligned} \tag{3.4}$$

In this decomposition, we see that there are two singlets under $SO(5)_{\text{diag}}$. The adjoint representation $\mathbf{10}^+$ and $\mathbf{10}^-$ will be used to construct the translational generators of $\mathbf{T}^{10}$.

The explicit form of the corresponding gauge generators are as follow. The $SO(5)_{\text{diag}}$ generators are given by

$$J^{ij} = T^{ij} + T^{i+5,j+9}, \qquad i,j = 1,\ldots,5 \tag{3.5}$$

while the $\mathbf{T}^{10}$ generators are found to be

$$t^{ij} = T^{ij} - T^{i+5,j+5} + \tilde{Y}^{ij}, \qquad ,i,j = 1,\ldots,5 \tag{3.6}$$

where $\tilde{Y}^{ij}$ are given in appendix A.

The embedding tensor is of the form

$$\Theta = g_1\Theta_{\text{ab}} + g_2\Theta_{\text{bb}} \tag{3.7}$$

where Θ_{ab} and Θ_{bb} are given by the Cartan-Killing form of SO(5). The supersymmetry constraint requires $g_2 = 0$ meaning that there is no coupling among $\mathbf{T}^{10}$ generators. This is similar to $N = 16$ and $N = 8$ theories with $SO(8) \ltimes \mathbf{T}^{28}$ gauge group studied in [10, 25].

We are now in a position to study the scalar potential of the resulting gauged supergravity. Following the technique of [37], we begin with scalar fields which are singlets under the semisimple part of the gauge group, SO(5). They are given by $\mathbf{1}^{\pm}$ in (3.4) and correspond to the non-compact generators

$$\begin{aligned} Y_{s1} &= Y_3 - Y_5 - Y_{12} + Y_{16} + Y_{17} - Y_{18} + Y_{27} + Y_{29}, \\ Y_{s2} &= Y_4 + Y_8 + Y_{11} + Y_{13} + Y_{22} - Y_{23} + Y_{28} - Y_{32}\,. \end{aligned} \tag{3.8}$$

Accordingly, the coset representative is parametrized by

$$L = e^{aY_{s1}}e^{bY_{s2}}\,. \tag{3.9}$$

Using the formulae (A.4) and (A.5), we can compute A_1^{IJ} and A_{2i}^{IJ} by using a computer program *Mathematica*. The scalar potential is computed to be

$$V = -6e^{4(a-b)}\left(1 + e^{8b}\right)g^2 \tag{3.10}$$

where we have denoted g_1 simply by g. The presence of the e^a factor implies that the potential has no critical point. We then expect the vacuum solution to be a domain wall.

To find a domain wall solution, we adopt the usual domain wall ansatz for the metric

$$ds^2 = e^{2A}dx_{1,1}^2 + dr^2\,. \tag{3.11}$$

The supersymmetry transformation of χ^{iI}, $\delta\chi^{iI} = 0$ from equation (2.18), gives the following equations

$$b'\gamma_r\epsilon^I + \frac{1}{2}g(1 - e^{4b})e^{2(a-b)}\epsilon^I = 0, \qquad I = 1,\ldots,5, \tag{3.12}$$

$$b'\gamma_r\epsilon^I - \frac{1}{2}g(1 - e^{4b})e^{2(a-b)}\epsilon^I = 0, \qquad I = 6,\ldots,10, \tag{3.13}$$

$$a'\gamma_r\epsilon^I - g\frac{e^{2(a+b)}\left(1+e^{4b}\right)}{1 + e^{8b}}\epsilon^I = 0, \qquad I = 1,\ldots,5, \tag{3.14}$$

$$a'\gamma_r\epsilon^I + g\frac{e^{2(a+b)}\left(1+e^{4b}\right)}{1 + e^{8b}}\epsilon^I = 0, \qquad I = 6,\ldots,10 \tag{3.15}$$

where we have used $'$ to denote the derivative $\frac{d}{dr}$ and $\phi^{A'} = \frac{1}{6}\mathrm{Tr}\left(L^{-1}L'Y^A\right)$. We will now impose the projection conditions $\gamma_r\epsilon^I = -\epsilon^I$ for $I = 1, \dots, 5$ and $\gamma_r\epsilon^I = \epsilon^I$ for $I = 6, \dots, 10$. ϵ^I has two real components. The projectors then reduce the supersymmetry by a fraction of $\frac{1}{2}$. With these two projectors, we end up with two independent equations

$$b' = \frac{1}{2}g(1 - e^{4b})e^{2(a-b)}, \tag{3.16}$$

$$a' = -g\frac{e^{2(a+b)}\left(1+e^{4b}\right)}{1 + e^{8b}}. \tag{3.17}$$

The supersymmetry variation of the gravitini ψ^I_μ, $\delta\psi^I_\mu = 0$ from equation (2.17) after using the above projectors, gives rise to

$$e^{4b} = 1, \tag{3.18}$$

$$A' = 2g\left(1 + e^{4b}\right)e^{2(a-b)} \tag{3.19}$$

where we have used the spin connection $\omega^{\hat{\nu}\hat{r}}_{\hat{\mu}} = A'\delta^{\hat{\nu}}_{\hat{\mu}}$ with $\hat{\mu}, \hat{\nu} = 0, 1$.

We see from (3.18) that supersymmetry demands $b = 0$. Equation (3.16) is now trivially satisfied, and equation (3.17) becomes

$$a' + e^{2a}g = 0\,. \tag{3.20}$$

The solution is easily obtained to be

$$a = -\frac{1}{2}\ln\left(2gr + C_1\right) \tag{3.21}$$

where C_1 is an integration constant. Substituting into equation (3.19) gives

$$A' = 4ge^{2a} = \frac{4g}{C_1 + 2gr} \tag{3.22}$$

whose solution is, with another integration constant C_2,

$$A = C_2 + 2\ln\left(2gr + C_1\right). \tag{3.23}$$

As in other solutions of this type, the residual supersymmetry is generated by the Killing spinors given by $\epsilon^i = e^{\frac{A}{2}}\epsilon^i_{0\pm}$, $i = 1, \dots, 5$ with the constant spinors $\epsilon^i_{0\pm}$ satisfying $\gamma_r\epsilon^i_{0\pm} = \pm\epsilon^i_{0\pm}$. The full symmetry of this solution is $I\mathrm{SO}(1,1) \times \mathrm{SO}(5)$ with the unbroken $N = (5,5)$ Poincare supersymmetry in notation of the dual two-dimensional field theory.

The two integration constants C_1 and C_2 can be set to zero by shifting the coordinate r and rescaling the coordinates x^μ. We can also write down the solution in the form of warped AdS_3 by introducing the new coordinate ρ via $\rho = -\frac{1}{4g^2r}$ in term of which the metric becomes

$$ds^2 = \frac{1}{(4g^2\rho)^2}\left(\frac{dx^2_{1,1} + d\rho^2}{\rho^2}\right). \tag{3.24}$$

We end this section by considering subgroups of $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ namely $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ and $(\mathrm{SO}(3) \ltimes \mathbf{T}^3) \times (\mathrm{SO}(2) \ltimes \mathbf{T}^1) \sim \mathrm{U}(2) \ltimes \mathbf{T}^4$. It can be checked that both of them are not admissible.

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3D fields	SO(5) representation	number of degrees of freedom
g_{33}	**1**	1
$g_{\mu 3}$	**1**	1
ϕ^i	**5**	5
ϕ_i	**5**	5
A^{ij}_μ	**10**	10
A^{ij}_3	**10**	10
ψ^i_3	**5**	10
χ^{678}	**1**	2
χ^{ijk}	**10**	20

Table 1. Representations of three dimensional fields resulted from S^1 reduction of $N = 5$ gauged supergravity in four dimensions.

4 Higher dimensional origin

In this section, we discuss higher dimensional origin of the $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ $N = 10$ gauged supergravity constructed in the previous section. By the general result of [5], this theory is on-shell equivalent to the SO(5) Yang-Mills gauged supergravity which can be obtained from S^1 reduction of $N = 5$ gauged supergravity in four dimensions with SO(5) gauge group. The four dimensional theory has been constructed in [14] and can be obtained as a truncation of the maximal $N = 8$ gauged supergravity. In the notation of [14], the field content of this theory contains one graviton e^a_M or g_{MN}, five gravitini ψ^i_M, eleven spin-$\frac{1}{2}$ fields χ^{ijk} and χ^{678}, ten scalars ϕ^i and ϕ_i living in the coset space $\mathrm{SU}(5,1)/\mathrm{U}(5)$ and ten vector fields A^{ij}_M being SO(5) gauge fields. Here, $M, N = 0, 1, 2, 3$ and $a, b = 0, 1, 2, 3$ are four dimensional space-time and tangent space indices respectively while $i, j = 1, \ldots, 5$ are SU(5) indices except for A^{ij}_M which transform in the adjoint representation of SO(5).

If we reduce this theory on S^1 along the x^3 direction, we find the following fields in three dimensions. The metric g_{MN} gives the non-dynamical three dimensional metric $g_{\mu\nu}$, the graviphoton $g_{\mu 3}$ and a scalar g_{33}. The SO(5) gauge fields result in the three dimensional gauge fields of the same gauge group A^{ij}_μ and ten scalars A^{ij}_3 transforming in the adjoint representation of SO(5). Finally, the ten scalars (ϕ^i, ϕ_i) obviously become the three dimensional scalars.

A spinor in four dimensions give rise to two spinors in three dimensions. We then obtain ten gravitini ψ^i_μ from ψ^i_M and ten spin-$\frac{1}{2}$ fields ψ^i_3. There are additional $20 + 2$ spin-$\frac{1}{2}$ fields from the reduction of χ^{ijk} and χ^{678}, respectively. In three dimensions, the metric and gravitini do not have any dynamics. We then find 32 fernionic on-shell degrees of freedom from $(\psi^i_3, \chi^{678}, \chi^{ijk})$. We can also dualize A^{ij}_μ and $g_{\mu 3}$ to $10 + 1$ scalars. All together, we end up with 32 scalars from $(\phi^i, \phi_i, g_{33}, g_{\mu 3}, A^{ij}_\mu, A^{ij}_3)$. This is the same as in $N = 10$ gauged supergravity.

We give $\mathrm{SO}(5)_{\mathrm{gauge}}$ representations of the reduced fields in table 1 from which we have omitted the non-dynamical fields $g_{\mu\nu}$ and ψ^i_μ. We have kept ϕ^i and ϕ_i separately to emphasize their four dimensional origin. We now consider the representation of the 32 scalars in $E_{6(-14)}/\mathrm{SO}(10)\times\mathrm{U}(1)$ coset space under the SO(5) part of the gauge group. Recall that under $\mathrm{SO}(10)\times\mathrm{U}(1)$, the scalars transform as $\mathbf{16}^+ + \mathbf{16}^-$. Under $\mathrm{SO}(10)\times\mathrm{U}(1)\supset\mathrm{SU}(5)\times\overline{\mathrm{U}(1)}\times\mathrm{U}(1)\supset\mathrm{SO}(5)$ in which the $\overline{\mathrm{U}(1)}$ is the U(1) subgroup of $\mathrm{U}(5)\subset\mathrm{SO}(10)$, we find

$$\begin{aligned}\mathbf{16}^+ + \mathbf{16}^- &\to (\mathbf{1}_{-5}+\bar{\mathbf{5}}_3+\mathbf{10}_{-1})^+ + (\mathbf{1}_{-5}+\mathbf{5}_{-3}+\overline{\mathbf{10}}_1)^- \\ &\to (\mathbf{1}+\mathbf{5}+\mathbf{10})+(\mathbf{1}+\mathbf{5}+\mathbf{10})\end{aligned}\tag{4.1}$$

We find perfect agreement with table 1. Reference [38] is very useful in this decomposition. In the formalism of [4], the fermions $\chi^{\dot{A}}$ transform as $\overline{\mathbf{10}}^+ + \mathbf{10}^-$ under $\mathrm{SO}(10)\times\mathrm{U}(1)$. Similar decomposition gives $2\times(\mathbf{1}+\mathbf{5}+\mathbf{10})$ under SO(5) gauge group. This is again the representations obtained from S^1 reduction shown in table 1. The result of [39] suggests that three dimensional supergravity with E_6 coset manifold can be obtained from dimensional reduction on a torus, S^1 in the present case, of a supergravity theory with A_5 coset manifold in four dimensions. Reference [39] consider only maximally non-compact E_6 and other types Lie groups. The result here should provide an example of a non-maximally non-compact E_6 ($E_{6(-14)}$) coset obtained from a non-maximally non-compact A_5 SU(5, 1) coset in four dimensions. Furthermore, the general formulae for toroidal reductions given in the appendix of [39] should also be applicable in this case.

5 Conclusions and discussions

In this paper, we have constructed $N=10$ $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauged supergravity in three dimensions. We have found that the resulting theory admits a $\frac{1}{2}$-BPS domain wall as a vacuum solution. The solutions preserves $N=(5,5)$ Poincare supersymmetry in two dimensions with ten supercharges. The solution is similar to the domain wall from the S^7 compactification of type II string theory discussed in [40]. This solution is the vacuum solution of the maximal $N=16$ $\mathrm{SO}(8)\ltimes\mathbf{T}^{28}$ gauged supergravity. The solution given here provides an example of a domain wall in non-maximal gauged supergravity and might be useful in the DW/QFT correspondence as well as its applications.

We have also discussed possible higher dimensional origin of this theory. This is given by S^1 reduction of $N=5$ SO(5) gauged supergravity in four dimensions. We have found that the spectrum of the reduction matches with the constructed three dimensional gauged supergravity. If the $N=5$ four dimensional theory is reduced on $S^1/\mathbb{Z}_2$, it could give rise to $N=5$ gauged supergravity in three dimensions. Indeed, the latter in general has scalar manifold $\mathrm{USp}(4,k)/\mathrm{USp}(4)\times\mathrm{USp}(k)$ [34]. We have seen that the $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauge group is embedded in $\mathrm{USp}(4,4)\subset E_{6(-14)}$. We then expect that $N=5$ SO(5) gauged supergravity in four dimensions reduced on $S^1/\mathbb{Z}_2$ should give $N=5$ $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauged supergravity in three dimensions with scalar manifold $\mathrm{USp}(4,4)/\mathrm{USp}(4)\times\mathrm{USp}(4)$ containing 16 scalars. It turns out that the latter theory admits $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauge group. The details will be reported in subsequent work [41]. Unlike the $N=10$ theory, the $N=5$ truncation admits maximally supersymmetric AdS_3 vacuum solution. This truncation should be similar to the case of $N=8$ $\mathrm{SO}(8)\ltimes\mathbf{T}^{28}$ gauged supergravity with $\mathrm{SO}(8,8)/\mathrm{SO}(8)\times\mathrm{SO}(8)$ scalar manifold studied in [25]. This theory is a truncation of $N = 16$ SO(8) $\ltimes$ $\mathbf{T}^{28}$ gauged supergravity with scalar manifold $E_{8(8)}/\mathrm{SO}(16)$.

Due to the similar structure as in the above examples, we would like to briefly discuss the case of $N = 12$ gauged supergravity. The scalar manifold is the 64-dimensional quaternionic manifold $E_{7(-5)}/\mathrm{SO}(12)\times\mathrm{SU}(2)$. The gauge group should be $\mathrm{SO}(6)\ltimes\mathbf{T}^{15}$ embedded in $\mathrm{SU}(4,4) \subset E_{7(-5)}$. The SO(6) is again identified as $\mathrm{SO}(6)_{\mathrm{diag}} \subset \mathrm{SO}(6)\times\mathrm{SO}(6) \subset \mathrm{SO}(12)$. The 64 scalars transform under $\mathrm{SO}(12)\times\mathrm{SU}(2)$ as $(\mathbf{32},\mathbf{2})$ and under $\mathrm{SO}(6)\times\mathrm{SO}(6)\times\mathrm{SU}(2)$ as $((\mathbf{4},\bar{\mathbf{4}})+(\mathbf{4},\bar{\mathbf{4}}),\mathbf{2})$. Then, under the SO(6) part of the gauge group, we find the representation for scalars $((\mathbf{4}\times\bar{\mathbf{4}}+\mathbf{4}\times\bar{\mathbf{4}}),\mathbf{2}) = (\mathbf{1}+\mathbf{15}+\mathbf{1}+\mathbf{15},\mathbf{2})$. The non-compact generators in the $\mathbf{15}$ should combine with $\mathrm{SO}(6)\times\mathrm{SO}(6)$ generators to form the $\mathbf{T}^{15}$ part of the gauge group. The fermions transform as $(\overline{\mathbf{32}},\mathbf{2})$ under $\mathrm{SO}(12)\times\mathrm{SU}(2)$ and $((\mathbf{4},\mathbf{4})+(\bar{\mathbf{4}},\bar{\mathbf{4}}),\mathbf{2})$ under $\mathrm{SO}(6)\times\mathrm{SO}(6)\times\mathrm{SU}(2)$. Under SO(6), they transform as $(\mathbf{10}+\mathbf{6}+\mathbf{10}+\mathbf{6},\mathbf{2})$.

We now consider S^1 reduction of $N = 6$ SO(6) gauged supergravity in four dimneions which is also a truncation of $N = 8$ SO(8) gauged supergravity [42]. The bosonic fields are $(g_{MN}, \phi^{AB}, \phi_{AB}, A^{AB}_M, A_M)$ where the 30 scalars (ϕ^{AB}, ϕ_{AB}) live in the coset space $SO^*(12)/\mathrm{U}(6)$ and $A, B = 1, \ldots, 6$, see [42] for more detail. The fermionic fields are given by $(\psi^A_M, \chi^A, \chi^{ABC})$. After S^1 reduction, the dynamical bosonic fields are given by $(g_{\mu 3}, g_{33}, \phi^{AB}, \phi_{AB}, A_\mu, A_3, A^{AB}_\mu, A^{AB}_3)$ transforming as $(\mathbf{1}+\mathbf{1}+\mathbf{15}+\mathbf{15}+\mathbf{1}+\mathbf{1}+\mathbf{15}+\mathbf{15})$ under SO(6) gauge group. After dualizing the vector fields, we end up with 64 scalars with correct SO(6) representations as in $N = 12$ gauged supergravity. The reduced dynamical fermionic fields are $(\psi^A_3, \chi^{ABC}, \chi^A)$ transforming under SO(6) as $2\times(\mathbf{6}+\mathbf{10}+\mathbf{10}+\mathbf{6})$ which are indeed the same as those in $N = 12$ theory. The factor of 2 comes from the fact that a four dimensional spinor gives two three dimensional spinors.

Finally, similar to the discussion in the $N = 5$ case, we expect that the $S^1/\mathbb{Z}_2$ reduction should give $N = 6$ $\mathrm{SO}(6)\ltimes\mathbf{T}^{15}$ gauged supergravity on three dimensions with scalar manifold $\mathrm{SU}(4,4)/S(\mathrm{U}(4)\times\mathrm{U}(4))$ whose compact and non-compact gauge groups have been explored in [43]. The possibility of non-semisimple gauge groups is under investigation [41].

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A Useful formulae and details

In this appendix, we give some details of $N = 10$ gauged supergravity with $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauge group constructed in the main text. First of all, the SO(10) R-symmetry generators

T^{IJ} are explicitly given by

$$
\begin{array}{llll}
T^{12} = c_1, & T^{13} = -c_2, & T^{23} = c_3, & T^{34} = c_6, \\
 & T^{14} = c_4, & T^{24} = -c_5, & \\
T^{15} = c_7, & T^{25} = -c_8, & T^{35} = c_9, & T^{45} = -c_{10}, \\
 & T^{56} = -c_{15}, & T^{16} = c_{11}, & \\
T^{26} = -c_{12}, & T^{46} = -c_{14}, & T^{36} = c_{13}, & T^{17} = c_{16}, \\
 & T^{27} = -c_{17}, & T^{47} = -c_{19}, & \\
T^{37} = c_{18}, & T^{67} = -c_{21}, & T^{57} = -c_{20}, & T^{78} = -c_{36}, \\
 & T^{18} = c_{30}, & T^{28} = -c_{31}, & \\
T^{48} = -c_{33}, & T^{38} = c_{32}, & T^{68} = -c_{35}, & T^{58} = -c_{34}, \\
 & T^{29} = -c_{46}, & T^{19} = c_{45}, & \\
T^{49} = -c_{48}, & T^{39} = c_{47}, & T^{69} = -c_{50}, & T^{59} = -c_{49}, \\
 & T^{89} = -c_{52}, & T^{79} = -c_{51}, & \\
T^{1,10} = -c_{71}, & T^{2,10} = c_{72}, & T^{3,10} = -c_{73}, & T^{4,10} = c_{74}, \\
 & T^{5,10} = c_{75}, & & \\
T^{6,10} = c_{76}, & T^{7,10} = c_{77}, & T^{8,10} = c_{78}, & T^{9,10} = -\tilde{c}_{53}
\end{array}
\tag{A.1}
$$

where $\tilde{c}_{53}$ and $\tilde{c}_{70}$ are defined by [36]

$$
\tilde{c}_{53} = \frac{1}{2}c_{53} + \frac{\sqrt{3}}{2}c_{70} \qquad \text{and} \qquad \tilde{c}_{70} = -\frac{\sqrt{3}}{2}c_{53} + \frac{1}{2}c_{70}\,. \tag{A.2}
$$

Also, notice a typo in the sign of $T^{9,10}$ in [17].

The $\tilde{Y}^{ij}$ part of the translational generators $\mathbf{T}^{10}$ is constructed from the following non-compact generators

$$
\begin{aligned}
\tilde{Y}^{12} &= \frac{1}{2}\left(Y_3 - Y_{12} + Y_{17} + Y_{29} + Y_5 - Y_{16} + Y_{18} - Y_{27}\right), \\
\tilde{Y}^{13} &= \frac{1}{2}\left(Y_2 + Y_{14} + Y_{21} - Y_{26} - Y_1 + Y_{15} - Y_{19} - Y_{25}\right), \\
\tilde{Y}^{14} &= \frac{1}{2}\left(Y_{31} - Y_7 - Y_6 - Y_{30} - Y_9 + Y_{10} + Y_{20} - Y_{24}\right), \\
\tilde{Y}^{15} &= \frac{1}{2}\left(Y_{15} - Y_{14} + Y_{25} - Y_{26} - Y_1 - Y_2 + Y_{19} + Y_{21}\right), \\
\tilde{Y}^{23} &= \frac{1}{2}\left(Y_1 + Y_2 + Y_{15} - Y_{14} + Y_{19} + Y_{21} - Y_{25} + Y_{26}\right), \\
\tilde{Y}^{24} &= \frac{1}{2}\left(Y_{10} + Y_9 - Y_{30} - Y_{31} + Y_6 - Y_7 - Y_{20} - Y_{24}\right), \\
\tilde{Y}^{25} &= \frac{1}{2}\left(Y_2 - Y_1 - Y_{25} - Y_{26} - Y_{14} - Y_{15} + Y_{19} - Y_{21}\right), \\
\tilde{Y}^{34} &= \frac{1}{2}\left(Y_8 - Y_4 - Y_{11} - Y_{28} + Y_{13} - Y_{32} + Y_{22} + Y_{23}\right), \\
\tilde{Y}^{35} &= \frac{1}{2}\left(Y_{18} + Y_{17} - Y_{12} + Y_{27} - Y_{29} - Y_{16} - Y_5 - Y_3\right), \\
\tilde{Y}^{45} &= \frac{1}{2}\left(Y_8 + Y_4 - Y_{11} - Y_{28} - Y_{13} + Y_{32} - Y_{23} + Y_{22}\right).
\end{aligned}
\tag{A.3}
$$

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This choice is of course not unique.

The scalar matrices for the moment maps are given by

$$\begin{aligned}
\mathcal{V}_{\mathrm{a}}^{ij,IJ} &= -\frac{1}{6}\mathrm{Tr}(L^{-1}J^{ij}LT^{IJ}), \\
\mathcal{V}_{\mathrm{b}}^{ij,IJ} &= -\frac{1}{6}\mathrm{Tr}(L^{-1}t^{ij}LT^{IJ}), \\
\mathcal{V}_{\mathrm{a}}^{ij,A} &= \frac{1}{6}\mathrm{Tr}(L^{-1}J^{ij}LY^{A}), \\
\mathcal{V}_{\mathrm{b}}^{ij,A} &= \frac{1}{6}\mathrm{Tr}(L^{-1}t^{ij}LY^{A})
\end{aligned} \tag{A.4}$$

from which the T-tensor follows

$$\begin{aligned}
T^{IJ,KL} &= g\left(\mathcal{V}_{\mathrm{a}}^{ij,IJ}\mathcal{V}_{\mathrm{b}}^{ij,KL}+\mathcal{V}_{\mathrm{b}}^{ij,IJ}\mathcal{V}_{\mathrm{a}}^{ij,KL}\right) \\
T^{IJ,A} &= g\left(\mathcal{V}_{\mathrm{a}}^{ij,IJ}\mathcal{V}_{\mathrm{b}}^{ij,A}+\mathcal{V}_{\mathrm{b}}^{ij,IJ}\mathcal{V}_{\mathrm{a}}^{ij,A}\right)
\end{aligned} \tag{A.5}$$

Using these together with (2.11), (2.12) and (2.13), we can find the tensors A_1^{IJ} and A_{2i}^{IJ} as well as the scalar potential.

B Non-semisimple gauging of $N = 9$ gauged supergravity in three dimensions

We will consider $N = 9$ gauged supergravity in three dimensions. The corresponding scalar manifold is given by the 16-dimensional $F_{4(-20)}/\mathrm{SO}(9)$ coset space. Some vacua of the compact and non-compact gaugings of this theory have been studied in [44]. In this appendix, we will explore the possibilities of non-semisimple gauge groups which are crucial for embedding the theory in higher dimensions. Notice that the construction of E_6 given in [36] is based on the F_4 group given in [35]. We can simply remove the last 26 matrices c_i, $i = 53, \ldots, 78$ from E_6 to get the group F_4 generated by c_i, $i = 1, \ldots, 52$ as has been used in [44]. All 52 matrices are effectively 26×26 matrices since all elements in the last row and last column are zero.

The SO(9) R-symmetry generators are T^{IJ} in (A.1) with $I, J = 1, \ldots, 9$, and non-compact generators are the first 16 generators of (3.1), Y^A, $A = 1, \ldots, 16$. In the case of $F_{4(4)}/\mathrm{USp}(6) \times \mathrm{SU}(2)$ which is a scalar manifold of $N = 4$ theory studied in [45], $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ can be gauged consistently with supersymmetry by the embedding of $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ in $\mathrm{SO}(5,4) \subset F_{4(4)}$. In the present case, the embedding of $\mathrm{SO}(3) \ltimes \mathbf{T}^3$ in $\mathrm{USp}(2,2) \subset \mathrm{USp}(4,2) \times \mathrm{SU}(2) \subset F_{4(-20)}$ should be possible.

To identify generators of this group, we first consider the $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ subgroup of the $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ in section 3. Obviously, the SO(4) part is generated by J^{ij}, $i, j = 1, \ldots, 4$. We then consider $\tilde{Y}^{ij}$ with $i, j = 1, \ldots, 4$. It can be verified that by removing Y_{17} to Y_{32}

form $\tilde{Y}^{ij}$, the resulting generators, see appendix A,

$$\begin{aligned}\tilde{Y}^{12} &= \frac{1}{2}(Y_3 - Y_{12} + Y_5 - Y_{16}), \\ \tilde{Y}^{13} &= \frac{1}{2}(Y_2 + Y_{14} - Y_1 + Y_{15}), \\ \tilde{Y}^{14} &= \frac{1}{2}(Y_{10} - Y_7 - Y_6 - Y_{30} - Y_9), \\ \tilde{Y}^{23} &= \frac{1}{2}(Y_1 + Y_2 + Y_{15} - Y_{14}), \\ \tilde{Y}^{24} &= \frac{1}{2}(Y_{10} + Y_9 + Y_6 - Y_7), \\ \tilde{Y}^{34} &= \frac{1}{2}(Y_8 - Y_4 - Y_{11} + Y_{13}) \end{aligned} \tag{B.1}$$

still transform in the adjoint representation of SO(4). It turns out that when combined into t^{ij}, the resulting generators do not commute. Therefore, it is not possible to find $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ subgroup of $F_{4(-20)}$. On the other hand, we can form two $\mathrm{SU}(2)_\pm$ subgroups from these generators by introducing the self-dual and anti-self-dual SO(4) generators

$$\begin{aligned} &J^1_+ = J^{12} + J^{34}, \qquad && J^2_+ = J^{13} - J^{24}, \qquad && J^3_+ = J^{14} + J^{23}, \\ &t^1_+ = t^{12} + t^{34}, && t^2_+ = t^{13} - t^{24}, && t^3_+ = t^{14} + t^{23} \end{aligned} \tag{B.2}$$

and

$$\begin{aligned} &J^1_- = J^{12} - J^{34}, \qquad && J^2_- = J^{13} + J^{24}, \qquad && J^3_- = J^{14} - J^{23}, \\ &t^1_- = t^{12} - t^{34}, && t^2_- = t^{13} + t^{24}, && t^3_- = t^{14} - t^{23}\,. \end{aligned} \tag{B.3}$$

It can be readily verified that each set of generators forms $\mathrm{SO}(3) \ltimes \mathbf{T}^3 \sim \mathrm{SU}(2) \ltimes \mathbf{T}^3$ algebra but generators $t^a_\pm$ from the two sets do not commute with eachMo other. Although this subgroup can be embedded in $F_{4(-20)}$, it is not admissible namely it cannot be gauged in a way that is consistent with supersymmetry. Embedding in higher dimensions aside, it seems to be difficult (if possible) to find non-semisimple gaugings of the $N = 9$ theory.

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New $N = 5, 6$, 3D gauged supergravities and holography

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Abstract: We study $N = 5$ gauged supergravity in three dimensions with compact, non-compact and non-semisimple gauge groups. The theory under consideration is of Chern-Simons type with $\mathrm{USp}(4, k)/\mathrm{USp}(4) \times \mathrm{USp}(k)$ scalar manifold. We classify possible semisimple gauge groups of the $k = 2, 4$ cases and identify some of their critical points. A number of supersymmetric AdS_3 critical points are found, and holographic RG flows interpolating between these critical points are also investigated. As one of our main results, we consider a non-semisimple gauge group $\mathrm{SO}(5) \ltimes \mathbf{T}^{10}$ for the theory with $\mathrm{USp}(4, 4)/\mathrm{USp}(4) \times \mathrm{USp}(4)$ scalar manifold. The resulting theory describes $N = 5$ gauged supergravity in four dimensions reduced on $S^1/\mathbb{Z}_2$ and admits a maximally supersymmetric AdS_3 critical point with $\mathrm{Osp}(5|2, \mathbb{R}) \times \mathrm{Sp}(2, \mathbb{R})$ superconformal symmetry. We end the paper with the construction of $\mathrm{SO}(6) \ltimes \mathbf{T}^{15}$ gauged supergravity with $N = 6$ supersymmetry. The theory admits a half-supersymmetric domain wall as a vacuum solution and may be obtained from an $S^1/\mathbb{Z}_2$ reduction of $N = 6$ gauged supergravity in four dimensions.

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Contents

1 Introduction

The duality between scalars and vectors together with the non-propagating nature of supergravity fields in three dimensions make three dimensional gauged supergravity substantially differs from its higher dimensional analogue. On one hand, only matter-coupled supergravity has propagating degrees of freedom in terms of scalars and spin-$\frac{1}{2}$ fields. Accordingly, the matter-coupled theory takes the form of a supersymmetric non-linear sigma model coupled to supergravity. On the other hand, recasting vectors to scalars, making the U-duality symmetry manifest, seems to create a trouble in any attempt to gauge the theory since the vector fields accompanying for the gauging are missing.

Special to three dimensions, vector fields can enter the gauged Lagrangian via Chern-Simons (CS) terms as opposed to the conventional Yang-Mills (YM) kinetic terms. Since CS terms do not lead to additional degrees of freedom, any number of gauge fields, or equivalently the dimension of the gauge group, can be introduced provided that the gauge group is a proper subgroup of the global symmetry group and consistent with supersymmetry. This gives rise to a very rich structure of gauged supergravity in three dimensions [1–5].

Additionally, the Chern-Simons form of gauged supergravity raises another difficulty namely the embedding of the resulting gauged theory in higher dimensions. This is due to the fact that all theories obtained from conventional dimensional reductions are of Yang-Mills form. It has been, however, shown that Yang-Mills gauged supergravity is on-shell equivalent to Chern-Simons gauged theory with a non-semisimple gauge group [6]. Up to now, there are many attempts to embed three dimensional gauged supergravity in higher dimensions and in string/M theory. These results would give rise to new string theory backgrounds with fluxes as well as new D-brane configurations [7]. However, it has been pointed out recently in [8] that there might exist supersymmetric string backgrounds which are not captured by gauged supergravities.

The rich structure and embedding in string/M theory aside, gauged supergravity proves to be a very useful tool in the AdS/CFT correspondence [9]. $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence can provide more insight not only to the AdS/CFT correspondence, including its generalizations such as the Domain Wall/Quantum Field Theory (DW/QFT) correspondence, but also to black hole physics [10, 11]. In holographic RG flows, AdS_3 vacua and domain walls interpolating between them interpreted as RG flows in the dual two dimensional field theories are of particular interest, see [12] for a thorough review. The deformations of a strongly coupled field theory can be understood in this framework. Some gauged supergravities do not admit a maximally supersymmetric AdS_3 but a half-supersymmetric domain wall as a vacuum solution. This class of gauged supergravities will be useful in the context of the DW/QFT correspondence [13–15].

In this work, we further explore the structure of gauged supergravity in three dimensions with $N = 5, 6$ supersymmetry. We begin with a study of compact and non-compact gaugings of the $N = 5$ theory with scalar manifolds $\mathrm{USp}(4, 2)/\mathrm{USp}(4) \times \mathrm{USp}(2)$ and $\mathrm{USp}(4, 4)/\mathrm{USp}(4)\times\mathrm{USp}(4)$. We will identify some supersymmetric AdS_3 critical points and study the associated RG flow solutions. This could be useful in $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence although the embedding in higher dimensions is presently not known. The result is

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similar to supersymmetric RG flows studied in [16–19] and in higher dimensions such as recent solutions of new maximal gauged supergravity in four dimensions given in [20].

We then move to non-semisimple gaugings of the $N = 5$ theory containing 16 scalars encoded in $\mathrm{USp}(4,4)/\mathrm{USp}(4)\times\mathrm{USp}(4)$ coset manifold with $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauge group. The gauge group is embedded in the global symmetry group $\mathrm{USp}(4,4)$. According to [6], the resulting theory is equivalent to SO(5) YM gauged supergravity. The latter might be obtained by a reduction of $N = 5$, SO(5) gauged supergravity in four dimensions on $S^1/\mathbb{Z}_2$ as pointed out in [21]. The theory may also be embedded in $N = 10$, $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauged supergravity via the embedding of the global symmetry group $\mathrm{USp}(4,4)\subset E_{6(-14)}$. The theory admits a maximally supersymmetric AdS_3 vacuum and provides another example of three dimensional gauged supergravities with known higher dimensional origin.

We finally turn to non-semisimple gauging of $N = 6$ theory with $\mathrm{SU}(4,4)/S(\mathrm{U}(4)\times\mathrm{U}(4))$ scalar manifold. The global symmetry $\mathrm{SU}(4,4)$ contains an $\mathrm{SO}(6)\ltimes\mathbf{T}^{15}$ subgroup that can be consistently gauged. Similar to $N = 5$ theory, this theory is equivalent to SO(6) YM gauged supergravity and could be obtained by an $S^1/\mathbb{Z}_2$ reduction of $N = 6$ gauged supergravity in four dimensions. Unlike $N = 5$ theory, the theory admits only a half-supersymmetric domain wall as a vacuum solution.

The paper is organized as follow. We give the construction of $N = 5$ theory in section 2. Relevant information and related formulae for general gauged supergravity in three dimensions are collected in appendix A. Vacua of compact and non-compact gauge groups are given in section 3 and 4, respectively. Section 5 deals with some examples of RG flows between critical points previously identified. Non-semisimple gaugings of $N = 5$ and $N = 6$ theories are constructed in sections 6 and 7, respectively. The maximally supersymmetric AdS_3 of $N = 5$ theory and a $\frac{1}{2}$-BPS domain wall of the $N = 6$ theory are explicitly given in these sections. We end the paper with some conclusions and discussions. Appendices B and C contain the explicit form of the relevant generators used in the main text as well as the scalar potential for $\mathrm{SO}(4)\times\mathrm{USp}(2)$ gauging in $N = 5$ theory.

2 $N = 5$ gauged supergravity in three dimensions

In $N = 5$ three dimensional gauged supergravity, scalar fields are described in term of $\mathrm{USp}(4,k)/\mathrm{USp}(4)\times\mathrm{USp}(k)$ coset manifold with dimensionality $4k$. The R-symmetry is given by $\mathrm{USp}(4)\sim\mathrm{SO}(5)_R$. All admissible gauge groups are embedded in the global symmetry group $\mathrm{USp}(4,k)$. In this paper, we will consider only the $k = 2$ and $k = 4$ cases.

We first introduce $\mathrm{USp}(4,k)$ generators constructed from a compact group $\mathrm{USp}(4+k)$ via the Weyl unitarity trick. In order to make contact with the $N = 6$ theory with global symmetry group $\mathrm{SU}(4,k)$ studied in section 7, we will construct the $\mathrm{USp}(4+k)$ generators by figuring out the $\mathrm{USp}(4+k)$ subgroup of $\mathrm{SU}(4+k)$, directly. The latter is generated by the well-known generalized Gell-Mann matrices given in, for example, [22]. We will denote $\mathrm{USp}(4+k)$ generators by J_i given explicitly in appendix B. The $\mathrm{SO}(5)_R$ R-symmetry generators, labeled by a pair of anti-symmetric indices $T^{IJ} = -T^{JI}$, can be identified as follow

$$\begin{aligned}
&T^{12} = \frac{1}{\sqrt{2}}(J_3 - J_6)\,, && T^{13} = -\frac{1}{\sqrt{2}}(J_1 + J_4)\,, && T^{23} = \frac{1}{\sqrt{2}}(J_2 - J_5)\,,\\
&T^{34} = \frac{1}{\sqrt{2}}(J_3 + J_6)\,, && T^{14} = \frac{1}{\sqrt{2}}(J_2 + J_5)\,, && T^{24} = \frac{1}{\sqrt{2}}(J_1 - J_4)\,,\\
&T^{15} = -J_9, && T^{25} = -J_{10}, && T^{35} = J_8,\\
&T^{45} = J_7\,. && &&
\end{aligned} \tag{2.1}$$

The non-compact generators Y^A are identified by

$$\begin{aligned}
Y^1 &= iJ_{14}, & Y^2 &= iJ_{15}, & Y^3 &= iJ_{16}, & Y^4 &= iJ_{17},\\
Y^5 &= iJ_{18}, & Y^6 &= iJ_{19}, & Y^7 &= iJ_{20}, & Y^8 &= iJ_{21},\\
Y^9 &= iJ_{25}, & Y^{10} &= iJ_{26}, & Y^{11} &= iJ_{27}, & Y^{12} &= iJ_{28},\\
Y^{13} &= iJ_{29}, & Y^{14} &= iJ_{30}, & Y^{15} &= iJ_{31}, & Y^{16} &= iJ_{32}\,.
\end{aligned} \tag{2.2}$$

For $k = 2$ case with 8 scalars, the associated non-compact generators are given by the first 8 generators, Y^A with $A = 1, \dots, 8$.

Admissible gauge groups are completely characterized by the symmetric gauge invariant embedding tensor $\Theta_{\mathcal{MN}}$, $\mathcal{M}, \mathcal{N} = 1, \dots, \dim G$. Viable gaugings are defined by the embedding tensor satisfying two constraints. The first constraint is quadratic in Θ and given by

$$\Theta_{\mathcal{PL}} f^{\mathcal{KL}}{}_{(\mathcal{M}}\Theta_{\mathcal{N})\mathcal{K}} = 0 \tag{2.3}$$

ensuring that a given gauge group G_0 is a proper subgroup of G. The other constraint due to supersymmetry takes the form of a projection condition

$$\mathbb{P}_{\boxplus} T^{IJ,KL} = 0 \tag{2.4}$$

where the T-tensor $T^{IJ,KL}$ is given by the moment map of the embedding tensor

$$T^{IJ,KL} \equiv \mathcal{V}^{\mathcal{M}\,IJ}\Theta_{\mathcal{MN}}\mathcal{V}^{\mathcal{N}\,KL}\,. \tag{2.5}$$

The $\boxplus$ denotes the Riemann tensor-like representation of $\mathrm{SO}(N)_R$. For symmetric scalar manifolds of the form G/H, the $\mathcal{V}$ maps can be obtained from the coset representative, see appendix A, and the constraint can be written in the form

$$\mathbb{P}_{R_0}\Theta_{\mathcal{MN}} = 0\,. \tag{2.6}$$

The representation R_0 of G contains the $\boxplus$ representation of $\mathrm{SO}(N)_R$.

We are now in a position to study gaugings of $N = 5$ supergravity. We will treat compact and non-compact gauge groups separately.

3 Compact gauge groups

In this section, we explore $N = 5$ gauged supergravity with compact gauge groups. The gauge groups are subgroup of $\mathrm{USp}(4) \times \mathrm{USp}(k)$ and takes the form $\mathrm{SO}(p) \times \mathrm{SO}(5-p) \times \mathrm{USp}(k)$, $p = 5, 4, 3$.

The $\mathrm{SO}(p) \times \mathrm{SO}(5-p)$ part is embedded in $\mathrm{SO}(5)_R$ as $\mathbf{5} \to (\mathbf{p}, \mathbf{1}) + (\mathbf{1}, \mathbf{5}-\mathbf{p})$. The corresponding embedding tensor is identified in [5] and takes the form

$$\Theta_{IJ,KL} = \theta \delta^{KL}_{IJ} + \delta_{[I[K}\Xi_{L]J]} \tag{3.1}$$

where

$$\Xi_{IJ} = \begin{cases} 2\left(1 - \frac{p}{5}\right)\delta_{IJ}, & I \leq p \\ -\frac{2p}{5}\delta_{IJ}, & I > p \end{cases}, \qquad \theta = \frac{2p-5}{5}. \tag{3.2}$$

The full embedding tensor for $\mathrm{SO}(p) \times \mathrm{SO}(5-p) \times \mathrm{USp}(k)$ is given by

$$\Theta = g_1 \Theta_{\mathrm{SO}(p)\times \mathrm{SO}(5-p)} + g_2 \Theta_{\mathrm{USp}(k)} \tag{3.3}$$

with two independent coupling constants. $\Theta_{\mathrm{USp}(k)}$ is given by the Killing form of $\mathrm{USp}(k)$. Together with the explicit form of the coset representative, the scalar potential is completely determined by the embedding tensor.

3.1 The $k = 2$ case

In this case, the theory contains 8 scalars parametrized by $\mathrm{USp}(4,2)/\mathrm{USp}(4) \times \mathrm{USp}(2)$ coset space. The full 8-dimensional manifold can be conveniently parametrized by the Euler angles of $\mathrm{SO}(5) \times \mathrm{USp}(2) \sim \mathrm{USp}(4) \times \mathrm{USp}(2)$. The details of the parametrization can be found in [23], and the application to $\mathrm{SU}(n,m)/S(\mathrm{U}(n) \times \mathrm{U}(m))$ coset can be found in [19].

3.1.1 $\mathrm{SO}(5) \times \mathrm{USp}(2)$ gauging

With $\mathrm{USp}(4) \times \mathrm{USp}(2)$ Euler angles, the full $\mathrm{USp}(4,2)/\mathrm{USp}(4) \times \mathrm{USp}(2)$ coset can be parametrized by the coset representative

$$L = e^{a_1 X_1} e^{a_2 X_2} e^{a_3 X_3} e^{a_4 J_7} e^{a_5 J_8} e^{a_6 J_9} e^{a_7 J_{15}} e^{b Y^7} \tag{3.4}$$

where X_i's are defined by

$$X_1 = \frac{1}{\sqrt{2}}(J_1 - J_{11}), \qquad X_2 = \frac{1}{\sqrt{2}}(J_2 - J_{12}), \qquad X_3 = \frac{1}{\sqrt{2}}(J_3 - J_{13}). \tag{3.5}$$

The resulting scalar potential is

$$\begin{aligned} V = \frac{1}{32}\Big[&64\left(g_2^2 - 12g_1^2 + 4g_1g_2\right)\cosh b - 1076g_1^2 - 180g_1g_2 - 45g_2^2 \\ &-4\left(52g_1^2 + 20g_1g_2 + 5g_2^2\right)\cosh(2b) + (2g_1 + g_2)^2\cosh(4b)\Big]. \end{aligned} \tag{3.6}$$

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	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(5,0)$	$\mathrm{SO}(5)\times\mathrm{USp}(2)$
II	$\cosh^{-1}\left[\frac{g_2-2g_1}{2g_1+g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{(2g_1+g_2)^2}$	$(4,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
III	$\cosh^{-1}\left[\frac{6g_1+g_2}{2g_1+g_2}\right]$	$-\frac{64g_1^2(3g_1+g_2)^2}{(2g_1+g_2)^2}$	$(1,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$

Table 1. Critical points of $\mathrm{SO}(5)\times\mathrm{USp}(2)$ gauging.

Note that the scalar fields associated to the gauge generators do not appear in the potential due to gauge invariance. We find some critical points as shown in table 1. V_0 is the value of the potential at each critical point. Unbroken supersymmetry is denoted by (n_-,n_+) where n_- and n_+ correspond to the number of supersymmetry in the dual two dimensional CFT. In three dimensional language, they correspond to the numbers of negative and positive eigenvalues of A_1^{IJ} tensor. As reviewed in appendix A, these eigenvalues, $\pm\tilde{\alpha}$, satisfy $V_0=-4\tilde{\alpha}^2$. Since, in our convention, the AdS_3 radius is given by $L=\frac{1}{\sqrt{-V_0}}$, we also have a relation $L=\frac{1}{2|\tilde{\alpha}|}$.

The maximally supersymmetric critical point at $L=\mathbf{I}$ preserves the full gauge symmetry. The two non-trivial critical points preserve $\mathrm{USp}(2)\times\mathrm{USp}(2)$ symmetry. We also give the A_1 tensors at each critical point:

$$\begin{aligned}
A_1^{(\mathrm{I})} &= -4g_1\mathbf{I}_{5\times5}, \\
A_1^{(\mathrm{II})} &= \mathrm{diag}\left(\alpha,\alpha,\alpha,\alpha,\frac{4g_1(g_1-g_2)}{2g_1+g_2}\right). \\
A_1^{(\mathrm{III})} &= \mathrm{diag}\left(\beta,\beta,\beta,\beta,\frac{-4g_1(3g_1+g_2)}{2g_1+g_2}\right).
\end{aligned} \tag{3.7}$$

where

$$\alpha=\frac{-4g_1(g_1+g_2)}{2g_1+g_2},\qquad \beta=\frac{-4g_1(5g_1+g_2)}{2g_1+g_2}. \tag{3.8}$$

The scalar mass spectrum at the trivial critical point is given in the table below.

m^2L^2	$\mathrm{SO}(5)\times\mathrm{USp}(2)$
$-\frac{3}{4}$	$(\mathbf{4},\mathbf{2})$

All scalars have the same mass $m^2L^2=-\frac{3}{4}$ with L being the AdS_3 radius at this critical point. The full symmetry of the background corresponds to $\mathrm{Osp}(5|2,\mathbb{R})\times\mathrm{Sp}(2,\mathbb{R})$ superconformal group. Notice that in finding critical points with constant scalars we can use the gauge symmetry and the composite $\mathrm{USp}(4)\times\mathrm{USp}(k)$ symmetry to fix the scalar parametrization as, for example, in the Euler angle parametrization. In determining scalar masses, we need to compute scalar fluctuations to quadratic order. In this case, only the the composite $\mathrm{USp}(4)\times\mathrm{USp}(k)$ symmetry can be used since the vector fields are set to zero, see the discussion in [24]. The scalar masses must accordingly be computed in the so-called unitary gauge with the coset representative

$$L=\prod_{i=1}^{8}e^{a_iY^i}\,. \tag{3.9}$$

The mass spectrum at $(4,0)$ critical point is shown below.

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
$\frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$

And, scalar masses at $(1,0)$ critical point are as follow.

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
$\frac{(4g_1+g_2)(10g_1+3g_2)}{(3g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$

Notice that there are seven massless Goldstone bosons corresponding to the symmetry breaking $\mathrm{SO}(5)\times\mathrm{USp}(2)\to\mathrm{USp}(2)\times\mathrm{USp}(2)$.

3.1.2 SO(4) × USp(2) gauging

We still use the same parametrization as in the previous case. The potential in this case turns out to be much more complicated although it dose not depend on a_1, a_2 and a_3. We give its explicit form in appendix C. The trivial critical point has $N=(4,1)$ supersymmetry and preserves the full $\mathrm{SO}(4)\times\mathrm{USp}(2)$ symmetry. The A_1 tensor and scalar masses at this point are given below.

$$A_1^{(\mathrm{I})}=-4g_1\mathrm{diag}\,(1,1,1,1,-1)\,, \tag{3.10}$$

m^2L^2	$\mathrm{SO}(4)\times\mathrm{USp}(2)\sim\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{USp}(2)$
$-\frac{3}{4}$	$(\mathbf{2},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{2},\mathbf{2})$

The corresponding superconformal symmetry is $\mathrm{Osp}(4|2,\mathbb{R})\times\mathrm{Osp}(1|2,\mathbb{R})$.

Other critical points with $a_4=a_5=a_6=a_7=0$ are shown in table 2. Critical points II and III preserve only $\mathrm{USp}(2)_{\mathrm{diag}}\times\mathrm{USp}(2)$ subgroup of $\mathrm{SO}(4)\times\mathrm{USp}(2)$. The $\mathrm{USp}(2)_{\mathrm{diag}}$ is a diagonal subgroup of one factor in $\mathrm{USp}(2)\times\mathrm{USp}(2)\sim\mathrm{SO}(4)$ and the $\mathrm{USp}(2)$ factor in the gauge group and is generated by J_1+J_{11},J_2+J_{12} and J_3+J_{13}. Critical point II has $(4,1)$ supersymmetry with the A_1 tensor

$$A_1^{(\mathrm{II})}=-\frac{4g_1(g_1+g_2)}{2g_1+g_2}\mathrm{diag}\,(1,1,1,1,-1)\,. \tag{3.11}$$

	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(4,1)$	$\mathrm{SO}(4)\times\mathrm{USp}(2)$
II	$\cosh^{-1}\left[\frac{g_2-2g_1}{2g_1+g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{(2g_1+g_2)^2}$	$(4,1)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
III	$\cosh^{-1}\left[\frac{6g_1+g_2}{2g_1+g_2}\right]$	$-\frac{64g_1^2(3g_1+g_2)^2}{(2g_1+g_2)^2}$	$(0,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$

Table 2. Critical points of $\mathrm{SO}(4)\times\mathrm{USp}(2)$ gauging.

The scalar mass spectrum is given in the table below.

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{1},\mathbf{3})$
$\frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$
$-\frac{g_1g_2(g_1+2g_2)}{(g_1+g_2)^2(2g_1+g_2)}$	$(\mathbf{2},\mathbf{2})$

Critical point III is non-supersymmetric with scalar masses given by

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{1},\mathbf{3})$
$\frac{(4g_1+g_2)(10g_1+3g_2)}{(3g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$
$-\frac{g_1(4g_1+g_2)(5g_1+2g_2)}{(2g_1+g_2)(3g_1+g_2)^2}$	$(\mathbf{2},\mathbf{2})$

.

We can now check its stability by comparing the above scalar masses with the Breitenlohner-Freedman bound $m^2L^2\geq -1$. At this critical point, the value of b is real for $g_1>0$ and $g_2>-2g_1$ or $g_1<0$ and $g_2<-2g_1$. For definiteness, we will consider the first possibility. The mass of the singlet scalar satisfies the BF bound for $g_1>0$ and $g_2>-3g_1$ while the mass of $(\mathbf{2},\mathbf{2})$ scalars requires $g_2>0.21432g_1$ for $g_1>0$ to satisfy to BF bound. Therefore, critical point III is stable for $g_1>0$ and $g_2>0.21432g_1$.

Note that both critical points II and III contain three massless scalars which are responsible for the symmetry breaking $\mathrm{SO}(4)\times\mathrm{USp}(2)\to\mathrm{USp}(2)\times\mathrm{USp}(2)$.

3.1.3 $\mathrm{SO}(3)\times\mathrm{SO}(2)\times\mathrm{USp}(2)$ gauging

Computing the scalar potential on the full 8-dimensional manifold turns out to be very complicated even with the Euler angle parametrization (3.4). In order to make things more tractable, we employ the technique introduced in [25] and consider a submanifold of $\mathrm{USp}(4,2)/\mathrm{USp}(4)\times\mathrm{USp}(2)$ invariant under $\mathrm{U}(1)_{\mathrm{diag}}$ symmetry generated by $T^{12}+T^{45}$. There are four singlets under this symmetry corresponding to the non-compact generators

$$X_1=\frac{1}{\sqrt{2}}(Y^1+Y^6),\qquad X_2=\frac{1}{\sqrt{2}}(Y^2+Y^8),$$
$$X_3=\frac{1}{\sqrt{2}}(Y^4-Y^3),\qquad X_4=\frac{1}{\sqrt{2}}(Y^7-Y^5).\tag{3.12}$$

	a_1	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(3,2)$	$SO(3)\times SO(2)\times USp(2)$
II	$\frac{1}{2}\ln\left[\frac{g_2-8g_1-4\sqrt{g_1(4g_1-g_2)}}{g_2}\right]$	$-\frac{64g_1^2(g_1-g_2)^2}{g_2^2}$	$(2,0)$	$U(1)\times U(1)$
III	$\frac{1}{2}\ln\left[\frac{g_2+8g_1-4\sqrt{g_1(4g_1+g_2)}}{g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{g_2^2}$	$(1,2)$	$U(1)\times U(1)$

Table 3. Critical points of $SO(3)\times SO(2)\times USp(2)$ gauging.

The coset representative can be parametrized by

$$L = e^{a_1X_1}e^{a_2X_2}e^{a_3X_3}e^{a_4X_4}. \tag{3.13}$$

The resulting potential is given by

$$\begin{aligned} V &= \frac{1}{128}\left[3+\cosh a_1\cosh a_2\cosh a_3\cosh a_4\right]\left[-2\left(512g_1^2+19g_2^2\right)\right.\\ &\quad +\left(99g_2^2-1024g_1^2\right)\cosh a_1\cosh a_2\cosh a_3\cosh a_4+3g_2^2\cosh(2a_1)\\ &\quad \times(\cosh a_1\cosh a_2\cosh a_3\cosh a_4)-2-12g_2^2\cosh^2 a_1\left[\cosh(2a_2)\right.\\ &\quad \left.+2\cosh^2 a_2\left(\cosh(2a_3)+2\cosh^2 a_3\cosh(2a_4)\right)\right]+2g_2^2\cosh^3 a_1\\ &\quad \times\cosh a_2\cosh a_3\left(3\left(\cosh(2a_2)+2\cosh^2 a_2\cosh(2a_3)\right)\cosh a_4\right.\\ &\quad \left.\left.+4\cosh^2 a_2\cosh^2 a_3\cosh(3a_4)\right)\right]. \end{aligned} \tag{3.14}$$

We find critical points as shown in table 3. We have given only the value of a_1 since, at all critical points, the four scalars are related by $a_2 = a_1$ and $a_3 = a_4 = 0$. As usual, when all scalars vanish, we have a maximally supersymmetric point with $N = (3,2)$ and $SO(3)\times SO(2)\times USp(2)$ symmetry. The corresponding A_1 tensor is

$$A_1^{(I)} = -4g_1\text{diag}\,(1,1,1,-1,-1)\,. \tag{3.15}$$

This background leads to the superconformal symmetry $Osp(3|2,\mathbb{R})\times Osp(2|2,\mathbb{R})$. The scalar masses at this point are shown below.

m^2L^2	$SO(2)\times SO(3)\times USp(2)$
$-\frac{3}{4}$	$(1,\mathbf{2},\mathbf{2})+(-1,\mathbf{2},\mathbf{2})$

The other two critical points preserve $U(1)\times U(1)$ symmetry. The corresponding A_1 tensor at these points is given by

$$\begin{aligned} A_1^{(II)} &= = \text{diag}\,(\alpha,\alpha,\beta,-\beta,-\beta)\,,\\ A_1^{(III)} &= \text{diag}\,(\gamma,\gamma,-\delta,\delta,\delta) \end{aligned} \tag{3.16}$$

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where

$$\alpha = \frac{4g_1(g_1 - g_2)}{g_2}, \qquad \beta = -\frac{4g_1(g_2 - 3g_1)}{g_2},$$
$$\gamma = -\frac{4g_1(3g_1 + g_2)}{g_2}, \qquad \delta = \frac{4g_1(g_1 + g_2)}{g_2}. \tag{3.17}$$

With some normalization of the U(1) charges, the scalar mass spectra can be computed as shown in the tables below. The original four singlets under $U(1)_{\text{diag}}$ correspond to one massless and three massive modes in the tables. The $U(1)_{\text{diag}}$ is given by a combination of the two U(1)'s in the unbroken symmetry $U(1) \times U(1)$. Therefore, the $(0, \pm 4)$ and $(\pm 4, 0)$ modes, which are singlets under one of the two U(1)'s, will not be invariant under $U(1)_{\text{diag}}$.

- $(2, 0)$ point:

m^2L^2	$U(1) \times U(1)$
0	$(0,4)+(0,-4)+(4,0)+(-4,0)+(0,0)$
$\frac{32g_1^2-32g_1g_2+6g_2^2}{(g_1-g_2)^2}$	$(0,0)$
$-\frac{2g_1(g_1-2g_2)}{(g_1-g_2)^2}$	$(-2,-2)+(2,2)$

- $(1, 2)$ point:

m^2L^2	$U(1) \times U(1)$
0	$(0,4)+(0,-4)+(4,0)+(-4,0)+(0,0)$
$\frac{32g_1^2+32g_1g_2+6g_2^2}{(g_1+g_2)^2}$	$(0,0)$
$\frac{2g_1(3g_1+2g_2)}{(g_1+g_2)^2}$	$(-2,-2)+(2,2)$

3.2 The $k = 4$ case

We now consider a bigger scalar manifold $\frac{USp(4,4)}{USp(4)\times USp(4)}$. Compact gauge groups in this case are $SO(5) \times USp(4)$, $SO(4) \times USp(4)$ and $SO(3) \times SO(2) \times USp(4)$. Analyzing the potential on the full 16-dimensional manifold would be very complicated. We then choose a particular submanifold invariant under a certain subgroup of the gauge group and study the potential on this restricted scalar manifold as in the $SO(3) \times SO(2) \times USp(2)$ gauge group of the previous case. The procedure is parallel to that of the $k = 2$ case, so we will omit some irrelevant details particularly the explicit form of the A_1 tensor at each critical point.

3.2.1 $SO(5) \times USp(4)$ gauging

We use the parametrization of a submanifold invariant under $USp(2) \subset USp(4)$. There are eight singlets under this USp(2) symmetry corresponding to non-compact generators of $USp(4, 2) \subset USp(4, 4)$. With the Euler angle parametrization, we can write the coset representative as

$$L = e^{a_1\tilde{X}_1}e^{a_2\tilde{X}_2}e^{a_3\tilde{X}_3}e^{a_4K_1}e^{a_5K_2}e^{a_6K_3}e^{a_7K_4}e^{bY^8} \tag{3.18}$$

	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(5,0)$	$\mathrm{SO}(5)\times\mathrm{USp}(4)$
II	$\cosh^{-1}\left[\frac{g_2-2g_1}{2g_1+g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{(2g_1+g_2)^2}$	$(4,0)$	$\mathrm{USp}(2)^3$
III	$\cosh^{-1}\left[\frac{6g_1+g_2}{2g_1+g_2}\right]$	$-\frac{64g_1^2(3g_1+g_2)^2}{(2g_1+g_2)^2}$	$(1,0)$	$\mathrm{USp}(2)^3$

Table 4. Critical points of $\mathrm{SO}(5)\times\mathrm{USp}(4)$ gauging.

where

$$\tilde{X}_1=\frac{1}{\sqrt{2}}(J_4-J_{11}),\qquad \tilde{X}_2=\frac{1}{\sqrt{2}}(J_5-J_{12}),\qquad \tilde{X}_3=\frac{1}{\sqrt{2}}(J_6-J_{13}),$$
$$K_1=J_{31},\qquad K_2=J_{32},\qquad K_3=J_{33},\qquad K_4=J_{36}\,. \tag{3.19}$$

The scalar potential turns out to be same as in (3.6). The critical points are shown in table 4. The critical points have the same structure as in the $k=2$ case but with bigger residual symmetry. The scalar mass spectra at each critical point are given in the tables below.

- $(5,0)$ point:

m^2L^2	$\mathrm{SO}(5)\times\mathrm{USp}(4)$
$-\frac{3}{4}$	$(\mathbf{4},\mathbf{4})$

- $(4,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3},\mathbf{1})$
$\frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{4g_1^2+8g_1g_2+3g_2^2}{4(g_1+g_2)^2}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$

- $(1,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3},\mathbf{1})$
$\frac{40g_1^2+22g_1g_2+3g_2^2}{(3g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{3\left(12g_1^2+8g_1g_2+g_2^2\right)}{4(3g_1+g_2)^2}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$

Notice that the number of massless Goldstone bosons agrees with the corresponding symmetry breaking in each case.

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	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(4,1)$	$\mathrm{SO}(4)\times\mathrm{USp}(4)$
II	$\cosh^{-1}\left[\frac{g_2-2g_1}{2g_1+g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{(2g_1+g_2)^2}$	$(4,1)$	$\mathrm{USp}(2)^3$
III	$\cosh^{-1}\left[\frac{6g_1+g_2}{2g_1+g_2}\right]$	$-\frac{64g_1^2(3g_1+g_2)^2}{(2g_1+g_2)^2}$	$(0,0)$	$\mathrm{USp}(2)^3$

Table 5. Critical points of $\mathrm{SO}(4)\times\mathrm{USp}(4)$ gauging.

3.2.2 SO(4) × USp(4) gauging

With the same coset representative, we find the same potential as shown in (C.1). The critical points with different unbroken symmetry are shown in table 5. The scalar mass spectra are given below.

- $(4,1)$ point:

m^2L^2	$\mathrm{SO}(4)\times\mathrm{USp}(2)\sim\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{USp}(4)$
$-\frac{3}{4}$	$(\mathbf{2},\mathbf{1},\mathbf{4})+(\mathbf{1},\mathbf{2},\mathbf{4})$

- $(4,1)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{1},\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3},\mathbf{1})$
$\frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{g_1g_2(g_1+2g_2)}{(g_1+g_2)^2(2g_1+g_2)}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$
$-\frac{(2g_1+g_2)(2g_1+3g_2)}{4(g_1+g_2)^2}$	$(\mathbf{2},\mathbf{2},\mathbf{1})$

- Non-supersymmetric point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{1},\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3},\mathbf{1})$
$\frac{40g_1^2+22g_1g_2+3g_2^2}{(3g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{3(2g_1+g_2)(6g_1+g_2)}{4(3g_1+g_2)^2}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$
$-\frac{g_1\left(20g_1^2+13g_1g_2+2g_2^2\right)}{(2g_1+g_2)(3g_1+g_2)^2}$	$(\mathbf{2},\mathbf{2},\mathbf{1})$

This critical point is stable for $g_1>0$ and $g_2>0.21432g_1$.

3.2.3 SO(3) × SO(2) × USp(4) gauging

In this case, we use the parametrization of L as in (3.13). The four scalars correspond to four singlets of $\mathrm{USp}(2)\times\mathrm{U}(1)_{\mathrm{diag}}$. The potential is the same as (3.14) with the critical points shown in table 6. The scalar mass spectra are given in the following tables.

	a_1	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g_1^2$	$(3,2)$	$\mathrm{SO}(3)\times\mathrm{SO}(2)\times\mathrm{USp}(4)$
II	$\frac{1}{2}\ln\left[\frac{g_2-8g_1-4\sqrt{g_1(4g_1-g_2)}}{g_2}\right]$	$-\frac{64g_1^2(g_1-g_2)^2}{g_2^2}$	$(2,0)$	$\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{USp}(2)$
III	$\frac{1}{2}\ln\left[\frac{g_2+8g_1-4\sqrt{g_1(4g_1+g_2)}}{g_2}\right]$	$-\frac{64g_1^2(g_1+g_2)^2}{g_2^2}$	$(1,2)$	$\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{USp}(2)$

Table 6. Critical points of $\mathrm{SO}(3)\times\mathrm{SO}(2)\times\mathrm{USp}(4)$ gauging.

- $(3,2)$ point:

m^2L^2	$\mathrm{SO}(3)\times\mathrm{USp}(4)$
$-\frac{3}{4}$	$(\mathbf{2},\mathbf{4})+(\mathbf{2},\mathbf{4})$

- $(2,0)$ point:

m^2L^2	$\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{USp}(2)$
0	$(4,0,\mathbf{1})+(-4,0,\mathbf{1})+(0,4,\mathbf{1})+(0,-4,\mathbf{1})+(0,0,\mathbf{1})$ $+(1,-1,\mathbf{2})+(-1,1,\mathbf{2})$
$\frac{32g_1^2-32g_1g_2+6g_2^2}{(g_1-g_2)^2}$	$(0,0,\mathbf{1})$
$-\frac{2g_1(g_1-2g_2)}{(g_1-g_2)^2}$	$(-2,-2,\mathbf{1})+(2,2,\mathbf{1})$
$-\frac{4g_1^2-8g_1g_2+3g_2^2}{4(g_1-g_2)^2}$	$(-1,-1,\mathbf{2})+(1,1,\mathbf{2})$

- $(1,2)$ point:

m^2L^2	$\mathrm{U}(1)\times\mathrm{U}(1)\times\mathrm{USp}(2)$
0	$(4,0,\mathbf{1})+(-4,0,\mathbf{1})+(0,4,\mathbf{1})+(0,-4,\mathbf{1})+(0,0,\mathbf{1})$ $+(1,-1,\mathbf{2})+(-1,1,\mathbf{2})$
$\frac{32g_1^2+32g_1g_2+6g_2^2}{(g_1+g_2)^2}$	$(0,0,\mathbf{1})$
$-\frac{2g_1(3g_1+2g_2)}{(g_1+g_2)^2}$	$(-2,-2,\mathbf{1})+(2,2,\mathbf{1})$
$-\frac{4g_1^2+8g_1g_2+3g_2^2}{4(g_1+g_2)^2}$	$(-1,-1,\mathbf{2})+(1,1,\mathbf{2})$

That critical points in the $k=4$ case are similar to those in the $k=2$ case should be related to the fact that the theory with $\mathrm{USp}(4,2)/\mathrm{USp}(4)\times\mathrm{USp}(2)$ scalar manifold can be embedded in the theory with $\mathrm{USp}(4,4)/\mathrm{USp}(4)\times\mathrm{USp}(4)$ scalar manifold. We have studied the potential on scalars which are singlets under $\mathrm{USp}(2)$. These singlets are precisely parametrized by non-compact directions of $\mathrm{USp}(4,2)\subset\mathrm{USp}(4,4)$, the global symmetry group of $k = 2$ case. This might explain the fact that this particular parametrization gives rise to the same potential as in the $k = 2$ case. Turning on more scalars would give more interesting structures.

4 Non-compact gauge groups

In this section, we classify admissible non-compact gauge groups. We will consider the $k = 2$ and $k = 4$ cases separately as in the previous section.

4.1 The $k = 2$ case

In this case, there is only one non-compact subgroup of $USp(4, 2)$ namely $USp(2, 2)$. The $USp(4, 2)$ itself can be gauged with the embedding tensor given by its Killing form, but the corresponding potential will become a cosmological constant. The subgroup of $USp(4, 2)$ that can be gauged is $USp(2) \times USp(2, 2) \subset USp(4, 2)$. The embedding tensor reads

$$\Theta = g_1 \Theta_{USp(2)} + g_2 \Theta_{USp(2,2)} \tag{4.1}$$

where g_1 and g_2 are two independent coupling constants. $\Theta_{USp(2,2)}$ and $\Theta_{USp(2)}$ are given by the Killing forms of $USp(2, 2)$ and $USp(2)$, respectively.

Generally, scalar fields corresponding to non-compact directions in the gauge group will drop out from the potential. Therefore, we do not need to include them in the coset representative. The remaining four scalars correspond to non-compact directions of another $USp(2, 2)$ in $USp(4, 2)$ and can be parametrized by the coset representative of $USp(2, 2)/USp(2) \times USp(2)$. We can use Euler angles of $USp(2) \times USp(2)$ to parametrize the coset representative as

$$L = e^{a_1 X_1} e^{a_2 X_2} e^{a_3 X_3} e^{b Y^7} \tag{4.2}$$

where X_i are given in (3.5). We find the following potential

$$\begin{aligned} V = &\frac{1}{16} \left[8(g_1 - g_2 + (g_1 + g_2)\cosh(b))^2 \sinh^2 b \right. \\ &\left. - (3g_1 + 11g_2 + 4(g_1 - g_2)\cosh b + (g_1 + g_2)\cosh(2b))^2 \right] . \end{aligned} \tag{4.3}$$

Some of the critical points are shown in table 7. The A_1 tensor at each supersymmetric critical point is given by

$$\begin{aligned} A_1^{(I)} &= (g_1 + g_2)\mathrm{diag}\left(-1, -1, -1, -1, 1\right), \\ A_1^{(II)} &= \mathrm{diag}\left(\beta, \beta, \beta, \beta, \frac{g_2(-2g_1 + g_2)}{g_1 + g_2}\right), \\ A_1^{(III)} &= \mathrm{diag}\left(\gamma, \gamma, \gamma, \gamma, -\frac{g_2(2g_1 + 3g_2)}{g_1 + g_2}\right) \end{aligned} \tag{4.4}$$

where

$$\beta = -\frac{g_2(2g_1 + g_2)}{g_1 + g_2}, \qquad \gamma = -\frac{g_2(2g_1 + 5g_2)}{g_1 + g_2} . \tag{4.5}$$

	b	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-4(g_1+g_2)^2$	$(4,1)$	$\mathrm{USp}(2)^3$
II	$\cosh^{-1}\left(\frac{g_2-g_1}{g_1+g_2}\right)$	$-\frac{4g_1^2(2g_1+g_2)^2}{(g_1+g_2)^2}$	$(4,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
III	$\cosh^{-1}\left(-\frac{g_1+3g_2}{g_1+g_2}\right)$	$-\frac{4g_1^2(2g_1+3g_2)^2}{(g_1+g_2)^2}$	$(1,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
IV	$\ln(2+\sqrt{3})$	$-\frac{1}{4}(27g_1^2+54g_1g_2+19g_2^2)$	$(0,0)$	$\mathrm{USp}(2)\times\mathrm{USp}(2)$

Table 7. Critical points of $\mathrm{USp}(2)\times\mathrm{USp}(2,2)$ gauging.

Critical point I preserves $N=(4,1)$ supersymmetry. The gauge group is broken down to its maximal compact subgroup $\mathrm{USp}(2)^3$. In this symmetry breaking, the four massless Goldstone bosons correspond to scalars associated to non-compact generators of the gauge group. The full symmetry at this point gives the superconformal symmetry $\mathrm{Osp}(4|2,\mathbb{R})\times\mathrm{Osp}(1|2,\mathbb{R})$ since the supercharges transform under $\mathrm{USp}(2)\times\mathrm{USp}(2)\subset\mathrm{SO}(5)_R$ as $(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$.

Scalar mass spectra at all critical points are given below.

- $(4,1)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{1},\mathbf{2},\mathbf{2})$
$-\frac{g_1(g_1+2g_2)}{(g_1+g_2)^2}$	$(\mathbf{2},\mathbf{1},\mathbf{2})$

- $(4,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$
$\frac{4g_1(3g_1+g_2)}{(2g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1})$

- $(1,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$
$\frac{4(g_1+2g_2)(3g_1+5g_2)}{(2g_1+3g_2)^2}$	$(\mathbf{1},\mathbf{1})$

- Non-supersymmetric point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$
$\frac{12(3g_1+g_2)(3g_1+5g_2)}{27g_1^2+54g_1g_2+19g_2^2}$	$(\mathbf{1},\mathbf{1})$

At non-trivial critical points, there are additional three massless scalars which are responsible for $USp(2) \times USp(2) \to USp(2)_{\rm diag}$ symmetry breaking. The non-supersymmetric critical point is stable for $g_2 > \frac{3}{79}(2\sqrt{210} - 45)g_1$.

4.2 The $k = 4$ case

There are three possible non-compact subgroups of $USp(4,4)$; $USp(2,2) \times USp(2,2)$, $USp(2) \times USp(4,2)$ and $USp(2) \times USp(2) \times USp(2,2)$. Only $USp(2,2) \times USp(2,2)$ can be gauged with the following embedding tensor

$$\Theta = g_1 \Theta_{USp(2,2)} + g_2 \Theta_{USp(2,2)} \,. \tag{4.6}$$

There are two independent coupling constants g_1 and g_2, and $\Theta_{USp(2,2)}$ is given by the Killing form of $USp(2,2)$. The relevant 8 scalars can be parametrized by $\left(\frac{USp(2,2)}{USp(2)\times USp(2)}\right)^2$ coset space with the two $USp(2,2)$ factors different from those appearing in the gauge group. With the Euler angle parametrization, the coset representative reads

$$L = e^{a_1 X_1} e^{a_2 X_2} e^{a_3 X_3} e^{b_1 Y^7} e^{a_4 X_4} e^{a_5 X_5} e^{a_6 X_6} e^{b_2 Y^{16}} \tag{4.7}$$

where

$$\begin{aligned}
X_1 &= \frac{1}{\sqrt{2}}(J_1 - J_{11}), & X_2 &= \frac{1}{\sqrt{2}}(J_2 - J_{12}), & X_3 &= \frac{1}{\sqrt{2}}(J_3 - J_{13}), \\
X_4 &= \frac{1}{\sqrt{2}}(J_4 - J_{22}), & X_5 &= \frac{1}{\sqrt{2}}(J_5 - J_{23}), & X_6 &= \frac{1}{\sqrt{2}}(J_6 - J_{24}).
\end{aligned} \tag{4.8}$$

The scalar potential is given by

$$\begin{aligned}
V = \frac{1}{16} & \big[(g_1 + g_2)(6 + \cosh(2b_1)) - (4(g_1 - g_2)\cosh b_1 + 4(g_2 - g_1)\cosh b_2 \\
& +(g_1 + g_2)\cosh(2b_2))^2 + 8(g_1 - g_2 + (g_1 + g_2)\cosh(b_1))^2 \sinh^2 b_1 \\
& +8(g_2 - g_1 + (g_1 + g_2)\cosh b_2)^2 \sinh^2 b_2\big] \,.
\end{aligned} \tag{4.9}$$

We find some critical points for $b_2 = 0$ as shown in table 8. Scalar masses at all critical points are given below.

- $(4, 1)$ point:

m^2L^2	$USp(2) \times USp(2) \times USp(2) \times USp(2)$
0	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}) + (\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{2})$
$-\frac{g_2(2g_1+g_2)}{(g_1+g_2)^2}$	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2})$
$-\frac{g_1(g_1+2g_2)}{(g_1+g_2)^2}$	$(\mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1})$

	b_1	V_0	unbroken SUSY	unbroken gauge symmetry
I	0	$-4(g_1+g_2)^2$	$(4,1)$	$\mathrm{USp}(2)^4$
II	$\cosh^{-1}\left(\frac{-g_1+g_2}{g_1+g_2}\right)$	$-\frac{4g_1^2(2g_1+g_2)^2}{(g_1+g_2)^2}$	$(4,0)$	$\mathrm{USp}(2)^3$
III	$\cosh^{-1}\left(\frac{-g_1-3g_2}{g_1+g_2}\right)$	$-\frac{4g_1^2(2g_1+3g_2)^2}{(g_1+g_2)^2}$	$(1,0)$	$\mathrm{USp}(2)^3$
IV	$\cosh^{-1}2$	$-\frac{1}{4}(27g_1^2+54g_1g_2+19g_2^2)$	$(0,0)$	$\mathrm{USp}(2)^3$

Table 8. Critical points of $\mathrm{USp}(2,2)\times\mathrm{USp}(2,2)$ gauging.

- $(4,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{2})+(\mathbf{3},\mathbf{1},\mathbf{1})$
$\frac{4g_1(3g_1+g_2)}{(2g_1+g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{(g_1+g_2)(3g_1+g_2)}{(2g_1+g_2)^2}$	$(\mathbf{1},\mathbf{2},\mathbf{2})$

- $(1,0)$ point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{2})+(\mathbf{3},\mathbf{1},\mathbf{1})$
$\frac{4\left(3g_1^2+11g_1g_2+10g_2^2\right)}{(2g_1+3g_2)^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{3\left(g_1^2+4g_1g_2+3g_2^2\right)}{(2g_1+3g_2)^2}$	$(\mathbf{1},\mathbf{2},\mathbf{2})$

- Non-supersymmetry point:

m^2L^2	$\mathrm{USp}(2)\times\mathrm{USp}(2)\times\mathrm{USp}(2)$
0	$(\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{2})+(\mathbf{3},\mathbf{1},\mathbf{1})$
$\frac{12(3g_1+g_2)(3g_1+5g_2)}{27g_1^2+54g_1g_2+19g_2^2}$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
$-\frac{24g_2(3g_1+g_2)}{27g_1^2+54g_1g_2+19g_2^2}$	$(\mathbf{1},\mathbf{2},\mathbf{2})$

At the trivial critical point, the $SO(5)_R$ R-symmetry is broken to $SU(2)\times SU(2)\sim USp(2)\times USp(2)$. The $N=5$ supercharges transform under this subgroup as $(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$. This gives rise to $\mathrm{Osp}(4|2,\mathbb{R})\times\mathrm{Osp}(1|2,\mathbb{R})$ superconformal symmetry. As in the previous case, the non-supersymmetric point is stable for $g_2>\frac{3}{79}(2\sqrt{210}-45)g_1$.

5 RG flow solutions

Given some AdS_3 critical points form the previous sections, we now consider domain wall solutions interpolating between these critical points. The solutions can be interpreted as

RG flows describing a perturbed UV CFT flowing to another CFT in the IR. Since the structure of critical points in both $k = 2$ and $k = 4$ cases is similar, we will consider only the flows in $k = 2$ case to simplify the algebra. The study of holographic RG flows is very similar to those in other gauged supergravities in three dimensions [16–19]. In this paper, we will give only examples of RG flows in compact $SO(5)\times USp(2)$ and non-compact $USp(2,2) \times USp(2)$ gauge groups.

We are interested only in supersymmetric flows connecting two supersymmetric critical points. The solution can be found by solving BPS equations arising from supersymmetry transformations of fermions $\delta\psi^I_\mu$ and $\delta\chi^{iI}$ which, for convenience, we will repeat them here from [5]

$$\begin{aligned} \delta\psi^I_\mu &= \mathcal{D}_\mu\epsilon^I + gA_1^{IJ}\gamma_\mu\epsilon^J, \\ \delta\chi^{iI} &= \frac{1}{2}(\delta^{IJ}\mathbf{1} - f^{IJ})^i{}_j \not{\mathcal{D}}\phi^j\epsilon^J - gNA_2^{JIi}\epsilon^J \end{aligned} \tag{5.1}$$

where $\mathcal{D}_\mu\epsilon^I = \left(\partial_\mu + \frac{1}{2}\omega^a_\mu\gamma_a\right)\epsilon^I$ for vanishing vector fields.

We now employ the standard domain wall ansatz for the metric

$$ds^2 = e^{2A(r)}dx^2_{1,1} + dr^2\,. \tag{5.2}$$

In order to preserve Poincare symmetry in two dimensions, all fields involving in the flow can only depend on the radial coordinate r identified with an energy scale in the dual field theory. BPS equations give rise to first order flow equations describing the dependence of active scalars on r. It can be verified that setting some of the scalars to zero satisfies their flow equations. We can then neglect all scalars that vanish at both UV and IR points.

5.1 An RG flow between $(5, 0)$ and $(4, 0)$ CFT's in $SO(5) \times USp(2)$ gauging

The flow involves only one active scalar parametrized by the coset representative

$$L = e^{b(r)Y^7}\,. \tag{5.3}$$

The BPS equation from $\delta\chi^{iI} = 0$ gives rise to the flow equation

$$\frac{db}{dr} = [2g_1 - g_2 + (2g_1 + g_2)\cosh b]\sinh b \tag{5.4}$$

where we have used the projection condition $\gamma_r\epsilon^I = \epsilon^I$. It is clearly seen from the above equation that there are two critical points at $b = 0$ and $b = \cosh^{-1}\frac{g_2-2g_1}{2g_1+g_2}$. This equation can be solved for r as a function of b, and the solution is given by

$$\begin{aligned} r = \frac{1}{8g_1g_2}\Bigg[&4g_1\ln\cosh\frac{b}{2} - (2g_1 + g_2)\ln[2g_1 - g_2 + (2g_1 + g_2)\cosh b] \\ &+2g_2\ln\sinh\frac{b}{2}\Bigg]\,. \end{aligned} \tag{5.5}$$

The integration constant has been neglected since we can shift the coordinate r to remove it.

The variation $\delta\psi^I_\mu = 0$ gives another equation for $A(r)$

$$\frac{dA}{dr} = \frac{1}{4}\left[4g_2\cosh b - 22g_1 - 3g_2 - 8g_1\cosh b \right.$$
$$\left. -2g_1\cosh(2b) - g_2\cosh(2b)\right] \tag{5.6}$$

or, in term of b,

$$\frac{dA}{db} = -\frac{[22g_1 + 3g_2 + (8g_1 - 4g_2)\cosh b + (2g_1+g_2)\cosh(2b)]\,\mathrm{csch} b}{8g_1 - 4g_2 + 4(2g_1+g_2)\cosh b}\,. \tag{5.7}$$

This equation is readily solved and gives A as a function of b

$$A = \frac{1}{g_2}\left[(g_1+g_2)\ln\left[2g_1 - g_2 + (2g_1+g_2)\cosh b\right] - (2g_1+g_2)\ln\cosh\frac{b}{2}\right.$$
$$\left. -2g_2\ln\sinh\frac{b}{2}\right]. \tag{5.8}$$

The additive integration constant can be absorbed by scaling $x^{0,1}$ coordinates. It can be verified that equation $\delta\psi^I_r = 0$ gives the Killing spinors of the unbroken supersymmetry $\epsilon^I = e^{\frac{A}{2}}\epsilon^I_0$ as usual, with constant spinors ϵ^I_0 satisfying $\gamma_r\epsilon^I_0 = \epsilon^I_0$.

Linearizing equation (5.5) near the UV point $b \approx 0$, we find

$$b(r) \sim e^{4g_1 r} = e^{-\frac{r}{2L_{\mathrm{UV}}}}, \qquad L_{\mathrm{UV}} = \frac{1}{8|g_1|}. \tag{5.9}$$

We have set $g_1 < 0$ to identify $r \to \infty$ as the UV point. The above behavior indicates that from a general result, see for example [12], the flow is driven by a relevant operator of dimension $\Delta = \frac{3}{2}$.

Near the IR point, we find

$$b(r) \sim e^{-\frac{8g_1g_2r}{2g_1+g_2}} = e^{\frac{g_2 r}{(g_1+g_2)L_{\mathrm{IR}}}}, \qquad L_{\mathrm{IR}} = -\frac{2g_1+g_2}{8g_1(g_1+g_2)} > 0\,. \tag{5.10}$$

The reality condition for b_{IR} requires $g_2 > -2g_1$ for $g_1 < 0$. From the above equation, we find $\frac{g_2}{g_2+g_1} > 0$, so in the IR the operator becomes irrelevant with dimension $\Delta_{\mathrm{IR}} = \frac{3g_2+2g_2}{g_1+g_2}$. This value of Δ_{IR} precisely gives the correct mass square $m^2L^2_{\mathrm{IR}} = \frac{g_2(2g_1+3g_2)}{(g_1+g_2)^2}$ given before.

The ratio of the central charges is computed to be

$$\frac{c_{\mathrm{UV}}}{c_{\mathrm{IR}}} = \frac{L_{\mathrm{UV}}}{L_{\mathrm{IR}}} = \sqrt{\frac{V_{0IR}}{V_{0UV}}} = \frac{g_1+g_2}{2g_1+g_2} > 1 \tag{5.11}$$

satisfying the holographic c-theorem for $g_1 < 0$ and $g_2 > -2g_1$.

5.2 An RG flow between (5, 0) and (1, 0) CFT's in SO(5) × USp(2) gauging

We then study another RG flow interpolating between $(5,0)$ and $(1,0)$ critical points. The coset representative is sill given by (5.3). Similar to the previous case, we obtain the following flow equations

$$\frac{db}{dr} = [6g_1 + g_2 - (2g_1+g_2)\cosh b]\sinh b,$$
$$\frac{dA}{dr} = \frac{1}{4}\left[3g_2 - 10g_1 - 4(6g_1+g_2)\cosh b + (2g_1+g_2)\cosh(2b)\right]. \tag{5.12}$$

The first equation gives a solution

$$r = -\frac{1}{8g_1(4g_1+g_2)}\left[4g_1 \ln\cosh\frac{b}{2} + (2g_1+g_2)\ln\left[(2g_1+g_2)\cosh b \right.\right.$$
$$\left.\left. -6g_1-g_2\right] - 2(4g_1+g_2)\ln\sinh\frac{b}{2}\right]. \qquad (5.13)$$

We can rewrite the second equation of (5.12) as

$$\frac{dA}{db} = \frac{[10g_1-3g_2+4(6g_1+g_2)\cosh b-(2g_1+g_2)\cosh(2b)]\,\mathrm{csch} b}{4(2g_1+g_2)\cosh b-4(6g_1+g_2)} \qquad (5.14)$$

whose solution can be found to be

$$A = \frac{1}{4g_1+g_2}\left[(3g_1+g_2)\ln\left((2g_1+g_2)\cosh b-6g_1-g_2\right)\right.$$
$$\left. -(2g_1+g_2)\ln\cosh\frac{b}{2} - 2(4g_1+g_2)\ln\sinh\frac{b}{2}\right]. \qquad (5.15)$$

The fluctuation around $b=0$ behaves as

$$b(r) \sim e^{4g_1 r} = e^{-\frac{r}{2L_{\mathrm{UV}}}}, L_{\mathrm{UV}} = \frac{1}{8|g_1|}. \qquad (5.16)$$

As in the previous case, we have chosen $g_1 < 0$ to make the UV point corresponds to $r \to \infty$. From the above equation, the flow is again driven by a relevant operator of dimension $\Delta_{\mathrm{UV}} = \frac{3}{2}$. Near the IR point, $b(r)$ becomes

$$b(r) \sim e^{-\frac{8g_1(4g_1+g_2)r}{2g_1+g_2}} = e^{\frac{(4g_1+g_2)r}{(3g_1+g_2)L_{\mathrm{IR}}}}, \qquad L_{\mathrm{IR}} = -\frac{2g_1+g_2}{8g_1(3g_1+g_2)}. \qquad (5.17)$$

We can verify that b_{IR} is real for $g_1 < 0$ and $g_2 < -2g_1$, the operator becomes irrelevant in the IR with dimension $\Delta_{\mathrm{IR}} = \frac{10g_1+3g_2}{3g_1+g_2}$. The ratio of the central charges is given by

$$\frac{c_{\mathrm{UV}}}{c_{\mathrm{IR}}} = \frac{3g_1+g_2}{2g_1+g_2} > 1, \qquad \text{for} \qquad g_1 < 0 \text{ and } g_2 < -2g_1\,. \qquad (5.18)$$

5.3 An RG flow between (4, 1) and (4, 0) CFT's in USp(2) × USp(2, 2) gauging

We next consider RG flows between critical points of non-compact USp(2) × USp(2, 2) gauge group. We will not give a non-supersymmetric flow to critical point IV in table 7 in this paper. It can be studied in the same procedure as [26] and [27]. Like in the compact case, it is consistent to truncate the full scalar manifold to a single scalar parametrized by

$$L = e^{b(r)Y^7}. \qquad (5.19)$$

The variation $\delta\chi^{iI} = 0$ gives

$$\frac{db}{dr} = (g_1 - g_2 + (g_1+g_2)\cosh b)\sinh b \qquad (5.20)$$

which is solved by the solution

$$r = \frac{1}{4g_1g_2}\left[2g_2\ln\sinh\frac{b}{2} + 2g_1\ln\cosh\frac{b}{2}\right.$$
$$\left. - (g_1+g_2)\ln\left[g_1 - g_2 + (g_1+g_2)\cosh b\right]\right]. \tag{5.21}$$

The equation from $\delta\psi^I_\mu = 0$ reads

$$\frac{dA}{dr} = -2\left[g_2 + g_1\cosh^4\frac{b}{2} + g_2\sinh^4\frac{b}{2}\right]. \tag{5.22}$$

The solution for A as a function of b can be found as in the previous cases. The result is given by

$$A = \frac{1}{2g_1}\left[(2g_1+g_2)\ln\left[g_1 - g_2 + (g_1+g_2)\cosh b\right] - 4g_1\ln\cosh\frac{b}{2}\right.$$
$$\left. -2(g_1+g_2)\ln\sinh\frac{b}{2}\right]. \tag{5.23}$$

Near the UV point, the b solution becomes

$$b(r) \sim e^{2g_1 r} = e^{\frac{g_1 r}{(g_1+g_2)L_{\text{UV}}}}, \qquad L_{\text{UV}} = \frac{1}{2(g_1+g_2)}. \tag{5.24}$$

b_{IR} is real for $g_1 < 0$ and $g_2 > -g_1$. With this range, $-\frac{g_1}{g_1+g_2} < 1$. The flow is then driven by a relevant operator of dimension $\Delta = \frac{3g_1+2g_2}{g_1+g_2} < 2$. At the IR point, we find the asymptotic behavior

$$b(r) \sim e^{-\frac{4g_1g_2 r}{g_1+g_2}} = e^{\frac{2g_2 r}{|2g_1+g_2|L_{\text{IR}}}}, \qquad L_{\text{IR}} = \frac{g_1+g_2}{2|g_1(2g_1+g_2)|} \tag{5.25}$$

corresponding to an irrelevant operator of dimension $\Delta = \frac{2g_2}{|2g_1+g2|} + 2$.

Finally, the ratio of the central charges is given by

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \frac{|g_1(2g_1+g_2)|}{(g_1+g_2)^2}. \tag{5.26}$$

5.4 An RG flow between $(4,1)$ and $(1,0)$ CFT's in $USp(2)\times USp(2,2)$ gauging

As a final flow solution, we quickly investigate a solution interpolating between $(4,1)$ and $(1,0)$ critical points. The flow equations are given by

$$\frac{db}{dr} = -\left[g_1 + 3g_2 + (g_1+g_2)\cosh b\right]\sinh b, \tag{5.27}$$
$$\frac{dA}{dr} = \frac{1}{4}\left[3g_1 - 5g_2 + 4(g_1+3g_2)\cosh b + (g_1+g_2)\cosh(2b)\right]. \tag{5.28}$$

The corresponding solutions take the form

$$r = -\frac{1}{4g_2(g_1+2g_2)}\left[(g_1+g_2)\ln\left[g_1+3g_2+(g_1+g_2)\cosh b\right]\right.$$
$$\left. +2g_2\ln\sinh\frac{b}{2}-2(g_1+2g_2)\ln\cosh\frac{b}{2}\right], \tag{5.29}$$

$$A = \frac{1}{2(g_1+2g_2)}\left[(2g_1+3g_2)\ln\left[g_1+3g_2+(g_1+g_2)\cosh b\right]\right.$$
$$\left. -4(g_1+2g_2)\ln\cosh\frac{b}{2}-2(g_1+g_2)\ln\sinh\frac{b}{2}\right]. \tag{5.30}$$

The fluctuations near the UV and IR points are given by

$$b(r)\sim e^{-2(g_1+2g_2)r} = e^{\frac{(g_1+2g_2)r}{(g_1+g_2)L_{\text{UV}}}}, \qquad L_{\text{UV}} = -\frac{1}{2(g_1+g_2)}, \tag{5.31}$$

$$b(r)\sim e^{-\frac{4g_2(g_1+2g_2)r}{g_1+g_2}} = e^{\frac{2g_2(g_1+2g_2)r}{|g_1(2g_1+3g2)|L_{\text{IR}}}}, \qquad L_{\text{IR}} = -\frac{(g_1+g_2)}{2|g_1(2g_1+3g_2)|}. \tag{5.32}$$

We have chosen a particular range of g_1 and g_2 namely $g_1<0$ and $-\frac{g_1}{2}<g_2<-g_1$ for which $g_1+g_2<0$. The flow is driven by a relevant operator of dimension $\Delta=\frac{3g_1+4g_2}{g_1+g_2}$. In the IR, the operator becomes irrelevant with dimension $\Delta=\frac{2g_2}{|2g_1+g2|}+2$.

The ratio of the central charges for this flow is

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \frac{|g_1(2g_1+3g_2)|}{(g_1+g_2)^2}. \tag{5.33}$$

6 $N=5$, SO(5) $\ltimes$ $\mathbf{T}^{10}$ gauged supergravity

In this section, we consider non-semisimple gauge groups in the form of $G_0\ltimes\mathbf{T}^{\dim G_0}$ in which G_0 is a semisimple group. $\mathbf{T}^{\dim G_0}$ constitutes a translational symmetry with $\dim G_0$ commuting generators transforming in the adjoint representation of G_0. We consider the $k=4$ case with USp(4, 4) global symmetry that admits a non-semisimple subgroup SO(5) $\ltimes\mathbf{T}^{10}$.

A general embedding of $G_0\ltimes\mathbf{T}^{\dim G_0}$ group is described by the embedding tensor of the form [6]

$$\Theta = g_1\Theta_{\text{ab}}+g_2\Theta_{\text{bb}}. \tag{6.1}$$

We have used the notation of [6] in denoting the semisimple and translational parts by a and b, respectively. The absence of aa coupling plays a key role in the equivalence of this theory and the Yang-Mills gauged supergravity with G_0 gauge group.

The next task is to identify SO(5) $\ltimes\mathbf{T}^{10}$ generators. The semisimple SO(5) is identified with the diagonal subgroup of SO(5) $\times$ SO(5) $\sim$ USp(4) $\times$ USp(4) $\subset$ USp(4, 4). The corresponding generators are given by

$$J^{ij} = T^{ij}+\tilde{T}^{ij}, \qquad i,j=1,2,\ldots,5. \tag{6.2}$$

T^{ij} are the SO(5) R-symmetry generators, and $\tilde{T}^{ij}$ are generators of USp(4). The translational generators are constructed from a combination of $T^{ij}-\tilde{T}^{ij}$ and non-compact generators. The 16 scalars transform as $(\mathbf{4},\mathbf{4})$ under $\mathrm{SO}(5)\times\mathrm{SO}(5)$. They accordingly transform as $\mathbf{1}+\mathbf{5}+\mathbf{10}$ under $\mathrm{SO}(5)_{\mathrm{diag}}$. Scalars in the $\mathbf{10}$ representation will be part of the $\mathbf{T}^{10}$ generators which are given by

$$t^{ij}=T^{ij}-\tilde{T}^{ij}+\tilde{Y}^{ij}, \qquad i,j=1,2,\ldots,5\,. \tag{6.3}$$

The explicit form of $\tilde{T}^{ij}$ and $\tilde{Y}^{ij}$ is given in appendix B.

In the present case, supersymmetry allows for any value of g_1 and g_2. Therefore, the embedding tensor contains two independent coupling constants. We begin with the scalar potential computed on the $\mathrm{SO}(5)_{\mathrm{diag}}$ singlet scalar. The above decomposition gives one singlet under this SO(5). We end up with a simple coset representative

$$L=e^{a(Y^7+Y^{16})}\,. \tag{6.4}$$

This results in the potential

$$V=-64g_1e^{-3a}\left(3e^a g_1+2g_2\right)\,. \tag{6.5}$$

The existence of a maximally supersymmetric critical point at $L=\mathbf{I}$ requires $g_2=-g_1$. This is the same as in $N=4,8$ gauged supergravities [28, 29]. With this condition and g_1 denoted by g, the potential becomes

$$V=-64g^2e^{-3a}\left(3e^a-2\right)\,. \tag{6.6}$$

Clearly, the only one critical point is given by $a=0$ with $V_0=-64g^2$ and $N=(5,0)$ supersymmetry. This critical point is a minimum of the potential as can be seen from figure 1. The vacuum is very similar to the AdS_3 vacuum found in $N=16$, $\mathrm{SO}(4)\times\mathrm{SO}(4)\ltimes(\mathbf{T}^{12},\hat{\mathbf{T}}^{34})$ gauged supergravity studied in [30]. The singlet has a positive mass square $m^2L^2=3$ as expected for a minimum point. In the dual CFT with superconformal symmetry $\mathrm{Osp}(5|2,\mathbb{R})\times\mathrm{Sp}(2,\mathbb{R})$, this scalar corresponds to an irrelevant operator of dimension $\Delta=3$. The full scalar masses are given below.

m^2L^2	SO(5)
3	$\mathbf{1}$
3	$\mathbf{5}$
0	$\mathbf{10}$

The ten massless scalars accompany for the symmetry breaking $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}\to\mathrm{SO}(5)$ at the vacuum.

To find other critical points, we reduce the residual symmetry of the scalar submanifold to $\mathrm{SO}(3)\subset\mathrm{SO}(5)$ under which the 16 scalars transform as $(\mathbf{2}+\mathbf{2})\times(\mathbf{2}+\mathbf{2})=4\times(\mathbf{1}+\mathbf{3})$. There are four singlets which can be parametrized by the coset representative

$$L=e^{a_1Y^4}e^{a_2Y^7}e^{a_3Y^9}e^{a_4Y^{16}}\,. \tag{6.7}$$

The resulting potential turns out to be very complicated. We, therefore, will not attempt to do the analysis of this potential in the present work.

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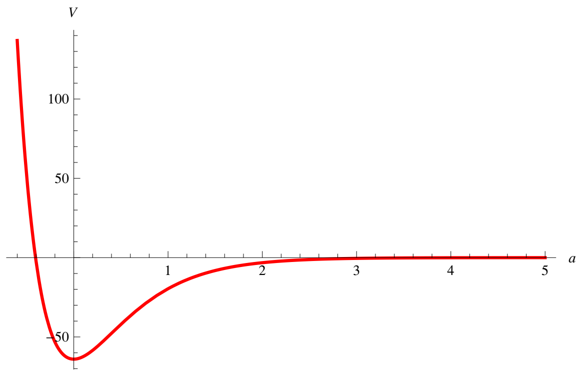

Figure 1. The scalar potential of $N = 5$, $SO(5) \ltimes \mathbf{T}^{10}$ gauged supergravity for SO(5) singlet scalar with $g = 1$.

7 $N = 6$, $SO(6) \ltimes \mathbf{T}^{15}$ gauged supergravity

In this section, we consider non-semisimple gauge groups of $N = 6$ theory. Compact and non-compact gauge groups in this theory together with their vacua and holographic RG flows have been studied in [19].

We are interested in $N = 6$ gauged supergravity with $\frac{SU(4,4)}{S(U(4)\times U(4))}$ scalar manifold. Most of our conventions here are parallel to those used in [19]. The global symmetry $SU(4,4)$ contains a non-semisimple subgroup $SO(6) \ltimes \mathbf{T}^{15}$. Similar to $N = 5$ theory, the $SO(6)$ part is given by the diagonal subgroup of $SO(6)\times SO(6) \sim SU(4)\times SU(4) \subset SU(4,4)$. The 32 scalars transform as $(\mathbf{4}, \bar{\mathbf{4}}) + (\bar{\mathbf{4}}, \mathbf{4})$ under $SU(4) \times SU(4)$. Under $SO(6)_{\text{diag}}$, they transform as

$$(\mathbf{4} \times \bar{\mathbf{4}}) + (\bar{\mathbf{4}} \times \mathbf{4}) = \mathbf{1} + \mathbf{15} + \mathbf{1} + \mathbf{15}. \tag{7.1}$$

The adjoint representations $\mathbf{15}$'s will be used to construct the translational generators $\mathbf{T}^{15}$. The full $SO(6) \ltimes \mathbf{T}^{15}$ generators are given in appendix B.

The embedding tensor is still given by (6.1), but in this case, the linear constraint $\mathbb{P}_{R_0}\Theta = 0$ requires $g_2 = 0$ similar to $N = 16, 10, 8$ theories [3, 21, 31]. The above decomposition gives two singlet scalars under SO(6) part of the gauge group. They correspond to non-compact generators

$$\begin{aligned} Y_{s1} &= \frac{1}{2}(Y^1 + Y^{11} + Y^{21} + Y^{31}), \\ Y_{s2} &= \frac{1}{2}(Y^2 + Y^{12} + Y^{22} + Y^{32}). \end{aligned} \tag{7.2}$$

Accordingly, the coset representative can be parametrized by

$$L = e^{\sqrt{2}b_1 Y_{s1}} e^{\sqrt{2}b_2 Y_{s2}} \tag{7.3}$$

where we have chosen a particular normalization for later convenience. The potential is, with $g = g_1$, given by

$$V = -224g^2 \left(\cosh b_1 \cosh b_2 - \sinh b_2\right)^2 . \tag{7.4}$$

The above potential does not admit any critical points, so the vacuum should be a half-supersymmetric domain wall. In the rest of this section, we will find this domain wall solution.

The supersymmetry transformations $\delta\psi^I_\mu$ and $\delta\chi^{iI}$ together with the domain wall ansatz (5.2) give rise to the following BPS equations

$$b_1' = 8g\text{sech}b_2 \sinh b_1, \tag{7.5}$$

$$b_2' = -8g\left(\cosh b_2 - \cosh b_1 \sinh b_2\right), \tag{7.6}$$

$$A' = -16g\left(\cosh b_1 \cosh b_2 - \sinh b_2\right) \tag{7.7}$$

where $'$ denotes $\frac{d}{dr}$. Equation (7.5) is readily solved by setting $b_1 = 0$. Equation (7.6) now becomes

$$b_2' = -8ge^{-b_2}\,. \tag{7.8}$$

The solution is given by

$$b_2 = \ln\left(-8gr + c_1\right) \tag{7.9}$$

where c_1 is an integration constant. With $b_1 = 0$ and b_2 given by (7.9), equation (7.7) becomes

$$A' = \frac{-16g}{c_1 - 8gr} \tag{7.10}$$

whose solution is easily found to be

$$A = 2\ln\left(-8gr + c_1\right) + c_2 \tag{7.11}$$

with another integration constant c_2. The two integration constants are not relevant because we can shift the coordinate r rescale $x^{0,1}$ to remove them. As in other domain wall solutions, the metric can be written in the form of a warped AdS_3 as

$$ds^2 = \frac{1}{(8g)^4\rho^2}\left(\frac{dx^2_{1,1} + d\rho^2}{\rho^2}\right) \tag{7.12}$$

where $\rho = -\frac{1}{(8g)^2 r}$.

8 Conclusions and discussions

In this paper, we have classified compact and non-compact gauge groups of $N = 5$ gauged supergravity in three dimensions with $\mathrm{USp}(4,2)/\mathrm{USp}(4)\times\mathrm{USp}(2)$ and $\mathrm{USp}(4,4)/\mathrm{USp}(4)\times\mathrm{USp}(4)$ scalar manifolds. We have also identified a number of supersymmetric AdS_3 vacua in each gauging and studied some examples of supersymmetric RG flows interpolating between these vacua in both compact and non-compact gauge groups. All of the solutions can be analytically found, and the flows describe deformations by relevant operators. They would be useful to the study of $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence such as the computation of correlation functions in the dual field theory similar to that studied in [32].

Among our main results, we have constructed $N = 5$, $\mathrm{SO}(5)\ltimes\mathbf{T}^{10}$ gauged supergravity. The theory is equivalent to $N = 5$ Yang-Mills gauged supergravity and could be obtained

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from $S^1/\mathbb{Z}_2$ reduction of $N = 5$ gauged supergravity in four dimensions as pointed out in [21]. The theory admits a maximally supersymmetric AdS_3 vacuum which should be dual to a superconformal field theory with $\mathrm{Osp}(5|2,\mathbb{R})\times\mathrm{Sp}(2,\mathbb{R})$ superconformal symmetry. We have also given all of the scalar masses at this vacuum. It is interesting to further study the scalar potential of this theory in order to find other critical points as well as the associated RG flow solutions. This could give some insight to the deformations in the dual CFT.

Similar construction has then been extended to $N = 6$ gauged supergravity with $\mathrm{SU}(4,4)/S(\mathrm{U}(4)\times\mathrm{U}(4))$ scalar manifold. The resulting theory is $N = 6$ gauged supergravity with $\mathrm{SO}(6)\ltimes\mathbf{T}^{15}$ gauge group. Like $N = 5$ theory, this is equivalent to SO(6) Yang-Mills gauged supergravity and should be obtained from $S^1/\mathbb{Z}_2$ reduction of $N = 6$ gauged supergravity in four dimensions. This has also been pointed out in [21] in which the spectrum of the S^1 reduction of four dimensional $N = 6$ gauged supergravity has been given. The theory admits a half-supersymmetric domain wall vacuum rather than a maximally supersymmetric AdS_3. We have also given the domain wall solution. This solution provides another example of domain walls in three dimensional gauged supergravity similar to the solutions of [21, 31] and might be useful in the study of DW/QFT correspondence.

The above non-semisimple gaugings are of importance for embedding the theories in higher dimensions. With the full embedding at hand, any solutions in a three dimensional framework, which are usually easier to find than higher dimensional ones, can be uplifted to string/M theory in which a full geometrical interpretation can be made. Other attempts to embed Chern-Simons gauged supergravities in three dimensions can be found in [28–30, 33–35]. In many cases, the precise reduction ansatz from ten or eleven dimensions remains to be done.

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A Useful formulae

For conveniences, we collect useful formulae used throughout this paper. The detailed discussion can be found in [5]. All of our discussions involve symmetric scalar manifolds of the form G/H. The G generators are denoted by $t^{\mathcal{M}} = (T^{IJ}, T^\alpha, Y^A)$ in which T^{IJ} and T^α are $\mathrm{SO}(N)\times H'$ generators and Y^A are non-compact generators. In the present cases, we have $H' = \mathrm{USp}(k)$ for $N = 5$ and $H' = \mathrm{U}(k)$ for $N = 6$ theories, respectively. $\mathrm{SO}(N)$ is the R-symmetry.

The coset manifold, consisting of d scalars ϕ^i, $i = 1,\dots,d = \dim(G/H)$, can be described by a coset representative L transforming by left- and right-multiplications of G

and H. Some useful relations are given by

$$L^{-1}t^{\mathcal{M}}L = \frac{1}{2}\mathcal{V}^{\mathcal{M}IJ}T^{IJ} + \mathcal{V}^{\mathcal{M}}{}_{\alpha}T^{\alpha} + \mathcal{V}^{\mathcal{M}}{}_{A}Y^{A}, \tag{A.1}$$

$$L^{-1}\partial_i L = \frac{1}{2}Q_i^{IJ}T^{IJ} + Q_i^{\alpha}T^{\alpha} + e_i^{\ A}Y^{A}. \tag{A.2}$$

The first relation gives scalar matrices $\mathcal{V}$ used in defining a moment map while the second gives $\mathrm{SO}(N)\times H'$ composite connections, Q^{IJ} and Q^{α}, and the vielbein on the manifold G/H, $e_i^{\ A}$. Accordingly, the metric on the scalar manifold is defined by

$$g_{ij} = e_i^{\ A}e_j^{\ B}\delta_{AB}, \qquad i,j,A,B = 1,\ldots,d. \tag{A.3}$$

The embedding tensor determines the fermionic mass-like terms and the scalar potential via the T-tensor defined by

$$T_{\mathcal{AB}} = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\Theta_{\mathcal{MN}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}}. \tag{A.4}$$

In the above equation, $\mathcal{A}$ and $\mathcal{B}$ label $\mathrm{SO}(N)\times H'$ representations.

The A_1^{IJ} and A_{2i}^{IJ} tensors appearing in the fermionic supersymmetry transformations and the scalar potential are given in terms of linear combinations of various components of $T_{\mathcal{AB}}$ by the following relations

$$\begin{aligned} A_1^{IJ} &= -\frac{4}{N-2}T^{IM,JM} + \frac{2}{(N-1)(N-2)}\delta^{IJ}T^{MN,MN}, \\ A_{2j}^{IJ} &= \frac{2}{N}T^{IJ}{}_{j} + \frac{4}{N(N-2)}f^{M(Im}{}_{j}T^{J)M}{}_{m} + \frac{2}{N(N-1)(N-2)}\delta^{IJ}f^{KL}{}_{j}{}^{m}T^{KL}{}_{m}. \end{aligned} \tag{A.5}$$

The f_{ij}^{IJ} tensor can be constructed from $\mathrm{SO}(N)$ gamma matrices or from the $\mathrm{SO}(N)$ generators in a spinor representation. In the present case, it is given in a flat basis by

$$f_{AB}^{IJ} = -2\mathrm{Tr}(Y^B\left[T^{IJ}, Y^A\right]). \tag{A.6}$$

The scalar potential can be computed from

$$V = -\frac{4}{N}\left(A_1^{IJ}A_1^{IJ} - \frac{1}{2}Ng^{ij}A_{2i}^{IJ}A_{2j}^{IJ}\right). \tag{A.7}$$

We end this section by noting the condition for unbroken supersymmetry. The associated Killing spinors correspond to the eigenvectors of A_1^{IJ} with eigenvalues $\pm\sqrt{-\frac{V_0}{4}}$.

B Relevant generators

In this appendix, we give generators of various groups used throughout the paper.

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B.1 $N = 5$ theory

J_i's are USp(8) generators written in terms of generalized Gell-Mann matrices λ_i generating the SU(8) group. They are explicitly given by

$$\begin{aligned}
&J_1 = \frac{i\lambda_1}{\sqrt{2}}, && J_2 = \frac{i\lambda_2}{\sqrt{2}}, && J_3 = \frac{i\lambda_3}{\sqrt{2}},\\
&J_4 = \frac{i\lambda_{13}}{\sqrt{2}}, && J_5 = \frac{i\lambda_{14}}{\sqrt{2}}, && J_6 = -\frac{i\lambda_8}{\sqrt{6}} + \frac{i\lambda_{15}}{\sqrt{3}},\\
&J_7 = \frac{i\lambda_6}{2} + \frac{i\lambda_9}{2}, && J_8 = -\frac{i\lambda_7}{2} + \frac{i\lambda_{10}}{2}, && J_9 = \frac{i\lambda_4}{2} - \frac{i\lambda_{11}}{2},\\
&J_{10} = -\frac{i\lambda_5}{2} - \frac{i\lambda_{12}}{2}, && J_{11} = \frac{i\lambda_{33}}{\sqrt{2}}, && J_{12} = \frac{i\lambda_{34}}{\sqrt{2}},\\
&J_{13} = -\frac{i\lambda_{24}}{\sqrt{5}} + \sqrt{\frac{3}{10}}i\lambda_{35}, && J_{14} = \frac{i\lambda_{18}}{2} + \frac{i\lambda_{25}}{2}, && J_{15} = -\frac{i\lambda_{19}}{2} + \frac{i\lambda_{26}}{2},\\
&J_{16} = \frac{i\lambda_{16}}{2} - \frac{i\lambda_{27}}{2}, && J_{17} = \frac{i\lambda_{22}}{2} + \frac{i\lambda_{29}}{2}, && J_{18} = -\frac{i\lambda_{23}}{2} + \frac{i\lambda_{30}}{2},\\
&J_{19} = \frac{i\lambda_{20}}{2} - \frac{i\lambda_{31}}{2}, && J_{20} = -\frac{i\lambda_{17}}{2} - \frac{i\lambda_{28}}{2}, && J_{21} = -\frac{i\lambda_{21}}{2} - \frac{i\lambda_{32}}{2},\\
&J_{22} = \frac{i\lambda_{61}}{\sqrt{2}}, && J_{23} = \frac{i\lambda_{62}}{\sqrt{2}}, && J_{24} = -\sqrt{\frac{3}{14}}i\lambda_{48} + \sqrt{\frac{2}{7}}i\lambda_{63},\\
&J_{25} = \frac{i\lambda_{38}}{2} + \frac{i\lambda_{49}}{2}, && J_{26} = -\frac{i\lambda_{39}}{2} + \frac{i\lambda_{50}}{2}, && J_{27} = \frac{i\lambda_{36}}{2} - \frac{i\lambda_{51}}{2},\\
&J_{28} = \frac{i\lambda_{42}}{2} + \frac{i\lambda_{53}}{2}, && J_{29} = -\frac{i\lambda_{43}}{2} + \frac{i\lambda_{54}}{2}, && J_{30} = \frac{i\lambda_{40}}{2} - \frac{i\lambda_{55}}{2},\\
&J_{31} = \frac{i\lambda_{46}}{2} + \frac{i\lambda_{57}}{2}, && J_{32} = -\frac{i\lambda_{47}}{2} + \frac{i\lambda_{58}}{2}, && J_{33} = \frac{i\lambda_{44}}{2} - \frac{i\lambda_{59}}{2},\\
&J_{34} = -\frac{i\lambda_{37}}{2} - \frac{i\lambda_{52}}{2}, && J_{35} = -\frac{i\lambda_{41}}{2} - \frac{i\lambda_{56}}{2}, && J_{36} = -\frac{i\lambda_{45}}{2} - \frac{i\lambda_{60}}{2}.
\end{aligned} \tag{B.1}$$

The USp(6) generators needed for constructing USp(4, 2) are given by the first 21 generators.

The SO(5) $\ltimes T^{10}$ generators are constructed as follow. The $SO(5)_{\text{diag}}$ is generated by $T^{ij} + \tilde{T}^{ij}$ in which

$$\begin{aligned}
&\tilde{T}^{12} = \frac{1}{\sqrt{2}}(J_{13} - J_{24}), && \tilde{T}^{13} = -\frac{1}{\sqrt{2}}(J_{11} + J_{22}), && \tilde{T}^{23} = \frac{1}{\sqrt{2}}(J_{12} - J_{23}),\\
&\tilde{T}^{34} = \frac{1}{\sqrt{2}}(J_{13} + J_{24}), && \tilde{T}^{14} = \frac{1}{\sqrt{2}}(J_{12} + J_{23}), && \tilde{T}^{24} = \frac{1}{\sqrt{2}}(J_{11} - J_{22}),\\
&\tilde{T}^{45} = J_{31}, && \tilde{T}^{15} = -J_{33}, && \tilde{T}^{25} = -J_{36},\\
&\tilde{T}^{35} = J_{32} \quad .
\end{aligned} \tag{B.2}$$

Generators $\tilde{Y}^{ij}$ in $\mathbf{T}^{10}$ are given by

$$\begin{aligned}
&\tilde{Y}^{12} = i(J_{16} - J_{30}), && \tilde{Y}^{13} = -i(J_{14} + J_{28}), && \tilde{Y}^{23} = i(J_{15} + J_{29}),\\
&\tilde{Y}^{34} = i(J_{16} + J_{30}), && \tilde{Y}^{14} = i(J_{15} + J_{29}), && \tilde{Y}^{24} = i(J_{14} - J_{28}),\\
&\tilde{Y}^{45} = i(J_{17} + J_{25}), && \tilde{Y}^{15} = -i(J_{19} + J_{27}), && \tilde{Y}^{25} = i(J_{21} - J_{34}),\\
&\tilde{Y}^{35} = i(J_{18} + J_{26}).
\end{aligned} \tag{B.3}$$

B.2 $\mathcal{N} = 6$ theory

For conveniences, we repeat non-compact generators of SU(4, 4) in terms of generalized Gell-Mann matrices, λ_i, $i = 1, \ldots, 63$, given in [19]

$$\bar{Y}^A = \begin{cases} \frac{1}{\sqrt{2}} c_{A+15}, & A = 1, \ldots, 8 \\ \frac{1}{\sqrt{2}} c_{A+16}, & A = 9, \ldots, 16 \\ \frac{1}{\sqrt{2}} c_{A+19}, & A = 17, \ldots, 24 \\ \frac{1}{\sqrt{2}} c_{A+24}, & A = 25, \ldots, 32 \end{cases} . \tag{B.4}$$

The $\mathrm{SO}(6)_R$ R-symmetry generators are identified to be

$$\begin{aligned}
\bar{T}^{12} &= \frac{1}{2}c_3 + \frac{1}{2\sqrt{3}}c_8 - \frac{1}{\sqrt{6}}c_{15}, & \bar{T}^{13} &= -\frac{1}{2}(c_2 + c_{14}), & \bar{T}^{23} &= \frac{1}{2}(c_1 - c_{13}), \\
\bar{T}^{34} &= \frac{1}{2}c_3 - \frac{1}{2\sqrt{3}}c_8 + \frac{1}{\sqrt{6}}c_{15}, & \bar{T}^{14} &= \frac{1}{2}(c_1 + c_{13}), & \bar{T}^{35} &= -\frac{1}{2}(c_6 + c_9), \\
\bar{T}^{56} &= \frac{1}{\sqrt{3}}c_8 + \frac{1}{\sqrt{6}}c_{15}, & \bar{T}^{36} &= -\frac{1}{2}(c_7 + c_{10}), & \bar{T}^{24} &= \frac{1}{2}(c_2 - c_{14}), \\
\bar{T}^{45} &= \frac{1}{2}(c_7 - c_{10}), & \bar{T}^{46} &= \frac{1}{2}(c_9 - c_6), & \bar{T}^{15} &= \frac{1}{2}(c_4 - c_{11}), \\
\bar{T}^{16} &= \frac{1}{2}(c_5 - c_{12}), & \bar{T}^{25} &= \frac{1}{2}(c_5 + c_{12}), & \bar{T}^{26} &= -\frac{1}{2}(c_4 + c_{11})
\end{aligned} \tag{B.5}$$

where $c_i = -i\lambda_i$.

The $\mathrm{SO}(6) \ltimes T^{15}$ generators are given by

$$\begin{aligned}
\mathrm{SO}(6): \quad & J_{\mathrm{a}}^{ij} = \bar{T}^{ij} + \tilde{\tilde{T}}^{ij}, \qquad i, j = 1, \ldots, 6 \\
\mathbf{T}^{15}: \quad & J_{\mathrm{b}}^{ij} = \bar{T}^{ij} - \tilde{\tilde{T}}^{ij} + \tilde{\tilde{Y}}^{ij}
\end{aligned} \tag{B.6}$$

where

$$\begin{aligned}
\tilde{\tilde{T}}^{12} &= i\left(\frac{1}{\sqrt{10}}\lambda_{24} - \sqrt{\frac{3}{20}}\lambda_{35} - \sqrt{\frac{3}{28}}\lambda_{48} + \frac{1}{\sqrt{7}}\lambda_{63}\right), \\
\tilde{\tilde{T}}^{34} &= i\left(\frac{1}{\sqrt{10}}\lambda_{24} - \sqrt{\frac{3}{20}}\lambda_{35} + \sqrt{\frac{3}{28}}\lambda_{48} - \frac{1}{\sqrt{7}}\lambda_{63}\right), \\
\tilde{\tilde{T}}^{56} &= i\left(\frac{1}{\sqrt{10}}\lambda_{24} + \frac{1}{\sqrt{15}}\lambda_{35} - \frac{2}{\sqrt{21}}\lambda_{48} - \frac{1}{\sqrt{7}}\lambda_{63}\right),
\end{aligned}$$

$$\begin{aligned}
\tilde{\tilde{T}}^{13} &= \frac{i}{2}\left(\lambda_{34} + \lambda_{62}\right), & \tilde{\tilde{T}}^{23} &= -\frac{i}{2}\left(\lambda_{33} - \lambda_{61}\right), & \tilde{\tilde{T}}^{14} &= -\frac{i}{2}\left(\lambda_{33} + \lambda_{61}\right), \\
\tilde{\tilde{T}}^{24} &= \frac{i}{2}\left(\lambda_{62} - \lambda_{34}\right), & \tilde{\tilde{T}}^{45} &= \frac{i}{2}\left(\lambda_{58} - \lambda_{47}\right), & \tilde{\tilde{T}}^{15} &= \frac{i}{2}\left(\lambda_{59} - \lambda_{44}\right), \\
\tilde{\tilde{T}}^{25} &= -\frac{i}{2}\left(\lambda_{45} + \lambda_{60}\right), & \tilde{\tilde{T}}^{35} &= \frac{i}{2}\left(\lambda_{46} + \lambda_{57}\right), & \tilde{\tilde{T}}^{16} &= \frac{i}{2}\left(\lambda_{60} - \lambda_{45}\right), \\
\tilde{\tilde{T}}^{26} &= \frac{i}{2}\left(\lambda_{44} + \lambda_{59}\right), & \tilde{\tilde{T}}^{36} &= \frac{i}{2}\left(\lambda_{47} + \lambda_{58}\right), & \tilde{\tilde{T}}^{46} &= \frac{i}{2}\left(\lambda_{46} - \lambda_{57}\right)
\end{aligned} \tag{B.7}$$

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and

$$\tilde{\tilde{Y}}^{12} = -\frac{1}{2}(\lambda_{27} - \lambda_{16} + \lambda_{40} - \lambda_{55}), \qquad \tilde{\tilde{Y}}^{34} = -\frac{1}{2}(\lambda_{55} - \lambda_{16} + \lambda_{27} - \lambda_{40}),$$
$$\tilde{\tilde{Y}}^{56} = -\frac{1}{2}(\lambda_{55} - \lambda_{16} - \lambda_{27} + \lambda_{40}), \qquad \tilde{\tilde{Y}}^{13} = -\frac{1}{2}(\lambda_{54} - \lambda_{19} + \lambda_{26} - \lambda_{43}),$$
$$\tilde{\tilde{Y}}^{23} = -\frac{1}{2}(\lambda_{53} - \lambda_{18} - \lambda_{25} + \lambda_{42}), \qquad \tilde{\tilde{Y}}^{14} = \frac{1}{2}(\lambda_{18} + \lambda_{25} + \lambda_{42} + \lambda_{53}),$$
$$\tilde{\tilde{Y}}^{24} = -\frac{1}{2}(\lambda_{19} - \lambda_{26} - \lambda_{43} + \lambda_{54}), \qquad \tilde{\tilde{Y}}^{45} = -\frac{1}{2}(\lambda_{50} - \lambda_{23} + \lambda_{30} - \lambda_{39}),$$
$$\tilde{\tilde{Y}}^{15} = -\frac{1}{2}(\lambda_{31} - \lambda_{20} - \lambda_{36} + \lambda_{51}), \qquad \tilde{\tilde{Y}}^{25} = -\frac{1}{2}(\lambda_{21} + \lambda_{32} - \lambda_{37} - \lambda_{52}),$$
$$\tilde{\tilde{Y}}^{35} = -\frac{1}{2}(\lambda_{22} + \lambda_{29} + \lambda_{38} + \lambda_{49}), \qquad \tilde{\tilde{Y}}^{16} = -\frac{1}{2}(\lambda_{21} - \lambda_{32} - \lambda_{37} + \lambda_{52}),$$
$$\tilde{\tilde{Y}}^{26} = -\frac{1}{2}(\lambda_{20} + \lambda_{31} + \lambda_{36} + \lambda_{51}), \qquad \tilde{\tilde{Y}}^{36} = -\frac{1}{2}(\lambda_{50} - \lambda_{23} - \lambda_{30} + \lambda_{39}),$$
$$\tilde{\tilde{Y}}^{46} = -\frac{1}{2}(\lambda_{29} - \lambda_{22} + \lambda_{38} - \lambda_{49}). \tag{B.8}$$

C Scalar potential for SO(4) × USp(2) gauging

The scalar potential for compact gauge group SO(4) × USp(2) is given by

$$\begin{aligned}
V \;=\; & 2g_2^2(3+\cosh b)\sinh^6\frac{b}{2} + \frac{1}{16}g_1 g_2\,[68 + 4\cos(2a_4) + 2\cos(2(a_4-a_5)) \\
& +4\cos(2a_5) + 2\cos(2(a_4+a_5)) + 2\cos(2(a_4-a_6)) + \cos(2(a_4-a_5-a_6)) \\
& +2\cos(2(a_5-a_6)) + \cos(2(a_4+a_5-a_6)) + 4\cos(2a_6) + 2\cos(2(a_4+a_6)) \\
& +\cos(2(a_4-a_5+a_6)) + 2\cos(2(a_5+a_6)) + \cos(2(a_4+a_5+a_6)) \\
& +32\cos^2 a_4\cos^2 a_5\cos^2 a_6\cos(2a_7)]\,(3+\cosh b)\sinh^6\frac{b}{2} \\
& -4g_1^2\Big[\cos^2 a_5\cos^2 a_6\cos^2 a_7\cosh^2\frac{b}{2}(3+\cosh b)^2\sin^2(2a_4) \\
& +64\cos^2 a_4\cosh^4\frac{b}{2}\sin^2 a_4\sin^2 a_5 + 64\cos^2 a_4\cos^2 a_5\cosh^4\frac{b}{2} \\
& \times\sin^2 a_4\sin^2 a_6 + 64\cos^2 a_4\cos^2 a_5\cos^2 a_6\cosh^4\frac{b}{2}\sin^2 a_4\sin^2 a_7 \\
& +\frac{1}{16384}\Big[51 + 259\cos(2a_4) + 4(-17+63\cos(2a_4))\cosh b + (17+\cos(2a_4)) \\
& \quad\times\cosh(2b) + 16\cos^2 a_4\cos(2a_5)\sinh^4\frac{b}{2} + 32\cos^2 a_4\cos^2 a_5\cos(2a_6)\sinh^4\frac{b}{2} \\
& \quad+64\cos^2 a_4\cos^2 a_5\cos^2 a_6\cos(2a_7)\sinh^4\frac{b}{2}\Big]^2 + \frac{1}{2}\Big[-4\cos^4 a_4\cos^2 a_5\cos^2 a_6 \\
& \times\cos^2 a_7\sin^2 a_5\sinh^6\frac{b}{2} - 4\cos^4 a_4\cos^4 a_5\cos^2 a_6\cos^2 a_7\sin^2 a_6\sinh^6\frac{b}{2} \\
& -4\cos^4 a_4\cos^4 a_5\cos^4 a_6\cos^2 a_7\sin^2 a_7\sinh^6\frac{b}{2} - 4\sin^2(2a_4)\sin^2 a_5\sinh^2 b
\end{aligned}$$

$$
\begin{aligned}
&-16\cos^2 a_4 \cos^2 a_5 \sin^2 a_4 \sin^2 a_6 \sinh^2 b - 16\cos^2 a_4 \cos^2 a_5 \cos^2 a_6 \sin^2 a_4 \\
&\times \sin^2 a_7 \sinh^2 b - \frac{1}{16}\cos^2 a_5 \cos^2 a_6 \cos^2 a_7 \sin^2(2a_4)\left[7\sinh\frac{b}{2} + 3\sinh\frac{3b}{2}\right]^2 \\
&-\frac{1}{4096}\Bigg[16\cos^2 a_4\left[\cos(2a_5) + 2\cos^2 a_5\left(\cos(2a_6) + 2\cos^2 a_6 \cos(2a_7)\right)\right] \\
&\quad \times \cosh\frac{b}{2}\sinh^3\frac{b}{2} + 2[63\cos(2a_4) + 17\cosh b - 17]\sinh b \\
&\quad + \cos(2a_4)\sinh(2b)\Bigg]^2\Bigg]\Bigg]. \qquad \text{(C.1)}
\end{aligned}
$$

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Deformations of large $N = (4,4)$ 2D SCFT from 3D gauged supergravity

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Abstract: Supersymmetric and non-supersymmetric deformations of large $N = (4,4)$ SCFT with superconformal symmetry $D^1(2,1;\alpha) \times D^1(2,1;\alpha)$ are explored in the gravity dual described by a Chern-Simons $N = 8$, $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauged supergravity in three dimensions. For $\alpha > 0$, the gauged supergravity describes an effective theory of the maximal supergravity in nine dimensions on $AdS_3 \times S^3 \times S^3$ with the parameter α being the ratio of the two S^3 radii. We consider the scalar manifold of the supergravity theory of the form $\mathrm{SO}(8,8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ and find a number of stable non-supersymmetric AdS_3 critical points for some values of α. These correspond to non-supersymmetric IR fixed points of the UV $N = (4,4)$ SCFT dual to the maximally supersymmetric critical point. We study the associated RG flow solutions interpolating between these fixed points and the UV $N = (4,4)$ SCFT. Possible supersymmetric flows to non-conformal field theories and half-supersymmetric domain walls within this gauged supergravity are also investigated.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Supergravity Models, dS vacua in string theory

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Contents

1 Introduction

$\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence is interesting in various aspects. Unlike in higher dimensional cases, much more insight to the AdS/CFT correspondence [1] is expected since both gravity and field theory sides are well under control. It is also useful in the study of black hole entropy, see for example [2] and [3]. Until now, various gravity backgrounds implementing $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence have been proposed. Some of them are obtained from Kaluza-Klein dimensional reductions of higher dimensional supergravities on spheres or other internal manifolds. The other are constructed directly within the three dimensional framework of Chern-Simons gauged supergravity, but, in some cases particularly for compact and non-compact gauge groups, higher dimensional origins are still mysterious.

One of the most interesting backgrounds for $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence is string theory on $AdS_3 \times S^3 \times S^3 \times S^1$. The background is half-supersymmetric and dual to large $N = (4,4)$ SCFT in two dimensions, see [4] for a classification of $N = 4$ SCFT in two dimensions. In string theory, this arises as a near horizon limit of the double D1-D5 brane system [5–7]. The Kaluza-Klein spectrum for small S^1 radius has been computed in [8]. Apart from the non-propagating supergravity multiplet in three dimensions, the spectrum contains massive multiplets of various spins. The full symmetry of $AdS_3 \times S^3 \times S^3$ is $D^1(2,1;\alpha) \times D^1(2,1;\alpha)$ whose bosonic subgroup is $\mathrm{SO}(2,2) \times \mathrm{SO}(4) \times \mathrm{SO}(4)$ corresponding to the isometry of $AdS_3 \times S^3 \times S^3$, respectively. Additionally, the holography of large $N = 4$ SCFT has recently been studied in the context of higher spin AdS_3 dual [9].

Like in higher dimensions, it would be useful to have an effective theory in three dimensions that describes the above $S^3 \times S^3$ dimensional reduction. The $AdS_3 \times S^3 \times S^3$ background will become an AdS_3 vacuum preserving sixteen supercharges and $\mathrm{SO}(4) \times \mathrm{SO}(4)$ gauge symmetry, which is the isometry of $S^3 \times S^3$. This can be achieved by a gauged matter-coupled supergravity in three dimensions [10–12]. The gauge group should contain the $\mathrm{SO}(4) \times \mathrm{SO}(4)$ factor. The natural construction should be the $N = 8$ gauged supergravity since the number of supersymmetry is exactly the same as that of the $AdS_3 \times S^3 \times S^3$ background. A theory describing supergravity coupled to massive spin-$\frac{1}{2}$ multiplets has been studied in [13] in which some critical points and a holographic RG flow have been discussed. The resulting theory is in the form of $N = 8$ gauged supergravity with compact $\mathrm{SO}(4) \times \mathrm{SO}(4)$ gauge group and $\mathrm{SO}(8,n)/\mathrm{SO}(8) \times \mathrm{SO}(n)$ scalar manifold.

When coupled to massive spin-1 multiplets, the theory needs to accompany for massive vector fields. For a theory coupled to two spin-1 multiplets, the corresponding gauge group is a non-semisimple group $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$. It has been argued that the effective theory is the $N = 8$ gauged supergravity with $\mathrm{SO}(8,8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ scalar manifold [14]. The gauging is a straightforward extension of the $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ gauging of [15] in which the effective theory of six-dimensional supergravity reduced on $AdS_3 \times S^3$ has been given. Some supersymmetric vacua of the $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauged theory have already been identified in [16]. All of these vacua are related to the maximally supersymmetric vacuum by marginal deformations. The theory with only the $\mathrm{SO}(4) \times \mathrm{SO}(4)$ semisimple part of the gauge group being gauged has been study in [17], and the solution corresponding to a marginal deformation from $N = (4,4)$ to $N = (3,3)$ SCFT, describing a D5-brane reconnection, has been explicitly given.

In this paper, we will reexamine the full $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauging and look for other deformations apart from the marginal ones. This could be relevant for $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence and black hole physics. The holographic study of the conformal symmetry $D^1(2,1;\alpha)$ is not only useful in the context of $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence but also in $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence. This is because the symmetry $D^1(2,1;\alpha)$ also arises in superconformal quantum mechanics [18–20]. The isometry of AdS_2 is $\mathrm{SO}(2,1)$ which is a subgroup of the AdS_3 isometry $\mathrm{SO}(2,2) \sim \mathrm{SO}(2,1) \times \mathrm{SO}(2,1)$. Accordingly, the superconformal symmetry in one dimension contains only a single $D^1(2,1;\alpha)$. The holographic study of $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence directly from two dimensional gauged supergravity has not been performed extensively. This is in part due to the lack of gauged supergravities in two dimensions. Until now, only the maximal gauged supergravity and its truncation have appeared [21, 22]. Since AdS_2 can be obtained by dimensional reduction of AdS_3 on S^1 via a very-near-horizon limit [23, 24], the results obtained here might be useful in the study of deformations in $D^1(2,1;\alpha)$ superconformal mechanics.

The paper is organized as follow. In section 2, we will give a brief review of $N = 8$, $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauged supergravity along with some relations to the $N = (4,4)$

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SCFT. Section 3 deals with a description of new critical points, and the stability condition for some of them is verified. In section 4, we study possible supersymmetric flows to non-conformal field theories and $\frac{1}{2}$-BPS domain walls. We also comment on some numerical RG flow solutions describing deformations of the $N = (4, 4)$ SCFT to other CFTs in the IR. We end the paper by giving some conclusions and discussions in section 5. The appendices summarize necessary ingredients needed in the construction of $N = 8$ theory and relevant formulae including the explicit form of some scalar potentials.

2 $N = 8$, $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauged supergravity in three dimensions

We now review the construction of $N = 8$ gauged supergravity with $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauge group. The theory has partially been studied before in [16]. We will explore the scalar potential of this theory in more details. Rather than follow the parametrization of $\mathrm{SO}(8, 8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ coset manifold as in [16], we will use the parametrization similar to that of [25]. In this parametrization, it is more convenient to determine the residual gauge symmetry while the parametrization used in [16] gives a simple action of the translation generators $\mathbf{T}^{12}$ on scalar fields.

It has been argued in [14] that this theory is an effective theory of ten dimensional supergravity on $AdS_3 \times S^3 \times S^3 \times S^1$, or nine dimensional supergravity on $AdS_3 \times S^3 \times S^3$ for small S^1 radius, and describes the coupling of two massive spin-1 multiplets, containing twelve vectors, to the non-propagating supergravity multiplet of the reduction. All together, the resulting theory is $N = 8$ gauged supergravity with the scalar manifold $\mathrm{SO}(8, 8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ and $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauge group.

The whole construction is similar to that given in [16] and [25]. We will work in the $\mathrm{SO}(8)$ R-symmetry covariant formulation of [12] with some relevant formulae and details explicitly given in appendix A. We first introduce the basis for a $GL(16, \mathbb{R})$ matrices

$$(e_{mn})_{pq} = \delta_{mp}\delta_{nq}, \qquad m, n, p, q = 1, \ldots, 16\,. \tag{2.1}$$

The compact generators of $\mathrm{SO}(8, 8)$ are then given by

$$\begin{aligned} &\mathrm{SO}(8)^{(1)}: \qquad && J_1^{IJ} = e_{JI} - e_{IJ}, && I, J = 1, \ldots, 8, \\ &\mathrm{SO}(8)^{(2)}: && J_2^{rs} = e_{s+8,r+8} - e_{r+8,s+8}, && r, s = 1, \ldots, 8\,. \end{aligned} \tag{2.2}$$

The non-compact generators corresponding to 64 scalars are identified as

$$Y^{Kr} = e_{K,r+8} + e_{r+8,K}, \qquad K, r = 1, \ldots, 8\,. \tag{2.3}$$

In the formulation of [12], scalars transform as a spinor under $\mathrm{SO}(8)_R$ R-symmetry. It can be easily seen from the above equation that Y^{Kr} transform as a vector under $\mathrm{SO}(8)_R$ identified with $\mathrm{SO}(8)^{(1)}$ with generators J_1^{IJ}. We define the following $\mathrm{SO}(8)_R$ generators in a spinor representation by

$$T^{IJ} = \begin{pmatrix} \Gamma^{IJ} & 0 \\ 0 & 0 \end{pmatrix} \tag{2.4}$$

constructed from the 8×8 SO(8) gamma matrices Γ^I. We have defined

$$\Gamma^{IJ} = -\frac{1}{4}\left(\Gamma^I(\Gamma^J)^T - \Gamma^J(\Gamma^I)^T\right) \tag{2.5}$$

with the 8×8 gamma matrices Γ^I are given in appendix A.

The gauge group $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ is embedded in SO(8, 8) as follow. We first form a diagonal subgroup of $\mathrm{SO}(8) \times \mathrm{SO}(8)$ with generators

$$\mathrm{SO}(8)_{\mathrm{diag}}: \qquad J^{AB} = J_1^{AB} + J_2^{AB}, \qquad A, B = 1, \ldots, 8\,. \tag{2.6}$$

The $\mathrm{SO}(4) \times \mathrm{SO}(4)$ part is generated by

$$\begin{aligned}
&\mathrm{SO}(4)^+: \qquad && j_1^{ab} = J^{ab}, && \\
&\mathrm{SO}(4)^-: \qquad && j_2^{\hat{a}\hat{b}} = J^{\hat{a}+4,\hat{b}+4}, \qquad && a, b, \hat{a}, \hat{b} = 1, \ldots, 4\,.
\end{aligned} \tag{2.7}$$

The "hat" indices refer to $\mathrm{SO}(4)^-$. We now construct the translational generators $\mathbf{T}^{28}$ as in [25]

$$t^{AB} = J_1^{AB} - J_2^{AB} + Y^{BA} - Y^{AB} \tag{2.8}$$

and identify $\mathbf{T}^{12} \sim \mathbf{T}^6 \times \mathbf{T}^6$ generators as

$$t_1^{ab} = t^{ab}, \qquad t_2^{\hat{a}\hat{b}} = t^{\hat{a}+4,\hat{b}+4}, \qquad a, b, \hat{a}, \hat{b} = 1, \ldots, 4\,. \tag{2.9}$$

The gauge group is embedded in SO(8, 8) with a specific form of the embedding tensor. As shown in [26], there is no coupling among the $\mathrm{SO}(4)^\pm$. The gauging is very similar to the $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ gauged supergravity constructed in [15] with two factors of $\mathrm{SO}(4) \ltimes \mathbf{T}^6$. The embedding tensor is simply given by two copies of that given in [15]. We end up with two independent coupling constants

$$\Theta = g_1\Theta_1 + g_2\Theta_2\,. \tag{2.10}$$

where $\Theta_{1,2}$ describe the embedding of each $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ factor of the full gauge group. We should note that supersymmetry allows for four independent couplings namely between the moment maps $g_1'(\mathcal{V}(j_1^{ab}), \mathcal{V}(t_1^{ab}))$, $g_2'(\mathcal{V}(t_1^{ab}), \mathcal{V}(t_1^{ab}))$, $g_3'(\mathcal{V}(j_2^{ab}), \mathcal{V}(t_2^{ab}))$ and $g_4'(\mathcal{V}(t_2^{ab}), \mathcal{V}(t_2^{ab}))$ in the T-tensor, see [15] and [16]. We have used a shorthand notation for $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$. However, the requirement that the theory admits a maximally supersymmetric vacuum at the origin of the scalar manifold imposes two conditions on the original four couplings. In more detail, the two conditions require $g_2' = -g_1'$ and $g_4' = -g_3'$. After rename the relevant couplings, we end up with the embedding tensor

$$\Theta_{abcd} = g_1\epsilon^+_{abcd} + g_2\epsilon^-_{\hat{a}\hat{b}\hat{c}\hat{d}}\,. \tag{2.11}$$

This embedding tensor together with the formulae in appendix A and an explicit parametrization of the coset representative of $\mathrm{SO}(8,8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ can be used to compute the scalar potential. We will analyze the resulting potential on submanifolds of $\mathrm{SO}(8,8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ invariant under some subgroups of $\mathrm{SO}(4) \times \mathrm{SO}(4)$ in the next section.

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Before looking at the critical points, we give a review of the relation between $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$, $N = 8$ gauged supergravity and $N = (4, 4)$ SCFT. The semisimple part of the gauge group $\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^-$ corresponds to the isometry of $S^3 \times S^3$. Together with the usual $\mathrm{SO}(2, 2)$ isometry of AdS_3, they constitute the bosonic subgroup $\mathrm{SO}(2, 1)_L \times \mathrm{SU}(2)^+_L \times \mathrm{SU}(2)^-_L \times \mathrm{SO}(2, 1)_R \times \mathrm{SU}(2)^+_R \times \mathrm{SU}(2)^-_R$ of the superconformal group $D^1(2, 1; \alpha) \times D^1(2, 1; \alpha)$ via the isomorphisms $\mathrm{SO}(2, 2) \sim \mathrm{SO}(2, 1)_L \times \mathrm{SO}(2, 1)_R$ and $\mathrm{SO}(4)^\pm \sim \mathrm{SU}(2)^\pm_L \times \mathrm{SU}(2)^\pm_R$. The α parameter is identified with the ratio of the coupling constant $g_2 = \alpha g_1$. For positive α, the theory describes the dimensional reduction of nine dimensional supergravity on $S^3 \times S^3$. For negative α, it may possibly describe the reduction on $S^3 \times H^3$ where H^3 is a hyperbolic space in three dimensions.

The translational part $\mathbf{T}^{12}$ of the gauge group describes twelve massive vector fields [26]. The massive vector fields will show up in the vacuum of the theory via twelve massless scalars in the adjoint representation of $\mathrm{SO}(4) \times \mathrm{SO}(4)$. These are Goldstone bosons for the $\mathbf{T}^{12}$ symmetry since the vacuum is invariant only under $\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^-$ not the full gauge group. We will see this when we compute the mass spectrum of scalar fields.

3 Some critical points of $N = 8$, $(\mathrm{SO}(4) \ltimes \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauged supergravity

We now look for critical points of the $N = 8$ gauged supergravity constructed in the previous section. Analyzing the scalar potential on the full 64-dimensional scalar manifold $\mathrm{SO}(8, 8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ is beyond our reach with the present-time computer. We then employ an effective method given in [27] to find some interesting critical points on a submanifold invariant under some subgroup of the gauge group. A group theoretical argument guarantees that the corresponding critical points are critical points of the scalar potential on the full scalar manifold. Even on these truncated manifolds, the explicit form of the potential is still very complicated. Therefore, in most cases, we refrain from giving the full expression for the potential.

At the trivial critical point with all scalars vanishing, the full gauge group $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ is broken down to its maximal compact subgroup $\mathrm{SO}(4) \times \mathrm{SO}(4)$ corresponding to the isometry of $S^3 \times S^3$. The 64 scalars transform under $\mathrm{SO}(8) \times \mathrm{SO}(8) \subset \mathrm{SO}(8, 8)$ as $(\mathbf{8}, \mathbf{8})$. Then, under the $\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^- \subset \mathrm{SO}(8)_{\mathrm{diag}}$, they transform as

$$\begin{aligned} \mathbf{8} \times \mathbf{8} &= \left[(\mathbf{4}^+, \mathbf{1}^+) + (\mathbf{1}^-, \mathbf{4}^-)\right] \times \left[(\mathbf{4}^+, \mathbf{1}^+) + (\mathbf{1}^-, \mathbf{4}^-)\right] \\ &= (\mathbf{1}^+ + \mathbf{6}^+ + \mathbf{9}^+, \mathbf{1}^+) + (\mathbf{1}^-, \mathbf{1}^- + \mathbf{6}^- + \mathbf{9}^-) + (\mathbf{4}^+, \mathbf{4}^-) + (\mathbf{4}^-, \mathbf{4}^+). \end{aligned} \tag{3.1}$$

We can further decompose the above representations into $\mathrm{SU}(2)^+_L \times \mathrm{SU}(2)^+_R \times \mathrm{SU}(2)^-_L \times \mathrm{SU}(2)^-_R$ representations labeled by $(\ell^+_L, \ell^+_R; \ell^-_L, \ell^-_R)$ as follow:

$$\begin{aligned} \mathbf{8} \times \mathbf{8} = &\ (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}; \mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{3}; \mathbf{1}, \mathbf{1}) \\ &+ (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{1}; \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{3}, \mathbf{3}) \\ &+ (\mathbf{2}, \mathbf{2}; \mathbf{2}, \mathbf{2}) + (\mathbf{2}, \mathbf{2}; \mathbf{2}, \mathbf{2}). \end{aligned} \tag{3.2}$$

h_R / h_L	$\frac{\alpha}{1+\alpha}$	$\frac{3\alpha+1}{2(1+\alpha)}$	$\frac{2\alpha+1}{1+\alpha}$
$\frac{\alpha}{1+\alpha}$	$(0,1;0,1)$	$(0,1;\frac{1}{2},\frac{1}{2})$	$(0,1;0,0)$
$\frac{3\alpha+1}{2(1+\alpha)}$	$(\frac{1}{2},\frac{1}{2};0,1)$	$(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2};0,0)$
$\frac{2\alpha+1}{1+\alpha}$	$(0,0;0,1)$	$(0,0;\frac{1}{2},\frac{1}{2})$	$(0,0,0,0)$

Table 1. The massive spin-1 multiplet $(0,1;0,1)_{\mathrm{S}}$.

h_R / h_L	$\frac{1}{1+\alpha}$	$\frac{3+\alpha}{2(1+\alpha)}$	$\frac{2+\alpha}{1+\alpha}$
$\frac{1}{1+\alpha}$	$(1,0;1,0)$	$(1,0;\frac{1}{2},\frac{1}{2})$	$(1,0;0,0)$
$\frac{3+\alpha}{2(1+\alpha)}$	$(\frac{1}{2},\frac{1}{2};1,0)$	$(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2};0,0)$
$\frac{2+\alpha}{1+\alpha}$	$(0,0;1,0)$	$(0,0;\frac{1}{2},\frac{1}{2})$	$(0,0,0,0)$

Table 2. The massive spin-1 multiplet $(1,0;1,0)_{\mathrm{S}}$.

$\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^-$	m^2L^2
$(\mathbf{1},\mathbf{1})$	$\frac{4g_1(2g_1+g_2)}{(g_1+g_2)^2}$
$(\mathbf{6},\mathbf{1})$	0
$(\mathbf{9},\mathbf{1})$	$-\frac{4g_1g_2}{(g_1+g_2)^2}$
$(\mathbf{1},\mathbf{1})$	$\frac{4g_2(2g_2+g_1)}{(g_1+g_2)^2}$
$(\mathbf{1},\mathbf{6})$	0
$(\mathbf{1},\mathbf{9})$	$-\frac{4g_1g_2}{(g_1+g_2)^2}$
$(\mathbf{4},\mathbf{4})$	$\frac{3g_2^2-2g_1g_2-g_1^2}{(g_1+g_2)^2}$
$(\mathbf{4},\mathbf{4})$	$\frac{3g_1^2-2g_1g_2-g_2^2}{(g_1+g_2)^2}$

Table 3. The mass spectrum of the trivial critical point.

The result precisely agrees with the representation content obtained from the $AdS_3 \times S^3 \times S^3$ reduction [8]. For conveniences, we also repeat the massive spin-1 supermultiplets $(0,1;0,1)_{\mathrm{S}}$ and $(1,0;1,0)_{\mathrm{S}}$ of the $AdS_3 \times S^3 \times S^3$ reduction in table 1 and 2.

We can now compute the scalar potential by using the formulae in appendix A. After expanding the potential around $L = \mathbf{I}$, we find the scalar mass spectrum at the maximally supersymmetric vacuum as shown in table 3. The AdS_3 radius is given by $L = \frac{1}{\sqrt{-V_0}}$, and the value of the potential at this point is $V_0 = -64(g_1+g_2)^2$. Using the relation $m^2L^2 = \Delta(\Delta-2)$ and $\Delta = h_L + h_R$, we can verify that the mass spectrum agrees with the values of h_R and h_L in table 1 and 2. As mentioned before, there are twelve massless Goldstone bosons transforming in the adjoint representation $(\mathbf{1},\mathbf{6})+(\mathbf{6},\mathbf{1})$ of $\mathrm{SO}(4)\times\mathrm{SO}(4)$. Note also that there is a Minkowski vacuum at $g_2 = -g_1$ or $\alpha = -1$.

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3.1 Critical points on the $SO(4)_{\rm diag}$ invariant manifold

We first consider scalars which are singlets under the diagonal subgroup $SO(4)_{\rm diag} \subset SO(4) \times SO(4)$. To obtain representations of the scalars under this subgroup, we take a tensor product in the last line of (3.1). We find that there are four singlets, two from the obvious ones $(\mathbf{1}^+ \times \mathbf{1}^+, \mathbf{1}^- \times \mathbf{1}^-)$ and the other two from the product $(\mathbf{4}^+ \times \mathbf{4}^-, \mathbf{4}^- \times \mathbf{4}^+)$. They correspond to the following non-compact generators

$$\begin{aligned} \tilde{Y}_1 &= Y^{11} + Y^{22} + Y^{33} + Y^{44}, & \tilde{Y}_2 &= Y^{55} + Y^{66} + Y^{77} + Y^{88}, \\ \tilde{Y}_3 &= Y^{51} + Y^{62} + Y^{73} + Y^{84}, & \tilde{Y}_4 &= Y^{15} + Y^{26} + Y^{37} + Y^{48}. \end{aligned} \tag{3.3}$$

The coset representative is accordingly parametrized by

$$L = e^{a_1 \tilde{Y}_1} e^{a_2 \tilde{Y}_2} e^{a_3 \tilde{Y}_3} e^{a_4 \tilde{Y}_4}. \tag{3.4}$$

Apart from the trivial critical point at $a_1 = a_2 = a_3 = a_4 = 0$, we find the following critical points.

- A non-supersymmetric AdS_3 is given by $a_1 = \frac{1}{2} \ln \frac{\sqrt{g_1 - 4g_3} - \sqrt{g_1}}{2\sqrt{g_1}}$ and $a_2 = a_3 = a_4 = 0$. The cosmological constant is

$$V_0 = -32 \left[g_1^2 + 4g_2^2 - 6g_1 g_2 + (4g_2 - g_1)\sqrt{g_1(g_1 - 4g_3)} \right]. \tag{3.5}$$

  a_1 is real for $g_1 > 0$ and $g_2 < 0$, and the critical point is AdS_3, $V_0 < 0$, for $g_1 > 0$ and $g_2 < -\frac{\sqrt{2}+1}{2} g_1$. An equivalent critical point is given by $a_2 \neq 0$ and $a_1 = a_3 = a_4 = 0$ but with $g_1 \leftrightarrow g_2$. For later reference, we will call this critical point P_1.

- Another non supersymmetric critical point is at $a_4 = \ln \frac{\sqrt{g_1} + \sqrt{3g_2}}{\sqrt{g_1} - \sqrt{3g_2}}$ with $g_2 = \frac{1}{9}\left(\sqrt{13} - 2\right) g_1$ and $V_0 = -\frac{8}{3}\left(43 + 13\sqrt{13}\right) g_1^2$. In this case, only a specific value of α gives a critical point. The residual gauge symmetry in this case is $SO(4)_{\rm diag}$. We will label this critical point as P_2.

The full scalar potential for the four scalars is given in appendix B.

We now analyze the scalar masses at the above critical points to check their stability. For critical point P_1, it is useful to classify the 64 scalars according to their representations under the residual symmetry $SO(4) \times SO(4)$. The result is shown in table 4. Similar to the trivial critical point, there are 12 massless scalars corresponding to the broken $\mathbf{T}^{12}$ symmetry. The stability bound, or BF bound $m^2 L^2 \geq -1$, is satisfied by $-\frac{13+9\sqrt{2}}{2} g_1 < g_2 < -\frac{1+\sqrt{1+\sqrt{2}}}{2} g_1$.

For critical point P_2, we can compute all scalar masses as shown in table 5. It is easily seen that all masses satisfy the BF bound. There are 18 massless Goldstone bosons corresponding to the symmetry breaking $(SO(4) \times SO(4)) \ltimes \mathbf{T}^{12} \to SO(4)$.

We end this subsection by noting an interesting result discovered in [17] but with a compact gauge group $SO(4) \times SO(4)$. This solution describes a marginal deformation of $N = (4,4)$ SCFT to $N = (3,3)$ SCFT and has an interpretation in term of a reconnection of D5-branes in the double D1-D5 system. The solution is also encoded in our present

$SO(4)^+ \times SO(4)^-$	m^2L^2
$(\mathbf{1},\mathbf{1})$	$\frac{12g_2}{g_2+\sqrt{g_1(g_1-4g_2)}}$
	$-\frac{16g_2^2+20g_1g_2-6g_1^2+2(g_1+2g_2)\sqrt{g_1(g_1-4g_2)}}{g_1^2-4g_1g_2-4g_2^2}$
	$\frac{4g_2^2+14g_1g_2-3g_1^2+(4g_2-g_1)\sqrt{g_1(g_1-4g_2)}}{2(g_1^2-4g_1g_2-4g_2^2)}$
	$-\frac{3g_1^2-30g_1g_2+12g_2^2+3(3g_1-4g_2)\sqrt{g_1(g_1-4g_2)}}{2(g_1^2-4g_1g_2-4g_2^2)}$
$(\mathbf{6},\mathbf{1})$	0
$(\mathbf{9},\mathbf{1})$	$\frac{8g_1g_2}{g_1^2-6g_1g_2+(2g_2-g_1)\sqrt{g_1(g_1-4g_2)}}$
$(\mathbf{1},\mathbf{1})$	$\frac{4g_2(2g_2+g_1)}{(g_1+g_2)^2}$
$(\mathbf{1},\mathbf{6})$	0
$(\mathbf{1},\mathbf{9})$	$-\frac{4g_1^2-24g_1g_2-8g_2^2+4(g_1-g_2)\sqrt{g_1(g_1-4g_2)}}{g_1^2-4g_1g_2-4g_2^2}$
$(\mathbf{4},\mathbf{4})$	$\frac{4g_2^2+14g_1g_2-3g_1^2+(4g_2-g_1)\sqrt{g_1(g_1-4g_2)}}{2(g_1^2-4g_1g_2-g_2^2)}$
$(\mathbf{4},\mathbf{4})$	$-\frac{12g_2^2-30g_1g_2+3g_1^2+(9g_1-12g_2)\sqrt{g_1(g_1-4g_2)}}{2(g_1^2-4g_1g_2-g_2^2)}$

Table 4. The scalar mass spectrum of the $SO(4) \times SO(4)$ critical point P_1.

$SO(4)$	m^2L^2
$\mathbf{1}$	13.6358, 6.0931, 3.3703, 3.1180
$\mathbf{6}$	$0_{(\times 18)}$
$\mathbf{9}$	$\frac{8}{29}(7\sqrt{13}-12)_{(\times 9)}$, $\frac{4}{29}(5\sqrt{13}-21)_{(\times 9)}$, $\frac{4}{29}(8+5\sqrt{13})_{(\times 9)}$, $\frac{4}{87}(19\sqrt{13}-74)_{(\times 9)}$

Table 5. The scalar mass spectrum of the $SO(4)$ critical point P_2 for $g_2 = \frac{\sqrt{13}-2}{9} g_1$.

framework. In this case, we must set $g_2 = g_1$, or equivalently setting $\alpha = 1$ in order to get massless (marginal) scalars preserving the $SO(4)$ diagonal subgroup of $SO(4) \times SO(4)$.

Follow [17], we further truncate the four scalars to two via

$$a_2 = a_1, \qquad a_4 = -a_3 \,. \tag{3.6}$$

This is a consistent truncation for $g_2 = g_1$ since it corresponds to a fixed point of an inner automorphism that leaves the embedding tensor invariant [17]. We find a critical point at

$$e^{a_1+a_3} = 1 + \sqrt{1-e^{2a_1}}, \qquad V_0 = -256 g_1^2 \tag{3.7}$$

with the corresponding A_1 tensor given by

$$A_1^{IJ} = \text{diag}\left(-8g_1, -8g_1, -8g_1, 8g_1, 8g_1, 8g_1, -8g_1\sqrt{4e^{-2a_1}-3}, 8g_1\sqrt{4e^{-2a_1}-3}\right). \tag{3.8}$$

We can see that as long as $a_1 \neq 0$, the $N = (4,4)$ supersymmetry is broken to $N = (3,3)$. We refer the reader to [17] for the full discussion of this vacuum.

3.2 Critical points on the $\mathrm{SO(2)_{diag}} \times \mathrm{SO(2)_{diag}}$ invariant manifold

We now proceed to consider a smaller residual symmetry $\mathrm{SO}(2)_{\mathrm{diag}} \times \mathrm{SO}(2)_{\mathrm{diag}} \subset \mathrm{SO}(4)_{\mathrm{diag}}$. Under $\mathrm{SO}(2) \times \mathrm{SO}(2)$, the $\mathrm{SO}(4)$ fundamental representation $\mathbf{4}$ decomposes according to $\mathbf{4} \to (\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{2})$. Substituting this decomposition for $\mathbf{4}^+$ and $\mathbf{4}^-$ in (3.1) and taking the product to form a diagonal subgroup, we find that there are sixteen singlets given by the non-compact generators

$$\begin{aligned}
&\bar{Y}_1 = Y^{11} + Y^{22}, \quad &&\bar{Y}_2 = Y^{33} + Y^{44}, \quad &&\bar{Y}_3 = Y^{55} + Y^{66}, \quad &&\bar{Y}_4 = Y^{77} + Y^{88}, \\
&\bar{Y}_5 = Y^{15} + Y^{26}, &&\bar{Y}_6 = Y^{37} + Y^{48}, &&\bar{Y}_7 = Y^{51} + Y^{62}, &&\bar{Y}_8 = Y^{73} + Y^{84}, \\
&\bar{Y}_9 = Y^{12} - Y^{21}, &&\bar{Y}_{10} = Y^{34} - Y^{43}, &&\bar{Y}_{11} = Y^{56} - Y^{65}, &&\bar{Y}_{12} = Y^{78} - Y^{87}, \\
&\bar{Y}_{13} = Y^{16} - Y^{25}, &&\bar{Y}_{14} = Y^{38} - Y^{47}, &&\bar{Y}_{15} = Y^{52} - Y^{61}, &&\bar{Y}_{16} = Y^{74} - Y^{83}.
\end{aligned} \tag{3.9}$$

The coset representative can be parametrized by

$$L = \prod_{i=1}^{16} e^{a_i \bar{Y}_i}. \tag{3.10}$$

Unlike the previous case, the scalar potential is so complicated that it is not possible to make the full analysis. However, with some ansatz, we find one non-trivial critical point at

$$\begin{aligned}
&a_1 = a_2 = \frac{1}{2}\ln 2, \qquad\qquad a_3 = -a_4 = \frac{1}{2}\ln\frac{g_2 - 6g_1 + \sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}}{2g_2}, \\
&V_0 = 64(8g_1^2 - g_2^2).
\end{aligned} \tag{3.11}$$

a_3 and a_4 are real for $g_1 > 0$ and $g_2 \geq -6g_1$. In this range, we find $V_0 < 0$ if $g_2 < -2\sqrt{2}g_1$. Therefore, it is possible to have an AdS_3 critical point. The residual symmetry is $\mathrm{SO}(4) \times \mathrm{SO}(2) \times \mathrm{SO}(2)$. We will denote this critical point by P_3 for later reference.

The stability of this critical point can be verified from the scalar mass spectrum given in table 6 in which α_i are eigenvalues of the submatrix

$$\frac{1}{8g_1^2 - g_2^2}\begin{pmatrix} -80g_1^2 & x_1 & x_2 \\ x_1 & -\frac{g_2^2}{3} & -\frac{2g_2^2}{3} \\ x_2 & -\frac{2g_2^2}{3} & -\frac{g_2^2}{3} \end{pmatrix} \tag{3.12}$$

with the following elements

$$\begin{aligned}
&x_1 = 2\sqrt{2}g_1\left(6g_1 + g_2 - \sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}\right) \\
\text{and} \qquad &x_2 = 2\sqrt{2}g_1\left(6g_1 + g_2 + \sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}\right).
\end{aligned} \tag{3.13}$$

Their numerical values can be obtained upon specifying the values of g_1 and g_2.

For all but $(\mathbf{1},\mathbf{1},\mathbf{1})$ and $(\mathbf{1},\mathbf{1},\mathbf{2})$ scalars, the masses are above the BF bound for $-6g_1 < g_2 < -2\sqrt{2}g_1$. The mass squares of $(\mathbf{1},\mathbf{1},\mathbf{1})$ scalars are above the BF bound for $-6g_1 < g_2 < -4.47g_1$. For $(\mathbf{1},\mathbf{1},\mathbf{2})$ scalars, the mass squares are above the BF bound

$SO(4)\times SO(2)\times SO(2)$	m^2L^2
$(\mathbf{4},\mathbf{2},\mathbf{1})$	$-\frac{60g_1^2-14g_1g_2+g_2^2+(6g_1-3g_2)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{16g_1^2-2g_2^2}$
$(\mathbf{4},\mathbf{1},\mathbf{2})$	$-\frac{60g_1^2-24g_1g_2+g_2^2+(3g_2-6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{16g_1^2-2g_2^2}$
$(\mathbf{4},\mathbf{2},\mathbf{1})$	$-\frac{124g_1^2-3g_2^2+(g_2+6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{16g_1^2-2g_2^2}$
$(\mathbf{4},\mathbf{1},\mathbf{2})$	$-\frac{124g_1^2-3g_2^2-(g_2+6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{16g_1^2-2g_2^2}$
$(\mathbf{1},\mathbf{2},\mathbf{1})$	$\frac{6g_2^2+24g_1g_2-72g_1^2+2(g_2-6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{8g_1^2-g_2^2}$
$(\mathbf{1},\mathbf{1},\mathbf{2})$	$\frac{6g_2^2+24g_1g_2-72g_1^2-2(g_2-6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{8g_1^2-g_2^2}$
$(\mathbf{9},\mathbf{1},\mathbf{1})$	$\frac{48g_1^2}{g_2^2-8g_1^2}$
$(\mathbf{6},\mathbf{1},\mathbf{1})$	0
$2\times(\mathbf{1},\mathbf{2},\mathbf{2})$	0
$2\times(\mathbf{1},\mathbf{1},\mathbf{1})$	0
$(\mathbf{1},\mathbf{1},\mathbf{1})$	$\alpha_1,\ \alpha_2,\ \alpha_3$

Table 6. The scalar mass spectrum of the $SO(4)\times SO(2)\times SO(2)$ critical point P_3.

for $-6g_1<g_2<\mathcal{X}$ with $\mathcal{X}$ being the first root of $p(\mathcal{X})=1088g_1^4-384g_1^3\mathcal{X}+352g_1^2\mathcal{X}^2-144g_1\mathcal{X}^3-37\mathcal{X}^4=0$. This can be translated to the value of α by setting $\mathcal{X}=\alpha g_1$. The equation $p(\mathcal{X})=0$ gives the value of $\alpha=-5.93479$. The stability is obtained in the range $-6g_1<g_2<-5.93479g_1$ which is very narrow. Notice that for $g_2=-6g_1$, we find $a_3=a_4=0$, and the symmetry is enhanced to $SO(4)\times SO(4)$. It can be checked that this critical point indeed becomes critical point P_1 with $g_2=-6g_1$.

3.3 Critical points on the $SU(2)^+_L\times SU(2)^-_L$ invariant manifold

One interesting deformation of $N=(4,4)$ SCFT is the chiral supersymmetry breaking $(4,4)\to(4,0)$. The realization of this breaking in the D1-D5 system has been studied in [28]. Another gravity dual of $N=(4,0)$ SCFT from string theory has been studied in [29], and the marginal perturbation driving $N=(4,4)$ SCFT to the $N=(4,0)$ SCFT has been identified in [30]. This supersymmetry breaking is not possible in the compact $SO(4)\times SO(4)$ gauging of [13] since there are no scalars which are singlets under a non-trivial subgroup of $SO(4)\times SO(4)$ in order to become the R-symmetry of $N=(4,0)$.

This is however possible in the present gauging. According to (3.2), we see that there are eight singlets under $SU(2)^+_L\times SU(2)^-_L$ given by

$$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{3};\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{3}). \tag{3.14}$$

They correspond to the following non-compact generators

$$\begin{aligned}
\hat{Y}_1&=Y^{11}+Y^{22}+Y^{33}+Y^{44}, & \hat{Y}_2&=Y^{12}-Y^{21}+Y^{34}-Y^{43},\\
\hat{Y}_3&=Y^{13}-Y^{31}-Y^{24}+Y^{42}, & \hat{Y}_4&=Y^{14}-Y^{41}+Y^{23}-Y^{32},\\
\hat{Y}_5&=Y^{55}+Y^{66}+Y^{77}+Y^{88}, & \hat{Y}_6&=Y^{56}-Y^{65}+Y^{78}-Y^{87},\\
\hat{Y}_7&=Y^{57}-Y^{75}-Y^{68}+Y^{86}, & \hat{Y}_8&=Y^{58}-Y^{85}+Y^{67}-Y^{76}\,.
\end{aligned}\tag{3.15}$$

We can parametrize the coset representative accordingly

$$L = e^{b_1\hat{Y}_1}e^{a_2\hat{Y}_2}e^{a_3\hat{Y}_3}e^{a_4\hat{Y}_4}e^{b_5\hat{Y}_5}e^{a_6\hat{Y}_6}e^{a_7\hat{Y}_7}e^{a_8\hat{Y}_8} \tag{3.16}$$

in which b_1 and b_5 denote the $SO(4)\times SO(4)$ singlets. We find one non-supersymmetric AdS_3 critical point characterized by

$$\begin{aligned} a_2 &= \cosh^{-1}\sqrt{\frac{g_1+\sqrt{g_1(g_1-4g_2)}}{4g_1}}, \\ V_0 &= -32\left[g_1^2+4g_2^2-6g_1g_2+(4g_2-g_1)\sqrt{g_1(g_1-4g_2)}\right]. \end{aligned} \tag{3.17}$$

The cosmological constant is the same as P_1, but the residual gauge symmetry is just $SO(4)^-\times SU(2)^+_L\times U(1)^+_R$ in which $U(1)^+_R\subset SU(2)^+_R$.

3.4 Critical points on the $SU(2)_{L\text{diag}}$ invariant manifold

We further reduce the residual symmetry to $SU(2)_{L\text{diag}}\subset SU(2)^+_L\times SU(2)^-_L$. Under $SO(4)_{\text{diag}}$, we already know that the 64 scalars transform as four copies of $\mathbf{1}+\mathbf{6}+\mathbf{9}$. We can then further truncate to $SU(2)_{L\text{diag}}$ and find sixteen singlets given by four copies of $(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{3})$ under $SU(2)_{L\text{diag}}\times SU(2)_{R\text{diag}}$. They can be parametrized by the coset representative

$$L=\prod_{i=1}^{16}e^{a_i\mathcal{Y}_i} \tag{3.18}$$

in which the non-compact generators are defined by

$$\begin{aligned}
\mathcal{Y}_1 &= \frac{1}{2}\left(Y^{15}+Y^{26}+Y^{37}+Y^{48}\right), & \mathcal{Y}_2 &= \frac{1}{2}\left(Y^{16}-Y^{25}+Y^{38}-Y^{47}\right),\\
\mathcal{Y}_3 &= \frac{1}{2}\left(Y^{17}-Y^{35}-Y^{28}+Y^{46}\right), & \mathcal{Y}_4 &= \frac{1}{2}\left(Y^{18}-Y^{45}+Y^{27}-Y^{36}\right),\\
\mathcal{Y}_5 &= \frac{1}{2}\left(Y^{51}+Y^{62}+Y^{73}+Y^{84}\right), & \mathcal{Y}_6 &= \frac{1}{2}\left(Y^{52}-Y^{61}+Y^{74}-Y^{83}\right),\\
\mathcal{Y}_7 &= \frac{1}{2}\left(Y^{53}-Y^{71}-Y^{64}+Y^{82}\right), & \mathcal{Y}_8 &= \frac{1}{2}\left(Y^{54}-Y^{81}+Y^{63}-Y^{72}\right),\\
\mathcal{Y}_9 &= \frac{1}{2}\left(Y^{11}+Y^{22}+Y^{33}+Y^{44}\right), & \mathcal{Y}_{10} &= \frac{1}{2}\left(Y^{12}-Y^{21}+Y^{34}-Y^{48}\right),\\
\mathcal{Y}_{11} &= \frac{1}{2}\left(Y^{13}-Y^{31}-Y^{24}+Y^{42}\right), & \mathcal{Y}_{12} &= \frac{1}{2}\left(Y^{14}-Y^{41}+Y^{23}-Y^{32}\right),\\
\mathcal{Y}_{13} &= \frac{1}{2}\left(Y^{55}+Y^{66}+Y^{77}+Y^{88}\right), & \mathcal{Y}_{14} &= \frac{1}{2}\left(Y^{56}-Y^{65}+Y^{78}-Y^{87}\right),\\
\mathcal{Y}_{15} &= \frac{1}{2}\left(Y^{57}-Y^{75}-Y^{68}+Y^{86}\right), & \mathcal{Y}_{16} &= \frac{1}{2}\left(Y^{58}-Y^{85}+Y^{67}-Y^{76}\right).
\end{aligned} \tag{3.19}$$

From a very complicated potential, we find one non-supersymmetric AdS_3 critical point given by

$$\begin{aligned} a_6 &= \ln\frac{\sqrt{g_2}-\sqrt{3g_1}}{\sqrt{g_2}+\sqrt{3g_1}}, \qquad g_2=(2+\sqrt{13})g_1, \\ V_0 &= -8(469+131\sqrt{13})g_1^2 \end{aligned} \tag{3.20}$$

which is invariant under $SU(2)\times U(1)$ symmetry.

Apart from P_1, P_2 and P_3, we have not given the complete mass spectra for other AdS_3 critical points since the computation is much more involved. A partial check shows that at least the scalar masses for the singlets in each sector satisfy the BF bound. It could happen that some other scalars might have masses violating the bound. However, similar to the three stable critical points studied above, it is likely that the other critical points are stable for some values of α or $g_{1,2}$.

4 Deformations of the $N = (4, 4)$ SCFT

In this section, we will study supersymmetric flows of the maximally supersymmetric $SO(4) \times SO(4)$ critical point in the UV to non-conformal field theories in the IR and half-supersymmetric domain walls. At the end of this section, we will discuss some RG flow solutions interpolating between the UV $N = (4, 4)$ SCFT and some of the non-supersymmetric critical points identified in the previous section.

4.1 Supersymmetric deformations

We begin with supersymmetric solutions which can be obtained by finding solutions of the associated BPS equations. We have not found any supersymmetric critical point apart from the trivial one at $L = \mathbf{I}$, so we only expect to find flow solutions to non-conformal field theories. In these flows, the solutions interpolate between the UV point at which all scalars vanish and the IR with infinite values of scalar vev's [31]. Since supersymmetric solutions are of interest here, we need the supersymmetry transformations of fermions which in the present case are given by the non-propagating gravitini ψ^I_μ and the spin-$\frac{1}{2}$ fields χ^{iI}. Their supersymmetry transformations are given by, see [12] for more details and conventions,

$$\delta\psi^I_\mu = \mathcal{D}_\mu\epsilon^I + gA_1^{IJ}\gamma_\mu\epsilon^J, \tag{4.1}$$

$$\delta\chi^{iI} = \frac{1}{2}(\delta^{IJ}\mathbf{1} - f^{IJ})^i{}_j \not{\mathcal{D}}\phi^j\epsilon^J - gNA_2^{JIi}\epsilon^J\,. \tag{4.2}$$

These equations will be used to find supersymmetric solutions in the next subsections.

4.1.1 A supersymmetric flow to $SO(4) \times SO(4)$ non-conformal field theory

We first look for a simple solution preserving $SO(4) \times SO(4)$ symmetry. Accordingly, only a_1 and a_2 in equation (3.4) are turned on in order to preserve the full $SO(4) \times SO(4)$. Using the standard domain wall ansatz for the metric

$$ds^2 = e^{2A}dx^2_{1,1} + dr^2 \tag{4.3}$$

with A depending only on the radial coordinate r, we find the BPS equations

$$a_1' + 8g_1e^{2a_1}\left(e^{2a_1} - 1\right) = 0, \tag{4.4}$$

$$a_2' + 8g_2e^{2a_2}\left(e^{2a_2} - 1\right) = 0, \tag{4.5}$$

$$A' + 8\left[g_1e^{2a_1}\left(e^{2a_1} - 2\right) + g_2e^{2a_2}\left(e^{2a_2} - 2\right)\right] = 0 \tag{4.6}$$

where we have imposed the projector $\gamma_r\epsilon^I = -\epsilon^I$, $I = 2, 4, 5, 8$ and $\gamma_r\epsilon^I = \epsilon^I$, $I = 1, 3, 6, 7$. The $'$ denotes the r-derivative. The resulting solution is then half-supersymmetric with $N =$

(4, 4) Poincare supersymmetry in the dual two dimensional field theory. Equations (4.4) and (4.5) can be solved for a_1 and a_2 as an implicit function of r. The result is

$$r = c_1 - \frac{1}{16g_1}\left[e^{-2a_1} + \ln\left(1 - e^{-2a_1}\right)\right], \tag{4.7}$$

$$r = c_2 - \frac{1}{16g_2}\left[e^{-2a_2} + \ln\left(1 - e^{-2a_2}\right)\right] \tag{4.8}$$

with integration constants c_1 and c_2. Equation (4.6) can immediately be integrated to give A as a function of a_1 and a_2. The result is

$$A = 2(a_1 + a_2) - \frac{1}{2}\ln(1 - e^{2a_1}) - \frac{1}{2}\ln(1 - e^{2a_2}). \tag{4.9}$$

In the UV, the dual field theory is conformal with $a_1 = a_2 = 0$. Near this point, the scalars behave as $a_1 \approx e^{-16g_1 r} = e^{-\frac{2g_1}{g_1+g_2}\frac{r}{L_{UV}}}$ and $a_2 \approx e^{-16g_2 r} = e^{-\frac{2g_2}{g_1+g_2}\frac{r}{L_{UV}}}$. We see that $a_{1,2} \to 0$ as $r \to \infty$. In this limit, we find $A' \approx 8(g_1 + g_2) = \frac{1}{L_{UV}}$ or $A \approx \frac{r}{L_{UV}}$ which gives the maximally supersymmetric AdS_3.

As $a_1, a_2 \to \infty$, we find $r \to$ constant as it should. Near $a_1, a_2 \to \infty$, equations (4.7) and (4.8) give $a_1 \approx -\frac{1}{4}\ln(32g_1 r)$ and $a_2 \approx -\frac{1}{4}\ln(32g_2 r)$. From equation (4.9), we find $A \approx a_1 + a_2 = -\frac{1}{4}\ln\left[(32r)^2 g_1 g_2\right]$. Accordingly, the metric becomes a domain wall in the IR

$$ds^2 = \frac{1}{32r\sqrt{g_1 g_2}}dx_{1,1}^2 + dr^2. \tag{4.10}$$

The full bosonic symmetry is ISO(1, 1) × SO(4) × SO(4) corresponding to non-comformal field theory with $N = (4, 4)$ supersymmetry.

However, flows of this type generally involve singularities. Various types of possible singularities have been classified in [32]. According to the result of [32], physical singularities are the ones at which the scalar potential is bounded from above. However, with the solution given above, the potential becomes infinite in this case. Therefore, the corresponding flow solution is not physically acceptable by the criterion of [32]. Since the framework we have used could be uplifted to ten dimensions via $S^3 \times S^3 \times S^1$ reduction, it is interesting to investigate whether this singularity is resolved in the full string theory.

4.1.2 A half-supersymmetric domain wall

We then look for a more general supersymmetric solution. The scalar sector of interest here is the $SU(2)^+_L \times SU(2)^-_L$ invariant one given in (3.16). We first relabel the scalars $(a_2, a_3, a_4, a_6, a_7, a_8)$ to $(b_2, b_3, b_4, b_6, b_7, b_8)$ in order to work with a uniform notation.

We begin with the BPS equations given by $\delta\chi^{iI} = 0$

$$b_1' = -16g_1 e^{b_1}\left(e^{b_1} - \text{sech}b_2\text{sech}b_3\text{sech}b_4\right), \tag{4.11}$$

$$b_2' = -16g_1 e^{b_1}\left(e^{b_1}\cosh b_2 - \text{sech}b_3\text{sech}b_4\right)\sinh b_2, \tag{4.12}$$

$$b_3' = -16g_1 \cosh b_2 \sinh b_3 e^{b_1}\left(e^{b_1}\cosh b_2\cosh b_3 - \text{sech}b_4\right), \tag{4.13}$$

$$b_4' = -16g_1 \cosh b_2 \cosh b_3 \sinh b_4 e^{b_1}\left(e^{b_1}\cosh b_2\cosh b_3\cosh b_4 - 1\right), \tag{4.14}$$

$$b_5' = -16g_2 e^{b_5}\left(e^{b_5} - \text{sech}b_6\text{sech}b_7\text{sech}b_8\right), \tag{4.15}$$
$$b_6' = -16g_2 \sinh b_6 e^{b_5}\left(e^{b_5}\cosh b_6 - \text{sech}b_7\text{sech}b_8\right), \tag{4.16}$$
$$b_7' = -16g_2 \cosh b_6 \sinh b_7 e^{b_5}\left(e^{b_5}\cosh b_6\cosh b_7 - \text{sech}b_8\right), \tag{4.17}$$
$$b_8' = -16g_2 \cosh b_6 \cosh b_7 \sinh b_8 e^{b_5}\left(e^{b_5}\cosh b_6\cosh b_7\cosh b_8 - 1\right). \tag{4.18}$$

where we have used the projection conditions $\gamma_r\epsilon^I = -\epsilon^I$, $I = 2,4,5,8$ and $\gamma_r\epsilon^I = \epsilon^I$, $I = 1,3,6,7$ as in the previous case. The gravitino variation $\delta\psi^I_\mu$, $\mu = 0,1$, gives

$$\begin{aligned} A' = & -8g_1 e^{b_1}\cosh b_2\cosh b_3\cosh b_4\left(e^{b_1}\cosh b_2\cosh b_3\cosh b_4 - 2\right) \\ & -8g_2 e^{b_5}\cosh b_6\cosh b_7\cosh b_8\left(e^{b_5}\cosh b_6\cosh b_7\cosh b_8 - 2\right). \end{aligned} \tag{4.19}$$

From these equations, we see that apart from the maximally supersymmetric point at $b_i = 0$, $i = 1,\ldots,8$, there is a flat direction of the potential given by

$$e^{-b_1} = \cosh b_2\cosh b_3\cosh b_4, \qquad e^{-b_5} = \cosh b_6\cosh b_7\cosh b_8 \tag{4.20}$$

which leads to $V_0 = -64(g_1+g_2)^2$. Equation (4.19) gives $A' = 8(g_1+g_2)$ or $A = 8(g_1+g_2)r$ which is the AdS_3 solution with radius $L = \frac{1}{8(g_1+g_2)}$. It can also be verified that the full $(4,4)$ supersymmetry is preserved. This should correspond to a marginal deformation of the $N = (4,4)$ SCFT. There are no other supersymmetric critical points in this sector. Therefore, the flow breaking supersymmetry from $(4,4)$ to $(4,0)$ is not possible.

However, there is a half-supersymmetric domain wall solution similar to the dilatonic p-brane solutions of $N = 1$, $D = 7$ and $N = 2$, $D = 6$ gauged supergravities studied in [33]. It is remarkable that the full set of the above equations admits an analytic solution. The strategy to find the solution is as follow. We first determine $b_{2,3,4}$ as functions of b_1 and similarly determine $b_{6,7,8}$ as functions of b_5. b_1 and b_5 are determined as functions of r and can be solved explicitly. From (4.11) and (4.12), we find

$$\frac{db_2}{db_1} = \cosh b_2 \sinh b_2 \tag{4.21}$$

which can be solved for b_2 as a function of b_1 giving rise to

$$b_2 = \coth^{-1} e^{-b_2 - 2c_1}. \tag{4.22}$$

Using (4.11) and (4.13) together with b_2 solution from (4.22), we find

$$\frac{db_3}{db_1} = \frac{\sinh(2b_3)}{2\left(1 - e^{2b_1+4c_1}\right)} \tag{4.23}$$

whose solution is given by

$$b_3 = \tanh^{-1}\frac{e^{b_1+2c_2}}{\sqrt{1 - e^{2b_1+4c_1}}}. \tag{4.24}$$

Combining (4.11) and (4.14) and substituting for b_2 and b_3 solutions give

$$\frac{db_4}{db_1} = -\frac{\cosh b_4\sinh b_4}{\left(e^{4c_1} + e^{4c_2}\right)e^{b_1} - 1}. \tag{4.25}$$

We then find the solution for b_4

$$b_4 = \tanh^{-1} \frac{e^{b_1+2c_3}}{\sqrt{1-e^{2b_1}\left(e^{4c_1}+e^{4c_2}\right)}}. \tag{4.26}$$

With solutions for b_2, b_3 and b_4, equation (4.11) becomes

$$b_1' = 16g_1 e^{b_1}\left(\sqrt{1-e^{2b_1}\left(e^{4c_1}+e^{4c_2}+e^{4c_3}\right)} - e^{b_1}\right). \tag{4.27}$$

This can be solved for b_1 as an implicit function of r. The solution is

$$\begin{aligned} r = & -\frac{1}{32g_1}\left[2e^{-b_1}\sqrt{1-\beta_1 e^{2b_1}} + \ln\left[e^{-2b_1}\left((\beta_1-1)e^{2b_1} - 1 + 2e^{b_1}\sqrt{1-\beta_1 e^{2b_1}}\right)\right]\right] \\ & +\text{constant} \end{aligned} \tag{4.28}$$

where $\beta_1 = e^{4c_1}+e^{4c_2}+e^{4c_3}$.

We can solve (4.15) to (4.18) by the same procedure. The resulting solutions are given by

$$\begin{aligned} b_6 &= \tanh^{-1} e^{b_5+2c_4}, \qquad b_7 = \tanh^{-1}\frac{e^{b_5+2c_5}}{\sqrt{1-e^{2b_5+4c_4}}}, \\ b_8 &= \tanh^{-1}\frac{e^{b_5+3c_6}}{\sqrt{1-e^{b_5}\left(e^{4c_4}+e^{4c_5}\right)}}, \\ r &= -\frac{1}{32g_2}\left[2e^{-b_5}\sqrt{1-\beta_2 e^{2b_5}} + \ln\left[e^{-2b_5}\left((\beta_2-1)e^{2b_5} - 1 + 2e^{b_5}\sqrt{1-\beta_2 e^{2b_5}}\right)\right]\right] \\ & \quad +\text{constant} \end{aligned} \tag{4.29}$$

where $\beta_2 = e^{4c_4}+e^{4c_5}+e^{4c_6}$.

After substituting all of the b_i solutions for $i = 2,3,4,6,7,8$ in (4.19), we obtain

$$A' = \frac{16g_1 e^{b_1}}{\sqrt{1-\beta_1 e^{2b_1}}} - \frac{8g_1 e^{2b_1}}{1-\beta_1 e^{2b_1}} + \frac{16g_2 e^{b_5}}{\sqrt{1-\beta_2 e^{2b_5}}} - \frac{8g_2 e^{2b_5}}{1-\beta_2 e^{2b_5}} \tag{4.30}$$

whose solution in terms of b_1 and b_5 is readily found by a direct integration using (4.11) and (4.15) including the solutions for the other b_i's. The resulting solution is given by

$$\begin{aligned} A = & \; b_1 + b_5 + \frac{1}{2}\tanh^{-1}\frac{e^{b_1}}{\sqrt{1-\beta_1 e^{2b_1}}} + \frac{1}{2}\tanh^{-1}\frac{e^{b_5}}{\sqrt{1-\beta_2 e^{2b_5}}} - \ln\left[1-\beta_1 e^{2b_1}\right] \\ & - \ln\left[1-(1+\beta_1)e^{2b_1}\right] - \ln\left[1-\beta_2 e^{2b_5}\right] - \ln\left[1-(1+\beta_2)e^{2b_5}\right]. \end{aligned} \tag{4.31}$$

As $b_1, b_5 \to 0$, other scalars do not vanish for finite c_i. We then find that the solution will not have an interpretation in terms of the usual holographic RG flows. The solution is rather of the 1-brane soliton type, see [33] for a general discussion of $(D-2)$-brane solitons in D dimensions. It can also be verified that the $\delta\psi^I_r = 0$ condition precisely gives the Killing spinors for the unbroken supersymmetry $\epsilon^I = e^{\frac{A}{2}}\epsilon^I_0$ with the constant spinor ϵ^I_0 satisfying $\gamma_r\epsilon^I_0 = -\epsilon^I_0$, $I = 2,4,5,8$ and $\gamma_r\epsilon^I_0 = \epsilon^I_0$, $I = 1,3,6,7$.

4.2 Non-supersymmetric deformations

We now briefly discuss non-supersymmetric RG flow solutions interpolating between the $N = (4, 4)$ SCFT in the UV and some critical points found in the previous section. The solutions are essentially non-supersymmetric since they connect a supersymmetric to a non-supersymmetric critical point. Finding the corresponding solutions involve solving the full second order field equations for both the scalars and the metric in contrast to solving the first order BPS equations in the supersymmetric case. Although there are some examples of analytic supersymmetric flow solutions in three dimensions, in general, analytic solutions with many active scalars, even for the supersymmetric case, can be very difficult to find. Therefore, we will not expect to find any analytic solutions in the non-supersymmetric case but rather look for numerical flow solutions.

In all cases, the interpolating solutions generally exist and can be obtained by a similar procedure used in [34]. In solving the second-order field equations for scalars and the metric function, two types of asymptotic behavior of scalars arise near the UV fixed point. One of them corresponds to a deformation by turning on a dual operator while the other corresponds to a vacuum expectation value (vev). The second-order equations lead to an ambiguity between these two possibilities. One way to solve this ambiguity is to recast the second-order field equations into a first-order form by introducing the generating function W [35, 36]. Like supersymmetric solutions obtained from first-order BPS equations, only one possibility is singled out from these new first-order equations.

In the present case, numerical analyses show that non-supersymmetric flows to P_1 and P_2 are driven by turning on relevant operators. These describe true deformations of the UV SCFT rather than vev deformations. The flow to P_3 involves four active scalars and is more difficult to find. However, the flow is expected to be driven by a scalar transforming as $(\mathbf{1}, \mathbf{1})$ under $\mathrm{SO}(4) \times \mathrm{SO}(4)$ at the UV point. From the value of g_1 and g_2 in the stability range, it can be checked that only the deformation dual to this scalar is relevant. The deformations corresponding to the remaining active scalars are given by vacuum expectation values of irrelevant operators since these scalars have positive mass squares.

5 Conclusions and discussions

In this paper, we have studied $N = 8$ gauged supergravity in three dimensions with a non-semisimple gauge group $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$. The ratio of the coupling constants of the two SO(4)'s is given by a parameter α. For positive α, the theory describes an effective theory of ten dimensional supergravity reduced on $S^3 \times S^3 \times S^1$. For negative α, on the other hand, the theory may describe a similar reduction on $S^3 \times H^3 \times S^1$ in which H^3 is a three-dimensional hyperbolic space. With $\alpha = -1$, the cosmological constant is zero. This solution should describe a ten dimensional background $M_3 \times S^3 \times H^3 \times S^1$ where M_3 is the three-dimensional Minkowski space.

We have studied the scalar potential and found a number of non-supersymmetric AdS_3 critical points. The trivial critical point with maximal supersymmetry is identified with the dual large $N = (4, 4)$ SCFT in two dimensions. We have explicitly checked the stability of some non-supersymmetric critical points by computing the full scalar mass spectra at these critical points. They are perturbatively stable for some values of α parameter in the sense that all scalar masses are above the BF bound. It is also interesting to see whether other critical points are stable or not. We have investigated RG flows, interpolating between the large $N = (4, 4)$ SCFT in the UV and non-supersymmetric IR fixed points with $SO(4) \times SO(4)$, $SO(4) \times SO(2) \times SO(2)$ and $SO(4)$ symmetries, and also commented on the operators driving these flows.

Another result of this paper is half-supersymmetric domain wall solutions to $N = 8$ gauged supergravity. For the domain wall preserving $SO(4) \times SO(4)$ symmetry, the solution describes an RG flow from $N = (4, 4)$ SCFT in the UV to a non-conformal $N = (4, 4)$ field theory in the IR. The solution has however a bad singularity according to the criterion of [32]. For the solution preserving $SU(2) \times SU(2)$ symmetry, the holographic interpretation is not clear. In the point of view of a $(D - 2)$-brane soliton, the solution should describe a 1-brane soliton in three dimensions according to the general discussion in [33]. When uplifted to ten dimensions, the solution might describe some configuration of D1-branes. Hopefully, the solutions obtained in this paper might be useful in string/M theory context, black hole physics and the AdS/CFT correspondence. The uplifted solution of the non-conformal flow preserving $SO(4) \times SO(4)$ symmetry is also necessary for the resolution of its singularity if the full ten-dimensional solution turns out to be non-singular.

Finally, the chiral supersymmetry breaking $(4, 4) \to (4, 0)$ found in [28] cannot be implemented in the framework of $N = 8$ gauged supergravity studied here. It would probably require a larger theory of $N = 16$ gauged supergravity with $(SO(4) \times SO(4)) \ltimes (\mathbf{T}^{12}, \hat{\mathbf{T}}^{34})$ gauge group studied in [14]. It would be very interesting to find the flow solution of [28] explicitly in the three dimensional framework. We hope to come back to these issues in future research.

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A Useful formulae and details

For completeness, we include a short review of gauged supergravity in three dimensions in the formulation of [12]. The theory is a gauged version of a supersymmetric non-linear sigma model coupled to non-propagating supergravity fields. N-extended supersymmetry requires the presence of $N - 1$ almost complex structures f^P, $P = 2, \ldots, N$ on the scalar manifold. The tensors $f^{IJ} = f^{[IJ]}$, generating the $SO(N)$ R-symmetry in a spinor representation under which scalar fields transform, play an important role. In the case of symmetric scalar manifolds of the form $G/SO(N) \times H'$, they can be written in terms of

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SO(N) gamma matrices. In our case, we use the 16×16 Dirac gamma matrices of SO(8)

$$\gamma^I = \begin{pmatrix} 0 & \Gamma^I \\ (\Gamma^I)^T & 0 \end{pmatrix}. \tag{A.1}$$

The 8×8 gamma matrices are explicitly given by

$$\begin{aligned} \Gamma_1 &= \sigma_4 \otimes \sigma_4 \otimes \sigma_4, & \Gamma_2 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_4, \\ \Gamma_3 &= \sigma_4 \otimes \sigma_1 \otimes \sigma_3, & \Gamma_4 &= \sigma_3 \otimes \sigma_4 \otimes \sigma_1, \\ \Gamma_5 &= \sigma_1 \otimes \sigma_2 \otimes \sigma_4, & \Gamma_6 &= \sigma_4 \otimes \sigma_1 \otimes \sigma_2, \\ \Gamma_7 &= \sigma_2 \otimes \sigma_4 \otimes \sigma_1, & \Gamma_8 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \end{aligned} \tag{A.2}$$

where

$$\begin{aligned} \sigma_1 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & \sigma_2 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, & \sigma_4 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \end{aligned} \tag{A.3}$$

According to our normalization, we find

$$f^{IJ}_{Kr,Ls} = -\mathrm{Tr}(Y_{Ls}\left[T^{IJ}, Y_{Kr}\right]). \tag{A.4}$$

Generally, the $d = \dim(G/H)$ scalar fields ϕ^i, $i = 1, \ldots, d$ can be described by a coset representative L. The useful formulae for a coset space are

$$L^{-1} t^{\mathcal{M}} L = \frac{1}{2} \mathcal{V}^{\mathcal{M}}{}_{IJ} T^{IJ} + \mathcal{V}^{\mathcal{M}}{}_{\alpha} X^\alpha + \mathcal{V}^{\mathcal{M}}{}_{A} Y^A, \tag{A.5}$$

$$L^{-1} \partial_i L = \frac{1}{2} Q_i^{IJ} T^{IJ} + Q_i^\alpha X^\alpha + e_i^A Y^A \tag{A.6}$$

where e_i^A, Q_i^{IJ} and Q_i^α are the vielbein on the coset manifold and SO(N) $\times H'$ composite connections, respectively. X^α's denote the H' generators.

Any gauging can be described by a symmetric and gauge invariant embedding tensor satisfying the so-called quadratic constraint

$$\Theta_{\mathcal{PL}} f^{\mathcal{KL}}{}_{(\mathcal{M}} \Theta_{\mathcal{N})\mathcal{K}} = 0, \tag{A.7}$$

and the projection constraint

$$\mathbb{P}_{R_0} \Theta_{\mathcal{MN}} = 0\,. \tag{A.8}$$

The first condition ensures that the gauge symmetry forms a proper symmetry algebra while the second condition guarantees the consistency with supersymmetry.

The T-tensor given by the moment map of the embedding tensor by scalar matrices $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$, obtained from (A.5), is defined by

$$T_{\mathcal{AB}} = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} \Theta_{\mathcal{MN}} \mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}}\,. \tag{A.9}$$

Only the components $T^{IJ,KL}$ and $T^{IJ,A}$ are relevant for computing the scalar potential. With our SO(8, 8) generators, we obtain the following $\mathcal{V}$ maps

$$
\begin{aligned}
\mathcal{V}^{ab,IJ}_{\mathcal{A}1} &= -\frac{1}{2}\mathrm{Tr}(L^{-1}J^{ab}_1 T^{IJ}), & \mathcal{V}^{ab,IJ}_{\mathcal{B}1} &= -\frac{1}{2}\mathrm{Tr}(L^{-1}t^{ab}_1 T^{IJ}),\\
\mathcal{V}^{ab,Kr}_{\mathcal{A}1} &= \frac{1}{2}\mathrm{Tr}(L^{-1}J^{ab}_1 Y^{Kr}), & \mathcal{V}^{ab,Kr}_{\mathcal{B}1} &= \frac{1}{2}\mathrm{Tr}(L^{-1}t^{ab}_1 Y^{Kr}),\\
\mathcal{V}^{\hat{a}\hat{b},IJ}_{\mathcal{A}2} &= -\frac{1}{2}\mathrm{Tr}(L^{-1}J^{\hat{a}\hat{b}}_2 T^{IJ}), & \mathcal{V}^{\hat{a}\hat{b},IJ}_{\mathcal{B}2} &= -\frac{1}{2}\mathrm{Tr}(L^{-1}t^{\hat{a}\hat{b}}_2 T^{IJ}),\\
\mathcal{V}^{\hat{a}\hat{b},Kr}_{\mathcal{A}2} &= \frac{1}{2}\mathrm{Tr}(L^{-1}J^{\hat{a}\hat{b}}_2 Y^{Kr}), & \mathcal{V}^{\hat{a}\hat{b},Kr}_{\mathcal{B}2} &= \frac{1}{2}\mathrm{Tr}(L^{-1}t^{\hat{a}\hat{b}}_2 Y^{Kr})
\end{aligned}
\tag{A.10}
$$

where we have followed the convention of calling the semisimple part SO(4) × SO(4) and the nilpotent part $\mathbf{T}^{12} \sim \mathbf{T}^6 \times \mathbf{T}^6$ as $\mathcal{A}$ and $\mathcal{B}$ types, respectively. We then compute the T-tensor components

$$
\begin{aligned}
T^{IJ,KL} &= g_1\left(\mathcal{V}^{ab,IJ}_{\mathcal{A}1}\mathcal{V}^{cd,KL}_{\mathcal{B}1} + \mathcal{V}^{ab,IJ}_{\mathcal{B}1}\mathcal{V}^{cd,KL}_{\mathcal{A}1} - \mathcal{V}^{ab,IJ}_{\mathcal{B}1}\mathcal{V}^{cd,KL}_{\mathcal{B}1}\right)\epsilon_{abcd}\\
&\quad + g_2\left(\mathcal{V}^{\hat{a}\hat{b},IJ}_{\mathcal{A}2}\mathcal{V}^{\hat{c}\hat{d},KL}_{\mathcal{B}2} + \mathcal{V}^{\hat{a}\hat{b},IJ}_{\mathcal{B}2}\mathcal{V}^{\hat{c}\hat{d},KL}_{\mathcal{A}2} - \mathcal{V}^{\hat{a}\hat{b},IJ}_{\mathcal{B}2}\mathcal{V}^{\hat{c}\hat{d},KL}_{\mathcal{B}2}\right)\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}, \qquad \text{(A.11)}\\
T^{IJ,Kr} &= g_1\left(\mathcal{V}^{ab,IJ}_{\mathcal{A}1}\mathcal{V}^{cd,Kr}_{\mathcal{B}1} + \mathcal{V}^{ab,IJ}_{\mathcal{B}1}\mathcal{V}^{cd,Kr}_{\mathcal{A}1} - \mathcal{V}^{ab,IJ}_{\mathcal{B}1}\mathcal{V}^{cd,Kr}_{\mathcal{B}1}\right)\epsilon_{abcd}\\
&\quad + g_2\, \mathcal{V}^{\hat{a}\hat{b},IJ}_{\mathcal{A}2}\mathcal{V}^{\hat{c}\hat{d},Kr}_{\mathcal{B}2} + \mathcal{V}^{\hat{a}\hat{b},IJ}_{\mathcal{B}2}\mathcal{V}^{\hat{c}\hat{d},Kr}_{\mathcal{A}2} - \mathcal{V}^{\hat{a}\hat{b},IJ}_{\mathcal{B}2}\mathcal{V}^{\hat{c}\hat{d},Kr}_{\mathcal{B}2}\Big)\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}\,. \qquad \text{(A.12)}
\end{aligned}
$$

The scalar potential can be computed by using the formula

$$
V = -\frac{4}{N}\left(A^{IJ}_1 A^{IJ}_1 - \frac{1}{2}N g^{ij} A^{IJ}_{2i} A^{IJ}_{2j}\right) \tag{A.13}
$$

in which the metric g_{ij} is related to the vielbein by $g_{ij} = e^A_i e^A_j$. The A_1 and A_2 tensors appearing in the gauged Lagrangian as fermionic mass-like terms are given by

$$
\begin{aligned}
A^{IJ}_1 &= -\frac{4}{N-2}T^{IM,JM} + \frac{2}{(N-1)(N-2)}\delta^{IJ}T^{MN,MN}, & \text{(A.14)}\\
A^{IJ}_{2j} &= \frac{2}{N}T^{IJ}{}_j + \frac{4}{N(N-2)}f^{M(Im}{}_j T^{J)M}{}_m + \frac{2}{N(N-1)(N-2)}\delta^{IJ} f^{KL}{}_j{}^m T^{KL}{}_m\,. & \text{(A.15)}
\end{aligned}
$$

Finally, we repeat the condition for supersymmetric critical points. The residual supersymmetry is generated by the eigenvectors of the A^{IJ}_1 tensor with eigenvalues equal to $\pm\sqrt{\frac{-V_0}{4}}$.

B Explicit forms of the scalar potential

For $\mathrm{SO}(4)_{\mathrm{diag}}$ invariant scalars, the potential is given by

$$
\begin{aligned}
V &= 4e^{6a_1}g_1^2\cosh^2(a_3-a_4)\cosh^2(a_3+a_4)\,[5\cosh[2(a_1-2a_3)]+8\cosh(4a_3)\\
&\quad +5\cosh[2(a_1+2a_3)]-4\cosh(2a_1)\,(7+2\cosh(2a_3)\cosh(2a_4))+2\cosh(4a_4)\times\\
&\quad (\cosh a_1-3\sinh a_1)^2-6\,(\cosh(4a_3)-4\cosh(2a_3)\cosh(2a_4)-6)\sinh(2a_1)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+4e^{6a_2}g_2^2\cosh^2(a_3-a_4)\cosh^2(a_3+a_4)\,[5\cosh[2(a_2-2a_3)]-8\cosh(4a_3)\\
&+5\cosh[2(a_2+2a_3)]-4\cosh(2a_2)\,(7+2\cosh(2a_3)\cosh(2a_4))+2\cosh(4a_4)\times\\
&(\sinh a_2-3\cosh a_2)^2-6\,(\cosh(4a_3)-4\cosh(2a_3)\cosh(2a_4)-6)\sinh(2a_2)\Big]\\
&-2e^{a_1+a_2+6(a_3+a_4)}g_1g_2\,[86\cosh(a_1+a_2)-64\cosh(a_1-a_2)\cosh(2a_3)+\cosh(2a_3)\times\\
&\cosh(6a_4)\,(\cosh a_1-3\sinh a_1)\,(3\cosh a_2-\sinh a_2)+16\cosh a_1\cosh(4a_3)\sinh a_2\\
&+\cosh(2a_4)\,[-64\cosh(a_1-a_2)+\cosh(6a_3)\,(3\cosh a_1-\sinh a_1)\times\\
&(\cosh a_2-3\sinh a_2)+2\cosh(2a_3)\,(37\cosh(a_1+a_2)-19\sinh(a_1+a_2))]\\
&-66\sinh(a_1+a_2)+2\cosh(4a_4)\,[8\cosh a_2\sinh a_1+\cosh(4a_3)\,(\sinh(a_1+a_2)\\
&-3\cosh(a_1+a_2))]+[25\cosh(a_1+a_2)-27\cosh a_2\sinh a_1+2\cosh(4a_3)\times\\
&(3\cosh a_1-\sinh a_1)\,(\cosh a_2-3\sinh a_2)-35\cosh a_1\sinh a_2]\sinh(2a_3)\sinh(2a_4)\\
&+2\,(\sinh(a_1+a_2)-3\cosh(a_1+a_2))\sinh(4a_3)\sinh(4a_4)+\sinh(2a_3)\sinh(6a_4)\times\\
&(3\cosh a_2-\sinh a_2)\,(\cosh a_1-3\sinh a_1)]\,. \qquad\qquad (B.1)
\end{aligned}
$$

The potential for $\mathrm{SU}(2)_L^+\times\mathrm{SU}(2)_L^-$ invariant scalars is given by, in notation of section 4,

$$
\begin{aligned}
V\;=\;128\Big[&g_1^2e^{2b_1}\cosh^2 b_2\cosh^2 b_3\cosh^2 b_4\left(e^{b_1}\cosh b_2\cosh b_3\cosh b_4-1\right)^2\\
&+g_2^2e^{2b_5}\cosh^2 b_6\cosh^2 b_7\cosh^2 b_8\left(e^{b_5}\cosh b_6\cosh b_7\cosh b_8-1\right)^2\Big]\,. \qquad (B.2)
\end{aligned}
$$

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Gravity duals of 5D $N = 2$ SYM theory from $F(4)$ gauged supergravity

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We study gravity duals of the minimal $N = 2$ super Yang-Mills gauge theories in five dimensions using the matter coupled $F(4)$ gauged supergravity in six dimensions. The $F(4)$ gauged supergravity coupled to n vector multiplets contains $4n + 1$ scalar fields, parametrized by $\mathbb{R}^+ \times SO(4, n)/SO(4) \times SO(n)$ coset manifold. Maximally supersymmetric vacua of the gauged supergravity with $SU(2) \times G$ gauge group, with G being an n-dimensional subgroup of $SO(n)$, correspond to five-dimensional superconformal field theories (SCFTs) with $SU(2)_R$ R symmetry and G global symmetry. Deformations of the UV SCFTs for $G = SU(2)$ and $G = U(2) \sim SU(2) \times U(1)$ symmetries that lead to nonconformal $N = 2$ super Yang-Mills with various unbroken global symmetries are studied holographically.

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I. INTRODUCTION

Much insight to strongly coupled gauge theories can be gained from studying their gravity duals via the AdS/CFT correspondence [1] and its generalization to nonconformal field theories [2–4]. One consequence of the AdS/CFT correspondence which has been extensively studied is holographic RG flows. These flows describe deformations of an UV conformal field theory (CFT) to another conformal fixed point or to a nonconformal field theory in the IR. On the gravity side, an RG flow in the dual field theory is described by an asymptotically anti–de Sitter (AdS) solution which becomes AdS space in a certain limit corresponding to the UV CFT. The gravity solutions interpolate between this AdS space and another AdS space in the case of flows to some IR fixed points. For flows to nonconformal field theories, gravity solutions in the IR will take the form of a domain wall [5]. Furthermore, in flows between CFTs, bulk scalar fields take finite constant values at both conformal fixed points while in flows to nonconformal theories, they are usually logarithmically divergent.

The above argument leads to gravity duals of various supersymmetric gauge theories in four dimensions, and many important characteristics of the gauge theories such as gaugino condensates and confinements can be successfully described by gravity solutions of five-dimensional gauged supergravity; see, for example, [6–8]. On the other hand, holographic duals of higher dimensional gauge theories have not much been explored in the literature. In this paper, we will carry out a similar study for $N = 2$ supersymmetric Yang-Mills (SYM) gauge theories in five dimensions using six-dimensional $F(4)$ gauged supergravity. This should provide the five-dimensional analogue of the four-dimensional results in [6–8].

Five-dimensional field theories are interesting in their own right. It has been discovered in [9–11] that five-dimensional gauge theories admit nontrivial fixed points with enhanced global symmetry. The five-dimensional (5D) field theory describes the dynamics of the D4/D8-brane system whose near horizon limit gives rise to AdS_6 geometry [12]. At the fixed points, the $SO(2N_f) \times U(1)$ global symmetry of the gauge theory with $N_f < 8$ flavors is enhanced to E_{N_f+1}. $E_{6,7,8}$ are the usual exceptional groups and other groups are defined by $E_1 = SU(2)$, $E_2 = SU(2) \times U(1)$, $E_3 = SU(3) \times SU(2)$, $E_4 = SU(5)$, and $E_5 = SO(10)$ [9]. This symmetry enhancement in the case of $SU(2)$ gauge theories has also been shown to appear in the superconformal indices [13].

By using AdS_6/CFT_5 correspondence, it has been proposed in [14] that five-dimensional superconformal field theories with global symmetry G should correspond to AdS_6 vacua of the matter coupled $F(4)$ gauged supergravity in the six-dimensional bulk with the $SU(2)_R \times G$ gauge group. The $SU(2)_R$ R symmetry is gauged by three of the four vector fields in the supergravity multiplet, while the G part of the gauge group is gauged by the vectors in the vector multiplets. The dual field theory has been identified with a singleton field theory on the boundary. A number of papers on gauge/gravity correspondence involving 5D gauge theories and the generalization to quiver gauge theories from the ten-dimensional point of view have appeared in [15–17]. RG flows between 5D quiver gauge theories with $N_f = 0$ have been studied recently in [18] in the ten-dimensional context. Holographic RG flows within the framework of $F(4)$ gauged supergravity have also been studied in [19] and [20]. In this paper, we will give another example of flow solutions to 5D nonconformal gauge theories in the framework of six-dimensional gauged supergravity. As in lower dimensions, this should be more convenient to work with than the ten-dimensional computation and could

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provide a useful tool in the holographic study of $N = 2$ 5D SYM.

Furthermore, the study of gravity duals of 5D gauge theories is not only important in $\mathrm{AdS}_6/\mathrm{CFT}_5$ correspondence but is also useful in the context of $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence [21,22]. This originates from the proposal that the less understood $N = (2,0)$ gauge theory in six dimensions could be defined in term of 5D SYM. Furthermore, it has been shown that 5D superconformal field theory (SCFT) could be an IR fixed point of $N = 2^*$ gauge theory in four dimensions [23]. Therefore, having gravity duals of 5D SYM could be very useful in understanding the dynamics of M5-branes and gauge theories in other dimensions as well.

The paper is organized as follows. In Sec. II, we review relevant information about matter coupled $F(4)$ gauged supergravity in six dimensions and formulas used throughout the paper. Holographic RG flows to nonconformal field theories from the UV fixed point identified with the maximally supersymmetric AdS_6 critical points will be given in Secs. III and IV. All of the solutions can be analytically obtained and would be more useful than the numerical solutions given in some other cases. We end the paper by giving some conclusions and comments in Sec. V.

II. MATTER COUPLED $F(4)$ GAUGED SUPERGRAVITY AND THE DUAL $N = 2$ SUPER YANG-MILLS THEORY

We begin with a brief review of the matter coupled $F(4)$ gauged supergravity in six dimensions. The theory is an extension of the pure $F(4)$ gauged supergravity, constructed a long time ago in [24], by coupling n vector multiplets to the $N = (1,1)$ supergravity multiplet. The resulting theory is elegantly constructed by using the superspace approach in [25–27]. In the present work, we will need only supersymmetry transformations of fermions and the bosonic Lagrangian involving the metric and scalars. Most of the notations and conventions are the same as those given in [25] and [26] but with the metric signature $(-+++++)$.

In half-maximal $N = (1,1)$ supersymmetry, the field content of the supergravity multiplet is given by

$$(e^a_\mu, \psi^A_\mu, A^\alpha_\mu, B_{\mu\nu}, \chi^A, \sigma),$$

where e^a_μ, χ^A, and ψ^A_μ denote the graviton, the spin-1/2 field, and the gravitini, respectively. Both χ^A and ψ^A_μ are eight-component pseudo-Majorana spinors with indices A, $B = 1, 2$ referring to the fundamental representation of the $SU(2)_R \sim USp(2)_R$ R symmetry. The remaining fields are given by the dilaton σ, four vectors A^α_μ, $\alpha = 0, 1, 2, 3$, and a two-form field $B_{\mu\nu}$.

A vector multiplet has component fields

$$(A_\mu, \lambda_A, \phi^\alpha).$$

Each multiplet will be labeled by an index $I = 1, \ldots, n$. The $4n$ scalars $\phi^{\alpha I}$ are described by a symmetric quaternionic manifold $SO(4,n)/SO(4) \times SO(n)$. The dilaton σ can also be regarded as living in the coset space $\mathbb{R}^+ \sim O(1,1)$. As in [25], it is convenient to decompose the α index into $\alpha = (0, r)$ in which $r = 1, 2, 3$. The $SU(2)_R$ R symmetry is identified with the diagonal subgroup of $SU(2) \times SU(2) \sim SO(4) \subset SO(4) \times SO(n)$. A general compact gauge group is then given by $SU(2) \times G$ with $\dim G = n$.

The $4n$ scalars living in the $SO(4,n)/SO(4) \times SO(n)$ coset can be parametrized by the coset representative $L^\Lambda{}_\Sigma$, $\Lambda, \Sigma = 0, \ldots, 3+n$. Using the index splitting $\alpha = (0, r)$, we can split $L^\Lambda{}_\Sigma$ into $(L^\Lambda{}_\alpha, L^\Lambda{}_I)$ and further to $(L^\Lambda{}_0, L^\Lambda{}_r, L^\Lambda{}_I)$. The vielbein of the $SO(4,n)/SO(4) \times SO(n)$ coset P^I_α can be obtained from the left-invariant 1-form of $SO(4,n)$

$$\Omega^\Lambda{}_\Sigma = (L^{-1})^\Lambda{}_\Pi \nabla L^\Pi{}_\Sigma, \qquad \nabla L^\Lambda{}_\Sigma = dL^\Lambda{}_\Sigma - f_\Gamma{}^\Lambda{}_\Pi A^\Gamma L^\Pi{}_\Sigma, \tag{1}$$

via

$$P^I_\alpha = (P^I{}_0, P^I{}_r) = (\Omega^I{}_0, \Omega^I{}_r). \tag{2}$$

The structure constants of the full gauge group $SU(2)_R \times G$ are denoted by $f^\Lambda{}_{\Pi\Sigma}$, which can be split into ϵ_{rst} and C_{IJK} for $SU(2)_R$ and G, respectively. The direct product structure of the gauge group $SU(2)_R \times G$ leads to two coupling constants, g_1 and g_2, which, in the above equation, are encoded in $f^\Lambda{}_{\Pi\Sigma}$.

In this paper, we are interested in $n = 3, 4$ cases with gauge groups $SU(2)_R \times SU(2)$ and $SU(2)_R \times SU(2) \times U(1)$. To describe $SO(4,n)/SO(4) \times SO(n)$, we introduce basis elements of $(4+n) \times (4+n)$ matrices by

$$(e^{xy})_{zw} = \delta_{xz}\delta_{yw}, \qquad w, x, y, z = 1, \ldots, n+4. \tag{3}$$

The $SO(4)$, $SU(2)_R$, and noncompact generators of $SO(4,n)$ are accordingly given by

$$\begin{aligned} SO(4)\colon\ & J^{\alpha\beta} = e^{\beta+1,\alpha+1} - e^{\alpha+1,\beta+1}, \quad \alpha, \beta = 0, 1, 2, 3, \\ SU(2)_R\colon\ & J^{rs} = e^{s+1,r+1} - e^{r+1,s+1}, \quad r, s = 1, 2, 3, \\ & Y^{\alpha I} = e^{\alpha+1,I+4} + e^{I+4,\alpha+1}, \quad I = 1, \ldots, n. \end{aligned} \tag{4}$$

Gaugings lead to fermionic mass–like terms and the scalar potential in the Lagrangian, as well as some modifications to the supersymmetry transformations at first order in the coupling constants. We will give only information relevant to the study of supersymmetric RG flows and refer the reader to [25] and [26] for more details and complete formulas. The bosonic Lagrangian for the metric and scalar fields is given by [26]

$$\mathcal{L} = \frac{1}{4}eR - e\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}eP_{I\alpha\mu}P^{I\alpha\mu} - eV, \qquad (5)$$

where $e = \sqrt{-g}$. The scalar kinetic term is written in term of $P^{I\alpha}_\mu = P^{I\alpha}_i\partial_\mu\phi^i$, $i = 1, \ldots, 4n$. For completeness, we also give the explicit form of the scalar potential

$$V = -e^{2\sigma}\left[\frac{1}{36}A^2 + \frac{1}{4}B^iB_i - \frac{1}{4}(C^I{}_tC_{It} + 4D^I{}_tD_{It})\right] - m^2e^{-6\sigma}N_{00} + me^{-2\sigma}\left[\frac{2}{3}AL_{00} - 2B^iL_{0i}\right], \qquad (6)$$

where N_{00} is the 00 component of the scalar matrix defined by

$$N_{\Lambda\Sigma} = L^0_\Lambda(L^{-1})_{0\Sigma} + L^i_\Lambda(L^{-1})_{i\Sigma} - L^I_\Lambda(L^{-1})_{I\Sigma}. \qquad (7)$$

Various quantities appearing in the scalar potential and in the supersymmetry transformations given below are defined as follows:

$$A = \epsilon^{rst}K_{rst}, \qquad B^i = \epsilon^{ijk}K_{jk0}, \qquad (8)$$

$$C^t_I = \epsilon^{trs}K_{rIs}, \qquad D_{It} = K_{0It}, \qquad (9)$$

where

$$\begin{aligned} K_{rst} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^mL^n_t + g_2C_{IJK}L^I{}_r(L^{-1})_s{}^JL^K_t, \\ K_{rs0} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^mL^n_0 + g_2C_{IJK}L^I{}_r(L^{-1})_s{}^JL^K_0, \\ K_{rIt} &= g_1\epsilon_{lmn}L^l{}_r(L^{-1})_I{}^mL^n_t + g_2C_{IJK}L^I{}_r(L^{-1})_I{}^JL^K_t, \\ K_{0It} &= g_1\epsilon_{lmn}L^l{}_0(L^{-1})_I{}^mL^n_t + g_2C_{IJK}L^I{}_0(L^{-1})_I{}^JL^K_t. \end{aligned} \qquad (10)$$

Finally, the supersymmetry transformations of χ^A, λ^I_A, and ψ^A_μ involving only scalars and the metric are given by

$$\delta\psi_{\mu A} = D_\mu\epsilon_A - \frac{1}{24}(Ae^\sigma + 6me^{-3\sigma}(L^{-1})_{00})\epsilon_{AB}\gamma_\mu\epsilon^B - \frac{1}{8}(B_te^\sigma - 2me^{-3\sigma}(L^{-1})_{t0})\gamma^7\sigma^t_{AB}\gamma_\mu\epsilon^B, \qquad (11)$$

$$\delta\chi_A = \frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon_{AB}\epsilon^B + \frac{1}{24}[Ae^\sigma - 18me^{-3\sigma}(L^{-1})_{00}]\epsilon_{AB}\epsilon^B - \frac{1}{8}[B_te^\sigma + 6me^{-3\sigma}(L^{-1})_{t0}]\gamma^7\sigma^t_{AB}\epsilon^B, \qquad (12)$$

$$\delta\lambda^I_A = P^I_{ri}\gamma^\mu\partial_\mu\phi^i\sigma^r{}_{AB}\epsilon^B + P^I_{0i}\gamma^7\gamma^\mu\partial_\mu\phi^i\epsilon_{AB}\epsilon^B - (2i\gamma^7D^I{}_t + C^I{}_t)e^\sigma\sigma^t_{AB}\epsilon^B - 2me^{-3\sigma}(L^{-1})^I{}_0\gamma^7\epsilon_{AB}\epsilon^B, \qquad (13)$$

where $\sigma^{tC}{}_B$ are Pauli matrices and $\epsilon_{AB} = -\epsilon_{BA}$. The space-time gamma matrices γ^a, with a being tangent space indices, satisfy

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \qquad \eta^{ab} = \mathrm{diag}(-1, 1, 1, 1, 1, 1), \qquad (14)$$

and $\gamma^7 = \gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5$.

We now give a short description of the UV SCFT which is identified with the AdS_6 vacuum preserving 16 supercharges. At this vacuum, all scalars vanish, and the full gauge group $SU(2)_R \times G$ is preserved. The bulk fields in the supergravity multiplet are dual to the operators in the energy-momentum tensor supermultiplet in the five-dimensional field theory, while the bulk vector multiplets correspond to the global current supermultiplets. The full spectrum of all supergravity fields can be found in [25] and [26]. $SU(2)_R$ singlet scalars in the adjoint representation of G are dual to operators of dimension four corresponding to the highest components of the global current supermultiplets. These scalars give supersymmetry preserving deformations, as discussed in [14]. On the other hand, the dilaton and $SU(2)_R$ triplet scalars are dual to operators of dimension three and correspond to supersymmetry breaking deformations.

III. RG FLOWS FROM $SU(2)_R \times SU(2)$ SCFT

We begin with the simplest possibility with $n = 3$ and the $SU(2)_R \times SU(2)$ gauge group. The gravity theory consists of 13 scalars parametrized by $O(1,1) \times SO(4,3)/SO(4) \times SO(3)$ coset space. We are interested in $SU(2)_R$ singlet scalars which are given by σ and an additional three scalars from $SO(4,3)/SO(4) \times SO(3)$. The latter correspond to the noncompact generators Y_{11}, Y_{12}, and Y_{13}. The coset representative is accordingly written as

$$L = e^{a_1Y_{11}}e^{a_2Y_{12}}e^{a_3Y_{13}}. \qquad (15)$$

The space-time metric is the standard domain wall ansatz

$$ds^2 = e^{2A(r)}dx^2_{1,4} + dr^2, \qquad (16)$$

in which five-dimensional Poincaré symmetry is manifest. From now on, the six-dimensional space-time indices will be split as (μ, r) with $\mu = 0, \ldots, 4$.

Using (11), (12), and (13), we find the following Bogomol'nyi-Prasad-Sommerfeld (BPS) equations:

$$a_1{}' = -2e^{-3\sigma}m\frac{\sinh a_1}{\cosh a_2\cosh a_3}, \qquad (17)$$

$$a_2{}' = -2e^{-3\sigma}m\frac{\cosh a_1\sinh a_2}{\cosh b_3}, \qquad (18)$$

$$a_3{}' = -2e^{-3\sigma}m\cosh a_1\cosh a_2\sinh a_3, \qquad (19)$$

$$\sigma' = -\frac{1}{2}[e^{\sigma}g_1 - 3e^{-3\sigma}m\cosh a_1\cosh a_2\cosh a_3], \quad (20)$$

$$A' = \frac{1}{2}[e^{\sigma}g_1 + e^{-3\sigma}m\cosh a_1\cosh a_2\cosh a_3], \quad (21)$$

where $'$ denotes $\frac{d}{dr}$ and we have used the projection $\gamma^r\epsilon^A = \epsilon^A$. The presence of γ^7 in $\delta\lambda^I_A$ does not impose any condition on ϵ^A since it appears as an overall factor in all of the BPS equations obtained from $\delta\lambda^I_A = 0$. That the bulk gravity solution preserves eight supercharges is to be expected because the minimal SYM in five dimensions has eight supercharges. The equation for the warp factor $A(r)$ is obtained from $\delta\psi^A_\mu$, $\mu = 0, 1, 2, 3, 4$. The $\delta\psi^A_r = 0$ equation would give the dependence of the Killing spinors on the r coordinate as in other cases. We now look at solutions of interest.

A. Flow to $SU(2)_R \times U(1)$ SYM

We first study the solution that breaks the $SU(2)$ global symmetry to $U(1)$. This corresponds to turning on only a_3 and σ. The latter is of course a singlet of the full gauge group $SU(2)_R \times SU(2)$. With $a_1 = a_2 = 0$, Eqs. (17) and (18) are trivially satisfied, and Eqs. (19), (20), and (21) become

$$a_3{}' = -2e^{-3\sigma}m\sinh a_3, \quad (22)$$

$$\sigma' = \frac{1}{2}(-g_1e^{\sigma} + 3e^{-3\sigma}m\cosh a_3), \quad (23)$$

$$A' = \frac{1}{2}(g_1e^{\sigma} + e^{-3\sigma}m\cosh a_3). \quad (24)$$

We can solve Eq. (22) by introducing a new radial coordinate $\tilde{r}$ such that $\frac{d\tilde{r}}{dr} = e^{-3\sigma}$. We then find the solution for a_3,

$$a_3 = \pm\ln\left[\frac{1 + e^{-2m\tilde{r}+C_1}}{1 - e^{-2m\tilde{r}+C_1}}\right]. \quad (25)$$

This form is very similar to the solution studied in [6] for the four-dimensional (4D) SYM. C_1 is an integration constant. There are two possibilities for the two signs. Combining Eqs. (22) and (23) gives an equation for $\frac{d\sigma}{da_3}$,

$$\frac{d\sigma}{da_3} = \frac{1}{4m}(e^{4\sigma}g_1\mathrm{csch}a_3 - 3m\coth a_3), \quad (26)$$

whose solution is given by

$$\sigma = -\frac{1}{4}\ln\left[\frac{g_1(3\cosh a_3 - \cosh(3a_3) + 18C_2\sinh^3 a_3)}{6m}\right], \quad (27)$$

with C_2 being another integration constant.

After changing to the $\tilde{r}$ coordinate and using the a_3 solution, we find that the combination of (24) and (23) becomes, with $'$ now being $\frac{d}{d\tilde{r}}$,

$$A' + \sigma' = \frac{2m(e^{4m\tilde{r}} + e^{2C_1})}{e^{2C_1} - e^{4m\tilde{r}}}. \quad (28)$$

The solution to this equation can be readily found to be

$$A = 2m\tilde{r} + \ln(1 - e^{C_1 - 2m\tilde{r}}) + \ln(1 + e^{C_1 - 2m\tilde{r}}) - \sigma, \quad (29)$$

where we have neglected the additive integration constant to A by absorbing it into the rescaling of the x^μ coordinates. To identify the maximally supersymmetric vacuum at $\sigma = a_3 = 0$ with the $N = 2$ SCFT, we have to set $g_1 = 3m$. In the above solutions, we have not done this in order to keep the solutions in a generic form. Note also that if we try to truncate σ out by setting $\sigma = 0$, Eq. (23) will imply $a_3 = 0$. Therefore, to obtain a nontrivial solution, we must keep σ nonvanishing.

An RG flow to a nonconformal field theory with only the dilaton σ in pure $F(4)$ gauged supergravity has been studied in [19]. The resulting solution is interpreted as the analogue of the Coulomb branch flow. We now have a more general flow solution in the case of matter coupled $F(4)$ gauged supergravity. As $r \to \infty$, σ, $a_3 \to 0$, we see that $\tilde{r} \sim r \to \infty$. In this limit, we obtain the maximally supersymmetric AdS_6 background with $A \sim 2mr = \frac{r}{L}$, where the AdS_6 radius in the UV is given by $L = \frac{1}{2m}$. According to the AdS/CFT correspondence, this is identified with the UV SCFT with $SU(2)_R \times SU(2)$ SCFT in five dimensions. From the above solutions, the behavior of σ and a_3 near the UV point with $g_1 = 3m$ is readily seen to be

$$a_3 \sim e^{-2mr} = e^{-\frac{r}{L}}, \qquad \sigma \sim a_3^3 \sim e^{-6mr} = e^{-\frac{3r}{L}}. \quad (30)$$

We see that a_3 corresponds to a deformation by a relevant operator of dimension $\Delta = 4$ while σ describes a deformation by a vacuum expectation value of operator of dimension $\Delta = 3$.

There is an issue of singularities in the IR which are typical in flows to nonconformal field theories. Physical and unphysical singularities can be classified by using the criterion given in [28]. From the solution, we see that a_3 is singular when $\tilde{r} \to \frac{C_1}{2m}$. We now consider the case with $a_3 > 0$ and $a_3 < 0$ separately. For $a_3 > 0$, we find $a_3 = -\ln(2m\tilde{r} - C_1) + \ln 2$, as $2m\tilde{r} \sim C_1$ and

$$\begin{aligned}\sigma = &\frac{3}{4}\ln(2m\tilde{r} - C_1) \\ &-\frac{1}{4}\ln[9C_2 - 2 + 3(2m\tilde{r} - C_1)^2(9C_2 - 2) \\ &+3(9C_2 + 2)(2m\tilde{r} - C_1)^4 + \cdots].\end{aligned} \quad (31)$$

The warp factor A near $\tilde{r} \to \frac{C_1}{2m}$ is given by

$$A = \ln(2m\tilde{r} - C_1) - \sigma. \tag{32}$$

For $C_2 = \frac{2}{9}$, we find that

$$\sigma \sim -\frac{1}{4}\ln(2m\tilde{r} - C_1), \qquad A \sim \frac{5}{4}\ln(2m\tilde{r} - C_1). \tag{33}$$

We can find the relation between $\tilde{r}$ and r in this limit by using $\frac{d\tilde{r}}{dr} = e^{-3\sigma}$. The relation is given by

$$2mr - C = 4(2m\tilde{r} - C_1)^4, \tag{34}$$

where C is a new integration constant. The metric becomes

$$ds^2 = (2mr - C)^{10} dx^2_{1,4} + dr^2, \tag{35}$$

where we have absorbed the multiplicative constant to the scaling of x^μ coordinates. According to the domain-wall/quantum field theory correspondence, this background is dual to a nonconformal SYM theory in five dimensions.

To determine whether the singularity in the solution is acceptable or not, we check the scalar potential on the solution. With $a_1 = a_2 = 0$ and $g_1 = 3m$, the potential is given by

$$V = e^{-6\sigma} m^2 [\cosh(2a_3) - 12e^{4\sigma}\cosh a_3 - 9e^{8\sigma}]. \tag{36}$$

It can be verified that $V \to -\infty$ as $a_3, \sigma \to \infty$. The singularity is then physical according to the criterion of [28]. For $a_3 < 0$, it can be easily checked that the singularity is acceptable for the choice $C_2 = -\frac{2}{9}$ which leads to

$$a_3 \sim \ln(2m\tilde{r} - C_1), \qquad \sigma - \frac{1}{4}\ln(2m\tilde{r} - C_1),$$
$$ds^2 = (2mr - C)^{10} dx^2_{1,4} + dr^2. \tag{37}$$

On the other hand, if $C_2 \neq \pm\frac{2}{9}$ for $a_3 \sim \pm\ln(2m\tilde{r} - C_1)$, respectively, the solution is asymptotic to

$$a_3 \sim \pm\ln(2m\tilde{r} - C_1), \qquad \sigma \sim \frac{3}{4}\ln(2m\tilde{r} - C_1),$$
$$ds^2 = (2mr - C)^{\frac{2}{13}} dx^2_{1,4} + dr^2, \tag{38}$$

where we have used the relation $(2m\tilde{r} - C_1)^{\frac{13}{4}} = \frac{13}{4}(2mr - C)$, near $\tilde{r} \sim \frac{C_1}{2m}$, with a constant C. The singularity in this case is, however, not acceptable since $V \to \infty$.

It is useful to comment on the IR singularities. Following the discussion in [5], the criterion of [28] is related to the fact that the divergence in a vacuum expectation value of an operator O dual to a canonical scalar ϕ is excluded. In the IR, the scalar bulk action is given by $S \sim \int e^{5A}(\partial\phi)^2$ since the potential is irrelevant due to the divergence of the scalar. The expectation value of O is then given by

$$\langle O \rangle \sim \frac{\delta S}{\delta \phi} \sim e^{5A}\partial_r\phi \sim (r - r_0)^{5\kappa - 1}, \tag{39}$$

where we have used the asymptotic behavior $\phi \sim \phi_0 \ln(r - r_0)$ and $A \sim \kappa \ln(r - r_0)$. The singularity occurs at $r = r_0$. We see that $\langle O \rangle$ diverges when $\kappa < \frac{1}{5}$. In the present case, the physical flow has $\kappa = 5$ while the unphysical one has $\kappa = \frac{1}{13}$. This is consistent with the finiteness of the expectation value of the dual operator.

B. Flow to $SU(2)_R$ SYM

If the other scalars, a_1 and a_2, are nonvanishing, the solution will break the $SU(2)$ global symmetry completely. It is now more difficult to solve all five BPS equations, but it turns out that these equations admit analytic solutions.

To obtain the solution, we consider A, σ, a_1, and a_2 as functions of a_3. Combining Eqs. (18) and (19), we find

$$\frac{da_2}{da_3} = \frac{\tanh a_2}{\sinh a_3 \cosh a_3}. \tag{40}$$

This is easily solved by

$$a_2 = \ln\left[\frac{e^{2a_3 + C_1} - e^{C_1} + \sqrt{(1 + e^{2a_3})^2 + e^{2C_1}(e^{2a_3} - 1)}}{1 + e^{2a_3}}\right]$$
$$= \sinh^{-1}(e^{C_1}\tanh a_3). \tag{41}$$

Similarly, by solving Eqs. (17) and (19), we obtain

$$a_1 = \sinh^{-1}\frac{e^{C_2}\sinh a_3}{\sqrt{1 - e^{2C_1} + (1 + e^{2C_1})\cosh(2a_3)}}. \tag{42}$$

Using the a_1 and a_2 solutions and the new radial coordinate $\tilde{r}$, we find the solution for a_3:

$$a_3 = \pm\frac{1}{2}\cosh^{-1}\left[\frac{e^{2C_2} + 2e^{2C_1} - 2 + 4\tanh^2(2m\tilde{r} - C_3)}{2 + 2e^{2C_1} + e^{2C_2}}\right]. \tag{43}$$

We can similarly solve for σ as a function of a_3. The solution is given by

$$\begin{aligned}\sigma = \frac{1}{4}\ln\,&[3m(\tilde{A}^2 + \tilde{B}^2)^2 \mathrm{csch}^6 a_3 (36\tilde{A}^2 C_4 (\tilde{A}^2 + \tilde{B}^2)^2 \sinh^3 a_3 (\tilde{A}^2\cosh(2a_3) + \tilde{B}^2)\\ &-2(3\tilde{A}^2 + \tilde{B}^2 - 2\tilde{A}^2\cosh(2a_3))(\tilde{A}^2\cosh(2a_3) + \tilde{B}^2)^{3/2})] - \frac{1}{4}\ln[1296\tilde{A}^4 C_4^2 g_1 (\tilde{A}^2 + \tilde{B}^2)^4 (\tilde{A}^2\cosh(2a_3) + \tilde{B}^2)\\ &-4g_1 \mathrm{csch}^6 a_3 (\tilde{A}^4\cosh(4a_3) + \tilde{A}^4 + \tilde{A}^2(\tilde{B}^2 - 3\tilde{A}^2)\cosh(2a_3) - 3\tilde{A}^2\tilde{B}^2 - \tilde{B}^4)^2].\end{aligned} \tag{44}$$

We have defined two new constants, $\tilde{A}=\sqrt{2+2e^{2C_1}+e^{2C_2}}$ and $\tilde{B}=\sqrt{2-2e^{2C_1}-e^{2C_2}}$, for convenience.

Finally, adding (20) to (21) and changing the variable from r to a_3, we find a simple equation for A:

$$\frac{dA}{da_3}+\frac{d\sigma}{da_3}=-\coth a_3, \tag{45}$$

whose solution is

$$A=-\sigma-\ln(\sinh a_3). \tag{46}$$

Near the UV point, we find $r\sim\tilde{r}\to\infty$, $a_1\sim a_2\sim a_3\sim e^{-\frac{r}{L}}$, and $\sigma\sim e^{-\frac{3r}{L}}$. The solution for A then gives $A\sim 2mr=\frac{r}{L}$. The flow is again driven by turning on operators of dimension four corresponding to $a_{1,2,3}$ and a vacuum expectation value (VEV) of a dimension three operator dual to σ.

It can be checked by expanding (43) that $a_3\to\pm\infty$ as $2m\tilde{r}\to\tilde{C}$, where we have collectively denoted all constant terms from the expansion by $\tilde{C}$. The behavior of a_3 near this point is $a_3\sim\pm\ln(2m\tilde{r}-\tilde{C})$. Although a_3 blows up when $2m\tilde{r}\sim\tilde{C}$, a_1 and a_2 remain finite, with $a_2\sim\sinh^{-1}e^{C_1}$ and $a_1\sim\sinh^{-1}\frac{e^{C_2}}{\sqrt{2+2e^{2C_1}}}$. Similar to the previous case, the criterion of [28] requires $C_4=\pm\frac{2\sqrt{2}\tilde{A}}{9(\tilde{A}^2+\tilde{B}^2)^2}$ for the singularity to be physical. This is true for both $a_3<0$ and $a_3>0$. We find that

$$a_3\sim\pm\ln(2m\tilde{r}-\tilde{C}),\qquad \sigma\sim-\frac{1}{4}\ln(2m\tilde{r}-\tilde{C}),$$
$$ds^2=(2mr-C)^{10}dx_{4,1}^2+dr^2. \tag{47}$$

It can be readily verified that there always exist the values of C_1 and C_2 at which this behavior gives $V\to-\infty$.

For $C_4\neq\pm\frac{2\sqrt{2}\tilde{A}}{9(\tilde{A}^2+\tilde{B}^2)^2}$, the solution near $2m\tilde{r}\sim\tilde{C}$ becomes

$$a_3\sim\pm\ln(2m\tilde{r}-\tilde{C}),\qquad \sigma\sim-\frac{3}{4}a_3=\frac{3}{4}\ln(2m\tilde{r}-\tilde{C}),$$
$$ds^2=(2mr-C)^{\frac{2}{13}}dx_{1,4}^2+dr^2. \tag{48}$$

This solution is not physical, as it can be checked that $V\to\infty$ for all values of C_1 and C_2.

C. Flow to $SU(2)_{\mathrm{diag}}$ SYM

In this subsection, we will look at an RG flow with $SU(2)_{\mathrm{diag}}\sim(SU(2)_R\times SU(2))_{\mathrm{diag}}$ singlet scalars. Some nonsupersymmetric AdS$_6$ vacua and holographic RG flows interpolating between these critical points and the maximally supersymmetric AdS$_6$ have been studied in [20]. In this work, we will give a supersymmetric flow to a nonconformal field theory.

There is only one singlet scalar under $SU(2)_{\mathrm{diag}}$ from $\frac{SO(4,3)}{SO(4)\times SO(3)}$; see the details in [20]. The coset representative can be written as

$$L=e^{a(Y_{21}+Y_{32}+Y_{43})}. \tag{49}$$

The supersymmetry transformations of ψ_μ^A, χ^A, and λ_A^I give the following BPS equations:

$$a'=-e^{\sigma}\sinh(2a)(g_1\cosh a-g_2\sinh a), \tag{50}$$

$$\sigma'=\frac{1}{2}e^{-3\sigma}[3m+e^{4\sigma}(g_2\sinh^3 a-g_1\cosh^3 a)], \tag{51}$$

$$A'=\frac{1}{2}e^{-3\sigma}[m+e^{4\sigma}(g_1\cosh^3 a-g_2\sinh^3 a)]. \tag{52}$$

Note that for nonsinglet scalars of $SU(2)_R$, the $SU(2)$ coupling g_2 appears.

In order to solve the above equations, we will treat σ and A as functions of a:

$$\frac{d\sigma}{da}=\frac{3me^{-4\sigma}-g_1\cosh^3 a+g_2\sinh^3 a}{2\sinh(2a)(g_1\cosh a-g_2\sinh a)}, \tag{53}$$

which can be solved by

$$\sigma=\frac{1}{4}\ln\left[\frac{6m\cosh(2a)+C_1\sinh(2a)}{2g_1\cosh a-2g_2\sinh a}\right]. \tag{54}$$

We can check that as $a\to0$ and $g_1=3m$, $\sigma\to0$ as expected for the UV point. This is the case for any value of C_1. To solve for a from Eq. (50), it is convenient to define a new coordinate $\tilde{r}$ via $e^{\sigma}=\frac{d\tilde{r}}{dr}$. In this case only is $\tilde{r}$ defined by $e^{\sigma}=\frac{d\tilde{r}}{dr}$. In all other cases, we have $e^{-3\sigma}=\frac{d\tilde{r}}{dr}$.

With this new variable, we can solve for $\tilde{r}$ as a function of a. The resulting solution is given by

$$2g_1g_2\tilde{r}=g_2\ln\coth\frac{a}{2}-2g_1\tan^{-1}\left[\tanh\frac{a}{2}\right]+2\sqrt{g_1^2-g_2^2}\tan^{-1}\left[\frac{g_1\tanh\frac{a}{2}-g_2}{\sqrt{g_1^2-g_2^2}}\right], \tag{55}$$

where we have neglected the additive integration constant.

Taking the combination (51) $-3\times$ (52) with (50), we can rewrite the equation for A as

$$\frac{d\sigma}{da}-3\frac{dA}{da}=\frac{g_1\sinh a+g_2(1-\cosh a)}{g_1\cosh a-g_2\sinh a}. \tag{56}$$

The solution is readily obtained to be

$$A=\frac{1}{3}[\sigma+\ln\sinh(2a)+\ln(g_1\cosh a-g_2\sinh a)]. \tag{57}$$

From the above solutions, we can find the behavior of a, σ, and A near the UV point, $a = \sigma = 0$. In this limit, $\tilde{r} \sim r \to \infty$, we find $a \sim \sigma \sim e^{-6mr} = e^{-\frac{3r}{L}}$ and $A \sim 2mr = \frac{r}{L}$. This indicates that the flow is driven by vacuum expectation values of operators of dimension three. This is to be expected since it has been pointed out in [20] that the flow driven by turning on the operators dual to σ and a corresponds to a nonsupersymmetric flow to a nonsupersymmetric IR fixed point. In the IR, there are a number of possibilities, depending on the values of g_2 and the integration constant C_1, since these lead to different IR behaviors of a and σ.

We begin with the $g_2 = g_1$ case and consider the solution for large $|a|$. For $a < 0$, we find by expanding the solution in (55) that a diverges as $a \sim \frac{1}{3}\ln(g_1\tilde{r} - \tilde{C})$. As in the previous case, we have collectively denoted all of the constants by $\tilde{C}$. When $C_1 = 6m$, the solutions for σ and A become

$$\sigma \sim \frac{1}{4}\ln(g_1\tilde{r} - \tilde{C}), \qquad A \sim \frac{7}{36}\ln(g_1\tilde{r} - \tilde{C}),$$
$$ds^2 = (3mr - C)^{\frac{14}{27}} dx^2_{1,4} + dr^2. \tag{58}$$

This leads to $V \to -\infty$, which is acceptable.

For $C_1 \neq 6m$, we find different behavior:

$$\sigma \sim -\frac{1}{12}\ln(g_1\tilde{r} - \tilde{C}), \qquad A \sim \frac{1}{12}\ln(g_1\tilde{r} - \tilde{C}),$$
$$ds^2 = (g_1 r - C)^{\frac{2}{13}} dx^2_{1,4} + dr^2, \tag{59}$$

which gives $V \to \infty$, as expected since in this case $\kappa < \frac{2}{5}$.

For $a > 0$, we find that $a \sim -\ln(g_1\tilde{r} - \tilde{C})$. There are two possibilities for $C_1 = -6m$ and $C_1 \neq -6m$ which give, respectively,

$$\sigma \sim \frac{1}{4}\ln(g_1\tilde{r} - \tilde{C}), \qquad A \sim \frac{13}{12}\ln(g_1\tilde{r} - \tilde{C}),$$
$$ds^2 = (g_1 r - C)^{\frac{26}{9}} dx^2_{1,4} + dr^2, \tag{60}$$

and

$$\sigma \sim -\frac{3}{4}\ln(g_1\tilde{r} - \tilde{C}), \qquad A \sim \frac{3}{4}\ln(g_1\tilde{r} - \tilde{C}),$$
$$ds^2 = (g_1 r - C)^{\frac{6}{7}} dx^2_{1,4} + dr^2. \tag{61}$$

Both of them give $V \to -\infty$. We then conclude that for $g_2 = g_1$, all flows with $a > 0$ are physical, but flows with $a < 0$ are physical only for $C_1 = 6m$.

We now move to the $g_1 \neq g_2$ case and quickly look at the $a > 0$ and $a < 0$ flows separately. With $a > 0$, the solution becomes

$$a \sim -\frac{1}{3}\ln[(g_1 - g_2)\tilde{r} - \tilde{C}],$$
$$\sigma \sim -\frac{1}{12}\ln[(g_1 - g_2)\tilde{r} - \tilde{C}],$$
$$ds^2 = [(g_1 - g_2)\tilde{r} - \tilde{C}]^{\frac{2}{13}} dx^2_{1,4} + dr^2, \tag{62}$$

for $C_1 \neq -6m$, and

$$a \sim -\frac{1}{3}\ln[(g_1 - g_2)\tilde{r} - \tilde{C}], \qquad \sigma \sim \frac{1}{4}\ln[(g_1 - g_2)\tilde{r} - \tilde{C}],$$
$$ds^2 = [(g_1 - g_2)\tilde{r} - \tilde{C}]^{\frac{14}{27}} dx^2_{1,4} + dr^2, \tag{63}$$

for $C_1 = -6m$. The former is unphysical, but the latter is physical provided that $-(5 + 4\sqrt{2})m < g_2 < (4\sqrt{2} - 5)m$.

Finally, for $a < 0$, we find the IR behavior

$$a \sim \frac{1}{3}\ln[(g_1 + g_2)\tilde{r} - \tilde{C}], \qquad \sigma \sim -\frac{1}{12}\ln[(g_1 + g_2)\tilde{r} - \tilde{C}],$$
$$ds^2 = [(g_1 + g_2)\tilde{r} - \tilde{C}]^{\frac{2}{13}} dx^2_{1,4} + dr^2, \tag{64}$$

for $C_1 \neq 6m$, and

$$a \sim \frac{1}{3}\ln[(g_1 + g_2)\tilde{r} - \tilde{C}], \qquad \sigma \sim \frac{1}{4}\ln[(g_1 + g_2)\tilde{r} - \tilde{C}],$$
$$ds^2 = [(g_1 + g_2)\tilde{r} - \tilde{C}]^{\frac{14}{27}} dx^2_{1,4} + dr^2, \tag{65}$$

for $C_1 = 6m$. Similar to the previous case, only the second possibility is physical, provided that $(5 - 4\sqrt{2})m < g_2 < (5 + 4\sqrt{2})m$. In summary, for $g_2 \neq g_1$, flows with $a > 0$ and $a < 0$ are physical for $C_1 = -6m$ and $C_1 = 6m$, respectively, for some appropriate values of g_2.

IV. RG FLOWS FROM $SU(2)_R \times U(2)$ SCFT

To give more examples, we consider $F(4)$ gauged supergravity coupled to four vector multiplets with the $SU(2)_R \times SU(2) \times U(1)$ gauge group. There are 16 scalars parametrized by the $SO(4,4)/SO(4) \times SO(4)$ coset. We will focus on $SU(2)_R$ singlet scalars which are the highest components of the global symmetry multiplet and correspond to supersymmetry preserving deformations. Together with the dilaton σ, there are five $SU(2)_R$ singlet scalars. The coset representative can be written as

$$L = e^{a_1 Y_{11}} e^{a_2 Y_{12}} e^{a_3 Y_{13}} e^{a_4 Y_{14}}. \tag{66}$$

Using the projector $\gamma^r \epsilon^A = \epsilon^A$, we can derive the following BPS equations:

$$a_1{}' = -\frac{2me^{-3\sigma}\sinh a_1}{\cosh a_2 \cosh a_3 \cosh a_4}, \tag{67}$$

$$a_2{}' = -\frac{2me^{-3\sigma}\sinh a_2 \cosh a_1}{\cosh a_3 \cosh a_4}, \tag{68}$$

$$a_3{}' = -\frac{2me^{-3\sigma}\cosh a_1 \cosh a_2 \sinh a_3}{\cosh a_4}, \qquad (69)$$

$$a_4{}' = -2me^{-3\sigma}\cosh a_1 \cosh a_2 \cosh a_3 \sinh a_4, \qquad (70)$$

$$\sigma' = \frac{1}{2}[3me^{-3\sigma}\cosh a_1 \cosh a_2 \cosh a_3 \cosh a_4 - g_1 e^{\sigma}], \qquad (71)$$

$$A' = \frac{1}{2}[me^{-3\sigma}\cosh a_1 \cosh a_2 \cosh a_3 \cosh a_4 + g_1 e^{\sigma}]. \qquad (72)$$

We are interested in the RG flows with the symmetry breaking patterns $U(2) \to SU(2)$, $U(2) \to U(1) \times U(1)$, and $U(2) \to U(1)$ and the completely broken $U(2)$. The procedure is essentially the same as in the previous section, so we will neglect some details and simply give the solutions.

A. Flow to $SU(2)_R \times SU(2)$ SYM

In order to preserve $SU(2) \subset SU(2) \times U(1)$, only a_4 is allowed to be nonvanishing. The above equations reduce to three simple equations:

$$a_4{}' = -2me^{-3\sigma}\sinh a_4, \qquad (73)$$

$$\sigma\prime = \frac{1}{2}(3me^{-3\sigma}\cosh a_4 - g_1 e^{\sigma}), \qquad (74)$$

$$A' = \frac{1}{2}(me^{-3\sigma}\cosh a_4 + g_1 e^{\sigma}). \qquad (75)$$

By introducing a new radial coordinate $\tilde{r}$ via $\frac{d\tilde{r}}{dr} = e^{-3\sigma}$ as in the previous section, we find the solutions

$$\begin{aligned} a_4 &= \pm \ln\left[\frac{1+e^{-2m\tilde{r}+C_1}}{1-e^{-2m\tilde{r}+C_1}}\right], \\ \sigma &= -\frac{1}{4}\ln\left[\frac{g_1(3\cosh a_4 - \cosh(3a_4) + 18C_2\sinh^3 a_4)}{6m}\right], \\ A &= 2m\tilde{r} + \ln(1-e^{C_1-2m\tilde{r}}) + \ln(1+e^{C_1-2m\tilde{r}}) - \sigma. \end{aligned} \qquad (76)$$

Near the UV point, a_4, σ, and A behave as

$$a_4 \sim e^{-2mr}, \qquad \sigma \sim e^{-6mr}, \qquad A \sim 2mr. \qquad (77)$$

Similar to the previous solutions, we find that the IR singularity at $\tilde{r} \sim \frac{C_1}{2m}$ is physical for $a_4 \sim \pm \ln(2m\tilde{r} - C_1)$ if we choose $C_2 = \pm\frac{2}{9}$. In both cases, the IR metric is given by

$$ds^2 = (2mr - C)^{10} dx_{1,4}^2 + dr^2. \qquad (78)$$

Other choices of C_2 lead to unacceptable singularities.

B. Flow to $SU(2)_R \times U(1) \times U(1)$ SYM

In this subsection, we will give the solution for the flow to SYM with $SU(2)_R \times U(1)^2$ symmetry. To find this solution, we set $a_1 = a_2 = a_4 = 0$. The BPS equations, which are similar to those in the previous subsection, give the following solutions, in terms of the $\tilde{r}$ coordinate:

$$\begin{aligned} a_3 &= \pm \ln\left[\frac{1+e^{-2m\tilde{r}+C_1}}{1-e^{-2m\tilde{r}+C_1}}\right], \\ \sigma &= -\frac{1}{4}\ln\left[\frac{g_1(3\cosh a_3 - \cosh(3a_3) + 18C_2\sinh^3 a_3)}{6m}\right], \\ A &= 2m\tilde{r} + \ln(1-e^{C_1-2m\tilde{r}}) + \ln(1+e^{C_1-2m\tilde{r}}) - \sigma. \end{aligned} \qquad (79)$$

Near the UV point, we find $a_3 \sim e^{-2mr}$, $\sigma \sim e^{-6mr}$, and $A \sim 2mr$. In the IR, $\tilde{r} \to \frac{C_1}{m}$, the physical solution with $C_2 = \pm\frac{2}{9}$ is given by

$$\begin{aligned} &a_4 \sim \pm \ln(2m\tilde{r} - C_1), \qquad \sigma \sim -\frac{1}{4}\ln(2m\tilde{r} - C_1), \\ &ds^2 = (2mr - C)^{10} dx_{1,4}^2 + dr^2. \end{aligned} \qquad (80)$$

C. Flow to $SU(2)_R \times U(1)$ SYM

We then consider the flow that breaks $SU(2) \times U(1)$ global symmetry to $U(1)$. In this case, we turn on both a_3 and a_4. This leads to more complicated equations due to the coupling between a_4 and a_3. We will regard a_4 as a new variable and find that the solutions for a_3, σ, and A are given by

$$\begin{aligned} a_3 &= \sinh^{-1}[e^{C_1}\tanh a_4], \\ \sigma &= -\frac{1}{4}\ln\bigg[\frac{g_1}{6\sqrt{2}m}[72C_2\sinh^3 a_4(1+e^{2C_1}) \\ &\quad - 2\cosh a_4[(1+e^{2C_1})\cosh(2a_4) \\ &\quad - e^{2C_1} - 2]\sqrt{2+2e^{2C_1}\tanh^2 a_4}]\bigg], \\ A &= -\sigma - \ln\sinh a_4. \end{aligned} \qquad (81)$$

The solution of a_4 in terms of $\tilde{r}$ is given by

$$\tilde{r} = \frac{1}{2m}\tanh^{-1}\sqrt{\frac{1+\cosh(2a_4) + 2e^{2C_1}\sinh^2 a_4}{2}}. \qquad (82)$$

At the UV point, we find the expected behavior $a_{3,4} \sim e^{-2mr}$, $\sigma \sim e^{-6mr}$, and $A \sim 2mr$. In the IR, we consider the behavior of the solutions as $a_4 \to \infty$. In this limit, the a_4 solution becomes $a_4 \sim -\ln(2m\tilde{r} - \tilde{C})$ for some constant $\tilde{C}$. We find that the requirement for the IR singularity to be acceptable is given by $C_2 = \frac{1}{9}\sqrt{\frac{1+e^{2C_1}}{2}}$. The behavior of a_3, σ, and A is given by

$$a_3 \sim \sinh^{-1} e^{C_1}, \qquad \sigma \sim -\frac{1}{4}\ln(2m\tilde{r} - \tilde{C}),$$
$$A \sim \frac{5}{4}\ln(2m\tilde{r} - \tilde{C}). \tag{83}$$

With the relation $2mr - C = 4(2m\tilde{r} - \tilde{C})^{\frac{1}{4}}$, the metric in the IR then takes the form of a domain wall

$$ds^2 = (2mr - C)^{10} dx_{1,4}^2 + dr^2. \tag{84}$$

D. Flow to $SU(2)_R$ SYM

We now quickly look at the flow breaking the $U(2)$ symmetry completely. Finding the solution in this case amounts to solving all of the six BPS equations. This, however, turns out not to be difficult. The resulting solutions for a_i, σ, and A are given by

$$\begin{aligned}
a_3 &= \sinh^{-1}(e^{C_1}\tanh a_4),\\
a_2 &= \sinh^{-1}\frac{e^{C_2}\sinh a_4}{\sqrt{1-e^{2C_1}+(1+e^{2C_1})\cosh(2a_4)}},\\
a_1 &= \sinh^{-1}\frac{e^{C_3}\sinh a_4}{\sqrt{2-2e^{2C_1}-e^{2C_2}+(2+2e^{2C_1}+e^{2C_2})\cosh(2a_4)}},\\
\sigma &= \frac{1}{4}\ln\left[96\sqrt{2}m\sqrt{4+\alpha^2-\alpha^2\mathrm{sech}^2 a_4}\right] - \frac{1}{4}\ln\Big[g_1(2304(\alpha^2+4)C_4\sinh^3 a_4\sqrt{\alpha^2+4-\alpha^2\mathrm{sech}^2 a_4}\\
&\quad - \sqrt{2}\,\mathrm{sech}\, a_4(3\alpha^4+(\alpha^2+4)^2\cosh(4a_4)+16\alpha^2-4(\alpha^4+6\alpha^2+8)\cosh(2a_4)-48))\Big],\\
A &= -\sigma - \ln\sinh a_4,\\
a_4 &= \frac{1}{2}\cosh^{-1}\left[\frac{8\tanh^2(2m\tilde{r}-C_5)+\alpha^2-4}{\alpha^2+4}\right],
\end{aligned} \tag{85}$$

where $\alpha = \sqrt{4e^{2C_1}+2e^{2C_2}+e^{2C_3}}$. At the UV fixed point, the solutions become

$$a_{1,2,3,4} \sim e^{-2mr}, \qquad \sigma \sim e^{-6mr}, \qquad A \sim 2mr. \tag{86}$$

In the IR, we have to set $C_4 = \frac{1}{144}\sqrt{\frac{4+\alpha^2}{2}}$ in order to obtain a physical solution. The solution is then given by

$$\begin{aligned}
a_4 &\sim -\ln(2m\tilde{r}-\tilde{C}), \qquad a_3 \sim \sinh^{-1} e^{C_1},\\
a_2 &\sim \sinh^{-1}\frac{e^{C_2}}{\sqrt{2+2e^{2C_1}}},\\
a_1 &\sim \sinh^{-1}\frac{e^{C_3}}{\sqrt{4+4e^{2C_1}+2e^{2C_2}}},\\
\sigma &\sim -\frac{1}{4}\ln(2m\tilde{r}-\tilde{C}), \qquad A \sim \frac{5}{4}\ln(2m\tilde{r}-\tilde{C}),\\
ds^2 &= (2mr-C)^{10}dx_{1,4}^2 + dr^2.
\end{aligned} \tag{87}$$

All of the flows given above are driven by turning on operators of dimension four and a VEV of a dimension three operator.

V. CONCLUSIONS

We have studied various holographic RG flows from matter coupled $F(4)$ gauged supergravity. These flows describe deformations of the UV $N=2$ SCFTs with $SU(2)$ and $SU(2)\times U(1)$ global symmetries in five dimensions to nonconformal $N=2$ SYM theories in the IR. We have explored various symmetry breaking patterns and interpreted the solutions as RG flows driven by turning on operators of dimension four in a vacuum with nonzero VEV of a dimension three operator dual to the six-dimensional dilaton, except for the flow to the $SU(2)_{\mathrm{diag}}$ SYM, which is driven by vacuum expectation values of dimension three operators. We have also identified physical flows which have acceptable IR singularities from the resulting solutions. Therefore, these solutions might be useful in the study of strongly coupled $N=2$ SYM in five dimensions. However, the identification of the dual five-dimensional SYM corresponding to these solutions in the IR is not clear. Accordingly, the precise physical interpretation of these solutions needs to be clarified.

It is interesting to holographically compute various characteristics of the 5D gauge theories such as the Wilson loops, as done in [29]. It could be useful to do this computation with the six-dimensional solutions given here, similar to the four-dimensional gauge theories studied in [6,7]. The solutions found in this paper would hopefully be useful in this aspect and for other holographic calculations. It will very interesting (if possible) to find a gravity solution describing the enhancement of the global symmetry $SO(2N_f) \times U(1)$ to the E_{N_f+1} fixed point in five dimensions. In this aspect, the six-dimensional framework considered here may not be able to accommodate this solution since the symmetry enhancement is not seen at the classical supergravity level, as remarked in [17].

It is not presently known how to embed the six-dimensional $F(4)$ gauged supergravity coupled to n vector multiplets to 10 or 11 dimensions, although the pure $F(4)$ gauged supergravity and the theory coupled to 20 vector multiplets are known to originate from massive type IIA compactification on warped S^4 and $K3$, respectively [30,31]. The embedding of $F(4)$ gauged supergravity in type IIB theory via the non-Abelian T duality has been proposed recently in [32]. This might also provide another mean to embed the six-dimensional gauged supergravity in higher dimensions. It would be interesting to find such an embedding, which in turn can be used to uplift the solutions found here and in [20] to ten dimensions. This could provide some insight to the dynamics of D4/D8-brane system. We hope to come back to these issues in future works.

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RG flows in 6D N = (1,0) SCFT from SO(4) half-maximal 7D gauged supergravity

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Abstract: We study $N = 2$ seven-dimensional gauged supergravity coupled to three vector multiplets with SO(4) gauge group. The resulting gauged supergravity contains 10 scalars consisting of the dilaton and 9 vector multiplet scalars parametrized by $\mathrm{SO}(3,3)/\mathrm{SO}(3)\times\mathrm{SO}(3)$ coset manifold. The maximally supersymmetric AdS_7 vacuum with unbroken SO(4) symmetry is identified with a $(1,0)$ SCFT in six dimensions. We find one new supersymmetric AdS_7 critical point preserving $\mathrm{SO}(3)_{\mathrm{diag}} \subset \mathrm{SO}(3)\times\mathrm{SO}(3) \sim \mathrm{SO}(4)$ and study a holographic RG flow interpolating between the SO(4) and the new SO(3) supersymmetric critical points. The RG flow is driven by a vacuum expectation value of a dimension-four operator and describes a deformation of the UV $(1,0)$ SCFT to another supersymmetric fixed point in the IR. In addition, a number of non-supersymmetric critical points are identified, and some of them are stable with all scalar masses above the BF bound. RG flows to non-conformal $N = (1,0)$ Super Yang-Mills with $\mathrm{SO}(2)\times\mathrm{SO}(2)$ and SO(2) symmetries are also investigated. Some of these flows have physically acceptable IR singularities since the scalar potential is bounded above. These provide physical RG flows from $(1,0)$ SCFT to non-conformal field theories in six dimensions.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Supergravity Models

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Contents

1 Introduction

The AdS/CFT correspondence has attracted a lot of attention during the past twenty years. The original proposal in [1] discussed many examples in various dimensions. These examples included the duality between M-theory on $AdS_7 \times S^4$ and $(2, 0)$ superconformal field theory (SCFT) in six dimensions. The $AdS_7 \times S^4$ geometry can arise from the near horizon limit of M5-brane. In term of $N = 4$ seven-dimensional gauged supergravity with SO(5) gauge group, the AdS_7 geometry corresponds to the maximally supersymmetric vacuum of the gauged supergravity, see for example [2].

In this paper, we will explore $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence with sixteen supercharges. The dual SCFT to the AdS_7 background in this case would be $(1, 0)$ six-dimensional SCFT. Six-dimensional gauge theories with $N = (1, 0)$ supersymmetry are interesting in many aspects. In [3], it has been shown that the theories admit non-trivial RG fixed points. Examples of these field theories also arise in string theory [4], see also a review in [5]. After the AdS/CFT correspondence, a supergravity dual of a $(1, 0)$ field theory with E_8 global symmetry has been proposed in [6]. The dual gravity background has been identified with the orbifolds of $AdS_7 \times S^4$ geometry in M-theory. The operator spectrum of the $(1, 0)$ six-dimensional SCFT has been matched with the Kaluza-Klein spectrum in [7, 8].

Like in lower dimensions, it is more convenient to study $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ correspondence in the framework of $(d+1)$-dimensional gauged supergravity. A consistent reduction ansatz can eventually be used to uplift the lower dimensional results to string/M theory in ten or eleven dimensions. A suitable framework in the holographic study of the above $(1, 0)$ field

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theories is the half-maximal gauged supergravity in seven dimensions coupled to n vector multiplets. The supergravity theory has $N = 2$ or sixteen supercharges in exact agreement with the number of supercharges in six-dimensional $(1, 0)$ superconformal symmetry. This has been proposed long time ago in [9]. With the pure gauged supergravity and critical points found in [10] and [11], holographic RG flows to a non-supersymmetric IR fixed point and to a non-conformal $(1, 0)$ gauge theory have been studied in [12] and [13].

Pure $N = 2$ gauged supergravity in seven dimensions admit only two AdS_7 vacua with one being maximally supersymmetric and the other one being stable non-supersymmetric. To obtain more AdS_7 critical points, matter coupled supergravity theory is needed. This has been constructed in [14] but without the topological mass term for the 3-form field which is a dual of the 2-form field in the supergravity multipet. Without this term, the scalar potential of the matter coupled gauged supergravity does not admit any critical point but a domain wall as can be verified by looking at the scalar potential explicitly given in [14]. Although mistakenly claimed in [15] that the topological mass term is not possible, the theory indeed admits this term as shown in [16] in which the full Lagrangian and supersymmetry transformations of this massive gauged supergravity have been given. This provides the starting point for the present work.

In this paper, we are interested in the gauged supergravity with SO(4) gauge group. This requires three vector multiplets since six gauge fields are needed in order to implement the SO(4) gauging. The theory can be obtained from a truncation of the maximal $N = 4$ gauged supergravity [17]. In addition to the dilaton, there are extra nine scalars from the vector multipets parametrized by $SO(3, 3)/SO(3) \times SO(3) \sim SL(4, \mathbb{R})/SO(4)$ coset manifold. We will explore the scalar potential of this theory in the presence of topological mass term and identify some of its critical points. The critical points will correspond to new IR fixed point of the $(1, 0)$ SCFT identified with the maximally supersymmetric critical point with SO(4) symmetry. We will also study RG flows between these critical points as well as RG flows to non-conformal field theories.

The paper is organized as follow. We briefly review the matter coupled gauged supergravity in seven dimensions and give relevant formulae which will be used throughout the paper in section 2. Some critical points of seven-dimensional gauged supergravity with SO(4) gauge group are explored in section 3. A number of supersymmetric and non-supersymmetric critical points and the corresponding scalar masses will also be given in this section. In section 4, we study supersymmetric deformations of the UV $N = (1, 0)$ SCFT to a new superconformal fixed point in the IR and to non-conformal SYM in six dimensions. Both types of the solutions can be analytically obtained. The paper is closed with some conclusions and comments on the results in section 5.

2 $N = 2$, SO(4) gauged supergravity in seven dimensions

We begin with a description of $N = 2$ gauged supergravity coupled to n vector multiplets. All notations are the same as those of [16]. The gravity multiplet in seven-dimensional $N = 2$ supersymmetry contains the following field content

$$\text{gravity multiplet}: \qquad (e^m_\mu, \psi^A_\mu, A^i_\mu, \chi^A, B_{\mu\nu}, \sigma). \tag{2.1}$$

A vector multiplet has the field content $(A_\mu, \lambda^A, \phi^i)$. Indices A, B label the doublet of the $\mathrm{USp}(2)_R \sim \mathrm{SU}(2)_R$ R-symmetry. Curved and flat space-time indices are denoted by $\mu, \nu, \ldots$ and $m, n, \ldots$, respectively. $B_{\mu\nu}$ and σ are a two-form and the dilaton fields. For supergravity theory coupled to n vector multiplets, there are n copies of $(A_\mu, \lambda^A, \phi^i)^r$ labeled by an index $r = 1, \ldots, n$, and indices $i, j = 1, 2, 3$ label triplets of $\mathrm{SU}(2)_R$. The $3n$ scalars ϕ^{ir} are parametrized by $\mathrm{SO}(3, n)/\mathrm{SO}(3)\times\mathrm{SO}(n)$ coset manifold. The corresponding coset representative will be denoted by

$$L = (L_I{}^i, L_I{}^r), \qquad I = 1, \ldots, n+3\,. \tag{2.2}$$

The inverse of L is given by $L^{-1} = (L^I{}_i, L^I{}_r)$ where $L^I{}_i = \eta^{IJ}L_{Ji}$ and $L^I{}_r = \eta^{IJ}L_{Jr}$. Indices i, j and r, s are raised and lowered by δ_{ij} and δ_{rs}, respectively while the full $\mathrm{SO}(3, n)$ indices I, J are raised and lowered by $\eta_{IJ} = \mathrm{diag}(---++\ldots+)$. There are some relations involving components of L and are given by

$$\begin{aligned} \eta_{IJ} &= -L_I{}^i L^i{}_J + L_I{}^r L_J{}^r, & L^i &= L_{Ii}, \\ L^i{}_I L^I{}_j &= -\delta^i_j, & L^i{}_I L^{Ij} &= -\delta^{ij}\,. \end{aligned} \tag{2.3}$$

Gaugings are implemented by promoting a global symmetry $\tilde{G} \subset \mathrm{SO}(3, n)$ to a gauge symmetry. Consistency of the gauging imposes a condition on the $\tilde{G}$ structure constants $f_{IJ}{}^K$

$$f_{IK}{}^L \eta_{LJ} + f_{JK}{}^L \eta_{LI} = 0 \tag{2.4}$$

meaning that η_{IJ} is invariant under the adjoint action of $\tilde{G}$. General semisimple gauge groups take the form of $\tilde{G} \sim G_0 \times H \subset \mathrm{SO}(3, n)$ with G_0 being one of the six possibilities: $\mathrm{SO}(3)$, $\mathrm{SO}(3, 1)$, $\mathrm{SL}(3, \mathbb{R})$, $\mathrm{SO}(2, 1)$, $\mathrm{SO}(2, 2)$ and $\mathrm{SO}(2, 2)\times\mathrm{SO}(2, 1)$ and H being compact with $\mathrm{dim}H \leq (n + 3 - \mathrm{dim}G_0)$.

In this paper, we are interested in the $\mathrm{SO}(4)$ gauged supergravity corresponding to $G_0 = \mathrm{SO}(3)$ and $H = \mathrm{SO}(3)$. To obtain AdS_7 vacua, we need to consider the gauged supergravity with a topological mass term for a 3-form potential. The 3-form field is a dual of the 2-form $B_{\mu\nu}$. With all modifications to the Lagrangian and supersymmetry transformations as given in [16], the bosonic Lagrangian involving only scalars and the metric can be written as

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{5}{8}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}P^{\mu ir}P_{\mu ir} - V \tag{2.5}$$

where the scalar potential is given by

$$V = \frac{1}{4}e^{-\sigma}\left(C^{ir}C_{ir} - \frac{1}{9}C^2\right) + 16h^2 e^{4\sigma} - \frac{4\sqrt{2}}{3}he^{\frac{3\sigma}{2}}C\,. \tag{2.6}$$

The constant h characterizes the topological mass term. The quantities appearing in the above equations are defined by

$$\begin{aligned} P^{ir}_\mu &= L^{Ir}\left(\delta^K_I\partial_\mu + f_{IJ}{}^K A^J_\mu\right)L^i{}_K, & &C_{rsi}f_{IJ}{}^K L^I{}_r L^J{}_s L_{Ki}, \\ C_{ir} &= \frac{1}{\sqrt{2}}f_{IJ}{}^K L^I{}_j L^J{}_k L_{Kr}\epsilon^{ijk}, & C &= -\frac{1}{\sqrt{2}}f_{IJ}{}^K L^I{}_i L^J{}_j L_{Kk}\epsilon^{ijk}\,. \end{aligned} \tag{2.7}$$

We also need fermionic supersymmetry transformations with all fields but scalars vanishing. These are given by

$$\delta\psi_\mu = 2D_\mu\epsilon - \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\gamma_\mu\epsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon, \tag{2.8}$$

$$\delta\chi = -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon + \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\epsilon - \frac{16}{5}e^{2\sigma}h\epsilon, \tag{2.9}$$

$$\delta\lambda^r = -i\gamma^\mu P^{ir}_\mu\sigma^i\epsilon - \frac{i}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{ir}\sigma^i\epsilon \tag{2.10}$$

where $\mathrm{SU}(2)_R$ indices on spinors are suppressed. σ^i are the usual Pauli matrices.

In the remaining of this section, we focus on $n = 3$ case with $\tilde{G} = \mathrm{SO}(4) \sim \mathrm{SO}(3) \times \mathrm{SO}(3)$. The first SO(3) factor is identified with the $\mathrm{SU}(2)_R$ R-symmetry. To give an explicit parametrization of $\mathrm{SO}(3,3)/\mathrm{SO}(3) \times \mathrm{SO}(3)$ coset, we define thirty-six 6×6 matrices

$$(e_{ab})_{cd} = \delta_{ac}\delta_{bd}, \qquad a, b \ldots = 1, \ldots 6\,. \tag{2.11}$$

Non-compact generators of $\mathrm{SO}(3,3)$ are identified as

$$Y_{ir} = e_{i,r+3} + e_{r+3,i}, \qquad r = 1, \ldots, 3\,. \tag{2.12}$$

Accordingly, $\mathrm{SO}(3) \times \mathrm{SO}(3)$ generators can be written as

$$\begin{aligned} \mathrm{SO}(3)_R : &\qquad J_{ij} = e_{ij} - e_{ji}, \\ \mathrm{SO}(3) : &\qquad J_{rs} = e_{rs} - e_{sr}\,. \end{aligned} \tag{2.13}$$

In this case, the structure constants for the gauge group are given by

$$f_{IJK} = (g_1\epsilon_{ijk}, g_2\epsilon_{rst}) \tag{2.14}$$

where g_1 and g_2 are coupling constants of $\mathrm{SO}(3)_R$ and SO(3), respectively.

3 Critical points of $N = 2$, SO(4) seven-dimensional gauged supergravity

In this section, we will compute the scalar potential of the SO(4) gauged supergravity and study some of its critical points. Although complicated, it is possible to compute the scalar potential for all of the ten scalars. However, the long expression would make any analysis more difficult. Consequently, we will proceed by studying the scalar potential on a subset of the ten scalars as originally proposed in [18]. In this approach, the scalar potential is computed on a scalar submanifold which is invariant under some subgroup H_0 of the full gauge symmetry SO(4). This submanifold consists of all scalars which are singlet under the unbroken subgroup H_0. All critical points found on this submanifold are essentially critical points of the potential on the full scalar manifold. This can be seen by expanding the full potential to first order in scalar fluctuations which in turn contain both H_0 singlets and H_0 non-singlets. By a simple group theory argument, the non-singlet fluctuations cannot lead to H_0 singlets at first order. Their coefficients, variations of the potential with respect to non-singlet scalars, must accordingly vanish. This proves to be more convenient and more efficient. However, the truncation is consistent only when all relevant H_0 singlet scalars are included on the chosen submanifold. With only some of these singlets, the consistency is not guaranteed.

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3.1 Critical points on $SO(3)_{\mathrm{diag}}$ scalars

We begin with the most simplest case namely the potential on $SO(3)_{\mathrm{diag}} \subset SO(3) \times SO(3)$ corresponding to the non-compact generator $Y_s = Y_{11} + Y_{22} + Y_{33}$. The coset representative is then parametrized by

$$L = e^{\phi Y_s}\,. \tag{3.1}$$

The scalar potential is given by

$$\begin{aligned} V &= \frac{1}{32}e^{-\sigma}\left[(g_1^2+g_2^2)\left(\cosh(6\phi)-9\cosh(2\phi)\right)-8g_1g_2\sinh^3(2\phi)\right. \\ &\qquad \left. +8\left[g_2^2-g_1^2+64h^2e^{5\sigma}+32e^{\frac{5\sigma}{2}}h\left(g_1\cosh^2\phi-g_2\sinh^3\phi\right)\right]\right]. \end{aligned} \tag{3.2}$$

Notice that there is no critical point when $h = 0$ as mentioned before. In this case, the SO(4) supergravity admits a half-supersymmetric domain wall as a vacuum solution. For $\phi = 0$, the above potential is the potential of pure $N = 2$ gauged supergravity with SO(3) gauge group studied in [10] and [11]. There are two critical points in the pure gauged supergravity. One of them preserves all of the supersymmetry while the other completely breaks supersymmetry. In our conventions, they are given by

$$\sigma = \frac{2}{5}\ln\left[-\frac{g_1}{16h}\right] \qquad \text{and} \qquad \sigma = \frac{2}{5}\ln\left[-\frac{g_1}{8h}\right]. \tag{3.3}$$

It can be readily verified by using supersymmetry transformations of ψ_μ, χ and λ^r that the first one is supersymmetric. We can bring the supersymmetric point to $\sigma = 0$ by choosing $g_1 = -16h$ and find that the two critical points are now given by

$$\begin{aligned} &\sigma = 0, && V_0 = -240h^2 \\ \text{and}\qquad &\sigma = \frac{2}{5}\ln 2, && V_0 = -160(2^{\frac{3}{5}})h^2 \end{aligned} \tag{3.4}$$

where V_0 denotes the value of the cosmological constant.

Although non-supersymmetric, the second critical point has been shown to be stable in [11]. In the presence of matter scalars, this is however not the case. This can be seen from the scalar masses given below.

$SO(3) \times SO(3)$	m^2L^2
$(\mathbf{1},\mathbf{1})$	12
$(\mathbf{3},\mathbf{3})$	−12

The AdS_7 radius L in our conventions is given by $L = \sqrt{\frac{-15}{V_0}} = \frac{1}{4h}$. The $(\mathbf{1},\mathbf{1})$ scalar correspond to σ, and $(\mathbf{3},\mathbf{3})$ is the nine scalars in $SO(3,3)/SO(3) \times SO(3)$. The BF bound in seven dimensions is $m^2L^2 \geq -9$. Therefore, the non-supersymmetric critical point of pure gauged supergravity is unstable in the matter coupled theory. This is very similar to the six-dimensional $N = (1,1)$ gauged supergravity pointed out in [19].

Scalar masses at the supersymmetric point are given in the table below.

$SO(3) \times SO(3)$	m^2L^2
$(\mathbf{1}, \mathbf{1})$	-8
$(\mathbf{3}, \mathbf{3})$	-8

In the dual $(1, 0)$ SCFT, these scalars correspond to dimension-4 operators via the relation $m^2L^2 = \Delta(\Delta - 6)$.

There is one non-trivial supersymmetric point at

$$\sigma = -\frac{1}{5}\ln\left[\frac{g_2^2 - 256h^2}{g_2^2}\right], \qquad \phi = \frac{1}{2}\ln\left[\frac{g_2 - 16h}{g_2 + 16h}\right],$$
$$V_0 = -\frac{240 g_2^{\frac{8}{5}} h^2}{(g_2^2 - 256h^2)^{\frac{4}{5}}}. \tag{3.5}$$

At this point, scalar masses are computed as follow.

$SO(3)_{\text{diag}}$	m^2L^2	Δ
$\mathbf{1}$	-8	4
$\mathbf{1}$	40	10
$\mathbf{3}$	0	6
$\mathbf{5}$	16	8

In the table, we have decomposed all of the ten scalars in representations of the $SO(3)_{\text{diag}}$ residual symmetry. This can be done by the following decomposition. Under $SO(3)_R \times SO(3)$, the nine scalars transform as $(\mathbf{3}, \mathbf{3})$. They then transform as $\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{3} + \mathbf{5}$ under $SO(3)_{\text{diag}}$. Notice that the $\mathbf{3}$ scalars are massless corresponding to Goldstone bosons of the symmetry breaking $SO(3) \times SO(3) \to SO(3)_{\text{diag}}$.

There is one non-supersymmetric critical point given by

$$\sigma = \frac{1}{5}\ln\left[\frac{4g_2^2}{g_2^2 - 256h^2}\right], \qquad \phi = \frac{1}{2}\ln\left[\frac{g_2 - 16h}{g_2 + 16h}\right],$$
$$V_0 = -\frac{160(2^{\frac{3}{5}}) g_2^{\frac{8}{5}} h^2}{(g_2^2 - 256h^2)^{\frac{4}{5}}}. \tag{3.6}$$

This critical point is stable as can be seen from the mass spectrum below.

$SO(3)_{\text{diag}}$	m^2L^2	Δ
$\mathbf{1}$	12	$3 + \sqrt{21}$
$\mathbf{1}$	36	$3 + 3\sqrt{5}$
$\mathbf{3}$	0	6
$\mathbf{5}$	0	6

For $g_2 = g_1$, we also find another non-supersymmetric critical point given by

$$\sigma = \frac{1}{10}\left[\sqrt{2}\ln 8 + 4\ln(1-2^{-\sqrt{2}})\right], \qquad \phi = -\frac{1}{2}\ln 2, \qquad V_0 = -246.675h^2\,. \tag{3.7}$$

This critical point is however unstable. Scalar masses at this point are given below.

$SO(3)_{\text{diag}}$	m^2L^2
1	-4.278
1	16.059
3	0
5	-14.282

We can see that the mass of **5** scalars violates the BF bound.

3.2 Critical points on scalar manifold with smaller residual symmetry

To find other critical points, we can consider smaller residual symmetries. Breaking $SO(3)_{\text{diag}}$ to $SO(2)_{\text{diag}}$, we find that there are two singlets from $SO(3,3)/SO(3)\times SO(3)$ with the coset representative

$$L = e^{\phi_1(Y_{11}+Y_{22})}e^{\phi_2 Y_{33}}\,. \tag{3.8}$$

This gives the scalar potential, with $g_1 = -16h$,

$$\begin{aligned} V = \frac{1}{8}e^{-\sigma}\Big[&2(g_2^2+64h^2(e^{5\sigma}-4)) - 2(g_2^2+256h^2)\cosh(2\phi_1) \\ &-64he^{\frac{5\sigma}{2}}\left(16h\cosh^2\phi_1\cosh\phi_2 + g_2\sinh^2\phi_1\sinh\phi_2\right) \\ &+\sinh^2(2\phi_1)\left[(g_2^2+256h^2)\cosh(2\phi_2)+32g_2h\sinh(2\phi_2)\right]\Big]\,. \end{aligned} \tag{3.9}$$

This potential does not admit any supersymmetric critical points unless $\phi_1 = \phi_2$ which is the previously found $SO(3)_{\text{diag}}$ point. When $\phi_1 = 0$, the above scalar submanifold preserves $SO(2)\times SO(2)$ symmetry, but there is no critical point except for $\phi_2 = 0$. We are not able to obtain any new critical points from the above potential.

We now move to scalar fields invariant under $SO(2)_R \subset SO(3)_R$. There are three singlets corresponding to Y_{11}, Y_{12} and Y_{13}. Denoting the associated scalars by ϕ_i, $i = 1,2,3$, we find a simple potential

$$V = -\frac{1}{2}g_1^2e^{-\sigma} + 16h^2e^{4\sigma} + g_1he^{\frac{3}{2}\sigma-\phi_1-\phi_2-\phi_3}(1+e^{2\phi_1})(1+e^{2\phi_2})(1+e^{2\phi_3}) \tag{3.10}$$

which does not admit any non-trivial critical points.

4 Supersymmetric RG flows

We now consider domain wall solutions interpolating between critical points identified in the previous section. These solutions will generally have an interpretation in terms of

RG flows in the dual field theories in six dimensions. We are mainly interested in supersymmetric RG flows which can be obtained from solving BPS equations coming from supersymmetry variations of fermionic fields ψ_μ, χ and λ^r. A stable non-supersymmetric AdS_7 critical point also admits a well-defined dual CFT, but in most cases, finding the corresponding flow solutions requires a numerical analysis. Accordingly, we will not consider non-supersymmetric flows in this paper.

4.1 An RG flow to a supersymmetric SO(3) fixed point

There is one supersymmetric AdS_7 critical point with SO(3) symmetry. In this subsection, we will find the domain wall solution interpolating between this point and the trivial critical point at $\sigma = \phi = 0$.

Using the standard domain wall metric

$$ds^2 = e^{2A(r)}dx_{1,5}^2 + dr^2 \tag{4.1}$$

where $dx_{1,5}^2$ is the flat metric in six-dimensional space-time and the projection condition $\gamma_r\epsilon = \epsilon$, we can derive the following BPS equations

$$\phi' = \frac{1}{8}e^{-\frac{\sigma}{2}-3\phi}(e^{4\phi}-1)\left(g_1+g_2+e^{2\phi}g_1-e^{2\phi}g_2\right), \tag{4.2}$$

$$\sigma' = \frac{1}{20}\left[e^{-\frac{\sigma}{2}-3\phi}\left(g_2(e^{2\phi}-1)^3-g_1(1+e^{2\phi})^3\right)-128he^{2\sigma}\right], \tag{4.3}$$

$$A' = \frac{1}{40}e^{-\frac{\sigma}{2}-3\phi}\left[g_2(e^{2\phi}-1)^3-g_1(1+e^{2\phi})^3\right]+\frac{4}{5}he^{2\sigma} \tag{4.4}$$

where $'$ denotes $\frac{d}{dr}$. The above equations do not involve $\delta\psi_r$ equation which will give the Killing spinor condition on ϵ as usual. The above equations clearly admit two critical points. To find the solution, we combine equations (4.2) and (4.3) to

$$\frac{d\sigma}{d\phi} = \frac{2\left[g_2(e^{2\phi}-1)^3-g_1(1+e^{2\phi})^3-128he^{\frac{\sigma}{2}+3\phi}\right]}{5(e^{4\phi}-1)\left(g_1+g_2+(g_1-g_2)e^{2\phi}\right)} \tag{4.5}$$

whose solution is given by

$$\sigma = \frac{2}{5}\ln\left[\frac{e^{\phi}\left(g_1+g_2+(g_1-g_2)e^{2\phi}\right)}{32h\left(12C_1(e^{2\phi}-1)-1\right)}\right]. \tag{4.6}$$

In order for the solution to interpolate between the two critical points, we need to fix the integration constant to be $C_1 = \frac{(g_1-g_2)^2}{48g_1g_2}$. We then find the solution for σ

$$\sigma = \frac{2}{5}\ln\left[-\frac{g_1g_2e^{\phi}}{8h\left(g_1+g_2+(g_2-g_1)e^{2\phi}\right)}\right]. \tag{4.7}$$

Introducing a new radial coordinate $\tilde{r}$ via $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$, we can solve equation (4.2) and find the solution for ϕ

$$g_1g_2\tilde{r} = 2g_1\tan^{-1}e^{\phi}+2\sqrt{g_2^2-g_1^2}\tanh^{-1}\left[e^{\phi}\sqrt{\frac{g_2-g_1}{g_2+g_1}}\right]+g_2\ln\left[\frac{1-e^{\phi}}{1+e^{\phi}}\right] \tag{4.8}$$

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where we have neglected an additive integration constant to $\tilde{r}$. Taking the combination (4.4)+$\frac{1}{8}\times$ (4.3) and changing the variable from r to ϕ, we find

$$\frac{dA}{d\phi}+\frac{1}{8}\frac{d\sigma}{d\phi}=\frac{g_2(e^{2\phi}-1)^3-g_1(1+e^{2\phi})^3}{4(e^{4\phi}-1)\,(g_1+g_2+(g_1-g_2)e^{2\phi})}\,. \tag{4.9}$$

The solution is easily found to be

$$A=\frac{1}{8}\left[2\phi-\sigma-2\ln\left(2-2e^{4\phi}\right)+2\ln\left(g_1+g_2+(g_1-g_2)e^{2\phi}\right)\right]. \tag{4.10}$$

Near the UV point $\sigma\sim 0$ and $\phi\sim 0$ with $g_1=-16h$, we find

$$\sigma\sim\phi\sim e^{-16hr}=e^{-\frac{4r}{L}},\qquad L=\frac{1}{4h} \tag{4.11}$$

since $\tilde{r}\sim r$ near $\sigma\sim 0$. The flow is then driven by vacuum expectation values (vev) of relevant operators of dimension $\Delta=4$. In the IR, we find that the solution behaves as

$$\sigma\sim e^{-\frac{4r}{L}},\qquad \phi\sim e^{\frac{4r}{L}},\qquad L=\frac{(g_2^2-256h^2)^{\frac{2}{5}}}{4hg_2^{\frac{4}{5}}}\,. \tag{4.12}$$

From this, we see that the operator dual to ϕ acquires an anomalous dimension and has dimension 10 in the IR. This is consistent with the value of m^2L^2 given previously.

4.2 RG flows to non-conformal field theories

A supersymmetric flow to non-conformal field theory in pure gauged supergravity has been studied in [13]. We will study similar solutions in the matter coupled gauged supergravity. These solutions would be a generalization of the solution given in [13].

4.2.1 Flows to SO(2) × SO(2), 6D Super Yang-Mills

We first consider $SO(2)_R$ singlets scalars. With $\gamma_r\epsilon=\epsilon$, the BPS equations for these three singlets, denoted by ϕ_i, $i=1,2,3$, σ and A are given by

$$\phi_1' = \frac{1}{2}e^{-\frac{\sigma}{2}-\phi_1}g_1(e^{2\phi_1}-1), \tag{4.13}$$

$$\phi_2' = \frac{1}{2}e^{-\frac{\sigma}{2}-\phi_2}g_1(e^{2\phi_2}-1), \tag{4.14}$$

$$\phi_3' = \frac{1}{2}e^{-\frac{\sigma}{2}-\phi_3}g_1(e^{2\phi_3}-1), \tag{4.15}$$

$$\sigma' = -\frac{1}{20}g_1e^{-\frac{\sigma}{2}-\phi_1-\phi_2-\phi_3}(1+e^{2\phi_1})(1+e^{2\phi_2})(1+e^{2\phi_3})-\frac{32}{5}he^{2\sigma}, \tag{4.16}$$

$$A' = -\frac{1}{40}g_1e^{-\frac{\sigma}{2}-\phi_1-\phi_2-\phi_3}(1+e^{2\phi_1})(1+e^{2\phi_2})(1+e^{2\phi_3})+\frac{4}{5}he^{2\sigma}\,. \tag{4.17}$$

The above equations clearly admit only one critical point at $\phi_i=0$.

For $\phi_1 = \phi_2 = 0$, the solution will preserve $SO(2)_R \times SO(2)$ symmetry. This is easily seen to be a consistent truncation. The solution to the above equations is given by

$$\begin{aligned}
\phi_3 &= \pm \ln \left[\frac{1+e^{g_1\tilde{r}+C_1}}{1-e^{g_1\tilde{r}+C_1}}\right], \\
\sigma &= \frac{2}{5}\phi_3 - \frac{2}{5}\ln\left[-\frac{16h}{g_1}\left[4C_2\left(e^{2\phi_3}-1\right)-1\right]\right], \\
A &= \frac{1}{8}\left[2\phi_3 - \sigma - 2\ln(e^{2\phi_3}-1)\right]
\end{aligned} \tag{4.18}$$

where as in the previous case $\tilde{r}$ is related to r via $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$.

Near the UV point, the asymptotic behavior of ϕ_3 and σ is given by

$$\phi_3 \sim \sigma \sim e^{-16hr}, \qquad A \sim 4hr \sim \frac{r}{L}. \tag{4.19}$$

In the IR, we will consider $\phi_3 > 0$ and $\phi_3 < 0$, separately. For $\phi_3 > 0$, there is a singularity when $\phi_3 \to \infty$ as $16h\tilde{r} \sim C_1$. With $C_2 \neq 0$, we find

$$\begin{aligned}
&\phi_3 \sim -\ln(16h\tilde{r}-C_1), \qquad \sigma \sim \frac{2}{5}\ln(16h\tilde{r}-C_1), \\
&A \sim -\frac{1}{8}(2\phi_3+\sigma) = \frac{1}{5}\ln(16h\tilde{r}-C_1).
\end{aligned} \tag{4.20}$$

As $16h\tilde{r} \sim C_1$, we find the relation between r and $\tilde{r}$ to be $16hr - C = \frac{5}{6}(16h\tilde{r}-C_1)^{\frac{6}{5}}$ with C being another integration constant. As expected from the general DW/QFT correspondence [20–22], the metric in the IR takes the form of a domain wall

$$ds^2 = (16hr - C)^{\frac{1}{3}} dx^2_{1,5} + dr^2 \tag{4.21}$$

where the multiplicative constant has been absorbed in the rescaling of the x^μ coordinates.

Flows to non-conformal field theories usually encounter singularities in the IR. As can be seen from the above metric, there is a singularity at $16hr \sim C$. The criterion for determining whether a given singularity is physical or not has been given in [23]. The condition rules out naked time-like singularities which are clearly unphysical. According to the criterion of [23], the IR singularity in the solution is acceptable if the scalar potential is bounded above. One way to understand this criterion has been given in [24] for four-dimensional gauge theories. We will follow this argument and briefly discuss the meaning of the criterion in [23] in the context of six-dimensional field theories. Near the IR singularity, scalars ϕ_i, assumed to be canonical ones, and the metric warped factor A behave as

$$\phi_i \sim B_i \ln(r-r_0), \qquad A \sim \kappa \ln(r-r_0) \tag{4.22}$$

where we have chosen the integration constant so that the singularity occurs at r_0. In the IR, the bulk action for these scalars mainly contains the kinetic terms since the potential is irrelevant. This is because the potential diverges logarithmically, but the kinetic terms go like $(r-r_0)^{-2}$. According to the AdS/CFT correspondence, the one point function or the vacuum expectation value of operators O_i dual to ϕ_i is given by $\langle O_i \rangle = \frac{\delta S}{\delta \phi_i}$. Using

$$S = \frac{1}{2}\int d^6x dr e^{6A}\partial_r\phi_i\partial^r\phi_i, \tag{4.23}$$

we find

$$\langle O_i \rangle = \frac{\delta S}{\delta \phi_i} \sim e^{6A} \partial_r \phi_i \sim B_i (r - r_0)^{6\kappa - 1} . \tag{4.24}$$

We can see that $\langle O_i \rangle$ diverges for $\kappa < \frac{1}{6}$. We then expect that solutions with $\kappa < \frac{1}{6}$ will be excluded. In four dimensions, it has been shown that this is related to the fact that the scalar potential becomes unbounded above. In the present case, we will see in the solutions given below that this is the case namely all solutions with $\kappa < \frac{1}{6}$ have $V \to \infty$.

It can be checked by using the scalar potential given in (3.10) that as $16h\tilde{r} \sim C_1$, the solution in (4.20) gives $V \to -\infty$. The solution is then physical and describes a supersymmetric RG flow from $(1,0)$ SCFT to six-dimensional SYM with $\mathrm{SO}(2) \times \mathrm{SO}(2)$ symmetry.

For $C_2 = 0$, the solution becomes

$$\begin{aligned} \phi_3 &\sim -\ln(16h\tilde{r} - C_1), \qquad\qquad \sigma \sim -\frac{2}{5}\ln(16h\tilde{r} - C_1), \\ ds^2 &= (16hr - C)^{\frac{3}{4}} dx^2_{1,5} + dr^2 . \end{aligned} \tag{4.25}$$

This is also physical since it leads to $V \to -\infty$.

For $\phi_3 < 0$ and $16h\tilde{r} \sim C_1$, the above solutions give, for any values of C_2,

$$\begin{aligned} \phi_3 &\sim \ln(16h\tilde{r} - C_1), \qquad\qquad \sigma \sim \frac{2}{5}\ln(16h\tilde{r} - C_1), \\ ds^2 &= (16hr - C)^{\frac{1}{3}} dx^2_{1,5} + dr^2 \end{aligned} \tag{4.26}$$

which give rise to $V \to -\infty$. This solution is then physically acceptable.

The solution with all $\phi_i \neq 0$ turns out to be very difficult to find although the above BPS equations suggest that $\phi_1 = \phi_2 = \phi_3$. Most probably, a numerical analysis might be needed. Therefore, we will not further investigate this case.

4.2.2 Flows to SO(2), 6D Super Yang-Mills

As a final example, we consider RG flows to non-conformal theories from $\mathrm{SO}(2)_{\mathrm{diag}}$ singlet scalars corresponding to $Y_{11} + Y_{22}$ and Y_{33}. The relevant BPS equations are given by

$$\phi_1' = \frac{1}{8} e^{-\frac{\sigma}{2} - 2\phi_1 - \phi_2} (e^{4\phi_1} - 1) \left[g_1 + g_2 + (g_1 - g_2) e^{2\phi_2} \right], \tag{4.27}$$

$$\phi_2' = \frac{1}{8} e^{-\frac{\sigma}{2} - 2\phi_1 - \phi_2} \left[g_1 (1 + e^{2\phi_1})^2 (e^{2\phi_2} - 1) - g_2 (1 + e^{2\phi_2})(e^{2\phi_1} - 1)^2 \right], \tag{4.28}$$

$$\begin{aligned} \sigma' = \frac{1}{20} e^{-\frac{\sigma}{2} - 2\phi_1 - \phi_2} \Big[& g_2 (e^{2\phi_2} - 1)(e^{2\phi_1} - 1)^2 - g_1 (1 + e^{2\phi_1})^2 (1 + e^{2\phi_2}) \\ & -128 h e^{\frac{5\sigma}{2} + 2\phi_1 + \phi_2} \Big], \end{aligned} \tag{4.29}$$

$$\begin{aligned} A' = \frac{1}{40} e^{-\frac{\sigma}{2} - 2\phi_1 - \phi_2} \Big[& g_2 (e^{2\phi_2} - 1)(e^{2\phi_1} - 1)^2 - g_1 (1 + e^{2\phi_1})^2 (1 + e^{2\phi_2}) \\ & +32 h e^{\frac{5\sigma}{2} + 2\phi_1 + \phi_2} \Big]. \end{aligned} \tag{4.30}$$

These equations reduce to the $\mathrm{SO}(3)_{\mathrm{diag}}$ case when $\phi_2 = \phi_1$. If we set $\phi_2 = 0$, consistency requires that $\phi_1 = 0$. For $\phi_1 = 0$, the solution has $\mathrm{SO}(2)_R \times \mathrm{SO}(2)$ symmetry. This gives rise to the same solution studied above.

Since there are no interesting truncations, we now consider a solution to the above equations with $\phi_1, \phi_2 \neq 0$. Finding the solution for a general value of g_2 turns out to be difficult. However, for $g_2 = g_1 = -16h$, we can find an analytic solution. The first step in finding this solution is to combine (4.27) and (4.28) into a single equation

$$\frac{d\phi_2}{d\phi_1} = \frac{1 + e^{4\phi_1} - 2e^{2\phi_1+\phi_2}}{1 - e^{4\phi_1}} \tag{4.31}$$

which is solved by

$$\phi_2 = \phi_1 - \frac{1}{2}\ln\left[\frac{8C_2 - 1 + e^{4\phi_1}}{8C_2}\right]. \tag{4.32}$$

Changing to a new radial coordinate $\tilde{r}$ via $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}-\phi_2}$, we obtain the solution to equation (4.27)

$$\phi_1 = \pm\frac{1}{2}\ln\left[\frac{1 + e^{C_1 - 16h\tilde{r}}}{1 - e^{C_1 - 16h\tilde{r}}}\right]. \tag{4.33}$$

To find the solution for σ, we change to another new coordinate R via $\frac{dR}{dr} = -e^{-\frac{\sigma}{2}-\phi_2-2\phi_1}$. Equations (4.27), (4.28) and (4.29) can be combined to

$$\frac{5}{2}\frac{d\sigma}{dR} + 2\frac{d\phi_1}{dR} + \frac{d\phi_2}{dR} = -16h\left(1 - e^{\frac{5}{2}\sigma+2\phi_1+\phi_2}\right) \tag{4.34}$$

which gives

$$\sigma = -\frac{2}{5}\left[2\phi_1 + \phi_2 + \ln\left(1 - C_3 e^{16hR}\right)\right]. \tag{4.35}$$

Combing (4.29) and (4.30), we find an equation for A as a function of R

$$\frac{dA}{dR} - \frac{1}{2}\frac{d\sigma}{dR} = -4e^{\frac{5}{2}\sigma+2\phi_1+\phi_2} \tag{4.36}$$

whose solution, after using σ solution, is given by

$$A = \frac{\sigma}{2} + \frac{1}{4}\ln\left[C_3 - e^{-16hR}\right]. \tag{4.37}$$

As in the previous case, we separately consider the two possibilities for $\phi_1 > 0$ and $\phi_1 < 0$.

For $\phi_1 > 0$, we can find the relation between R and $\tilde{r}$ by using the relation $\frac{dR}{d\tilde{r}} = -e^{-2\phi_1(\tilde{r})}$. This results in

$$8hR = 8h\tilde{r} - \ln\left[2(e^{C_1} + e^{16h\tilde{r}})\right]. \tag{4.38}$$

In term of $\tilde{r}$, the σ and A solutions become

$$\sigma = -\frac{2}{5}\left[2\phi_1 + \phi_2 + \ln\left[1 - \frac{C_3 e^{16h\tilde{r}}}{4(e^{C_1} + e^{16h\tilde{r}})^2}\right]\right], \tag{4.39}$$

$$A = \frac{\sigma}{2} + \frac{1}{4}\ln\left[C_3 - 4e^{-16h\tilde{r}}(e^{C_1} + e^{16h\tilde{r}})^2\right]. \tag{4.40}$$

Near the IR singularity at $16h\tilde{r} \sim C_1$, we have $\phi_2 \sim -\phi_1$ for all values of C_2. In the IR, the solution behaves differently for $C_3 = 16e^{C_1}$ and $C_3 \neq 16e^{C_1}$. This is because

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the logarithmic term in (4.39) and (4.40) diverges, in this limit, when $C_3 = 16e^{C_1}$. For $C_3 \neq 16e^{C_1}$, we find

$$\phi_1 \sim -\phi_2 \sim -\frac{1}{2}\ln(16h\tilde{r} - C_1), \qquad \sigma \sim -\frac{2}{5}\phi_1 \sim \frac{1}{5}\ln(16h\tilde{r} - C_1),$$
$$A \sim \frac{\sigma}{2} \sim \frac{1}{10}\ln(16h\tilde{r} - C_1), \qquad ds^2 = (16hr - C)^{\frac{1}{8}}dx_{1,5}^2 + dr^2 . \tag{4.41}$$

This gives rise to $V \to \infty$ which is physically unacceptable.

However, if $C_3 = 16e^{C_1}$, the solution becomes

$$\sigma \sim -\frac{3}{5}\ln(16h\tilde{r} - C_1), \qquad A \sim \frac{1}{5}\ln(1h\tilde{r} - C_1),$$
$$ds^2 = (16hr - C)^{\frac{1}{3}}dx_{1,5}^2 + dr^2 . \tag{4.42}$$

This gives $V \to -\infty$, so this singularity is acceptable. We see that flows with $\phi_1 > 0$ are physical provided that $C_3 = 16e^{C_1}$.

For $\phi_1 < 0$, the solution $\phi_1 = -\frac{1}{2}\ln\left[\frac{1+e^{C_1-16h\tilde{r}}}{1+e^{C_1-16h\tilde{r}}}\right]$ gives

$$8hR = 8h\tilde{r} - \ln\left[2(e^{C_1} - e^{16h\tilde{r}})\right] . \tag{4.43}$$

Accordingly, the solutions for σ and A become

$$\sigma = -\frac{2}{5}\left[2\phi_1 + \phi_2 + \ln\left[1 - \frac{C_3 e^{16h\tilde{r}}}{4(e^{C_1} - e^{16h\tilde{r}})^2}\right]\right], \tag{4.44}$$
$$A = \frac{\sigma}{2} + \frac{1}{4}\ln\left[C_3 - 4e^{-16h\tilde{r}}(e^{C_1} - e^{16h\tilde{r}})^2\right] . \tag{4.45}$$

In this case, the logarithmic term in (4.45) diverges as $16h\tilde{r} \sim C_1$ when $C_3 = 0$, but the logarithmic term in (4.44) vanishes. When $C_3 \neq 0$, the situation is reversed. Unlike the $\phi_1 > 0$ case, the value of C_2 is important since there are two possibilities $\phi_1 = \mp\phi_2$ depending $C_2 = \frac{1}{8}$ or $C_2 \neq \frac{1}{8}$.

We begin with the first case with $C_2 = \frac{1}{8}$ and $C_3 = 0$. The IR behavior of the solution is given by

$$\phi_1 \sim -\phi_2 \sim \frac{1}{2}\ln(16h\tilde{r} - C_1), \qquad \sigma \sim \frac{1}{5}\ln(16h\tilde{r} - C_1),$$
$$A \sim \frac{3}{5}\ln(16h\tilde{r} - C_1), \qquad 16hr - C = \frac{5}{3}(16h\tilde{r} - C_1)^{\frac{3}{5}} . \tag{4.46}$$

The metric becomes

$$ds^2 = (16hr - C)^2 dx_{1,5}^2 + dr^2 . \tag{4.47}$$

When $C_3 \neq 0$, the solution in the IR becomes

$$\phi_1 \sim -\phi_2 \sim \frac{1}{2}\ln(16h\tilde{r} - C_1), \qquad \sigma \sim \frac{3}{5}\ln(16h\tilde{r} - C_1),$$
$$A \sim \frac{3}{10}\ln(16h\tilde{r} - C_1), \qquad ds^2 = (16hr - C)^{\frac{3}{4}}dx_{1,5}^2 + dr^2 . \tag{4.48}$$

Both of them lead to $V \to -\infty$. Therefore, the solution with $\phi_1 < 0$ and $C_2 = \frac{1}{8}$ is physical for all values of C_3.

For $C_2 \neq \frac{1}{8}$, we find, with $C_3 = 0$, the IR behavior of the solution

$$\begin{aligned} \phi_1 \sim \phi_2 \sim \frac{1}{2}\ln(16h\tilde{r} - C_1), \qquad \sigma \sim -\frac{6}{5}\ln(16h\tilde{r} - C_1), \\ ds^2 = (16hr - C)^{-\frac{2}{9}} dx_{1,5}^2 + dr^2, \end{aligned} \tag{4.49}$$

and, for $C_3 \neq 0$,

$$\begin{aligned} \phi_1 \sim \phi_2 \sim \frac{1}{2}\ln(16h\tilde{r} - C_1), \qquad \sigma \sim \frac{1}{5}\ln(16h\tilde{r} - C_1), \\ ds^2 = (16hr - C)^{\frac{1}{8}} dx_{1,5}^2 + dr^2 \,. \end{aligned} \tag{4.50}$$

Both of them lead to $V \to \infty$. We then conclude that flows with $\phi_1 < 0$ and $C_2 \neq \frac{1}{8}$ are not physical for any C_3.

It could be very interesting to have interpretations of these results in terms of six-dimensional gauge theories.

5 Conclusions

We have studied some critical points of $N = 2$, SO(4) gauged supergravity in seven dimensions. We have found one new supersymmetric AdS_7 critical point with SO(3) symmetry. Recently, many new $AdS_7 \times M_3$ solutions have been identified in massive type IIA theory [25]. It would be interesting to see weather the new supersymmetric AdS_7 obtained here could be related to the classification in [25]. We have also found a number of non-supersymmetric AdS_7 critical points and checked their stability by computing all of the scalar masses. We have found that although the non-supersymmetric critical point originally found in pure gauged supergravity has been shown to be stable, it is unstable in the presence of vector multiplet scalars. On the other hand, new stable non-supersymmetric points are discovered here and should correspond to new non-trivial IR fixed points of the $(1, 0)$ SCFT.

An analytic RG flow solution interpolating between the SO(3) supersymmetric critical point and the trivial point with SO(4) symmetry has also been given. To the best of the author's knowledge, this is the first example of holographic RG flows between two supersymmetric fixed points of the $(1, 0)$ field theory in six dimensions. We have further studied supersymmetric flows to non-conformal field theories and identified the physical flows. These would provide more general flow solutions than those considered in [12] and [13] and could be useful in a holographic study of the dynamics of six-dimensional gauge theories similar to the analysis of [26]. Finding a field theory interpretation of the gravity solutions obtained in this paper is also interesting.

We end the paper with a short comment on a more general situation with n vector multiplets. The $(1, 0)$ field theory with E_8 symmetry considered in [6] would need $n = 248 + 3$ vector multiplets. The resulting gauge group in this case is $SO(4) \times E_8$. The total

$3 \times (248+3)$ scalars, living on $\mathrm{SO}(3,248+3)/\mathrm{SO}(3) \times \mathrm{SO}(248+3)$ coset manifold, and the dilaton transform as $(\mathbf{3},\mathbf{3},\mathbf{1})$, $(\mathbf{3},\mathbf{1},\mathbf{248})$ and $(\mathbf{1},\mathbf{1},\mathbf{1})$ under $\mathrm{SO}(3)_R \times \mathrm{SO}(3) \times E_8$. We have considered only $(\mathbf{3},\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{1},\mathbf{1})$ scalars which are E_8 singlets. It is also interesting to consider scalars in $(\mathbf{3},\mathbf{1},\mathbf{248})$ representation. Our solutions given in this paper are of course solutions of the theory with $\mathrm{SO}(4) \times E_8$ gauge group by the group theory argument of [18].

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$N = 2$ SO(4) 7D gauged supergravity with topological mass term from 11 dimensions

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Abstract: We construct a consistent reduction ansatz of eleven-dimensional supergravity to $N = 2$ SO(4) seven-dimensional gauged supergravity with topological mass term for the three-form field. The ansatz is obtained from a truncation of the S^4 reduction giving rise to the maximal $N = 4$ SO(5) gauged supergravity. Therefore, the consistency is guaranteed by the consistency of the S^4 reduction. Unlike the gauged supergravity without topological mass having a half-supersymmetric domain wall vacuum, the resulting 7D gauged supergravity theory admits a maximally supersymmetric AdS_7 critical point. This corresponds to $N = (1, 0)$ superconformal field theory in six dimensions. We also study RG flows from this $N = (1, 0)$ SCFT to non-conformal $N = (1, 0)$ Super Yang-Mills theories in the seven-dimensional framework and use the reduction ansatz to uplift this RG flow to eleven dimensions.

Keywords: Gauge-gravity correspondence, Supergravity Models, AdS-CFT Correspondence

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Contents

1 Introduction

Gauged supergravities in various dimensions play an important role in both string compactifications and in the AdS/CFT correspondence. In some cases, a consistent truncation can be made in such a way that a lower dimensional gauged supergravity is obtained via a dimensional reduction of a (gauged) supergravity in higher dimensions on spheres [1]. Embedding lower dimensional gauged supergravities is now of considerable interest since this provides a method to uplift lower dimensional solutions to string/M theory.

It is known that sphere reductions of 10 or 11 dimensional supergravities give rise to gauged supergravity in lower dimensions. Well-known examples of these consistent sphere reductions include S^7 and S^4 reductions of eleven-dimensional supergravity and S^5 reduction of type IIB theory giving rise to SO(8), SO(5) and SO(6) gauged supergravities in four, seven and five dimensions, respectively [2–4]. According to the AdS/CFT correspondence [5], seven-dimensional gauged supergravity is useful in the study of $N = (2, 0)$ and $N = (1, 0)$ field theories in six dimensions [6–10]. The latter describe the dynamics of M5-branes worldvolume in M-theory and are less-known on the field theory side. Therefore, seven-dimensional gauged supergravity is expected to give some insight to six-dimensional field theories via gauge/gravity correspondence.

In this paper, we are interested in obtaining $N = 2$ seven-dimensional gauged supergravity with SO(4) gauged group and topological mass term. In seven dimensions, the theory is obtained by coupling three vector multiplets to the pure SU(2) gauged supergravity constructed in [11]. This matter-coupled theory has been constructed in [12] and [13]. The SO(4) gauged supergravity has also been constructed in [14] by truncating the maximal $N = 4$ SO(5) gauged supergravity. All of these constructions have not included the

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topological mass term for the three-form field, and the resulting theory does not admit AdS_7 vacuum solutions. It has been shown in [15] that the topological mass term is possible. The massive gauged theory has been explored in [16] in which new AdS_7 vacua and the corresponding RG flow interpolating between these vacua have been given.

To give an interpretation to this solution in the string/M theory context, it is necessary to embed this solution to 10 or 11 dimensions. The reduction ansatz of eleven-dimensional supergravity giving rise to pure SU(2) gauged supergravity has been given in [17]. The SO(4) gauged theory without topological mass term from a dimensional reduction of eleven- and ten-dimensional supergravity has been given in [18] using the result of [19]. This result is clearly not sufficient to uplift the solution in [16]. The dimensionally reduced theory needs to include the topological mass term in order to admit AdS_7 vacua. We will give an extension to the result of [17, 18] by constructing SO(4) gauged theory including topological mass term from a truncation of S^4 reduction of eleven dimensional supergravity. This provides an ansatz to uplift the 7-dimensional solutions of massive $N = 2$ SO(4) gauged supergravity to eleven dimensions.

The paper is organized as follow. In section 2, we give relevant formulae for $N = 2$ SO(4) gauged supergravity in seven dimensions. The embedding of this theory in eleven dimensions is obtained via a consistent truncation of the S^4 reduction of eleven-dimensional supergravity in section 3. We then use the resulting ansatz to uplift RG flow solutions from the maximally supersymmetric AdS_7 vacuum with SO(4) symmetry to non-conformal SYM in section 4. We end the paper by giving some conclusions and comments in section 5.

2 SO(4) $N = 2$ gauged supergravity in seven dimensions

In this section, we give a description of SO(4) $N = 2$ gauged supergravity in seven dimensions with topological mass term. All of the notations are the same as those in [15] to which the reader is referred for further details.

The SO(4) gauged theory is obtained by coupling three vector multiplets to the $N = 2$ supergravity multiplet. The field contents are given respectively by

$$\begin{aligned} \text{Supergravity multiplet}: &\qquad (e^a_\mu, \psi^A_\mu, A^i_\mu, \chi^A, B_{\mu\nu}, \sigma) \\ \text{Vector multiplets}: &\qquad (A_\mu, \lambda^A, \phi^i)^r \end{aligned} \tag{2.1}$$

where an index $r = 1, 2, 3$ labels the three vector multiplets. Curved and flat space-time indices are denoted by $\mu, \nu, \ldots$ and $a, b, \ldots$, respectively. $B_{\mu\nu}$ and σ are a two-form and the dilaton fields. The two-form field will be dualized to a three-form field $C_{\mu\nu\rho}$. Indices $i, j = 1, 2, 3$ label triplets of $SU(2)_R$. The 9 scalars ϕ^{ir} are parametrized by $SO(3, 3)/SO(3) \times SO(3) \sim SL(4, \mathbb{R})/SO(4)$ coset manifold. The corresponding coset representative of $SO(3, 3)/SO(3) \times SO(3)$ will be denoted by

$$L = ({L_I}^i, {L_I}^r), \qquad I = 1, \ldots, 6\,. \tag{2.2}$$

whose inverse is given by $L^{-1} = ({L^I}_i, {L^I}_r)$ where ${L^I}_i = \eta^{IJ} L_{Ji}$ and ${L^I}_r = \eta^{IJ} L_{Jr}$. Indices i, j and r, s are raised and lowered by δ_{ij} and δ_{rs}, respectively while the full SO(3, 3) indices I, J are raised and lowered by $\eta_{IJ} = \text{diag}(- - - + + +)$.

The SO(4) $\sim$ SU(2)$\times$SU(2) gauging is implemented by promoting the SU(2)$\times$SU(2) $\sim$ SO(3) $\times$ SO(3) $\subset$ SO(3,3) to a gauge symmetry. The structure constants for the SU(2) $\times$ SU(2) gauge group, which will appear in various quantities, are given by

$$f_{IJK} = (g_1\epsilon_{ijk}, g_2\epsilon_{rst}). \tag{2.3}$$

To obtain SO(4) gauge group, we will later set $g_2 = g_1$. The bosonic Lagrangian can be written in a form language as

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}R*\mathbb{I} - \frac{1}{2}e^{\sigma}a_{IJ}*F^I_{(2)}\wedge F^J_{(2)} - \frac{1}{2}*H_{(4)}\wedge H_{(4)} - \frac{5}{8}*d\sigma\wedge d\sigma \\ &\quad -\frac{1}{2}*P^{ir}\wedge P_{ir} + \frac{1}{\sqrt{2}}H_{(4)}\wedge\omega_{(3)} - 4hH_{(4)}\wedge C_{(3)} - V*\mathbb{I}\end{aligned} \tag{2.4}$$

where the scalar potential is given by

$$V = \frac{1}{4}e^{-\sigma}\left(C^{ir}C_{ir} - \frac{1}{9}C^2\right) + 16h^2e^{4\sigma} - \frac{4\sqrt{2}}{3}he^{\frac{3\sigma}{2}}C\,. \tag{2.5}$$

The constant h describes the topological mass term for the three-form $C_{(3)}$ with $H_{(4)} = dC_{(3)}$. The quantities appearing in the above Lagrangian are defined by

$$\begin{aligned} P^{ir}_\mu &= L^{Ir}\left(\delta^K_I\partial_\mu + f_{IJ}{}^{K}A^J_\mu\right)L^i{}_K, \qquad & C_{rsi} &= f_{IJ}{}^{K}L^I{}_rL^J{}_sL_{Ki}, \\ C_{ir} &= \frac{1}{\sqrt{2}}f_{IJ}{}^{K}L^I{}_jL^J{}_kL_{Kr}\epsilon^{ijk}, & C &= -\frac{1}{\sqrt{2}}f_{IJ}{}^{K}L^I{}_iL^J{}_jL_{Kk}\epsilon^{ijk}, \\ a_{IJ} &= L^i{}_IL_{iJ} + L^r{}_IL_{rJ}\,. \end{aligned} \tag{2.6}$$

The Chern-Simons three-form satisfying $d\omega_{(3)} = F^I_{(2)}\wedge F^I_{(2)}$ is given by

$$\omega_{(3)} = F^I_{(2)}\wedge A^I_{(1)} - \frac{1}{6}f_{IJ}{}^{K}A^I_{(1)}\wedge A^J_{(1)}\wedge A_{(1)K} \tag{2.7}$$

with $F^I_{(2)} = dA^I_{(1)} + \frac{1}{2}f_{JK}{}^{I}A^J_{(1)}\wedge A^K_{(1)}$

It is also useful to give the corresponding field equations

$$d\left(e^{-2\sigma}*H_{(4)}\right) + 16hH_{(4)} - \frac{1}{\sqrt{2}}F^I_{(2)}\wedge F^I_{(2)} = 0, \tag{2.8}$$

$$\begin{aligned}&\frac{5}{4}d*d\sigma - \frac{1}{2}e^{\sigma}a_{IJ}*F^I_{(2)}\wedge F^J_{(2)} + e^{-2\sigma}*H_{(4)}\wedge H_{(4)} \\ &\quad + \left[\frac{1}{4}\left(C^{ir}C_{ir} - \frac{1}{2}C^2\right) + 2\sqrt{2}he^{\frac{3}{2}\sigma}C - 64h^2e^{4\sigma}\right]\epsilon_{(7)} = 0\end{aligned} \tag{2.9}$$

$$D(e^{\sigma}a_{IJ}*F^I_{(2)}) + \frac{1}{\sqrt{2}}H_{(4)}\wedge F^J_{(2)} + *P^{ir}f_{IJ}{}^{K}L^I{}_rL_{iK} = 0 \tag{2.10}$$

$$\begin{aligned}&D*P^{ir} - e^{\sigma}L^i{}_IL^r{}_J*F^I_{(2)}\wedge F^J_{(2)} \\ &\quad - *\mathbb{I}\left[\sqrt{2}e^{-\sigma}C_{jr}C^{rsk}\epsilon^{ijk} + 4\sqrt{2}he^{\frac{3\sigma}{2}}C_{ir}\right] = 0\,.\end{aligned} \tag{2.11}$$

The Yang-Mills equation (2.10) can be written in terms of C^{ir} and C^{irs} by using the relation

$$f_{IJ}{}^{K}L^I{}_rL_{iK} = -\frac{1}{2\sqrt{2}}\epsilon^{ijk}C^{jr}L^k{}_J - C^{irs}L_{sJ}\,. \tag{2.12}$$

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In obtaining the scalar equation (2.11), we have used the projections in the variations of scalars as in [12]

$$\begin{aligned}\delta L^i{}_I &= X^i{}_r L^r{}_I + X^i{}_j L^j{}_I,\\ \delta L^r{}_I &= X^{rs} L_{sI} + X^{ri} L_{iI}\end{aligned} \tag{2.13}$$

which lead to

$$\begin{aligned}\delta C^2 &= 6\sqrt{2} C C^{ir} X_{ir},\\ \delta(C^{ir}C_{ir}) &= 4\sqrt{2} C_{ir} C^{rsj} X^k{}_s \epsilon^{ijk} - \frac{2\sqrt{2}}{3} C_{ir} C X^i{}_r\,.\end{aligned} \tag{2.14}$$

We finally give supersymmetry transformations for fermions with all fermionic fields vanishing. These are given by

$$\begin{aligned}\delta\psi_\mu &= 2D_\mu\epsilon - \frac{\sqrt{2}}{30} e^{-\frac{\sigma}{2}} C\gamma_\mu\epsilon - \frac{1}{240\sqrt{2}} e^{-\sigma} H_{\rho\sigma\lambda\tau}\left(\gamma_\mu\gamma^{\rho\sigma\lambda\tau} + 5\gamma^{\rho\sigma\lambda\tau}\gamma_\mu\right)\epsilon\\ &\quad -\frac{i}{20} e^{\frac{\sigma}{2}} F^i_{\rho\sigma}\sigma^i\left(3\gamma_\mu\gamma^{\rho\sigma} - 5\gamma^{\rho\sigma}\gamma_\mu\right)\epsilon - \frac{4}{5} h e^{2\sigma}\gamma_\mu\epsilon,\end{aligned} \tag{2.15}$$

$$\begin{aligned}\delta\chi &= -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon - \frac{i}{10} e^{\frac{\sigma}{2}} F^i_{\mu\nu}\sigma^i\gamma^{\mu\nu}\epsilon - \frac{1}{60\sqrt{2}} e^{-\sigma} H_{\mu\nu\rho\sigma}\gamma^{\mu\nu\rho\sigma}\epsilon\\ &\quad +\frac{\sqrt{2}}{30} e^{-\frac{\sigma}{2}} C\epsilon - \frac{16}{5} e^{2\sigma} h\epsilon,\end{aligned} \tag{2.16}$$

$$\delta\lambda^r = -i\gamma^\mu P^{ir}_\mu \sigma^i\epsilon - \frac{1}{2} e^{\frac{\sigma}{2}} F^r_{\mu\nu}\gamma^{\mu\nu}\epsilon - \frac{i}{\sqrt{2}} e^{-\frac{\sigma}{2}} C^{ir}\sigma^i\epsilon \tag{2.17}$$

where $SU(2)_R$ doublet indices $A, B, \ldots$ on spinors are suppressed. σ^i are the usual Pauli matrices.

3 Seven dimensional $N = 2$ gauged supergravity from eleven dimensions

We now construct a reduction ansatz for embedding SO(4) $N = 2$ gauged supergravity mentioned in the previous section in eleven dimensions. The ansatz will be obtained from a consistent truncation of the S^4 reduction of eleven-dimensional supergravity giving rise to the maximal $N = 4$ SO(5) gauged supergravity in seven dimensions. To obtain the topological mass term, we will impose the so-called odd-dimensional self-duality as in [17].

3.1 $N = 4$ SO(5) gauged supergravity from seven dimensions

To set up the notations and make the paper self-contained, we briefly repeat the S^4 reduction of eleven-dimensional supergravity [3, 20]. We will work in the notations of [19] and deal mainly with bosonic fields. The field content of eleven-dimensional supergravity consists of the graviton $\hat{g}_{MN}$, gravitino $\hat{\psi}_M$ and a four-form field $\hat{F}_{(4)}$. Eleven-dimensional space-time indices are denoted by $M, N = 0, 1, \ldots, 10$.

The S^4 reduction is characterized by the following ansatz

$$d\hat{s}^2_{11} = \Delta^{\frac{1}{3}} ds^2_7 + \frac{1}{g^2}\Delta^{-\frac{2}{3}} T^{-1}_{ij} D\mu^i D\mu^j, \tag{3.1}$$

$$\hat{F}_{(4)} = \frac{1}{4!}\epsilon_{i_1\ldots i_5}\left[\frac{4}{g^3}\Delta^{-2}\mu^m\mu^n T^{i_1m}DT^{i_2n}\wedge D\mu^{i_3}\wedge D\mu^{i_4}\wedge D\mu^{i_5}\right.$$
$$\left. +\frac{6}{g^2}\Delta^{-1}T^{i_5j}\mu^j F^{i_1i_2}_{(2)}\wedge D\mu^{i_3}\wedge D\mu^{i_4} - \frac{1}{g^3}\Delta^{-2}U\mu^{i_1}D\mu^{i_2}\wedge\ldots\wedge D\mu^{i_5}\right]$$
$$-T_{ij} * S^i_{(3)}\mu^j + \frac{1}{g}S^i_{(3)}\wedge D\mu^i \tag{3.2}$$

where the quantities appearing in the above equations are defined by

$$U = 2T_{ij}T_{jk}\mu^i\mu^k - \Delta T_{ii}, \qquad \Delta = T_{ij}\mu^i\mu^j, \qquad \mu^i\mu^i = 1,$$
$$F^{ij}_{(2)} = dA^{ij}_{(1)} + gA^{ik}_{(1)}\wedge A^{kj}_{(1)}, \qquad D\mu^i = d\mu^i + gA^{ij}_{(1)}\mu^j,$$
$$DT_{ij} = dT_{ij} + gA^{ik}_{(1)}T_{kj} + gA^{jk}_{(1)}T_{ik}\,. \tag{3.3}$$

The symmetric matrix T_{ij}, $i, j = 1, \ldots, 5$ with unit determinant parametrize the $SL(5,\mathbb{R})/SO(5)$ coset manifold.

The bosonic field content of $N = 4$ gauged supergravity is given by the metric $g_{\mu\nu}$, ten vectors $A^{ij}_{(1)} = A^{[ij]}_{(1)}$ gauging the $SO(5)$ gauge group, five three-form fields $S^i_{(3)}$ and four-teen scalars T_{ij}. The corresponding field equations are given by

$$D(T_{ij} * S^j_{(3)}) = F^{ij}_{(2)}\wedge S^j_{(3)}, \tag{3.4}$$
$$H^i_{(4)} = gT_{ij} * S^j_{(3)} + \frac{1}{8}\epsilon_{ij_1\ldots j_4}F^{j_1j_2}_{(2)}\wedge F^{j_3j_4}_{(2)}, \tag{3.5}$$
$$D(T^{-1}_{ik}T^{-1}_{jl} * F^{ij}_{(2)}) = -2gT^{-1}_{i[k} * DT_{l]i} - \frac{1}{2g}\epsilon_{i_1i_2i_3kl}F^{i_1i_2}_{(2)}\wedge H^{i_3}_{(4)}$$
$$+\frac{3}{2g}\delta^{j_1j_2j_3j_4}_{i_1i_2kl}F^{i_1i_2}_{(2)}\wedge F^{j_1j_2}_{(2)}\wedge F^{j_3j_4}_{(2)} - S^k_{(3)}\wedge S^l_{(3)}, \tag{3.6}$$
$$D(T^{-1}_{ik} * DT_{kj}) = 2g^2\left(2T_{ik}T_{kj} - T_{kk}T_{ij}\right)\epsilon_{(7)} + T^{-1}_{im}T^{-1}_{kl} * F^{ml}_{(2)}\wedge F^{kj}_{(2)}$$
$$+T_{jk} * S^k_{(3)}\wedge S^i_{(3)} - \frac{1}{5}\delta_{ij}\left[2g^2\left(2T_{kl}T_{kl} - (T_{kk})^2\right)\epsilon_{(7)}\right.$$
$$\left. +T^{-1}_{nm}T^{-1}_{kl} * F^{ml}_{(2)}\wedge F^{kn}_{(2)} + T_{kl} * S^k_{(3)}\wedge S^l_{(3)}\right] \tag{3.7}$$

where

$$H^i_{(4)} = DS^i_{(3)} = dS^i_{(3)} + gA^{ij}_{(1)}\wedge S^j_{(3)}\,. \tag{3.8}$$

All of these equation can be obtained from the Lagrangian

$$\mathcal{L}_7 = R * \mathbb{I} - \frac{1}{4}T^{-1}_{ij} * DT_{jk}\wedge T^{-1}_{kl}DT_{li} - \frac{1}{4}T^{-1}_{ik}T^{-1}_{jl} * F^{ij}_{(2)}\wedge F^{kl}_{(2)} - \frac{1}{4}T_{ij} * S^i_{(3)}\wedge S^j_{(3)}$$
$$+\frac{1}{2g}S^i_{(3)}\wedge H^i_{(4)} - \frac{1}{8g}\epsilon_{ij_1\ldots j_4}S^i_{(3)}\wedge F^{j_1j_2}_{(2)}\wedge F^{j_3j_4}_{(2)} + \frac{1}{g}\Omega_{(7)} - V * \mathbb{I} \tag{3.9}$$

where $\Omega_{(7)}$ is the Chern-Simens three-form whose explicit form can be found in [22]. The scalar potential for T_{ij} is given by

$$V = g^2\left(T_{ij}T_{ij} - \frac{1}{2}(T_{ii})^2\right). \tag{3.10}$$

We have not given Einstein equation since we will not consider Einstein equation in this paper. The consistency of the full truncation, including the Einstein equation, to $N = 2$ SO(4) gauged supergravity is guaranteed from the consistency of the S^4 reduction.

For completeness, we also repeat supersymmetry transformations of fermionic fields ψ_μ and $\lambda_{\hat{i}}$. Indices $\hat{i}, \hat{j} = 1, \ldots, 5$ are vector indices of the composite $\mathrm{SO}(5)_c$ symmetry. Additionally, both ψ_μ and $\lambda_{\hat{i}}$ transform as a spinor under $\mathrm{SO}(5)_c$ with the condition $\Gamma^{\hat{i}}\lambda_{\hat{i}} = 0$, but we have omitted the $\mathrm{SO}(5)_c$ spinor indices to make the following expressions more compact. The $\mathrm{SO}(5)_c$ gamma matrices will be denoted by $\Gamma^{\hat{i}}$. The associated supersymmetry transformations are given by [22]

$$\begin{aligned}
\delta\psi_\mu &= D_\mu\epsilon - \frac{1}{20}gT_{\hat{i}\hat{i}}\gamma_\mu\epsilon - \frac{1}{40\sqrt{2}}\left(\gamma_\mu{}^{\nu\rho} - 8\delta^\nu_\mu\gamma^\rho\right)F^{\hat{i}\hat{j}}_{\nu\rho}\Gamma_{\hat{i}\hat{j}}\epsilon \\
&\quad -\frac{1}{60}\left(\gamma_\mu{}^{\nu\rho\sigma} - \frac{9}{2}\delta^\nu_\mu\gamma^{\rho\sigma}\right)S_{\hat{i}\nu\rho\sigma}\Gamma^{\hat{i}}\epsilon, \qquad (3.11)\\
\delta\lambda_{\hat{i}} &= \frac{1}{16\sqrt{2}}\gamma^{\mu\nu}\left(\Gamma_{\hat{k}\hat{l}}\Gamma_{\hat{i}} - \frac{1}{5}\Gamma_{\hat{i}}\Gamma_{\hat{k}\hat{l}}\right)F^{\hat{k}\hat{l}}_{\mu\nu}\epsilon + \frac{1}{2}\gamma^\mu\Gamma^{\hat{j}}P_{\mu\hat{i}\hat{j}}\epsilon \\
&\quad -\frac{1}{120}\gamma^{\mu\nu\rho}\left(\Gamma_{\hat{i}}{}^{\hat{j}} - 4\delta^{\hat{j}}_{\hat{i}}\right)S_{\hat{j}\mu\nu\rho}\epsilon + \frac{1}{2}g\left(T_{\hat{i}\hat{j}} - \frac{1}{5}T_{\hat{k}\hat{k}}\delta_{\hat{i}\hat{j}}\right)\Gamma^{\hat{j}}\epsilon \qquad (3.12)
\end{aligned}$$

where

$$\begin{aligned}
F^{\hat{i}\hat{j}}_{(2)} &= \Pi_i{}^{\hat{i}}\Pi_j{}^{\hat{j}}F^{ij}_{(2)}, & T_{\hat{i}\hat{j}} &= (\Pi^{-1})_{\hat{i}}{}^i(\Pi^{-1})_{\hat{j}}{}^j\delta^{ij}, \\
D\epsilon &= d\epsilon + \frac{1}{4}\omega_{ab}\gamma^{ab}\epsilon + \frac{1}{4}Q_{\hat{i}\hat{j}}\Gamma^{\hat{i}\hat{j}}\epsilon, & T^{ij} &= (\Pi^{-1})_{\hat{i}}{}^i(\Pi^{-1})_{\hat{j}}{}^j\delta^{\hat{i}\hat{j}}, \\
P_{(\hat{i}\hat{j})} + Q_{[\hat{i}\hat{j}]} &= (\Pi^{-1})_{\hat{i}}{}^i\left(\delta^j_i d + gA_{(1)i}{}^j\right)\Pi_j{}^{\hat{k}}\delta_{\hat{j}\hat{k}}, & S_{(3)\hat{i}} &= (\Pi^{-1})_{\hat{i}}{}^i S_{(3)i} \qquad (3.13)
\end{aligned}$$

with $\Pi_i{}^{\hat{i}}$ being the $\mathrm{SL}(5,\mathbb{R})/\mathrm{SO}(5)$ coset representative.

3.2 SO(4) $N = 2$ gauged supergravity from S^4 reduction

We now truncate the $N = 4$ gauged supergravity to $N = 2$ theory with topological mass term for the three-form field and SO(4) gauge group. In this process, the gauge group SO(5) is broken to SO(4). We will split the index i as $(\alpha, 5)$ with $\alpha = 1, \ldots, 4$. Furthermore, we will set $T_{5\alpha}$, S^α and $F^{5\alpha}$ to zero. The S^4 coordinates μ^i will be chosen to be $\mu^i = (\cos\xi\mu^\alpha, \sin\xi)$ in which μ^α satisfy $\mu^\alpha\mu^\alpha = 1$. Similar to μ^i, μ^α are coordinates on S^3. The scalar truncation is given by $T_{ij} = (T_{\alpha\beta}, T_{55}) = (X\tilde{T}_{\alpha\beta}, X^{-4})$ with $\tilde{T}_{\alpha\beta}$ being unimodular. The scalar field X will be related to the $N = 2$ dilaton.

With these truncations, the three-form field equations (3.4) and (3.5) become

$$D(X^{-4} * S^5_{(3)}) = 0 \qquad (3.14)$$

$$dS^5_{(3)} = gX^{-4} * S^5_{(3)} + \frac{1}{8}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}_{(2)} \wedge F^{\gamma\delta}_{(2)}\,. \qquad (3.15)$$

We have used $\epsilon_{5\alpha\beta\gamma\delta} = \epsilon_{\alpha\beta\gamma\delta}$. From (3.14), we see that the four-form $X^{-4} * S^5_{(3)}$ is closed. We will denote it by

$$X^{-4} * S^5_{(3)} = -F_{(4)} = -dC_{(3)} \qquad (3.16)$$

or

$$S^5_{(3)} = X^4 * F_{(4)}\,. \tag{3.17}$$

To satisfy equation (3.15), we impose the odd-dimensional self-duality condition

$$S^5_{(3)} = -gC_{(3)} + \omega_{(3)} \tag{3.18}$$

or

$$X^4 * F_{(4)} = -gC_{(3)} + \omega_{(3)} \tag{3.19}$$

where $\omega_{(3)}$, satisfying $d\omega_{(3)} = \frac{1}{8}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}_{(2)} \wedge F^{\gamma\delta}_{(2)}$, is the Chern-Simons term given by

$$\omega_{(3)} = \frac{1}{8}\epsilon_{\alpha\beta\gamma\delta}\left(F^{\alpha\beta}_{(2)} \wedge A^{\gamma\delta}_{(1)} - \frac{1}{3}gA^{\alpha\beta}_{(1)} \wedge A^{\gamma\kappa}_{(1)} \wedge A^{\kappa\delta}_{(1)}\right). \tag{3.20}$$

Equations for $S^\alpha_{(3)}$ are trivially satisfied.

For the Yang-Mills equations, it can be verified that setting $F^{5\alpha}_{(2)} = 0$ satisfies their field equations. For $F^{\alpha\beta}_{(2)}$, we find

$$D\left(X^{-2}\tilde{T}^{-1}_{\alpha\gamma}\tilde{T}^{-1}_{\beta\delta} * F^{\gamma\delta}_{(2)}\right) = -2g\tilde{T}^{-1}_{\gamma[\alpha} * D\tilde{T}_{\beta]\gamma} + \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}F^{\gamma\delta}_{(2)} \wedge F_{(4)} \tag{3.21}$$

where we have used the odd-dimensional self-duality condition.

We then consider scalar equations. Equations for $T_{5\alpha}$ are trivially satisfied while the T_{55} equation gives rise to the dilaton eqiation

$$\begin{aligned} d(X^{-1} * dX) = {} & \frac{1}{5}X^4 * F_{(4)} \wedge F_{(4)} - \frac{1}{20}X^{-2}\tilde{T}^{-1}_{\alpha\beta}\tilde{T}^{-1}_{\gamma\delta} * F^{\beta\delta}_{(2)} \wedge F^{\alpha\gamma}_{(2)} \\ & -\frac{1}{10}g^2\left[4X^{-8} - 3X^{-3}\tilde{T}_{\alpha\alpha} - 2X^2\left(\tilde{T}_{\alpha\beta}\tilde{T}_{\alpha\beta} - \frac{1}{2}(\tilde{T}_{\alpha\alpha})^2\right)\right]\epsilon_{(7)}. \end{aligned} \tag{3.22}$$

For $T_{ij} = T_{\alpha\beta}$, we find

$$\begin{aligned} D(\tilde{T}^{-1}_{\alpha\gamma} * D\tilde{T}_{\gamma\beta}) + \delta_{\alpha\beta}d(X^{-1} * dX) = {} & X^{-2}\tilde{T}^{-1}_{\alpha\gamma}\tilde{T}^{-1}_{\delta\kappa} * F^{\gamma\kappa}_{(2)} \wedge F^{\delta\beta}_{(2)} \\ & +2g^2\left[X^2\left(2\tilde{T}_{\alpha\gamma}\tilde{T}_{\gamma\beta} - \tilde{T}_{\gamma\gamma}\tilde{T}_{\alpha\beta}\right) - X^{-3}\tilde{T}_{\alpha\beta}\right]\epsilon_{(7)} \\ & +\delta_{\alpha\beta}\left[\frac{1}{5}X^4 * F_{(4)} \wedge F_{(4)} - \frac{1}{5}X^{-2}\tilde{T}^{-1}_{\gamma\delta}\tilde{T}^{-1}_{\kappa\lambda} * F^{\delta\lambda}_{(2)} \wedge F^{\kappa\gamma}_{(2)}\right. \\ & \left. -\frac{2}{5}g^2\left[2X^2\left(\tilde{T}_{\gamma\delta}\tilde{T}_{\gamma\delta} - \frac{1}{2}(\tilde{T}_{\gamma\gamma})^2\right) + X^{-8} - 2X^{-3}\tilde{T}_{\gamma\gamma}\right]\epsilon_{(7)}\right]. \end{aligned} \tag{3.23}$$

We can now use the X equation (3.22) and end up with

$$\begin{aligned} D(\tilde{T}^{-1}_{\alpha\gamma} * D\tilde{T}_{\gamma\beta}) = {} & 2g^2\left[2X^2\left(\tilde{T}_{\alpha\gamma}\tilde{T}_{\gamma\beta} - \frac{1}{2}\tilde{T}_{\gamma\gamma}\tilde{T}_{\alpha\beta}\right) - X^{-3}\tilde{T}_{\alpha\beta}\right]\epsilon_{(7)} \\ & +X^{-2}\tilde{T}^{-1}_{\alpha\gamma}\tilde{T}^{-1}_{\delta\kappa} * F^{\gamma\kappa}_{(2)} \wedge F^{\delta\beta}_{(2)} + \delta_{\alpha\beta}\left[\left\{\frac{5}{2}g^2X^2\left(\tilde{T}_{\gamma\delta}\tilde{T}_{\gamma\delta} - \frac{1}{2}(\tilde{T}_{\gamma\gamma})^2\right)\right.\right. \\ & \left.\left. +\frac{1}{2}g^2X^{-3}\tilde{T}_{\gamma\gamma}\right\}\epsilon_{(7)} - \frac{1}{4}X^{-2}\tilde{T}^{-1}_{\gamma\delta}\tilde{T}^{-1}_{\kappa\lambda} * F^{\delta\lambda}_{(2)} \wedge F^{\kappa\gamma}_{(2)}\right] \end{aligned} \tag{3.24}$$

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With all of the above truncations, we find the following ansatz for the metric and the four-form field

$$\begin{aligned} d\hat{s}_{11}^2 &= \Delta^{\frac{1}{3}} ds_7^2 + \frac{2}{g^2}\Delta^{-\frac{2}{3}} X^3 \left[X\cos^2\xi + X^{-4}\sin^2\xi \tilde{T}^{-1}_{\alpha\beta}\mu^\alpha\mu^\beta\right] d\xi^2 \\ &\quad -\frac{1}{g^2}\Delta^{-\frac{2}{3}}X^{-1}\tilde{T}^{-1}_{\alpha\beta}\sin\xi\mu^\alpha d\xi D\mu^\beta + \frac{1}{2g^2}\Delta^{-\frac{2}{3}}X^{-1}\tilde{T}^{-1}_{\alpha\beta}\cos^2\xi D\mu^\alpha D\mu^\beta, \end{aligned} \tag{3.25}$$

$$\begin{aligned} \hat{F}_{(4)} &= F_{(4)}\sin\xi + \frac{1}{g}X^4\cos\xi * F_{(4)}\wedge d\xi + \frac{1}{g^3}\Delta^{-2}U\cos^5\xi d\xi\wedge\epsilon_{(3)} \\ &\quad +\frac{1}{3!g^3}\epsilon_{\alpha\beta\gamma\delta}\Delta^{-2}X^{-3}\sin\xi\cos^4\xi\mu^\kappa\left[5\tilde{T}^{\alpha\kappa}X^{-1}dX + D\tilde{T}^{\alpha\kappa}\right]\wedge D\mu^\beta\wedge D\mu^\gamma\wedge D\mu^\delta \\ &\quad +\frac{1}{2g^3}\epsilon_{\alpha\beta\gamma\delta}\Delta^{-2}\cos^3\xi\mu^\kappa\mu^\lambda\Big[\cos^2\xi X^2\tilde{T}^{\alpha\kappa}D\tilde{T}^{\beta\lambda} - \sin^2\xi X^{-3}\delta^{\beta\lambda}D\tilde{T}^{\alpha\kappa} \\ &\quad -5\sin^2\xi\tilde{T}^{\alpha\kappa}X^{-4}\delta^{\beta\lambda}dX\Big]\wedge D\mu^\gamma\wedge D\mu^\delta\wedge d\xi + \frac{1}{2g^2}\cos\xi\epsilon_{\alpha\beta\gamma\delta}\times \\ &\quad \times\left[\frac{1}{2}\cos\xi\sin\xi X^{-4}D\mu^\gamma - \left(X^{-4}\sin^2\xi\mu^\gamma + X^2\cos^2\xi\tilde{T}^{\gamma\kappa}\mu^\kappa\right)d\xi\right]\wedge F^{\alpha\beta}_{(2)}\wedge D\mu^\delta \end{aligned} \tag{3.26}$$

where

$$\begin{aligned} U &= \sin^2\xi\left(X^{-8} - X^{-3}\tilde{T}_{\alpha\alpha}\right) + \cos^2\xi\mu^\alpha\mu^\beta\left(2X^2\tilde{T}_{\alpha\gamma}\tilde{T}_{\gamma\beta} - X^2\tilde{T}_{\alpha\beta}\tilde{T}_{\gamma\gamma} - X^{-3}\tilde{T}_{\alpha\beta}\right) \\ \epsilon_{(3)} &= \frac{1}{3!}\epsilon_{\alpha\beta\gamma\delta}\mu^\alpha D\mu^\beta\wedge D\mu^\gamma\wedge D\mu^\delta\,. \end{aligned} \tag{3.27}$$

All of the above equations reduce to the pure $N = 2$ gauged supergravity with SU(2) gauge group for $\tilde{T}_{\alpha\beta} = \delta_{\alpha\beta}$ after using various relations given in [21]. Note that for $\tilde{T}_{\alpha\beta} = \delta_{\alpha\beta}$, equation (3.24) gives

$$* F^{\alpha\gamma}_{(2)}\wedge F^{\gamma\beta}_{(2)} = \frac{1}{4}\delta_{\alpha\beta} * F^{\gamma\delta}_{(2)}\wedge F^{\delta\gamma}_{(2)} \tag{3.28}$$

which means that the SO(4) gauge fields $A^{\alpha\beta}_{(1)}$ must be truncated to those of SU(2) satisfying $F^{\alpha\beta}_{(2)} = \pm\frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}F^{\gamma\delta}_{(2)}$. This is expected since there are only three vector fields in the pure gauged supergravity which only admit SU(2) gauging.

The above equations can be obtained from the Lagrangian

$$\begin{aligned} \mathcal{L}_7 &= R*\mathbb{I} - \frac{1}{4}X^{-2}\tilde{T}^{-1}_{\alpha\gamma}\tilde{T}^{-1}_{\beta\delta} * F^{\alpha\beta}_{(2)}\wedge F^{\gamma\delta}_{(2)} - \frac{1}{4}\tilde{T}^{-1}_{\alpha\beta} * D\tilde{T}_{\beta\gamma}\wedge\tilde{T}^{-1}_{\gamma\delta}D\tilde{T}_{\delta\alpha} \\ &\quad -\frac{1}{2}X^4 * F_{(4)}\wedge F_{(4)} + \frac{1}{8}\epsilon_{\alpha\beta\gamma\delta}C_{(3)}\wedge F^{\alpha\beta}_{(2)}\wedge F^{\gamma\delta}_{(2)} - 5X^{-2} * dX\wedge dX \\ &\quad -\frac{1}{2}gF_{(4)}\wedge C_{(3)} - V*\mathbb{I} \end{aligned} \tag{3.29}$$

where the scalar potential is given by

$$V = \frac{1}{2}g^2\left[X^{-8} - 2X^{-3}\tilde{T}_{\alpha\alpha} + 2X^2\left(\tilde{T}_{\alpha\beta}\tilde{T}_{\alpha\beta} - \frac{1}{2}\tilde{T}^2_{\alpha\alpha}\right)\right]. \tag{3.30}$$

For $\tilde{T}_{\alpha\beta} = \delta_{\alpha\beta}$, we find $\tilde{T}_{\alpha\alpha} = \tilde{T}_{\alpha\beta}\tilde{T}_{\alpha\beta} = 4$. The above potential becomes

$$V = \frac{1}{2}g^2\left(X^{-8} - 8X^{-3} - 8X^2\right) \tag{3.31}$$

which is exactly the same as that given in [17] up to a redefinition of the coupling constant g.

We can also check another truncation namely to $\mathrm{U}(1)\times\mathrm{U}(1)$ gauged supergravity. To preserve $\mathrm{SO}(2)\times\mathrm{SO}(2)$ symmetry, we take the scalar matrix to be

$$\tilde{T}_{\alpha\beta} = \begin{pmatrix} e^{\frac{\phi_1}{\sqrt{2}}} & & & \\ & e^{\frac{\phi_1}{\sqrt{2}}} & & \\ & & e^{-\frac{\phi_1}{\sqrt{2}}} & \\ & & & e^{-\frac{\phi_1}{\sqrt{2}}} \end{pmatrix} \tag{3.32}$$

and define $X = e^{-\frac{\phi_2}{\sqrt{10}}}$. The potential (3.30) becomes

$$V = \frac{1}{2}g^2\left[e^{\frac{8\phi_2}{\sqrt{10}}} - 8e^{-\frac{2\phi_2}{\sqrt{10}}} - 4e^{\frac{3\phi_2}{\sqrt{10}}}\left(e^{\frac{\phi_1}{\sqrt{2}}} + e^{-\frac{\phi_1}{\sqrt{2}}}\right)\right] \tag{3.33}$$

which takes the same form as that given in [23]. Finally, it should be remarked that the three-form field equation coming from the Lagrangian (3.29) needs to be supplemented with the odd-dimensional self-duality condition as in the pure SU(2) gauged supergravity discussed in [17].

The nine scalars, parametrized by $\tilde{T}_{\alpha\beta}$, in the dimensionally reduced theory are encoded in the $\mathrm{SL}(4,\mathbb{R})/\mathrm{SO}(4)$ coset manifold. Therefore, in order to compare the result with gauged $N=2$ SO(4) supergravity given in the previous section, we need to use the relation between $\mathrm{SL}(4,\mathbb{R})/\mathrm{SO}(4)$ and $\mathrm{SO}(3,3)/\mathrm{SO}(3)\times\mathrm{SO}(3)$ coset manifolds. This is given in [15]. For the details of this mapping, the reader is referred to [15]. We will only give the $\mathrm{SO}(3,3)/\mathrm{SO}(3)\times\mathrm{SO}(3)$ coset representative $L^A_{\ I} = (L^i_{\ I}, L^r_{\ I})$ and that of $\mathrm{SL}(4,\mathbb{R})/\mathrm{SO}(4)$, $\mathcal{V}^\alpha_R$ with $R=1,\ldots,4$,

$$L^A_{\ I} = \frac{1}{4}\Gamma_I^{\alpha\beta}\eta^A_{RS}\mathcal{V}^R_{\ \alpha}\mathcal{V}^S_{\ \beta} \tag{3.34}$$

where Γ^I and η^A are chirally projected SO(3,3) gamma matrices.

It can be shown that the scalar potential can be written as

$$\begin{aligned} V &= \frac{1}{4}e^{-\sigma}\left(C^{ir}C_{ir} - \frac{1}{9}C^2\right) + 16h^2e^{4\sigma} - \frac{4\sqrt{2}}{3}he^{\frac{3\sigma}{2}}C \\ &= \frac{1}{8}e^{-\sigma}\left(T_{\alpha\beta}T_{\alpha\beta} - \frac{1}{2}T_{\alpha\alpha}^2\right) + 2T_{\alpha\alpha}he^{\frac{3\sigma}{2}} + 16h^2e^{4\sigma} \end{aligned} \tag{3.35}$$

This form is similar to the potential (3.30) if $\tilde{T}_{\alpha\beta}$ is identified with $T_{\alpha\beta}$. Note that $T_{\alpha\beta}$ and C, C^{ir} contain the gauge coupling g_1 and g_2. In order to compare the Lagrangian of the two theories, we need to multiply the Lagrangian (2.4) by two and separate the coupling constants g_1 and g_2 from the structure constants $f_{IJK} = (g_1\epsilon_{ijk}, g_2\epsilon_{rst})$. With these, the two scalar potentials are exactly the same if we identify

$$g_2 = g_1 = -16h = -2g\,. \tag{3.36}$$

We also need to redefine the following fields in the Lagrangian (2.4):

$$H_{(4)} \to \frac{F_{(4)}}{\sqrt{2}}, \qquad\qquad C_{(3)} \to \frac{C_{(3)}}{\sqrt{2}},$$

$$F^I = \frac{1}{4}\Gamma^I_{\alpha\beta}F^{\alpha\beta}_{(2)} \qquad \text{or} \qquad F^{\alpha\beta}_{(2)} = -\frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}\Gamma^I_{\gamma\delta}F^I$$
$$X = e^{-\frac{\sigma}{2}}\,. \tag{3.37}$$

By using (3.34), it can also be checked that

$$\tilde{T}^{-1}_{\alpha\gamma}\tilde{T}^{-1}_{\beta\delta} = \frac{1}{4}\Gamma^I_{\alpha\beta}\Gamma^J_{\gamma\delta}\left(L^i{}_I L_{iJ} + L^r{}_I L_{rJ}\right). \tag{3.38}$$

The field equations from the two theories also match.

We now move to supersymmetry transformations of fermions. The maximal $N = 4$ theory contains the gravitini ψ_μ and the spin-$\frac{1}{2}$ fields $\lambda_{\hat{i}}$. The latter is decomposed into (λ_R, λ_5). The $\mathrm{SO}(5)_c$ $\Gamma^{\hat{i}}$ gamma matrices are accordingly decomposed as $\Gamma^{\hat{i}} = (\Gamma^R, \Gamma^5)$. $\Gamma^5 = \Gamma^1\Gamma^2\Gamma^3\Gamma^4$ acts as the chirality matrix of SO(4). Following [18], we make the truncation

$$\epsilon^- = \psi^-_\mu = \lambda^-_5 = \lambda^+_\alpha = 0\,. \tag{3.39}$$

$\epsilon^\pm$ satisfy $\Gamma^5\epsilon^\pm = \pm\epsilon^\pm$ with $\epsilon = \epsilon^+ + \epsilon^-$. We will now drop $\pm$ superscript from ϵ, λ and ψ_μ.

In accordance with the bosonic truncation $T^{ij} = (T^{\alpha\beta}, T^{55}) = (X\tilde{T}^{\alpha\beta}, X^{-4})$, we truncate the $\mathrm{SL}(5,\mathbb{R})/\mathrm{SO}(5)$ coset representative as $\Pi_i{}^{\hat{i}} = (\Pi_\alpha{}^R, \Pi_5{}^{\hat{5}})$. With the identification $\Pi_\alpha{}^R = X^{-\frac{1}{2}}\mathcal{V}_\alpha{}^R$ and $\Pi_5{}^{\hat{5}} = X^2$, we can write $\tilde{T}^{\alpha\beta}$ in term of $\mathrm{SL}(4,\mathbb{R})/\mathrm{SO}(4)$ coset representative $\mathcal{V}_\alpha{}^R$ as

$$\tilde{T}^{\alpha\beta} = (\mathcal{V}^{-1})^\alpha_R(\mathcal{V}^{-1})^\beta_S\delta^{RS} \qquad \text{and} \qquad \tilde{T}_{RS} = (\mathcal{V}^{-1})^\alpha_R(\mathcal{V}^{-1})^\beta_S\delta_{\alpha\beta}\,. \tag{3.40}$$

We then find that equations (3.11) and (3.12) become

$$\delta\psi_\mu = D_\mu\epsilon - \frac{1}{20}g(X\tilde{T}_{RR} + X^{-4})\gamma_\mu\epsilon - \frac{1}{40\sqrt{2}}X^{-1}\left(\gamma_\mu{}^{\nu\rho} - 8\delta^\nu_\mu\gamma^\rho\right)\Gamma_{RS}F^{RS}_{\nu\rho}\epsilon$$
$$-\frac{1}{60}X^{-2}\left(\gamma_\mu{}^{\nu\rho\sigma} - \frac{9}{2}\delta^\nu_\mu\gamma^{\rho\sigma}\right)S^5_{\nu\rho\sigma}\epsilon, \tag{3.41}$$
$$\delta\lambda_R = \frac{1}{4}\gamma^\mu\Gamma_R X^{-1}\partial_\mu X\epsilon + \frac{1}{2}\Gamma^S\gamma^\mu P_{RS}\epsilon + \frac{1}{16\sqrt{2}}X^{-1}\gamma^{\mu\nu}\left(\Gamma_{ST}\Gamma_R - \frac{1}{5}\Gamma_R\Gamma_{ST}\right)F^{ST}_{\mu\nu}\epsilon$$
$$-\frac{1}{10}gX^{-4}\Gamma_R\epsilon - \frac{1}{2}gX\left(\tilde{T}_{RS} - \frac{1}{5}\tilde{T}_{TT}\delta_{RS}\right)\Gamma^S\epsilon - \frac{1}{120}X^{-2}\gamma^{\mu\nu\rho}\Gamma_R S^5_{\mu\nu\rho}\epsilon\,. \tag{3.42}$$

The constraint $\Gamma^{\hat{i}}\lambda_{\hat{i}} = 0$ imposes the condition $\lambda^+_5 = -\Gamma^R\lambda^-_R$. Therefore, the independent fields will be ψ_μ and λ_R. This is the reason for excluding $\delta\lambda_5$ in the above equations. We then identify $\Gamma^R\lambda_R$ with χ and $\hat{\lambda}_R = \lambda_R - \frac{1}{4}\Gamma_R\Gamma^S\lambda_S$ with λ^r in (2.17). Note that $\hat{\lambda}_R$ has only three independent components due to the condition $\Gamma^R\hat{\lambda}_R = 0$.

With these and the odd-dimensional self-duality, we end up with, after some gamma matrix algebra,

$$\delta\psi_\mu = D_\mu\epsilon - \frac{1}{20}gX\tilde{T}\gamma_\mu\epsilon - \frac{1}{40\sqrt{2}}X^{-1}\left(\gamma_\mu{}^{\nu\rho} - 8\delta^\nu_\mu\gamma^\rho\right)\Gamma_{RS}F^{RS}_{\nu\rho}\epsilon$$
$$-\frac{1}{20}gX^{-4}\gamma_\mu\epsilon - \frac{1}{480}X^2\left(3\gamma_\mu{}^{\nu\rho\sigma\tau} - 8\delta^\nu_\mu\gamma^{\rho\sigma\tau}\right)F_{\nu\rho\sigma\tau}\epsilon, \tag{3.43}$$

$$\delta\chi = X^{-1}\gamma^\mu\partial_\mu X\epsilon - \frac{2}{5}gX^{-4}\epsilon + \frac{1}{10}gX\tilde{T}_{RR}\epsilon$$
$$-\frac{1}{120}X^2\gamma^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}\epsilon - \frac{1}{20\sqrt{2}}X^{-1}\gamma^{\mu\nu}\Gamma_{RS}F^{RS}_{\mu\nu}\epsilon, \tag{3.44}$$
$$\delta\hat{\lambda}_R = -\frac{1}{2}\gamma^\mu\Gamma^S P_{\mu RS}\epsilon - \frac{1}{8}gX\tilde{T}_{SS}\Gamma_R\epsilon + \frac{1}{2}gX\tilde{T}_{RS}\Gamma^S\epsilon$$
$$-\frac{1}{8\sqrt{2}}X^{-1}\gamma^{\mu\nu}\Gamma_S\left(F^{RS}_{\mu\nu} + \frac{1}{2}\epsilon_{RSTU}F^{TU}_{\mu\nu}\right)\epsilon\,. \tag{3.45}$$

In the above equations, we have used the following definitions

$$P_{RS} = (\mathcal{V}^{-1})^\alpha_{(R}\left(\delta^\beta_\alpha d + gA_{(1)\alpha}{}^\beta\right)\mathcal{V}_\beta{}^T\delta_{S)T},$$
$$Q_{RS} = (\mathcal{V}^{-1})^\alpha_{[R}\left(\delta^\beta_\alpha d + gA_{(1)\alpha}{}^\beta\right)\mathcal{V}_\beta{}^T\delta_{S]T},$$
$$D\epsilon = d\epsilon + \frac{1}{4}\omega_{ab}\gamma^{ab} + \frac{1}{4}Q_{RS}\Gamma^{RS}\,. \tag{3.46}$$

Notice that with our convention for $\Gamma^5\epsilon = \epsilon$, Γ_{RS} is anti-self dual. The field strength $F^{RS}_{(2)}$ appearing in (3.43) and (3.44) must be accordingly anti-self dual. This should be identified with the SU(2) field strength $F^i_{(2)}$ in (2.15) and (2.16). On the other hand, the self dual part of $F^{RS}_{(2)}$ appears in (3.45) and should be identified with $F^r_{(2)}$ in (2.17).

In more detail, after using gamma matrix identities such as $\gamma_\mu\gamma^{\nu\rho} = \gamma_\mu{}^{\nu\rho} + 2\delta^{[\nu}_\mu\gamma^{\rho]}$, we can rewrite equation (2.15) as

$$\delta\psi_\mu = 2D_\mu\epsilon - \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\gamma_\mu\epsilon - \frac{1}{120\sqrt{2}}e^{-\sigma}H_{\rho\sigma\lambda\tau}\left(3\gamma_\mu{}^{\rho\sigma\lambda\tau} - 8\delta^\rho_\mu\gamma^{\sigma\lambda\tau}\right)\epsilon$$
$$-\frac{i}{10}e^{\frac{\sigma}{2}}F^i_{\rho\sigma}\sigma^i\left(\gamma_\mu{}^{\rho\sigma} - 8\delta^\rho_\mu\gamma^\sigma\right)\epsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon\,. \tag{3.47}$$

Using the relation $C = -\frac{3}{2\sqrt{2}}g_1\tilde{T}$ given in [15] with the relation $g_2 = g_1 = -2g$ and identifying $F_{RS}\Gamma^{RS} = 2\sqrt{2}iF^i\sigma^i$, we find that equation (3.43) matches with (3.47). Similarly, equation (3.44) matches with (2.16). Note that in order to match the gravitino variation, we need to multiply (3.43) by two.

Comparing (2.17) and (3.45) is more complicated since various terms are not related to each other in a simple way. For example, we should write the anti-self dual part of Γ_{RS} in terms of the anti-self dual t' Hooft symbols $\bar{\eta}^i_{RS}$ and Pauli matrices σ^i

$$\Gamma^{(-)}_{RS} = i\sigma^i\bar{\eta}_{iRS} \tag{3.48}$$

and similarly for the self dual part

$$\Gamma^{(+)}_{RS} = i\sigma^r\eta_{rRS}\,. \tag{3.49}$$

Accordingly, we should identify

$$F^i = \frac{1}{2}\bar{\eta}^i_{RS}F^{RS} \qquad \text{and} \qquad F^r = \frac{1}{2}\eta^r_{RS}\left(F^{RS}_{\mu\nu} + \frac{1}{2}\epsilon_{RSTU}F^{TU}_{\mu\nu}\right). \tag{3.50}$$

Equation (3.45) should then match with (2.17), but we refrain from giving the full detail here due to the complicated algebra.

4 Embedding seven-dimensional RG flow to eleven dimensions

In this section, we will use the reduction ansatz obtained in the previous section to uplift some seven-dimensional solutions. The dimensional reduction gives rise to the condition $g_2 = g_1$. This makes the supersymmetric AdS_7 critical point with $\mathrm{SO}(3)_{\mathrm{diag}}$ symmetry found in [16] disappears. Accordingly, the flow solution given in [16] cannot be uplifted to eleven dimensions with the present reduction ansatz. However, to give examples of the uplifted solutions, we will study other solutions in the case of $g_2 = g_1$.

4.1 Uplifting AdS_7 solutions

We now further truncate the nine scalars given by $\tilde{T}_{\alpha\beta}$ to one scalar invariant under $\mathrm{SO}(3)_{\mathrm{diag}} \subset \mathrm{SO}(3)\times\mathrm{SO}(3) \sim \mathrm{SO}(4)$. This scalar sector has already been studied in [16]. We will give more solutions in this section. Under $\mathrm{SO}(3)_{\mathrm{diag}}$, the nine scalars transform as $\mathbf{1}+\mathbf{3}+\mathbf{5}$. There is only one singlet. It can be checked that the $\mathrm{SO}(3)_{\mathrm{diag}}$ singlet correspond to

$$\mathcal{V}^R{}_\alpha = \begin{pmatrix} e^{\frac{\phi}{2}} & & & \\ & e^{\frac{\phi}{2}} & & \\ & & e^{\frac{\phi}{2}} & \\ & & & e^{-\frac{3\phi}{2}} \end{pmatrix} \qquad \text{or} \qquad \tilde{T}_{\alpha\beta} = \begin{pmatrix} e^{\phi} & & & \\ & e^{\phi} & & \\ & & e^{\phi} & \\ & & & e^{-3\phi} \end{pmatrix}. \tag{4.1}$$

$\tilde{T}_{\alpha\beta}$ can be written more compactly as $\tilde{T}_{\alpha\beta} = (\delta_{ab}e^{\phi}, e^{-3\phi})$ for $a, b = 1, 2, 3$. By using (3.34) and the explicit form of Γ^I and η^A given in [15], it is easy to verify that this $\mathcal{V}$ precisely gives the $\mathrm{SO}(3,3)/\mathrm{SO}(3)\times\mathrm{SO}(3)$ coset representative L used in [16].

Using this and the relation $X = e^{-\frac{\sigma}{2}}$, we find the scalar potential

$$V = \frac{1}{2}g^2 e^{-\sigma}\left[e^{5\sigma+e^{-6\phi}} - 6e^{-2\phi} - 3e^{2\phi} - 2e^{\frac{5}{2}\sigma-3\phi}\left(1+3e^{4\phi}\right)\right]. \tag{4.2}$$

This potential admits two AdS_7 critical points given by

$$\sigma = \phi = 0, \qquad V_0 = -480h^2 \tag{4.3}$$

$$\sigma = -\frac{1}{10}\ln 2, \qquad \phi = -\frac{1}{4}\ln 2, \qquad V_0 = -160\times 2^{\frac{3}{5}}h^2 \tag{4.4}$$

where we have used $g = 8h$ or equivalently $g_1 = -16h$ as given in [16]. By using the BPS equations given in [16], which are repeated below, we see that the second critical point is non-supersymmetric. Scalar masses at this critical point can be computed to be

$\mathrm{SO}(3)_{\mathrm{diag}}$	m^2L^2
$\mathbf{1}$	-12
$\mathbf{1}$	12
$\mathbf{3}$	0
$\mathbf{5}$	-12

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where the AdS_7 radius is given by $L = \sqrt{-\frac{15}{V_0}}$. The three massless scalars are the expected Goldstone bosons corresponding to the symmetry breaking of SO(4) to SO(3). One of the **1** and **5** scalars have masses below the BF bound $m^2L^2 = -9$, so this critical point is unstable.

The first critical point is the trivial point preserving all supersymmetries and the full SO(4) gauge symmetry. The scalar masses can be found in [16]. We will now uplift this AdS_7 vacuum to eleven dimensions. We begin with the coordinates $\mu^\alpha = (\cos\psi\hat{\mu}^a, \sin\psi)$ in which $\hat{\mu}^a\hat{\mu}^a = 1$. Since $\sigma = \phi = 0$, we then find $\Delta = 1$ and

$$ds^2_{11} = e^{\frac{2r}{L_{UV}}} dx^2_{1,5} + dr^2 + \frac{1}{32h^2}\left[d\xi^2 + \frac{1}{4}\cos^2\xi\left(d\psi^2 + \cos^2\psi d\Omega^2_2\right)\right] \tag{4.5}$$

$$\hat{F}_{(4)} = -\frac{3}{256h^3}\cos^5\xi d\xi\wedge\epsilon_{(3)} \tag{4.6}$$

where $d\Omega^2_2$ is the metric on the two-sphere. The eleven dimensional geometry is given by $AdS_7 \times S^4$. Turning on the dilaton σ would deform the four-sphere but leave the S^3 inside invariant. If $\phi, \sigma \neq 0$, the metric would be further deformed in such a way that the S^2 part described by $d\Omega^2_2$ is invariant. The unbroken symmetry in this case is the SO(3) isometry of this S^2 identified with the unbroken $\mathrm{SO}(3)_{\mathrm{diag}}$. The SO(3) critical point is however unstable. Therefore, we will not consider AdS_7 solution with SO(3) symmetry.

4.2 Uplifting RG flows to non-conformal SO(3) super Yang-Mills

To give more examples, we will study RG flow solutions to non-conformal Super Yang-Mills theories in the IR. We will work in the theory of section 2. With $g_2 = g_1$ and the standard domain wall metric ansatz $ds^2_7 = e^{A(r)}dx^2_{1,5} + dr^2$, the BPS equations taken from [16] become

$$\phi' = -4e^{-\frac{\sigma}{2}-3\phi}\left(e^{4\phi}-1\right)h, \tag{4.7}$$

$$\sigma' = \frac{8}{5}e^{-\frac{\sigma}{2}-3\phi}\left(1+3e^{4\phi}-4e^{\frac{5}{2}\sigma+3\phi}\right)h, \tag{4.8}$$

$$A' = \frac{4}{5}e^{-\frac{\sigma}{2}-3\phi}\left(1+3e^{4\phi}+e^{\frac{5}{2}\sigma+3\phi}\right) \tag{4.9}$$

in which $\frac{d}{dr}$ is denoted by $'$. After changing to the new coordinate $\tilde{r}$ given by $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$, we find the solution

$$16h\tilde{r} = \ln\left[\frac{1+e^\phi}{1-e^\phi}\right] - 2\tan^{-1}\phi + C_1, \tag{4.10}$$

$$\sigma = \frac{2}{5}\left[\phi - \ln\left[1 + 12C_2 - 12C_2e^{4\phi}\right]\right], \tag{4.11}$$

$$A = \frac{1}{4}\left[\phi - 2\ln(1-e^{4\phi})\right] - \frac{1}{8}\sigma\,. \tag{4.12}$$

The solution interpolates between an AdS_7 in the UV, $\tilde{r} \sim r \to \infty$, and a domain wall in the IR, $4h\tilde{r} \to \tilde{C}$, for a constant $\tilde{C}$.

At the UV, the solution becomes

$$\sigma \sim \phi \sim e^{-16hr} \sim e^{-\frac{4r}{L_{UV}}}, \qquad A \sim 4hr \sim \frac{r}{L_{UV}}\,. \tag{4.13}$$

The eleven-dimensional metric is given by (4.5).

In the IR, we find that ϕ blows up as

$$\phi \sim -\ln(4h\tilde{r} - \tilde{C}) \tag{4.14}$$

for a constant $\tilde{C}$. The behaviour of σ depends on the value of the integration constant C_2.

For $C_2 = 0$, we find

$$\sigma \sim -\frac{2}{5}\ln(4h\tilde{r} - \tilde{C}) \sim -\frac{1}{2}\ln(4hr - C) \tag{4.15}$$

where we have used the relation between $\tilde{r}$ and r in the IR limit with C being another integration constant. The seven-dimensional metric is given by

$$ds_7^2 = (4hr - C)^2 dx_{1,5}^2 + dr^2\,. \tag{4.16}$$

For $C_2 \neq 0$, the solution becomes

$$\begin{aligned}\sigma &\sim \frac{6}{5}\ln(4h\tilde{r} - \tilde{C}) \sim \frac{3}{4}\ln(4hr - C),\\ ds_7^2 &= (4hr - C)^{\frac{3}{4}} dx_{1,5}^2 + dr^2\,.\end{aligned} \tag{4.17}$$

Both cases give $V \to -\infty$, so the solution is physical by the criterion of [24].

We now look at the eleven-dimensional geometry. For $C_2 = 0$ and $C_2 \neq 0$, the eleven dimensional metric is given respectively by

$$\begin{aligned}ds_{11}^2 &= (1 - \sin^2\xi\cos^2\psi)^{-\frac{1}{3}}\left[\left(\frac{14}{3}h\rho\right)^2 dx_{1,5}^2 + d\rho^2\right] + \frac{1}{32h^2}(1 - \sin^2\xi\cos^2\psi)^{-\frac{2}{3}} \times\\ &\quad\times\left[\left(\frac{14}{3}h\rho\right)^{-\frac{27}{7}}\sin^2\xi\cos^2\psi d\xi^2 + \frac{1}{4}\sin\xi\sin(2\psi)\left(\frac{14}{3}h\rho\right)^{-\frac{1}{2}} d\psi d\xi\right.\\ &\quad\left. + \frac{1}{4}\left(\frac{14}{3}h\rho\right)^{-\frac{20}{7}} d\psi^2 + \frac{1}{4}\cos^2\psi\left(\frac{14}{3}h\rho\right)^{\frac{10}{7}} d\Omega_2^2\right],\end{aligned} \tag{4.18}$$

$$\begin{aligned}ds_{11}^2 &= (\cos\xi\cos\psi)^{-\frac{2}{3}}\left[\left(\frac{14}{3}h\rho\right)^{\frac{13}{14}} dx_{1,5}^2 + d\rho^2\right] + \frac{1}{32h^2}(\cos\xi\cos\psi)^{-\frac{4}{3}} \times\\ &\quad\times\left[\left(\frac{14}{3}h\rho\right)^{\frac{17}{14}}(1 - \sin^2\xi\cos^2\psi)\, d\xi^2 - \frac{1}{4}\sin\xi\sin(2\psi)\left(\frac{14}{3}h\rho\right)^{\frac{7}{4}} d\xi d\psi\right.\\ &\quad\left. + \frac{1}{4}\cos^2\xi\left(\frac{14}{3}h\rho\right)^{\frac{10}{7}}(\sin^2\psi d\psi^2 + \cos^2\psi d\Omega_2^2)\right]\end{aligned} \tag{4.19}$$

where $\left(\frac{14}{3}h\rho\right)^{\frac{6}{7}} = 4hr - C$.

As expected, when turning on ϕ and σ, the warped factors involve coordinates (ξ, ψ). The S^4 is then deformed leaving the S^2 intact. If only $\sigma \neq 0$, the S^3 part of the internal metric would be invariant as pointed in [17]. The deformation with only $\phi \neq 0$ is not possible since the BPS equation for σ would imply $\phi = 0$ as pointed out in [16].

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5 Conclusions

In this paper, we have constructed $N = 2$ SO(4) gauged supergravity in seven dimensions with topological mass term. The resulting theory admit AdS_7 vacua and could be useful in the context of the AdS/CFT correspondence. The resulting reduction ansatz has been found by truncating the S^4 reduction leading to $N = 4$ SO(5) gauged supergravity and can be used to uplift seven-dimensional solutions to eleven dimensions. We have also constructed new seven-dimensional RG flow solutions and uplifted the resulting solutions to eleven dimensions. The flows can be interpreted as deformations of the UV $N = (1, 0)$ SCFT in six dimensions with SO(4) symmetry to non-conformal SYM with $\mathrm{SO}(3)_{\mathrm{diag}}$ symmetry. These deformations are driven by vacuum expectation values of dimension 4 operators. Additionally, the result of this paper can be used to uplift flows to SO(2) non-conformal gauge theories studied in [16] for $g_2 = g_1$.

However, the RG flow between two supersymmetric AdS_7 critical points recently found in [16] cannot be uplifted by using the reduction ansatz constructed here. It would be interesting to find an embedding of this solution in 10 or 11 dimensions. It is also interesting to extend the reduction ansatz given here to non-compact gauge groups SO(3, 1) and SO(2, 2). The internal manifold should involve hyperbolic spaces $H^{3,1}$ and $H^{2,2}$, respectively. Other possible non-compact gauge groups are $\mathrm{SL}(3, \mathbb{R})$, SO(2, 1) and $\mathrm{SO}(2, 2) \times \mathrm{SO}(2, 1)$. It would be very interesting to find higher dimensional origins for these gauge groups as well. Finally, more insight to six-dimensional gauge theories might be gained from studying these seven-dimensional gauged supergravities via $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence. We hope to come back to these issues in future works.

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Noncompact gauging of N=2 7D supergravity and AdS/CFT holography

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Abstract: Half-maximal gauged supergravity in seven dimensions coupled to n vector multiplets contains $n+3$ vectors and $3n+1$ scalars parametrized by $\mathbb{R}^+\times \mathrm{SO}(3,\mathrm{n})/\mathrm{SO}(3)\times \mathrm{SO}(\mathrm{n})$ coset manifold. The two-form field in the gravity multiplet can be dualized to a three-form field which admits a topological mass term. Possible non-compact gauge groups take the form of $G_0\times H\subset \mathrm{SO}(3,\mathrm{n})$ with a compact group H. G_0 is one of the five possibilities; $\mathrm{SO}(3,1)$, $\mathrm{SL}(3,\mathbb{R})$, $\mathrm{SO}(2,2)$, $\mathrm{SO}(2,1)$ and $\mathrm{SO}(2,2)\times\mathrm{SO}(2,1)$. We investigate all of these possible non-compact gauge groups and classify their vacua. Unlike the gauged supergravity without a topological mass term, there are new supersymmetric AdS_7 vacua in the $\mathrm{SO}(3,1)$ and $\mathrm{SL}(3,\mathbb{R})$ gaugings. These correspond to new $N=(1,0)$ superconformal field theories (SCFT) in six dimensions. Additionally, we find a class of $AdS_5\times S^2$ and $AdS_5\times H^2$ backgrounds with $\mathrm{SO}(2)$ and $\mathrm{SO}(2)\times\mathrm{SO}(2)$ symmetries. These should correspond to $N=1$ SCFTs in four dimensions obtained from twisted compactifications of six-dimensional field theories on S^2 or H^2. We also study RG flows from six-dimensional $N=(1,0)$ SCFT to $N=1$ SCFT in four dimensions and RG flows from a four-dimensional $N=1$ SCFT to a six-dimensional SYM in the IR. The former are driven by a vacuum expectation value of a dimension-four operator dual to the supergravity dilaton while the latter are driven by vacuum expectation values of marginal operators.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Supergravity Models

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Contents

1 Introduction

Gauged supergravities play an important role in string/M theory compactification and gauge/gravity correspondence. Generally, a gauge supergravity theory admits many types of gauge groups namely compact, non-compact and non-semisimple groups, and different types of gauge groups give rise to different vacuum structures. Gauged supergravity theories may be accordingly classified into two categories by the vacua they admit. AdS supergravities are theories admitting a maximally supersymmetric AdS space as a vacuum solution while those with a half-maximally supersymmetric domain wall vacuum are called domain-wall supergravities. The former is useful in the context of the AdS/CFT correspondence [1], and the latter is relevant in the DW/QFT correspondence [2, 3].

The study of $N = (1, 0)$ superconformal field theories (SCFT) in the context of $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence has originally done by orbifolding the $AdS_7 \times S^4$ geometry of M-theory giving rise to the gravity dual of $N = (2,0)$ SCFT [4–6]. And, recently, many AdS_7 solutions to type IIA string theory have been identified in [7]. These backgrounds are dual to $N = (1,0)$ SCFTs in six dimensions, and the holographic study of these SCFTs has been given in [8]. Furthermore, a number of $N = (1,0)$ SCFTs in six dimensions have been found and classified in the context of F-theory in [9]. It would be desirable to have a description of these SCFT in terms of the gravity solutions to seven-dimensional gauged supergravity. However, it has been pointed out in [10] that AdS_7 solutions found in [7] cannot be obtained from seven-dimensional gauged supergravity.

In the framework of seven-dimensional gauged supergravity, there are only a few results in the holography of $N = (1,0)$ SCFTs. It has been proposed in [11] that the $N = (1,0)$ SCFTs arising in the M5-brane world-volume theories should be described by $N = 2$ seven-dimensional gauged supergravity and its matter-coupled version. A non-supersymmetric holographic RG flow within pure $N = 2$ gauged supergravity has been studied in [12], and recently, new supersymmetric AdS_7 critical points and holographic RG flows between these critical points have been explored in [13]. The gauged supergravity considered in [13] is the $N = 2$ gauged supergravity coupled to three vector multiplets resulting in $SO(4) \sim SU(2) \times SU(2)$ gauge group with two coupling constants for the two SU(2)'s. When these couplings are equal, the theory can be embedded in eleven dimensions by using the reduction ansatz recently obtained in [14].

To find more supersymmetric AdS_7 backgrounds, in this paper, we will consider the $N = 2$ gauged supergravity in seven dimensions coupled to a number of vector multiplets with non-compact gauge groups. The gauged supergravity is obtained from coupling pure $N = 2$ supergravity constructed in [15] to vector multiplets [16]. Furthermore, the two-form field in the supergravity multiplet can be dualized to a three-form field [17]. It turns out to be possible to add a topological mass term to this three-form field resulting in a gauged supergravity with a massive three-form field [18]. The latter differs considerably from the theory without topological mass in the sense that it is possible to have maximally supersymmetric AdS_7 backgrounds.

We will see that there are new AdS_7 critical points for non-compact gauging of the $N = 2$ supergravity with topological mass term. These provide more examples of AdS_7 solutions with sixteen supercharges. We will also find that some non-compact gauge groups admit $AdS_5 \times S^2$ and $AdS_5 \times H^2$ geometries as a background solution. In the context of twisted field theories, these solutions should describe a six-dimensional SCFT wrapped on a two-dimensional Riemann surface. In the IR, the six-dimensional SCFT would flow to another SCFT in four dimensions. These results give new AdS_5 backgrounds dual to $N = 1$ four-dimensional SCFTs.

The holographic study of twisted field theories has originally been applied to $N = 4$ SYM [19]. Until now, the method has been applied to other dimensions, see for example [20–23]. In [23], AdS_5 solutions from a truncation of the maximal $N = 4$ gauged supergravity in seven dimensions have been found. These AdS_5 geometries correspond to a class of $N = 1$ SCFTs in four dimensions obtained from M5-branes wrapped on complex curves. In this paper, we will give more examples of these $N = 1$ SCFTs by finding new AdS_5 geometries with eight supercharges in the half-maximal $N = 2$ gauged supergravity. We

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also give some examples of RG flows from six-dimensional SCFTs to these four-dimensional SCFTs. Furthermore, we find an RG flow from a four-dimensional $N=1$ SCFT in the UV to a six-dimensional $N=(1,0)$ SYM in the IR. This flow gives another example of the flows considered in [24] in which the flows from $N=4$ SYM to six-dimensional $N=(2,0)$ SCFT and $N=2^*$ theory to five dimensional $N=2$ SCFT have been studied.

The paper is organized as follow. In section 2, we describe $N=2$ gauged supergravity in seven dimensions to set up the notation and discuss all possible non-compact gauge groups. These gauge groups will be studied in detail in section 3, 4, 5 and 6 in which possible vacua and RG flow solutions will be given. In section 7, we give a summary of the results and some conclusions.

2 Seven-dimensional $N=2$ gauged supergravity coupled to n vector multiplets

In this section, we give a description of the matter-coupled minimal $N=2$ gauged supergravity in seven dimensions with topological mass term. All of the notations are the same as those in [18] to which the reader is referred to for further details.

A general matter-coupled theory is constructed by coupling n vector multiplets to pure $N=2$ supergravity constructed in [15]. The supergravity multiplet $(e^m_\mu, \psi^A_\mu, A^i_\mu, \chi^A, B_{\mu\nu}, \sigma)$ consists of the graviton, two gravitini, three vectors, two spin-$\frac{1}{2}$ fields, a two-form field and a real scalar, the dilaton. The only matter mutiplet is the vector multiplet $(A_\mu, \lambda^A, \phi^i)$ consisting of a vector field, two gauginos and three scalars. We use the convention that curved and flat space-time indices are denoted by $\mu, \nu, \ldots$ and $m, n, \ldots$, respectively. Spinor fields, ψ^A_μ, χ^A, λ^A, and the supersymmetry parameter ϵ^A are symplectic-Majorana spinors transforming as doublets of the R-symmetry $\mathrm{USp}(2)_\mathrm{R} \sim \mathrm{SU}(2)_\mathrm{R}$. From now on, the $\mathrm{SU}(2)_\mathrm{R}$ doublet indices $A, B = 1,2$ will be dropped. Indices $i, j = 1,2,3$ label triplets of $\mathrm{SU}(2)_\mathrm{R}$.

The supergravity theory coupled to n vector multiplets has $\mathrm{SO}(3,\mathrm{n})$ global symmetry. The n vector multiplets will be labelled by an index $r = 1, \ldots n$. There are then $n+3$ vector fields in total. Accordingly, only a subgroup G of the global symmetry $\mathrm{SO}(3,\mathrm{n})$ of dimension $\dim G \leq n+3$ can be gauged. Possible gauge groups with structure constants $f_{IJ}{}^K$ and gauge algebra

$$[T_I, T_J] = f_{IJ}{}^K T_K \tag{2.1}$$

can be gauged provided that the $\mathrm{SO}(3,\mathrm{n})$ Killing form η_{IJ}, $I, J = 1, \ldots n+3$, is invariant under G

$$f_{IK}{}^L \eta_{LJ} + f_{JK}{}^L \eta_{LI} = 0\,. \tag{2.2}$$

Since η_{IJ} has only three negative eigenvalues, any gauge group can have three or less compact generators or three or less non-compact generators. It follows from (2.2) that the part of η_{IJ} corresponding to each simple subgroup G_α of G must be a multiple of the G_α Killing form. Therefore, possible non-compact gauge groups take the form of $G_0 \times H$ with a compact group $H \subset \mathrm{SO}(3,\mathrm{n})$ of dimension $\dim H \leq (n+3-\dim G_0)$ [18]. The G_0 factor can only be one of the five possibilities: $\mathrm{SO}(3,1)$, $\mathrm{SL}(3,\mathbb{R})$, $\mathrm{SO}(2,1)$, $\mathrm{SO}(2,2) \sim \mathrm{SO}(2,1) \times \mathrm{SO}(2,1)$ and $\mathrm{SO}(2,2) \times \mathrm{SO}(2,1)$.

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Apart from the dilaton σ which is a singlet under the gauge group, there are $3n$ scalar fields ϕ^{ir} parametrized by $\mathrm{SO(3,n)/SO(3) \times SO(n)}$ coset manifold. The associated coset representative $L = (L_I{}^i, L_I{}^r)$ transforms under the global $\mathrm{SO(3,n)}$ and the local $\mathrm{SO(3) \times SO(n)}$ by left and right multiplications, respectively. Its inverse is denoted by $L^{-1} = (L^I{}_i, L^I{}_r)$ with the relations $L^I{}_i = \eta^{IJ} L_{Ji}$ and $L^I{}_r = \eta^{IJ} L_{Jr}$.

The two-form field $B_{\mu\nu}$ can be dualized to a three-form field $C_{\mu\nu\rho}$ which admits a topological mass term

$$\frac{h}{36} \epsilon^{\mu_1 \ldots \mu_7} H_{\mu_1 \ldots \mu_4} C_{\mu_5 \ldots \mu_7} \tag{2.3}$$

where the four-form field strength is defined by $H_{\mu\nu\rho\sigma} = 4\partial_{[\mu} C_{\nu\rho\sigma]}$.

The bosonic Lagrangian of the $N = 2$ massive-gauged supergravity is then given by

$$\begin{aligned} e^{-1}\mathcal{L} = & \frac{1}{2}R - \frac{1}{4}e^{\sigma} a_{IJ} F^I_{\mu\nu} F^{J\mu\nu} - \frac{1}{48} e^{-2\sigma} H_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma} - \frac{5}{8}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} P^{ir}_\mu P^\mu_{ir} \\ & - \frac{1}{144\sqrt{2}} e^{-1} \epsilon^{\mu_1 \ldots \mu_7} H_{\mu_1 \ldots \mu_4} \omega_{\mu_5 \ldots \mu_7} + \frac{1}{36} h e^{-1} \epsilon^{\mu_1 \ldots \mu_7} H_{\mu_1 \ldots \mu_4} C_{\mu_5 \ldots \mu_7} - V \end{aligned} \tag{2.4}$$

where the scalar potential is given by

$$V = \frac{1}{4} e^{-\sigma} \left(C^{ir} C_{ir} - \frac{1}{9} C^2 \right) + 16 h^2 e^{4\sigma} - \frac{4\sqrt{2}}{3} h e^{\frac{3\sigma}{2}} C \,. \tag{2.5}$$

The Chern-Simons term is defined by

$$\omega_{\mu\nu\rho} = 3\eta_{IJ} F^I_{[\mu\nu} A^J_{\rho]} - f_{IJ}{}^K A^I_\mu \wedge A^J_\nu \wedge A_{\rho K} \tag{2.6}$$

with $F^I_{\mu\nu} = 2\partial_{[\mu} A^I_{\nu]} + f_{JK}{}^I A^J_\mu A^K_\nu$.

We are going to find supersymmetric bosonic background solutions, so the supersymmetry transformations of fermions are needed. Since, in the following analysis, we will set $C_{\mu\nu\rho} = 0$, we will accordingly give the supersymmetry transformations with all fermions and the three-form field vanishing. These are given by

$$\delta\psi_\mu = 2D_\mu \epsilon - \frac{\sqrt{2}}{30} e^{-\frac{\sigma}{2}} C \gamma_\mu \epsilon - \frac{i}{20} e^{\frac{\sigma}{2}} F^i_{\rho\sigma} \sigma^i \left(3\gamma_\mu \gamma^{\rho\sigma} - 5\gamma^{\rho\sigma}\gamma_\mu \right) \epsilon - \frac{4}{5} h e^{2\sigma} \gamma_\mu \epsilon, \tag{2.7}$$

$$\delta\chi = -\frac{1}{2}\gamma^\mu \partial_\mu \sigma \epsilon - \frac{i}{10} e^{\frac{\sigma}{2}} F^i_{\mu\nu} \sigma^i \gamma^{\mu\nu} \epsilon + \frac{\sqrt{2}}{30} e^{-\frac{\sigma}{2}} C \epsilon - \frac{16}{5} e^{2\sigma} h \epsilon, \tag{2.8}$$

$$\delta\lambda^r = -i\gamma^\mu P^{ir}_\mu \sigma^i \epsilon - \frac{1}{2} e^{\frac{\sigma}{2}} F^r_{\mu\nu} \gamma^{\mu\nu} \epsilon - \frac{i}{\sqrt{2}} e^{-\frac{\sigma}{2}} C^{ir} \sigma^i \epsilon \,. \tag{2.9}$$

The covariant derivative of ϵ is defined by

$$D_\mu \epsilon = \partial_\mu \epsilon + \frac{1}{4} \omega^{ab}_\mu \gamma_{ab} + \frac{i}{4} \sigma^i \epsilon^{ijk} Q_{\mu jk} \tag{2.10}$$

where γ^a are space-time gamma matrices.

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The quantities appearing in the Lagrangian and the supersymmetry transformations are defined by

$$\begin{aligned}
P^{ir}_\mu &= L^{Ir}\left(\delta^K_I\partial_\mu + f_{IJ}{}^K A^J_\mu\right)L^i{}_K, \qquad Q^{ij}_\mu = L^{Ij}\left(\delta^K_I\partial_\mu + f_{IJ}{}^K A^J_\mu\right)L^i{}_K, \\
C_{ir} &= \frac{1}{\sqrt{2}}f_{IJ}{}^K L^I{}_j L^J{}_k L_{Kr}\epsilon^{ijk}, \qquad C = -\frac{1}{\sqrt{2}}f_{IJ}{}^K L^I{}_i L^J{}_j L_{Kk}\epsilon^{ijk}, \\
C_{rsi} &= f_{IJ}{}^K L^I{}_r L^J{}_s L_{Ki}, \qquad a_{IJ} = L^i{}_I L_{iJ} + L^r{}_I L_{rJ}, \\
F^i_{\mu\nu} &= L_I{}^i F^I, \qquad F^r_{\mu\nu} = L_I{}^r F^I\,.
\end{aligned} \tag{2.11}$$

In the following sections, we will study all possible non-compact gauge groups G_0 without the compact H factor. This is a consistent truncation since all scalar fields we retain are H singlets. All of the solutions found here are automatically solutions of the gauged supergravity with $G_0 \times H$ gauge group according to the result of Schur's lemma as originally discussed in [25].

Before going to the computation, we will give a general parametrization of the $SO(3,n)/SO(3)\times SO(n)$ coset. We first introduce $(n+3)^2$ basis elements of a general $(n+3)\times(n+3)$ matrix as follow

$$(e_{IJ})_{KL} = \delta_{IK}\delta_{JL}\,. \tag{2.12}$$

The composite $SO(3)\times SO(n)$ generators are given by

$$\begin{aligned}
SO(3):&\qquad J^{(1)}_{ij} = e_{ji} - e_{ij}, \qquad i,j = 1,2,3, \\
SO(n):&\qquad J^{(2)}_{rs} = e_{s+3,r+3} - e_{r+3,s+3}, \qquad r,s = 1,\ldots,n\,.
\end{aligned} \tag{2.13}$$

The non-compact generators corresponding to the $3n$ scalars are given by

$$Y^{ir} = e_{i,r+3} + e_{r+3,i}\,. \tag{2.14}$$

The coset representative in each case will be given by an exponential of the relevant Y^{ir} generators.

3 SO(3, 1) gauge group

The minimal scalar coset for embedding $SO(3,1)$ gauge group is $SO(3,3)/SO(3)\times SO(3)$. We will choose the gauge structure constants to be

$$f_{IJK} = -g(\epsilon_{ijk}, \epsilon_{rsi}), \qquad i,j,r,s = 1,2,3 \tag{3.1}$$

from which we find $f_{IJ}{}^K = \eta^{KL}f_{IJL}$ with $\eta^{IJ} = (-1,-1,-1,1,1,1)$. Together with the dilaton σ, there are ten scalars in this case. At the vacuum, the full $SO(3,1)$ gauge symmetry is broken down to its the maximal compact subgroup $SO(3)$. The ten scalars transform as $\mathbf{1}+\mathbf{1}+\mathbf{3}+\mathbf{5}$ with the first singlet being the dilaton.

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Critical point	σ	V_0	L
I	0	$-240h^2$	$\frac{1}{4h}$
II	$\frac{2}{5}\ln 2$	$-160(2^{\frac{3}{5}})h^2$	$\frac{\sqrt{3}}{2(2^{\frac{4}{5}})h}$

Table 1. Supersymmetric and non-supersymmetric AdS_7 critical points in SO(3,1) gauging.

$\mathrm{SO}(3)_{\mathrm{diag}}$	m^2L^2	Δ
1	-8	4
1	40	10
3	0	6
5	16	8

Table 2. Scalar masses at the supersymmetric AdS_7 critical point in SO(3,1) gauging.

3.1 AdS_7 critical points

We now investigate the vacuum structure of the $N = 2$ gauged supergravity with SO(3,1) gauge group. We simplify the task by restricting the potential to the two $\mathrm{SO}(3) \subset \mathrm{SO}(3,1)$ singlet scalars. This truncation is consistent in the sense that all critical points found on this restricted scalar manifold are automatically critical points of the potential computed on the full scalar manifold as pointed out in [25].

The scalar potential on these SO(3) singlets is given by

$$V = \frac{1}{16}e^{-\sigma-6\phi}\left[\left(1+8e^{2\phi}+3e^{4\phi}-32e^{6\phi}+3e^{8\phi}+8e^{10\phi}+e^{12\phi}\right)g^2 \right. \\ \left. -32e^{\frac{5}{2}\sigma+3\phi}\left(1+e^{2\phi}+e^{4\phi}+e^{6\phi}\right)gh+256h^2e^{5\sigma+6\phi}\right]. \tag{3.2}$$

The scalar ϕ is an SO(3) singlet coming from $\mathrm{SO}(3,3)/\mathrm{SO}(3)\times\mathrm{SO}(3)$. It can be easily checked that this potential admits two critical points at $\phi = 0$ and

$$\sigma = \frac{2}{5}\ln\frac{g}{16h}, \qquad \text{and} \qquad \sigma = \frac{2}{5}\ln\frac{g}{8h}\,. \tag{3.3}$$

As in the SO(4) gauging studied in [13], the second critical point is non-supersymmetric as can be checked by computing the supersymmetry transformations of fermions. We will shift the dilaton field so that the supersymmetric AdS_7 occurs at $\sigma = 0$. This is effectively achieved by setting $g = 16h$. The gauge group SO(3,1) is broken down to its maximal compact subgroup SO(3), so the two critical points have SO(3) symmetry. At these critical points, the values of the cosmological constant (V_0) and the AdS_7 radius (L) are given in table 1.

In our convention, the relation between V_0 and L is given by $L = \sqrt{-\frac{15}{V_0}}$. We can compute scalar masses at the trivial critical point, $\sigma = 0$, as shown in the table 2.

In the table, we have given the representations under the unbroken $\mathrm{SO}(3) \subset \mathrm{SO}(3,1)$ symmetry. The conformal dimension Δ of the dual operators in the six-dimensional SCFT is also given. The three scalars in the **3** representation correspondence to the Goldstone bosons in the symmetry breaking SO(3,1) to SO(3). These scalars correspond to marginal

SO(3)	m^2L^2	Δ
1	12	$3+\sqrt{21}$
1	36	$3(1+\sqrt{5})$
3	0	6
5	0	6

Table 3. Scalar masses at the non-supersymmetric AdS_7 critical point in $SO(3,1)$ gauging.

operators of dimension six. From the table, we see that only the operator dual to the dilaton is relevant. The other are either marginal or irrelevant.

Unlike in the SO(4) gauging in which the non-supersymmetric AdS_7 is unstable, we find that, in $SO(3,1)$ gauging, it is indeed stable as can be seen from the scalar masses given in table 3. From the table, we see that the operator dual to σ becomes irrelevant at this critical point. We then expect that there should be an RG flow driven by this operator from the $N = 2$ supersymmetric fixed point to this CFT. The gravity solution would involve the metric $g_{\mu\nu}$ and σ. Since the flow is non-supersymmetric, the flow solution has to be found by solving the full second-order field equations. In general, these equations do not admit an analytic solution. We will not go into the detail of this flow here and will not give the corresponding numerical flow solution. A similar study in the case of pure $N = 2$ SU(2) gauged supergravity can be found in [12].

3.2 AdS_5 critical points

We now look for a vacuum solution of the form $AdS_5 \times S^2$. In this case, an abelian gauge field is turned on. There are six gauge fields A^I, $I = 1,\ldots,6$, of $SO(3,1)$ in which the first three gauge fields are those of the compact subgroup SO(3). We will choose the non-zero gauge field to be A^3. The seven-dimensional metric is given by

$$ds^2 = e^{2f(r)}dx^2_{1,3} + dr^2 + e^{2g(r)}(d\theta^2 + \sin^2 d\phi^2) \tag{3.4}$$

where $dx^2_{1,3}$ is the flat metric on the four-dimensional Minkowski space. The ansatz for the gauge field is given by

$$A^3 = a\cos\theta d\phi, \qquad F^3 = -a\sin\theta d\theta \wedge d\phi\,. \tag{3.5}$$

From the metric, we can compute the following spin connections

$$\begin{aligned} \omega^{\hat{\phi}}{}_{\hat{\theta}} &= e^{-g(r)}\cot\theta e^{\hat{\phi}}, & \omega^{\hat{\phi}}{}_{\hat{r}} &= g(r)' e^{\hat{\phi}}, \\ \omega^{\hat{\theta}}{}_{\hat{r}} &= g(r)' e^{\hat{\theta}}, & \omega^{\hat{\mu}}{}_{\hat{r}} &= f' e^{\hat{\mu}}\,. \end{aligned} \tag{3.6}$$

From $SO(3,3)/SO(3)\times SO(3)$ coset, there are three singlets under this $SO(2) \subset SO(3)$. One of them is the SO(3) singlet mentioned before. The other two come from **3** and **5** representations of SO(3) with the former being one of the three Goldstone bosons. We can then set up relevant BPS equations by computing the supersymmetry transformations of ψ_μ, χ and λ^r. We will not give $\delta\psi_r = 0$ equation here. This will give rise to the equation for the Killing spinors as a function of r.

We then impose the projections

$$\gamma_r \epsilon = \epsilon \qquad \text{and} \qquad i\gamma^{\hat{\theta}\hat{\phi}}\sigma^3\epsilon = \epsilon \tag{3.7}$$

where hatted indices are tangent space indices. By imposing the twist condition

$$ag = 1, \tag{3.8}$$

we find that equation $\delta\psi_\theta = 0$ is the same as $\delta\psi_\phi = 0$. The Killing spinors are then given by constant spinors on S^2. Equations $\delta\psi_\mu$, $\mu = 0, 1, 2, 3$ lead to a single equation for $f(r)$. With all these, we find the following set of the BPS equations

$$\phi_1' = \frac{e^{-\frac{\sigma}{2}-2\phi_1+2\phi_2-\phi_3}\left(1+e^{2\phi_3}\right)\left(e^{2\phi_3}-1\right)g}{2\left(1+e^{4\phi_2}\right)}, \tag{3.9}$$

$$\phi_2' = 0, \tag{3.10}$$

$$\begin{aligned}\phi_3' = &-\frac{1}{4}e^{-\frac{\sigma}{2}-2\phi_1-\phi_3-2g(r)}\left[2ae^{\sigma+2\phi_1}\left(e^{2\phi_3}-1\right)\right.\\ &\left.-e^{2g(r)}\left(2e^{2\phi_1}+e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}-1\right)g\right],\end{aligned} \tag{3.11}$$

$$\begin{aligned}\sigma' = &\frac{1}{10}e^{-\frac{\sigma}{2}-2\phi_1-\phi_3-2g(r)}\left[2ae^{\sigma+2\phi_1}\left(1+e^{2\phi_3}\right)+64he^{\frac{5}{2}\sigma+2\phi_1+\phi_3+2g(r)}\right.\\ &\left.-e^{2g(r)}\left(1-2e^{2\phi_1}-e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}\right)g\right],\end{aligned} \tag{3.12}$$

$$\begin{aligned}g(r)' = &-\frac{2}{5}ae^{\frac{\sigma}{2}-\phi_3-2g(r)}\left(1+e^{2\phi_3}\right)+\frac{4}{5}he^{2\sigma}\\ &+\frac{1}{20}e^{-\frac{\sigma}{2}-2\phi_1-\phi_3}\left(1-2e^{2\phi_1}-e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}\right)g,\end{aligned} \tag{3.13}$$

$$\begin{aligned}f' = &\frac{1}{10}ae^{\frac{\sigma}{2}-\phi_3-2g(r)}\left(1+e^{2\phi_3}\right)+\frac{4}{5}he^{2\sigma}\\ &+\frac{1}{20}e^{-\frac{\sigma}{2}-2\phi_1-\phi_3}\left(1-2e^{2\phi_1}-e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}\right)g\end{aligned} \tag{3.14}$$

where ϕ_i, $i = 1, 2, 3$ are the three singlets from $SO(3,3)/SO(3)\times SO(3)$. The $'$ denotes $\frac{d}{dr}$. To avoid the confusion with the gauge coupling g, we have explicitly written the S^2 warp factor as $g(r)$.

ϕ_2, being one of the Goldstone bosons, disappears entirely from the scalar potential which, for these SO(2) singlets, is given by

$$\begin{aligned}V = &\frac{1}{16}e^{-\sigma-4\phi_1-2\phi_3}\left[\left(1+2e^{4\phi_1}+e^{4\phi_3}+2e^{4(\phi_1+\phi_3)}-16e^{4\phi_1+2\phi_3}+e^{8\phi_1+4\phi_3}\right)g^2\right.\\ &+32ghe^{\frac{5\sigma}{2}+2\phi_1+\phi_3}\left(1-2e^{2\phi_1}-e^{4\phi_1}-e^{2\phi_3}-2e^{2(\phi_1+\phi_3)}+e^{4\phi_1+2\phi_3}\right)\\ &\left.+256h^2e^{5\sigma+4\phi_1+2\phi_3}\right].\end{aligned} \tag{3.15}$$

When $\phi_3 = \phi_1$, this reduces to the SO(3) invariant potential (3.2). Equation (3.10) implies that ϕ_2 is a constant. We will choose $\phi_2 = 0$ from now on in order to be consistent with the supersymmetric AdS_7 critical point.

The $AdS_5 \times S^2$ geometry is characterized by the fixed point solution of $g(r)' = \phi_i' = \sigma' = 0$. From the above equations, there is a solution only for $\phi_i = 0$ and

$$\sigma = \frac{2}{5}\ln\frac{g}{12h}, \qquad g(r) = -\frac{1}{2}\ln\frac{g}{3a} + \frac{1}{5}\ln\frac{g}{12h}\,. \tag{3.16}$$

Near this fixed point with $g = 16h$, we find $f \sim \left(\frac{512}{9}\right)^{\frac{2}{5}} hr$. Therefore, the AdS_5 radius is given by $L_{AdS_5} = \frac{1}{h}\left(\frac{9}{512}\right)^{\frac{2}{5}}$. At this fixed point, the projection $\gamma_r\epsilon = \epsilon$ is not needed, so the number of unbroken supercharges is eight. According to the AdS/CFT correspondence, we will identify this AdS_5 solution with an $N = 1$ SCFT in four dimensions.

3.3 RG flows from 6D $N = (1, 0)$ SCFT to 4D $N = 1$ SCFT

The existence of $AdS_5 \times S^2$ geometry indicates that the $N = (1, 0)$ SCFT in six dimensions corresponding to AdS_7 critical point can undergo an RG flow to a four-dimensional $N = 1$ SCFT. We begin the study of this RG flow solution by rewriting the BPS equations for $\phi_i = 0$

$$\sigma' = \frac{2}{5}e^{-\frac{\sigma}{2}}\left(ae^{\sigma-2g(r)} + g - 16he^{\frac{5\sigma}{2}}\right), \tag{3.17}$$

$$g(r)' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(g - 4ae^{\sigma-2g(r)} + 4he^{\frac{5\sigma}{2}}\right), \tag{3.18}$$

$$f' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(g + ae^{\sigma-2g(r)} + 4he^{\frac{5\sigma}{2}}\right). \tag{3.19}$$

Near the IR AdS_5 fixed point, we find

$$\begin{aligned}\sigma \sim g(r) &\sim e^{(\sqrt{7}-1)\frac{r}{L_{AdS_5}}}, \\ f &\sim \frac{r}{L_{AdS_5}}\,.\end{aligned} \tag{3.20}$$

We then conclude that the operators dual to σ and $g(r)$ become irrelevant in four dimensions with dimension $\Delta = 3 + \sqrt{7}$. We are not able to find an analytic solution to the above equations. We therefore give an example of numerical solutions in figure 1.

At the IR fixed point, the value of σ does not depend on a, but different values of a give rise to different solutions for $g(r)$. In figure 1, we have given some examples of the $g(r)$ solutions with three different values of a, $a = 1, 2, 3$ with $g = 16h$ and $h = 1$. From the solutions, we see that, at large r, $g(r) \sim r$ and $\sigma \sim 0$. Furthermore, as $g(r) \sim r \to \infty$, we find $f(r) \sim g(r) \sim r$. The UV geometry is AdS_7 corresponding to the six-dimensional $N = (1, 0)$ SCFT. The behavior of σ near the UV point is given by

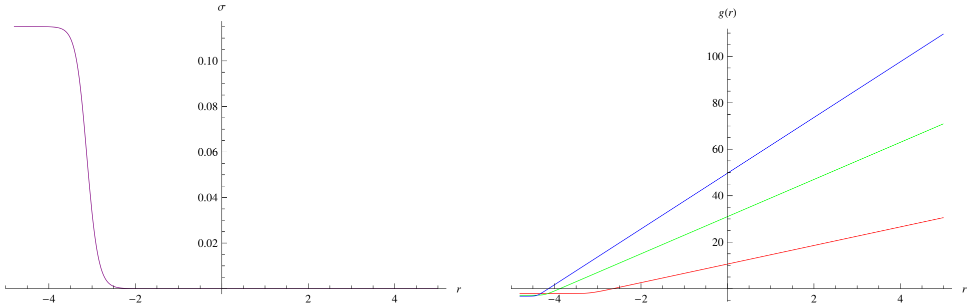
$$\sigma \sim e^{-\frac{4r}{L_{AdS_7}}} \tag{3.21}$$

which indicates that the flow is driven by a VEV of a dimension-four operator.

3.4 $AdS_5 \times H^2$ geometry

We now consider a fixed point of the form $AdS_5 \times H^2$ with H^2 being a genus $g > 1$ Riemann surface. In this case, we take the metric ansatz to be

$$ds^2 = e^{2f(r)}dx_{1,3}^2 + dr^2 + \frac{e^{2g(r)}}{y^2}(dx^2 + dy^2). \tag{3.22}$$



(a) A solution for σ. **(b)** Solutions for $g(r)$.

Figure 1. RG flow solutions from $N = (1, 0)$ SCFT in six dimensions to four-dimensional $N = 1$ SCFT with the $g(r)$ solution given for three different values of a; $a = 1$ (red),$a = 2$ (green), $a = 3$ (blue).

The SO(2) gauge field is then given by

$$A = \frac{a}{y}dx, \qquad F = \frac{a}{y^2}dx \wedge dy\,. \tag{3.23}$$

The spin connections computed from the above metric are given by

$$\omega^{\hat{x}}{}_{\hat{r}} = g(r)'e^{\hat{x}}, \qquad \omega^{\hat{y}}{}_{\hat{r}} = g(r)'e^{\hat{y}}, \qquad \omega^{\hat{x}}{}_{\hat{y}} = -e^{-g(r)}e^{\hat{x}}\,. \tag{3.24}$$

The twisted condition is still given by $ga = 1$. The BPS equations change by some signs, and it is still true that the AdS_5 is possible only for $\phi_i = 0$. The BPS equations, for $\phi_i = 0$, are then given by

$$\sigma' = \frac{2}{5}e^{-\frac{\sigma}{2}}\left(-ae^{\sigma-2g(r)} + g - 16he^{\frac{5\sigma}{2}}\right), \tag{3.25}$$

$$g(r)' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(g + 4ae^{\sigma-2g(r)} + 4he^{\frac{5\sigma}{2}}\right), \tag{3.26}$$

$$f' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(g - ae^{\sigma-2g(r)} + 4he^{\frac{5\sigma}{2}}\right). \tag{3.27}$$

The fixed point conditions $\sigma' = g(r)' = 0$ have the solution

$$\sigma = \frac{2}{5}\ln\frac{g}{12h}, \qquad g(r) = -\frac{1}{2}\ln\left[-\frac{g}{3a}\right] + \frac{1}{5}\ln\frac{g}{12h}\,. \tag{3.28}$$

In this case, there is no real solution for $g(r)$ since the twisted condition requires that g must have the same sign as a. Therefore, we conclude that there is no supersymmetric $AdS_5 \times H^2$ solution for SO(3, 1) gauging.

4 SL(3, ℝ) gauge group

In this section, we consider the SL(3, ℝ) gauge group. The minimal scalar manifold to accommodate this eight-dimensional gauge group is SO(3, 5)/SO(3)×SO(5). The structure constants can be obtained from the generators $T_I = (i\lambda_2, i\lambda_5, i\lambda_7, \lambda_1, \lambda_3, \lambda_4, \lambda_6, \lambda_8)$ with $I = 1, \ldots, 8$. λ_i are the usual Gell-mann matrices.

SO(3)	m^2L^2	Δ
1	-8	4
3	112	14
5	0	6
7	72	12

Table 4. Scalar masses at the supersymmetric AdS_7 critical point in $SL(3,\mathbb{R})$ gauging.

SO(3)	m^2L^2	Δ
1	12	$3+\sqrt{21}$
3	96	$3+\sqrt{105}$
5	0	6
7	36	$3(1+\sqrt{5})$

Table 5. Scalar masses at the non-supersymmetric AdS_7 critical point in $SL(3,\mathbb{R})$ gauging.

Under $SL(3,\mathbb{R})$, the adjoint representation of $SO(3,5)$ decomposes as

$$\mathbf{28} \to \mathbf{8} + \mathbf{10} + \mathbf{10}'\,.$$

At the vacuum, the $SL(3,\mathbb{R})$ symmetry is broken down to $SO(3)$ with the embedding $\mathbf{3} \to \mathbf{3}$. Therefore, under $SO(3)$, the $\mathbf{28}$ of $SO(3,5)$ further decomposes as

$$\mathbf{28} \to \mathbf{3} + \mathbf{5} + \mathbf{3} + \mathbf{7} + \mathbf{3} + \mathbf{7}\,.$$

The fifteen scalars transform under $SO(3)$ as $\mathbf{3}+\mathbf{5}+\mathbf{7}$. The other representations $\mathbf{3}+\mathbf{3}+\mathbf{7}$ combine into the adjoint representation of the composite local $SO(3) \times SO(5)$ symmetry.

4.1 AdS_7 critical points

By computing the scalar potential, we find that there are two AdS_7 critical points with $SO(3)$ symmetry as in the $SO(3,1)$ gauging for vanishing vector multiplet scalars. One of them is supersymmetric, and the other one is non-supersymmetric. We will similarly set $g = 16h$ to bring the supersymmetric AdS_7 to $\sigma = 0$. The characteristics of these two critical points are the same as in $SO(3,1)$ gauging, so we will not repeat them here. However, scalar masses at these two critical point are different and are given in table 4 and 5.

As in the previous case, the $SO(3)$ singlet is the dilaton. In this case, there are five Goldstone bosons from the $SL(3,\mathbb{R}) \to SO(3)$ symmetry breaking. The non-supersymmetric AdS_7 is stable as in the $SO(3,1)$ gauging and can be interpreted as a unitary six-dimensional CFT. We then expect that there should be an RG flow from the supersymmetric AdS_7 to the non-supersymmetric one. As in the previous case, the flow is driven by a VEV of the operator dual to the dilaton σ. In the IR, the operator becomes irrelevant with dimension $\Delta = 3 + \sqrt{21}$.

4.2 AdS_5 critical points

We now study possible AdS_5 fixed points. We will turn on a gauge field of SO(2) which is a subgroup of the compact subgroup $SO(3) \subset SL(3,\mathbb{R})$. Among the fifteen scalars, there are three singlets under this SO(2), and we will denote them by ϕ_i, $i = 1, 2, 3$. Each of the three SO(3) representations, $\mathbf{3} + \mathbf{5} + \mathbf{7}$, gives one SO(2) singlet.

We again use the metric ansatz (3.4) and the gauge field $A^3 = a\cos\theta d\phi$. With the twisted condition $ga = 1$ and the projectors $\gamma_r\epsilon = \epsilon$ and $i\gamma^{\hat{\theta}\hat{\phi}}\sigma^3\epsilon = \epsilon$, we obtain a system of complicated BPS equations. Since these equations might be useful for other applications, we explicitly give them here

$$\phi_1' = \frac{\sqrt{3}ge^{-\frac{\sigma}{2}-2\phi_1-\frac{2}{\sqrt{3}\phi_3}}\left(e^{4\phi_1}-1\right)\left(e^{4\phi_2}-1\right)\left(e^{\frac{4\phi_3}{\sqrt{3}}}-1\right)}{4\left(1+e^{4\phi_2}\right)}, \tag{4.1}$$

$$\phi_2' = \frac{\sqrt{3}}{4}ge^{-\frac{\sigma}{2}-2\phi_2-\frac{2\phi_3}{\sqrt{3}}}\left(1+e^{4\phi_2}\right)\left(e^{\frac{4\phi_3}{\sqrt{3}}}-1\right), \tag{4.2}$$

$$\begin{aligned}\phi_3' = {} & \frac{1}{16}e^{-\frac{\sigma}{2}-2\phi_1-2\phi_2-\frac{2\phi_3}{\sqrt{3}}-2g(r)}\left[4\sqrt{3}ae^{\sigma+2\phi_1+2\phi_2}\left(1-e^{\frac{4\phi_3}{\sqrt{3}}}\right)\right.\\ & +ge^{g(r)}\left(3e^{4\phi_1+4\phi_2+\frac{4\phi_3}{\sqrt{3}}}+3e^{4\phi_2+\frac{4\phi_3}{\sqrt{3}}}-4\sqrt{3}e^{2\phi_1+2\phi_2+\frac{4\phi_3}{\sqrt{3}}}-3e^{4\phi_1+\frac{4\phi_3}{\sqrt{3}}}-3e^{\frac{4\phi_3}{\sqrt{3}}}\right.\\ & \left.\left.+3e^{4(\phi_1+\phi_2)}+4\sqrt{3}e^{2(\phi_1+\phi_2)}+3e^{4\phi_2}-3e^{4\phi_1}-3\right)\right],\end{aligned} \tag{4.3}$$

$$\begin{aligned}\sigma' = {} & \frac{1}{20}e^{-\frac{\sigma}{2}-2\phi_1-2\phi_2-\frac{2\phi_3}{\sqrt{3}}-2g(r)}\left[4ae^{\sigma+2(\phi_1+\phi_2)}\left(1+e^{\frac{4\phi_3}{\sqrt{3}}}+128he^{\frac{5\sigma}{2}+2\phi_1+2\phi_2+\frac{2\phi_3}{\sqrt{3}}+2g(r)}\right)\right.\\ & ge^{2g(r)}\left(\sqrt{3}\left(1+e^{4\phi_1}\right)-\sqrt{3}e^{4\phi_2}-4e^{2(\phi_1+\phi_2)}-\sqrt{3}e^{4(\phi_1+\phi_2)}-\sqrt{3}e^{\frac{4\phi_3}{\sqrt{3}}}\right.\\ & \left.\left.-\sqrt{3}e^{4\phi_1+\frac{4\phi_3}{\sqrt{3}}}-4e^{2\phi_1+2\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_1+4\phi_2+\frac{4\phi_3}{\sqrt{3}}}\right)\right],\end{aligned} \tag{4.4}$$

$$\begin{aligned}g(r)' = {} & -\frac{2}{5}ae^{\frac{\sigma}{2}-\frac{2\phi_3}{\sqrt{3}}-2g(r)}\left(1+e^{\frac{4\phi_3}{\sqrt{3}}}\right)+\frac{4}{5}he^{2\sigma}\\ & -\frac{1}{40}ge^{-\frac{\sigma}{2}-2\phi_1-2\phi_2-\frac{2\phi_3}{\sqrt{3}}}\left[\sqrt{3}\left(1+e^{4\phi_1}\right)-\sqrt{3}e^{4\phi_2}-4e^{2(\phi_1+\phi_2)}-\sqrt{3}e^{4(\phi_1+\phi_2)}\right.\\ & \left.-\sqrt{3}e^{\frac{4\phi_3}{\sqrt{3}}}\left(1+e^{4\phi_1}\right)-4e^{2\phi_1+2\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_1+4\phi_2+\frac{4\phi_3}{\sqrt{3}}}\right],\end{aligned} \tag{4.5}$$

$$\begin{aligned}f' = {} & \frac{1}{10}ae^{\frac{\sigma}{2}-\frac{2\phi_3}{\sqrt{3}}-2g(r)}\left(1+e^{\frac{4\phi_3}{\sqrt{3}}}\right)+\frac{4}{5}he^{2\sigma}\\ & -\frac{1}{40}ge^{-\frac{\sigma}{2}-2\phi_1-2\phi_2-\frac{2\phi_3}{\sqrt{3}}}\left[\sqrt{3}\left(1+e^{4\phi_1}\right)-\sqrt{3}e^{4\phi_2}-4e^{2(\phi_1+\phi_2)}-\sqrt{3}e^{4(\phi_1+\phi_2)}\right.\\ & \left.-\sqrt{3}e^{\frac{4\phi_3}{\sqrt{3}}}\left(1+e^{4\phi_1}\right)-4e^{2\phi_1+2\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_2+\frac{4\phi_3}{\sqrt{3}}}+\sqrt{3}e^{4\phi_1+4\phi_2+\frac{4\phi_3}{\sqrt{3}}}\right].\end{aligned} \tag{4.6}$$

It can be easily verified that the first three equations have a fixed point solution only when $\phi_i = 0$ for all $i = 1, 2, 3$. The remaining equations then reduce to the same form as in the SO(3, 1) case. The RG flow solutions can also be studied in a similar manner, and we will not repeat it here.

As a final remark, we note here that similar to the previous case, it is not possible to have an $AdS_5 \times H^2$ solution.

5 SO(2, 2) gauge group

Unlike the previous two cases, this gauging does not admit a maximally supersymmetric AdS_7. The vacuum is rather a half-supersymmetric domain wall. This is not unexpected since the minimal superconformal algebra in six dimensions has $\mathrm{SU}(2)_\mathrm{R}$ R-symmetry, but the vacuum of this gauging has only SO(2)×SO(2) symmetry. The minimal scalar manifold for embedding this gauge group is SO(3, 3)/SO(3) × SO(3). The embedding of SO(2, 2) in SO(3, 3) is given by the following structure constants

$$f_{IJ}{}^K = (g_1\epsilon_{\bar{i}\bar{j}\bar{l}}\eta^{\bar{k}\bar{l}}, g_2\epsilon_{\bar{r}\bar{s}\bar{t}}\eta^{\bar{q}\bar{t}}) \tag{5.1}$$

with $\bar{i} = 1, 2, 6$, $\bar{r} = 3, 4, 5$, $\eta_{\bar{i}\bar{j}} = (-1, -1, 1)$ and $\eta_{\bar{r}\bar{s}} = (-1, 1, 1)$.

5.1 Domain wall solutions

The vacuum of this gauging will have SO(2) × SO(2) symmetry. Among the nine scalars from SO(3, 3)/SO(3) × SO(3), there is one SO(2) × SO(2) singlet which will be denoted by ϕ. The scalar potential for SO(2) × SO(2) singlet scalars is given by

$$V = \frac{1}{2}g_1 e^{-\sigma} + 4g_1 h e^{\frac{3\sigma}{2}}\left(e^{-\phi} - e^{\phi}\right) + 16h^2 e^{4\sigma}\,. \tag{5.2}$$

It can be checked that this potential does not admit any critical points unless $h = g_1 = 0$. The vacuum is then a domain wall.

To study the domain wall solution, we write down the associated BPS equations by setting all the fields but the metric and scalars to zero. The metric is given by the domain wall ansatz

$$ds^2 = e^{2A(r)}dx^2_{1,5} + dr^2\,. \tag{5.3}$$

With the projection $\gamma_r\epsilon = \epsilon$, the relevant BPS equations read

$$\phi' = -\frac{1}{2}g_1 e^{-\frac{\sigma}{2}-\phi}\left(1 + e^{2\phi}\right), \tag{5.4}$$

$$\sigma' = \frac{1}{5}e^{-\frac{\sigma}{2}-\phi}\left[g_1\left(e^{2\phi} - 1\right) - 32he^{\frac{5\sigma}{2}+\phi}\right], \tag{5.5}$$

$$A' = \frac{1}{10}e^{-\frac{\sigma}{2}-\phi}\left[g_1\left(e^{2\phi} - 1\right) + 8he^{\frac{5\sigma}{2}+\phi}\right]. \tag{5.6}$$

By changing the radial coordinate from r to $\tilde{r}$ with the relation $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$, it is not difficult to find the solutions for ϕ, σ and A. These are given by

$$\phi = \ln\left[\tan\frac{C_1 - g_1\tilde{r}}{2}\right], \tag{5.7}$$

$$\sigma = \frac{2}{5}\phi - \frac{2}{5}\ln\left[\frac{16h}{g_1}\left(4C_2(1 + e^{2\phi}) - 1\right)\right], \tag{5.8}$$

$$A = \frac{1}{5}\phi - \frac{1}{4}\ln(1 + e^{2\phi}) + \frac{1}{20}\ln\left[1 - 4C_2\left(1 + e^{2\phi}\right)\right] \tag{5.9}$$

where C_1 and C_2 are integration constants. We have omitted the additive constant to A since this can be removed by rescaling $dx^2_{1,5}$ coordinates. According to the general DW/QFT correspondence, this solution should be dual to a non-conformal $N = (1, 0)$ gauge theory in six dimensions. As $\tilde{r} \to \frac{C_1}{g_1}$, the two scalars are logarithmically divergent. After changing the coordinate from $\tilde{r}$ back to r, we find the behavior of ϕ and σ as $\tilde{r} \sim \frac{C_1}{g_1}$, which is equivalent to $r \sim \frac{C}{g_1}$,

$$\phi \sim \frac{5}{6} \ln \left[\frac{C - g_1 r}{2}\right], \qquad \sigma \sim \frac{1}{3} \ln \left[\frac{C - g_1 r}{2}\right] \tag{5.10}$$

where C is a new integration constant coming from solving for $\tilde{r}$ in term of r. After rescaling $dx^2_{1,5}$ coordinates, the metric in this limit is given by

$$ds^2 = (C - g_1 r)^{\frac{1}{3}} dx^2_{1,5} + dr^2 . \tag{5.11}$$

5.2 AdS_5 critical points

We now look for a vacuum solution of the form $AdS_5 \times S^2$. In this case, there are two abelian SO(2) gauge groups. The corresponding gauge fields are denoted by

$$A^3 = a \sin\theta d\phi, \qquad A^6 = b \sin\theta d\phi . \tag{5.12}$$

The metric is still given by (3.4). In order to find the BPS equations, we impose the projectors $\gamma_r \epsilon = \epsilon$ and $i\gamma^{\hat{\theta}\hat{\phi}}\sigma^3 \epsilon = \epsilon$. The twisted condition is now given by

$$g_1 b = 1 . \tag{5.13}$$

Proceed as in the previous cases but with one more gauge field, we find the following BPS equations

$$\begin{aligned}
\phi' &= \frac{1}{2} e^{-\frac{\sigma}{2} - \phi - 2g(r)} \left[a e^{\sigma} \left(1 - e^{2\phi}\right) - \left(1 + e^{2\phi}\right) \left(b e^{\sigma} + e^{2g(r)} g_1\right)\right], && (5.14)\\
\sigma' &= \frac{1}{5} e^{-\frac{\sigma}{2} - \phi - 2g(r)} \Big[(a - b) e^{\sigma} + (a + b) e^{\sigma + 2\phi} \\
&\quad + e^{2g(r)} \left[\left(e^{2\phi} - 1\right) g_1 - 32 h e^{\frac{5\sigma}{2} + \phi}\right]\Big], && (5.15)\\
g(r)' &= \frac{1}{10} e^{-\frac{\sigma}{2} - \phi - 2g(r)} \Big[e^{2g(r)} \left[\left(e^{2\phi} - 1\right) g_1 + 8 h e^{\frac{5\sigma}{2} + \phi}\right] \\
&\quad + 4(b - a) e^{\sigma} - 4(a + b) e^{\sigma + 2\phi}\Big], && (5.16)\\
f' &= \frac{1}{10} e^{-\frac{\sigma}{2} - \phi - 2g(r)} \Big[e^{2g(r)} \left[\left(e^{2\phi} - 1\right) g_1 + 8 h e^{\frac{5\sigma}{2} + \phi}\right] \\
&\quad + (a - b) e^{\sigma} + (a + b) e^{\sigma + 2\phi}\Big] && (5.17)
\end{aligned}$$

where ϕ is the $SO(2) \times SO(2)$ singlet scalar from $SO(3, 3)/SO(3) \times SO(3)$.

The equations $\phi' = \sigma' = g(r)' = 0$ admit a fixed point solution given by

$$\begin{aligned}
\phi &= \frac{1}{2}\ln\left[\frac{\sqrt{4b^2-3a^2}-a}{2(a+b)}\right], \\
\sigma &= \frac{1}{5}\ln\left[\frac{a^2 g_1^2\left(\sqrt{4b^2-3a^2}-a\right)}{32(a+b)h^2\left(2b-3a+\sqrt{4b^2-3a^2}\right)}\right], \\
g(r) &= \frac{1}{10}\ln\left[\frac{(a+b)^4\left(a-2b+\sqrt{4b^2-3a^2}\right)^5\left(3a-2b-\sqrt{4b^2-3a^2}\right)^3}{1024a^3 g_1^3 h^2\left(a-\sqrt{4b^2-3a^2}\right)^4}\right].
\end{aligned} \tag{5.18}$$

It can be checked that the solution exists for $g_1 < 0$ and $a < 0$ with $b > -a$ or $g_1 < 0$ with $a > 0$ and $b > a$. This in turn implies that g_1 and b always have opposite sign in contradiction with the twisted condition $g_1 b = 1$. Therefore, the SO(2, 2) gauging does not admit $AdS_5 \times S^2$ geometry.

However, there exists an $AdS_5 \times H^2$ geometry. In this case, we have the metric (3.22) with the gauge fields given by

$$A^3 = \frac{a}{y}dx, \qquad A^6 = \frac{b}{y}dx\,. \tag{5.19}$$

The twisted condition is still given by $g_1 b = 1$. The BPS equations are given by (5.14), (5.15), (5.16) and (5.17) but with (a, b) replaced by $(-a, -b)$. The values of scalar fields at the AdS_5 fixed point solution are real for $g_1 < 0$ and $a < 0$ with $b < a$ in compatible with the twisted condition. Furthermore, it is not possible to have an AdS_5 fixed point with $a = \pm b$. This rules out the possibility of AdS_5 fixed point with $SO(2)_{\text{diag}} \subset SO(2) \times SO(2)$ symmetry. For $a = 0$, only one SO(2) gauge field turned on, it can also be checked that the AdS_5 fixed point does not exist. The $b = 0$ case is not possible since this is not consistent with the twisted condition with finite g_1.

5.3 RG flows from $N = 1$ 4D SCFT to 6D $N = (1, 0)$ SYM

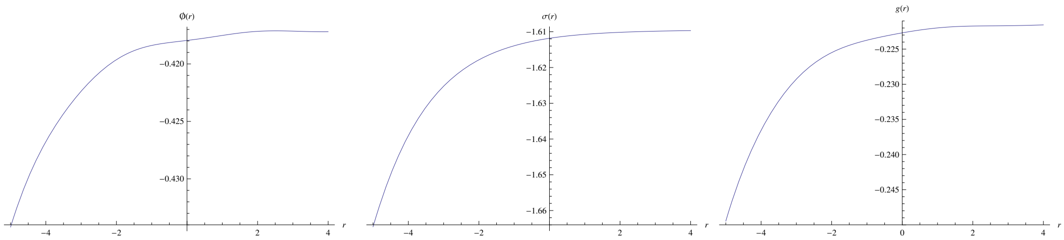
According to the AdS/CFT correspondence, the existence of AdS_5 fixed point implies a dual $N = 1$ SCFT in four dimensions. Near this AdS_5 critical point, the linearized BPS equations give

$$\phi \sim \sigma \sim g(r) \sim e^{-\frac{4r}{L}} \tag{5.20}$$

where L is the AdS_5 radius. We see that the AdS_5 should appear in the UV identified with $r \to \infty$. This UV SCFT in four dimensions undergoes an RG flow to a six-dimensional $N = (1, 0)$ SYM corresponding to the domain wall solution given by equations (5.7), (5.8) and (5.9). In the IR, the warped factors behave as $f(r) \sim g(r) \sim \ln(C - g_1 r)^{\frac{1}{3}}$ while the behavior of the scalars σ and ϕ is given in (5.10). The flow is then driven by vacuum expectations value of marginal operators dual to ϕ, σ and $g(r)$. We give an example of numerical flow solutions to the BPS equations in figure 2. This solution is found for particular values of $a = -1$, $b = -2$, $g = -\frac{1}{2}$ and $h = 1$ which give

$$\phi = -0.4171, \qquad \sigma = -1.6095, \qquad g(r) = -0.2214 \tag{5.21}$$

at the AdS_5 fixed point.



(a) A solution for ϕ. **(b)** A solution for σ. **(c)** A solution for $g(r)$.

Figure 2. An RG flow solution from $N = 1$ SCFT in four dimensions to six-dimensional $N = (1, 0)$ SYM.

As usual in flows to non-conformal field theories, the domain wall geometry in the IR is singular. We have checked that the domain wall solution given in equation (5.10) gives rise to a good singularity according to the criterion of [26]. Given the behavior of σ and ϕ in (5.10), we find that the scalar potential is bounded above $V \to -\infty$. Therefore, the IR domain wall corresponds to a physical gauge theory in six dimensions.

6 SO(2, 1) and SO(2, 2) × SO(2, 1) gauge groups

In this section, we consider the last two possible non-compact gauge groups SO(2, 1) and SO(2, 2) × SO(2, 1). We will see that both of them admit a vacuum solution in the form of a domain wall.

6.1 Vacua of SO(2, 1) gauging

In this case, the minimal scalar manifold is given by SO(3, 1)/SO(3). There are three scalars in this manifold. The structure constants of the SO(2, 1) gauge group can be chosen to be

$$f_{IJK} = (g\epsilon_{\bar{i}\bar{j}\bar{k}}, 0), \qquad \bar{i} = 1, 2, 4 \,. \tag{6.1}$$

This corresponds to choosing the SO(2, 1) generators to be (T_{41}, T_{42}, T_{12}) from the SO(3, 1) generators (T_{ij}, T_{4i}), $i, j = 1, 2, 3$.

The scalar potential does not have any critical points. Therefore, we expect that the vacuum is a domain wall. Using the domain wall ansatz for the metric and the projector $\gamma_r \epsilon = \epsilon$, we find the BPS equations for all of the four scalars

$$\phi_1' = -\frac{e^{-\frac{\sigma}{2}-\phi_1}\left(e^{2\phi_1}-1\right)\left(e^{2\phi_3}-1\right)g}{2\left(1+e^{2\phi_3}\right)}, \tag{6.2}$$

$$\phi_2' = -\frac{e^{-\frac{\sigma}{2}-\phi_2}\left(e^{2\phi_2}-1\right)\left(e^{2\phi_3}-1\right)g}{2\left(1+e^{2\phi_3}\right)}, \tag{6.3}$$

$$\phi_3' = -\frac{1}{2}e^{-\frac{\sigma}{2}-\phi_3}\left(1+e^{2\phi_3}\right)g, \tag{6.4}$$

$$\sigma' = \frac{1}{20}e^{-\frac{\sigma}{2}-\phi_1-\phi_2-\phi_3}\left(1+e^{2\phi_1}\right)\left(1+e^{2\phi_2}\right)\left(e^{2\phi_3-1}\right)g - \frac{32}{5}he^{2\sigma}, \tag{6.5}$$

$$A' = \frac{1}{40}e^{-\frac{\sigma}{2}-\phi_1-\phi_2-\phi_3}\left(1+e^{2\phi_1}\right)\left(1+e^{2\phi_2}\right)\left(e^{2\phi_3-1}\right)g + \frac{4}{5}he^{2\sigma} \,. \tag{6.6}$$

In these equations, ϕ_i, $i = 1, 2, 3$ are scalars in SO(3, 1)/SO(3).

It is difficult to find an exact solution with all scalars non-vanishing. On the other hand, a numerical solution could be obtained by the same procedure as in the previous sections. Since analytic solutions might be more interesting, we consider only a domain wall solution preserving $SO(2) \subset SO(2,1)$ symmetry. Among these ϕ_i's, ϕ_3 is an $SO(2)$ singlet. It turns out that on this scalar submanifold the solution is the same as that given in (5.7), (5.8) and (5.9) with ϕ replaced by ϕ_3.

6.2 Vacua of $SO(2,2) \times SO(2,1)$ gauging

The last gauge group to be considered is $SO(2,2)\times SO(2,1) \sim SO(2,1)\times SO(2,1)\times SO(2,1)$. The minimal scalar manifold in this case is $SO(3,6)/SO(3) \times SO(6)$ with the embedding of $SO(2,2) \times SO(2,1)$ in $SO(3,6)$ given by the following structure constants

$$f_{IJ}{}^{K} = (g_1\epsilon_{\bar{i}\bar{j}\bar{k}}\eta^{\bar{k}\bar{l}}, g_2\epsilon_{\bar{r}\bar{s}\bar{t}}\eta^{\bar{t}\bar{q}}, g_3\epsilon_{\tilde{i}\tilde{j}\tilde{k}}\eta^{\tilde{k}\tilde{l}}), \qquad \bar{i} = 1,4,5, \quad \bar{r} = 2,6,7, \quad \tilde{i} = 3,8,9\,. \tag{6.7}$$

The Killing metrics are given by $\eta_{\bar{i}\bar{j}} = (-1,1,1)$, $\eta_{\bar{r}\bar{s}} = (-1,1,1)$ and $\eta_{\tilde{i}\tilde{j}} = (-1,1,1)$, and g_1, g_2 and g_3 are gauge couplings of the three $SO(2,1)$ factors.

Apart from the dilaton, there are no scalars which are singlet under the maximal compact subgroup $SO(2) \times SO(2) \times SO(2)$. However, it can be shown that the potential does not have any critical points for $g_i, h \neq 0$. A simple domain wall solution can be obtained by solving the BPS equations for σ and the metric. There might be other solutions with non-vanishing scalars from $SO(3,6)/SO(3) \times SO(6)$, but we have not found any of them. Therefore, we will restrict ourselves to the domain wall with only σ and the metric non-vanishing. Using the projector $\gamma_r\epsilon = \epsilon$ as usual, we find the following BPS equations

$$\sigma' = -\frac{32}{5}e^{2\sigma}h, \tag{6.8}$$

$$A' = = \frac{4}{5}e^{2\sigma}h\,. \tag{6.9}$$

These equations can be readily solved for the solution

$$\sigma = -\frac{1}{2}\ln\left[\frac{64hr}{5}+C\right], \tag{6.10}$$

$$A = \frac{1}{16}\ln\left[\frac{64hr}{5}+C\right] \tag{6.11}$$

where C is an integration constant. The seven-dimensional metric is given by

$$ds^2 = (64hr+5C)^{\frac{1}{8}}dx_{1,5}^2 + dr^2 \tag{6.12}$$

where we have rescaled the $dx_{1,5}^2$ coordinates by $\frac{1}{5}$.

For $h = 0$, there is a Minkowski vacuum with $V_0 = 0$. All scalar masses at this critical point are given in table 6. The $SO(2)^3$ singlet is the dilaton which is massless while the other six massless scalars are Goldstone bosons of the symmetry breaking $SO(2,1)^3 \to SO(2)^3$.

m^2	$\mathrm{SO}(2)\times\mathrm{SO}(2)\times\mathrm{SO}(2)$ representation
0	$(\mathbf{1},\mathbf{1},\mathbf{1})$
0	$(\mathbf{1},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{2},\mathbf{1})+(\mathbf{2},\mathbf{1},\mathbf{1})$
g_1^2	$2\times(\mathbf{2},\mathbf{1},\mathbf{1})$
g_2^2	$2\times(\mathbf{1},\mathbf{2},\mathbf{1})$
g_3^2	$2\times(\mathbf{1},\mathbf{1},\mathbf{2})$

Table 6. Scalar masses at the supersymmetric Minkowski vacuum in $\mathrm{SO}(2,2)\times\mathrm{SO}(2,1)$ gauging.

7 Conclusions

We have studied $N = 2$ gauged supergravity in seven dimensions with non-compact gauge groups. In $\mathrm{SO}(3,1)$ and $\mathrm{SL}(3,\mathbb{R})$ gaugings, we have found new supersymmetric AdS_7 critical points. These should correspond to new $N = (1,0)$ SCFTs in six dimensions. We have also found that there exist $AdS_5\times S^2$ solutions to these gaugings. The solutions preserve eight supercharges and should be dual to some $N = 1$ four-dimensional SCFT with $\mathrm{SO}(2)\sim\mathrm{U}(1)$ global symmetry identified with the R-symmetry. We have then studied RG flows from the six-dimensional $N = (1,0)$ SCFT to the $N = 1$ SCFT in four dimensions and argued that the flow is driven by a vacuum expectation value of a dimension-four operator dual to the supergravity dilaton. A numerical solution for an example of these flows has also been given. In addition, we have shown that both of the gauge groups admit a stable non-supersymmetric AdS_7 solution which should be interpreted as a unitary CFT. This is not the case for the compact $\mathrm{SO}(4)$ gauging studied in [13] in which the non-supersymmetric critical point has been shown to be unstable.

In the $\mathrm{SO}(2,2)$ gauging, we have given a domain wall vacuum solution preserving half of the supersymmetry. According to the DW/QFT correspondence, this is expected to be dual to a non-conformal SYM in six dimensions. This $\mathrm{SO}(2,2)$ gauging does not admit an $AdS_5\times S^2$ solution but an $AdS_5\times H^2$ geometry with eight supercharges. The latter corresponds to an $N = 1$ SCFT in four dimensions with $\mathrm{SO}(2)\times\mathrm{SO}(2)$ global symmetry. It is likely that the a-maximization [27–29] is needed in order to identify the correct $\mathrm{U}(1)_\mathrm{R}$ symmetry out of the $\mathrm{SO}(2)\times\mathrm{SO}(2)$ symmetry. We have studied an RG flow from this SCFT to a non-conformal SYM in six dimensions, dual to the seven-dimensional domain wall, and argued that the flow is driven by vacuum expectation values of marginal operators. We have also investigated $\mathrm{SO}(2,1)$ and $\mathrm{SO}(2,2)\times\mathrm{SO}(2,1)$ gaugings. Both of them admit a half-supersymmetric domain wall as a vacuum solution. For vanishing topological mass, the $\mathrm{SO}(2,2)\times\mathrm{SO}(2,1)$ gauging admits a seven-dimensional Minkowski vacuum preserving all of the supersymmetry and $\mathrm{SO}(2)\times\mathrm{SO}(2)\times\mathrm{SO}(2)$ symmetry.

Due to the existence of new supersymmetric AdS_7 critical points, the results of this paper might be useful in $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence within the framework of seven-dimensional gauged supergravity. The new AdS_5 backgrounds could be of interest in the context of $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence. RG flows across dimensions described by gravity solutions connecting these geometries would provide additional examples of flows in twisted field theories. It is also interesting, if possible, to identify these AdS_5 critical points with the known four-dimensional SCFTs.

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Until now, only the embedding of the SO(4) gauging of $N = 2$ supergravity coupled to three vector multiplets in eleven-dimensional supergravity has been given [14]. The embedding of non-compact gauge groups in ten or eleven dimensions in the presence of topological mass term is presently not known. It would be of particular interest to find such an embedding so that the results reported here would be given an interpretation in terms of brane configurations in string/M theory.

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RG flows from (1,0) 6D SCFTs to N=1 SCFTs in four and three dimensions

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ABSTRACT: We study $AdS_5 \times \Sigma_2$ and $AdS_4 \times \Sigma_3$ solutions of $N = 2$, $SO(4)$ gauged supergravity in seven dimensions with $\Sigma_{2,3}$ being $S^{2,3}$ or $H^{2,3}$. The $SO(4)$ gauged supergravity is obtained from coupling three vector multiplets to the pure $N = 2$, $SU(2)$ gauged supergravity. With a topological mass term for the 3-form field, the $SO(4) \sim SU(2) \times SU(2)$ gauged supergravity admits two supersymmetric AdS_7 critical points, with $SO(4)$ and $SO(3)$ symmetries, provided that the two $SU(2)$ gauge couplings are different. These vacua correspond to $N = (1, 0)$ superconformal field theories (SCFTs) in six dimensions. In the case of Σ_2, we find a class of $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions preserving eight supercharges and $SO(2) \times SO(2)$ symmetry, but only $AdS_5 \times H^2$ solutions exist for $SO(2)$ symmetry. These should correspond to some $N = 1$ four-dimensional SCFTs. We also give RG flow solutions from the $N = (1, 0)$ SCFTs in six dimensions to these four-dimensional fixed points including a two-step flow from the $SO(4)$ $N = (1, 0)$ SCFT to the $SO(3)$ $N = (1, 0)$ SCFT that eventually flows to the $N = 1$ SCFT in four dimensions. For $AdS_4 \times \Sigma_3$, we find a class of $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions with four supercharges, corresponding to $N = 1$ SCFTs in three dimensions. When the two $SU(2)$ gauge couplings are equal, only $AdS_4 \times H^3$ are possible. The uplifted solutions for equal $SU(2)$ gauge couplings to eleven dimensions are also given.

KEYWORDS: AdS-CFT correspondence, Gauge/Gravity Correspondence and Supergravity Models.

1. Introduction

Six-dimensional superconformal field theories (SCFTs) are interesting in various aspects. In the context of M-theory, these SCFTs arise as a worldvolume theory of M5-branes in the near horizon limit. The correspondence between a six-dimensional $N = (2,0)$ SCFT and M-theory on $AdS_7 \times S^4$ is one of the examples given in the AdS/CFT correspondence originally proposed in [1]. This $\text{AdS}_7/\text{CFT}_6$ correspondence has been explored in great details both from the M-theory point of view and the effective $N = 4$ $SO(5)$ gauged supergravity in seven dimensions.

In this paper, we are interested in the half-maximal $N = (1,0)$ SCFTs in six dimensions. It has been shown in [2] that $N = (1,0)$ field theory possesses a non-trivial fixed point, and recently many $N = (1,0)$ SCFTs have been classified in [3, 4] and [5]. The holographic study of this $N = (1,0)$ theory has mainly been investigated by orbifolding the $AdS_7 \times S^4$ geometry of eleven-dimensional supergravity, see for example [6, 7, 8]. Recently, many new AdS_7 geometries from massive type IIA string theory have been found in [9], and the dual SCFTs of these AdS_7 vacua have been studied in [10].

We are particularly interested in studying $N = (1,0)$ SCFTs within the framework of seven-dimensional gauged supergravity. These SCFTs should be dual to AdS_7 solutions of $N = 2$ gauged supergravity in seven dimensions [11]. Pure $N = 2$ gauged supergravity with $SU(2)$ gauge group admits both supersymmetric and non-supersymmetric AdS_7 vacua [12]. The two vacua can be interpreted as a supersymmetric and a non-supersymmetric CFT, respectively. A domain wall solution interpolating between these vacua has been studied in [13]. This solution describes a non-supersymmetric deformation of the UV $N = (1,0)$ SCFT to another non-supersymmetric CFT in the IR.

When coupled to vector multiplets, the $N = 2$ gauged supergravity with many possible gauge groups can be obtained [14, 15, 16]. Although the resulting matter-coupled theory can support only a half-supersymmetric domain wall vacuum, supersymmetric AdS_7 vacua are possible if a topological mass term for the 3-form field, dual to the 2-form field in the gravity multiplet, is introduced. These supersymmetric AdS_7 critical points with $SO(4)$ and $SO(3)$ symmetries together with analytic RG flows interpolating between them have been studied in [17] in the case of $SO(4)$ gauge group. And recently, AdS_7 vacua including compactifications to AdS_5 of non-compact gauge groups have been explored in [18]. The latter type of solutions generally describe twisted compactifications of $N = (1,0)$ six-dimensional field theories to four dimensions.

In this paper, we are interested in holographic description of twisted compactifications of $N = (1,0)$ SCFTs on two-manifolds $\Sigma_2 = (S^2, H^2)$ and three-manifold $\Sigma_3 = (S^3, H^3)$. The corresponding gravity solutions will take the form of $AdS_5 \times \Sigma_2$

and $AdS_4 \times \Sigma_3$, respectively. The dual field theories will be SCFTs in four or three dimensions. Gravity solutions interpolating between above mentioned AdS_7 vacua and these AdS_5 or AdS_4 geometries will describe RG flows from $N = (1, 0)$ SCFTs to lower dimensional SCFTs. Previously, this type of solutions has mainly been studied within the framework of the maximal $N = 4$ gauged supergravity. The solutions provide gravity duals of twisted compactifications of the $N = (2, 0)$ SCFTs. A number of these AdS_5 solutions together with the uplift to eleven-dimensional supergravity by using the reduction ansatz given in [19] and [20] have been studied previously in [21, 22, 23, 24]. In addition, compactifications of $N = (1, 0)$ SCFT has recently been explored from the point of view of massive type IIA theory in [25].

We will give another new solution to this class from $N = 2$ $SO(4)$ gauged supergravity. It has been pointed out in [22] that the $AdS_5 \times S^2$ solution preserving $SO(2) \times SO(2)$ symmetry and $N = 2$ supersymmetry in five dimensions, eight supercharges, cannot be obtained from pure minimal $N = 2$ gauged supergravity. We will show that this solution is a solution of $N = 2$ $SO(4)$ gauged supergravity obtained from coupling pure $N = 2$ gauged supergravity to three vector multiplets. We will additionally give new $AdS_5 \times H^2$ solutions which are different from those given in [22] and [23] in the sense that the two $SU(2)$ gauge couplings are different, and the residual symmetry is only the diagonal subgroup of $SO(2) \times SO(2)$. This case is not a truncation of the $N = 4$ $SO(5)$ gauged supergravity, and the embedding of these solutions in higher dimensions are presently unknown. We will also study holographic RG flow solutions interpolating between AdS_7 vacua and these AdS_5 fixed points. The solutions describe deformations of $N = (1, 0)$ SCFTs in six dimensions to the IR $N = 1$ SCFT in four dimensions.

On AdS_4 solutions from seven-dimensional gauged supergravity, a class of $AdS_4 \times H^3$ and $AdS_4 \times S^3$ solutions have been obtained in [26]. A number of extensive studies of these solutions in terms of wrapped M5-branes on various supersymmetric cycles in special holonomy manifolds have been given in [27, 28, 29]. In particular, the solution studied in [29] has been obtained from the maximal gauged supergravity and preserves $N = 2$ superconformal symmetry in three dimensions. In this work, we will look for AdS_4 solutions in the $N = 2$ $SO(4)$ gauged supergravity preserving only four supercharges. The corresponding solutions should then correspond to some $N = 1$ SCFTs in three dimensions. We will show that there exist $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions in this $SO(4)$ gauged supergravity with four supercharges when the two $SU(2)$ gauge couplings are different. For equal $SU(2)$ gauge couplings, only $AdS_4 \times H^3$ solutions exist and can be uplifted to eleven dimensions using the reduction ansatz given in [30].

The paper is organized as follow. In section 2, relevant information on $N = 2$ $SO(4)$ gauged supergravity in seven dimensions and supersymmetric AdS_7 critical points are reviewed. $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions together with holographic RG flows from AdS_7 critical points to these AdS_5 fixed points will be given in section 3. We present $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions in section 4 and give the embedding of some $AdS_5 \times \Sigma_2$ and $AdS_4 \times \Sigma_3$ solutions in eleven dimensions in section 5. We finally give some comments and conclusions in section 6.

2. Seven-dimensional $N = 2$ $SO(4)$ gauged supergravity and AdS_7 critical points

In this section, we give a description of the $SO(4)$ $N = 2$ gauged supergravity in seven dimensions and the associated supersymmetric AdS_7 critical points. These critical points preserve $N = 2$ supersymmetry in seven dimensions and correspond to six-dimensional $N = (1, 0)$ SCFTs. All of the notations used throughout the paper are the same as those in [16] and [17].

2.1 $SO(4)$ gauged supergravity

The $SO(4)$ $N = 2$ gauged supergravity in seven dimensions is constructed by gauging the half-maximal $N = 2$ supergravity coupled to three vector multiplets. The supergravity multiplet $(e^m_\mu, \psi^A_\mu, A^i_\mu, \chi^A, B_{\mu\nu}, \sigma)$ consists of the graviton, two gravitini, three vectors, two spin-$\frac{1}{2}$ fields, a two-form field and the dilaton. We will use the convention that curved and flat space-time indices are denoted by $\mu, \nu, \ldots$ and $m, n, \ldots$, respectively. Each vector multiplet $(A_\mu, \lambda^A, \phi^i)$ contains a vector field, two gauginos and three scalars. The bosonic field content of the matter coupled supergravity then consists of the graviton, six vectors and ten scalars parametrized by the $\mathbb{R}^+ \times SO(3, 3)/SO(3) \times SO(3) \sim \mathbb{R}^+ \times SL(4, \mathbb{R})/SO(4)$ coset manifold. In the following, we will consider the supergravity theory in which the two-form field $B_{\mu\nu}$ is dualized to a three-form field $C_{\mu\nu\rho}$. The latter admits a topological mass term, so the resulting gauged supergravity admits an AdS_7 vacuum.

The $SO(4)$ gauged supergravity is obtained by gauging the $SO(4) \sim SO(3)\times SO(3)$ subgroup of the global symmetry group $SO(3, 3)$. One of the $SO(3)$ in the gauge group $SO(3) \times SO(3)$ is the $SO(3)_R \sim USp(2)_R \sim SU(2)_R$ R-symmetry. All spinor fields, including the supersymmetry parameter ϵ^A, are symplectic-Majorana spinors transforming as doublets of the $SU(2)_R$ R-symmetry. From now on, the $SU(2)_R$ douplet indices $A, B = 1, 2$ will not be shown explicitly. The $SU(2)_R$ triplets are labeled by indices $i, j = 1, 2, 3$ while indices $r, s = 1, 2, 3$ are the triplet indices of the other $SO(3)$ in $SO(3)_R \times SO(3)$.

The 9 scalar fields in the $SO(3, 3)/SO(3) \times SO(3)$ coset are parametrized by the coset representative $L = ({L_I}^i, {L_I}^r)$ which transforms under the global $SO(3, 3)$ and the local composite $SO(3)\times SO(3)$ by left and right multiplications, respectively. The inverse of L is denoted by $L^{-1}=(L^I{}_i, L^I{}_r)$ satisfying the relations $L^I{}_i=\eta^{IJ}L_{Ji}$ and $L^I{}_r=\eta^{IJ}L_{Jr}$.

The bosonic Lagrangian of the $N=2$ gauged supergravity is given by

$$\begin{aligned}e^{-1}\mathcal{L} &= \frac{1}{2}R-\frac{1}{4}e^{\sigma}a_{IJ}F^I_{\mu\nu}F^{J\mu\nu}-\frac{1}{48}e^{-2\sigma}H_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma}-\frac{5}{8}\partial_\mu\sigma\partial^\mu\sigma-\frac{1}{2}P^{ir}_\mu P^\mu_{ir}\\ &\quad -\frac{1}{144\sqrt{2}}e^{-1}\epsilon^{\mu_1\ldots\mu_7}H_{\mu_1\ldots\mu_4}\omega_{\mu_5\ldots\mu_7}+\frac{1}{36}he^{-1}\epsilon^{\mu_1\ldots\mu_7}H_{\mu_1\ldots\mu_4}C_{\mu_5\ldots\mu_7}-V\end{aligned}\tag{2.1}$$

where the scalar potential and the Chern-Simons term are given by

$$V = \frac{1}{4}e^{-\sigma}\left(C^{ir}C_{ir}-\frac{1}{9}C^2\right)+16h^2e^{4\sigma}-\frac{4\sqrt{2}}{3}he^{\frac{3\sigma}{2}}C,\tag{2.2}$$

$$\omega_{\mu\nu\rho} = 3\eta_{IJ}F^I_{[\mu\nu}A^J_{\rho]}-f_{IJ}{}^KA^I_\mu\wedge A^J_\nu\wedge A_{\rho K}\tag{2.3}$$

with the gauge field strength defined by $F^I_{\mu\nu}=2\partial_{[\mu}A^I_{\nu]}+f_{JK}{}^IA^J_\mu A^K_\nu$. The structure constants $f_{IJ}{}^K$ of the gauge group include the gauge coupling associated to each simple factor in a general gauge group $G_0\subset SO(3,3)$.

We are mainly interested in supersymmetric solutions. Therefore, the supersymmetry transformations of fermions are necessary. However, we will not consider bosonic solutions with the three-form field turned on. We will accordingly set $C_{\mu\nu\rho}=0$ throughout. The fermionic supersymmetry transformations, with all fermions and the three-form field vanishing, are given by

$$\delta\psi_\mu = 2D_\mu\epsilon-\frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\gamma_\mu\epsilon-\frac{i}{20}e^{\frac{\sigma}{2}}F^i_{\rho\sigma}\sigma^i\left(3\gamma_\mu\gamma^{\rho\sigma}-5\gamma^{\rho\sigma}\gamma_\mu\right)\epsilon-\frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon,\tag{2.4}$$

$$\delta\chi = -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon-\frac{i}{10}e^{\frac{\sigma}{2}}F^i_{\mu\nu}\sigma^i\gamma^{\mu\nu}\epsilon+\frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\epsilon-\frac{16}{5}e^{2\sigma}h\epsilon,\tag{2.5}$$

$$\delta\lambda^r = -i\gamma^\mu P^{ir}_\mu\sigma^i\epsilon-\frac{1}{2}e^{\frac{\sigma}{2}}F^r_{\mu\nu}\gamma^{\mu\nu}\epsilon-\frac{i}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{ir}\sigma^i\epsilon\,.\tag{2.6}$$

Various quantities appearing in the Lagrangian and supersymmetry transformations are defined by the following relations

$$\begin{aligned}D_\mu\epsilon &= \partial_\mu\epsilon+\frac{1}{4}\omega^{mn}_\mu\gamma_{mn}+\frac{i}{4}\sigma^i\epsilon^{ijk}Q_{\mu jk},\\ P^{ir}_\mu &= L^{Ir}\left(\delta^K_I\partial_\mu+f_{IJ}{}^KA^J_\mu\right)L^i{}_K, \qquad Q^{ij}_\mu = L^{Ij}\left(\delta^K_I\partial_\mu+f_{IJ}{}^KA^J_\mu\right)L^i{}_K,\\ C_{ir} &= \frac{1}{\sqrt{2}}f_{IJ}{}^KL^I{}_jL^J{}_kL_{Kr}\epsilon^{ijk}, \qquad C=-\frac{1}{\sqrt{2}}f_{IJ}{}^KL^I{}_iL^J{}_jL_{Kk}\epsilon^{ijk},\\ C_{rsi} &= f_{IJ}{}^KL^I{}_rL^J{}_sL_{Ki}, \qquad a_{IJ}=L^i{}_IL_{iJ}+L^r{}_IL_{rJ},\\ F^i_{\mu\nu} &= L_I{}^iF^I, \qquad F^r_{\mu\nu}=L_I{}^rF^I\end{aligned}\tag{2.7}$$

where γ^m are space-time gamma matrices satisfying $\{\gamma^m, \gamma^n\} = 2\eta^{mn}$ with $\eta^{mn} = \mathrm{diag}(-1, 1, 1, 1, 1, 1, 1)$.

2.2 Supersymmetric AdS_7 critical points

We will now briefly review supersymmetric AdS_7 critical points found in [17]. There are two critical points preserving the full $N = 2$ supersymmetry in seven dimensions. The two critical points however have different symmetries namely one critical point, at which all scalars vanishing, preserves the full $SO(4)$ gauge symmetry while the other is only invariant under the diagonal subgroup $SO(3)_{\mathrm{diag}} \subset SO(3) \times SO(3)$.

For $SO(3) \times SO(3)$ gauge group, the gauge structure constants can be written as [16]

$$f_{IJK} = (g_1 \epsilon_{ijk}, -g_2 \epsilon_{rst}). \tag{2.8}$$

Before discussing the detail of the two critical points, we give an explicit parametrization of the $SO(3,3)/SO(3) \times SO(3)$ coset as follow. With the 36 basis elements of a general 6×6 matrix

$$(e_{IJ})_{KL} = \delta_{IK}\delta_{JL}, \qquad I, J, \ldots = 1, \ldots, 6 \tag{2.9}$$

the generators of the composite $SO(3) \times SO(3)$ symmetry are given by

$$\begin{aligned} SO(3)_R : \qquad & J^{(1)}_{ij} = e_{ji} - e_{ij}, \qquad i, j = 1, 2, 3, \\ SO(3) : \qquad & J^{(2)}_{rs} = e_{s+3,r+3} - e_{r+3,s+3}, \qquad r, s = 1, 2, 3\,. \end{aligned} \tag{2.10}$$

The non-compact generators corresponding to 9 scalars take the form of

$$Y^{ir} = e_{i,r+3} + e_{r+3,i}\,. \tag{2.11}$$

Accordingly, the coset representative can be obtained by an exponentiation of the appropriate Y^{ir} generators. Y^{ir} generators and the 9 scalars transform as $(\mathbf{3}, \mathbf{3})$ under the $SO(3) \times SO(3)$ local symmetry.

The supersymmetric AdS_7 critical points preserve at least $SO(3)$ symmetry. Therefore, we will consider only the coset representative invariant under $SO(3)$ symmetry. The dilaton σ is an $SO(3) \times SO(3)$ singlet. From the 9 scalars in $SO(3,3)/SO(3) \times SO(3)$, there is one $SO(3)_{\mathrm{diag}}$ singlet from the decomposition $\mathbf{3} \times \mathbf{3} \to \mathbf{1} + \mathbf{3} + \mathbf{5}$. The singlet corresponds to the non-compact generator

$$Y_s = Y^{11} + Y^{22} + Y^{33}\,. \tag{2.12}$$

The coset representative is then given by

$$L = e^{\phi Y_s}\,. \tag{2.13}$$

The scalar potential for the dilaton σ and the $SO(3)_{\text{diag}}$ singlet scalar ϕ can be straightforwardly computed. Its explicit form reads [17]

$$V = \frac{1}{32}e^{-\sigma}\Big[(g_1^2+g_2^2)\left(\cosh(6\phi)-9\cosh(2\phi)\right)+8g_1g_2\sinh^3(2\phi) \\ +8\Big[g_2^2-g_1^2+64h^2e^{5\sigma}+32e^{\frac{5\sigma}{2}}h\left(g_1\cosh^2\phi+g_2\sinh^3\phi\right)\Big]\Big]. \quad (2.14)$$

There are two supersymmetric AdS_7 vacua given by

$$SO(4)-\text{critical point}\ : \qquad \sigma=\phi=0, \qquad V_0=-240h^2, \quad (2.15)$$

$$SO(3)-\text{critical point}\ : \qquad \sigma=-\frac{1}{5}\ln\left[\frac{g_2^2-256h^2}{g_2^2}\right],$$

$$\phi=\frac{1}{2}\ln\left[\frac{g_2+16h}{g_2-16h}\right], \qquad V_0=-\frac{240g_2^{\frac{8}{5}}h^2}{(g_2^2-256h^2)^{\frac{4}{5}}} \quad (2.16)$$

where we have chosen $g_1=-16h$ in order to make the $SO(4)$ critical point occurs at $\sigma=0$. This is achieved by shifting σ. The value of the cosmological constant has been denoted by V_0.

The two critical points correspond to $N=(1,0)$ SCFTs in six dimensions with $SO(4)$ and $SO(3)$ symmetries, respectively. An RG flow solution interpolating between these two critical points has already been studied in [17]. In the next sections, we will study supersymmetric RG flows from these SCFTs to other SCFTs in four and three dimensions providing holographic descriptions of twisted compactifications of these $N=(1,0)$ SCFTs.

3. Flows to $N=1$ SCFTs in four dimensions

In this section, we look for solutions of the form $AdS_5\times S^2$ or $AdS_5\times H^2$ in which S^2 and H^2 are a two-sphere and a two-dimensional hyperbolic space, respectively.

In the case of S^2, we take the seven-dimensional metric to be

$$ds_7^2=e^{2F(r)}dx_{1,3}^2+dr^2+e^{2G(r)}(d\theta^2+\sin^2 d\phi^2) \quad (3.1)$$

with $dx_{1,3}^2$ being the flat metric on the four-dimensional spacetime. By using the vielbein

$$e^{\hat{\mu}}=e^Fdx^\mu, \qquad e^{\hat{r}}=dr, \\ e^{\hat{\theta}}=e^Gd\theta, \qquad e^{\hat{\phi}}=e^G\sin\theta d\phi, \quad (3.2)$$

we can compute the following spin connections

$$\omega^{\hat{\phi}}{}_{\hat{\theta}}=e^{-G}\cot\theta e^{\hat{\phi}}, \qquad \omega^{\hat{\phi}}{}_{\hat{r}}=G'e^{\hat{\phi}}, \\ \omega^{\hat{\theta}}{}_{\hat{r}}=G'e^{\hat{\theta}}, \qquad \omega^{\hat{\mu}}{}_{\hat{r}}=F'e^{\hat{\mu}}\,. \quad (3.3)$$

where $'$ denotes the r-derivative. Hatted indices are tangent space indices.

In the case of H^2, we take the matric to be

$$ds_7^2 = e^{2F(r)}dx_{1,3}^2 + dr^2 + \frac{e^{2G(r)}}{y^2}(dx^2 + dy^2). \tag{3.4}$$

With the vielbein

$$\begin{aligned} e^{\hat{\mu}} &= e^F dx^\mu, \qquad & e^{\hat{r}} &= dr, \\ e^{\hat{x}} &= \frac{e^G}{y}dx, \qquad & e^{\hat{y}} &= \frac{e^G}{y}dy, \end{aligned} \tag{3.5}$$

the spin connections are found to be

$$\begin{aligned} \omega^{\hat{x}}{}_{\hat{r}} &= G'e^{\hat{x}}, \qquad & \omega^{\hat{y}}{}_{\hat{r}} &= G'e^{\hat{y}}, \\ \omega^{\hat{\mu}}{}_{\hat{r}} &= F'e^{\hat{\mu}}, \qquad & \omega^{\hat{x}}{}_{\hat{y}} &= -e^{-G(r)}e^{\hat{x}}\,. \end{aligned} \tag{3.6}$$

3.1 AdS_5 solutions with $SO(2) \times SO(2)$ symmetry

We now construct the BPS equations from the supersymmetry transformations of fermions. We first consider the S^2 case. In order to preserve supersymmetry, we make a twist by turning on the $SO(2) \times SO(2) \subset SO(4)$ gauge fields, among the six gauge fields A^I,

$$A^3 = a\cos\theta d\phi \qquad \text{and} \qquad A^6 = b\cos\theta d\phi \tag{3.7}$$

such that the spin connections on S^2 is cancelled by these gauge connections. The Killing spinor corresponding to the unbroken supersymmetry is then a constant spinor on S^2.

We begin with the solutions preserving the full $SO(2) \times SO(2)$ residual gauge symmetry generated by $J_{12}^{(1)}$ and $J_{12}^{(2)}$. Scalars which are singlet under $SO(2) \times SO(2)$ are the dilaton and the scalar corresponding to the $SO(3,3)$ non-compact generators Y^{33}. We will denote this scalar by Φ. By considering the variation of the gravitino along S^2 directions, we find that the cancellation between the spin and gauge connections imposes the twist condition

$$ag_1 = 1\,. \tag{3.8}$$

Using the projection conditions

$$\gamma_r\epsilon = \epsilon, \qquad \text{and} \qquad i\sigma^3\gamma^{\hat{\theta}\hat{\phi}}\epsilon = \epsilon, \tag{3.9}$$

we find the following BPS equations

$$\begin{aligned}
\Phi' &= \frac{1}{2}e^{-\frac{\sigma}{2}-\Phi-2G}\left[e^{2G}g_1(e^{2\Phi}-1)-ae^{\sigma}(e^{2\Phi}-1)-be^{\sigma}(e^{2\Phi}+1)\right], &(3.10)\\
\sigma' &= \frac{1}{5}e^{-\frac{\sigma}{2}-\Phi-2G}\left[e^{\sigma}\left[a-b+(a+b)e^{2\Phi}\right]-e^{2G}\left(g_1+g_1e^{2\Phi}+32he^{\frac{5\sigma}{2}+\Phi}\right)\right], &(3.11)\\
G' &= -\frac{1}{10}e^{-\frac{\sigma}{2}-\Phi-2G}\left[4e^{\sigma}\left[a-b+(a+b)e^{2\Phi}\right]+e^{2G}\left(g_1+g_1e^{2\Phi}-8he^{\frac{5\sigma}{2}+\Phi}\right)\right], &(3.12)\\
F' &= \frac{1}{10}e^{-\frac{\sigma}{2}-\Phi-2G}\left[e^{\sigma}\left[a-b+(a+b)e^{2\Phi}\right]-e^{2G}\left(g_1+g_1e^{2\Phi}-8he^{\frac{5\sigma}{2}+\Phi}\right)\right]. &(3.13)
\end{aligned}$$

In the H^2 case, we choose the gauge fields to be

$$A^3=\frac{a}{y}dx \qquad \text{and} \qquad A^6=\frac{b}{y}dx \tag{3.14}$$

which can be verified that the spin connection $\omega^{\hat{x}\hat{y}}$ in (3.6) is cancelled by virtue of the twist condition (3.8) and the projection conditions

$$\gamma_r\epsilon=\epsilon \qquad \text{and} \qquad i\sigma^3\gamma^{\hat{x}\hat{y}}\epsilon=\epsilon\,. \tag{3.15}$$

By an analogous computation, we find a similar set of BPS equations as in (3.10), (3.11), (3.12) and (3.13) with (a,b) replaced by $(-a,-b)$.

At large r, solutions to the above BPS equations should approach the $SO(4)$ AdS_7 critical point with $\Phi\sim\sigma\sim 0$ and $F\sim G\sim r$. This is the UV (1,0) SCFT. As $r\to-\infty$, we look for the solution of the form $AdS_5\times S^2$ or $AdS_5\times H^2$ such that $\phi'=\sigma'=G'=0$ and $F'=$ constant. We find that there is an AdS_5 solution given by

$$\begin{aligned}
\Phi &= \frac{1}{2}\ln\left[\frac{b\pm\sqrt{4a^2-3b^2}}{2(a+b)}\right],\\
\sigma &= \frac{1}{5}\ln\left[\frac{g_1^2b^2(b\pm\sqrt{4a^2-3b^2})}{32(a+b)h^2(3b-2a\pm\sqrt{4a^2-3b^2})}\right],\\
G &= \frac{1}{10}\ln\left[\frac{b^2(a+b)^4(b\pm\sqrt{4a^2-3b^2})(2a-3b\mp\sqrt{4a^2-3b^2})^3}{32g_1^3h^2(2a+b\mp\sqrt{4a^2-3b^2})^5}\right],\\
L_{AdS_5} &= \left[\frac{(a+b)^2(2a-3b\pm\sqrt{4a^2-3b^2})^4}{b^4g_1^4h(b\mp\sqrt{4a^2-3b^2})^2}\right]^{\frac{1}{5}}. \qquad (3.16)
\end{aligned}$$

This solution is given for $\Sigma_2=S^2$. The solution in the H^2 case is given similarly by flipping the signs of a and b.

It should be noted that, in this fixed point solution with $SO(2)\times SO(2)$ symmetry, the coupling g_2 does not appear. The solution can then be taken as a solution of the gauged supergravity with $g_2 = g_1$. Therefore, the solution can be uplifted to eleven dimensions by using the reduction ansatz in [30]. This will be done in section 5. The uplifted solution is however not new since similar solutions have been found previously in [22, 23], and supergravity solutions interpolating between AdS_7 and $AdS_5 \times S^2$ or $AdS_5 \times H^2$ have also been investigated. The solutions have an interpretation in terms of RG flows from the UV SCFT in six dimensions to four-dimensional SCFTs with $SO(2) \times SO(2)$ symmetry.

Note also that, in this case, it is not possible to find an RG flow from the $SO(3)$ AdS_7 point to any of these four-dimensional SCFTs since this AdS_7 critical point is not accessible from the BPS equations given above.

3.2 AdS_5 solutions with $SO(2)$ symmetry

We now consider AdS_5 solutions with $SO(2)$ symmetry. We will study two possibilities namely the $SO(2)_{\text{diag}} \subset SO(2) \times SO(2) \subset SO(3) \times SO(3)$ and $SO(2)_R \subset SO(3)_R$.

3.2.1 Flows with $SO(2)_{\text{diag}}$ symmetry

We begin with the $SO(2)_{\text{diag}}$ symmetry generated by $J^{(1)}_{12} + J^{(2)}_{12}$. Among the 9 scalars in $SO(3,3)/SO(3) \times SO(3)$, there are three singlets under $SO(2)_{\text{diag}}$ corresponding to the following decomposition of $SO(3) \times SO(3)$ representations under $SO(2)_{\text{diag}}$

$$\mathbf{3} \times \mathbf{3} = (\mathbf{2} + \mathbf{1}) \times (\mathbf{2} + \mathbf{1}) = \mathbf{1} + \mathbf{1} + \mathbf{2} + \mathbf{2} + \mathbf{2} + \mathbf{1}\,. \tag{3.17}$$

The three singlets correspond to the non-compact generators

$$Y^{11} + Y^{22}, \qquad Y^{33}, \qquad Y^{12} - Y^{21}\,. \tag{3.18}$$

The coset representative describing these singlets can be written as

$$L = e^{\Phi_1(Y^{11}+Y^{22})} e^{\Phi_2 Y^{33}} e^{\Phi_3(Y^{12}-Y^{21})}\,. \tag{3.19}$$

Since we have not found any $AdS_5 \times S^2$ solution, we will give only the result for the H^2 case. The $SO(2)_{\text{diag}}$ gauge field can be obtained from the $SO(2) \times SO(2)$ gauge fields in (3.7) with the condition that

$$bg_2 = ag_1\,. \tag{3.20}$$

As in the previous case, the twist imposes the condition $g_1 a = 1$ which in the present case also implies $g_2 b = 1$.

Using the projection conditions (3.15), we find the following BPS equations

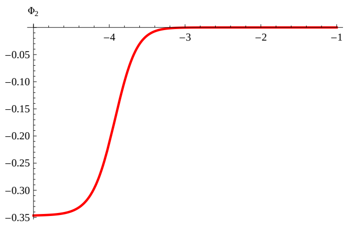
$$
\begin{aligned}
\Phi_1' &= \frac{1}{8}e^{-\frac{\sigma}{2}-2\Phi_1-\Phi_2}(e^{4\Phi_1}-1)\left[g_1-g_2+(g_1+g_2)e^{2\Phi_2}\right], && (3.21)\\
\Phi_2' &= \frac{1}{16g_2}e^{-\frac{\sigma}{2}}\left[8g_1a\left[g_1-g_2+(g_1+g_2)e^{\Phi_2}\right]\right.\\
&\quad +g_2\left[e^{-2\Phi_1-\Phi_2-2\Phi_3}(1+e^{4\Phi_1})(1+e^{4\Phi_3})\left[g_2-g_1+(g_1+g_2)e^{2\Phi_2}\right]\right.\\
&\quad \left.\left.+4(g_1-g_2)e^{\Phi_2}-(g_1+g_2)e^{-\Phi_2}\right]\right], && (3.22)\\
\Phi_3' &= \frac{1}{8}e^{-\frac{\sigma}{2}-\Phi_2-2\Phi_3}(e^{4\Phi_3}-1)\left[g_1-g_2+(g_1+g_2)e^{2\Phi_2}\right], && (3.23)\\
\sigma' &= \frac{1}{40g_2}e^{-\frac{\sigma}{2}-2\Phi_1-\Phi_2-2\Phi_3}\left[8ae^{\sigma+2\Phi_1+2\Phi_3-2G}\left[g_1-g_2-(g_1+g_2)e^{2\Phi_2}\right]\right.\\
&\quad -g_2\left[g_1(1+e^{2\Phi_2})(1+e^{4\Phi_1}+e^{4\Phi_3}+4e^{2\Phi_1+2\Phi_3}+e^{4\Phi_1+4\Phi_3})\right.\\
&\quad +g_2(e^{2\Phi_2}-1)(1+e^{4\Phi_1}+e^{4\Phi_3}-4e^{2\Phi_1+2\Phi_3}+e^{4\Phi_1+4\Phi_3})\\
&\quad \left.\left.+256he^{\frac{5\sigma}{2}+2\Phi_1+\Phi_2+2\Phi_3}\right]\right], && (3.24)\\
G' &= \frac{1}{20}e^{-\frac{\sigma}{2}}\left[16he^{\frac{5\sigma}{2}}-g_1(e^{\Phi_2}+e^{-\Phi_2})+g_2(e^{\Phi_2}-e^{-\Phi_2})\right.\\
&\quad -\frac{1}{4}e^{-2\Phi_1-\Phi_2-2\Phi_3}(1+e^{4\Phi_1})(1+e^{4\Phi_3})[g_1-g_2+(g_1+g_2)e^{2\Phi_2}]\\
&\quad \left.+\frac{8a}{g_2}e^{\sigma-\Phi_2-2G}[g_2-g_1+(g_1-g_2)e^{2\Phi_2}]\right], && (3.25)\\
F' &= \frac{1}{20}e^{-\frac{\sigma}{2}}\left[16he^{\frac{5\sigma}{2}}-g_1(e^{\Phi_2}+e^{-\Phi_2})+g_2(e^{\Phi_2}-e^{-\Phi_2})\right.\\
&\quad -\frac{1}{4}e^{-2\Phi_1-\Phi_2-2\Phi_3}(1+e^{4\Phi_1})(1+e^{4\Phi_3})[g_1-g_2+(g_1+g_2)e^{2\Phi_2}]\\
&\quad \left.-\frac{2a}{g_2}e^{\sigma-\Phi_2-2G}[g_2-g_1+(g_1-g_2)e^{2\Phi_2}]\right]. && (3.26)
\end{aligned}
$$

In this case, there are a number of possible AdS_5 fixed point solutions, and it is possible to have a solution interpolating between the $SO(3)$ AdS_7 critical points and the AdS_5 in the IR. We will investigate each of them in the following discussion.

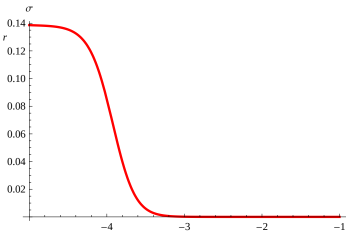
We first look at the $AdS_5\times H^2$ critical point with $g_2=g_1$ since this can be uplifted to eleven dimensions. When $g_2=g_1$, the fixed point solution exists only for $\Phi_1=\Phi_3=0$, and the corresponding solution is given by

$$
\begin{aligned}
&\Phi_2=-\frac{1}{2}\ln 2, \sigma=\frac{1}{5}\ln 2,\\
&G=\frac{3}{5}\ln 2-\frac{1}{2}\ln\left[\frac{g_1}{a}\right], \qquad L_{AdS_5}=\frac{1}{2^{\frac{12}{5}}h} \qquad (3.27)
\end{aligned}
$$

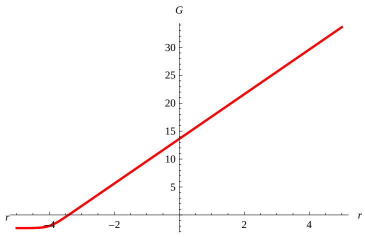
The AdS_5 solution preserves eight supercharges corresponding to $N=1$ superconformal field theory in four dimensions with $SO(2)$ symmetry. A flow solution interpolating



(a) Φ_2 solution



(b) σ solution



(c) G solution

Figure 1: RG flows from $SO(4)$ $N=(1,0)$ SCFT in six dimensions to four-dimensional $N=1$ SCFT with $SO(2)_{\text{diag}}$ symmetry for $g_1=g_2$.

between this $AdS_5\times H^2$ fixed point and the $SO(4)$ AdS_7 given in (2.15) for $h=1$ is shown in Figure 1.

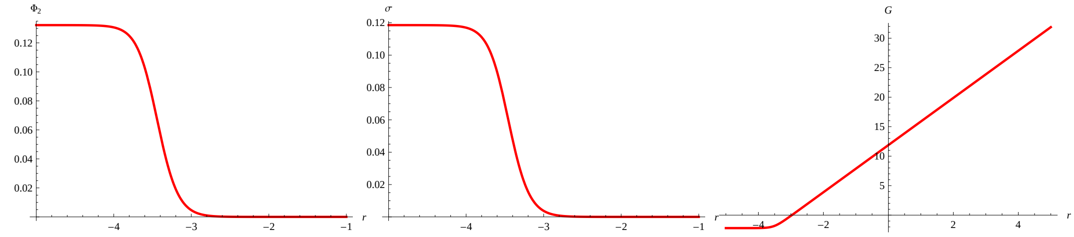
It should be noted here that this fixed point can be obtained from the $SO(2)\times SO(2)$ fixed points given in the previous section by setting the parameter $b=a$. It can be readily verified that, for $b=a$, solution in (3.16) is valid only for the upper sign and $\Sigma_2=H^2$. The resulting solution is precisely that given in (3.27).

We now move to solutions with $g_2\neq g_1$. The solution given in (3.27) is a special case of a more general solution, with $\Phi_1=\Phi_3=0$ and $g_2\neq g_1$, which is given by

$$
\begin{aligned}
\Phi_2 &= \frac{1}{2}\ln\left[\frac{g_1\pm\sqrt{4g_2^2-3g_1^2}}{2(g_1+g_2)}\right], \qquad \Phi_1=\Phi_3=0,\\
\sigma &= \frac{1}{5}\left[\frac{1024h^2(\sqrt{g_2^2-192h^2}\mp 8h)}{(g_2-16h)(g_2+24h\mp\sqrt{g_2^2-192h^2})}\right],\\
G &= \frac{1}{10}\ln\left[\frac{a^5(g_2-16h)^4(\sqrt{g_2^2-192h^2}\mp 8h)(g_2+24h\mp\sqrt{g_2^2-192h^2})^3}{1024g_2^5h^3(g_2-8h\mp\sqrt{g_2^2-192h^2})^5}\right],\\
L_{AdS_5} &= \frac{1}{2}\left[\frac{(g_2-16h)^2(g_2+24h\mp\sqrt{g_2^2-192h^2})^4}{2h^9(8h\mp\sqrt{g_2^2-192h^2})}\right]^{\frac{1}{5}}
\end{aligned}
\tag{3.28}
$$

where we have used the relation $g_1=-16h$ in the solutions for σ and G to simplify the expressions. An example of the corresponding flow solutions from the UV $N=(1,0)$ $SO(4)$ SCFT to this critical point, with $g_2=-2g_1$ and $h=1$, is given in Figure 2.

In all of the above solutions, it is not possible to have a flow from the $SO(3)$ AdS_7 critical point (2.16). To find this type of flows, we look for AdS_5 fixed points with



(a) Φ_2 solution (b) σ solution (c) G solution

Figure 2: RG flows from $SO(4)$ $N = (1, 0)$ SCFT in six dimensions to four-dimensional $N = 1$ SCFT with $SO(2)_{\text{diag}}$ symmetry for $g_1 \neq g_2$.

$\Phi_3 = 0$ but $\Phi_1 \neq 0$ and $\Phi_2 \neq 0$. In this case, the $AdS_5 \times H^2$ solution is given by

$$\begin{aligned}
\Phi_2 &= \frac{1}{2}\ln\left[\frac{g_2 - g_1}{g_2 + g_1}\right], \qquad \Phi_1 = \pm\Phi_2, \\
\sigma &= \frac{1}{5}\ln\left[\frac{g_1^2 g_2^2}{144h^2(g_2^2 - g_1^2)}\right], \qquad G = \frac{1}{2}\ln\left[\frac{2^{\frac{4}{5}}(3^{\frac{3}{5}})a(g_2^2 - 256h^2)^{\frac{4}{5}}}{g_2^{\frac{8}{5}} g_1}\right], \\
L_{AdS_5} &= \frac{3^{\frac{4}{5}}(g_2^2 - 256h^2)^{\frac{2}{5}}}{2^{\frac{18}{5}} g_2^{\frac{4}{5}} h}\,. \qquad (3.29)
\end{aligned}$$

Note that at the values of Φ_1 and Φ_2 are the same as the $SO(3)$ AdS_7 point. In equation (2.16), we have

$$\Phi_1 = \Phi_2 = \frac{1}{2}\ln\left[\frac{g_2 - g_1}{g_2 + g_1}\right] \equiv \Phi_0\,. \qquad (3.30)$$

Actually, there are two equivalent values of Φ_1 namely either $\Phi_1 = \Phi_0$ or $\Phi_1 = -\Phi_0$. The two choices are equivalent in the sense that they give rise to the same value of the cosmological constant and the same scalar masses. The difference between the two is the generators of $SO(3)$ under which the $SO(3)$ singlet scalar ϕ in (2.16) is invariant. For $\Phi_1 = \Phi_0$, we have $\Phi_1 = \Phi_2$ which is invariant under the $SO(3)$ generated by $J^{(1)}_{ij} + J^{(2)}_{ij}$. The alternative value of $\Phi_1 = -\Phi_0$ gives $\Phi_1 = -\Phi_2$ which is invariant under $SO(3)$ generators $J^{(1)}_{12} + J^{(2)}_{12}$, $J^{(1)}_{13} - J^{(2)}_{13}$ and $J^{(1)}_{23} - J^{(2)}_{23}$. This difference does not affect the result discussed here since, in both cases, the residual $SO(2)_{\text{diag}}$ is still generated by $J^{(1)}_{12} + J^{(2)}_{12}$.

The flow from $SO(3)$ $N = (1, 0)$ SCFT would be driven only by the dilaton σ which has different values at the $SO(3)$ AdS_7 and the AdS_5 fixed points. This is expected since at $SO(3)$ AdS_7 critical point only σ corresponds to relevant operators, see the scalar masses in [17].

We now consider RG flows from $N = (1,0)$ SCFTs in six dimensions to four-dimensional SCFTs identified with the critical point (3.29). In order to give some explicit examples, we choose particular values of the two couplings g_1 and g_2. In the following solutions, we will set $g_2 = -2g_1$ and $h = 1$. With these, the IR $AdS_5 \times H^2$ is given by

$$\Phi_1 = \Phi_2 = \frac{1}{2}\ln 3 \approx 0.5493, \qquad \sigma = \frac{1}{5}\ln\left[\frac{64}{27}\right] \approx 0.1726,$$
$$G = \frac{1}{10}\ln\left[\frac{3^7}{2^{44}}\right] \approx -2.2808\,. \tag{3.31}$$

The $SO(4)$ UV point (2.15) is given by

$$\Phi_1 = \Phi_2 = \sigma = 0 \tag{3.32}$$

while the $SO(3)$ AdS_7 point (2.16) occurs at

$$\sigma = \frac{1}{5}\ln\frac{4}{3} \approx 0.0575, \qquad \Phi_2 = \Phi_1 = \frac{1}{2}\ln 3 \approx 0.5493\,. \tag{3.33}$$

We have chosen $\Phi_1 = \Phi_2$ at the IR fixed points for definiteness.

There exist an RG flow from the $SO(4)$ $N = (1,0)$ SCFT in the UV to the $N = 1$ four-dimensional SCFT in the IR as shown in Figure 3. With a particular boundary condition, we can find an RG flow from the $SO(4)$ AdS_7 to the $SO(3)$ AdS_7 critical points and then to the AdS_5 critical point as shown in Figure 4. This solution is similar to the flow from $SO(6)$ AdS_5 to Khavaev-Pilch-Warner (KPW) AdS_5 critical point and continue to a two-dimensional $N = (2,0)$ SCFT in [31].

3.2.2 Flows with $SO(2)_R$ symmetry

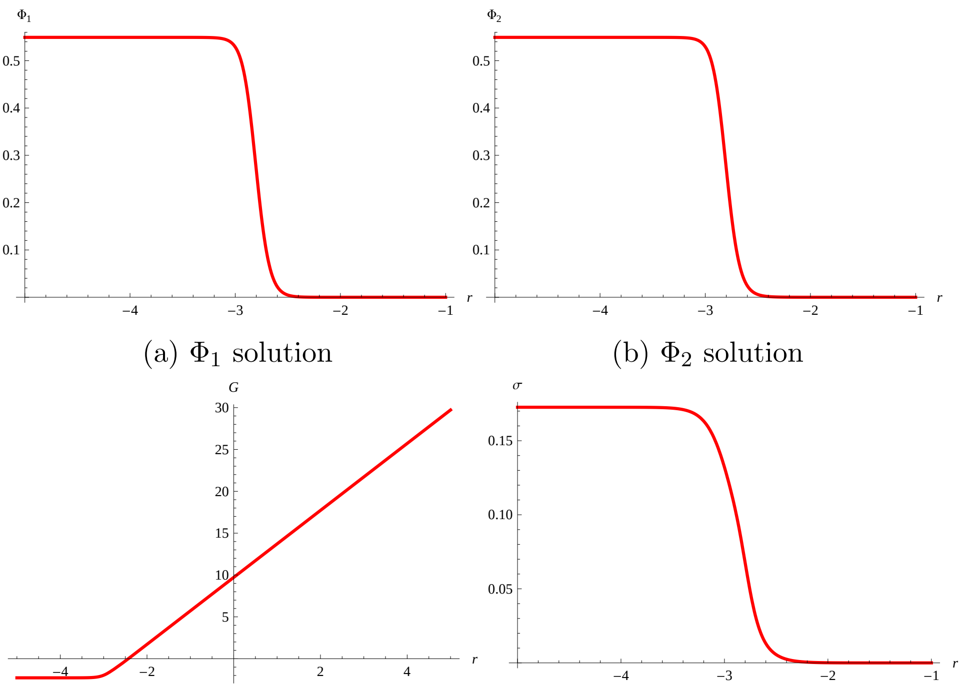
We then move on and briefly look at the $SO(2)_R$ symmetry. There are three singlet scalars from the $SO(3,3)/SO(3) \times SO(3)$ coset. These scalars will be denoted by Φ_1, Φ_2 and Φ_3 corresponding to the non-compact generators Y_{31}, Y_{32} and Y_{33}, respectively.

In this case, the gauge field corresponding the $SO(2)_R$ generator is given by

$$A^3 = a\cos\theta d\phi\,. \tag{3.34}$$

By using the same procedure, we find that, in order to have a fixed point, all of the Φ_i's must vanish, and only $AdS_5 \times H^2$ solutions exist. The solution again preserves eight supercharges corresponding to $N = 1$ superconformal symmetry in four dimensions. The fixed point solution is given by

$$\sigma = \frac{2}{5}\ln\frac{4}{3}, \qquad G = \frac{1}{5}\ln\frac{4}{3} - \frac{1}{2}\ln\frac{g_1}{3a}, \qquad F = \frac{16h}{9^{\frac{2}{5}}}r \tag{3.35}$$



(a) Φ_1 solution

(b) Φ_2 solution

(c) G solution

(d) σ solution

Figure 3: An RG flow from $SO(4)$ $N = (1,0)$ SCFT in six dimensions to four-dimensional $N = 1$ SCFT with $SO(2)_{\text{diag}}$ symmetry.

There exist RG flows from the $SO(4)$ $N = (1,0)$ SCFT to these four-dimensional SCFTs. The BPS equations describing theses flows are given by

$$\sigma' = \frac{2}{5}e^{-\frac{\sigma}{2}}\left(ae^{\sigma-2G} - g_1 - 16he^{\frac{5\sigma}{2}}\right), \tag{3.36}$$

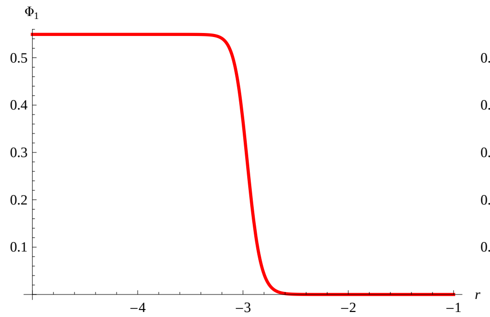
$$G' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(4he^{\frac{5\sigma}{2}} - g_1 - 4ae^{\sigma-2G}\right), \tag{3.37}$$

$$F' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(4he^{\frac{5\sigma}{2}} - g_1 + ae^{\sigma-2G}\right). \tag{3.38}$$

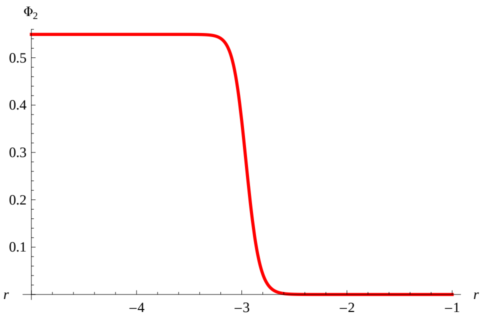
Examples of the solutions with some values of the parameter a are shown in Figure 5. This critical point is also a solution of pure $N = 2$ gauged supergravity studied in [21].

4. Flows to $N = 1$ SCFTs in three dimensions

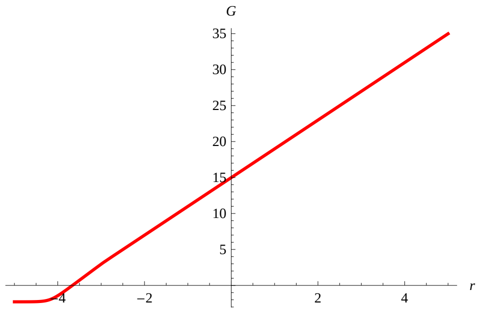
In this section, we look for AdS_4 vacua of the form $AdS_4 \times S^3$ or $AdS_4 \times H^3$ with S^3 and H^3 being a three-sphere and a three-dimensional hyperbolic space, respectively. These



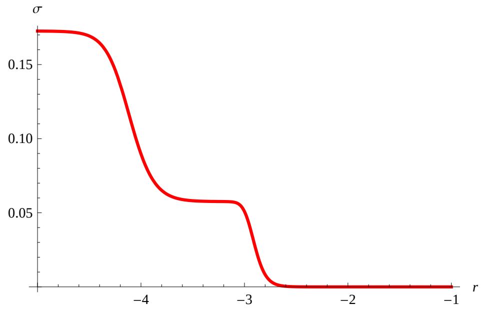
(a) Φ_1 solution



(b) Φ_2 solution

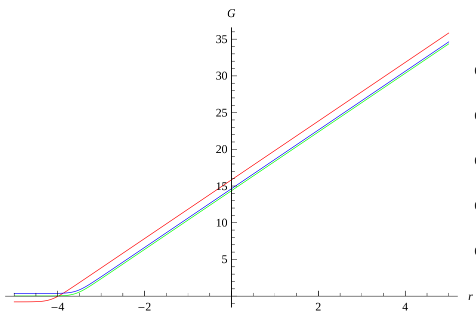


(c) G solution

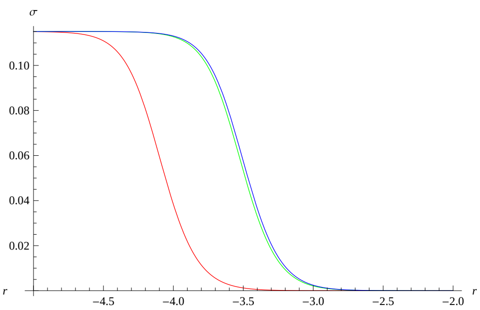


(d) σ solution

Figure 4: An RG flow from $SO(4)$ $N = (1, 0)$ SCFT to $SO(3)$ $N = (1, 0)$ SCFT in six dimensions and then to $N = 1$ four-dimensional SCFT with $SO(2)_{\text{diag}}$ symmetry.



(a) Φ solution



(b) σ solution

Figure 5: RG flows from $SO(4)$ $N = (1, 0)$ SCFT in six dimensions to four-dimensional $N = 1$ SCFT with $SO(2)_R$ symmetry for $a = 1, 5, 10$ (red, green, blue).

solutions will correspond to some SCFTs in three dimensions. In order to identify these AdS_4 vacua with the IR fixed points of the six-dimensional SCFTs corresponding to both of the AdS_7 vacua given in (2.15) and (2.16), we consider the scalars which are singlets under $SO(3)_{\text{diag}}$ subgroup of the full $SO(4)$ gauge group. The relevant scalar from the $SO(3, 3)/SO(3) \times SO(3)$ coset is the one corresponding to the generator (2.12) with the coset representative given in (2.13).

In the S^3 case, we will take the metric ansatz to be

$$ds_7^2 = e^{2F}dx_{1,2}^2 + dr^2 + e^{2G}\left[d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{4.1}$$

From the above metric, we find the spin connections

$$\begin{aligned}
&\omega^{\hat{\mu}}{}_{\hat{r}} = F'e^{\hat{\mu}}, \qquad \omega^{\hat{\psi}}{}_{\hat{r}} = G'e^{\hat{\psi}}, \qquad \omega^{\hat{\theta}}{}_{\hat{r}} = G'e^{\hat{\theta}},\\
&\omega^{\hat{\phi}}{}_{\hat{r}} = G'e^{\hat{\phi}}, \qquad \omega^{\hat{\phi}}{}_{\hat{\theta}} = e^{-G}\frac{\cot\theta}{\sin\psi}e^{\hat{\phi}},\\
&\omega^{\hat{\phi}}{}_{\hat{\psi}} = e^{-G}\cot\psi e^{\hat{\phi}}, \qquad \omega^{\hat{\theta}}{}_{\hat{\psi}} = e^{-G}\cot\psi e^{\hat{\theta}}
\end{aligned} \tag{4.2}$$

which accordingly suggest to turn on the following $SO(3)_{\text{diag}}$ gauge fields

$$\begin{aligned}
A^1 &= \frac{g_2}{g_1}A^4 = a\cos\psi d\theta,\\
A^2 &= \frac{g_2}{g_1}A^5 = a\cos\theta d\phi,\\
A^3 &= \frac{g_2}{g_1}A^6 = a\cos\psi\sin\theta d\phi.
\end{aligned} \tag{4.3}$$

Note that at the beginning, the parameter a of each gauge field needs not be equal. However, the twist condition

$$ag_1 = 1 \tag{4.4}$$

requires that all of the parameters in front of A^i must be equal. The corresponding field strengths are, after using (4.4),

$$\begin{aligned}
F^1 &= -ae^{-2G}e^{\hat{\psi}}\wedge e^{\hat{\theta}},\\
F^2 &= -ae^{-2G}e^{\hat{\theta}}\wedge e^{\hat{\phi}},\\
F^3 &= -ae^{-2G}e^{\hat{\psi}}\wedge e^{\hat{\phi}}.
\end{aligned} \tag{4.5}$$

To set up the BPS equations, we impose the projection conditions

$$\gamma_r\epsilon = \epsilon, \qquad i\sigma^1\gamma_{\hat{\theta}\hat{\psi}}\epsilon = \epsilon, \qquad i\sigma^2\gamma_{\hat{\phi}\hat{\theta}}\epsilon = \epsilon, \qquad i\sigma^3\gamma_{\hat{\phi}\hat{\psi}}\epsilon = \epsilon. \tag{4.6}$$

For the H^3 case, we take the metric to be

$$ds_7^2 = e^{2F}dx_{1,2}^2 + dr^2 + \frac{e^{2G}}{y^2}(dx^2 + dy^2 + dz^2) \tag{4.7}$$

with the spin connections given by

$$\begin{aligned}
&\omega^{\hat{z}}{}_{\hat{r}} = G'e^{\hat{z}}, \qquad \omega^{\hat{y}}{}_{\hat{r}} = G'e^{\hat{y}}, \qquad \omega^{\hat{x}}{}_{\hat{r}} = G'e^{\hat{x}},\\
&\omega^{\hat{x}}{}_{\hat{y}} = -e^{-G}e^{\hat{x}}, \qquad \omega^{\hat{z}}{}_{\hat{y}} = -e^{-G}e^{\hat{z}}, \qquad \omega^{\hat{\mu}}{}_{\hat{r}} = F'e^{\hat{\mu}}.
\end{aligned} \tag{4.8}$$

We then turn on the following gauge fields, to cancel the above spin connections on H^3,

$$A^1 = \frac{a}{y}dx, \qquad A^2 = 0, \qquad A^3 = \frac{a}{y}dz \tag{4.9}$$

with $A^{i+3} = \frac{g_1}{g_2}A^i$, $i = 1, 2, 3$. These gauge fields then become $SO(3)_{\text{diag}}$ gauge fields.

We will also impose the projection conditions

$$\gamma_r \epsilon = \epsilon, \qquad i\sigma^1\gamma_{\hat{x}\hat{y}}\epsilon = -\epsilon, \qquad i\sigma^2\gamma_{\hat{x}\hat{z}}\epsilon = -\epsilon, \qquad i\sigma^3\gamma_{\hat{z}\hat{y}}\epsilon = -\epsilon\,. \tag{4.10}$$

The twist condition is still given by (4.4).

In both cases, the last projector in (4.6) and (4.10) is not independent from the second and the third ones, so the fixed point solution will preserve four supercharges corresponding to $N = 1$ superconformal symmetry in three dimensions.

With all of the above conditions, we find the following BPS equations, for the H^3 case,

$$\phi' = -\frac{1}{8g_2}e^{-\frac{\sigma}{2}-3\phi-2G}\left[e^{2G}(e^{4\phi}-1)g_2 - 4ae^{\sigma+2\phi}\right]\left[g_1 - g_2 + (g_1+g_2)e^{2\phi}\right], \tag{4.11}$$

$$\begin{aligned}\sigma' &= -\frac{1}{20}e^{-\frac{\sigma}{2}-3\phi-2G}\left[\frac{12a}{g_2}e^{\sigma+2\phi}\left[(e^{2\phi}-1)g_1 + (1+e^{2\phi})g_2\right]\right.\\ &\quad\left.+e^{2G}g_2\left[g_2(e^{2\phi}-1)^3 + g_1(e^{2\phi}+1)^3 + 128he^{\frac{5\sigma}{2}+3\phi}\right]\right],\end{aligned} \tag{4.12}$$

$$\begin{aligned}G' &= \frac{1}{40}e^{-\frac{\sigma}{2}-3\phi-2G}\left[\frac{28a}{g_2}e^{\sigma+2\phi}\left[(e^{2\phi}-1)g_1 + (1+e^{2\phi})g_2\right]\right.\\ &\quad\left.-e^{2G}g_2\left[g_2(e^{2\phi}-1)^3 + g_1(e^{2\phi}+1)^3 - 32he^{\frac{5\sigma}{2}+3\phi}\right]\right],\end{aligned} \tag{4.13}$$

$$\begin{aligned}F' &= -\frac{1}{40}e^{-\frac{\sigma}{2}-3\phi-2G}\left[\frac{12a}{g_2}e^{\sigma+2\phi}\left[(e^{2\phi}-1)g_1 + (1+e^{2\phi})g_2\right]\right.\\ &\quad\left.+e^{2G}g_2\left[g_2(e^{2\phi}-1)^3 + g_1(e^{2\phi}+1)^3 - 32he^{\frac{5\sigma}{2}+3\phi}\right]\right].\end{aligned} \tag{4.14}$$

The corresponding equations for the S^3 case are similar with a replaced by $-a$.

We now look for a fixed point solution at which $G' = \phi' = \sigma' = 0$ and $F' =$ constant. For $g_2 = g_1$, only $AdS_4 \times H^3$ solutions exist and are given by

$$\begin{aligned}\phi &= \frac{1}{4}\ln 2, \qquad \sigma = \frac{3}{10}\ln 2,\\ G &= \frac{1}{10}\ln\left[\frac{64a^5}{g_1^3h^2}\right], \qquad L_{AdS_5} = \frac{1}{2^{\frac{13}{5}}h}\,.\end{aligned} \tag{4.15}$$

This solution can be uplifted to eleven dimensions using the ansatz of [30].

When $g_2 \neq g_1$, we also find $AdS_4 \times H^3$ solutions

$$\begin{aligned}
\phi &= \frac{1}{2}\ln\left[\frac{g_2 - g_1}{g_2 + g_1}\right], \qquad \sigma = \frac{1}{5}\ln\left[\frac{g_1^2 g_2^2}{100h^2(g_2^2 - g_1^2)}\right], \\
G &= \frac{1}{2}\ln\left[\frac{5a(g_2^2 - g_1^2)}{g_1 g_2^2}\right] + \frac{1}{5}\ln\left[\frac{-g_1 g_2}{10h\sqrt{g_2^2 - g_1^2}}\right], \\
L_{AdS_4} &= \frac{1}{2^{\frac{6}{5}}h}\left[\frac{25h^2(g_2^2 - g_1^2)}{g_1^2 g_2^2}\right]^{\frac{2}{5}}. \qquad (4.16)
\end{aligned}$$

This solution can be connected to both AdS_7 critical points in (2.15) and (2.16) by some RG flows.

In this $g_2 \neq g_1$ case, there can be both $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions. The solution however takes a more complicated form depending on the values of g_1 and g_2. The $AdS_4 \times H^3$ and $AdS_4 \times S^3$ solutions are given respectively by

$$G = \frac{1}{2}\ln\left[\frac{4ae^{\sigma+2\phi_0}}{g_2(e^{4\phi_0} - 1)}\right], \qquad (4.17)$$

$$\sigma = \frac{2}{5}\ln\left[\frac{e^{-3\phi_0}\left[g_2(1 - e^{6\phi_0}) - g_1(e^{6\phi_0} + 1)\right]}{32h}\right] \qquad (4.18)$$

and

$$G = \frac{1}{2}\ln\left[\frac{4ae^{\sigma+2\phi_0}}{g_2(1 - e^{4\phi_0})}\right], \qquad (4.19)$$

$$\sigma = \frac{2}{5}\ln\left[\frac{e^{-3\phi_0}\left[g_2(1 - e^{6\phi_0}) - g_1(e^{6\phi_0} + 1)\right]}{32h}\right]. \qquad (4.20)$$

In both cases, the scalar ϕ_0 is a solution to the equation

$$g_1(1 - 2e^{2\phi_0} - 2e^{4\phi_0} + e^{6\phi_0}) - g_2(1 + 2e^{2\phi_0} - 2e^{4\phi_0} - e^{6\phi_0}) = 0\,. \qquad (4.21)$$

The explicit form of ϕ_0 can be obtained but will not be given here due to its complexity. There are many possible solutions for ϕ_0 depending on the values of g_1, g_2 and a. An example of $AdS_4 \times S^3$ solutions is, for $g_2 = \frac{1}{2}g_1$, given by

$$\phi = -0.9158, \qquad \sigma = 0.5493, \qquad G = 0.4116 + \frac{1}{2}\ln\left[\frac{a}{g_1}\right]. \qquad (4.22)$$

One of the $AdS_4 \times H^3$ solutions is, for $g_2 = \frac{1}{2}g_1$, given by

$$\phi = 0.2706, \qquad \sigma = 0.2351, \qquad G = 1.0936 + \frac{1}{2}\ln\left[\frac{a}{g_1}\right]. \qquad (4.23)$$

Numerical solutions for RG flows from the UV $N = (1, 0)$ SCFTs in six dimensions to these three-dimensional $N = 1$ SCFTs can be found in the same way as those given in the previous section. And, with suitable boundary conditions, the flow from $SO(4)$ AdS_7 point to the $SO(3)$ AdS_7 point and then to $AdS_4 \times S^3$ or $AdS_4 \times H^3$ in the case of $g_2 \neq g_1$ should be similarly obtained. We will however not give these solutions here.

5. Uplifting the solutions to eleven dimensions

In this section, we will uplift some of the AdS_5 and AdS_4 solutions found in the previous sections to eleven dimensions using a reduction ansatz given in [30]. Only solutions with equal $SU(2)$ gauge couplings, $g_2 = g_1$, can be uplifted by this ansatz. Therefore, we will consider only this case in the remaining of this section.

The reduction ansatz given in [30] is naturally written in terms of $SL(4, \mathbb{R})/SO(4)$ scalar manifold rather than the $SO(3, 3)/SO(3)\times SO(3)$ we have considered throughout the previous sections. It is then useful to change the parametrization of scalars from the $SO(3, 3)/SO(3)\times SO(3)$ to $SL(4, \mathbb{R})/SO(4)$ cosets. For convenience, we will repeat the supersymmetry transformations of fermions with the three-form field and fermions vanishing

$$\begin{aligned}
\delta\psi_\mu &= D_\mu\epsilon - \frac{1}{20}gX\tilde{T}\gamma_\mu\epsilon - \frac{1}{20}X^{-4}\gamma_\mu\epsilon \\
&\quad + \frac{1}{40\sqrt{2}}X^{-1}\left(\gamma_\mu{}^{\nu\rho} - 8\delta_\mu^\nu\gamma^\rho\right)\Gamma_{RS}F^{RS}_{\nu\rho}\epsilon, && (5.1)\\
\delta\chi &= -X^{-1}\gamma^\mu\partial_\mu X\epsilon - \frac{2}{5}gX^{-4}\epsilon + \frac{1}{10}gX\tilde{T} - \frac{1}{20\sqrt{2}}X^{-1}\gamma^{\mu\nu}\Gamma_{RS}F^{RS}_{\mu\nu}\epsilon, && (5.2)\\
\delta\hat{\lambda}_R &= -\frac{1}{2}\gamma^\mu\Gamma^S P_{\mu RS}\epsilon - \frac{1}{8}gX\tilde{T}\Gamma_R\epsilon + \frac{1}{2}gX\tilde{T}_{RS}\Gamma^S\epsilon \\
&\quad - \frac{1}{8\sqrt{2}}X^{-1}\gamma^{\mu\nu}\Gamma_S\left(F^{RS}_{\mu\nu} + \frac{1}{2}\epsilon_{RSTU}F^{TU}_{\mu\nu}\right)\epsilon && (5.3)
\end{aligned}$$

where

$$\begin{aligned}
P_{RS} &= (\mathcal{V}^{-1})^\alpha_{(R}\left(\delta^\beta_\alpha d + gA_{(1)\alpha}{}^\beta\right)\mathcal{V}_\beta{}^T\delta_{S)T}, \\
Q_{RS} &= (\mathcal{V}^{-1})^\alpha_{[R}\left(\delta^\beta_\alpha d + gA_{(1)\alpha}{}^\beta\right)\mathcal{V}_\beta{}^T\delta_{S]T}, \\
D\epsilon &= d\epsilon + \frac{1}{4}\omega_{ab}\gamma^{ab} + \frac{1}{4}Q_{RS}\Gamma^{RS} \\
\tilde{T}_{RS} &= (\mathcal{V}^{-1})_R{}^\alpha(\mathcal{V}^{-1})_S{}^\beta\delta_{\alpha\beta}, \qquad \tilde{T} = \tilde{T}_{RS}\delta^{RS}\,. && (5.4)
\end{aligned}$$

In the above equations, $\mathcal{V}^R_{\ \alpha}$ denotes the $SL(4, \mathbb{R})/SO(4)$ coset representative.

For the explicit form of the eleven-dimensional metric and the four-form field including the notations used in the above equations, we refer the reader to [30]. We now consider the AdS_5 and AdS_4 solutions separately.

5.1 Uplifting the AdS_5 solutions

For AdS_5 solutions, the seven-dimensional metric is given by (3.1) and (3.4). We will restrict ourselves to AdS_5 fixed points with $SO(2)\times SO(2)$ symmetry. The non-zero gauge fields are $A^{\alpha\beta}=(A^{12},A^{34})$ whose explicit form is given by

$$A^{12}=a\cos\theta d\phi \qquad \text{and} \qquad A^{34}=b\cos\theta d\phi\,. \tag{5.5}$$

The $U(1)\times U(1)$ singlet scalar from $SL(4,\mathbb{R})/SO(4)$ coset is parametrized by the coset representative

$$\mathcal{V}_\alpha^{\ R}=\text{diag}(e^{\frac{\Phi}{2}},e^{\frac{\Phi}{2}},e^{-\frac{\Phi}{2}},e^{-\frac{\Phi}{2}}) \tag{5.6}$$

from which the $\tilde{T}_{RS}=\text{diag}(e^{-\Phi},e^{-\Phi},e^{\Phi},e^{\Phi})$ follows. Note that the parameter a and b here are different from those in section 3 since the gauge fields A^i and A^r correspond respectively to the anti-self-dual and self-dual parts of the $SO(4)$ gauge fields $A^{\alpha\beta}$.

Using the above supersymmetry transformations and imposing the projection conditions $\gamma_{\hat{r}}\epsilon=\epsilon$ and $\gamma^{\hat{\theta}\hat{\phi}}\Gamma_{12}\epsilon=\epsilon$, we obtain the BPS equations

$$X^{-1}X'-\frac{2}{5}gX^{-4}+\frac{1}{5}gX(e^{\Phi}+e^{-\Phi})+\frac{1}{5\sqrt{2}}X^{-1}e^{-2G}(ae^{\Phi}-be^{-\Phi})=0, \tag{5.7}$$

$$-\Phi'-gX(e^{\Phi}-e^{-\Phi})+\frac{1}{\sqrt{2}}X^{-1}e^{-2G}(ae^{\Phi}+be^{-\Phi})=0, \tag{5.8}$$

$$F'-\frac{1}{5}gX(e^{\Phi}+e^{-\Phi})-\frac{1}{10}gX^{-4}-\frac{1}{10\sqrt{2}}X^{-1}e^{-2G}(ae^{\Phi}-be^{-\Phi})=0, \tag{5.9}$$

$$G'-\frac{1}{5}gX(e^{\Phi}+e^{-\Phi})-\frac{1}{10}gX^{-4}+\frac{4}{5\sqrt{2}}X^{-1}e^{-2G}(ae^{\Phi}-be^{-\Phi})=0\,. \tag{5.10}$$

In the above equations, we have used $\Gamma_{34}\epsilon=-\Gamma_{12}\epsilon$ which follows from the condition $\Gamma_{1234}\epsilon=\epsilon$. The latter is part of the truncation from the maximal $SO(5)$ gauged supergravity to the half-maximal $SO(4)$ gauged supergravity studied in [30]. We have also used the twist condition given by

$$g(a-b)+1=0\,. \tag{5.11}$$

which comes from the requirement that the gauge connection cancels the spin connection. Note that this condition differs from (3.8) since the gauge fields are different. In condition (3.8), the $SU(2)_R$ gauge fields are given by the A^I with $I=1,2,3$, and the $SO(2)_R\subset SU(2)_R$ gauge field has been chosen to be A^3. On the other hand, the condition (5.11) involves $A^{12}-A^{34}$ corresponding to the $SO(2)_R$ subgroup of the

$SU(2)_R$ R-Symmetry for which the corresponding gauge fields are identified with the anti-self-dual part of the $SO(4)$ gauge fields $A^{\alpha\beta}$ in the convention of [30].

For large r, the solution should approach $X = 1$, $\Phi = 0$ and $F \sim G \sim r$ giving AdS_7 background with $SO(4)$ symmetry. This corresponds to the UV $N = (1,0)$ SCFT in six dimensions. In the IR with the boundary condition $F \sim r$ and $G, \Phi, \sigma \sim$ constant, there is a class of solutions given by

$$\begin{aligned}
\Phi &= \frac{1}{2}\ln\left[\frac{a+b\pm\sqrt{a^2+ab+b^2}}{a}\right], \\
G &= \frac{1}{2}\ln\left[\frac{a\left(a+2b\pm\sqrt{a^2+ab+b^2}\right)}{\sqrt{2}gX^2\left(b\pm\sqrt{a^2+ab+b^2}\right)}\right], \\
X^{10} &= \frac{a\left(a+2b\pm\sqrt{a^2+ab+b^2}\right)^2}{4(a+b)^2\left(a+b\pm\sqrt{a^2+ab+b^2}\right)}, \\
L_{AdS_5} &= \frac{a2^{\frac{1}{5}}}{g}\left[\frac{a+2b\pm\sqrt{a^2+ab+b^2}}{(a+b)^2\left(a+b\pm\sqrt{a^2+ab+b^2}\right)}\right]^{\frac{2}{5}}.
\end{aligned} \tag{5.12}$$

This gives $AdS_5 \times S^2$ background preserving $U(1) \times U(1)$ symmetry and eight supercharges since only the projector $\gamma^{\hat{\theta}\hat{\phi}}\Gamma_{12}\epsilon = \epsilon$ is needed at the fixed point. Therefore, this solution corresponds to $N = 1$ SCFT in four dimensions. This solution is the same as in [22] with the identification $(m_1, m_2) \to (-b, a)$ up to some field redefinitions. So, we conclude that the $AdS_5 \times \Sigma_2$ solutions found in [22] is a solution of the $N = 2$ $SO(4)$ gauged supergravity.

For the H^2 case, the above analysis can be repeated in a similar manner. The resulting BPS equations are, as expected, given by (5.7), (5.8), (5.9) and (5.10) with (a, b) replaced by $(-a, -b)$. It can also be verified that for both $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions given in (5.12), solutions with the positive sign are valid for $g > 0$ and $a > 0$ while solutions with the negative sign are valid for $g < 0$ and $a < 0$.

It should also be noted that we can truncate the above BPS equations to those of $SO(2)_R$ symmetry, generated by the anti-selfdual gauge field $A^{12} - A^{34}$, by setting $b = -a$. Since the twist condition in this case becomes $2ga = -1$ which implies that $ga < 0$, only the $AdS_5 \times H^2$ exists. This precisely agrees with the result of section 3.2.2. The corresponding solution is given by

$$X = \left(\frac{3}{4}\right)^{\frac{1}{5}}, \qquad G = -\frac{1}{2}\ln\left[-\frac{g}{2^{\frac{3}{10}}3^{\frac{3}{5}}a}\right], \qquad L_{AdS_5} = \frac{3^{\frac{4}{5}}}{2^{\frac{3}{5}}g}. \tag{5.13}$$

The $AdS_5 \times H^2$ with $SO(2)_{\text{diag}}$ symmetry found in section 3.2.1 for $g_2 = g_1$ can also be uplifted using the formulae given here by truncating the $SO(2) \times SO(2)$ symmetry to $SO(2)_{\rm diag}$ as remarked previously in section 3.2.1. The $SO(2)_{\rm diag}$ corresponds to the gauge field A^{12} since the A^3 and A^6, in section 3.2, are related to the anti-self-dual, $\frac{1}{2}(A^{12}-A^{34})$, and self-dual, $\frac{1}{2}(A^{12}+A^{34})$, fields, respectively. So, the $SO(2)_{\rm diag}$ gauge field is given by A^{12}. As in section 3.2, only solutions with the upper sign in the solution (5.12) and $AdS_5\times H^2$ are possible. The result is given by

$$\Phi=\frac{1}{2}\ln 2,\qquad X^{10}=\frac{1}{8},\qquad G=\frac{1}{2}\ln\left[-\frac{a2^{\frac{11}{10}}}{g}\right]. \tag{5.14}$$

This is consistent with the twist condition (5.11) which, for $b=0$, becomes $ga=-1$.

We now move to the uplift of these AdS_5 solutions. Both $AdS_5\times S^2$ and $AdS_5\times H^2$ solutions can be uplifted in a similar way. For definiteness, we will only give the uplifted $AdS_5\times S^2$ solution. Using the reduction ansatz given in [30], we find the eleven-dimensional metric

$$\begin{aligned}
ds^2_{11} &= \Delta^{\frac{1}{3}}\left[e^{\frac{2r}{L_{AdS_5}}}dx^2_{1,3}+dr^2+e^{2G_0}(d\theta^2+\sin^2\theta d\phi^2)\right]\\
&+\frac{2}{g}\Delta^{-\frac{2}{3}}X_0^3\left[X_0\cos^2\xi+X_0^{-4}\sin^2\xi\left(e^{-\Phi_0}\sin^2\psi+e^{\Phi_0}\cos^2\psi\right)\right]d\xi^2\\
&+\frac{1}{2g^2}\Delta^{-\frac{2}{3}}X_0^{-1}\cos^2\xi\left[e^{-\Phi_0}\left[\cos^2\psi d\phi^2+\sin^2\psi(d\alpha-ag\cos\theta d\phi)^2\right]\right.\\
&\left.+\,e^{\Phi_0}\left[\cos^2\psi d\phi^2+\sin^2\psi(d\beta-bg\cos\theta d\phi)^2\right]\right]\\
&-\frac{1}{2g^2}\Delta^{-\frac{2}{3}}X_0^{-1}\sin\xi\sin(2\psi)\left(e^{-\Phi_0}-e^{\Phi_0}\right)d\xi d\psi
\end{aligned} \tag{5.15}$$

where we have used the coordinates μ^α, satisfying $\mu^\alpha\mu^\alpha=1$, as follow

$$\begin{aligned}
\mu^1 &= \sin\psi\cos\alpha, \qquad \mu^2=\sin\psi\sin\alpha,\\
\mu^3 &= \cos\psi\cos\beta, \qquad \mu^4=\cos\psi\sin\beta\,.
\end{aligned} \tag{5.16}$$

The quantities X_0, Φ_0 and G_0 are the values of the corresponding fields at the fixed point (5.12). The quantity Δ is defined by

$$\Delta=X^{-4}\sin^2\xi+X\tilde{T}_\alpha\mu^\alpha\mu^\beta\cos^2\xi \tag{5.17}$$

which, in the present case, gives

$$\Delta=X_0\cos^2\xi\left(e^{-\Phi_0}\sin^2\psi+e^{\Phi_0}\cos^2\psi\right)+X_0^{-4}\sin^2\xi\,. \tag{5.18}$$

The 4-form field, at the fixed point, is given by

$$
\begin{aligned}
\hat{F}_{(4)} = {} & \frac{1}{g^3} U \Delta^{-2} \cos^3 \xi d\xi \wedge \epsilon_{(3)} + \frac{a}{g^2} \cos\theta \cos\xi \left[\sin\xi \cos\xi \sin\psi \cos\psi X_0^{-4} d\psi \right. \\
& \left. \cos^2 \psi \left(X_0^{-4} \sin^2 \xi + e^{\Phi_0} X_0^2 \cos^2 \xi \right) d\xi \right] \wedge d\beta \wedge d\theta \wedge d\phi \\
& - \frac{b}{g^2} \sin\theta \cos\xi \left[\sin\xi \cos\xi \sin\psi \cos\psi X_0^{-4} d\psi \right. \\
& \left. - \left(X_0^{-4} \sin^2 \xi + X_0^2 \cos^2 \xi e^{-\Phi_0} \right) \sin^2 \psi d\xi \right] \wedge d\alpha \wedge d\theta \wedge d\phi
\end{aligned}
\tag{5.19}
$$

where

$$
\begin{aligned}
U = {} & \sin^2 \xi \left[X_0^{-8} - 2 X_0^{-3} \left(e^{\Phi_0} + e^{-\Phi_0} \right) \right] \\
& - \cos^2 \xi \left[2 X_0^2 + X_0^{-3} \left(e^{-\Phi_0} \sin^2 \psi + e^{\Phi_0} \cos^2 \psi \right) \right] .
\end{aligned}
\tag{5.20}
$$

The uplifted solutions for some particular values of a and b have already been given in [23].

5.2 Uplifting the AdS_4 solutions

We now consider the embedding of the $AdS_4 \times H^3$ solution given in (4.15) in eleven dimensions. The $SL(4,\mathbb{R})/SO(4)$ coset representative, invariant under $SO(3)_{\text{diag}}$, is given by

$$
\mathcal{V}_\alpha^{\ R} = (\delta_{ab} e^{\frac{\phi}{2}}, e^{-\frac{3\phi}{2}})
\tag{5.21}
$$

which gives $\tilde{T}_{RS} = (\delta_{ab} e^{-\phi}, e^{3\phi})$. We have split the α index as follow $\alpha = (a, 4)$, $a = 1, 2, 3$.

To set up the associated BPS equations, we use the seven-dimensional metric (4.7) and the following gauge fields

$$
A^{12} = \frac{a}{y} dz, \qquad A^{31} = 0, \qquad A^{23} = \frac{a}{y} dx .
\tag{5.22}
$$

The twist condition is given by $ga = 1$. We will also impose the projection conditions

$$
\Gamma_{23} \gamma_{\hat{x}\hat{y}} \epsilon = -\epsilon, \qquad \Gamma_{13} \gamma_{\hat{z}\hat{x}} \epsilon = -\epsilon, \qquad \Gamma_{12} \gamma_{\hat{z}\hat{y}} \epsilon = -\epsilon, \qquad \Gamma_{\hat{r}} \epsilon = \epsilon .
\tag{5.23}
$$

With all of the above conditions, we obtain the following BPS equations

$$
-\phi' + \frac{1}{2} g X (e^{-\phi} - e^{3\phi}) + \sqrt{2} a X^{-1} e^{\phi - 2G} = 0,
\tag{5.24}
$$

$$
-X^{-1} X' - \frac{2}{5} g X^{-4} + \frac{1}{10} g X (3 e^{-\phi} + e^{3\phi}) + \frac{3}{5\sqrt{2}} a X^{-1} e^{\phi - 2G} = 0,
\tag{5.25}
$$

$$
G' - \frac{1}{10} g X (3 e^{-\phi} + e^{3\phi}) - \frac{1}{10} g X^{-4} + \frac{7}{5\sqrt{2}} a X^{-1} e^{\phi - 2G} = 0,
\tag{5.26}
$$

$$
F' - \frac{1}{10} g X (3 e^{-\phi} + e^{3\phi}) - \frac{1}{10} g X^{-4} - \frac{3}{5\sqrt{2}} a X^{-1} e^{\phi - 2G} = 0 .
\tag{5.27}
$$

These equations admit a fixed point solution

$$\begin{aligned} \phi_0 &= \frac{1}{4}\ln\frac{11}{3}, \qquad X_0^{20} = \frac{11(3^3)}{2^{12}}, \\ G_0 &= \frac{1}{10}\ln\left[\frac{3(11^2)}{2\sqrt{2}}\right] - \frac{1}{2}\ln\left[\frac{g}{a}\right], \qquad L_{AdS_4} = \frac{1}{g}\left(\frac{11(3^3)}{2^7}\right)^{\frac{1}{5}}. \end{aligned} \tag{5.28}$$

The parametrization of the μ^α coordinates can be chosen to be

$$\mu^\alpha = (\cos\Psi\hat{\mu}^a, \sin\Psi) \tag{5.29}$$

with $\hat{\mu}^a$ satisfying $\hat{\mu}^a\hat{\mu}^a = 1$. The $SO(3)_{\text{diag}}$ symmetry corresponds to the gauge fields A^{ab}. In the following, we accordingly set $A^{4a} = 0$ for $a = 1, 2, 3$ and find that

$$D\mu^a = \cos\Psi D\hat{\mu}^a - \sin\Psi\hat{\mu}^a d\Psi, \qquad D\mu^4 = \cos\Psi d\Psi \tag{5.30}$$

where

$$D\hat{\mu}^a = d\hat{\mu}^a + gA^{ab}\hat{\mu}^b\,. \tag{5.31}$$

With all these results, the eleven-dimensional metric is given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{1}{3}}\left[e^{\frac{r}{L_{AdS_4}}}dx_{1,2}^2 + dr^2 + \frac{e^{2G_0}}{y^2}\left[dx^2+dy^2+dz^2\right]\right] \\ &+\frac{2}{g^2}\Delta^{-\frac{2}{3}}X_0^3\left[X_0\cos^2\xi + X_0^{-4}\sin^2\xi\left(\cos^2\Psi e^{\phi_0} + \sin^2\Psi e^{-3\phi_0}\right)\right]d\xi^2 \\ &+\frac{1}{2g^2}\Delta^{-\frac{2}{3}}X_0^{-1}\cos^2\xi\left[\cos^2\Psi e^{\phi_0}D\hat{\mu}^a D\hat{\mu}^a + \left(\sin^2\Psi e^{\phi_0} + \cos^2\Psi e^{-3\phi_0}\right)d\Psi^2\right] \\ &-\frac{1}{g^2}\Delta^{-\frac{2}{3}}X_0^{-1}\sin\xi\left(e^{-3\phi_0} - e^{\phi_0}\right)\sin\Psi\cos\Psi d\Psi d\xi\,. \end{aligned} \tag{5.32}$$

The S^2 coordinates $\hat{\mu}^a$ can be parametrized by

$$\hat{\mu}^1 = \sin\beta\cos\alpha, \qquad \hat{\mu}^2 = \sin\beta\sin\alpha, \qquad \hat{\mu}^3 = \cos\beta\,. \tag{5.33}$$

The warped factor Δ is given by

$$\Delta = X_0^2 e^{-\phi_0}\cos^2\xi\cos^2\Psi + X_0^{-4}\sin^2\xi + X_0 e^{3\phi_0}\sin^2\Psi\cos^2\xi\,. \tag{5.34}$$

The four-form field on the $AdS_4\times H^3$ background can be written as

$$\begin{aligned} \hat{F}_{(4)} &= \frac{1}{g^3}U\cos^3\xi\cos^2\Psi d\xi\wedge d\Psi\wedge\epsilon_{(2)} \\ &+\frac{1}{2g^2}\cos\xi\epsilon_{abc}\left[\hat{\mu}^c\left[X_0^{-4}\sin^2\xi(\sin^2\Psi - \cos^2\Psi)\right.\right. \\ &\left. +X_0^2(e^{3\phi_0}\sin^2\Psi - e^{-\phi_0}\cos^2\Psi)\right]d\xi\wedge F^{ab}\wedge d\Psi \\ &-\left[(X_0^{-4}\sin^2\xi + X_0^2\cos^2\xi e^{3\phi_0})\sin\Psi\cos\Psi d\xi\right. \\ &\left.\left. +X_0^{-4}\cos\xi\sin\xi\cos^2\Psi d\Psi\right]\wedge F^{ab}\wedge D\hat{\mu}^c\right] \end{aligned} \tag{5.35}$$

where

$$
\begin{aligned}
\epsilon_{(2)} &= \frac{1}{2}\epsilon_{abc}\hat{\mu}^a D\hat{\mu}^b \wedge D\hat{\mu}^c, \\
U &= \cos^2\xi\left[X_0^2\left[e^{6\phi_0}\sin^2\Psi - e^{-2\phi_0}\cos^2\Psi - e^{2\phi_0}(2\sin^2\Psi+1)\right]\right. \\
&\quad \left. -X_0^{-3}(e^{-\phi_0}\cos^2\Psi + e^{3\phi_0}\sin^2\Psi)\right] \\
&\quad +\sin^2\xi X_0^{-3}(X_0^{-5} - 3e^{-\phi_0} - e^{3\phi_0}).
\end{aligned} \tag{5.36}
$$

6. Conclusions

We have studied $AdS_5 \times \Sigma_2$ and $AdS_4 \times \Sigma_3$ solutions of $N = 2$ gauged supergravity in seven dimensions with $SO(4)$ gauge group. We have found that there exist both $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions with the gauge fields for $SO(2) \times SO(2)$ turned on. With $SO(2)_R$ or $SO(2)_{\text{diag}}$ gauge fields, only $AdS_5 \times H^2$ solution is possible. This is consistent with the results given in [21] and [23]. We recover $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions studied in [22] and [23] with $SO(2) \times SO(2)$ symmetry. In the case of equal $SU(2)$ gauge couplings, the solutions can be uplifted to eleven dimensions, and the uplifted solutions have explicitly given.

We have also considered RG flow solutions interpolating between supersymmetric AdS_7 critical points in the UV and these AdS_5 solutions in the IR. In the case of $SO(2)_{\text{diag}}$ symmetry, there exist flow solutions from $SO(4)$ AdS_7 critical point to AdS_5 as well as flows from $SO(4)$ AdS_7 to $SO(3)$ AdS_7 and then continue to AdS_5 fixed points similar to the flows from four-dimensional SCFTs to two-dimensional $N = (2, 0)$ SCFTs studied in [31]. Other results of this paper are a number of new $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions for unequal $SU(2)$ gauge couplings. With equal $SU(2)$ couplings, only $AdS_4 \times H^3$ geometry is possible, and the resulting solutions can be uplifted to eleven dimensions.

The results obtained in this paper should be relevant in the holographic study of $N = (1, 0)$ SCFTs in six dimensions. These would also provide new AdS_5 and AdS_4 solutions, corresponding to new SCFTs in four and three dimensions, within the framework of seven-dimensional gauged supergravity. The embedding of the solutions in the case of unequal $SU(2)$ gauge couplings (if possible) would be interesting to explore. It would also be interesting to compare the AdS_5 and AdS_4 solutions obtained here and the solutions found recently in [32, 33] in the context of massive type IIA theory. Finally, it is of particular interest to find an interpretation of all these solutions in terms of wrapped M5-branes on Σ_2 and Σ_3. Along this line, it would also be useful to find an implication of the AdS_4 solutions in terms of the M2-brane worldvolume theories.

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